

Mathematics Society

Weekly Questions (Week 3)

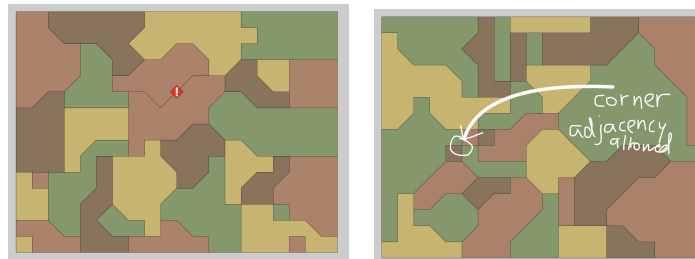
November 13, 2023

Note that questions are not ordered by difficulty. You can do them in any order you like (even the parts of the questions, unless stated otherwise). They are completely independent of each other.

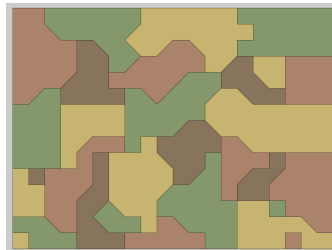
Question 1 (Puzzle) *Map*

The famous four-color theorem states that in order to color a map so that *no two adjacent regions have the same color*, you need at most 4 colors.

Some examples are shown below to demonstrate the rules:

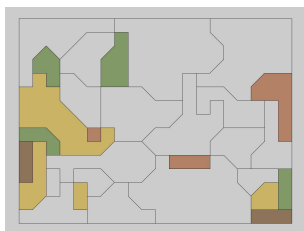


No two adjacent regions have the same color. Corner adjacency is allowed.

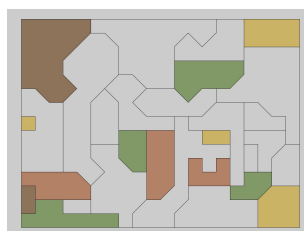


Correctly Solved

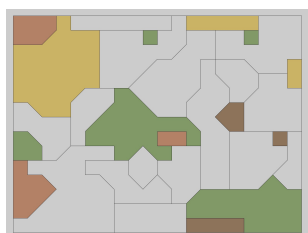
Below are select incomplete maps for you to fill in. Some of the regions in the maps below have been colored. Can you color the remaining regions so that no two adjacent regions have the same color, while only using 4 colors?



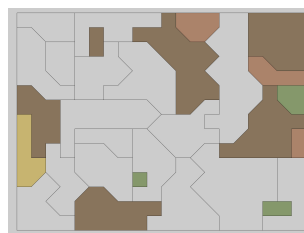
Easy Task 1



Easy Task 2



Easy Task 3



Medium Task

Question 2 (Logic Puzzle) *Parity problems*

All of the following parts can be solved with the same underlying principle: parity. It means "whether something is even or odd".

- a There are 7 boxes that you need to click to remove the checks from all of them. But, if you click one of the boxes, then all the other boxes "flip": that is, all unchecked boxes will become checked, and all checked boxes will become unchecked. Only the box you clicked will remain unchanged. Here's a demonstration:
Suppose you click the box in the middle(outlined in black), in the first state shown below.



After the box outlined in black is clicked, the boxes are checked in this way:



If, at the start, all 7 boxes are checked, is it possible to click some boxes so that all of the boxes become unchecked? If it is possible, which boxes should be checked, and in what order? If it is not possible, why?

Your goal:



Hint: *How many blue ticks are there after and before each click?*

Question 2 Cont'd

- b A bored student writes out all the whole numbers from 1 to 2023 on a blackboard. Then, she erases two numbers from the board, and writes their difference on the blackboard.(So, for example, if she erased 5 and 2018, she would write 2013 on the blackboard.) She continues this procedure of erasing two numbers and writing their difference, until only 1 number is left. **Is that number odd or even?**

Question 3 (Olympiad) *Squares*¹

Let quadrilaterals ABCD and DEFG be squares, with E lying on CD. If the side length of square ABCD is a , and the side length of square DEFG is b , what is the area of $\triangle BEF$?

¹idk what to call this question