STAT 2857A – Lecture 17 Examples and Exercises

Example 17.1

Let X denote the total amount of precipitation on a randomly selected rainy day in London. Suppose that

$$X \sim \text{Gamma}((0.628, 8.662).$$

a) What is the pdf of X?
b) What are the mean and variance?
c) What is the probability that the total precipitation is more than 10~mm given that it rains at all?

Example 17.2

Radioactive decay is well modelled by the exponential distribution. Uranium-235 has a half-life of 703,800,000 years. Let T be time in billions of years to decay of a single atom of Uranium-235.

a) What is the pdf of T?
b) What are the mean and variance of T?
c) What is the probability that $T < 1$?
d) What is the probability that $T > 2$ given $T > 1$?
e) What is the probability that $T > 100,001$ given $T > 100,000$?

Example 17.3

Suppose that $Z \sim \text{Normal}(0, 1)$ and $X \sim \chi^2_1$.

Confirm that

$$P(Z^2 \leq 2) = P(X \leq 2).$$

Exercise 17.1

In a Poisson process with rate λ, the time from one event to the next event, T, is exponentially distributed with mean $1/\lambda$. Let T_k denote the time from one event until the k-th following event. It turns out that T_k follows a gamma distribution with parameters $\alpha = k$ and $\beta = 1/\lambda$.

a) Suppose that $\lambda = .1$ events per second. What are the mean and variance of T_5?
b) What is the probability that the 5-th event occurs after 60 seconds?
c) Compare the the 5-th, 50-th, and 95-th percentiles of T_5 and of a normal random variable with the same mean and variance.
d) Repeat the previous step for T_25, T_50, and T_100. Plot the ratio of the percentiles of the two distributions vs k. What happens?