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The Forty-second International Conference on Machine Learning



# CoPINN: Cognitive Physics-Informed Neural Networks

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Github

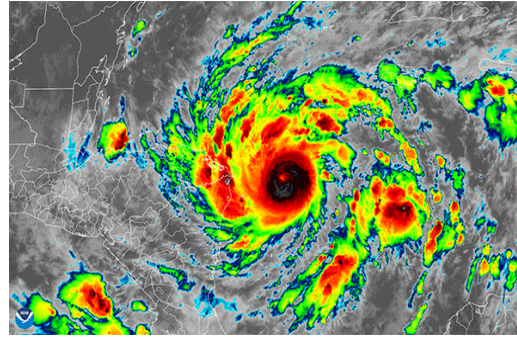


Survey on Fuzzy Deep Learning

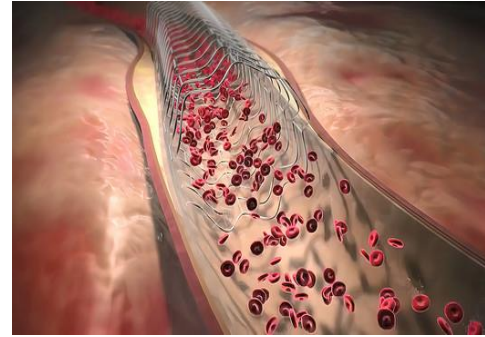
# Fluid in Life



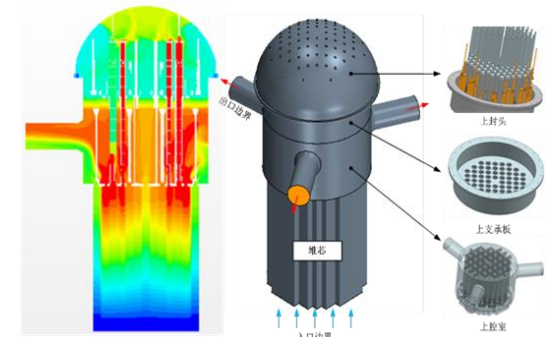
Turbulence



Meteorological



Biology Fluid

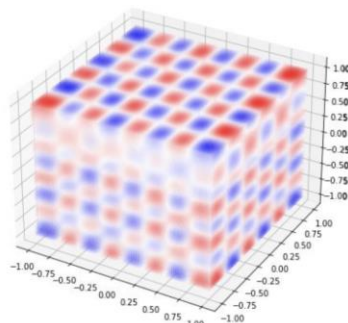


Reactor

How to understand the fluid

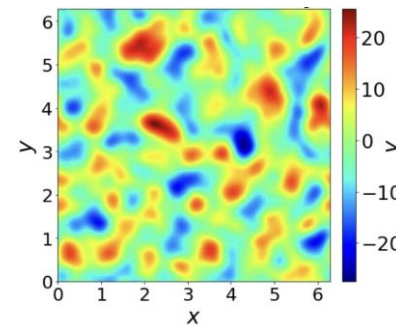


# Partial Differential Equations



## Helmholtz Equation

$$\begin{aligned}\Delta u + k^2 u &= q, & x \in \Omega, \\ u(x) &= 0, & x \in \partial\Omega,\end{aligned}$$



## Navier-Stokes Equation

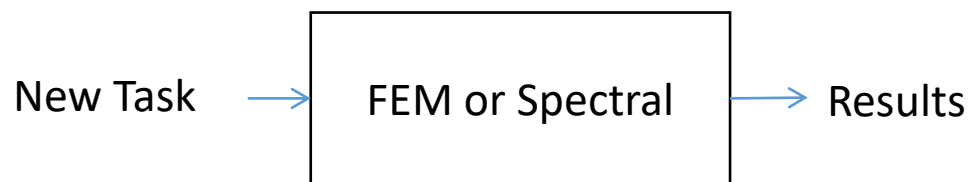
$$\begin{aligned}\partial_t \omega + u \cdot \nabla \omega &= \nu \Delta \omega, & x \in \Omega, t \in \Gamma, \\ \nabla \cdot u &= 0, & x \in \Omega, t \in \Gamma, \\ \omega(x, 0) &= \omega_0(x), & x \in \Omega,\end{aligned}$$

How to understand the PDEs



# PDE Solvers

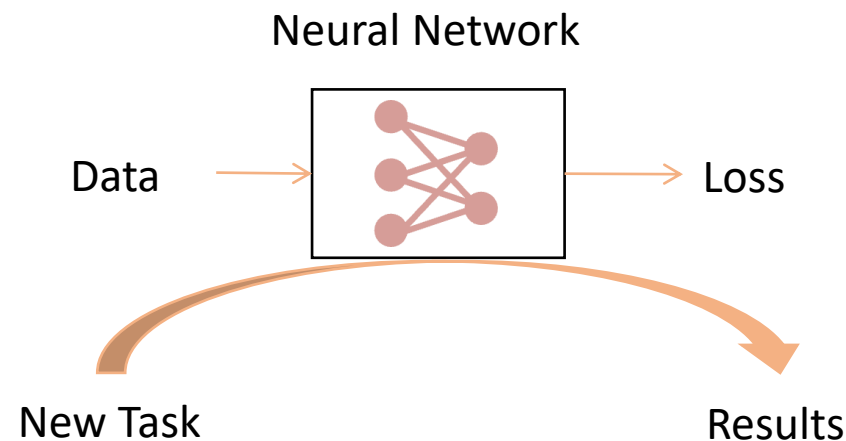
## Classic Numerical Methods



- Recalculation for every new sample
- Each round will take hours or even days for precise simulation

*Huge computation costs*

## Neural Network Methods

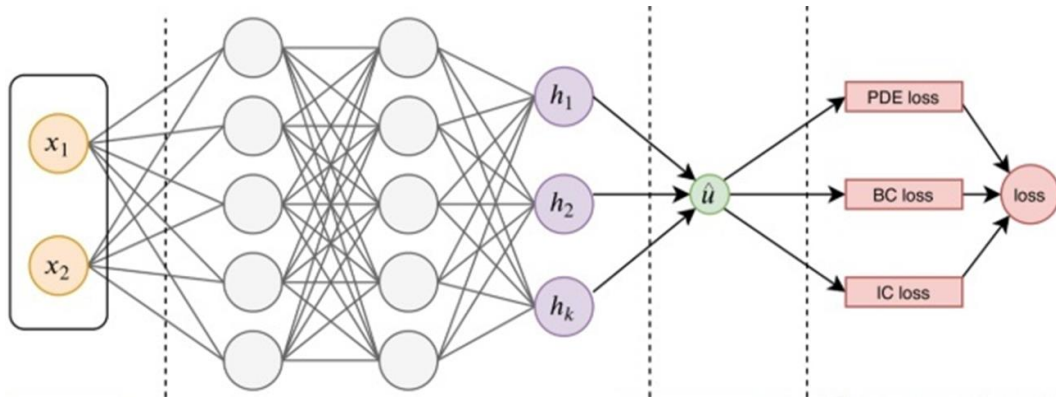


- Training once, inference a lot
- Each inference needs several seconds

*An efficient surrogate tool !*

# Physics-Informed Neural Networks (PINN)

## Framework

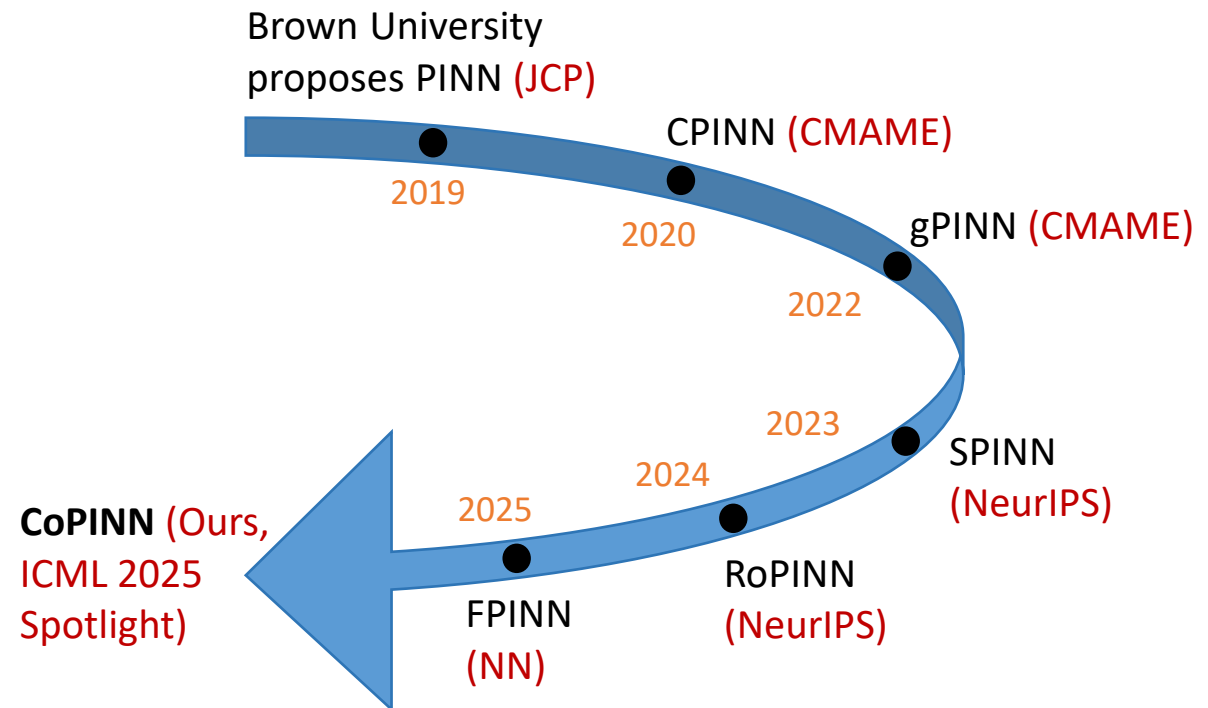


$$\text{Loss} = \text{PDE loss} + \text{BC loss} + \text{IC loss}$$

Boundary Condition

Initial Condition

## History of PINN



# Motivations

## Phenomenon

The physical boundary error is larger and has greater learning difficulty.

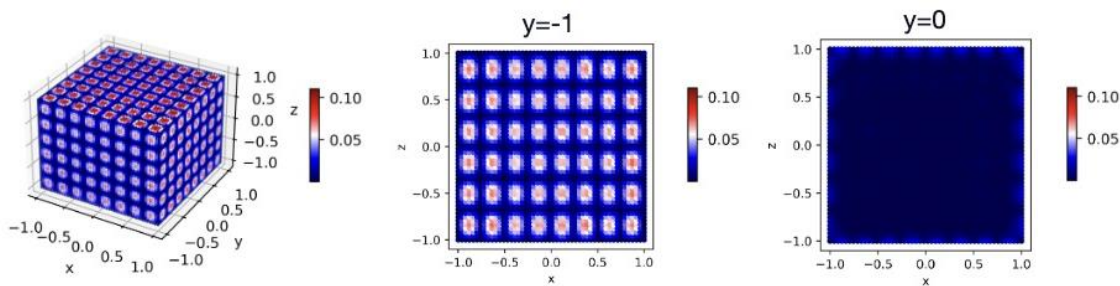


Fig1. Absolute error visualization of SOTA methods on the Helmholtz equation

## ➤ Motivation

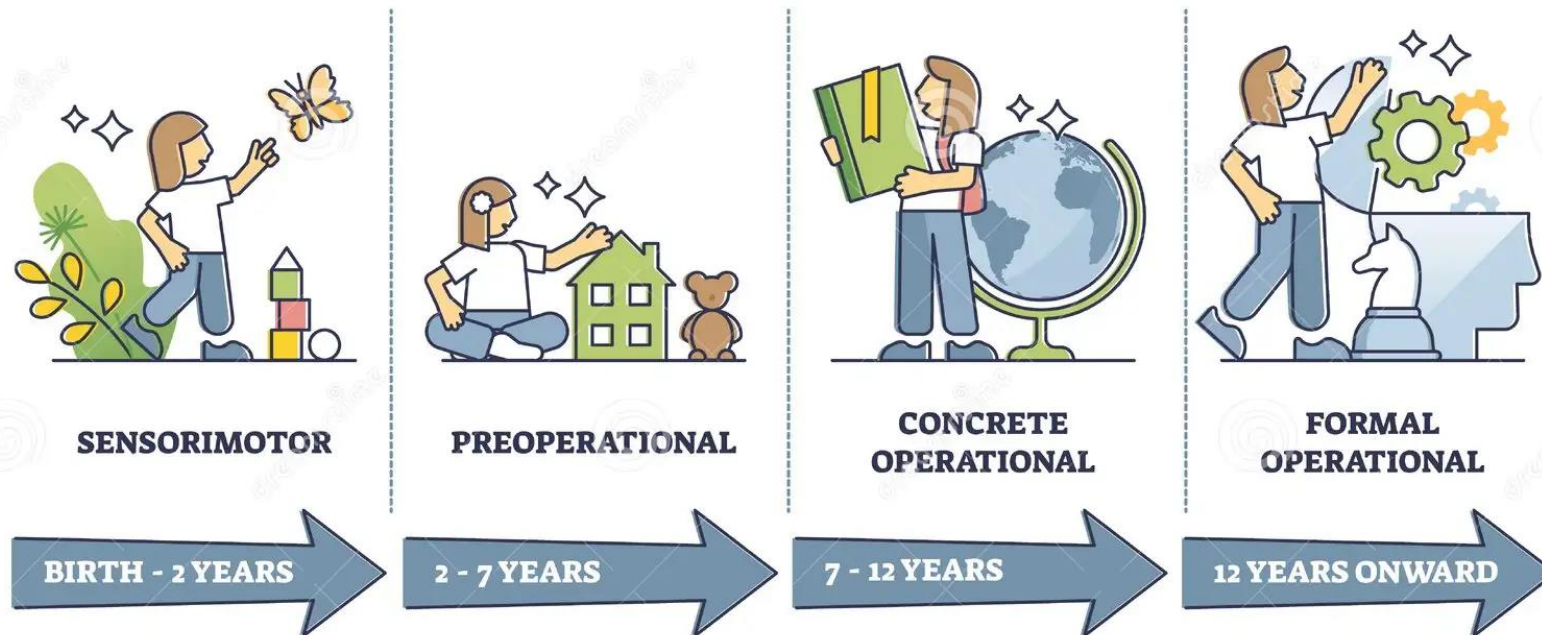
- The error values in stubborn regions (such as physical boundaries) are much larger than those in the middle smooth regions.
- Existing studies have ignored the "difficult sample points" generated in difficult-to-learn areas such as physical boundary areas, leading to the **unbalanced prediction problem (UPP)**.



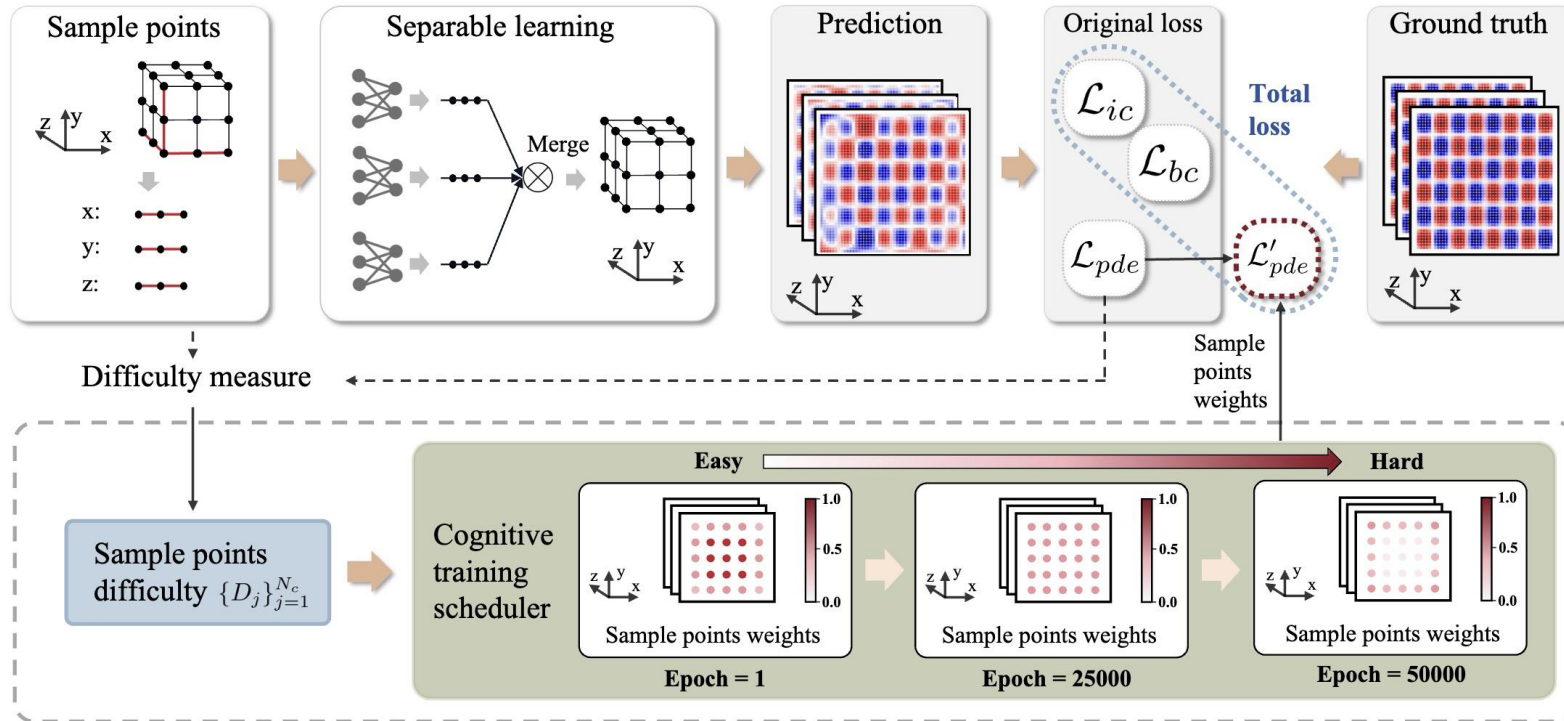
# Motivations

Can the model learn simple knowledge first and then learn these difficult points like humans?

## COGNITIVE DEVELOPMENT



# Method-Overall framework



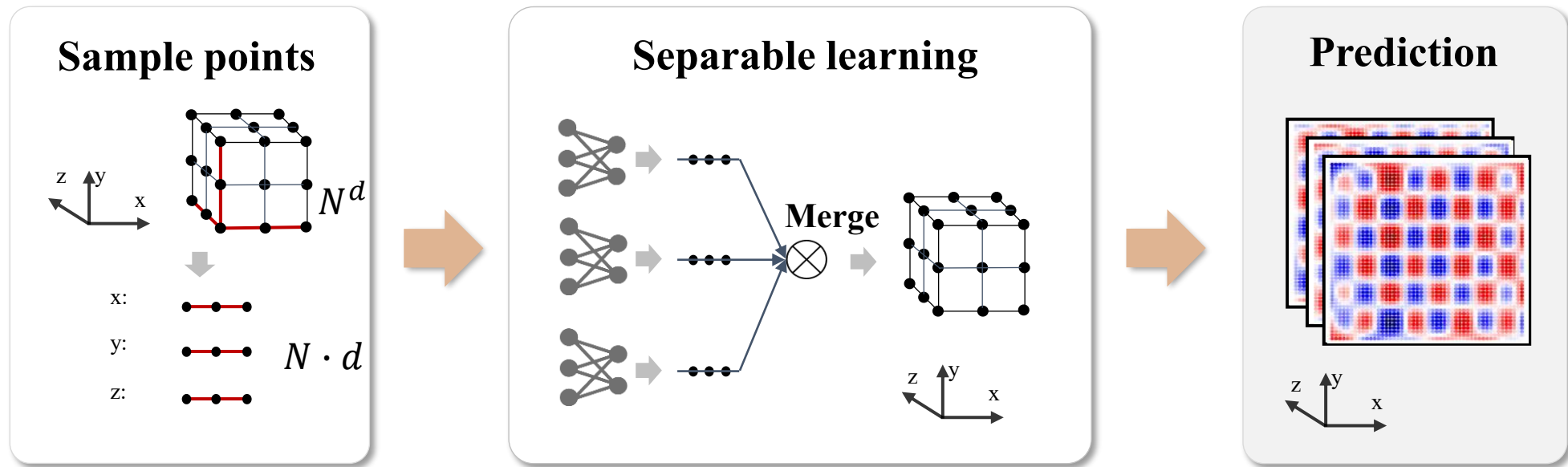
- Separable learning
- Cognitive Training Scheduler



# Method-Separable learning

## How to reduce computing cost?

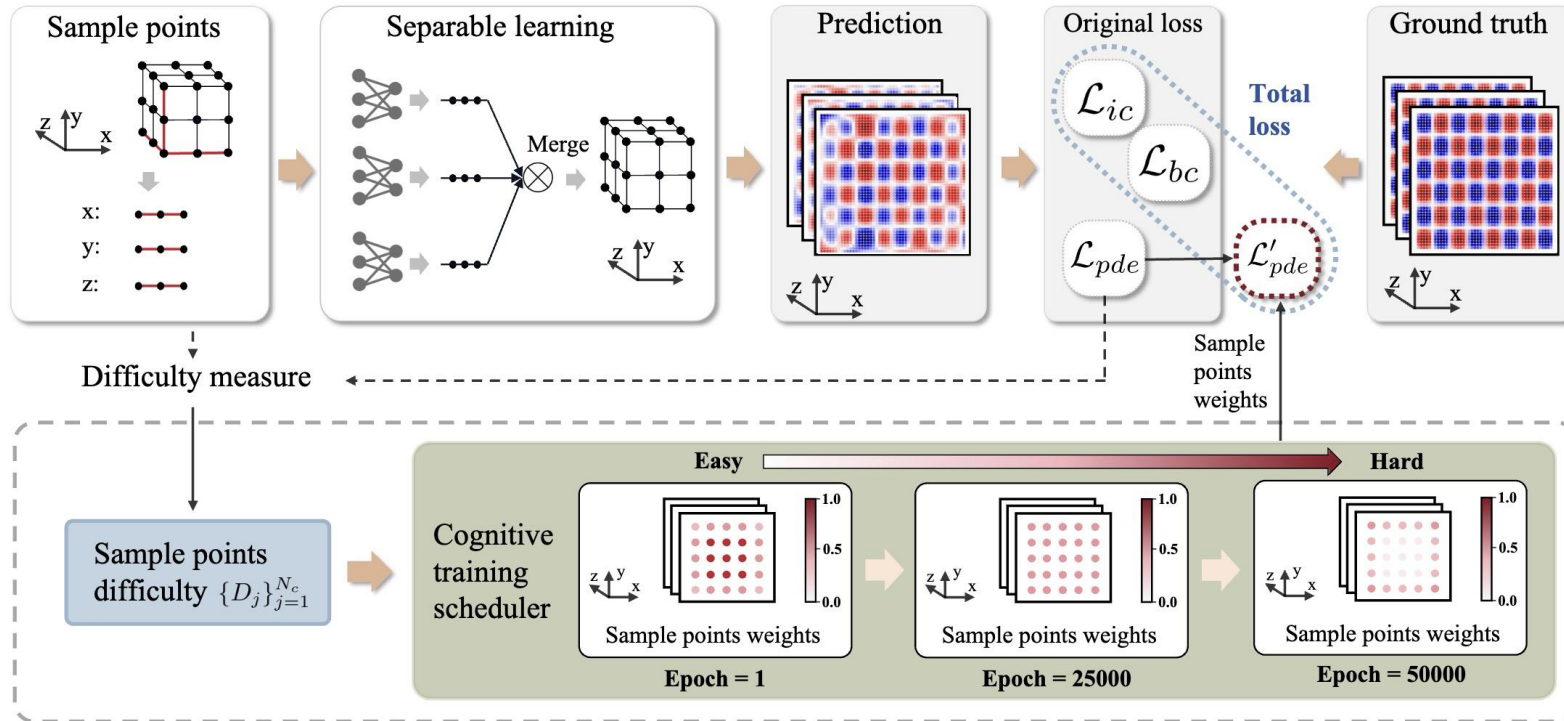
- Use separable sub-networks to independently encode the one-dimensional coordinates of each dimension (space or time);
- Use Merge strategies to obtain multi-dimensional predicted physical quantities.



$$\hat{u}(x_1, x_2, \dots, x_d) = \sum_{j=1}^r \prod_{i=1}^d f_j^{(\theta_i)}(x_i)$$

$$\hat{U}(X_{:,1}, X_{:,2}, \dots, X_{:,d}) = \sum_{j=1}^r \bigotimes_{i=1}^d F_{:,j,i}$$

# Method-Cognitive Training Scheduler



- Dynamically evaluate the learning difficulty of samples based on the gradient of PDEs residuals;
- Imitating the human cognitive process, learn the boundary physical quantities and mutation physical quantities in PDEs from easy to hard to perform adaptive optimization.

# Method-Cognitive training scheduler

Weight of the  $j$ -th easy sample point in the  $i$ -th epoch:

$$v_j^i = v_e^1 - \tau_e \cdot (i - 1) - \beta \cdot \delta_j^i$$

Epoch change component:  $\tau_e \cdot (i - 1)$

$$\tau_e = \frac{v_e^1 - v_e^{N_e}}{N_e} = \frac{1}{N_e} \rightarrow \text{Epoch count}$$

$$v_e^i = v_e^1 - \tau_e \cdot (i - 1) = 1 - \frac{i - 1}{N_e}$$

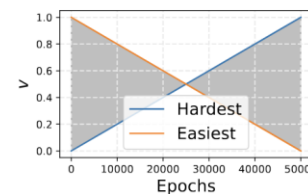
$$v_h^i = v_h^1 + \tau_e \cdot (i - 1) = 0 + \frac{i - 1}{N_e}$$

$e$  means the easiest,  $h$  means the hardest

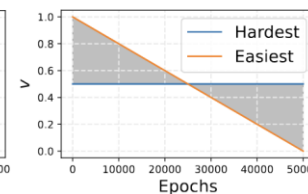
Sample change component:  $\beta \cdot \delta_j^i$

$$\delta_j^i = (v_e^i - v_h^i) \cdot \frac{D_j^i - D_e^i}{D_h^i - D_e^i}$$

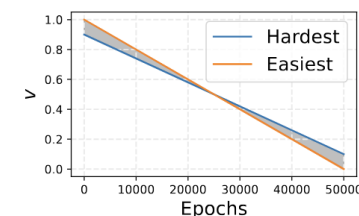
Difficulty Measure:  $D_k^i = \left\| \frac{\partial \mathcal{L}_{pde}^i}{\partial \mathbf{x}_k^i} \right\|_2$



(a)  $\beta = 1$



(b)  $\beta = 0.5$



(c)  $\beta = 0.1$



# Method-Cognitive training scheduler

$$\begin{aligned} \min_{\theta} \mathcal{L}(\hat{u}^{(\theta)}) = & \min_{\theta} \left( \lambda_{pde} \int_{\Gamma} \int_{\Omega} \boxed{v_i} \cdot ||\mathcal{N} [\hat{u}^{(\theta)}] (x, t)||^2 dx dt \right. \\ & + \lambda_{ic} \int_{\Omega} ||\hat{u}^{(\theta)}(x, 0) - u_{ic}(x)||^2 dx \\ & \left. + \lambda_{bc} \int_{\Gamma} \int_{\Omega} ||\mathcal{B} [\hat{u}^{(\theta)}] (x, t) - u_{bc}(x, t)||^2 dx dt \right), \end{aligned}$$

# Experiments-Datasets

## Flow Mixing Equation

$$u(t, x, y) = -\tanh\left(\frac{y}{2} \cos(\omega t) - \frac{x}{2} \sin(\omega t)\right), \quad t \in [0, 4], x \in [-4, 4], y \in [-4, 4],$$

$$\omega = \frac{1}{r} \frac{v_t}{v_{t,max}}$$

$$u_t + \alpha u_x + \beta u_y = 0,$$

## Diffusion Equation

$$u_t = \alpha(\|\nabla u\|^2 + u\Delta u), \quad x \in \Omega, t \in \Gamma,$$

$$u(x, 0) = u_{ic}(x), \quad x \in \Omega,$$

$$u(x, t) = 0, \quad x \in \partial\Omega, t \in \Gamma,$$

## Klein-Gordon Equation

$$u_{tt} - \Delta u + u^2 = f, \quad x \in \Omega, t \in \Gamma,$$

$$u(x, 0) = x_1 + x_2, x \in \Omega, \quad x \in \Omega,$$

$$u(x, t) = u_{bc}(x), \quad x \in \partial\Omega, t \in \Gamma,$$

## Convection Equation

$$\frac{\partial u}{\partial t} + \xi \frac{\partial u}{\partial x} = 0, \quad x \in [0, 2\pi], \quad t \in [0, 1]$$

$$u(x, 0) = h(x)$$

$$u(0, t) = u(2\pi, t)$$

## Helmholtz Equation

$$\Delta u + k^2 u = q, \quad x \in \Omega,$$

$$u(x) = 0, \quad x \in \partial\Omega,$$

# Experiments-Results

**Table 1. Experimental results**

Equation	Evaluation Metric		$RL_2$					$RMSE$				
	Methods	$N_c$	$16^3$	$32^3$	$64^3$	$128^3$	$256^3$	$16^3$	$32^3$	$64^3$	$128^3$	$256^3$
		Ref.										
Helmholtz	PINN	JCP'2019	0.9819	0.9757	0.9723	O/M	O/M	0.3420	0.3398	0.3386	O/M	O/M
	gPINN	CMAME'2022	0.3852	0.3255	0.4008	O/M	O/M	0.1342	0.1133	0.1396	O/M	O/M
	AHD-PINN	IJCAI'2024	0.2108	0.1903	0.1871	O/M	O/M	0.0734	0.0663	0.0652	O/M	O/M
	SPINN	NeurIPS'2023	0.1177	0.0809	0.0592	0.0449	0.0435	0.0410	0.0282	0.0206	0.0156	0.0151
	SPINN (m)	NeurIPS'2023	<u>0.1161</u>	<u>0.0595</u>	<u>0.0360</u>	<u>0.0300</u>	<u>0.0311</u>	<u>0.0404</u>	<u>0.0207</u>	<u>0.0125</u>	<u>0.0104</u>	<u>0.0108</u>
	RoPINN	NeurIPS'2024	0.4059	0.3338	O/M	O/M	O/M	0.1414	0.1162	O/M	O/M	O/M
	FPINN	NN'2025	0.3862	0.3502	0.3097	O/M	O/M	0.1345	0.1220	0.1079	O/M	O/M
	CoPINN	Ours	<b>0.0172</b>	<b>0.0050</b>	<b>0.0016</b>	<b>0.0007</b>	<b>0.0006</b>	<b>0.0040</b>	<b>0.0015</b>	<b>0.0004</b>	<b>0.0002</b>	<b>0.0002</b>
	IMP.	-	85%	92%	96%	98%	98%	90%	93%	97%	98%	98%
(2+1)-d Klein-Gordon	PINN	JCP'2019	0.0343	0.0281	0.0299	O/M	O/M	0.0218	0.0178	0.0190	O/M	O/M
	gPINN	CMAME'2022	0.0108	0.0025	O/M	O/M	O/M	0.0069	0.0016	O/M	O/M	O/M
	SPINN	NeurIPS'2023	0.0193	0.0060	0.0045	0.0040	0.0039	0.0123	0.0038	0.0029	0.0025	0.0025
	SPINN (m)	NeurIPS'2023	<u>0.0062</u>	<u>0.0020</u>	<u>0.0013</u>	<u>0.0008</u>	<u>0.0009</u>	<u>0.0039</u>	<u>0.0013</u>	<u>0.0008</u>	<u>0.0005</u>	<u>0.0006</u>
	AHD-PINN	IJCAI'2024	0.0133	0.0082	0.0147	O/M	O/M	0.0084	0.0052	0.0093	O/M	O/M
	RoPINN	NeurIPS'2024	0.1925	0.1806	0.1740	O/M	O/M	0.1223	0.1147	0.1105	O/M	O/M
	FPINN	NN'2025	0.0331	0.0213	O/M	O/M	O/M	0.0210	0.0136	O/M	O/M	O/M
	CoPINN	Ours	<b>0.0016</b>	<b>0.0006</b>	<b>0.0004</b>	<b>0.0003</b>	<b>0.0002</b>	<b>0.0010</b>	<b>0.0005</b>	<b>0.0002</b>	<b>0.0002</b>	<b>0.0002</b>
	IMP.	-	74%	70%	69%	63%	78%	74%	62%	62%	60%	67%
(3+1)-d Klein-Gordon	PINN	JCP'2019	0.0129	O/M	O/M	O/M	O/M	0.0096	O/M	O/M	O/M	O/M
	gPINN	CMAME'2022	0.0144	O/M	O/M	O/M	O/M	0.0107	O/M	O/M	O/M	O/M
	SPINN	NeurIPS'2023	0.0151	0.0086	<u>0.0073</u>	O/M	O/M	0.0112	0.0064	<u>0.0054</u>	O/M	O/M
	SPINN (m)	NeurIPS'2023	<u>0.0078</u>	<u>0.0065</u>	<u>0.0077</u>	O/M	O/M	<u>0.0058</u>	<u>0.0048</u>	<u>0.0057</u>	O/M	O/M
	AHD-PINN	IJCAI'2024	0.0109	0.0107	O/M	O/M	O/M	0.0081	0.0080	O/M	O/M	O/M
	RoPINN	NeurIPS'2024	0.0135	O/M	O/M	O/M	O/M	0.0100	O/M	O/M	O/M	O/M
	FPINN	NN'2025	0.0152	O/M	O/M	O/M	O/M	0.0113	O/M	O/M	O/M	O/M
	CoPINN	Ours	<b>0.0041</b>	<b>0.0027</b>	<b>0.0017</b>	<b>O/M</b>	<b>O/M</b>	<b>0.0031</b>	<b>0.0019</b>	<b>0.0012</b>	<b>O/M</b>	<b>O/M</b>
	IMP.	-	47%	58%	78%	/	/	47%	60%	79%	/	/
Diffusion	PINN	JCP'2019	0.0095	0.0082	0.0081	O/M	O/M	0.00082	0.00071	0.00070	O/M	O/M
	gPINN	CMAME'2022	0.0099	0.0087	0.0090	O/M	O/M	0.00086	0.00076	0.00078	O/M	O/M
	SPINN	NeurIPS'2023	0.0447	0.0115	0.0075	0.0061	0.0061	0.00387	0.00100	0.00065	0.00053	0.00053
	SPINN (m)	NeurIPS'2023	0.0390	0.0067	<u>0.0041</u>	<u>0.0036</u>	<u>0.0036</u>	0.00338	0.00058	<u>0.00036</u>	<u>0.00031</u>	<u>0.00031</u>
	AHD-PINN	IJCAI'2024	0.0102	0.0080	0.0074	O/M	O/M	0.00088	0.00069	0.00064	O/M	O/M
	RoPINN	NeurIPS'2024	0.0412	0.0308	0.0306	O/M	O/M	0.00357	0.00267	0.00265	O/M	O/M
	FPINN	NN'2025	<b>0.0058</b>	0.0048	O/M	O/M	O/M	<u>0.00050</u>	<u>0.00042</u>	O/M	O/M	O/M
	CoPINN	Ours	<b>0.0058</b>	<b>0.0038</b>	<b>0.0035</b>	<b>0.0033</b>	<b>0.0033</b>	<b>0.00047</b>	<b>0.00033</b>	<b>0.00030</b>	<b>0.00028</b>	<b>0.00028</b>
	IMP.	-	0%	21%	12%	8%	8%	6%	21%	17%	10%	10%

Table 4. Full results of the flow Mixing 3D equation.  $N_c$  is the number of collocation points. The best and the second-best results are highlighted in **boldface** and underlined, respectively.

Evaluation Metric		$RL_2$					$RMSE$				
Methods	$N_c$	$16^3$	$32^3$	$64^3$	$128^3$	$256^3$	$16^3$	$32^3$	$64^3$	$128^3$	$256^3$
	Ref.										
SPINN	NeurIPS'2023	0.1352	0.1054	0.0861	0.0787	0.0672	0.1087	0.0875	0.0711	0.0575	0.0488
SPINN (m)	NeurIPS'2023	0.0884	0.0263	0.0080	0.0032	0.0023	0.0342	0.0241	0.0063	0.0022	0.0016
CoPINN	Ours	<b>0.0853</b>	<b>0.0230</b>	<b>0.0063</b>	<b>0.0032</b>	<b>0.0022</b>	<b>0.0323</b>	<b>0.0190</b>	<b>0.0060</b>	<b>0.0022</b>	<b>0.0015</b>
IMP.	-	4%	11%	21%	0%	4%	6%	21%	5%	0%	6%

Table 5. Relative L2 errors of comparative methods over 1D-convection equation.

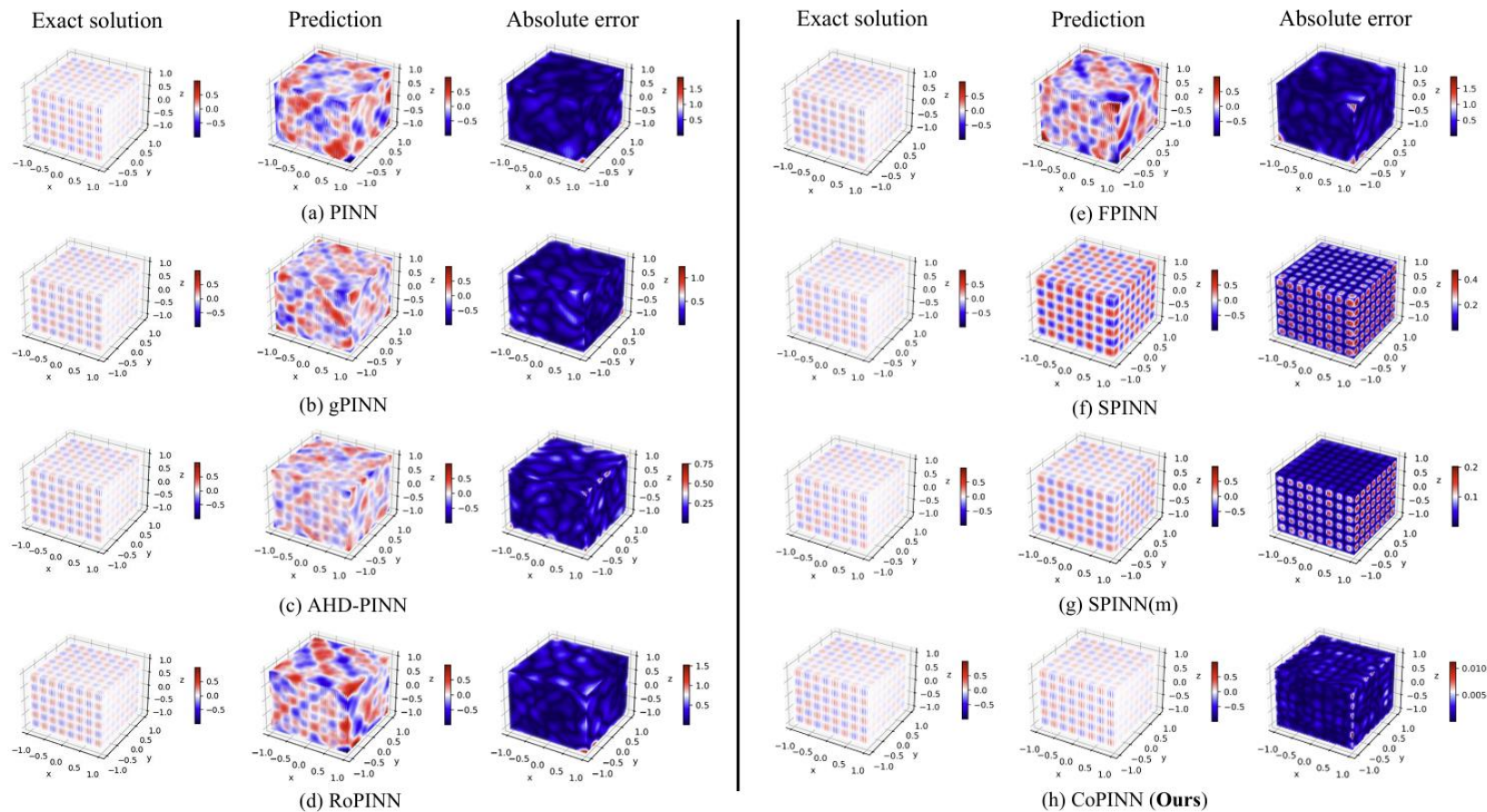
Methods	$\xi$	30	50
RAD (Wu et al., 2023)		0.0418	0.6547
R3 (Daw et al., 2023)		0.0160	0.5794
Causal R3 (Daw et al., 2023)		0.0073	<b>0.5471</b>
Ours		<b>0.0055</b>	0.5811

- CoPINN Outperforms comparison methods on all datasets.
- The improvement on the difficult dataset is higher than that on the simple dataset, proving that CoPINN can model complex regions (mutation regions such as boundaries) more effectively.
- By leveraging separable architecture, CoPINN mitigates the burden of representing the entire 2/3/4D function and completes training.



# Experiments-Visualization results

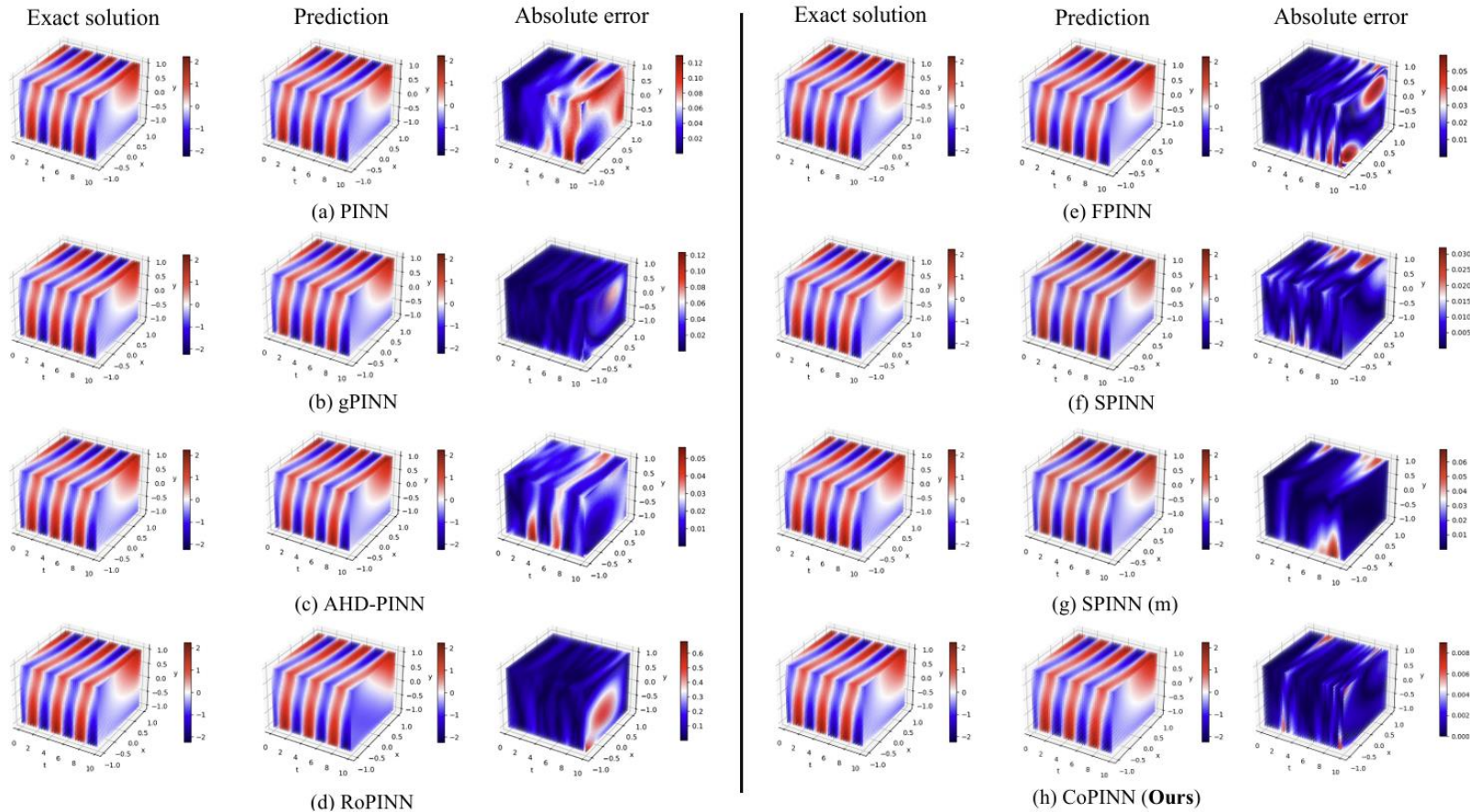
## Results on Helmholtz Equation



- The visualization results show that CoPINN has better learned the distribution of the physical field controlled by PDE and has a smaller absolute error.

# Experiments-Visualization results

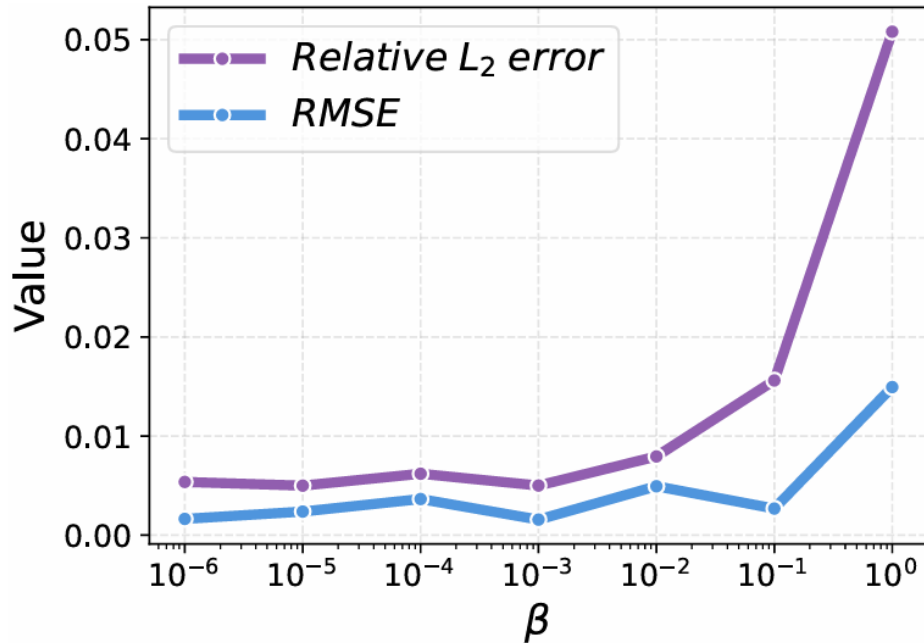
## Results on (2+1)-d Klein-Gordon equation



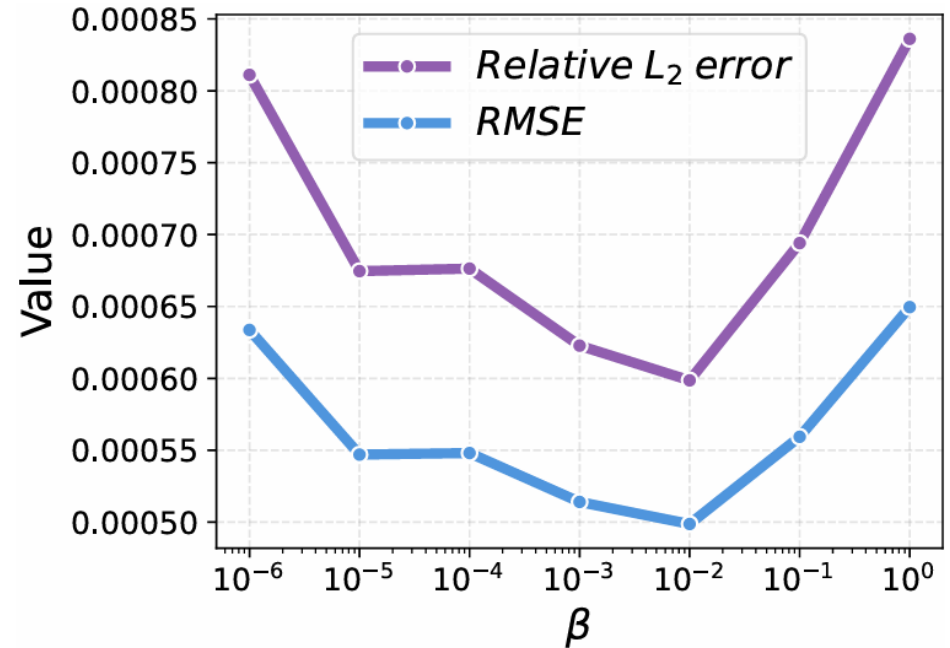
- The visualization results show that CoPINN has better learned the distribution of the physical field controlled by PDE and has a smaller absolute error.

# Experiments-Parameter sensitivity analysis

Results on Helmholtz Equation



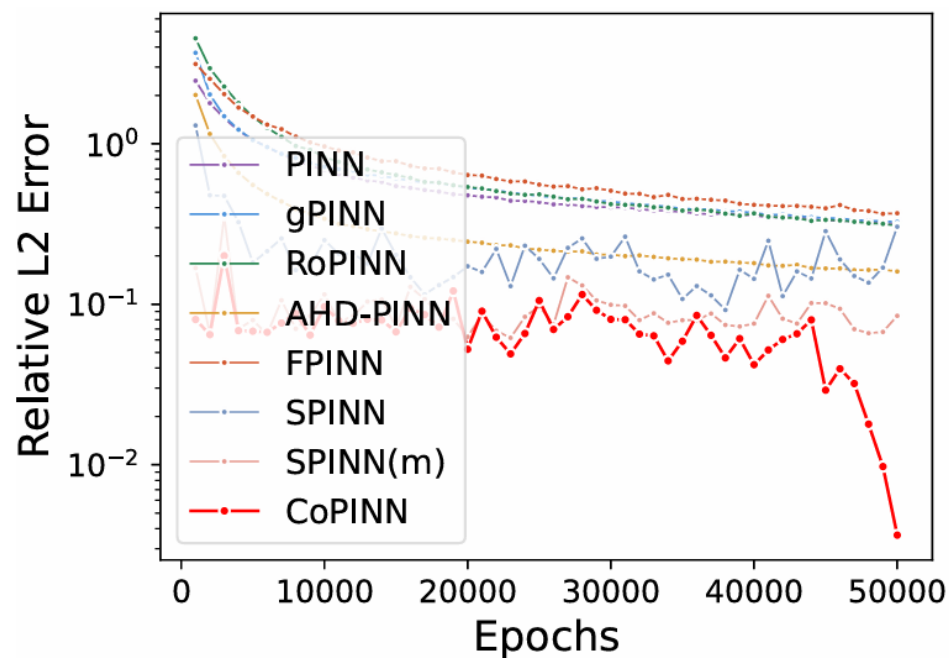
Results on (2+1)-d Klein-Gordon equation



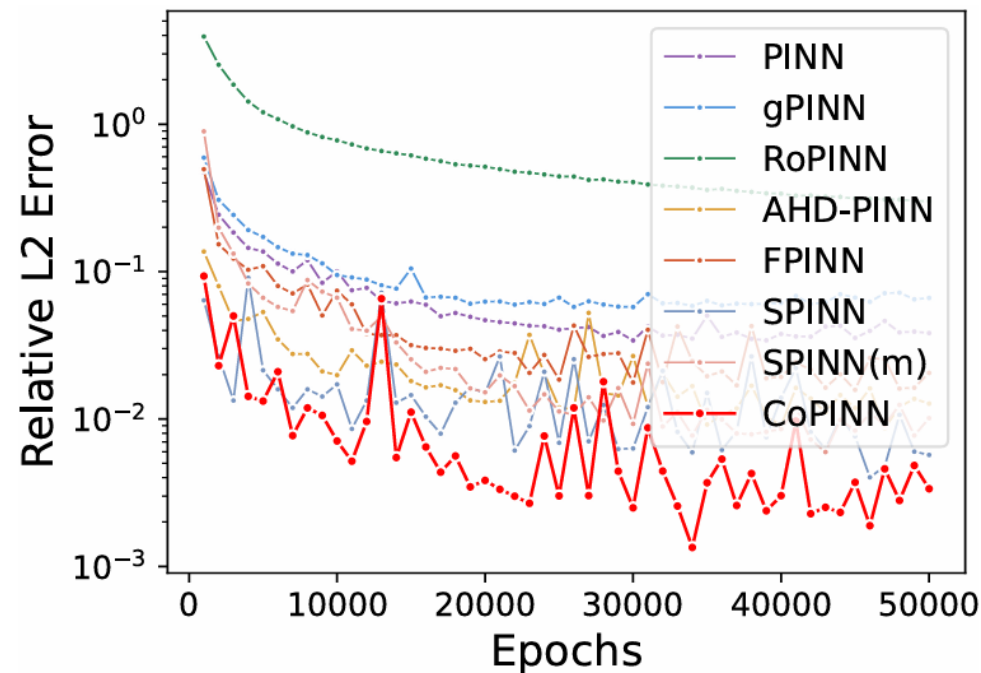
- Performance of CoPINN with varying  $\beta$  on the Helmholtz equation and the (2+1)-d Klein-Gordon equation with  $N_c=32^3$ .

# Experiments-Error analysis

Results on Helmholtz Equation



Results on (2+1)-d Klein-Gordon equation



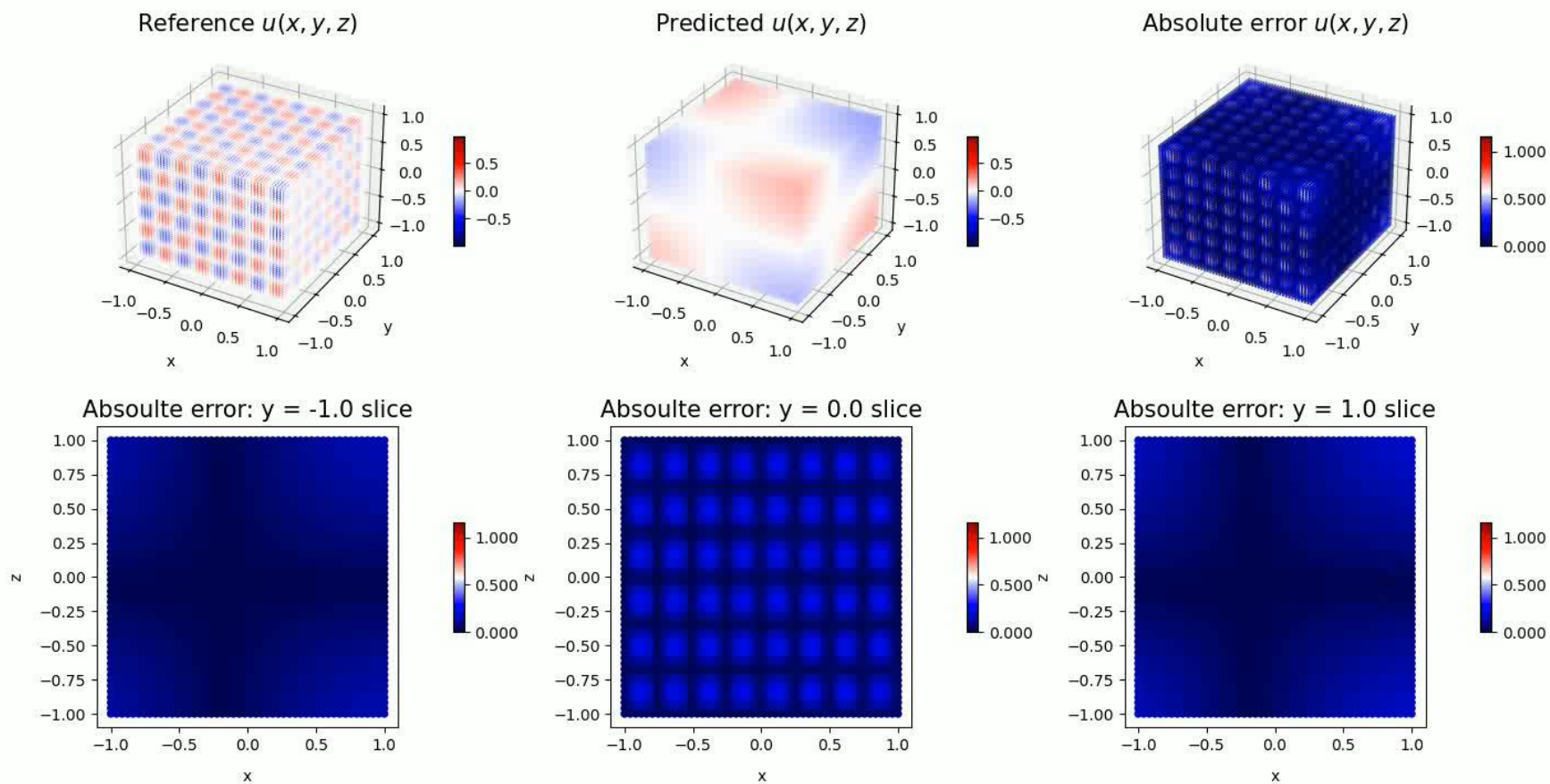
➤ Variation curve of relative L2 with the increase of epoch under  $N_c=32^3$ .



# Experiments-Error analysis

## Results on Helmholtz Equation

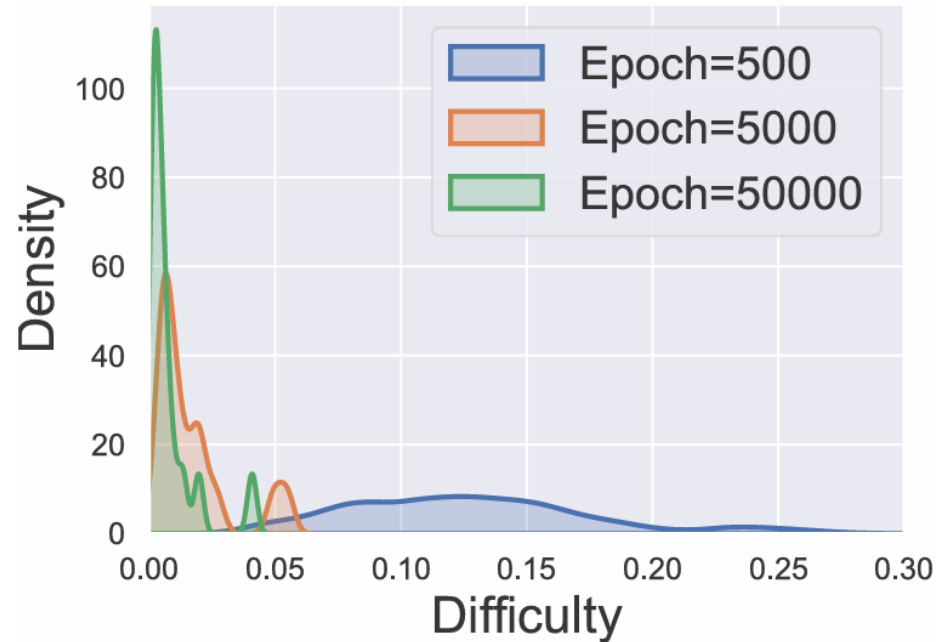
Epoch = 100



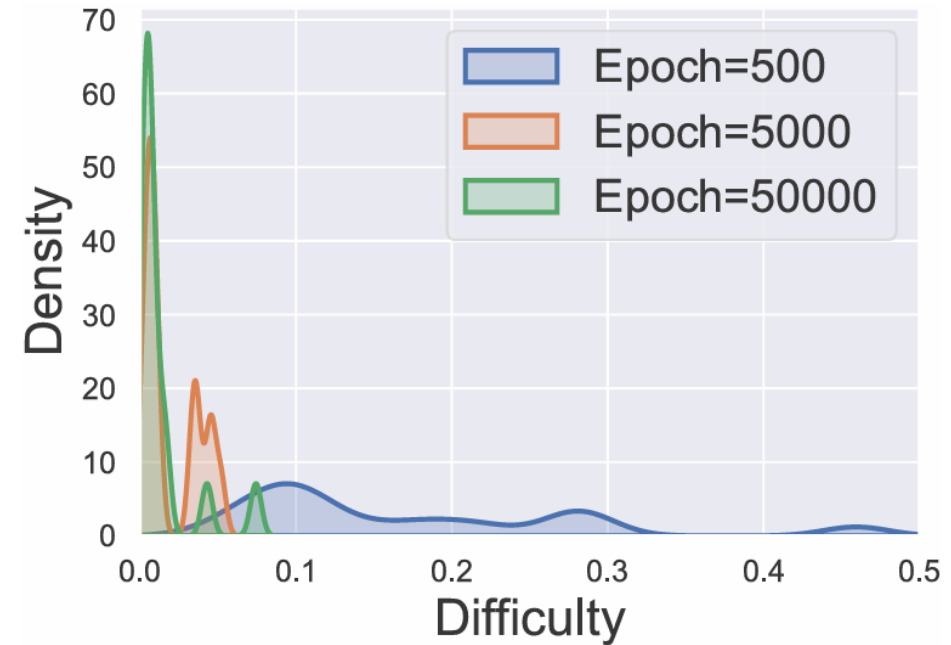
➤ Variation of absolute error with the increase of epoch under  $N_c=32^3$ .

# Experiments-Difficulty analysis

Cognitive training



w/o Cognitive training



- Difficulty density on different epochs. Experiments are conducted on the (3+1)-d Klein-Gordon equation PDE systems with  $16^3$  collocation points.



# Contributions

## CoPINN: Cognitive Physics-Informed Neural Networks

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### Abstract

Physics-informed neural networks (PINN) aim to constrain the outputs and gradients of deep learning models to satisfy specified governing physics equations, which have demonstrated significant potential for solving partial differential equations (PDEs). Although existing PINN methods have achieved pleasing performance, they always treat both easy and hard sample points indiscriminately, especially ones in the physical boundaries. This easily causes the PINN model to fall into undesirable local minima and unstable learning, thereby resulting in an Unbalanced Prediction Problem (UPP). To deal with this daunting problem, we propose a novel framework named Cognitive Physics-Informed Neural Networks (CoPINN) that imitates the human cognitive learning manner from easy to hard. Specifically, we first employ separable subnetworks to encode independent one-dimensional coordinates and apply an aggregation scheme to generate multi-dimensional predicted physical variables. Then, during the training phase, we dynamically evaluate the difficulty of each sample according to the gradient of the PDE residuals. Finally, we propose a cognitive training scheduler to progressively optimize the entire sampling regions from easy to hard, thereby embracing robustness and generalization against predicting physical boundary regions. Extensive experiments demonstrate that our CoPINN achieves state-of-the-art performance, particularly significantly reducing prediction errors in stubborn regions. The code is available at this repository: <https://github.com/siyanncnd/CoPINN>.

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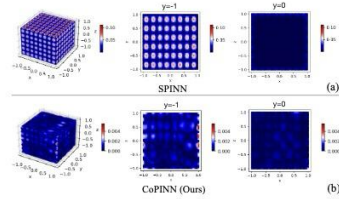


Figure 1. The 2D and 3D visualization of the absolute error between the predicted and exact values. The left graph illustrates the absolute error of the entire three-dimensional space. The middle graph demonstrates the absolute error of the boundary when  $y = -1$ , while the right graph displays the absolute error of the cross-section when  $y = 0$ . (a) The absolute error of SPINN (SOTA) (Cho et al., 2023) on the Helmholtz equation. These results indicate that SPINN exhibits significantly larger errors near the physical boundary region compared to the middle region, which reveals the Unbalanced Prediction Problem (UPP). (b) The absolute error of our CoPINN on the Helmholtz equation, which shows that CoPINN maintains consistent small absolute errors both near the physical boundary and in the middle region.

### 1. Introduction

As a type of equation in mathematics that describes the relationship between functions of multiple variables (Evans, 2022), partial differential equations (PDEs) usually involve partial derivatives of unknown functions and describe the laws of change of natural phenomena in disciplines such as physics, chemistry, and economics. In recent years, solving PDEs has a wide range of applications in many fields of science and engineering (Roubíček, 2013; Li et al., 2025), which are crucial for understanding and uncovering the underlying physical laws. Therefore, a large number of traditional numerical methods have been developed to solve PDEs, such as finite element methods (Jagota et al., 2013), finite difference methods (Thomas, 2013), and finite volume methods (Barth et al., 2018). Unfortunately, these methods are resource-intensive and mesh-dependent, thus often requiring a huge time cost to solve such complex systems, for example, the Navier-Stokes equations (Zou et al., 2024a). Thanks to the powerful nonlinear representation capabilities of deep learning, they are also widely used to solve partial

- We reveal and study an untouched yet pervasive significant problem in PINN, termed the unbalanced prediction problem (UPP). Unlike previous PINN methods that always treat both easy and hard samples equally, we propose a novel cognitive PINN framework to alleviate the negative effect of the hard samples in stubborn regions during the learning process.
- We propose a PINN model with SPL that measures the sample-level difficulty and promotes the neural network to fit samples from easy to hard in solving PDEs. To the best of our knowledge, our CoPINN could be the first work that leverages SPL to enhance the PINN performance in difficult regions.
- We conduct a series of experiments on multiple widely used PDE equations. These numerical results demonstrate that the proposed CoPINN can consistently outperform seven state-of-the-art PINN methods by a considerable margin under different numbers of collocation points.



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