

Answers to JTOW #3: Basic Inequalities

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1. n positive even: $x \in (-\infty, -1) \cup (1, \infty)$;
 n positive odd: $x \in (1, \infty)$;
 $n = 0$: $x \in \emptyset$;
 n negative even: $x \in (-1, 0) \cup (0, 1)$;
 n negative odd: $x \in (0, 1)$.
2. $x \in \mathbb{R}$.
3. $x \in (-\infty, -2) \cup (-1, 1) \cup (2, +\infty)$.
4. $x \in [-2, 2)$.
5. $x \in (0, 2]$.
6. $x \in [-15, -6) \cup (10, 19]$.
7. $x \in (-\infty, -4] \cup [-3, -2) \cup (-1, +\infty)$.
8. $t \in (-\infty, -\frac{1}{4})$.
9. WLOG, assume that $a \leq b, c$. Then $2a \leq b + c$ so $3a + 2b + c \leq a + 3b + 2c$, so $a^{3a+2b+c} \leq b^{3a+2b+c} \leq b^{a+3b+2c}$, where both inequalities hold if and only if $a = b$ and $2a = b + c$, in other words $a = b = c$. But $a^{3a+2b+c} = b^{a+3b+2c}$, so we conclude that $a = b = c$.