$f(s) \mid X$

X, $f(s)$ jointly Gaussian

$(f(s_1) \cdots f(s_k))^T$

$f(s) \mid X \sim N\left(\sigma_X^{-2} \Sigma^{*-1}_{11} X, \ldots \right)$

$\mathrm{cov}\left(\begin{pmatrix} f(s_1) \\ \vdots \\ f(s_k) \end{pmatrix}, X \right)$

$\mathrm{cov}(X, f(s)) = \Sigma^*(s)$

Claim: $f(s) - \sigma_X^{-2} \Sigma^{*}(s)^{-1} X$ is independent of X.

proof: $(f(s_1) \cdots, f(s_k)) \mid X$

Σ_{A_1}

$\Sigma_{A,B} = \left(\Sigma^*(s_1), \ldots, \Sigma^*(s_k) \right)^T$

So $\sigma_X^{-2} \Sigma_{A,B} X = \sigma_X^{-2} \begin{pmatrix} \Sigma^*(s_1) X \\ \vdots \end{pmatrix}$

So $f(s) - \sigma_X^{-2} \Sigma(s)$ has no dependence on X.

only dependence is through the covariance