$$\mu|_A(f_n\, \mathbb{1}[A]) = \cancel{\sum h_i\, \mathbb{1}[A\cap B_i]}$$

$$\sum h_i \frac{\mu(A\cap B_i)}{\mu(A)} \nearrow \cancel{\mathbb{E}[X|A]}$$

$$\boxed{(A\cap B)^c}$$

$$= \boxed{A^c \cap B^c}$$

$$= \frac{1}{\mu(A)}\left(\sum h_i \frac{\cancel{\mu(A\cap B_i)}}{\mu(A\cap B_i)}\right)$$

$$\| $$

$$\mathbb{E}[X\mathbb{1}[A]]$$

$$|f_n| \le |X|$$

$$\boxed{= \frac{\mathbb{E}[X\mathbb{1}[A]]}{\mu(A)=\mathbb{P}(A)}}$$

$\#$ $\mu(X) = \mu(X^+) - \mu(X^-)$

~~Then~~ Define $\mathbb{E}|_A[\mathbb{E}[X \mid A, \mathcal{G}] =: \mathbb{E}|_A[(X|A) \mid \mathcal{G}\cap A]$

Claim: This equals $\dfrac{\mathbb{E}[X\mathbb{1}[A] \mid \mathcal{G}]}{\mathbb{P}(A)}$

Proof: Given $B \in \mathcal{G}\cap A$

we have: $\mathbb{E}|_A[(X|A)\mathbb{1}[B]] = \mathbb{E}|_A[X\mathbb{1}[B]|A]$ as $B \subseteq A$!

$$= \frac{\mathbb{E}[X\mathbb{1}[B]\mathbb{1}[A]]}{\mathbb{P}(A)}$$

So $(X|A)\mathbb{1}(B) = X\mathbb{1}[B]|A$ see def^n of $|A$