this u^p cancels out any difficulties.

$$\det(2\Lambda) \int_{u=V+w}^{\infty} u^p p_{u|Vu}(u|0)\, du$$

$$+ \int_{u=V+w}^{\infty} u^{\frac{D+k}{2}} p_{u|Vu}(u|0)\, du$$

$k < D$

$$p_{u|Vu}(u|0) \propto u^{\frac{n-D-2}{2}} e^{-u/2\sigma}$$

$$\Rightarrow u^p p_{u|Vu}(u|0) \propto u^{\frac{n-2}{2}} e^{-u/2\sigma}$$

pdf of a χ^2_n distrb.