

~~we~~ have

$$\mathbb{E}[f(x, y) \mathbb{1}[A]]$$

$$= \mathbb{E}[\mathbb{E}[f(x, y) \mathbb{1}[A] \mid x] \mathbb{1}[A]]$$

$$\mathbb{E}[R \mathbb{1}[A]]$$

$$g(y)$$

$$\mathbb{1}[A]$$

$$A \in \sigma(X)$$

||

$$\sigma\left(\left\{X^{-1}(A) : A \in \mathcal{B}\right\}\right)$$

Claim:

If g is a bounded $\sigma(X)$ measurable f^n ,
then $g = f(X)$ some bounded, and measurable f !

certainly any such works.

and here indicators e.g. let $f(x) = \mathbb{1}[x \in A]$

$$\text{then } g(x) = \mathbb{1}[x \in A]$$

all that matters is that this involves the indicators!