

STAT 2857A – Lecture 8 Examples and Exercises

Solutions – Revised 02/10/24

Example 8.1

- a) There are $\binom{5}{x}$ outcomes for which $X = x$. E.g., there is one sample in which no people have brown eyes, 5 samples in which one person has brown eyes, etc. The probability of any outcome in which x people have brown eyes is $.79^x(.21)^{(5-x)}$. Hence the pmf is:

$$p(x) = \binom{5}{x}.79^x(.21)^{(5-x)}$$

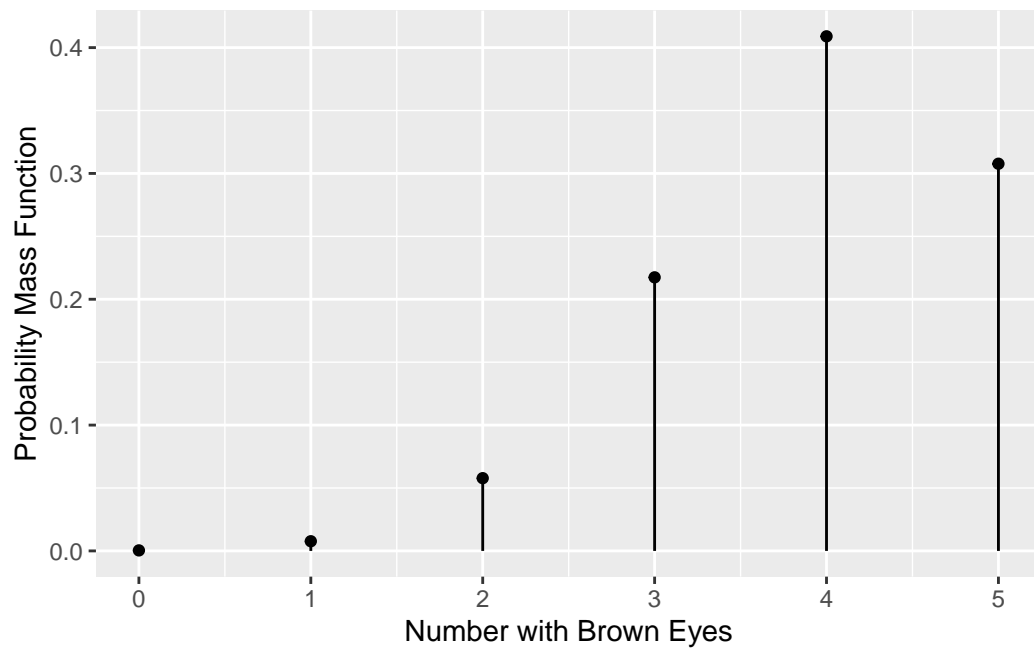
for $x = 0, 1, 2, 3, 4, 5$.

Strictly speaking, we should denote that $p(x) = 0$ for all other $x \in \mathbb{R}$. However, it is common not to do this. We know it must be true because $p(0) + p(1) + \dots + p(5) = 0$.

We can list these values out in a table:

x	p(x)
0	0.00041
1	0.00768
2	0.05780
3	0.21743
4	0.40898
5	0.30771

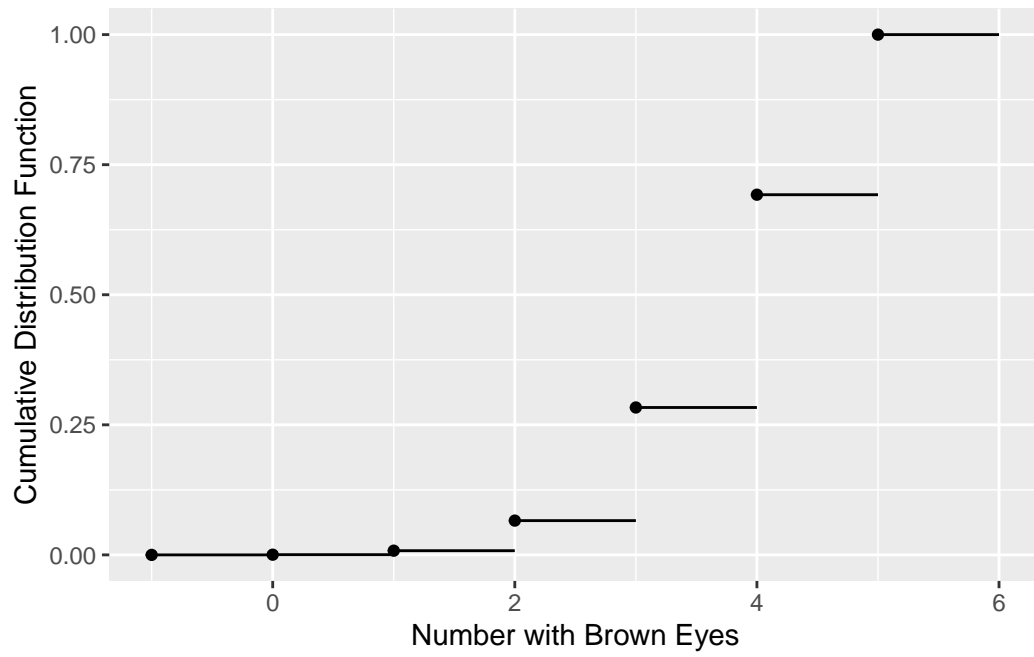
- b) Here is a plot of the pmf:



c) The cdf is defined as $F(x) = P(X \leq x)$ for any $x \in \mathbb{R}$. The values are:

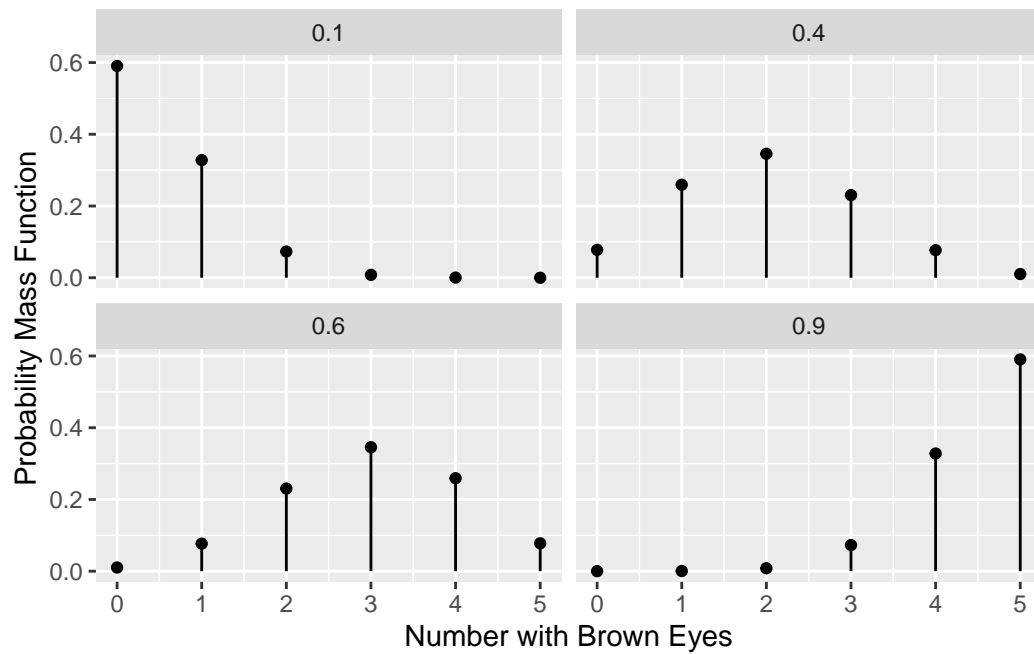
$$F(x) = \begin{cases} 0 & x < 0 \\ p(0) = .00041 & 0 \leq x < 1 \\ p(0) + p(1) = .00809 & 1 \leq x < 2 \\ p(0) + p(1) + p(2) = .065989 & 2 \leq x < 3 \\ p(0) + p(1) + p(2) + p(3) = .28332 & 3 \leq x < 4 \\ p(0) + p(1) + p(2) + p(3) + p(4) = .69229 & 4 \leq x < 5 \\ p(0) + p(1) + p(2) + p(3) + p(4) + p(5) = 1 & 5 \leq x \end{cases}$$

d) Here is the plot of the cdf:



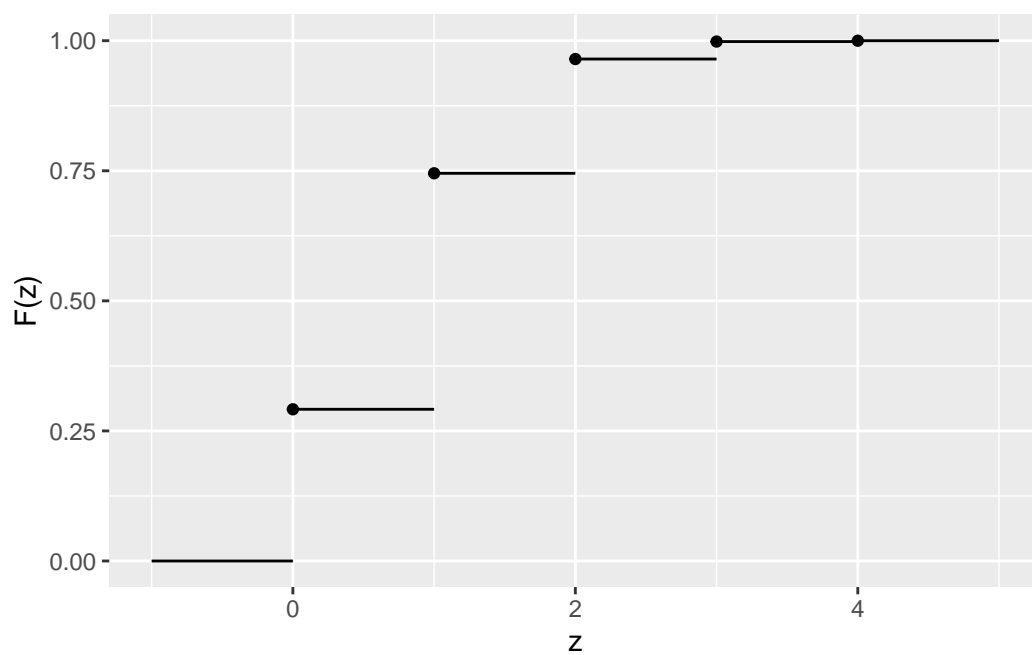
Example 8.2

The probability mass function will tend to favour small values of X when p is close to 0 and big values of X when p is close to 1. When p is close to .50, the pmf will favour either 2 or 3 of the people having brown eyes. Here are some examples of the pmf for different values of p :



Exercise 8.1

a)



b) The possible values of Z are found where the steps occur. These are 0, 1, 2, 3, 4.

c) The probability mass function is

x	P(X=x)
0	0.2916
1	0.4536
2	0.2196
3	0.0336
4	0.0016

d)

