$$\boxed{AA^T + AA^T = -A^2 \neq -A^2 ??}$$

they are clearly pos definite

these go cancel away I think.

only need to check the AA^T, A^TA terms

$$\begin{pmatrix} 1 & 1+b(s-t)^2 & 1+b(r-t)^2 \\ 1+b(s-t)^2 & 1 & 1+b((s-t)^2+(r-t)^2) \\ 1+b(r-t)^2 & 1+b((s-t)^2+(r-t)^2) & 1 \end{pmatrix}$$

In the general case,

$B = B^T$ (as is ... as of 2nd deriv)

& $A + A^T = 0$ as first

So get $\begin{pmatrix} I & I+Ax+Bx^2 \\ I+A^Tx+Bx^2 & I \end{pmatrix}$

so the A, A^T terms in the det are the same and must cancel (so compare the As!

$$\begin{pmatrix} 1 & 1+x & 1+y \\ 1+x & 1 & 1+x+y \\ 1+y & 1+x+y & 1 \end{pmatrix}$$

$$\det = -(1+x+y)^2 - (1+x)\left((1+x) - (1+y)(1+x+y)\right)$$

$$\mapsto \begin{pmatrix} 1 & 1+x & 1+y \\ x & -x & x \\ 1+y & 1+x+y & 1 \end{pmatrix} \mapsto \begin{pmatrix} 1 & 1+x & 1+y \\ x & -x & x \\ y & y & -y \end{pmatrix}$$

$$\mapsto \begin{pmatrix} 1 & 1+x & 1+y \\ x & & \\ 1 & & \end{pmatrix}$$

$\det = \cancel{xy - yx}\ xy - (1+x)(-xy - xy) + (1+y)(\underbrace{xy + xy}_{=2xy})$ $\quad = 2xy$

$= \cancel{xy} + xy + 2xy + 2x^2y + 2xy + 2y^2x$

$$xy = b(s-t)^2\, b(r-t)^2 = \boxed{b^2(s-t)^2(r-t)^2}$$

= leading term!