$$\hat{\beta} = (X^TX)^{-1}X^TY$$

$$\operatorname{Var}(\hat{\beta}) = \sigma^2 (X^TX)^{-1}$$

} for fixed X!

All assumes fixed X.

for non-fixed X,

$$\operatorname{var}(\hat{\beta} \mid X) = \sigma^2 (X^TX)^{-1}$$

Under Gaussi

given X.

$$\hat{\beta} \sim N(0, \sigma^2 (X^TX)^{-1})$$

$$\hat{\beta}_j \sim N(0, \sigma^2 (X^TX)_{jj})$$

and $\hat{\sigma} \sim \sigma^2 \chi^2_{n-p}$ independently of $\hat{\beta}$.

$$\Rightarrow \frac{\hat{\beta}_j}{\hat{\sigma}} \sim \boxed{t_{n-p}}$$

$$\left((X^TX)^{-1}_{jj}\right)^{1/2} t_{n-p}$$

$$\Rightarrow \boxed{\frac{\hat{\beta}_{n,j}}{\hat{\sigma}_n (X^TX)^{-1/2}_{jj}} \sim t_{n-p}}$$