STAT 2857A – Lecture 5 Examples and Exercises

Example 5.1

Suppose that A and B are disjoint events with positive probability ($P(A) > 0$ and $P(B) > 0$). Can they be independent?

Example 5.2

Which pairs of events do you think are independent? Explain.

a) A: It rains in London, Ontario, on October 1.\ B: It rains in London, Ontario, on October 2.
b) A: It rains in London, Ontario, on October 1.\ B: It rains in London, England, on October 1.
c) A: Erin scores $> 80\%$ on an exam.\ B: Jonah scores $> 80\%$ on the same exam.
d) A: The Yankees win the baseball World Series.\ B: The Royals win the baseball World series.

Example 5.3

Let A and B be two events such that

a) $P(A \cap B') = .15$
b) $P(A' \cap B') = .35$
c) $P(A' \cap B) = .35$

Are A and B independent?

Example 5.4

Show that if A' and B' are independent then A and B are also independent.

Exercise 5.1

Suppose that you toss a fair coin n times and count the number of heads.

a) Let H_i be the event that the coin lands heads side up on the i-th toss. What does it mean for H_1 and H_2 to be independent?
b) Does independence necessarily mean that the coin is fair?
c) What does it mean for the events $H_1, \ldots, H_n$ to be mutually independent?
d) What is the probability that the coin lands heads-side up on every one of $n = 10$ tosses?
e) What is the probability that the tosses alternate between landing heads-side up first then tails-side up etc if $n = 10$?
f) What is the probability that the coin lands heads-side up 5 times if $n = 10$?