

$$\int \mathbb{E}[|\det f''| \mathbb{1}[f'' < 0] \mid f' = 0] p_t(0) dt$$

$$= \int \mathbb{E} \left[|f''(t)| \mathbb{1}[f''(t) < 0] \mid f(t) = 0, \mathbb{1}[f(t) > u] \right] p_t(0) dt$$

$$\begin{matrix} f' \\ f'' \end{matrix} \sim \text{[Diagram showing a wavy line with points and labels } \Delta, \delta, \Sigma, \Delta^T \text{]}$$

$$\text{[Diagram showing a wavy line with points and labels } \Delta, \delta, \Sigma, \Delta^T \text{]} \quad N \left(\begin{matrix} \Delta^T \\ \Sigma \end{matrix} \right) \begin{matrix} B. \\ A \end{matrix}$$

$$\Rightarrow f'' \mid f' \sim N(\Delta \Lambda^{-1} f', \Sigma - \Delta \Lambda^{-1} \Delta^T)$$

$$\rightarrow f'' \mid f' = 0 \sim N(0, \underbrace{\Sigma - \Delta \Lambda^{-1} \Delta^T}_{=k(t)})$$

$$\mathbb{E}[-f'' \mathbb{1}[f'' < 0] \mid f(t) = x, f'(t) = 0]$$

is the expected value of a truncated Gaussian which should have a closed form