Claim: X PRDS $\Leftrightarrow$ P (set $P_i = 1 - F_i(X_i)$)

for strictly increasing F_i

is PRDS

$\Leftarrow$

Proof: Given an increasing set D.

$$\mathbb{P}(X \in D \mid X_i = x) = \mathbb{P}(P \in 1 - F(D) \mid P_i = p) \quad (*)$$

where $1 - F(D) = \left\{ \begin{pmatrix} 1 - F_1(d_1) \\ \vdots \\ 1 - F_m(d_m) \end{pmatrix} : (d_1, \dots, d_m)^T \in D \right\}$, $p = 1 - F_i(x)$

F strictly increasing so ~~$y \in 1-F(D)$~~

$1 - F(D)$ is decreasing $\Rightarrow$ $(*)$ is decreasing in p.

so increasing in x. $\Rightarrow$ X PRDS.

similarly vice versa. (using $A \mapsto F^{-1}(1 - A)$)

using the Lemma

Lemma ~~X PRDS $\Leftrightarrow$ $\forall$~~

~~for~~ X PRDS $\Leftrightarrow$ $\forall$ decreasing sets D.

$$\mathbb{P}(X \in D \mid X_i = x) \text{ is decreasing in } x$$

Proof: D decreasing $\Leftrightarrow$ D^c increasing.

~~as~~ ~~if~~ see $(*)$ (P1) for proof.