CMPUT 175
Introduction to Foundations of Computing

Searching

Objectives

- Introduce two techniques for searching for an element in a collection
- Learn sequential search algorithm.
- Learn the binary search algorithm for ordered collections
- Learn how to evaluate the complexity of an algorithm and compare between algorithms

Outline of Lecture

- Review the simple list examples
- Sequential search approach
- Complexity of sequential search
- Binary search approach
- Complexity of binary search
- Compare sequential search and binary search

Array Example

```python
# Find the largest element in a list of
numbers

markList = [50, 37, 71, 99, 63]
max = markList[0]
for index in range(1,len(markList)):
        if (markList[index] > max):
                max = markList[index]
print("highest mark=",max)
```

markList

50	0
37	1
71	2
99	3
63	4

index=4

max

99

Array Example2

```
# Find the index of the largest element in a list of numbers
markList = [50, 37, 71, 99, 63]
indexOfMax = 0
for index in range(1,len(markList)):
    if (markList[index] > markList[indexOfMax]):
        indexOfMax = index
print("index of highest mark=",indexOfMax)
```

markList

markList	index
50	0
37	1
71	2
99	3
63	4

index = 4

indexOfMax

3

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The Search Problem

- Given a container, find the index of a particular element, called the key.
- Technique applies for lists, arrays, files, etc.
- Applications: information retrieval, database querying, etc.

0	1	2	3	4	5	6	7	8	9
25	50	10	95	75	30	70	55	60	80

Collection

30

Element sought for

Sequential Search

- Compare the key to each element in turn, until the correct element is found, and return its index.

0	1	2	3	4	5	6	7	8	9
25	50	10	95	75	30	70	55	60	80
30	30	30	30	30	30				

Sequential Search Code

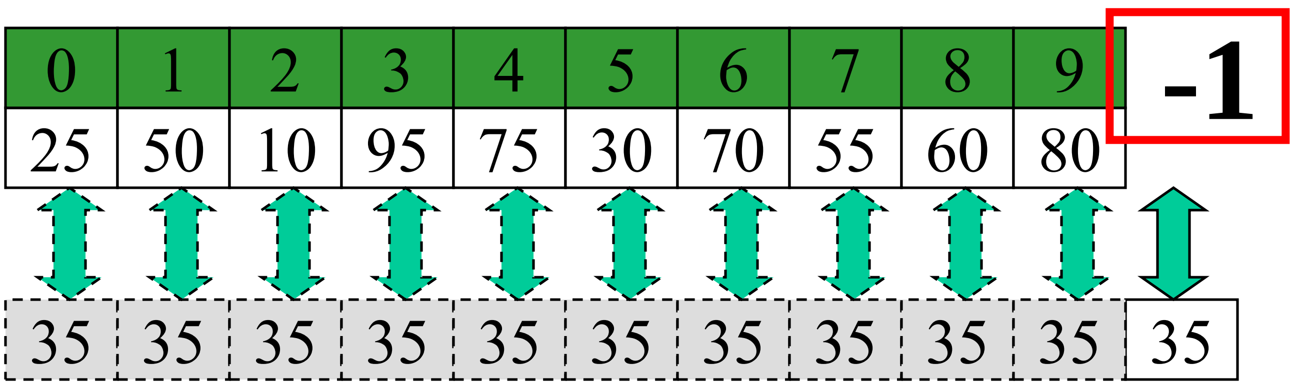
Compare all elements of the collection until we find the key.

```
#  a sequential search code (first tentative)
def sequential_search(data, key):
    found = False
    index = 0

    while ( not found ) :
        if ( key == data[index] ):
            found = True
        else:
            index = index + 1

    return index
```

Could be a problem here

Element not found

- We must take into account that the key we are searching for may not be in the list.
- In this case we must return a special index, say -1.



Search Algorithm

INPUT: data: list of int; key: int;

OUTPUT: index : an int such that

data[index] == key if key is in data,

or -1 if key is not stored in data.

Method:

1. index = 0; found=false;
2. While (not found and index < data.length)

   check similarity data[index] and key

   index = index + 1
3. if not found then index = -1;

Revised Sequential Search Code

```
#   a sequential search method
def sequential_search(data, key) :
    found = False
    index = 0

    while ( not found and index < len(data) ):
        if ( key == data[index] ):
            found = True
        else:
            index = index + 1

    if  ( not found):
        index = -1
    return index
```

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Complexity Analysis

- How efficient is this algorithm?
- In general if we have an algorithm that does something with ***n*** objects, we want to express the time efficiency of the algorithm as a function of ***n***.
- Such an expression is called the **time complexity** of the algorithm.
- In the case of search, we can count the number of comparison operations between the key and the elements.

Worst, Best and Average cases

- In fact, we usually have multiple expressions:
  - the worst case complexity,
  - the best case complexity
  - the average case complexity.

Complexity of Sequential Search

- How many comparison operations are required for a sequential search of an n-element container?

- In the worst case ➔ n.
- In the best case ➔ 1.
- In the average case: $\frac{1+2+3+\ldots+n}{n} = \frac{n(n+1)}{2n} = \frac{(n+1)}{2}$

Key not in container or last position

Key is in first position in container

- In this case, we say the complexity of Search is in the order of *n*, denoted as ***O(n)***.
- **Can we improve this algorithm?**

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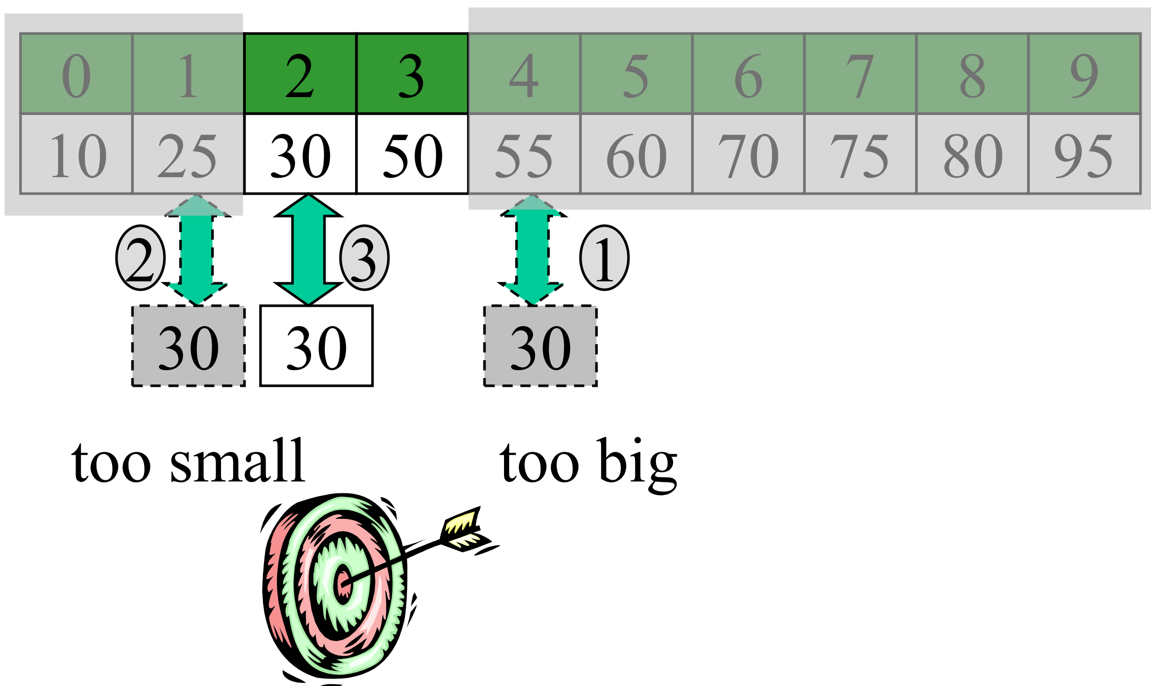
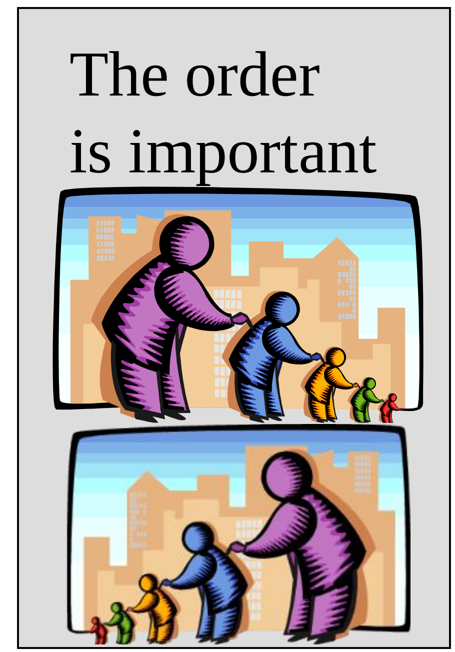
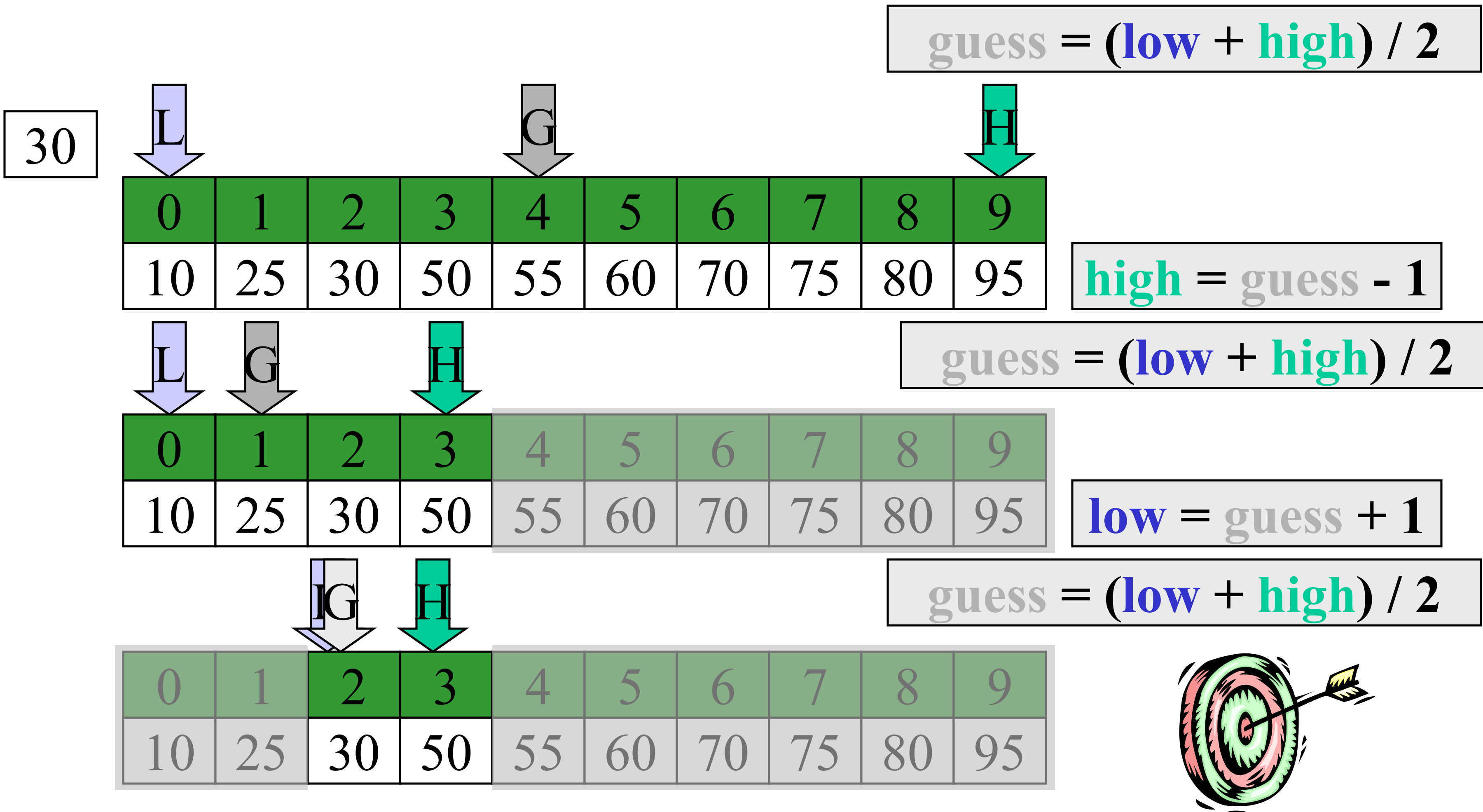
Binary Search

- If the elements are <u>ordered</u>, we can do better.
- Guess the middle and adjust accordingly.



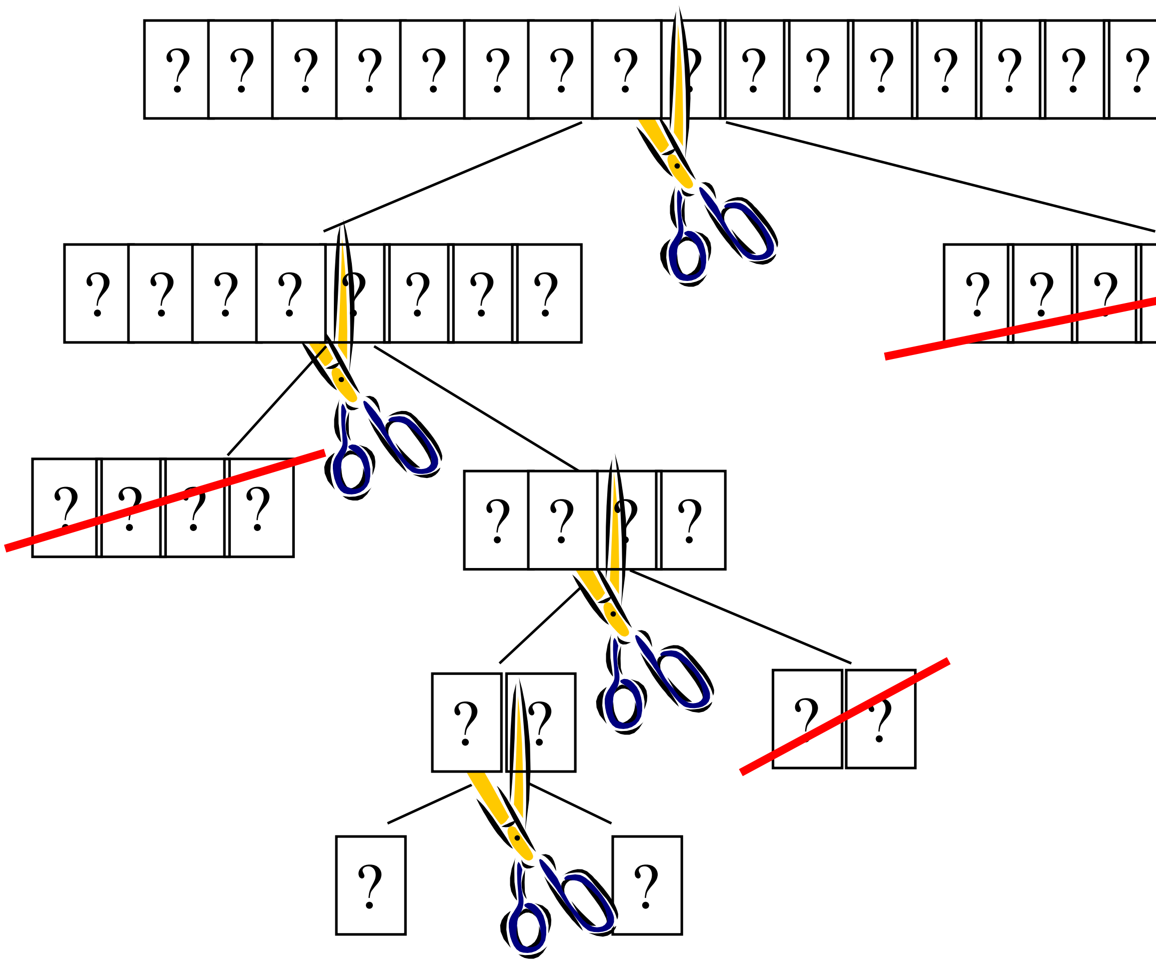


Binary Search Algorithm



Strategy of Binary Search:

Given an ordered list of integers, and a value of integer, search for the value in the array using an approach of **Divide and Conquer .**



Like searching in a dictionary, calendar or white pages.
Similar strategy as guess game in Lab exercise 8.

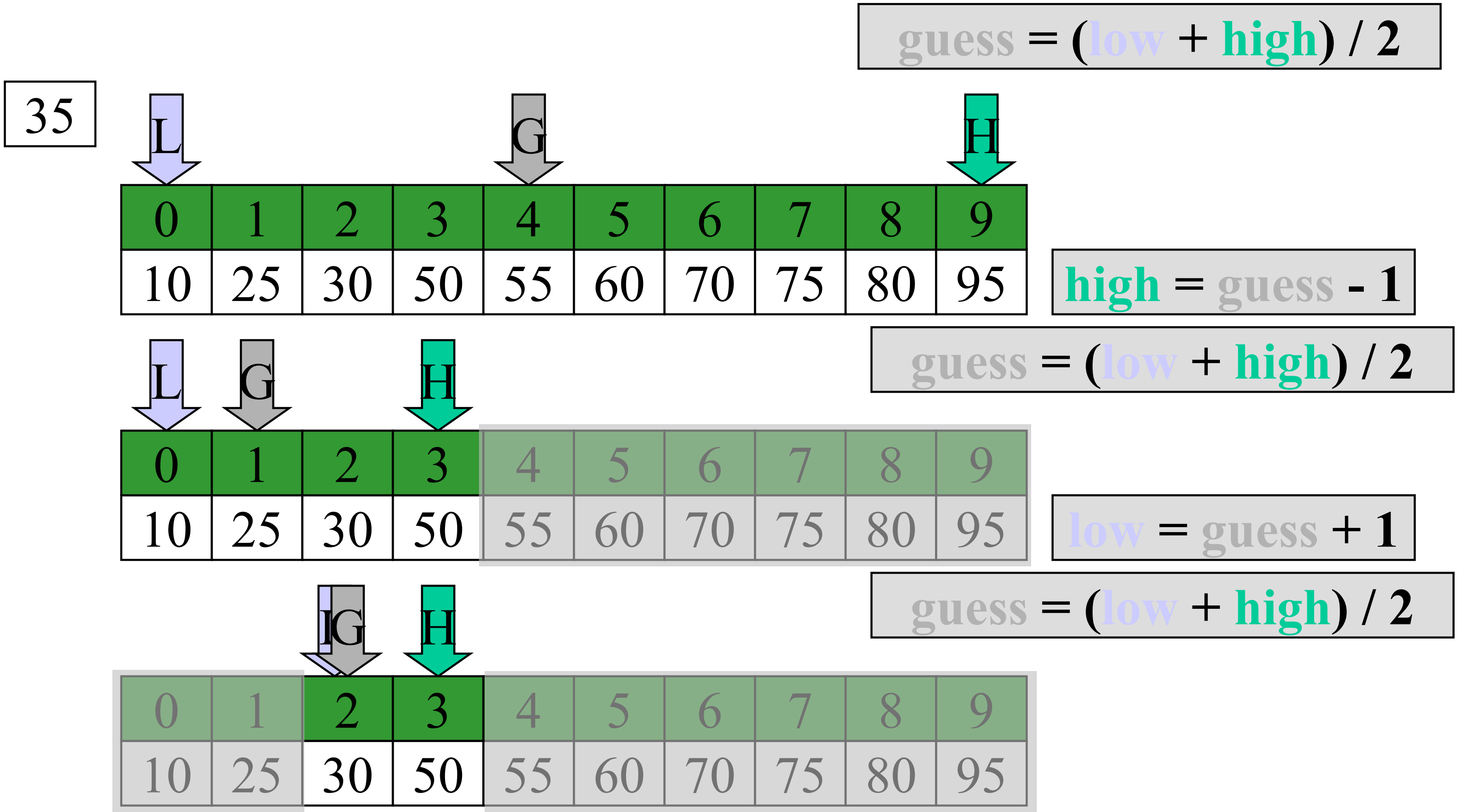
Binary Search Code

Divide in 2 between lower and upper bounds until we find the key.

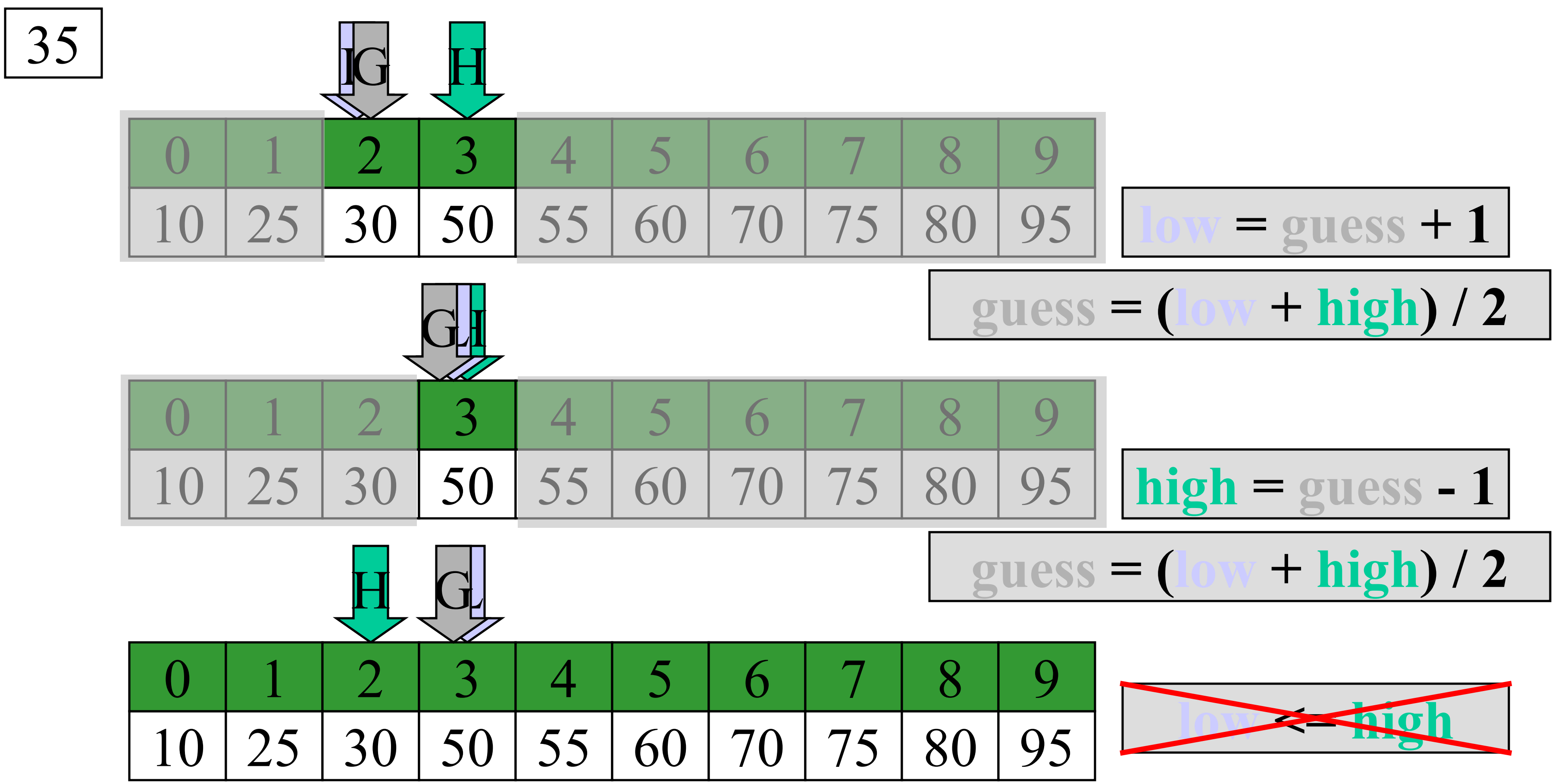
```
# a binary search code of ordered array (first tentative)
def binary_search( data, key ) :
    found = False
    low = 0
    high=len(data)-1
    while ( not found ) :
        guess = (high+low)//2
        if ( key == data[guess] ):
            found = True
        else:
            if (key < data[guess]):
                high=guess-1
            else:
                low = guess+1
    return guess
```

Could be a problem here

Element not found



Element not found (con’t)



Binary Search Algorithm

INPUT: data: list of ordered int; key: int;
OUTPUT: index : an int such that
data[index] = = key if key is in data,
or -1 if key is not stored in data.

Method:

```
1.  lower = 0;  upper = length;
2.  While ( not found && low < =upper )
        index = (lower + upper) /2;
        check similarity data[index] and key
        if similar then found, otherwise
            if  key  < data[index]
                    upper =  index-1;
            else   lower = index +1;
3.  If  ( data[index] != key )  index = -1;
```

```python
#   a binary search code of ordered array
def binary_search( data, key ) :
    found = False
    low = 0
    high=len(data)-1
    while ( not found and low<=high) :
        guess = (high+low)//2
        if ( key == data[guess] ):
            found = True
        else:
            if (key < data[guess]):
                high=guess-1
            else:
                low = guess+1
    if (not found):
        guess=-1
    return guess
```

Revised Binary Search Code

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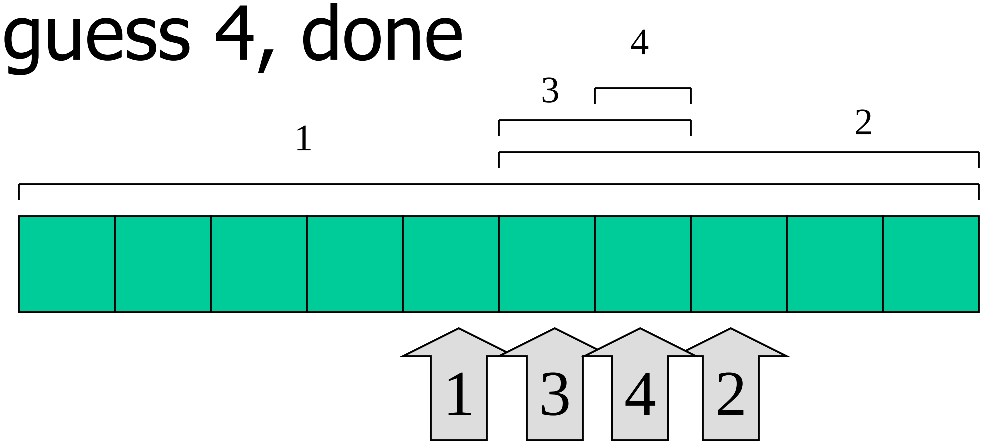
Worst-case Binary Search

- Each time we guess, we divide the list in half:
- In the worst case:
  - 10 elements, make guess 1, then
  - 5 elements, make guess 2, then
  - 2 elements, make guess 3, then
  - 1 element, make guess 4, done

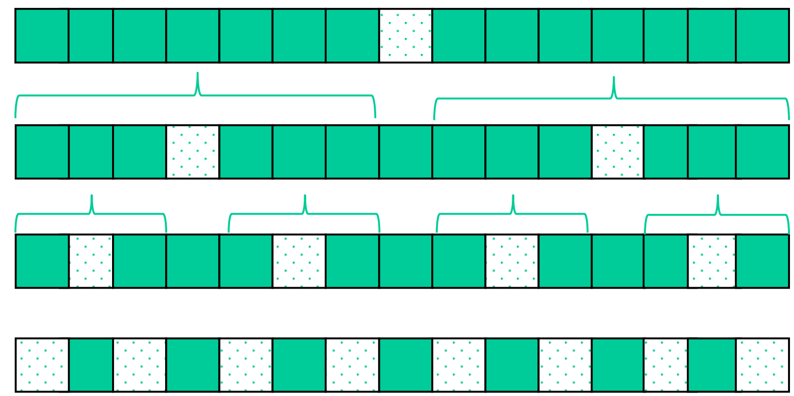


Worst-case Binary Search (con't)

- With 10 elements we needed 4 guesses
- If there were 15 elements:
  - 15 elements, make guess 1, then
  - 7 elements, make guess 2, then
  - 3 elements, make guess 3, then
  - 1 elements, make guess 4, done
- These results are the same, but if we have from 16 to 31 elements it takes 5 guesses.
- This formula is: $\lfloor log_2(n) + 1 \rfloor$
- $\log_2 (n)$ is the number of times you have to divide n by 2 to get 1

Average-case Binary Search

- If there were 15 elements:
  - 1 element takes 1 guess
  - 2 elements take 2 guesses
  - 4 elements take 3 guesses
  - 8 elements take 4 guesses



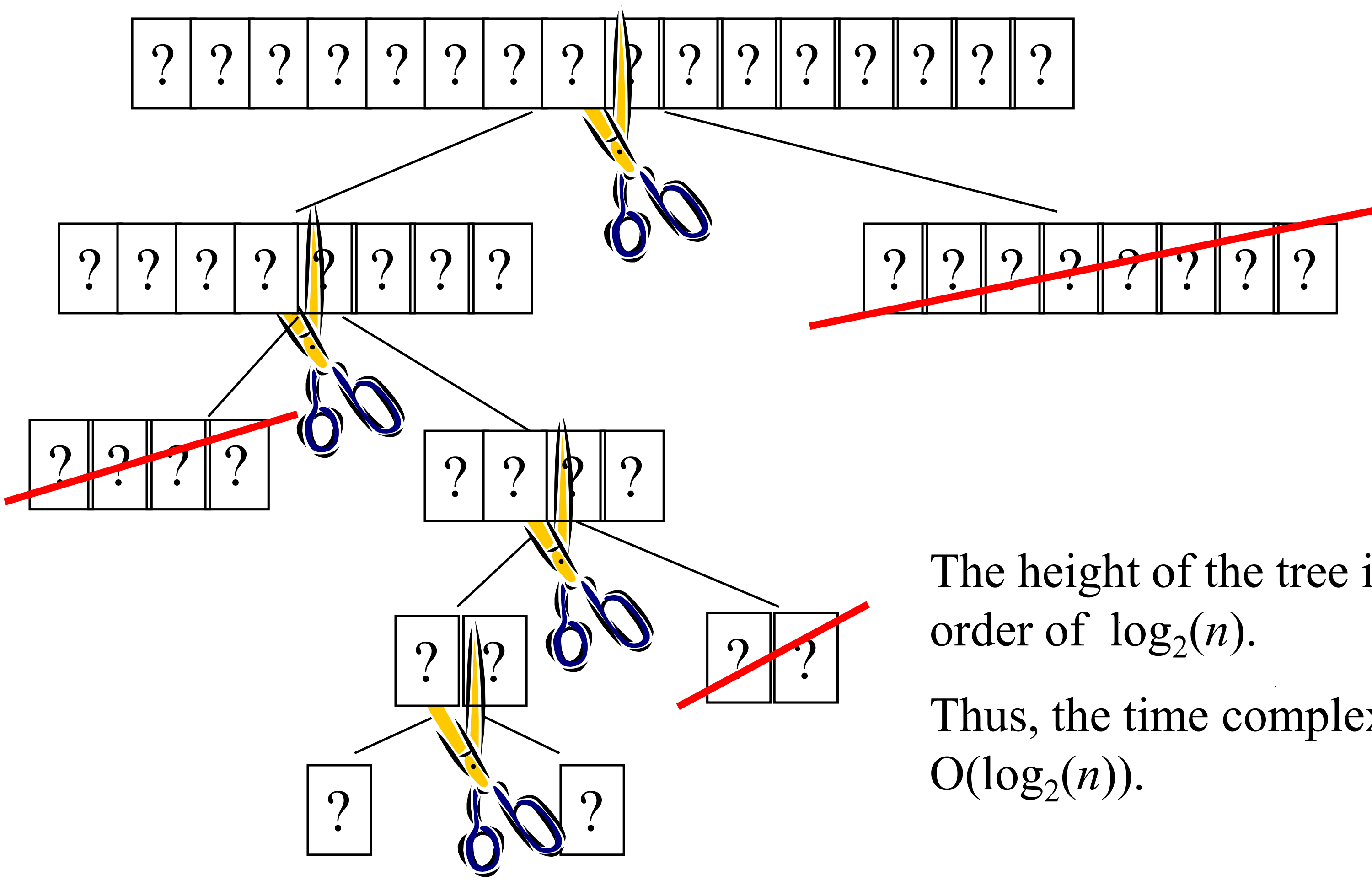
- The average is:

$$\frac{(1*1)+(2*2)+(4*3)+(8*4)}{15} = \frac{49}{15} \approx 3$$

- The average case is about one less than the worst case, so this is: $\lfloor log_2(n) \rfloor$

Time Complexity of Binary Search

The number of comparisons is proportional to the height of the following search tree:



The height of the tree is in the order of $\log_2(n)$.

Thus, the time complexity is $O(\log_2(n))$.

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Sequential and Binary Search

- For average and worst case sequential search, it takes: $\frac{(n+1)}{2}$ and n.

- For average and worst case binary search, it takes: $\lfloor log_2(n) \rfloor$ and $\lfloor log_2(n) + 1 \rfloor$

list size	Sequential average	Sequential worst	Binary average	Binary worst	Ratio
10	6	10	3	4	2
100	51	100	6	7	8
1000	501	1000	9	10	55
10000	5001	10000	13	14	384