[10] B. A. Francis and T. T. Georgiou, "Stability theory for linear time-invariant plants with periodic digital controllers," *IEEE Trans. Automat. Contr.*, vol. AC-33, no. 9, pp. 820–832, Sept. 1988.
[11] R. Lozano, J.-M. Dion, and L. Dugard, "Singularity-free adaptive pole placement using periodic controllers," *IEEE Trans. Automat. Contr.*, vol. 38, no. 1, pp. 104–108, Jan. 1993.
[12] P. P. Khargonekar, K. Poolla, and A. Tannenbaum, "Robust control of linear time-invariant plants using periodic compensation," *IEEE Trans. Automat. Contr.*, vol. AC-30, no. 11, pp. 1088–1096, Nov. 1985.
[13] R. Ortega, "On periodic adaptive stabilization of non-minimum phase systems," in *Proc. 30th IEEE Conf. Decision and Contr.*, Brighton, England, Dec. 1991.
[14] D. S. Bayard and D. Boussalis, "Instrument printing, tracking, and vibration suppression using zero annihilation periodic control," *Optical Engineering*, vol. 33, no. 3, pp. 737–745, March 1994.

Disturbance Decoupled Observer Design: A Unified Viewpoint

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***Abstract*—This note is concerned with the observer design for linear systems with unknown inputs. An equivalent system, which is free of unknown inputs, is derived for the purpose of the observer design. Based on the equivalent system description, one can design an observer (full order, reduced order, minimal order or functional) for linear systems with unknown inputs using well-known techniques. It is shown that the equivalent system exists as long as there exists a disturbance decoupled observer. Two sets of the known existence conditions of disturbance decoupled observers are proved to be equivalent.**

I. Introduction

In modeling, different types of system uncertainties such as nonlinearities, parameter changes, and unknown external excitation can conveniently be represented as unknown inputs which are also termed disturbances.

We consider linear time-invariant systems of the form

$$\dot{x} = Ax + Ed \tag{1}$$

$$y = Cx + Fd \tag{2}$$

where $x \in \mathbf{R}^n$, $d \in \mathbf{R}^q$, $y \in \mathbf{R}^m$ are the state vector, the disturbance vector and the measurement vector, respectively. In the literature, d is usually termed unknown inputs. A, C, E, and F are known constant matrices of appropriate dimensions. Without loss of generality, we assume that C has full row rank. For brevity we have omitted known input terms in (1), (2). It is well known this does not affect the generality of the discussion on the observer design. Notice that the disturbances are also included in the measurement equation (2), which makes the system description more general than the one often adopted in the related research.

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The objective of the disturbance decoupled observer design is to design a Luenberger-type observer of the form

$$\dot{z} = Gz + Ly, \qquad z \in \mathbf{R}^p \tag{3}$$

$$\hat{x} = Hz + Jy \tag{4}$$

where $\hat{x}$ is the estimation of the state x in (1), $e = x - \hat{x} \to 0$ as $t \to \infty$, and e is independent of the disturbance d. If the state of the observer z has dimension n, the observer is called a full-order observer, otherwise a reduced order if $p_{\min} < p < n$ or a minimal order one when $p = p_{\min}$. We shall prove that the minimal order of the disturbance decoupled observer is given by $p_{\min} = n - m + \operatorname{rank} F$. An estimation of the disturbance d can also be obtained by solving d from (1), (2) and substituting $\hat{x}$ for x.

In the last years, considerable attention has been devoted to the above problem, see, for example, reduced and minimal-order observer design [1]–[9], and full-order observer design [10]–[13]. Various methods such as algebraic, geometric, inversion approaches, generalized inverse, singular value decomposition, and the Kronecker canonical form techniques have been used in the observer design. The disturbance decoupled observer technique has shown its successful applications in fault detection and isolation observer design [15]–[18] and in decentralized observer design for interconnected large-scale systems [19]–[22]. It is worth pointing out that, from somewhat different viewpoint, the disturbance decoupled observer problem was rigorously dealt with in [23] using frequency domain method. In that work, the resulting observer within frequency domain description was called a strong-state observer and the corresponding necessary and sufficient existence conditions, known as the strong* detectability conditions, were provided in a nice form. It is also worth noting that from the point of view of realization, a minimal-order observer is favourable. As shown in [13], however, in some special cases a full-order observer may have a better rate of convergence of partial state estimates than a minimal order observer. This is a reason why the full-order observer design should also be considered.

Recent investigation, see [18], [24] for the reduced order and minimal order observer cases and [25] for the full-order observer case, indicates evidently in some way that the design of disturbance decoupled observers is equivalent to that of usual observers. As a matter of fact, this point has been implied more or less in previous works. It hints to us that there should exist an equivalent disturbance free system behind different kinds of disturbance decoupled observer design methods. If this equivalent system can be found, the design of disturbance decoupled observers of full order, reduced order as well as minimal order will be able to be treated in a unifying framework. The objective of this note is to find out the equivalent system and show the simplicity of designing disturbance decoupled observers using the equivalent system.

In Section II, an equivalent disturbance decoupled system is derived, and the design of disturbance decoupled observers is provided. The necessary and sufficient existence conditions of the disturbance decoupled observer are discussed in Section III, and the conclusion follows in Section IV.

II. Equivalent System and Observer Design

This section derives firstly, through disturbance eliminating, a disturbance decoupled system which is equivalent to the original system (1), (2) in the sense of observer design. Then an observer design is provided based on the disturbance decoupled system description. In the next section, the conditions required in the derivation of the equivalent system and in the design of the observer will be proved to be necessary and sufficient for the existence of disturbance decoupled observers.

Consider the measurement equation at first. Since y is a measurement, disturbance d always satisfies the measurement equation (2). According to the well-known matrix equation solution theory [26], the general solution of d to (2) is

$$d = F^-(y - Cx) + (I_q - F^-F)\overline{d} \tag{5}$$

where F^- is an arbitary generalized inverse of F satisfying $FF^-F = F$, $\overline{d}$ is a certain vector of dimension q; $\overline{d}$ can be considered as a new disturbance.

Substituting (5) into (1) yields

$$\dot{x} = (A - EF^-C)x + EF^-y + E(I_q - F^-F)\overline{d}. \tag{6}$$

Premultiplying (2) by $(I_m - FF^-)$ leads to

$$\overline{y} = \overline{C}x \tag{7}$$

with $\overline{y} = (I_m - FF^-)y$ and $\overline{C} = (I_m - FF^-)C$. Equation (7) describes a disturbance decoupled new measurement.

To express $\overline{d}$ in terms of x, y, and $\dot{y}$ with the hope that $\overline{d}$ will be eliminated, pre multiplying (6) by $\overline{C}$ results in

$$\dot{\overline{y}} = \overline{C}(A - EF^-C)x + \overline{C}EF^-y + \overline{C}E(I_q - F^-F)\overline{d}. \tag{8}$$

In view of (5) and for the similar reason, the solution $\overline{d}$ to (8) exists, and the general solution is given by

$$\begin{aligned}\overline{d} = {} & [\overline{C}E(I_q - F^-F)]^-[\dot{\overline{y}} - \overline{C}(A - EF^-C)x - \overline{C}EF^-y] \\ & + \{I_q - [\overline{C}E(I_q - F^-F)]^-[\overline{C}E(I_q - F^-F)]\}\tilde{d}\end{aligned} \tag{9}$$

where $\tilde{d}$, again a new disturbance, is a certain vector of dimension q.

Substituting (9) into (6), one gets

$$\begin{aligned}\dot{x} = {} & \overline{A}x + \overline{B}(I_m - FF^-)\dot{y} \\ & + (I_n - \overline{BC})EF^-y + (I_n - \overline{BC})E(I_q - F^-F)\tilde{d}\end{aligned} \tag{10}$$

where

$$\overline{A} = (I_n - \overline{BC})(A - EF^-C), \qquad \overline{C} = (I_m - FF^-)C \tag{11}$$

$$\overline{B} = E(I_q - F^-F)[\overline{C}E(I_q - F^-F)]^-. \tag{12}$$

Obviously, if $(I_n - \overline{BC})E(I_q - F^-F) = 0$ which, as will be shown in the next section, is necessary for the existence of disturbance decoupled observers, (10) appears to be a disturbance decoupled dynamical equation. Combining it with the disturbance decoupled measurement equation (7) results in the following disturbance decoupled system

$$\dot{x} = \overline{A}x + \overline{B}(I_m - FF^-)\dot{y} + (I_n - \overline{BC})EF^-y \tag{13}$$

$$\overline{y} = \overline{C}x. \tag{14}$$

We can see that the main idea of the derivation of the equivalent system (13), (14) is to find an expression of the disturbance d in terms of x, y, and $\dot{y}$ (but not $\dot{x}$) to eliminate d in the original disturbance coupled dynamical equation (1). The new measurement equation (14) is obtained by extracting disturbance decoupled measurements from the original measurement equation (2).

Remark 2.1: Since $y = Cx + Fd$, $\dot{y}$ may not exist due to unknown behaviour of the disturbance d. This yields no effect on (13), however, because $(I_m - FF^-)\dot{y} = \overline{C}\dot{x}$ always exists. It is also worth noting that, as will be seen below, $(I_m - FF^-)\dot{y}$ is not explicitly contained in the disturbance decoupled observer.

Now, under the assumption $(I_n - \overline{BC})E(I_q - F^-F) = 0$, if, furthermore, the pair $\{\overline{A}, \overline{C}\}$ is detectable, one can indeed design a state observer for the original system using the equivalent system (13), (14). It will be seen in the next section that the detectability of the pair $\{\overline{A}, \overline{C}\}$ is necessary and sufficient for the existence of disturbance decoupled observers under the necessary condition $(I_n - \overline{BC})E(I_q - F^-F) = 0$. We conclude the following result.

Theorem 2.1: Provided that $(I_n - \overline{BC})E(I_q - F^-F) = 0$ and the pair $\{\overline{A}, \overline{C}\}$ is detectable, the system described by (13), (14) is equivalent to the system (1), (2) in the sense of observer design. A disturbance decoupled observer of full order is given by

$$\begin{aligned}\dot{z} = {} & (\overline{A} - L\overline{C})z + [(I_n - \overline{BC})EF^- + L(I_m - FF^-) \\ & + (\overline{A} - L\overline{C})\overline{B}(I_m - FF^-)]y\end{aligned} \tag{15}$$

$$\hat{x} = z + \overline{B}(I_m - FF^-)y \tag{16}$$

where the matrix $\overline{A} - L\overline{C}$ has eigenvalues with negative real parts.

Proof: Treating y and $\dot{y}$ in (13) formally as inputs and noting the detectability of the pair $\{\overline{A}, \overline{C}\}$, one can readily design a full-order state observer for the system (13), (14) as

$$\dot{\hat{x}} = (\overline{A} - L\overline{C})\hat{x} + \overline{B}(I_m - FF^-)\dot{y} + (I_n - \overline{BC})EF^-y + L\overline{y} \tag{17}$$

where the gain matrix L is designed such that $\overline{A} - L\overline{C}$ has eigenvalues with negative real parts, which is always possible if the pair $\{\overline{A}, \overline{C}\}$ is detectable.

Let $\hat{x}$ be defined by (16). Substitution of (16) into (17) leads to (15). □

It is interesting to verify that $e = x - \hat{x} \to 0$ and e is independent of the disturbance d. In fact, one has

$$\begin{aligned}\dot{x} - \dot{\hat{x}} &= Ax + Ed - \dot{z} - \overline{BC}\dot{x} \\ &= Ax + Ed \\ &\quad - \{(\overline{A} - L\overline{C})z + [(I_n - \overline{BC})EF^- + L(I_m - FF^-) \\ &\quad + (\overline{A} - L\overline{C})\overline{B}(I_m - FF^-)]y\} \\ &\quad - \{\overline{BC}Ax + \overline{BC}Ed\} \\ &= \{A - (I_n - \overline{BC})EF^-C - L\overline{C} \\ &\quad - (\overline{A} - L\overline{C})\overline{BC} - \overline{BC}A\}x \\ &\quad - (\overline{A} - L\overline{C})z + \{E - \overline{BC}E - (I_n - \overline{BC})EF^-F\}d \\ &= \{A - (I_n - \overline{BC})EF^-C - L\overline{C} - \overline{BC}A\}x - (\overline{A} - L\overline{C})\hat{x} \\ &\quad + \{E - \overline{BC}E - (I_n - \overline{BC})EF^-F\}d \\ &= (\overline{A} - L\overline{C})(x - \hat{x}) + (I_n - \overline{BC})E(I_q - F^-F)d \\ &= (\overline{A} - L\overline{C})(x - \hat{x}).\end{aligned}$$

Remark 2.2: If in the system (1), (2) the measurements are free of disturbances, i.e., $F = 0$, then the equivalent system (13), (14) and the corresponding full-order observer (15), (16) will have the simple forms which can readily be obtained by setting F and F^- to zeros in (13), (14) and (15), (16).

Remark 2.3: By treating y and $\dot{y}$ in (13) formally as inputs, following the similar way as shown in the above proof, one can also design a reduced or minimal-order disturbance decoupled observer. Since the estimation error of any reduced or minimal-order observer is a certain linear combination of the estimation error of the full-order observer, i.e., $e_{\text{red}} = Te$ for some matrix T, $e_{\text{red}} \to 0$ and its independence of the disturbance d are guaranteed by the corresponding properties of e.

For the design of the reduced or minimal order observer, it is convenient to use the following equivalent form of (13), (14)

$$\dot{\xi} = \tilde{A}\xi + \tilde{B}y \tag{18}$$

$$\tilde{y} = \tilde{C}\xi \tag{19}$$

with the relation $x = \xi + \tilde{B}(I_m - FF^-)y$ and $\tilde{y} = P(I_m - \overline{CB})(I_m - FF^-)y$, $\tilde{A} = \overline{A}$, $\tilde{B} = \overline{AB}(I_m - FF^-) + (I_n - \overline{BC})EF^-$, $\tilde{C} = P\overline{C}$. Here, F^- is a generalized inverse of F satisfying $FF^-F = F$ and $F^-FF^- = F^-$ (thus, $\operatorname{rank} F = \operatorname{rank} F^-$, [27]), $P \in \boldsymbol{R}^{\operatorname{rank}\overline{C} \times m}$ is an arbitrary matrix of full row rank satisfying $\operatorname{rank}\begin{bmatrix} F^- \\ P \end{bmatrix} = m$. It is ready to use the well-established procedures [28], [29] to design the minimal-order disturbance decoupled observer based on the equivalent system description (18), (19) by treating y in (18) formally as inputs.

Remark 2.4: In view of the relation $\operatorname{rank}\tilde{C} = \operatorname{rank}\overline{C} = \operatorname{rank} C - \operatorname{rank} F$, the minimal order of the disturbance decoupled observer is given by

$$p_{\min} = n - m + \operatorname{rank} F. \tag{20}$$

III. Existence Condition

This section examines the existence condition of disturbance decoupled observers. Two sets of the known necessary and sufficient existence conditions are outlined. One will see that one of them is just the same as those required in Theorem 2.1. These two sets of conditions are in quite different forms and obtained independently in [11] and [23], respectively. These two results are very important, but sufficient attention has not been riveted on them, especially on the latter. We shall provide a straightforward proof of the equivalence between them.

Theorem 3.1 [11]: There exists a disturbance decoupled observer if and only if

$$RE(I_q - F^-F) = 0 \quad \text{and} \tag{21}$$

$$\text{the pair } \{R(A - EF^-C),\ (I_m - FF^-)C\} \quad \text{is detectable} \tag{22}$$

where R is defined as

$$R = I_n - E(I_q - F^-F)\big[(I_m - FF^-) \cdot CE(I_q - F^-F)\big]^-(I_m - FF^-)C. \tag{23}$$

Using the notations (11), (12), one has $R = I_n - \overline{BC}$. Thus, it can be seen that the conditions (21), (22) are exactly the same as those required in Theorem 2.1.

Theorem 3.2 [23]: There exists a disturbance decoupled observer if and only if

$$\operatorname{rank}\begin{bmatrix} F & CE \\ 0 & F \end{bmatrix} = \operatorname{rank} F + \operatorname{rank}\begin{bmatrix} E \\ F \end{bmatrix}; \tag{24}$$

$$\operatorname{rank}\begin{bmatrix} -sI_n + A & E \\ C & F \end{bmatrix} = n + \operatorname{rank}\begin{bmatrix} E \\ F \end{bmatrix}, \qquad \forall s \in C, \quad Re(s) \geq 0. \tag{25}$$

Theorem 3.3: The two sets of existence conditions stated in Theorem 3.1 and Theorem 3.2 are equivalent.

The proof of this theorem needs the following simple lemmas which are obvious according to matrix algebraic theory.

Lemma 3.1: For arbitrary matrices M and N of appropriate dimensions

$$N(I - M^-M) = 0 \qquad \text{iff } \operatorname{rank}\begin{bmatrix} N \\ M \end{bmatrix} = \operatorname{rank} M. \tag{26}$$

Lemma 3.2: For arbitrary matrices M, U, and V of appropriate dimensions, where U is of full column rank and V is of full row rank, then

$$\operatorname{rank} M = \operatorname{rank} UMV. \tag{27}$$

Proof of Theorem 3.3: We prove firstly the equivalence between conditions (21) and (24). It was shown in [11] the condition

$$\begin{bmatrix} 0 & E \end{bmatrix}\begin{bmatrix} 0 & F \\ F & CE \end{bmatrix}^- \begin{bmatrix} 0 & F \\ F & CE \end{bmatrix} = \begin{bmatrix} 0 & E \end{bmatrix} \tag{28}$$

is an equivalent expression of (21). On the other hand, an equivalent condition of (24) is

$$\operatorname{rank}\begin{bmatrix} 0 & E \\ 0 & F \\ F & CE \end{bmatrix} = \operatorname{rank}\begin{bmatrix} 0 & F \\ F & CE \end{bmatrix}. \tag{29}$$

Applying Lemma 3.1 to (28) and (29) confirms the equivalence between (28) and (29) and therefore between (21) and (24).

Now we prove the equivalence between (22) and (25) under the condition (21) or equivalently under (24).

Since

$$\begin{bmatrix} I_n - \overline{BC} & -s\overline{B}(I_m - FF^-) - (I_n - \overline{BC})EF^- \\ 0 & I_m - FF^- \\ \overline{BC} & s\overline{B}(I_m - FF^-) \\ 0 & FF^- \end{bmatrix} \cdot \begin{bmatrix} -sI_n + A & E \\ C & F \end{bmatrix} = \begin{bmatrix} -sI_n + \overline{A} & (I_n - \overline{BC})E(I_q - F^-F) \\ \overline{C} & 0 \\ \overline{BC}A & \overline{BC}E \\ FF^-C & F \end{bmatrix} \tag{30}$$

and $(I_n - \overline{BC})E(I_q - F^-F) = 0$, on account of Lemma 3.2, we know

$$\operatorname{rank}\begin{bmatrix} -sI_n + A & E \\ C & F \end{bmatrix} = \operatorname{rank}\begin{bmatrix} -sI_n + \overline{A} & 0 \\ \overline{C} & 0 \\ \overline{BC}A & \overline{BC}E \\ FF^-C & F \end{bmatrix}. \tag{31}$$

Evidently, if

$$\operatorname{rank}\begin{bmatrix} \overline{BC}A & \overline{BC}E \\ FF^-C & F \end{bmatrix} = \operatorname{rank}\begin{bmatrix} \overline{BC}E \\ F \end{bmatrix} = \operatorname{rank}\begin{bmatrix} E \\ F \end{bmatrix} \tag{32}$$

one can conclude that (22) holds if and only if (25) holds.

The remaining part of the proof is to show that (32) does hold. Since $(I_n - \overline{BC})E(I_q - F^-F) = 0$, one has

$$\begin{bmatrix} \overline{BC}A & \overline{BC}E \\ FF^-C & F \end{bmatrix}\begin{bmatrix} I_n & 0 & 0 \\ 0 & I_q - F^-F & F^-F \end{bmatrix} = \begin{bmatrix} \overline{BC}A & E(I_q - F^-F) & \overline{BC}EF^-F \\ FF^-C & 0 & F \end{bmatrix} \tag{33}$$

and thus, according to Lemma 3.2 and in view of (12),

$$\begin{aligned} \operatorname{rank}\begin{bmatrix} \overline{BC}A & \overline{BC}E \\ FF^-C & F \end{bmatrix} &= \operatorname{rank}\begin{bmatrix} \overline{BC}A & E(I_q - F^-F) & \overline{BC}EF^-F \\ FF^-C & 0 & F \end{bmatrix} \\ &= \operatorname{rank}\begin{bmatrix} 0 & E(I_q - F^-F) & 0 \\ FF^-C & 0 & F \end{bmatrix} \\ &= \operatorname{rank}\begin{bmatrix} E(I_q - F^-F) & 0 \\ 0 & F \end{bmatrix} \\ &= \operatorname{rank}\begin{bmatrix} \overline{BC}E \\ F \end{bmatrix}. \end{aligned} \tag{34}$$

Furthermore, because

$$\begin{bmatrix} I_n & -EF^- \\ 0 & I_m \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix} [I_q - F^-F \quad F^-F] = \begin{bmatrix} E(I_q - F^-F) & 0 \\ 0 & F \end{bmatrix} \tag{35}$$

and using again Lemma 3.2, one has

$$\operatorname{rank} \begin{bmatrix} \overline{BC}E \\ F \end{bmatrix} = \operatorname{rank} \begin{bmatrix} E(I_q - F^-F) & 0 \\ 0 & F \end{bmatrix} = \operatorname{rank} \begin{bmatrix} E \\ F \end{bmatrix}. \tag{36}$$

This completes the proof. □

Remark 3.1: The equivalence of the existence conditions in Theorem 3.1 and Theorem 3.2 was unknown before [18]. In that work, the equivalence was proved through the minimal order disturbance decoupled observer design, and therefore, the proof was very involved. The present proof is straightforward and neat.

IV. Conclusion

The design of disturbance decoupled observers for linear time-invariant systems with unknown inputs can easily be dealt with using an equivalent system description. No unknown inputs appear in the equivalent system description. The equivalent system description exists if and only if there exists a disturbance decoupled observer. In the derivation no coordinate transformations are needed. Owing to the existence of the equivalent system, the design of disturbance decoupled observers of full order, reduced order and minimal order can be treated in a unifying framework by using well-known techniques.

References

[1] S.-H. Wang, E. J. Davison and P. Dorato, "Observing the states of systems with unmeasurable disturbance," *IEEE Trans. Automat. Contr.*, vol. AC-20, pp. 716–717, 1975.

[2] S. P. Bhattacharyya, "Observer design for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.*, vol. AC-23, pp. 483–484, 1978.

[3] P. Kudva, N. Viswanadham, and A. Ramakrishna, "Observers for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 113–115, 1980.

[4] N. Kobayashi and T. Nakamizo, "An observer design for linear systems with unknown inputs," *Int. J. Contr.*, vol. 35, pp. 605–619, 1982.

[5] R. J. Miller and R. Mukundan, "On designing reduced-order observers for linear time-invariant systems subject to unknown inputs," *Int. J. Contr.*, vol. 35, pp. 183–188, 1982.

[6] F. W. Fairman, S. S. Mahil, and L. Luk, "Disturbance decoupled observer design via singular value decomposition," *IEEE Trans. Automat. Contr.*, vol. AC-29, pp. 84–86, 1984.

[7] S. Engell and D. Konik, "Zustandsermittlung bei unbekanntem Eingangssignal," *Automatisierungstechnik at*, vol. 34, pp. 38–42, pp. 247–251, 1986.

[8] Y. Park and J. L. Stein, "Closed-loop, state and inputs observer for systems with unknown inputs," *Int. J. Contr.*, vol. 48, pp. 1121–1136, 1988.

[9] Y.-P. Guan and M. Saif, "A novel approach to the design of unknown input observers," *IEEE Trans. Automat. Contr.*, vol. 36, pp. 632–635, 1991.

[10] J. E. Kurek, "The observation of the state vector of linear systems with unknown inputs," *Int. J. Contr.*, vol. 36, pp. 511–515, 1982.

[11] ——, "The state vector reconstruction for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.*, vol. AC-28, pp. 1120–1122, 1983.

[12] P. Ficklscherer and P. C. Müller, "Robuste Zustandsbeobachter für lineare Mehrgrössenregelungssysteme mit unbekannten Störeingängen," *Automatisierungs-technik at*, vol. 33, pp. 173–179, 1985.

[13] F. Yang and R. W. Wilde, "Observers for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.*, vol. AC-33, pp. 677–681, 1988.

[14] N. Viswanadham and R. Srichander, "Fault detection using unknown-input observers," *Contr. Theory and Advanced Techn.*, vol. 3, pp. 91–101, 1987.

[15] M. S. Phatak and N. Viswanadham, "Actuator fault detection and isolation in linear systems," *Int. J. Syst. Sci.*, vol. 19, pp. 2593–2603, 1989.

[16] J. Wünnenberg, "Observer-based fault detection in dynamic systems," Ph.D. dissertation, University of Duisburg, Duisburg, Germany, 1990.

[17] P. M. Frank, "Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy—A survey and some new results," *Automatica*, vol. 26, pp. 459–474, 1990.

[18] H. Hou and P. C. Müller, "Fault detection and isolation observers," *Int. J. Contr.*, to be published.

[19] N. Viswanadham and A. Ramakrishna, "Decentralized estimation and control for interconnected systems," *Large Scale Systems*, vol. 3, pp. 255–266, 1982.

[20] P. Ficklscherer and P. C. Müller, "Decentralized state observers for large-scale systems," in *Proc. 9th IFAC World Cong.*, 1984, pp. 1153–1158.

[21] M. Saif and Y.-P. Guan, "Decentralized state estimation in large-scale interconnected dynamical systems," *Automatica*, vol. 28, pp. 215–219, 1992.

[22] M. Hou and P. C. Müller, "On the design of decentralized observers," in *Automatica*, to be published.

[23] M. L. J. Hautus, "Strong detectability and observers," *Linear Algebra Appl.*, vol. 50, pp. 353–368, 1983.

[24] M. Hou and P. C. Müller, "Design of observers for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 871–875, 1992.

[25] M. Darouach, M. Zasadzinski and S. J. Xu, "Full-order observers for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 606–609, 1994.

[26] C. R. Rao and S. K. Mitra, *Generalized Inverse of Matrices and its Applications.* New York: Wiley, 1971.

[27] P. M. Pringle and A. A. Payner, *Generalized Inverse Matrices with Applications to Statistics.* London: Griffin, 1971.

[28] C. T. Chen, *Linear System Theory and Design.* New York: Holt Rinehart, Winson, 1984.

[29] J. O'Reilly, *Observers for Linear Systems.* London: Academic, 1983.

Stability Analysis of Linear Systems with Time Delay

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Abstract— **This paper presents new asymptotic stability criteria for linear systems with time delay. The results not only improve previous results, but also provide a bound for the delay time such that if the system is asymptotically stable when the delay does not exist, it retains the asymptotic stability when the delay time is within the bound.**

I. Introduction

Recently, stability analysis of linear systems with time delay in the system dynamics has received considerable attention [1]–[7], [11]–[13]. This is because time delay is encountered in many control problems, especially in process control systems. Current results in this topic can be divided into two categories. One category is to provide delay-independent stability criteria [16], and the other is to provide delay-dependent stability criteria [1], [7], [8], [13]. Although delay-independent stability criteria are easy to check, it is usually not reasonable to ask a time delay system to be asymptotically stable

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