

Homework 1: let $V = \{(x, y) \mid y = k_0 x + k_1 x^2 + k_2 x^3, k_0, k_1, k_2 \in \mathbb{R}, x, y \in \mathbb{R}\}$

$$V_1 + V_2 = (x_1 + x_2, y_1 + y_2)$$

$$\begin{aligned} y_1 + y_2 &= k_0 x_1 + k_1 x_1^2 + k_2 x_1^3 + k_0 x_2 + k_1 x_2^2 + k_2 x_2^3 \\ &= k_0(x_1 + x_2) + k_1(x_1^2 + x_2^2) + k_2(x_1^3 + x_2^3) \quad (1) \end{aligned}$$

if V is a subspace of \mathbb{R}^2 ,

$$y_1 + y_2 = k_0(x_1 + x_2) + k_1(x_1 + x_2)^2 + k_2(x_1 + x_2)^3 \quad (2)$$

but (1) is not identically equal to (2).

So the V is not a subspace of \mathbb{R}^2 .

Homework 2:

answer: let $W_1 = a_1 V_1 + b_1 V_2$ $W_2 = a_2 V_1 + b_2 V_2, W_1, W_2 \in W$

$$W_1 + W_2 = \underbrace{(a_1 + a_2)}_{\in \mathbb{R}} V_1 + \underbrace{(b_1 + b_2)}_{\in \mathbb{R}} V_2 \in W$$

$$\alpha W_1 = \underbrace{\alpha a_1}_{\in \mathbb{R}} V_1 + \underbrace{\alpha b_1}_{\in \mathbb{R}} V_2 \in W \quad (\alpha \in \mathbb{F})$$

Thus, W is a subspace of \mathbb{R}^2

Homework 3:

answer: let $B_1 \in W_2, B_1 \in \mathbb{R}^{n \times n}, \det(B_1) \neq 0$

$$\because \det(B_1) \neq 0 \Rightarrow B_1 \neq 0$$

$\therefore W_2$ is not a subspace of $\mathbb{R}^{n \times n}$.