WTS:

$$\cancel{\int_{B_r(t_0)} \mathbb{E}\left[\delta_\varepsilon(\nabla Y(t))\,|\det \nabla^2 Y(t_0)|\,\mathbb{I}[A(t_0)]\right]}$$

$$\cancel{\to \int_{B_r(t_0)} \mathbb{E}\left[|\det \nabla^2 Y(t_0)|\,\mathbb{I}[A(t_0)] \,\middle|\, \nabla Y(t)=0\right] p_{\nabla Y(t)}(0)\,dt}$$

~~also need p(v|w) bounded same for the next bit~~ ~~Not this bit~~

~~Let V, W be random vars s.t. p(v|w) is cts ∀w, B(0) ⊂ V ⊂ $\mathbb{R}^N$~~

Claim: $\mathbb{E}[\delta_\varepsilon(V)\, g(W)] = \mathbb{E}[g(W) \mid V=0]\, p_V(0)$

Proof: LHS $=$

$$\int_{W \times V} \delta_\varepsilon(v)\, g(w)\, p(v,w)\, dv\, dw$$

$$= \cancel{\int_W g(w)\, p(w|v)}$$

$$= \int_W g(w)\, p(w) \underbrace{\int_V \delta_\varepsilon(v)\, p(v|w)\, dv}_{\to\, p(0|w) \text{ as cts}}$$

by Lebesgue's cty thm

Need boundedness to apply DCT!

$$= \int_W g(w)\, p(0,w) = \int_W g(w)\, p(w|0)\, p_V(0) \quad \square$$