

$$\begin{pmatrix} \nabla f \\ \nabla^2 f \end{pmatrix} \sim N \left(0, \begin{pmatrix} \Lambda(t) & \cancel{A(t)} \\ A^T(t) & B(t) \end{pmatrix} \right) \begin{matrix} B \\ A \end{matrix}$$

~~*~~

$$\nabla^2 f | \nabla f \sim \text{Normal}$$

$$\text{mean: } \begin{pmatrix} \cancel{A^T(t)} & A(t)^T \end{pmatrix} \begin{pmatrix} \Lambda(t)^{-1} \\ \cancel{B(t)^{-1}} \end{pmatrix} \nabla f$$

$$\text{Variance: } \Lambda(t) - \cancel{A(t)^T}$$

$$B(t)^{-1} - A(t)^T B(t)^{-1} A(t)$$

$$\text{so } \nabla^2 f | \nabla f = 0 \sim N \left(0, \underbrace{B(t)^{-1} - A(t)^T B(t)^{-1} A(t)}_{= K(t)} \right)$$

