

$$\mathbb{E}[G^T A G G^T B G]$$

$$= \mathbb{E}[\text{tr}(G^T A G G^T B G)]$$

$$= \mathbb{E}[\text{tr}(A G G B G G^T)]$$

$$= \text{tr}(\mathbb{E}[A G G B G G^T])$$

$$= \text{tr}(A \mathbb{E}[G G^T B G G^T])$$

$$= \text{tr}(A (\Sigma (B + B^T) \Sigma + \text{tr}(B \Sigma) \Sigma))$$

$$= \text{tr}(A \Sigma (B + B^T) \Sigma) + \text{tr}(A \Sigma) \text{tr}(B \Sigma)$$

$$\mathbb{E}[X X^T A X X^T]$$

$\stackrel{\text{unip}}{=} \Sigma(A + A^T) + \text{tr}(A \Sigma) \Sigma$
fun above