- Governing equation (Young-Laplace):

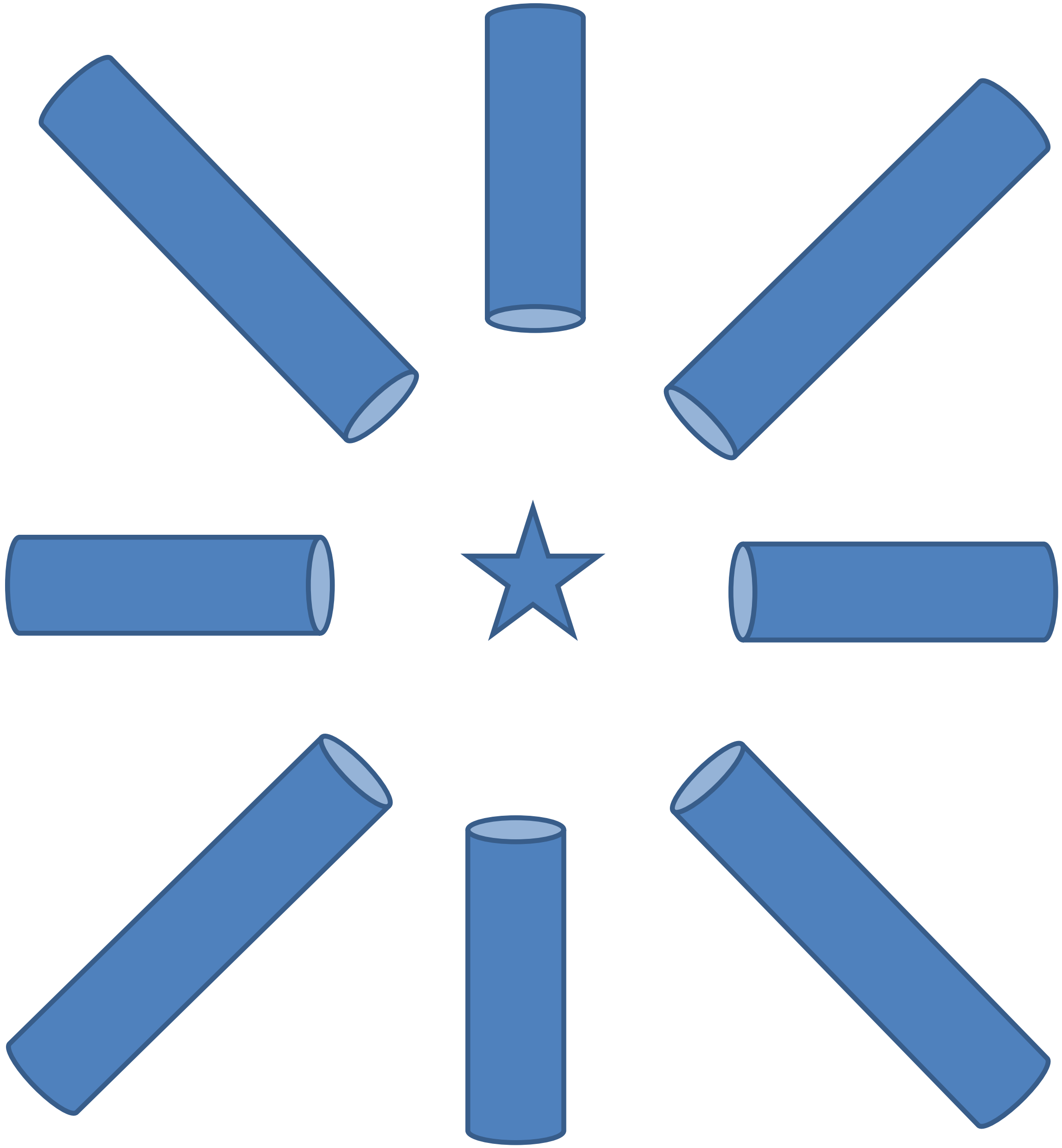
$$P_t = P_c + P_g = \frac{-2\sigma * \cos(\alpha)}{R} - (\rho_{def} - \rho_{inv})gz$$

P_t => Total pressure
P_c => Capillary pressure
P_g => Hydrostatic pressure
σ => Interfacial tension
α => fluid/fluid/solid contact angle (90 to 180 for nonwetting invasion)
R => capillary radius
ρ_{def} => density of the defending fluid
ρ_{inv} => density of the invading fluid
g => acceleration of gravity
z => distance into the network

$$P_t = \frac{-2\sigma * \cos(\alpha)}{R} - 0.5*(\rho_{def}-\rho_{inv})gz$$

$$P_t = \frac{-2\sigma * \cos(\alpha)}{R} - (\rho_{def}-\rho_{inv})gz$$

$$P_t = \frac{-2\sigma * \cos(\alpha)}{R} - 0.5*(\rho_{def}-\rho_{inv})gz$$



$$P_t = \frac{-2\sigma * \cos(\alpha)}{R}$$

$$P_t = \frac{-2\sigma * \cos(\alpha)}{R}$$

$$P_t = \frac{-2\sigma * \cos(\alpha)}{R} + 0.5*(\rho_{def}-\rho_{inv})gz$$

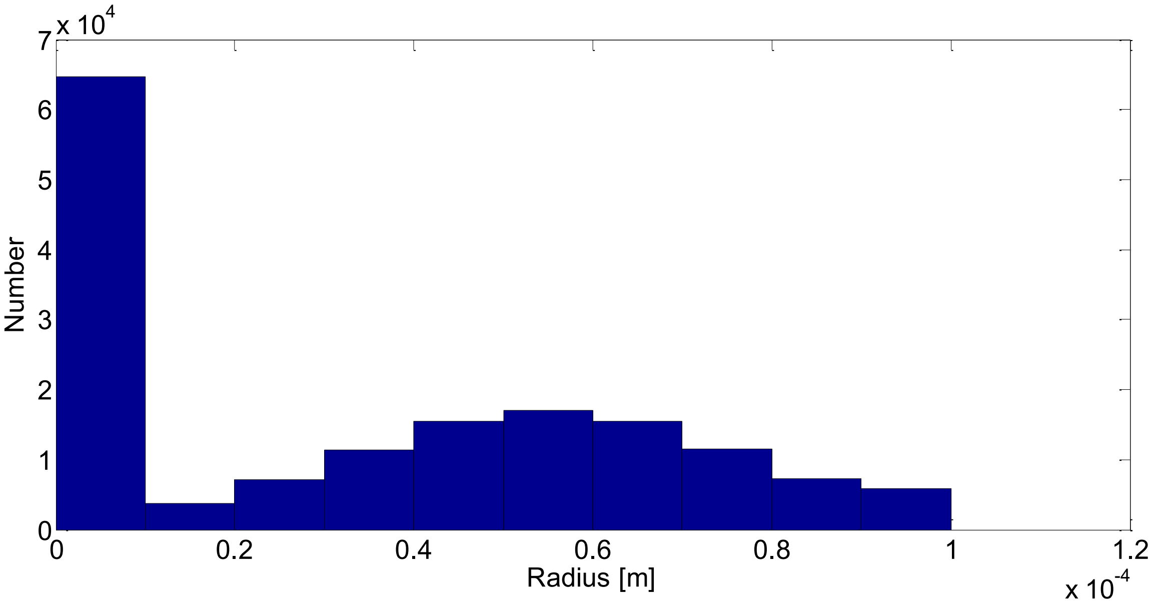
$$P_t = \frac{-2\sigma * \cos(\alpha)}{R} + (\rho_{def}-\rho_{inv})gz$$

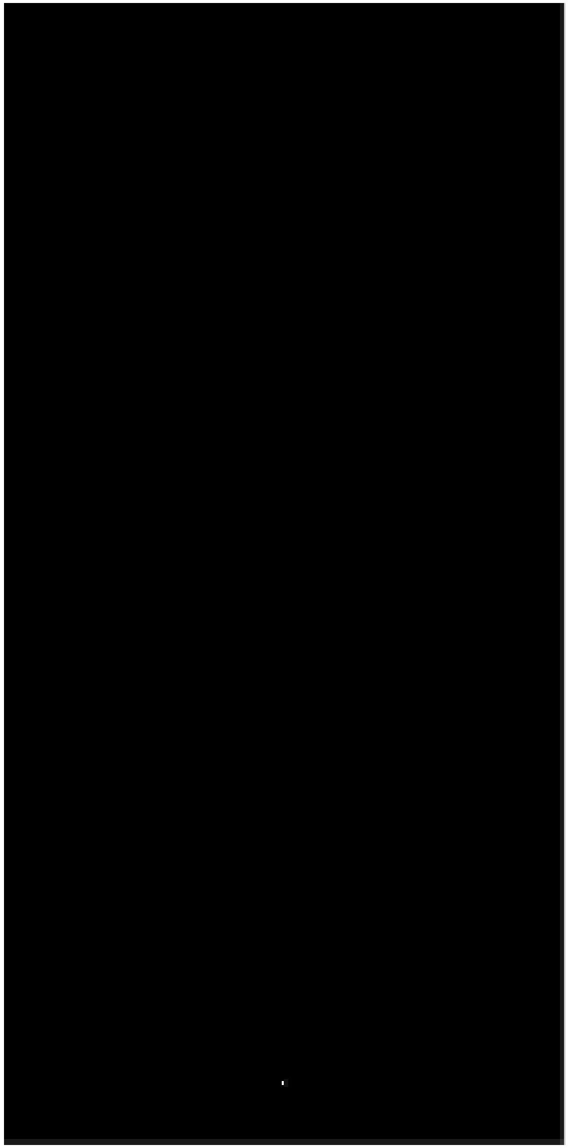
$$P_t = \frac{-2\sigma * \cos(\alpha)}{R} + 0.5*(\rho_{def}-\rho_{inv})gz$$

- Pseudocode
  - Assign parameter values for fluids (next slide)
  - Create a random network of radii, normally distributed, with a specified maximum.
  - Define a percentage of radii to be zero and distribute them randomly
  - Transform radii to capillary pressures (P_c)
  - Define injection point
  - While the injected gas has not reached the edge of the domain
    - Calculate P_t at each of the non-invaded nodes surrounding the current location.
    - Add these values to a list that contains P_t values at every possible invasion move.
    - Find the smallest P_t in the list and invade at its location
    - Move to that location and flag as invaded
  - end

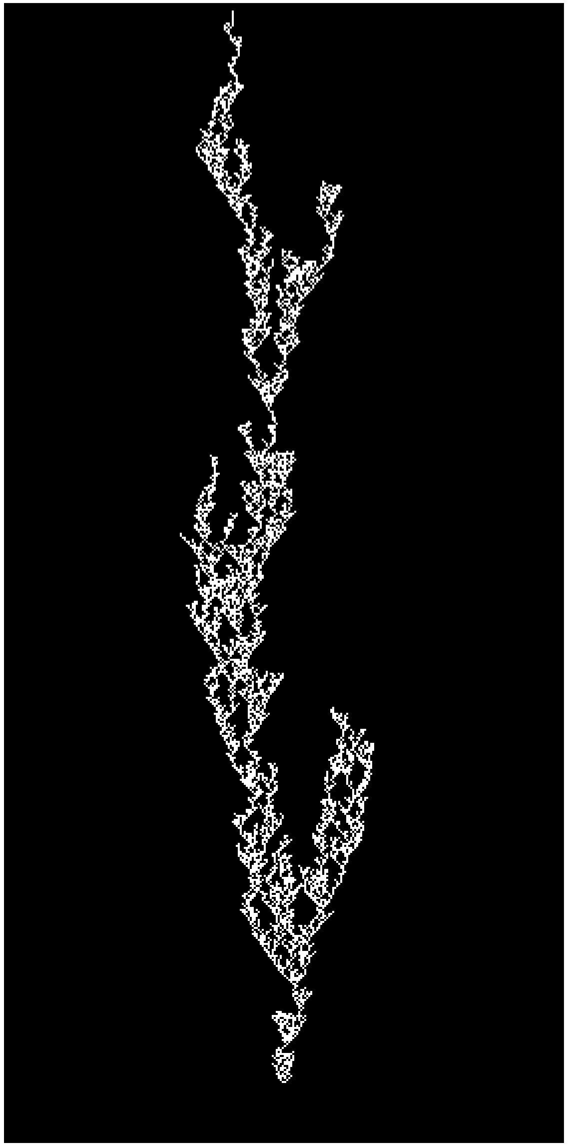
Parameter values (air/water in uniform sand):

- Minimum radius = 0.00001 m
- Maximum radius = 0.0001 m
- Fraction of radii equal to zero = 0.5
- Interfacial tension = 0.05 N/m
- Contact angle = 110 deg
- Density of defending (wetting) fluid = 1000 kg/m^3
- Density of invading (nonwetting) fluid = 1.2 kg/m^3
- Acceleration of gravity = 9.8 m/s^2
- System height = 0.57 m
- System width = 0.28 m

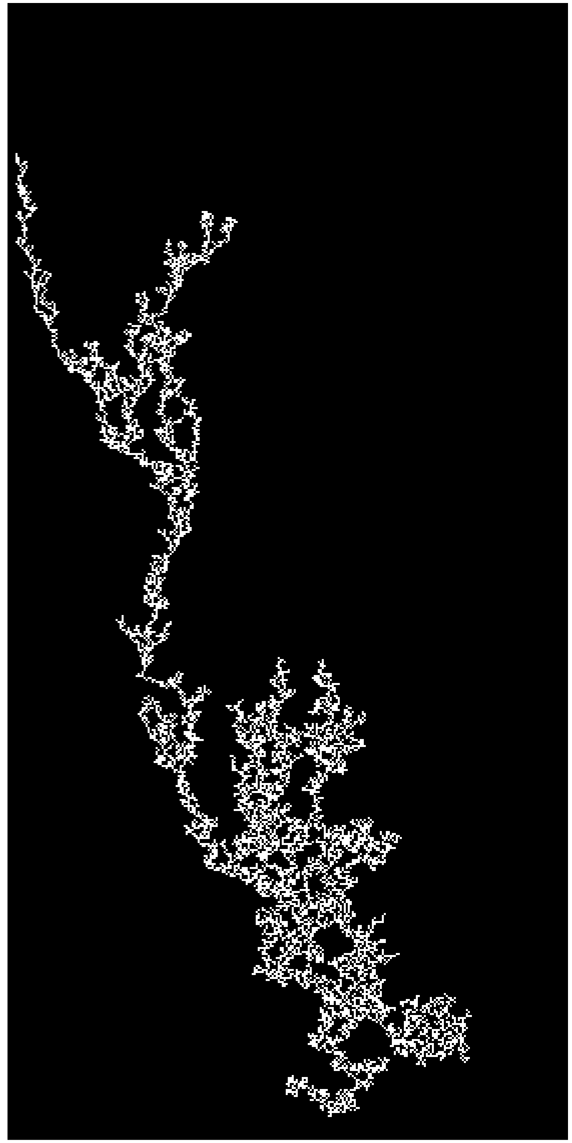




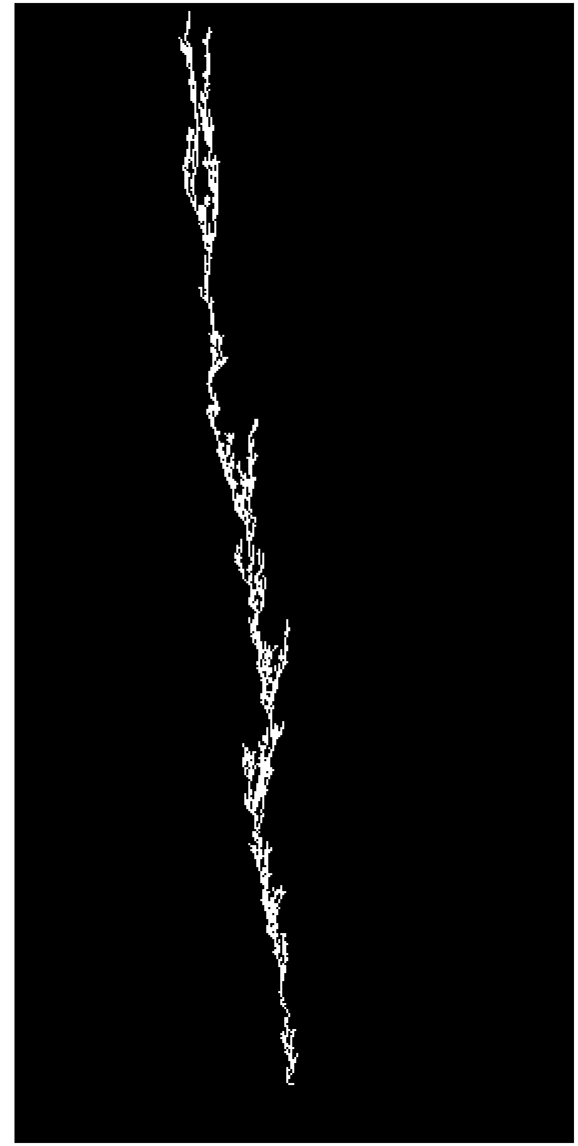
- Array size = 570x280



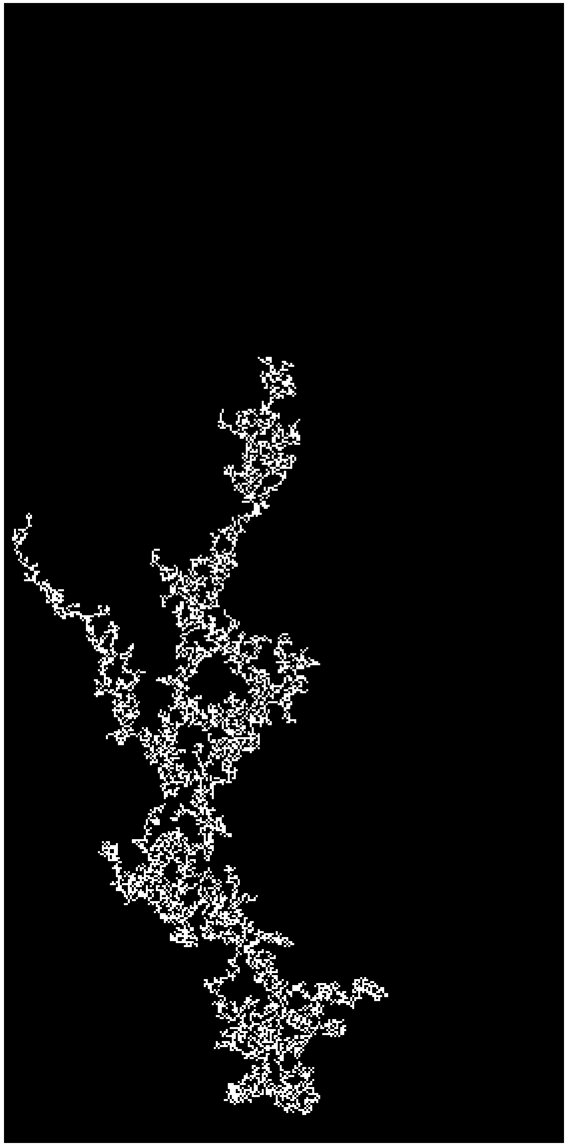
Parameters as before



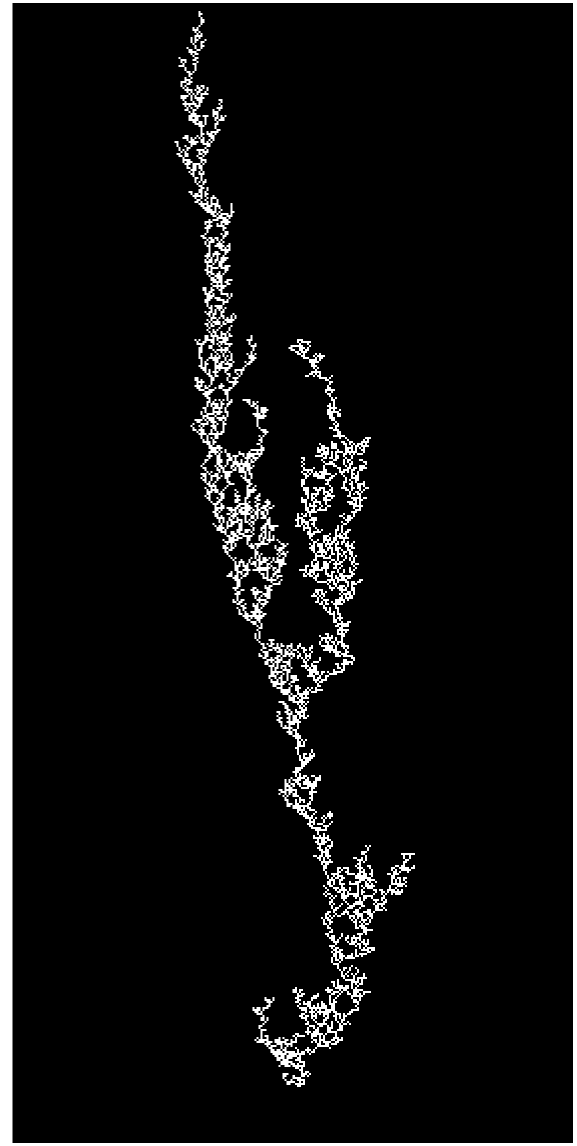
Smaller density contrast (DensInv = 800 kg/m^3)



Fraction Closed = 0.1



- Larger interfacial tension (0.5 N/m)



- Larger contact angle (160 deg)

- References:

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    - Glass, R. J., Conrad, S. H., Yarrington, L., *Gravity-destabilized nonwetting phase invasion in macroheterogeneous porous media: near-pore-scale macro modified invasion percolation simulation of experiments*, Water Resources Research, 37 (2001).
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