$$\mathbb{E}\left[z_j^k \, \mathbb{1}\left[\|z\| > u^{1/2} \eta \right] \right]$$

$$\|z\| = \|u^{-1/2} P + 2H\|$$

$$z = b_n\left(u^{-1/2} P + 2H \right)$$

$$\mathbb{E}\left[\mathbb{E}\left[b_n(z) \, \mathbb{1}\left[\|u^{-1/2} P + 2H\| > u^{1/2} \eta \right] \right] \right]$$

$$\leq \|u^{-1/2} P\| + 2\|H\|$$

$$\leq \mathbb{1}\left[2\|H\| > u^{1/2} \eta \right]$$

$$= \sum \mathbb{E}\left[(u^{-1/2} P_j)^k \right] \leq O\left(e^{-u^{1/2}\eta} \right)$$