$$\rho(s,t)_{ij} = \mathbb{E}\Big[\frac{\partial f}{\partial s_i}(s$$

$$\|$$

$$\mathbb{E}\big[\nabla^T f(t)\, \nabla f(s)\big]_{ij}$$

$$= \mathbb{E}\Big[\frac{\partial f}{\partial s_i}(s)\, \frac{\partial f}{\partial t_j}(t)\Big].$$

$$\Rightarrow \frac{\partial}{\partial s_k}\rho(s,t)\Big|_{s=t} = \mathbb{E}\Big[\frac{\partial^2 f}{\partial s_i \partial s_k}(s)\, \frac{\partial f}{\partial t_j}(t)\Big]$$

$$\mathbb{E}\Big[\quad \det\begin{pmatrix} 0 & a & b \\ -a & 0 & \\ -b & & \ddots \end{pmatrix}$$

$\cancel{\mathbb{E}[f_i(s) f_i}$

$$\mathbb{E}\Big[\frac{\partial}{\partial s_i}(s)\frac{\partial}{\partial t_j}(t)\Big]$$

$$\det\begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}^2 \cancel{a = a^2?}$$

$$\det\Big(I - \big(I + \cancel{A+B}\, A(s-t) + B(s-t)^2\big)\Big)^2$$

$$= \det\big(\cancel{2A(s-t)}\; 2A(s-t) + B(s-t)^2\big).$$
