SS2857 Probability and Statistics 1
Fall 2024

Lecture 1

2.1 Sample Spaces and Events

EXAMPLE 1.1

Happy Birthmonth!

Consider the month of birth for three students selected randomly from the class (call them Alexandria, Braden, and Chen).

  a) Describe the outcomes.
  b) How many outcomes are in the sample space?
  c) Identify one simple event?
  d) Identify one compound event?

Statistics is concerned with **experiments**: actions or activties whose outcome is uncertain.

An **outcome** is any possibility that might be observed an experiment.

Sample Space

The **sample space** of an experiment, *often* denoted by $\mathcal{S}$, is the set of all possible outcomes of that experiment.

EVENTS

An **event** is any collection (subset) of outcomes contained in the sample space $\mathcal{S}$.
An event is said to be **simple** if it consists of exactly one outcome and **compound** if it consists of more than one outcome.

Events are commonly denotes by uppercase letters, A, B, C etc.

Similar events are often identified by subscripts, A_1, A_2, A_3 etc.

Since events are sets, we can apply set operations:

- Complement: A' is the set of all outcomes not in A.
- Intersection: $A \cap B$ is the set of all outcomes that are in A and in B.
- Union: $A \cup B$ is the set of all outcomes in either A, B, or both.
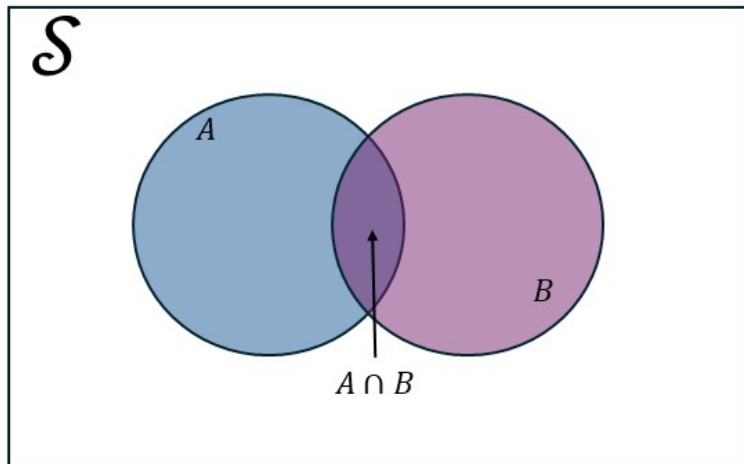
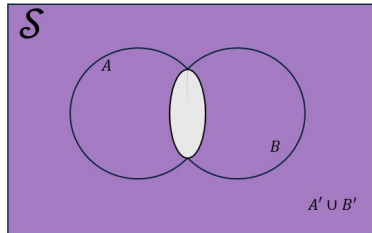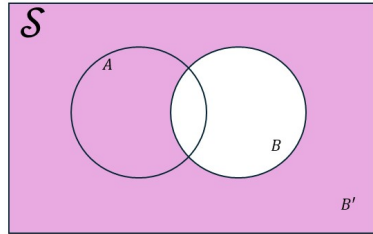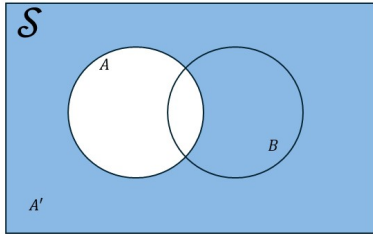# De Morgan's Laws

Let A and B be any two events. Then

- $(A \cap B)' = A' \cup B'$
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Note: such laws are useful, but it is better that you understand them then memorize them.

$A \cap B$

Proof of Part B

$x \in (A \cup B)'$. Then:

$$
\begin{aligned}
x \in (A \cup B)' \quad &\textit{iff} \quad x \notin A \cup B \\
&\textit{iff} \quad x \in A' \quad \text{and} \quad x \in B' \\
&\textit{iff} \quad x \in A' \cap B'.
\end{aligned}
$$

Therefore, $(A \cup B)' = A' \cap B'$

EXAMPLE 1.1 PART 2

Happy Birthmonth!

Consider the month of birth for three students selected randomly from the class (call them Alexandria, Braden, and Chen).

Let A_i denote the event that Alexandria is born in month i. Let B_i denote the event that Braden is born in month i. Let C_i denote the event that Chen is born in month i.

Describe each of the following events in words?

a) $E_1 = A_1 \bigcap B_1 \bigcap C_1$

b) $E_2 = \bigcup_{i=1}^{12}(A_i \bigcap B_i \bigcap C_i)$

c) $E_3 = \bigcup_{i=1}^{12}(A_i \bigcap B_i \bigcap C_i')$

Two events A and B are said to be **disjoint** or **mutually exclusive** if they share no outcomes:

$$A \cap B = \emptyset.$$

EXAMPLE 1.1 PART 3

Happy Birthmonth!

Consider the month of birth for three students selected randomly from the class (call them Alexandria, Braden, and Chen).

Let A_i denote the event that Alexandria is born in month i. Let B_i denote the event that Braden is born in month i. Let C_i denote the event that Chen is born in month i.

a) Identify two events that are disjoint/mutually exclusive.
b) Identify two events that are *not* disjoint/mutually exclusive.

SET BUILDER NOTATION

Set builder notation is an easy way to describe events (sets) by characterizing the properties of its outcomes (elements) rather than listing all possible outcomes (elements):

$$\mathcal{S} = \{\text{type}|\text{restrictions}\}.$$

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Examples

1. $\mathbb{Q} = \left\{x \in \mathbb{R}|x = \frac{a}{b} \text{ for some } a, b \in \mathbb{Z}, b \neq 0\right\}$ is the set of rational numbers.
2. $\mathcal{P} = \{x \in \mathbb{Z}|x = 2^c \text{ for some } c \in \mathbb{Z}^+\}$ is the set of powers of 2.
3. $\mathcal{S} = \{(a, b, c)|a, b, c \in \{1, \dots, 12\}\}$ is the sample space of the birthday experiment.

EXAMPLE 1.1 PART 4

Happy Birthmonth!

Consider the events E_1, E_2, and E_3 from part 2.

a) What is the probability of each event?

b) What do these probabilities mean?

EXERCISE 1.1

An unfortunate student has tests in biology, chemistry, and statistics all in one day (a cruel experiment). Thankfully, each test is graded on a pass/fail basis.

1. List all of the outcomes in the sample space.
2. Write the sample space in set builder notation.
3. Identify i) two events that are mutually exclusive and ii) two events that are not mutually exclusive.
4. Let E_1 be the event the student passes one test and E_2 the event they pass two test. List the outcomes in and describe the event $(E_1 \cup E_2)'$.

Questions?