

1)

X_1 = Atlanta - Los Angeles

X_2 = Atlanta - New York

X_3 = Tulsa - Los Angeles

X_4 = Tulsa - New York

X_5 = Seattle - Los Angeles

X_6 = Seattle - New York

X_7 = Baltimore- Los Angeles

X_8 = Baltimore - New York

Objective Function: Minimize $\$8X_1 + \$5X_2 + \$4X_3 + \$7X_4 + \$5X_5 + \$6X_6 + \$4X_7 + \$6X_8$

$X_1 + X_2 \leq 600$

$X_3 + X_4 \leq 900$

$X_5 + X_6 \leq 500$

$X_7 + X_8 \leq 500$

$X_1 + X_3 + X_5 + X_7 \leq 800$

$X_2 + X_4 + X_6 + X_8 \leq 1200$

$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8 \geq 0$

	A	B	C	D	E	F
1						
2	Retail Stores	Manufacturing Plant			Demand	
3		Atlanta	Tulsa	Seattle		
4	Los Angeles	\$8.00	\$4.00	\$5.00	800	
5	New York	\$5.00	\$7.00	\$6.00	1200	
6	Capacity	600	900	500		
7						
8	Retail Stores	Manufacturing Plant			Demand	
9		Atlanta	Tulsa	Seattle		
10	Los Angeles				0	
11	New York				0	
12	Capacity	0	0	0		
13						
14						
15	Retail Stores	Manufacturing Plant			Demand	
16		Atlanta	Tulsa	Seattle		
17	Los Angeles	\$0.00	\$0.00	\$0.00		
18	New York	\$0.00	\$0.00	\$0.00		
19	Capacity				\$0.00	Total cost

	A	B	C	D	E	F
1						
2	Retail Stores	Manufacturing Plant			Demand	
3		Atlanta	Tulsa	Seattle		
4	Los Angeles	\$8.00	\$4.00	\$5.00	800	
5	New York	\$5.00	\$7.00	\$6.00	1200	
6	Capacity	600	900	500		
7						
8	Retail Stores	Manufacturing Plant			Demand	
9		Atlanta	Tulsa	Seattle		
10	Los Angeles	0	800	0	800	
11	New York	600	100	500	1200	
12	Capacity	600	900	500		
13						
14						
15	Retail Stores	Manufacturing Plant			Demand	
16		Atlanta	Tulsa	Seattle		
17	Los Angeles	\$0.00	\$3,200.00	\$0.00		
18	New York	\$3,000.00	\$700.00	\$3,000.00		
19	Capacity				\$9,900.00	Total cost

The total cost in choosing Seattle is \$9900.

	A	B	C	D	E	F
1						
2	Retail Stores	Manufacturing Plant			Demand	
3		Atlanta	Tulsa	Baltimore		
4	Los Angeles	\$8.00	\$4.00	\$4.00	800	
5	New York	\$5.00	\$7.00	\$6.00	1200	
6	Capacity	600	900	500		
7						
8	Retail Stores	Manufacturing Plant			Demand	
9		Atlanta	Tulsa	Baltimore		
10	Los Angeles	0	800	0	800	
11	New York	600	100	500	1200	
12	Capacity	600	900	500		
13						
14	Retail Stores	Manufacturing Plant			Demand	
15		Atlanta	Tulsa	Baltimore		
16	Los Angeles	\$0.00	\$3,200.00	\$0.00		
17	New York	\$3,000.00	\$700.00	\$3,000.00		
18	Capacity				\$9,900.00	Total cost

The total cost in choosing Baltimore is \$9900.

The cost is same in both the cases. Therefore, **the company can select either of the location.**

2)

Variables: $X1 = \text{Beef}$ $X2 = \text{Pork}$ $X3 = \text{Chicken}$ $X4 = \text{Turkey}$ **Objective function:**

Objective is to provide most economical combination of four meats to make this hot dog i.e.

$$\text{MinZ} = 0.76X1 + 0.82X2 + 0.64X3 + 0.58X4$$

Constraints:

$X1 + X2 + X3 + X4 = 0.125$ Constraints for Weight of each hot dog (2 ounce = 0.125 pound)

$32.5X1 + 54X2 + 25.6X3 + 6.4X4 \leq 6$ Constraints for grams of Fat

$210X1 + 205X2 + 220X3 + 172X4 \leq 27$ Constraints for grams of Cholesterol

$640X1 + 1055X2 + 780X3 + 528X4 \leq 100$ Constraints for grams of Calorie

$X1 \geq 0.25(X1 + X2 + X3 + X4)$ Constraint for each 2-ounce hot dog to be at least 25% beef

$X2 \geq 0.25(X1 + X2 + X3 + X4)$ Constraint for each 2-ounce hot dog to be at least 25% pork

$X1, X2, X3, X4 \geq 0$ Non-negativity constraints

	Cost/Pound	Calories/Pound	Fat (G/lb)	Cholesterol (G/lb)	Lb of meat/Hotdog
Beef(X1)	\$ 0.76	640	32.5	210	0.03125
Pork(X2)	\$ 0.82	1055	54	205	0.03125
Chicken(X3)	\$ 0.64	780	25.6	220	0
Turkey(X4)	\$ 0.58	528	6.4	172	0.0625
Objective Function (MinZ)	0.085625				
Constraints					
Weight Constraint	0.125	=	0.125		
Fat Constraint	3.103125	<=	6		
Cholesterol Constraint	23.71875	<=	27		
Calorie Constraint	85.96875	<=	100		
Constraints 25% beef	0.03125	>=	0.03125		
Constraints 25% pork	0.03125	>=	0.03125		

sij	1	2	3	4	5	Sub-Schedule Elimination Constraints				
1	0	20	15	8	6					
2	15	0	18	9	28	Sub-Schedule: 1-5				
3	24	23	0	13	13	x1,5	x5,1			
4	15	27	8	0	14	0	1	1	<=	1
5	8	17	24	15	0					
Total Surgical Setup Time						58				
xij	1	2	3	4	5					
1	0	1	0	0	0	1	1-2-4-3-5-1	Optimal Schedule (no Sub-Schedules)		
2	0	0	0	1	0	1				
3	0	0	0	0	1	1				
4	0	0	1	0	0	1				
5	1	0	0	0	0	1				
	1	1	1	1	1					

sij	1	2	3	4	5	6	7	8	9	10	Sub-Schedule Elimination Constraints									
1	0	9	12	26	11	24	12	13	17	15										
2	24	0	28	23	22	5	7	18	9	23	Sub-Schedule: 1-2-6-5									
3	19	30	0	30	15	22	25	15	28	15	x1,2	x2,1	x2,6	x6,2	x6,5	x5,6	x5,1	x1,5		
4	18	10	27	0	28	12	16	19	22	7	1	0	1	0	0	0	1	0	3	<= 3
5	5	16	11	7	0	25	27	30	23	15										
6	7	26	6	17	6	0	28	10	13	28	Sub-Schedule: 1-2-9									
7	23	26	20	20	24	30	0	16	18	27	x1,2	x2,1	x2,9	x9,2	x9,1	x1,9				
8	23	20	22	8	18	10	14	0	14	12	1	0	0	0	0	0	1	<=	2	
9	7	13	9	19	29	27	18	23	0	30										
10	16	10	11	11	28	26	6	11	12	0										
Total Surgical Setup Time											92									
xij	1	2	3	4	5	6	7	8	9	10										
1	0	1	0	0	0	0	0	0	0	0	1	1-2-6-8-4-10-7-9-3-5-1	Optimal Schedule (no Sub-Schedules)							
2	0	0	0	0	0	1	0	0	0	0	1									
3	0	0	0	0	1	0	0	0	0	0	1									
4	0	0	0	0	0	0	0	0	0	0	1									
5	1	0	0	0	0	0	0	0	0	0	1									
6	0	0	0	0	0	0	0	0	1	0	1									
7	0	0	0	0	0	0	0	0	0	1	0									
8	0	0	0	1	0	0	0	0	0	0	1									
9	0	0	1	0	0	0	0	0	0	0	1									
10	0	0	0	0	0	0	1	0	0	0	1									
	1	1	1	1	1	1	1	1	1	1										

4)

Goals	Description	Rank	Wt
Goal 1	Minimize overutilization of plastic	2	0.364
Goal 2	Minimize overutilization of metals	1	0.182

Goal 3	Minimize overutilization of rubber	1	0.182
Goal 4	Minimize overutilization of budget	0.5	0.091
Goal 5	Minimize underutilization of budget	0.5	0.091
Goal 6	Maximize available hours usage (i.e. Minimize underutilization)	0.5	0.091

Decision Variables:

X_1 = No. of Tiny Tanks produced

X_2 = No. of Tiny Trucks produced

X_3 = No. of Tiny Turtle produced

' O_i ' = Excess of right side of the goal w.r.t. the goal 'i' and

' U_i ' = Deficit of right side of the goal w.r.t. the goal 'i' for all $i = 1, 2, \dots, 6$

Objective Function:

Min $(0.364O_1 + 0.182O_2 + 0.182O_3 + 0.091O_4 + 0.091U_5 + 0.091U_6)$

Subject to,

$$1.5X_1 + 2.0X_2 + 1.0X_3 + U_1 - O_1 = 16000$$

$$0.5X_1 + 0.5X_2 + 1.0X_3 + U_3 - O_3 = 5000$$

$$0.3X_1 + 0.6X_2 + 0.0X_3 + U_2 - O_2 = 9000$$

$$2.0X_1 + 2.0X_2 + 1.0X_3 + U_6 - O_6 = 40$$

$$7.0X_1 + 5.0X_2 + 4.0X_3 + U_4 - O_4 = 164000$$

$$7.0X_1 + 5.0X_2 + 4.0X_3 + U_5 - O_5 = 164000$$

$$X_1, X_2, X_3, O_i, U_i \geq 0$$

5)

Let X_1 = thousands of dollars spent on TV ads

X_2 = thousands of dollars spent on radio ads

$$\text{Min } P_1(d_1^-) + P_2(d_2^+) + P_3(d_3^+) + P_4(d_4^-) + P_5(d_5^- + 2d_6^-)$$

Subject to:

$$10,000X_1 + 7,500X_2 + d_1^- - d_1^+ = 750,000 \quad (\text{At least 750,000 exposures})$$

$$X_1 + X_2 + d_2^- - d_2^+ = 100 \quad (\text{At most \$100,000 spent})$$

$$X_1 + d_3^- - d_3^+ = 70 \quad (\text{At most \$70,000 spent on TV ads})$$

$$10,000X_1 + 7,500X_2 + d_4^- - d_4^+ = 1,000,000 \quad (\text{Achieve 1 million exposures})$$

$$2,500X_1 + 3,000X_2 + d_5^- - d_5^+ = 250,000 \quad (\text{At least 250,000 18-21 exposures})$$

$$3,000X_1 + 1,500X_2 + d_6^- - d_6^+ = 250,000 \quad (\text{At least 250,000 25-30 exposures})$$

$$d_i^+ \geq 0 \quad \text{and} \quad d_i^- \geq 0 \quad \forall i \quad i = 1, 2, 3, 4, 5, 6$$

$$X_j \geq 0 \quad \forall j \quad j = 1, 2$$

Extra Credit:

a)

Pattern	3-ft	4-ft	5-ft
1	3	0	0
2	2	1	0
3	1	0	1
4	0	1	1
5	0	2	0
6	0	0	2

b)

Define x_i as the number of 10-ft boards cut into pattern type i , $i = 1, \dots, 6$. Then:

$$\begin{aligned}
 \min \quad & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \\
 \text{subject to} \quad & 3x_1 + 2x_2 + 1x_3 \geq 90 \\
 & 1x_2 + 1x_4 + 2x_5 \geq 60 \\
 & 1x_3 + 1x_4 + 2x_6 \geq 60 \\
 & x_1, x_2, x_3, x_4, x_5, x_6 \in \mathbb{Z}_+
 \end{aligned}$$

The minimal number of 10-ft boards to be cut is 83. This can be done by cutting 2 boards into pattern 1, 42 into pattern 2, 9 into pattern 5, and 30 into pattern 6 (among other ways -- there are multiple optimal solutions to this integer program). Incidentally, this solution produces 20 feet of scrap.

Original Length	10						
	Board width (in feet)						
Pattern #	3	4	5	Waste (ft)	Pattern	To Cut	
1	3	0	0	1	1	2	
2	2	1	0	0	2	42	
3	1	0	1	2	3	0	
4	0	1	1	1	4	0	
5	0	2	0	2	5	9	
6	0	0	2	0	6	30	
	3	4	5		Min Total Cut	83	
	90	60	60		Excess Scrap	20	
	>=	>=	>=		Excess Inventory	0	
Demand	90	60	60		Total Excess	20	
Leftover Inventory	0	0	0				

c)

Optimize a model with 3 rows, 6 columns and 9 nonzeros

Coefficient statistics:

Matrix range [1e+00, 3e+00]

Objective range [1e+00, 1e+00]

Bounds range [0e+00, 0e+00]

RHS range [6e+01, 9e+01]

Found heuristic solution: objective 120

Presolve time: 0.00s

Presolved: 3 rows, 6 columns, 9 nonzeros

Variable types: 0 continuous, 6 integer (0 binary)

Root relaxation: objective 8.250000e+01, 3 iterations, 0.00 seconds

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	82.50000	0	1	120.00000	82.50000	31.3%	-	0s
H	0	0			83.0000000	82.50000	0.60%	-	0s

Explored 0 nodes (3 simplex iterations) in 0.01 seconds

Thread count was 8 (of 8 available processors)

Optimal solution found (tolerance 1.00e-04)

Best objective 8.300000000000e+01, best bound 8.300000000000e+01, gap 0.0%

Minimum # of Rolls = 83.0

Cut 45.0 of pattern2

Cut 8.0 of pattern5

Cut 30.0 of pattern6

|

```

3 nWidths = 3;
4 nPatterns = 6;
5
6 widthsInPatterns = [
7 [3, 2, 1, 0, 0, 0],
8 [0, 1, 0, 1, 2, 0],
9 [0, 0, 1, 1, 0, 2]
10 ];
11 demand = [90, 60, 60];
12
13 model = Model("lumberyard")
14
15 patternsCut = []
16
17 for i in range(nPatterns):
18     patternsCut.append(model.addVar(vtype=GRB.INTEGER, obj=1, lb=0,
19     name="pattern%d" % (i+1)))
20
21 model.modelSense = GRB.MINIMIZE
22 model.update()
23
24 expr = LinExpr()
25 for i in range(nWidths):
26     expr = 0
27     for j in range(nPatterns):
28         expr += widthsInPatterns[i][j]*patternsCut[j]
29     model.addConstr(expr, GRB.GREATER_EQUAL, demand[i] , name="sum%d" % (i))
30 model.update()
31 model.write('lumberyard.lp')
32 model.optimize()
33
34 if model.Status == GRB.OPTIMAL:
35     print "\nMinimum # of Rolls =", model.ObjVal
36     for v in model.getVars():
37         if v.X > 0:
38             print "Cut", v.X, "of", v.VarName
39 |

```