

STAT 2857A – Lecture 14 Examples and Exercises

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Example 14.1

Let

$$f(x) = \begin{cases} 0 & x \leq 0 \\ cx & 0 < x \leq 1 \\ 0 & x > 1 \end{cases}$$

- a) Find the value of c such that $f(x)$ is a valid probability density function (pdf).
- b) Find the associated cumulative density function (cdf).
- c) Compute the probabilities of the following events:
 - i) $X \leq .5$
 - ii) $X = .5$
 - iii) $X < .5$
 - iv) $.25 \leq X \leq .75$
 - v) $X < .25$ or $X > .75$
- d) Prove that X satisfies the definition of a continuous random variable.

Example 14.2

Let

$$f(x) = \begin{cases} 0 & x \leq 0 \\ 2x & 0 < x \leq 1 \\ 0 & x > 1 \end{cases}$$

- a) Find the median of X .
- b) Find the 5-th and 95-th percentiles of X .
- c) What is the shortest interval, (x_1, x_2) , such that $P(x_1 < X < x_2) = .90$?

Exercise 14.1

Consider the distribution with cdf

$$F(X) = \begin{cases} 0 & x < 0 \\ \log_{10}(x+1) & 0 \leq x < 9 \\ 1 & 9 \leq x \end{cases}$$

- a) Plot $F(x)$.
- b) Compute the pdf, $f(x)$.
- c) Plot $f(x)$.
- d) Compute the following probabilities:
 - i) $(X \leq \sqrt{10}-1)$
 - ii) $P(X < \sqrt{10}-1)$
 - iii) $P(X = \sqrt{10}-1)$
 - iv) $P(X > \sqrt{10}-1)$
 - v) $P(X \geq \sqrt{10}-1)$