SS2857 Probability and Statistics 1
Fall 2024

Lecture 3

Revised 18/09/24

2.3 Counting Techniques

Equally Likely Outcomes

If all outcomes in the sample space are equally likely, the probability of any simple event is $1/N$, then the probability of any event is

$$P(A) = \frac{n(A)}{A}.$$

Example 3.1

Suppose that you roll a (fair) three-sided die[1] three times.

a) What is the probability that you never roll a three?
b) What is the probability that the sum is greater than 4?
c) What is the probability that the number rolled is less than three every time or the sum greater than 4?

[1] https://www.cnet.com/news/this-3d-printed-3-sided-die-is-a-work-of-modern-art/

Permutations

Count the number of ways to draw k of n distinct objects *without replacement* when order matters. I.e., two outcomes are considered the same if they contain the same elements *in the same order*.

$$ {}_nP_k = n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!} $$

Combinations

Count the number of ways to draw k of n distinct objects *without replacement* when order does not matter. I.e., two outcomes are considered the same if they contain the same elements *in any order*.

$$ {}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{P_{k,n}}{k!} $$

Example 3.2

Identify whether each of the following is a permutation or a combination. What are the values?

The number of:

a) hands of 5 cards that can be dealt from a standard deck of 52 unique cards.

b) ways for four people to line up.

c) selections on the Lotto 6/49.

d) teams of 10 students that are possible in a class of 150.

e) create 2 lines of 10 from a class of 30 students.

Example 3.3

A standard deck of cards contains 13 cards (A,2,3,...,10,J,Q,K) in each of 4 suits (Clubs, Diamonds, Hearts, Spades).

a) What is the probability that you are dealt a royal flush (10, J, Q, K, A) in the same suit)?

b) What is the probability that you are dealt a royal flush in order?

c) c) What is the probability that you are dealt a pair (two cards of one face value and three cards of other non-matching face values)?

d) What is the probability of getting a full-house (two cards of one face value and three of another?

Formatting Solutions

1. Define your events (or any other variables)
   Write:
   Let C be the event that the sky is blue. Then $P(C) = \cdots$
   Not:
   $$P(\textit{the sky is blue}) = \cdots$$

2. Keep extra (5 or 6) significant figures in your work to ensure that you don't commit rounding errors:
   $$P(C) = \frac{3744}{2,598,960} = .001441.$$

3. Finish with a clear conclusion so we don't need to guess which number is your final answer. Round your final answer to 2 or 3 significant figures.
   The probability of the event is .0014.

Questions?

Exercise 3.1

Consider Example 3.3. Show the following:

a) The probability of being dealt a flush (all 5 cards from one suit) is .00198.

b) The probability of being dealt two pairs (a pair of one value, a pair of another, and one other card) is .0475.

c) The probability of being dealt three of a kind (three of one value, one of another value, and one of yet another) is .0211.