$$\sum_{i=1}^{k_n} a_{ni} \, \mathbb{E}\left[ \varepsilon_{ni}(u) \varepsilon_{ni}(v) \right] a_{ni}^T$$

if $\varepsilon_{ni}$ are identically distributed, then ~~[illegible]~~ i.d.

$$\mathbb{E}\left[ \varepsilon_{ni}(u) \varepsilon_{ni}(v) \right] = c_n(u,v)$$

So this equals $c_n(u,v) \underbrace{\sum_{i=1}^{k_n} a_{ni} a_{ni}^T}_{= A_n^T A_n}$

which converges to $c(u,v)\Sigma$. $\boxed{\hat{\varepsilon} = (I-P)\varepsilon Y}$

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$a_{ni} = \frac{X_i}{\sqrt{n}}$

$$\sum \hat{\varepsilon}_i(u) \hat{\varepsilon}_i(v) a_{ni} a_{ni}^T$$

$$= \frac{1}{n} \sum_{i=1}^{n} \boxed{\hat{\varepsilon}_i(u) \hat{\varepsilon}_i(v)} X_i X_i^T$$

need a triangular law as $\hat{\varepsilon}$ ~~$\hat{\varepsilon}$~~ the law of $\hat{\varepsilon}$ is changing!

Note: assume $X_i, \varepsilon_i$ are indep!

$$\mathbb{E}\left[ \hat{\varepsilon}_i(u) \hat{\varepsilon}_i(v) \right]$$

$$= \mathrm{Var}\left( (I-P)\varepsilon(u)_i \, (I-P)\varepsilon(v)_j \right)$$

note $\mathbb{E}\varepsilon = 0$ so can ignore the mean.