Want $\displaystyle\int_T \mathbb{E}\Big[\cancel{\ } |\det \nabla^2 f(t)| \, \mathbb{1}[\text{index}(\nabla^2 f(t)) = k] \, \mathbb{1}[f(t)>u]$

$\qquad\qquad \Big| \nabla f(t) = 0 \Big]$

Then $\begin{pmatrix} f \\ \nabla^2 f \end{pmatrix} \Big| \nabla f(t) = \cancel{\ } \, a \nabla f(t) + \cancel{\ } N\big(0, (\sigma^2 K$

$\qquad\qquad N(0, K(t))\big)$

Let $K = K(t)$ some fixed t.

Claim: $\exists$ a stat'y field g on T such that

$$\begin{pmatrix} f \\ \nabla^2 f \end{pmatrix} \sim N(0, K \cancel{(t)}) \quad \cancel{\ }$$

Given this,

$$\boxed{\mathbb{E}\Big[\overset{\text{Rep } f}{f}\Big]}$$

$$\mathbb{E}\Big[\sum (-1)^i \mu_i \Big] = \sum (-1)^i \, \mathbb{E}\mu_i$$

$$= \sum (-1)^i \int_T \mathbb{E}\Big[|\det \nabla^2 g(t)| \, \mathbb{1}[\text{index } \nabla^2 g(t) = k] \; \mathbb{1}[g(t) > u] \Big) \; p_t(0)$$