$$\mathbb{E}\left[|Z_{ij}|^k \, 1\left[\|2u^{-1/2}P + 2H\| \|Z\| > v^{1/2}\eta_{1/2} \right] \right.$$

$$= \mathbb{E}\left[\left| 2u^{-1/2}P_{ij} + 2H_{ij} \right|^k 1\left[\|2u^{-1/2}P + 2H\| \leq v^{1/2}\eta_{1/2} \right] \right. \quad (\otimes)$$

$$\|2u^{-1/2}P + 2H\| \leq \|2H\| + \|2u^{-1/2}P\|$$

& so $\|2u^{-1/2}P + 2H\| > v^{1/2}\eta_{1/2}$

$$\Rightarrow \|2H\| > v^{1/2}\eta_{1/2} \quad \text{or} \quad \|2u^{-1/2}P\| > v^{1/2}\eta_{1/2}$$

& $(\otimes)$ is $\leq$

$$\mathbb{E}\left[\left| 2u^{-1/2}P_{ij} + 2H_{ij} \right|^k 1\left[\|2H\| > v^{1/2}\eta_{1/2} \right] \right]$$

$$+ \mathbb{E}\left[\left| 2u^{-1/2}P_{ij} + 2H_{ij} \right|^k 1\left[\|2u^{-1/2}P\| > v^{1/2}\eta_{1/2} \right] \right.$$