So $\left(\frac{1}{N}\sum_{n=1}^{N} X_n(1), \quad \frac{1}{N}\sum_{n=1}^{N} (-1)^{p_n} X_n(2) \right)$

$$=_d \left(\frac{1}{N} \sum (-1)^{p_n} X_n(1), \quad \frac{1}{N} \sum X_n(2) \right)$$

but $\left(\frac{1}{N}\sum_{n=1}^{N} (-1)^{p_n} X_n(1), \quad \mu \right)$

$$\neq_d \left(\frac{1}{N} \sum_{n=1}^{N} X_n(1), \quad \frac{1}{N} \sum (-1)^{p_n} \mu \right)$$

ie as this is random!

~~must~~ can't cond^n on p_n btw as else this is not equality in distribution!

Other thing to write up: If $(\tilde{P}, Q) \overset{d}{=} (\tilde{P}^{\pi}, Q)$

then $\mathbb{P}\left(\tilde{P}_{(k)} \geq Q_k^{(\alpha)} \ \forall k \in \{1, \ldots, m\} \right)$

$$= \mathbb{P}\left(\tilde{P}^{\pi}_{(k)} \geq Q_k^{(\alpha)} \ \forall k \in \{\ldots, m\} \right)$$

$$\geq \mathbb{P}\left(\tilde{P}^{\pi}_{(k)} \geq \hat{Q}_k^{(\alpha)} \ \forall k \in \{1, \ldots, m\} \right)$$

but ~~IDK~~ this is necessarily true.