$\mathbb{E}[\phi(A_u)]$

just a 2D integral so easy to evaluate!

$$\mathbb{E}[M_u] = \int_{t\in M}\int_{u}^{\infty} p_{f(t)\mid f'(t)}(x\mid 0)\, f(x,t)\, dx$$

$$\begin{array}{c} f \\ f' \end{array} \sim N\begin{array}{c|cc} & A & B \\ \hline A & \sigma^2 & \Gamma^T \\ B & \Gamma & \Lambda \end{array}$$

$$\Rightarrow f\mid f' \sim N\left(\Gamma^T\Lambda^{-1}f',\ \sigma^2 - \Gamma^T\Lambda^{-1}\Gamma\right)$$

$$\Rightarrow f\mid f'=0 \sim N\left(0,\ \sigma^2(t) - \Gamma^T(t)\Lambda^{-1}(t)\Gamma(t)\right)$$

$$\left[e^{-(x-\mu)^2/2\sigma^2} \times -\sigma^2\right]_{-\infty}^{0} = e^{-\mu^2/2\sigma^2} \times -\sigma^2$$

for high u maybe can approximate this!

integral $\int_u^\infty p_{f(t)\mid f'(t)}(x\mid 0)\, e^{-\mu^2/2\sigma^2} - \sigma^2$ has a closed form!

other integral: $\int_u^\infty p_{f(t)\mid f'(t)}(x\mid 0) \times \mu \int_{-\infty}^{0} e^{-(x-\mu)^2/2\sigma^2}\, dx$

dunno IDK.