$\hat{\beta}$

$var\left((X^T X)^{-1} X^T Y \right)$

Y_i

X_i

$(\;) = x_i$

$c^T \hat{\beta} = c^T \underbrace{(X^T X)^{-1}}_{p \times p} \overbrace{\sum X_i Y_i}^{p \times 1}$

$$\Rightarrow \boxed{var(\hat{\beta} | X) = c^T (X^T X)^{-1} \underbrace{var\left(\sum X_i Y_i | X \right)}_{} (X^T X)^{-1} c}$$

$= (X^T X)^{-1} \sigma^2$

$= \boxed{c^T (X^T X)^{-1} \sigma^2}$

$\sum X$

$var\left(\sum_{i=1}^{n} X_i Y_i \right)$

$\sum_i x_i^2 \, Var(Y_i)$

$\hat{\beta}_n \left(\sum X_i^T Y_i \right) \hat{\beta}_n$

$E\left[\sum_i X_i X_i^T Y_i^2 | X \right]$

$E \sum$

$= (X^T X)$

$\sigma^2 c^T (X^T X)^{-1} \sum X_i X_i^T$

$\hat{\beta}_n = (X^T X)^{-1} \sum X_i Y_i$

$= (X^T X)^{-1} \sum_{k=1}^{K} \boxed{\sum_{i \in} X_i Y_i}$

$E\left[\left(\sum_i X_i Y_i \right) \left(\sum_i X_i^T Y_i \right) \right]$

$E \sum X_i X_i^T \, E Y_i^2$

as cross term ~~come~~ $= 0$.