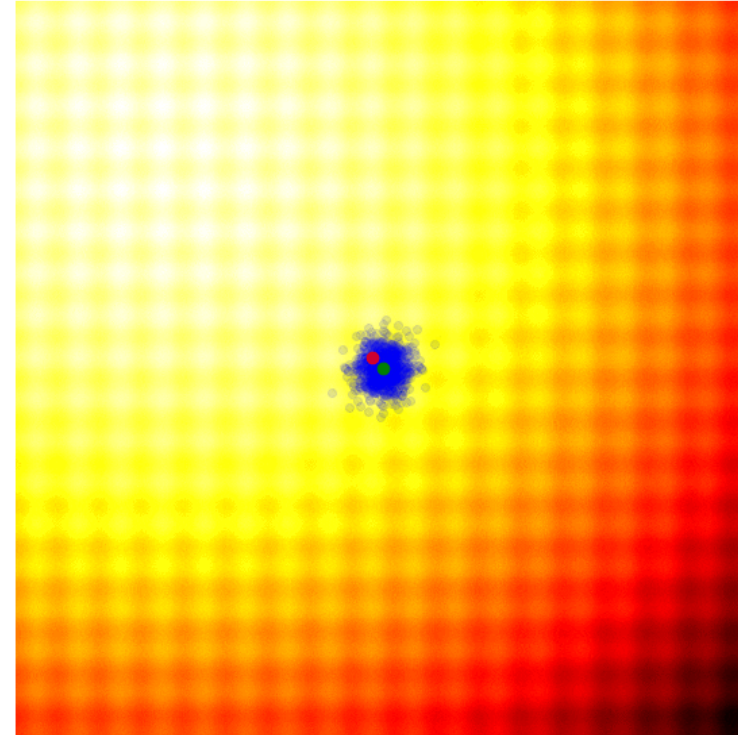
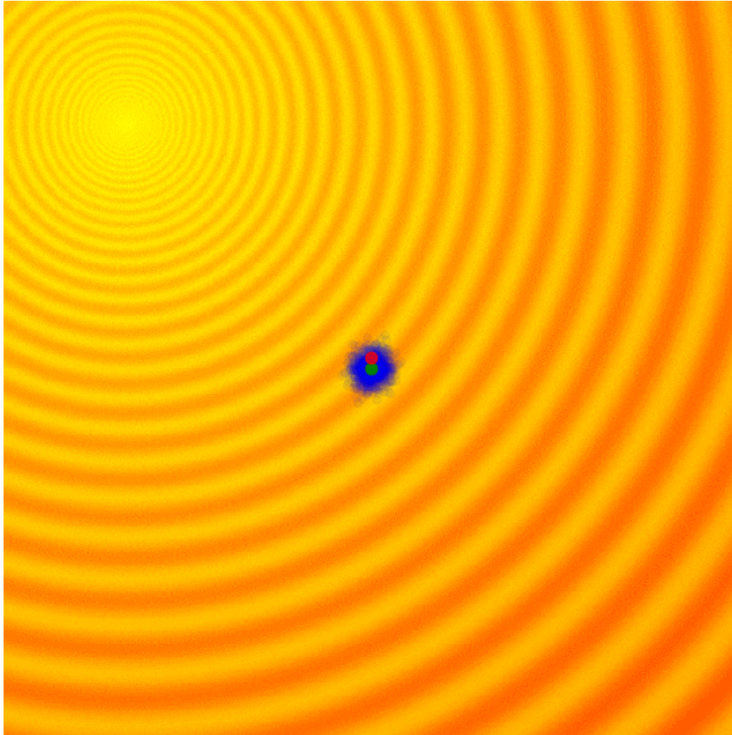


Reinforcement Learning

Lecture 5. Population-based Methods

Sungjoon Choi, Korea University

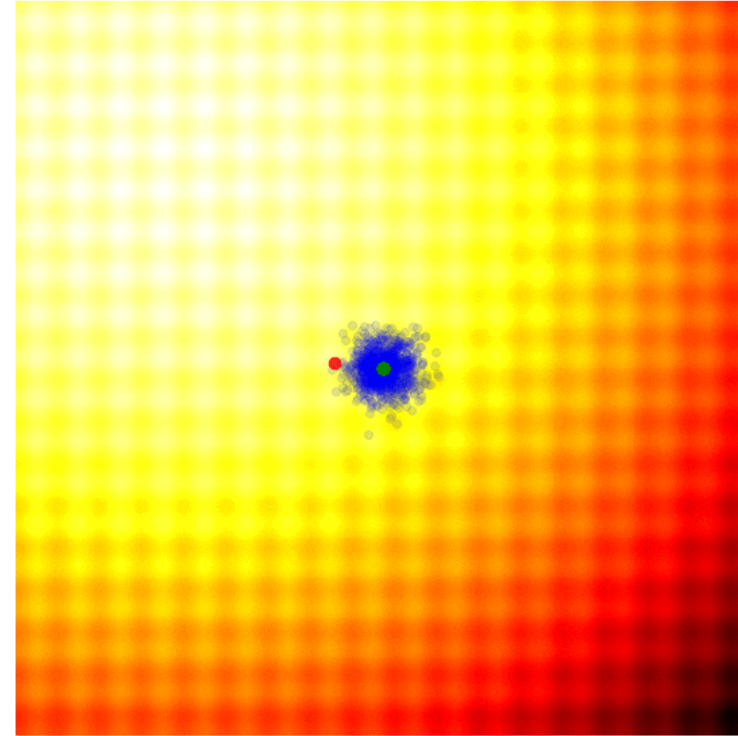
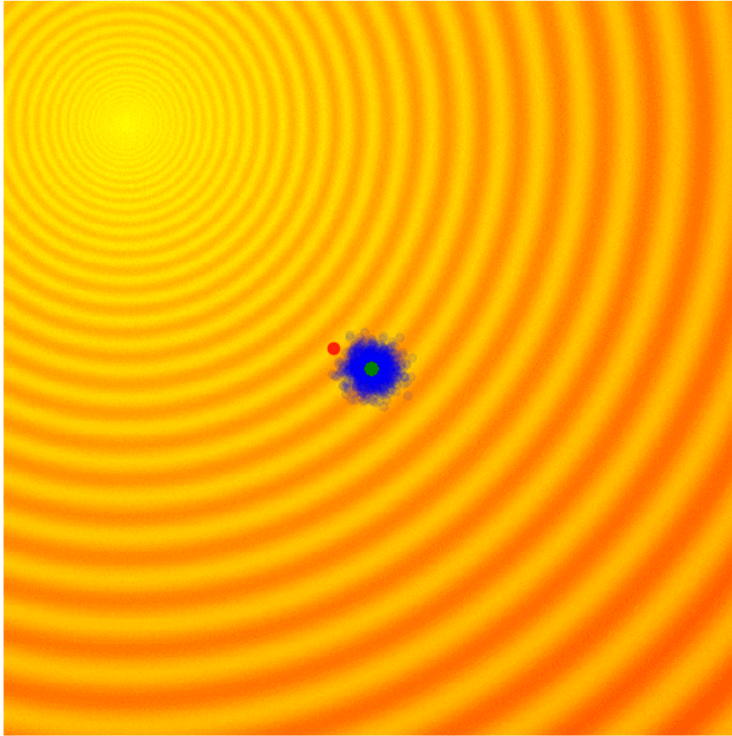
Population-based Search



- The **green dot** indicates the mean of the distribution at each generation, the **blue dots** are the sampled solutions, and the **red dot** is the best solution so far.

Cross Entropy Method (CEM)

Genetic Algorithm



- The **green dot** indicates the **elite population** from the previous generation, the **blue dots** are the offsprings to form the set of candidate solutions, and the **red dot** is the best solution so far.

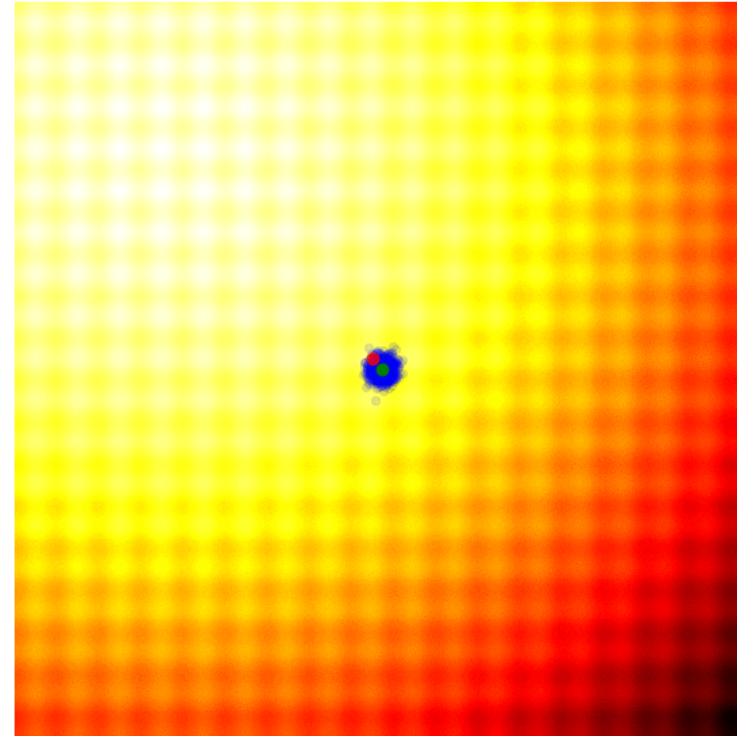
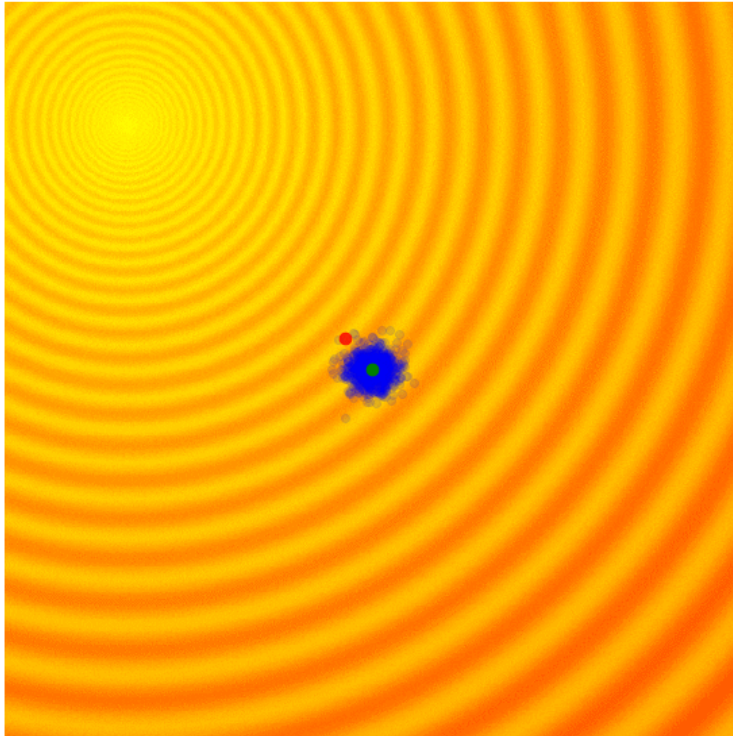
Cross Entropy Method (CEM)

- Initialize $\mu \in \mathbb{R}^d$ and $\sigma \in \mathbb{R}^d$
- **for** iteration = 1, 2, ...
 - Collect n samples of $\theta_i \sim \mathcal{N}(\mu, \text{diag}(\sigma))$
 - Perform a noisy evaluation $R_i \sim \theta_i$
 - Select the top $p\%$ of samples (aka the **elite set**)
 - Fit a Gaussian distribution (with diagonal covariance) to the **elite set**, obtaining a new μ and σ
- **end for**
- Return the final μ

CMA-ES

"The CMA Evolution Strategy: A Tutorial," 2016

CMA-ES

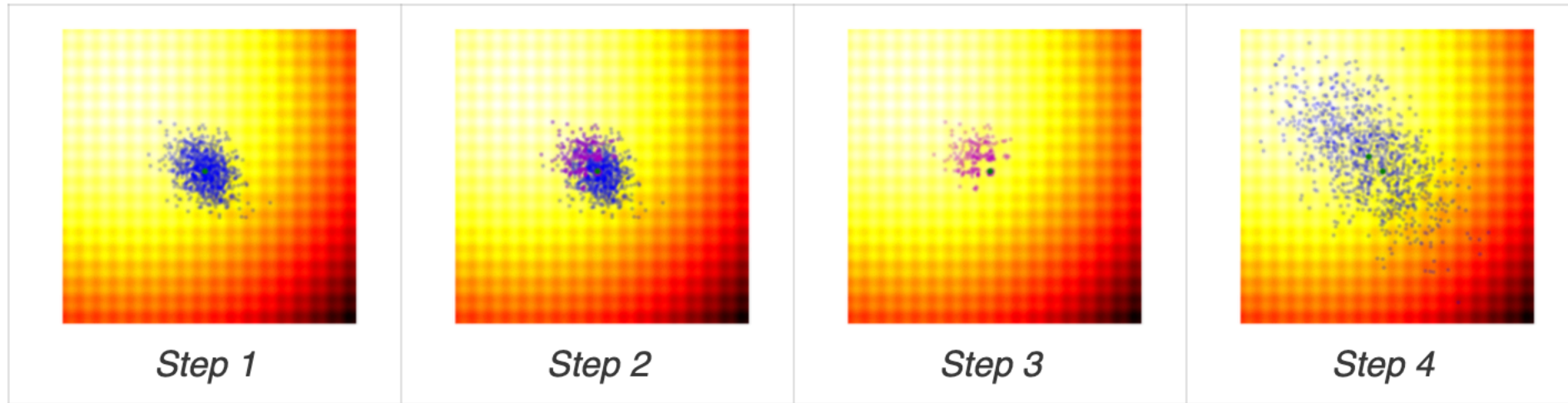


- The **green dot** indicates the **elite population** from the previous generation, the **blue dots** are the offsprings to form the set of candidate solutions, and the **red dot** is the best solution so far.
- CMA-ES can **adaptively** increase or decrease the search space!

CMA-ES

- Initialize $\mu \in \mathbb{R}^d$ and $\sigma \in \mathbb{R}^d$
- **for** iteration $g = 1, 2, \dots$
 - Collect n samples of $\theta_i \sim \mathcal{N}(\mu, \text{diag}(\sigma))$
 - Perform a noisy evaluation $R_i \sim \theta_i$
 - Select the top $p\%$ of samples (aka the **elite set**)
 - Fit $\mu^{(g)}$ to the **elite set** by $\mu^{(g)} = \frac{1}{N_{\text{best}}} \sum_i^{N_{\text{best}}} \theta_i$
 - The trick is to update the covariance using the previous mean $\mu^{(g-1)}$.
 - For 1-dimensional case, $\sigma_x^{2,(g+1)} = \frac{1}{N_{\text{best}}} \sum_{i=1}^{N_{\text{best}}} (x_i - \mu_x^{(g)})^2$
- **end for**
- Return the final μ

The Effect of CMA



Augmented Random Search (ARS)

"Simple random search provides a competitive approach to reinforcement learning," 2018

Augmented Random Search



- Augmented Random Search (ARS) is a population-based, derivative-free reinforcement learning method.
 - Derivative-free methods such as a **cross-entropy method** (CEM) or **covariance matrix adaptation** (CMA) treat the return as as a black box function to be optimized in terms of the policy parameters.
 - From the current policy parameter θ^t , multiple parameters $\{\tilde{\theta}_i^t\}_{i=1}^N$ are sampled and the returns $\{\eta(\pi_{\tilde{\theta}_i^t})\}_{i=1}^N$ of the corresponding policies $\{\pi_{\tilde{\theta}_i^t}\}_{i=1}^N$ evaluated by rollouts.
- ARS utilizes a simple linear policy and searches over the space of matrices.

Basic Random Search

Algorithm 1 Basic Random Search (BRS)

- 1: **Hyperparameters:** step-size α , number of directions sampled per iteration N , standard deviation of the exploration noise ν
- 2: **Initialize:** $\theta_0 = \mathbf{0}$, and $j = 0$.
- 3: **while** ending condition not satisfied **do**
- 4: Sample $\delta_1, \delta_2, \dots, \delta_N$ of the same size as θ_j , with i.i.d. standard normal entries.
- 5: Collect $2N$ rollouts of horizon H and their corresponding rewards using the policies

$$\pi_{j,k,+}(x) = \pi_{\theta_j + \nu \delta_k}(x) \quad \text{and} \quad \pi_{j,k,-}(x) = \pi_{\theta_j - \nu \delta_k}(x),$$

Positively perturbed policy Negatively perturbed policy

with $k \in \{1, 2, \dots, N\}$.

- 6: Make the update step:

$$\theta_{j+1} = \theta_j + \underbrace{\frac{\alpha}{N}}_{\text{Stepsize}} \sum_{k=1}^N \underbrace{[r(\pi_{j,k,+}) - r(\pi_{j,k,-})]}_{\text{Score of the perturbation}} \underbrace{\delta_k}_{\text{Perturbation}}.$$

- 7: $j \leftarrow j + 1$.
- 8: **end while**

.

Augmented Random Search



- Drawbacks of Basic Random Search
 - Input normalization is often necessary for high-dimensional inputs.
 - Not all perturbations (directions) are useful.

Augmented Random Search

Algorithm 2 Augmented Random Search (ARS): four versions **V1**, **V1-t**, **V2** and **V2-t**

- 1: **Hyperparameters:** step-size α , number of directions sampled per iteration N , standard deviation of the exploration noise ν , number of top-performing directions to use b ($b < N$ is allowed only for **V1-t** and **V2-t**) Use top b search directions.
- 2: **Initialize:** $M_0 = \mathbf{0} \in \mathbb{R}^{p \times n}$, $\mu_0 = \mathbf{0} \in \mathbb{R}^n$, and $\Sigma_0 = \mathbf{I}_n \in \mathbb{R}^{n \times n}$, $j = 0$.
- 3: **while** ending condition not satisfied **do**
- 4: Sample $\delta_1, \delta_2, \dots, \delta_N$ in $\mathbb{R}^{p \times n}$ with i.i.d. standard normal entries.
- 5: Collect $2N$ rollouts of horizon H and their corresponding rewards using the $2N$ policies

$$\begin{aligned} \mathbf{V1}: \quad & \begin{cases} \pi_{j,k,+}(x) = (M_j + \nu\delta_k)x \\ \pi_{j,k,-}(x) = (M_j - \nu\delta_k)x \end{cases} \\ \mathbf{V2}: \quad & \begin{cases} \pi_{j,k,+}(x) = (M_j + \nu\delta_k) \text{diag}(\Sigma_j)^{-1/2}(x - \mu_j) \\ \pi_{j,k,-}(x) = (M_j - \nu\delta_k) \text{diag}(\Sigma_j)^{-1/2}(x - \mu_j) \end{cases} \end{aligned} \quad \text{Input normalization}$$

for $k \in \{1, 2, \dots, N\}$.

- 6: Sort the directions δ_k by $\max\{r(\pi_{j,k,+}), r(\pi_{j,k,-})\}$ denote by $\delta_{(k)}$ the k -th largest direction, and by $\pi_{j,(k),+}$ and $\pi_{j,(k),-}$ the corresponding policies.
- 7: Make the update step:

$$M_{j+1} = M_j + \frac{\alpha}{b\sigma_R} \sum_{k=1}^b [r(\pi_{j,(k),+}) - r(\pi_{j,(k),-})] \delta_{(k)},$$

where σ_R is the standard deviation of the $2b$ rewards used in the update step.

- 8: **V2** : Set μ_{j+1} , Σ_{j+1} to be the mean and covariance of the $2NH(j+1)$ states encountered from the start of training.²
- 9: $j \leftarrow j + 1$
- 10: **end while**



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