1. Mathematical Logic

① **Proposition Calculus:-**

Sol:- Proposition calculus, which deals with the statements 'T' and 'F' and is concerned with analysis of propositions.

② **Predicate Calculus:-**

Sol:- Predicate calculus, which deals with the predicates which are propositions containing variables.

③ **Proposition (or) Statement:-**

Sol:- A proposition (or) statement is a declarative sentence that is either 'T' (or) F, but not both.

④ **Proposition Variables / Notations:-**

Sol:- Propositions (or) statements are usually denoted by the letters p, q, r... known as proposition variables.

⑤ **Atomic Proposition:-**

A proposition consisting of only a single propositional variable (or) a single propositional constant is called an atomic (primary, primitive) proposition (or) simply proposition.

⑥ **Molecular Proposition:-**

A proposition obtained from the combinations of two (or) more propositions by means of logical operators (or) connectives of two (or) more propositions (or) by negating a single proposition is referred to molecular (or) composite (or) compound proposition.

7) **Logical Connectives:-**

The words and phrases are used symbols to form compound proposition are called connectives.

There are 5 basic connectives

$\sim, \wedge, \vee, \rightarrow, \leftrightarrow$

8) **Conditional Proposition:-**

The compound proposition, "if p then q" denoted by $p \rightarrow q$ is called a conditional proposition.

Here, p is called antecedent (or) hypothesis.

q is called consequent (or) conclusion.

9) **Tautology:-**

A compound proposition that is always true for all possible truth values of its variables (or) It contains only 'T' in the last column of its truth table is called a tautology (T_0).

10) **Contradiction:-**

A compound proposition that is always false for all possible truth values of its variables (or) It contains only 'F' in the last column of its truth table is called a contradiction.

11) **Contingency:-**

A compound proposition that is neither a tautology nor a contradiction is called a contingency.

12) **Well formed formula (WFF):-**

A statement which is gramatically correct is called well defined formula (or) WFF.

(13) Duality Law:-

It can be obtained by changing (or replacing "AND" by "OR" & "OR" by "AND".

Ex:- $P \wedge Q$ is changed to $P \vee Q$.

(14) Functionally Complete set of Connectives:-

Any set of connectives, in which every formula can be expressed in terms of an equivalent formula containing the connectives from the set is called functionally complete set of connectives.

(15) Minimal functionally complete set of Connectives:-

A functionally complete set of connectives which does not contain a connective that can be expressed in terms of the other connectives of the set is called "minimal fcc".

(16) Normal Form:-

A connection which connects more statements then it is called normal form.

Normal form can be 2 types

(i) DNF (ii) CNF

(17) DNF - Disjunctive Normal Form:-

A group of conjunctions are connected with disjunction then it is called DNF.

Ex:- $(P \wedge Q) \vee (Q \wedge R) \vee (R \wedge P)$

(18) CNF - Conjunctive Normal Form:-

A group of disjunctions are connected with conjunction then it is called CNF.

(19) Pdnf – Principal DNF :-

Pdnf can be defined as an equivalent formula consisting of disjunctions of minterms only. This is called the sum of products (SOP) canonical form.

(20) Pcnf – Principal CNF :-

Pcnf can be defined as an equivalent formula consisting of conjunctions of maxterms only. This is called the product of sum (POS) canonical form

(21) Valid Argument :-

If a conclusion is derived from a set of premises by using the accepted rules of reasoning then such a process of derivation is called a deduction (or) a formal proof.
The argument is called a valid argument and the conclusion is called a valid conclusion.

(22) Premises :-

All statements except the conclusion is called premise (or) hypothesis.

(23) Valid Args :-

An arg is said to be valid, if the conclusion is true, whenever the arg is true.

(24) Open Statement :-

$x+2=5$, $x^2<13$, x divides 5.
The above sentences are not propositions, because we can not decisively say whether it is true (or

false. unless the symbol 'x' is specified.
sentences of this kind are called "open statements" or open sentences.
The unspecified symbols such as 'x' in the above sentences are called "free variables".

(25) Predicate Logic:-

A part of declarative sentence describing the properties of an object or relation among object is called predicate.
The logic based on the analysis of predicate in any statement is called predicate logic.

(26) Quantifier:-

The statement which indicates the quantity is called quantifier.
Ex:- $\forall$, $\exists$

(27) Quantified statement:-

A statement involving quantifiers is called quantified statement.
There are 2 types of quantified statements.
(i) Universal Quantifier
(ii) Existance Quantifier.

(28) Universal Quantifier:-

The statement "for all" ($\forall$) is called "Universal Quantifier".

(29) Existance Quantifier:-

The statement "There exist" ($\exists$) is called "Existance Quantifier".

(10) Implication:-
Let p and q are propositions. The proposition "p implies q" denoted by $p \to q$ is called implication.

(11) Bounded Variables:-
The variables which are bounded by the quantifiers are called bounded variables.

Ex:- $\forall x$, $P(x)$, $\exists x P(x)$.

(12) Free Variables:-
The variables which are not bounded by the quantifiers are called the free variables.

2. Set Theory.

(1) Finite & Infinite Sets:-
A set with finite number of elements in it, is called a finite set.
A set with infinite number of elements in it, is called a infinite set.

(2) Null Set:-
A set which contains no element at all is called the null set. Also known as empty set (or) void set. It is denoted by ϕ.

$$\phi = \{x \mid P(x) \wedge \neg P(x)\}.$$

(3) Singleton Set:-
A set which has only one element is called a singleton set. Ex:- $S = \{a\}$ (or) $\{\phi\}$ (or) $\{0\}$.

④ Subset:-

A set within a set. The contained set is called a subset of the containing set.

Let A and B are sets such that every element of A is also an element of B. then A is said to be a subset of B.

⑤ Power set:-

A collection of all subsets of A is called the power set of A. P(A). 2^n.

⑥ Super Set:-

If 'A' is a subset of B then, B is called the superset of A. $A \subseteq B$

$B \supseteq A$.

⑦ Proper subset:- $A \subset B$.

If A is a subset of B, but there is at least one element of B, which does not belong to A. It is called proper subset.

⑧ Equal sets:-

Two sets A and B are said to be equal, if and only if (iff) every element of A is an element of B consequently every element of B is an element of A.

⑨ Universal set:-

All the sets under investigation are likely to be considered as subsets of particular set. This set is called "Universal set."

⑩ Cardinality of a set:-

The cardinal number of set-A is the number of elements in the set-A.

⑪ Collection of sets:-

If the elements of a set are set themselves, then such a set is said to be a collection of sets (or) class of sets (or) family of sets.

⑫ Relation:- Any set of ordered pairs defines a binary relation (or) simply a relation.

⑬ n-ary relation:-

$A_1 \times A_2 \times \dots \times A_n$ is called an n-ary relation.

If $R = \phi$ then R is called void (or) empty relation.

$R = A_1 \times A_2 \times \dots \times A_n$ then, R is called the universal relation.

For $n = 1, 2, 3, \dots$ R is called a unary, binary, ternary relations respectively.

⑭ Functions:-

A particular class of relations called functions.

⑮ One-one:-

For every element in set-A has a unique element in set-B. (or)

If distinct elements of A are mapped into distinct elements of B.

(16) Many-one :-
A function 'f' is said to be many-one iff, two (or) more elements of A have same image in B.

(17) Into :-
A function f is called into function iff, there exists atleast one element in B which is not the image of any element in A.

(18) Onto :-
If every element of B is the image of some element in A.

(or)

If every element in B has a pre-image in A.

(19) Bijective :-
A function 'f' is both one-one and on-to then it is bijective.

(20) Identity Function :-
The function $f : A \to A$ defined by $f(x) = x$ for every $x \in A$ is called the identity of A. (I_A).

(21) Disjoint sets :-
If two sets A and B have no common elements then they are called disjoint sets.
$A \cap B = \phi$.

(22) Binary Relation :-
Binary relation is used to indicate the relation b/w the pairs of two objects. Relation b/w two pairs of objects is called Binary Relation.

(23) Constant Function:-

A function 'f' is a constant function, iff all the elements of set-A have the same Image in set-B.

(24) Hasse Diagram:-

A partial ordering relation $\leq$ is represented as a diagram is called the Hasse diagram.

(25) Properties:-

(i) Each element is represented by small (or dot) circle.

(ii) We does not put arrows on edges and, we does not draw self loops at vertices

(iii) If there is a edge from A to B and there is a edge from B to C and also edge from A to C.

(26) Binary Operation:-

Let 's' be the non-empty set, then the cartesian product $S \times S$ be the set of all ordered pairs of elements in S, then the function $f:(S \times S) \to S$ is called a binary operation on 's'.

(27) Properties:-

Let $*$ be the binary operation on set-S.

(i) Closure property $\Rightarrow$ $a \in S$, $a * a \in S$

(ii) Commutative $\Rightarrow$ $a * b = b * a$, $a, b \in S$

(iii) Associative $\Rightarrow$ $a*(b*c) = (a*b)*c$, $a, b, c \in S$

(iv) Identity $\Rightarrow$ $a \cdot e = e \cdot a = a$.

(v) Inverse $\Rightarrow$ $a * a^{-1} = a^{-1} \cdot a = e$.

(27) **Semi group :-**

A algebraic system (S, *) consisting of a non-empty set S and it is associative and closure ~~in~~ binary operation * is defined on S. Then it is called semi group under the operation *.

Ex: (Z, +) and (Z, *)

(28) **Monoid :-**

Let (S, *) be a semi-group. The semi-group (S, *) is said to be monoid, if 'S' contains the identity element 'e' with respect to *. Then it is called monoid.

(29) **Pigeon hole principle :-**

If n+1 (or) more objects are placed in to n boxes then there is atleast one box containing two (or) more objects.

(31) **Generalized Pigeon hole principle :-**

If 'n' pigeon holes are occupied by kn+1 (or) more pigeons then atleast one pigeon hole is occupied by k+1 (or) more pigeon.

(32) **Applications :-**

This principle is applicable in many fields like

→ Number Theory → Algorithms
→ Probability → Geometry etc.

(33) Lattice :-

A lattice is a partially ordered set (poset) in which every pair of elements has a GLB - Greatest Lower Bound and a LUB - Least Upper Bound.

$LUB = a \vee b$ (or $a \oplus b$) (join)

$GLB = a \wedge b$ (or $a * b$) (Meet).

(34) Properties :-

Let $(L, \leq)$ be a lattice

(i) Idempotent → $a \vee a = a$

(ii) Commutative → $a \vee b = b \vee a$

(iii) Associative → $a \vee (b \vee c) = (a \vee b) \vee c$

(iv) Absorption → $a \vee (b \wedge c) = a$, $a \wedge (b \vee c) = a$

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(35) Algebraic System :-

A system consisting of a non-empty set and one or more n-ary operations on the set is called an "algebraic system".

(36) Properties :-

Let $\langle S, *, + \rangle$ be an algebraic system.

where, $*, +$ are binary operations

- → closure
- → Associative
- → Commutative
- → Identity
- → Inverse
- → Distributive
- → Cancellation → $a * b = a * c$, $b = c$.
- → Idempotent → $a * a = a$
  $0 + 0 = 0$, $1 + 1 = 1$

Groups:-

Let 'S' be a non-empty set and '+' be a binary operation on S, the the algebraic system <S, +> is called a group, iff it satisfies the following properties.

→ Closure
→ Associative
→ Identity
→ Inverse

(37) Subgroup:-

Let <G, *> be a group, if H be a finite subset of group 'G', then H is a sub group of G, iff it satisfies the following properties

→ Closure
→ Associative
→ Identity
→ Inverse

(39) Homomorphism:-

Let G and G' be any two groups with binary operations, '*' and 'Δ' respectively then a mapping $f: G \to G'$ said to be homomorphism. if $f(a * b) = f(a) \, \Delta \, f(b)$. , $\forall \, a, b \in G$.

(40) Isomorphism:-

Let <G, +> and <G', *> are two groups, A function $f: G \to G'$ is called isomorphism iff $G \cong G'$

→ f is one-one → f is onto → f is homomorphism.

3. Elementary Combinatorics

① Binomial Theorem.

$$(x+y)^n = \sum_{r=0}^{n} {}^nC_r \, x^{n-r} \cdot y^r.$$

$$(1+x)^n = \sum_{r=0}^{n} C(n,r) \cdot x^r = \sum_{r=0}^{n} {}^nC_r \cdot x^r.$$

$$(1+x)^{-n} = \sum_{r=0}^{\infty} (-1)^n \, {}^{n+r-1}C_r \cdot x^r.$$

$$(1-x)^{-n} = \sum_{r=0}^{\infty} {}^{n+r-1}C_r \cdot x^r.$$

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②. Newton's Identity.

$$ {}^nC_r \cdot {}^rC_k = {}^nC_k \cdot {}^{n-k}C_{r-k}.$$

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③ Pascals Identity.

$$ {}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$$

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④ Vandermondels Identity

$$ {}^{m+n}C_r = \sum_{k=0}^{\infty} {}^mC_{r-k} \cdot {}^nC_k .$$

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⑤. Multinomial Theorem.

$$(x_1+x_2+\cdots+x_k)^n = \frac{n!}{n_1! \cdot n_2! \cdots n_k!} \cdot (x_1)^{n_1} (x_2)^{n_2} \cdots (x_k)^{n_k}.$$

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⑥ Extended Pigeonhole Principle

Sol: If n pigeons are assigned to m pegionholes and $n > m$, then one of the ~~pegs~~ pigeonholes must contain atleast $\left[\frac{n-1}{m}\right]+1$ pigeons.

⑦ Pigeonhole Principle.

Sol:- If n pigeons are assigned to m pigeonholes then at least one pigeonhole contains two (or more pigeons.

⑧ Generalized Pigeonhole Principle.

Sol:- If n pigeonholes are occupied by $kn+1$ (or more pigeons where k is a positive integer, then at least one pigeon hole is occupied by $k+1$ (or more pigeons.

Q. What is the method of undetermined coefficients?

Sol:- In mathematics, the method of undetermined coefficients is an approach to finding a particular solution to certain non-homogeneous recurrence relation

Q. What is Fallacy?

Sol:- An argument which is not true, then it is called Fallacy.