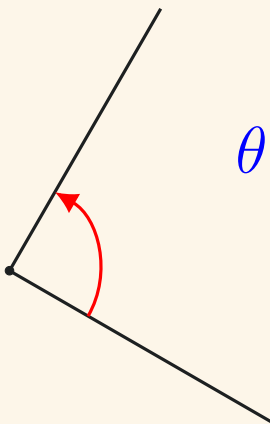


The Comprehensive List of
References in Geometry

AN ILLUSTRATED MANUAL



DECEMBER 2020





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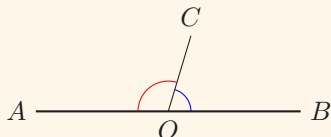
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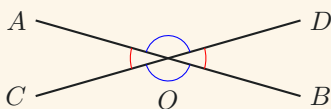
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Lines



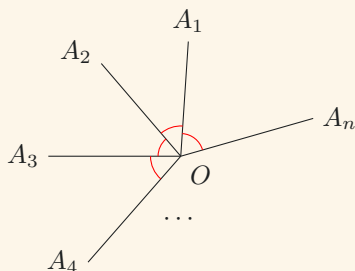
Adj. \angle s on st. line

$$\angle AOC + \angle COB = 180^\circ.$$



Vert. oppo. \angle s

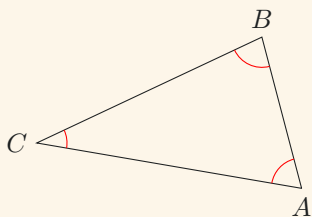
$$\begin{aligned}\angle AOC &= \angle BOD, \\ \angle AOD &= \angle COB.\end{aligned}$$



\angle s at a pt.

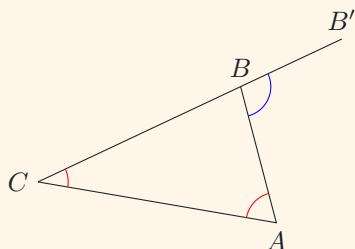
$$\begin{aligned}\angle A_1OA_2 + \angle A_2OA_3 \\ + \cdots + \angle A_{n-1}OA_n = 360^\circ.\end{aligned}$$

Triangles



\angle *sum of $\triangle s$*

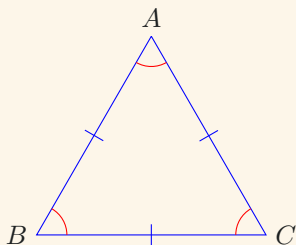
$$\angle CAB + \angle ABC + \angle BCA = 180^\circ.$$



Ext. \angle *of $\triangle s$*

$$\angle CAB + \angle BCA = \angle B'BA.$$

Equilateral Triangles

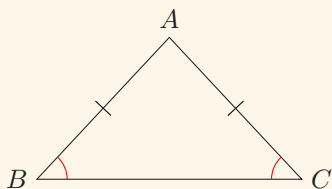


Prop. of equil. $\triangle s$

$$AB = BC = CA,$$

$$\angle CAB = \angle ABC = \angle BCA = 60^\circ.$$

Isosceles Triangles

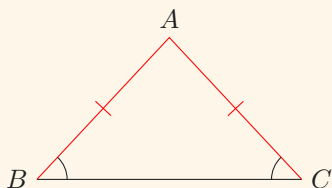


Base \angle s, isos. \triangle

$$AB = AC$$



$$\angle ABC = \angle ACB.$$

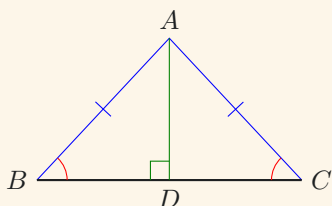


Sides oppo. eq. \angle s

$$\angle ABC = \angle ACB$$



$$AB = AC.$$



Prop. of isos. \triangle s

$$\angle ABC = \angle ACB$$

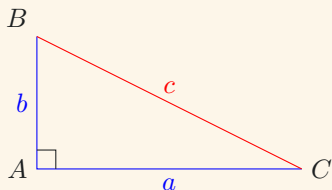


$$AB = AC$$



$$AD \perp BC.$$

Pythagoras's Theorem

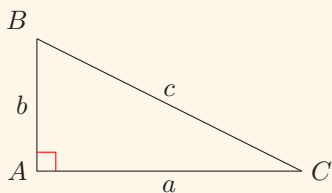


Pyth. thrm.

$$BA \perp AC$$



$$a^2 + b^2 = c^2.$$



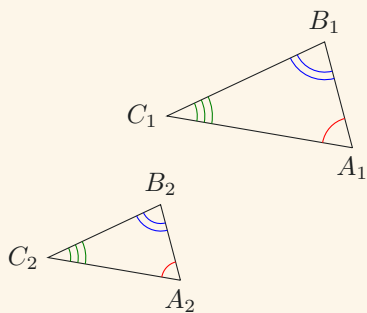
Conv. of Pyth. thrm.

$$a^2 + b^2 = c^2$$



$$BA \perp AC.$$

Similar Triangles



Corr. $\angle s \mid \sim \triangle s$

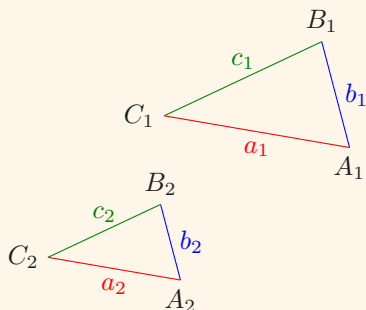
$$\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2$$



$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$

$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$

$$\angle B_1C_1A_1 = \angle B_2C_2A_2.$$



Corr. sides | $\sim \triangle s$

$$\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2$$

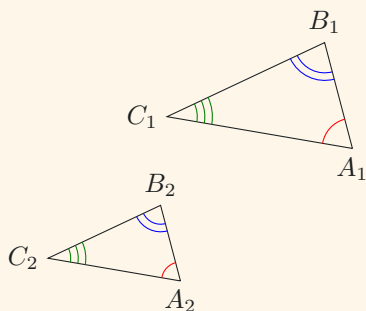
$$\Downarrow$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2},$$

$$\frac{b_1}{c_1} = \frac{b_2}{c_2},$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}.$$

Conditions for Proving Similar Triangles



AAA

$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$

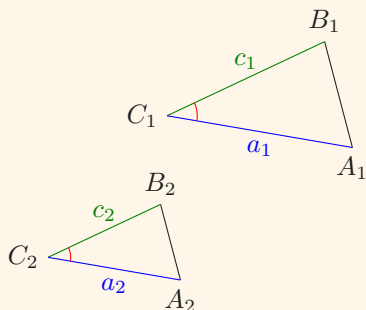
$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$

$$\angle B_1C_1A_1 = \angle B_2C_2A_2$$

◀ Any two of the three is sufficient.

$$\Downarrow$$

$$\underline{\underline{\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.}}$$



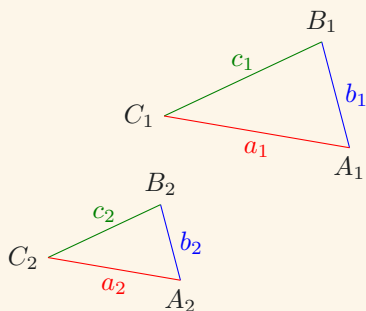
*Ratio of two sides,
incl. \angle*

$$\frac{a_1}{c_1} = \frac{a_2}{c_2},$$

$$\angle B_1C_1A_1 = \angle B_2C_2A_2$$

$$\Downarrow$$

$$\underline{\underline{\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.}}$$



*Three sides
proportional*

$$\frac{a_1}{b_1} = \frac{a_2}{b_2},$$

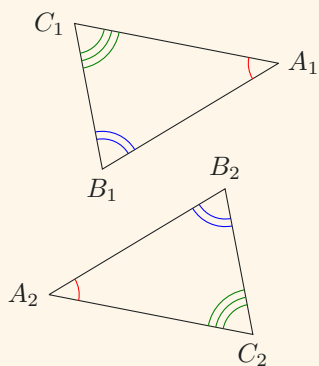
$$\frac{b_1}{c_1} = \frac{b_2}{c_2},$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

$$\Downarrow$$

$$\underline{\underline{\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.}}$$

Congruent Triangles



Corr. $\angle s$ | $\cong \triangle s$

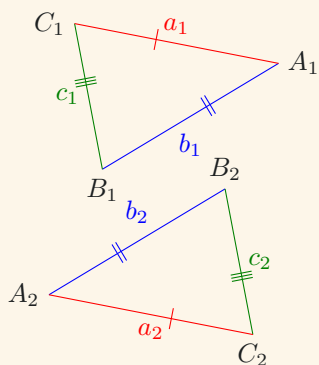
$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2$$



$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$

$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$

$$\angle B_1C_1A_1 = \angle B_2C_2A_2.$$



Corr. $sides$ | $\cong \triangle s$

$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2$$

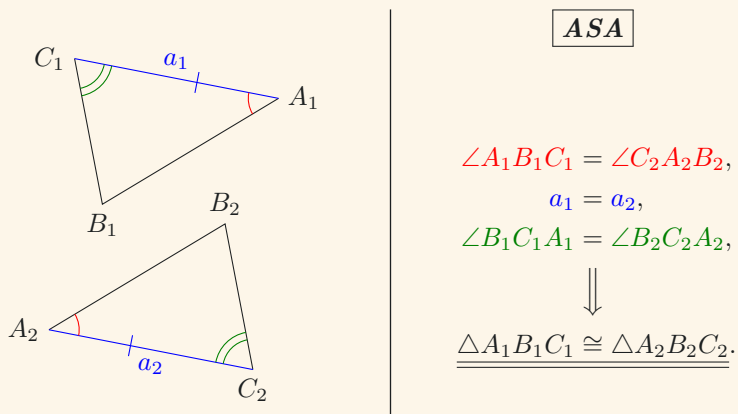
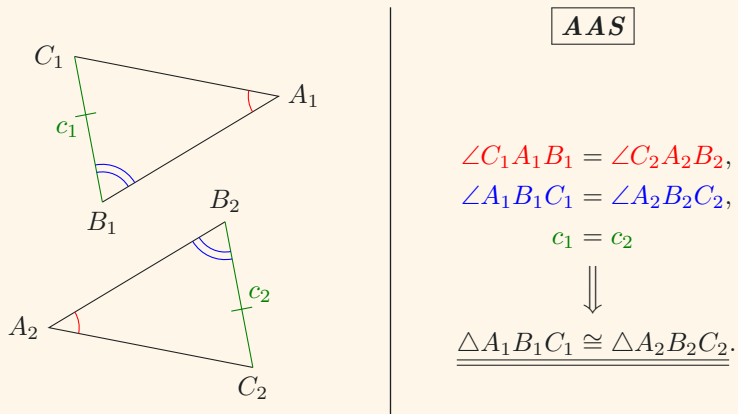


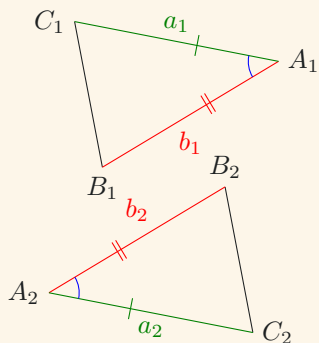
$$a_1 = a_2,$$

$$b_1 = b_2,$$

$$c_1 = c_2.$$

Conditions for Proving Congruent Triangles



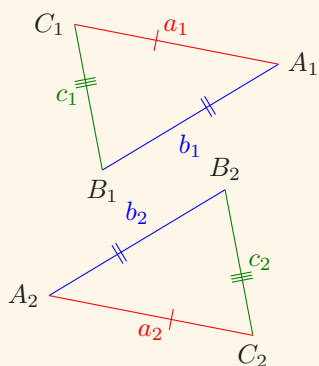


SAS

$$\begin{aligned} a_1 &= b_1, \\ \angle B_1A_1C_1 &= \angle B_2A_2C_2, \\ b_1 &= b_2 \end{aligned}$$



$$\underline{\underline{\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.}}$$

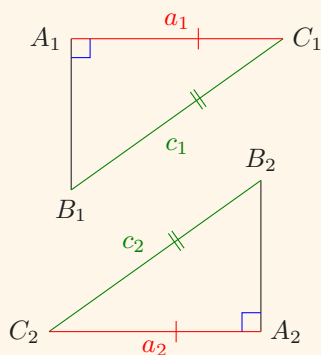


SSS

$$\begin{aligned} a_1 &= a_2, \\ b_1 &= a_2, \\ c_1 &= a_2 \end{aligned}$$



$$\underline{\underline{\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.}}$$



RHS

$$\angle B_1A_1C_1 = \angle B_2A_2C_2$$

$$= 90^\circ,$$

$$c_1 = c_2,$$

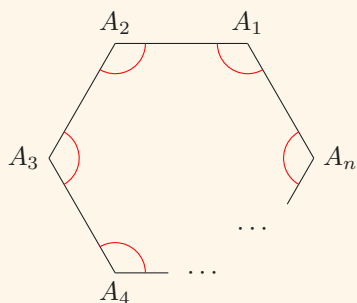
$$a_1 = a_2$$



$$\underline{\underline{\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.}}$$

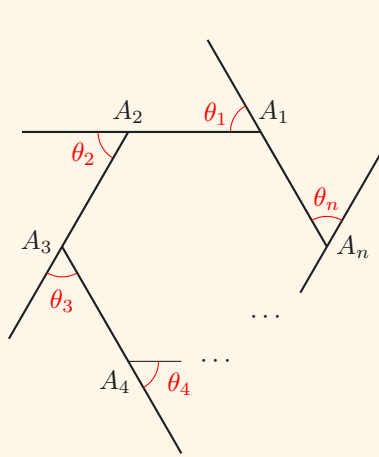
◀ One of the sides must be the hypotenuse.

Polygons



∠ sum of polygons

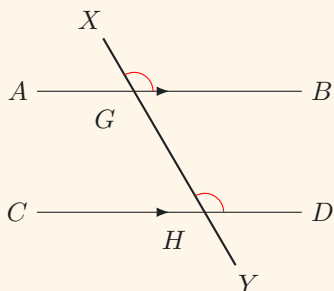
$$\begin{aligned} & \angle A_nA_1A_2 + \angle A_1A_2A_3 + \dots \\ & + \angle A_{n-1}A_nA_1 = (n-2) \cdot 180^\circ. \end{aligned}$$



Ext. \angle s of polygons

$$\theta_1 + \theta_2 + \cdots + \theta_n = 360^\circ.$$

Parallel Lines

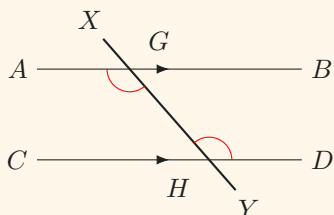


Corr. \angle s | $AB \parallel CD$

$$AB \parallel CD$$



$$\angle XGB = \angle XHD.$$

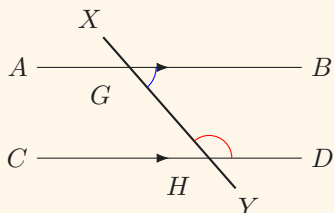


Alt. $\angle s \mid AB \parallel CD$

$$AB \parallel CD$$



$$\angle YGA = \angle XHD.$$



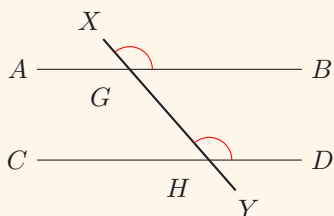
Int. $\angle s \mid AB \parallel CD$

$$AB \parallel CD$$



$$\angle YGB + \angle XHD = 180^\circ.$$

Conditions for Proving Parallelism

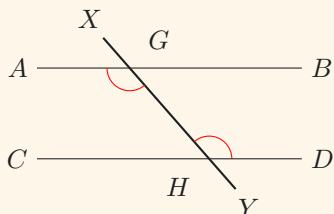


Int. $\angle s eq.$

$$\angle YGB + \angle XHD = 180^\circ$$



$$\underline{\underline{AB \parallel CD.}}$$

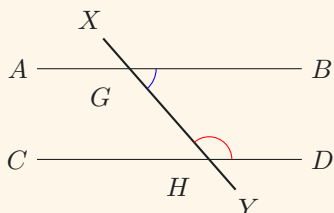


Alt. \angle s eq.

$$\angle YGA = \angle XHD$$



$$\underline{\underline{AB \parallel CD.}}$$



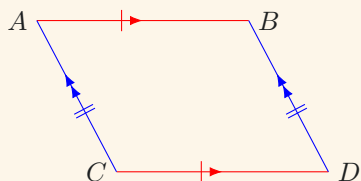
Int. \angle s eq

$$\angle YGB + \angle XHD = 180^\circ$$



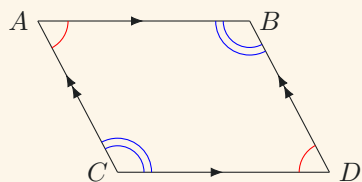
$$\underline{\underline{AB \parallel CD.}}$$

Parallelograms



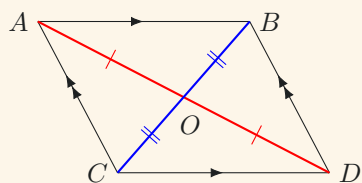
Oppo. sides of \parallel -gram

$$\begin{aligned} AB &= CD, \\ AC &= BD. \end{aligned}$$



Oppo. \angle s of \parallel -gram

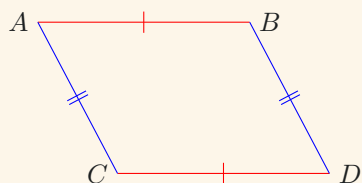
$$\begin{aligned} \angle BAC &= \angle BDC, \\ \angle ACD &= \angle ABD. \end{aligned}$$



Diags. of \parallel -gram

$$\begin{aligned} OA &= OD, \\ OB &= OC. \end{aligned}$$

Conditions for Identifying Parallelograms

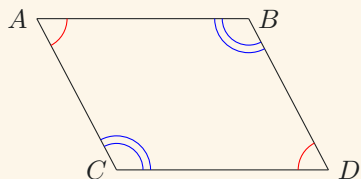


Oppo. sides eq.

$$\begin{aligned} AB &= CD, \\ AC &= BD \end{aligned}$$



$ABCD$ is a \parallel -gram.



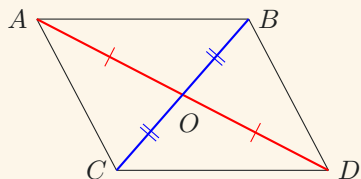
Oppo. \angle s eq.

$$\angle BAC = \angle BDC,$$

$$\angle ACD = \angle ABD$$



$ABCD$ is a \parallel -gram.



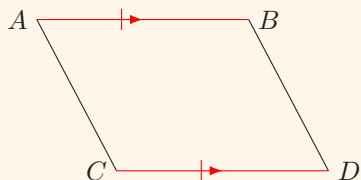
Diags. bisect each other

$$OA = OD,$$

$$OB = OC$$



$ABCD$ is a \parallel -gram.



Two sides eq. and \parallel

$$AB = CD,$$

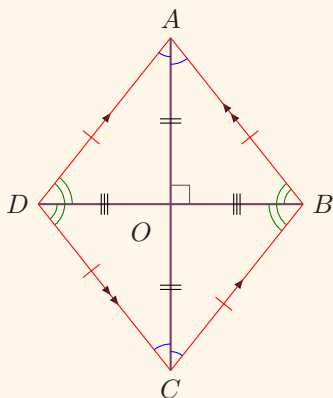
$$AB \parallel CD$$



$ABCD$ is a \parallel -gram.

Other Quadrilaterals

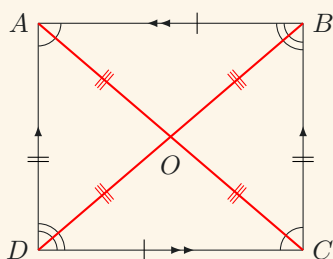
Rhombus - a quadrilateral with four equal sides



Prop. of rhombuses

$$\begin{aligned}
 AB &\parallel CD & AD &\parallel BC, \\
 OA &= OC & OB &= OD, \\
 AB &= BC = CD = DA, \\
 AC &\perp BD, \\
 \angle OAD &= \angle OAB \\
 &= \angle OCD = \angle OCB, \\
 \angle ODA &= \angle ODC \\
 &= \angle OBA = \angle OBC.
 \end{aligned}$$

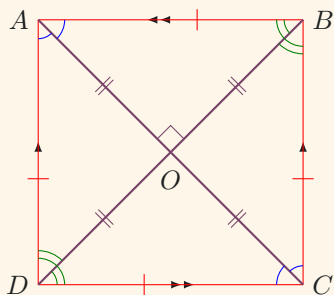
Rectangle - a quadrilateral with four equal interior angles



Prop. of rectangles

$$\begin{aligned}
 AB &\parallel CD & AD &\parallel BC, \\
 AB &= CD & AD &= BC, \\
 \angle BAD &= \angle BCD, \\
 \angle ABC &= \angle ADC, \\
 OA &= OB = OC = OD.
 \end{aligned}$$

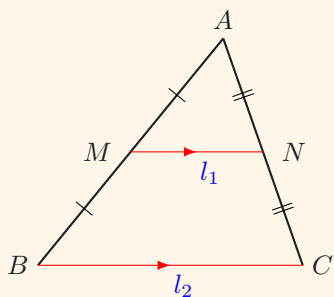
Square - a quadrilateral with four equal sides and interior angles



Prop. of squares

$AB \parallel CD \quad AD \parallel BC,$
 $AB = BC = CD = DA,$
 $OA = OB = OC = OD,$
 $AC \perp BD,$
 $\angle OAD = \angle OAB = 90^\circ$
 $\quad = \angle OCD = \angle OCB,$
 $\angle ODA = \angle ODC = 90^\circ$
 $\quad = \angle OBA = \angle OBC.$

Miscellaneous Results



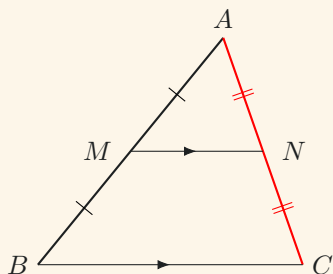
Mid-pt. thrm.

$$MA = MB \quad NA = NC$$



$$2l_1 = l_2,$$

$$MN \parallel BC.$$

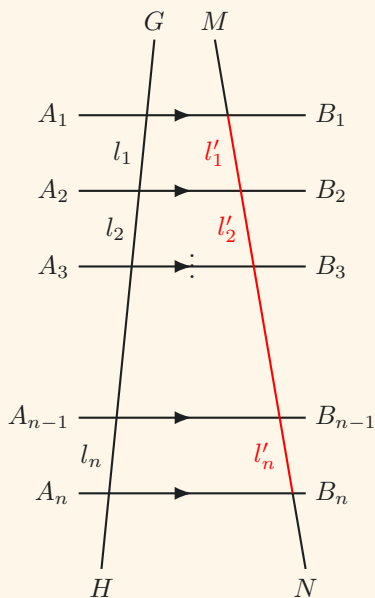


Intercept thrm. variant i

$$MA = NA \quad MN \parallel BC$$



$$MB = NC.$$



Intercept thrm. variant ii

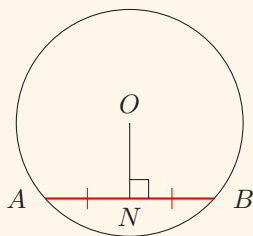
$$A_1B_1 \parallel A_2B_2 \parallel \cdots \parallel A_nB_n$$

$$l_1 = l_2 = \cdots = l_n$$



$$l'_1 = l'_2 = \cdots = l'_n.$$

Circles

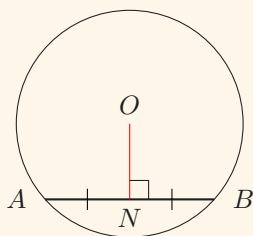


*Line from cen. \perp
chord bisects chord*

$$ON \perp AB$$



$$NA = NB.$$

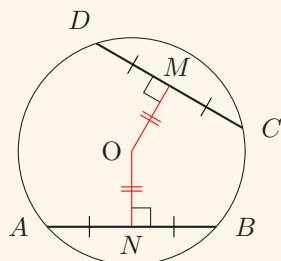


*Line joining cen.
to mid-pt. of
chord \perp chord*

$$NA = NB$$



$$ON \perp AB.$$



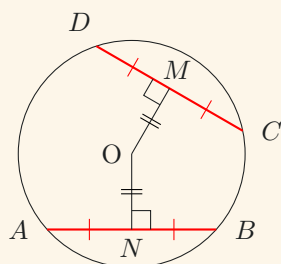
*Eq. chords
equidis. from cen.*

$$ON \perp AB \quad OM \perp CD,$$

$$AB = CD$$



$$ON = OM.$$



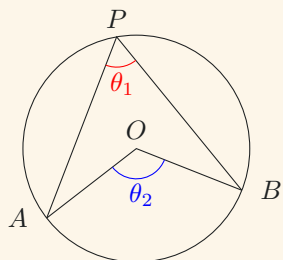
*Chords equidis. from
cen. are eq.*

$$ON \perp AB \quad OM \perp CD,$$

$$ON = OM$$

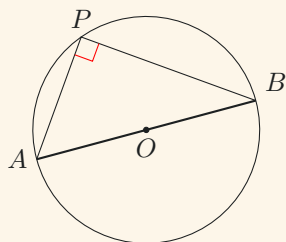


$$AB = CD.$$



\angle at cen. twice
 \angle at \odot^{ce}

$$2\theta_1 = \theta_2.$$

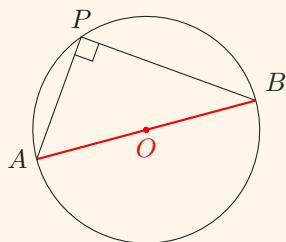


\angle in semi-circ.

AB is a diameter



$$\angle APB = 90^\circ.$$

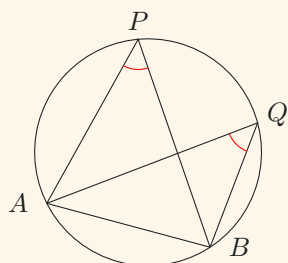


Converse of
 \angle in semi-circ.

$$\angle APB = 90^\circ$$

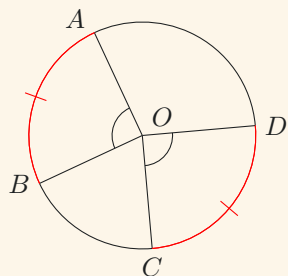


AB is a diameter.



$\angle s$ in the same segment

$$\angle APB = \angle AQB.$$

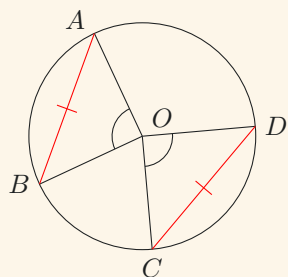


Eq. $\angle s$, eq. arcs

$$\angle AOB = \angle COD$$



$$\widehat{AB} = \widehat{CD}.$$

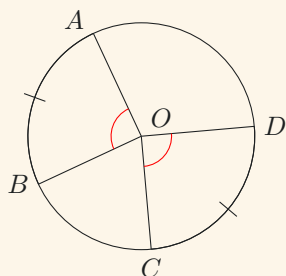


Eq. $\angle s$, eq. chords

$$\angle AOB = \angle COD$$



$$AB = CD.$$

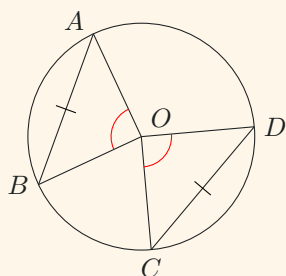


Eq. arcs, eq. \angle s

$$\widehat{AB} = \widehat{CD}$$



$$\angle AOB = \angle COD.$$

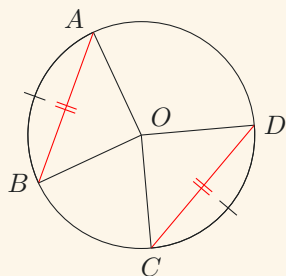


Eq. chords, eq. \angle s

$$AB = CD$$



$$\angle AOB = \angle COD.$$

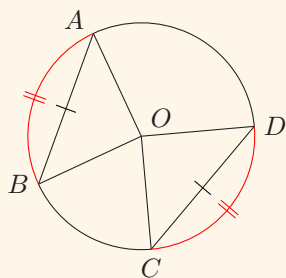


Eq. arcs, eq. chords

$$\widehat{AB} = \widehat{CD}$$



$$AB = CD.$$

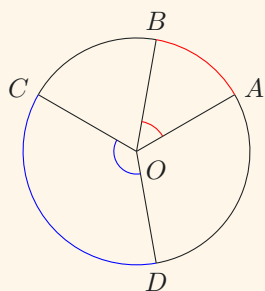


Eq. chords, eq. arcs

$$AB = CD$$

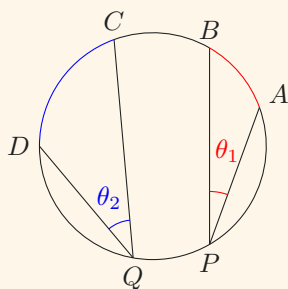


$$\widehat{AB} = \widehat{CD}.$$



Arcs prop. to \angle s at cen.

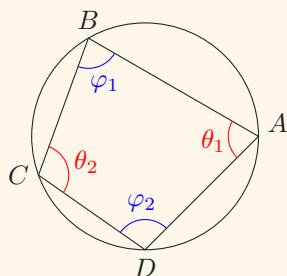
$$\frac{\widehat{AB}}{\widehat{CD}} = \frac{\theta_1}{\theta_2}.$$



Arcs prop. to \angle s at \odot^{ce}

$$\frac{\widehat{AB}}{\widehat{CD}} = \frac{\theta_1}{\theta_2}.$$

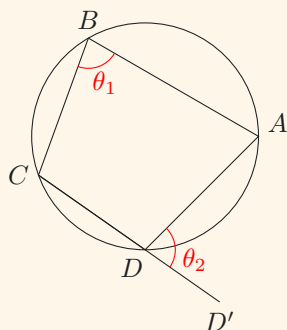
Cyclic Quadrilaterals



Oppo. \angle s, cyclic quad.

$$\theta_1 + \theta_2 = 180^\circ,$$

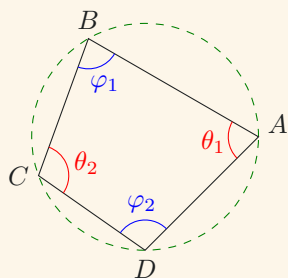
$$\varphi_1 + \varphi_2 = 180^\circ.$$



Ext. \angle s, cyclic quad.

$$\theta_1 = \theta_2.$$

Conditions for Identifying Cyclic Quadrilaterals

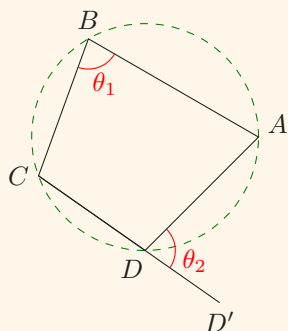


Oppo. \angle s supp.

$$\begin{aligned}\theta_1 + \theta_2 &= 180^\circ, \\ \varphi_1 + \varphi_2 &= 180^\circ\end{aligned}$$



A, B, C and D are concyclic.

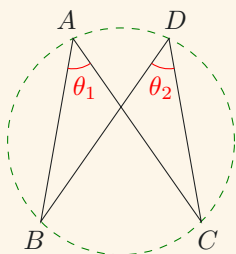


Ext. \angle = int. oppo. \angle

$$\theta_1 = \theta_2$$



A, B, C and D are concyclic.



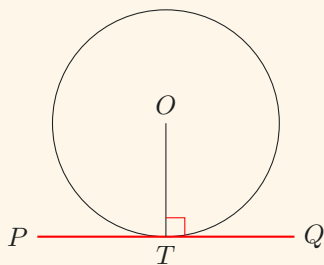
*Converse of $\angle s$
in the same segment*

$$\theta_1 = \theta_2$$



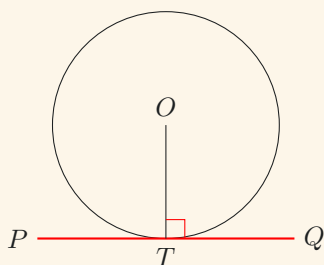
A, B, C and D are concyclic.

Tangents



Tan. \perp radius

$$OT \perp PQ.$$

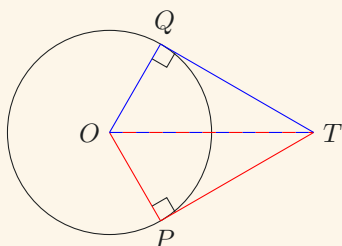


*Converse of
tan. \perp radius*

$$OT \perp PQ.$$

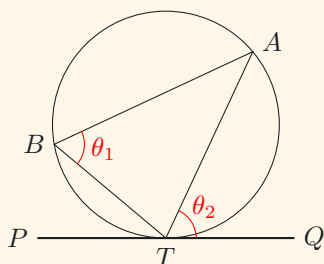


PQ is tangent
to the circle at T .



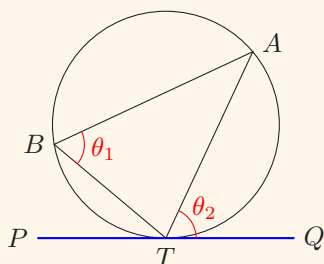
Tan. prop.

$$\triangle OPT \cong \triangle OQT.$$



∠ in alt. segment

$$\theta_1 = \theta_2.$$



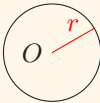
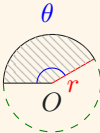
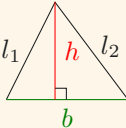
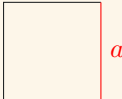
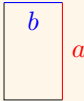
*Converse of
∠ in alt. segment*

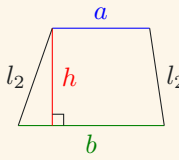
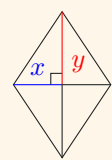
$$\theta_1 = \theta_2$$



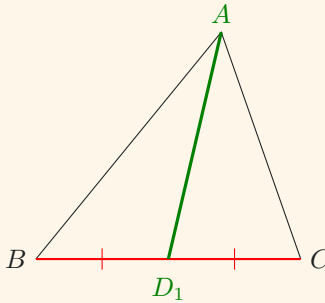
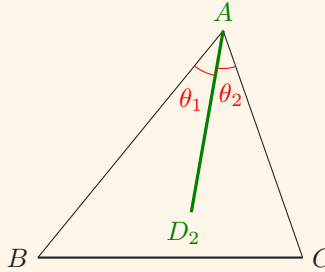
PQ is tangent
to the circle at *T*.

Appendix A: Area and Perimeters of Common Plane Figures

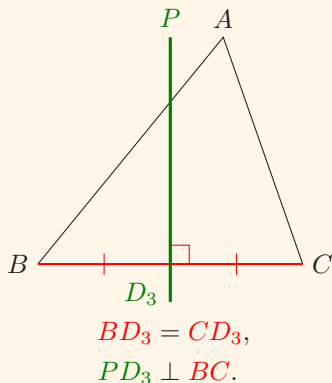
FIGURE	AREA	PERIMETER	
	πr^2	$2\pi r$	◀ CIRCLES
	$\frac{\pi \theta r^2}{360^\circ}$	$\frac{2\pi \theta r}{360^\circ} + 2r$	◀ SECTORS
	$\frac{hb}{2}$	$b + l_1 + l_2$	◀ TRIANGLES
	a^2	$4a$	◀ SQUARES
	ab	$2(a + b)$	◀ RECTANGLES

	$\frac{h(a+b)}{2}$	$a+b+l_1+l_2$	◀ TRAPEZIUMS
	$\frac{xy}{2}$	$4\sqrt{x^2+y^2}$	◀ RHOMBUSES

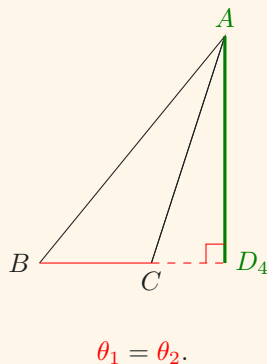
Appendix B: Special Points and Lines in \triangle s

<div>AD_1 is the <i>median</i> of BC</div>  $BD_1 = CD_1.$	<div>AD_2 is an <i>angle bisector</i> of $\angle BAC$</div>  $\theta_1 = \theta_2.$
--	---

PD_3 is a *perpendicular bisector* of BC

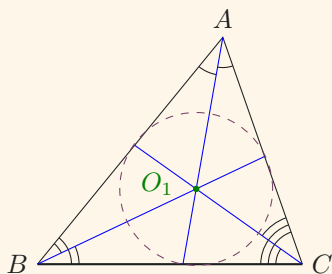


AD_2 is the *altitude* of BC



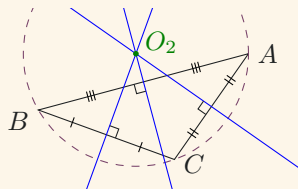
◀ May not necessarily lie in \triangle .

O_1 is the *in-centre* of $\triangle ABC$



The *in-centre* is the point of intersection of the three **angle bisectors** (of a triangle). It is the center of the triangle's **inscribed circle**.

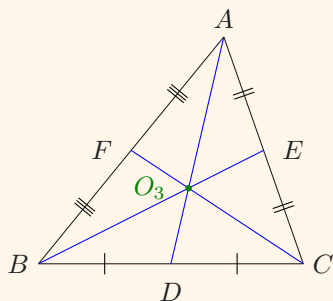
O_2 is the *circumcentre* of $\angle BAC$



◀ May not necessarily lie in \triangle .

The *circumcentre* is the point of intersection of the three **perpendicular bisectors**. It is the center of the triangle's **circumcircle**.

O_1 is the *centroid* of $\triangle ABC$

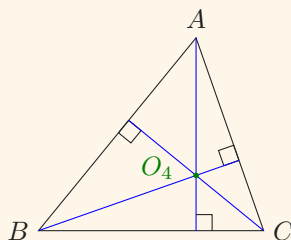


The *centroid* is the point of intersection of the three *medians* (of a triangle).

It divides each median in the ratio of 2 : 1, i.e.,

$$\frac{O_3A}{O_3D} = \frac{O_3B}{O_3E} = \frac{O_3C}{O_3F} = 2.$$

O_4 is the *orthocentre* of $\triangle ABC$



The *orthocentre* is the point of intersection of the three *altitudes*.

◀ May not necessarily lie in \triangle .

These four points are generally distinct from one another.