

# Permutation and Combination

1. The number of ways of selecting two numbers from the set  $\{1, 2, \dots, 12\}$  whose sum is divisible by 3 is  
 (A) 66 (B) 16  
 (C) 6 (D) 22
  
1. **(D)**  
 Any natural number is either of the form  $3k$  or  $3k - 1$  or  $3k + 1$ . Sum of two numbers will be divisible by 3 if and only if either both are of the form  $3k$  or one is of the form  $3k - 1$  and other is of the form  $3k + 1$ . This can be done in  ${}^4C_2 + {}^4C_1 \times {}^4C_1 = 6 + 16 = 22$
  
2. The number of flags with three strips in order that can be formed using 2 identical red, 2 identical blue and 2 identical white strips is  
 (A) 24 (B) 20  
 (C) 90 (D) 8
  
2. **(A)**  
 No. required flags =  $3! \times \text{coefficient of } x^3 \text{ in } \left(1 + x + \frac{x^2}{2!}\right)^3 = 6 \times 4 = 24$
  
3. If  ${}^nP_r = {}^nP_{r+1}$  and  ${}^nC_r = {}^nC_{r-1}$ , then  $(n, r)$  are  
 (A) (2,3) (B) (3,2)  
 (C) (4,2) (D) (4,3)
  
3. **(B)**  

$$\begin{aligned} {}^np_r &= {}^np_{r+1} &\Rightarrow & n-r = 1 &\dots (1) \\ {}^nC_r &= {}^nC_{r-1} &\Rightarrow & 2r - 1 = n &\dots (2) \end{aligned}$$
 Solving (1) & (2) we get  $n = 3, r = 2$
  
4. The number of 9 digit numbers that can be formed by using the digits 1,2,3,4 and 5 is  
 (A)  ${}^9C_1 \times {}^8C_2$  (B)  $5^9$   
 (C)  ${}^9C_5$  (D)  $9!$
  
4. **(B)**
  
5. The number of diagonals that can be drawn by joining the vertices of an octagon is  
 (A) 28 (B) 48  
 (C) 20 (D) None of these
  
5. **(C)**  
 ${}^8C_2 - 8 = 20$
  
6. Number of ways in which 5 identical objects can be distributed in 8 persons such that no person gets more than one object is  
 (A) 8 (B)  ${}^8C_5$   
 (C)  ${}^8P_5$  (D) None of these
  
6. **(B)**  
 No. of ways = Coefficient of  $x^5$  in  $(x^0 + x')^8 = {}^8C_5$

7. Number of ways in which 7 girls & 7 boys can be arranged such that no two boys and no two girls are together is  
 (A)  $12!(2!)^2$  (B)  $7! 8!$   
 (C)  $2(7!)^2$  (D) None of these

7. **(C)**  
 Corresponding to one arrangement of the boys, there are two ways in which the girls can be arranged; position (1) remaining vacant is position (2) remaining vacant  
 (1) B – B – B – B – B – B (2)  $\Rightarrow 2(7!) (7!)$  ways.

8. The number of ordered triplets (a, b, c),  $a, b, c \in \mathbb{N}$ , such that  $a + b + c \leq 20$  is  
 (A) less than 100 (B) less than 1000  
 (C) equal to 1000 (D) more than 1000

8. **(D)**  
 $a + b + c \leq 20$   
 $\Rightarrow a + b + c + d = 20, a, b, c \geq 1, d \geq 0$   
 $\Rightarrow a^1 + b^1 + c^1 + d = 17, a^1, b^1, c^1, d \geq 0$   
 No. of solutions =  $^{17+4-1}C_{4-1} = ^{20}C_3 = 1140$

9. In a hockey tournament, a total of 153 matches were played. If each team played one match with every other team, the total number of teams that participated in the tournament were  
 (A) 20 (B) 18  
 (C) 16 (D) 14

9. **(B)**  
 Given  ${}^nC_2 = 153$   
 $\Rightarrow n^2 - n - 306 = 0 \Rightarrow n = 18.$

10. In how many ways can we distribute 5 different balls in 4 different boxes when order is not consider inside the boxes and empty boxes are not allowed  
 (A) 120 (B) 150  
 (C) 240 (D) None of these

10. **(C)**  
 ${}^5C_2 (4!) = 240$

11. The number of rectangles that you can find on a chessboard is  
 (A) 144 (B) 1296  
 (C) 256 (D) None of these.

11. **B**

12. The number of even divisors of 1008 is  
 (A) 23 (B) 21  
 (C) 20 (D) None of these.

12. **A**

13. If  $\frac{{}^nP_{r-1}}{a} = \frac{{}^nP_{r-1}}{b} = \frac{{}^nP_{r+1}}{c}$ , then

- (A)  $ab, b, ac$  are in A.P.  
(C)  $b^2 = a(b + c)$

- (B)  $ab, b, ac$  are in G.P.  
(D) None of these.

13. **C**

14. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is  
(A) at least 30 (B) at least 20  
(C) exactly 25 (D) None of these.

14. **C**

15. The number of arrangements of the letters of the word BANANA in which two N's do not appear adjacently.  
(A) 40 (B) 60  
(C) 80 (D) 100

15. **A**

16. The number of triangles which can be formed from 12 points out of which 7 are collinear is  
(A) 105 (B) 210  
(C) 175 (D) 185

16. **B**

17. The number of ways in which 5 male and 2 female members of a committee can be seated around a round table so that the two females are not seated together is  
(A) 480 (B) 600  
(C) 720 (D) 840

17. **B**

18. A set contains  $(2n + 1)$  elements. The number of subsets of the set which contain at most  $n$  elements is  
(A)  $2^n$  (B)  $2^{n+1}$   
(C)  $2^{n-1}$  (D)  $2^{2n}$

18. **D**

19. A polygon has 44 diagonals. The number of its sides is  
(A) 9 (B) 10  
(C) 11 (D) 12

19. **C**

20. Everybody in a room shakes hand with everybody else. The total number of hand shakes is 153. The total number of persons in the room is  
(A) 16 (B) 17  
(C) 18 (D) 19

20. C

21. Eight chairs are numbered from 1 to 8. Two women and three men wish to occupy one chair each. First the women chose the chairs from amongst chairs marked 1 to 4; then the men select the chairs from amongst the remaining. The number of possible arrangement is  
(A)  ${}^6C_3 \times {}^4C_4$  (B)  ${}^4P_2 \times {}^4P_3$   
(C)  ${}^4C_3 \times {}^4P_3$  (D)  ${}^4P_2 \times {}^6P_3$
22. In an examination there are 3 multiple choice questions and each question has 4 choices. Number of sequences in which a student can fail to get all answers correct is  
(A) 11 (B) 15  
(C) 80 (D) 63
23. A box contains two white balls, three black balls and four red balls. The number of ways in which three balls can be drawn from the box so that atleast one of the balls is black is  
(A) 74 (B) 84  
(C) 64 (D) 20
24. Number of subsets of a set containing n distinct objects is  
(A)  ${}^nC_1 + {}^nC_2 + \dots + {}^nC_n$  (B)  ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n$   
(C)  $2^n - 1$  (D)  $2^n + 1$
25. In a group of boys, two boys are brothers and in this group 6 more boys are there. In how many ways can they sit if the brothers are not to sit along with each other?  
(A)  $2 \times 6!$  (B)  ${}^7P_2 \times 6!$   
(C)  ${}^7C_2 \times 6!$  (D) none of these
26. In a 12 storey building 3 persons enter a lift cabin, It is known that they will leave the lift at different storeys. In how many ways can do so if the lift does not stop at the second storey.  
(A) 720 (B) 240  
(C) 120 (D) 36
27. The number of five digits telephone numbers having atleast one of their digits repeated is  
(A) 90000 (B) 100000  
(C) 30240 (D) 69760
28. The number of arrangement of the letters of the word 'BANANA' in which two N's donot appear adjacent is  
(A) 40 (B) 60  
(D) 80 (D) 100
29. The number of straight lines that can be formed by joining 20 points of which 4 points are collinear is  
(A) 183 (B) 186  
(C) 197 (D) 190
30. Number of numbers greater than 1000 but less than 4000 that can be formed by using the digit 0, 1, 2, 3, 4 when repetition is allowed is  
(A) 125 (B) 105  
(C) 375 (D) 625

31. There are 'n' seats round a table marked 1, 2, 3, ....., n. The number of ways in which m ( $\leq n$ ) persons can take seats is;  
 (A)  ${}^n p_m$  (B)  ${}^n C_m (m-1)!$   
 (C)  ${}^{n-1} C_m (m)!$  (D)  ${}^{n-1} p_{m-1}$
32. Number of divisors of the form  $4n + 2$ ,  $n \geq 0$  of the integer 240 is;  
 (A) 4 (B) 8  
 (C) 10 (D) none of these
33. Six identical coins are arranged in a row. The total number of ways in which the number of heads is equal to the number of tails is;  
 (A) 40 (B) 20  
 (C) 9 (D) 18
34. How many different nine digit numbers can be formed from the number 227788558 by rearranging it's digits so that odd digits occupy the even positions?  
 (A) 16 (B) 36  
 (C) 60 (D) none of these
35. The number of proper divisors of 1800 which are also divisible by 10 is;  
 (A) 16 (B) 18  
 (C) 17 (D) none of these
36. Let  $A = \{x : x \text{ is a prime and } x \leq 31\}$ . The number of different rational numbers whose numerator and denominator belong to A is;  
 (A) 110 (B) 109  
 (C) 111 (D) none of these
37. Let  $n_1$  and  $n_2$  be two, four digit numbers. How many such pairs can be there so that  $n_2$  can be subtracted from  $n_1$  without borrowing?  
 (A)  $45^3 \cdot 36$  (B)  $45^4$   
 (C)  $55^3 \cdot 45$  (D) none of these
38. Consider a rectangle ABCD. Three, four, five and six points are marked respectively on the sides AB, BC, CD and DA (none of them being the vertex of the rectangle). Number of triangles that can be formed with these points as vertices, so that there is atmost one angular point of the triangle on any side of rectangle ABCD is;  
 (A) 232 (B) 342  
 (C) 282 (D) none of these
39. Brijesh has 10 friends among who two are married to each other. She wishes to invite 5 of them for a party. If the married couple don't accept to attend the party, if invited together, then the number of different ways in which she can invite 5 friends is;  
 (A)  ${}^8 C_5 + 2 \cdot {}^9 C_5$  (B)  ${}^8 C_5 + {}^9 C_5 + {}^8 C_4$   
 (C)  ${}^8 C_5 + 2 \cdot {}^8 C_4$  (D) none of these
40. Let  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{r}$  be any vector such that  $\vec{r} \cdot \hat{i}$ ,  $\vec{r} \cdot \hat{j}$  and  $\vec{r} \cdot \hat{k}$  are positive integers. If  $3 \leq \vec{r} \cdot \vec{a} \leq 10$ , then number of all such vectors  $\vec{r}$  is;  
 (A)  ${}^{12} C_7$  (B)  ${}^{10} C_7$   
 (C)  ${}^{11} C_7$  (D) none of these

41. The number of distinct rational numbers  $x$  such that  $0 < x < 1$  and  $x = \frac{p}{q}$ , where  $p, q \in \{1, 2, 3, 4, 5, 6\}$  is  
 (A) 15 (B) 13  
 (C) 12 (D) 11
42. The total number of 5-digit numbers of different digits in which the digit in the middle is the largest is 9  
 (A)  $\sum_{n=4}^9 {}^nP_4$  (B)  $33 (3!)$   
 (C)  $30 (3!)$  (D) none of these
43. The number of 6-digit numbers in which the sum of digits is divisible by 5 is  
 (A) 180000 (B) 540000  
 (C)  $5 \times 10^5$  (D) none of these
44. The number of divisors of the form  $(4n+2)$  ( $n \geq 0$ ) of the integer 240 is  
 (A) 4 (B) 8  
 (C) 10 (D) 3
45. The number of non-negative integral solutions of  $a + b + c = n$ ,  $n \in \mathbb{N}$ ,  $n \geq 3$ , is  
 (A)  ${}^{(n-1)}C_2$  (B)  ${}^{(n-1)}P_2$   
 (C)  $n(n-1)$  (D) none of these
46. The number of ways to give 20 apples to 3 boys, each receiving at least 4 apples, is  
 (A)  ${}^{10}C_8$  (B) 90  
 (C)  ${}^{20}C_{20}$  (D) none of these
47. The position vector of a point  $P$  is  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ , where  $x, y, z \in \mathbb{N}$  and  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ . If  $\vec{r} \cdot \vec{a} = 10$ , the number of possible positions of  $P$  is  
 (A) 36 (B) 72  
 (C) 66 (D) none of these
48. In a plane there are two families of lines  $y = x + r$ ,  $y = -x + r$ , where  $r \in \{0, 1, 2, 3, 4\}$ . The number of squares of diagonals of the length 2 formed by the lines is  
 (A) 9 (B) 16  
 (C) 25 (D) none of these
49. There are  $n$  seats round a table numbered  $1, 2, 3, \dots, n$ . The number of ways in which  $m$  ( $\leq n$ ) person can take seats is  
 (A)  ${}^nP_m$  (B)  ${}^nC_m (m-1)!$   
 (C)  ${}^{(n-1)}P_{(m-1)}$  (D)  ${}^nC_{m+1} \times m!$
50. The rank of the word RACE if the words formed by letters of word RACE are arranged in the dictionary order is \_\_\_\_\_
51. The number of  $n$ -digit numbers, no two consecutive digits being the same, is  
 (A)  $n!$  (B)  $9!$   
 (C)  $9^n$  (D)  $n^9$
51. (C)

The first digit can be chosen in 9 ways (other than zero), the second can be chosen in 9 ways (any digit other than the first digit), the third digit can be chosen in 9 ways (any digit other than the second digit) and so on. Hence required number of numbers is  $9 \times 9 \times \dots \times 9$  (n times)  $= 9^n$ .

52. The number of divisors of 3630, which have a remainder of 1 when divided by 4, is  
 (A) 12 (B) 6  
 (C) 4 (D) none of these.

52. (B)

$$3630 = 2 \times 3 \times 5 \times 11^2.$$

Now a divisor will be of the form  $(4n+1)$  if divisor is form the help of  $(4n+1)$  type number or by  $(4n+3)$  types number taken even times.

Hence divisors are 1, 5,  $3 \times 11$ ,  $11^2$ ,  $5 \times 11^2$ ,  $5 \times 3 \times 11$ , i.e., 6.

53. The number of solutions of the inequation  ${}^{10}C_{x-1} > 3 \cdot {}^{10}C_x$  is  
 (A) 0 (B) 1  
 (C) 2 (D) 9

53. (C)

$${}^{10}C_{x-1} > 3 \cdot {}^{10}C_x \Rightarrow \frac{1}{11-x} > \frac{3}{x} \Rightarrow 4x > 33 \Rightarrow x \geq 9, \text{ but } x \leq 10.$$

So  $x = 9, 10$ . Hence there are two solutions

54. Triplet  $(x, y, z)$  is chosen from the set  $\{1, 2, 3, \dots, n\}$ , such that  $x \leq y < z$ . The number of such triplets is  
 (A)  $n^3$  (B)  ${}^nC_3$   
 (C)  ${}^nC_2$  (D) none of these

54. (D)

Any three numbers  $x, y, z$  from  $\{1, 2, 3, \dots\}$  can be chosen in  ${}^nC_3$  ways and we get unique triplet  $(x, y, z)$ ,  $x < y < z$ . Again any two numbers  $x, z$  can be chosen from  $\{1, 2, 3, \dots, n\}$  in  ${}^nC_2$  ways and we get the triplet  $(x, x, z)$ ,  $x < z$ . Hence total number of required triplets is  ${}^nC_2 + {}^nC_3$ .

55. If  $m$  and  $n$  are positive integers more than or equal to 2,  $m > n$ , then  $(mn)!$  is divisible by  
 (A)  $(m!)^n$  (B)  $(n!)^m$   
 (C)  $(m+n)!$  (D)  $(m-n)!$

55. (A), (B), (C), (D)

$\frac{(mn)!}{(m!)^n}$  is the number of ways of distributing  $mn$  distinct objects in  $n$  persons equally.

Hence  $\frac{(mn)!}{(m!)^n}$  is an integer  $\Rightarrow (m!)^n \mid (mn)!$ . Similarly  $(n!)^m \mid (mn)!$ . Further  $m+n < 2m \leq$

$mn \Rightarrow (m+n)! \mid (mn)!$  and  $m-n < m < mn$   
 $\Rightarrow (m-n)! \mid (mn)!$

56. Let  $S$  be the set of 6-digit numbers  $a_1a_2a_3a_4a_5a_6$  (all digits distinct) where  $a_1 > a_2 > a_3 > a_4 < a_5 < a_6$ . Then  $n(S)$  is equal to  
 (A) 210 (B) 2100  
 (C) 4200 (D) 420

56. (B)

First, 6 distinct digits can be selected in  ${}^{10}C_6$  ways. Now the position of smallest digit in them is fixed i.e. position 4. Of the remaining 5 digits, two digits can be selected in  ${}^5C_2$  ways. These two digits can be placed to the right of 4<sup>th</sup> position in one way only. The remaining three digits to the left of 4<sup>th</sup> position are in the required order automatically. So  $n(S) = {}^{10}C_6 \times {}^5C_2 = 210 \times 10 = 2100$ .

57. The number of positive integral solutions of the equation  $x_1 x_2 x_3 = 60$  is

- (A) 54 (B) 27  
(C) 81 (D) None of these.

57. (A)

Here  $x_1 x_2 x_3 = 2^2 \times 3 \times 5$ . Let number of two's given to each of  $x_1, x_2, x_3$  be  $a, b, c$ . Then  $a+b+c = 2, a, b, c \geq 0$

The number of integral solutions of this equations is equal to coefficient of  $x^2$  in  $(1-x)^{-3}$  i.e.  ${}^4C_2$  i.e. the available 2 two's can be distributed among  $x_1, x_2$  and  $x_3$  in  ${}^4C_2 = 6$  ways. Similarly, the available 1 three can be distributed among  $x_1, x_2, x_3$  in  ${}^3C_2 = 3$  ways (= coefficient of  $x$  in  $(1-x)^{-3}$ )

$\therefore$  Total number of ways =  ${}^4C_2 \times {}^3C_2 \times {}^3C_2 = 6 \times 3 \times 3 = 54$  ways.

58. For the series 21, 22, 23, . . . ,  $k-1, k$ ; the A.M. and G.M. of the first and last number exist in the given series. If 'k' is a three digit number, then 'k' can attain

- (A) 5 values (B) 6 values  
(C) 2 values (D) 4 values

58. (C)

21, 22, 23, . . . ,  $k-1, k$

$$\text{A.M.} = \frac{21+k}{2}, \text{G.M.} = \sqrt{21.k}$$

$\Rightarrow k = 21.\lambda^2, \lambda \in \mathbb{I}$  also  $100 \leq k \leq 999$  and  $k$  should be odd

$\Rightarrow \frac{100}{21} \leq \lambda^2 \leq \frac{999}{21} \Rightarrow 4.76 \leq \lambda^2 \leq 47.57 \Rightarrow \lambda = 3, 4, 5, 6$  but  $\lambda$  should be odd  $\Rightarrow$  odd  $\lambda = 3, 5 \Rightarrow$  'k' can assume 2 different values.

59. Consider a set  $\{1, 2, 3, \dots, 100\}$ . The number of ways in which a number can be selected from the set so that it is of the form  $x^y$ , where  $x, y \in \mathbb{N}$  and  $\geq 2$ , is

- (A) 12 (B) 16  
(C) 5 (D) 11

59. (A)

Perfect square =  $\left[ \sqrt{100} \right] - 1 = 9$  (excluding one)

Perfect cubes =  $\left[ 100^{1/3} \right] - 1 = 3$

Perfect 4<sup>th</sup> powers =  $\left[ 100^{1/4} \right] - 1 = 3$

Perfect 5<sup>th</sup> powers =  $\left[ 100^{1/5} \right] - 1 = 1$

Perfect 6<sup>th</sup> powers =  $\left[ 100^{1/6} \right] - 1 = 1$



Now, perfect  $4^{\text{th}}$  powers have already been counted in perfect squares and perfect  $6^{\text{th}}$  powers have been counted with perfect squares as well as with perfect cubes. Hence the total ways =  $9 + 3 + 1 - 1 = 12$ .

60. Number of natural numbers  $< 2 \cdot 10^4$  which can be formed with the digits 1, 2, 3 only is equal to

(A)  $\frac{3^6 + 2 \cdot 3^4 - 3}{2}$

(B)  $\frac{3^6 - 2 \cdot 3^4 + 3}{2}$

(C)  $\frac{3^7 - 1}{2}$

(D) none of these

60. (A)

Total number of numbers will be equal to the sum of numbers of all possible 1-digit, 2-digit, 3-digit, 4-digit and 5-digit numbers.  $\Rightarrow$  Total number of numbers =  $3 + 3^2 + 3^3 + 3^4 + 3^5$   
 $= \frac{3(3^5 - 1)}{2} + 3^4 = \frac{3^6 + 2 \cdot 3^4 - 3}{2}$ .

61. The sum of the factors of  $7!$ , which are odd and are of the form  $3t + 1$  where  $t$  is a whole number, is

(A) 10

(B) 8

(C) 9

(D) 15

61. (B)

$$7! = 2^4 \times 3^2 \times 5 \times 7$$

Since the factor should be odd as well as of the form  $3t + 1$ , the factor cannot be a multiple of either 2 or 3. So the factors may be 1, 5, 7 and 35 of which only 1 and 7 are of the form  $3t + 1$ , whose sum is 8.

62. Number of positive integers  $n$  less than 15, for which  $n! + (n+1)! + (n+2)!$  is an integral multiple of 49, is

(A) 3

(B) 4

(C) 5

(D) 6

62. (A)

$$n! + (n+1)! + (n+2)! = n! \{ 1 + n + 1 + (n+2)(n+1) \} = n!(n+2)^2$$

$\Rightarrow$  Either 7 divides  $n+2$  or 49 divides  $n! \Rightarrow n = 5, 12, 14$ .

63. Let  $n$  be a positive integer with  $f(n) = 1! + 2! + 3! + \dots + n!$  and  $P(x), Q(x)$  be polynomials in  $x$  such that  $f(n+2) = P(n)f(n+1) + Q(n)f(n)$  for all  $n \geq 1$ . Then

(A)  $P(x) = x + 3$

(B)  $Q(x) = -x - 2$

(C)  $P(x) = -x - 2$

(D)  $Q(x) = x + 3$

63. (A), (B)

$$f(n) = 1! + 2! + 3! + \dots + n!$$

$$f(n+1) = 1! + 2! + 3! + \dots + (n+1)!$$

$$f(n+2) = 1! + 2! + 3! + \dots + (n+2)!$$

$$f(n+2) - f(n+1) = (n+2)! = (n+2)(n+1)! = (n+2)[f(n+1) - f(n)]$$

$$\Rightarrow f(n+2) = (n+3)f(n+1) - (n+2)f(n) \Rightarrow P(x) = x + 3, Q(x) = -x - 2$$

64. The number of ordered pairs  $(m, n)$  ( $m, n \in \{1, 2, \dots, 20\}$ ) such that  $3^m + 7^n$  is a multiple of 10, is

- (A) 100 (B) 200  
(C)  $4! \times 4!$  (D) none of these

64. (A)

The last digit of powers of 3 will be 3, 9, 7, 1 and it repeats in the same order. The last digit of powers of 7 will be 7, 9, 3, 1 and it repeats in same order. Now  $3^m + 7^n$  will be a multiple of 10 as  $3+7$ ,  $9+1$ ,  $7+3$ ,  $1+9$ .

$\Rightarrow (m, n)$  will be of the form  $(4t+1, 4k+1)$ ,  $(4t+2, 4k)$ ,  $(4t+3, 4k+3)$  and  $(4t, 4k+2)$ .

So total number of ways =  $5 \times 5 + 5 \times 5 + 5 \times 5 + 5 \times 5 = 100$

65. The number of four-digit natural numbers in which odd digits occur at even places and even digits occur at odd places and digits are in increasing order from left to right,

- (A) is less than 36 (B) is greater than 100  
(C) lies between 60 and 100 (D) none of these.

65. (A)

I	II	III	IV
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Two distinct odd digits for the second and fourth places can be selected in  ${}^4C_2 = 6$  ways (since we cannot take 1, as first digit will be at least 2). Now these can be arranged in increasing order in one way only. Similarly two distinct even digits for the first and third places can be selected in  ${}^4C_2 = 6$  ways (since we cannot take 0). Now these can be arranged in increasing order in one way only.

Now total number of ways of filling the four places is  $6 \times 6 = 36$ .

But this contains the numbers of the type 6385 which are not needed. So number of such numbers will be less than 36.

66. The number of permutations of the letters of the word HINDUSTAN such that neither the pattern 'HIN' nor 'DUS' nor 'TAN' appears, are

- (A) 166674 (B) 169194  
(C) 166680 (D) 181434

66. (B)

Total number of permutations =  $\frac{9!}{2!}$

Number of those containing 'HIN' =  $7!$

Number of those containing 'DUS' =  $\frac{7!}{2!}$

Number of those containing 'TAN' =  $7!$

Number of those containing 'HIN' and 'DUS' =  $5!$

Number of those containing 'HIN' and 'TAN' =  $5!$

Number of those containing 'TAN' and 'DUS' =  $5!$

Number of those containing 'HIN', 'DUS' and 'TAN' =  $3!$

Required number =  $\frac{9!}{2!} - \left( 7! + 7! + \frac{7!}{2} \right) + 3 \times 5! - 3! = 169194$ .

67. Nine hundred distinct N-digit numbers are to be formed by using 6, 8 and 9 only. The smallest value of N for which this is possible, is

- (A) 6 (B) 7  
(C) 8 (D) 9

67. **(B)**  
 $(3)^6 = 729 < 900$  and  $(3)^7 = 2187 > 900$
68.  $y = x + r$  and  $y = -x + r$  where  $r$  takes all decimal digits. Then the number of squares in  $xy$  plane formed by these lines with diagonals of 2 units length are  
 (A) 81 (B) 100  
 (C) 64 (D) 49
68. **(C)**  
 Draw all ten lines  $y = x + r$  and other ten lines  $y = -x + r$ . We can observe that required squares are  $8^2 = 64$
69. Let  $y$  be an element of the set  $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$  and  $x_1, x_2, x_3$  be positive integers such that  $x_1 x_2 x_3 = y$ , then the number of positive integral solutions of  $x_1 x_2 x_3 = y$  is  
 (A) 64 (B) 27  
 (C) 81 (D) None of these
69. **(A)**  
 The number of solutions of the given equation is the same as the number of solution of the equation  $x_1 x_2 x_3 x_4 = 30 = 2 \times 3 \times 5$  ( here  $x_4$  is dummy variable )  
 Hence number of solutions is  $4^3 = 64$ .
70. The number of ways in which we can choose 2 distinct integers from 1 to 100 such that difference between them is at most 10 is  
 (A)  $^{100}C_2 - ^{90}C_2$  (B)  $^{100}C_{98} - ^{90}C_{88}$   
 (C)  $^{100}C_2 - ^{90}C_{88}$  (D) None of these
70. **(A), (B), (C)**  
 Let the chosen integers be  $x_1$  and  $x_2$ .  
 Let there be  $a$  integer before  $x_1$ ,  $b$  integer between  $x_1$  and  $x_2$  and  $c$  integer after  $x_2$   
 $\therefore a + b + c = 98$ . Where  $a \geq 0$ ,  $b \geq 10$ ,  $c \geq 0$   
 Now if we consider the choices where difference is at least 11, then the number of solution is  $^{88+3-1}C_{3-1} = ^{90}C_2$   
 $\therefore$  Number of ways in which  $b$  is less than 10 is  $^{100}C_2 - ^{90}C_2$  which is equal to (A), (B) and (C) option.
71. How many words can be formed by taking four different letters of the word MATHEMATICS?  
 (A) 796 (B) 1680 (C) 2454 (D) 18
72. In an examination there are 3 multiple choice questions and each question has 4 choices. Number of sequences in which a student can fail to get all answers correct is  
 (A) 11 (B) 15 (C) 80 (D) 63
73. Number of ways in which 6 persons can be seated in a row so that two particular persons are never seated together is equal to  
 (A) 480 (B) 72 (C) 120 (D) 240
74. The number of ways in which  $N$  positive signs and  $n$  negative sign ( $N \geq n$ ) may be placed in a row so that no two negative signs are together is  
 (A)  $^N C_n$  (B)  $^{N+1} C_n$  (C)  $N!$  (D)  $^{N+1} P_n$

75. The number of diagonals of hexagon is  
 (A) 3 (B) 6 (C) 9 (D) 12
76. The number of 10 digits that can be written by using the digits 1 and 2 is  
 (A)  $10 \times 10$  (B)  ${}^{10}P_2$  (C)  $2^{10}$  (D)  $10!$
77. The number of all the odd divisors of 3600 is  
 (A) 45 (B) 4 (C) 18 (D) 9
78. Number of all four digit numbers having different digits formed of the digits 1, 2, 3, 4 and 5 and divisible by 4 is  
 (A) 24 (B) 30 (C) 125 (D) 100
79. Let A be the set of 4-digit numbers  $a_1a_2a_3a_4$  where  $a_1 > a_2 > a_3 > a_4$ , then  $n(A)$  is equal to  
 (A) 126 (B) 84 (C) 210 (D) none of these
80. A polygon has 44 diagonals, then  $n$  is equal to  
 (A) 10 (B) 11 (C) 12 (D) 13
81.  ${}^nC_r + 2 \cdot {}^nC_{r+1} + {}^nC_{r+2}$  is equal to ( $2 \leq r \leq n$ )  
 (A)  $2 \cdot {}^nC_{r+2}$  (B)  ${}^{n+1}C_{r+1}$  (C)  ${}^{n+2}C_{r+2}$  (D) none of these
82. Number of ways in which 6 persons can be seated around a table so that two particular persons are never seated together is equal to  
 (A) 480 (B) 72 (C) 120 (D) 240
83. How many words can be made from the letters of the word INSURANCE, if all vowels come together  
 (A) 18270 (B) 17280 (C) 12780 (D) none of these
84. If  $a, b, c, d, e$  are prime integers, then the number of divisors of  $ab^2c^2de$  excluding 1 as a factor, is  
 (A) 94 (B) 72 (C) 36 (D) 71
85. The number of 5-digit numbers in which no two consecutive digits are identical is  
 (A)  $9^2 \times 8^3$  (B)  $9 \times 8^4$  (C)  $9^5$  (D) None of these