Multiple Choice

1. The value of $3 \times 2020 + 2 \times 2020 - 4 \times 2020$ is

Following bedmas:

3x2020 + 2x2020 - 4x2020
=6060 + 4040 - 8080
=10100 - 8080
=2020

(Fermat 2020, #2)

2. The sum of the first 9 positive integers is 45; in other words,

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$$

What is the sum of the first 9 positive multiples of 5? In other words, what is the value of:

$$5 + 10 + 15 + \cdots + 40 + 45\,?$$

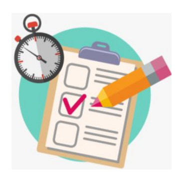
Given

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$$

(5+10+15+ … + 40 + 45) = 5(1+2+3+4+5+6+7+8+9) = 5*45 = 225

(Cayley 2020, #11)

3. A multiple choice test has 10 questions on it. Each question answered correctly is worth 5 points, each unanswered question is worth 1 point, and each question answered incorrectly is worth 0 points. How many of the integers between 30 and 50, inclusive, are not possible total scores?



For each of the 10 questions, each correct answer is worth 5 points, each unanswered question is worth 1 point, and each incorrect answer is worth 0 points. If 10 of 10 questions are answered correctly, the total score is 10 × 5 = 50 points. If 9 of 10 questions are answered correctly, either 0 or 1 questions can be unanswered. This means that the total score is either 9 × 5 = 45 points or 9 × 5 + 1 = 46 points. If 8 of 10 questions are answered correctly, either 0 or 1 or 2 questions can be unanswered. This means that the total score is either 8 × 5 = 40 points or 8 × 5 + 1 = 41 points or 8 × 5 + 2 = 42 points. If 7 of 10 questions are answered correctly, either 0 or 1 or 2 or 3 questions can be unanswered. This means that the total score is one of 35, 36, 37, or 38 points. If 6 of 10 questions are answered correctly, either 0 or 1 or 2 or 3 or 4 questions can be unanswered. This means that the total score is one of 30, 31, 32, 33, or 34 points. So far, we have seen that the following point totals between 30 and 50, inclusive, are possible: 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 45, 46, 50 which means that 39, 43, 44, 47, 48, 49 are not possible. If 5 or fewer questions are answered correctly, is it possible to obtain a total of at least 39 points? The answer is no, because in this case, the number of correct answers is at most 5 and the number of unanswered questions is at most 10 (these both can't happen at the same time) which together would give at most $5 \times 5 + 10 = 35$ points. Therefore, there are exactly 6 integers between 30 and 50, inclusive, that are not possible total scores.

(Cayley 2020, #19)

4. The y-intercepts of three parallel lines are 2, 3 and 4. The sum of the x-intercepts of the three lines is 36. What is the slope of these parallel lines?

Let the lines with y-intercepts 2, 3, and 4 have a slope of m (parallel, therefore same slope).

Let the x-intercepts of the lines with y-intercepts of 2, 3, 4 be x1, x2, x3 respectively

Solve for x1 when y=0:
$0 = m(x1) + 2$
$x1 = -2/m$

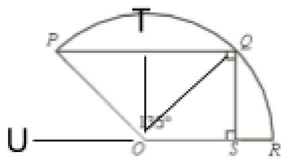
Similarly, $x2=-3/m$ and $x3=-4/m$

Since:

$x1+x2+x3=36$
$-2/m - 3/m - 4/m = 36$
$(-2-3-4) = 36m$
$m=-9/36$
$m=-\frac{1}{4}$

(2015 Fermat, #15)

5. In the diagram, points **P**, **Q**, and **R** lie on a circle with center **O** and radius 12. Point **S** lies on **OR**, if $\angle POR = 135^\circ$, the area of trapezoid **OPQS** is closest to:



i) Find ∠POU

∠POU = 180° - ∠PQS = 180°-135° = 45°

ii) Find ∠OPT

By Z-pattern:
∠OPT = ∠POU = 45°

iii) Find ∠TOP

∠TOP = ∠POS - ∠TOS = 135° - 90° = 45°

iv) Find lengths of PT and OT

Since ∠TOP = ∠OPT = 45, △POT is a right isosceles triangle, since OP is a radius, OP=12, thus OT=PT=12/ $\sqrt{2}$

v) Find length of OS

OS = $\sqrt{OQ^2 - QS^2}$ = $\sqrt{OQ^2 - OT^2}$ (TOSQ is a rectangle, as ∠TQS = ∠QSO = 90°, thus QS=OT) = $\sqrt{12^2 - (\frac{12}{\sqrt{2}})^2}$ = $\sqrt{144 - 144/2}$ = $\sqrt{72}$ = $6\sqrt{2}$

vi) Find the length of TQ

TQ = OS = 6 $\sqrt{2}$

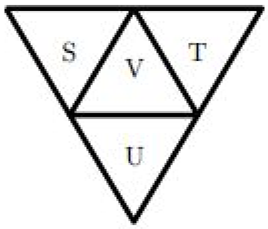
vii) FInd the area of the trapezoid OPQS

A_{OPQS} = $0.5(OT)(PQ + OS)$
= 0.5 (OT) (PT + TQ + OS)
= 0.5 (12/ $\sqrt{2}$)(12/ $\sqrt{2}$ + 6 $\sqrt{2}$+6 $\sqrt{2}$)
= 0.5 ((144/2) + (12*6) + (12*6))
A_{OPQS}= 108

(Cayley #23, 2012)

Word Problems

1. In the picture below, there are four triangles labelled S, T, U, and V. Two of the triangles will be coloured red and the other two triangles will be coloured blue. How many ways can the triangles be coloured such that the two blue triangles have a common side?



If two blue triangles have to share a common side, one of the sections that must be colored is section V, as it is the only section that shares a side with any other section. As for the second blue section, you can color sections S, T, U, which means that there are 3 different ways to color the triangle such that the blue triangles share a side. Where V&S, V&T, V&U are colored blue.

(COMC 2015, A2)

2. Suppose that n is positive integer and that a is the integer equal to $\frac{10^{2n}-1}{3(10^n+1)}$. If the sum of the digits of a is 567, what is the value of n?

a = $\frac{10^{2n}-1}{3(10^n+1)}$

= $\frac{(10^n)^2-1}{3(10^n+1)}$

= $\frac{(10^n-1)(10^n+1)}{3(10^n+1)}$

= $\frac{(10^n-1)}{3}$; 10^n - 1 = 9999...n times (ex. 10^5 - 1 = 99999 (5 times))

= $\frac{999...(n\ times)}{3}$

a = 333...(n times)

Sum of the digits of a = 3 + 3 + 3 … (n times)

567 = 3*n

n = 189

(CSMC 2018, #4)

3. A positive integer is said to be bi-digital if it uses two different digits, with each digit used exactly twice. For example, 1331 is bi-digital, whereas 1113, 1111, 1333, and 303 are not. Determine the exact value of the integer b, the number of bi-digital positive integers.

Sol'n 1: There are 9 choices for what the left-most digit of the number is (it cannot be 0) and there are 3 choices for where the second copy of this digit is. There are 9 possibilities for the digit that fills the remaining positions. Thus, b = 9 × 3 × 9 = 243.

Sol'n 2: We consider two cases. Either 0 is one of the digits, or it is not. If 0 is not one of the digits, then we have (9 choose 2) = 36 ways to choose 2 digits which are not 0. There are 4!/((2!)2) = 6 ways to arrange these digits, for a total of 216 numbers. If 0 is one of the digits, it cannot be the first digit of the number, since then the number would have fewer than 4 digits. In this case, there are (9 choose 1) = 9 ways to choose the other digit. The first digit must be the non-zero digit and there are 3 places for the other non-zero digit, so there are 27 such numbers. Thus, b = 216 + 27 = 243

(CSMC 2013, A4)