

$$\text{det} \begin{pmatrix} A \\ B+J \end{pmatrix}$$

$$= \text{det} \begin{pmatrix} A \\ B+J \end{pmatrix} = \begin{pmatrix} \text{cov } A & \text{cov}(A, B) + \text{cov}(A, J) \\ \text{cov } A, B + \text{cov } A, J & \text{cov}(B) + \text{cov}(J) + \text{cov}(B, J) \end{pmatrix}$$

$$= \text{cov} \begin{pmatrix} A \\ B \end{pmatrix} + \text{cov} \begin{pmatrix} 0 & \text{cov}(A, J) \\ \text{cov}(A, J) & \text{cov}(J) + \text{cov}(B, J) \end{pmatrix}$$

$$\text{cov}(A, J)^2 \leq \text{var } A \text{ var } J$$

$$\begin{vmatrix} 0 & \text{cov}(A, J) \\ \text{cov}(A, J) & \text{cov}(J) + \text{cov}(B, J) \end{vmatrix}$$

$$\leq \begin{pmatrix} 0 & (\text{var } A \text{ var } J)^{1/2} \\ (\text{var } A \text{ var } J)^{1/2} & \text{var}(J) + (\text{var } J \text{ var } B)^{1/2} \end{pmatrix}$$

$$\text{cov}(A, J)_j \leq (\text{var } A_i \text{ var } J_j)^{1/2}$$

$$\leq \left(\sum \text{var } A_i^{1/2} \right) \left(\sum \text{var } J_j^{1/2} \right) \mathbf{1} \mathbf{1}^T$$

~~$$\leq \left(\sum \text{var } A_i^{1/2} \right) \left(\sum \text{var } J_j^{1/2} \right) \begin{pmatrix} 0 & \mathbf{1} \mathbf{1}^T \\ \mathbf{1} \mathbf{1}^T & \mathbf{I} \end{pmatrix}$$~~