A Comparison of GARCH Models for VaR Estimation

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Abstract

This paper studies 4 GARCH models in Value at Risk (VaR) prediction. We choose SPDR S&P 500 ETF price data as our data set, which is provided by *Yahoo Finance*. For the robustness and comparison, both volatile (including 2008 financial crisis) and stable market period are included in the research. We choose the best order for each GARCH model by AIC and BIC. By one-year rolling window cross-validation, we then forecast the VaR one trading day ahead based on the predicted conditional mean and variance derived from GARCH models. The prediction accuracy for each model is assessed by the distance between the hit rate and the confidence level (5% in our study). The results indicate that in both market conditions, IGARCH model outperforms other GARCH models in estimating 5% VaR.

Executive Summary

- Title: A comparison of GARCH models for VaR estimation
- Dataset name: SPDR S&P 500 ETF price/return
- Data sourse: *https://finance.yahoo.com/quote/SPY/profile?p=SPY*
- Final model: IGARCH(2,1) for 2006-2009 and IGARCH(1,1) for 2013-2016
- Conclusion: Although the eGARCH model demonstrates the best in-sample estimation according to BIC, in the rolling prediction for VaR, IGARCH model outperforms the other GARCH models based on the hit rates during the both volatile and stable periods. This might result from that fitting extra parameters for eGARCH often leads to the overfitting problem with small training dataset. So we conclude that IGARCH model show a good balance between the in-sample estimation and the prediction for VaR due to its sparseness.

Contents

1 Introduction

Value at Risk (VaR) is one of the most important measures of the market risk that has been widely used for financial risk management. The need for VaR stems from the fact that the past few decades have witnessed tremendous volatility in financial instruments for use in managing the tail risks, especially during the financial crisis. The concept and use of VaR is relatively recent. VaR was first used by major financial firms in the late 1980s to measure the risks of their trading portfolio. Since then, the use of VaR has exploded. J.P. Morgan's attempt to establish a market standard through its RiskMetrics system in 1994 provided a tremendous impetus to its growth. VaR is now widely used by other financial institutions, nonfinancial corporations, and institutional investors. VaR model assumes that the returns of a financial asset follow a conditional Normal distribution with its mean and variance. However, this model has a drawback that it assumes actual returns volatilities independent with previous data. Yet empirical studies found that many financial return series exhibit time-dependence. In other words, financial data often exhibit volatility clustering and leverage effect.

With the fact that time-varying and clustering volatility is more common than constant volatility, we applied Heteroskedastic volatility model to fix these issues. Models of Autoregressive Conditional Heteroskedastic (ARCH) give the most popular way of parameterizing this dependence. In applications, the ARCH model has been replaced by the so-called generalized ARCH (GARCH) model that Bollerslev (1986) and Taylor (1986)[1] proposed independently, which includes the past volatility as well.

In this paper we use S&P 500 ETF Data to train different GARCH model and compare their in-sample fitness by the information criteria and the residual diagnose analysis. Then we predict VaR using each GARCH model with rolling window, and compare the forecast accuracy of 4 different GARCH models based on the hit rate.

2 Data Description

In order to study the behavior of financial time series in the U.S. stock market in recent years with different GARCH models and VaR model, we choose SPDR S&P 500 ETF price data as our data source. As one of the most popular financial products in the U.S., S&P 500 becomes an ideal data source for our project. The data used in this paper are daily and can be obtained from *Yahoo Finance*[1]. With R code *get.hist.quote(instrument="spy", start="2006-01-01", end="2009-12-31", quote="AdjClose", provider="yahoo",compression="d", retclass="zoo")* with package *tseries*, we can also directly get access to the data set.

The entire data set covers the period from January 1, 2006 to December 31, 2016. The length of the realization is reasonable due to the following several reasons: first, data is appropriate for our purpose of this project, that is, to study the recent stock market price and volatility performance using GARCH models, and to calculate VaR prediction. Second, by covering the 2008 Financial Crisis, observations in this period of time reflect various stages of the economic cycle.

We divide the data sets into two periods: the first period is from January 1, 2006 to

[1] https://finance.yahoo.com/quote/SPY/profile?p=SPY

December 31, 2009, while the second period covers January 1, 2013 to December 31, 2016. The reason for this partition is that we want to study the stock price performance in both volatile and relatively stable period. Four-year time period for each data set enables us to estimate parameters in different models properly. In order to estimate roughly ten parameters in GARCH model and more than fifteen parameters in other GARCH models, around one thousand data points is appropriate, meaning the four-year time window is an optimal choice for this purpose.

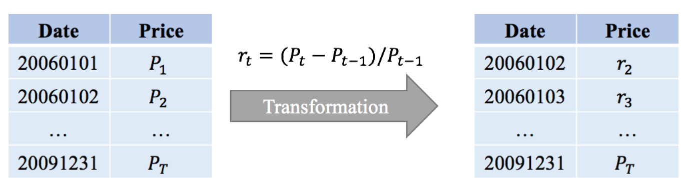
Since the data we obtained are prices of the stock market, scaling is not an issue any more, and the data preliminary transformation is introduced in the next section.

3 Preliminary Transformation

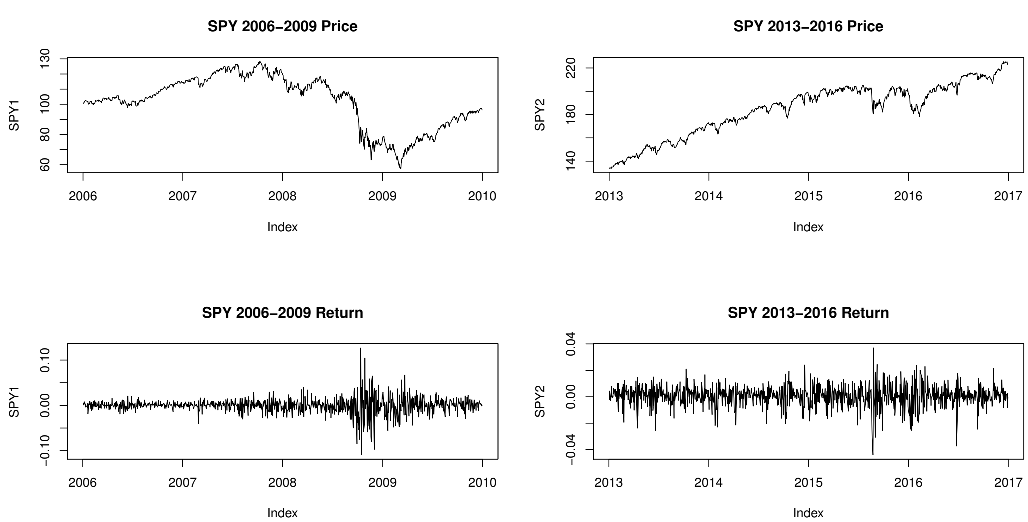
The original dataset we use is SPDR S&P 500 ETF price. The SPDR S&P 500 trust is an exchange-traded fund which trades on the NYSE Arca. For a long time, this fund was the largest ETF in the world. Thus, the dataset is complete and clean, so there is no missing observation problem with this dataset.

The goal for our project is using GARCH model to predict rolling Value-at-Risk for S&P 500 return . So we need to conduct preliminary transformation, changing our data from price unit to return unit.

Take one time series data for example, we calculate the return series by:



We can visualize the non-stationary price series and stationary return series as follows.



The return of S&P 500 ETF summary statistics are shown below:

Index	Period	n	mean	standard deviation	Skewness	Kurtosis
SPY	2006-2009	1005	-0.0001725	0.01658368	-0.2507273	13.57764
SPY	2013-2016	1006	0.00047462	0.008063443	-0.4917587	5.507785

Table 3.1 Statistics Summary for SPY

4 Model Identification and Diagnostic Analysis

After the transformation of the original price data, we get the return data for our underlying. From the plot we can see that the return series is stationary. Thus, for our mean model, we choose ARMA(1,1). And we focus on the research of the performances of different GARCH models. There are four chosen GARCH families for our research, GARCH, IGARCH, eGARCH and GJR-GARCH.

We study the fitness of our four models to the whole dataset to find the order of different GARCH models. We vary our p order from 1 to 3 and q order from 1 to 3, and therefore there are 9 models for a specific GARCH model. We fit the data with all the nine models and choose the best model for each family based on BIC(AIC) and residual analysis. Because we assume the innovations are all white noise multiplied by its corresponding conditional variance (see model specification below), if our model fits the data well, we should have uncorrelated and normally distributed normalized residuals. Thus for our residual analysis, we mainly focus on ACF, PACF and Ljung-Box test for autocorrelation detection and Q-Q plot for Normality test.

4.1 GARCH Model

The GARCH model is firstly brought up in 1982 by Robert F. Engle[2]. Similar to the idea of generalization of AR and MA model to ARMA model, the ARCH model is generalized to the GARCH model by including the conditional variance of past periods. The financial datasets hardly show constant volatility. Thus GARCH families can usually improve the fitness of the financial time series compared to other time series models.

The GARCH(p, q) model (p $>$ 0 and q $\geq$ 0 are integers) is defined as

$$r_t = \varphi_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \tag{1}$$

$$\sigma_t^2 = \omega + \alpha(B)\sigma_t^2 + \beta(B)\varepsilon_t^2 \tag{2}$$

$$\varepsilon_t = \sigma_t e_t \tag{3}$$

$$e_t \overset{i.i.d.}{\sim} N(0,1) \tag{4}$$

where $\omega > 0$, $\alpha(B) = \alpha_1 B + ... + \alpha_p B^p$ and $\beta(B) = \beta_1 B + ... + \beta_q B^q$, with $\alpha_i \geq 0$ for $i = 1, ..., p$ and $\beta_j \geq 0$ for $j = 1, ..., q$.

(1) is the mean equation. As we stated earlier, we think the return data is stationary. Thus we choose ARMA model for our mean process. And usually the best order of ARMA model would not be greater than 1. So we choose the simplest ARMA(1,1) model so that we can focus on the performance of the GARCH models. (2) is the core of the GARCH

model, the conditional variance equation, where ε_t represents the residual from (1) and σ_t represents the conditional variance. (3) and (4) show the relationship between the residual and conditional variance. We assume the residual has the conditional distribution as $N(0, \sigma_t^2)$. Usually we divide the residual of the model by its corresponding conditional variance and we get a WN (called normalized residuals). Our residual analysis then focus on the research of the normalized residuals to see whether they are uncorrelated and normally distributed.

We fit the ARMA(1,1)+GARCH(p,q) model with the two datasets, and then we choose the best GARCH model by the following AIC and BIC results.

(p,q)	1		2		3	
	AIC	BIC	AIC	BIC	AIC	BIC
1	-6.076476	-6.047147	-6.073405	-6.039187	-6.070277	-6.031171
2	-6.088412	-6.054194	-6.086422	-6.047316	-6.083399	-6.039405
3	-6.086817	-6.047711	-6.084827	-6.040833	-6.082837	-6.033954

Table 4.1.1 AIC and BIC for GARCH model in 2006-2009

(p,q)	1		2		3	
	AIC	BIC	AIC	BIC	AIC	BIC
1	-6.964961	-6.935654	NA	NA	-6.960001	-6.920926
2	-6.962454	-6.928263	-6.963703	-6.924627	-6.95825	-6.91429
3	-6.963795	-6.924719	-6.961883	-6.917923	-6.959099	-6.910255

Table 4.1.2 AIC and BIC for GARCH model in 2013-2016

From Table 4.1.1 and Table 4.1.2, we find that for the volatile period from 2006 to 2009, the best GARCH model we choose is GARCH(2,1) and for the relatively stable period from 2013 to 2016, the best GARCH model we choose is GARCH(1,1). Then we perform the residual diagnose analysis as follows.

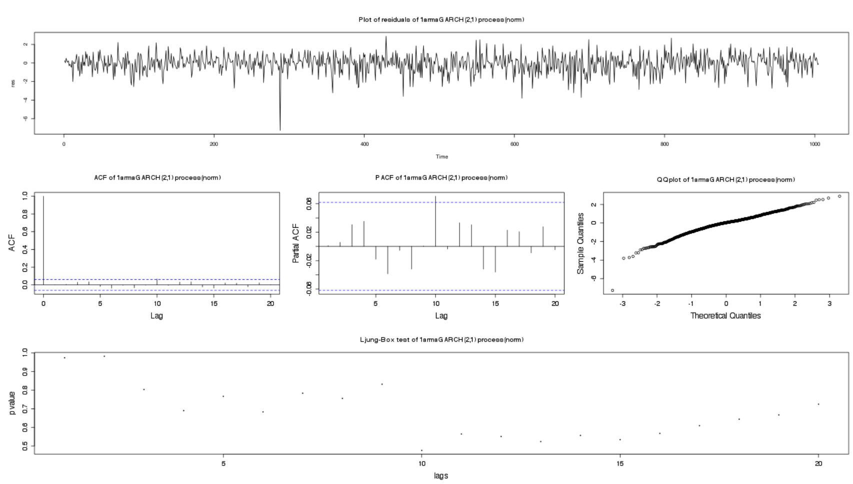


Figure 4.1.1 Residual Diagnose Plot for GARCH(2,1) in 2006-2009

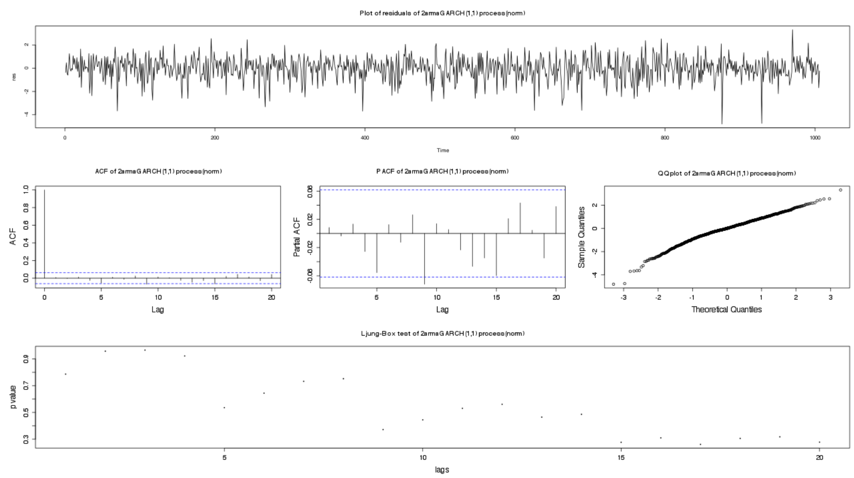


Figure 4.1.2 Residual Diagnose Plot for GARCH(1,1) in 2006-2009

The normalized residual plot shows that it resembles the white noise process. From ACF, PACF plots and Ljung-Box test result, we find the residual series is uncorrelated. The Q-Q plot is nearly a straight line, so it follows the Normal distribution. Therefore, we have the confidence that the GARCH models we fit capture the most information from the series leaving the residual a WN.

4.2 IGARCH Model

Note that the variance equation of GARCH model can be written as

$$(1-\alpha(B)-\beta(B))\varepsilon_t^2 = \omega + (1-\beta(B))v_t, \quad v_t = \varepsilon_t^2 - \sigma_t^2$$

According to the empirical studies in Engle and Bollerslev (1986), Chou (1988)[3], the estimated lag polynomial $(1-\alpha(B)-\beta(B))$ is found to have a significant unit root in some applications of GARCH models. Factoring this polynomial as $(1-\alpha(B)-\beta(B)) = (1-B)\phi(B)$, where φ(B) has all the roots outside the unit circle, Engle and Bollerslev (1986)[2] proposed the following integrated GARCH, or IGARCH(p, q) model:

$$\phi(B)(1-B)\varepsilon_t^2 = \omega + (1-\beta(B))v_t, \quad v_t = \varepsilon_t^2 - \sigma_t^2 \tag{5}$$

where $\phi(B) = 1 - \phi_1 B - ... - \phi_q B^q$. As many empirical studies using GARCH(1,1) models give $\alpha_1 + \beta_1$ very close to 1 implying high persistent volatility, the impact of past information on future volatility forecasts decays very slowly. Therefore, we believe that the IGARCH(1,1) model given by

$$r_t = \varphi_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \tag{6}$$

$$\sigma_t^2 = \omega + \alpha_1 \sigma_t^2 + (1-\alpha_1)\varepsilon_t^2 \tag{7}$$

is a good alternative to GARCH(1,1) model. When $\mu = 0$, the IGARCH(1,1) model is reduced to RiskMetrics model with $\lambda = \beta_1$. From the good performance of RiskMetrics for some α

such as 5% or 10% documented in the literature, it is anticipated that IGARCH(1,1) can also be a good model for VaR estimation.

We fit the ARMA(1,1)+IGARCH(p,q) model with the two datasets, and then we choose the best IGARCH model by the following AIC and BIC results.

(p,q)	1		2		3	
	AIC	BIC	AIC	BIC	AIC	BIC
1	-6.07637	-6.051926	-6.074926	-6.045596	-6.080673	-6.046455
2	-6.087671	-6.058342	-6.08628	-6.052062	-6.087911	-6.048805
3	-6.085925	-6.051717	-6.684353	-6.045247	-6.086021	-6.042026

Table 4.2.1 AIC and BIC for IGARCH model in 2006-2009

(p,q)	1		2		3	
	AIC	BIC	AIC	BIC	AIC	BIC
1	-6.94859	-6.924176	-6.949161	-6.919854	-6.944072	-6.909881
2	-6.949051	-6.919744	-6.947453	-6.913262	-6.942114	-6.903038
3	-6.947356	-6.913156	-6.942942	-6.903867	-6.942109	-6.898749

Table 4.2.2 AIC and BIC for IGARCH model in 2013-2016

From Table 4.2.1 and Table 4.2.2, we find that for the volatile period from 2006 to 2009, the best IGARCH model we choose is IGARCH(2,1) and for the relatively stable period from 2013 to 2016, the best IGARCH model we choose is IGARCH(1,1). Then we perform the residual diagnose analysis as follows.

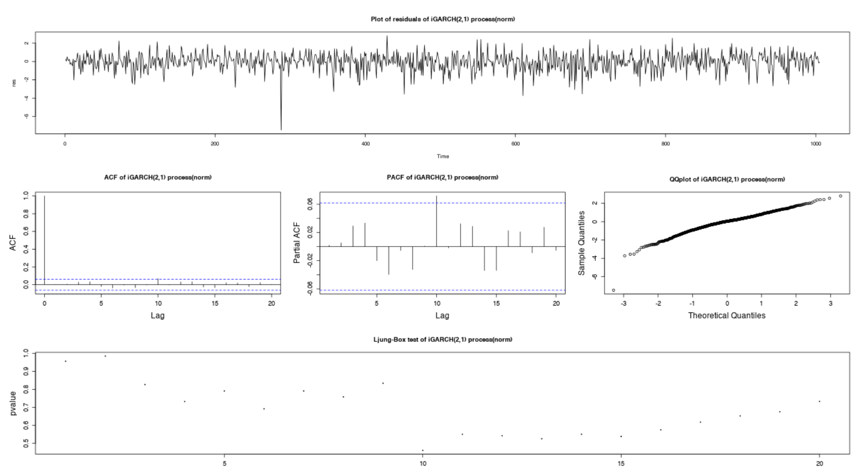


Figure 4.2.1 Residual Diagnose Plot for IGARCH(2,1) in 2006-2009

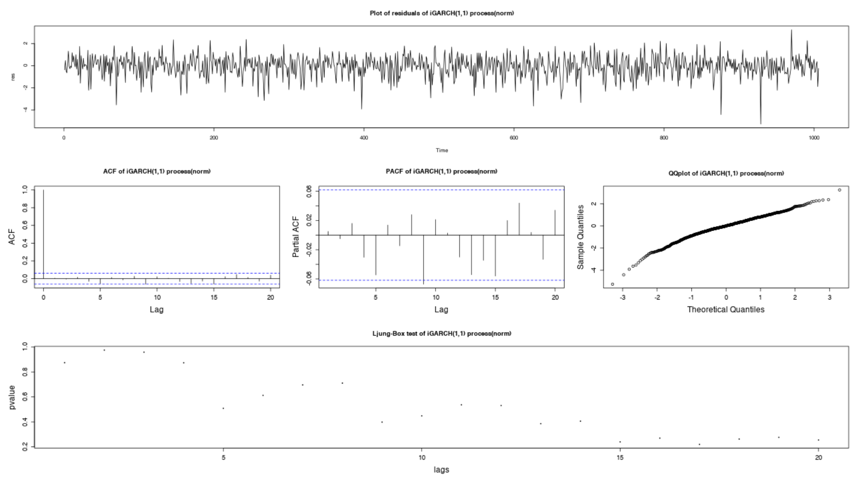


Figure 4.2.2 Residual Diagnose Plot for IGARCH(2,1) in 2013-2016

The normalized residual plot shows that it resembles the white noise process. From ACF, PACF plots and Ljung-Box test result, we find the residual series is uncorrelated. The Q-Q plot is nearly a straight line, so it follows the Normal distribution. Therefore, we have the confidence that the IGARCH models we fit capture the most information from the series leaving the residual a WN.

4.3 eGARCH Model

The eGARCH model was proposed by Nelson (1991)[4], and it is short for exponential generalized autoregressive conditional heteroskedastic model. Nelson and Cao (1992)[5] argue that the nonnegativity constraints in the linear GARCH model are too restrictive. The GARCH model imposes the nonnegative constraints on the parameters, α_i and β_j, while there are no restrictions on these parameters in the eGARCH model. In the eGARCH model, the conditional variance, σ_t^2, is an asymmetric function of lagged disturbances.

The conditional variance equation of eGARCH(p,q) is defined as

$$\log \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \log \sigma_{t-k}^2 + \sum_{j=1}^{q} \beta_j g(Z_{t-j}) \tag{8}$$

$$g(Z_t) = \theta Z_t + \lambda(|Z_t| - E(|Z_t|)) \tag{9}$$

where σ_t^2 is the conditional variance. Z_t may be a standard normal variable or come from a generalized error distribution. The formulation for $g(Z_t)$ allows the sign and the magnitude of Z_t to have separate effects on the volatility. This is particularly useful in an asset pricing context.

We fit the ARMA(1,1)+eGARCH(p,q) model with the two datasets, and then we choose the best eGARCH model by the following AIC and BIC results.

(p,q)	1		2		3	
	AIC	BIC	AIC	BIC	AIC	BIC
1	-6.115297	-6.081079	-6.111956	-6.07285	-6.111956	-6.07285
2	-6.142063	-6.098068	-6.14103	-6.092147	-6.14103	-6.092147
3	-6.155418	-6.101647	-6.153793	-6.095133	-6.153793	-6.095133

Table 4.3.1 AIC and BIC for eGARCH model in 2006-2009

(p,q)	1		2		3	
	AIC	BIC	AIC	BIC	AIC	BIC
1	-7.042749	-7.008558	-7.041042	-7.001966	-7.039145	-6.995186
2	-7.045369	-7.001409	-7.049923	-7.001079	-7.046922	-6.993193
3	-7.0435	-6.989771	-7.048043	-6.98943	-7.048726	-6.985228

Table 4.3.2 AIC and BIC for eGARCH model in 2013-2016

From Table 4.3.1 and Table 4.3.2, we find that for the volatile period from 2006 to 2009, the best eGARCH model we choose is eGARCH(3,3) and for the relatively stable period from 2013 to 2016, the best eGARCH model we choose is eGARCH(1,1). Then we perform the residual diagnose analysis as follows.

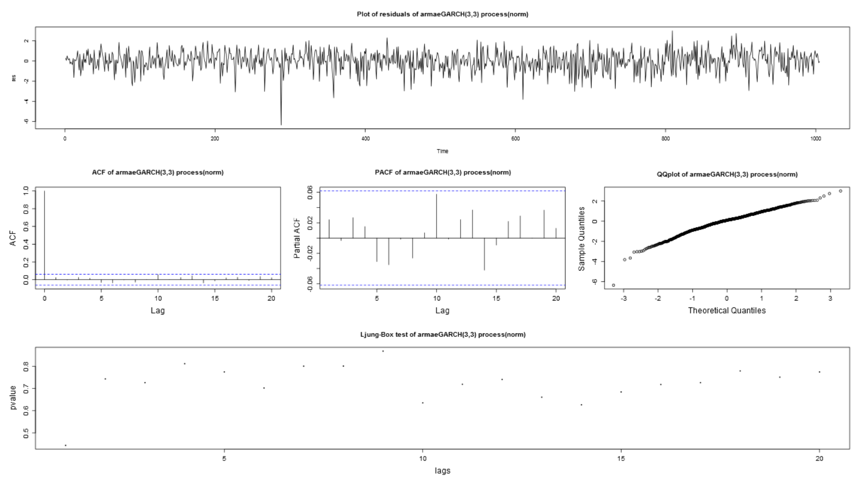


Figure 4.3.1 Residual Diagnose Plot for eGARCH(3,3) in 2006-2009

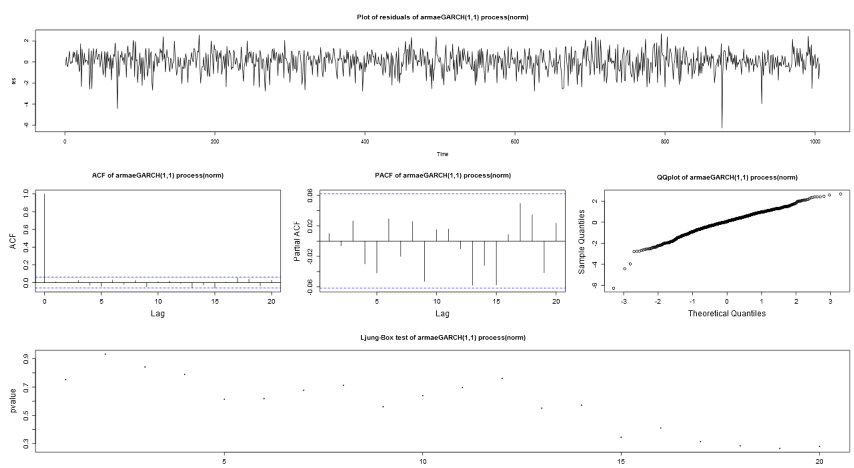


Figure 4.3.2 Residual Diagnose Plot for eGARCH(1,1) in 2013-2016

The normalized residual plot shows that it resembles the white noise process. From ACF, PACF plots and Ljung-Box test result, we find the residual series is uncorrelated. The Q-Q plot is nearly a straight line, so it follows the Normal distribution. Therefore, we have the confidence that the eGARCH models we fit capture the most information from the series leaving the residual a WN.

4.4 GJR-GARCH Model

In some financial time series, large negative returns appear to increase volatility more than do positive returns of the same magnitude. This is called the leverage effect. The standard GARCH model cannot model the leverage effect because it model σ_t as a function of past values of ε_t^2, whether the past values of ε_t are positive or negative is not taken into account. The problem is that the square function x^2 is symmetric in x. Therefore, the standard GARCH model show symmetric in both positive and negative returns scenarios.

The solution from symmetric to asymmetric is to replace the square function with a flexible asymmetric functions. The GJR-GARCH could conduct this by additionally capturing asymmetry in return volatility. The conditional variance equation of GJR-GARCH (p, q) model is defined as[6]:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \sigma_{t-k}^{\delta} + \sum_{j=1}^{q} (\beta_j + \gamma_j I_{t-j}) \varepsilon_{t-j}^2$$

where

$$I_{t-i} = \begin{cases} 1 & \varepsilon_{t-i} < 0 \\ 0 & \varepsilon_{t-i} \geq 0 \end{cases}$$

The effect of ε_{t-i}^2, upon σ_t is through the function $g_y(x) = \alpha_i + \gamma_i I_{t-i}$. When $\gamma > 0$, $g_y(-x) > g_y(x)$ for any $x > 0$, so there is a leverage effect in negative return. If $\gamma < 0$, then there is a leverage effect in the opposite direction that positive past values of ε_t increase volatility more than negative past values of the same magnitude.

We fit the ARMA(1,1)+GJR-GARCH (p, q) model with the two datasets, and then we choose the best GJR-GARCH model by the following AIC and BIC results.

(p,q)	1		2		3	
	AIC	BIC	AIC	BIC	AIC	BIC
1	-6.110811	-6.076593	-6.110811	-6.068552	-6.104522	-6.060527
2	-6.115971	-6.071976	-6.115971	-6.070443	-6.116168	-6.062396
3	-6.112655	-6.058883	-6.112655	-6.058288	-6.114957	-6.051409

Table 4.4.1 AIC and BIC for GJR-GARCH model in 2006-2009

(p,q)	1		2		3	
	AIC	BIC	AIC	BIC	AIC	BIC
1	-7.005846	-6.971655	-7.003367	-6.964292	-6.998363	-6.954403
2	-7.001486	-6.957526	-7.00555	-6.956706	-7.002923	-6.949195
3	-6.994518	-6.940789	-6.99783	-6.939217	-6.999935	-6.936437

Table 4.4.2 AIC and BIC for GJR-GARCH model in 2013-2016

From Table 4.3.1 and Table 4.3.2, we find that for both the volatile period from 2006 to 2009 and the relatively stable period from 2013 to 2016, the best eGARCH model we choose are eGARCH(1,1). Then we perform the residual diagnose analysis as follows.

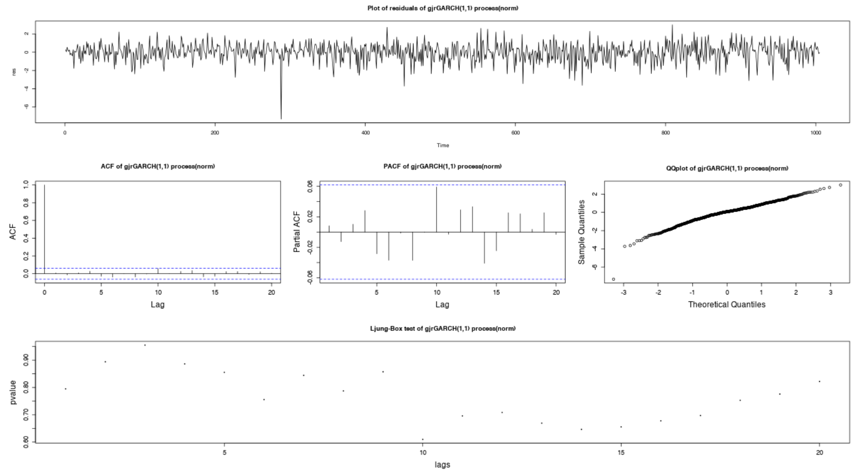


Figure 4.4.1 Residual Diagnose Plot for GJR-GARCH(1,1) in 2006-2009

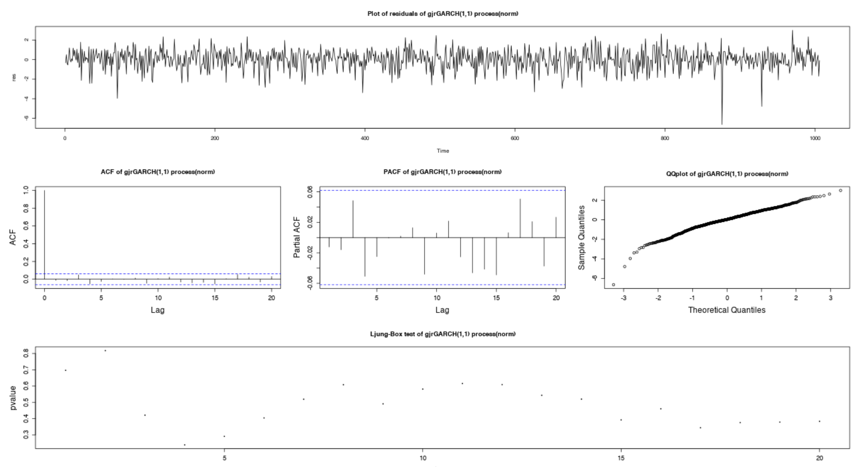


Figure 4.4.2 Residual Diagnose Plot for GJR-GARCH(1,1) in 2013-2016

The normalized residual plot shows that it resembles the white noise process. From ACF, PACF plots and Ljung-Box test result, we find the residual series is uncorrelated. The Q-Q plot is nearly a straight line, so it follows the Normal distribution. Therefore, we have the confidence that the GJR-GARCH models we fit capture the most information from the series leaving the residual a WN.

5 Value-at-Risk Forecast Using GARCH Family

5.1 VaR Calculation

As we have mentioned earlier, VaR is widely used for risk management in financial industry. In this part, we shift our attention to the performance of our models for VaR predictions. Firstly, we need to explain how we calculate VaR based on our GARCH model families. As shown earlier, we assume Normal innovations for all our models. And our models can produce predicted conditional mean and predicted conditional variance, based on which we can derive the conditional distribution of the return data. And then the VaR is defined as (with $\alpha\%$ confidence level),

$$VaR_{\alpha} = -\hat{\mu} - \hat{\sigma} * \Phi^{-1}(\alpha)$$

where $\hat{\mu}$ is the predicted mean, $\hat{\sigma}$ is the predicted volatility and α is the confidence level. Note that we take the absolute value of the above VaR calculation as the industry convention.

5.2 Forecast VaR Using Cross-Validation with GARCH Models

In this section, we use the rolling-window cross validation method for the VaR prediction tests. For both datasets, we choose one year as the training set and then make predictions for the following one trading day ahead. We estimate the VaR for next day based on the method stated earlier. And then we compare the return for that day with the estimated VaR to see whether the return data falls below the VaR. Going through the whole dataset, we calculate the hit rate which denotes the ratio that the return data falling below the VaR. We then assess the VaR prediction accuracy by comparing the hit rate with our prespecified confidence level $\alpha\%$. The closer the hit rate is to the confidence level (5% for our case), the better performance of the model is to predict the VaR.

Table 5.2.1 summaries the hit rates for the best models we choose from Section 4 for each GARCH models in two periods. We can see that due to the unpredictability of the 2007-2008 financial crisis, the hit rate for each model is higher than the confidence level. However, in the relatively stable period, the hit rate is much closer to 5% for each GARCH model.

06 -- 09	GARCH(2,1)	iGARCH(2,1)	eGARCH(3,3)	gjrGARCH(1,1)
	0.082	0.081	0.083	0.086
13 -- 16	GARCH(1,1)	iGARCH(1,1)	eGARCH(1,1)	gjrGARCH(1,1)
	0.068	0.059	0.068	0.073

Table 5.2.1 Hit Rates Summary

Also, we plot the predicted VaR series for both datasets as follows. From the plot, we could find that during the volatile period, the predicted VaR is at peak. The hiking of VaR is coincident with higher risk exposure in financial crisis, accurately measuring the potential risk in the market.

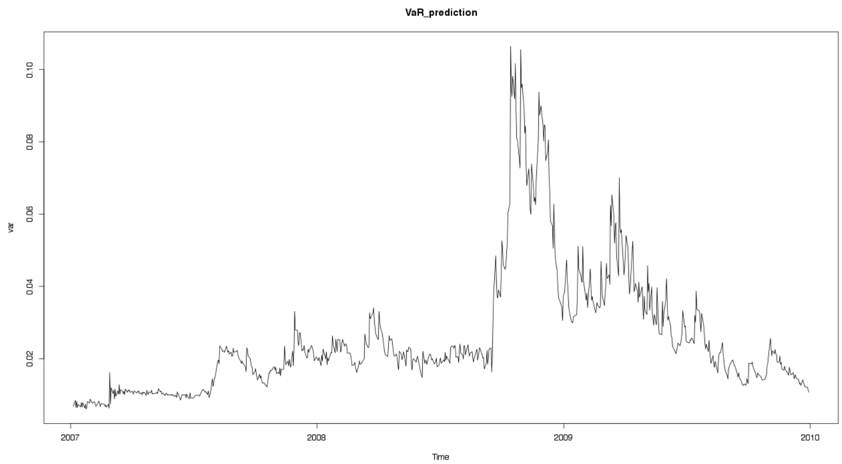


Figure 5.2.1 Predicted VaR by IGARCH(2,1) in 2007-2009

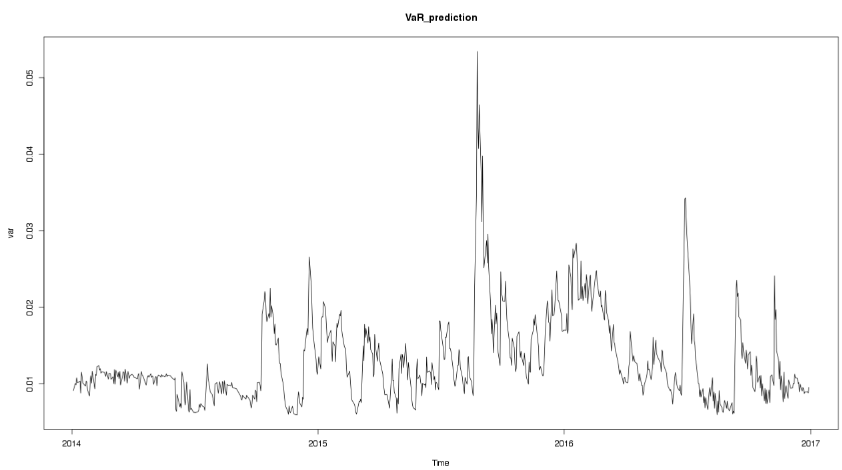


Figure 5.2.2 Predicted VaR by IGARCH(1,1) in 2014-2016

6 Comparison between GARCH Models

6.1 In-sample Estimation

In the procedure of choosing the best order for each GARCH model during the whole period, we compare "Goodness of Fit" among our four different GARCH models. Based on AIC and BIC, the eGARCH model and GJR-model give the best fitness on both volatile and stable periods. One of possible interpretations is that eGARCH and GJR-model include more parameters, thus rendering more accurate fitted model than the other two GARCH models. Besides, eGARCH model and GJR-model could also incorporate asymmetric effect of asset returns on volatility, better descripting the financial asset returns especially in financial crisis.

6.2 Rolling Prediction

Based on hit rates, the most accurate prediction model is IGARCH model, and then GARCH model. One deficiency of GARCH model by the empirical study is that the square returns of financial assets comply with "unit root" characteristic. And IGARCH incorporates this feature, thus giving a better prediction of VaR. However, contrary to AIC and BIC result above, the best two GARCH models do not perform well during the cross-validation period. One of the possible reasons is that the eGARCH and GJR-GARCH model are overfitted during the order selection procedure, which produce better result in AIC, BIC but underperform for predicting VaR. In addition to overfitting, the discrepancy between information criteria and prediction results might be explained by the incorporation of mean AMRA (1,1) model, introducing mean forecast errors that could not interpreted by GARCH models.

7 Conclusion

In this paper, we explore the performance of four GARCH models: GARCH, iGARCH, eGARCH and GJR-GARCH on VaR estimation of S&P 500 ETF returns. The dataset covers both the volatile period (2006-2009) and the stable period (2013-2016). Fitting four GARCH models in the two datasets, we choose the best order for each GARCH model based on BIC(AIC) and residual analysis in each period.

With the best models chosen from four GARCH families, we then use the rolling-window cross validation method for the VaR prediction tests. We choose one year as the training set and then make predictions for the following one trading day ahead. Going through the whole dataset, we calculate the hit rate which denotes the ratio that the true return data falling below the predicted VaR. VaR prediction accuracy is assessed by comparing the hit rate with our prespecified confidence level $\alpha\%$.

Although the eGARCH model demonstrates the best in-sample estimation according to BIC, in the rolling prediction for VaR, IGARCH model outperforms the other GARCH models based on the hit rates during the both volatile and stable periods. This might result from that fitting extra parameters for eGARCH often leads to the overfitting problem with small training dataset. So we conclude that IGARCH model show a good balance between the in-sample estimation and the prediction for VaR due to its sparseness.

8 Reference

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