Claim: If ~~F is~~ $F: M \to N$ is a diffeomorphism,

then $(F^{-1})_* = (F_*)^{-1}$

(note both are from $T_p(N) \to T_p(M)$.

Proof: ~~Suffice Since~~ suffices to show that

for all $X \in T_pN$ and $f \in C^\infty(M)$,

we have $(F_*^{-1}X)f = ((F^{-1})_* X)f$.

Now, ~~let Y = F~~ let $g = f \circ F^{-1}$, and $Y = F_*^{-1}X$, ~~Then~~.

~~LHS =~~ $\cancel{(F_*^{-1}X)(g \circ f) =}$

Then LHS $= Y(g \circ F) = (F_* Y)g$

$= X \circ g = X(f \circ F^{-1}) = ((F^{-1})_* X)f$.

□
