

$$\begin{aligned}
 \text{So } \rho(s, t) &= 1 - \mathbb{E}(f''(t)^2)^{\frac{1}{2}} \text{cor}(f''(t)) \\
 &= 1 - \text{cor}(f''(t)) (s-t)^2 \\
 &\quad + o((s-t)^2)
 \end{aligned}$$

$$\begin{aligned}
 \text{So } \text{d} \rho &= \\
 &1 - (1 - \text{cor} f''(t) (s-t)^2)^{\frac{1}{2}} \\
 &= 2 \text{cor} f''(t) (s-t)^2
 \end{aligned}$$

Note that for a variance field,

$$\mathbb{E} f' f'' = 0, \quad \mathbb{E} (f')^2 = 1$$

$$\text{So } \text{d} \text{cor}(f', f'') = \text{cor} f''(t) !!$$
