

Similarly, $\left\| \frac{y_i - \mu}{\hat{\sigma}} \right\| \leq \left\| \frac{y_i - \hat{\mu}}{\hat{\sigma}} \right\| + \left\| \frac{\hat{\mu} - \mu}{\hat{\sigma}} \right\|$

$$\Rightarrow \left\| \frac{y_i - \hat{\mu}}{\hat{\sigma}} \right\| \geq \left\| \frac{y_i - \mu}{\hat{\sigma}} \right\| - \left\| \frac{\hat{\mu} - \mu}{\hat{\sigma}} \right\|$$

$$\begin{aligned} z_i &= \left| \frac{\hat{\sigma}}{\sigma} \right| \left(\left\| \frac{y_i - \mu}{\sigma} \right\| - \left\| \frac{\hat{\mu} - \mu}{\sigma} \right\| \right) \\ &\geq (1 - \varepsilon) \left(\left\| \frac{y_i - \mu}{\sigma} \right\| - \varepsilon \right) = z_i^* \end{aligned}$$

$\Rightarrow \frac{1}{n} \sum \mathbb{1}[z_i > L] < \alpha$

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then $z_i > L \Rightarrow f(y_i) > L \Rightarrow x_i > L$

so $\frac{1}{n} \sum \mathbb{1}[z_i > L] \leq \frac{1}{n} \sum \mathbb{1}[f(x_i) > L] \leq \frac{1}{n} \sum \mathbb{1}[x_i > L]$

$\Rightarrow \frac{1}{n} \sum \mathbb{1}[z_i > L] \leq \frac{1}{n} \sum \mathbb{1}[x_i > L] \leq \frac{1}{n} \sum \mathbb{1}[x_i > L]$