STAT 2857A – Lecture 15 Examples and Exercises

Example 15.1

Consider the random variable, X, from Example 14.1. This random variable represents the distance that an object dropped from a height of 1~m falls in a randomly selected time between 0 and 1 second on the home planet of Emperor Zurg where the force of gravity is only 2~m/s^2. The pdf and cdf are

$$f(x) = \begin{cases} 0 \leq 0 & \\ 2x & 0 < x < 1 \\ 0 & 1 \leq x \end{cases} \quad \text{and } F(x) = \int_{-\infty}^{x} f(u)\, du = \begin{cases} 0 & x \leq 0 \\ x^2 & 0 < x < 1 \\ 1 & 1 \leq x \end{cases}$$

a) Find the mean of X.
b) Compute the variance of X.
c) Provide an interpretation for the mean.
d) Let Y be the distance traveled in inches not metres ($Y = 39.37X$). Find the mean and variance of Y.
e) Let $Z = X^2$. Find the mean and variance of Z.

Exercise 15.2

Suppose that the random variable X has pdf

$$f(x) = \frac{3}{4}\left[2x - x^2\right], \quad 0 \leq x \leq 2.$$

a) Confirm that $f(x)$ is a valid pdf.
b) Find the mean and variance of X.
c) Find the mean and variance of $Y = 3X + 2$.
d) Find the mean and variance of $Z = X^2$.