Why is $\mathbb{P}(X+Y>u \mid X=b) = \mathbb{P}(b+Y>u)$?

$\mathbb{P}(\cancel{f(X)}\ f(X,Y) > u \mid X=b) =$

$$:= \int \mathbb{1}[f(b,y) > u]\ p_{Y|X}(y|b)\, dy \qquad = \frac{p(b,y)}{p(b)}$$

$$= \mathbb{P}(f(b,Y) > u).$$

$\mathbb{E}[X \mid Y(\omega)] := \mathbb{E}[X \mid Y](\omega).$ by defn.

Given X, Y.

$$\mathbb{E}[X \mid Y=y] := \mathbb{E}[X \mid Y = Y^{-1}_*(y)]$$

i.e. $Y: \Omega \to \mathbb{R}$, let $Y^{-1}_*(y)$ be any element of the set $Y^{-1}(y)$

Claim: if $Y(\omega_1) = Y(\omega_2)$,

then $\mathbb{E}[X \mid Y(\omega_1)] = \mathbb{E}[X \mid Y(\omega_2)]$

Proof: