# Rate of yield

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My issue was with how the rate of catch is calculated. In order to illustrate the problem I will consider a model without growth and with all mortality being fishing mortality, hence all dead fish are the catch.

## 1 Continuous time model

The McKendrick-von Foerster equation with no growth, $\frac{\partial g(w)N(w)}{\partial w} = 0$ (and $\frac{dw}{dt} = 0$), or non-fishing mortality is

$$\frac{dN(w)}{dt} = -FN(w) \tag{1}$$

as $\mu(w) = F$ for all sizes. Integrating both sides to get biomass then this becomes

$$\frac{dB}{dt} = -FB,$$

which has the exact solution for time $\Delta t$

$$B(t + \Delta t) = B(t)\exp(-F\Delta t),$$

where $B(t)$ is the biomass at time $t$. All lost biomass is catch, i.e.

$$B(t) - B(t + \Delta t) = (\exp(F\Delta t) - 1)B(t + \Delta t)$$

hence the rate of catch is

$$\frac{(\exp(F\Delta t) - 1)B(t + \Delta t)}{\Delta t} \neq FB(t + \Delta t),$$

except $\lim_{\Delta t \to 0}$, which is currently how it is currently calculated.

## 2 Discrete time model

The model was a discrete time model that moves with time step $\delta t$,

$$B(t + \delta t) = \frac{B(t)}{1 + F\delta t}, \tag{2}$$

as calculated in mizer currently, then the catch would be

$$\begin{aligned} B(t) - B(t + \delta t) &= B(t + \delta t)(1 + F\delta t) - B(t + \delta t) \\ &= B(t + \delta t)F\delta t \end{aligned}$$

which as a rate becomes $B(t + \delta t)F$, exactly as currently done in mizer and the getYield function. Therefore the getYield function is the solution to the discrete time model (equation 1) and not the McKendrick-von Foerster model (equation 2), although as the time step gets smaller it does.

We can calculate the actual fishing mortality rate, $\mu_f$, that was put on the stock as a rate parameter by combining the two models,

$$\frac{B(t)}{1 + F\delta t} = B(t)\exp(-\mu_F \delta t)$$

which means

$$\mu_F = -\frac{1}{\delta t}\log\left(\frac{1}{1 + F\delta t}\right) \neq F,$$

except $\lim_{\Delta t \to 0}$. $\mu_F$ is equivilant to the fishing mortality that come out of the assessments. I'm not sure if this only holds when growth is zero though.