Similarly, $\left\|\frac{Y_i-\mu}{\hat\sigma}\right\| \le \left\|\frac{Y_i-\hat\mu}{\hat\sigma}\right\| + \left\|\frac{\hat\mu-\mu}{\hat\sigma}\right\|$

$$\Rightarrow \left\|\frac{Y_i-\hat\mu}{\hat\sigma}\right\| \ge \left\|\frac{Y_i-\mu}{\hat\sigma}\right\| - \left\|\frac{\hat\mu-\mu}{\hat\sigma}\right\|$$

$$\cancel{\ge} = \left|\frac{\sigma}{\hat\sigma}\right|\left(\left\|\frac{Y_i-\mu}{\sigma}\right\| - \left\|\frac{\hat\mu-\mu}{\sigma}\right\|\right)$$

$$\ge (1-\varepsilon)\left(\left\|\frac{Y_i-\mu}{\sigma}\right\| - \varepsilon\right) = Z_i$$

$l_\alpha^{n,\varepsilon}$

$\Rightarrow \cancel{q_\alpha^n}$

$$l_\alpha^{n,\varepsilon} = \arg\min_{l} \frac{1}{n}\sum 1[Z_i > l] < \alpha$$

then $Z_i > l \Rightarrow f(Y_i) > l \Rightarrow X_i > l$.

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so $\frac{1}{n}\sum 1[Z_i > l] \le \frac{1}{n}\sum 1[f(Y_i) > l] \le \frac{1}{n}\sum 1[X_i > l]$.

$\Rightarrow \cancel{q_\alpha}$

$$\Rightarrow l_\alpha^{n,\varepsilon} \le q_\alpha^n \le u_\alpha^{n,\varepsilon}$$