$\sup_{B} \underbrace{\nabla^3 Y \mid \nabla Y(t), \nabla^2 Y(t)}_{Z_t(s)}$ is the supremum of a mean-zero Gaussian process

as $\det \nabla^2 Y(t)$ is in the integral.

as such it has a pdf p_t that exists and is continuous by ~~XXX~~ Theorem 1.

It remains to show that p_t is bounded over t.

technically the result is in 1D but can write ~~XXX~~ the term in Taylor's theorem

is $\sum_{ijk} t_i^* t_j^* t_k^* \nabla^3 Y(t^*)_{ijk}$.

so on each of the sums obtain a 1D Gaussian

note that. $\sup \left| \sum_{ijk} t_i^* t_j^* t_k^* \nabla^3 Y(t^*)_{ijk} \right|$

$\leq \sum_{ijk} t_i t_j t_k \sup \left| \nabla^3 Y(t^*)_{ijk} \right|$.

and so integrating over this, the sum comes out of the integral.

To show this, let $F_t(u) = \mathbb{P}(\sup_s Z_t(s) > u)$.

~~Additionally~~, Taking $t=0$, $\Phi(0) = 1/2$.

and set $b_t = F_t^{-1}(1/2)$.

PTO