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Exercise – 7.2

Write a **MATLAB m-code** to find the time domain specifications for a 2nd order under-damped system, if given the closed loop transfer function parameters. Plot the step response for the same and find the final value of response.

Initial Calculations:

Closed loop transfer function of a 2nd order system is of the form:

$$\frac{G(s)}{1+G(s)H(s)} = \frac{{\omega_n}^2}{s^2 + 2\delta\omega_n s + {\omega_n}^2}$$

Comparing the above with

$$\frac{G(s)}{1+G(s)H(s)} = \frac{c}{as^2 + bs + c} = \frac{c/a}{s^2 + (b/a)\,s + (c/a)}$$

i. Accept from the user, values of ***a***, ***b*** and ***c.***

ii. Calculate the un-damped natural frequency, ω_n

$$\omega_n = \sqrt{\frac{c}{a}}$$

iii. Calculate the damping ratio, δ

$$\delta = \frac{b}{2a\omega_n}$$

iv. Verify if it is an under-damped system (0<δ<1) and proceed

v. Calculate damped frequency, ω_d

$$\omega_d = \omega_n\sqrt{1-\delta^2}$$

vi. Calculate θ

$$\theta = cos^{-1}\delta$$

vii. Delay time, t_d

$$t_d = \frac{1+0.7\delta}{\omega_n}$$

viii. Rise time, t_r

$$t_r = \frac{\pi - \theta}{\omega_d}$$

ix. Peak time, t_p

$$t_p = \frac{n\pi}{\omega_d}$$

x. Peak overshoot, M_p

$$M_p = e^{-\delta\pi / \sqrt{1-\delta^2}} \times 100\ \%$$

xi. Settling time, t_s

For a 2% steady state error

$$t_s = \frac{4}{\delta\omega_n}$$

For a 5% steady state error

$$t_s = \frac{3}{\delta\omega_n}$$

M-code:

```
% Ex-7.2 (Control Systems)
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clc;
clear all;
close all;

fprintf('Program to calculate the time-domain specifications of a 2nd order under-
damped system\n\n')
while 1
a = input('Enter the coefficient of s^2 in the closed loop transfer function: ');
b = input('Enter the coefficient of s^1 in the closed loop transfer function: ');
c = input('Enter the coefficient of s^0 in the closed loop transfer function: ');

wn = sqrt(c/a);
del = b/(2*a*wn);
if del<1 && del>0
    break
end
del
fprintf('This is not an under-damped system. Please re-enter.\n\n')
end
wd = wn*sqrt(1-del^2);
theta = acos(del);

disp('Damping ratio: ');
del
disp('Un-damped natural frequency wn (rad/s): ');
wn
disp('Damped frequency wd (rad/s): ');
wd
disp('Delay time (s): ');
td = (1+0.7*del)/wn
disp('Rise time (s): ');
tr = (pi-theta)/wd
disp('Peak time (s): ');
tp = pi/wd
disp('Peak overshoot (%): ');
Mp = exp(-del*pi/(sqrt(1-del^2)))*100
disp('Settling time for a 2% steady state error (s): ');
ts = 4/(del*wn)
disp('Settling time for a 5% steady state error (s): ');
ts = 3/(del*wn)

t = 0:0.01:ts+1;
out = 1-exp(-del*wn.*t).*sin(wd.*t+theta)/sqrt(1-del^2);
plot(t,out);
grid on;
xlabel('Time(s) ---->');
ylabel('Response ---->');
title('Step response of a 2nd order under-damped system');
```
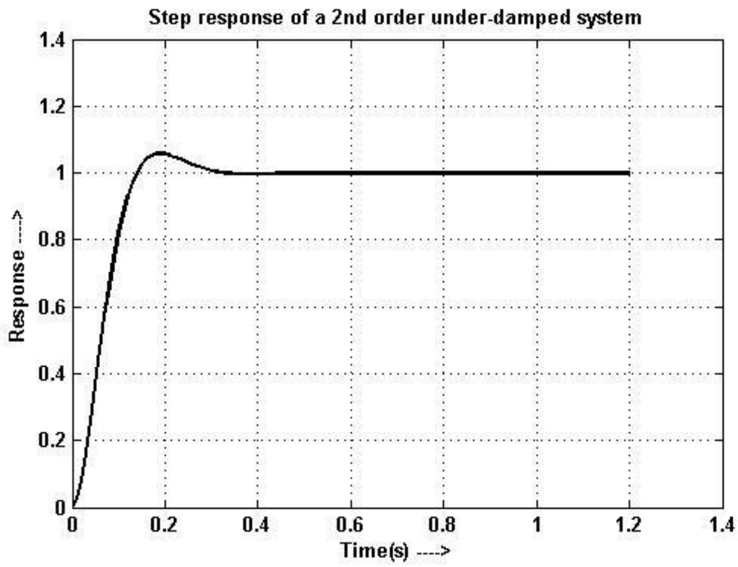

Terminal Display:

```
Program to calculate the time-domain specifications of a 2nd order under-damped
system
Enter the coefficient of s^2 in the closed loop transfer function: 1
Enter the coefficient of s^1 in the closed loop transfer function: 60
Enter the coefficient of s^0 in the closed loop transfer function: 500

del =

    1.3416

This is not an under-damped system. Please re-enter.

Enter the coefficient of s^2 in the closed loop transfer function: 1
Enter the coefficient of s^1 in the closed loop transfer function: 30
Enter the coefficient of s^0 in the closed loop transfer function: 500
Damping ratio:

del =

    0.6708

Un-damped natural frequency wn (rad/s):

wn =

   22.3607

Damped frequency wd (rad/s):

wd =

   16.5831

Delay time (s):

td =

    0.0657

Rise time (s):

tr =

    0.1391

Peak time (s):

tp =

    0.1894

Peak overshoot (%):

Mp =

    5.8328
```

```
Settling time for a 2% steady state error (s):

ts =

    0.2667

Settling time for a 5% steady state error (s):

ts =

    0.2000
```

Waveform:



Results:

Hence MATLAB is used to determine the time domain specifications of a 2nd order under-damped system imposed with a step input. The response is plotted.