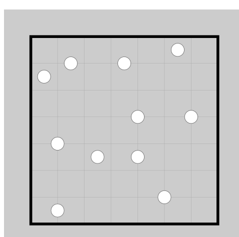
# Mathematics Society
# Weekly Questions (Week 4)

November 24, 2023

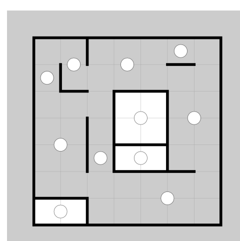
**Question 1 (Puzzle)** *Galaxies* [1]
You have a rectangular grid containing a number of dots. Your aim is to draw edges along the grid lines which divide the rectangle into regions in such a way that every region is 180° rotationally symmetric, and contains exactly one dot which is located at its centre of symmetry.

You could draw temporary arrows in the boxes to help you decide which boxes must belong to which white centers.

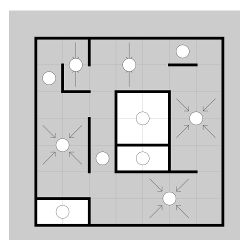
Examples:



A blank grid

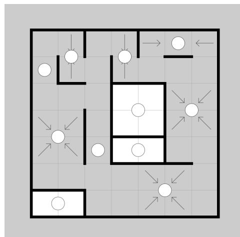


Find known edges by rotation about the white centers.

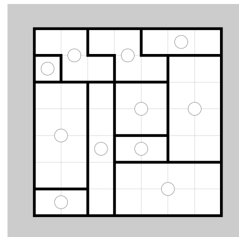


Need to use arrows

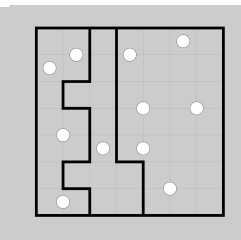
[1] Origin: https://www.nikoli.co.jp/ja/puzzles/astronomical_show/
Fun fact: this company coined the name "Sudoku", and popularized it!



More arrows figured out

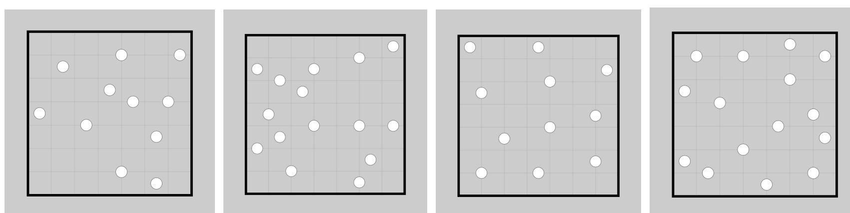


Solved!

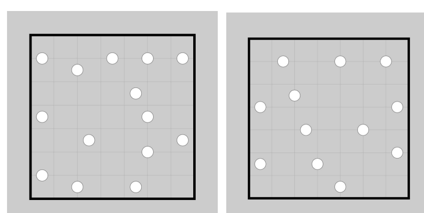


Not allowed(not rotationally symmetric)

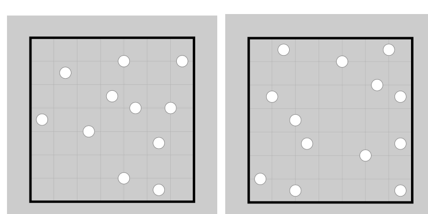
Problem Set:



Easy



Medium



Hard

**Question 2 (Brainteasers and Problem-solving Techniques)** *Cheryl's Birthday*

This logic puzzle went viral in 2015 in days, and was originally aimed at finding above-average students in an Olympiad for 14-year-old students.

Albert and Bernard just became friends with Cheryl, and they want to know when her birthday is. Cheryl gives them a list of 10 possible dates, which her birthday is definitely on. They are:

May 15, May 16, May 19
June 17, June 18
July 14, July 16
August 14, August 15, August 17

Cheryl then whispers the month of her birthday to Albert, which Bernard can't hear, and whispers the day of her birthday to Bernard, which Albert can't hear.

Albert: I don't know when Cheryl's birthday is, but I know for sure that Bernard doesn't know either.

Bernard: At first I did not know when Cheryl's birthday is, but after Albert said that, I've figured it out.

Albert: Since you were able to get it from what I said, I have deduced when it is as well.

So when is Cheryl's birthday?

**You are reminded to consider the information received from both Albert and Bernard's perspectives. You might want to make a grid of dates.**

**Question 3 (Olympiad)** *Last digits of an expression*

a Prove that $(a+b)(a^2-ab+b^2) = a^3+b^3$, by expanding $(a+b)(a^2-ab+b^2)$.

b By using part a, find the last seven digits of

$$\left\lfloor \frac{10^{2022}}{10^{674}+2022} \right\rfloor$$

**Question 4 (Bonus Question)** *Seemingly Impossible Puzzle*

It seems like this puzzle doesn't have enough information for there to be a unique solution. But, in fact, the things that the logicians say, and their certainty in saying them, are enough to completely determine the numbers they are talking about. It's like Cheryl's Birthday, but a bit harder, and with more calculation.

**The puzzle:**
X and Y are two whole numbers greater than 1, and $Y > X$. *(So, $X + Y$ must be greater than 5.)*
Their sum is not greater than 100.
Sue and Peter are two perfect logicians.
Sue knows the sum, $X + Y$ and Peter knows the product $X \times Y$. Both Sue and Peter know all the information in this paragraph.

In the following conversation, both participants are always telling the truth:

Sue says, "Peter does not know X and Y."
Peter says, "Now I know X and Y."
Sue says, "Now I also know X and Y."

What are X and Y?

**Hint 1:**
Sue is completely sure that Peter doesn't know X and Y for certain, because she knows the sum of the numbers. For example, if she got 7 as the sum of the numbers, then the possible pairs would be (2,5) and (3,4), because of the condition Y > X > 1. If the numbers were really 2 and 5, then their product would be 10, and because of the conditions, Peter would be able to determine that the numbers must be 2 and 5. So, the sum can't be 10, or otherwise Sue would not be certain that Peter doesn't know the numbers.