$$\mathbb{P}(X\in A, Y\le y) = \int_A\int_{-\infty}^{y} f_{X,Y}(x,y)\,dx\,dy.$$

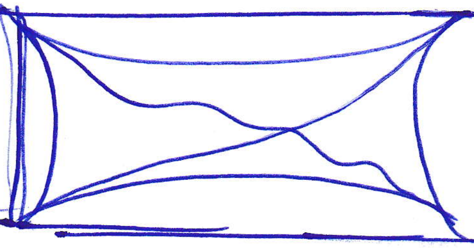
$$\mathbb{E}[Z\mathbb{1}[C]] = \mathbb{E}[\hat{Z}\mathbb{1}[C]]$$

$$\mathbb{E}[\mathbb{1}[B]\,|\,Y]$$

$$= \mathbb{P}(B|Y) = \mathbb{P}(X\in B\,|\,Y)$$

So $\mathbb{E}[\mathbb{1}[B]\,|\,Y=y]$ is well defined!!

$$= \int_B f_{X|Y}(x,y)\,dx$$



$X\in B$

B: X event

C: Y event

$$\mathbb{E}[\mathbb{1}[B]\mathbb{1}[C]] = \mathbb{E}\left[\int_B f_{X|Y}(x,y)\,dx\,\mathbb{1}[C]\right]$$

$$\int_B\int_{C\cap B_{Y}} f_{X,Y}(x,y)\,dx\,dy = \int_C\int_B \frac{f_{X,Y}(x,y)}{f_Y(x,y)}\,dx\,f_Y\,dy$$

$$B = X^{-1}(0)$$

$f_{X,Y}$

f_X defined so that $\forall$ measurable.