

let ∇

$$(t, t) \nabla^2 C(t, t)$$

$$\text{cov} \left(\frac{dX(t)}{dt}, \frac{dX(s)}{ds} \right)$$

$$\left(\begin{matrix} s \\ t \end{matrix} \right) \mapsto \frac{\partial}{\partial t} \frac{\partial}{\partial s} \text{cov}(X(t), X(s))$$

$$\left(\begin{matrix} \text{cov}(\nabla X(t), \nabla^2 X(s)) \\ \downarrow \\ \text{cov}(\nabla X(s), \nabla^2 X(t)) \end{matrix} \right)$$

$$\nabla^2 \left(\begin{matrix} \frac{\partial^3}{\partial t^2 \partial s} \text{cov}(X(t), X(s)) \\ \frac{\partial^3}{\partial t^2 \partial s} \text{cov}(X(t), X(s)) \end{matrix} \right) = \nabla^2 C$$

$$\left(\begin{matrix} \frac{\partial^4}{\partial t^2 \partial s} \text{cov}(X(t), X(s)) \\ \frac{\partial^4}{\partial t^2 \partial s} \text{cov}(X(t), X(s)) \end{matrix} \right) = \nabla^2 C$$