So $\lim_{r\to 0} \frac{\text{num}}{\text{denom}} \leq \lim_{r\to 0} \frac{\mathbb{P}\left(M_1 \sum_l |X(l)| \frac{h^{3/2}}{v^{3/4}} > \kappa/3,\ \mathcal{M}(B_r(0)),\ Y(0) > v\right)}{\mathbb{P}(\mathcal{M}(B_r(0)),\ Y(0) > v)}$ (5)

$$= \lim_{r\to 0} \frac{\mathbb{E}[\mu(B_r(0))\, \mathbb{1}[B]] + o(r^D)}{\mathbb{E}[\mu(B_r(0))\, \mathbb{1}[Y(0) > v]] + o(r^D)}$$

where $B = \left\{ M \sum_l |X(l)| \frac{h^{3/2}}{v^{3/4}} > \kappa/3,\ Y(0) > v \right\}$.

applying Lemma 2.2.
in the numerator & denominator.

As in the proof of ~~Lemma~~ Proposition 2.1,

$$\mathbb{E}[\mu(B_r(0))\, \mathbb{1}[B]] \leq \lim_{\varepsilon\to 0} \mathbb{E}[N_\varepsilon \mathbb{1}[B]]$$

$$\mathbb{E}[N_\varepsilon \mathbb{1}[B]] \leq \int_{B_r(0)} \mathbb{E}[\delta_\varepsilon(\nabla Y(t))\, |\det \nabla^2 Y(t)|\, \mathbb{1}[B]]\, dt$$

These terms satisfy the cond^n's of Lemma XXX,
So this converges to

$$\int_{B_r(0)} \mathbb{E}[|\det Y^2(t)|\, \mathbb{1}[B] \mid \nabla Y(t) = 0]\, p_{\nabla Y(t)}(0)\, dt$$

as $\varepsilon \to 0$.