1.

e) clustering coeff: $c_i = \dfrac{\delta \, |\overset{(*)}{\{e_{jk} : v_j, v_k \in N_i,\ e_{jk} \in E\}}|}{k_i(k_i - 1)}$,

(*) is a number of connections between person's neighbours.

k_i is a number of person's neighbours.

$k_i = |N_i|$

$N_i = \{v_j : e_{ij} \in E \lor e_{ji} \in E\}$

$\delta = 2$ for undirected connection (1 for directed)

E - closest neighbours

Carl	$c_i = \frac{2\cdot 2}{3\cdot 2} = \frac{2}{3}$	Bob	$c_i = \frac{2\cdot 0}{2\cdot 1} = 0$
Frank	$c_i = \frac{2\cdot 2}{3\cdot 2} = \frac{2}{3}$	Gail	$c_i = \frac{2\cdot 3}{4\cdot 3} = \frac{1}{2}$
Alice	$c_i = \frac{2\cdot 3}{5\cdot 4} = \frac{3}{10}$	Harry	$c_i = \frac{2\cdot 3}{3\cdot 2} = 1$
Ernst	$c_i = \frac{2\cdot 1}{2\cdot 1} = 1$	Jen	$c_i = \frac{2\cdot 3}{3\cdot 2} = 1$
David	$c_i = \frac{2\cdot 1}{2\cdot 1} = 1$	Irene	$c_i = \frac{2\cdot 3}{3\cdot 2} = 1$

$$\bar{c_i} = \frac{2/3 + 2/3 + 3/10 + 1 + 1 + 0 + 1/2 + 1 + 1 + 1}{10} = \frac{107}{150}$$

f) closeness centrality: $C(v) = \dfrac{\overset{= 9 \text{ in our case}}{N-1}}{\sum_y d(y,v)}$, $d(y,v)$ is the shortest path between y and v.

$C(\text{Carl}) = 9/(1+2+1+2+1+3+4+4+4) = 9/22$ $C(\text{Bob}) = 9/(1+2+2+2+2+1+2+2+2) = 9/16$

$C(\text{Frank}) = 9/(1+2+1+2+1+3+4+4+4) = 9/22$ $C(\text{Gail}) = 9/(2+1+3+3+3+3+1+1+1) = 9/18$

$C(\text{Alice}) = 9/(1+1+1+1+1+2+3+3+3) = 9/16$ $C(\text{Harry}) = 9/(3+2+4+4+4+4+1+1+1) = 9/24$

$C(\text{Ernst}) = 9/(1+2+2+2+1+3+4+4+4) = 9/23$ $C(\text{Jen}) = 9/(3+2+4+4+4+4+1+1+1) = 9/24$

$C(\text{David}) = 9/(1+2+1+2+2+3+4+4+4) = 9/23$ $C(\text{Irene}) = 9/(3+2+4+4+4+4+1+1+1) = 9/24$

The most central node is for the highest value of closeness centrality so Alice and Bob.

g) betweenness centrality: $B(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$

$\sigma_{st}(v)$ is number of σ_{st}'s that pass node V.

σ_{st} is a number of shortest paths between s and t.

For Carl: (let $B = \frac{\sigma_{st}(v)}{\sigma_{st}}$)

Alice - Bob	0/1
Alice - David	0/1
Alice - Ernst	0/1
Alice - Frank	0/1
Alice - Gail	0/1
Alice - Harry	0/1
Alice - Irene	0/1
Alice - Jen	0/1
Bob - David	0/1
Bob - Ernst	0/1
Bob - Frank	0/1
Bob - Gail	0/1
Bob - Harry	0/1
Bob - Irene	0/1
Bob - Jen	0/1
David - Ernst	0/1
David - Frank	1/2
David - Gail	0/1
David - Harry	0/1
David - Irene	0/1
David - Jen	0/1
Ernst - Frank	0/1
Ernst - Gail	0/1
Ernst - Harry	0/1
Ernst - Irene	0/1
Ernst - Jen	0/1
Frank - Gail	0/1
Frank - Harry	0/1
Frank - Irene	0/1
Frank - Jen	0/1
Gail - Harry	0/1
Gail - Irene	0/1
Gail - Jen	0/1
Harry - Irene	0/1
Harry - Jen	0/1
Irene - Jen	0/1

$B(V) = 0,5$

The same way as for Carl I calculated other nodes and got results:

Node	B(v)
Carl	0,5
Frank	0,5
Alice	22
Ernst	0
David	0
Bob	20
Gail	18
Harry	0
Jen	0
Irene	0

The most central node is Alice because she has the highest value of betweenness centrality.

3.

	Carl	Frank	Alice	Ernst	David	Bob	Gail	Harry	Jen	Irene
Carl	0	1	1	0	1	0	0	0	0	0
Frank	1	0	1	1	0	0	0	0	0	0
Alice	1	1	0	1	1	1	0	0	0	0
Ernst	0	1	1	0	0	0	0	0	0	0
David	1	0	1	0	0	0	0	0	0	0
Bob	0	0	1	0	0	0	1	0	0	0
Gail	0	0	0	0	0	1	0	1	1	1
Harry	0	0	0	0	0	0	1	0	1	1
Jen	0	0	0	0	0	0	1	1	0	1
Irene	0	0	0	0	0	0	1	1	1	0

MATRIX A

a) $k = (3, 3, 5, 2, 2, 2, 4, 3, 3, 3)^T$

Degree is a number of edges that arrive or depart from a node.

$k = A \cdot e$

b) $L = \frac{1}{2} \cdot k^T \cdot e = \frac{1}{2} \cdot 30 = 15$ (Actually 30 is $\det[k^T \cdot e]$)

c) $N = A^2$ - we use a square of A to find values of N, because we want to find where two people have the same neighbours - then we calculate this and we do not have 0 when two people have value 1 with a neighbour.

d) $T = \frac{1}{6} tr(A^3)$ - we use trace of A^3 because we want to find connections between 3 people and we divide it by 6 because in undirected connection we count every triangle 6 times.

c)

	Carl	Frank	Alice	Ernst	David	Bob	Gail	Harry	Jen	Irene
Carl	3	1	2	2	1	1	0	0	0	0
Frank	1	3	2	1	2	1	0	0	0	0
Alice	2	2	5	1	1	0	0	0	0	0
Ernst	2	1	1	2	1	1	0	0	0	0
David	1	2	1	1	2	1	0	0	0	0
Bob	1	1	0	1	1	2	0	1	1	1
Gail	0	0	0	0	0	0	4	2	2	2
Harry	0	0	0	0	0	1	2	3	2	2
Jen	0	0	0	0	0	1	2	2	3	2
Irene	0	0	0	0	0	1	2	2	2	3

d) $T = 7$

e) When matrix A is full of 1 (except main diagonal where we have 0) it means network is connected.