STS2005(09) **미적분학 I(HW2)** **Due Date(2020. 3/28)**

No : ______ 학과 : 경제학/컴퓨터공학 학번 : 20160563 이름 : 송진아

1. Use the $\varepsilon - \delta$ definition of a limit to prove:

$\boxed{\text{2.4-\#17}}$ $\lim_{x\to 1}(2x+3) = 5$

Proof. Now show that for any $\varepsilon > 0$, there is $\delta > 0$ such that for all $x \in \mathbb{R}$, if $0 < |x-1| < \delta$, then $|(2x+3)-5| < \varepsilon$. Let $\varepsilon > 0$ be given. Note that

$$\left|(2x+3)-5\right| = \boxed{2}\,|x-1|.$$

Let $\delta = \boxed{\frac{1}{2}\varepsilon}$. Then $\delta > 0$ and for all $x \in \mathbb{R}$, if $0 < |x-1| < \delta$, then

$$\left|(2x+3)-5\right| = \boxed{2}\,|x-1| < \boxed{2}\,\delta = \boxed{2\cdot\frac{1}{2}\varepsilon} = \varepsilon$$

$\boxed{\text{2.4-\#32}}$ $\lim_{x\to 2} x^3 = 8$

Proof. Now show that for any $\varepsilon > 0$, there is $\delta > 0$ such that for all $x \in \mathbb{R}$, if $0 < |x-2| < \delta$, then $|x^3-8| < \varepsilon$. Let $\varepsilon > 0$ be given. Note that

$$\left|x^3-8\right| = (x^2+2x+4)\,|x-2|.$$

$x=2$

???

If $|x-2| < 1$, then

$9+6+4$

$(x\neq 2)$

$$1 < x < 3 \quad \text{and so} \quad (x^2+2x+4) < \boxed{19}, \qquad (x^2+2x+4)\,|x-2| \underline{\le} \boxed{19}\,|x-2|.$$

Let $\delta = \min\{1, \boxed{\frac{1}{19}\varepsilon}\}$. Then $\delta > 0$ and for all $x \in \mathbb{R}$, if $0 < |x-2| < \delta$, then

$$\begin{aligned}\left|x^3-8\right| = (x^2+2x+4)\,|x-2| &< \boxed{19}\,|x-2| \quad (\because |x-2| < \delta \leqslant 1)\\ &< \boxed{19}\,\delta \quad (\because |x-2| < \delta)\\ &\leqslant \boxed{19\cdot\frac{1}{19}\varepsilon} = \varepsilon.\end{aligned}$$

$\boxed{\text{2.4-\#37}}$ $\lim_{x\to a}\sqrt{x} = \sqrt{a}$ if $a \geq 0$. → Theorem 처럼 등호 포함되면?

Proof. Now show that for any $\varepsilon > 0$, there is $\delta > 0$ such that for all $x \geqslant 0$, if $0 < |x-a| < \delta$, then $|\sqrt{x}-\sqrt{a}| < \varepsilon$. Let $\varepsilon > 0$ be given. Note that

$$\left|\sqrt{x}-\sqrt{a}\right| = \frac{1}{\boxed{|\sqrt{x}+\sqrt{a}|}}\,|x-a| \leqslant \frac{1}{\sqrt{a}}|x-a|.$$

Let $\delta = \boxed{\sqrt{a}\cdot\varepsilon}$. Then $\delta > 0$ and for all $x \geqslant 0$, if $0 < |x-1| < \delta$, then

$$\left|\sqrt{x}-\sqrt{a}\right| \leqslant \frac{1}{\sqrt{a}}|x-a| < \frac{1}{\sqrt{a}}\delta = \boxed{\frac{1}{\sqrt{a}}\cdot\sqrt{a}\,\varepsilon} = \varepsilon$$

No : 학과 : 경제학/컴퓨터공학 학번 : 20160563 이름 : 송진아

2.4 EXERCISES

18. $\lim_{x\to 3}\left(1+\frac{x}{3}\right)=2$

Let $\varepsilon>0$

Note that $\left|\left(1+\frac{x}{3}\right)-2\right|=\frac{1}{3}|x-3|$

Let $\delta=3\cdot\varepsilon$. Then $\delta>0$ and

for all $x\in\mathbb{R}$, if $0<|x-3|<\delta$ then

$\left|\left(1+\frac{x}{3}\right)-2\right|=\frac{1}{3}|x-3|<\frac{1}{3}\cdot 3\varepsilon=\varepsilon$

30. $\lim_{x\to 2}(x^2+2x-7)=1$

Let $\varepsilon>0$

Note that $|(x^2+2x-7)-1|=|(x-2)(x+4)|=|x+4|\cdot|x-2|$

If $0<|x-2|<1$, then

$1<x<2$ or $2<x<3$ and so $|x+4|<7$.

Let $\delta=\min\{\frac{1}{7}\varepsilon,1\}$. Then $\delta>0$ and

for all $x\in\mathbb{R}$, if $0<|x-2|<\delta$ then

$|(x^2+2x-7)-1|=|x+4|\cdot|x-2|<7\cdot|x-2|<7\delta\le 7\cdot\frac{1}{7}\varepsilon=\varepsilon$

No : 학과 : 경제학/컴퓨터공학 학번 : 20160563 이름 : 송진아

2.5 EXERCISES

45. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geqslant 2 \end{cases}$$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (cx^2 + 2x) = 4c + 4$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x^3 - cx) = 8 - 2c$

$f(2) = 4c + 4$

$\Rightarrow 4c + 4 = 8 - 2c$

$6c = 4$

$\therefore c = \frac{2}{3}$

IVT, EVT

52. Suppose f is continuous on $[1, 5]$ and the only solutions of the equation $f(x) = 6$ are $x = 1$ and $x = 4$. If $f(2) = 8$, explain why $f(3) > 6$.

Suppose to the countrary that $f(3) \le 6$. Then

$\exists c \in [2, 3]$ such that $f(c) = 6$ (by IVT)

However it contradicts to the assumption that

the only solutions of the equation $f(x) = 6$ are $x = 1$ and $x = 4$. ⊗

$\therefore f(3) > 6$

53–56 Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

56. $\sin x = x^2 - x, \quad (1, 2)$ (continuous, continuous)

Let $f(x) = \sin(x) - x^2 + x$.

Note that a root of the given equation is c such that $f(c) = 0$.

Since $f(x)$ is a continuous function, $f(x)$ is continuous on $[1, 2]$.

Note that $f(1) = \sin(1) > 0$ and $f(2) = \sin(2) - 2 < 0$ ⋯ (1)

Then $\exists c \in (1, 2)$ such that $f(c) = 0$ (by (1) & IVT)