

$$\mathbb{E}(\varphi(A))^2$$

$$\mathbb{E}((\varphi(A) + \varphi(B) - \varphi(A \cap B))^2)$$

$$= \mathbb{E}[\varphi(A)^2 + \varphi(B)^2 - \varphi(A)\varphi(A \cap B) - \varphi(B)\varphi(A \cap B) + \varphi(A \cap B)^2 + \varphi(A)\varphi(B)]$$

$$\text{Let } g(t) = C_{tt}^{-1/2} f(t)$$

$$\begin{aligned} \text{Then } \text{cov}(g(t)) &= \mathbb{E}[C_{tt}^{-1/2} f(t) f(t)^T C_{tt}^{-1/2}] \\ &= C_{tt}^{-1/2} \mathbb{E}[f(t) f(t)^T] C_{tt}^{-1/2} \\ &= I \end{aligned}$$

$$\text{and } \text{cov}(g(t), g(s))$$

$$\begin{aligned} &= \mathbb{E}[C_{tt}^{-1/2} f(t) f(s)^T C_{ss}^{-1/2}] \\ &= C_{tt}^{-1/2} \mathbb{E}[f(t) f(s)^T] C_{ss}^{-1/2} \\ &= C_{tt}^{-1/2} C_{ts} C_{ss}^{-1/2} \end{aligned}$$