so adding these terms (minding the $-(s-t)^2$

you get: $c(t_0,t_0)\left(\mathbb{E} f''(t_0)f''(t_0) - \mathbb{E} f'(t_0)f'''(t_0)\right)$

$$- \frac{d}{ds}\Big|_{s=t_0} c(s,s) \times \mathbb{E} f'(s) f''(s)$$

$2\mathbb{E} f''(t_0) f'(t_0)$

$$\frac{d}{ds} c(s,s) = \frac{d}{ds} \mathbb{E} f'(s) f'(s)$$

$$= \mathbb{E}\left[\frac{d}{ds} f'(s) f'(s)\right] = 2\mathbb{E}\left[f''(s) f'(s)\right]$$

$$\frac{d^2}{ds^2} c(ss) = \mathbb{E}\left[\frac{d^2}{ds^2} f'(s) f'(s)\right] = 2\mathbb{E}\frac{d}{ds} f''(s) f'(s)$$

$$= 2\mathbb{E}\left[f'''(s) f'(s)\right] + \mathbb{E}\left[f''(s) f''(s)\right]$$

$$= c(t_0,t_0)\left(\mathbb{E} f''(t_0) f''(t_0) - \overbrace{\mathbb{E} f'(t_0) f'''(t_0)}^{= \mathbb{E} f''(t_0) f''(t_0)}\right)$$

$$- 2\left(\mathbb{E} f'(t_0) f''(t_0)\right)^2$$

$= \cancel{\text{detcov}(f'(t_0), f''(t_0))}$ ≥ 0!

$$= \cancel{c(t_0,t_0)\mathbb{E} f''(t_0) f''(t_0) - \left(\mathbb{E} f'(t_0) f''(t_0)\right)^2}$$

$$= 2c(t_0,t_0)\mathbb{E} f''(t_0) f''(t_0) - 2\left(\mathbb{E} f'(t_0) f''(t_0)\right)$$

$$= 2\,\text{detcov}(f'(t_0), f''(t_0)) > 0$$! Yay! ☺