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## 5.2. Exercises #23

Use the form of the definition of the integral given in Theorem 4 to evaluate the integral.

$$\int_{-2}^{0} (x^2 + x)\, dx$$

Let $f(x) = x^2+x$.

Note that $f(x)$ is integrable on $[-2,0]$ since $f(x)$ is continuous on $[-2,0]$.

$$\int_{-2}^{0} f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = \frac{0-(-2)}{n}$ and $x_i = -2 + i\cdot\Delta x$ by Theorem 4

Therefore, $\int_{-2}^{0}(x^2+x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-2+i\cdot\frac{2}{n}\right)\cdot\frac{2}{n}$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2}{n}i-2\right)\left(\frac{2}{n}i-1\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty}\frac{4}{n}\underline{\sum_{i=1}^{n}\left(\frac{2}{n^2}\cdot i^2-\frac{3}{n}i+1\right)}$$

$$= \lim_{n\to\infty}\frac{4}{n}\left(\frac{2}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}-\frac{3}{n}\cdot\frac{n(n+1)}{2}+n\right)$$

$$= 4\left(\frac{4}{6}-\frac{3}{2}+1\right)$$

$$= \frac{2}{3}$$

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## 5.2. Exercises #57

Use the properties of integrals to verify the inequality without evaluating the integrals.

$$2 \leqslant \int_{-1}^{1} \sqrt{1 + x^2}\, dx \leqslant 2\sqrt{2}$$

Note that if $m \le f(x) \le M$ for $a \le x \le b$, then

$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$ ; Comparison property 8

Let $m=1$ and $M=\sqrt{2}$.

Since $0 \le x^2 \le 1$ for $-1 \le x \le 1$,

We obtain an inequality $1 \le f(x) \le \sqrt{2}$ for $-1 \le x \le 1$.

Therefore, $2 \le \int_{-1}^{1} \sqrt{1+x^2}\, dx \le 2\sqrt{2}$ by comparison property 8

## 5.3. Exercise #13

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$h(x) = \int_{1}^{e^x} \ln t \, dt$$

Let $f(x) = \int_1^x \ln t \, dt$, $g(x) = e^x$.

Then, we have $h(x) = f \circ g(x)$ and so

$h'(x) = g'(x) \cdot f'(g(x))$ by chain rule.

Note that $\ln t$ is continuous on $(0, \infty)$.

$f'(x) = \ln x$ by FTC part 1.

Therefore, $h'(x) = e^x \cdot \ln e^x = e^x \cdot x$

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## 5.3. Exercise #64

If $f(x) = \int_0^x (1 - t^2)e^{t^2}\,dt$, on what interval is $f$ increasing?

Note that $(1-t^2)e^{t^2}$ is continuous on $\mathbb{R}$.

Hence, $f'(x) = (1-x^2)e^{x^2}$ by FTC.

Since $1-x^2 > 0$ only for $-1<x<1$ and $e^{x^2} > 0$ $\forall x \in \mathbb{R}$,

we have $f'(x) > 0$ only for $-1<x<1$.

Therefore, $f$ is increasing on $(-1, 1)$

※ 강의자료 및 교재 정의에 따라 increasing ≡ strictly increasing로 간주

## 5.3. Exercise #83

Find a function f and a number a such that

$$6 + \int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x} \quad \text{for all } x > 0$$

continuous ?

For $x=a$, we have $6+0 = 2\sqrt{a}$ and so $a=9$.

Differentiating both sides w.r.t. $x$, we get

$$\frac{f(x)}{x^2} = \frac{1}{\sqrt{x}}$$ by FTC part1 and so $f(x) = x^{\frac{3}{2}}$

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# 5.5. Exercise #3

Evaluate the integral by making the given substitution.

$$\int x^2\sqrt{x^3+1}\,dx, \quad u = x^3+1 \geq 0$$

Differenciating both sides of $u = x^3+1$ w.r.t. $x$,
we have $\frac{du}{dx} = 3x^2$ and so $3x^2\,dx = du$.

Using Substitution Rule, we get

$$\int x^2\sqrt{x^3+1}\,dx = \int \frac{\sqrt{u}}{3}\,du = \frac{2}{9}u^{\frac{3}{2}} + C = \frac{2}{9}(x^3+1)^{\frac{3}{2}} + C$$

# 5.5. Exercise #25

Evaluate the indefinite integral.

$$\int e^x\sqrt{1+e^x}\,dx$$

Let $u = 1+e^x$.

Differentiating both sides w.r.t. $x$, we get
$du/dx = e^x$ and so $e^x\,dx = du$.

Using Substitution Rule, we have

$$\int e^x\sqrt{1+e^x}\,dx = \int \sqrt{u}\,du = \frac{3}{2}u^{\frac{3}{2}} + C = \frac{3}{2}(1+e^x)^{\frac{3}{2}} + C$$

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## 5.5. Exercise #59

Evaluate the definite integral.

$$\int_1^2 \frac{e^{1/x}}{x^2}\,dx \qquad x \neq 0$$

Let $u = \frac{1}{x}$.

Differenciating both sides w.r.t. $x$, we have

$du/dx = -x^{-2}$ and so $-x^{-2}dx = du$.

Using Substitution Rule, we get

$$\int_1^2 \frac{e^{1/x}}{x^2}dx = \int_1^{\frac{1}{2}}(-e^u)du = \int_{\frac{1}{2}}^1 e^u du = \left[e^u\right]_{\frac{1}{2}}^1 \quad (\because \text{FTC part2})$$

$$= e^1 - e^{\frac{1}{2}}$$

## Chapter 5. Review #70

Find $\displaystyle\lim_{h\to 0}\frac{1}{h}\int_2^{2+h}\sqrt{1+t^3}\,dt$

Let $f(h) = \int_2^h \sqrt{1+t^3}\,dt$ and $g(h) = h$. continuous

$$\lim_{h\to 0}\frac{1}{h}\int_2^{2+h}\sqrt{1+t^3}\,dt = \lim_{h\to 0}\frac{\overset{\to 0}{f(h+2)}}{\underset{\to 0}{g(h)}} = \lim_{h\to 0}\frac{(f(h+2))'}{g'(h)} \quad (\because \text{l'Hospital's Rule})$$

$$= \lim_{h\to 0}\frac{f'(h+2)}{g'(h)} \quad (\because \text{Chain Rule})$$

$$= \lim_{h\to 0}\frac{\sqrt{1+(h+2)^3}}{1} \quad (\because \text{FTC part1})$$

$$= 3$$

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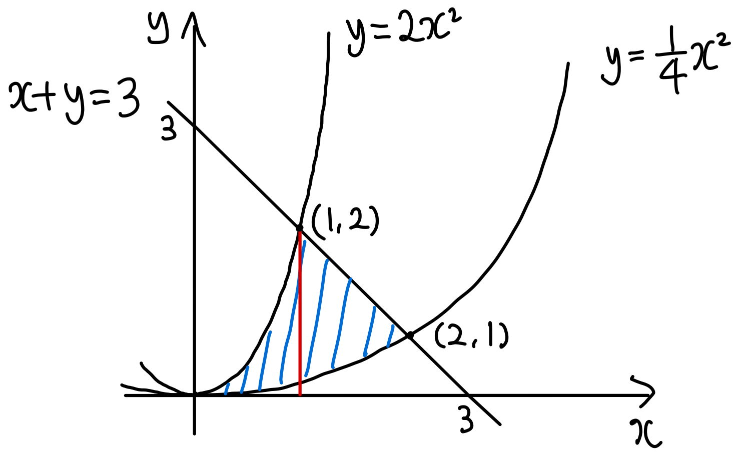
## 6.1. Exercises #28

Sketch the region enclosed by the given curves and find its area.

$y = \frac{1}{4}x^2, \quad y = 2x^2, \quad x + y = 3, \quad x \geqslant 0$

$y = 3 - x$




Note that the given functions are all continuous on $\mathbb{R}$.
Let A be the area we are looking for.

Then, $A = \int_0^1 (2x^2 - \frac{1}{4}x^2)\,dx + \int_1^2 (-x + 3 - \frac{1}{4}x^2)\,dx$

$$= \left[\frac{7}{4}\cdot\frac{1}{3}\cdot x^3\right]_0^1 + \left[-\frac{1}{4}\cdot\frac{1}{3}\cdot x^3 - \frac{1}{2}x^2 + 3x\right]_1^2$$

$$= \frac{7}{12} + \left(-\frac{2}{3} - 2 + 6\right) - \left(-\frac{1}{12} - \frac{1}{2} + 3\right)$$

$$= \frac{7}{12} + \frac{10}{3} - \frac{29}{12}$$

$$= \frac{3}{2}$$

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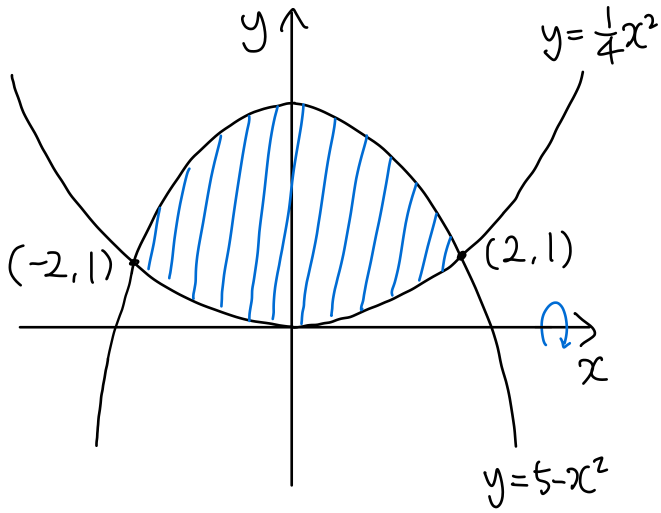
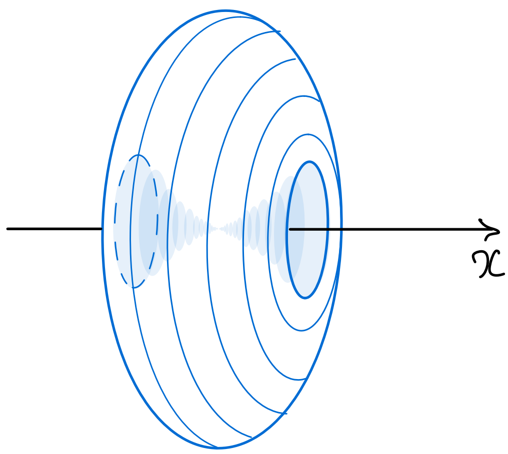
## 6.2. Exercises #9

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

$$y = \frac{1}{4}x^2, y = 5 - x^2 \quad ; \quad \text{about the x-axis}$$




Let $A(x)$ be the area of the solid at $x$ and $V$ be the volume of the solid.

Then, $A(x) = (5-x^2)^2\pi - (\frac{1}{4}x^2)^2\pi = (x^4 - \frac{1}{16}x^4 - 10x^2 + 25)\pi$

$= (\frac{15}{16}x^4 - 10x^2 + 25)\pi$

Since $A(x)$ is continuous function,

$V = \int_{-2}^{2} A(x)\,dx = \int_{-2}^{2} (\frac{15}{16}x^4 - 10x^2 + 25)\pi\,dx$

$= \pi \cdot \left[\frac{\cancel{15}^{3}}{16}\cdot\frac{1}{\cancel{5}}x^5 - 10\cdot\frac{1}{3}x^3 + 25x\right]_{-2}^{2}$

$= \pi(6 - \frac{80}{3} + 50)\cdot 2 = \frac{176}{3}\pi$

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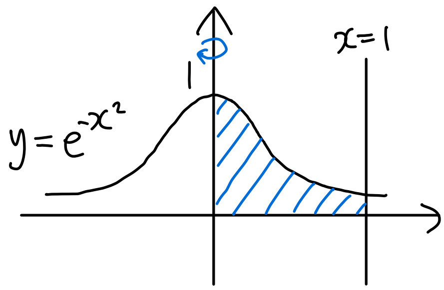
## 6.3. Exercises #5

Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis.

$y = e^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 1$




Note that $y=e^{-x^2}$ is continuous nonnegative function on $\mathbb{R}$.

Let $V$ be the volume of the solid.

Using the method of cylindrical shells, we get

$$V = \int_0^1 2\pi x e^{-x^2}\,dx.$$

Substitute $u=x^2$ with $du=2x\,dx$. Then,

$$\int_0^1 2\pi x e^{-x^2}\,dx = \int_0^1 \pi e^{-u}\,du = \pi\left[-e^{-u}\right]_0^1 = \pi(-e^{-1}+1)$$

## 6.5. Exercises #13

If $f$ is continuous and $\int_1^3 f(x)\,dx = 8$, show that $f$ takes on the value 4 at least once on the interval [1, 3].

Note that $f$ is continuous on $[1,3]\subset\mathbb{R}$

By MVT for Integral,

$$\exists c\in[1,3] \quad \text{s.t. } f(c) = \frac{1}{3-1}\int_1^3 f(x)\,dx = 4$$

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# 8.1. Exercises #17

Find the exact length of the curve.

$$y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \quad 1 \leqslant x \leqslant 2$$

Note that $y$ is diff$\underline{\text{ble}}$ on $[1,2]$ and

$y' = \frac{1}{2}x - \frac{1}{2}x^{-1}$ is continuous on $[1,2]$.

Let $L$ be the length of the curve.

Then, $L = \int_1^2 \sqrt{1 + \left(\frac{1}{2}x - \frac{1}{2}x^{-1}\right)^2}\, dx$

$$= \int_1^2 \sqrt{\frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4}x^{-2} + 1}\, dx$$

$$= \int_1^2 \frac{1}{2}\left|x + x^{-1}\right| dx$$

$$= \int_1^2 \frac{1}{2}(x + x^{-1})\, dx$$

$$= \frac{1}{2}\left[\frac{1}{2}x^2 + \ln x\right]_1^2$$

$$= \frac{1}{2}\left(2 + \ln 2 - \frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{\ln 2}{2}$$