$$\sup_{|s-t|<\eta} |f(s)-f(t)| \leq$$

Lemma: ~~All real r.v.s~~ X real r.v. ⇒ X tight.

Proof: $\cancel{\mathbb{P}(X\in[n,n+1)}$

$$1 = \mathbb{P}(X\in\mathbb{R}) = \mathbb{P}\left(X\in\bigcup_n [n,n+1)\right)$$

need r.v. for this as use ~~measurability~~

$$= \mathbb{P}\left(X^{-1}\left(\bigcup_n [n,n+1)\right)\right) = \sum_n \mathbb{P}(X\in[n,n+1))$$

so $\forall\varepsilon \Rightarrow \exists\, n$ s.t. $\sum_{-N}^{N} \mathbb{P}(X\in[n,n+1)) > 1-\varepsilon$

so take $C = B = [-N, N+1]$

compact and so done. $\square$

Lemma: f separable, T compact ⇒ $\sup_{t\in T} f(t)$ is tight!

So f separable ⇒ $\sup_t f$ msble

⇒ $\sup_t f$ is ~~compact~~ tight!

note need $\sup_t f<\infty$

~~so need~~ compactness of T and cty of f.

Need to show cfields are separable