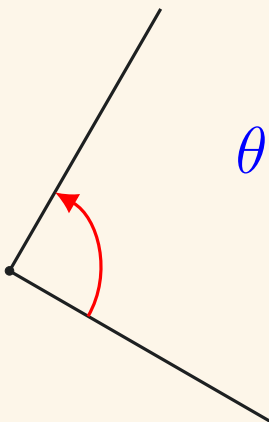


The Comprehensive List of  
**References in Geometry**

AN ILLUSTRATED MANUAL



Isaac Li  
Mathematics Society

DECEMBER 2020





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## Appendix A

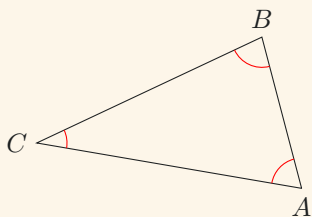
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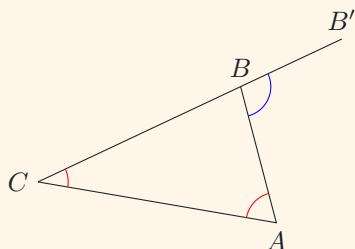


## Triangles



$\angle$  *sum of  $\triangle s$*

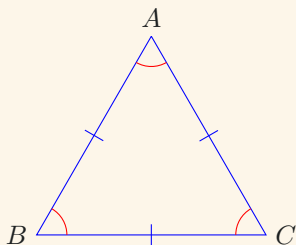
$$\begin{aligned} \angle CAB + \angle ABC \\ + \angle BCA = 180^\circ. \end{aligned}$$



*Ext.*  $\angle$  *of  $\triangle s$*

$$\angle CAB + \angle BCA = \angle B'BA.$$

## Equilateral Triangles

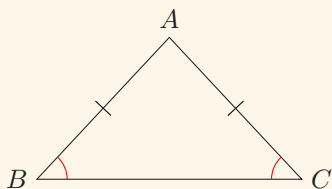


*Prop. of equil.  $\triangle s$*

$$AB = BC = CA,$$

$$\begin{aligned} \angle CAB = \angle ABC \\ = \angle BCA = 60^\circ. \end{aligned}$$

## Isosceles Triangles

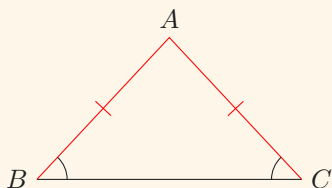


*Base  $\angle$ s, isos.  $\triangle$*

$$AB = AC$$



$$\angle ABC = \angle ACB.$$

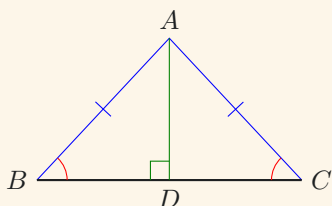


*Sides oppo. eq.  $\angle$ s*

$$\angle ABC = \angle ACB$$



$$AB = AC.$$



*Prop. of isos.  $\triangle$ s*

$$\angle ABC = \angle ACB$$



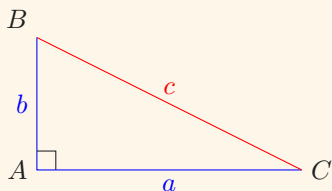
$$AB = AC$$



$$AD \perp BC.$$



## Pythagoras's Theorem

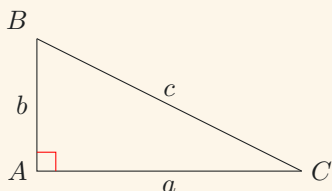


*Pyth. thrm.*

$$BA \perp AC$$



$$a^2 + b^2 = c^2.$$



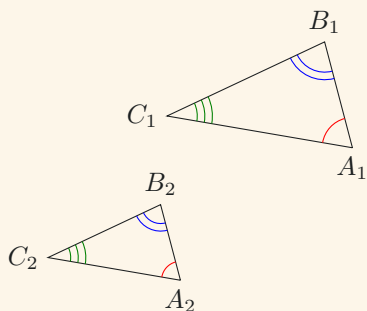
*Conv. of Pyth. thrm.*

$$a^2 + b^2 = c^2$$



$$BA \perp AC.$$

## Similar Triangles



*Corr.*  $\angle s \mid \sim \triangle s$

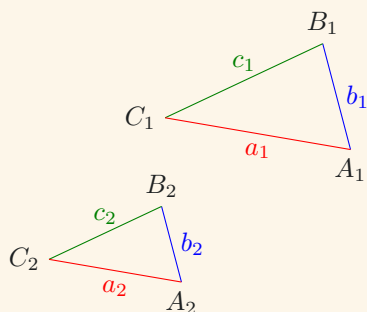
$$\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2$$



$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$

$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$

$$\angle B_1C_1A_1 = \angle B_2C_2A_2.$$



**Corr. sides** |  $\sim \triangle s$

$$\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2$$

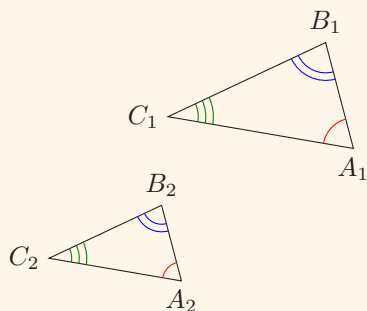


$$\frac{a_1}{b_1} = \frac{a_2}{b_2},$$

$$\frac{b_1}{c_1} = \frac{b_2}{c_2},$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}.$$

## Conditions for Proving Similar Triangles



**AAA**

$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$

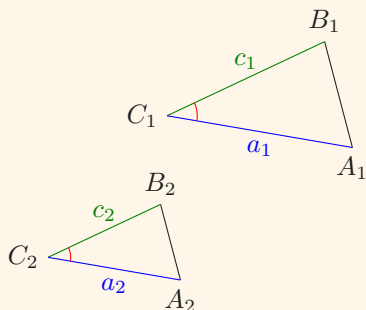
$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$

$$\angle B_1C_1A_1 = \angle B_2C_2A_2$$

◀ Any two of the three is sufficient.



$$\underline{\underline{\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.}}$$



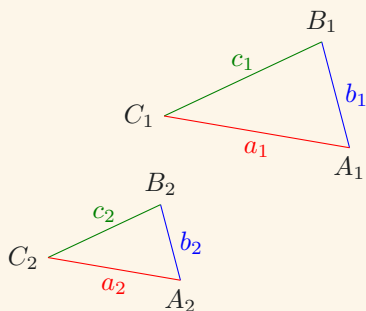
*Ratio of two sides,  
incl.  $\angle$*

$$\frac{a_1}{c_1} = \frac{a_2}{c_2},$$

$$\angle B_1C_1A_1 = \angle B_2C_2A_2$$

$$\Downarrow$$

$$\underline{\underline{\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.}}$$



*Three sides  
proportional*

$$\frac{a_1}{b_1} = \frac{a_2}{b_2},$$

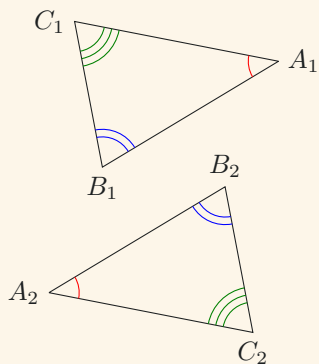
$$\frac{b_1}{c_1} = \frac{b_2}{c_2},$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

$$\Downarrow$$

$$\underline{\underline{\triangle A_1B_1C_1 \sim \triangle A_2B_2C_2.}}$$

## Congruent Triangles



**Corr.  $\angle s$  |  $\cong \triangle s$**

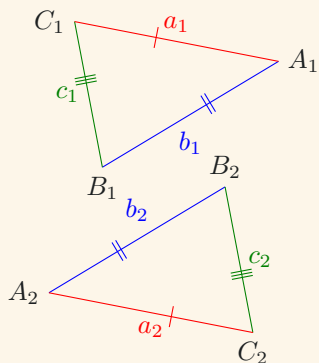
$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2$$



$$\angle C_1A_1B_1 = \angle C_2A_2B_2,$$

$$\angle A_1B_1C_1 = \angle A_2B_2C_2,$$

$$\angle B_1C_1A_1 = \angle B_2C_2A_2.$$



**Corr.  $sides$  |  $\cong \triangle s$**

$$\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2$$

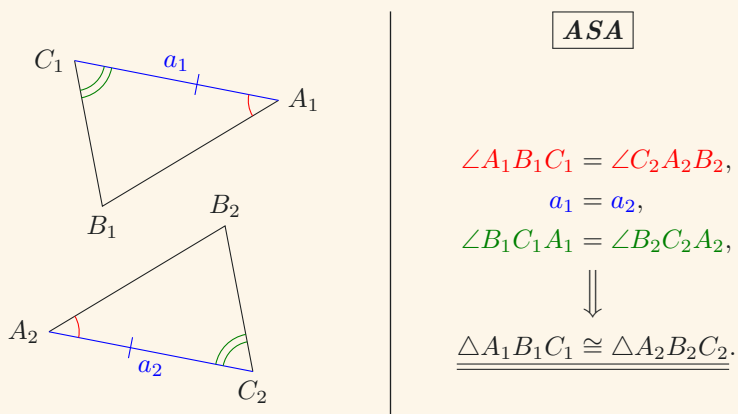
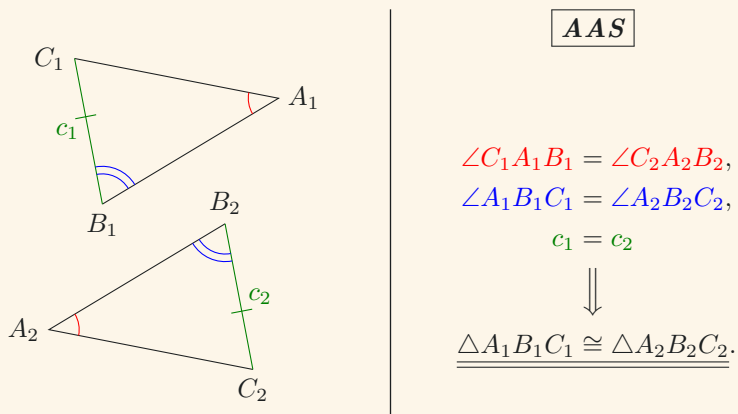


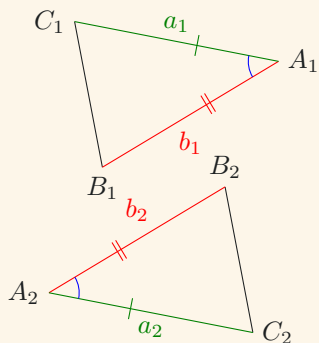
$$a_1 = a_2,$$

$$b_1 = b_2,$$

$$c_1 = c_2.$$

## Conditions for Proving Congruent Triangles



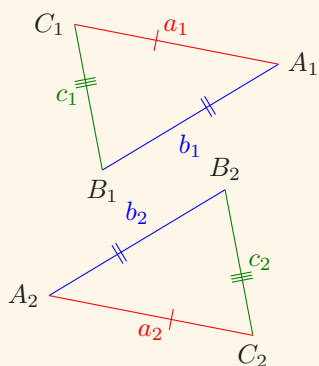


**SAS**

$$\begin{aligned} a_1 &= b_1, \\ \angle B_1A_1C_1 &= \angle B_2A_2C_2, \\ b_1 &= b_2 \end{aligned}$$



$$\underline{\underline{\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.}}$$

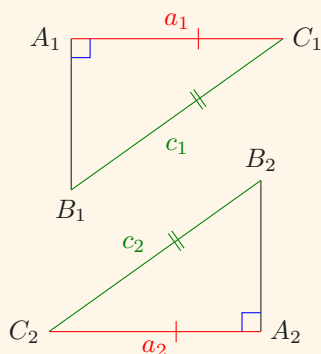


**SSS**

$$\begin{aligned} a_1 &= a_2, \\ b_1 &= a_2, \\ c_1 &= a_2 \end{aligned}$$



$$\underline{\underline{\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.}}$$

***RHS***

$$\angle B_1A_1C_1 = \angle B_2A_2C_2$$

$$= 90^\circ,$$

$$c_1 = c_2,$$

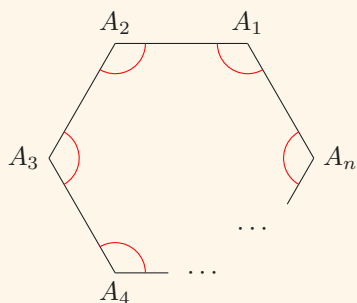
$$a_1 = a_2$$



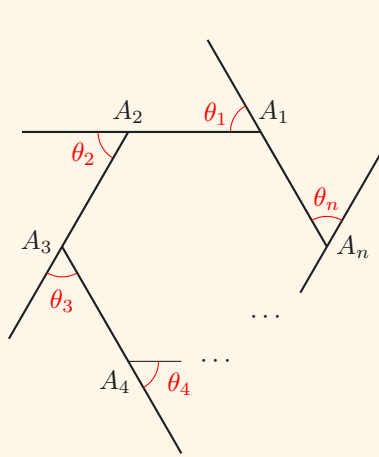
$$\underline{\underline{\triangle A_1B_1C_1 \cong \triangle A_2B_2C_2.}}$$

◀ One of the sides must be the hypotenuse.

## Polygons

***∠ sum of polygons***

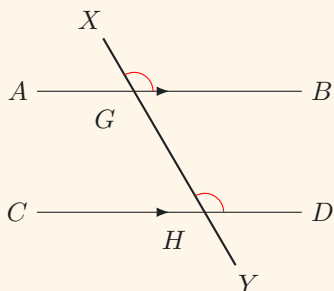
$$\begin{aligned} & \angle A_nA_1A_2 + \angle A_1A_2A_3 + \dots \\ & + \angle A_{n-1}A_nA_1 = (n-2) \cdot 180^\circ. \end{aligned}$$



*Ext.  $\angle$ s of polygons*

$$\theta_1 + \theta_2 + \cdots + \theta_n = 360^\circ.$$

## Parallel Lines



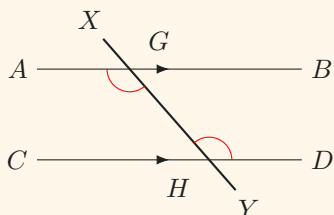
*Corr.  $\angle$ s* |  $AB \parallel CD$

$$AB \parallel CD$$



$$\angle XGB = \angle XHD.$$



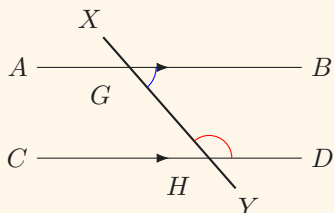


**Alt.**  $\angle s \mid AB \parallel CD$

$$AB \parallel CD$$



$$\angle YGA = \angle XHD.$$



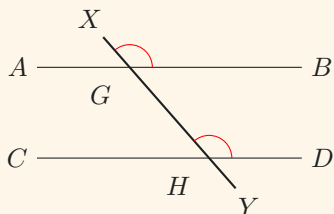
**Int.**  $\angle s \mid AB \parallel CD$

$$AB \parallel CD$$



$$\angle YGB + \angle XHD = 180^\circ.$$

## Conditions for Proving Parallelism

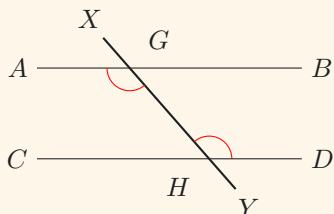


**Int.**  $\angle s \text{ eq.}$

$$\angle YGB + \angle XHD = 180^\circ$$



$$\underline{\underline{AB \parallel CD.}}$$

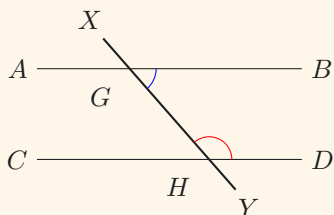


*Alt.  $\angle$ s eq.*

$$\angle YGA = \angle XHD$$



$$\underline{\underline{AB \parallel CD.}}$$



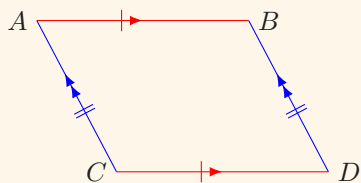
*Int.  $\angle$ s eq*

$$\angle YGB + \angle XHD = 180^\circ$$



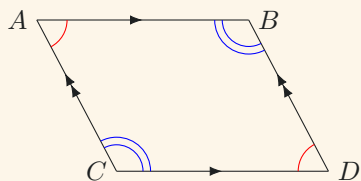
$$\underline{\underline{AB \parallel CD.}}$$

## Parallelograms



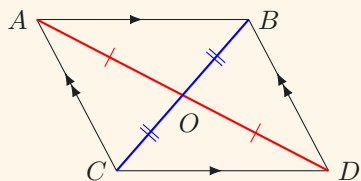
*Oppo. sides of  $\parallel$ -gram*

$$\begin{aligned} AB &= CD, \\ AC &= BD. \end{aligned}$$



*Oppo.  $\angle$ s of  $\parallel$ -gram*

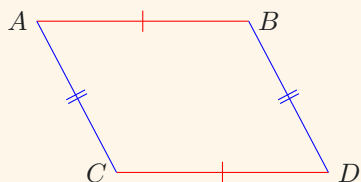
$$\begin{aligned} \angle BAC &= \angle BDC, \\ \angle ACD &= \angle ABD. \end{aligned}$$



*Diags. of  $\parallel$ -gram*

$$\begin{aligned} OA &= OD, \\ OB &= OC. \end{aligned}$$

## Conditions for Identifying Parallelograms

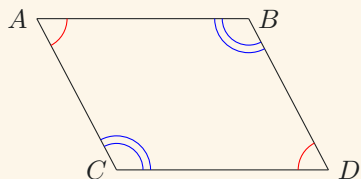


*Oppo. sides eq.*

$$\begin{aligned} AB &= CD, \\ AC &= BD \end{aligned}$$



$ABCD$  is a  $\parallel$ -gram.



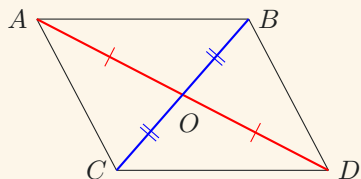
***Oppo.  $\angle$ s eq.***

$$\angle BAC = \angle BDC,$$

$$\angle ACD = \angle ABD$$



$ABCD$  is a  $\parallel$ -gram.



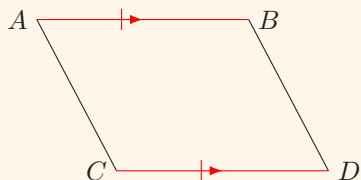
***Diags. bisect each other***

$$OA = OD,$$

$$OB = OC$$



$ABCD$  is a  $\parallel$ -gram.



***Two sides eq. and  $\parallel$***

$$AB = CD,$$

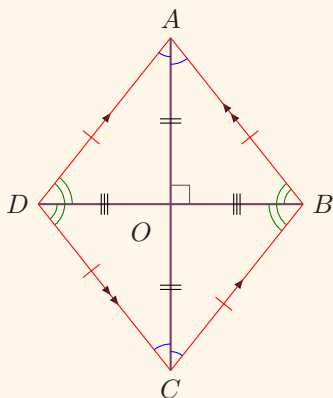
$$AB \parallel CD$$



$ABCD$  is a  $\parallel$ -gram.

## Other Quadrilaterals

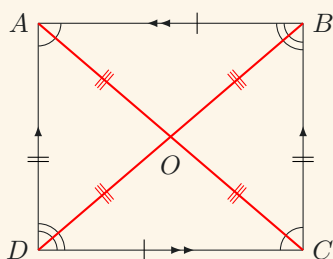
**Rhombus** - a quadrilateral with four equal sides



### *Prop. of rhombuses*

$$\begin{aligned}
 AB &\parallel CD & AD &\parallel BC, \\
 OA &= OC & OB &= OD, \\
 AB &= BC = CD = DA, \\
 AC &\perp BD, \\
 \angle OAD &= \angle OAB \\
 &= \angle OCD = \angle OCB, \\
 \angle ODA &= \angle ODC \\
 &= \angle OBA = \angle OBC.
 \end{aligned}$$

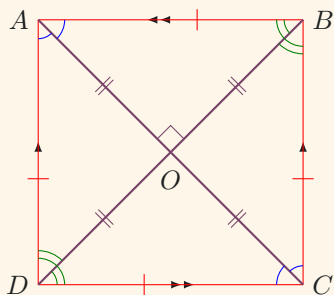
**Rectangle** - a quadrilateral with four equal interior angles



### *Prop. of rectangles*

$$\begin{aligned}
 AB &\parallel CD & AD &\parallel BC, \\
 AB &= CD & AD &= BC, \\
 \angle BAD &= \angle BCD, \\
 \angle ABC &= \angle ADC, \\
 OA &= OB = OC = OD.
 \end{aligned}$$

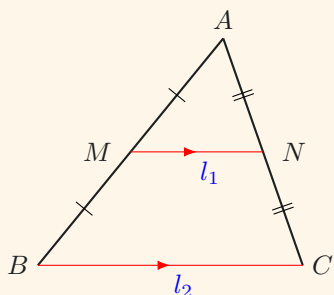
**Square** - a quadrilateral with four equal sides and interior angles



**Prop. of squares**

$AB \parallel CD \quad AD \parallel BC,$   
 $AB = BC = CD = DA,$   
 $OA = OB = OC = OD,$   
 $AC \perp BD,$   
 $\angle OAD = \angle OAB = 90^\circ$   
 $\quad = \angle OCD = \angle OCB,$   
 $\angle ODA = \angle ODC = 90^\circ$   
 $\quad = \angle OBA = \angle OBC.$

**Miscellaneous Results**



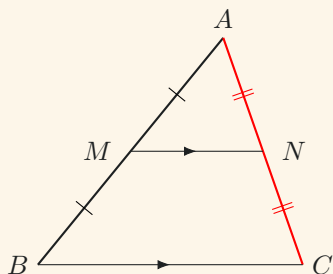
**Mid-pt. thrm.**

$$MA = MB \quad NA = NC$$



$$2l_1 = l_2,$$

$$MN \parallel BC.$$

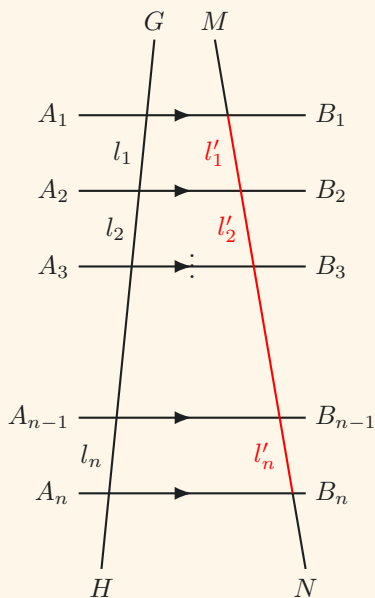


**Intercept thrm.** variant i

$$MA = NA \quad MN \parallel BC$$



$$MB = NC.$$



**Intercept thrm.** variant ii

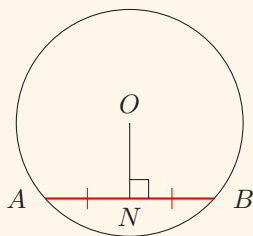
$$A_1B_1 \parallel A_2B_2 \parallel \cdots \parallel A_nB_n$$

$$l_1 = l_2 = \cdots = l_n$$



$$l'_1 = l'_2 = \cdots = l'_n.$$

# Circles

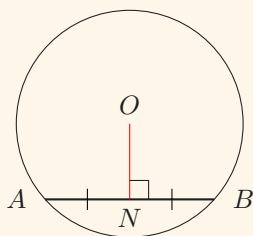


*Line from cen.  $\perp$   
chord bisects chord*

$$ON \perp AB$$



$$NA = NB.$$



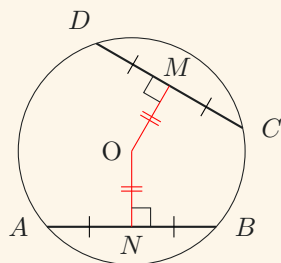
*Line joining cen.  
to mid-pt. of  
chord  $\perp$  chord*

$$NA = NB$$



$$ON \perp AB.$$





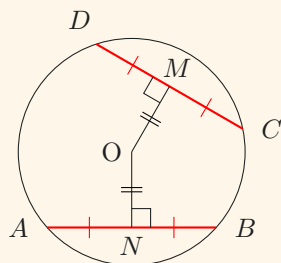
*Eq. chords  
equidis. from cen.*

$$ON \perp AB \quad OM \perp CD,$$

$$AB = CD$$



$$ON = OM.$$



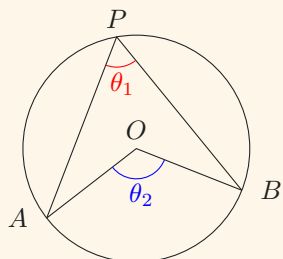
*Chords equidis. from  
cen. are eq.*

$$ON \perp AB \quad OM \perp CD,$$

$$ON = OM$$

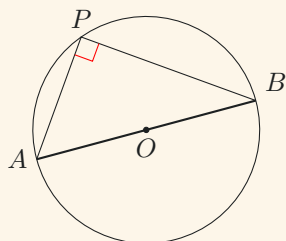


$$AB = CD.$$



$\angle$  at cen. twice  
 $\angle$  at  $\odot^{ce}$

$$2\theta_1 = \theta_2.$$

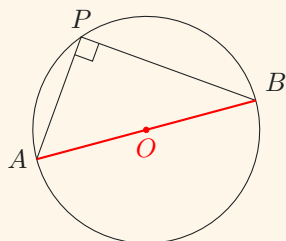


$\angle$  in semi-circ.

$AB$  is a diameter



$$\angle APB = 90^\circ.$$

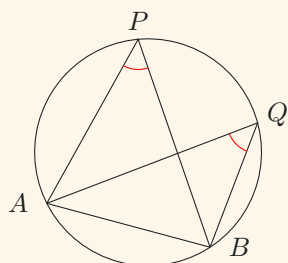


*Converse of*  
 $\angle$  in semi-circ.

$$\angle APB = 90^\circ$$

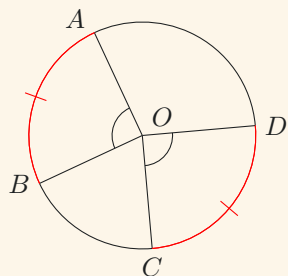


$AB$  is a diameter.



*$\angle$ s in the same segment*

$$\angle APB = \angle AQB.$$

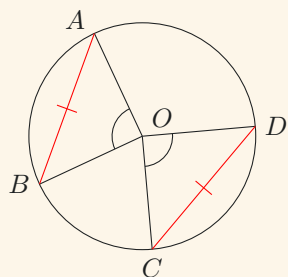


*Eq.  $\angle$ s, eq. arcs*

$$\angle AOB = \angle COD$$



$$\widehat{AB} = \widehat{CD}.$$

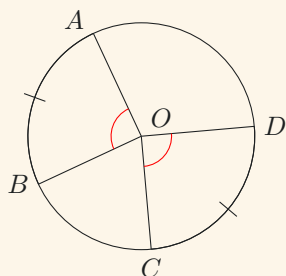


*Eq.  $\angle$ s, eq. chords*

$$\angle AOB = \angle COD$$



$$AB = CD.$$

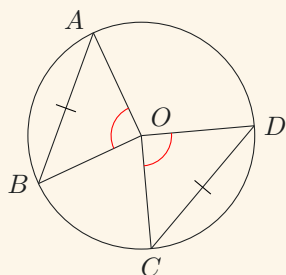


*Eq. arcs, eq.  $\angle$ s*

$$\widehat{AB} = \widehat{CD}$$



$$\angle AOB = \angle COD.$$

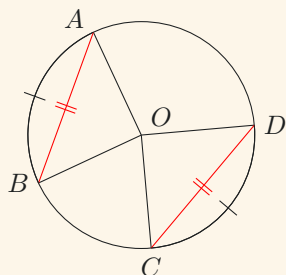


*Eq. chords, eq.  $\angle$ s*

$$AB = CD$$



$$\angle AOB = \angle COD.$$

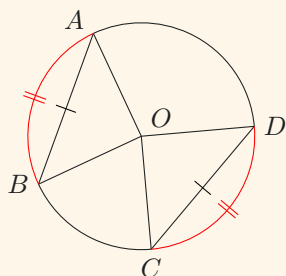


*Eq. arcs, eq. chords*

$$\widehat{AB} = \widehat{CD}$$



$$AB = CD.$$

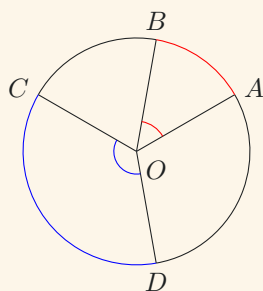


*Eq. chords, eq. arcs*

$$AB = CD$$

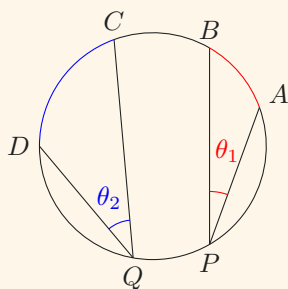


$$\widehat{AB} = \widehat{CD}.$$



*Arcs prop. to  $\angle$ s at cen.*

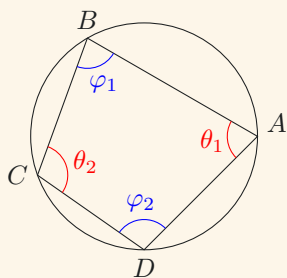
$$\frac{\widehat{AB}}{\widehat{CD}} = \frac{\theta_1}{\theta_2}.$$



*Arcs prop. to  $\angle$ s at  $\odot^{ce}$*

$$\frac{\widehat{AB}}{\widehat{CD}} = \frac{\theta_1}{\theta_2}.$$

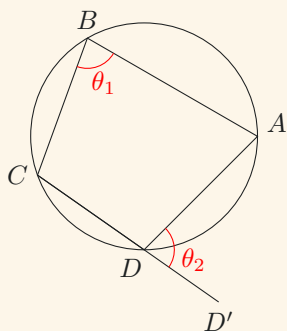
## Cyclic Quadrilaterals



*Oppo.  $\angle$ s, cyclic quad.*

$$\theta_1 + \theta_2 = 180^\circ,$$

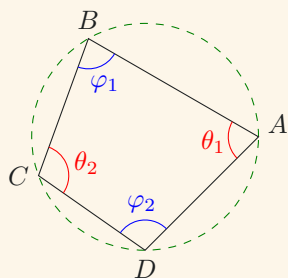
$$\varphi_1 + \varphi_2 = 180^\circ.$$



*Ext.  $\angle$ s, cyclic quad.*

$$\theta_1 = \theta_2.$$

## Conditions for Identifying Cyclic Quadrilaterals



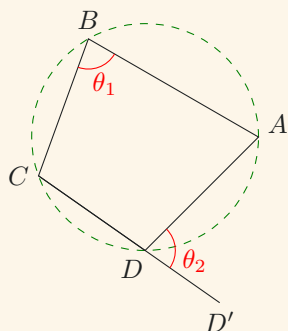
*Oppo.  $\angle$ s supp.*

$$\theta_1 + \theta_2 = 180^\circ,$$

$$\varphi_1 + \varphi_2 = 180^\circ$$



$A, B, C$  and  $D$  are concyclic.

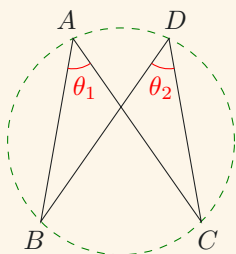


*Ext.  $\angle =$  int. oppo.  $\angle$*

$$\theta_1 = \theta_2$$



$A, B, C$  and  $D$  are concyclic.



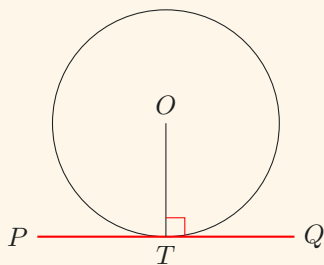
*Converse of  $\angle s$   
in the same segment*

$$\theta_1 = \theta_2$$



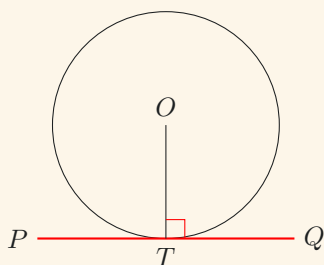
$A, B, C$  and  $D$  are concyclic.

## Tangents



*Tan.  $\perp$  radius*

$$OT \perp PQ.$$



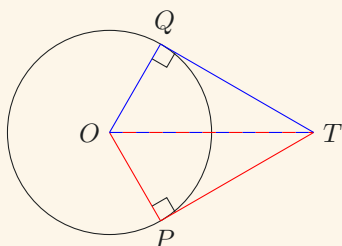
*Converse of  
tan.  $\perp$  radius*

$$OT \perp PQ.$$



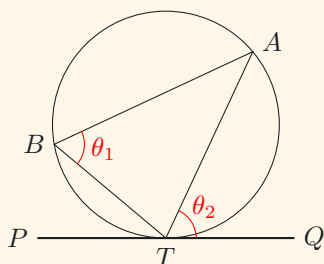
$PQ$  is tangent  
to the circle at  $T$ .





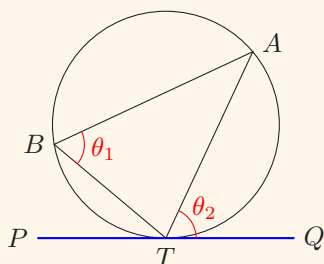
*Tan. prop.*

$$\triangle OPT \cong \triangle OQT.$$



*∠ in alt. segment*

$$\theta_1 = \theta_2.$$



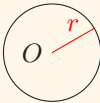
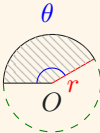
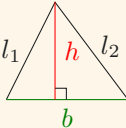
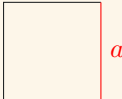
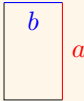
*Converse of  
∠ in alt. segment*

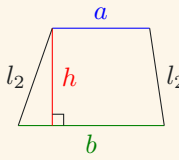
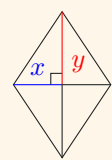
$$\theta_1 = \theta_2$$



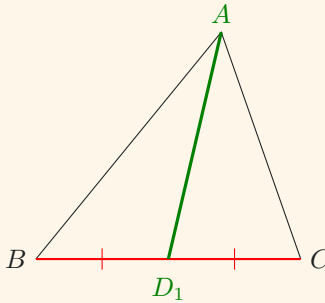
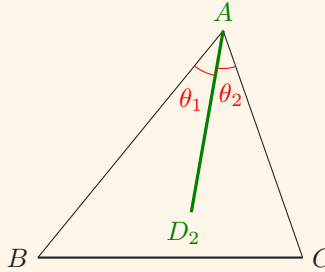
$PQ$  is tangent  
to the circle at  $T$ .

Appendix A: Area and Perimeters of Common Plane Figures

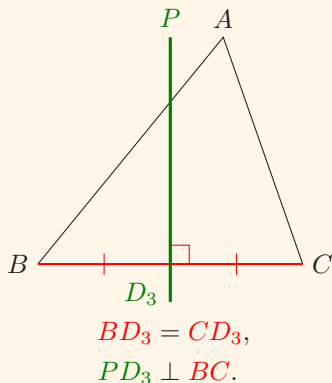
FIGURE	AREA	PERIMETER	
	$\pi r^2$	$2\pi r$	◀ CIRCLES
	$\frac{\pi \theta r^2}{360^\circ}$	$\frac{2\pi \theta r}{360^\circ} + 2r$	◀ SECTORS
	$\frac{hb}{2}$	$b + l_1 + l_2$	◀ TRIANGLES
	$a^2$	$4a$	◀ SQUARES
	$ab$	$2(a + b)$	◀ RECTANGLES

	$\frac{h(a+b)}{2}$	$a+b+l_1+l_2$	◀ TRAPEZIUMS
	$\frac{xy}{2}$	$4\sqrt{x^2+y^2}$	◀ RHOMBUSES

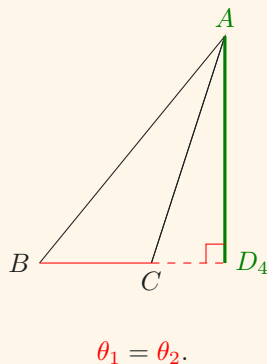
## Appendix B: Special Points and Lines in $\triangle$ s

<div><math>AD_1</math> is the <i>median</i> of <math>BC</math></div>  $BD_1 = CD_1.$	<div><math>AD_2</math> is an <i>angle bisector</i> of <math>\angle BAC</math></div>  $\theta_1 = \theta_2.$
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$PD_3$  is a *perpendicular bisector* of  $BC$

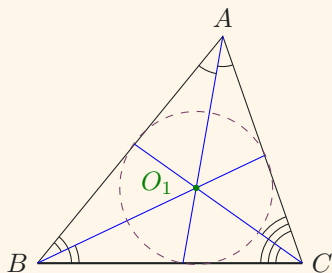


$AD_2$  is the *altitude* of  $BC$



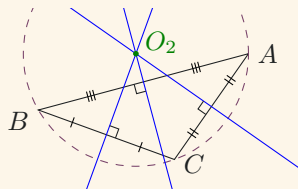
◀ May not necessarily lie in  $\triangle$ .

$O_1$  is the *in-centre* of  $\triangle ABC$



The *in-centre* is the point of intersection of the three **angle bisectors** (of a triangle). It is the center of the triangle's **inscribed circle**.

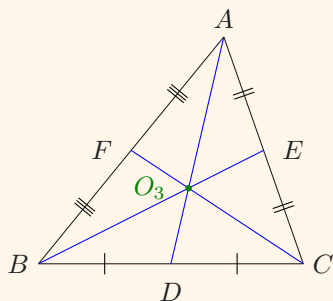
$O_2$  is the *circumcentre* of  $\angle BAC$



◀ May not necessarily lie in  $\triangle$ .

The *circumcentre* is the point of intersection of the three **perpendicular bisectors**. It is the center of the triangle's **circumcircle**.

$O_1$  is the *centroid* of  $\triangle ABC$

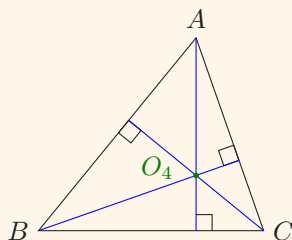


The *centroid* is the point of intersection of the three *medians* (of a triangle).

It divides each median in the ratio of 2 : 1, i.e.,

$$\frac{O_3A}{O_3D} = \frac{O_3B}{O_3E} = \frac{O_3C}{O_3F} = 2.$$

$O_4$  is the *orthocentre* of  $\triangle ABC$



The *orthocentre* is the point of intersection of the three *altitudes*.

◀ May not necessarily lie in  $\triangle$ .

These four points are generally distinct from one another.