Latent Class Model Analysis: A Computational Pipeline

Siyuan Tang | STATS 607 | December 2025

A. Motivation

Problem

Analyzing multivariate categorical data becomes intractable as the number of variables grows. With $m = 20$ binary variables, there are $2^{20} \approx$ 1 million possible patterns. Latent class models solve this by assuming $K \ll 2^m$ latent classes within which variables are conditionally independent.

Key challenges:

1. How many classes (K) exist?
2. How to fit models efficiently?

Impact

Latent class analysis is used in social sciences, medicine, marketing, and psychology for clustering and segmentation. This project provides researchers with automated, validated, and computationally efficient tools.

B. Project Description

What I Built

For my methodology of latent class modeling, see `docs/methodology.pdf`.

1. Core Pipeline (`src/`)

- **Vectorized EM algorithm** for parameter estimation
- **BIC-based model selection** with smart parallelization
- **Synthetic data generation** for validation

2. Simulation Studies (`simulation/`)

- Monte Carlo validation: $M = 50$ simulations $\times$ 5 sample sizes $\times$ 4 K values

- Tests: BIC selection accuracy, parameter estimation errors, classification performance
- Metrics: Success rates, MAE/RMSE for π and θ, confusion matrices

3. Visualization

- BIC curves, parameter error plots, confusion matrices, accuracy trends

4. Makefile

Course Concepts Used

Vectorization: Eliminated nested loops using NumPy broadcasting

Parallel Computing: Adaptive parallelization strategy

- Few K values → parallelize initializations
- Many K values → parallelize across K

Numerical Stability: Log-space computations with log-sum-exp trick

Simulation Studies: Structured Monte Carlo framework for validation

Progress Tracking: via tqdm

Makefile Commands

Software Engineering: Modular design, CLI interfaces, comprehensive documentation

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C. Results

Main Pipeline Demo

Synthetic data analysis:

```
python main.py --generate-synthetic --n-samples 2000 --k-true 3
```

Output: BIC correctly selects $K = 3$

```
Mixture Weights:
  True π:      [0.3359 0.3342 0.3299]
  Estimated π: [0.341  0.3298 0.3292]
  Mean Absolute Error (MAE):  0.003426
  Root Mean Squared Error (RMSE): 0.003939
```

```
Categorical Probabilities (θ_rkc):
  Mean Absolute Error (MAE):  0.013091
  Root Mean Squared Error (RMSE): 0.016334

Per-Class θ Errors:
  Class 0: MAE = 0.013441
  Class 1: MAE = 0.012655
  Class 2: MAE = 0.013177

  ✓ True parameters saved to: results/analysis_true_params.npz
```

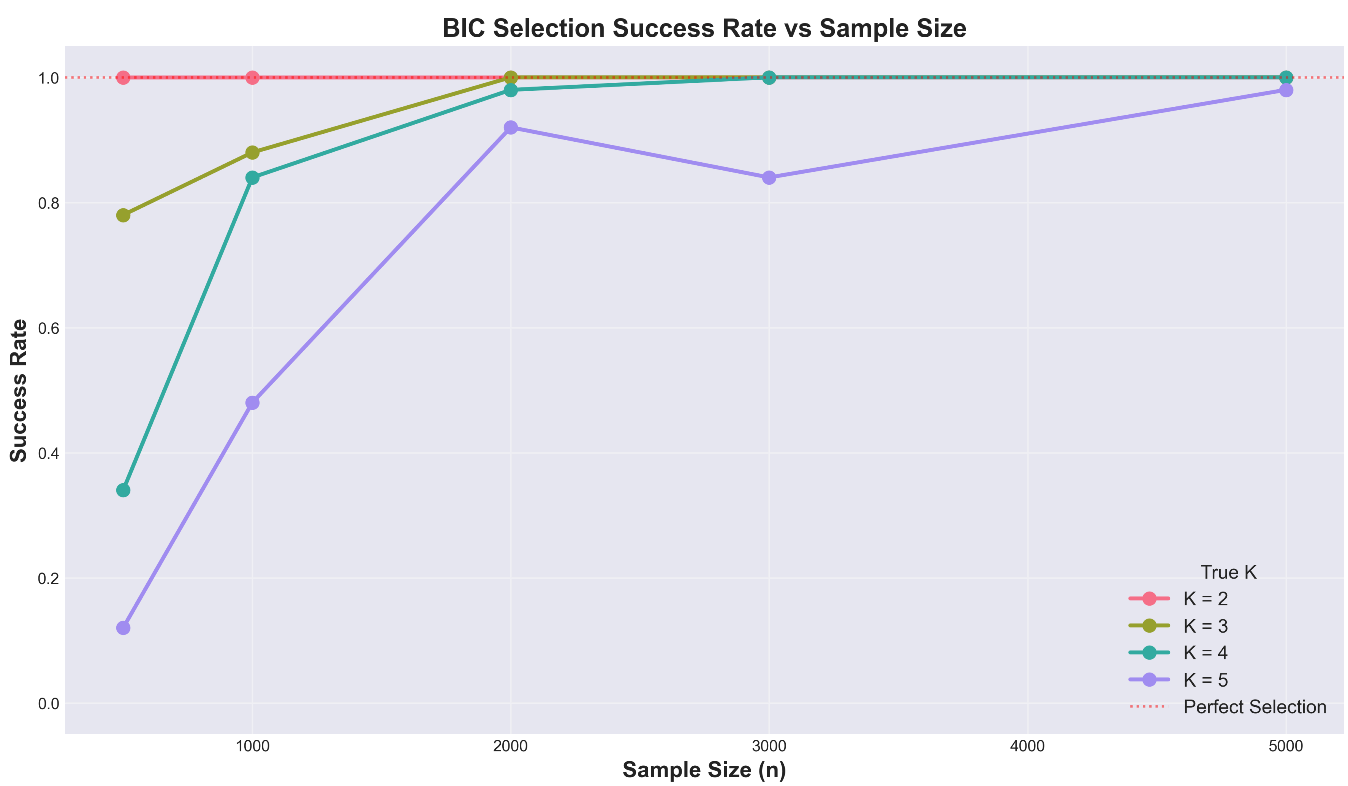
User's dataset:

```
# Suppose the user's dataset is `data/mydata.csv`)
python main.py --data data/mydata.csv --output-prefix my_analysis
```

Simulation Results

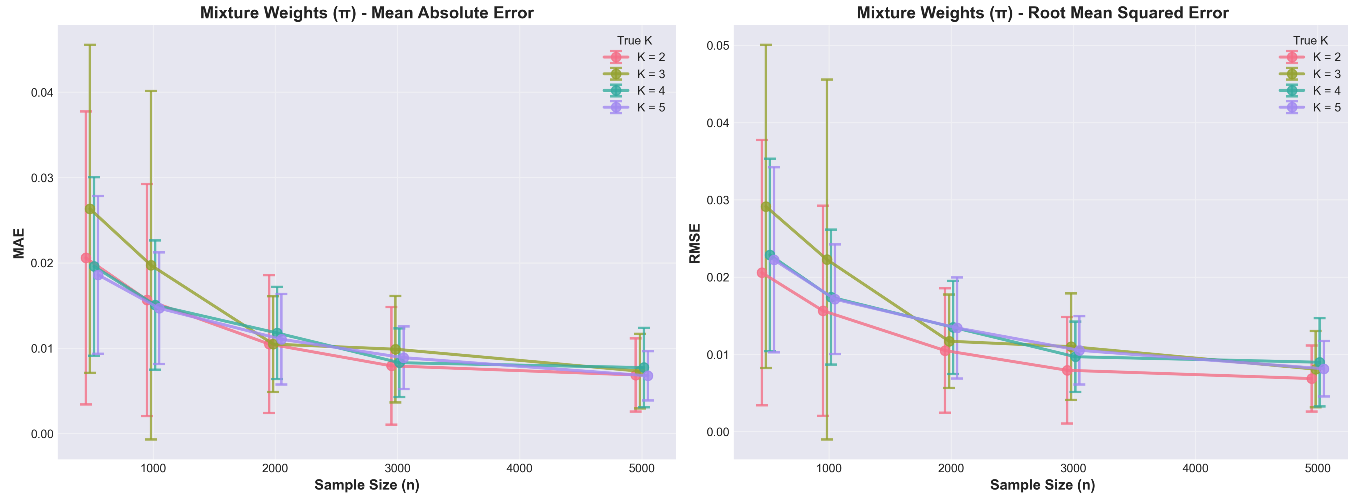
1. BIC Selection Accuracy

See `simulation/results/figures/bic_success_rates_combined.png`.



2. Parameter Estimation

See `simulation/results/figures/pi_estimation_errors.png`.

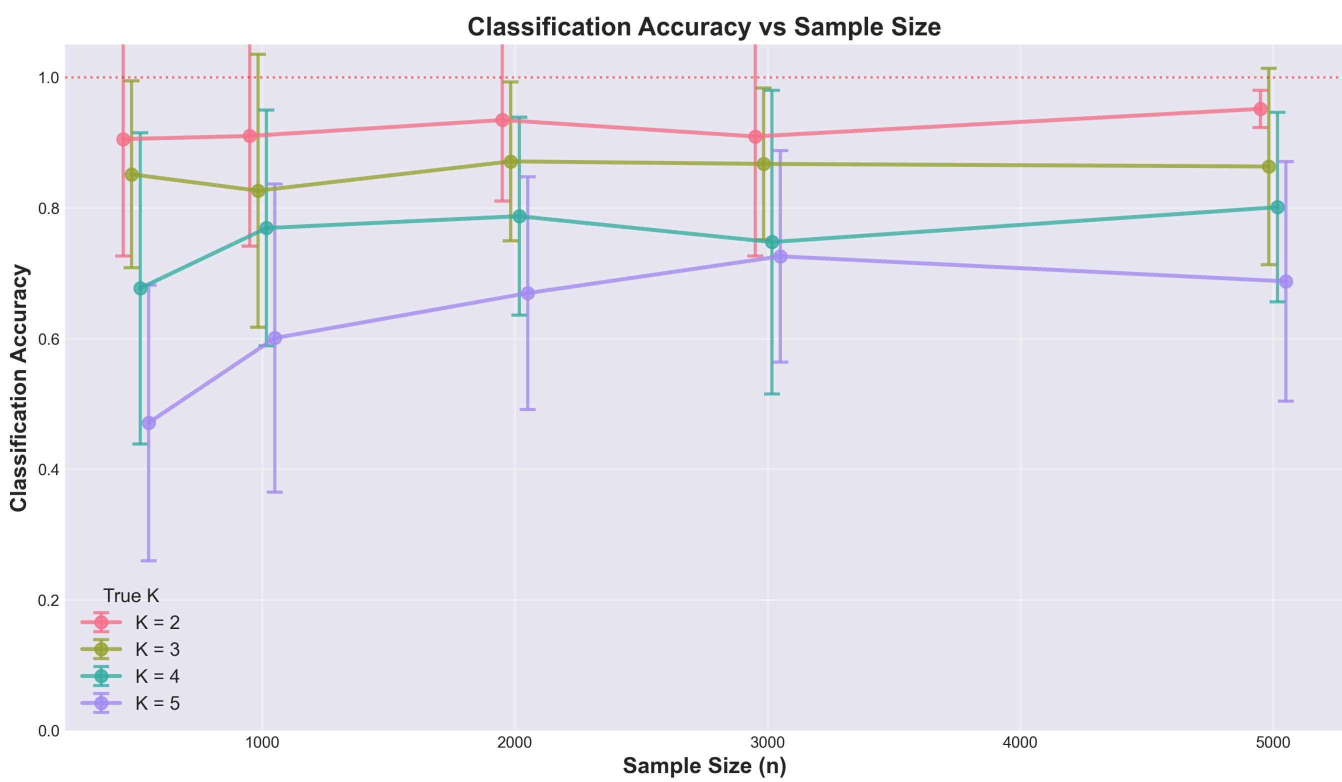


And `simulation/results/figures/theta_estimation_errors.png`. (Not shown here)

3. Classification Performance

See `simulation/results/figures/confusion_matrices_K{2,3,4,5}.png`. (Not shown here)

See `simulation/results/figures/classification_accuracy_vs_n.png`.



4. Computation Time

See `simulation/results/figures/bic_computation_time.png`. (Not shown here)

D. Lessons Learned

Challenges

1. Numerical Instability

- Problem: Probability products can cause underflow
- Solution: Log-space + log-sum-exp trick

2. Label Switching

- Problem: EM produces arbitrary class labels
- Solution: Post-hoc sorting by mixture weights

3. Parallelization Trade-offs

- Problem: Naive parallelization → memory issues and slow
- Solution: Adaptive strategy based on K range size

How My Approach Changed

Before STATS 607:

- Nested loops
- Sequential processing only
- Ad-hoc scripts

After STATS 607:

- Vectorization and parallelization
- Modular and reusable code
- Comprehensive validation