have

$$\mathbb{E}\underbrace{\sup_t f(t)}_{>0} = \mathbb{E}\Big[\overbrace{\sup f \; \mathbb{I}[\sup f > 0]}^{a}\Big] + \mathbb{E}\Big[\underbrace{\sup f \; \mathbb{I}[\sup f < 0]}_{b}\Big]$$

$\Rightarrow$ ~~$\mathbb{E}$~~ $\Rightarrow |b| \leq |a|$ as

$\Rightarrow \mathbb{E}\sup f(t) \geq$

$|x+y| \leq |x| + |y|$

$\Rightarrow |x| \geq$ ~~$|y|$~~ $|x+y| - |y|$

Replace u with $Y(0)$, $C_{u,v} \mapsto C_v$

and define $C_v =$ connected component containing origin

have that

$$\mathbb{P}(\quad C_v \quad\quad \| \quad) \to 1$$