Bias in fMRI

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fMRI Model

1st level model

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This gives us a least squares estimate of β_j:

$$\hat{\beta}_j = (X_j^T X_j)^{-1} X_j^T Y_j$$

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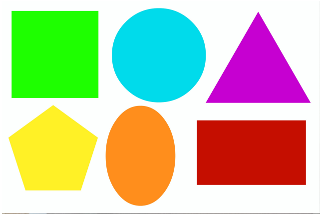

2nd level model

We have a vector of contrasts: $\beta_c = \beta_c = [c^T\beta_1, \ldots, c^T\beta_m]^T$. We would like to identify differences across groups of subjects, so we fit the model:

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and do the regression

$$\hat{\beta}_c = X_g\beta_g + \eta + (\hat{\beta}_c - \beta_c) = X_g\beta_g + \epsilon_g.$$

One and Two Sample Tests

Using the model $\hat{\beta}_c = X_g \beta_g + \epsilon_g$ we get an estimate: $\hat{\beta}_g$ for β. In the case that $X_g = 1_m$ is a vector of ones, we have

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$$\hat{\beta}_g = \frac{1}{n_1}\sum_{j=1}^{n_1} c^T\beta_j - \frac{1}{n_2}\sum_{j=n_1+1}^{n_1+n_2} c^T\beta_j.$$

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Either way we have $\hat{\beta}_g$ and we would like to use it to make inferences on β_g.

Winner's Curse

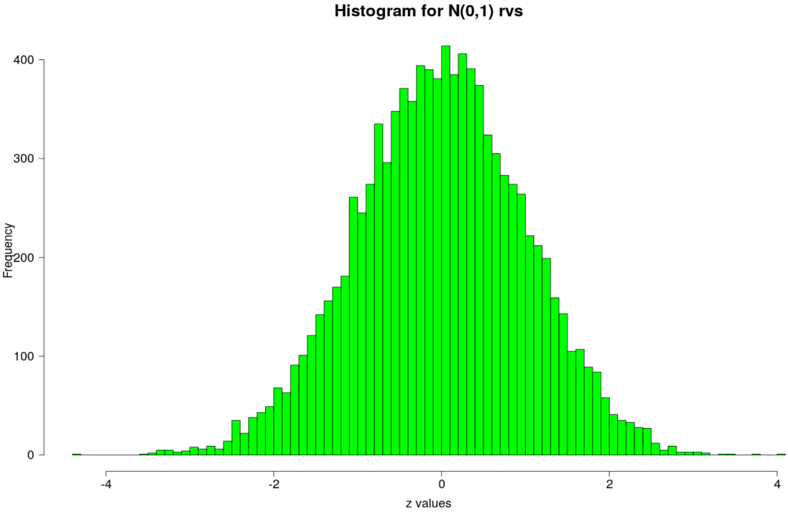
Examples

Dice Example: Imagine you roll 10 fair dice and at random some of them show a 6. If you rolled them again would you expect them still to be 6?

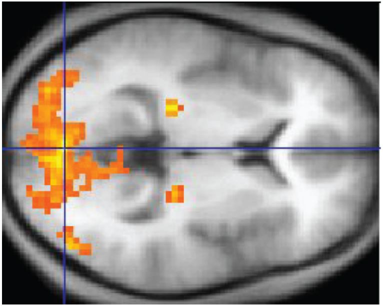
Figure 1: Some Dice

Mean 0 example

Suppose for now that we have 10000 independent N(0,1) random variables. Then the largest are biased estimates for the true mean.



The Winner's Curse in fMRI



Choose significant voxels based on some statistic and its maxima.

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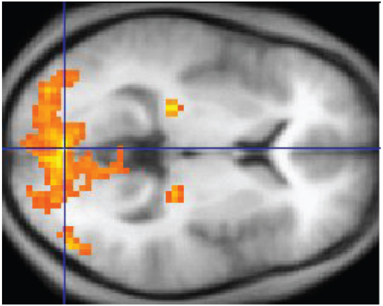

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(Vul et al., 2009)

Double dipping - circular inference

Split your subjects into two groups.

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Issues: Less data to estimate so higher variance.

Ideally would like to have a method where you didn't have to sacrifice this loss of the data. Seems like magic but is possible.!

Bootstrapping Algorithm

The bias

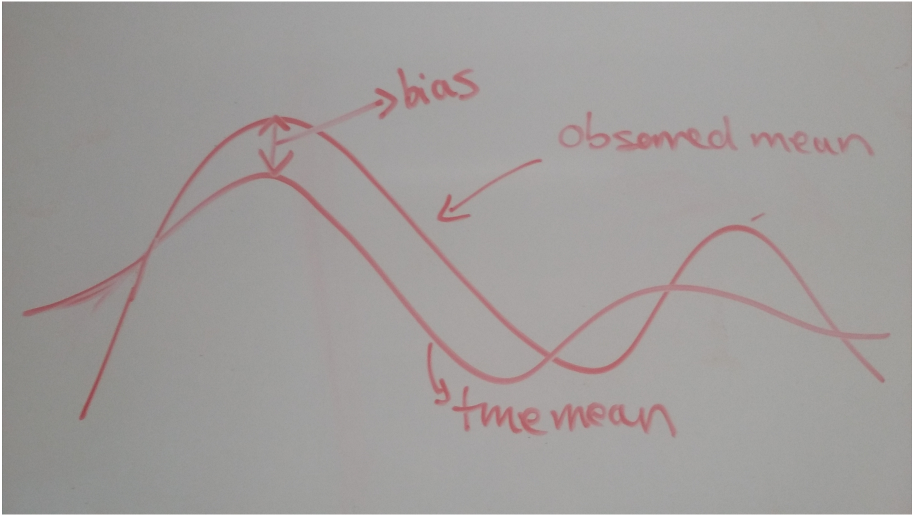

Figure 2: Comparing the observed mean against the true mean.

True Mean

meanimg.jpg

groupimg.jpg

meanimg.jpg

groupimg.jpg

The slices of the means plotted side by side. Note how you can match up the peaks reasonably well. Also note that the largest peak of the group mean is cut off indicating the winner's curse.

Algorithm

Algorithm 1 Bootstrap Bias Calculation

1: **Input**: Images: $Z_1, \dots, Z_m$ and some number of bootstrap iterations: B.

Algorithm

Algorithm 2 Bootstrap Bias Calculation

1: **Input**: Images: $Z_1, \ldots, Z_m$ and some number of bootstrap iterations: B.
2: Let $z = \frac{1}{m} \sum_{j=1}^{m} Z_j$.

Algorithm

Algorithm 3 Bootstrap Bias Calculation

1: **Input**: Images: $Z_1, \ldots, Z_m$ and some number of bootstrap iterations: B.
2: Let $z = \frac{1}{m}\sum_{j=1}^{m} Z_j$.
3: **for** $b = 1, \ldots, B$ **do**
4: Simulate $Z_1^*, \ldots, Z_m^*$ independently with replacement from $Z_1, \ldots, Z_m$.

Algorithm

Algorithm 4 Bootstrap Bias Calculation

Input: Images: $Z_1, \ldots, Z_m$ and some number of bootstrap iterations: B.
Let $z = \frac{1}{m}\sum_{j=1}^{m} Z_j$.
for $b = 1, \ldots, B$ **do**
    Simulate $Z_1^*, \ldots, Z_m^*$ independently with replacement from $Z_1, \ldots, Z_m$.
    Let $y = \frac{1}{m}\sum_{j=1}^{m} Z_j^*$.

Algorithm

Algorithm 5 Bootstrap Bias Calculation

1: **Input**: Images: $Z_1, \dots, Z_m$ and some number of bootstrap iterations: B.
2: Let $z = \frac{1}{m} \sum_{j=1}^{m} Z_j$.
3: **for** $b = 1, \dots, B$ **do**
4: Simulate $Z_1^*, \dots, Z_m^*$ independently with replacement from $Z_1, \dots, Z_m$.
5: Let $y = \frac{1}{m} \sum_{j=1}^{m} Z_j^*$.
6: Find the maximum y and let y_{max} be its value. Let i be a 3D vector of the coordinates of this maxima such that $y(i) = y_{max}$.
7: Let $\text{bias}(b) = y(i) - z(i)$.
8: **end for**

Algorithm

Algorithm 6 Bootstrap Bias Calculation

Input: Images: $Z_1, \ldots, Z_m$ and some number of bootstrap iterations: B.
Let $z = \frac{1}{m} \sum_{j=1}^{m} Z_j$.
for $b = 1, \ldots, B$ **do**
    Simulate $Z_1^*, \ldots, Z_m^*$ independently with replacement from $Z_1, \ldots, Z_m$.
    Let $y = \frac{1}{m} \sum_{j=1}^{m} Z_j^*$.
    Find the maximum y and let y_{max} be its value. Let i be a 3D vector of the coordinates of this maxima such that $y(i) = y_{max}$.
    Let $\text{bias}(b) = y(i) - z(i)$.
end for
Calculate $\hat{\delta} := \frac{1}{B} \sum_{b=1}^{B} \text{bias}(b)$.
end for
return $z_{max} - \hat{\delta}$.

The bias estimate

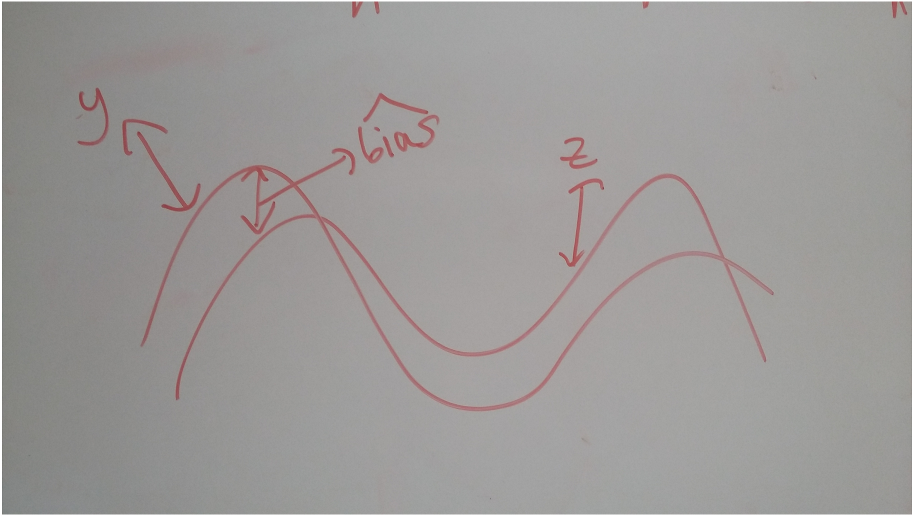

Figure 5: Comparing the bootstrapped mean against the sample mean.

Results

estsvstrue100.jpg

Bias

posterbiasplot.jpg

MSE

posterMSEplot.jpg

That's all folks.

Figure 9: Questions? :)

Bibliography