Continuing similarly we have:

$$\prod_{n=1}^{K} i_1 \cdots i_k - 1$$

need an inductive method!

$$= i_1$$

$$\cancel{(i_k - 1)\prod i_1 \cdots i_{k-1} + \prod i_1 \cdots i_{k-1}}$$

$$= (i_k - 1)\left(\prod i_1 \cdots i_{k-1} - 1\right)$$

$$= \prod i_1 \cdots i_k - \prod i_1 \cdots i_{k-1} - i_k + 1$$

$$= \prod i_1 \cdots i_k - \big(\underbrace{\prod i_1 \cdots i_{k-1} - 1}_{= o(\Sigma^N)}\big) - \big(\underbrace{i_k - 1}_{o(\Sigma^N)}\big) - 1$$

get

$$(i-1)(j-1)(k-1)$$

fine as $\underline{0 = (1-1)^N}$!

$$= ijk - ij - ik - jk$$

need to check the # of 1s works out!

$$\prod (i_1 - 1) \cdots (i_N - 1)$$

$$= \prod i_1 \cdots i_N - \sum_{\binom{N}{1}} i_{j_1} \cdots i_{j_{N-1}} + \sum_{\binom{N}{2}} i_{j_1} \cdots i_{j_{N-2}} \ldots$$

use induction on $\sum_{\binom{N}{1}} \underline{\prod i \cdots i_N -}$