EE 362 – Lab 2

Digital Signal Processing Fundamentals[1]

Pre-Lab: The first part of this lab addresses a MATLAB limitation first examined in Lab 1. Therefore, as preparation, you should review Lab 1, Exercise 3.3. Conceptually, it should also be helpful to review the corresponding sections in your textbook, i.e., from Chapters 2 and 3.

Verification: Except for Exercise 5.1 , all sections in this lab should be completed **during your assigned lab time**, and the steps marked **Instructor Verification** must also be signed off **during the lab time**. One of the laboratory instructors must verify the appropriate steps by signing on the **Instructor Verification** line. When you have completed a step that requires verification, simply demonstrate the step to the TA or instructor. Turn in the completed verification sheet to your TA at the end of the lab.

Lab Report: You are only required to turn in a report on Exercise 5.1 with graphs and appropriate explanations/calculations. You are asked to **label** the axes of your plots and include a title for every plot. In order to keep track of plots, include your plot inlined within your report. If you are unsure about what is expected, ask the TA who will grade your report. Completed lab report must be submitted in one week since your lab day in the EE362 lab reports box (second floor, beside Room 2C61). Late lab reports will not be marked, and will be given a mark of zero.

1 Introduction

This Lab 2 introduces you to the fundamentals of digital signal processing (DSP), particularly using MATLAB as a software implementation tool. We will focus on three main topics:

- Convolution;
- Discrete-Time Fourier Transform (DTFT);
- Z-Transform.

Throughout the lab, you will encounter both conceptual and computer-based exercises. Not all exercises will be formally marked or verified; only those specifically noted with "Instructor Verification" will have formal attention. However, you should be able to do all of the included exercises.

2 Convolution

As seen in Lab 1, the built-in MATLAB convolution function does not track the sample indices, making it difficult to interpret the resulting sequences. In this section, you will address this missing gap.

Exercise 2.1 In this exercise, you will implement an "improved" version of convolution.

[1]Please report typos and send comments to francis.bui@usask.ca

(a) Given the sequences nx and nh (which represent sample indices for the input and the impulse response, respectively), write a MATLAB function which takes four input sequences $x[n]$, nx, $h[n]$ and nh, and produces two outputs: the convolution result $y[n]$ and the corresponding sample indices ny.

(b) Test your function at least with the sequences given in Lab 1, Section 3.1. You should obtain an output similar to that shown in Fig. 1, for $y[n]$ with respect to its indices nh.

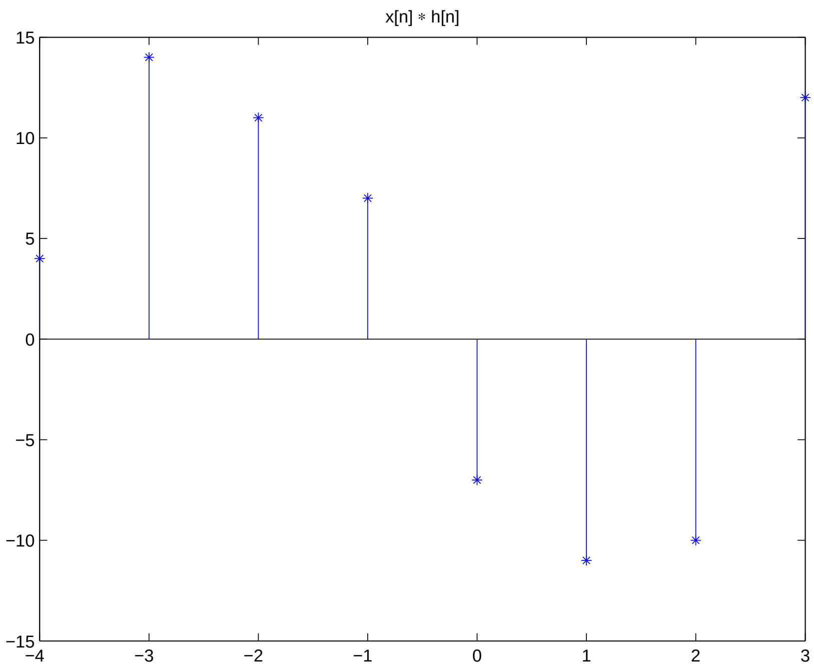

Figure 1: Convolution output

(c) Using the convolution function, verify the linear property of convolution by computing the following two outputs :

(i) $h[n] * (x_1[n] + x_2[n])$

(ii) $h[n] * x_1[n] + h[n] * x_2[n]$.

In this case, let $h[n] = u[n] - u[n-2]$, $x_1[n] = \delta[n] - \delta[n-2]$, and $x_2[n] = \delta[-n+4] - 3\delta[n-4]$ (*Hint*: you may wish to review Lab 1, Exercise 2.2).

Instructor Verification: Exercise 2.1 part c.

3 Discrete Time Fourier Transform

The DTFT of a sequence $x[n]$ is given by:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\; e^{-j\omega n} \tag{1}$$

Generally, the DTFT is a complex sequence, consisting of both real and imaginary parts. For analytical purposes, the magnitude and phase responses of the DTFT are typically plotted as separate graphs.

Exercise 3.1 Equation (1) contains an infinite summation, with respect to n. How can this be addressed in implementation?

As discussed in the lectures, one of the challenges in discrete-time signal processing is the *uniqueness* problem, in which complex sinusoids separated by $2\pi r$, $r \in \mathbb{Z}$, are indistinguishable. Consequently, the resulting periodicity property of DTFT states that $X(e^{j\omega})$ is periodic with period 2π. Then, the DTFT can be completely determined for any plot over ω values spanning a 2π-range. For example, ω can be chosen in the range $[0, 2\pi]$ or $[-\pi, \pi]$.

Furthermore, in certain cases, the symmetry property of the resulting DTFT is such that the magnitude response of $X(e^{j\omega})$ has even symmetry, while the phase response of $X(e^{j\omega})$ has odd symmetry. In such cases, it's sufficient to plot the DTFT for $\omega \in [0, \pi]$.

Exercise 3.2 What conditions are needed for the symmetry just described? *Hint*: consider the type of input sequence.

In many cases, it is possible to evaluate the DTFT analytically to obtain a closed-form solution, as shown in the following Example 3.1.

Example 3.1 Evaluate the DTFT for the sequence, $x[n] = (0.5)^n u[n]$. Also obtain its magnitude and phase responses.

Solution. The DTFT of $x[n]$ is given by:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (0.5)^n u[n]\ e^{-j\omega n}. \tag{2}$$

It can be shown that:

$$X(e^{j\omega}) = \frac{e^{j\omega}}{e^{j\omega} - 0.5}. \tag{3}$$

Then, the following MATLAB code computes the magnitude and phase responses of $x[n]$.

```
% ----------------------------------------------------------------
% DTFT of x[n] = ((0.5)^n) . u[n]
w = 0 : pi/100 : pi;
x_dtft = exp(1i*w) ./ (exp(1i*w) - 0.5);
% Magnitude Response
mag_x_dtft = abs(x_dtft);
% Phase Response
phs_x_dtft = angle(x_dtft);
% ----------------------------------------------------------------
```

Exercise 3.3 Continuing with Example 3.1,

(a) Plot the magnitude and phase responses with respect to ω.

(b) Normalize the x-axis to π units, and plot the magnitude and phase responses.

(c) Plot the magnitude response for a different range of ω (can be from -4π to $+4\pi$), such that the periodicity property of DTFT appears clearly.

(d) Plot the magnitude response for $\omega \in [-\pi, \pi]$. Is it even or odd symmetric?

(e) Plot the phase response for $\omega \in [-\pi, \pi]$. Is it even or odd symmetric?

Instructor Verification: Exercise 3.3 parts c, d, e.

4 Matrix-Based Formulation of DTFT

If $x[n]$ is finite with $n \in [0, N-1]$, then the DTFT of $x[n]$ at $\omega = \omega_k = \frac{\pi}{M}k$ is given by:

$$X(e^{j\omega_k}) = \sum_{n=0}^{N-1} x[n]\; e^{-j\omega_k n}; \qquad k = 0, 1, 2, \ldots, M. \tag{4}$$

Furthermore, a matrix-based formulation can be used to obtain the corresponding DTFT sequence, i.e., a row vector, $\mathbf{X} = \{X(e^{j\omega_k})\}$, $k = 0, 1, 2, \ldots, M$, with

$$\mathbf{X} = \mathbf{x}\, e^{-j\dfrac{\pi}{M}\mathbf{n}^T\mathbf{k}} \tag{5}$$

where $(\cdot)^T$ denotes the vector transposition.

Exercise 4.1 Determine the various vector variables (i.e., $\mathbf{x}$, $\mathbf{n}$, and $\mathbf{k}$) in Eq. (5). *Hint*: you may wish to look at the following Example 4.1 first.

More generally, for the matrix-based approach, n does not necessarily need to start from 0, as shown in the following Example 4.1.

Example 4.1 Evaluate the DTFT for $x[n] = [1, \underline{2}, 3, 4, 5, 6, 7, 8]$ at 501 equispaced frequencies between $[0, \pi]$.

Solution. The following MATLAB code uses the matrix-based formulation to compute the DTFT.

```
% -------------------------------------------------------------------
% DTFT using the Matrix based equation
x = [1, 2, 3, 4, 5, 6, 7, 8];
n = -1:6;
M = 500;
k = 0:M;
w = (pi/M)*k;

% DTFT
x_dtft = x * exp(-1i * (pi/M) * n' * k);

% Magnitude Response
mag_x_dtft = abs(x_dtft);

% Phase Response
phs_x_dtft = angle(x_dtft);
% -------------------------------------------------------------------
```

Exercise 4.2 Continuing with Example 4.1,

(a) Plot the magnitude and phase response with respect to ω.

(b) Normalize the x-axis to π and plot the magnitude and phase responses.

(c) Plot the magnitude response for a different range of ω such that the periodicity property of DTFT appears clearly.

(d) Plot the magnitude response for $\omega \in [-\pi, \pi]$. Is it even or odd symmetric?

(e) Plot the phase response for $\omega \in [-\pi, \pi]$. Is it even or odd symmetric?

(f) Evaluate the DTFT using the standard Eq. (1) for $\omega \in [0, \pi]$. Plot the magnitude and phase responses. Compare the result with that of the given MATLAB code.

(g) Write a MATLAB function which returns the matrix-based DTFT of the input sequence. The function should take three inputs: $x[n]$, n and M. Compare the result with that of the given MATLAB code.

(h) Write a MATLAB script to compute the DTFT of $\cos\frac{\pi}{5}n$ for $n \in [0, 200]$, $M = 400$ and $k \in [-M, M]$. Plot the magnitude and phase responses. The plots should be similar to those shown in Figure 2.

Instructor Verification: Exercise 4.2 parts g, h.

5 Z-Transform

The z-transform (ZT) of a sequence $x[n]$ is given by:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\ z^{-n}. \tag{6}$$

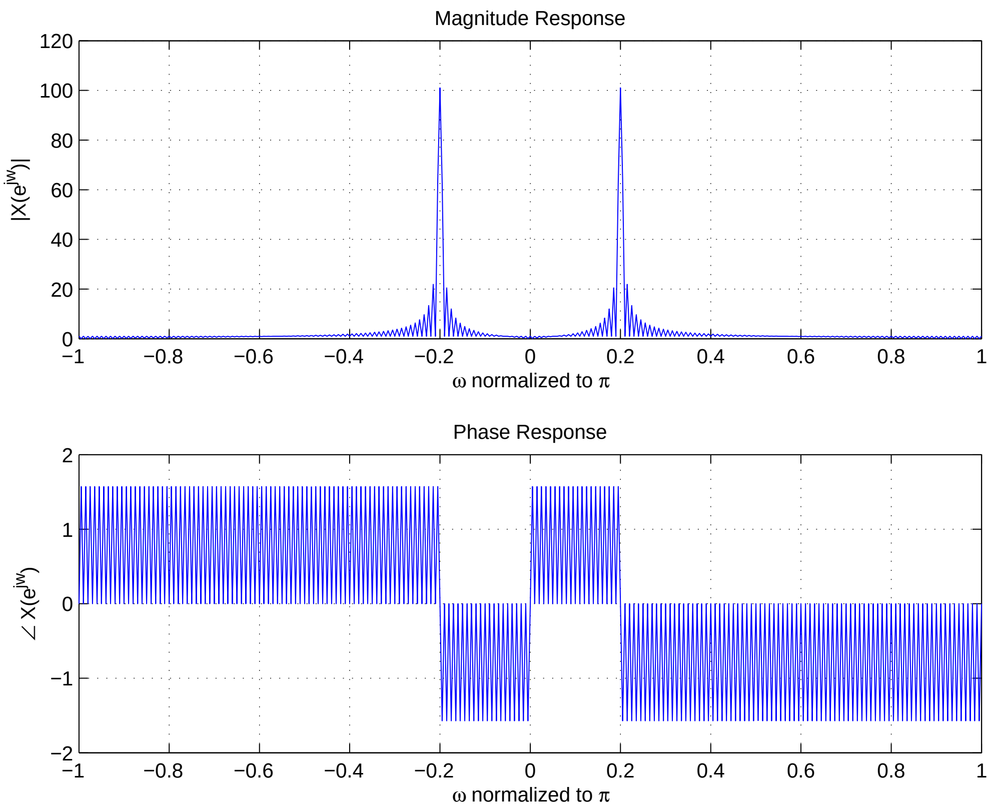

Figure 2: DTFT responses

The region of convergence (ROC) is defined as the set of values of z for which $X(z)$ exists.

Generally, $X(z)$ is a complex-number sequence similar to the DTFT, and as such is analyzed by plotting the magnitude and phase responses separately.

z is called the complex frequency and is represented in its polar form as:

$$z = |z|\ e^{jw}. \tag{7}$$

The DTFT of $x[n]$ can be viewed as a special case of the ZT of $x[n]$, by substituting $z = e^{j\omega}$. Then the complex frequency z is given by:

$$z = e^{j\omega} = 1\ e^{j\omega}, \tag{8}$$

which represents a unit circle in the z-plane.

For analytical purposes, the ZT can also be represented as:

$$X(z) = \frac{B(z)}{A(z)}, \tag{9}$$

where $B(z)$ and $A(z)$ are respectively the numerator and denominator polynomials. The roots of $B(z)$ are called the zeros of z-transform and the roots of $A(z)$ are called the poles of z-transform.

5.1 LTI System Analysis Using Z-Transform

The relation between the input and output of an LTI system is given by:

$$y[n] = h[n] * x[n], \tag{10}$$

where $x[n]$ and $y[n]$ represent the input and output respectively, and $h[n]$ represents the impulse response of the LTI system.

Using the convolution property of z-transform, the convolutive form of (10) can be written in a multiplicative form:

$$Y(z) = H(z)\ X(z). \tag{11}$$

The transfer function of the LTI system is then given by:

$$H(z) = \frac{Y(z)}{X(z)}. \tag{12}$$

5.2 Z-Transform in MATLAB

Given $H(z)$ in terms of numerator and denominator polynomials. How can MATLAB be used to compute and analyze the ZT?

- There are several different ways to sketch the pole-zero plot in MATLAB:
  - The pole-zero plot can be drawn using the MATLAB function "zplane(b,a)", where **b** and **a** are the row vectors containing the coefficients of numerator and denominator polynomials respectively.
  - If H(z) is of the form $k\frac{Y(z)}{X(z)}$, where k represents the gain, then the MATLAB function "tf2zpk(b,a)" produces the zeros, poles and gain of the transfer function. Its usage is of the form: $[z, p, k] = tf2zpk(b, a)$. We can then use the MATLAB function "zplane(z,p)" to draw the pole-zero plot.
  - The MATLAB function "roots" can be used to find the zeros and poles of the transfer function. The MATLAB statement $z = roots(b)$ returns the zeros while the statement $z = roots(a)$ returns poles. The pole-zero plot can then be plotted using "zplane(z,p)".
- Given the numerator polynomial coefficients (vector **b**) and denominator coefficients (vector **a**), we can use the MATLAB function "freqz(b,a)" to obtain the frequency response. Type "help freqz" in MATLAB prompt to know more about this function. This function also plots the magnitude and phase responses, with ω normalized to π, if not assigned to any variable.
- Alternatively, the MATLAB statement "[h,w] = freqz(b,a)" returns frequency response vector **h** and angular frequency vector **w**. The magnitude response can then be obtained as "$20 * \log 10(\text{abs}(h))$" and the phase response as "$(180/pi) * \text{angle}(h)$". By default, the frequency response will be calculated for 512 samples.
- The impulse response can be obtained using the MATLAB function "impz(b,a)". This function plots the impulse response if not assigned to any variable.

5.3 Computer-Based Exercises for Z-Transform

Exercise 5.1 Given an LTI system, $y[n] = x[n] - \frac{1}{3}x[n-1] + \frac{1}{12}y[n-1] + \frac{1}{3}y[n-2]$,

(a) Mathematically, show that the z-transform $H(z) = \dfrac{z^2 - \dfrac{1}{3}z}{z^2 - \dfrac{1}{12}z - \dfrac{1}{3}}$.

(b) Plot the pole-zero plot.

(c) Plot the magnitude and phase responses for 256 samples.

(d) Plot the impulse response of the system.

Lab report: This Exercise 5.1 is to be turned in.

EE 362 Lab 2 - Instructor Verification Sheet

Turn this page in to your grading TA

Name:

Date of Lab:

Instructor Verification: Exercise 2.1 part c.	
Verified:	Date/Time:

Instructor Verification: Exercise 3.3 parts c, d, e.	
Verified:	Date/Time:

Instructor Verification: Exercise 4.2 parts g, h.	
Verified:	Date/Time: