~~B~~ LCT (1)

Suppose $p_{s_n}(y, \nabla^2 y \mid \nabla y = 0)$ is cts $\forall y, \nabla^2 y$

then $s_n \to s \Rightarrow p_{s_n}(y, \nabla^2 y \mid \nabla y) \longrightarrow p_s(y, \nabla^2 y \mid \nabla y = 0)$

have

$$|\det \nabla^2 y| \, p_{s_n}(y, \nabla^2 y \mid \nabla y = 0) \, \mathbb{1}[y > u] \, \mathbb{1}[\nabla^2 y < 0]$$

$$\leq |\det \nabla^2 y| \, p_{s_n}(y, \nabla^2 y \mid \nabla y = 0) = \textbf{RHS}$$

and $\int \textbf{RHS} = \cancel{\mathbb{E}[\,|\det \nabla^2 y|}$

$$= \mathbb{E}\left[\, |\det \nabla^2 y(s_n)| \;\middle|\; \begin{matrix} \nabla y(s_n) = 0 \\ \cancel{\nabla y = 0} \end{matrix} \right]$$

(can say finite by cty of the covariance fns, subnited on a ball around s.)

$\nabla^2 y(s_n) \mid \nabla y(s_n) = 0$ ~~is cts~~

$$\begin{pmatrix} \nabla y(s_n)^{B} \\ \nabla^2 y(s_n)_{A} \end{pmatrix} \sim N\begin{pmatrix} \overset{\Sigma_{BB}}{\Lambda} & \overset{\Sigma_{BA}}{\Delta^T} \\ \underset{\Sigma_{AB}}{\Delta} & \underset{\Sigma_{AA}}{\Omega} \end{pmatrix}$$

$$\Rightarrow \nabla^2 y(s_n) \mid \nabla y(s_n) \sim N\left(\Delta \Lambda^{-1} \nabla y(s_n),\ \Omega - \Delta \Lambda^{-1} \Delta^T \right)$$

$$\Rightarrow \nabla^2 y(s_n) \mid \nabla y(s_n) = 0 \sim N\left(0, \Omega_{(s_n)} - \Delta_{(s_n)} \Lambda^{-1}_{(s_n)} \Delta^T_{(s_n)}\right)$$