Let $(B_i)_{i\in\mathbb{N}}$ be a nested neighbourhood basis of x. (exists by lemma 2.47)

Then choose $x_i \in B_i \cap A$

Then $x_i \to x = \lim_{i\to\infty} x_i$

i.e. as given U neighbourhood of x $\exists i$ s.t.

$B_i \subseteq U \Rightarrow x_i \in U$!

□

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need to prove (b) & (c)

2.48 (b) ~~x~~

~~2.50 (b) what does dense mean here?~~

X ~~separable~~

Proof of Thm 2.50(b)

Let $(B_i)_{i\in\mathbb{N}}$ and for each i choose $x_i \in B_i$ let $A = \{x_i\}_{i\in\mathbb{N}}$

~~Then for~~ Given C closed s.t. $A \subseteq C$.

C^c open $\Rightarrow$ $\exists i$ s.t. $B_i \subseteq C^c$ $\Rightarrow x_i \in C^c$ #

□

Thm 2.55, ~~why is any singleton~~

proof, for $x \in M$, $\exists U$ open, φ invertible s.t. $x \in U$,

$\varphi(U)$ is a point i.e. $p \in \mathbb{R}^0$

then $U = \varphi^{-1}(p)$ is a point set

so $U = \{x\}$.

$\Rightarrow \{x\}$ is open.