$\mathbb{E}X^k$

$$\mathbb{E}\left[Z_{ij}^k \, 1\left[\|Z\| > u^{1/2} \eta \right] \right]$$

$$Z = \left(H + u^{-1/2} P \right)$$

$$\mathbb{E}\left[Z_{ij} \right]$$

$$\mathbb{E}X^k \le \left(\mathbb{E}X\right)^k.$$

P known

$$= \mathbb{E}\left[\mathbb{E}\left[Z \; \cdots \; | P \right] \right]$$

$$= 1\left[\sum_{ij} Z_{ij}^2 < u^{1/2} \eta \right) \qquad \sum Z_{ij}^2 > u^{1/2} \eta$$

$$= \qquad Z_{ij}^2 > \frac{u^{1/2} \eta}{D^2} \quad \text{same } ij$$

$$Z_{ij}^2 > \frac{u^{1/2} \eta}{D} \quad \forall ij$$

$$\text{so} \quad 1\left[Z_{ij}^2 > \frac{u^{1/2} \eta}{D} \; \forall ij \right] \le 1\left[\sum Z_{ij}^2 > u^{1/2} \eta \right]$$

$$1\left[Z_{ij}^2 > u^{1/2} \eta \right]$$