

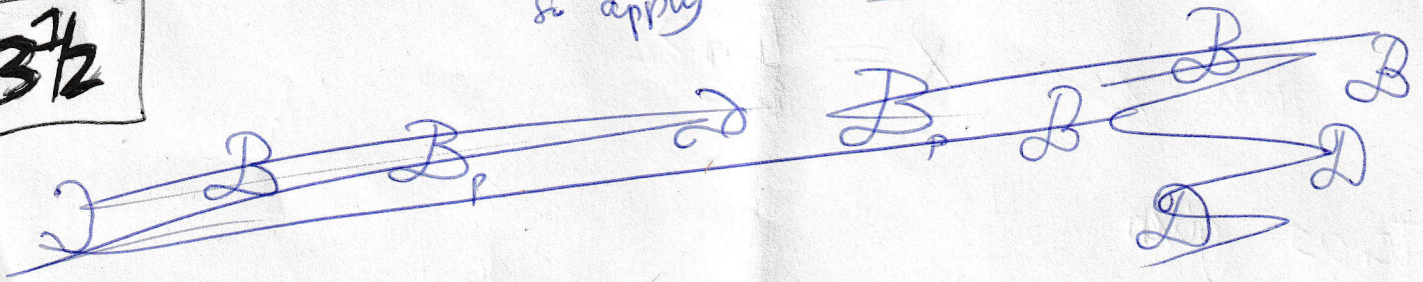
Proof of Prop 2.44 / Ex 2.45

①

$B_1, B_2$  open  $\Rightarrow B_1 \cap B_2$  open.

so apply Ex 2.40

3 1/2



Lemma 2.47 I guess a finite neighborhood basis can exist if for instance the space is finite.

~~$X$  first countable,  $A \subseteq X$ ,  $x \in X$~~

Lemma 2.48. (a)  $x \in \text{cl} A$   
 $\Rightarrow$  Given open  $U$ ,  $\exists N$  s.t.  $x_i \in U \quad \forall i \geq N$ .

So  $(\Leftarrow)$  Suppose  $x \in \bar{A}^c \rightarrow$  open then. get a neighborhood  
 (as  $\bar{A}^c$  contains no points of  $A$ )  
 (can't need first countable for this) ~~but~~

$(\Rightarrow)$

Given open  $U \ni x$ ,

~~$x \in \bar{A}$  then~~ all? ask Tom for thoughts on this.

(b)  ~~$x \in \text{Int}(A)$~~   ~~$x \in \bar{A}$~~ ,  ~~$x \in U$~~  open

then  $U \cap A \neq \emptyset$  as if it were then.

$\bar{A} \cap U^c$  would be a smaller (than  $\bar{A}$ ) closed set  
 containing  $A$  which is a contradiction. (of def<sup>n</sup> of  $\bar{A}$ )