

Go over why  $C_k^2 < \infty$  is necessary.

2020

$$\begin{aligned} \rho(s, t) &= \text{cov}(f'(s), f'(t)) \\ &= \mathbb{E}[f'(s) f'(t)] \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{d}{ds} \rho(s, t) &= \left. \frac{d}{ds} \mathbb{E}[f'(s) f'(t)] \right|_{s=t} \\ &= \mathbb{E}[f''(s) f'(t)] \\ &= \mathbb{E}[f''(t) f'(t)] = 0 \end{aligned}$$

as  $\rho(t, t) = \text{variance}$

$$\Rightarrow \mathbb{E}[f'(t)^2] = 0$$

$$\Rightarrow 2\mathbb{E}[f'(t) f'(t)] = 0$$

$$\text{So } \rho(s, t) = \rho(t, t) - \mathbb{E}[f''(t)] \frac{(s-t)^2}{2} + o(s-t)$$

$$\begin{aligned} \frac{d^2}{ds^2} \rho(s, t) \Big|_{s=t} &= \mathbb{E}[f'''(s) f'(t)] = \mathbb{E}[f'''(s) f'(t)] \\ &= -\mathbb{E}[f''(t)^2] \end{aligned}$$