# Report for Comprehensive Exam

# Siyuan Zhao

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# 1 Topic 1: Ways to Personalized

# 1.1 Q1: Estimating Treatment Effect

Question: You have explored at least one way to personalize. That is to try to learn in an RCT which conditions that should be given to which types of students. Review the methods you are aware (1) Susan Athey's Random Causal Forest and 2) Counterfactual inference for causal inference/individual treatment effect, , 3) individual treatment rules from the medical literature and 4) Susan Murph's methods) of and how are they different and similar.

Strong Ignorability The treatment assignment is defined to be strongly ignorable [Rosenbaum and Rubin, 1983] if the following two conditions hold: 1).  $(Y(1),Y(0)) \perp t \mid X$  and 2).  $0 < \Pr(t=1 \mid X) < 1$ . The first condition says that treatment assignment is independent of the potential outcomes conditional on the observed features of subjects. The second condition says that every subject has a nonzero probability to receive either treatment. The aforementioned first condition is also referred to as the "no-hidden confounding variables" assumption that all factors determining the outcome of each condition are observed.

#### 1.1.1 Random Causal Forests

[Wager and Athey, 2015] proposed random causal forest (RCF) to infer treatment effect. From the conceptual point of view, causal trees can be viewed as nearest neighbor methods with an adaptive neighborhood metric. Assume that there are n observed independent samples  $(X_i, Y_i, W_i)_{i=1}^n$ . The workflow of the RCF is that a causal tree is first built by recursively splitting the feature space until all samples are partitioned into a set of leaves L, each of which contains a few training samples. Then, for a data point x, the predicted outcome  $\hat{\mu}(x)$  is evaluated by identifying the leaf L(x) containing x and calculating

$$\hat{\mu} = \frac{1}{|\{i : X_i \in L(x)\}|} \sum_{\{i : X_i \in L(x)\}} Y_i$$

Given a test point x, the closest points to x are those fall in the same leaf as it. The authors believe that the leaf is small enough that the responses  $Y_i$  are roughly identically distributed. Then the treatment effect  $\hat{\tau}$  for any  $x \in L(x)$  is estimated as following:

$$\hat{\tau}(x) = \frac{1}{|\{i : W_i = 1, X_i \in L(x)\}|} \sum_{\{i : W_i = 1, X_i \in L(x)\}} Y_i$$

$$-\frac{1}{|\{i : W_i = 0, X_i \in L(x)\}|} \sum_{\{i : W_i = 0, X_i \in L(x)\}} Y_i$$
(1)

RCF assumes that there is overlapping in the data, i.e., for some  $\epsilon > 0$  and all  $x \in [0,1]^d$ ,

$$\epsilon < \mathbb{P}[W = 1 \mid X = x] < 1 - \epsilon$$

This condition effectively guarantees that, for large enough n, there will be enough treatment and control units near any test point x for local methods to work.

Honest Trees and Forests The authors also define the Honest Trees and Forests in [Wager and Athey, 2015]. A tree is honest if, for each training example i, it only uses the response  $Y_i$  to estimate the within-leaf treatment effect  $\tau$  using Equation 1 or to decide where to place the splits, but not both. There are two causal forest algorithms proposed from [Wager and Athey, 2015] that satisfy this condition.

The first algorithm, which is called double-sample trees, achieves honesty by dividing its training data into two halves  $\mathcal{I}$  and  $\mathcal{J}$ . Then, it uses the  $\mathcal{J}$ -sample to place the splits, while holding out the  $\mathcal{I}$ -sample to do within-leaf estimation. The details of the algorithm is shown in Algorithm 1.

# Algorithm 1: Double-sample Causal Trees

**Input:** n training examples of  $(X_i, Y_i, W_i)$ , where  $X_i$  are features,  $Y_i$  is the response, and  $W_i$  is the treatment assignment.

A minimum leaf size k.

- 1 Draw a random subsample of size s from  $\{1, \ldots, n\}$  without replacement, and then divide it into two disjoint sets of size  $|\mathcal{I}| = \lfloor s/2 \rfloor$  and  $|\mathcal{J}| = \lceil s/2 \rceil$
- **2** Grow a tree via recursive partitioning. The splits are chosen using any data from the  $\mathcal{J}$  sample and X- or W-observations from the  $\mathcal{I}$  sample, but without using Y-observations from  $\mathcal{I}$ -sample.
- 3 Estimate leaf-wise response using only the  $\mathcal{I}$ -sample observations.

The algorithm estimates  $\hat{\tau}(x)$  using Equation 1 on the  $\mathcal{I}$  sample. The splitting criteria is to maximize the variance of  $\hat{\tau}(X_i)$  for  $i \in \mathcal{J}$ . Each leaf of the tree must contain k or more  $\mathcal{I}$ -sample observations of each treatment class.

Another approach to build honest trees is inspired by the idea of propensity score matching, which is called propensity trees. The idea behind this approach is to train a classification tree to predict the propensity score, which is the treatment assignments  $W_i$ , and ignore the outcome  $Y_i$  when placing splits. This method is useful in observational studies since propensity score is aimed to reduce the selection bias by equating groups based on these features. The details of the algorithm is shown in Algorithm 2.

#### Algorithm 2: Propensity Trees

**Input:** n training examples of  $(X_i, Y_i, W_i)$ , where  $X_i$  are features,  $Y_i$  is the response, and  $W_i$  is the treatment assignment. A minimum leaf size k.

- 1 Draw a random subsample  $\mathcal{I} \in \{1, \ldots, n\}$  of size  $|\mathcal{I}| = s$  (no replacement).
- **2** Train a classification tree using sample  $\mathcal{I}$  where the outcome is the treatment assignment, i.e., on the  $(X_i, W_i)$  pairs with  $i \in \mathcal{I}$ . Each leaf of the tree must have k or more observations of each treatment class.
- **3** Estimate  $\tau(x)$  using Equation 1 on the leaf containing x.

#### 1.1.2 Counterfactual Inference

The problem of causal inference is often framed in terms of counterfactual problems such as "Would this particular student benefit more from the video hint or the text hint when the student cannot solve a problem?" In the binary intervention set, there are two possible interventions  $\mathcal{T} = \{0,1\}$ , where intervention 1 is often referred as the "treated" and intervention 0 is the "control." Given a sample of subjects and a treatment, each subject has a pair of potential outcomes:  $Y_0$  and  $Y_1$ , the outcomes under the control and the treatment, respectively. Let t be an indicator variable denoting the treatment received (t=0) for the control and t=1 for the treatment). Only one outcome, which is called factual outcome,  $y_F(\mathbf{x}) = t \cdot Y_1(\mathbf{x}) + (1-t) \cdot Y_0(\mathbf{x})$ , is observed for the subject  $\mathbf{x}$ , where  $\mathbf{x} \in \mathcal{X}$  are the observed features for the subject. The unobserved outcomes are referred to as the counterfactual outcome, denoted as  $y_{CF}(\mathbf{x}) = (1-t) \cdot Y_1(\mathbf{x}) + t \cdot Y_0(\mathbf{x})$ . In other words, when a subject  $\mathbf{x}$  is assigned to the "control" (t=0),  $y_F(\mathbf{x})$  is equal to  $Y_1(\mathbf{x})$ , and  $y_{CF}(\mathbf{x})$  is equal to  $Y_0(\mathbf{x})$ . The other way around,  $y_F(\mathbf{x})$  is equal to  $Y_0(\mathbf{x})$ , and  $y_{CF}(\mathbf{x})$  is equal to  $Y_1(\mathbf{x})$ . The estimated individual treatment effect (ITE) is then calculated by ITE( $\mathbf{x}$ ) =  $Y_1(\mathbf{x}) - Y_0(\mathbf{x})$ . The goal of counterfactual inference is to predict the counterfactual outcome given the observed data  $D_n = \{(\mathbf{x}_i, t_i, y_F^i)_{i=1}^n\}$  from

either RCTs or observational studies in order to estimate the ITE. We assume n samples  $\left\{(x_i,t_i,y_F^i)\right\}_{i=1}^n$  form an empirical distribution  $\hat{p}^F = \left\{(x_i,t_i)\right\}_{i=1}^n$ . We call this empirical distribution  $\hat{p}^F \sim p^F$  the empirical factual distribution. In order to calculate ITE, we need to infer the counterfactual outcome which is dependent on the empirical distribution  $\hat{p}^{CF} = \left\{(x_i,1-t_i)\right\}_{i=1}^n$ , which is called the empirical counterfactual distribution  $\hat{p}^{CF} \sim p^{CF}$ . The  $p^F$  and  $p^{CF}$  may not be equal because the distributions of the control and the treated populations may be different. The inequality of two distributions may cause the counterfactual inference over a different distribution than the one observed from the experiment. In machine learning terms, this scenario is usually referred to as domain adaptation, where the distribution of features in test data are different than the distribution of features in training data.

[Johansson et al., 2016] proposed Balancing Neural Networks (BNN) which

can be applied to solve the counterfactual inference problem. They used a form of regularizer to enforce the similarity between the distributions of representations learned for populations with different interventions, for example, the representations for students who received text hints versus those who received video hints. This reduces the variance from fitting a model on one distribution and applying it to another. Because of random assignment to the interventions in RCTs, the distributions of the populations within different interventions are highly likely to be identical. However, in the observational study, we may end up with the situation where only male students receive video hints and female students receive text hints. Without enforcing the similarity between the distributions of representations for male and female students, it is not safe to make a prediction of the outcome if male students receive text hints.

The Counterfactual Regression (CFR) [Shalit et al., 2016] is built on the BNN. The important difference between these two models is that the CFR uses a more powerful distribution metric in the form of IPMs to learn a balancing representation.

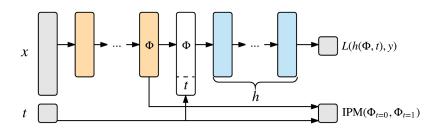


Figure 1: CFR for ITE estimation. L is a loss function, IPM is an integral probability metric

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The structure of CFR is illustrated in Figure 1. To learn a representation of deep features  $\Phi$ , the CFR uses fully connected layers with ReLu activation function, where Relu(z) = max(0,z). We need to generalize from factual distribution to counterfactual distribution in the feature representation  $\Phi$  to obtain accurate estimation of counterfactual outcome. The common successful approaches for domain adaptation encourage similarity between the latent feature representations w.r.t the different distributions. This similarity is often enforced by minimizing a certain distance between the domain-specific hidden features. The distance between two distributions is usually referred to as the discrepancy distance, introduced by [Mansour et al., 2009], which is a hypothesis class dependent distance measure tailored for domain adaptation.

Integral Probability Metric (IPM) are used to measure the distance between two distributions  $p_0 = p(x|t=0)$ , and  $p_1 = p(x|t=1)$ , also known as the

control and treated distributions. The IPM for  $p_0$  and  $p_1$  is defined as

$$IPM_{\mathcal{F}}(p_0, p_1) := \sup_{f \in \mathcal{F}} \left| \int_S f dp_0 - \int_S f dp_1 \right|,$$

where  $\mathcal{F}$  is a class of real-valued bounded measurable functions on S.

The choice of functions is the crucial distinction between IPMs [Sriperumbudur et al., 2009]. Two specific IPMs are used in our experiments: the Maximum Mean Discrepancy (MMD), and the Wasserstein distance. IPM $_{\mathcal{F}}$  is called MMD, when  $\mathcal{F} = \{f: \|f\|_{\mathcal{H}} \leq 1\}$ , where  $\mathcal{H}$  represents a reproducing kernel Hilbert space (RKHS) with k as its reproducing kernel. In other words, the family of norm-1 reproducing kernel Hilbert space (RKHS) functions lead to the MMD. The family of 1-Lipschitz functions  $\mathcal{F} = \{f: \|f\|_L \leq 1\}$ , where  $\|f\|_L$  is the Lipschitz semi-norm of a bounded continuous real-valued function f, make IPM the Wasserstein distance. Both the Wasserstein and MMD metrics have consistent estimators which can be efficiently computed in the finite sample case [Sriperumbudur et al., 2012]. The important property of IPM is that

$$p_0 = p_1 \text{ iff } IPM_{\mathcal{F}}(p_0, p_1) = 0.$$

The representation with reduction of the discrepancy between the control and the treated populations helps the model to focus on balancing features across two populations when inferring the counterfactual outcomes. For instance, if in an experiment, almost no male student ever received intervention A, inferring how male students would react to intervention A is highly prone to error and a more conservative use of the gender feature might be warranted.

After obtaining the representation for subject  $\mathbf{x_i}$ , CFR concatenates the treatment assignment  $t_i$  to the representation  $\Phi(\mathbf{x}_i)$  and feeds  $[\Phi(\mathbf{x}_i), t_i]$  to another two fully connected layers to generate the predicted outcome.

#### 1.1.3 Individualized Treatment Rules

An individual treatment rule (ITR)  $d: \mathcal{X} \to \mathcal{T}$  is a deterministic decision rule from subject  $\mathbf{x} \in \mathcal{X}$  into the intervention space  $\mathcal{T}$ . In experiments, we observe a triplet  $(\mathbf{x}, t, y)$  from each subject, where  $\mathbf{x} = (x_1, x_2, \dots, x_n)^T \in \mathcal{X}$  denotes the participant's observed features,  $t \in \mathcal{T} = -1, 1$  denotes the treatment assignment, and  $y \in Y$  is the observed outcome, also called the "reward" in the literature on reinforcement learning. Note that t = -1 means that the subject is assigned to the control and t = 1 mean that the subject is assigned to the treatment in the context of ITR. Let  $p(t|\mathbf{x})$  be the probability of assigning the subject with features  $\mathbf{x}$  to the intervention t. [Qian and Murphy, 2011] showed that the value of d satisfies

$$V(d) = E\left[\frac{y}{p(t|x)}\mathbb{I}_{t=d(\mathbf{x})}\right],$$

where  $\mathbb{I}(\cdot)$  is an indicator function. The goal of the ITR is to find an optimal ITR,  $d^*$ , which is a rule that has the maximal value such that,

$$d^* \in \arg\max_{d} V(d).$$

[Qian and Murphy, 2011] also showed that finding  $d^*$  is equivalent to minimizing the following equation:

$$d^* \in \operatorname*{arg\,min}_{d} E\left[\frac{y}{p(t|x)} \mathbb{I}_{t \neq d(\mathbf{x})}\right]$$
 (2)

Assume that the observed data  $\{(\mathbf{x}_i,t_i,y_i),\ i=1,\ldots,n\}$  are collected independently. For any decision function  $f(\mathbf{x})$ , let  $d_f(\mathbf{x}) = sign(f(\mathbf{x}))$  be the associated rule, where sign(u) = 1 for u > 0 and -1 otherwise. The particular choice of the value of sign(0) is not important. With the observed data, the weighted classification error in Equation 2 can be approximated by the empirical risk

$$\frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{p(t_i|x_i)} \mathbb{I}_{t_i \neq d_f(\mathbf{x_i})}.$$
(3)

[Qian and Murphy, 2011] proposed a two-step procedure that first train a model using the observed data to estimate a conditional mean for the outcome  $E(y|\mathbf{x},t)$ , and then determines the treatment rule by comparing the predicted value  $E(y|\mathbf{x},t=1)$  between  $E(y|\mathbf{x},t=-1)$ .

In contrast, [Zhao et al., 2012] proposed one-step procedure by showing that find an optimal ITR is equivalent to a classification task where we want to classify t with  $\mathbf{x}$  as the input. Outcome weighted learning (OWL) proposed by [Zhao et al., 2012] use the hinge loss function and the regularization technique aiming to minimize

$$\frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{p(t_i|x_i)} (1 - t_i f(\mathbf{x}_i))_+ + \lambda \|f\|^2 , \qquad (4)$$

where  $(u)_{+} = \max(u, 0)$  is the positive part of u, ||f|| is some norm for f, and  $\lambda$  is a tuning parameter controlling the trade-off between empirical risk and complexity of the decision function f.

[Zhou et al., ] showed that decision rule in Equation 8 is affected by a simple shift of the outcome y and proposed Residual weighted learning (RWL) to relieve this issue. The idea behind RWL is to introduce a function g to reduce the variance of  $\frac{y-g(\mathbf{x})}{p(t|\mathbf{x})}\mathbb{I}_{t\neq d(\mathbf{x})}$  and a reasonable candidate of g is

$$g^*(\mathbf{x}) = \frac{\mathbb{E}(y|\mathbf{x}, t=1) + \mathbb{E}(y|\mathbf{x}, t=-1)}{2} = \mathbb{E}\left(\frac{y}{2p(t, \mathbf{x})}|\mathbf{x}\right). \tag{5}$$

RWL thus is to minimize the following empirical risk:

$$\frac{1}{n} \sum_{i=1}^{n} \frac{y_i - \hat{g}^*(\mathbf{x}_i)}{p(t_i | \mathbf{x_i})} \mathbb{I}_{t_i \neq d_f(\mathbf{x_i})}$$

$$\tag{6}$$

where  $\hat{g}^*$  is an estimate of  $g^*$ . For simplicity, let  $\hat{r}_i = y_i - \hat{g}^*(\mathbf{x}_i)$ . As in OWL, [Zhou et al., ] consider a surrogate loss function T to replace the 0-1 loss in Equation 6. The non-convex loss T has the following form:

$$T(u) = \begin{cases} 0 & \text{if } u \ge 1, \\ (1-u)^2 & \text{if } 0 \le u < 1, \\ 2 - (1+u)^2 & \text{if } -1 \le u < 0, \\ 2 & \text{if } u < -1 \end{cases}$$
 (7)

It is called the smoothed ramp loss in [Zhou et al., ]. By incorporating the regularization, RWL is eventually aimed to minimize the following empirical risk:

$$\frac{1}{n} \sum_{i=1}^{n} \frac{\hat{r}_i}{p(t_i | x_i)} T(t_i f(\mathbf{x}_i))_+ + \lambda \|f\|^2 , \qquad (8)$$

where ||f|| is the norm for f, and  $\lambda$  is a tuning parameter.

#### 1.1.4 Comparison

All three aforementioned methods (RCF, Counterfactual Inference, ITR) can be trained on the data from either RCTs or observational studies and can be used to assign a subject to a predicted optimal condition. Propensity trees from RCF [Wager and Athey, 2015] and BNN from counterfactual inference [Johansson et al., 2016] are specifically optimized for observational studies. Propensity tress uses the idea from propensity score matching to reduce possible variance in features of subjects between two conditions, while BNN uses deep learning approaches for domain adaptation to enforce the similarity between the distributions of representations of subjects from two condition.

Since ITR is a deterministic rule aimed to assign a subject to optimal condition based on the observed features, ITR does not focus on estimating the treatment effect. BNN predicts two potential outcomes (the treatment outcome and the control outcome) for each subject, so BNN is designed to estimate the individual treatment effect. RCF assumes that subjects on the same leaf have the same potential outcomes, so RCF usually estimates the treatment effect on a small subset of populations from two conditions.

#### 1.2 Q2: Bayesian Optimization

**Question:** You have been doing stuff with Bandits. When might be learning a function to predict goodness of hints across students be a good idea? Look into Bayesian Optimization. How is that different than what you are planning on doing. How might you use such methods?

Bayesian optimization is a sequential model-based approach for global optimization of an unknown objective function f. The problem can be mathematically expressed as:

$$\mathbf{x}^* = \arg\max_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}) \tag{9}$$

where  $\mathcal{X}$  is the design space of interest. Bayesian optimization is the combination of two main components: a surrogate probabilistic model which captures our beliefs about the behavior of the unknown objective function f and is updated once new observation is gathered, and an acquisition function  $\alpha: \mathcal{X} \to \mathbb{R}$ which performs the selection of the optimal sequence of queries based on the previous model. Since the objective function f is unknown, the Bayesian strategy is to place a prior over it which captures our beliefs about the behavior of the function. After gathering the function evaluations, which are treated as data, the prior is updated to form the posterior distribution over the objective function. Equipped with the posterior distribution, an acquisition function  $\alpha$ is induced to determines what the next query point should be. The trade-off between the exploration and the exploitation is determined by the acquisition function leveraging the uncertainty in the posterior. Examples of acquisition functions includes probability of improvement, expected improvement, Bayesian expected losses, upper confidence bounds (UCB), and mixtures of these. The purpose of the trade-off is to minimize the number of function evaluations. As such, Bayesian optimization is well suited for functions that are very expensive to evaluate. The details of Bayesian Optimization is shown in Algorithm 3.

The Gaussian process (GP) is the most popular surrogate model. The objective function is modeled as a sample from the GP distribution. Attributes of the GP such as mean and variance are used by the acquisition function to decide the successive query points. More information about GP in the bandit setting can be found in Section 2.3.

```
Algorithm 3: Bayesian optimization
```

```
Input: n observations \mathcal{D} = \{(\mathbf{x}_i, y_i)\}_{i=1}^n from objective function f for t = n+1, n+2, \ldots do

select new \mathbf{x}_t by maximizing acquisition function \alpha

\mathbf{x}_t = \underset{\mathbf{x}}{\arg\max} \alpha(\mathbf{x}; \mathcal{D})
query objective function to observe y_t = f(\mathbf{x}_t)
augment data \mathcal{D} = \{\mathcal{D}, (\mathbf{x}_t, y_t)\}
update statistical model

end
```

## 1.2.1 Thompson Sampling

Thompson sampling for Beta-Bernoulli bandit perhaps is the simplest non-trivial multi-armed bandit strategy in Bayesian optimization [Shahriari et al., 2016]. Bernoulli bandit problem is the bandit problem when the rewards are

either 0 or 1, and for arm i the probability of success (reward=1) is  $\mu_i$ . The Thompson sampling maintains Bayesian priors on the Bernoulli means  $\mu_i$ 's. Beta distribution turns out to be a very convenient choice of priors for Bernoulli rewards. The pdf of Beta $(\alpha, \beta)$  with parameters  $\alpha > 0, \beta > 0$  is given by

$$f(x; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha - 1} (1 - x)^{\beta - 1}.$$
 (10)

The mean of  $\operatorname{Beta}(\alpha,\beta)$  is  $\alpha/(\alpha+\beta)$ ; and higher the  $\alpha,\beta$ , tighter is the concentration of  $\operatorname{Beta}(\alpha,\beta)$  around the mean. If the prior for each arm is a  $\operatorname{Beta}(\alpha,\beta)$  distribution, then after observing a Bernoulli trial, the posterior distribution is simply  $\operatorname{Beta}(\alpha+1,\beta)$  or  $\operatorname{Beta}(\alpha,\beta+1)$ , depending on whether the trial resulted in a success or failure, respectively. Thompson sampling then samples from these posterior distributions across all arms and chooses the arm with largest sample value. This procedure is summarized in Algorithm 4.

#### Algorithm 4: Thompson sampling for Beta-Bernoulli bandit

```
Input: \alpha, \beta: hyperparameters of the beta prior
Initialize n_{a,0} = n_{a,1} = i = 0 for all K arms a

repeat

for a = 1, ..., K do

w_a \sim \text{Beta}(\alpha + n_{a,1}, \beta + n_{a,0})

end

a_i = \underset{a}{\text{arg max }} w_a

Observe y_i by pulling arm a_i

if y_i = 0 then

n_{a_i,0} = n_{a_i,0} + 1

else

n_{a_i,1} = n_{a_i,1} + 1

end

n_{a_i,1} = n_{a_i,1} + 1

end
```

In summary, Thompson sampling is the simplest non-trivial multi-armed bandit strategy in Bayesian Optimization. Since the Thompson sampling models the arms as independent, Thompson sampling must try every arm at least once. It will be an issue if the number of arms becomes large. In this case, Bayesian optimization with GP is an alternative to the Thompson sampling and alleviate this issue.

# 1.2.2 Choice between function predictor and multi-armed bandit algorithm

In many experiments, the designs space available to the designers have components that can be varied independently. For example, in designing an advertisement, one has choices such arkwork, font style, and size. If there are five choices for each, the total number of possible configurations is 125.

In general, this number grows combinatorially in the number of components. This presents challenges for approaches such as the Thompson sampling, since the Thompson sampling models the arms as independent, which will lead to strategies that must try every arm at least once. This rapidly becomes infeasible in the large design spaces.

Even if the total number of possible design configuration is relatively small, it is still challenging for multi-armed bandit algorithms, such as Thompson sampling, when the number of available subjects is limited for these algorithms to figure out the optimal arm.

To alleviate this issue, a commonly used approach is to learn a function, whose input is a feature vector of the arm and output is associated reward of the arm, capturing dependence between the arms. Assume that each possible arm a has an associated feature vector  $\mathbf{x}_a \in \mathbb{R}^d$ . The expected reward of each arm can be expressed as a function of this feature vector, such as  $f(a) = f(\mathbf{x}_a)$ . The goal is to learn this function  $f: \mathbb{R}^d \to \mathbb{R}$  from the experiment data in order to choose the arm with the highest reward among all possible arms.

## 1.3 Q3: Contextual Bandit vs. Causal Inference

**Question:** In what conditions to use different ways to personalize: Bandits (contextual) or Causal inference (Including Susan Athey's Random Causal Forest and things like counterfactual inference for personalized learning)

The ultimate goal of contextual bandit algorithms and causal inference is the same: find the good intervention for each subject based on the observed features. The contextual bandit algorithms are widely used in the sequential experiments. In this setting, single interventions from a pre-defined set are repeatedly performed to evaluate their goodness via observed feedback. But explicit experiments may be difficult to be conducted in some research areas. Thus, contextual bandit algorithms cannot be directly applied and causal inference models are commonly used here. The idea behind causal inference models is to predict the outcome of an intervention from the observed data without explicitly performing it.

# 1.4 Q4: Online Learning vs. Batch Learning

**Question:** Why can BKT not update itself in real-time while Thompson sampling can update itself online?

There are four parameters in BKT that are learned from enough training data. Before deploying BKT online, we need to collect reliable training data and learn these four parameters offline. On the contrary, Thompson sampling is non-parametric model, so collecting training data is not necessary for it. Thompson sampling is a sequential method, which means it makes use of observations one

at a time, or in small batches. Thus, it can be used in real-time sequential experiments.

Four parameters in BKT are usually learned via Expectation-Maximization (EM) algorithm. EM algorithm is an iterative method to find the optimal values for parameters. An iterative method is a sequence of improving approximate solutions and requires computational time and resources. It will take some amount of time to update BKT parameters in real-time. On the contrary, Thompson sampling has a simple close form to update its posterior distribution after receiving new observations.

# 1.5 Q5: Semi-Supervised Learning

**Question:** What is Semi-supervised learning? How is that related to Mitchel's CoLearning? Is the same idea under a different name? Can it be used to help solve your problems related to PeerASSIST?

In the setting of semi-supervised learning, the training data consists of l labeled instances  $\{(\mathbf{x}_i, y_i)\}_{i=1}^l$  and u unlabeled instances  $\{(\mathbf{x}_j)\}_{j=l+1}^{l+u}$ , often with  $l \ll u$ . The goal of semi-supervised learning is to learn a model with better performance from the combination of labeled data and unlabeled data than from labeled data alone. The benefit of semi-supervised learning is that the labeled data can be hard and expensive to obtain since labels may require human experts, and the unlabeled data is often cheap in large quantity. Figure 2 illustrates how unlabeled data help models achieve a better performance.

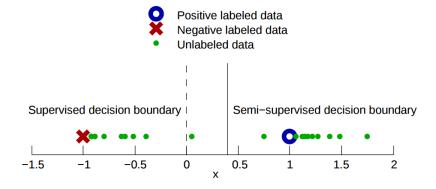


Figure 2: An illustrative example on how semi-supervised learning helps the model have a better performance.

#### 1.5.1 Co-training

Co-training [Blum and Mitchell, 1998] is a technique in semi-supervised learning that requires two views of the data. It assumes that each example is described using two different feature sets that provide different, complementary information about the example. Co-training first learns a separate classifier for each view using any labeled examples. The most confident predictions of each classifier on the unlabeled data are then used to iteratively construct additional labeled training data. The details of co-training algorithm is described in Algorithm 5.

Algorithm 5: Co-training Algorithm for Semi-Supervised Learning

```
Input: labeled data \{(\mathbf{x}_i, y_i)\}_{i=1}^l, unlabeled data \{\mathbf{x}_j\}_{j=l+1}^{l+u} each instance has two views \mathbf{x}_i = \left[\mathbf{x}_i^{(1)}, \mathbf{x}_i^{(2)}\right] and a learning speed k

Let L_1 = L_2 = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_l, y_l)\}.

repeat

Train view-1 f^{(1)} from L_1, view-2 f^{(2)} from L_2
Classify unlabeled data with f^{(1)} and f^{(2)} separately.

Add f^{(1)}'s top k most-confident predictions (\mathbf{x}, f^{(1)}(\mathbf{x})) to L_2
Add f^{(2)}'s top k most-confident predictions (\mathbf{x}, f^{(2)}(\mathbf{x})) to L_1
Remove these from the unlabeled data.

until unlabeled data is used up;
```

There are three important assumptions which guarantee the performance of co-training algorithm: 1). feature split  $x = [x^{(1)}; x^{(2)}]$  exists; 2).  $x^{(1)}$  or  $x^{(2)}$  alone is sufficient to train a good classifier; 3).  $x^{(1)}$  and  $x^{(2)}$  are conditionally independent given the class.

## 1.5.2 Application in PeerASSIST

Semi-supervised learning approaches come in handy in PeerASSIST when we try to learn a function to predict the effectiveness of peer explanations. In the setting of PeerASSIST, not every peer explanation will be received by students, and thus, some of the explanations have no observed data on their effectiveness (e.g., the popularity, next problem correctness). When learning the function, these peer explanations without observed data are discarded, which is a waste of the data. Moreover, if there are not enough observed data, it becomes difficult to accurately learn the function. Semi-supervised learning approaches can be applied in this scenario to learn a better function.

# 2 Topic 2: Application of Deep Learning to EDM and related issues

# 2.1 Q1: Natural Language Processing for Enhancing Learning

Question: You have done some work in using memory network to predict NLP classifications in a Kaggle Data set. What are some applications you can put those skills to that will drive new designs that could improve student learning? I think I have heard to throw out a few ideas (1) the evaluation of the quality of open response from students, or 2) Which kids are improving or slacking? Or 3) Sentiment analysis of posts or comments from students. Is there a reasonable way to think about how to use NLP in your PeerASSIST feature? Pitch a complete idea of something you think practical (you don't have to want to do it, but if you did that would be nice).

#### 2.1.1 Confusion Detector

Given the enormous student/instructor ratio in a MOOC's discussion forums, it is difficulty for an instructor to read all posts in a MOOC's discussion forums. To address this issue, [Agrawal et al., 2015] collected and created the Stanford MOOCPosts dataset: a corpus composed of 29,604 anonymized learner forum posts from eleven Stanford University public online classes. Each post in the MOOCPosts dataset was scored across six dimensions – confusion, sentiment, urgency, question, answer, and opinion – and subsequently augmented with additional metadata. Then the authors built a confusion classifier from the Stanford MOOCPost dataset to automatically detect confusion from students posts and proposed a recommendation algorithm, which takes the student confusion as input, to automatically recommended a video lecture from a collection of several video lectures related to the course.

### 2.1.2 Automated Essay Scoring

Automated grading is a critical part of Massive Open Online Courses (MOOCs) system and any intelligent tutoring systems (ITS) at scale. Essay writing is usually a common student assessment process in schools and universities. In this task, students are required to write essays of various length, given a prompt or essay topic. Some standard tests, such as Test of English as a Foreign Language (TOEFL) and Graduate Record Examination (GRE), assess student writing skills. Manually grading these essay will be time-consuming. Thus automated essay scoring (AES) systems has been used in these tests to reduce the time and cost of grading essays. Moreover, as massive open online courses (MOOCs) become widespread and the number of students enrolled in one course increases, the need for grading and providing feedback on written assignments are ever

critical.

AES has employed numerous efforts to improving its performance. AES uses statistical and Natural Language Processing (NLP) techniques to automatically predict a score for an essay based on the essay prompt and rubric. Most existing AES systems are built on the basis of predefined features, e.g. number of words, average word length, and number of spelling errors, and a machine learning algorithm [Chen and He, 2013]. It is normally a heavy burden to find out effective features for AES. Moreover, the performance of the AES systems is constrained by the effectiveness of the predefined features. Recently another kind of approach has emerged, employing neural network models to learn the features automatically in an end-to-end manner [Taghipour and Ng, 2016]. By this means, a direct prediction of essay scores can be achieved without performing any feature extraction. The model based on long short-term memory (LSTM) networks in [Taghipour and Ng, 2016] has demonstrated promise in accomplishing multiple types of automated grading tasks.

#### 2.1.3 NLP in PeerASSIST

The goal of PeerASSIST is to crowdsource the explanation (in the format of texts) of how to solve a given problem from students and apply multi-armed bandit algorithm to efficiently select the most helpful and useful peer explanation from all collected peer explanations for a given problem. The effectiveness of each peer explanations is measured in three dimensions: teacher's rating (thumb up or thumb down), student's rating (thumb up or thumb down), and the student next problem correctness after receiving the peer explanation.

A potential application of NLP techniques to PeerASSIST is to build a classifier to predict the probability of a peer explanation being an effective one based on the content of the explanation. As mentioned above, NLP techniques, especially deep learning approaches for NLP, have achieved promising results on text classification tasks. The predicted probabilities can potentially used to select the optimal explanation when one is needed to be delivered to the students. In this scenario, we can deliver the explanation with the highest probability to students. Another possible use of the predicted probabilities is to treat these values as a part of the contextual information for the explanations and incorporate this contextual information into contextual multi-armed bandit algorithms.

The explanations collected from students are not always of good quality. Since the classifier can automatically detect explanations with bad quality (i.e., the explanations with low probability values), one can quickly filter out these potential bad explanations, which saves multi-armed bandit algorithm from wasting attempts trying these explanations. When the explanations that students type in are detected as bad ones by the classifier, we can also build an intervention by telling students that the algorithm thinks that their explanations need some improvement. Then the students have a second chance to submitting new explanations. We hope that there will be observable improvement on the quality of peer explanations for these students.

# 2.2 Q2a: Reliable Crowdsourcing

Question: Besides assessing the learning gains associated with each learning artifacts, it can also be useful to characterize them for certain features. Suppose you ask many students to rate a bunch of learning artifacts along different dimensions, e.g., media format, pedagogical approach, difficulty to understand, etc. How do you know which learners to trust, and how do you combine their opinions into aggregate labels? What kinds of algorithms exist for this purpose and how do they work?

During this process of asking students to rate the learning artifacts, we might gather noisy rating from students due to the fact that students may have a wide ranging level of rating expertise which are unknown, and in some cases may be adversarial. To learn the ground truth of these learning artifacts, we need to aggregate their opinions to recover the true, unknown label of each learning artifacts.

## 2.2.1 Majority Voting

Given each item is labeled by different workers, it is a straightforward approach to take the majority label as the true label. From reported experimental results on real crowdcourcing data [Snow et al., 2008], majority voting performs significantly better on average than individual workers. However, majority voting considers each item independently and gives the same weight across all workers who label the item when aggregating true label.

#### 2.2.2 Dawid-Skene Model

[Dawid and Skene, 1979] were among the first to consider such a problem setup. They assume each workers are conditionally independent given the true labels and each worker is associated with a probabilistic confusion matrix that generates her labels. Each entry of the matrix indicates the probability that items in one class are labeled as another. Given the observed responses, the true labels for each items and the confusion matrices for each worker can be jointly estimated by a maximum likelihood method. The optimization can be implemented by the expectation-maximization (EM) algorithm.

#### 2.2.3 GLAD

[Whitehill et al., 2009] proposed a richer graphic model which includes item difficulty and the expertise of the worker. The difficulty of item is modeled by the parameter  $1/\beta_j \in [0,\infty)$ . Here  $1/\beta_j = \infty$  means the image is very ambiguous and hence the most proficient worker has a random chance of labeling it correctly.  $1/\beta_j = 0$  means the item is so easy that even the most obtuse worker will always label it correctly.

The expertise of each worker i is modeled using the parameter  $\alpha_i \in (-\infty, +\infty)$ . Here  $\alpha = +\infty$  means the worker always labels items correctly;  $-\infty$  means the worker always labels items incorrectly.

The labels given by worker i to item j are denoted as  $L_{ij}$  and are generated as follows:

$$p(L_{ij} = Z_j | \alpha_i, \beta_j) = \frac{1}{1 + e^{-\alpha_i \beta_j}}$$
(11)

As the difficulty  $1/\beta_j$  of an item increases, the probability of the label being correct moves toward 0.5. Similarly, as the worker's expertise decreases (lower  $\alpha_i$ ), the chance of correctly labeling drops to 0.5. Figure 3 shows the structure of the graphical model.

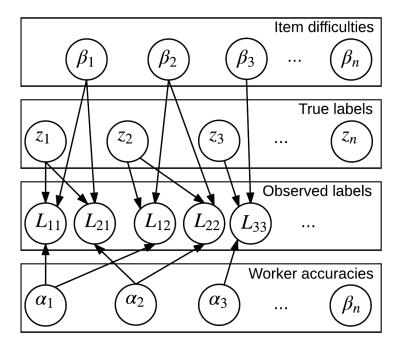


Figure 3: An illustrative example on how semi-supervised learning helps the model have a better performance.

# 2.2.4 Deep Learning Approach

[Shaham et al., 2016] showed that the Dawid and Skene model is equivalent to a Restricted Boltzmann Machine (RBM) with a single hidden node under the

assumption that all workers are conditionally independent. Thus the posterior probabilities of the true labels can be estimated via a trained RBM.

A RBM is an undirected bipartite graphical model, consisting of a visible layer X and a hidden layer H. The visible layer consists of d binary random variables and the hidden layer m binary random variables. These two layers are fully connected to each other. A RBM is parametrized by  $\lambda = (W, a, b)$ , where W is the weight matrix of the connections between the visible and hidden units, and a, b are the bias vectors of the visible and hidden layers, respectively. An illustration of a RBM is depicted in Figure 4.

A RBM implies the conditional probabilities

$$p_{\lambda}(X_i = 1|H) = \sigma(a_i + W_i.H)$$
  
$$p_{\lambda}(H_i = 1|X) = \sigma(b_i + X^T W_{i.i}),$$

where  $\sigma(z)$  is the sigmoid function,  $W_i$  is the *i*-th row of W and  $W_{ij}$  is its *j*-th column.

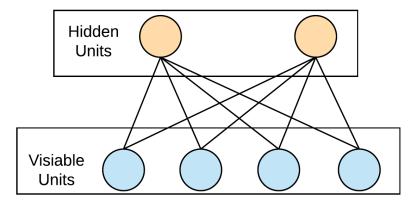


Figure 4: Diagram of a restricted Boltzmann machine with four visible units and three hidden units (no bias units)

To relax the assumption on the conditional independence of the variables  $X_1, \ldots, X_d$ , a RBM-based Deep Neural Net (DNN) is proposed to estimate the posterior probabilities  $p_{\theta}(Y|X)$ . The mechanism to stack multiple RBMs is that the hidden layer of each RBM is the input for the successive RBM. The RBMs are trained one at a time from bottom to top. Specifically, given training data  $x^{(1)}, \ldots, x^{(n)} \in \{0, 1\}^d$ , the bottom RBM is trained first, and then obtain the hidden representation of the first layer by sampling  $h^{(i)}$  from the conditional RBM distribution  $p_{\lambda}(H|X=x^{(i)})$ . The vector  $h^{(1)}, \ldots, h^{(n)}$  are then used as a

training set for the second RBM and so on. An illustration of a RBM is depicted in Figure 5.

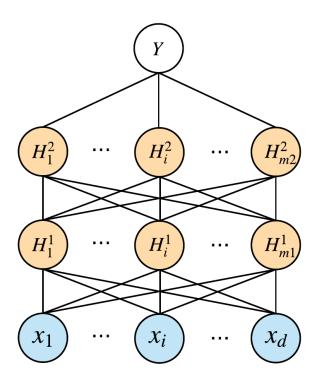


Figure 5: Diagram of RBM-based DNN with two hidden layers.

# 2.2.5 Minimax Entropy

[Zhou et al., 2012] proposed a minimax entropy principle to estimate the ground truth given the observed labels by workers. The model is illustrated in Figure 6. Each row corresponds to a worker indexed by i (from 1 to m). Each column corresponds to an item to be labeled, indexed by j (from 1 to n). Each item has an unobserved label represented as a vector  $y_{jl}$ , which is 1 when item j is in class l (from 1 to l), and 0 otherwise. Observed data is a tensor of labels l0 otherwise. We assume that l1 when the item l2 is labeled as class l3 by the worker l4, and 0 otherwise. We assume that l2 are drawn from l3, l4 l5 as class l6. The proposed model will estimate l3 from the observed l4.

 $\pi_{ij}$  is modeled through the principle of maximum entropy with the con-

	item 1	item 2	 item n
worker 1	$z_{11}$	$z_{12}$	 $z_{1n}$
worker 2	$z_{21}$	$z_{22}$	 $z_{2n}$
worker $m$	$z_{m1}$	$z_{m2}$	 $z_{mn}$

	item 1	item 2	 item n
worker 1	$\pi_{11}$	$\pi_{12}$	 $\pi_{1n}$
worker 2	$\pi_{21}$	$\pi_{22}$	 $\pi_{2n}$
worker $m$	$\pi_{m1}$	$\pi_{m2}$	 $\pi_{mn}$

Figure 6: Left: observed labels. Right: underlying distributions. Highlights on both tables indicate that rows and columns of the distributions are constrained by sums over observations.

straints from the ideas of majority voting and Dawid and Skene's model. Using the principle of maximum entropy to estimate the probability distribution provides the largest remaining uncertainty (i.e., the maximum entropy) consistent with the constraints. Majority voting indicates that the number of observed votes per class per item  $\sum_i z_{ijk}$  should match  $\sum_i \pi_{ijk}$ . Dawid and Skene's model indicates that the observed confusion matrix per worker  $\sum_j y_{jl} z_{ijk}$  should match  $\sum_i y_{jl} \pi_{ijk}$ . Thus, the maximum entropy model for  $\pi_{ij}$  given  $y_{jl}$  is as follows:

$$\max_{\pi} - \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{c} \pi_{ijk} \ln \pi_{ijk}$$
s.t. 
$$\sum_{i=1}^{m} \pi_{ijk} = \sum_{i=1}^{m} z_{ijk}, \forall j, k, \quad \sum_{j=1}^{n} y_{jl} \pi_{ijk} = \sum_{j=1}^{n} y_{jl} z_{ijk} \forall i, k, l,$$

$$\sum_{k=1}^{c} \pi_{ijk} = 1, \forall i, j, \pi_{ijk} \ge 0, \forall i, j, k.$$
(12)

To infer  $y_{jl}$ , the authors proposed to choose  $y_{jl}$  to minimize the entropy in Equation 12, which leaves  $z_{ij}$  the least random given  $y_{jl}$ .

$$\min_{y} \max_{\pi} - \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{c} \pi_{ijk} \ln \pi_{ijk}$$
s.t. 
$$\sum_{i=1}^{m} \pi_{ijk} = \sum_{i=1}^{m} z_{ijk}, \forall j, k, \quad \sum_{j=1}^{n} y_{jl} \pi_{ijk} = \sum_{j=1}^{n} y_{jl} z_{ijk} \forall i, k, l,$$

$$\sum_{k=1}^{c} \pi_{ijk} = 1, \forall i, j, \pi_{ijk} \ge 0, \forall i, j, k, \sum_{l=1}^{c} y_{jl} = 1, \forall j, y_{jl} \ge 0, \forall j, l. \quad (13)$$

#### 2.2.6 Control Items

Control items with known answers can be used to evaluation workers' reliability and bias, and hence weight their answers accordingly on target items with

unknown answers. There is a trade-off in this scenario: we can better estimate workers' reliability and bias by letting them answer more control items, but there are fewer opportunities left for the target items. On the other hand, using fewer control items produces poor estimates of worker's reliability and bias, which leads to a bad result when aggregating workers' answers. [Liu et al., 2013] examined the effectiveness of control items and provided insights on how many control items are enough under different scenarios, which can help crowdsourcing practitioners make their own decisions. Aforementioned methods (except for majority voting) evaluate workers' reliability by comparing their agreement with other workers and control items can be incorporated into these methods.

# 2.3 Q2b: Gaussian Processes in Contextual Bandit

**Question:** For the purposes of identifying the most effective learning artifact, how do contextual bandits relate to Gaussian processes? How are they different/similar?

The reward function in contextual bandit setting is usually an unknown function and expensive to evaluate (performing an arm on a subject). In this case, Gaussian process (GP) is an ideal method to model our belief on the behavior of the reward function. The reward function is modeled as a sample from GP.

The Gaussian process  $GP(\mu_0, k)$  is a non-parametric model that is fully characterized by its prior mean function  $\mu_0 : \mathcal{X} \to \mathbb{R}$  and its positive-definite kernel, or covariance function,  $k : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ .

Let  $\mathcal{D}_n = \{(\mathbf{x}_i, y_i)\}$  denote a set of n observations and  $\mathbf{x}$  denote the arbitrary test point. The random variable  $f(\mathbf{x})$  is also a GP distribution conditioned on observations  $\mathcal{D}_n$  with following mean  $\mu_n(\mathbf{x})$  and variance  $\sigma_n^2(\mathbf{x})$ :

$$\mu_n(\mathbf{x}) = \mu_0(\mathbf{x}) + \mathbf{k}(\mathbf{x})^T (\mathbf{K} + \sigma^2 \mathbf{I})^{-1} (\mathbf{y} - \mathbf{m})$$
(14)

$$\sigma_n^2(\mathbf{x}) = k(\mathbf{x}, \mathbf{x}) - \mathbf{k}(\mathbf{x})^T (\mathbf{K} + \sigma^2 \mathbf{I})^{-1} \mathbf{k}(\mathbf{x})$$
(15)

where  $m_i := \mu_0(\mathbf{x}_i)$ ,  $K_{i,j} := k(\mathbf{x_i}, \mathbf{x_j})$ , and  $\mathbf{k}(\mathbf{x})$  is a vector of covariance terms between  $\mathbf{x}$  and  $\mathbf{x}_{1:n}$ .

The posterior mean and variance evaluated at any point  $\mathbf{x}$  represent the model's prediction and uncertainty in the objective function at the point  $\mathbf{x}$  [Shahriari et al., 2016]. In order to apply GP under the setting of multi-armed bandit, we need a acquisition function to select the next query point given the posterior model. The acquisition function should be carefully design to trade off exploration of the search area and exploitation of current promising areas [Shahriari et al., 2016].

## 2.3.1 GP-UCB

To decrease uncertainty globally, one strategy could be to pick the point which maximums the variance  $\mathbf{x}_{n+1} = \underset{\mathbf{x} \in D}{\arg \max} \sigma_n(\mathbf{x})$ . However, this strategy is not

well suited for multi-armed bandit problem since it can be wasteful. Another idea is to select the point which maximizes the expected reward given the posterior model  $\mathbf{x}_{n+1} = \arg\max_{\mathbf{x} \in D} \mu_n(\mathbf{x})$ . However, this idea is too greedy and tends

to end up with a local optima. To balance exploration and exploitation, a combined strategy is to choose

$$\mathbf{x}_{n+1} = \arg\max_{\mathbf{x} \in D} \mu_n(\mathbf{x}) + \beta_n \sigma_n(\mathbf{x}), \tag{16}$$

where  $\beta_n$  are constants. There are theoretically motivated guidelines for setting and scheduling the hyperparameter  $\beta_n$  to achieve optimal regret.

Since Equation 2.3.1 is an upper confidence bound of the marginal posterior  $P(f(\mathbf{x})|\mathbf{y}_n)$ , a natural interpretation of this strategy is that it selects the point  $\mathbf{x}$  such that  $f(\mathbf{x})$  is a reasonable upper bound on  $f(\mathbf{x})$ . This algorithm is called Gaussian process upper confidence bound (GP-UCB), introduced by [Srinivas et al., 2010]. The GP-UCB selection rule is motivated by the UCB algorithm for the classical multi-armed bandit problem.

#### 2.3.2 Contextual GP-UCB

Motivated by GP-UCB algorithm mentioned above, [Krause and Ong, 2011] extended the generalization of this algorithm by incorporating contextual information

$$\mathbf{x}_{n+1} = \arg\max_{\mathbf{x} \in D} \mu_n(\mathbf{x}, \mathbf{z}_n) + \beta_n \sigma_n(\mathbf{x}, \mathbf{z}_n), \tag{17}$$

where  $z_n \in Z$  is the contextual information from a set Z of contexts,  $\mu_n(\cdot)$  and  $\sigma_n^2(\cdot)$  are the posterior mean and variance of the GP conditioned on observations  $\mathcal{D}_n = \{(\mathbf{x}_i, \mathbf{z}_i, y_i)\}$ . The authors called the selection rule the contextual Gaussian process UCB algorithm (GCP-UCB).

To derive the kernel k on the product space  $Z \times X$  of contexts and actions, a natural approach to start with kernel functions  $k_Z : Z \times Z \to \mathbb{R}$  and  $k_X : X \times X \to \mathbb{R}$  on the space of contexts and actions. [Krause and Ong, 2011] proposed two possibilities of constructing composite kernel k from context kernel  $k_Z$  and action kernel  $k_X$ . One is to calculate a product kernel  $k = k_Z \otimes k_X$ , by setting

$$(k_Z \otimes k_X)((\mathbf{z}, \mathbf{x}), (\mathbf{z}', \mathbf{x}')) = k_Z(\mathbf{z}, \mathbf{z}')k_X(\mathbf{x}, \mathbf{x}'). \tag{18}$$

An alternative is to calculate the additive kernel  $k = k_Z \oplus k_X$ , by setting

$$(k_Z \oplus k_X)((\mathbf{z}, \mathbf{x}), (\mathbf{z}', \mathbf{x}')) = k_Z(\mathbf{z}, \mathbf{z}') + k_X(\mathbf{x}, \mathbf{x}'). \tag{19}$$

# 2.4 Q2c: Selection Bias & Propensity Score Matching

**Question:** how precisely are estimates of the effectiveness of a learning artifact biased when the student herself can choose which artifact she receives (selection bias)? How can methods such as propensity score matching partially reduce this bias?

In RCTs, students are randomly assigned to conditions so treatment is assigned by randomization. As a consequence of randomization, an unbiased estimate of the ATE can be directly computed from the data. An unbiased estimate of the ATE is  $E[Y_i(1)-Y_i(0)]=E[Y(1)]-E[Y(0)]$ . When the student can choose which artifact she receives, randomization of treatment assignment does not exist. As a consequence, the treated students often differ systematically from untreated students. In general,  $E[Y(1) \mid t=1] \neq E[Y(1)]$  holds. Thus, an unbiased estimate of the ATE cannot be obtained by directly comparing outcomes between the the treated and the control groups.

A propensity score [Rosenbaum and Rubin, 1983] is the probability of a unit (e.g., student, classroom, school) being assigned to a particular treatment given a set of observed features. Propensity scores are used to reduce selection bias by equating groups based on these features.

Suppose that we have a binary treatment  $T = \{0, 1\}$ , an outcome Y, and observed features X. The propensity score is defined as the conditional probability of treatment given background variables:

$$p(x) := \Pr(T = 1 \mid X = x)$$

Due to random treatment assignment in RCT, the propensity score is known. However, the actual propensity score in observational experiments is unknown and it can be estimated using statistical or machine learning models from the experiment data. The most commonly used model is a logistic regression model, in which treatment assignment T is the dependent variable and features X are independent variables. Beyond logistic regression, the use of bagging or boosting [Lee et al., 2010], tree-based model (Propensity trees) and causal random forests [Wager and Athey, 2015], and neural networks [Setoguchi et al., 2008] have been proposed to estimate the propensity score.

The purpose of Propensity score matching (PSM) [Austin, 2011] is to form matched sets of the treated and the control subjects who share a similar value of the predicted propensity score. The most common implementation of PSM is one-to-one or pair matching, in which pairs of the control and the treated subjects are formed, such that matched subjects have similar values of the propensity score. Once a matched sample has been formed, the treatment effect can be estimated by directly comparing outcomes between treated and untreated subjects in the matched sample. PSM attempts to mimic randomization by creating a sample of units that received the treatment that is comparable on all observed covariates to a sample of units that did not receive the treatment. In summary, the analysis of a propensity score matched sample can mimic that of an RCT: one can directly compare outcomes between the treated and the control subjects within the propensity score matched sample.

The aforementioned methods, propensity trees [Wager and Athey, 2015] in section 1.1.1 and balancing neural networks (BNN) [Johansson et al., 2016] in section 1.1.2, can also be used to reduce the selection bias. The propensity trees is inspired by the idea of propensity score matching. The goal of propensity

trees is to train a classification tree to predict the treatment assignments. The subjects on the same leaf are considered as matched subjects. Due to the selection bias, two distributions of observed covariates for the control and the treated groups may be different. Thus, the problem of counterfactual inference may require inference over a different distribution than the one from which samples are given. In other words, the features distribution of the test data differs from that of the training data. This is a special case of domain adaptation. BNN uses deep learning approaches for domain adaptation to learn a balanced representation between the control and the treatment groups to reduce the selection bias.

# 3 Topic 3: Crowdsourcing for Education. More on the HCI side and HCOMP ideas

# 3.1 Q1: Peer Assessment

Question: Chris Schunn works on peer grading. What literature in that is relevant to your dissertation? Do people make good reviewers? Do they learn from reviewing? Do authors benefit from getting reviews? (Do they essay they right get better if they get a review?)

After going through Chris Schunn's publications, I find these papers [Patchan and Schunn, 2016, Patchan and Schunn, 2015, Schunn et al., 2016, Cho et al., 2006] related to my dissertation. The summary of research findings from these paper is listed below.

In [Cho et al., 2006, Schunn et al., 2016], the authors run a set of studies to investigate the reliability and validity of peer assessment of writing on college level and high school level, respectively. The authors use term reliability to refer to the extent to which students' assessment of writing correlate with each other and use validity to describe the extent to which students can accurately judge the quality of the writing. These two papers reach the conclusion that the assessments from students at college level and high school level have strong reliability and validity. In other words, peers in general are capable of providing valid ratings for others' writing, and their feedback is usually thought to be helpful by other students. Students improved their writing ability from peer assessment, and specifically from providing feedback to peers.

[Patchan and Schunn, 2016] investigated the relationship between writer ability and reviewer ability and found several interesting interactions between these two factors. Often lower-ability writers were more willing to improve their writing using feedback from other low-ability reviewers than from high-ability reviewers, while higher-ability writers benefited equally from receiving feedback from lower-ability and higher-ability reviewers, which is inconsistent with student beliefs that only feedback from high-ability peers is worthwhile.

In terms of the style of the feedback, [Patchan and Schunn, 2015] found that

low reviewer provided more praise than high reviewers whereas high reviewers provided more criticism than low reviewers. This criticism described more problems and offered more solutions. There was one interesting interaction between reviewer ability and text quality – high reviewers described more problems in the low-quality texts than in the high-quality texts, whereas low reviewers did not make this distinction.

# 3.2 **Q2: DALITE**

Question: How would that be adapted to deal with DALITE objects?

Distributed Active Learning Technology Integrated Environment (DALITE) [Bhatnagar et al., 2015] is a web-based application for an asynchronous peer instruction. In DALITE, students are asked to submit their explanations for their choice after answering a multiple-choice question. Then they are prompted with several explanations from other students. Half of these peer explanations are for the choice that students select, and the other half of the peer explanations are for a different choice. Students have a chance to reflect their thinking by comparing their explanations with prompted explanations and make a second choice. Students can also select the best explanation among those displayed.

Similarly to [Cho et al., 2006, Schunn et al., 2016], we should first investigate in DALITE whether students can learn from providing their own explanations and from reflecting their thinking by comparing their explanations with other students' explanations. Another investigation should be focused on the quality of peer explanations and students' perceptions about the asynchronous peer instruction workflow.

[Patchan and Schunn, 2016] found that lower-ability writers were more willing to improve their writing using feedback from other low-ability reviewers than from high-ability reviewers, while higher-ability writers benefited equally from receiving feedback from lower-ability and higher-ability reviewers. This inspires us to investigate whether this conclusion holds in the context of DALITE. We usually believe that high-ability students produce high-quality explanations and both high-ability and low-ability students benefit from these explanations. If the aforementioned conclusion holds in DALITE, an adaptive strategy can be applied to select the prompted explanations for students based on their ability. In other words, for low-ability students, explanations from other low-ability students should be selected.

Another interesting topic inspired by [Patchan and Schunn, 2015] is the style of peer explanations. There are two factors of the students which play an important role in this setting: the ability of solving a question and the ability of providing a high-quality explanations. We can look at the interactions between the style and these two factors.

# 3.3 Q3: Learnersourcing Subgoal Labels

**Question:** How is the work by Rob Miller and Juho about learning subgoal related to PeerASSIST?

[Weir et al., 2015] presented a three-stage workflow for learners to generate subgoal labels for how-to video while they are learning from the videos. The goal of the first stage (Generation) is to let learners submit the subgoal labels after a certain interval during watching the videos. The goal of the second stage (Evaluation) is to let learners select the most relevant answer for the video section, as well as discard potential spam answers. The goal of the third stage (Proofreading) is to let learners evaluate the most popular subgoal labels from stage 2 for quality and eventually agree on a final label for that subgoal. During stage 2 and stage 3, learners always have a option to refine the label.

In PeerASSIST, only students who receive 0 on a problem are prompted with a peer explanation from one of students who answer the same problem correctly. Then these students have a option to give a rating (thumb up or thumb down) to the peer explanation that they receive. The students, who answer the problem correctly in the first attempt, do not have a chance seeing and rating the peer explanations. If a certain problem is relatively easy, there will be few students who receive 0 and it is difficult to find the optimal peer explanations based on students' ratings. Even if the problem is relatively difficult, we miss the contribution from the students with correct answer in the first attempt when deciding the optimal peer explanations. Motivated by [Weir et al., 2015], we can apply two-stage workflow to collect ratings from students in terms of the quality of the peer explanations after they submit an answer. Assume that there are several peer explanations available for a given problem and the ranking of these peer explanations are already decided by some algorithm (e.g., multi-armed bandit algorithm) based on the history data.

On stage 1, students are prompted with several (i.e., 3) peer explanations and have a option to rate these explanations after they submit an answer. The goal of this stage is to collect students' opinions on these explanations and use these data as one of the indicators to determine the most effective explanation and discard spam explanations. In terms of which explanations should be prompted to students on this stage, several simple approaches can be applied: random selection from all available explanations or newly added explanations. Exploration strategy can be applied in this scenario. For the purpose of exploration, explanations with high uncertainty are prompted with students. This strategy can potentially reduce the uncertainty of these explanations and help multi-armed bandit algorithm efficiently find the optimal explanations.

On stage 2, student are prompted with the currently optimal peer explanation from the ranking algorithm and asked to evaluate whether this peer explanation is highly effective. The goal of this stage is to let students to evaluate the optimal explanation for quality and analog to the goal of exploration strategy in multi-armed bandit algorithm.

# 3.4 Q4: HCOMP & CSCW

Question: Look at the two most recent HCOMP conference and the CSCL conferences. What do you see in those conferences that relate to what you want to propose for your dissertation? I assume some will be methodological, while others will be on the design side. For the purpose of this question let's look at the design side.

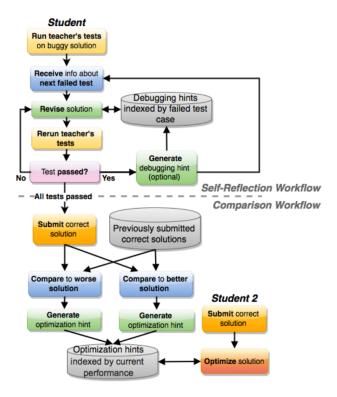


Figure 7: Reprinted from [Glassman et al., 2016]. In the self-reflection work-flow, students generate hints by reflecting on an obstacle they themselves have recently overcome. In the comparison workflow, students compare their own solutions to those of other students, generating a hint as a byproduct of explaining how one might get from one solution to the other.

The aforementioned paper, Learnersoucring Subgoal Labels for How-to Videos [Weir et al., 2015], is published in CSCW 2015. In this paper, the authors showed that students were qualified to generate subgoal labels for how-to videos based on their new expertise. Motivated by this prior work, another paper, Learnersourcing Personalized Hints [Glassman et al., 2016], is published in CSCW

2016. In [Glassman et al., 2016], the authors designed two workflows aimed to engage students in creating personalized hints. Results showed that students can create helpful hints for their peers that augment or even replace teachers' personalized assistance, when that assistance is not available.

In the self-reflection workflow, students iteratively work on their solutions to pass as many teacher-created tests as possible. There are hints available for any verification. Students can ask for these hints to help them pass the test case. After fixing a bug in their own solution, students can generate a new hint for others struggling with the same error.

In the comparison workflow, students compare their solution to alternative solution submitted by other students or teachers. If their solution performs better than a solution W, they are asked to submit an optimization hint for students who have solutions like W. If their solution performs worse than a solution B, they are prompted to generate an optimization hint for students who have solutions like their own. Figure 7 illustrates both workflows.

[Yoon et al., 2016] developed a new multi-modal annotation (e.g., gesture and waveform) tool, called RichReview<sup>++</sup>. The aim of this tool is to texts, audios and gestures to mimic face-to-face communication experience for collaborators reviewing and providing feedback on documents. It is also extended to support peer discussion.

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