

Let  $F(x) = P(X \leq x)$   $X \sim N(0,1)$

(6)

and  $G(x) = P(\sup(Y, Z) \leq x)$

$\begin{pmatrix} Y \\ Z \end{pmatrix} \sim \text{Joint Gaussian}$   
marginally  $N(0,1)$

Then, claim for

$$x < 1/2 \quad f(x) > g(x)$$

Proof:  $f(x) = \lim_{h \rightarrow 0} \frac{F(x+h) - F(x)}{h}$

small h ie  
set  
 ~~$x+h < 1/2$~~ !

$$= \frac{1}{h} P(X \in (x, x+h)) \geq \frac{1}{h} P(\sup(Y, Z) \in (x, x+h))$$

$$= \frac{1}{h} P(Y \in (x, x+h))$$

$$> \frac{1}{h} P(\max(Y, Z) \in (x, x+h))$$

QED