

$$\log S_t = \mu + \varepsilon_t \quad \Rightarrow \quad S_t = e^{\mu} e^{\varepsilon_t}$$

$$\Rightarrow S_t / S_0 = e^{\varepsilon_t - \varepsilon_0}$$

Then $\log S_t - \log S_{t-1} = \varepsilon_t - \varepsilon_{t-1}$ So μ cancels!

$$\log \left(\frac{S_t}{S_{t-1}} \right)$$

(assuming μ is constant).

$$\text{So } \sum \log(S_t) \text{ diffs} = \varepsilon_T - \varepsilon_0$$

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$$\log(S_T) - \log(S_0) = \varepsilon_T - \varepsilon_0$$

$$\Rightarrow S_T = S_0 e^{\varepsilon_T - \varepsilon_0}$$

$$\varepsilon_T - \varepsilon_0 = \sum \varepsilon_t - \varepsilon_{t-1}$$