

Multiple Rank Notations

$$\bar{y}-1 = (i-1)(j-1) + j-1 + i-1$$

$$\Rightarrow \sum_{i,j \geq 1} (i-1) p_{ij}$$

$$\sum_{i,j \geq 1} p_{ij} = P(\geq 1, \geq 1)$$

$$\sum_{i,j \geq 1} (i-1)(j-1) p_{ij} + \sum_{i,j \geq 1} (j-1) p_{ij} + \sum_{i,j \geq 1} (i-1) p_{ij}$$

$$\leq \sum_{i,j \geq 1} i(j-1)(j-1) p_{ij} \quad \left\{ \sum_{i,j} = \sum_{i,j \geq 0} ! \right\}$$

$$+ \sum_{i,j} (j-1) p_{ij} + \sum_{i \geq 1, j \geq 0} (i-1) p_{ij}$$

$$= E[(\mu_N^1 (\mu_N^1 - 1) \mu_N^2 (\mu_N^2 - 1))] +$$

$$\sum_{j \geq 1} p_j (j-1) p_j + \sum_{i \geq 1} (i-1) p_i$$

$$\leq E[\mu_N^2 (\mu_N^2 - 1)] \quad \leftarrow E[\mu_N^1 (\mu_N^1 - 1)]$$