

$$f(s) | X$$

$X, f(s)$  jointly Gaussian.

$$\begin{pmatrix} f(s_1) \dots f(s_k) \end{pmatrix}^T | X \sim N \left( \begin{pmatrix} \sigma_X^{-2} \sum_{i=1}^k X_i \dots \end{pmatrix}, \begin{pmatrix} \sigma_X^{-2} \sum_{i=1}^k \sum_{j=1}^k \Sigma^*(s_i, s_j) \end{pmatrix} \right)$$

$$\text{cov}(X, f(s)) = \Sigma^*(s)$$

Claim:  $f(s) - \sigma_X^{-2} \sum_{i=1}^k \Sigma^*(s, s_i) X_i$  is independent of  $X$ .

proof:  $(f(s_1), \dots, f(s_k)) | X$

$$\Sigma_{A|B}$$

$$\Sigma_{A|B} = \begin{pmatrix} \Sigma^*(s_1) & \dots & \Sigma^*(s_k) \end{pmatrix}^T$$

$$\text{So } \sigma_X^{-2} \sum_{A|B} X = \sigma_X^{-2} \begin{pmatrix} \Sigma^*(s_1) X \\ \vdots \end{pmatrix}$$

so  $f(s) - \sigma_X^{-2} \sum \Sigma^*(s, s_i) X_i$  has no dependence on  $X$ .

only dependence is through the covariance