Now have two terms. ③

For the 2nd term:

$$\mathbb{E}\Big[\int_T \underbrace{\delta_\varepsilon(\nabla X(t))}_{\text{controlled}}\, |L_{i\sigma(i)}|\, \underbrace{\|t-t_0\|}_{\le r}\, \mathbb{I}[A]\, dt\Big]$$

can take $\mathbb{E}$ inside as L is integrable and $\mathbb{I}[A] \le 1$.

$$\le \int_T \mathbb{E}\big[\underbrace{\delta_\varepsilon(\nabla X(t))\,|L_{i\sigma(i)}|}_{\|}\big]\, dt \times \|t-t_0\|$$

now ∇t_j.

$$\mathbb{E}\big[\delta_\varepsilon(\nabla X(t))\, \underline{\mathbb{E}[L_{i\sigma(i)} \mid \nabla X(t)]}\big]$$ by the tower rule

actually have a product here but same story!

claim this is bounded.

$X' = f$

$X \mid X =$ lattice data

Proof: $f(t) = \sum_l K(t-l)\, X(l)$

$$\Rightarrow \nabla^2 f(t) = \sum_l \nabla^2 K(t-l)\, X(l)$$

$$\Rightarrow \big|(\nabla^2 f(t))_{ij} - \nabla^2 f(s)_{ij}\big| \le (L_{K''})_{ij} \underbrace{\sum_l |X(l)|}_{= L_{ij}}$$

So Now, ~~$X(t)$~~, $f(t) = \sum_l K(t-l)\, X(l)$

$\Rightarrow X(l), f(t)$ are jointly Gaussian

and as such, $X(l) \mid f(t)$ is Gaussian $\Rightarrow \mathbb{E}[|X(l)| \mid \nabla f(t)] < \infty$

as has finite moments