

$$\begin{aligned}
 \mathbb{E}[S_0] &= \mathbb{E}\left[\sum_i S_i\right] = \mathbb{E}\left[\mathbb{E}[S_i | C]\right] \\
 &= \mathbb{E}\left[C \mathbb{E}[S_i | C=1]\right] \\
 &= \mathbb{E}C \mathbb{E}S_i
 \end{aligned}$$

If $X \sim \text{Exp}(\theta)$ ^{mean} $\mathbb{E}X = \theta$.

then $X^\tau \sim \text{Weibull}(\theta^\tau, 1/\tau)$

$$\mathbb{E}[X^\tau] = \theta^\tau \Gamma(1+\tau)$$

$$S^{2/D} \sim \text{Exp}\left(\frac{2\pi\omega^2}{\omega^2 \Gamma(1+D/2)^{2/D}}\right) \quad \begin{matrix} \text{mean} \\ \text{not rate!} \end{matrix}$$

$$\Rightarrow S_i \sim \text{Weib}\left(\left(\frac{2\pi\omega^2}{\omega^2}\right)^{D/2} \Gamma(1+D/2)^{-1}, 2/D\right)$$

$$\mathbb{E}\left[\omega(\lambda_1^{2/D})\right] \Rightarrow \lambda = \frac{\mathbb{E}S}{\mathbb{E}C} \Gamma^{-1}(1+D/2)$$