

LUCC ^{disjoint} additivity (general D) (3)

$$\mathbb{E}[\chi(\mathcal{A}_n(A \cup B))] = \mathbb{E}[\chi(\mathcal{A}_n(A))] + \mathbb{E}[\chi(\mathcal{A}_n(B))]$$

$$\Rightarrow \sum_{j=0}^D \mathcal{L}_j(A \cup B) p_j(u) = \sum_{j=0}^D (\mathcal{L}_j(A) p_j(u) + \mathcal{L}_j(B) p_j(u)) \quad \forall u$$

hermite polys are orthogonal.

$$\Rightarrow \mathcal{L}_j(A \cup B) = \mathcal{L}_j(A) + \mathcal{L}_j(B) !$$

Actually: $\forall A, B, \forall j$

$$\boxed{\mathcal{L}_j(A \cup B) = \mathcal{L}_j(A) + \mathcal{L}_j(B) - \mathcal{L}_j(A \cap B)}$$

$$\mathbb{E}[\chi(\mathcal{A}_n(A \cup B))] = \mathbb{E}[\chi(\mathcal{A}_n(A))] + \mathbb{E}[\chi(\mathcal{A}_n(B))]$$

$$- \mathbb{E}[\chi(\mathcal{A}_n(A \cap B))]$$

$$\Rightarrow \sum_{j=0}^D \mathcal{L}_j(A \cup B) p_j(u) = \sum_{j=0}^D (\mathcal{L}_j(A) + \mathcal{L}_j(B) - \mathcal{L}_j(A \cap B)) p_j(u) \quad \forall u$$

hermite polys.