$$= \sum_k (s_k - t_k)(\nabla g)_{ki}$$

$$= \sum_k (s_k - t_k) \frac{\partial g_k}{\partial t_i} g_j$$

Now

So the sum is $\sum_k (s_k - t_k)\left(g_i \frac{\partial g_k}{\partial t_j} + g_j \frac{\partial g_k}{\partial t_i} \right)$

and $F[g_i g_j] = a \cdot \delta_{ij}$

$\Rightarrow F\left[g_i \frac{\partial g_j}{\partial t_k} + \frac{\partial g_i}{\partial t_k} g_j \right] = 0.$