1

Prop: ~~$L_1(A \dot\cup B) = L_1(A) + L_1(B)$~~

$$\mathbb{E}[\chi(\mathcal{A}_u(A \dot\cup B))] = \mathbb{E}[\chi(\mathcal{A}_u(A))] + \mathbb{E}[\chi(\mathcal{A}_u(B))] - \mathbb{E}[\chi(\mathcal{A}_u(A \cap B))]$$

$$\boxed{\mathcal{A}_u(T) = \{t \in T : f(t) > u\}}$$

Proof:

$$\mathbb{E}[\chi(\mathcal{A}_u(A \cup B))]$$

$$= \mathbb{E}[\chi(\mathcal{A}_u(A)) + \chi(\mathcal{A}_u(B)) - \chi(\mathcal{A}_u(A \cap B))]$$

as $\mathcal{A}_u(A \dot\cup B) = \mathcal{A}_u(A) \dot\cup \mathcal{A}_u(B)$

$$= \mathbb{E}[\chi(\mathcal{A}_u(A))] + \mathbb{E}[\chi(\mathcal{A}_u(B))] - \mathbb{E}[\chi(\mathcal{A}_u(A \cap B))]$$

Claim: $L_1(A \dot\cup B) = L_1(A) + L_1(B)$

Proof:

$$\mathbb{E}[\chi(\mathcal{A}_u(A \dot\cup B))] = \sum_{j=0}^{3} \mathcal{L}_j(A \dot\cup B)\rho_j(u)$$

$$= \sum_{j \neq 1} (\mathcal{L}_j(A) + \mathcal{L}_j(B))\rho_j(u) + \mathcal{L}_1(A \cup B)\rho_1(u)$$