STAT 2857A – Lecture 20 Examples and Exercises

Example 21.1

The simplest possible joint distribution is that for two Bernoulli random variables. Suppose that X and Y take the values 0 and 1 according to the following joint pmf:

x	0		1	
y	0	1	0	1
$p(x,y)$	p_{00}	p_{01}	p_{10}	$p_{11}.$

a) What is the expected value of XY?
b) What are the covariance and correlation of X and Y?
c) What are the mean and variance of $Z = 2X + 4Y$?
d) Under what conditions are X and Y independent? What is are the mean XY in this case?

Example 21.2

Each of the following plots represents the joint pmf of two random variables, X and Y. The points, (x,y) represent the possible values of (X,Y). The distribution places equal probability, $1/6$, at each point.

A

B

C

D

x
y
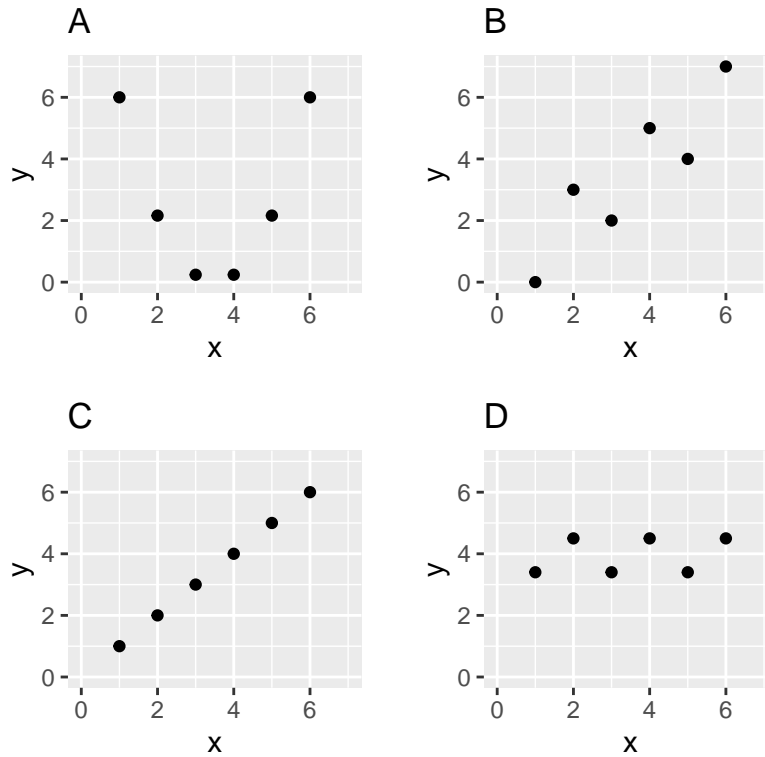
Exercise 21.1

Consider rolling two fair, three-sided die. Let S denote the sum of the values showing on the two die and D the absolute value of the difference. E.g., if one die shows the value 1 and the second shows the value 2 then $S = 3$ and $D = 1$, regardless of which was thrown first.

a) Construct a table showing the joint pmf of S and D.
b) Compute the marginal pmf of both S and D.
c) Compute the expected value and variance of S and D.
d) Compute the covariance and correlation of S and D.
e) Are S and D independent? Justify your answer.