$$\frac{s-t}{\|s-t\|} = \pm 1$$

$$\mathrm{detvar}\big(\nabla f(t),\ \nabla^2 f(t)\, e_{t,s} + \cancel{\|s-t\|^{1/2}} z_{t,s},$$
$$\nabla^2 f(t)\, e_{r,s} + \nabla^3 f(t), \ldots$$

$+z_{t,t}$

In 1D, $\mathrm{detvar}\big(\nabla f(t),\ \nabla^2 f(t)\, e_{t,s},\ \cancel{\nabla^3 f(t)}\big)$

$\nabla^3 f(t) e$

$\frac{s-t}{\|s-t\|} \to 1$ as $s-t \to 0$. ie can assume all entries of $e_{r,s}$ are bounded away from 0.

have $\cancel{\nabla f(t) =}$

$e_{r,t} - e_{s,t}$

$\cancel{\nabla f(s) =}$

$$= \frac{r-t}{\|r-t\|} - \frac{s-t}{\|s-t\|}$$

$$\nabla f(r) = \nabla f(t) + \nabla^2 f(t)(r-t)^T + \cancel{c\nabla}\,(r-t)\nabla^3 f(t)(r-t)^T + z^2$$

Then $\mathrm{detvar}\big((\nabla^T f(t), \nabla^T f(s), \nabla^T f(r)\big)$

$$= \mathrm{detvar}\big(\nabla^T f(t),\ \nabla^2 f(t)\, e_{t,s},$$
$$\nabla^2 f(t)\, e_{r,t} + e_{r,t} \nabla^3 f(t) \times e_{r,t}\big)$$
$$\times (1 + o(1))$$

$$= \mathrm{detvar}\big(\nabla^T f(t),\ \nabla^2 f(t) e_{t,s},\ e_{r,t}\nabla^3 f(t) e_{r,t}\big)(1+o(1)) \; ?$$