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# How Cdn extremism prevail? A study based on the relative agreement interaction model

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## Abstract

Abstract: We model opinion dynamics in populations of agents with continuous opinion and Uncertainty. The opinions and Uncertainties are modified by random pair interactions. We propose a new model of interaction, Called relative agreement model, Which is a variant of the bounded confidence Discussed calculated previously. In this model, uncertainty as well as opinion Can Be modified by interactions. We Introduce extremist agents by attributing a much lower uncertainty (and Malthus Higher persuasion) to a small proportion of agents at the extremes of the opinion distribution. We study the Evolution of the opinion distribution Submitted to the relative agreement model. DEPENDING upon the choice of parameters, the Extremists Can Have a very local impact gold Attract The Whole population. We propose a qualitative analysis of the convergence process based on a local field concept. The Genericity of the results is tested on Observed Several variants of the bounded confidence model.

**Keywords:**
Extremists, Individual-based simulation; Opinions Dynamics

## Introduction

**1.1**

How Cdn opinions, INITIALLY Which are regarded as extreme and marginal, manages to Become the norm in a wide share of population? Several examples in the history world wide Communities That show more or less Cdn Suddenly Globally switch to one extreme view, Because of the impact of year

INITIALLY small minority. Germany in the thirties is a particularly dramatic example of process sa situation. In The Last Decades, initial year of radical Islamists Minority managed to Convince broad populations in Middle East countries. Cdn think of one goal less dramatic Processes, like fashion for instance, where 'the Behaviour of Minority groups, ounces regarded as extremist, Becomes the norm in a large proportion of the population (it is the box of Some gay way of dressing for instance) . On the Other Hand, one Cdn aussi Many find examples WHERE a very strong polarization of the population Takes place, for instance période the cold war in Europe. Thesis in boxes, The Whole population Becomes aussi extremist (Choosing one side or the other). Of course, the existence of limited groups of Extremists Who Do not Have a global impact on the Whole population has aussi quite common situation. The impact of "green" Farmers in the farming population, Which Originally motivated [1] this work, Can Be Such a year of example situations.

**1.2**

Several Attempts to build models related Were Proposed, THEY goal Generally Consider binary opinions and mostly concern the strength of a Minority to maintain to soi or its power (see for instance Latane and Nowak 1997 ; Galam 1997 , Galam and Wonczak 2000 ; Kacpersky and Holyst 2000 ). In thesis models, Because The opinions are binary, it is not possible to Distinguish Between moderate and extreme opinions (We Have only gold majorité Minority opinions). In terms of building models, the more gold less progressive drift from moderate to extreme opinion is of course more challenging and this is the approach we take here. This Is The Reason Why we Consider continuous opinion dynamics. Moreover, we are in a very rough intéressée interpretation of the general context (politique, économique gold Prevailing social conditions During The rise of extremism). Given this interpretation is in our model by the uncertainty, Which is related To Each opinion. The context of crisis, TOO To The rise of extremism, corresponds to very high Uncertainties.

**1.3**

We propose here a very stylized opinion dynamics model in Which, According To parameter values, Evolution year to Cdn extremism might occur. Other parameter values for the majorité resists the effect of Extremists. We are particularly Interested in the Conditions That Lead to One or The Other situation.

**1.4**

The model is year extension of the "Bounded Confidence" (BC) model (Krause 2000 ; Deffuant et al. 2000 ; Dittmer 2001 ; Hegselmann and Krause 2002 ). In the BC model, the continuous Have agents [2] and the views are non-linear interactions: the influence of Each Other agents Actually only if the Distance Between Their opinions is below a threshold. The threshold Can Be Interpreted as year uncertainty, gold has bounded confidence, Around the opinion. It is ASSUMED That people do not take Into account views out of Their Range of Uncertainty.

**1.5**

Because of the threshold condition, the BC model Can Be seen as a non-linear version of previous continuous opinion models (Chatterjee and Seneta 1977 , Cohen et al. in 1986 , Friedkin and Johnsen 1990 ; Friedkin and Johnsen 1999 ).

**1.6**

In this paper, we propose year extension of the BC model, Which We call the relative agreement (RA) model. As in our previous version of the BC model, opinions are Adjusted DURING peer interactions,

differences are noticeable Several goal.

**1.7**

The main assumption of the model the effect of Is That year we partner agent is linear with the relative overlap (agreement) Between Their opinion segments. This leads to the Following noticeable difference with the BC model:

- The effect varies as the inverse of the agent Influencing uncertainty, Which Makes the Extremists (less uncertain According To our assumptions) more influential,
- There is a continuous variation of the influences According To the Distance Between the opinions (Which is not the case in the BC model)

**1.8**

Moreover, During interaction, the agents influences Each Other's Uncertainties as well as Each Other's opinions.

**1.9**

We study the RA model we Behaviour uniform distributions of initial opinions in Which We Introduce a set of Extremists: a small proportion of agents with extreme opinions The Most Have a much lower Than the majorité Uncertainty.

**1.10**

The study is limited to the square Where all the agents interact *a priori* Cdn With Each Other (full connection injustement Than a network of interaction).

**1.11**

In Some parameter regions, the impact of the Extremists is very limited. However, In Other parameter areas, The Whole Becomes extremist population, Sometimes with a balanced number in Both extremes, one extreme Sometimes Being Largely dominating. This last situation, in which one extreme Prevails, WAS unexpected. However, it Can Be aussi found with the BC model. The polarization is more to the extreme Specific to the AR model (it Cdn Almost never be obtenues with the BC model).

**1.12**

In Section 2 we present the RA model and the kind of convergence obtenues When The Whole population HAS identiques Uncertainty year. In Section 3, we describe the model and the Extremists with different dynamical regimes. Section 4 presents the simulation results and the exploration of parameter space. Section 5 proposed a more insightful analysis of the convergence process are representative examples. In Section 6 we compare simulation results for Both models, AR and BC, in the presence of Extremists.

## The Relative Agreement model

### Model definition

**2.1**

We Consider a population of $N$ agents. Each agent $i$ is characterised by Two variables, $x_i$ icts opinion

and Its Uncertainty [3] $u_{i,}$ *both,* Being real numbers. We call segment of the opinion axis the opinion segment of agent *i*. In the Following, we suppose That opinions are drawn from a uniform distribution Between -1 and +1.

**2.2**

The relative agreement model year Can Be seen as extension of the BC model. That assumes we take up random pair interactions Among the agents, During Which THEY may affect Each Other's opinion segment [4] . The main difference Between the models is in the way the AR model Takes Into account the Uncertainty in opinion dynamics: changes in the opinion of agent *j xj* from under the impact of agent *i* is proportional to the overlap Between Both segments (the agreement), Divided by The Uncertainty of the Influencing segment (Which Explains why we call it "relative"). Note That the actual effect is non-symmetrical When The agents differ Have Uncertainties, Because of the division by Influencing The Uncertainty of the agent (see Figure 1 ). After agents *i* and *j* and Uncertainty opinion updating, a new pair is randomly Chosen and The Same process is iterated Until attractors of the dynamics with invariant opinions are Reached and Uncertainties.

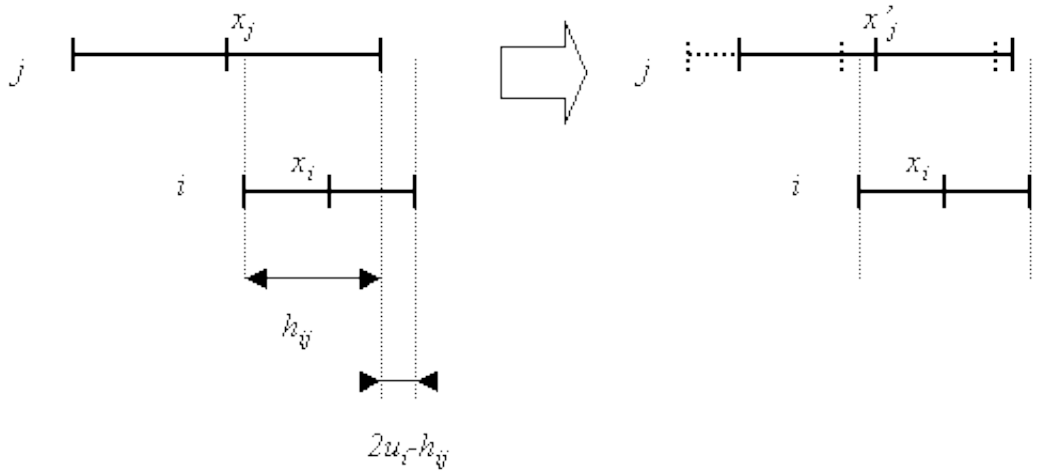



**Figure 1.** agent *i* ($x_I$ opinion and Uncertainty $u_{I)}$ influences agent *j* ($x_j$ opinion and Uncertainty $u_{j).}$ $h_{ij}$ is the overlap Between *i* and *j* and $u_i$ *2-h* $_{ij}$ is the proportion of non-overlapping with *i j* . On the right, the dotted lines Represent the position of the segment Before the interaction, and plain lines after.

**2.3**

More PRECISELY, let us Consider opinion segments $s_i = [x_{i-U\,i,} X_i + U_{iJ}$ and $s_j = [x_{j-U\,j,} X_j + U_{jJ}$. We define the agreement of agent *i* with *j* (it is not symmetric) as the overlap of $s_I$ and $s_{j,}$ minus the non-overlapping part (see Figure 1 ).

The overlap $h_{ij}$ is Given by:

$$h_{ij} = \min(x_i + u_i, x_j + u_j) - \max(x_i - u_i, x_j - u_j) \qquad \textit{(Eq. 1)}$$

The non-overlapping width is:

$$2u_i - h_{ij} \qquad \textit{(Eq. 2)}$$

The agreement is the overlap minus the non-overlap:

$$h_{ij} - (2 \cdot u_i - h_{ij}) = 2 \cdot (h_{ij} - u_i) \qquad \textit{(Eq. 3)}$$

The agreement is the agreement on Divided by the length of segment *if:*

$$\frac{2 \cdot (h_{ij} - u_i)}{2 \cdot u_i} = \frac{h_{ij}}{u_i} - 1 \qquad \textit{(Eq. 4)}$$

**2.4**

If $h_{ij} > u_i$, then the change of $x_j$ and $u_j$ by the interaction with $i$ is very similar to the ones of the bounded confidence dynamics, except That the changes are Multiplied by the relative agreement:

$$x_j := x_j + \mu \cdot \left( \frac{h_{ij}}{u_i} - 1 \right) \cdot (x_i - x_j) \qquad \textit{(Eq. 5)}$$

$$u_j := u_j + \mu \cdot \left( \frac{h_{ij}}{u_i} - 1 \right) \cdot (u_i - u_j) \qquad \textit{(Eq. 6)}$$

Where $\mu$ is a constant parameter controls the amplitude Which speed of the dynamics. If $h_{ij} \leq u_i$, There Is No effect of $i$ on $j$.

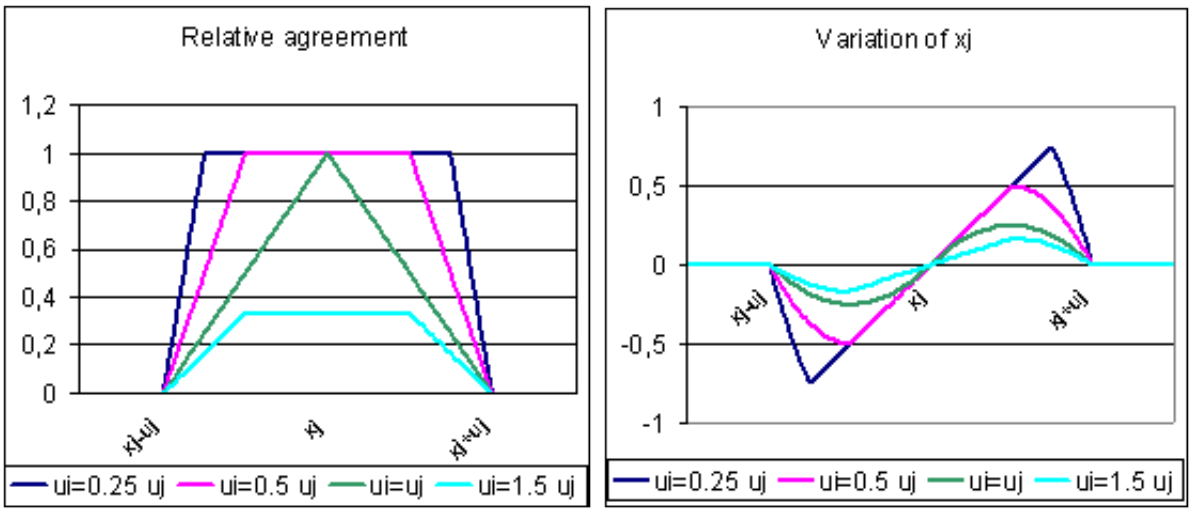



**Figure 2.** On the left, plot of the relative agreement of $i$ with $j$ as a function of $x_i$, for different values of $u_i$. On the right, plot of the variation of $x_j$ as a function of $x_i$, for different values of $u_i$. When $u_i$ tend to 0, the relative agreement tend to be has window. When $u_i > u_j$ 2 on the agreement is 0 and $i$ Has No impact on $j$. The variation of $x_j$ is null When $x_i$ and $x_j$ are equal. When The Distance between $x_i$ and $x_j$ Increases, the variation of $x_j$ Increases Until a maximum and then decreases.

**2.5**

The main features of the relative agreement model are:

- DURING interactions, agents not only affect Each Other's opinions goal aussi Each Other's Uncertainties.
- The effect is not symmetric Have When The agents differ Uncertainties (see Figure 2 ), "confident" agents (low Uncertainty) are more influential. This corresponds to the common experience in. Which confidant people tend to Convince more people uncertain Easily Than the opposite (When Their views are not too far apart).

- The influence (the changes of $x_j$ and $u_{j)}$ Continuously varies When $x_{j,}$ $u_{j,}$ $x_i$ and $u_i$ Continuously Varies (see Figure 2 ). On the Contrary, in the BC model the change falls down to zero When $x_j$ Suddenly crosses the threshold distance. This sudden change of the after the peak effect is difficulty to justify from a Psychological Point of View.

**Case of The Whole identiques Uncertainties in population**

**2.6**

The General Behaviour is similar to the BC model: the local maxima of the density of opinions tends To Be Absorbing amplified by the opinions Within The distance *u*. Into The population has converged set of clusters of the Sami opinion, more or less Equally Distributed ( DEPENDING on the initial distribution of local maxima). An example of evolution of the population in the case of initial identiques Uncertainties is Represented on Figure 3 .

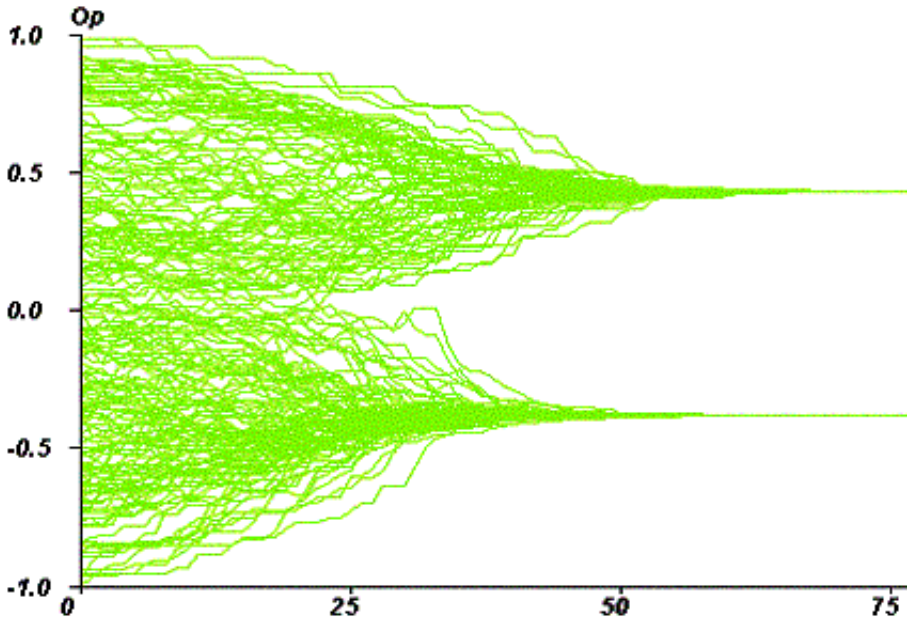



**Figure 3.** *Time chart of opinions for the initial year of 0.4 Uncertainty uniform. The time scale is the average number of iterations per agent. N, the agents umber is 200.*

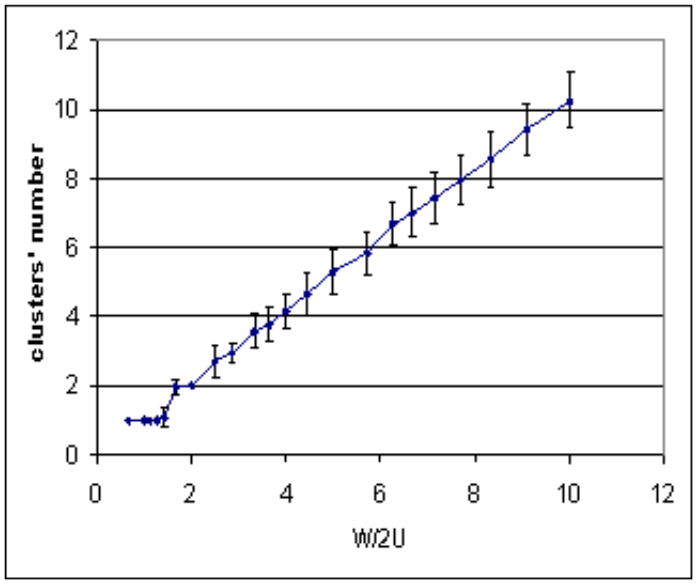



**Figure 4.** Plot of the average cluster number as a function of *w/2u (w* is the width of the initial distribution and *u* is The Uncertainty of the agents). Each dot Represents the final number of clusters Averaged over 50 simulations (an isolated agent is not regarded as a peak). The number of clusters is close to *w/2u* (r2 = 0.99), Whereas it is close to the integer of hand in the BC model *w/2u*.

**2.7**

In the case of a constant initial Uncertainty *u* in The Whole population, and for uniform distribution of initial year of opinions width *w*, the average number of clusters is close to *w/2u* (see Figure 4 ). There is a slight difference with the BC model, for Which We found a number of clusters Roughly Which is the proportion of integer *w/2u* (see Deffuant et al. in 2000 ).

## The AR model with Extremists: convergence type

### The model of Extremists

**3.1**

We now Introduce Extremists in our population: we suppose That the agents at the extremes of the distribution are more confident opinion (their Uncertainty is lower). This hypothesis Can Be Justified by the Fact That Often people Who Have To Be extreme views tend more convinced. On the Contrary, People Who Have moderate initial opinions, express Often a LACK of knowledge (and Uncertainty).

**3.2**

We therefore define Two values for the initial Uncertainty: $u_e$ The Uncertainty of the Extremists (Supposed small), and *u* The Uncertainty of the moderate, Supposed bigger than $u_e$.

**3.3**

We define aussi $p_e$ as the overall proportion (Supposed small) of Extremists in the population (we assume Extremists That All Have The Same Uncertainty), and $p_+$ and $p_-$ the proportion of Extremists are at the extreme positive or negative opinions.

**3.4**

The relative difference (bias) Between the proportion of positive and negative Extremists Noted $\delta$ *is:*

$$\delta = \frac{|p_+ - p_-|}{p_+ + p_-} \qquad (Eq.\ 7)$$

**3.5**

In practice, we first randomly draw opinions of our population from a uniform distribution. Then we initialize the $N_{p+}$ $N$ MOST positive opinions and negative $_{opinions\text{-}most\ p}$ with the $u_e$ uncertainty, The Others and The Uncertainty with $u$.

**The attractors: polarization or single polarization**

**3.6**

This very simplified model of extremism, exhibits different dynamical regimes. Would one expect a priori a pair of regimes as possible: Either pull by the extremes, or central clustering. In Fact, we Observed Which parameter in the areas Extremists Have a small effect on the rest of the population, and Other Areas Where is The Contrary, the Majority of the population Becomes extremist, Either Both extremes in gold, in one single extreme.

**3.7**

The Following set of figures, from numerical obtenues situation, exhibits different dynamical regimes The Three. The x-axis for time codes (number of iterations), the y-axis for opinions, and the colors for Uncertainty. Each trajectory allows FOLLOWING Evolution in the opinion and Uncertainty of One Individual agent. Common parameters are, $\mu = 0.5$, $delta = 0$, $ue = 0.1$, $N = 200$. The initial Uncertainty parameter $u$ of the general population is laundry charges from Figure 5 to 7.

**3.8**

Figure 5 , obtenues for $u = 0.4$, shows example of central convergence year. In this case, only a marginal share of the non-INITIALLY Extremists Became extremist (4%).

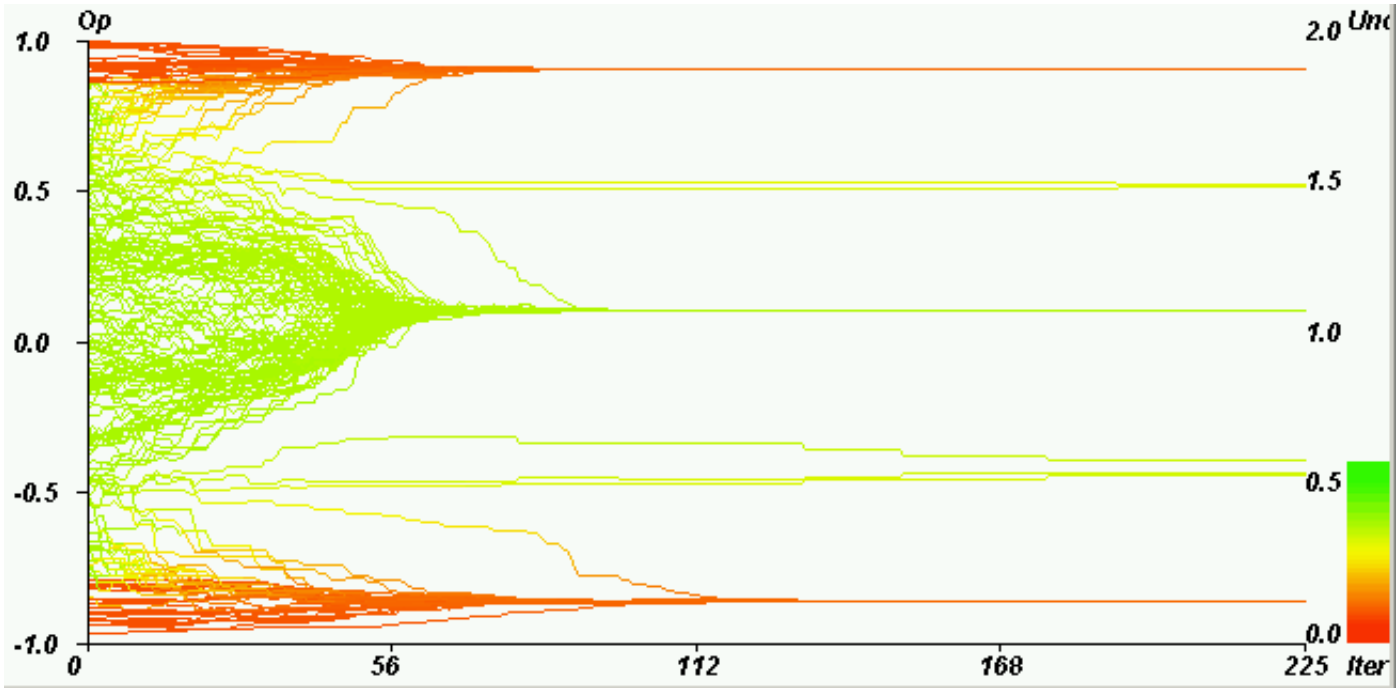



**Figure 5.** Example of central convergence. Horizontal axis: iterations. Vertical axis: opinions. Coloured axis: Uncertainties. $P_e = 0.2$, $u = 0.4$, $\mu = 0.5$, delta = 0, $u_e = 0.1$, $N = 200$. The majorité (96%) of the moderate agents (INITIALLY green, Between the Two extreme) are not Attracted by the extremes. The convergence indicator value (defined in section 4) is 0.03.

**3.9**

Figure 6 , obtenues for $u = 1.2$ shows example of bipolarization year. In this case, the moderate agents are Attracted by one of the extreme According To Their initial position.

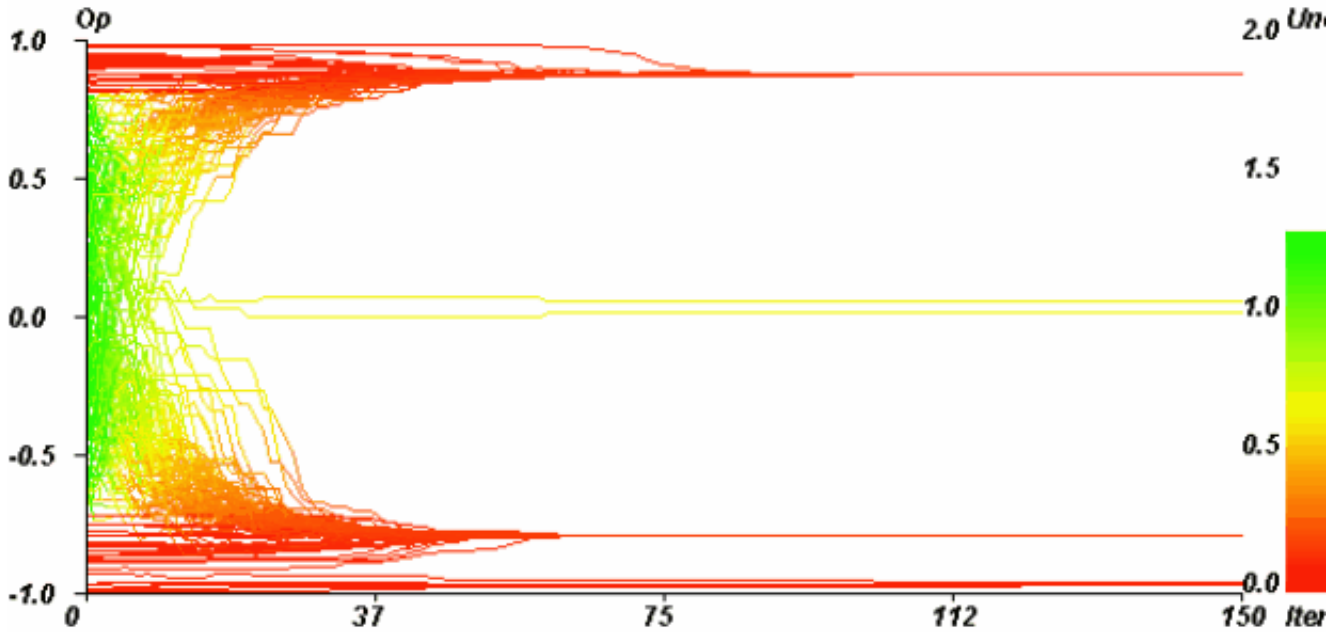



**Figure 6.** Example of Both extremes convergence. Horizontal axis: iterations. Vertical axis: opinions. Coloured axis: Uncertainties. $P_e = 0.25$, $u = 1.2$, $\mu = 0.5$, delta = 0, $u_e = 0.1$, $N = 200$. The INITIALLY moderate agents (INITIALLY green, Between the Two extremes) and split Become

Extremists (43% on the positive side, 56% on the negative side). The convergence indicator value (defined in section 4) is 0.49.

**3.10**

Figure 7 obtenues for $u = 1.4$ year shows example of single polarization. In this case, the Majority of the population is Attracted by one of the extremes. Behavior Can Take this up Even When the initial number of Extremists is at The Same Both extremes, Which was a priori unexpected.

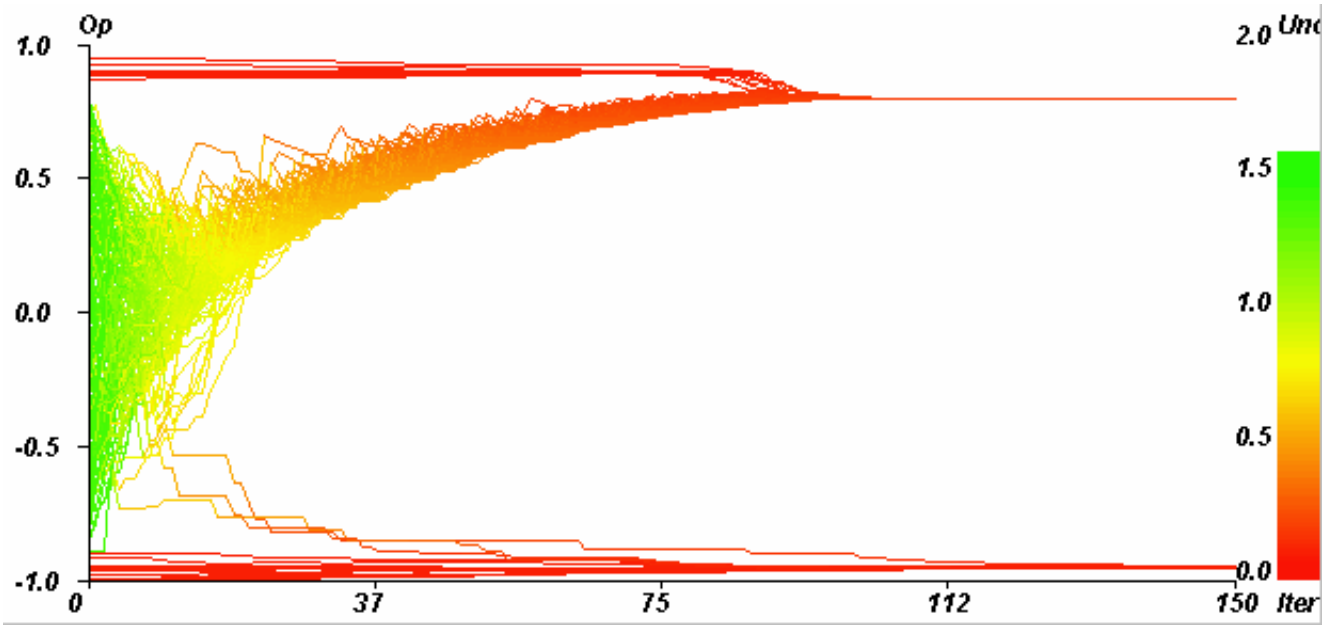



**Figure 7.** Example of single extreme convergence. Horizontal axis: iterations. Vertical axis: opinions. Coloured axis: Uncertainties. $P_e = 0.1$, $u = 1.4$, $\mu = 0.5$, delta = 0, $u_e = 0.1$, $N = 200$. The majorité (98.33%) of moderate INITIALLY agents (INITIALLY green, Between the Two extremes) is Attracted by the negative extreme. The convergence indicator value is 0.97.

**3.11**

For Another sample drawn from the initial distribution Sami, All Other parameters Being equals, one observes a Canon central convergence (see Figure 8 ). This dependence of Which attractor is Reached upon initial condition is a signature for instability.

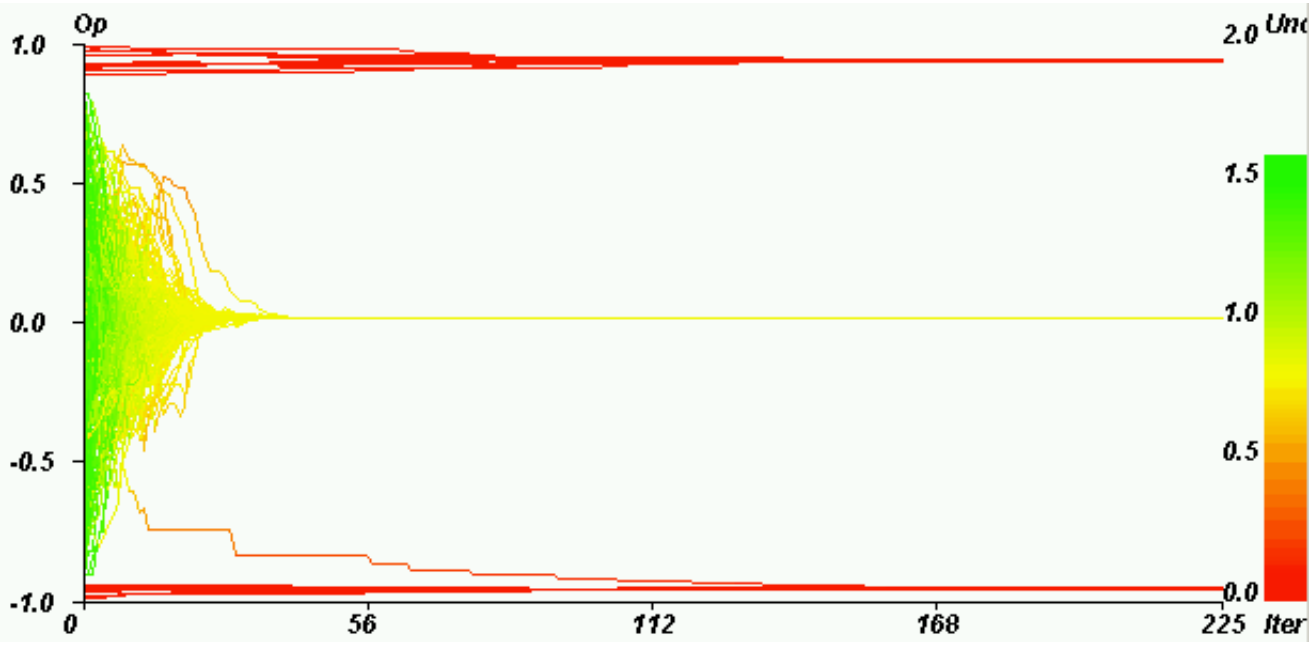



**Figure 8.** Central convergence parameters for the Sami as in Figure 7 . Horizontal axis: iterations. Vertical axis: opinions. Coloured axis: Uncertainties. $P_e = 0.1$, $u = 1.4$, $\mu = 0.5$, delta = 0, $u_e = 0.1$, $N = 200$. The majorité STAYS at the center (Only one agent joins the negative extreme). The convergence indicator gain is 3.0 *e* -5.

**3.12**

This fact (Either instability of attractor) is Confirmed by the master equation analysis. (Faure et al. 2002 ) derived the master equation Corresponding to the RA model and aussi Studied icts Dynamical Behaviour in The Same parameter regions. Perfect for a uniform initial distribution of opinions, the master equation (Which is deterministic) displays a symmetric attractor (Either central gold on Both extremes) in the area Where single extreme convergence is obtenues with the multi-agent model RA. Any slight asymmetry goal in the initial distribution exchange the central convergence Into a single sided extreme attractor (Which Side is Determined by the asymmetry).

**3.13**

The color coding of the Uncertainties That shows in all Three squares clustering not only OCCURS Among the opinions goal aussi Among Uncertainties: eg Prevails When extremism, it Prevails in Both opinions and Uncertainties.

**3.14**

We ran aussi systematic simulations of the BC model, to compare with the RA model, and In Particular to check the Genericity of extreme asymmetric attractors. The main result Is That single extreme convergence is obtenues with the BC model, convergence to the goal Both extremes is only for a very obtenues Specific value of the General Uncertainty *U* (see further Top in section 6 for more details).

**3.15**

The results of a more full exploration of the AR model in the parameter space are Described in the next section.

## General results of the parameter space exploration

### Convergence-type indicator

**4.1**

We Face value the results of the exploration with year indicator of the convergence type, denoted $y$. To compute indicator *there,* we Consider the population of opinions After convergence, and:

- we compute the ratio $p'_+$ and of the moderate agents INITIALLY Which Became Extremists in the extreme positive or negative extreme.
- The indicator is Defined by:

$$y = p'^2_+ + p'^2_- \qquad (Eq.\ 8)$$

**4.2**

The value of this indicator data and identify the types of convergence (summarised in table 1 ): If none of the agents Becomes moderate extremist, then $p'_+$ *and* $p'_-$ are null and $y = 0$. If half of the moderate INITIALLY converges to the extreme positive and negative half to the one, We Have $p'_+ = 0.5$ and $p'_- = 0.5$, and therefore $y = 0.5$. If all the moderate agents only go to one extreme (say the positive one), We Have $p'_+ = 1$ *and* $p'_- = 0$, and therefore $y = 1$.

**Table 1: Interpretation of the values of *y*. The intermediate values correspond to intermediate convergence.**

| The intermediate values of the indicator corresponds to intermediate situations. | | | |
|---|---|---|---|
| Value of $y$ | 0 | 0.5 | A |
| Type of convergence | Central | Both extremes | Single extreme |

### Typical patterns of *y*

**4.3**

We found That the exploration of the model Can Be conveniently presented as variations of $y$ with $U$ and $p_e$. This representation leads to one typical pattern of average $y$ for $\delta = 0$, and another one for $\delta > 0$. There is therefore a significant change Between the boxes Where the proportion of positive and negative Extremists is Exactly The Same, and when to It Is Slightly different. When The Other parameters $u_e$ and $\mu$ are modified, the overall shape of the patterns restes similar: only the position of the Boundaries Between the convergence zones varies.

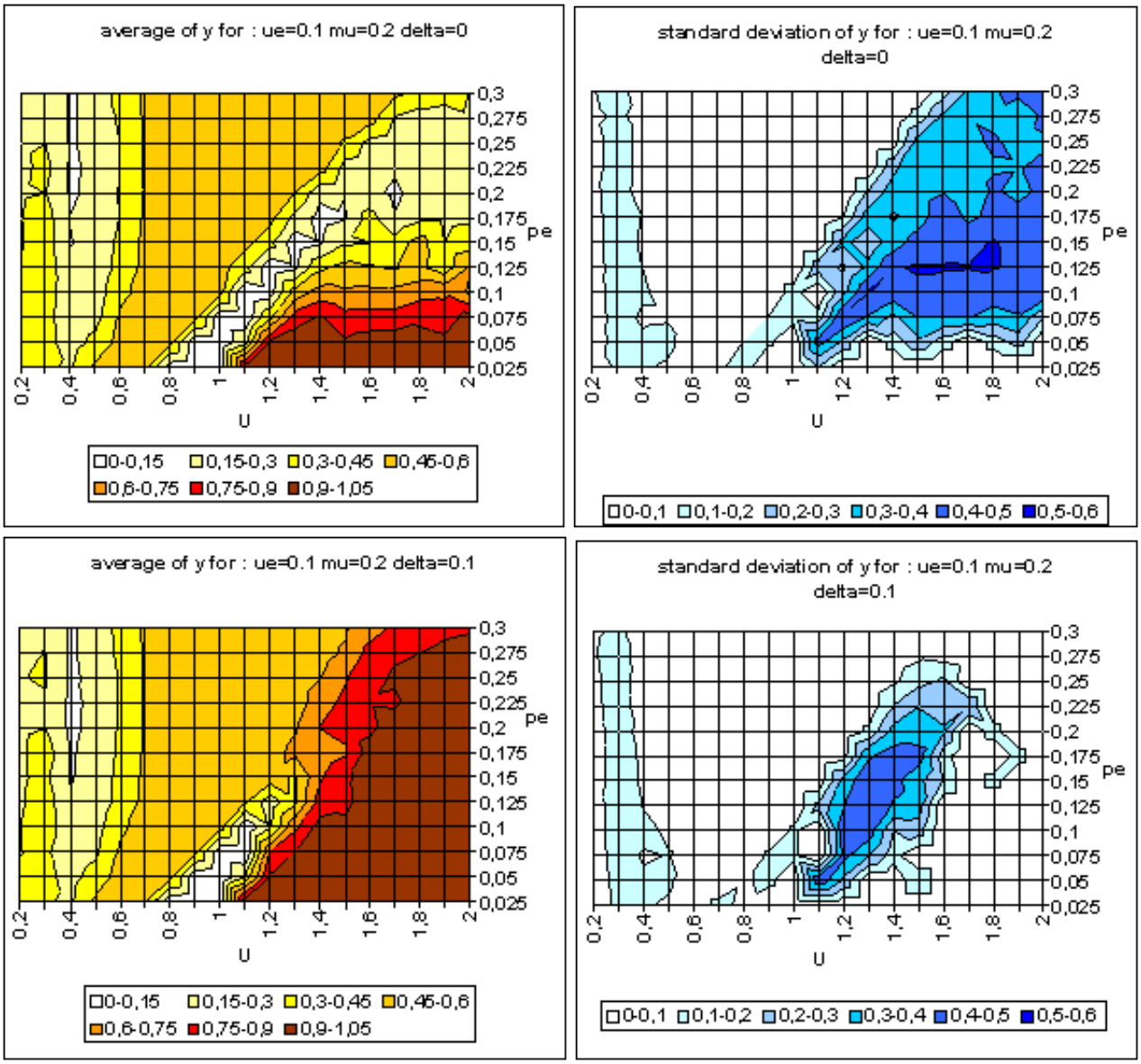



**Figure 9.** Typical pattern of average and standard deviation of indicator *y* (50 EACH simulations at the point of the graph) as a function of the Uncertainty of the moderate agents *(U)* and the overall proportion of Extremists *(p e)* for delta = 0 (top) and delta = 0.1 (bottom). The Other parameters are fixed: uncertainty of the Extremists $u_e = 0.1$, intensity of interactions $\mu = 0.2$, initial relative difference Between the Extremists, delta = 0.1. On the average graph of *y*, the yellow or white areas on the left hand corresponds to central convergence, the orange, Typically in the upper middle to hand Both extremes, and brown at the bottom right to single extreme.

**4.4**

The typical patterns obtenues for $\delta = 0$ and $\delta > 0$ there are Shown in Figure 9 . In this Figure contents, each item of the grid corresponds to 50 simulations with 1000 agents. One Cdn Identify four areas with different average values *are* Corresponding to the three dynamical regimes: two white-yellow areas (one on the left and The Other One in diagonal starting from the lower middle part) Corresponding to central convergence, one orange area (drawing a triangle in the middle area) for double extreme convergence and one brown area (at the bottom right): single extreme convergence). The dynamical regimes diagrams of Figure 9 display broad regions of intermediate values *are* "pure" dynamical

regimes, Corresponding to $y = 0$ or 1 or 0.5, are separated by "crossover" Where intermediate regions *are* average values and high standard deviation due Could Be to:

- Either a bimodal distribution of "pure" attractors DEPENDING ON random sampling of initial condition and pairing;
- Or to a unimodal distribution of more complex attractors with varying number of agents in different clusters.

**4.5**

The histogram of $y$ values taken along the horizontal line $p_e = 0125$ (see Figure 10 ) Removes the Ambiguity about the Nature of the attractors in the crossover region. We See That *U* the low crossover area corresponds to a unimodal distribution *there* Between 0 (central convergence) and 0.5 (both extremes convergence), while the broad *U* crossover corresponds to a bimodal distribution of central gold Either single extreme convergence.

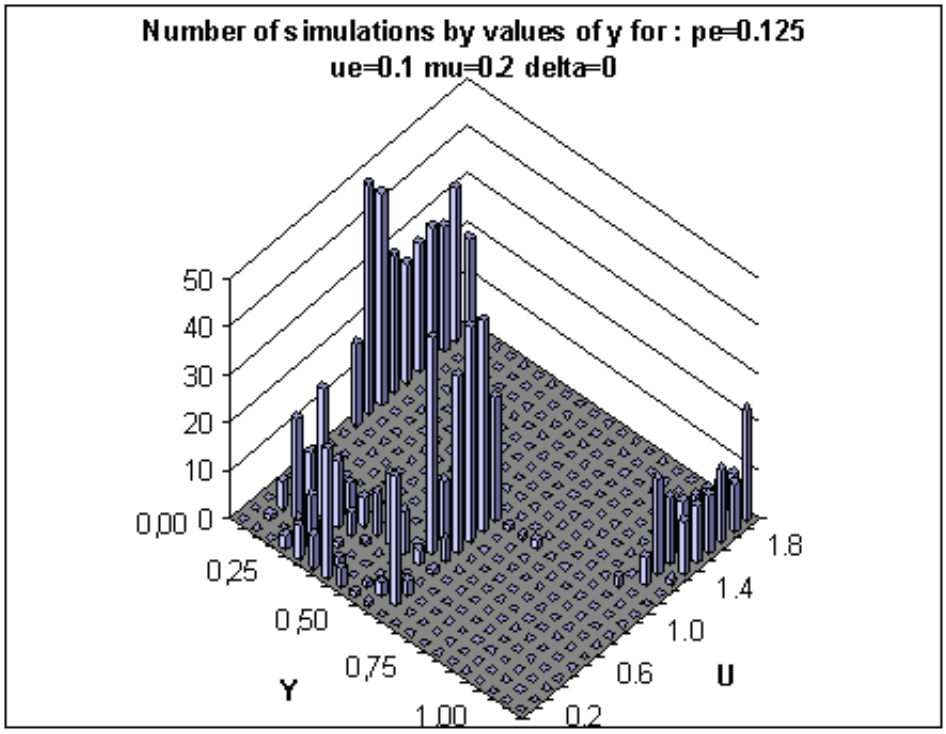



**Figure 10.** Histograms of the values of the indicator *are* taken along the line $p = 0.125_e$ of Figure 9 with $\delta = 0$. *U* In the broad crossover region *(u*> 1) The Same parameters lead to central Either single or extreme clustering *(y* close to 0 or close to 1). In the medium *u* (0.5 <U <1) we only get Both extreme clustering *(y* close to 0.5). *U* in the small crossover region *(u* below 0.5), the histogram shows convergences Which are different Intermediary Between central and extreme dual (with varying proportions of agents HAVING joined the extremes).

**4.6**

We are now considering how thesis typical patterns are modified with the variation of Other parameters.

**Influence of the parameter variations on the typical patterns**

**4.7**

We Performed a systematic exploration of the parameter space, in order to Identify the impact of the different parameters on the kind of convergence. All the simulations repeated 10 times Were For Each set of parameters, Which makes a total of 84 000 simulations. All the simulations Were Performed with a population of 1000 agents.

**Table 2: tested values of the parameter space.**

| Description | Symbol | Tested values |
|---|---|---|
| Overall proportion of Extremists | $p_e$ | 0025, 0.5, 0.1, 0.15, 0.2, 0.25, 0.3 |
| Uncertainty of the initial moderate agents | $U$ | 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6,1.8, 2 |
| Uncertainty of the initial Extremists | $u_e$ | 0.05, 0.1, 0.15, 0.2 |
| Difference between the relative proportion of positive and negative Extremists: | $\delta$ | 0, 0.1, 0.2, 0.3, 0.4, 0.5 |
| $\delta = \frac{\lvert p_+ - p_- \rvert}{p_+ + p_-}$ *(Eq. 9)* | | |
| Intensity of interaction | $\mu$ | 0.1, 0.2, 0.3, 0.4, 0.5 |

**4.8**

The exploration led to the Following results:

- When the intensity of interactions *(μ)* Increases, the area Increases Both extremes convergence and single extreme convergence the area decreases. Moreover, in the case of a number of Sami EACH Extremists are extreme *(δ* = 0), the central focal area Between the extreme Both convergence and single extreme zones decreases (see Figure 11 ).
- When the initial bias Between the Extremists *(δ)* Increases the single extreme convergence area Increases, And The Both extremes convergence area decreases (see Figure 12 ).
- We Did not find any significant effect of the Uncertainty of the Extremists *($u_{e)}$* on the patterns (see Figure 13 )

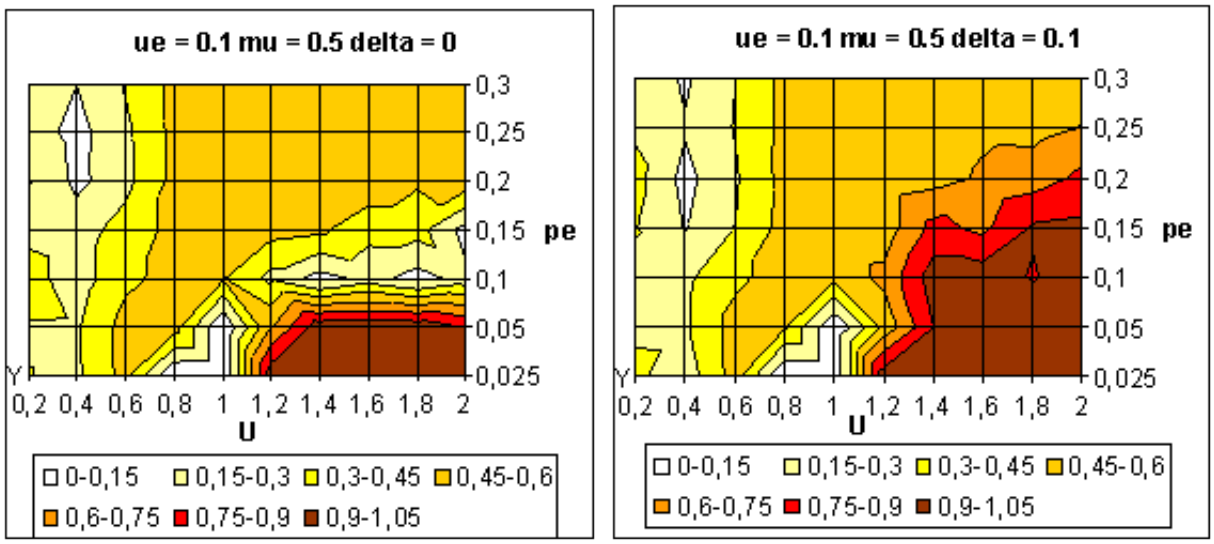



**Figure 11.** Average value of y over 10 simulations. Influence of intensity parameter (μ) on the dynamics. These figures must Be Compared with Figure 9 .

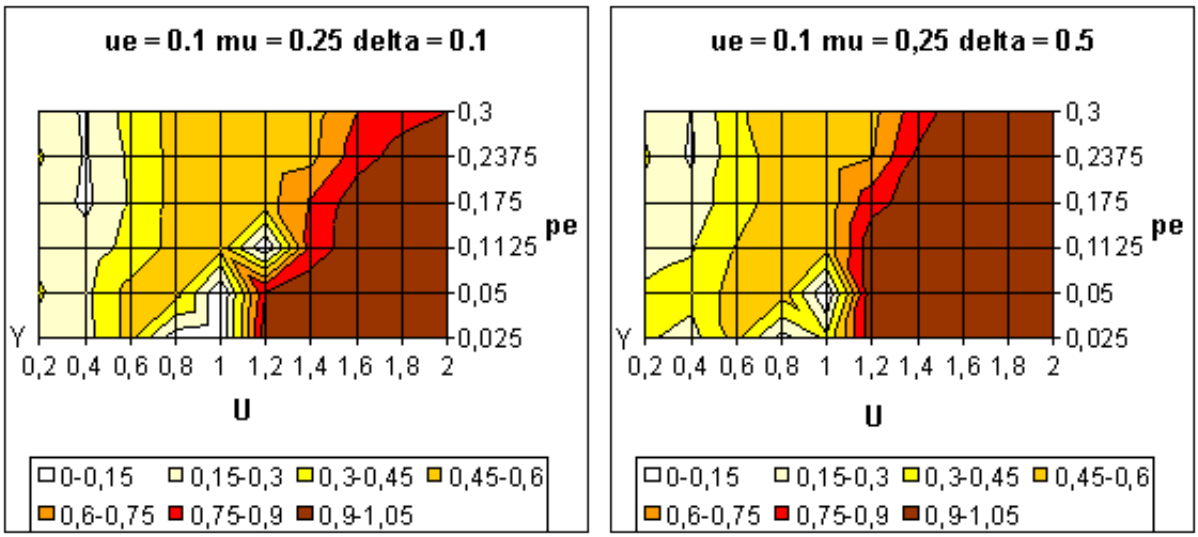



**Figure 12.** Average value of y over 10 simulations. Influence of the Bias Between the extremes (delta) on the dynamics.

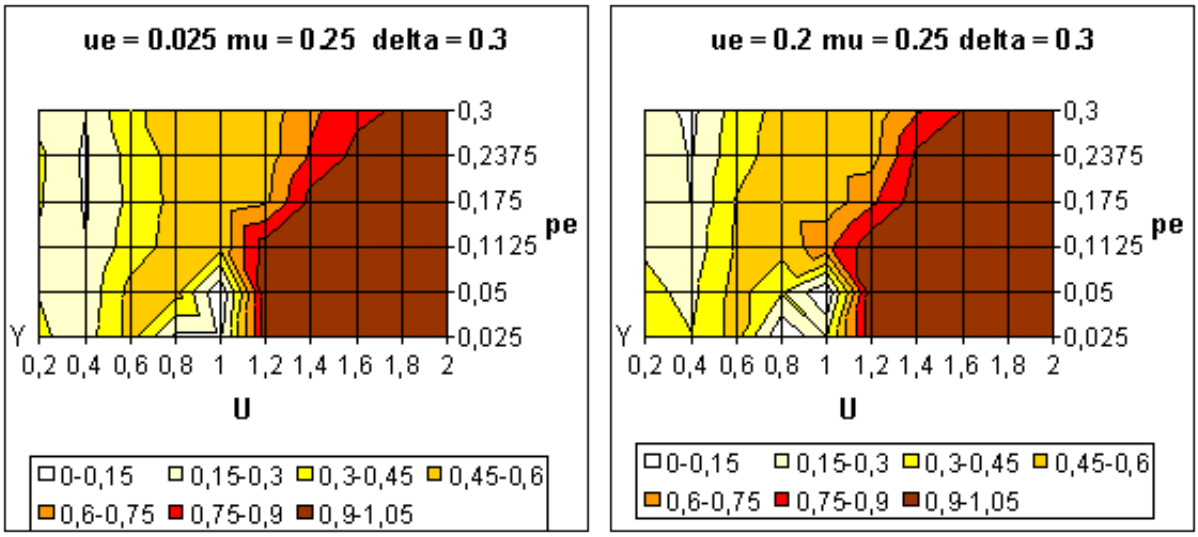



**Figure 13.** Average value of *y* over 10 simulations. Influence of the Extremists' uncertainty $(u_e)$ on the dynamics.

## Analysis of the convergence process

**5.1**

In this section, we propose a deeper analysis of the convergence process are representative examples.

**Method of analysis**

**5.2**

Our analysis is based on the computation of the mean effect of the population on the opinion of Each Agent, and the contribution of the Extremists to this overall mean influence.

**5.3**

More PRECISELY, For Each agent *j* and For Each time step, we compute *Ej*, Which is the mean effect due to the Extremists Divided by the overall mean effect:

$$E_j = \frac{\sum_{i-extremist} \delta x_{ij}}{\sum_{i} \delta x_{ij}} \qquad (Eq.\ 10)$$

**5.4**

Where is the "virtual" change of agent *j* 's view from under the impact of agent *i*. In Fact, At Any time step, only one interaction with a randomly Chosen agent Cdn Actually take place.

**5.5**

In the Following, WE REPRESENT the mean contribution of the Extremists to the overall mean effect as a color, and we Artificially bounded by the negative values -1. When $Dx_{ij}$ values are close to 1, the overall mean effect is Almost Exclusively due to the Extremists. When They are close to -1, the year Extremists Have influence, Which is opposite to the overall mean influence.

**Central convergence for small initial Uncertainty of the moderate *(U)***

**5.6**

Figure 14 shows example of opinion Year Evolution in the small ranks of $U$ ($U = 0.4$), Which leads to a central convergence.

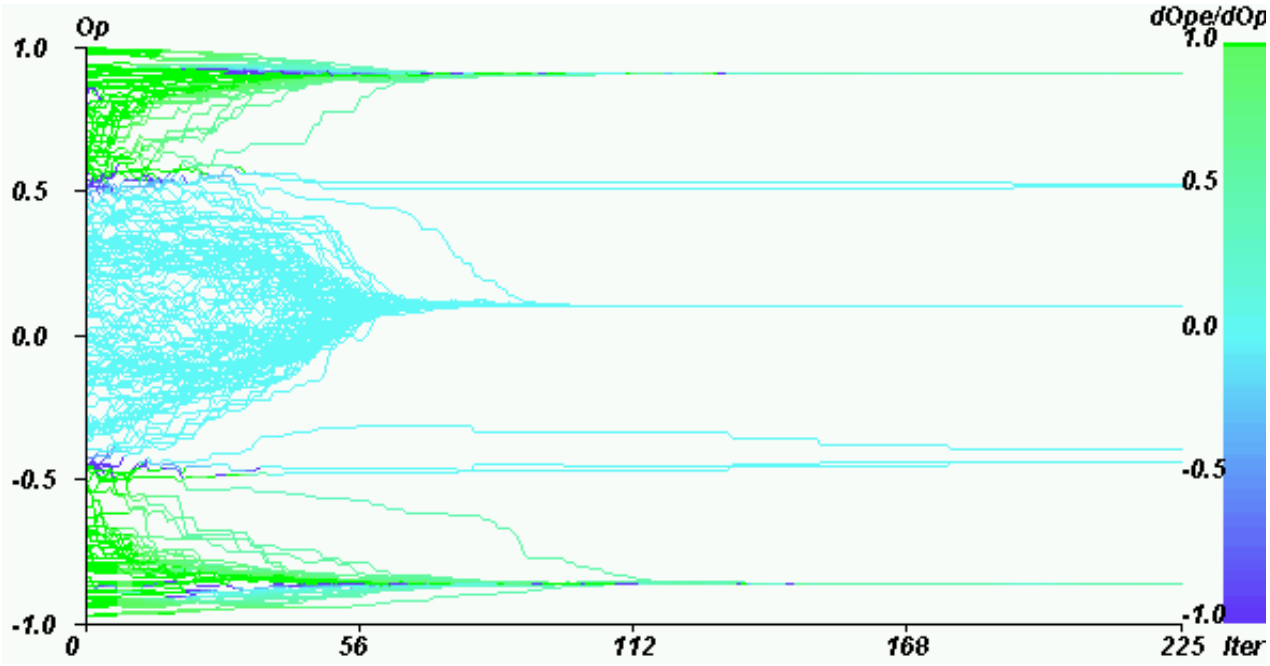



**Figure 14.** Weight of the Extremists in the mean effect on the opinion for $U = 0.4$, $p_e = 0.2$, delta = 0, $N = 200$. Green dots close to the extreme Both Indicate That the effect is Mainly the effect of the Extremists. Turquoise dots in the middle Indicate That the the impact of the Extremists is negligible. Blue points, just in between, Indicate That the impact of the Extremists is Contrary to the global influence.

**5.7**

The Figure shows Roughly Two kinds of trajectories: the green ones, close to the Both extremes, are Mainly due to the impact of the Extremists, Extremists Whereas the not at all affect the trajectories turquoise in the middle. At the limit of the Extremists influence, There are Some dark blue dots That Indicate That the overall mean and the Extremists influences are opposite, and neutralizes Cdn Each Other. This Explains why A Few agents continues Horizontally from thesis areas.

**5.8**

This position corresponds to the intuitive Where the box Extremists Attract the share of the population, Which Their opinions are close to. In this case, Their impact is limited to closed thesis INITIALLY share of the population.

**Both extremes convergence**

**5.9**

Both the extreme convergence Cdn take up with different values of the Uncertainty of the moderate

INITIALLY agents. Figure 15 shows annually example of Both extremes convergence with a Relatively small initial officers Uncertainty of the moderate ($U = 0.8$), and Figure 16 , a box Where this initial Uncertainty is higher ($U = 1.6$).

**5.10**

Would one expect the convergence to Both extremes corresponds to year extension of the extremist attraction Observed on Figure 14 . The analysis of Figure 15 shows That It Is Not Exactly the box. The agents mostly That Are Attracted by the extreme (green trajectories) Have INITIALLY medium views, and on the Contrary, the agents Which Are The Closest to the extreme, tends to Be Attracted more by the center (dark blue trajectories).

**5.11**

The Reason Is That for a moderate initial value of the Uncertainty of 0.8, the agents are close to Which the extreme, Have a large part of Their segment Which is out of the width of the initial distribution. There Is A Higher therefore influences of the central population, Which is not compensated by the presence of the Extremists. On the Contrary, agents with opinion year Closer to the center Extremists Have the site location is one extremity of Their segment of uncertainty, agents and only moderate in the rest of the segment. Furthermore, Decrease Before the final, the attraction gets Stronger When The Distance Between Increases agents (see Figure 2 ). The impact of the Higher Comparatively Extremists HAS kind of leverage effect, Which is decisive.

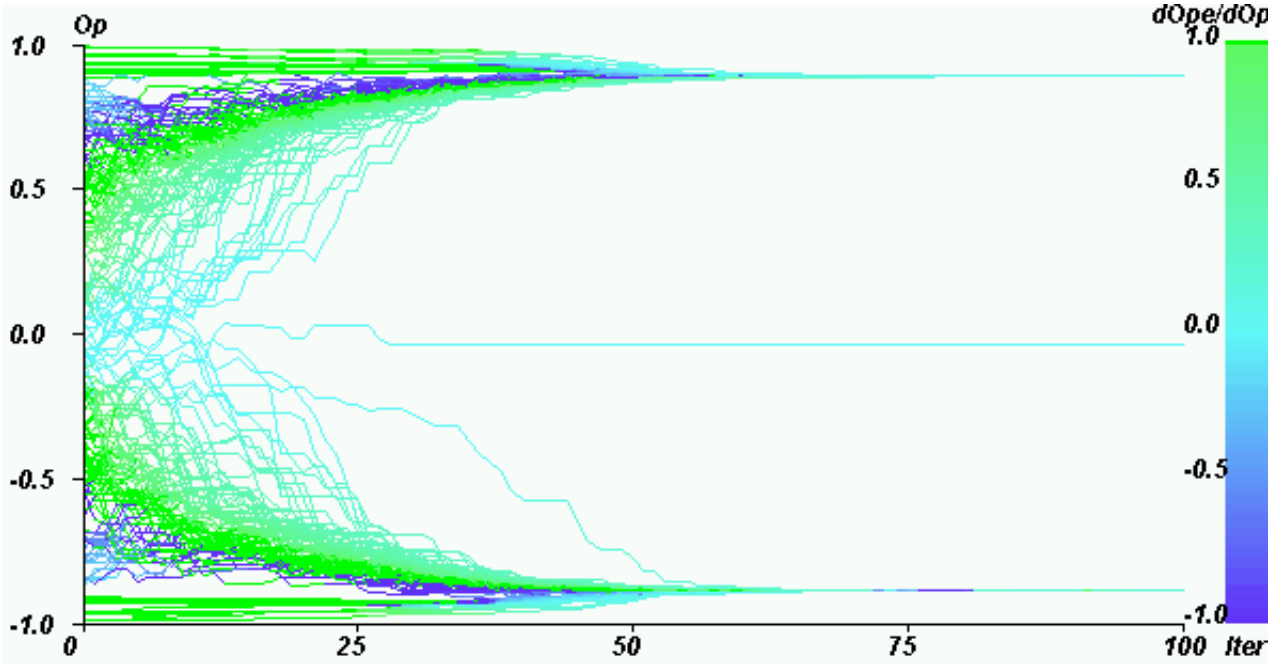



**Figure 15.** Mean effect in a box of Both extreme convergence. The values of the parameters are: $p_e = 0.15$, $U = 0.8$, $\mu = 0.5$, delta $= 0$, $N = 200$. We noted the Extremists That Have No significant effect on the Closest To The Extreme (dark blue points). On the Contrary, the Extremists Have a preponderant effect in a region of more moderate views INITIALLY (green parts).

**5.12**

Figure 16 shows to Both extreme convergence for a larger initial uncertainty (and a larger proportion of Extremists). One observes quite Cdn different processes:

- At The Beginning, the moderate agents tend to cluster at the center, and the impact of the Extremists is small (turquoise green color).
- Then, in a second phase, There Is A Competition Between the strong influences of the Extremists and the impact of the moderate (dark blue trajectories of shares). The Reason Is That the Uncertainties of the agents and Decreased Globally Reached values for the agents Which Have only one extreme at the extremity of Their opinion segment, Which Enhances the Extremists influence.
- Finally, the Extremists win this competition and split the population in Two almost equal shares. If the proportion of lower Extremists HAD beens, the clustering Towards the center Could have prevailed (see next paragraph).

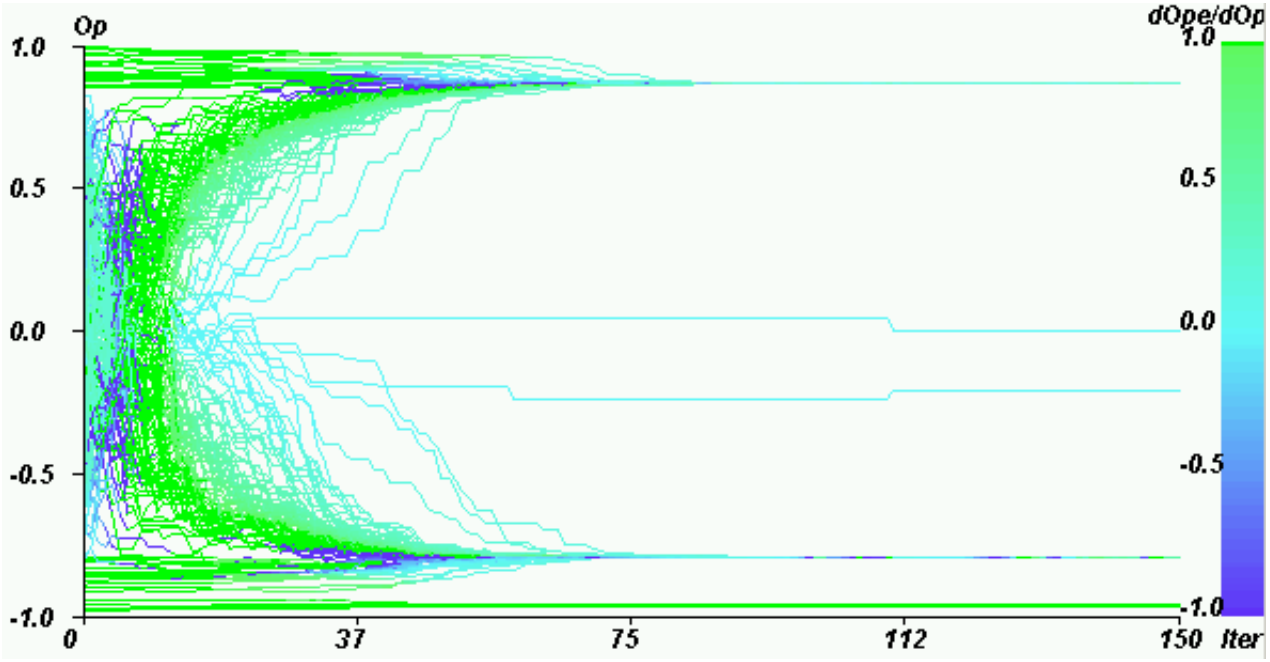



**Figure 16.** Weight of the Extremists in the mean effect for $U = 1.6, p_e = 0.2, \mu = 0.5$, delta = 0, $N$ = 200. At the very Beginning, the Extremists Have a low impact and the population tend to cluster at the center. Then, a competition Between the Extremists influence and the Central Tendency clustering OCCURS (Because The Uncertainty of the agents Decreased).

**Single extreme convergence**

**5.13**

Figure 17 shows annually example of single extreme convergence. In this case, there is a low proportion of extremists, and their role is very small at the very beginning of the process (mostly turquoise trajectories), except at the centre where some green parts appear.

**5.14**

Then a competition between clustering and dispersion occurs (dark blue parts), which is won by the central clustering process. During this phase a limited number of agents (5) join the extremes.

**5.15**

One can notice that the process is not perfectly symmetric, because of randomness in the initial

distribution and in the sampling of interaction pairs. In particular, at the end of the central clustering phase, there is a bottom part which is turquoise, indicating that the negative extremists are not in contact any more with the central cluster. Therefore, only the positive extremists have an influence on the cluster, and attract it progressively.

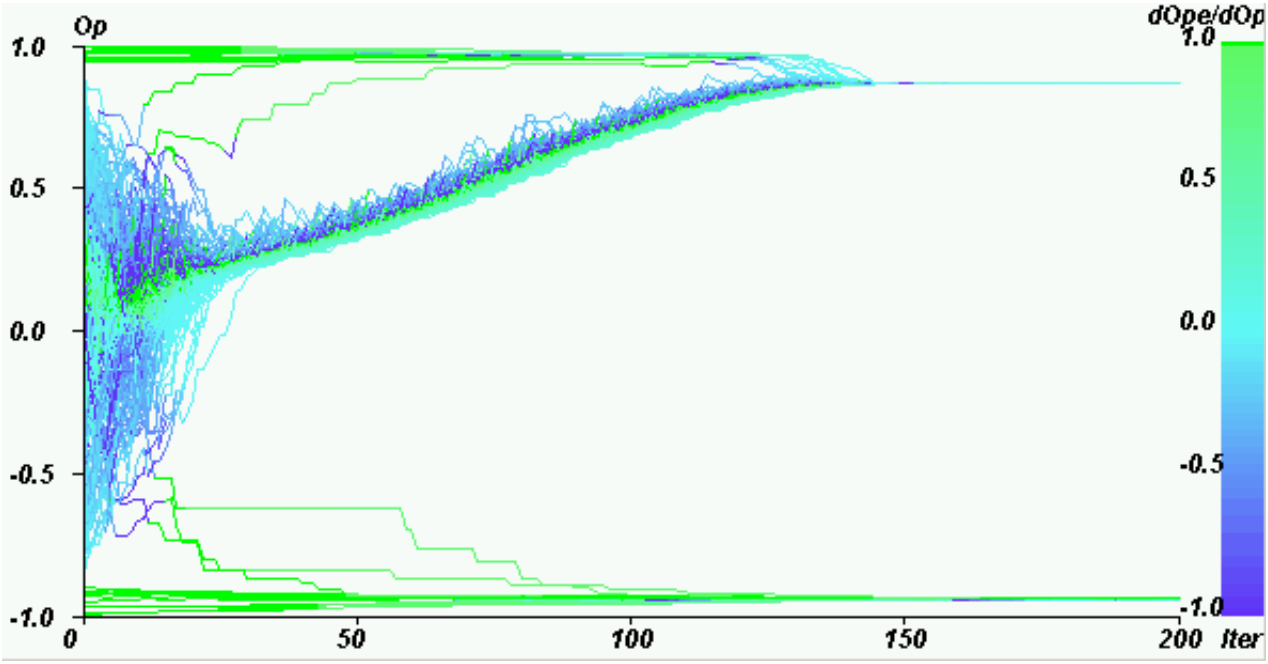



**Figure 17** . Weight of the extremists in the mean influence for $U = 1.4, p_e = 0.1, \mu = 0.5$, delta $= 0$. Case of single extreme convergence.

**5.16**

During the progressive drift to the positive extreme, we notice that the trajectories can roughly be separated into three parts: green at the bottom, dark blue at the top and middle, and a set of blue turquoise jumps in the upper part (see figure 17 ). The interpretation is the following:

- The agents, which are at the bottom half of the cluster, tend to go up, because of the influence of the extremists and of the clustering tendency. The weight of the extremists is high because the density is higher at the bottom, and most of the agents are very close to each other. The influence of the majority is therefore low (on average) compared to the influence of the extremists.
- The agents, which are at the top half of the cluster, are submitted to contradictory influences: they tend to go down to the gravity centre of the cluster and the extremists can also attract them up. This is why they are dark blue.
- When the agents are attracted by the extremists and have deviated from the majority, the influence of this majority becomes higher (because they are further), and they tend to go back to the centre of the cluster. However, doing so, they also attract up some agents. This leads to the progressive drift to the extreme (see figure 18 ).

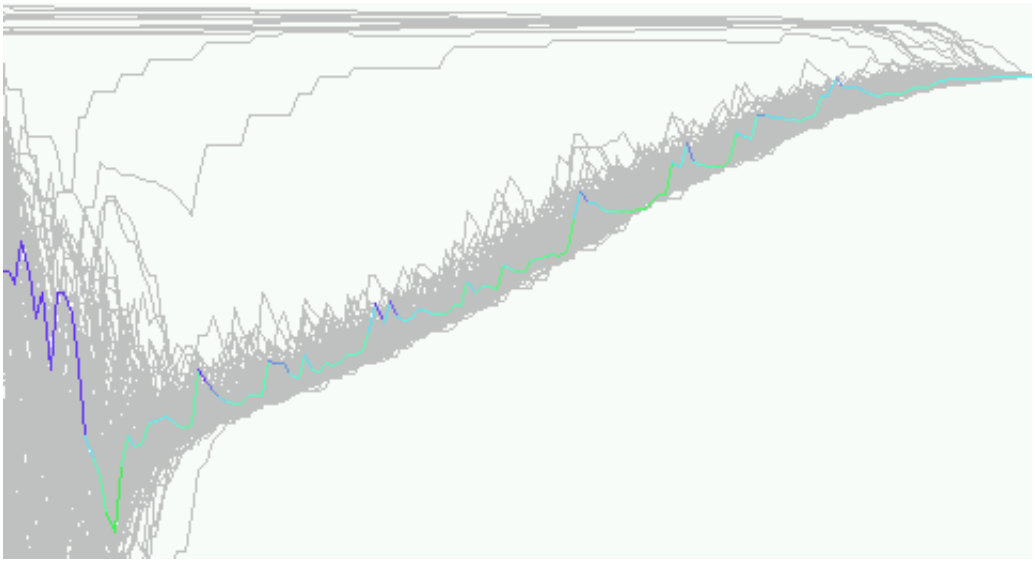


**Figure 18** . Zoom on an individual trajectory in the run of figure 17 , all the trajectories are in grey, except one, for which the colour indicate the weight of the extremists in the mean influence. After the central clustering process, the agents fluctuate during the drift to the extreme. From times to times they go suddenly up under the influence of an extremist. Then they go down under the attraction of the close majority. This leads to the progressive drift of the clustered majority to the extreme.

**Central convergence at the boundary between both extremes and single extreme convergence**

**5.17**

Figure 19 shows an example of central convergence obtained with the same parameters as on figure 17 (Large uncertainty $U = 1.4$, and a low number of extremists $p_e = 10\%$).

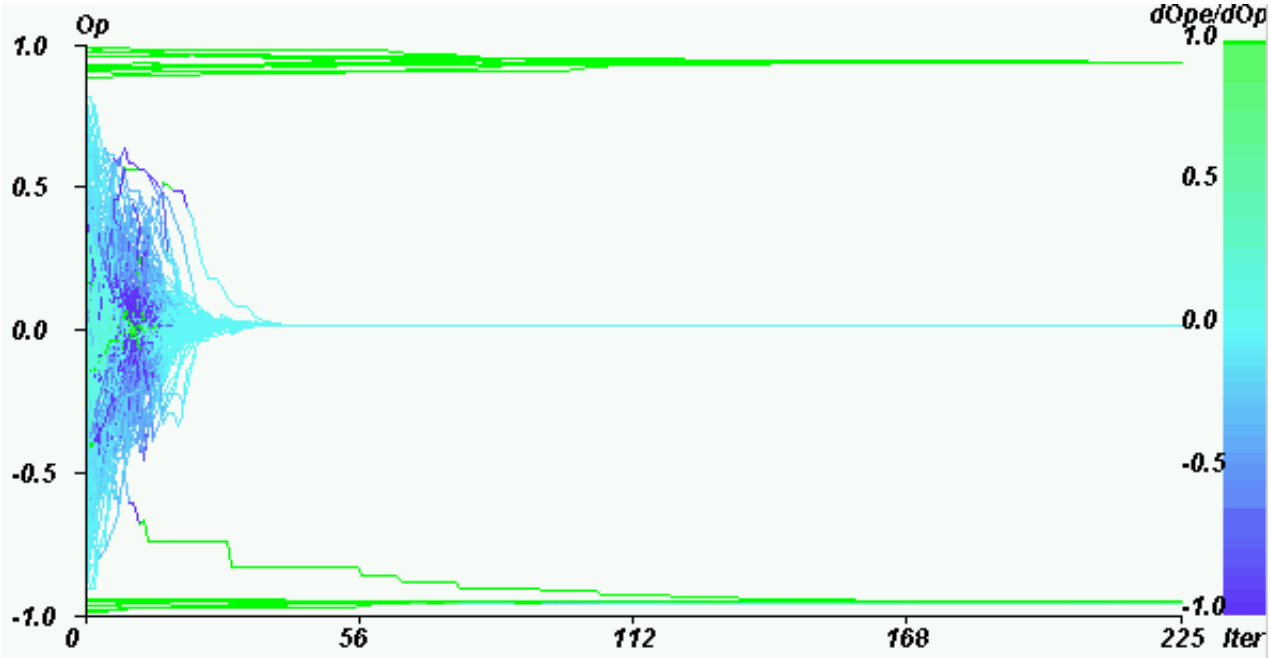



**Figure 19** . Weight of the extremists in the mean influence for $U = 1.4, p_e = 0.1, \mu = 0.5$, delta = 0. For the same parameters as on figure 17 , we obtain a central convergence.

**5.18**

We notice a beginning of the process, which is similar to the one of figure 17 : initially the extremists have a low influence, and then, when the uncertainty gets smaller, there is a competition between the attraction toward the extremes and toward the centre (dark blue parts). During this phase, one agent actually joins the negative extreme. Finally, the central clustering process wins. In this case, the central cluster is very close to the centre and the contact with the extremists is lost because the uncertainties are too small (the trajectories become turquoise). Therefore, the cluster remains at the centre.

**5.19**

The fluctuations on the position of the central cluster explain therefore why for the same parameter values, there can be either a single extreme or a central convergence.

## Convergence patterns for BC model variants

**6.1**

When observing unexpected phenomena, such as the evolution towards single-sided extremism, it is important to check whether the new behaviour is generic, ie whether it depends on some specific assumption of our model or whether it can be observed for a large class of models, equivalent at least in terms of this dynamical property. We mentioned earlier that the same dynamics is observed with the master equation formalism of the RA model as soon as any small asymmetry is introduced in the initial conditions; we pursue here our search for genericity by checking three versions of the BC model.

### The constant uncertainty version

**6.2**

In its simplest version the BC model varies from the RA model by having a square window interaction function rather than a progressive one: Let $x$ and $x'$ be a pair of opinions, and $u'$ the uncertainty of opinion $x'$ . If $|x - x'| < u'$ the influence of $x'$ on $x$ is given by:

$$x = x + \mu(x' - x) \qquad \textit{(Eq. 11)}$$

**6.3**

6.3 The uncertainties remain constant throughout the dynamics for all agents. A sample program in C is available as melsimp.c . Figure 20 shows the patterns of results obtained with the constant uncertainty model for populations with extremists and the same set-up that we used with the RA model ( figure 9 ).

**6.4**

All three types of convergence observed for the RA model also appear with the BC model. However, several major differences with the RA model are noticeable:

- The both extremes convergence takes place only in a very limited parameter zone, with an uncertainty of the population $U$ close to 0.4 or 0.5, corresponding to a convergence to 2 clusters in the case of a population without extremists.
- The single extreme convergence is limited to a zone of parameters around an uncertainty of the population $U = 1$ (larger for delta = 0.1). In this region, the agents which are attracted by one

extreme, loose the contact with the other extreme. Any fluctuation of the density in favour of one or the other extreme leads therefore to the convergence of the whole population to this extreme.

- When $U > 1.2$, we only get central convergence. In this parameter zone, there is a central cluster and some fluctuations of the agents around an average position because of the attraction of the extremists on both sides. However, the moderate agents never go to the extremes.
- The standard deviation of $y$ is high for values which are close to the transition between single extreme convergence and central convergence. There is also a zone of relatively high standard deviation for high percentages of extremists and $U$ around 0.4, because the convergence to both extremes can occasionally take place in this zone.

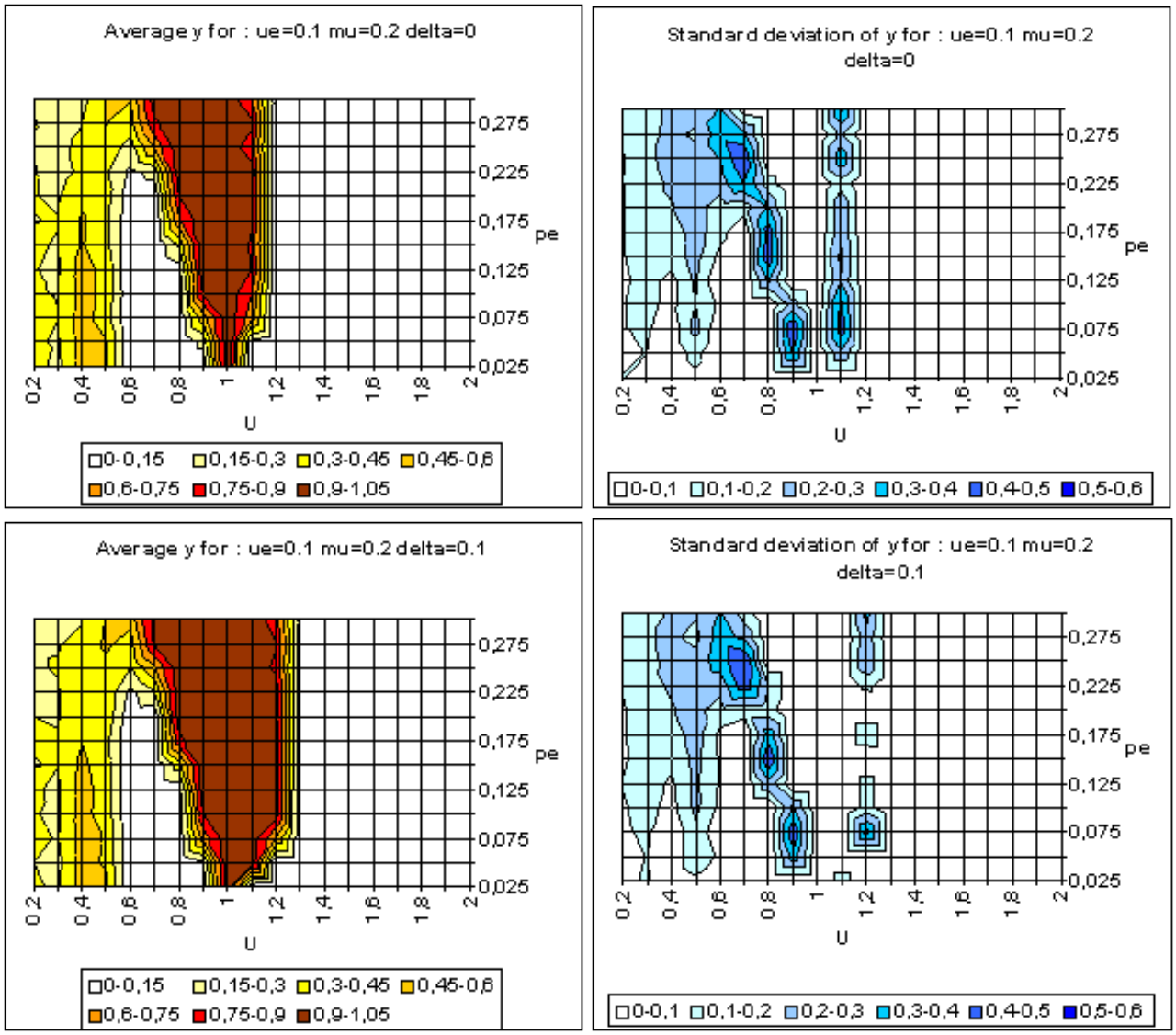



**Figure 20** . Simple BC model. Average and standard deviation of $y$ for 50 simulations on each point of the grid, for delta = 0 (above) and delta = 0.1 (bottom).

### 6.5 Bounded confidence with averaging uncertainties

In this version of the BC model the uncertainties of the two interacting agents get also closer during the

interaction according to:

$$u = u + \mu\left(u' - u\right) \qquad \textit{(Eq. 12)}$$

**6.6**

This model is closer to the RA model. Figure 21 displays the dynamical regimes of this model in the same parameter region as figure 9 .

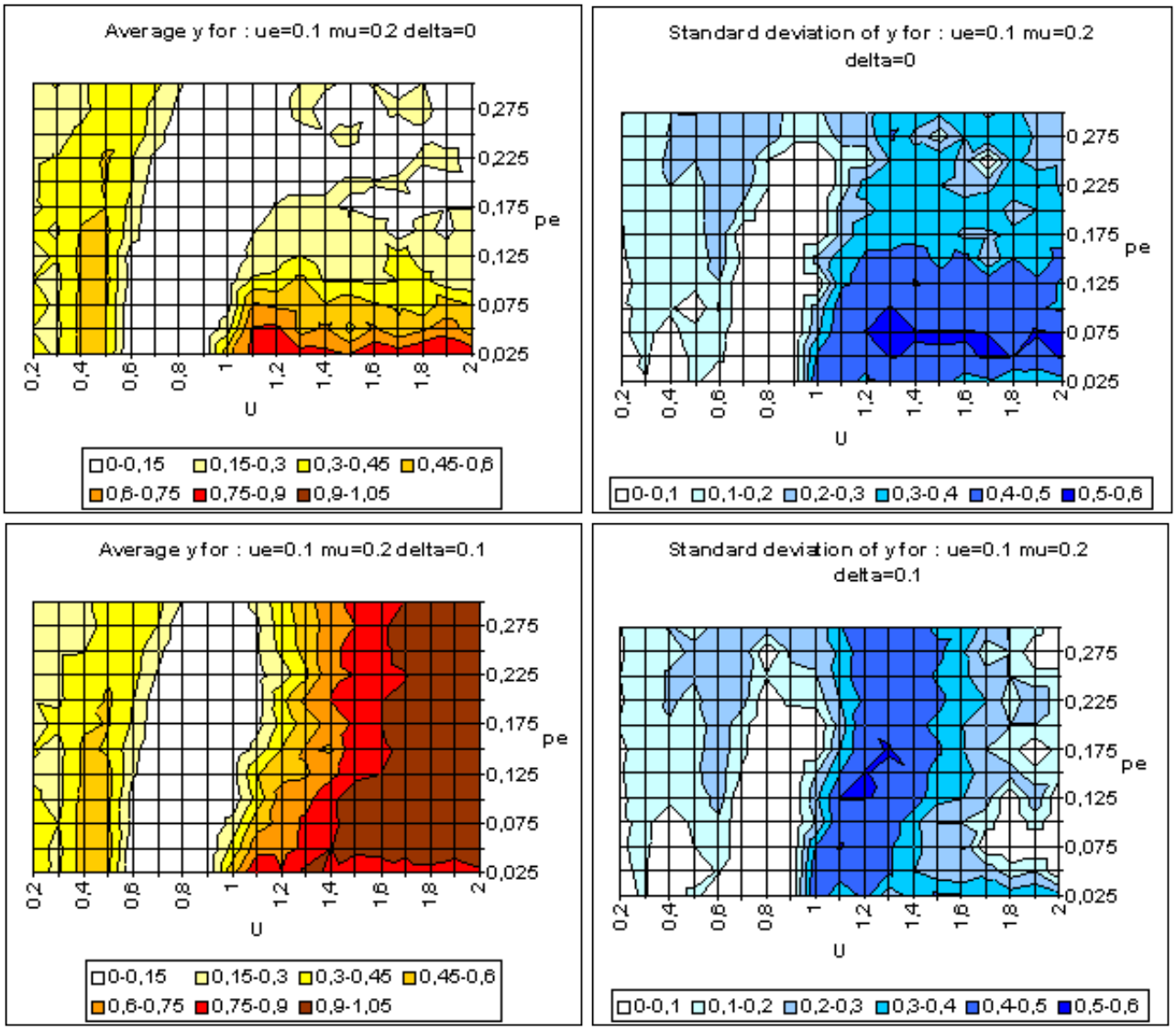



**Figure 21.** BC model with averaging Uncertainties. Average and standard deviation of *y* for 50 simulations on each point of the grid, for delta = 0 (above) and delta = 0.1 (bottom).

**6.7**

The BC with averaging uncertainties leads also to the three types of convergence. The analysis of figure 21 yields the following noticeable points:

- Like with the simple BC model, the both extremes convergence takes place in a very limited parameter zone (the orange region close to $U = 0.4$ on the left graphs). Again, it corresponds to the value for which the population converges to 2 clusters in the absence of extremists.

- There is band of central convergence for $U$ close to 0.8. This corresponds to a central clustering which looses the contact with the extremes because of a rapid decreasing of the uncertainty.
- When the initial uncertainty $U$ is higher than 1.1, we tend to get a single extreme convergence or central convergence. If $\delta > 0$, the initial process of central clustering is likely to be closer to one extreme, which leads to a single extreme convergence: the cluster gets closer and closer to the extreme, and looses the contact with the other extreme because the uncertainty decreases. If $\delta = 0$, the central cluster is more likely to be very close to the centre (because of the initial symmetry), and the central convergence is more likely (especially for high percentages of extremists which lead to a faster decrease of the uncertainty and loss of contact with any of the extremes).

**Bounded confidence with uncertainty based on variance evaluation**

**6.8**

The model of bounded confidence with an evolution of the uncertainty based on variance estimation was proposed in (Weisbuch et al. 2002 ). The evolution of the opinions and uncertainties is driven by (in a simple version):

$$x = \alpha \cdot x + (1-\alpha) \cdot x' \qquad \textit{(Eq. 13)}$$

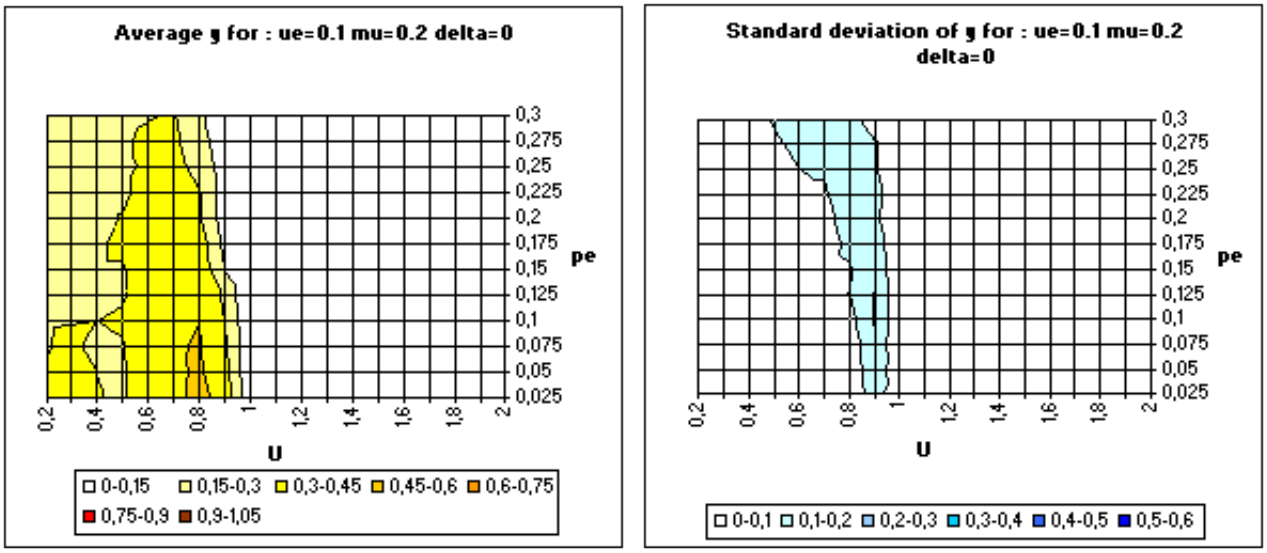
$$u^2 = \alpha \cdot u^2 + \alpha \cdot (1-\alpha) \cdot (x - x')^2 \qquad \textit{(Eq. 14)}$$

**6.9**

Where $\alpha$ is a positive parameter below 1. This expression gives rise to an approximately exponential decay of uncertainties. As opposed to the BC model with averaging uncertainties, the agents are influenced only by the opinion, and not by the uncertainty of their partner.

**6.10**

Figure 22 displays the dynamical regimes of this model in the same parameter region as figure 9 .



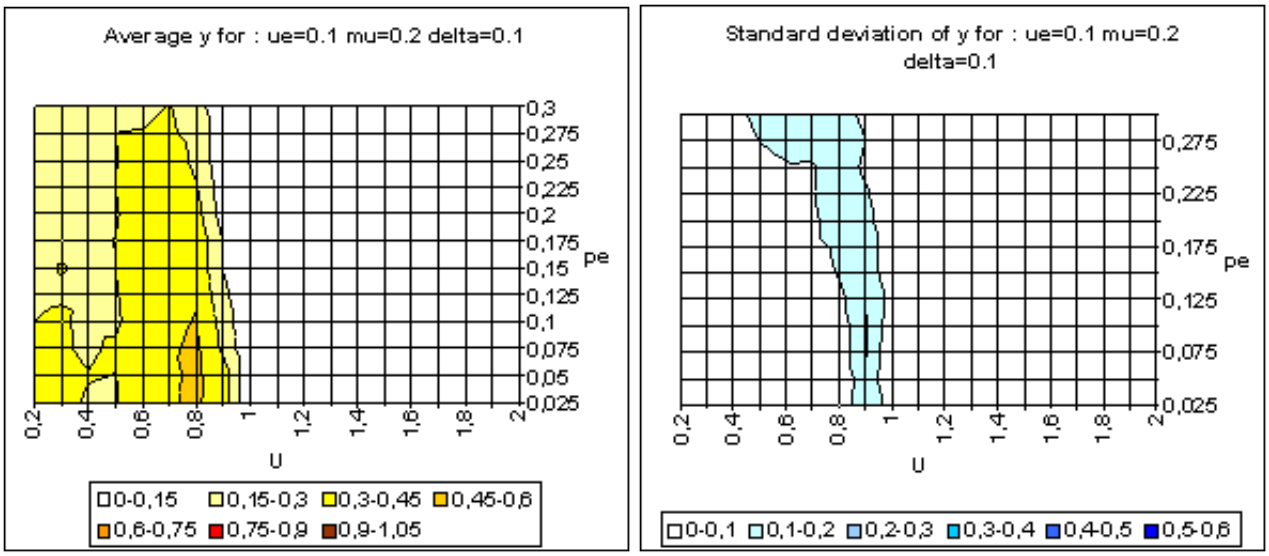



**Figure 22** . BC model with uncertainty based on variance estimation. Average and standard deviation of *y* for 50 simulations on each point of the grid, for delta = 0 (above) and delta = 0.1 (bottom).

**6.11**

The striking features of figure 22 are the absence of single extreme convergence, and the very rare presence of double extreme convergence. The reason is the exponential decrease of uncertainty in this model, which rapidly puts the extremists out of uncertainty range of moderate agents.

## Discussion and conclusion

**7.1**

The behaviour of the relative agreement model can be roughly summarized as follows:

- For small general uncertainty, the influence of the extremists is limited to those agents which were initially close to them, and we get a central convergence.
- For high general uncertainty, the extremes tend to prevail, leading either to a bipolarisation or to the convergence to a single extreme. When the extremists are numerous and of equal number in both extremes, there is an instability between central and single extreme convergence. This instability is due to variations in the position of a central cluster which is formed in a first phase of the process (jointly with a decrease of the uncertainty).

**7.2**

Some aspects of the RA model seem relevant as a metaphor for social dynamics. The fact that extremism tends to prevail more easily when the population is initially very uncertain corresponds to common sense expectations. More specifically, in the case of central convergence with small initial uncertainties, the attraction by the extremes of those agents initially closer to the extremes was also expected.

**7.3**

Other aspects of the RA behaviour are more surprising. The fact that the single extreme convergence occurs after a phase of strong consensus at a moderate opinion, with an almost equal interest for both extremes is somehow less intuitive. Moreover, in the case of large uncertainties, it is rather the more

central agents which tend to drive the drift to the extremes, whereas the closest to the extremes tend to go to the centre. This was not expected either.

**7.4**

Comparing the RA to other variants of the BC model shows that the bipolarisation to the extremes is specific to the RA model. The evolution towards single sided extremism is also exhibited both variants of the BC model. The genericity of this dynamical behaviour is a significant result of this analysis.

**7.5**

Future work is necessary for assessing how these behaviours could correspond to observed social processes, and to assess the limitative character of the many approximations of the models, among which:

- All the agents can communicate with each other (no specific social networks); different structures of social networks can modify the convergence and result in more clusters.
- Agents' opinions are disconnected from any specific information about "the real world out there" and from any strictly personal interest (economic for instance).
- We have not taken into account any power structure or any competition in this power structure, which can have a dramatic impact on the influence of extremism.

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We thank David Neau, Edmund Chattoe, Rainer Hegselmann and Nigel Gilbert for helpful discussions and the anonymous referees who made a number of helpful suggestions. We acknowledge the support of the FET-IST department of the European Community Grant IST-2001-33555 COSIN.

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## Notes

[1] This work was originally related to the EU funded IMAGES project (FAIR 3 96 2092), modelling the diffusion of green practises in the European farming population. See (Deffuant 2001 ) for information.

[2] Integer and vector opinions dynamics have also been studied in the literature, especially as "cultural dynamics", see eg Axelrod R. ( 1997 ) "Disseminating cultures" pp. 145-177, in Axelrod R., The complexity of cooperation, Princeton University Press.

[3] We don't use here uncertainty with the idea that it represents some maximum discrepancy between the agent opinion and some objective reality as it is done in the literature on information cascade. We here use uncertainty since agents with high uncertainty easily change opinion (while those with low uncertainty on the contrary almost never change their opinion). However, this choice is not totally satisfactory because the interpretation of the model includes also the notions of conviction, confidence, tolerance or openness to others influence. Unfortunately, there is no word mixing together all these notions, we chose the one which appeared the most general. Since the model is explicitly defined here, we expect that wording is of less importance than in the traditional sociological literature.

[4] A difference with the approach of Hegselmann and Krause ( 2002 ) and our is that they use a parallel updating of opinions as if at each time step, all agents would survey the opinions of all the others to whom they are connected and average over these opinions. We use random sequential updating based on the idea of pair interactions. Several variants can be imagined for the updating process, random sequential updating seeming a natural one. The only rationales we can imagine for parallel updating is that at the origin (Chatterjee and Seneta 1977 ; Cohen et al. 1986 ) the model was built to describe decisions in committees and that the linear (no threshold) model is soluble via matrix techniques.

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