$$\mathbb{E}[\varphi(A_u)]$$

$$\cancel{p_f}\ p_{f|\nabla f}$$

$$= (2\pi)^{-(N+1)/2} \int_u^\infty p_f(\cancel{x} \mid \nabla f = 0)\, dx$$

$$\times \int_M \mathbb{E}[\det(-\nabla^2 f) \mid \nabla f = 0,\ f = x]\ \mathrm{Vol}_g$$

$$= (2\pi)^{-(N+1)/2} \int_u^\infty \cancel{p_f(x \mid \nabla f = 0}\ p_{f|\nabla f}(x \mid 0)\, dx$$

$$\times \int_M \sum_{l=0}^{\lfloor D/2 \rfloor} \frac{(-1)^l}{l!} \mathrm{Tr}^M(R^l) H_{D-2l}(x)$$

this in general depends on M!

$$= \sum_{l=0}^{\lfloor D/2 \rfloor} (2\pi)^{-(N+1)/2} \int_u^\infty p_{f|\nabla f}(x \mid 0)\, H_{D-2l}(x)\, dx$$

$$\times \frac{(-1)^l}{l!} \times \int_M \mathrm{Tr}^M(R^l)\ \mathrm{Vol}_g.$$

so can't separate these integrals like in the constant variance case!

$$\mathrm{cov}(f(x), \nabla f(x)) = \nabla \mathrm{cov}(f(x), f(x)) = \nabla \sigma^2$$