Advanced Algorithm HW10

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1 Q1

Since the number of edges in the sparse graph is equivalent to number of vertex, we can conclude that $E = V$. This means the running time of "all-pair shortest path Dijkstra's algorithm" in terms of edges is $E^3 * lgE$.

The running time of all-pair shortest path with Dijkstra's algorithm in dense graph is O(V^4). In the dense graph, the number of edges is equivalent to the square of the number of vertex($E = V^2$). Therefore, the running time will be O(E^2) terms of dense graph.

2 Q2

No, they are different. If you were to implement the Dijkstra's algorithm for all-part shortest path in dense graph, the running time will be O(V^4) since the Dijkstra's algorithm should run on every single vertex, which will be O($V * V^3$). However, the Floyd-Warshall algorithm checks every element in adjacency matrix, compare and update the value depending on the if statement. In other words, the Floyd-Warshall in nature itself will solve all-pair shortest path, which its running time is O(V^3).

3 Q3

The assumption of this question is that Mary wants to solve the single-source shortest path problem on a dense graph with **negative weight**. Since the graph has a negative weight, she cannot use the Dijkstra's algorithm, but she can use the slow-version of bellman-ford algorithm. The running time of bellman-ford algorithm on dense graph is O(V^3) in terms of single-source path algorithm. The running time of Floyd-Warshall algorithm in all-pair shortest path algorithm is $\theta(V^3)$. In terms of time of execution they are same, but bellman-ford algorithm only loop through $|V| - 1$, and depending on sorting the all edges might not be the accurate answer because this is greedy algorithm. But, the Floyd-Warshall Algorithm may output better and accurate solution. So I think she should use the Floyd-Warshall Algorithm.

4 Q4

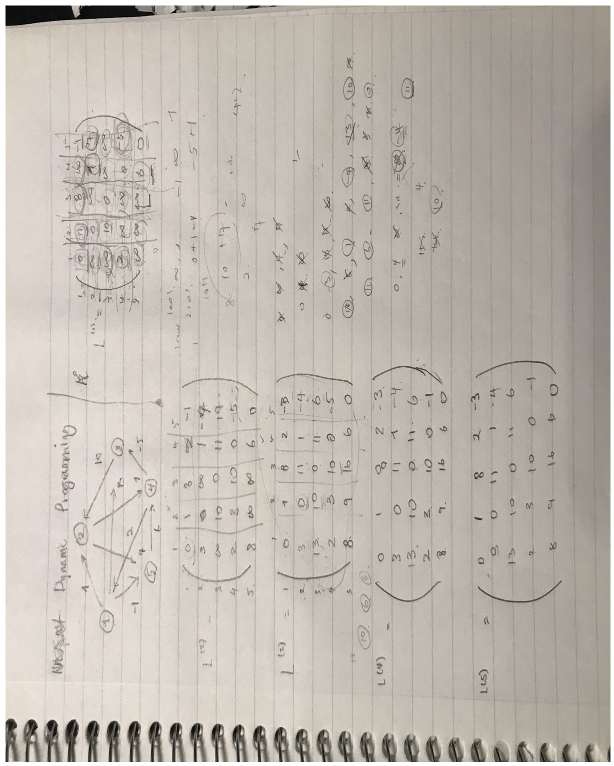

Figure 1: Q4

5 Q5

Q5 is submitted in the other file.

6 Q6

$https://colab.research.google.com/drive/10Q7nsb3EHPDsjNYDBNGXJ_nO_0DoOq4x?usp=sharing$