

Probability and Statistics I

10. The Binomial Probability Distribution

3.5 The Binomial Probability Distribution



Example 10.1

What is the probability of each of these events?

- a) You toss a fair coin 5 times and it lands heads side up 3 times?
- b) You draw 5 cards from a standard deck *with replacemnt* and draw 3 red cards?
- c) You guess the answer to 5 true or false questions on a quiz and get three correct.
- d) You roll a fair die 5 times and the number shown is odd on 3 rolls.

Binomial Experiment

An experiment is a binomial experiment if

- a) It consists of a fixed number of n trials.
- b) Each trial can result in two outcomes (success and failure).
- c) The trials are mutually independent.
- d) The probability of success is on each trial a constant, denoted by p .

Example 10.2: The Binomial Experiment

Approximately 79% of the world's population has brown eyes ¹.

Suppose that we sample 5 people from the population at random and record their eye-colour as brown or not brown. Let X represent the number of people in our sample with brown eyes.

Is this a binomial experiment?

¹<https://www.worldatlas.com>

Binomial Random Variables

Let X count the number of successes in a binomial experiment with n trials and probability of success p .

Then X is said to be a binomial random variable. Alternatively, we say that X has a binomial distribution.

Mathematically we write

$$X \sim \text{Binomial}(n, p).$$

THE BINOMIAL DISTRIBUTION

Suppose that

$$X \sim \text{Binomial}(n, p).$$

PMF and CDF

PMF: $b(x; n, p) = \binom{n}{x} p^x (1 - p)^{n-x}$, $x = 0, \dots, n$

CDF: requires special functions

Properties

Mean: $\mu = np$

Variance: $\sigma^2 = np(1 - p)$

Calculator

<https://stattrek.com/online-calculator/binomial>

THE BINOMIAL DISTRIBUTION

Example 10.3: The Binomial Distribution

A standard roulette wheel has 37 pockets in which the ball may land. Of these, 18 pockets are red, 18 are black, and 1 is green.



Example 10.3 ctd: The Binomial Distribution

A standard roulette wheel has 37 pockets in which the ball may land. Of these, 18 pockets are red, 18 are black, and 1 is green.

Suppose that you place \$1 bets that the ball will land in a black pocket on 20 consecutive games. Let X be the number of times you win.

- a) Why is this a binomial experiment?
- b) What is the distribution of X ?
- c) What is the pmf of X ?
- d) What is the probability that you win exactly half the games?

Example 10.3 ctd: The Binomial Distribution

A standard roulette wheel has 37 pockets in which the ball may land. Of these, 18 pockets are red, 18 are black, and 1 is green.

Suppose that you place \$1 bets that the ball will land in a black pocket on 20 consecutive games. Let X be the number of times you win.

- e) What is the probability that you win more than half the games?
- f) What are the mean, variance, and standard deviation of X ?
- g) Let $\$v$ be the amount you win on a \$1 bet if the ball lands in a black pocket. What value of $\$v$ makes this a fair game?

Exercise 10.1

The shooting percentage in hockey records the percentage of shots on goal taken in a season on which a player scores. The highest shooting percentage in the 2023-2024 NHL season, 24.5%, was claimed by Sam Reinhart of the Florida Panthers.

Suppose that Sam takes 200 shots on net in the 2024-2025 season with the same shooting percentage. Let S be the number of goals Sam scores.

- Explain why it makes sense to consider this a binomial experiment.
- State the distribution of S .
- What is the pmf of S ?
- What are the expected value and standard deviation of S ?
- What is the probability that Sam scores at least 50 goals?