$$\mathbb{E}\left[\hat{\varepsilon}_i(u)\,\hat{\varepsilon}_i(v)\right]$$

4 LM permutation I think

$$= \mathbb{E}\Big[\underbrace{((I-P)\varepsilon(u))_i\,((I-P)\varepsilon(v))_i}\Big]$$

$$(I-P)\varepsilon = \varepsilon - P\varepsilon$$

$$\mathbb{E}\left[\varepsilon_i^4(u)\,\varepsilon_i^4(v)\right] < \infty$$

$$= \cancel{\mathbb{E}\left[(\varepsilon(u) - P\varepsilon(u))\right]}$$

$$\mathbb{E}\left[\left(\varepsilon_i(u) - (P\varepsilon(u))_i\right)\left(\varepsilon_i(v) - (P\varepsilon(v))_i\right)\right]$$

$$\mathbb{E}\left[\|P\varepsilon\|^2\right] = \sigma^2 p$$

$$\mathbb{E}\left[\|(P\varepsilon)_i\|^2\right] = \frac{\sigma^2 p}{n}$$

(as they really dependent on $n$!

$$\mathbb{E}\left[\varepsilon_{ni}(u)\,\varepsilon_{ni}(v)\right]$$

$$= C_n(u,v) - \underbrace{\mathbb{E}\left[(P\varepsilon)_i\,\varepsilon_i(v)\right]} \;\cdots$$

$$\leq \underbrace{\mathbb{E}\left[(P\varepsilon)_i^2\right]}^{1/2}\left(\mathbb{E}\,\varepsilon_i^2(v)\right)^{1/2}$$

$$\longrightarrow 0 \text{ as } n \to \infty$$