Calibration of a Visible-Light Prototype for the CORSAIR Polarimeter

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Abstract

The coronal magnetic field creates the structure of the corona and is the source of heat and energy for CMEs & solar flares. Studying the polarized light emitted by the corona allows for a better understanding of the coronal magnetic field. If a magnetic field is present, the saturated Hanle effect and the Zeeman effect produce measurable signals in the linear and circular polarization. These signals are very difficult to measure because they are 10 to 10,000 times weaker than the intensity.

The Coronal Spectropolarimeter for Airborne Infrared Research (CORSAIR) is an instrument that contains a coronagraph, polarimeter, and grating spectrometer, and provides two-dimensional spectropolarimetric imaging up to one solar radius from the limb. It is designed to measure the solar corona's full polarization state integrated along the line of sight. The CORSAIR polarimeter comprises a rotating wave plate followed by a fixed linear polarizer (analyzer). The multi-order wave plate provides high polarimetric efficiency across five lines with wavelengths between 1 and 4 microns.

In order to establish a calibration scheme for the CORSAIR polarimeter and explore the issues related to using a multi-order wave plate, we have prototyped the polarimeter in visible light. The prototype operates at 532 nm and 670 nm with a wave plate retardance of just over 17 waves and 13 waves, similar to the retardance of the IR wave plate at 1074.7 nm (Fe XIII) and 1430.5 nm (Si X). The prototyped calibration unit consists of a linear polarizer and a rotating quarter wave plate (QWP) at each wavelength.The polarimeter is calibrated by rotating the QWP through 180 degrees for each rotation angle of the polarimeter wave plate, producing a set of modulation and demodulation matrices and associated efficiencies. Due to manufacturing imperfections and other real-world effects, we see small differences between the calibrated and modeled modulation matrices and efficiencies, which have the potential to completely change the resulting linear and circular polarization. We use a model of the full coronal polarization state to explore the effect of polarimeter calibration on the measured linear and circular polarization and define the required calibration accuracy for the IR polarimeter.

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1 Introduction

The sun's corona is the hottest and outermost area around the sun, extending for millions of miles before fading out into what is called solar wind. The magnetic field is the source of heat and energy for solar flares and coronal mass ejections (CMEs), which spew highly energized particles into the corona and around the sun. The solar magnetic field defines the structure of the sun's corona, including plasma and energetic particles ejected by CMEs and solar weather events.

Studying the electrons emitted by the sun is an important step in understanding characteristics of the magnetic field. When electrons lose energy, they release energy in the form of wavelengths of light. In the presence of a magnetic field, like in the solar corona, the wavelengths experience a very slight "splitting," called Zeeman splitting, which is a split or shift of the light into two unique wavelengths.

Measuring this splitting is crucial for understanding magnetic field behavior, but it is difficult without the use of a polarimeter.

1.1 Theory

Polarimeters use a combination of polarizers and retarders (wave plates) to analyze the Stokes vectors of incoming light from the sun.

Stokes vectors are representation of light's polarization, made up of (I, Q, U, V), representing intensity (I), vertical & horizontal linear polarization (Q & U), and circular polarization (V). Compartmetalizing light into these four elements is necessary to view the splitting of wavelengths discussed prior. Our eyes and instrumentation, like cameras, can only actually measure intensity, but due to the shape of the graph of intensity (a singular curve), it is very difficult to measure splitting based just on this element. Wavelength splitting occurs on a microscopically small scale, making it difficult to discern without obvious changes. When light is split into its Stokes parameters, this becomes less challenging due to the specific graph of circularly polarized light; the circular polarization element, V, has a two-curve graph that allows for a much easier detection of splitting than a single curve. Separating the light into its Stokes parameters allows for the detection of splitting, even on a small scale, by observing the characteristics of the light's circular polarization element.

The creation and calibration of a polarimeter is based mostly on matrix operations and linear algebra. The first piece to determining the properties of a polarimeter is creating a Mueller Matrix, a 4x4 matrix in which the first row describes the mathematical transformation to use on a Stokes vector to output intensity. The format for creating a Mueller Matrix is given by:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & G + H\cos(4\Psi) & H\sin(4\Psi) & -\sin(\tau)\sin(2\Psi) \\ 0 & H\sin(4\Psi) & G - H\cos(4\Psi) & \sin(\tau)\cos(2\Psi) \\ 0 & \sin(\tau)\sin(2\Psi) & -\sin(\tau)\cos(t\Psi) & \cos(\tau) \end{bmatrix}$$

with Ψ representing position angle, τ is retardance, and

$$G = \frac{1}{2(1+\cos(\tau)} \tag{1}$$

$$H = \frac{1}{2(1-\cos(\tau)}.\text{[Man]} \tag{2}$$

With each Mueller Matrix and set of data, one can create a modulation matrix M, an nx4 matrix created from the first row of each Mueller Matrix. Therefore,

$$I = MS \tag{3}$$

where I is intensity, M is the modulation matrix, and S is the Stokes vector.

Each modulation matrix can be used to find D, the demodulation matrix, by finding the inverse of the M matrix. Since the M matrix is a size of nx4 elements, it is only square when $n = 4$. However, often times n is a much larger number, depending on the amount of data taken, so the M matrix is not a square matrix, and therefore not invertible. This problem can be solved by using a pseudo-inverse operation, which allows us to find the inverse of a matrix without it being square. Using the pseudo-inverse, D is defined by:

$$D = (M^T M)^{-1} M^T. \tag{4}$$

$$M = A \cdot B^T (B \cdot B^T)^{-1} \tag{5}$$

When taking real data from the sun, a vector of measured intensities, or I, is observed from the instrument. Using the demodulation matrix to find the Stokes vector S, we put the measured intensity into the equation

$$S = DI, \tag{6}$$

allowing us to work backwards to find the Stokes vector from just the measured intensity [ea10].

While in theory, these calculations can be done using just a few equations and matrix multiplications, imperfections in instrument materials manufacturing (particularly in the wave plates and linear polarizers), can cause significant error from the theoretical values. When calibrated prior to launch, these errors can preemptively be compensated for, ensuring accurate measurements of the magnetic field.

2 Polarimetry

2.1 CORSAIR

The Coronal Spectropolarimeter for Airborne Infrared Research (CORSAIR) is a solar instrument contatining a coronagraph, polarimeter, and grading spectrometer that provides two-dimensional spectropolarimetric imaging at a maximum of one solar radius from the limb. The full instrument will measure the solar corona's polarization state in full integrated along the line of sight, as well as estimate magnetic field strength, direction, and plasma thermodynamics. It has a planned one-day flight in September of 2024, followed by a long flight over Antarctica in 2026 or 2027. CORSAIR's development is led by the Smithsonian Astrophysical Observatory along with the NCAR High Altitude Observatory and the University of Hawaii Institute for Astronomy.

2.2 Setup

The CORSAIR polarimeter prototype is made up of a lamp with a decagon-shaped aperture (meant to simulate distant light that enters the system as a collimated beam), a rotating calibration linear polarizer, a rotating calibration quarter wave plate, CORSAIR rotating wave plate, a CORSAIR fixed linear polarizer, an interchangable red & green light filter, and a camera & lens.

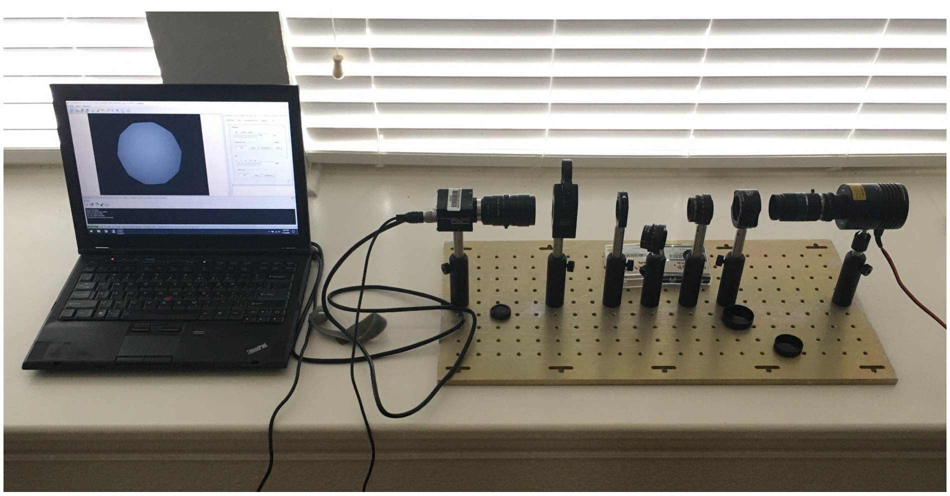

Figure 1: Polarimeter prototype setup; elements are listed from right to left, as light travels from the lamp on the right to the camera on the left.

The light starts as a collimated beam in the lamp on the right, and runs through the elements of the setup until being captured by the camera and fed into the computer imaging software.

When data is taken, unless the images are taken in a completely dark room, some background light is detected along with the light from the lamp. To eliminate this interference, each data set is taken with a "dark" photo; the lamp is turned off or covered, and an image is taken that only includes background light. Then, any time the data is used in the polarimeter software, the brightness of the dark image is subtracted from the brightness of the data so that the background light can be ignored. For this reason, it is important to take all elements of one data set at the same time, as the background light changes depending on the light level in the room. A dark photo must be taken for each new data set to ensure the new background brightness will not interfere with the data.

To use the polarimeter in the visible light spectrum, we use the birefringent material quartz.

2.3 Calibration

Calibrating each element of the polarizer is important to find angle offsets and imperfections in manufactured materials. Each calibration requires a process of isolating each element of the polarizer in order to check for inaccuracy. For every calibration, the item being calibrated is rotated while all other elements are fixed or eliminated from the system to isolate one piece of equipment. The calibrations build on each other, so the linear polarizer calibration is done first, followed by the calibration quarter wave plate, and then the CORSAIR wave plate.

2.3.1 Linear Polarizers

Linear polarizers work by changing a light wave's magnetic field so that it oscillates only in a specific orientation. If one linear polarizer is fixed and one is rotating, the amount of light intensity let through will change based on the angles. When both polarizers are aligned, it means the magnetic field is already oscillating in the same orientation, so theoretically, all light should get through. When they are crossed, or perpendicular to each other, no light should be allowed through.

The linear polarizer calibration setup includes only the rotating calibration linear polarizer and the fixed CORSAIR linear polarizer in the system. Starting at $0°$, the calibration linear polarizer is rotated in steps of $10°$ until it reaches $180°$. Using MATLAB, we created an imaging function to find the brightness of each of these images. Each image is run through the function, which identifies a 21x21 pixel center of the image (the brightest spot) and averages the brightness in that square. This number is used as the determined brightness for each image. After finding the brightness of the images, the averages are sent into an nx1 matrix (19x1 in this case, since $0°$ to $180°$ is 19 data points) to store the measured intensities; this matrix of intensities can then be plotted against the rotation angles to show a graph of the measured intensities.

To calibrate this data with respect to the theoretical intensity values, we must use curve fitting to find the x-shift, y-shift, and y-scalars to match the measured data with the theoretical graph. Since the rotation angles on the stage of the rotating polarizers are only different with respect to each other, starting angle is arbitrary; therefore, curve fitting is used to find the x-shift necessary to match the measured graph with the theoretical graph. In addition, while theoretical intensities have a maximum value of 1, MATLAB reads imaging on different scaling, so it is necessary to find the scaling factor for the theoretical values to change the stretch of the graph. To do this, we created a simulation function to simulate a rotating linear polarizer; using MATLAB's lsqcurvefit function, which takes the simulation function and coefficient guesses as parameters, we are able to find the scaling factor, y-shift, and x-shift of the graphs.

Simulation and curve fitting functions must be given an equation that relates the coefficients to the original data. In our case, this equation is represented by:

$$I_{mod} = A(I_{meas} + C) + B \tag{7}$$

where I_{mod} is the modeled intensity, I_{meas} is measured intensity, A is the y-scalar, B is y-shift, and C is x-shift.

The most important data to take from the linear polarizer calibration are the data points with maximum intensity and minimum intensity. When the image is at maximum intensity, or a local maximum, this means that the fixed and rotating linear polarizers are aligned, parallel with each other. At the local minimum of intensity, the linear polarizers are crossed. These values will be important later for calibrating the quarter wave plate, as well as checking maximum and minimum intensity values.

The linear polarizer data should remain the same in both red and green wavelengths.

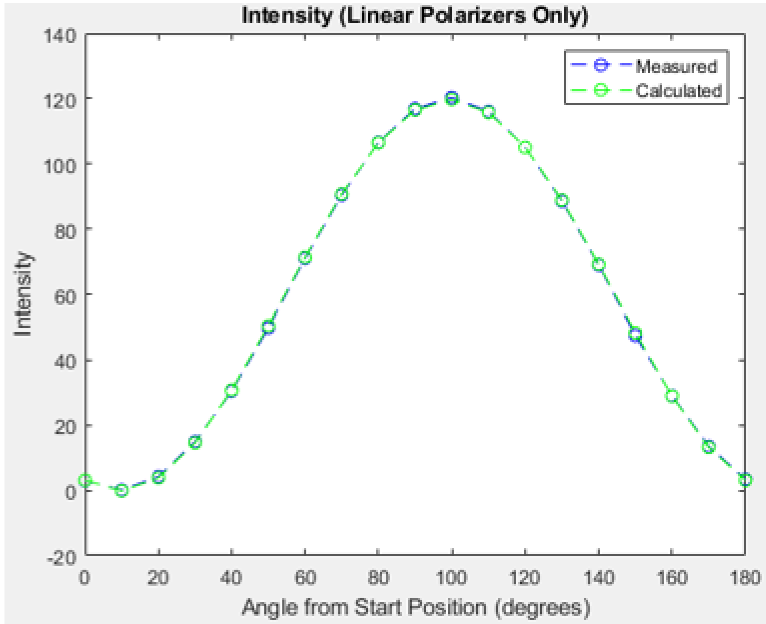

Figure 2: Graph of the linear polarizers' measured intensity fitted with calculated intensity.

2.3.2 Quarter Wave Plate

Following a similar process to the linear polarizers, the quarter wave plate is set up in a system with the fixed CORSAIR linear polarizer, the rotating quarter wave plate, and the calibration linear polarizer. The calibration linear polarizer, which was rotating in the previous step, is now fixed at an angle, either the aligned or crossed position (the calibration process is repeated at both the aligned and crossed position to ensure that they match). The quarter wave plate is rotated from 0° to 90° with a step size of 5°. This data is run through the imaging function and lsqcurvefit in the same way as the linear polarizer, and plotted in the same way.

The output coefficients from lsqcurvefit are important because they tell us the fast or slow axis of the quarter wave plate. The output x-shift value from lsqcurvefit represents one of these axes. It is not possible to tell just based on the output whether the slow or fast axis was found, so we had to check using our equipment. To do this, we set the quarter wave plate fast axis marking horizontal, and added the x-shift value from the code. If the resulting angle is 0° or 180°, then that is the fast axis; if the resulting angle is 90° or 270°, then the slow axis was found.

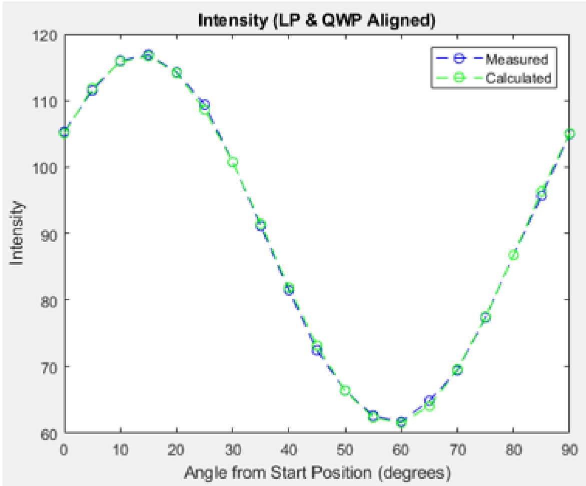

Figure 3: Fitted intensity for rotating quarter wave plate with linear polarizers aligned, green wavelength (0.532 µm).

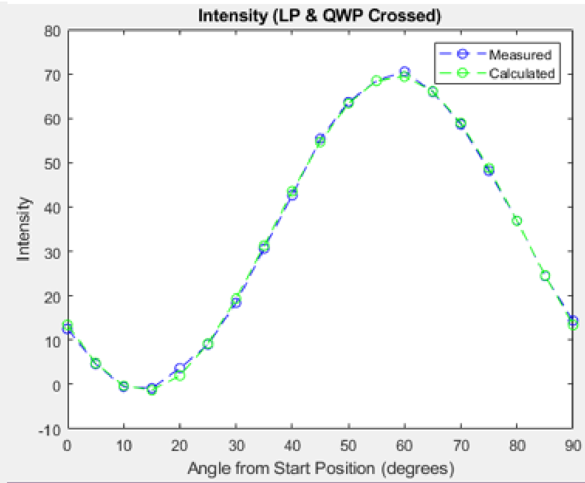

Figure 4: Fitted intensity for rotating quarter wave plate with linear polarizers crossed, green wavelength (0.532 µm).

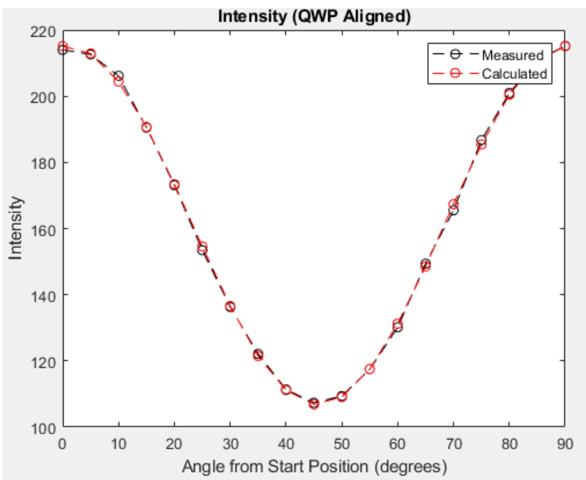

Figure 5: Fitted intensity for rotating quarter wave plate with linear polarizers aligned, red wavelength (0.670 µm).

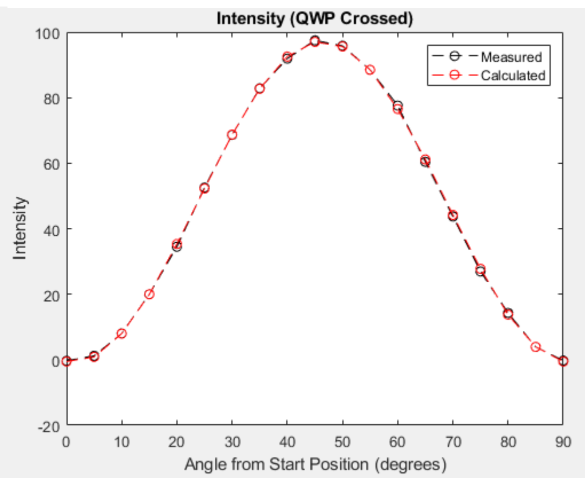

Figure 6: Fitted intensity for rotating quarter wave plate with linear polarizers crossed, red wavelength (0.670 µm).

2.3.3 CORSAIR Wave Plate

Calibration of the CORSAIR wave plate is very similar to the calibration of the quarter wave plate. The data is taken from 0° to 90° with a step size of 5°, and run through the imaging function like usual.

The CORSAIR wave plate presents a different challenge from this point forward comparatively to the quarter wave plate because while the quarter wave plate has a known retardance of 90°, the CORSAIR wave plate has an unknown retardance. Knowing this, it is necessary to add another variable to the curve fitting function. Rather than only calculating the x-shift, y-shift, and y-scalar, the function must now calculate a coefficient to represent the fitted retardation of the CORSAIR wave plate. This function can be checked for accuracy by using it on the quarter wave plate; the output coefficient for retardation should be close to 90°, plus or minus a few tenths of a degree.

Like the linear polarizers, the CORSAIR wave plate characterization should remain the same in both red and green wavelengths.

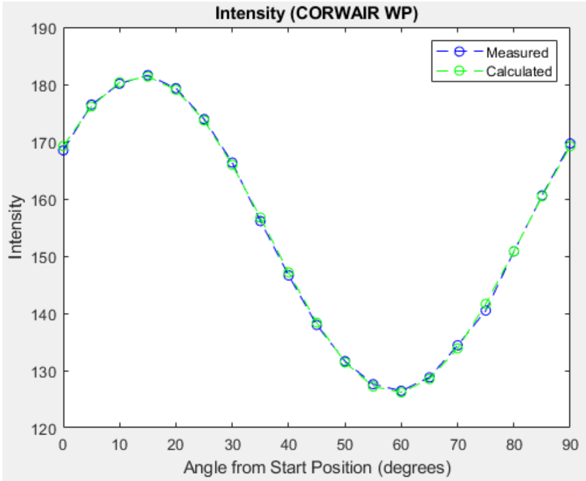

Figure 7: Fitted intensity for rotating CORSAIR wave plate with linear polarizers aligned, green wavelength (0.532 µm).

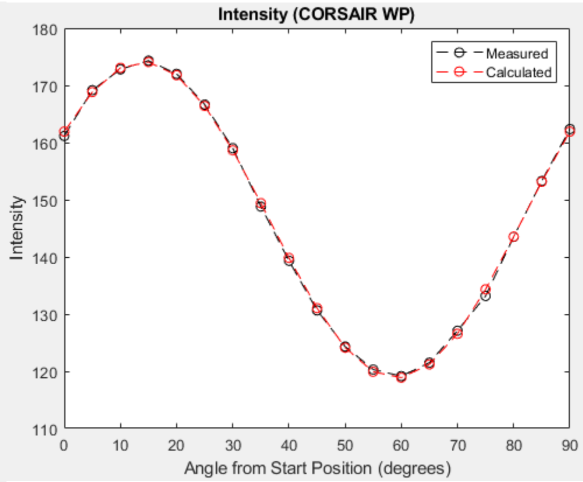

Figure 8: Fitted intensity for rotating CORSAIR wave plate with linear polarizers aligned, red wavelength (0.670 µm).

The differences in maximum brightness of the red and green spectrum are due to the brightness settings of the camera. When taking data, the red light filter let through significantly more light than the green, so we had to decrease exposure for the red spectrum in order to avoid saturation and loss of accuracy on brighter photos. This does not affect the calibration process.

2.4 CORSAIR Wave Plate Characterization

After calibrating all parts of the prototype, the next step is characterizing the CORSAIR wave plate. The characterization process includes all elements in the system (both linear polarizers, the quarter wave plate, and the CORSAIR wave plate). Characterizing the instrument wave plate allows us to create a matrix of intensities, which we call A in our code (this is referred to as I in equation (3)); from here, we use A to find our measured M matrix, also from equation (3).

To characterize the wave plate, we must take a matrix of data by rotating both the quarter wave plate and the CORSAIR wave plate. Depending on the desired accuracy and preciseness of the data set, the matrix should end up as any size square matrix. In our case, we did not have an automated data system, so taking large sets of data would be extensively time consuming; for this reason, we chose a 6x6 matrix, or 36 total data points. To achieve this, while the quarter wave plate is set to 0°, the CORSAIR wave plate is rotated from 0° to 150° with a step size of 30°. Then, the quarter wave plate is moved up to 30°, and the CORSAIR set is repeated. This goes on until the quarter wave plate also reaches 150° with a step size of 30°, creating a 6x6 matrix of points.

When creating this matrix in code, it is important to be sure the nested loops are in the correct position. The outer loop should be the quarter wave plate, while the inner loop should be the CORSAIR wave plate. We may also create our Stokes vector matrix, which we will call B in our code, within the outer loop of our code; the B matrix is an nx4 matrix of Stokes vectors stacked next to each other.

Once the data has been formed into a matrix using a coded function, the intensity matrix A has been created. Our next necessary step is to map the matrix A to the desired scale using a linear mapping technique. For this project, we used the generic linear mapping equation:

$$V_{out} = \frac{(c+d) + (d-c) * (\frac{2*v_{in} - (a+b)}{(b-a)})}{2} \tag{8}$$

where V_{out} is the mapped matrix, v_{in} is the input matrix, a and b are the original extrema, and c and d are the desired extrema.

Using this equation, the matrix can be scaled using a linear combination to the desired size, which should be around [0, 1]. Now, the A and B matrices can be used in accordance to equations (3) and (4) to find the modulation and demodulation matrices, respectively.

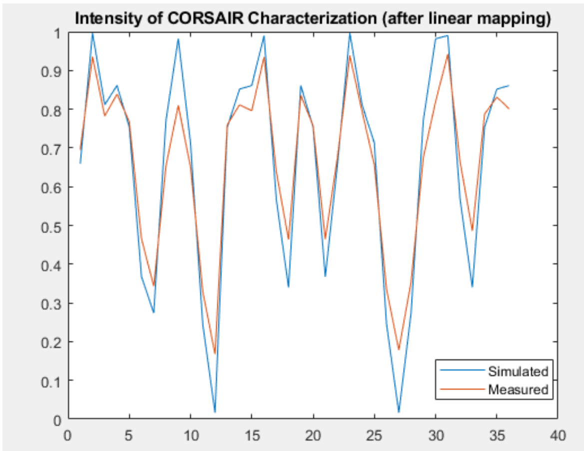

Figure 9: CORSAIR characterization results for intensity after linear mapping, green wavelength (0.532 µm).

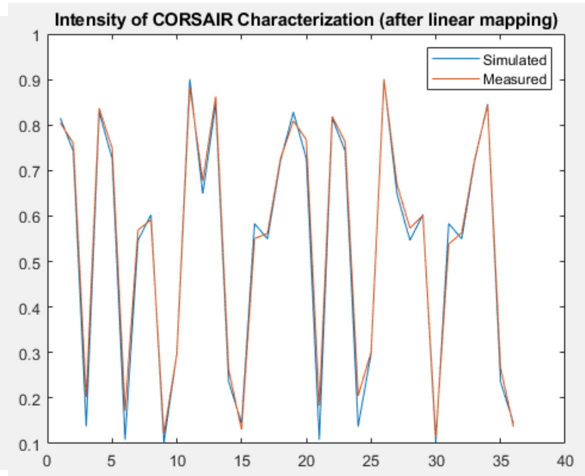

Figure 10: CORSAIR characterization results for intensity after linear mapping, red wavelength (0.670 µm).

The final step of the CORSAIR characterization is to calculate efficiency. Efficiency is calculated by:

$$\epsilon = (n \sum_{j=1}^{n} D_{ij}^2)^{1/2}, \tag{9}$$

where n is size of the measured matrix, and D is the demodulation matrix.

3 Modeled Usage

The purpose of prototyping the CORSAIR polarimeter is to prepare for real data taken of the sun. As the final step of prototyping, we take a simulated image of the corona comprised of a 400x400 pixel set of data meant to model actual solar data.[1] In a real life application, the goal is to use measured intensity values from the sun to determine Stokes vectors.

To check the effectiveness of the polarimeter, we work somewhat circularly since we are using simulated data. The modeled data is generated with Stokes vectors to go along with it; in reality, we would need to find these ourselves. We use the generated modeled Stokes vectors to create a matrix of measured intensities by multiplying these measured intensities by the polarimeter's output measured M modulation matrix. To find the simulated intensities, we multiply the calculated modulation matrix from the polarimeter's code by the generated Stokes vectors. Finally, go backwards again to find our measured Stokes (S_{meas}) by multiplying the polarimeter's measured demodulation matrix by our intensity matrix V_{meas}, and our simulated Stokes (S_{sim}) by multiplying the polarimeter's simulated demodulation matrix by V_{meas}.

The goal is to get these final S_{meas} and/or S_{sim}, which are representative of the polarimeter code's accuracy, to be as close as possible to the generated Stokes vectors, which we know are the correct ones.

[1]The application to model coronal imaging data was created by Maxim Kramar, University of Hawaii Institute for Astronomy.

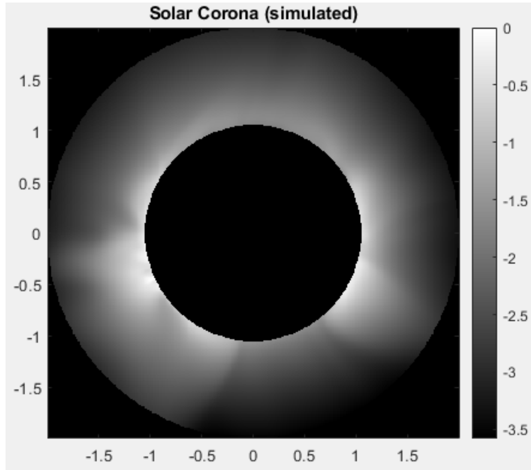

Figure 11: Simulated imaging of the solar corona in greyscale. Axes are in units of solar radii.

The findings of the final step using the modeled solar corona imaging showed the importance of prototyping the equipment before usage. After outputting the Stokes vectors found by the polarimeter code, S_{meas} was vastly closer to the true Stokes parameters than S_{sim}. This is because S_{meas} takes into account calibration and adjusts for imperfection, while S_{sim} is the set of Stokes vectors that were merely calculated theoretically. Without calibration using the polarimeter prototype, data collected on the CORSAIR flight would not take into account the manufacturing generic imperfections of polarimetry.

3.1 Error

The need for calibration comes from imperfections in the manufacturing of wave plates, linear polarizers, etc. It is necessary to have very high levels of precision when analyzing solar imaging and polarized light, particularly with respect to the circular polarization element V. To observe the importance of calibration when taking solar data, we look at the difference between true Stokes vectors and Stokes vectors calculated using an un-calibrated machine.

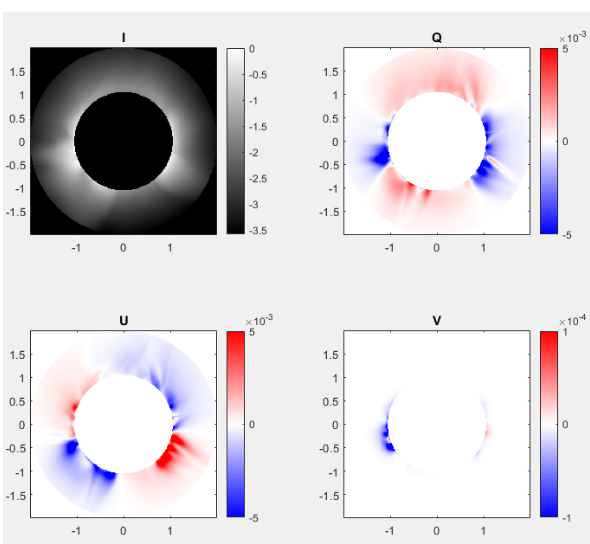

Figure 12: True Stokes vectors generated along with coronal imaging.

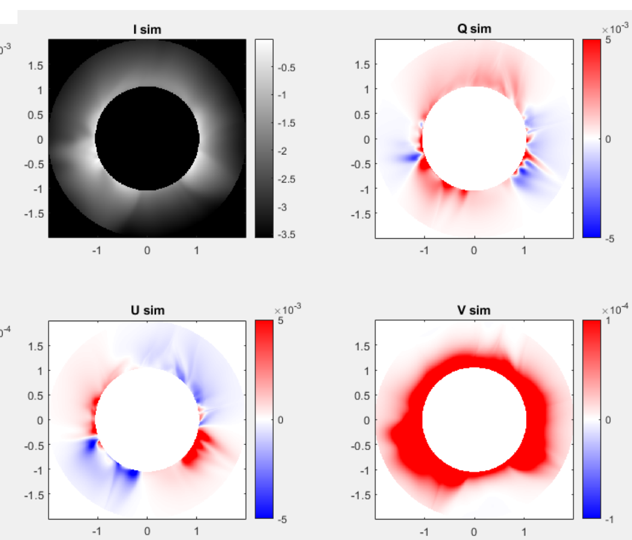

Figure 13: Calculated Stokes vectors using only calculated matrices (i.e. without prototype calibration).

Figure 12 shows the Stokes vectors generated in our coronal imaging model; these are the true and correct Stokes values we will use to test accuracy. In Figure 13, we see the Stokes vectors found when using only the theoretical matrices from section 1.1. These are the results we would get if we assumed our manufactured setup is perfect and follows our ideal mathematical calculations fully; however, this is obviously not the case. One can visually see the most difference in circularly polarized light, and these errors will become more obvious when plotting percent error of an un-calibrated polarimeter.

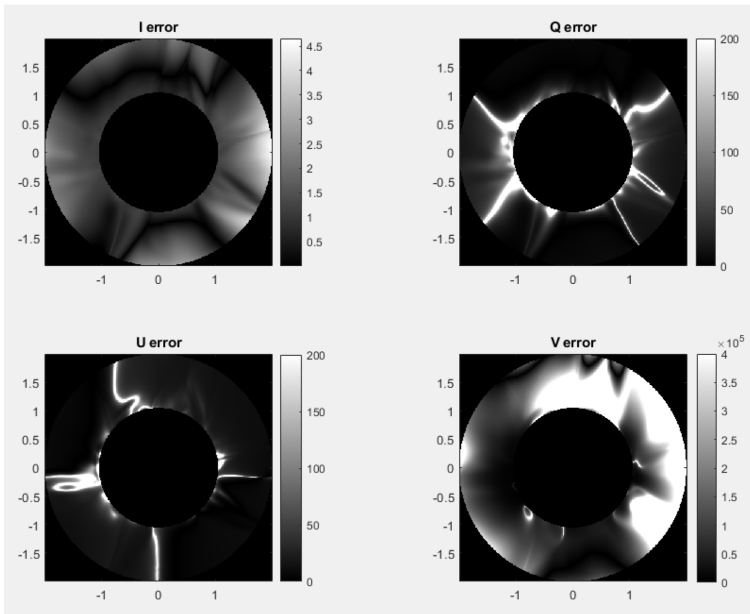

Figure 14: Plotted percent error of Figures 12 & 13. This represents the error present when Stokes vectors are calculated without machine calibration.

Figure 14 shows actual percentage error for each pixel of generated data using an un-calibrated machine. Error in intensity is small compared to other elements, around 4.5% at a maximum. Moving to Q and U, most error is localized around some thin regions; these high-error regions occur where, as seen in figures 12 & 13, the values for Q and U are very close to 0. This phenomenon of string-like high-error regions occurs because the closer values are to 0, the harder it is to get a high accuracy.

This problem carries over into the circular polarization element V. Since V has the smallest values to begin with of any Stokes element, V experiences the most problems with an un-calibrated machine. Compared to the other Stokes elements, error for V without calibration is massive, with a maximum of over 4 x 10^5 %. As V is also the most important Stokes parameter when observing the coronal magnetic field, minimizing error in V is essential; so, it is clear from the massive errors in V that using an un-calibrated polarimeter to study the sun's magnetic field will simply not work.

It is important not to neglect errors that can come from the calibration process itself. Small inaccuracies in data-taking steps are the cause of imperfections in Stokes calculations, although despite these errors, a calibrated polarimeter still maintains a much lower percent error than one using only theoretical calculations. When taking data, like with the quarter wave plate or CORSAIR wave plate calibrations in section 2.3, human error in rotation angle can cause slight discrepancies in data. If the angles are set slightly over or under intended (which is nearly unavoidable on at least a small scale, given that for this experiment we did not have automated stage rotation available), calibration accuracy is slightly decreased. Another piece of calibration that could be improved is the step size in each calibration. If each calibration could be done in steps of only a few degrees, or if our 6x6 matrix from section 2.4 became much larger (say 20x20), calibration accuracy could be greatly improved. Simply taking larger data sets decreases room for error; however, since our data was taking manually instead of using automated rotation, making a large matrix (like a 20x20, which would have 400 images) was simply too time-consuming for our prototyping purposes. However, when calibrating the actual CORSAIR polarimeter, it would be important to increase those data set sizes to minimize error as much as possible.

References

[ea10] Steven Tomczyk et al. Wavelength-diverse polarization modulators for stokes polarimetry. *High Altitude Observatory, National Center for Atmospheric Research*, 2010.

[Man] N. Manset. Polarization of light: from basics to instruments (in less than 100 slides). *Canada France Hawaii Telescope*.