First term is:

$$\cancel{\mathbb{E}\left[\delta_\varepsilon(\nabla X\right.} \int_{B_r(t_0)} \mathbb{E}\Big[\delta_\varepsilon(\nabla X(t))\,|\det \nabla^2 X(t_0)|\,\mathbb{1}[A(t_0)]\Big]\,dt$$

$$= \cancel{\int_{B_r(t_0)} \mathbb{E}\Big[|\det \nabla^2 X(t_0)|\ \mathbb{E}\big[\delta_\varepsilon(\nabla X(t) \mid \nabla^2 X(t_0))\big]\Big]\,dt}$$

$$= \int_{B_r(t_0)} \int_{\mathbb{R}^{D(D+1)/2} \times \mathbb{R}^D} \mathbb{1}[(x,x'')\in A(t_0)] \times |\det X''|\; p_{\nabla^2 X(t_0)\mid X(t_0)}(x'')$$

$$\times \int \delta_\varepsilon(x')\, p_{\nabla X(t)\mid \nabla^2 X(t_0), X(t_0)}(x' \mid x'', x)\,dx'\,dx''$$

p bounded as Gaussian and cts

$$\to \int_{B_r(t_0)} \int_{\mathbb{R}^{D(D+1)/2}} |\det x''|\; p_{\nabla^2 X(t_0)\mid X(t_0)}(x'') \quad \text{as } \varepsilon \to 0$$

$$\times\ p_{\nabla X(t)\mid \nabla^2 X(t_0), X(t_0)}(0 \mid x, x'')\,dx''\,dt.$$

$$= \cancel{\int_{B_r(t_0)} \mathbb{E}\big[|\det \nabla^2 X(t_0)| \mid \nabla X(t) = 0\big]\,dt}$$

$$= \int_{B_r(t_0)} \mathbb{E}\big[|\det \nabla^2 X(t_0)|\,\mathbb{1}[A(t_0)] \mid \nabla X(t) = 0\big] \times p_{\nabla X(t)}(0)\,dt$$