

Now  $Y$  is a convolution field (by assumption), (3)

So ~~convolution field~~

$$\begin{aligned}
 Y(s) &= \sum_{l \in L} K(s-l) X(l) \\
 &= \sum_l \overbrace{K(s_0-l) X(l)}^{= Y(s_0)} + \underbrace{(s-s_0)^T \sum_l \nabla K^*(s_0-l) X(l)}_{= \nabla Y(s_0)} \\
 &\quad + \sum_l \frac{1}{2} (s-s_0)^T \sum_l \overbrace{\nabla^2 K(s_0-l) X(l)}^{\nabla^2 Y(s_0)} (\cancel{s-s_0}) \\
 &\quad + \cancel{\sum_l \nabla^3 K(s_0-l) X(l)} \\
 &\quad + \sum_l \sum_{ijk} (\cancel{s-s_0})_i (\cancel{s-s_0})_j (\cancel{s-s_0})_k \times \nabla^3 K(\cancel{s-s_0}-l)_{ijk} X(l).
 \end{aligned}$$

check terms of Taylor's error!

$s^* \in B_{s_0}(\|s\|)?$

$$\begin{aligned}
 &= Y(s_0) + (s-s_0)^T \nabla Y(s_0) + \frac{1}{2} (s-s_0)^T \nabla^2 Y(s_0) (s-s_0) \\
 &\quad + \sum_l \sum_{ijk} (s^*-s_0)_i (s^*-s_0)_j (s^*-s_0)_k \nabla^3 K(s^*-l)_{ijk} X(l)
 \end{aligned}$$

As such, (taking  $s = t/\sqrt{r}$ ,

lap  $\downarrow \downarrow$