① ~~②~~

have numerator =

$$\mathbb{E}\Big[\, |\det \nabla^2 X(\tau)|\, \mathbb{1}[X(\tau) > v]\, \mathbb{1}\Big[\Big|\frac{\nabla^2 X(\tau)}{X(\tau)} + \frac{\Lambda(\tau)}{\sigma^2}\Big| > \eta\Big] \,\Big|\, \nabla X(\tau) = 0\Big] \times \cancel{p_{\nabla X(\tau)}(0)}$$

as no $\mathbb{1}[\epsilon K]$ term!

$$= \int_v^\infty \int \mathbb{1}\Big[\Big|\frac{X''}{X} + \frac{\Lambda(\tau)}{\sigma^2}\Big| > \eta\Big]\, p(X, X'' \mid X' = 0) \times |\det X''|$$

$$= \int_v^\infty p(X) \int \mathbb{1}\Big[\Big|\frac{X''}{X} + \frac{\Lambda(\tau)}{\sigma^2}\Big| > \eta\Big]\, p(X'' \mid X' = 0, X) \times |\det X''|\, dX\, dX''$$

We have,

$$X'' \mid X' = 0, X = -\frac{X\, V(\Lambda)}{\sigma^2} + N(0, \Sigma)$$

fixed!

$\Rightarrow$ substitute $Z = X'' + \dfrac{X V(\Lambda)}{\sigma^2}$

$\left|\dfrac{Z}{X}\right|$

$X > v$

$\Rightarrow \dfrac{1}{X} < \dfrac{1}{v}$

$\Rightarrow \left|\dfrac{Z}{X}\right| < \left|\dfrac{Z}{v}\right|$

$\left|\dfrac{Z}{X}\right| > \eta \Rightarrow \left|\dfrac{Z}{v}\right| > \eta.$

$$\Rightarrow \mathbb{1}\Big[\Big|\frac{Z}{X}\Big| > \eta\Big] \le \underbrace{\mathbb{1}\Big[\Big|\frac{Z}{v}\Big| > \eta\Big]}_{\to 0}$$