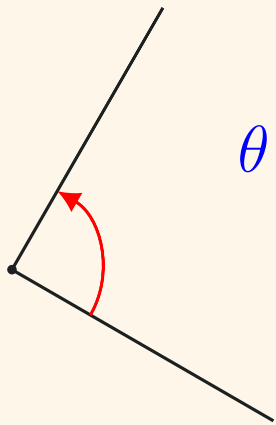
The Comprehensive List of

References in Geometry

AN ILLUSTRATED MANUAL



Mathematics Society

DECEMBER 2020

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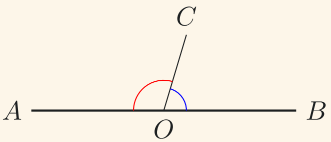
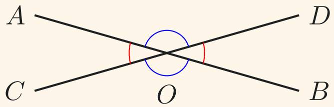
Lines



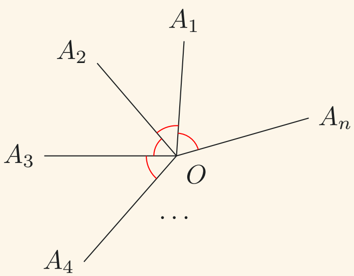
$\boxed{\textbf{\textit{Adj. } } \angle \textbf{\textit{s on st. line}}}$

$$\angle AOC + \angle COB = 180^\circ.$$



$\boxed{\textbf{\textit{Vert. oppo. } } \angle \textbf{\textit{s}}}$

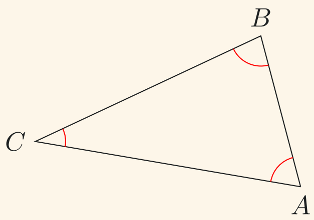
$$\angle AOC = DOB,$$
$$\angle AOD = COB.$$



$\boxed{\angle \textbf{\textit{s at a pt.}}}$

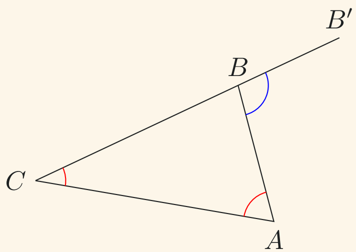
$$\angle A_1OA_2 + \angle A_2OA_3 + \cdots + \angle A_{n-1}OA_n = 360^\circ.$$

Triangles



∠ sum of △s

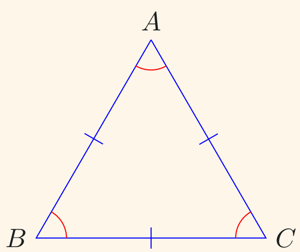
$$\color{red}{\angle CAB} + \color{red}{\angle ABC} + \color{red}{\angle BCA} = 180^\circ.$$



Ext. ∠ of △s

$$\color{red}{\angle CAB} + \color{red}{\angle BCA} = \color{blue}{\angle B'BA}.$$

Equilateral Triangles

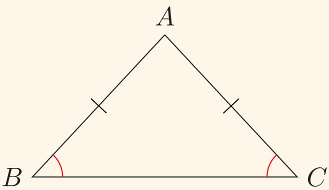


Prop. of equil. △s

$$\color{blue}{AB} = \color{blue}{BC} = \color{blue}{CA},$$

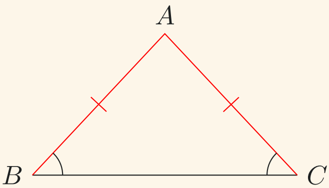
$$\color{red}{\angle CAB} = \color{red}{\angle ABC} = \color{red}{\angle BCA} = 60^\circ.$$

Isosceles Triangles



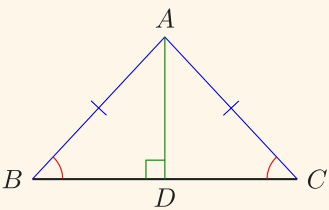
$$\boxed{\textbf{\textit{Base }}\angle\textbf{\textit{s, isos. }}\triangle}$$

$$AB = AC$$
$$\Downarrow$$
$$\angle ABC = \angle ACB.$$



$$\boxed{\textbf{\textit{Sides oppo. eq. }}\angle\textbf{\textit{s}}}$$

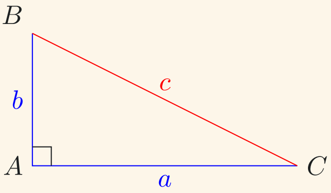
$$\angle ABC = \angle ACB$$
$$\Downarrow$$
$$AB = AC.$$



$$\boxed{\textbf{\textit{Prop. of isos. }}\triangle\textbf{\textit{s}}}$$

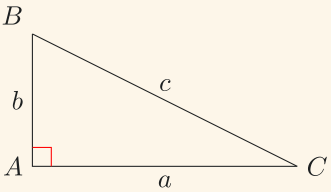
$$\angle ABC = \angle ACB$$
$$\Updownarrow$$
$$AB = AC$$
$$\Updownarrow$$
$$AD \perp BC.$$

Pythagoras's Theorem



Pyth. thrm.

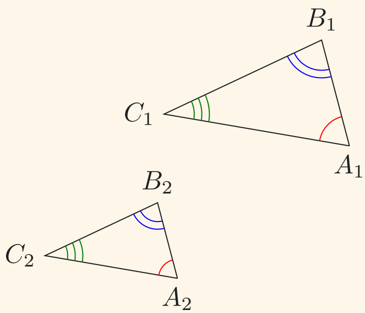
$$BA \perp AC$$
$$\Downarrow$$
$$a^2 + b^2 = c^2.$$



Conv. of Pyth. thrm.

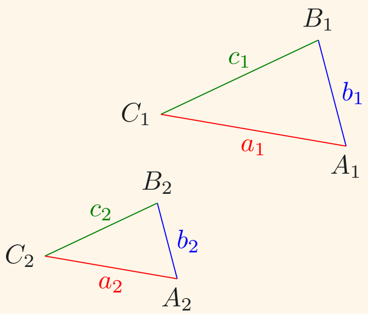
$$a^2 + b^2 = c^2$$
$$\Downarrow$$
$$BA \perp AC.$$

Similar Triangles



Corr. $\angle$s | $\sim\triangle$s

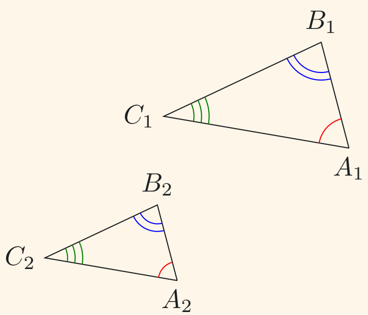
$$\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2$$
$$\Downarrow$$
$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$
$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$
$$\angle B_1C_1A_1 = \angle B_2C_2A_2.$$



$\boxed{\textbf{\textit{Corr. sides}} \mid \sim\triangle\textbf{\textit{s}}}$

$$\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2$$
$$\Downarrow$$
$$\frac{a_1}{b_1} = \frac{a_2}{b_2},$$
$$\frac{b_1}{c_1} = \frac{b_2}{c_2},$$
$$\frac{a_1}{b_1} = \frac{a_2}{b_2}.$$

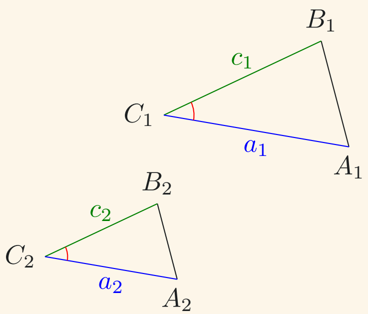
Conditions for Proving Similar Triangles



$\boxed{\textbf{\textit{AAA}}}$

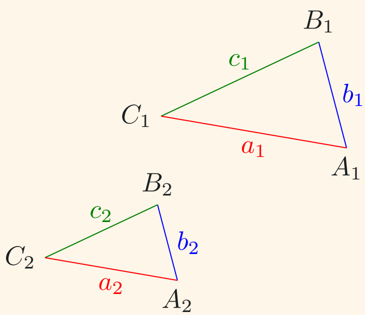
$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$
$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$
$$\angle B_1C_1A_1 = \angle B_2C_2A_2$$
$$\Downarrow$$
$$\underline{\underline{\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.}}$$

◀ Any two of the three is sufficient.



Ratio of two sides, incl. ∠

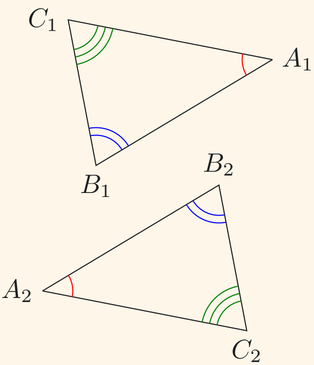
$$\frac{a_1}{c_1} = \frac{a_2}{c_2},$$
$$\angle B_1C_1A_1 = \angle B_2C_2A_2$$
$$\Downarrow$$
$$\underline{\underline{\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.}}$$



Three sides proportional

$$\frac{a_1}{b_1} = \frac{a_2}{b_2},$$
$$\frac{b_1}{c_1} = \frac{b_2}{c_2},$$
$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$
$$\Downarrow$$
$$\underline{\underline{\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.}}$$

Congruent Triangles

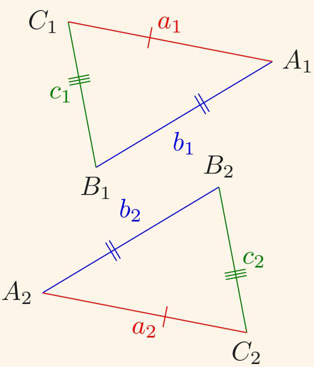


$\boxed{\textbf{\textit{Corr. }} \angle \boldsymbol{s} \mid \cong \triangle \boldsymbol{s}}$

$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2$$

$$\Downarrow$$

$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$
$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$
$$\angle B_1C_1A_1 = \angle B_2C_2A_2.$$



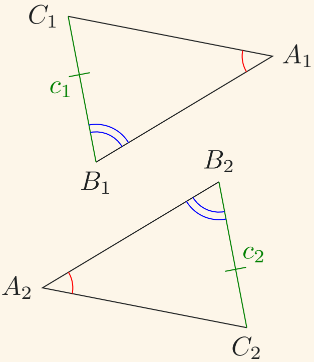
$\boxed{\textbf{\textit{Corr. sides}} \mid \cong \triangle \boldsymbol{s}}$

$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2$$

$$\Downarrow$$

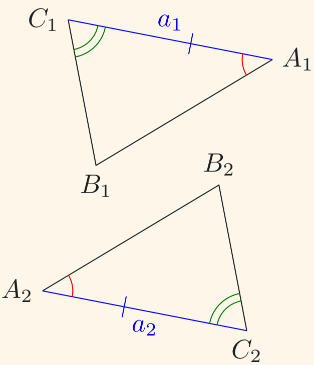
$$a_1 = a_2,$$
$$b_1 = a_2,$$
$$c_1 = a_2.$$

Conditions for Proving Congruent Triangles



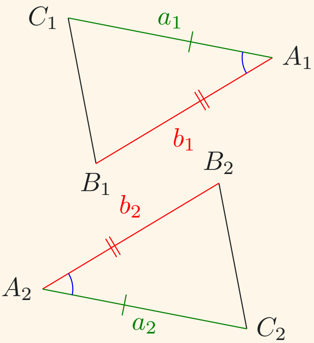
$\boxed{\boldsymbol{AAS}}$

$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$
$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$
$$c_1 = c_2$$
$$\Downarrow$$
$$\underline{\underline{\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.}}$$



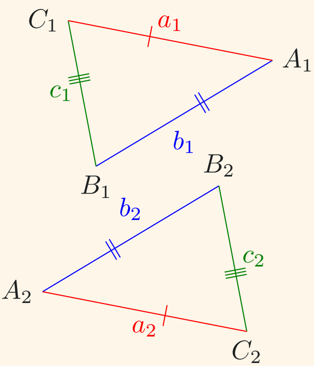
$\boxed{\boldsymbol{ASA}}$

$$\angle A_1B_1C_1 = \angle C_2A_2B_2,$$
$$a_1 = a_2,$$
$$\angle B_1C_1A_1 = \angle B_2C_2A_2,$$
$$\Downarrow$$
$$\underline{\underline{\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.}}$$



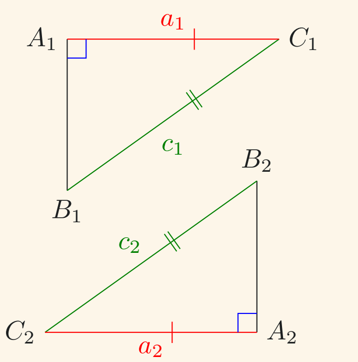
$\boxed{\textbf{\textit{SAS}}}$

$$a_1 = b_1,$$
$$\angle B_1A_1C_1 = \angle B_2A_2C_2,$$
$$b_1 = b_2$$
$$\Downarrow$$
$$\underline{\underline{\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.}}$$



$\boxed{\textbf{\textit{SSS}}}$

$$a_1 = a_2,$$
$$b_1 = a_2,$$
$$c_1 = a_2$$
$$\Downarrow$$
$$\underline{\underline{\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.}}$$

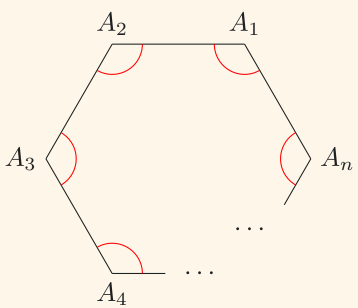


$\boxed{\textbf{\textit{RHS}}}$

$$\angle B_1A_1C_1 = \angle B_2A_2C_2 = 90^\circ,$$
$$c_1 = c_2,$$
$$a_1 = a_2$$
$$\Downarrow$$
$$\underline{\underline{\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.}}$$

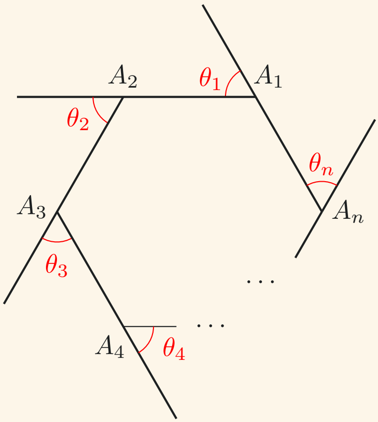
◀ One of the sides must be the hypotenuse.

Polygons



$\boxed{\angle \textbf{\textit{ sum of polygons}}}$

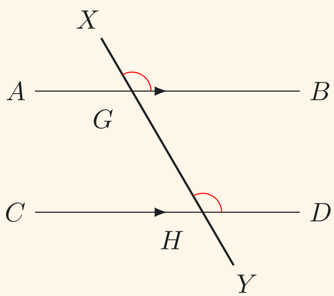
$$\angle A_nA_1A_2 + \angle A_1A_2A_3 + \cdots + \angle A_{n-1}A_nA_1 = (n-2)\cdot 180^\circ.$$



Ext. $\angle$s of polygons

$$\theta_1 + \theta_2 + \cdots + \theta_n = 360^\circ.$$

Parallel Lines

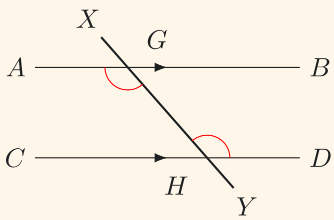


***Corr. $\angle$s** | $AB /\!/ CD$*

$$AB /\!/ CD$$

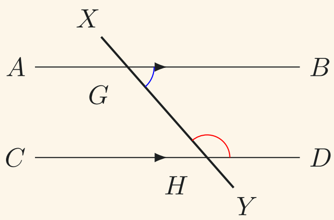
$$\Downarrow$$

$$\angle XGB = \angle XHD.$$



$\boxed{\textbf{\textit{Alt. }} \angle \textbf{\textit{s}} \mid AB \mathbin{/\mkern-5mu/} CD}$

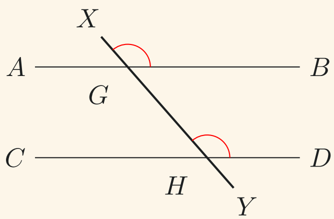
$$AB \mathbin{/\mkern-5mu/} CD$$
$$\Downarrow$$
$$\angle YGA = \angle XHD.$$



$\boxed{\textbf{\textit{Int. }} \angle \textbf{\textit{s}} \mid AB \mathbin{/\mkern-5mu/} CD}$

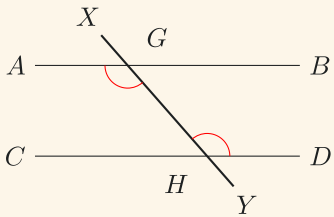
$$AB \mathbin{/\mkern-5mu/} CD$$
$$\Downarrow$$
$$\angle YGB + \angle XHD = 180^\circ.$$

Conditions for Proving Parallelism



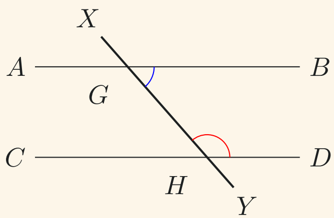
$\boxed{\textbf{\textit{Int. }} \angle \textbf{\textit{s eq.}}}$

$$\angle YGB + \angle XHD = 180^\circ$$
$$\Downarrow$$
$$\underline{\underline{AB \mathbin{/\mkern-5mu/} CD}}.$$



$\boxed{\textbf{\textit{Alt. }}\angle \textbf{\textit{s eq.}}}$

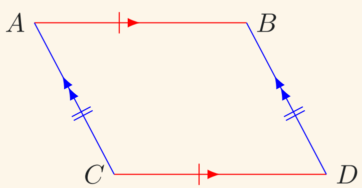
$$\angle YGA = \angle XHD$$
$$\Downarrow$$
$$\underline{\underline{AB \parallel CD}}.$$



$\boxed{\textbf{\textit{Int. }}\angle \textbf{\textit{s eq}}}$

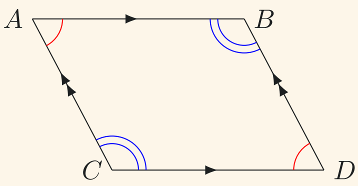
$$\angle YGB + \angle XHD = 180^\circ$$
$$\Downarrow$$
$$\underline{\underline{AB \parallel CD}}.$$

Parallelograms



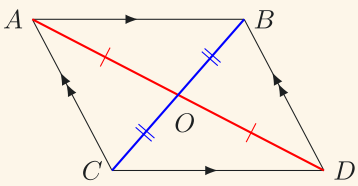
$\boxed{\textbf{\textit{Oppo. sides of }} \parallel\textbf{\textit{-gram}}}$

$$AB = CD,$$
$$AC = BD.$$



Oppo. ∠s of //-gram

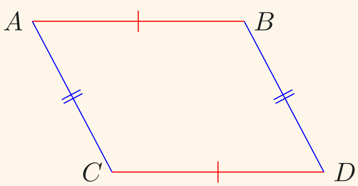
$$\angle BAC = \angle BDC,$$
$$\angle ACD = \angle ABD.$$



Diags. of //-gram

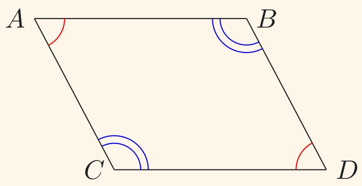
$$OA = OD,$$
$$OB = OC.$$

Conditions for Identifying Parallelograms



Oppo. sides eq.

$$AB = CD,$$
$$AC = BD$$
$$\Downarrow$$
$ABCD$ is a //-gram.

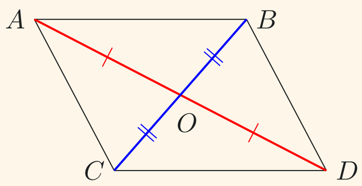


$\boxed{\textbf{\textit{Oppo. } \angle \textbf{\textit{s eq.}}}}$

$\angle BAC = \angle BDC,$
$\angle ACD = \angle ABD$

$$\Downarrow$$

$\underline{\underline{ABCD \text{ is a } /\!/\text{-gram.}}}$

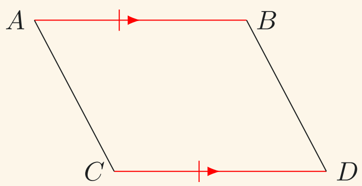


$\boxed{\textbf{\textit{Diags. bisect each other}}}$

$OA = OD,$
$OB = OC$

$$\Downarrow$$

$\underline{\underline{ABCD \text{ is a } /\!/\text{-gram.}}}$



$\boxed{\textbf{\textit{Two sides eq. and }} /\!/}$

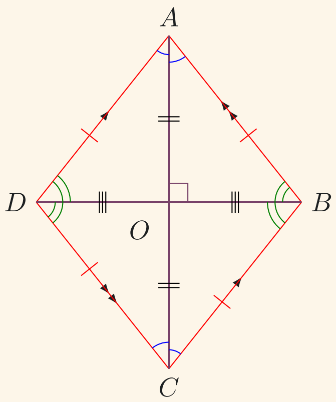
$AB = CD,$
$AB /\!/ BD$

$$\Downarrow$$

$\underline{\underline{ABCD \text{ is a } /\!/\text{-gram.}}}$

Other Quadrilaterals

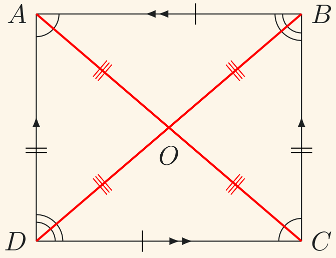
Rhombus - a quadrilateral with four equal sides



Prop. of rhombuses

$$AB \parallel CD \qquad AD \parallel BC,$$
$$OA = OC \qquad OB = OD,$$
$$AB = BC = CD = DA,$$
$$AC \perp BD,$$
$$\angle OAD = \angle OAB = \angle OCD = \angle OCB,$$
$$\angle ODA = \angle ODC = \angle OBA = \angle OBC.$$

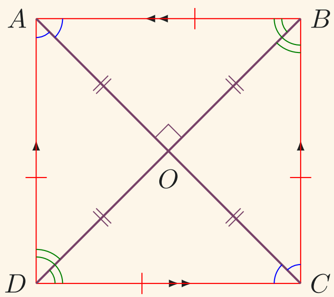
Rectangle - a quadrilateral with four equal interior angles



Prop. of rectangles

$$AB \parallel CD \qquad AD \parallel BC,$$
$$AB = CD \qquad AD = BC,$$
$$\angle BAD = \angle BCD,$$
$$\angle ABC = \angle ADC,$$
$$OA = OB = OC = OD.$$

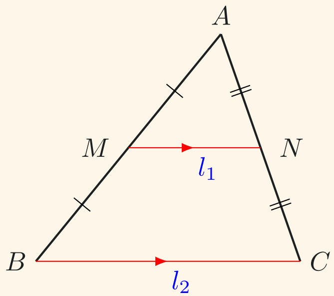
Square - a quadrilateral with four equal sides and interior angles



Prop. of squares

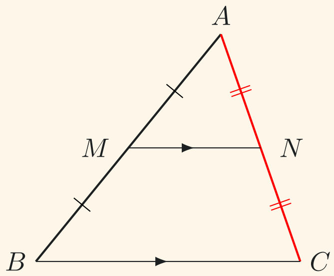
$$AB \parallel CD \qquad AD \parallel BC,$$
$$AB = BC = CD = DA,$$
$$OA = OB = OC = OD,$$
$$AC \perp BD,$$
$$\angle OAD = \angle OAB = 90^\circ$$
$$= \angle OCD = \angle OCB,$$
$$\angle ODA = \angle ODC = 90^\circ$$
$$= \angle OBA = \angle OBC.$$

Miscellaneous Results



Mid-pt. thrm.

$$MA = MA \quad NA = NC$$
$$\Downarrow$$
$$2l_1 = l_2,$$
$$MN \parallel BC.$$

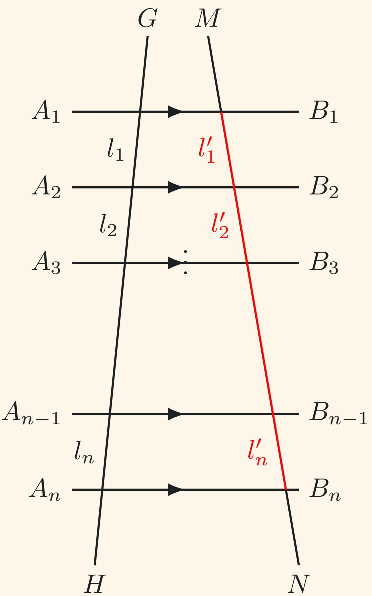


Intercept thrm. variant i

$$MA = MA \quad MN \parallel BC$$

$$\Downarrow$$

$$NA = NC.$$



Intercept thrm. variant ii

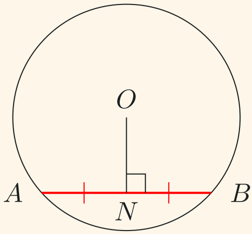
$$A_1B_1 \parallel A_2B_2 \parallel \cdots \parallel A_nB_n$$

$$l_1 = l_2 = \cdots = l_n$$

$$\Downarrow$$

$$l'_1 = l'_2 = \cdots = l'_n.$$

Circles

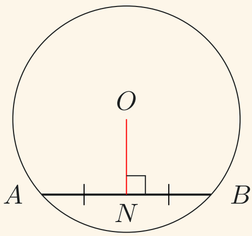


$$\boxed{\textbf{\textit{Line from cen.}} \perp \textbf{\textit{ chord bisects chord}}}$$

$$ON \perp AB$$

$$\Downarrow$$

$$NA = NB.$$

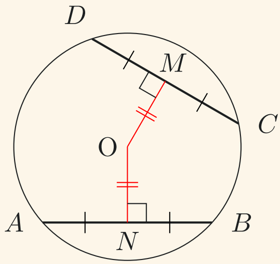


$$\boxed{\textbf{\textit{Line joining cen. to mid-pt. of chord}} \perp \textbf{\textit{ chord}}}$$

$$NA = NB$$

$$\Downarrow$$

$$ON \perp AB.$$



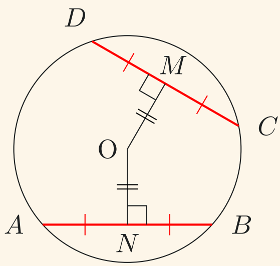
$$\boxed{\textbf{\textit{Eq. chords equidis. from cen.}}}$$

$$ON \perp AB \quad OM \perp CD,$$

$$AB = CD$$

$$\Downarrow$$

$$ON = OM.$$



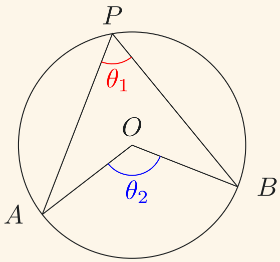
$$\boxed{\textbf{\textit{Chords equidis. from cen. are eq.}}}$$

$$ON \perp AB \quad OM \perp CD,$$

$$ON = OM$$

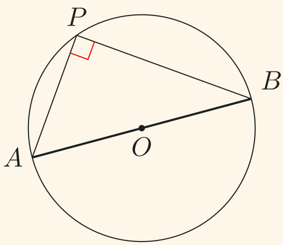
$$\Downarrow$$

$$AB = CD.$$



$\boxed{\angle \textit{ at cen. twice } \angle \textit{ at } \odot^{ce}}$

$$2\theta_1 = \theta_2.$$

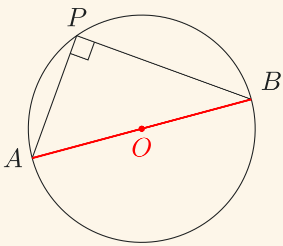


$\boxed{\angle \textit{ in semi-circ.}}$

AB is a diameter

$$\Downarrow$$

$$\angle APB = 90^\circ.$$

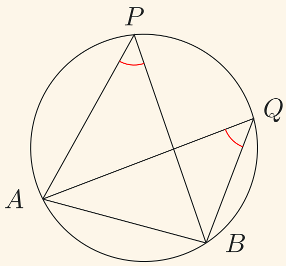


$\boxed{\textit{Converse of } \angle \textit{ in semi-circ.}}$

$$\angle APB = 90^\circ$$

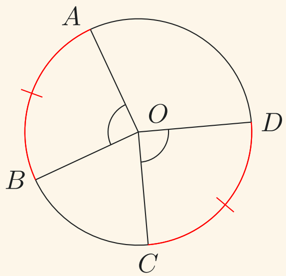
$$\Downarrow$$

AB is a diameter.



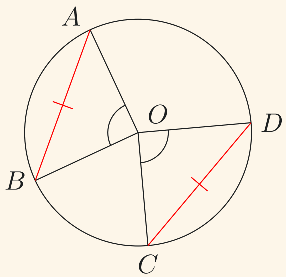
$\boxed{\angle s \text{ in the same segment}}$

$\angle APB = \angle AQB.$



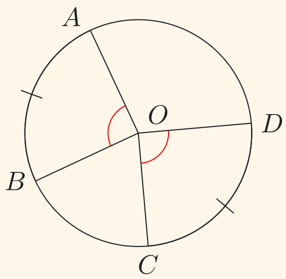
$\boxed{\text{Eq. } \angle s\text{, eq. arcs}}$

$$\angle AOB = \angle COD$$
$$\Downarrow$$
$$\overset{\frown}{AB} = \overset{\frown}{CD}.$$



$\boxed{\text{Eq. } \angle s\text{, eq. chords}}$

$$\angle AOB = \angle COD$$
$$\Downarrow$$
$$AB = CD.$$

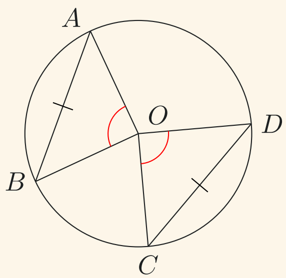


$\boxed{\textbf{\textit{Eq. arcs, eq. }}\angle\textbf{\textit{s}}}$

$$\overset{\frown}{AB} = \overset{\frown}{CD}$$

$$\Downarrow$$

$$\angle AOB = \angle COD.$$

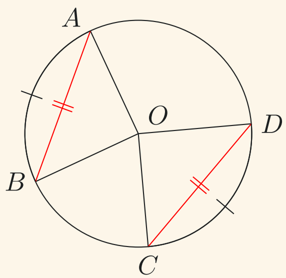


$\boxed{\textbf{\textit{Eq. chords, eq. }}\angle\textbf{\textit{s}}}$

$$AB = CD$$

$$\Downarrow$$

$$\angle AOB = \angle COD.$$

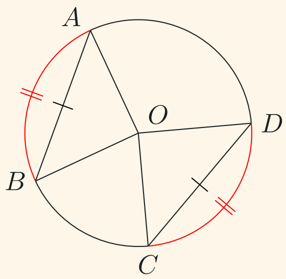


$\boxed{\textbf{\textit{Eq. arcs, eq. chords}}}$

$$\overset{\frown}{AB} = \overset{\frown}{CD}$$

$$\Downarrow$$

$$AB = CD.$$

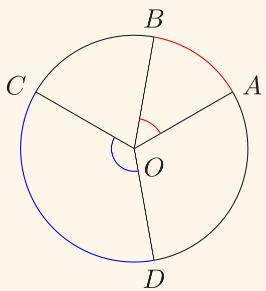


Eq. chords, eq. arcs

$$AB = CD$$

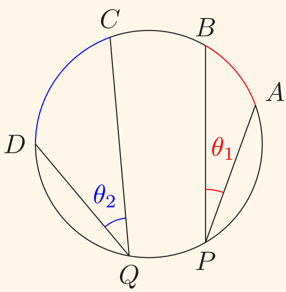
$$\Downarrow$$

$$\overparen{AB} = \overparen{CD}.$$



Arcs prop. to $\angle$s at cen.

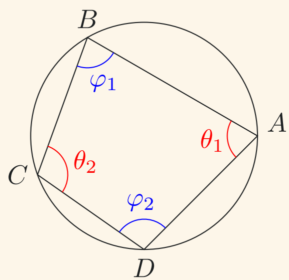
$$\frac{\overparen{AB}}{\overparen{CD}} = \frac{\theta_1}{\theta_2}.$$



Arcs prop. to $\angle$s at $\odot^{ce}$

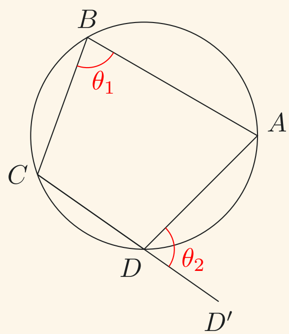
$$\frac{\overparen{AB}}{\overparen{CD}} = \frac{\theta_1}{\theta_2}.$$

Cyclic Quadrilaterals



Oppo. ∠s, cyclic quad.

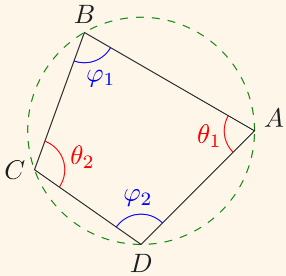
$$\theta_1 + \theta_2 = 180^\circ,$$
$$\varphi_1 + \varphi_2 = 180^\circ.$$



Ext. ∠s, cyclic quad.

$$\theta_1 = \theta_2.$$

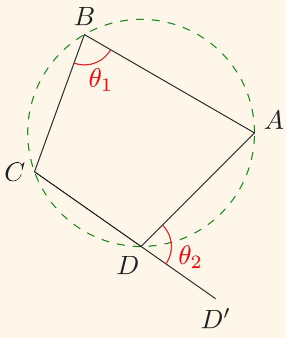
Conditions for Identifying Cyclic Quadrilaterals



Oppo. ∠s supp.

$$\theta_1 + \theta_2 = 180^\circ,$$
$$\varphi_1 + \varphi_2 = 180^\circ$$
$$\Downarrow$$

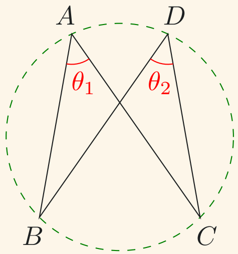
A, B, C and D are concylic.



Ext. ∠ = int. oppo. ∠

$$\theta_1 = \theta_2$$
$$\Downarrow$$

A, B, C and D are concylic.



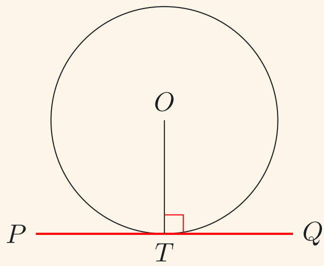
Converse of $\angle$s in the same segment

$\theta_1 = \theta_2$

$\Downarrow$

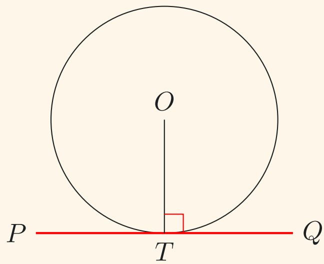
A, B, C and D are concylic.

Tangents



Tan. $\perp$ radius

$OT \perp PQ$.

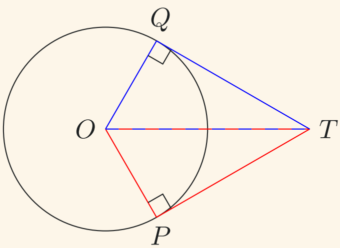


Converse of tan. $\perp$ radius

$OT \perp PQ$.

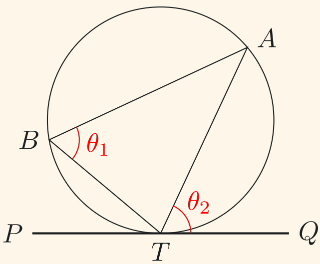
$\Downarrow$

PQ is tangent to the circle at T.



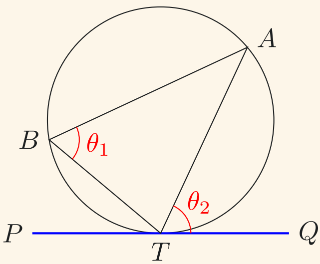
$\boxed{\textbf{\textit{Tan. prop.}}}$

$\triangle OPT \cong \triangle OQT$.



$\boxed{\angle \textbf{\textit{ in alt. segment}}}$

$\theta_1 = \theta_2$.



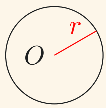
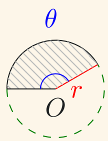
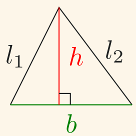
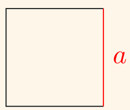
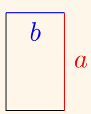
Converse of
∠ in alt. segment

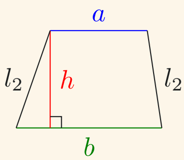
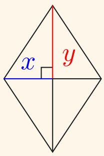
$$\theta_1 = \theta_2$$

$$\Downarrow$$

PQ is tangent to the circle at T.

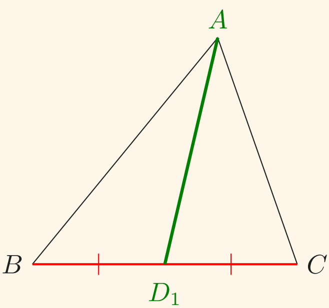
Appendix A: Area and Perimeters of Common Plane Figures

Figure	Area	Perimeter	
	πr^2	$2\pi r$	◀ Circles
	$\dfrac{\pi\theta r^2}{360°}$	$\dfrac{2\pi\theta r}{360°} + 2r$	◀ Sectors
	$\dfrac{hb}{2}$	$b + l_1 + l_2$	◀ Triangles
	a^2	$4a$	◀ Squares
	ab	$2\,(a+b)$	◀ Rectangles

	$\frac{h\,(a+b)}{2}$	$a + b + l_1 + l_2$	◀ TRAPEZIUMS
	$\frac{xy}{2}$	$4\sqrt{x^2 + y^2}$	◀ RHOMBUSES

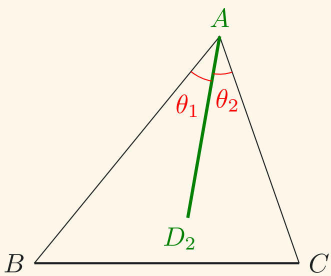
Appendix B: Special Points and Lines in △s

$\boxed{\text{$\boldsymbol{AD_1}$ is the \textit{median} of BC}}$



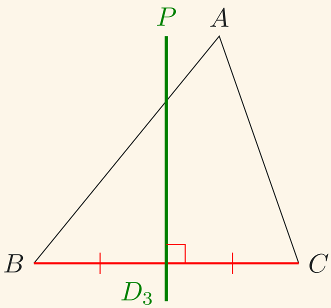
$BD_1 = CD_1.$

$\boxed{\text{$\boldsymbol{AD_2}$ is an \textit{angle bisector} of $\angle BAC$}}$



$\theta_1 = \theta_2.$

PD_3 is a *perpendicular bisector* of BC

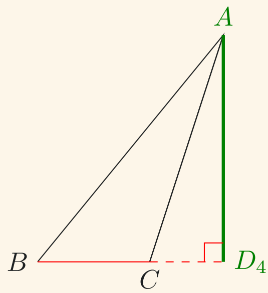


$BD_3 = CD_3$,

$PD_3 \perp BC$.

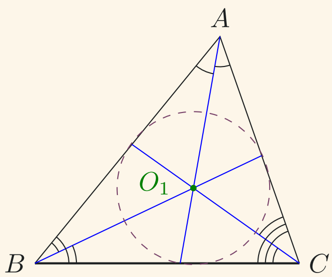
AD_2 is the *altitude* of BC

◀ May not necessarily lie in △.



$\theta_1 = \theta_2$.

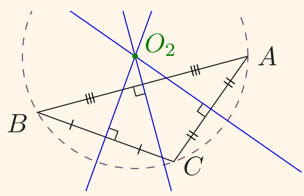
O_1 is the *in-centre* of $\triangle ABC$



The *in-centre* is the point of intersection of the three angle bisectors (of a triangle).
It is the center of the triangle's inscribed circle.

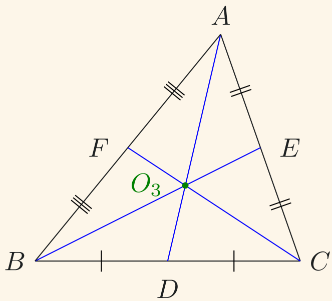
O_2 is the *circumcentre* of $\angle BAC$

◀ May not necessarily lie in △.



The *circumcentre* is the point of intersection of the three perpendicular bisectors.
It is the center of the triangle's circumcircle.

$\boxed{\boldsymbol{O_1} \text{ is the } \textit{centroid} \text{ of } \triangle ABC}$

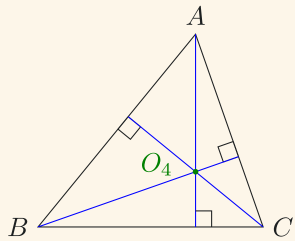


The *centroid* is the point of intersection of the three medians (of a triangle).
It divides each median in the ratio of 2 : 1, i.e.,

$$\frac{O_3A}{O_3D} = \frac{O_3B}{O_3E} = \frac{O_3C}{O_3F} = 2.$$

$\boxed{\boldsymbol{O_4} \text{ is the } \textit{orthocentre} \text{ of } \triangle ABC}$

◄ May not necessarily lie in $\triangle$.



The *orthocentre* is the point of intersection of the three altitudes.

These four points are generally distinct from one another.