Unit-2:- Set Theory

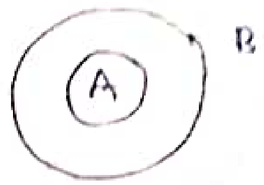
Basic Concepts of Set Theory, Relations and Ordering, the principle of Inclusion - Exclusion, Pigeon hole principle and its application, Functions composition of functions, Inverse Functions, Recursive Functions, Lattices and its properties, Algebraic structures: Algebraic Systems: Examples and General properties, Semigroups and Monoids, groups, sub groups, homomorphism, Isomorphism.

Set:- A set is a collection of well defined objects (or) A collection of elements.

Ex:- $A = \{1, 2, 3, 4\}$, $N = \{1, 2, 3, 4, 5, \ldots\}$

Subset:-

Let A, B are two sets, If every element of A is an element of B, then A is called the subset of B. It is denoted by $A \subseteq B$.



Ex:- If $B = \{1, 2, 3, 4, 5\}$

$A = \{2, 3\}$

Equality of Sets:-

If A and B are two sets, every element of A is an element of B then A and B are said to be equality sets (or) The two sets A and B are equality sets, if it contains same elements.

Ex:- If $A = \{1, 2, 3\}$

$B = \{1, 2, 3\}$ $\boxed{A = B}$

Power set:-

A set of all sub sets of A is called the power set and it is denoted by $P(A)$ (or) 2^A

Let A be the any set then a collection of all sub sets of A is called a power set of A.
If A contains 'n' elements then we get 2^n sub sets.

Eg: If $A = \{1, 2, 3\}$ then the power set of A is

$$P(A) = \{\{\phi\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}\}$$

$$A = \{a, b\} \text{ then } P(A) = \{\{\phi\}, \{a\}, \{b\}, \{a, b\}\}$$

Empty set:-

A set does not contains any element is called "Empty set" or "Null set".
It is denoted by $A = \{\}$ or ϕ.

Finite set:-

If the number of elements in the set is finite then the set is called finite set.

Eg: The set of students in our class.

Infinite Set:-

If the number of elements in the set is infinite then the set is called infinite set.

Ex: The stars in the sky.
The leafs of the tree.

Single Set:-

A set having only one element is called single set.

Eg: $A = \{1\}$.

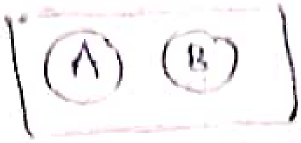
Universal Set :-

It is a set, which includes every set under discussion. It is denoted by E.

The set containing (or) all the sets in the given context is called the universal set.

E

Disjoint Set :-



If two sets A and B have no common elements then they are called disjoint sets. It is denoted by $A \cap B = \phi$.

Ex :- $A = \{2, 4, 6\}$, $B = \{1, 3, 5\}$ $\boxed{A \cap B = \phi}$

Cardinality of the set (or) size of the set :-

A number of distinct elements in a finite set A is called as cardinality of the set (or) size of the set.

It is denoted by $n(A)$ (or) $|A|$.

Ex :- $A = \{1, 2, 3, 4, 5\}$

$n(A) = 5$ (or) $|A| = 5$.

Some operations on a set :-

Union :- Let A, B are two sets, a set which contains all the elements of A and B then it is called as union of sets A and B. It is denoted as $A \cup B$.

Ex :- $A \cup B = \{x \mid x \in A \vee x \in B\}$

$A \cup B = \{x \mid x \in A \vee x \in B\}$

$A = \{1, 2, 3\}$

$B = \{4, 5\}$

$A \cup B = \{1, 2, 3, 4, 5\}$

Intersection:-

Let A and B are two sets, The set which contains common elements of A and B then it is called intersection of sets A and B. It is denoted by $A \cap B$.

Ex:- Let $A = \{1,2,3,4\}$

$B = \{2,4\}$

$A \cap B = \{2,4\}$

Relative Compliment (or) Difference of sets:-

Let A, B are two sets the relative compliment of B in A written as $A-B$, and it is defined as the set of consisting of all elements of A which are not elements in B

(or)

The difference of two sets A and B is the set of all elements of A, which are not elements of B. The difference of sets A and B is denoted by $A-B$.

Ex:- $A-B = \{x \mid x \in A \wedge x \notin B\}$

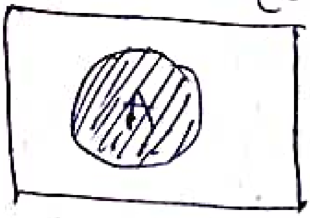
Let $A = \{1,2,3\}$, $B = \{3,4,7\}$

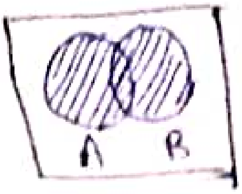
$A-B = \{1,2\}$, $B-A = \{4,7\}$

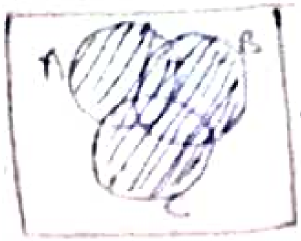
Venn Diagrams:

A venn diagram is a pictorial representation of sets, which are used to show representation b/w sets

Ex:- Universal set =

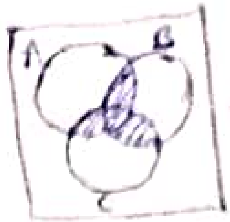


$A \cup B =$ 

, 

$= A \cup B \cup C$

$A \cap B =$ 

, 

$= A \cap B \cap C$

Cartesian Product:

Let A and B are two sets, the cartesian product of A and B denoted by $A \times B$ and it is defined as

$$A \times B = \{ x, y \mid x \in A, y \in B \}$$

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① Let $A = \{\alpha, \beta\}$, $B = \{1, 2, 3\}$ then find

(i) $A \times B$ (ii) $B \times A$ (iii) $A \times A$, $B \times B$ (iv) $(A \times B) \cap (B \times A)$

Sol: given $A = \{\alpha, \beta\}$, $B = \{1, 2, 3\}$

(i) $A \times B = \{(\alpha, 1), (\alpha, 2), (\alpha, 3), (\beta, 1), (\beta, 2), (\beta, 3)\}$

(ii) $B \times A = \{(1, \alpha), (1, \beta), (2, \alpha), (2, \beta), (3, \alpha), (3, \beta)\}$

(iii) $A \times A = \{(\alpha, \beta)\} \times \{\alpha, \beta\}$

$= \{(\alpha, \alpha), (\alpha, \beta), (\beta, \alpha), (\beta, \beta)\}$

(iii) $B \times B = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

(v) $(A \times B) \cap (B \times A) = \phi$

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② If $A = \phi$, $B = \{1, 2, 3\}$ then find $A \times B$ and $B \times A$

Sol: $A \times B = \phi$, $B \times A = \phi$

③ Let $E = \{1,2,3,4,5,6,7,8,9\}$

$A = \{1,2,4,6,8\}$, $B = \{2,4,5,9\}$

Find $\bar{A}$, $\bar{B}$, $\bar{A} \cup \bar{B}$, $\overline{A \cap B}$, $B-A$, $A-B$, $A \cap B$, $A \cup B$

Sol: given, $E = \{1,2,3,4,5,6,7,8,9\}$

$A = \{1,2,4,6,8\}$, $B = \{2,4,5,9\}$

Now, $\bar{A} = E - A = \{3,5,7,9\}$

$\bar{B} = E - B = \{1,3,6,7,8\}$

$\bar{A} \cup \bar{B} = \{1,3,5,6,7,8,9\}$

$A \cap B = \{2,4\}$

$\overline{A \cap B} = E - (A \cap B) = \{1,3,5,6,7,8,9\}$

$A \cup B = \{1,2,4,5,6,8,9\}$

$A - B = \{1,6,8\}$

$B - A = \{5,9\}$

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④ Prove that $A \cup (B \cup C) = (A \cup B) \cup C$

Sol: Let $x \in [A \cup (B \cup C)]$

$= (x \in A)$ or $x \in (B \cup C)$

$= (x \in A)$ or $(x \in B)$ or $(x \in C)$

$= x \in (A \cup B)$ or $x \in C$

$= x \in [(A \cup B) \cup C]$

$\therefore A \cup (B \cup C) = (A \cup B) \cup C$

⑤ Let $A, B, C \subseteq R^2$ where, $A = \{(x,y) \mid y = 2x+1\}$,
$B = \{(x,y) \mid y = 3x\}$, $C = \{(x,y) \mid x-y = 7\}$

find $A \cap B$, $B \cap C$, $\overline{A \cup B}$, $\bar{B} \cup \bar{C}$

Sol: Let $R = \{1, 2, 3, 4, \ldots\}$

$$\epsilon = R^2 = R \times R = \{(1,1), (1,2), (1,3), \ldots \\ (2,1), (2,2), (2,3) \ldots \\ (3,1), (3,2), (3,3) \ldots \\ (4,1), (4,2), (4,3) \ldots \\ \ldots \}$$

$A = \{(1,3), (2,5), (3,7), (4,9)\}$

$B = \{(1,3), (2,6), (3,9), (4,12)\}$

$C = \{(1,-6), (2,-5), (3,-4), (4,-3)\}$

$A \cap B = \{(1,3)\}$

$B \cap C = \phi$

$A \cup B = \{(1,3), (2,5), (2,6), (3,7), (3,9), (4,9), (4,12)\}$

$\overline{A \cup B} = \epsilon - (A \cup B) = \{(1,1), (1,2), (2,1), (2,2) \ldots\}$

$\bar{B} = \epsilon - (B) = \{(1,1), (1,2), (2,1), (2,2), \ldots\}$

$\bar{C} = \epsilon - (C) = \{(1,1), (1,2), (1,3), \ldots\}$

$\bar{B} \cup \bar{C} = \{(1,1), (1,2), (2,1), (2,2) \ldots\}$

Principle of Inclusion and Exclusion:-

(i) For any two sets A and B then

$$|A \cup B| = |A| + |B| - |A \cap B|$$

(ii) For A, B, C are any zero three sets then

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

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① A discrete mathematics class has 25 contains majors in computer science. 13 students majors in mathematics and 8 students joined computer science and mathematics. How many students are in the class, if every student in the class major in mathematics, computer science (or both mathematics and computer science.

Sol:- given $n(CS) = 25$

$n(M) = 13$

$n(M \cap CS) = 8$

$$n(M \cup CS) = n(M) + n(CS) - n(M \cap CS)$$

$$= 13 + 25 - 8$$

$$= 38 - 8 = 30$$

∴ The total number of students in the class is 30

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② A total of 1232 students has taken course in Spanish, 879 have taken a course in French, 114 have taken a course in Russian. Further 103 have taken courses in both Spanish and French. 23 have taken courses in both Spanish and Russian and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French and Russian. How many students have taken a course in all three languages.

Sol: $n(S)=1232$, $n(F)=879$, $n(R)=114$

$n(S\cap F)=103$

$n(S\cap R)=23$ $n(S\cup F\cup R)=2092$

$n(F\cap R)=14$ $n(S\cap F\cap R)=?$

wkt, $n(S\cup F\cup R)=n(S)+n(F)+n(R)-n(S\cap F)-n(S\cap R)-n(F\cap R)+n(S\cap F\cap R)$

$$2092 = 1232+879+114-103-23-14+n(S\cap F\cap R)$$

$$n(S\cap F\cap R)=2092+103+23+14-1232-879-114$$

$$\therefore n(S\cap F\cap R)=7$$

Relations:-

Let A, B be the sets, if R is relation from A to B, then R is a set of ordered pairs A, B. where $a\in A$ and $b\in B$

(or)

Any set of ordered pairs defined as a relation.

(or)

Let A, B are two sets then the relation R can be defined as $(a,b)\in R$ where $a\in A, b\in B$.

Eg: $A=\{1,2,5\}$, $B=\{2,4\}$

$\therefore R=\{(1,2),(2,4)\}$

Domain and Range:

If there are two sets A and B and R is the relation order of pairs x, y then

The domain of R is $\{x \mid (x,y)\in R$ for some y in B$\}$

Range of R is $\{y \mid (x,y)\in R$ for some x in A$\}$

Ex: Let $A = \{2, 3, x\}$
$B = \{4, 6, 7\}$

$R = \{(2,4), (3,6), (x,7)\}$

Domain of $R = \{2, 3, x\}$

Range of $R = \{4, 6, 7\}$

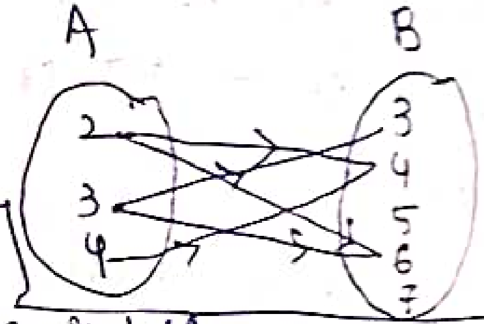
Ex: Let $A = \{2, 3, 4\}$
$B = \{3, 4, 5, 6, 7\}$

Define a relation R from A to B. $A, B \in R$, if A divides B and also find domain and range.

$R = \{(2,4), (2,6), (3,3), (3,6), (4,4)\}$

Domain of $R = \{2, 3, 4\}$

Range of $R = \{3, 4, 6\}$ (or)



Binary Relation:-

Relation is used to indicate the relation b/w the pairs of two objects. Relation b/w the pairs of objects is called Binary Relation.

Properties of Relations:-

(i) Reflexive:- A binary relation R in a set A is reflexive, if every $a \in A$ then $(a, a) \in R$.

Ex: $A = \{1, 2, 3\}$ then $R = \{(1,1), (2,2), (3,3)\}$ is reflexive on A.

(ii) Irreflexive:- A relation R on a set A is irreflexive, if for every $a \in A$ then $(a, a) \notin R$.

Ex:- $A = \{1,2,3\}$ then $R = \{(1,2),(2,3),(3,1)\}$ is a irreflexive on A then $(1,1) \notin R$, $(2,2),(3,3) \notin R$.

(iii) Symmetric :-

A relation R on a set A is said to be symmetric, if $\forall\, a,b \in A$ whenever $(a,b) \in R$ then $(b,a) \in R$.

Ex:- $A = \{1,2,3\}$ then $R = \{(1,2),(2,1),(2,3),(3,2),(1,3),(3,1)\}$ is a symmetric relation of A.

Since, $(1,2) \in R$ then $(2,1) \in R$,

$(2,3) \in R$ then $(3,2) \in R$

$(1,3) \in R$ then $(3,1) \in R$.

(iv) Anti-Symmetric :-

A relation R on a set A is said to be anti-symmetric, if $\forall\, a,b \in A$ whenever $(a,b) \in R$ then $(b,a) \notin R$.

Ex:- $A = \{1,2,3\}$ then $R = \{(1,2),(3,1),(2,3)\}$ is a anti-symmetric on A.

(v) Transitive:-

A relation R on a set A is said to be transitive, if $\forall\, a,b,c \in A$ whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$.

Ex:- $A = \{1,2,3\}$ then $R = \{(1,2),(2,3),(1,3)\}$ is a transitive on A.

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① The relation R_1, R_2, R_3 are defined $A = \{1,2,3\}$ as follows

$R_1 = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

$R_2 = \{(1,2),(2,1),(1,3),(3,1)\}$

$R_3 = \{(1,2),(2,3),(3,1)\}$

Find whether each of R_1, R_2, R_3 reflexive, symmetric and transitive.

Sol:- R_1 is reflexive and symmetric and transitive.

R_2 is symmetric.

R_3 is ~~not transitive~~ not transitive, It is irreflexive

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(2). Given $S = \{1,2,3,\dots,10\}$ and a relation R on S. Where $R = \{(x,y) \mid x+y=10\}$. What are the properties on the relation R.

Sol:- given $S = \{1,2,3,\dots,10\}$

$R = \{(x,y) \mid x+y=10\}$

$R = \{(1,9),(2,8),(3,7),(4,6),(5,5),(6,4),(7,3),(8,2),(9,1)\}$

R is not a reflexive

R is a irreflexive

R is a symmetric

R is not anti-symmetric

R is not a transitive.

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(3). Let $A = \{1,2,3,4\}$ and a relation R define on A by $R = \{(1,2),(1,3),(2,4),(2,3)\}$. Thus here $A = \{a_1, a_2, a_3, a_4\} = B$.

Where, $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $a_4 = 4$. Then Find the matrix relation.

Sol:- given $A = \{1,2,3,4\}$

$B = \{1,2,3,4\}$

$R = \{(1,2),(1,3),(2,4),(2,3)\}$

$$A \times B = \{(1,1),(1,2),(1,3),(1,4),\\ (2,1),(2,2),(2,3),(2,4),\\ (3,1),(3,2),(3,3),(3,4),\\ (4,1),(4,2),(4,3),(4,4)\}$$

$$\text{Matrix Relation} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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(4). Let $X = \{1,2,3,4\}$ and $R = \{(x,y) \mid x > y\}$ then find the MR and also its graph of the relation.

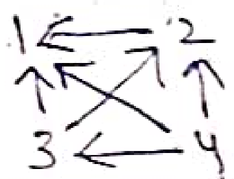
Sol:- given $X = \{1,2,3,4\}$

$R = \{(x,y) \mid x > y\}$

$R = \{(4,1),(4,2),(4,3),(3,1),(3,2),(2,1)\}$

$$MR = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Graph of R



Types of the Relations:-

(i) Equalence Relation:-

The relation R is said to be equalence relation on set-X. If it is reflexive, symmetric and transitive. $\boxed{R \quad S \quad T}$

(ii) Compartibility Relation:-

The relation R is said to be compartibility relation on set-X. If it is reflexive and symmetric. $\boxed{R \quad S}$

(iii) Partial Ordering Relation:-

The relation R is said to be partial ordering relation on set-X, if it is reflexive, anti-symmetric and transitive. $\boxed{R \quad A \quad T}$

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① Let $X = \{1,2,3,4\}$ and $R = \{(1,1), (1,4), (4,1), (4,4), (2,2), (2,3), (3,2), (3,3)\}$. Show that R is equalence relation and also write the matrix of R.

Sol:- given $X = \{1,2,3,4\}$

$R = \{(1,1), (1,4), (2,2), (2,3), (3,2), (3,3), (4,1), (4,4)\}$

R is reflexive. Since $(1,1), (2,2), (3,3) \in R$.

R is symmetric on X. $(1,4), (4,1) \in R$

$(2,3), (3,2) \in R$

$(1,4), (4,1) \in R$.

R is transitive on X. $(2,3), (3,2), (2,2)$ | $(1,4), (4,1), (1,1)$

$(4,1), (1,4), (4,4)$

$\therefore$ R is Reflexive, Symmetric and Transitive ~~then~~
So, R is equalence relation.

$$\text{Matrix Relation} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

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(2). Let $X = \{1,2,3,4,5,6,7,8\}$ and $R = \{(x,y) \mid (x-y)$ is divisible by $3\}$. Show that R is a equalence relation and also write the matrix relation.

Sol:- given $X = \{1,2,3,4,5,6,7,8\}$

$R = \{(x,y) \mid (x-y)$ is divisible by $3\}$.

$R = \{(8,5), (8,2), (8,8), (7,7), (6,6), (5,5), (4,4), (3,3), (2,2), (1,1), (7,4), (7,1), (6,3), (5,2), (4,1)\}$

$R = \{(1,1), (2,2), (3,3), (4,1), (4,4), (5,2), (5,5), (6,3), (6,6), (7,1), (7,4), (7,7), (8,2), (8,5), (8,8)\}$
$(5,8), (2,8), (4,7), (1,7), (2,5), (1,4)\}$

R is reflexive. Since $(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8) \in R$.

R is symmetric on X. Since $(1,4), (4,1)$ | $(4,7), (7,4)$
$(2,5), (5,2)$ | $(2,8), (8,2)$
$(1,7), (7,1)$ | $(5,8), (8,5)$

R is transive on X. Since, $(4,1), (1,4) \Rightarrow (4,4)$
$(2,5), (5,2) \Rightarrow (2,2)$
$(1,7), (7,1) \Rightarrow (1,1)$
$(7,4), (4,7) \Rightarrow (7,7)$ ..... etc.

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

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③ Verify the following relation R on $X = \{1,2,3,4\}$ is an equivalence relation (or) not ?

$R = \{(1,1), (1,4), (4,1), (2,2), (2,3), (3,4), (3,3), (3,2), (4,3), (4,4)\}$

sol: given $X = \{1,2,3,4\}$

$R = \{(1,1), (1,4), (2,2), (2,3), (3,2), (3,3), (3,4), (4,1), (4,3), (4,4)\}$

(i) R is a reflexive on X

since, $1 \in X \Rightarrow (1,1) \in R$

lly, $(2,2), (3,3), (4,4) \in R$.

(ii) R is a symmetric on X.

since, $(1,4) \in R \Rightarrow (4,1) \in R$

$(2,3) \in R \Rightarrow (3,2) \in R$

$(3,4) \in R \Rightarrow (4,3) \in R$.

(iii) R is not transitive on X

since, $(2,3), (3,4) \in R \Rightarrow (2,4) \notin R$.

$\therefore$ R is not a equvalence relation.

④ If $A = \{1,2,3,4,5,6,\ldots,14\}$ and $R = \{(x,y) \mid 3x-y=0\}$ check whether R is equivalence relation.

Sol:- Given $A = \{1,2,3,\ldots,14\}$.

$R = \{(x,y) \mid 3x-y=0\}$.

if $x=1$, $y=3$

$\parallel^{ly}$, $x=2$, $y=6$

$x=3$, $y=9$. ....

$R = \{(1,3),(2,6),(3,9),(4,12)\}$

(i) R is not a reflexive on X. $(1,1),(2,2) \notin R$.

(ii) R is not a symmetric. $(1,3) \in R \Rightarrow (3,1) \notin R$.

(iii) R is not a transitive. $(1,3),(3,9) \in R \Rightarrow (1,9) \notin R$

$\therefore$ R is not a equalence relation.

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⑤ $R = \{(x,y) \mid y = x+5,\ x<4 \text{ and } x,y \in N\}$. Check whether R is equalence relation (or) not.

Sol:- given $R = \{(x,y), \mid y = x+5,\ x<4 \text{ and } x,y \in N\}$

if $x=1$, $y=6$

$x=2$, $y=7$

$x=3$, $y=8$.

$\therefore R = \{(1,6),(2,7),(3,8)\}$.

R is not a reflexive, symmetric and transitive.

$\therefore$ R is not a equalence relation.

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gslod Gslody Gslody Gslody Gslody Gslody Gslody

6. If $A = \{1,2,3,4,5\}$ and $R = \{(x,y) \mid y$ is [illegible]
Check whether R is equivalence relation or not.

Sol: given that $A = \{1,2,3,4,5\}$

$R = \{(1,2), (1,3), (1,4), (1,5), (2,4), (1,1), (2,2), (3,3), (4,4), \ldots$

(i) R is a reflexive

Since, $(1,1), (2,2), \ldots (5,5) \in R$.

(ii) R is not a symmetric and transitive.

$\therefore$ R is not a equivalence relation

---

7. Let $X = \{1,2,3,\ldots,25\}$ and
$R = \{(x,y) \mid x-y$ is divisible by $5\}$.
Show that R is an equivalence relation.

Sol: given $X = \{1,2,3,\ldots,25\}$

$R = \{(x,y) \mid x-y$ is divisible by $5\}$.

$R = \{(25,25), (20,20), (15,15), (10,10), (5,5), (25,5),$
$(25,10), (25,15), (25,20), (20,5), (20,10), (20,15),$
$(15,5), (15,10), (10,5) \ldots\ldots \}$

(i) R is a reflexive.

Since, $(25,25), (20,20), (15,15), (10,10) \ldots\ldots \in R$.

(ii) R is a symmetric.

Since, $(25,5) \in R \Rightarrow (5,25) \in R$

$(25,10) \in R \Rightarrow (10,25) \in R$

$\vdots \qquad \vdots$

$(5,10) \in R \Rightarrow (10,5) \in R$.

(iii) R is a transitive:

Since, $(25,5), (5,10) \in R \Rightarrow (25,10) \in R$

$(5,10), (10,20) \in R \Rightarrow (5,20) \in R$.

$\vdots \quad \vdots \quad \vdots$

$\therefore$ R is a equalence relation.

Composition of binary relation:-

Let R is a relation from X to Y and S be the relation from Y to Z then the composition of binary relation is denoted by ROS (or) R.S and it is defined by the relation consisting of ordered pairs (x, z).

where $x \in X, z \in Z$.

---

① If $A = \{1,2,3,4\}$ and R and S are two relations on set A defined by $R = \{(1,2), (1,3), (2,4), (4,4)\}$

$S = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4)\}$. Find ROS, SOR, ROR, SOS, RO(SOR), (ROS)OR, (ROR)OR, (SOS)OS.

Sol:- given $A = \{1,2,3,4\}$

$R = \{(1,2), (1,3), (2,4), (4,4)\}$

$S = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4)\}$

$ROS = \{(1,3), (1,4)\}$

$SOR = \{(1,2), (1,3), (1,4), (2,4)\}$

$ROR = \{(1,4), (2,4), (4,4)\}$

$SOS = \{(1,1),(1,2),(1,3),(1,4)\}$

$RO(SOR) = \{(1,4)\}$

$(ROS)OR = \{(1,4)\}$

$(ROR)OR = \{(1,4),(2,4),(4,4)\}$

$(SOS)OS = \{(1,3),(1,4)\}$

$\therefore (ROS)OR = RO(SOR)$

---

② Let $A = \{a,b\}$ and $R = \{(a,b),(b,a),(b,b)\}$ and $S = \{(a,a),(b,a),(b,b)\}$ be the relations in A. Find R.S , S.R .

Sol:- given $A = \{a,b\}$

$R = \{(a,b),(b,a),(b,b)\}$

$S = \{(a,a),(b,a),(b,b)\}$

$R.S = ROS = \{(a,a),(a,b),(b,a),(b,b)\}$

$S.R = SOR = \{(a,b),(b,b),(b,a)\}$

---

③ Let $R = \{(1,2),(3,4),(2,2)\}$ and $S = \{(4,2),(2,5),(3,1),(1,3)\}$ Then find ROS, SOR, ROR, SOS, RO(SOR), (ROS)OR, (ROR)OR, (SOS)OS

Sol: given $R = \{(1,2),(3,4),(2,2)\}$

$S = \{(4,2),(2,5),(3,1),(1,3)\}$

$ROS = \{(1,5),(3,2),(2,5)\}$

$SOR = \{(4,2),(3,2),(1,4)\}$

$ROR = \{(1,2),(3,4),(2,2)\} \circ \{(1,2),(3,4),(2,2)\}$

$= \{(1,2),(2,2)\}$

$SoS = \{(4,2),(2,5),(3,1),(1,3)\} \circ \{(4,2),(2,5),(3,1),(1,3)\}$
$= \{(4,5),(3,3),(1,1)\}$

$Ro(SoR) = \{(3,2)\}$

$(RoS)oR = \{(3,2)\}$

$(RoR)oR = \{(1,2),(2,2)\}$

$(SoS)oS = \{(3,1),(1,3)\}$

---

④ Let $f(x) = x+2$, $g(x) = x-2$, $h(x) = 3x$. for $x \in R$ where R is the set of real numbers. Find gof, fog, fof, gog, foh, hog, $gofoh$, $fogoh$.

Sol: given $f(x) = x+2$
$g(x) = x-2$
$h(x) = 3x$

Now, $gof(x) = g[f(x)]$
$= g(x+2)$
$= x+\cancel{2}-\cancel{2}$

$\therefore gof(x) = x.$

$fog(x) = f[g(x)] = f[x-2]$
$= x-\cancel{2}+\cancel{2} = x$

$fof(x) = f[f(x)] = f[x+2] = x+2+2 = x+4$

$gog(x) = g[g(x)] = g[x-2] = x-2-2 = x-4.$

$foh(x) = f[h(x)] = f[3x] = 3x+2$

$hog(x) = h[g(x)] = h[x-2] = 3(x-2) = 3x-6$

$gofoh(x) = g[f[h(x)]] = g[f[3x]] = g[3x+2] = 3x+\cancel{2}-\cancel{2}$
$= 3x$

$$fogoh(x) = f[g(h(x))]$$
$$= f[g[3x]]$$
$$= f[3x-2]$$
$$= 3x - \cancel{2} + \cancel{2} = 3x /.$$

$\therefore\ fog = gof$

$\therefore\ fogoh = gofoh.$

---

(5) Let f and g be the functions from R to R defined by $f(x) = x^2$ and $g(x) = x+5$. Prove that $gof \neq fog$. Also find $gof(3)$, $fog(4)$.

Sol:- given $f(x) = x^2$
$g(x) = x+5$

given $gof \neq fog$.

$LHS = gof(x)$	$RHS = fog(x)$
$= g[f(x)]$	$= f[g(x)]$
$= g[x^2]$	$= f[x+5]$
$= x^2+5$	$= (x+5)^2$
	$= x^2+10x+25$

$\therefore\ gof \neq fog.$

Now, $gof(3) = g[f(3)]$
$$= g[9] = 9+5 = 14$$

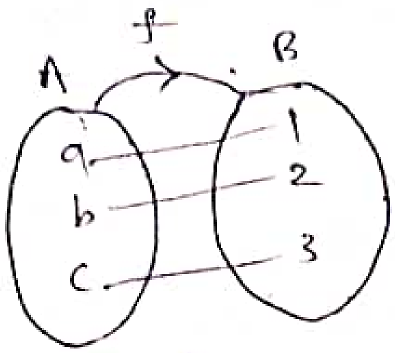
$fog(4) = f[g(4)]$
$$= f[4+5]$$
$$= f(9) = 9^2 = 81 //.$$

Functions :-

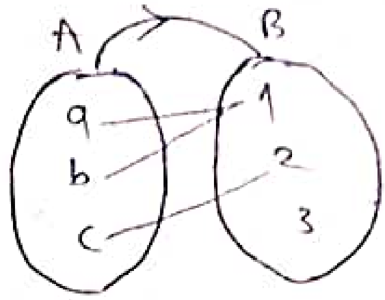
Let A and B two non empty sets then 'f' is a relation from A to B such that for each element in set-A there is a unique element in set-B is called the function.

Here, $a, b \in F$ and $a \in A$, $b \in B$.

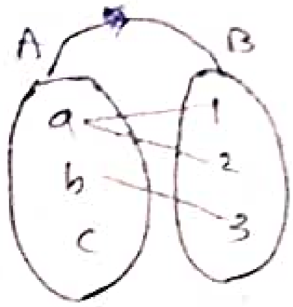
Here 'B' is called the image of 'A'. and 'A' is called the pre-image of 'B'.



It is a function.



It is a function.



It is not a function.

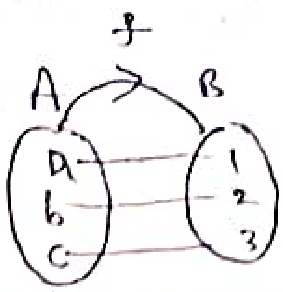
Types of Functions :-

(i) One to One Function :-

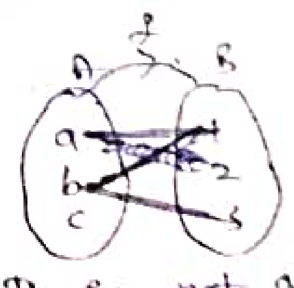
A function $f : A \to B$ is said to be one-to-one function, if every element of set-A has a unique element in set-B.

A one-one function is also called "Injective".

Ex :-



It is one-one function.

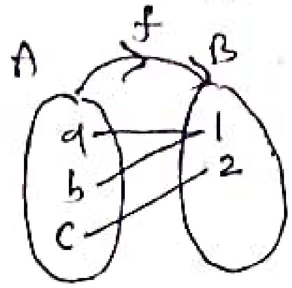


It is not a one-one function.

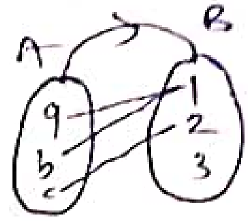
(ii) On to Function:-

Let $f: A \to B$ is said to be on-to function, if every element in B has a pre-image in A. An on to function is also called "surjective".

Ex:-



It is an on-to function.

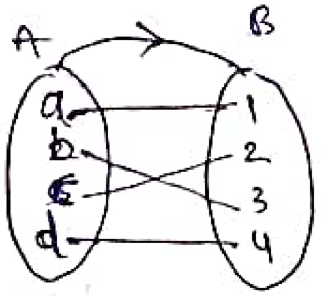


It is not an on to function.

(iii) Bi-jective:-

A function $f: A \to B$ is said to be bijective, if 'f' is both one-one and on-to then 'f' is called Bijective.

Ex:-

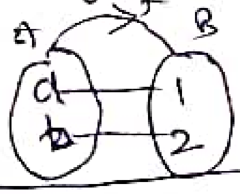


It is a bijective function.

(iv) Identity Function:-

A function $f: A \to A$ is said to be identity funct if the image of every element of A is it self.

Ex:-

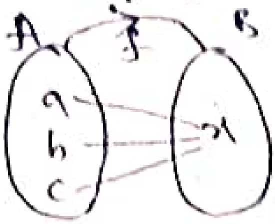


It is a identity function.

(v) Constant Function:-

A function $f: A \to B$ is said to be constant function if all the elements of set-A have the same image in set-B.

Eg:



It is a constant function.

(vi) Inverse of a function:-

Let A, B are two sets, if a function $f: A \to B$ is said to be inverse of a function, it is defined by B to A. It is denoted by $f^{-1}: B \to A$.

Finding inverse of a function:-

(i) Replace $f(x)$ with y

(ii) Interchange 'x' and 'y' and solve for y value.

(iii) Replace 'y' with $f^{-1}(x)$.

① Find the inverse of the function. (i) $f(x) = \frac{3x+2}{2x+1}$

Sol:- given that $f(x) = \frac{3x+2}{2x+1}$

$f(x) = y$

$x = f^{-1}(y)$.

$$y = \frac{3x+2}{2x+1}$$

$$y(2x+1) = 3x+2$$

$$2xy + y = 3x + 2$$

$$2xy - 3x = 2 - y$$

$$x(2y-3) = 2-y \Rightarrow x = \frac{2-y}{2y-3}$$

$\therefore\ y = f(x) \Rightarrow x = f^{-1}(y)$

$$x = \frac{2-y}{2y-3}$$

$$f^{-1}(y) = \frac{2-y}{2y-3}$$

put $y = x \Rightarrow f^{-1}(x) = \frac{2-x}{2x-3}$ //

(6) $f(x) = \sqrt{x+4} - 3$

Sol :-

$$y = f(x)$$

$$y = \sqrt{x+4} - 3$$

$$y+3 = \sqrt{x+4}$$

S.O.B.S

$$y^2 + 9 + 6y = x+4$$

$$x = y^2 + 6y + 5$$

$$\therefore f^{-1}(y) = y^2 + 6y + 5$$

$$f^{-1}(x) = x^2 + 6x + 5 \text{ //}$$

Hasse Diagram :-

A partial ordering relation $\leq$ is represented as a diagram is called the Hasse Diagram.

Properties of Hasse Diagram :-

(i) In Hasse diagram each element represented by small circle (or) dot circle.

(ii) In Hasse diagram we represent the vertices by small circles (or) dot circle. But we does not put arrows on edges and we does not draw self loops at vertices.

(iii) In diagram of partial ordering relation there is a edge from A to B and there is a edge from B to C then there is a edge from A to C. such as we need not exhibit an edge from A to C. It will automatically expressed edge from A to C.

---

① Let $A = \{1,2,3,4,6,12\}$ defined the relation R if b divides a. Prove that R is a partial ordered relation on A and draw the Hasse diagram.

Sol: given $A = \{1,2,3,4,6,12\}$.

$R = \{(a,b) \mid b \text{ divides } a\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,12), (2,2), (2,4),$
$(2,6), (2,12), (3,3), (3,6), (3,12), (4,4), (4,12),$
$(6,6), (6,12), (12,12)\}$

(i) R is reflexive on A.

since, $(1,1), (2,2), \ldots (12,12) \in R$.

(ii) R is a anti-symmetric on A

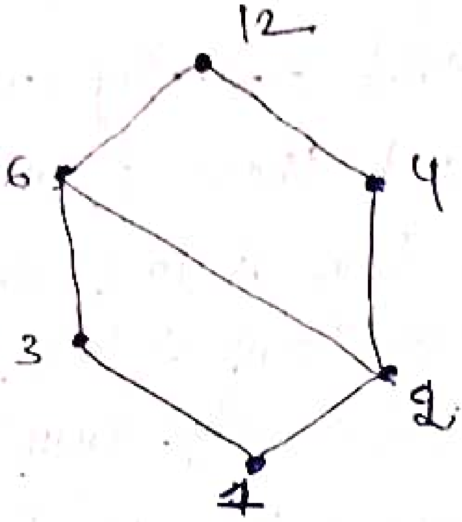
since, $(1,2) \in R \Rightarrow (2,1) \notin R$.

(iii) R is a transitive on A

since, $(1,2), (2,6) \in R \Rightarrow (1,6) \in R$
$(2,6), (6,12) \in R \Rightarrow (2,12) \in R$

$\therefore$ R is a partial ordered relation [RAT]

The Hasse diagram is



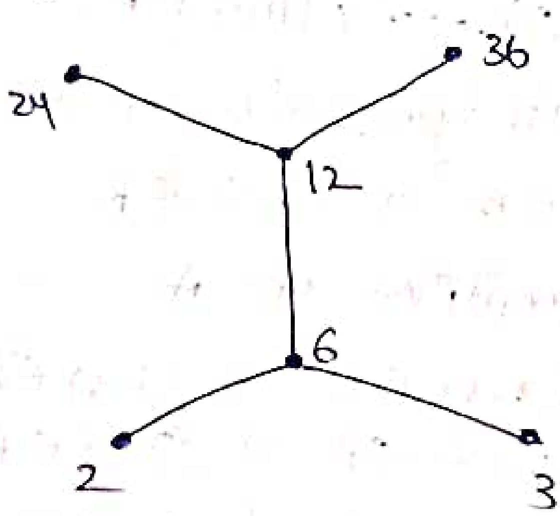
② Let $X = \{2,3,6,12,24,36\}$ and the relation defined by $x \leq y$ and y divides x and draw the hasse diagram.

Sol:- given $X = \{2,3,6,12,24,36\}$

$R = \{(a,b) \mid a \leq b,\ b \text{ divides } a\}$

$R = \{(2,6),(2,12),(2,24),(2,36),(3,6),(3,12),(3,24),$
$(3,36),(6,12),(6,36),(2,2),(3,3),(6,6),(12,12),$
$(24,24),(36,36),(3,24),(6,24),(12,24),(12,36)\}$

Hasse diagram is



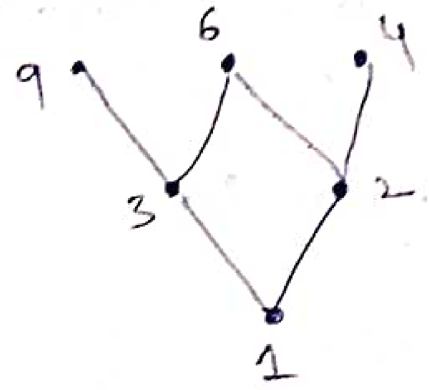
③ $[\{1,2,3,4,6,9\} \mid]$

Sol: given $A = \{1,2,3,4,6,9\}$

$R = \{(a,b) \mid b \mid a\}$

$R = \{(1,2),(1,3),(1,4),(1,6),(1,9),(2,4),(2,6),(3,6),(3,9),$
$(1,1),(2,2),(3,3),(4,4),(6,6),(6,9)\}$

Hasse diagram is.



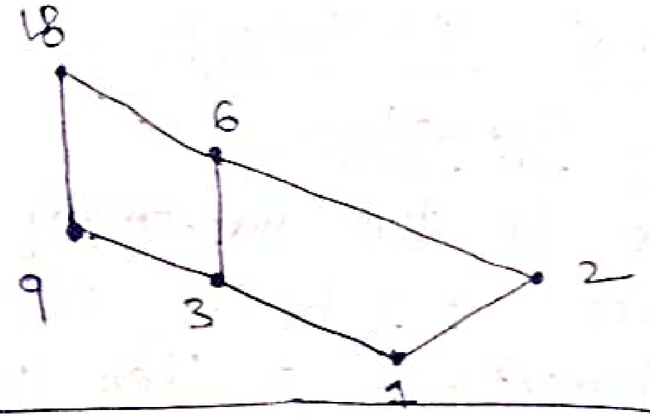
④ $[\{1,2,3,6,9,18\} \mid]$

Sol: given $A = \{1,2,3,6,9,18\}$

$R = \{(a,b) \mid b \mid a\}$

$R = \{(1,2),(1,3),(1,6),(1,9),(1,18),(2,6),(2,18),(3,6),$
$(3,9),(3,18),(6,18),(9,18),(1,1),(2,2),(3,3),$
$(6,6),(9,9),(18,18)\}$

Hasse diagram is.



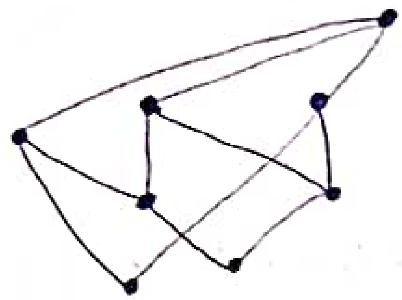
⑥ [{1,2,3,5,6,10,15,30}|]

Sol: given $A = \{1,2,3,5,6,10,15,30\}$

$R = \{(a,b) \mid b|a\}$

$R = \{(1,2), (1,3), (1,5), (1,6), (1,10), (1,15), (1,30),$
$(2,6), (2,10), (2,30), (3,6), (3,15), (3,30),$
$(5,10), (5,15), (5,30), (6,30), (10,30), (15,30),$
$(1,1), (2,2), (3,3), (5,5), (6,6), (10,10), (15,15), (30,30)\}$

The Hasse diagram is;



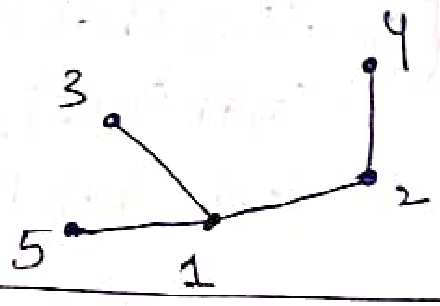
⑦ [{1,2,3,4,5}|]

Sol: given $A = \{1,2,3,4,5\}$

$R = \{(a,b) \mid b|a\}$

$R = \{(1,2), (1,3), (1,4), (1,5), (2,4), (1,1), (2,2), (3,3), (4,4), (5,5)\}$

The Hasse diagram is.



Algebraic Structures:-

Binary Operation:-

Let 's' be the non-empty set then the cartisian product $S \times S$ be the set of all ordered pairs of elements in S, then a function $f: (S \times S) \to S$

is called a binary operation on S.

Properties of binary operation:-

Let "$*$" be the binary operation on non-empty set S.

(i) **Closure Property:-**

$\forall a \in S \Rightarrow a * a \in S$

(ii) **Commutative Property:-**

$a, b \in S \Rightarrow a * b = b * a$

(iii) **Associative Property:-**

$a, b, c \in S \Rightarrow a * (b * c) = (a * b) * c$

(iv) **Identity Property:-**

If $a \in S$ then $\exists$ 'e' in S such that $a \cdot e = e \cdot a = a$ where, e is the identity element.

(v) **Inverse:-**

If $a \in S$ $\exists$ $b \in S$ such that $a * b = b * a = e$ where, b is called inverse of a.

Semi-group:-

A algebraic system $(S, *)$ consisting of a non-empty S and associative and closure binary operation $*$ is defined on S then it is called semi-group under the operation $*$

Ex:- Z is a set of integers. where $(Z, +)$ and $(Z, \cdot)$ are the semi groups.

Monoid:-

Let $(S, *)$ be a semi group. The semi group $(S, *)$ is said to be monoid, if 'S' contains the identity element 'e' with respect to $*$.

Evidently every monoid is a semigroup but a semi group need not be the monoid.

---

① The algebraic structure a set of integers. Prove that $(Z, +)$ and $(Z, \times)$ is a semi group and monoid.

Sol:- Consider the algebraic structure $(Z, +)$ and $(Z, \times)$

Closure Property:-

If $a, b \in Z \Rightarrow a * b \in Z$.

If $2 \in Z$ then $2+2=4 \in Z$

If $3 \in Z$ then $3 * 3 = 9 \in Z$

$\therefore (Z, +)$ and $(Z, \times)$ satisfies the closure property.

Associative:-

If $a, b, c \in Z$ then $a * (b * c) = (a * b) * c$

Let $2, 3, 5 \in Z$

$2 * (3 * 5) = (2 * 3) * 5$

$\Rightarrow 2 + (3+5) = (2+3)+5$ $\Rightarrow$ $2 \times (3 \times 5) = (2 \times 3) \times 5$

$2+8 = 5+5$ $\qquad$ $2 \times 15 = 6 \times 5$

$10 = 10$ $\qquad$ $30 = 30$

$\therefore (Z, +)$ and $(Z, *)$ are satisfies associative property.

Identity:-

$a * e = e * a = a$

where, e is the identity element.

Let $3 \in Z$.

$\Rightarrow 3 + 0 = 0 + 3 = 3$.

$\Rightarrow 3 \times 1 = 1 \times 3 = 3$.

$\therefore (Z, +)$ and $(Z, \times)$ satisfies the identity property.

$\therefore (Z, +)$ and $(Z, \times)$ are the semi groups and monoids.

---

②. In each of the following cases a binary operation $*$ on A is defined through a multiplication table. Determine whether $(A, *)$ is a semi group or a monoid.

where, $A = (a, b)$, $A = (a, b, c, d)$.

(a)

*	a	b
a	a	a
b	b	b

(b)

*	a	b
a	a	b
b	a	a

(c)

*	a	b	c	d
a	a	a	a	a
b	a	a	b	a
c	a	b	c	d
d	a	a	d	a

Sol: given $A = (a, b)$, $A = (a, b, c, d)$

(a) $a * a = a$

$a * b = a$

$b * a = b$

$b * b = b$

Closure $\Rightarrow$ $a \in A$, $a * a = a \in A$

$b \in A$, $b * b = b \in A$.

Associative $\Rightarrow$ $a, b \in A$, $a * b = b * a$

So, (a) is not semigroup and monoid. $a = b$