STS2005(09) 미적분학I (HW5) DueDate(2020/5/4)

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7.1. Exercises #8

Evaluate the integral.

$$\int \underbrace{t}_{u} \underbrace{\sec^2 2t}_{v'} dt = t \cdot \tan(2t)\frac{1}{2} - \int 1 \cdot \tan(2t)\frac{1}{2}\, dt$$

$$= t \cdot \tan(2t)\frac{1}{2} - \frac{1}{2}\left(-\ln|\cos 2t| + C_0\right)\cdot\frac{1}{2}$$

$$= \frac{1}{2}\cdot t \cdot \tan(2t) + \frac{1}{4}\ln|\cos 2t| + C$$

7.1. Exercises #24

Evaluate the integral.

$$\int_0^1 \underbrace{(x^2+1)}_{u}\underbrace{e^{-x}}_{v'}\, dx = \left[(x^2+1)e^{-x}(-1)\right]_0^1 + \int_0^1 \underbrace{2x}_{u} \cdot \underbrace{e^{-x}}_{v'} dx$$

$$= \left[-(x^2+1)e^{-x} + 2x\, e^{-x}(-1)\right]_0^1 + \int_0^1 2e^{-x} dx$$

$$= \left[-(x^2+2x+1)e^{-x}\right]_0^1 + 2\left[-e^{-x}\right]_0^1$$

$$= \left[-(x^2+2x+3)e^{-x}\right]_0^1$$

$$= -6\cdot e^{-1} + 3$$

7.2. Exercises #1

Evaluate the integral.

$$\int \sin^2 x \cos^3 x\, dx = \int u^2(1-u^2)'\, du \quad (\text{where } u = \sin x)$$

$$= \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$$

$$= \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$$

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7.2. Exercises #19

Evaluate the integral.

$$\int t\sin^2 t\,dt = \int t\cdot\frac{1-\cos(2t)}{2}\,dt$$

$$= \int \frac{t}{2}\,dt - \int \underbrace{t}_{u}\cdot\underbrace{\frac{\cos(2t)}{2}}_{v'}\,dt$$

$$= \frac{1}{4}t^2 + C_0 - \frac{1}{4}t\sin(2t) + \int 1\cdot\frac{1}{4}\sin(2t)$$

$$= \frac{1}{4}t^2 - \frac{1}{4}t\sin(2t) - \frac{1}{8}\cos(2t) + C$$

7.2. Exercises #27

Evaluate the integral.

$$\int \tan^3 x\sec x\,dx = \int (u^2-1)^1 u^0\,du \quad (\text{where } u = \sec x)$$

$$= \frac{1}{3}u^3 - u + C$$

$$= \frac{1}{3}\sec^3 x - \sec x + C$$

7.2. Exercises #41

Evaluate the integral.

$$\int \sin 8x\cos 5x\,dx = \int \frac{1}{2}(\sin 13x + \sin 3x)\,dx$$

$$= \frac{1}{2}\left(-\frac{\cos 13x}{13} - \frac{\cos 3x}{3}\right) + C$$

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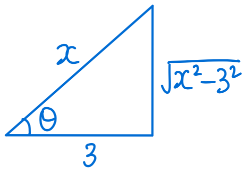
7.3. Exercises #1

Evaluate the integral using the indicated trigonometric substitution. Sketch and label the associated right triangle.

$$\int \frac{1}{\underbrace{x^2}_{9\sec^2\theta}\underbrace{\sqrt{x^2-9}}_{3\tan\theta}}dx \qquad x = 3\sec\theta$$



$\left(0 \le \theta < \frac{1}{2}\pi\right)$

$$\frac{dx}{d\theta} = 3\tan\theta\sec\theta \Rightarrow dx = 3\tan\theta\sec\theta\, d\theta$$

$\tan\theta = \frac{\sqrt{x^2-9}}{3}$ $\sec\theta = \frac{x}{3}$

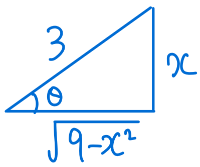
$$\int \frac{1}{x^2\sqrt{x^2-9}}dx = \int \frac{\cancel{3\tan\theta\sec\theta}}{\underset{9}{\cancel{27}}\sec^{\cancel{2}}\theta\cdot\cancel{\tan\theta}}d\theta = \frac{1}{9}\int\cos\theta\, d\theta$$

$$= \frac{1}{9}\sin\theta + C = \frac{\sqrt{x^2-9}}{9x} + C$$

7.3. Exercises #2

Evaluate the integral using the indicated trigonometric substitution. Sketch and label the associated right triangle.

$$\int \underbrace{x^3}_{27\sin^3\theta}\underbrace{\sqrt{9-x^2}}_{3\cos\theta}dx \qquad x = 3\sin\theta$$



$$\frac{dx}{d\theta} = 3\cos\theta \Rightarrow dx = 3\cos\theta\, d\theta$$

$$\int x^3\sqrt{9-x^2}dx = \int 3^5\sin^3\theta\cos^2\theta\, d\theta = -3^5\int(1-u^2)u^2\,du \quad (\text{where } u=\cos\theta)$$

$$= -3^5\left(\frac{1}{3}u^3 - \frac{1}{5}u^5 + C_0\right) = -\underset{5}{\overset{1}{\cancel{3^5}}}\left(\frac{(15)\sqrt{9-x^2}^3}{\cancel{3^4}} - \frac{\sqrt{9-x^2}^5}{\cancel{5\cdot3^5}}\right) + C$$

$$= -\frac{1}{5}\sqrt{9-x^2}^3(15-9+x^2) + C = -\frac{1}{5}\sqrt{9-x^2}^3(6+x^2) + C$$

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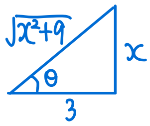
7.3. Exercises #3

Evaluate the integral using the indicated trigonometric substitution. Sketch and label the associated right triangle.

$$\int \frac{x^3}{\sqrt{x^2+9}}dx \qquad x = 3\tan\theta$$



$$dx = 3\sec^2\theta\, d\theta$$

$$\int \frac{x^3}{\sqrt{x^2+9}}dx = \int \frac{1}{3}\cos\theta \cdot 3^3\tan^3\theta \cdot 3\sec^2\theta\, d\theta = 3^3\int \tan^3\theta\sec\theta\, d\theta$$

$$= 3^3\int (u^2-1)\,du \ (\text{where } u = \sec\theta) = 3^2u^3 - 3^3u + C$$

$$= \frac{\sqrt{x^2+9}^{\,3}}{3} - 3^2\sqrt{x^2+9} + C = \frac{1}{3}\sqrt{x^2+9}\,(x^2+9-27) + C$$

$$= \frac{1}{3}\sqrt{x^2+9}\,(x^2-18) + C$$

7.4. Exercises #12

$$\frac{1}{AB} = \frac{1}{B-A}\left(\frac{1}{A} - \frac{1}{B}\right)$$

Evaluate the integral. 분수적분 → ln + 부분분수

$$\int_0^1 \frac{x-4}{x^2-5x+6}dx = \frac{1}{2}\int_0^1 \frac{2x-8}{x^2-5x+6}dx$$

$$= \frac{1}{2}\int_0^1 \left(\frac{2x-5}{x^2-5x+6} - \frac{3}{\underbrace{x^2-5x+6}_{(x-2)(x-3)}}\right)dx$$

$$= \frac{1}{2}\Big[\ln|x^2-5x+6|\Big]_0^1 - \frac{3}{2}\int_0^1\left(\frac{1}{x-3} - \frac{1}{x-2}\right)dx$$

$$= \frac{1}{2}\Big[\ln|x^2-5x+6| - 3\ln|x-3| + 3\ln|x-2|\Big]_0^1$$

$$= \frac{1}{2}\left(\cancel{\ln 2} - \cancel{\ln 6} - (3\ln 2 - \overset{2}{\cancel{3}}\ln 3) + (-3\ln 2)\right)$$

$$= \ln 3 - 3\ln 2$$

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7.4. Exercises #23

Evaluate the integral.

$$\int \frac{10}{(x-1)(x^2+9)}\,dx = \int \frac{3x^2-2x+9}{(x-1)(x^2+9)}\,dx - \int \frac{\overset{3x+1}{\cancel{3x^2-2x-1}}}{(\cancel{x-1})(x^2+9)}\,dx$$

$\frac{d}{dx}$ $x^2+9+(x-1)2x$

$$= \ln|(x-1)(x^2+9)| + C_0 - \frac{3}{2}\int\frac{2x}{x^2+9}\,dx + \int\frac{1}{x^2+9}\,dx$$

$$= \ln|(x-1)(x^2+9)| - \frac{3}{2}\ln|x^2+9| + \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

7.8. Exercises #7

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_{-\infty}^{0} \frac{1}{3-4x}\,dx = \lim_{t\to-\infty}\int_t^0 \frac{1}{3-4x}\,dx = \lim_{t\to-\infty} -\frac{1}{4}\Big[\ln|3-4x|\Big]_t^0$$

$$= -\frac{1}{4}\lim_{t\to-\infty}\left(\ln 3 - \ln|3-t|\right) = \infty \Rightarrow \text{divergent}$$

7.8. Exercises #13

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_{-\infty}^{\infty} xe^{-x^2}\,dx = \lim_{t\to-\infty}\int_t^0 xe^{-x^2}\,dx + \lim_{t\to\infty}\int_0^t xe^{-x^2}\,dx$$

↘ 기함수 → 0

$$= \lim_{t\to\infty} -\frac{1}{2}\left(\Big[e^{-x^2}\Big]_{-t}^{0} - \Big[e^{-x^2}\Big]_0^t\right)$$

$$= -\frac{1}{2}\lim_{t\to\infty}\left(1 - e^{-t^2} - 1 + e^{-t^2}\right) = 0 \Rightarrow \text{convergent}$$

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7.8. Exercises #31

Determine whether each integral is convergent or divergent.
Evaluate those that are convergent.

$$\int_{-2}^{3} \frac{1}{x^4}\,dx = \lim_{t\to 0^-}\int_{-2}^{t}\frac{1}{x^4}dx + \lim_{t\to 0^+}\int_{t}^{3}\frac{1}{x^4}dx \quad \left(\frac{1}{x^4} \text{ is discontinuous at } x=0\right)$$

$$= \lim_{t\to 0^-}\left[-\frac{1}{3}x^{-3}\right]_{-2}^{t} + \lim_{t\to 0^+}\left[-\frac{1}{3}x^{-3}\right]_{t}^{3}$$

$$= -\frac{1}{3}\left(\lim_{t\to 0^-}(\underbrace{t^{-3}}_{-\infty}+\frac{1}{8}) + \lim_{t\to 0^+}(\frac{1}{27}\underbrace{-t^{-3}}_{-\infty})\right)$$

$$= \infty \Rightarrow \text{divergent}$$