Topic Models

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Outline

Probabilistic topic models

Topic Models

- Topic models are algorithms for discovering the main "themes" in an unstructured corpus
- Requires no prior information, training set, or special annotation of the texts
  – only a decision on K (number of topics)
- A probabalistic, generative advance on several earlier methods, "Latent Semantic Analysis" (LSA) and "probabalistic latent semantic indexing" (pLSI)

Probabilistic topic models

- Topic modeling allows us to automatically organize, understand, and summarize large archives of text data.
- Uncover hidden themes.
- Annotate the documents according to themes.
- Organize the collection using annotations.

Latent Dirichlet allocation (LDA)

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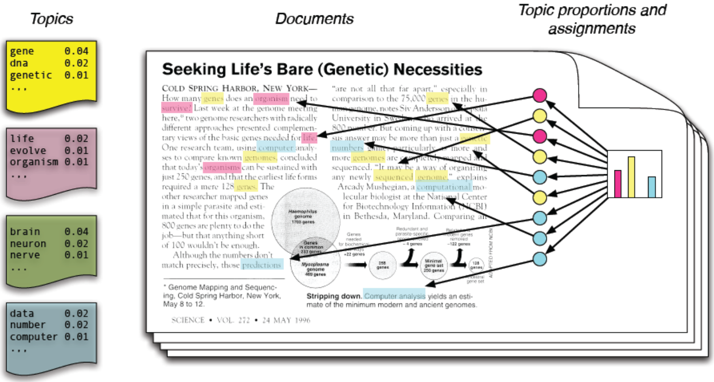
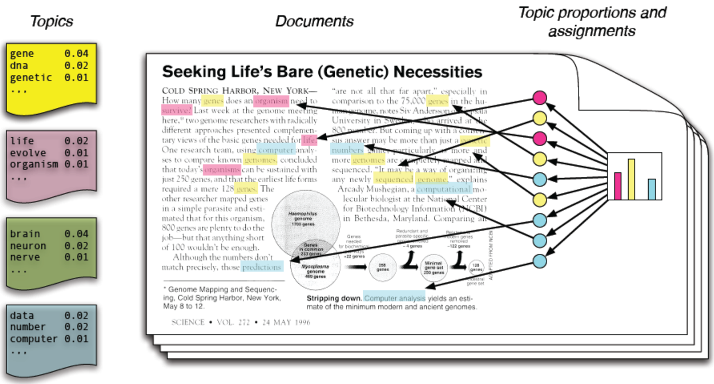


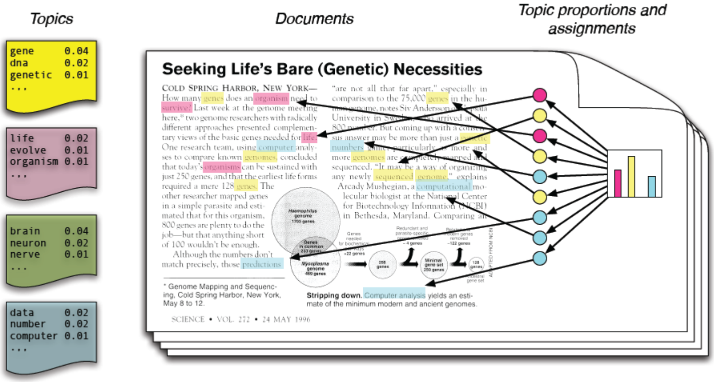
- Each topic is a distribution over words
- Each document is a mixture of corpus-wide topics
- Each word is drawn from one of those topics

Latent Dirichlet allocation (LDA)



- In reality, we only observe the documents
- The other structure are hidden variables

Latent Dirichlet allocation (LDA)



- Our goal is to infer the hidden variables
- I.e., compute their distribution conditioned on the documents

$$p(topics, proportions, assignments | documents)$$

Latent Dirichlet Allocation

- The LDA model is a Bayesian mixture model for discrete data where topics are assumed to be uncorrelated.
- LDA provides a generative model that describes how the documents in a dataset were created.
- Each of the K *topics* is a distribution over a fixed vocabulary.
- Each document is a collection of words, generated according to a multinomial distribution, one for each of K topics.
- Inference consists of estimating a posterior distribution from a joint distribution based on the probability model from a combination of what is observed (words in documents) and what is hidden (topic and word parameters).

Latent Dirichlet Allocation: Details

- For each document, the LDA generative process is:
  1. randomly choose a distribution over topics (a multinomial of length K)
  2. for each word in the document
     - 2.1 Probabilistically draw one of the K topics from the distribution over topics obtained in (a), say topic β_k (each document contains topics in different proportions)
     - 2.2 Probabilistically draw one of the V words from β_k (each individual word in the document is drawn from one of the K topics in proportion to the document's distribution over topics as determined in previous step)
- The goal of inference in LDA is to discover the topics from the collection of documents, and to estimate the relationship of words to these, *assuming this generative process*

LDA generative model

How to generate

1. Term distribution β for each topic is drawn:

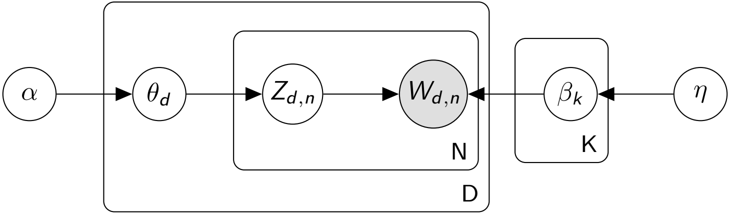
$$\beta \sim \text{Dirichlet}(\delta)$$

   β is the term distribution of topics and contains the probability of a word occurring in a given topic
2. proportions θ of the topic distribution for the document are drawn by

$$\theta \sim \text{Dirichlet}(\alpha)$$

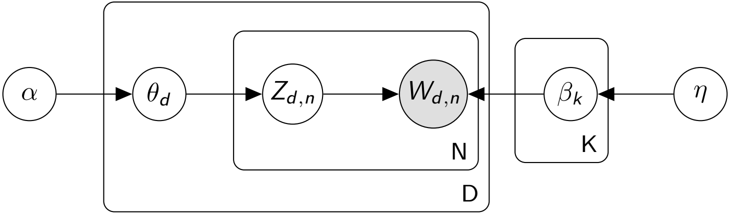
3. For each of the N words in each document
   - choose a topic $x_i \sim \text{Multinomial}(\theta)$
   - choose a word $w_i \sim \text{Multinomial}(p(w_i|z_i, \beta))$

LDA as a graphical model



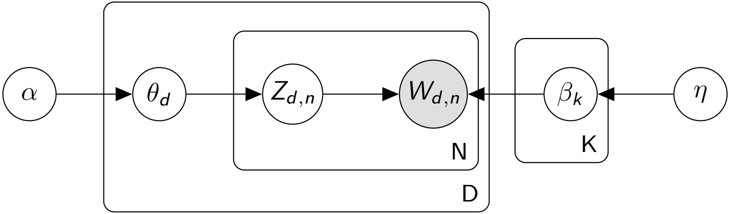
- Encodes assumptions
- Defines a factorization of the joint distribution
- Connects to algorithms for computing with data

LDA as a graphical model



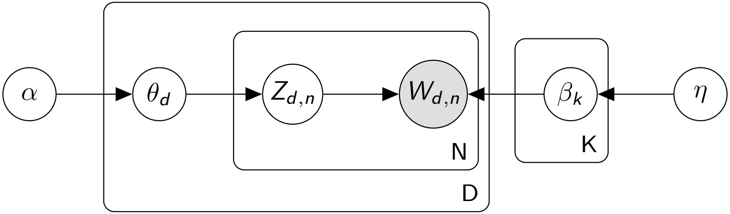
- Nodes are random variables; edges indicate dependence.
- Shaded nodes are observed; unshaded nodes are hidden.
- Plates indicate replicated variables.

LDA as a graphical model



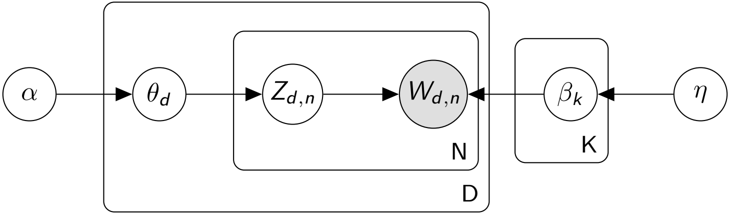
- α proportions parameter
- θ_d per-document topic proportions
- $Z_{d,n}$ per-word topic assignment
- $W_{d,n}$ observed word
- β_k topics
- η topic parameter

LDA as a graphical model



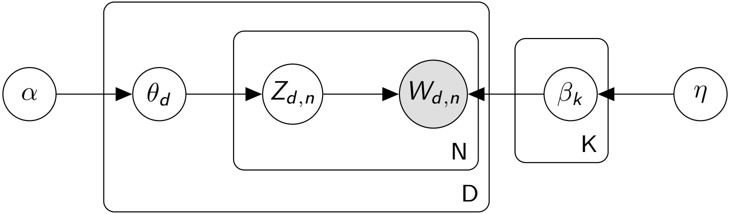
$$p(\beta, \theta, \mathbf{z}, \mathbf{w}) = \left(\prod_{i=1}^{K} p(\beta_i | \eta) \right) \left(\prod_{d=1}^{D} p(\theta_d | \alpha) \prod_{n=1}^{N} p(z_{d,n} | \theta_d) p(w_{d,n} | \beta_{1:k}, z_{d,n}) \right)$$

LDA as a graphical model



- This joint defines a posterior, $p(\theta, z, \beta | w)$.
- From a collection of documents, infer
  - Per-word topic assignment $z_{d,n}$
  - Per-document topic proportions θ_d
  - Per-corpus topic distributions β_k
- Then use posterior expectations to perform the task at hand: information retrieval, document similarity, exploration, and others.

LDA as a graphical model



- $\beta_k \sim \text{Dirichlet}(\eta)$
- $\theta_d \sim \text{Dirichlet}(\alpha)$
- $Z_{d,n} \sim \text{Multinomial}(\theta_\text{d})$
- $W_{d,n} \sim \text{Multinomial}(\text{p}(\text{w}_\text{i}|\text{z}_\text{i}, \beta_\text{k}))$

The Dirichlet distribution

- The Dirichlet distribution is an exponential family distribution over the simplex, i.e., positive vectors that sum to one

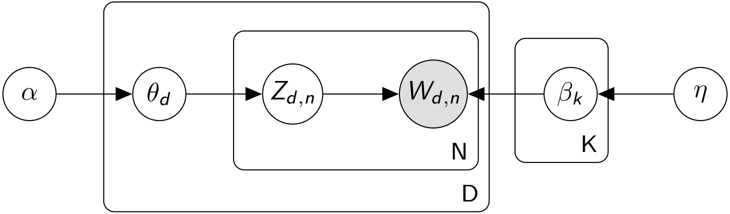
$$p(\theta|\vec{\alpha}) = \frac{\Gamma(\sum_i \alpha_i)}{\prod_i \Gamma(\alpha_i)} \prod_i \theta_i^{\alpha_i - 1}.$$

- It is conjugate to the multinomial.
- The Dirichlet is the conjugate prior distribution for the multinomial, and is used in the Bayesian inference required to estimate these parameters.
- The Dirichlet is used twice in LDA:
  - The topic proportions (θ) are a K dimensional Dirichlet
  - The topics (β) are a V dimensional Dirichlet.
- The parameter α controls the mean shape and sparsity of θ.
- Estimation is performed using (collapsed) Gibbs sampling and/or Variational Expectation-Maximization (VEM)

Why does LDA "work"?

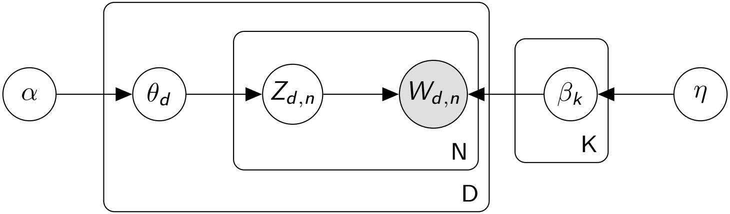
- LDA trades off two goals.
    1. For each document, allocate its words to as few topics as possible.
    2. For each topic, assign high probability to as few terms as possible.
- These goals are at odds.
    - Putting a document in a single topic makes (2) hard: All of its words must have probability under that topic.
    - Putting very few words in each topic makes (1) hard: To cover a document's words, it must assign many topics to it.
- Trading off these goals finds groups of tightly co-occurring words.

LDA summary



- LDA is a probabilistic model of text. It casts the problem of discovering themes in large document collections as a posterior inference problem.
- It lets us visualize the hidden thematic structure in large collections, and generalize new data to fit into that structure.
- Builds on latent semantic analysis (Deerwester et al., 1990; Hofmann, 1999). It is a mixed-membership model (Erosheva, 2004). It relates to PCA and matrix factorization (Jakulin and Buntine, 2002). It was independently invented for genetics (Pritchard et al., 2000).

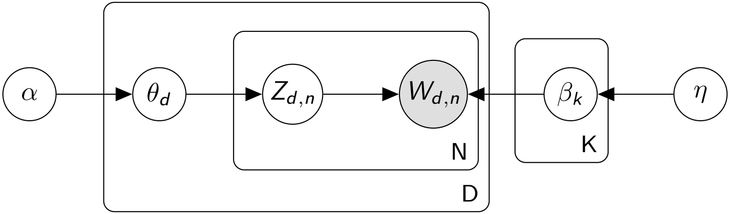
LDA summary



- LDA is a simple building block that enables many applications.
- It is popular because organizing and finding patterns in data has become important in the sciences, humanities, industry, and culture.
- Further, algorithmic improvements let us fit models to massive data.

Beyond Latent Dirichlet Allocation

LDA summary



- LDA is a simple topic model.
- It can be used to find topics that describe a corpus.
- Each document exhibits multiple topics.
- There are several ways to extend this model.

Extending LDA

- LDA can be embedded in more complicate models, embodying further intuitions about the structure of the texts.
- E.g., it can be used in models that account for syntax, authorship, word sense, dynamics, correlation, hierarchies, and other structure.
- The data generating distribution can be changed. We can apply mixed-membership assumptions to many kinds of data.
- E.g., we can build models of images, social networks, music, purchase histories, computer code, genetic data, and other types.
- The posterior can be used in creative ways.
- E.g., we can use inferences in information retrieval, recommendation, similarity, visualization, summarization, and other applications.

Extending LDA

- These different kinds of extensions can be combined.
- To give a sense of how LDA can be extended, we'll look at several examples of major extensions.
- We will discuss
  - Correlated topic models
  - Dynamic topic models
  - Supervised topic models
  - Relational topic models
  - Ideal point topic models
  - Collaborative topic models

Correlated and Dynamic Topic Models

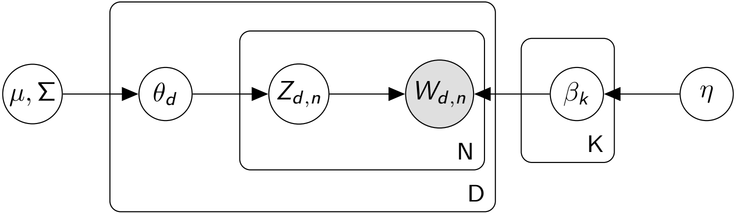
Correlated topic models

- The Dirichlet is a distribution on the simplex, positive vectors that sum to 1.
- It assumes that components are nearly independent.
- In real data, an article about fossil fuels is more likely to also be about geology than about genetics.
- The logistic normal is a distribution on the simplex that can model dependence between components (Aitchison, 1980).
- The log of the parameters of the multinomial are drawn from a multivariate Gaussian distribution,

$$X \sim N_k(\mu, \Sigma)$$

$$\theta_i \propto exp\{x_i\}.$$

Correlated topic models



where the first node is logistic normal prior.

- Draw topic proportions from a logistic normal.
- This allows topic occurrences to exhibit correlation.
- Provides a "map" of topics and how they are related
- Provides a better fit to text data, but computation is more complex

Dynamic topic models

- LDA assumes that the order of documents does not matter.
- Not appropriate for sequential corpora (e.g., that span hundreds of years)
- Further, we may want to track how language changes over time.
- Dynamic topic models let the topics drift in a sequence.

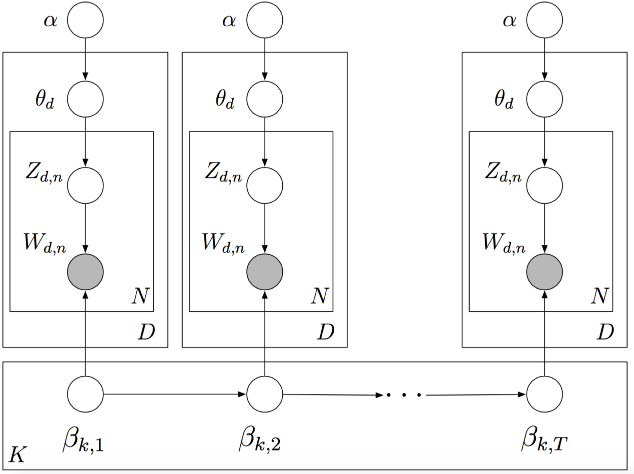


Plate (K) is topics drift through time.

Dynamic topic models

$$\beta_{k,1} \rightarrow \beta_{k,2} \rightarrow \cdots \rightarrow \beta_{k,T}$$

- Use a logistic normal distribution to model topics evolving over time.
- Embed it in a state-space model on the log of the topic distribution

$$\beta_{t,k} | \beta_{t-1,k} \sim N(\beta_{t-1,k}, I\sigma^2)$$

$$p(w|\beta_{t,k}) \propto exp\{\beta_{t,k}\}$$

- As for CTMs, this makes computation more complex. But it lets us make inferences about sequences of documents.

Dynamic topic models

- Time-corrected similarity shows a new way of using the posterior.
- Consider the expected Hellinger distance between the topic proportions of two documents,

$$d_{ij} = E\left[\sum_{k=1}^{K} (\sqrt{\theta_{i,k}} - \sqrt{\theta_{j,k}})^2 | \mathbf{w}_i, \mathbf{w}_j\right]$$

- Uses the latent structure to define similarity
- Time has been factored out because the topics associated to the components are different from year to year.
- Similarity based only on topic proportions

Summary: Correlated and dynamic topic models

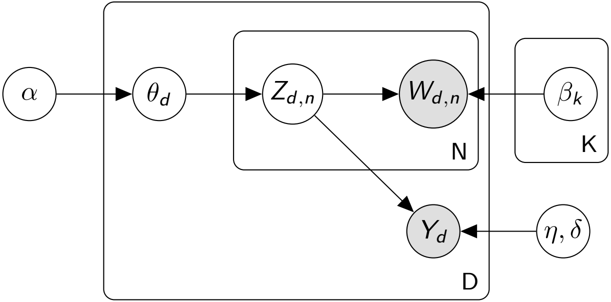
- The Dirichlet assumption on topics and topic proportions makes strong conditional independence assumptions about the data.
- The correlated topic model uses a logistic normal on the topic proportions to find patterns in how topics tend to co-occur.
- The dynamic topic model uses a logistic normal in a linear dynamic model to capture how topics change over time.
- What's the catch? These models are harder to compute.

Supervised Topic Models

Supervised LDA

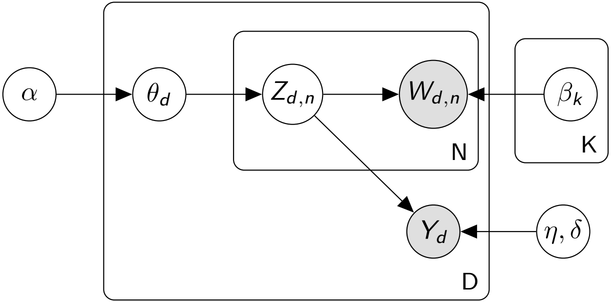
- LDA is an unsupervised model. How can we build a topic model that is good at the task we care about?
- Many data are paired with response variables.
  - User reviews paired with a number of stars
  - Web pages paired with a number of "likes"
  - Documents paired with links to other documents
  - Images paired with a category
- Supervised LDA are topic models of documents and responses. They are fit to find topics predictive of the response.

Supervised LDA



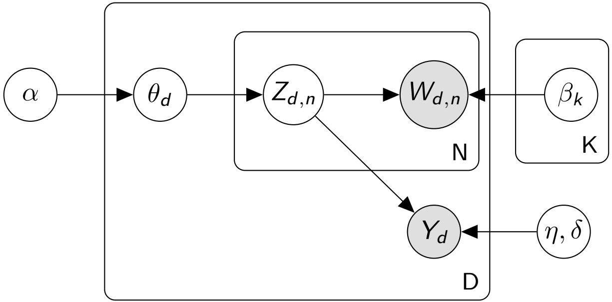
- Y_d is document response
- η, δ regression parameters

Supervised LDA



1. Draw topic proportions $\theta|\alpha \sim Dir(\alpha)$
2. For each word
   - Draw topic assignment $z_n|\theta \sim Mult(\theta)$.
   - Draw word $w_n|z_n, \beta_{1:K} \sim Mult(\beta_{z_n})$
3. Draw response variable $y|z_{1:N}, \eta, \sigma^2 \sim N(\eta^T \bar{z}, \sigma^2)$ where $\bar{z} = (1/N) \sum_{n=1}^{N} z_n$.

Supervised LDA

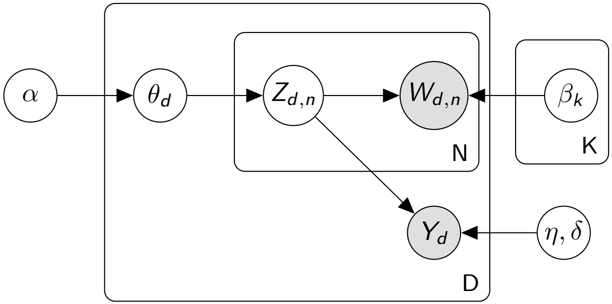


- Fit sLDA parameters to documents and responses. This gives: topics $\beta_{1:K}$ and coefficients $\eta_{1:K}$.
- Given a new document, predict its response using the expected value:

$$E\left[Y | w_{1:N}, \alpha, \beta_{1:K}, \eta, \sigma^2\right] = \eta^T E[\bar{Z} | w_{1:N}]$$

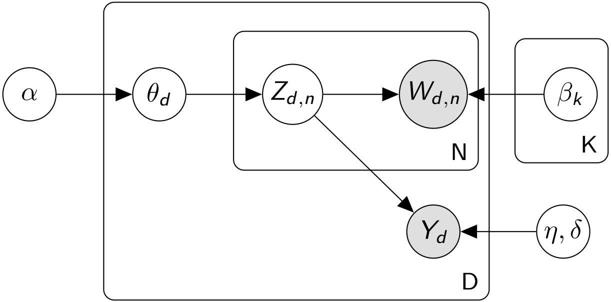
- This blends generative and discriminative modeling.

Supervised LDA



- sLDA enables model-based regression where the predictor is a document.
- It can easily be used wherever LDA is used in an unsupervised fashion (e.g., images, genes, music).
- sLDA is a supervised dimension-reduction technique, whereas LDA performs unsupervised dimension reduction.

Supervised LDA



- sLDA has been extended to generalized linear models, e.g., for image classification and other non-continuous responses.
- We will discuss two extensions of sLDA
  - Relational topic models: Models of networks and text
  - Ideal point topic models: Models of legislative voting behavior

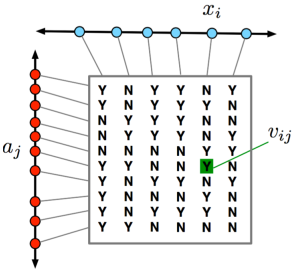
Relational topic models

- Many data sets contain connected observations.
- For example:
  - Citation networks of documents
  - Hyperlinked networks of web-pages.
  - Friend-connected social network profiles

Relational topic models

- Research has focused on finding communities and patterns in the link-structure of these networks. But this ignores content.
- sLDA was adapted to pairwise response variables. This leads to a model of content and connection.
- Relational topic models find related hidden structure in both types of data.

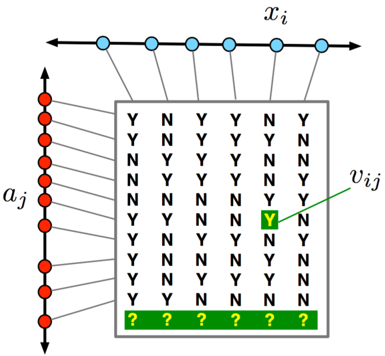
Ideal point topic models



$$p(v_{ij}) = f(d(x_i, a_j))$$

- The ideal point model uncovers voting patterns in legislative data.
- We observe roll call data v_{ij}.
- Bills attached to discrimination parameters a_j. Senators attached to ideal points x_i.
- Posterior inference reveals the political spectrum of senators.
- Widely used in quantitative political science.

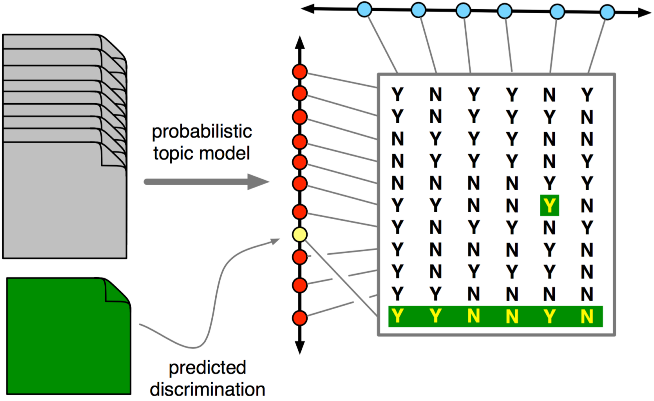
Ideal point topic models



$$p(v_{ij}) = f(d(x_i, a_j))$$

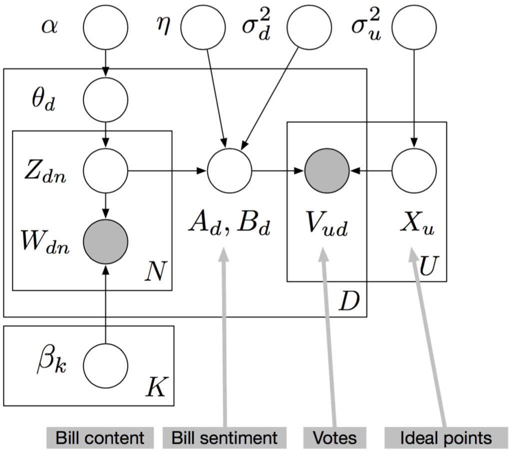
- We can predict a missing vote.
- But we cannot predict all the missing votes from a bill.
- Cf. the limitations of collaborative filtering

Ideal point topic models



- Use supervised LDA to predict bill discrimination from bill text.
- But this is a latent response.

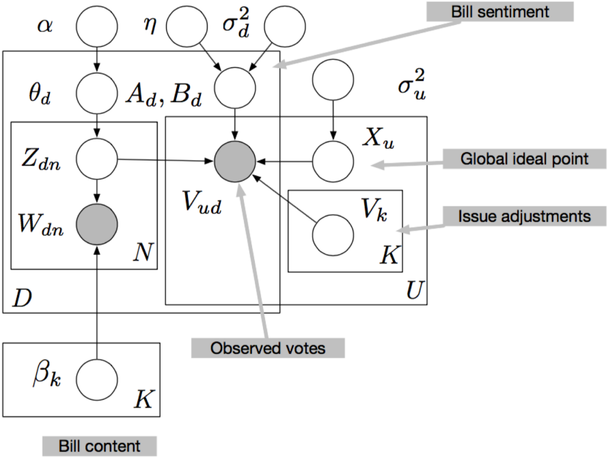
Ideal point topic models



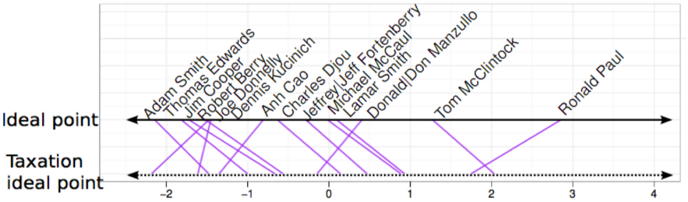
Issue-adjusted ideal points

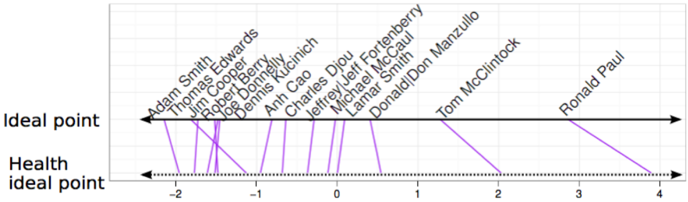
- Ideal point model uses topics to predict votes from new bills.
- Alternatively, we can use the text to characterize how legislators diverge from their usual ideal points.
- For example: A senator might be left wing, but vote conservatively when it comes to economic matters.

Issue-adjusted ideal points



Issue-adjusted ideal points





Summary: Supervised topic models

- Many documents are associated with response variables.
- Supervised LDA embeds LDA in a generalized linear model that is conditioned on the latent topic assignments.
- Relational topic models use sLDA assumptions with pair-wise responses to model networks of documents.
- Ideal point topic models demonstrates how the response variables can themselves be latent variables. In this case, they are used downstream in a model of legislative behavior.
- (sLDA, the RTM, and others are implemented in the R package "lda.")

Structural Topic Model

Structural Topic Model

- We may be interested in how some covariate is associated with the prevalence of topic usage (Gender, date, political party, etc).
- The Structural Topic Model (STM) allows for the inclusion of arbitrary covariates of interest into the generative model
- The addition of covariates provides structure to the prior distributions
  - Benefit 1: improves the estimation of the topics by allowing documents to share information according to the covariates (known as 'partial pooling' of parameters)
  - Benefit 2: the relationship between covariates and latent topics is most frequently the estimand of interest, so we should include this in the estimation procedure

Structural Topic Model

- As with the CTM, topics within the STM can be correlated
- Topic prevalence is allowed to vary according to the covariates X
  - Each document has its own prior distribution over topics, which is defined by its covariates, rather than sharing a global mean.
- Topical content can also vary according to the covariates Y
  - Word use *within* a topic can differ for different groups of speakers/writers

Structural Topic Model

Topic prevalence model:

- Draw topic proportions from a logistic normal generalised linear model based on covariates X
- This allows the expected document-topic proportions to vary by covariates, rather than from a single shared prior

Structural Topic Model

Topical content model:

- The β coefficients, which indicate the distribution over words for a given topic, are allowed to vary according to the covariates Y.
- This allows us to estimate how different covariates affect the words used *within a given topic*.

Extending LDA

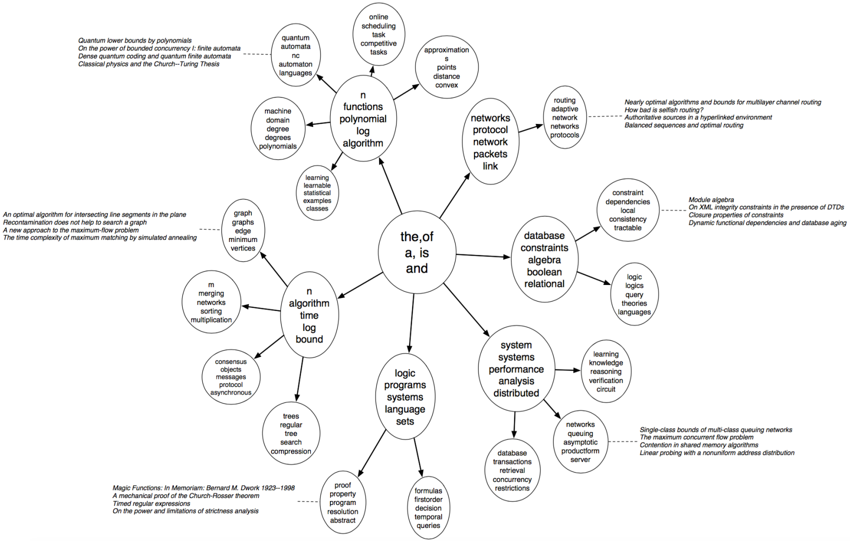
- Syntactic topic models
- Topic models on images
- Topic models on social network data
- Topic models on music data
- Topic models for recommendation systems
- Spike and slab priors
- Models of word contagion
- N-gram topic models

Bayesian Nonparametric Models

Bayesian Nonparametric Models

- Topic models assume that the number of topics is fixed.
- It is a type of regularization parameter. It can be determined by cross validation and other model selection techniques.
- Bayesian nonparametric methods skirt model selection:
  - The data determine the number of topics during inference.
  - Future data can exhibit new topics.
- (This is a field unto itself, but has found wide application in topic modeling.)

Hierarchical topic model (Blei et al. 2010)



Summary: Bayesian nonparametrics

- Bayesian nonparametric modeling is a growing field (Hjort et al., 2011).
- BNP methods can define priors over latent combinatorial structures.
- In the posterior, the documents determine the particular form of the structure that is best for the corpus at hand.
- Recent innovations:
    - Improved inference (Blei and Jordan, 2006, Wang et al. 2011)
    - BNP models for language (Teh, 2006; Goldwater et al., 2011)
    - Dependent models, such as time series models (MacEachern 1999, Dunson 2010, Blei and Frazier 2011)
    - Predictive models (Hannah et al. 2011)
    - Factorization models (Griffiths and Ghahramani, 2011)

Evaluating LDA performance

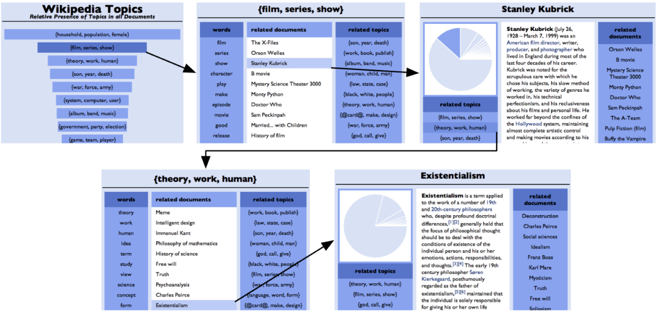
Using and Checking Topic Models

- We have collected data, selected a model, and inferred the posterior.
- How do we use the topic model?
- Using a model means doing something with the posterior inference.
- E.g., visualization, prediction, assessing document similarity, using the representation in a downstream task (like IR).

Using and Checking Topic Models

- Questions we ask when evaluating a model:
  - Does my model work? Is it better than another model?
  - Which topic model should I choose? Should I make a new one?
- These questions are tied up in the application at hand.
- Sometimes evaluation is straightforward, especially in prediction tasks.

Using and Checking Topic Models



- But a promise of topic models is that they give good exploratory tools. Evaluation is complicated, e.g., is this a good navigator of my collection?
- And this leads to more questions:
  - How do I interpret a topic model?
  - What quantities help me understand what it says about the data?

Using and Checking Topic Models

- How to interpret and evaluate topic models is an active area of research.
  - Visualizing topic models
  - Naming topics
  - Matching topic models to human judgements
  - Matching topic models to external ontologies
  - Computing held out likelihoods in different ways

Perplexity

- Perplexity: can be computed as (using VEM):

$$\text{perplexity}(w) = \exp\left\{-\frac{\sum_{d=1}^{M} \log p(w_d)}{\sum_{d=1}^{M} N_d}\right\}$$

- lower perplexity score indicates better performance

Evaluating model performance: human judgment

(Chang, Jonathan et al. 2009. "Reading Tea Leaves: How Humans Interpret Topic Models." *Advances in neural information processing systems*.)

Uses human evaluation of:

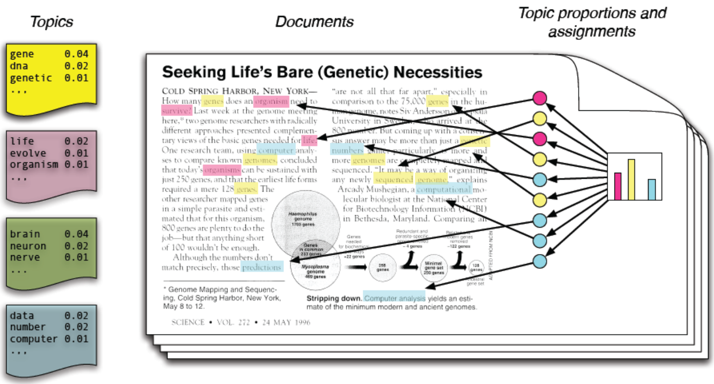
- whether a topic has (human-identifiable) semantic coherence: word intrusion, asking subjects to identify a spurious word inserted into a topic
- whether the association between a document and a topic makes sense: topic intrusion, asking subjects to identify a topic that was not associated with the document by the model

- Often the quality measures from human benchmarking were negatively correlated with traditional quantitiative diagnostic measures.

Summary

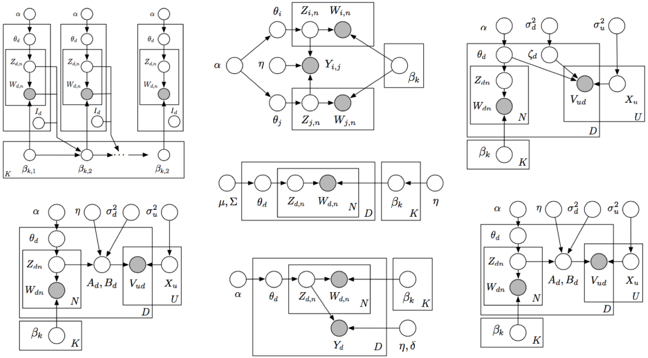
- What are topic models?
- What kinds of things can they do?
- How do I compute with a topic model?
- How do I evaluate and check a topic model?
- What are some unanswered questions in this field?
- How can I learn more?

Summary



- LDA assumes that there are K topics shared by the collection.
- Each document exhibits the topics with different proportions.
- Each word is drawn from one topic.
- We discover the structure that best explain a corpus.

Summary



Topic models can be adapted to many settings

- relax assumptions
- combine models
- model more complex data

Implementations of topic models in R

Incomplete list:

- lda
- topicmodels
- stm
- mallet
- textmineR
- text2vec
- LDAvis