so

taking $$\beta = \left[\pi \Gamma(D/2+1) \overset{\approx \mathbb{E}n}{\frac{\mathbb{E}m}{\mathbb{E}N}} \right]^{2/D}$$

$$\Rightarrow \quad \mathbb{E} n^{2/D} = \frac{1}{\beta} = \cancel{\frac{(\mathbb{E}m/\mathbb{E}N)^{D/2}}{\Gamma(D/2+1)^{2/D}}}$$

$$= \frac{\left(\overset{\approx \mathbb{E}n}{\mathbb{E}N/\mathbb{E}m}\right)^{2/D}}{\Gamma(D/2+1)^{2/D}}$$

flips

$$= \frac{2\pi W^2}{u^2 \Gamma(D/2+1)^{2/D}}$$! as required.

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ie assuming that $\beta n^{2/D} \sim \exp(1)$
and $\beta =$ (12)

get the "right form" of the $\mathbb{E}n^{2/D}$ asymptotics.