$\mathbb{E}[X|Y]$ $\mathbb{E}[\cancel{X|Y(\omega)}]$

define $\mathbb{E}[X|Y(\omega)] = \mathbb{E}[X|Y](\omega)$ as $\mathbb{E}[X|Y]$ is a r.v.

$\mu_{Y(\omega)}$
$\shortparallel$
is $\mathbb{E}[\,\cdot\,|Y(\omega)]$ a measure?

well, ~~E~~ let $\mu(A) = \mathbb{E}[1_A | Y(\omega)]$

Then

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \mathbb{E}\left[\bigcup_{n=1}^{\infty} A_n \,\Big|\, Y(\omega)\right]$$

$$= \lim_{N\to\infty} \underbrace{\mathbb{E}\left[\bigcup_{n=1}^{N} 1[A_n] \,\Big|\, Y(\omega)\right]}$$

$$\shortparallel$$

$$\mathbb{E}\left[\bigcup_{n=1}^{N} 1[A_n] \,\Big|\, Y\right](\omega) = \sum_{n=1}^{N} \mathbb{E}[1[A_n]|Y](\omega)$$

basically $\nearrow$ by CMCT.

$\sum^{\infty} \mathbb{E}[\quad]$.

so μ is a measure!