$$(t^T \Lambda t) = \sum_{kl} t_k t_l \lambda_{kl}$$

$$= \text{vech}(T)^T \, \text{vech}(\Lambda) \qquad \boxed{2.}$$

Weird other stuff could be useful?

$$\Rightarrow -\frac{1}{2}\lambda_{ij}(t^T \Lambda t) = \frac{1}{2}\text{vech}(T)^T(\lambda_{ij}\,\text{vech}(\Lambda))$$

$$\Rightarrow \ddot r_{ij} - \lambda_{ij} r(t) = \frac{1}{2}\text{vech}(T)^T\left(\text{vech}(\nabla^2 \ddot r_{ij}(0)) - \lambda_{ij}\,\text{vech}(\Lambda)\right)$$

and equals

$\ddot r_{ij} - \lambda_{ij} r(t)$

ij th entry of

$\boxed{V(\nabla^2 r(t))^T + r(t) V(\Lambda)^T}$

So $V(\nabla^2 r(t))^T + r(t)\, V(\Lambda)$

$$= \frac{1}{2}\text{vech}(T)^T M + o(t^2)$$

and $\frac{1}{2}\text{vech}(T)^T$ ~~vech~~ Z

$$= \frac{1}{2}\text{vech}(T)^T\left(V(\nabla^2 X(0)) + V(\Lambda) X(0)\right)$$

$$= \frac{1}{2} t^T \nabla^2 X(0)\, t + \frac{1}{2} t^T \Lambda t\, X(0)$$