Unit-1:- Mathematical Logic

Introduction, statements and Notation, Connectives, Well-formed formulas, Tautology, Duality law, Equivalence, Implication, Normal Forms, Functionally complete set of connectives, Inference theory of statement calculus, Predicate Calculus, Inference theory of predicate calculus.

Conjunction:-

($\wedge$) "AND" symbol.

Let p, q are two propositions.
The conjunction $p \wedge q$ is true, only p is true and q is true then all other cases are false.

Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

True - 1
False - 0

p	q	p·q
1	1	1
1	0	0
0	1	0
0	0	0

Disjunction:-

$\vee$, "OR" symbol.

Let p, q are two propositions.
The disjunction $p \vee q$ is false, only, p is false and q is false then all other cases are true.

Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	p+q
1	1	1
1	0	1
0	1	1
0	0	0

Conditional :-

Let p, q are two propositions.
The conditional $p \to q$ is false only 'p' is true and q is false, remaining all cases are true

Truth Table

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	p then q
1	1	1
1	0	0
0	1	1
0	0	1

Bi-Conditional :-

Let p, q are two propositions.
The bi-conditional is denoted by $p \Leftrightarrow q$ (or $p \rightleftarrows q$)
(or $(p \to q) \wedge (q \to p)$) $\wedge$, "AND" symbol.

Truth Table

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T.	T	T

p	q	$p \Leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

(X-NOR).

$x \odot y$

Exclusive Disjunction:- (X-OR).

- Let p, q are two propositions.

The compound propositions $p \veebar q$ (read as either p or q, but not both)

Truth Table

p	q	$p \veebar q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$p \veebar q$
1	1	0
1	0	1
0	1	1
0	0	0

---

① Construct the truth table for the following compound propositions

$\neg$, (NOT) symbol

(i) $p \wedge \neg q$

$\wedge$, "and". $(x \cdot y)$.

Sol:- Truth Table

p	q	$\neg q$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

p	q	$\neg q$	$p \wedge \neg q$
1	1	0	0
1	0	1	1
0	1	0	0
0	0	1	0

(2). Prove that for any propositions p, q, r. the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$

Sol:-

p	q	r	$p \to q$	$q \to r$	(A) $(p \to q) \wedge (q \to r)$	(B) $p \to r$	$A \to B$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	F	T	T
T	F	F	F	T	F	F	T
F	F	F	T	T	T	T	T

"$\neg$" = negation. $\wedge = (\cdot)$, conjunction AND

(3). Prove that for any propositions p & q. The compound proposition $[(\neg q) \wedge (p \to q)] \to \neg p$

Sol:-

p	q	$\neg q$	$p \to q$	(A) $(\neg q) \wedge (p \to q)$	$\neg p$	$A \to \neg p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

Tautology:-

A compound proposition which is "True" for all possible truth values of its components is called a "Tautology". It is denoted by "T_0".

Contradiction:-

A compound proposition which is "False" for all possible truth values of its components, it is called a "contradiction". It is denoted by "F_0".

Contingency:-

A compound proposition that can be "True" (or) "False" is called a "Contingency."

(4) Show that for any two propositions p and q.

(i) $(p \veebar q) \vee (p \leftrightarrow q)$ is a tautology.

Sol:- $(p \veebar q) \vee (p \leftrightarrow q)$

p	q	$p \veebar q$	$p \leftrightarrow q$	$(p \veebar q) \vee (p \leftrightarrow q)$
T	T	F	T	T
T	F	T	F	T
F	T	T.	F	T
F	F	F.	T	T

(ii) $(p \veebar q) \wedge (p \leftrightarrow q)$ is a contradiction.

Sol:-

p	q	$p \veebar q$	$p \leftrightarrow q$	$(p \veebar q) \wedge (p \leftrightarrow q)$
T	T	F	T	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

(iii) $(p \veebar q) \wedge (p \rightarrow q)$ is a contingency

Sol:-

p	q	$p \veebar q$	$p \rightarrow q$	$(p \veebar q) \wedge (p \rightarrow q)$
T	T	F	T	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

(5). Prove that for any propositions p and q, the compound proposition $p \veebar q$ and $(p \vee q) \wedge (\neg p \vee \neg q)$ are logically equivalent. given $p \veebar q \equiv (p \vee q) \wedge (\neg p \vee \neg q)$.

Sol:- Now, $p \veebar q$

			(A)			(B)	
p	q	$p \veebar q$	$p \vee q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$A \wedge B$
T	T	F	T	F	F	F	F
T	F	T	T	F	T	T	T
F	T	T	T	T	F	T	T
F	F	F	F	T	T	T	F

$\therefore\ p \veebar q \equiv (p \vee q) \wedge (\neg p \vee \neg q)$

(6) Prove that for any propositions p, q, r

$[p \to (q \wedge r)] \Leftrightarrow (p \to q) \wedge (p \to r)$

Sol:-

p	q	r	$q \wedge r$	$p \to (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
F	T	T	F	T
F	F	F	F	T
F	F	T	F	T
F	T	F	F	T
T	F	F	F	F

$\therefore\ p \to (q \wedge r)$.

Now, $(p \to q) \wedge (p \to r)$

$\to$ (then) $\wedge$ "and" $(\cdot)$

$\vee$ "OR" $(+)$.

p	q	r	$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
F	T	T	T	T	T
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	T	T
T	F	F	F	F	F

$\therefore [p \to (q \wedge r)] \Leftrightarrow (p \to q) \wedge (p \to r)$

(7) p, q, r three propositions

$[(p \vee q) \to r] \Leftrightarrow [(p \to r) \wedge (q \to r)]$

Sol:-

p	q	r	$p \vee q$ (A)	$A \to r$	$p \to r$ (C)	$q \to r$ (D)	$C \wedge D$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	F
T	F	F	T	F	F	T	F

(8) Show that for any propositions p and q, the compound proposition $p \to (p \vee q)$ is a tautology and the compound proposition $p \wedge (\neg p \wedge q)$ is a contradiction.

Sol:-

p	q	$p \vee q$	$p \to (p \vee q)$	$\neg p$	$\neg p \wedge q$	$p \wedge (\neg p \wedge q)$
T	T	T	T	F	F	F
T	F	T	T	F	F	F
F	T	T	T	T	T	F
F	F	F.	T	T	F	F

$\therefore$ $p \to (p \vee q)$ is a tautology. and

$p \wedge (\neg p \wedge q)$ is a contradiction.

(9) p, q are two propositions

(i) $\neg(p \to q) \equiv (p \wedge \neg q)$

Sol:-

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$p \wedge \neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

(ii) $p \to q \equiv \neg\neg(p \to q) \equiv \neg(p \wedge \neg q) \equiv \neg p \vee q$

Sol:-

p	q	$p \to q$	$\neg(p \to q)$	$\neg\neg(p \to q)$	$\neg q$	$\neg p$	$\neg p \vee q$	A $(p \wedge \neg q)$	$\neg A$
T	T	T	F	T	F	F	T	F	T
T	F	F	T	F	T	F	F	T	F
F	T	T	F	T	F	T	T	F	T
F	F	T	F	T	T	T	T	F	T

(10). Prove the following is a tautology.

(i) $[p \wedge (p \to q)] \to q$

Sol:-

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(ii) $p \to [q \to (p \wedge q)]$

Sol:

p	q	$p \wedge q$	$q \to (p \wedge q)$	$p \to [q \to (p \wedge q)]$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	T	T

(iii) $[p \vee (q \wedge r)] \vee \neg [p \vee (q \wedge r)]$

Sol:

p	q	r	$q \wedge r$ (A)	$p \vee A$ (B)	$\neg B$	$B \vee \neg B$
T	T	T	T	T	F	T
T	T	F	F	T	F	T
T	F	T	F	T	F	T
F	T	T	T	T	F	T
F	F	F	F	F	T	T
F	F	T	F	F	T	T
F	T	F	F	F	T	T
T	F	F	F	T	F	T

(iv) $[(p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))] \vee [(\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)]$

Sol: ~~[illegible]~~

turn the page for answer.

Sol:

p	q	r	$p\vee q$	$\neg p$	$\neg q$	$\neg r$	(A) $\neg q\vee\neg r$	$\neg p\wedge r$	B $\neg(\neg p\wedge r)$	E $(p\vee q)\wedge B$	C $(\neg p\wedge\neg q)$	D $(\neg p\wedge\neg r)$	F $C\vee D$	$E\vee F$
T	T	T	T	F	F	F	F	F	T	T	F	F	F	T
T	T	F	T	F	F	T	T	F	T	T	F	F	F	T
T	F	T	T	F	T	F	T	F	T	T	F	F	F	T
F	T	T	T	T	F	F	F	F	T	T	F	F	F	T
F	F	F	F	T	T	T	T	T	F	F	T	T	T	T
F	F	T	T	T	T	F	T	T	F	F	T	F	T	T
F	T	F	T	T	F	T	T	T	F	F	F	T	T	T
T	F	F	T	F	T	T	T	F	T	T	F	F	F	T

Formulas:-

(i) $\neg(p\vee q) = \neg p\wedge\neg q$

(ii) $\neg(p\wedge q) = \neg p\vee\neg q$

(iii) $\neg(p\to q) = p\wedge\neg q$

(iv) $p\to q \equiv \neg\neg(p\to q)$

$\equiv \neg(p\wedge\neg q)$

$\equiv \neg p \vee q$

(v) $\neg\neg p \equiv p$

(vi) $p\vee(q\vee r) = (p\vee q)\vee r$

(vii) $p\wedge(q\wedge r) = (p\wedge q)\wedge r$

(viii) $p\wedge(q\vee r) = (p\wedge q)\vee(p\wedge r)$

(ix) $p\vee(q\wedge r) = (p\vee q)\wedge(p\vee r)$

⑤ $\neg\neg p = p$

Sol:-

p	$\neg p$	$\neg\neg p$
T	F	T
F	T	F

① Prove that $\neg(p \vee q) = \neg p \wedge \neg q$

Sol:

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

② $\neg(p \wedge q) = \neg p \vee \neg q$

Sol:

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

③ $\neg(p \to q) = p \wedge \neg q$

Sol:

p	q	$\neg q$	$p \to q$	$\neg(p \to q)$	$p \wedge \neg q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

④ $p \to q = \neg\neg(p \to q) = \neg(p \wedge \neg q) = \neg p \vee q$

Sol:

p	q	$p \to q$	$\neg(p \to q)$	$\neg\neg(p \to q)$	$\neg p$	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$\neg p \vee q$
T	T	T	F	T	F	F	F	T	T
T	F	F	T	F	F	T	T	F	F
F	T	T	F	T	T	F	F	T	T
F	F	T	F	T	T	T	F	T	T

(6). $p \vee (q \vee r) = (p \vee q) \vee r$

Sol :-

p	q	r	$p \vee q$	$q \vee r$	$p \vee (q \vee r)$	$(p \vee q) \vee r$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
F	T	T	T	T	T	T
F	F	F	F	F	F	F
F	F	T	F	T	T	T
F	T	F	T	T	T	T
T	F	F	T	F	T	T

(7) $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

Sol :-

p	q	r	$q \wedge r$	$p \wedge q$	$p \wedge (q \wedge r)$	$(p \wedge q) \wedge r$
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	T	F	F	F	F
F	T	T	T	F	F	F
F	F	F	F	F	F	F
F	F	T	F	F	F	F
F	T	F	F	F	F	F
T	F	F	F	F	F	F

Some Properties and Formulas :-

(i) Double Negation : $\neg(\neg p) = p$

(ii) Idempotent Law : $p \vee p = p$; $p \wedge p = p \Rightarrow$ తత్సమానం

(iii) Commutative property : $p \vee q = q \vee p$

స్థిత్యంతర / వినిమయ ధర్మం $\quad p \wedge q = q \wedge p$

⑧. $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

Sol:

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
F	T	T	T	F	F	F	F
F	F	F	F	F	F	F	F
F	F	T	T	F	F	F	F
F	T	F	T	F	F	F	F
T	F	F	F	F	F	F	F

⑨. $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

Sol:

p	q	r	$q \wedge r$	$p \vee q$	$p \vee r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	F	F	F	F	F	F
F	F	T	F	F	F	F	F
F	T	F	F	T	T	F	T
T	F	F	F	T	T	T	T

(iv) Absorption law : $[p \vee (p \wedge q)] = p$

$[p \wedge (p \vee q)] = p$

(v) De Morgan law : $\neg (p \vee q) = \neg p \wedge \neg q$

$\neg (p \wedge q) = \neg p \vee \neg q$

(vi) Associative law : $p \vee (q \vee r) = (p \vee q) \vee r$

సహచర ధర్మం

$$p \wedge (q \wedge r) = (p \wedge q) \wedge r$$

(vii) Distributive law : $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

విభాగన్యాయం

$$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$

(viii) Law for the negation of a conditional

$$\neg (p \to q) = p \wedge \neg q$$

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① Double Negation :- $\neg(\neg p) = p$

Sol :

p	$\neg p$	$\neg(\neg p)$
T	F	T
F	T	F

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② Idempotent law :-

$$p \vee p = p$$

p	p	$p \vee p$
T	T	T
F	F	F

$$p \wedge p = p$$

p	p	$p \wedge p$
T	T	T
F	F	F

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③. Commutative Property :-

$$p \vee q = q \vee p$$

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

$$p \wedge q = q \wedge p$$

p	q	$q \wedge p$	$p \wedge q$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

④ Absorption law :-

$[p \vee (p \wedge q)] = p$

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

$[p \wedge (p \vee q)] = p$

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

⑤ Demorgans law :-

$\neg(p \vee q) = \neg p \wedge \neg q$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

$\neg(p \wedge q) = \neg p \vee \neg q$:-

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

⑥. Associative law:-

$p \vee (q \vee r) = (p \vee q) \vee r$

p	q	r	$q \vee r$	$p \vee q$	$p \wedge (q \vee r)$	$(p \vee q) \vee r$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
F	T	T	T	T	T	T
F	F	F	F	F	F	F
F	F	T	T	F	T	T
F	T	F	T	T	T	T
T	F	F	F	T	T	T

$p \wedge (q \wedge r) = (p \wedge q) \wedge r$

p	q	r	$q \wedge r$	$p \wedge q$	$p \wedge (q \wedge r)$	$(p \wedge q) \wedge r$
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	T	F	F	F	F
F	T	T	T	F	F	F
F	F	F	F	F	F	F
F	F	T	F	F	F	F
F	T	F	F	F	F	F
T	F	F	F	F	F	F

(7). Distributive law:-

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	F	F	F	F	F	F
F	F	T	F	F	F	T	F
F	T	F	F	F	T	F	F
T	F	F	F	T	T	T	T

$$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
F	T	T	T	F	F	F	F
F	F	F	F	F	F	F	F
F	F	T	T	F	F	F	F
F	T	F	T	F	F	F	F
T	F	F	F	F	F	F	F

⑧. Law for two negation of a condition

$$\neg(p \to q) = p \wedge \neg q$$

p	q	$p \to q$	$\neg(p \to q)$	$\neg p$	$p \wedge \neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

Converse, Inverse, Contrapositive, Implication:-

Conditional: $p \to q$

Converse : $q \to p$

Inverse : $\neg p \to \neg q$

Contrapositive : $\neg q \to \neg p$

① If 2 is an integer, then 9 is a multiplier.

Sol:- Let p = 2 is an integer and

q = 9 is a multiplier.

Condition : $p \to q$

Converse : If 9 is a multiplier, Then 2 is an integer

Inverse : If 2 is not an integer, then 9 is not a multiplier

Contrapositive : If 9 is not an multiplier, then 2 is not an integer

②. If 4 is an odd number, then New Delhi is in USA.

Sol:- Let p = 4 is an odd number and

q = New Delhi is in USA.

Condition : $p \to q$

Converse : If New Delhi is in USA, Then 4 is an odd number

Inverse : If 4 is not an odd number, then New Delhi is not in USA

Contrapositive : If New Delhi is not in USA, then 4 is not an odd number.

③. If a quadrilateral is a parallelogram, then it's diagonals bisect each other.

Sol:- Let p = quadrilateral is a parallelogram.

q = It's diagonals bisect each other.

Condition : $p \to q$

Converse : If its diagonals bisect each other then a quadrilateral is a parallelogram.

Inverse : If a quadrilateral is not a parallelogram, then its diagonals doesn't bisect each other.

Contrapositive : It's diagonals doesn't bisect each other, then a quadrilateral is not a parallelogram.

---

④. If $4+3=7$, then $0!=1$

Sol:- Let $p = 4+3=7$

$q = 0! = 1$

Condition : $p \to q$

Converse : If $0!=1$, then $4+3=7$

Inverse : If $4+3 \neq 7$, then $0! \neq 1$

Contrapositive : If $0! \neq 1$, then $4+3 \neq 7$.

Consistent and Inconsistent:-

The premises $P_1, P_2, P_3, \dots, P_n$.

The conjunction of all premises $P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_n$ is true, is called consistenece.

The conjunction of all premises $P_1 \wedge P_2 \wedge P_3 \dots \wedge P_n$ is false, is called inconsistence.

Rule of Inference :-

(i) Rule of conjunctive simplification :

p is true, $p \wedge q$ is true, $p \wedge q \Rightarrow p$

(ii) Rule of Disjunctive amplification :

p is true, $p \vee q$ is true, $p \Rightarrow p \vee q$

(iii) Rule of Syllogism :-

Let p, q, r are propositions

$p \to q$ is true and $q \to r$ is true then $p \to r$ is true.

$\therefore (p \to q) \wedge (q \to r) \Rightarrow p \to r$.

(iv) Modus Ponens :-

Let p is true and $p \to q$ is true then q is true.

$p \wedge (p \to q) \Rightarrow q$

(v) Modus Tollens :-

Let $p \to q$ is true and q is false then p is false

$(p \to q) \wedge \neg q \to \neg p$

(vi) Disjunctive Syllogism :-

Let $p \vee q$ is T and p is F then q is T

$(p \vee q) \wedge \neg p \Rightarrow q$.

Rule of Inferences :-

① Rule of conjunctive simplification :-

Let p, q are two propositions.

p is true, $p \wedge q$ is true

$\boxed{p \wedge q = p}$

② Rule of Disjunctive amplication:-

Let p, q are two propositions.

q is true, then p∨q is true.

$$\boxed{P = P \vee q}$$

③ Rule of Syllogism:-

Let p, q, r are three propositions

p→q is true and q→r is true then

p→r is true.

$$\boxed{[(p \to q) \wedge (q \to r)] \Rightarrow (p \to r)}$$

Tabular Form:-

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

④ Modus Pones:-

Let p, q are two propositions

p is true and p→q is true and q is true.

$$\boxed{[p \wedge (p \to q)] \Rightarrow q}$$

Tabular Form:-

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

⑤ Modus Tollens:-

Let p, q are two propositions

(p→q) is true and q is False then p is False.

$$\boxed{(p \to q) \wedge \neg q \Rightarrow \neg p}$$

Tabular Form:-

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

⑥ Disjunctive Syllogism:-

Let p, q are propositions

p∨q is true and p is F then q is T.

$$\boxed{(p \vee q) \wedge \neg p \Rightarrow q}$$

Tabular form:-

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Q. Test whether the following is a valid argument.

(1) If ~~sachin~~ sachin hit century, then he gets a free car.

Sachin hits a century

∴ Sachin gets a free car

Sol:- Here, p: Sachin hits a century.

q: Sachin gets a free car.

$$\begin{array}{l} p \to q \\ \underline{p} \\ \therefore q \end{array}$$

∴ In view of modus pones.

Given argument is valid.

(2). If I study, then I do not fail in the examination.

If I do not fail in the examination, my father gifts a two wheelar to me.

∴ If I study then my father gifts a two-wheelar to me

Sol:- Here, p: I study.

q: I do not fail in the examination.

r: My father gifts a two-wheelar to me.

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ p \to r \end{array}$$

In view of Rule of syllogism.

Given argument is valid.

(3). If sachin hits a century, then he gets a free car.

Sachin gets a free car

∴ Sachin has hit a century

Sol:- Here, p: Sachin hits a century.

q: Sachin gets a free car.

$$\begin{array}{l} p \to q \\ q \\ \hline p. \end{array}$$

$\therefore$ The given argument is invalid.

$\boxed{p \to p \wedge q \Rightarrow q.}$ It is valid.

---

(4). Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \to \neg H$, $\neg H \to (A \wedge \neg B)$ and $(A \wedge \neg B) \to R \vee S$

Sol: Given premises,

$C \vee D, (C \vee D) \to \neg H, \neg H \to (A \wedge \neg B)$ and $(A \wedge \neg B) \to R \vee S$

$= [C \vee D] \wedge [(C \vee D) \to \neg H] \wedge [\neg H \to (A \wedge \neg B)] \wedge [(A \wedge \neg B) \to R \vee S]$

$(P) \wedge (P \to q) = q.$

$= \neg H \wedge [\neg H \to (A \wedge \neg B)] \wedge [(A \wedge \neg B) \to R \vee S]$

$P \wedge P \to q = q$

$= (A \wedge \neg B) \wedge [(A \wedge \neg B) \to R \vee S]$

$P \wedge P \to q = q$

$= R \vee S$ //.

only one formula modus pones

---

(5) Test the validity of the following arguments.

If a person is poor, he is unhappy.
If a person is unhappy, he dies young.

$\therefore$ Poor person die young.

Sol: Let P: A person is poor.
q: he is unhappy.
r: he dies young.

$$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r. \end{array}$$

In view of the rule of syllogism.
Given argument is valid.

⑥. If there is strike by students, the examinations will be postponed.
The examinations was not postponed.

∴ There was no strike by students

Sol:- Let p: There is strike by students
q: The examinations will be postponed.

modus tollens

$$\begin{array}{c} p \rightarrow q \\ \neg q \\ \hline \neg p \end{array}$$

∴ In view of modus Tollens.
∴ Given statement is valid.

⑦ If Ravi studies, then he will pass in discrete mathematics paper.
If Ravi does not play cricket, then he will study.
Ravi failed in Discrete Mathematics paper.

∴ Ravi played cricket.

Sol:- Let p: Ravi studies
q: Ravi will pass in discrete mathematics paper.
r: Ravi play cricket

$$\begin{array}{c} p \rightarrow q \\ \neg r \rightarrow p \\ \neg q \\ \hline \therefore r \end{array}$$

$= (p \rightarrow q) \land (\neg r \rightarrow p) \land \neg q$

$= (p \rightarrow q) \land \neg q \land (\neg r \rightarrow p)$

$= \neg p \land (\neg r \rightarrow p)$ $\quad \neg r \rightarrow p \equiv \neg p \rightarrow r$

$= \neg p \land (\neg p \rightarrow r)$

$= r$

∴ Given argument is valid.

⑧. If I drive to work, then I will arrive tired.
I do not drive to work

∴ I will not arrive tired.

Sol:- Let p: I drive to work
q: I will arrive tired.

$$\begin{array}{l} p \to q \\ \underline{\neg p} \\ \neg q \end{array}$$

$(p \to q) \land \neg p$

Given argument is invalid.

⑨ If I have talent and hard work, then I will become successful in life.
If I become successful in life, then I will be happy.

∴ If I will not be happy, then I do not work hard (or) I do not have talent.

Sol:- Let p: I have talent and hard work.
q: I will become successful in life.
r: I will be happy.

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \neg r \to \neg p \end{array}$$

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ p \to r \end{array}$$

∴ Given argument is valid.

$$p \to r \equiv \neg r \to \neg p$$

Statement (or) Proposition:-

The collection of well defined words with meaningful is called a statement (or) proposition.
The statement which is either true (or) false.

Ex:- The sun rises in the east.
Smoking is injurious to health.
$(a+b)^2 = a^2+b^2+2ab$

Connectives (or) Notations:-

The words which connects symbols is called a connections (or) notations

(i) Negation:-

The negation which is applicable for one sentence. The opposite statement is called negation. Mathematically and logically represented by NOT and Symbolically denoted by $\sim$ (or) $\neg$ (or) Γ

Ex:- p : Today is sunday.
$\sim p$: Today is not sunday.

Ex:- $p : x+y=1$
$\sim p : x+y \neq 1$.

Truth-Table

p	$\sim p$
T	F
F	T

(ii) Conjunction:-

A connection which connects both statements is called conjunction. Mathematically and logically represented by "AND". Symbolically it is denoted by "$\wedge$".

Ex:- $p : x+y=21$
$q : x>2, y>0$.
$p \wedge q : x+y=21$ and $x>0, y>0$

Ex: p : It is cold
q : It is raining.
$p \wedge q$: It is cold and raining.

Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(iii) **Disjunction:-**

A connective which connects any one statement is called disjunction. Mathematically and logically represented by "OR". Symbolycally denoted by "$\vee$".

Ex: p : Today is thursday.
q : Today is sunday.
$p \vee q$: Today is thursday (or) sunday

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(iv) **Conditional Connective:-**

Let p and q be the any two statements, if p then q is called conditional connective. It is denoted by $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(v) **Biconditional connective:-**

Let p and q be the any two statements then if and only if is called Bi-conditional connective. It is denoted by $p \leftrightarrow q$ (or) $p \rightleftarrows q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Logical Equalence for the table:-

Let A, B are the any two given statements the statements A, B are said to be logically equalence if $A \equiv B$.

Tautology:-

In the given statement the output values are true then it is called tautology.

Contoradiction:-

In the given statement the output values are false then it is called contoradiction.

Contingency:-

In the given statement the o/p values are either true (or) false then it is called contingency.

① write the negation of the following statements.

(i) It is raining then the game is canceled.

Sol:- Let p = It is raining

q = The game is canceled. $\neg(p \to q) \equiv p \wedge \neg q$

The given statement can be written as $p \to q$

$\therefore$ The negation of given statement $(p \to q)$ is $\sim(p \to q)$.

(ii) If he studies then he will pass the examinations.

Sol:- Let p = He studies

q = He will pass the examinations.

The given statement can be written as $p \to q$

$\therefore$ The negation of given statement $(p \to q)$ is $\sim(p \to q)$

(iii) John will take a job in industory (or) go to graduate school.

Sol:- Let p = John will take a job in industory.

q = John will go to graduate school.

The given statement can be written as $p \vee q$.

∴ The negation of given statement $(p \vee q)$ is $\sim(p \vee q)$.

(iv) James will bicycle or run tomorrow.

Sol. Let p = James will bicycle.

q = James will run tomorrow.

The given statement can be written as $p \vee q$.

∴ The negation of given statement $(p \vee q)$ is $\sim(p \vee q)$.

(v) If the professor is fast then the printer is slow.

Sol. Let p = The professor is fast

q = the printer is slow.

The given statement can be written as $p \to q$.

∴ The negation of given statement is $\sim(p \to q)$.

---

(2) Let p, q, and r be the statements.

p = You have the flee

q = You miss the final examination.

r = You pass the course.

(i) $p \to q$

Sol. Let p = If you have the flee

q = You miss the final examination.

If you have the flee then you miss the final examination.

(ii) $\neg p \to r$

Sol. If you don't have the flee then you pass the course.

(iii) $q \to \neg r$:

Sol. If you miss the final examination then you don't pass the course.

(iv) $p \vee q \vee r$

Sol:- If you have the flee (or) you miss the final examination (or) you pass the course.

(v) $(p \to \sim r) \vee (q \to \sim r)$

Sol:- If you have the flee then you don't pass the course. (or) we miss the final examination then you don't pass the course.

(vi) $p \wedge q \vee (\neg q \wedge r)$

Sol:- If you have the flee and you miss the final examination (or) you don't miss the final examination and we pass the course.

Derived Connectives:-

NAND:-

The negation of conjunction of two statements is called NAND.

Let p, q be the two statements then the NAND of p and q is false, when both p and q are true. Otherwise remaining o/ps are true it is denoted by $p \uparrow q$.

Truth Table.

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

NOR:-

The negation of disjunction of two statements is called "NOR".

Let p and q be the two statements NOR of p and q is true when p and q are false. Otherwise true.

The NOR of p and q can be denoted by $p \downarrow q$.

Truth Table

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

XOR:-

Truth Table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Assume that p and q be the any two statements the X-OR of p and q denoted by $p \oplus q$ $p \underline{\vee} q$

The X-OR of p and q is true, when exactly one of p and q is true, otherwise it is false.

Well defined formula (or) Well formed formulas:-

A statement which is expressed grammatically correct then it is called well formed formula.

Properties:-

(i) If p is well defined formula then $\sim p$ is also well defined formula.

(ii) If p and q are well formed formula then $p \wedge q$, $p \vee q$, $p \rightarrow q$, $p \leftrightarrow q$ are also well formed formulas.

(iii) All variables and constants are well formed formulas.

Duality law:-

Let A and B are said to be duality law if and only if one can be obtained by changing "AND" to "OR" (or) "OR" to "AND".

Ex:- $p \wedge q$ is changed to $p \vee q$.

Logical Identities:-

(i) DeMorgan's law

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

(ii) Associative law

$p \vee (q \vee r) \equiv (p \vee q) \vee r$

$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

(iii) Commutative law

$p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$

(iv) Idempotent law

$p \vee p \equiv p$

$p \wedge p \equiv p$

(v) Double law

$\sim(\sim p) \equiv p$

(vi) Distributive law

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(vii) $p \rightarrow q \equiv \sim p \vee q$

Normal Form:-

A connection which connects more statements then it is called normal form. The normal form can be classified in to two types

(i) Disjunctive Normal Form (DNF)

(ii) Conjunctive Normal Form (CNF)

DNF:-

The group of conjunctions are connected with disjunction then it is called DNF.

Ex:- $(p \wedge q) \vee (q \wedge r) \vee (r \wedge p)$

CNF:-

The group of disjunctions are connected with conjunction then it is called CNF.

Ex:- $(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$

Premises:-

All statements except the conclusion is called premises (or) hypothesis.

Valid Arguments:-

An argument is a sequence of statements.

An argument is said to be a valid argument if and only if, the premises are all true, the conclusion must be true.

Rules of Valid Arguments (or) Inferences

Rule-1: If the statement 'p' is true and the statement $p \to q$ is accepted true then q must be true.

Symbolically, it is written as

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

This argument is called "Rule of detachment".

Rule-2:

If the statements $p \to q$ and $q \to r$ are accepted true then the statement $p \to r$ is accepted true.

Symbolically, it is represented as

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

① Verify the validity of the following arguments.

If it rains today then we will not have a party today.

If we do not have party today then we will have a party tomorrow.

∴ If it rains today then we will have a party tomorrow.

Sol: Let p : It rains today

q : We will not have a party today.

r : We will have a party tomorrow.

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

∴ The argument is valid.

② If I study hard then I get job.
I study hard

∴ I get job.

Sol: Let p : I study hard
q : I get job.

$$\begin{array}{c} p \to q \\ p \\ \hline q \\ \hline \end{array}$$

∴ The argument is valid.

③ If Rahul hits a century then he gets a benz car.
Rahul hits a century

∴ Rahul gets a benz car.

Sol: Let p : Rahul hits a century.
q : He gets a benz car

$$\begin{array}{c} p \to q \\ p \\ \hline \therefore q \\ \hline \end{array}$$

∴ Given argument is valid.

④. Prove that the validity of the following statement.
If I get the job and work hard then I will get promoted.
If I get promoted then I will be happy.
I will not be happy.

∴ Either I will not get the job or I will not work hard.

Sol: p : I get the job,
q : I work hard.
r : I will get promoted.
s : I will be happy

$$\begin{array}{c} (p \wedge q) \to r \\ r \to s \\ \sim s \\ \hline \therefore \sim p \vee \sim q \\ \hline \end{array}$$

$\Rightarrow (P\wedge Q) \to r \wedge (r \to s) \wedge \sim s$

$\Rightarrow (P\wedge Q) \to s \wedge \sim s$

$\Rightarrow (P\wedge Q) \to \sim(P\wedge Q)$

$\Rightarrow \sim P \vee \sim Q.$

Modus Ponens
$P\wedge(P\to Q) \Rightarrow Q$

$(P\to Q)\wedge \sim Q = \sim P$
Modus Tollens

$\sim(P\wedge Q) = \sim P \vee \sim Q$

---

(5) Prove that the following statements without using truth table.

(i) $P \to (Q\to r) \equiv (P\wedge Q) \to R$

Sol: $P\to(Q\to r) \equiv \sim P \vee (Q\to r)$ $\quad$ $P\to Q = \sim P\vee Q$

$\equiv \sim P \vee (\sim Q \vee r)$

$= (\sim P \vee \sim Q) \vee r$ $\quad$ Associative law

$= \sim(P\wedge Q) \vee r$

$\equiv (P\wedge Q) \to r$

(ii) $(P\to r)\wedge(Q\to r) \equiv (P\vee Q) \to r$

Sol: $(P\to r)\wedge(Q\to r) \equiv (\sim P\vee r)\wedge(\sim Q\vee r)$ $\quad$ $P\to Q \equiv \sim P\vee Q$

$\equiv (\sim P\wedge \sim Q)\vee r$

$\equiv \sim(P\vee Q)\vee r$ $\quad$ By distributive law

$= (P\vee Q)\to r$

---

(6) Find DNF for $P\wedge(P\to Q)$

Sol: $P\wedge(P\to Q) \equiv P\wedge(\sim P\vee Q)$

$\equiv (P\wedge\sim P)\vee(P\wedge Q)$

which is required DNF

(7) Find DNF for $(p \to q) \wedge (\sim p \wedge q)$

Sol: $(p \to q) \wedge (\sim p \wedge q) \equiv (\sim p \vee q) \wedge (\sim p \wedge q)$

$\equiv [\sim p \wedge (\sim p \wedge q)] \vee [q \wedge (\sim p \wedge q)]$

$\equiv [(\sim p \wedge \sim p) \wedge q] \vee [(q \wedge q) \wedge \sim p]$

$\equiv (\sim p \wedge q) \vee (q \wedge \sim p)$

which is required DNF

---

(8) Find the normal form of $\sim(p \to (q \wedge r))$

Sol: $\sim(p \to (q \wedge r)) = \sim(\sim p \vee (q \wedge r))$

$\equiv p \wedge \sim(q \wedge r)$

$\equiv p \wedge (\sim q \vee \sim r)$

$= (p \wedge \sim q) \vee (p \wedge \sim r) \to$ DNF

---

(9) Find the normal form of $[q \vee (p \wedge r)] \wedge \sim[(p \vee r) \wedge q]$

Sol: $\equiv [q \vee (p \wedge r)] \wedge [\sim(p \vee r) \vee \sim q]$

$\equiv [(q \vee p) \wedge (q \vee r)] \wedge [(\sim p \wedge \sim r) \vee \sim q]$

$\equiv [(q \vee p) \wedge (q \vee r)] \wedge [(\sim p \vee \sim q) \wedge (\sim r \vee \sim q)]$

$= (q \vee p) \wedge (q \vee r) \wedge (\sim p \vee \sim q) \wedge (\sim r \vee \sim q)$

which is required CNF

---

(10) Find the normal form of $p \vee (\sim p \to (q \vee (\sim q \to r)))$

Sol: $\equiv p \vee (\sim p \to (q \vee (\sim\sim q \vee r)))$

$\equiv p \vee (\sim p \to (q \vee (q \vee r)))$

$\equiv p \vee (\sim p \to (q \vee r))$

$\equiv p \vee (\sim\sim p \vee (q \vee r))$

$\equiv p \vee (p \vee (q \vee r))$

$\equiv (p \vee p) \vee (p \vee (q \vee r))$

$\equiv p \vee (p \vee q \vee r)$

$= p \vee q \vee r$

Problems on Inference:-

① Find that R is valid inference from the premises $p \to q$, $q \to r$ and p.

Sol:-

$p \to q$	(Rule P)	
$q \to r$	(Rule P)	
$p \to r$	(Rule T)	(1) (2)
	$= \{p \to q, q \to r, p \to r\}$	
p	(Rule P)	
R	(Rule T)	(3) (4)
	$= \{p \to q, p \Rightarrow q\}$	

modus ponens

$= (p \to q) \wedge (q \to r) \wedge p$

$= p \wedge (p \to q) \wedge (q \to r)$

$= q \wedge (q \to r)$

$= r$ //

② Show that $s \vee r$ is tautology implied by $(p \vee q) \wedge (p \to r) \wedge (q \to s)$

Sol:-

$p \to r$	(Rule P)	
$p \vee q$	(Rule P)	
$\sim p \to q$	(Rule T)	
$q \to s$	(Rule P)	
$\sim p \to s$	(Rule T)	(3), (4)
	$= \{p \to q, q \to r, p \to r\}$	
$\sim s \to p$	(Rule T)	(5)
$\sim s \to r$	(Rule T)	(1)
$s \vee r$	(Rule T)	(7).

$(p \vee q) \wedge (p \to r) \wedge (q \to s)$

$(\neg p \to q) \wedge (q \to s) \wedge (P \to R)$

$(\neg p \to s) \wedge (P \to R)$

$(\neg s \to p) \wedge (P \to R)$

$\neg S \to R$

$S \vee R$ //

Rule of syllogism //

③. Show that $r \to s$ can be derived from the premises ($p \to (q \to s)$, $\sim r \vee p$, and q)

Sol: Assume r is a premise. then $\to$ ①

$$r \wedge (p \to (q \to s)) \wedge (\sim r \vee p) \wedge q$$

we know that $p \wedge (p \to q) = q$

$$p \to q = \sim p \vee q$$

$$r \wedge (r \to p) \wedge (p \to (q \to s)) \wedge q$$

$$p \wedge (p \to (q \to s)) \wedge q$$

$$(q \to s) \wedge q \quad \downarrow q \wedge (q \to s) = s$$

$$s \to ②$$

Now, from ① & ②

$$r \to s.$$

---

④. $[p \to (q \to R)] \wedge [q \to (R \to S)]$, Ans: $\boxed{p \to (q \to s)}$

Sol: Assume 'p' is a premise Then $\to$ ①

$$p \wedge (p \to (q \to R)) \wedge (q \to (R \to S)) \quad \downarrow p \wedge (p \to q) = q$$

$$(q \to R) \wedge (q \to (R \to S))$$

$$(\neg q \vee R) \wedge (\neg q \vee (\neg R \vee S)) \quad \downarrow \text{Taking } \neg q \text{ as common}$$

$$\neg q \vee [R \wedge (\neg R \vee S)]$$

$$\neg q \vee [R \wedge (R \to S)]$$

$$\neg q \vee S$$

$$q \to s \to ②$$

from ① & ②

$$p \to (q \to s)$$

Predicate Logic:-

A part of declarative sentence describing the properties of an object in relation among object is called predicate.

The logic based on the analysis of predicate in any statement is called predicate logic.

Ex:- (i) Ramu is intelligent.
Gopi is intelligent.

(ii) Sita is a student. Rakesh is a student.

There are two statements. There is a common property "is intelligent" and "is a student". We can write these statements together as 'x' is intelligent. The variable 'x' can be replaced by Ramu (or) Gopi

If 'p' is predicate and suppose that 'x' is Ramu and 'y' is Gopi. "is intelligent" is the predicate.

$\therefore$ $p(x)$ = Ramu is intelligent.
$p(y)$ = Gopi is intelligent.

Quantifier (or) Quantified:-

The statement which indicates the quantity is called a quantifier.

The quantifiers are "for all ($\forall$)" and "There exist ($\exists$)".

$\forall(x)\, p(x)$ is the predicate 'p' satisfied for all 'x'.

$\exists(x)\, p(x)$ is the predicate 'p' is satisfied for some 'x'.

Quantifier Statement:-

A statement involving quantifiers is called quantified statement. There are two types of quantified statements.

namely (i) Universal Quantifier
(ii) Existance Quantifier.

Universal Quantifier:-

The statement "for all" ($\forall$) is called universal quantifier. The universal quantifier denoted by the symbol $\forall(x)$. Which is read as for all 'x'. (or) for every 'x' (or) every 'x' such that.

Ex:- Consider the statement, "All AI students are intelligent". Let P(x) denote that 'x' is a intelligent then the statement can be written as $\forall(x)\, p(x)$.

Existance Quantifier:-

The statement "there exist" is called existance quantifier. It is denoted by $\exists(x)$. Which is read as for some 'x' (or) there is an 'x' such that (or) there is at least one 'x'.

Ex:- Consider the statement, there exist 'x' such that $x^2=5$.

The statement can be written as $\exists(x)\, p(x)$.

where $p(x) = x^2=5$.

Negation of Quantified Statement:-

To find the negation of a quantified statement change the quantifier from universal to existance (or) existance to universal.

Ex:- $\sim(\forall(x)\, p(x)) \equiv \exists(x) \sim p(x)$

$\sim(\exists(x)\, p(x)) \equiv \forall(x) \sim p(x)$.

① Let the universe be the set of integers, $p(x) = x$ is a even, $q(x) = x$ is a prime $r(x) = 5$ divides x. Write the following symbols as quantified statements.

(i) $\exists x\,(p(x) \wedge q(x))$

Sol: Some integers are even and prime numbers

(ii) $\forall x\,(p(x) \wedge q(x))$

Sol: Every integer (or) all integers are even and prime numbers.

(iii) $\forall x\,[p(x) \wedge q(x)] \rightarrow r(x)$

Sol: 5 divides every even and prime integers.

(or)

All even and prime integers divisible by 5.

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②. Write each of the following in symbolic form.

(i) All flowers are beautiful.

Sol: Let $\forall x$ is x is flowers Then x is beautiful.

Let $p(x) = x$ is a flower.

$q(x) = x$ is a beautiful.

The given statement can be written as

$$\forall x\,[p(x) \rightarrow q(x)].$$

(ii) Every person is precious.

Sol: Let $p(x) = x$ is a person.

$q(x) = x$ is preccous.

then $\forall x\,[p(x) \rightarrow q(x)]$.

(iii) Some students of this college can speak English and knows Hindi.

Sol:- $\exists x\, [p(x) \wedge q(x)]$

Let $p(x)$ = Speak English

$q(x)$ = Speak Hindi