SS2857 Probability and Statistics 1
Fall 2021

Lecture 8

Revised 02/10/24

3.2 Probability Distributions for Discrete Random Variables

Probability Distribution

The probability distribution (aka the distribution) of a random variable identifies:

1) The possible values of the random variable.
2) How the probability is distributed to these values.

Probability Mass Function

The probability mass function (pmf) of a discrete random variable is the function

$$p(x) = P(X = x), x \in \mathbb{X}.$$

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We commonly define the probability mass function where it is positive and implicitly assume that the function is equal to zero for all other values. E.g.,

$$p(x) = P(X = x) = .5, x \in \{0, 1\}$$

implies $P(X = x) = 0$ for all other $x \in \mathbb{R}$.

Example 8.1: Probability Mass Functions

Approximately 79% of world's population has brown eyes.

Suppose that we sample 5 people from the population at random with replacement and record their eye-colour as brown or not brown. Let X be the number of people in our sample with brown eyes.

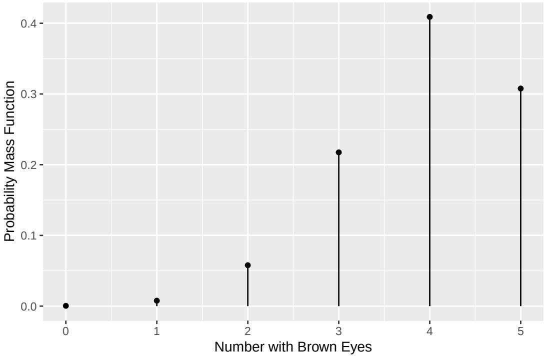
a) Compute the pmf of X.
b) Draw a figure showing the pmf of X.

Example 8.2: Probability Mass Functions

x	$p(x)$
0	0.00041
1	0.00768
2	0.05780
3	0.21743
4	0.40898
5	0.30771

Example 8.2 ctd: Probability Mass Functions

The pmf of X looks like this:



Cumulative Distribution Function

The cumulative distribution function (cdf) of a discrete random variable is the function

$$F(x) = P(X \leq x) = \sum_{y: y \in D, y \leq x} p(y), x \in \mathbb{X}.$$

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Cumulative distribution functions may either be defined as a function or in a table.

The cdf must be defined for all values along the real line. E.g.,

$$F(x) = P(X \leq x) \begin{cases} 0 & x < 0 \\ .5 & 0 \leq x < 1 \\ 1 & x \geq 1. \end{cases}$$

Example 8.2 ctd: Cumulative Distribution Functions

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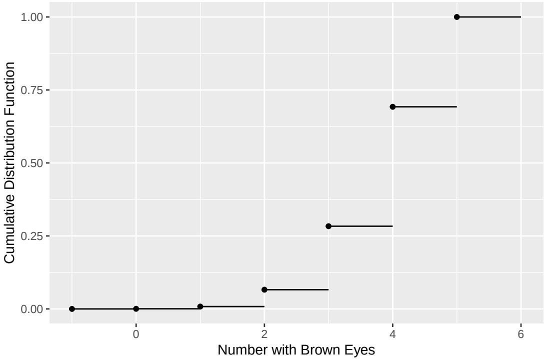
d) Compute the cdf of X.
e) Draw a figure showing the cdf of X.

Example 8.2 ctd: Cumulative Distribution Functions

$$F(x) = \begin{cases} 0 & x < 0 \\ p(0) = .00041 & 0 \leq x < 1 \\ p(0) + p(1) = .00809 & 1 \leq x < 2 \\ p(0) + p(1) + p(2) = .065989 & 2 \leq x < 3 \\ p(0) + p(1) + p(2) + p(3) = .28332 & 3 \leq x < 4 \\ p(0) + p(1) + p(2) + p(3) + p(4) = .69229 & 4 \leq x < 5 \\ p(0) + p(1) + p(2) + p(3) + p(4) + p(5) = 1 & 5 \leq x \end{cases}$$

Example 8.2 ctd:Cumulative Distribution Functions



Cumulative Distribution Functions

Any cumulative distribution, $F(x)$, must satisfy the following properties:

- Tends to 0 as x decreases: $\lim_{x \to -\inf} = 0$.
- Tends to 1 as x increases: $\lim_{x \to \inf} = 1$.
- Non-decreasing.
- Continuous from the right: $\lim_{x^* \downarrow x} F(x^*) = F(x)$.

Cumulative distributions for discrete random variables are step functions.

Parameter

A parameter is a quantity that can be assigned different possible values to identify one specific distribution within a family of distributions.
E.g., the family of Bernoulli distributions is defined by the pmf

$$p(x) = \begin{cases} 1-p & x=0 \\ p & x=1 \end{cases} = p^x(1-p)^{(1-x)}.$$

The parameter is p.

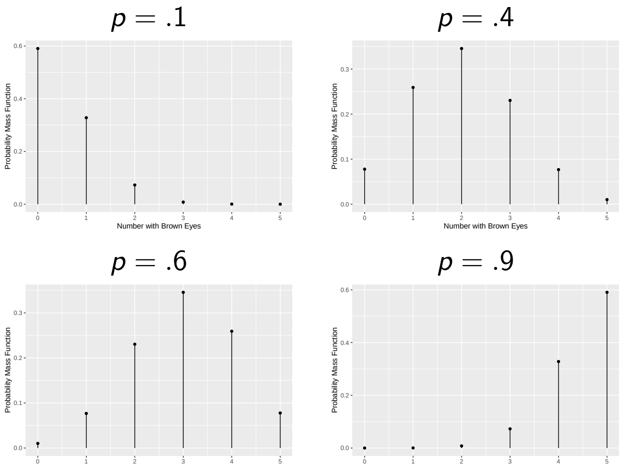
Example 8.3

Let p be the proportion of the world's population with brown eyes.

Suppose that we sample 5 people from the population at random with replacement and record their eye-colour as brown or not brown. Let X be the number of people in our sample with brown eyes.

How would the distribution of X change if p was varied between 0 and 1?

Example 8.3 ctd



Questions?

Exercise 8.1

Consider a discrete random variable, Z, with the cdf:

$$F(z) = \begin{cases} 0 & x < 0 \\ 0.292 & 0 \leq x < 1 \\ 0.745 & 1 \leq x < 2 \\ 0.965 & 2 \leq x < 3 \\ 0.998 & 3 \leq x < 4 \\ 1 & x \geq 4 \end{cases}$$

a) Sketch the cdf.
b) What are the possible values Z (i.e., for what values of z is $P(Z = z) > 0$)?
c) What is the probability mass function?
d) Sketch the pmf.