$$\int_{\mathbb{R}^N} \delta_\varepsilon(x) \, P_{\nabla X(t)}(x) \, dx \leq \sup_X P_{\nabla X(t)}(x) \underbrace{\int_{\mathbb{R}^N} \delta_\varepsilon(x)}_{=1}$$

so clearly things are nice and bounded!