STAT 2857A – Lecture 25 Examples and Exercises

Example 25.1

According to Burmaster and Murray (1998), the height of men between the ages of 50 and 80 is normally distributed with a mean of 174.20 cm and a variance of 42.36 cm^2. Suppose that we collect a random sample of 25 men from the population. Let their heights be denoted by $X_1, \dots, X_{25}$.

a) What is the sampling distribution of the total height, $T_{25} = \sum_{i=1}^{25} X_i$?
b) What is the sampling distribution of the average height, $\bar{X}_{25} = \sum_{i=1}^{25} X_i/25$?
c) What is the sampling distribution of $Z = \frac{(\bar{X}_{25} - 174.20)}{\sqrt{1.69}}$?

Example 25.2

Suppose that $X_1, \dots, X_n$ are independent and identically distributed Bernoulli random variables such that

$$P(X_i = 0) = 1 - p \text{ and } P(X_i = 1) = p$$

for all $i = 1, \dots, n$.

a) What is the pmf of $\bar{X}_n$?
b) What is the approximate cdf of

$$Z = \frac{\bar{X}_n - np}{\sqrt{p(1-p)/n}}$$

when n is large?

Example 25.3

See slides.

Example 25.4

The Acme string company produces spools of string advertised to have a length of 100 m. However, the length of string on a randomly selected ball actually has a mean of 101 m and a standard deviation of .2 m. Approximate the 95-th percentile of the total amount of string in a box containing 50 spools.

Exercise 25.1

According to Burmaster and Murray (1998), the log weight in kilograms of men between the ages of 50 and 80 is normally distributed with a mean of 4.41 and variance .46. It can be shown that the weight then follows a log-normal distribution with mean $\mu_W = 91.45$ kg and variance $\sigma_W^2 = 1970.83$ kg. The pdf is shown on the next slide. The vertical dashed line represents the mean.

a) Describe the shape of the density.
b) Approximate the distribution of $\bar{W}_n$. What conditions need to be satisfied?
c) Explain what the approximation in the previous part means.
d) Use the approximation to show that $\lim_{n\to\infty} P(\mu_W - \epsilon < \bar{W}_n < \mu_W + \epsilon) = 1$ for any $\epsilon > 0$. Explain what this means.