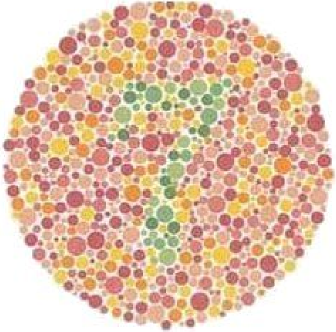
SS2857 Probability and Statistics 1
Fall 2024

Lecture 4

2.4 Conditional Probability

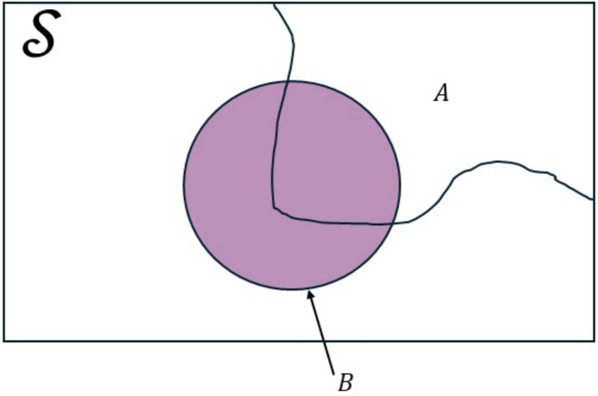


Conditional Probability

For any two events A and B with $P(B) > 0$, the conditional probability of A given that B has occurred is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Conditional probabilities essentially reduce the sample space from $\mathcal{S}$ to some event $B \subset \mathcal{S}$.



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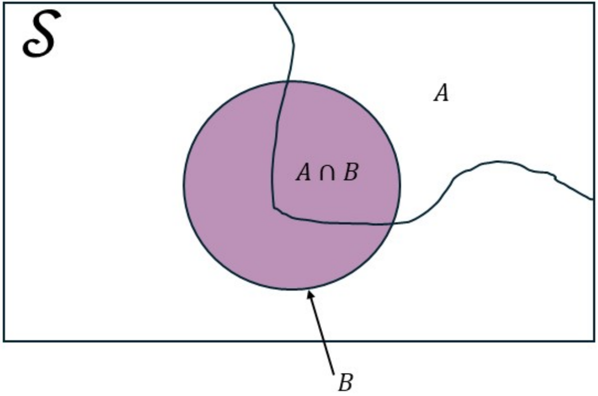


The multiplication rule is simply a rearrangement of the definition of conditional probability.
If

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

then

$$P(A \cap B) = P(A|B)P(B).$$

Example 4.1

According to `http://https://www.colourblindawareness.org/`, colour blindness affects 1 in 12 people assigned to be male at birth and 1 in 200 people assigned to be female at birth. Approximately 48.78% of all babies born in the world are assigned to be female at birth.

a) Identify the conditional probability(ies) in this statement and define them in terms of the events F a person is assigned to be female at birth and B a person is colour blind.

b) What is the probability that a randomly selected newborn is female and colour blind/male and colour blind?

c) What is the probability that a randomly selected newborn is colour blind?

Law of Total Probability

Let $A_1, \ldots, A_k$ be exhaustive,

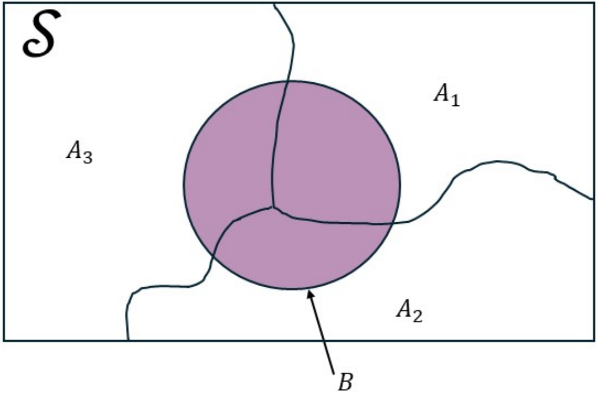
- mutually exclusive: $A_i \cap A_j = \emptyset$ for all $i, j = 1, \ldots, k$, and
- exhaustive: $\cup_{i=1}^{k} A_i = \mathcal{S}$,

Then for any other event B

$$P(B) = \sum_{i=1}^{k} P(B|A_i)P(A_i) \left(= \sum_{i=1}^{k} P(A_i \cap B) \right).$$

Note: We can also say that $A_1, \ldots, A_k$ partition $\mathcal{S}$.

The law of total probability allows us to compute the probability of an event by summing the probabilities of individual pieces:



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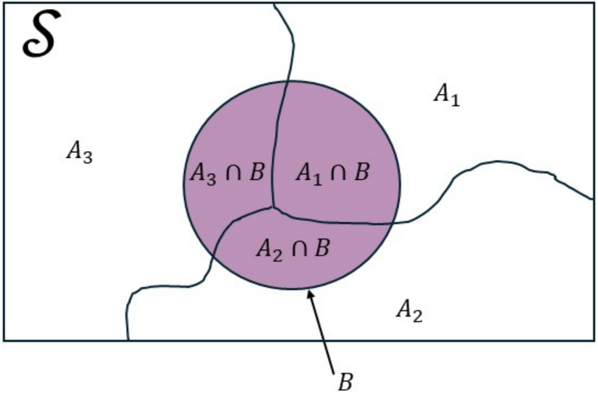

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According to http://https://www.colourblindawareness.org/, colour blindness affects 1 in 12 people assigned to be male at birth and 1 in 200 people assigned to be female at birth. Approximately 48.78% of all babies born in the world are assigned to be female at birth.

d) What is the probability that a baby is assigned to be male at birth given that it is colour blind?

Let $A_1, \ldots, A_k$ be a collection of mutually exclusive and exhaustive events (i.e., a partition of $\mathcal{S}$) with $P(A_i) > 0$ for $i = 1, \ldots, k$. Then for any other event B for which $P(B) > 0$,

$$P(A_j|B) = \frac{P(B|A_j)P(A_j)}{P(B)} = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{k} P(B|A_j)P(A_j)} \quad j = 1, \ldots, k.$$

Bayes' rule allows us to switch the direction of conditional probabilities so that we can compute the probability of $A_j|B$ from the probability of $B|A_j$:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}.$$

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e) What is the probability that someone answers yes to the following statement:

   I am colour blind **OR**[1] my birthday falls on an odd numbered day of the month.

[1] We will always use the inclusive or so that the statement is true if one or both are true.

Questions?

Exercise 4.1

The 2024 US Presidential election will be held on November 5 and provides a lot of opportunity for interesting statistics. Current polls put Kamala Harris in the lead nationally with 48% of the vote versus Donald Trump's 46% (the remaining 6% are undecided or voting for other candidates). However, Kamala's support by state varies from 28% in Utah (population 3,417,734) to 70% in Vermont (population 647,464). The total population of the 50 US states is 334,235,923.

a) Identify the conditional probabilities defined in this question.
b) What is the probability that a randomly selected US voter is from Utah and is planning to vote for Kamala Harris?
c) What is the probability that someone not from Utah plans to vote for Kamala Harris?
d) Suppose that a randomly selected person is planning to vote for Kamala Harris. Is this person more likely to be from Utah or from Vermont?
e) What is the key assumption to answering these questions?