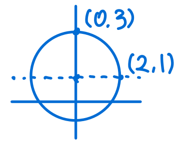
STS2005(09) 미적분학I (HW8) DueDate(2020/6/10)

학과 : 경제학과 학번 : 20160563 이름 : 송진아

10.1 Exercises #33

Find parametric equations for the path of a particle that moves along the circle $x^2 + (y - 1)^2 = 4$ in the manner described.



(a) Once around clockwise, starting at (2, 1)
(b) Three times around counterclockwise, starting at (2, 1)
(c) Halfway around counterclockwise, starting at (0, 3)

(a) $x = 2\cos(-\theta) = 2\cos\theta,\ y - 1 = 2\sin(-\theta) \Rightarrow y = -2\sin\theta + 1.\ \ \theta \in [0, 2\pi]$

(b) $x = 2\cos\theta,\ \ y = 2\sin\theta + 1.\ \ \theta \in [0, 6\pi]$

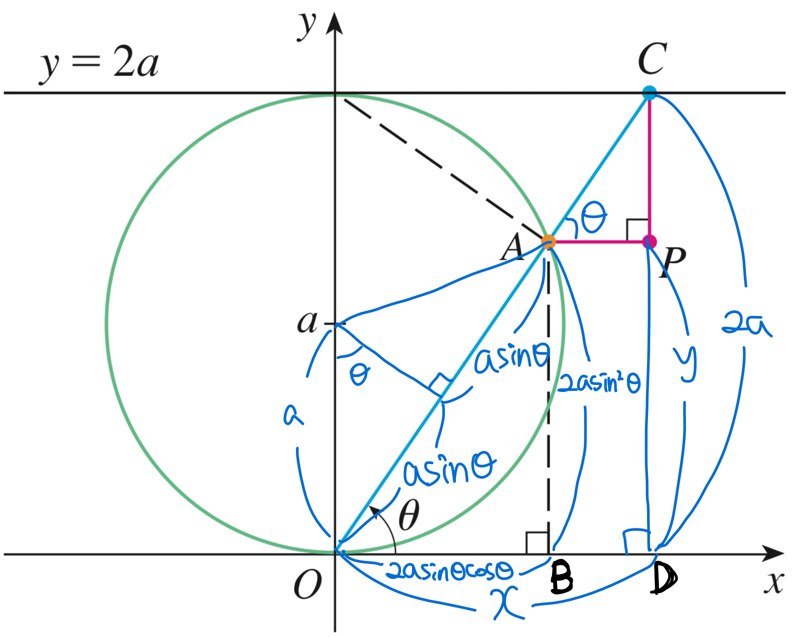
(c) $x = 2\cos\theta,\ y = 2\sin\theta + 1.\ \ \theta \in [\frac{1}{2}\pi, \frac{3}{2}\pi]$

10.1 Exercises #43

A curve, called a witch of Maria Agnesi, consists of all possible positions of the point P in the figure. Show that parametric equations for this curve can be written as

$$x = 2a \cot\theta \qquad y = 2a \sin^2\theta$$

Sketch the curve.



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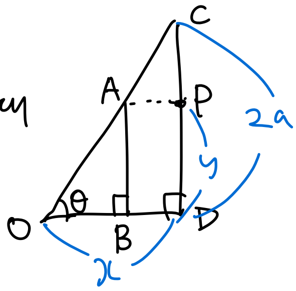
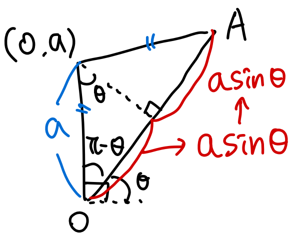
△OAB와 △OCD는 닮음이다.

P의 좌표가 (x, y)일 때



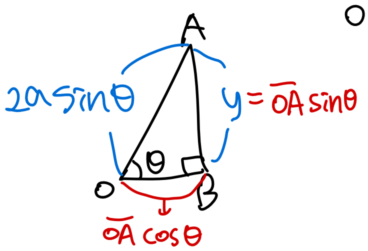
가 주어져 있다.

원의 중심에서 A로 보조선을 그으면,



$\overline{OA} = 2a\sin\theta$ 이다.

△OAB에서



$y = 2a\sin\theta \cdot \sin\theta = 2a\sin^2\theta$ 이다.

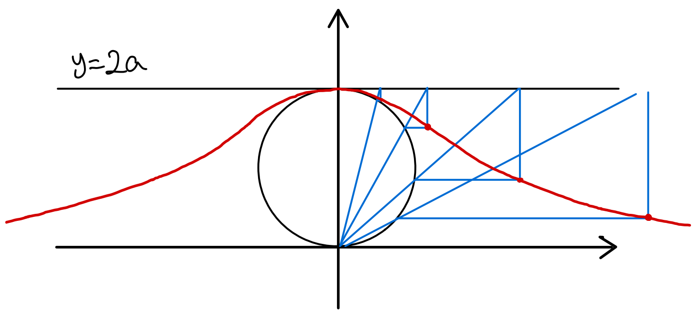
닮음비는 $y : 2a = 2a\sin^2\theta : 2a = \sin^2\theta : 1$ 이다.

$\overline{OB} : \overline{OD} = \overline{OA}\cos\theta : x = 2a\sin\theta\cos\theta : x = \sin^2\theta : 1$

$\Rightarrow x\sin^2\theta = 2a\sin\theta\cos\theta$

$\Rightarrow x = 2a\dfrac{\cos\theta}{\sin\theta} = 2a\cot\theta$

Curve는 아래와 같다.



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10.2 Exercises #11

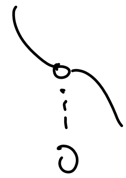
Find dy/dx and d^2y/dx^2. For which values of t is the curve concave upward?

$x = t^2 + 1, \quad y = t^2 + t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t+1}{2t} \quad \forall t \in \mathbb{R}-\{0\}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}} = \frac{\frac{\cancel{4t-4t}-2}{4t^2}}{2t} = \frac{-1}{4t^3} \quad \forall t \in \mathbb{R}-\{0\}$$

$$\frac{dy}{dx} \begin{cases} >0 & \text{for } t>0 \\ <0 & \text{for } t<0 \end{cases}, \quad \frac{d^2y}{dx^2} \begin{cases} >0 & \text{for } t<0 \\ <0 & \text{for } t>0 \end{cases} \Rightarrow$$



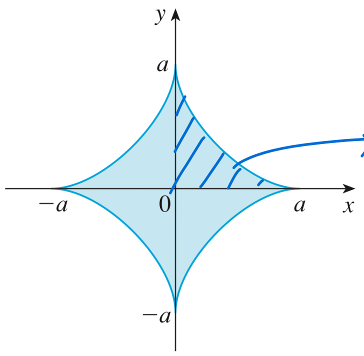
the curve concave upward on $(-\infty, 0)$

10.2 Exercises #34

Find the area of the region enclosed by the astroid

$x = a\cos^3\theta,\ y = a\sin^3\theta$

(Astroids are explored in the Laboratory Project on page 649.)



$\int_0^a y\,dx$

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$$\int_0^a y\,dx = \int_{\frac{\pi}{2}}^{0} (a\sin^3\theta)(-3a\cos^2\theta\sin\theta)\,d\theta$$

$$= -3a^2\int_{\frac{\pi}{2}}^{0} \sin^4\theta\cos^2\theta\,d\theta$$

$$= -3a^2\int_{\frac{\pi}{2}}^{0} \left(\frac{1-\cos(2\theta)}{2}\right)^2\left(\frac{1+\cos(2\theta)}{2}\right)d\theta$$

$$= -\frac{3a^2}{8}\int_{\frac{\pi}{2}}^{0}\left(\underline{\cos^3(2\theta)} - \underline{\cos^2(2\theta)} - \cos(2\theta) + 1\right)d\theta$$

$$= -\frac{3a^2}{8}\int_{\frac{\pi}{2}}^{0}\left(\underline{\cancel{\cos(2\theta)} - \cos(2\theta)\sin^2(2\theta)} - \underline{\frac{1+\cos(4\theta)}{2}} - \cancel{\cos(2\theta)} + 1\right)d\theta$$

$$= -\frac{3a^2}{8}\left[-\frac{1}{6}\sin^3(2\theta) - \frac{1}{2}\theta - \frac{1}{8}\sin(4\theta) + \theta\right]_{\frac{\pi}{2}}^{0}$$

$$= -\frac{3a^2}{8}\left(0 - \left(-\frac{\pi}{4} + \frac{\pi}{2}\right)\right) = \frac{3a^2\pi}{32}$$

$$\text{Area} = 4 \times \int_0^a y\,dx = \frac{3a^2\pi}{8}$$

10.2 Exercises #44

Find the exact length of the curve.

$x = e^t + e^{-t}$, $y = 5 - 2t$, $0 \le t \le 1$

$$L = \int_0^1 \sqrt{(e^t - e^{-t})^2 + (-2)^2}\,dt = \int_0^1 \sqrt{e^{2t} + e^{-2t} - 2 + 4}\,dt$$

$$= \int_0^1 (e^t + e^{-t})\,dt = \left[e^t - e^{-t}\right]_0^1 = e - \frac{1}{e} - \cancel{(1-1)} = e - \frac{1}{e}$$

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10.3 Exercises #18

Identify the curve by finding a Cartesian equation for the curve.

$r = tan\theta sec\theta$

$x = r\cos\theta = \tan\theta = \frac{y}{x} \quad (x \neq 0)$

$\therefore y = x^2$

10.3 Exercises #25

Find a polar equation for the curve represented by the given Cartesian equation.

$x^2 + y^2 = 2cx$

$(r\cos\theta)^2 + (r\sin\theta)^2 = 2c(r\cos\theta) \Rightarrow r = 2c\cos\theta$

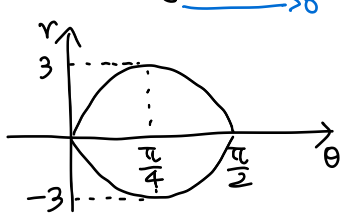
$\therefore (2c\cdot\cos\theta, \theta)$

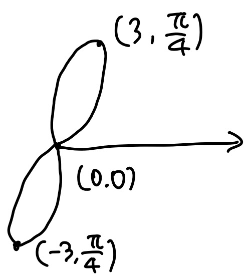
10.3 Exercises #41

Sketch the curve with the given polar equation by first sketching the graph of r as a function of in Cartesian coordinates.

$r^2 = 9\sin 2\theta$ $\quad r = \pm 3\sqrt{\sin 2\theta}$

$r = \pm 3\underbrace{\sqrt{\sin 2\theta}}_{>0} \Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$





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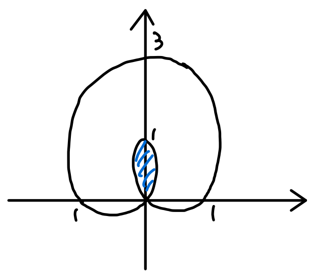
10.4 Exercises #21

Find the area of the region enclosed by one loop of the curve.

$r = 1 + 2\sin\theta$ (inner loop) $r<0$



$r = 1+2\sin\theta = 0 \Rightarrow \sin\theta = -\frac{1}{2} \Rightarrow \theta = \frac{7}{6}\pi$ or $\frac{11}{6}\pi$

$r<0$ for $\theta \in \left(\frac{7}{6}\pi, \frac{11}{6}\pi\right)$

$$\int_{\frac{7}{6}\pi}^{\frac{11}{6}\pi} \frac{1}{2}(1+2\sin\theta)^2 d\theta = \frac{1}{2}\int_{\frac{7}{6}\pi}^{\frac{11}{6}\pi} (4\sin^2\theta + 4\sin\theta + 1)d\theta$$

$$= \frac{1}{2}\int_{\frac{7}{6}\pi}^{\frac{11}{6}\pi} \left(2(1-\cos(2\theta)) + 4\sin\theta + 1\right)d\theta$$

$$= \frac{1}{2}\Big[2\theta - \sin(2\theta) - 4\cos\theta + \theta\Big]_{\frac{7}{6}\pi}^{\frac{11}{6}\pi}$$

$$= \frac{1}{2}\left(\frac{11}{3}\pi + \frac{\sqrt{3}}{2} - 4\frac{\sqrt{3}}{2} + \frac{11}{6}\pi - \left(\frac{7}{3}\pi - \frac{\sqrt{3}}{2} + 4\frac{\sqrt{3}}{2} + \frac{7}{6}\pi\right)\right)$$

$$= \frac{1}{2}\cdot(-3\sqrt{3} + 2\pi) = \frac{-3\sqrt{3}}{2} + \pi$$

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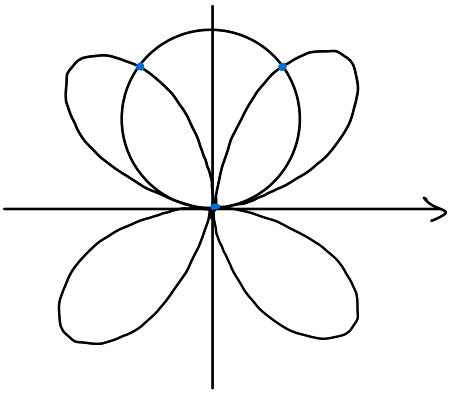
10.4 Exercises #41

Find all points of intersection of the given curves.

$r = \sin\theta, \quad r = \sin 2\theta$



i) $\sin\theta = \sin 2\theta = 2\sin\theta\cos\theta$

$\Rightarrow \theta = 0$ or $1 = 2\cos\theta$

$\Rightarrow \theta = 0, \frac{\pi}{3}, \frac{5}{3}\pi$

ii) $\sin(\pi+\theta) = -\sin 2\theta$

$\Rightarrow +\sin\theta = +2\sin\theta\cos\theta$

(same as i))

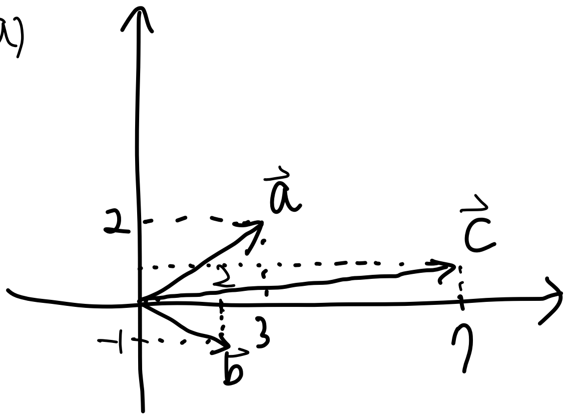
iii) we already have $(0,0)$

$\therefore (0,0), \left(\frac{\sqrt{3}}{2}, \frac{\pi}{3}\right), \left(-\frac{\sqrt{3}}{2}, \frac{5}{3}\pi\right)$

12.2 Exercises #45

(a) Draw the vectors a = <3, 2>, b = <2, -1>, and c = <7, 1>.
(b) Show, by means of a sketch, that there are scalars s and t such that c = sa + tb.
(c) Use the sketch to estimate the values of s and t.
(d) Find the exact values of s and t.

(a)



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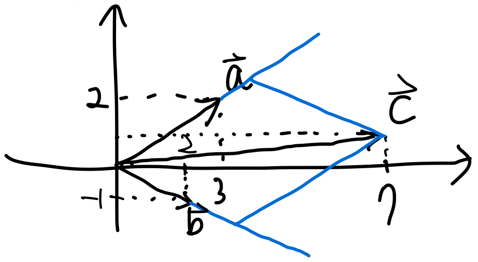
(b) 아래와 같이 $\vec{a}, \vec{b}$를 변의 일부로 가지고 $\vec{c}$를 꼭짓점으로 가지는 평행사변형을 그릴 수 있다.



(c) 약 $1 < s, t < 2$

(d) $s\langle 3,2\rangle + t\langle 2,-1\rangle = \langle 7,1\rangle$

$$\begin{cases} 3s+2t=7 \\ 2s-t=1 \end{cases} \quad t=2s-1, \quad 3s+4s-2=7 \Rightarrow t=\frac{11}{7},\ s=\frac{9}{7}$$

12.3 Exercises #19

Find the angle between the vectors. (First find an exact expression and then approximate to the nearest degree.)

$\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} - \mathbf{k}$

$\vec{a}\cdot\vec{b} = 8-1 = 7$

$\|\vec{a}\| = \sqrt{26},\ \|\vec{b}\| = \sqrt{5}$

$$\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|}\right) = \cos^{-1}\left(\frac{7}{\sqrt{130}}\right) \approx 0.91 \text{ rad} \approx 52^\circ$$

12.3 Exercises #47

If $\mathbf{a} = \langle 3, 0, -1\rangle$, find a vector $\mathbf{b}$ such that $\operatorname{comp}_{\mathbf{a}} \mathbf{b} = 2$.

Let $\vec{b} = \frac{2}{\|\vec{a}\|}\vec{a}$. Then $\operatorname{comp}_a b = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|} = \frac{2\|\vec{a}\|}{\|\vec{a}\|} = 2$

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12.4 Exercises #19

Find two unit vectors orthogonal to both <3, 2, 1> and <-1, 1, 0>.

$$\langle 3,2,1\rangle \times \langle -1,1,0\rangle = \begin{vmatrix} e_1 & e_2 & e_3 \\ 3 & 2 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

$$= ((0-1), -(0+1), 3+2)$$

$$= (-1,-1,5)$$

$$\text{Unit vectors} = \pm \frac{1}{\sqrt{27}}(-1,-1,5)$$

12.4 Exercises #53

Suppose that $\mathbf{a} \neq \mathbf{0}$.

(a) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

(b) If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

(c) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

(a) No. 반례 : $\vec{a} = e_1, \vec{b} = e_2, \vec{c} = e_3$

$\vec{a}\cdot\vec{b} = 0,\ \vec{a}\cdot\vec{c} = 0.$ but $\vec{b} \neq \vec{c}$

(b) No. 반례 : $\vec{a} = \langle 1,0,0\rangle,\ \vec{b} = \langle 1,0,0\rangle,\ \vec{c} = \langle -1,0,0\rangle$

$\vec{a}\times\vec{b} = (0,0,0)$, $\vec{a}\times\vec{c} = (0,0,0)$ but $\vec{b} \neq \vec{c}$

(c) Yes. pf) $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} \Rightarrow \vec{a}\cdot(\vec{b}-\vec{c}) = 0 \Leftrightarrow \vec{a},\ \vec{b}-\vec{c}$: orthogonal

$\vec{a}\times\vec{b} = \vec{a}\times\vec{c} \Rightarrow \vec{a}\times(\vec{b}-\vec{c}) = 0 \Rightarrow \underset{\neq 0}{\|\vec{a}\|}\,\|\vec{b}-\vec{c}\|\,\underset{=1}{\sin\theta} = 0$

$\therefore \|\vec{b}-\vec{c}\| = 0$ 즉, $\vec{b} = \vec{c}$