Mathematics Society
Weekly Questions (Week 7 = 0)

February 2, 2024

Can you find the flaws in the following proofs?

Proof 1 Let $a = b$. Then

$$\begin{aligned} a &= b \\ a^2 &= ab \\ a^2 - b^2 &= ab - b^2 \\ (a-b)(a+b) &= (a-b)b \\ a+b &= b \\ a+b-b &= b-b \\ a &= 0 \end{aligned}$$

Since we didn't put any restrictions on what a could be, substitute in $a = 1$ to get $1 = 0$.

Proof 2 This time, no variables are needed.

$$
\begin{aligned}
-20 &= -20 \\
25 - 45 &= 16 - 36 \\
5^2 - 5 \times 9 &= 4^2 - 4 \times 9 \\
5^2 - 5 \times 9 + \frac{81}{4} &= 4^2 - 4 \times 9 + \frac{81}{4} \\
(5 - \frac{9}{2})^2 &= (4 - \frac{9}{2})^2 \\
5 - \frac{9}{2} &= 4 - \frac{9}{2} \\
5 &= 4 \\
5 - 4 &= 4 - 4 \\
1 &= 0
\end{aligned}
$$

Proof 3 *requires calculus*
Recall this integral from calculus:

$$\int \frac{1}{\sqrt{1-x^2}}\mathrm{dx} = \arcsin x$$

But this is also true:

$$-\int \frac{1}{\sqrt{1-x^2}}\mathrm{dx} = \int -\frac{1}{\sqrt{1-x^2}}\mathrm{dx} = \arccos x$$

So, from these we can conclude that $\arcsin x = -\arccos x$.
But $\arcsin 1 = \frac{\pi}{2}$ and $\arccos 1 = 0$. So $\frac{\pi}{2} = 0$, and dividing by $\frac{\pi}{2}$, we get $1 = 0$.

More false proofs to follow next week.