

LCD

1. $\lim_{x \rightarrow 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ is
 (A) 2 (B) -2 (C) 1/2 (D) -1/2
2. $f(x) = \begin{cases} ax^2 + bx + c, & |x| > 1 \\ x + 1, & |x| \leq 1 \end{cases}$. If $f(x)$ is continuous for all values of x , then;
 (A) $b = 1, a + c = 0$ (B) $b = 0, a + c = 2$
 (C) $b = 1, a + c = 1$ (D) none of these
3. The equation of the tangent to the curve $f(x) = 1 + e^{-2x}$ where it cuts the line $y = 2$ is
 (A) $x + 2y = 2$ (B) $2x + y = 2$
 (C) $x - 2y = 1$ (D) $x - 2y + 2 = 0$
4. $\lim_{x \rightarrow \infty} \frac{x + \sin x}{x - \sin x} = \dots\dots\dots$
5. $\lim_{x \rightarrow \infty} \left(\frac{x+3}{1+x} \right)^{x+3} = \dots\dots\dots$
6. $\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$, then $a = \dots\dots\dots$ $b = \dots\dots\dots$
7. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$ is equal to
 (A) π (B) 1/4
 (C) 1/2 (D) 1
8. $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 1}}{2x + 1}$ is equal to
 (A) 1 (B) 0
 (C) -1 (D) 1/2
9. If $f(x) = (1 - x^n)^{1/n}$, $0 < x < 1$, n being an odd positive integer and $h(x) = f(f(x))$, then $h'\left(\frac{1}{2}\right)$ is equal to
 (A) 2^n (B) 2
 (C) $n \cdot 2^{n-1}$ (D) 1
10. Among $\lim_{x \rightarrow 0} \sec^{-1}\left(\frac{x}{\sin x}\right) \dots\dots (1)$
 and $\lim_{x \rightarrow 0} \sec^{-1}\left(\frac{\sin x}{x}\right) \dots\dots (2)$
 (A) (1) exists, (2) does not exist (B) (1) does not exist, (2) exists
 (C) both (1) and (2) exist (D) neither (1) nor (2) exists

11. A function $f(x)$ is defined as $f(x) = \begin{cases} x^2 - 3x + a, & x < 1 \\ -2, & x = 1 \\ bx + 3, & x > 1 \end{cases}$
- What are the values of a and b respectively such that $f(x)$ is continuous at $x = 1$.
 (A) 1, -2 (B) 0, -5 (C) -1, 0 (D) 2, -3
12. Given a function $f(x)$ continuous $\forall x \in \mathbb{R}$ such that $\lim_{x \rightarrow 0} \left[f(x) + \log \left(1 - \frac{1}{e^{f(x)}} \right) - \log(f(x)) \right] = 0$, then $f(0)$ is
 (A) 0 (B) 1 (C) 2 (D) 3
13. The value of $\lim_{x \rightarrow \infty} x \cos \left(\frac{\pi}{4x} \right) \sin \left(\frac{\pi}{4x} \right)$ is
 (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 1 (D) π
14. The value of $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$ is
 (A) $\log_e \left(\frac{a}{b} \right)$ (B) $\log_e \left(\frac{b}{a} \right)$ (C) $\log_e(ab)$ (D) none of these
15. If $f(x) = \begin{cases} mx + 1, & x \leq \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then
 (A) $m = 1, n = 0$ (B) $m = \frac{n\pi}{2} + 1$ (C) $n = \frac{m\pi}{2}$ (D) $m = n = \frac{\pi}{2}$
16. The value of $\lim_{x \rightarrow \infty} \frac{\sqrt{1+x^4} - (1+x^2)}{x^2}$ is equal to
 (A) 0 (B) -1
 (C) 2 (D) 1
17. $\lim_{x \rightarrow 0} \frac{e^{\tan x} - e^x}{\tan x - x}$ is equal to
 (A) 1 (B) e
 (C) -1 (D) 0
18. The function $f(x) = \frac{\tan(\pi - [x - \pi])}{1 + [x]^2}$, where $[.]$ denotes greatest integer function, is
 (A) discontinuous at some x
 (B) continuous at all x , but $f'(x)$ does not exist for some x
 (C) $f'(x)$ exists for all x
 (D) none of these
19. If the function $f(x) = \begin{cases} Ax - B, & x \leq 1 \\ 3x, & 1 < x < 2 \\ Bx^2 - A, & x \geq 2 \end{cases}$ be continuous at $x = 1$ and discontinuous at $x = 2$, then
 (a) $A = 3 + B, B \neq 3$ (b) $A = 3 + B, B = 3$ (c) $A = 3 + B$ (d) none of these

41. If $f(x) = \begin{cases} ax^2 + b & , x \leq 1 \\ bx^2 + ax + c & , x > 1 \end{cases}$, $b \neq 0$. Then $f(x)$ is continuous and differentiable at $x = 1$ if
 (a) $c = 0, a = 2b$ (b) $a = b, c \in \mathbb{R}$
 (c) $a = b, c = 0$ (d) $a = b, c \neq 0$.
42. If $f(x) = x^3 \operatorname{sgn} x$, then
 (a) f is derivable at $x = 0$ (b) f is continuous, but not derivable at $x = 0$
 (c) LHD at $x = 0$ is 1 (d) RHD at $x = 0$ is 0.
43. If $f(x) = (x - x_0) \phi(x)$ and $\phi(x)$ is continuous at $x = 0$, then $f'(x_0)$ is equal to
 (a) $\phi'(x_0)$ (b) $\phi(x_0)$
 (c) $x_0 \phi(x_0)$ (d) none of these.
44. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]} & \text{for } [x] \neq 0 \\ 0 & \text{for } [x] = 0 \end{cases}$ where $[x]$ denotes greatest integer function, then $\lim_{x \rightarrow 0} f(x) =$
 (A) 1 (B) 0
 (C) -1 (D) doesn't exist
45. If the function $f(x) = \begin{cases} \frac{\sin(2x)^2}{x^2} + e^{-x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous, then k is
 (A) 2 (B) 3
 (C) 4 (D) 5.
46. For a function $y = f(x)$, $\frac{dy}{dx} = (x-1)(x+2)$. Find the point of local maximum and minimum for the function $y = f(x)$.

47. Find the function $y = f(x)$ for the above function if it is given that $y = 2$ at $x = 0$.

48. The value of derivative of $f(x) = |x-1| + |x-3|$ at $x = 2$ is
 (A) -2 (B) 0
 (C) 2 (D) not defined
49. The function $f(x) = |\sin x| - 1$ is
 (A) continuous everywhere (B) not differentiable at $x = \frac{\pi}{3}$
 (C) differentiable at $x = 0$ (D) differentiable everywhere
50. Let $f(x) = \begin{cases} 3x - 4, & 0 \leq x \leq 2 \\ 2x + \lambda, & 2 < x \leq 3 \end{cases}$, if $f(x)$ is continuous at $x = 2$, then λ is
 (A) -1 (B) -2
 (C) 2 (D) none of these
51. The number of points at which the function $f(x) = \frac{x}{\log|x|}$ is discontinuous is
 (A) 1 (B) 2

(C) 3

(D) 4

52. The number of values of $x \in [0, 2]$ at which the real function $f(x) = |x - 1/2| + |x - 1| + \tan x$ is not differentiable is

(A) 2

(B) 3

(C) 1

(D) 0

LEVEL-II

1. The function $(x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$ is not differentiable at
 (A) -1 (B) 0 (C) 1 (D) 2
2. For $x \in \mathbb{R}$, $\lim_{x \rightarrow \infty} \left(\frac{x-3}{x+2} \right)^x$ is
 (A) e (B) e^{-1} (C) e^{-5} (D) e^5
3. $\lim_{x \rightarrow \frac{\pi}{2}} \left[\frac{6 \cos x}{2x - \pi} \right]$, where $[.]$ denotes the greatest integer function, is equal to;
 (A) -3 (B) -4 (C) -2 (D) none of these
4. Let $f(x) = (\tan x)^{\frac{1}{x - \frac{\pi}{4}}}$ $\forall x \in (0, \pi/2) \sim \{\pi/4\}$, then the value of $f(\pi/4)$ such that $f(x)$ becomes continuous at $x = \frac{\pi}{4}$ is equal to;
 (A) e (B) \sqrt{e} (C) $\frac{1}{\sqrt{e}}$ (D) e^2
5. Let $f(x) = [5 + 3 \sin x] \forall x \in \mathbb{R}$. Then total number of points of discontinuity of $f(x)$ in $[0, \pi]$ is equal to;
 (A) 5 (B) 6 (C) 7 (D) 4
6. $f(x) = \sin^{-1}(\sin x)$, $x \in [-2\pi, 2\pi]$. Total number of critical points of $f(x)$ is ;
 (A) 3 (B) 4 (C) 5 (D) 2
7. If the line $ax + by + c = 0$ is normal to the curve $x^2 y + 5 = 0$ then
 (A) $a > 0, b > 0$ (B) $b > 0, a < 0$
 (C) $a < 0, b < 0$ (D) $b < 0, a > 0$
8. The maximum value of $f(x) = |x \ln x|$ in $x \in (0, 1)$ is;
 (A) $1/e$ (B) e
 (C) 1 (D) none of these
9. $f(x) = 3x^3 + 4e^x - k$ is always increasing then value of $k =$
 (A) 2 (B) $-4/9$
 (C) $4/9$ (D) all of these
10. $\lim_{x \rightarrow 2} \{ [2 - x] + [x - 2] - x \}$ is
 (A) 0 (B) 3
 (C) -3 (D) does not exist
11. Let $f(x)$ be a twice differentiable function and $f''(0) = 2$ then $\lim_{x \rightarrow 2} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is
 (A) 6 (B) 1
 (C) 12 (D) 3
12. Let $h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$ for all real values of x then
 (A) h is \uparrow whenever $f(x)$ is \downarrow (B) h is \uparrow whenever $f(x)$ is \downarrow 0

(C) h is \downarrow whenever f is \downarrow

(D) nothing can be said in general

13. Let $f'(x) > 0$, $g'(x) < 0$ for all $x \in \mathbb{R}$, then

(A) $f\{g(x)\} > f\{g(x+1)\}$

(B) $f\{g(x)\} > f\{g(x-1)\}$

(C) $g\{f(x)\} > g\{f(x+1)\}$

(D) $g\{f(x)\} > g\{f(x-1)\}$

14. $\lim_{x \rightarrow \infty} \frac{\ln[x]}{1+x} = \dots\dots\dots$

[.] G. I. F

15. $\lim_{n \rightarrow \infty} (3^n + 5^n + 7^n)^{\frac{1}{n}} = \dots\dots\dots$

16. If α, β are the roots of $ax^2 + bx + c = 0$ then $\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2} = \dots\dots\dots$

17. $\lim_{x \rightarrow 1} (1 - x + [x - 1] + [1 - x]) = \dots\dots\dots$

18. $f(x) = \sin^{-1}(\cos x)$ then points of nondifferentiability between $[0, 2\pi] = \dots\dots\dots$

19. Let $f(x+y) = f(x) \cdot f(y)$ for all x & y , if $f(5) = 2$ and $f'(0) = 3$, then $f'(5) = \dots\dots\dots$

20.
$$f(x) = \begin{cases} \frac{a|x^2 - x - 2|}{2 + x - x^2}, & x < 2 \\ b, & x = 2 \text{ (where [.] denotes the greatest integer function).} \\ \frac{x - [x]}{x - 2}, & x > 2 \end{cases}$$
 If $f(x)$

is continuous at $x = 2$, then

(A) $a = 1, b = 2$

(B) $a = 1, b = 1$

(C) $a = 0, b = 1$

(D) $a = 2, b = 1$

21. Let $f(x) = \begin{cases} -1, & x \leq 0 \\ 0, & x = 0 \text{ and } g(x) \sin x + \cos x, \\ 1, & x > 0 \end{cases}$ then points of discontinuity of $f\{g(x)\}$ in $(0,$

$2\pi)$ is

(A) $\left\{\frac{\pi}{2}, \frac{3\pi}{4}\right\}$

(B) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

(C) $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$

(D) $\left\{\frac{5\pi}{4}, \frac{7\pi}{3}\right\}$

22. If α and β are the roots of $ax^2 + bx + c = 0$ then $\lim_{x \rightarrow \alpha} (1 + ax^2 + bx + c)^{1/(x-\alpha)}$ is

(A) $a(\alpha - \beta)$

(B) $\ln|a(\alpha - \beta)|$

(C) $e^{a(\alpha - \beta)}$

(D) $e^{a|\alpha - \beta|}$

23. $\lim_{x \rightarrow \pi/4} \frac{\sqrt{2} \cos x - 1}{\cot x - 1}$ is equal to

(A) $1/\sqrt{2}$
(C) $\frac{1}{2\sqrt{2}}$

(B) $1/2$
(D) 1

24. The function $f(x) = [x]^2 - [x^2]$ where $[y]$ denotes the greatest integer less than or equal to y , is discontinuous at

(A) all integers
(B) all integers except 0 and 1
(C) all integers except 0
(D) all integers except 1

25. If the derivative of $f(x)$ w.r. to x is $\frac{1 - \sin^2 x}{2f(x)}$, then $f(x)$ is a periodic function with period

(A) π
(B) 2π
(C) $\pi/2$
(D) none of these.

26. $\lim_{x \rightarrow 0} \left[(\min(y^2 + 2y + 7)) \frac{\sin x}{x} \right] = ?$ (where $[.]$ denotes greatest integer function)

(A) 4
(B) 5
(C) 6
(D) none of these

27. $\lim_{x \rightarrow 0} \left[100 \frac{\tan x}{x} \right] = ?$ (where $[.]$ denotes greatest integer function)

(A) 100
(B) 99
(C) 101
(D) 0

28. If $f(x) = |\cos 2x|$, then $f' \left(\frac{\pi}{4} + 0 \right)$ is equal to

(A) 2
(B) 0
(C) -2
(D) doesn't exist

29. $\lim_{x \rightarrow \pi/2} (\sin x)^{1/\cos x} =$

(A) 0
(B) e
(C) 1
(D) doesn't exist

30. $\lim_{x \rightarrow 0} \frac{1 - \cos(1 - \cos x)}{x^4}$ equals to

(A) $\frac{1}{2}$
(B) $\frac{1}{8}$
(C) $\frac{1}{4}$
(D) $\frac{1}{16}$

31. $\lim_{x \rightarrow \pi/4} (2 - \tan x)^{\frac{1}{\ln(\tan x)}}$ equals to

(A) e
(B) 1
(C) 0
(D) e^{-1}

32. $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals to

(A) 0
(B) π
(C) $-\pi$
(D) not exist

33. $f(x) = \max\{x, x^3\}$, then the number of points where $f(x)$ is not differentiable, are

(A) 1
(B) 2
(C) 3
(D) 4

34. $\lim_{x \rightarrow 2} \frac{\tan(e^{x-2} - 1)}{\ln(x-1)}$
 (A) 2 (B) -2 (C) 1 (D) -1
35. The function defined by $f(x) = \begin{cases} \frac{\sin x^2}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is
 (A) continuous and derivable at $x = 0$ (B) neither continuous nor derivable at $x = 0$
 (C) continuous but not derivable at $x = 0$ (D) none of these
36. $\lim_{x \rightarrow 0} (1 + \tan^2 \sqrt{x})^{1/2x}$ is equal to
 (A) 1 (B) 0 (C) $e^{1/2}$ (D) $e^{-1/2}$
37. The left hand derivative of $f(x) = [x] \sin(\pi x)$ at $x = k$, k is an integer is
 (A) $(-1)^k(k-1)\pi$ (B) $(-1)^{k-1}(k-1)\pi$ (C) $(-1)^k k\pi$ (D) $(-1)^{k-1} k\pi$
38. $\lim_{x \rightarrow y} \frac{x^y - y^x}{x^x - y^y}$ is
 (A) $\frac{\log y}{1 - \log y}$ (B) $\frac{1 - \log y}{\log y}$ (C) $\frac{1 - \log y}{1 + \log y}$ (D) $(1 - \log y) \log ey$
39. $\lim_{x \rightarrow 0} (\sin x)^x$ is
 (A) 1 (B) 0 (C) ∞ (D) does not exist
40. If $f(x)$ is a continuous function $\forall x \in \mathbb{R}$ and the range of $f(x) = (2, \sqrt{26})$ and $g(x) = \left[\frac{f(x)}{a} \right]$ is continuous $\forall x \in \mathbb{R}$ ($[.]$ denotes the greatest integer function), then the least positive integral value of a is
 (A) 2 (B) 3 (C) 6 (D) 5
41. Let $f(x) = \lim_{n \rightarrow \infty} \frac{1}{\left(\frac{3}{\pi} \tan^{-1} 2x \right)^{2n} + 5}$. then the set of values of x for which $f(x) = 0$, is
 (A) $|2x| > \sqrt{3}$ (B) $|(2x)| < \sqrt{3}$ (C) $|2x| \geq \sqrt{3}$ (D) $|2x| \leq \sqrt{3}$
42. If $f(x) = \frac{\sin(e^{x-2} - 1)}{\log(x-1)}$, then $\lim_{x \rightarrow 2} f(x)$ is equal to
 (A) -2 (B) -1 (C) 0 (D) 1
43. If $f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ and $f(x)$ is continuous at $x = 0$, the value of k is
 (A) $a - b$ (B) $a + b$ (C) $\log a + \log b$ (D) none of these
44. The expression of $\frac{dy}{dx}$ of the function $y = a^{x^a}$ is
 (A) $\frac{y^2}{x(1 - y \log x)}$ (B) $\frac{y^2 \log y}{x(1 - y \log x)}$ (C) $\frac{y^2 \log y}{x(1 - y \log x \log y)}$ (D) $\frac{y^2 \log y}{x(1 + y \log x \log y)}$

45. The value of $\lim_{x \rightarrow \infty} \frac{x^3 \sin(1/x) - 2x^2}{1 + 3x^2}$ is
 (A) 0 (B) $-1/3$ (C) -1 (D) $-2/3$
46. $\lim_{x \rightarrow 2} \frac{\tan(e^{x-2} - 1)}{\ln(x-1)}$
 (A) 2 (B) -2 (C) 1 (D) -1
47. Let $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$. Then $f(x) \cdot \sin x$ is
 (A) differentiable at $x = 0$ (B) continuous at $x = 0$
 (C) not continuous at $x = 0$ (D) none of these
48. The function $f(x) = \frac{\cos x - \sin x}{\sin 4x}$ is not defined at $x = \pi/4$. The value which should be assigned to f at $x = \pi/4$, so that it is continuous there, is
 (A) 0 (B) 1 (C) -1 (D) none of these
49. $\lim_{x \rightarrow \infty} \frac{\ln x - [x]}{[x]}$, ($[.]$ denotes the greatest integer function)
 (A) has value -1 (B) has value 0 (C) has value 1 (D) does not exist
50. The function $\frac{e^{\tan x} - 1}{e^{\tan x} + 1}$ is discontinuous
 (A) at $n\pi$, $n \in \mathbb{I}$ (B) at $(2n+1)\frac{\pi}{2}$, $n \in \mathbb{I}$ (C) No where (D) Every where
51. If a, b, c, d are positive, then $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{a + bx}\right)^{c+dx} =$
 (A) $e^{d/b}$ (B) $e^{c/a}$
 (C) $e^{(c+d)/(a+b)}$ (D) e
52. The length of the largest interval in which the function $3 \sin x - 4 \sin^3 x$ is increasing, is
 (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$
 (C) $\frac{3\pi}{2}$ (D) π
53. The interval in which $f(x) = e^{|x^2 - 6x + 8|}$ increases, is
 (A) $(-\infty, 2) \cup (3, 4)$ (B) \mathbb{R}
 (C) $(2, 3) \cup (4, \infty)$ (D) $(2, 4)$
54. If $x + |y| = 2y$, then y as a function of x is
 (A) continuous but not differentiable at $x = 0$ (B) continuous and differentiable at $x = 0$
 (C) differentiable for all x (D) none of these
55. If $y = a \log |x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then
 (A) $a = 2$, $b = -1$ (B) $a = 2$, $b = -1/2$
 (C) $a = -1/2$, $b = 1/2$ (D) none of these

56. The points of extremum of the function $\phi(x) = \int_1^x e^{-t^2/2}(1-t^2)dt$, is/are
 (A) $x = 0$ (B) $x = 1/2$
 (C) $x = -2$ (D) $x = \pm 1$
57. Let $f(x) = x^{n+1} + a \cdot x^n$, where 'a' is a positive real number. Then $x = 0$ is a point of
 (A) local minimum for any integer n (B) local maximum for any integer n
 (C) local minimum if n is an even integer (D) local minimum if n is an odd integer
58. Least natural number 'a' for which $x + ax^{-2} > 2 \forall x \in (0, \infty)$ is
 (A) 1 (B) 2 (C) 5 (D) none of these
59. Let $f(x) = \lim_{n \rightarrow \infty} (\sin x)^{2n}$, then f is
 (a) continuous at $x = \pi/2$, (b) discontinuous at $x = \pi/2$
 (c) discontinuous at $x = -\pi/2$
 (d) discontinuous at infinite number of points.
60. Let $f(x) = \begin{cases} x^n \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then f(x) is continuous, but not differentiable at $x = 0$, if
 (a) $n \in (0, 1]$ (b) $n \in [1, \infty)$ (c) $n \in (-\infty, 0)$ (d) $n = 0$
61. If $f(x) = \{ |x| - |x-1| \}^2$, then $f'(x)$ equals
 (a) 0 for all x (b) $2 \{ |x| - |x-1| \}$
 (c) $\begin{cases} 0 & \text{for } x < 0 \text{ and for } x > 1 \\ 4(2x-1) & \text{for } 0 < x < 1 \end{cases}$ (d) $\begin{cases} 0 & \text{for } x < 0 \\ 4(2x-1) & \text{for } x > 0 \end{cases}$
62. If the function $f(x) = \begin{cases} (1 + |\sin x|)^{\frac{a}{|\sin x|}}, & -\frac{\pi}{6} < x < 0 \\ b, & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \frac{\pi}{6} \end{cases}$ is continuous at $x = 0$, then
 (a) $a = \log_e b$, $a = \frac{2}{3}$ (b) $\log_e a$, $a = \frac{2}{3}$
 (c) $a = \log_e b$, $b = 2$ (d) none of these
63. The function $f(x) = \begin{cases} \sin\left(\frac{\pi x}{2}\right), & x < 1 \\ |2x-3|[x], & x \geq 1 \end{cases}$
 (a) is continuous at $x = 1$ (b) is differentiable at $x = 1$
 (c) is continuous but not differentiable at $x = 1$ (d) none of these

64. The value of p for which the function $f(x) = \begin{cases} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{p}\right) \log\left\{1 + \frac{x^2}{3}\right\}}, & x \neq 0 \\ 12(\log 4)^3, & x = 0 \end{cases}$ is continuous at $x = 0$ is
 (a) 1 (b) 2
 (c) 3 (d) 4
65. $\lim_{x \rightarrow \infty} \left(\frac{1}{1-n^2} + \frac{2}{1-n^2} + \dots + \frac{n}{1-n^2} \right)$ is equal to
 (A) 0 (B) $-1/2$
 (C) $1/2$ (D) none of these
66. $\lim_{x \rightarrow 2} (-1)^{[x]}$, where $[x]$ is the greatest integer function, is equal to
 (A) 1 (B) -1
 (C) ± 1 (D) doesn't exist
67. $\lim_{x \rightarrow \infty} \left(\frac{x}{2+x} \right)^{2x} =$
 (A) e^{-4} (B) e^{-6}
 (C) e^{-2} (D) none of these
68. If $f(x) = [x \sin \pi x]$ { where $[x]$ denotes greatest integer function}, then $f(x)$ is
 (A) continuous at $x = 0$ (B) continuous in $(-1, 0)$
 (C) differentiable at $x = 1$ (D) differentiable in $(-1, 1)$
69. In order that function $f(x) = (x+1)^{\cot x}$ is continuous at $x = 0$, $f(0)$ must be defined as
 (A) 0 (B) e
 (C) $1/e$ (D) none of these
70. $\lim_{x \rightarrow \infty} \left(\frac{x^n}{e^x} \right) = 0$, (n is integer), for
 (A) no value of n (B) all value of n
 (C) only negative value of n (D) only positive value of n
71. $\lim_{n \rightarrow \infty} (4^n + 5^n)^{\frac{1}{n}}$ is equal to
 (A) 4 (B) 5
 (C) e (D) none of these
72. $\lim_{x \rightarrow \frac{\pi}{2}} [\sin^{-1}(\sin x)]$ equals, where $[.]$ denotes the greatest integer function
 (A) $\frac{\pi}{2}$ (B) 0
 (C) 1 (D) does not exist

73. The value of derivative of $f(x) = |x - 1| + |x - 3|$ at $x = 2$ is
 (A) -2 (B) 0
 (C) 2 (D) not defined
74. The number of points where the function $f(x) = x^2 - 1 + |\ln |x||$ is not differentiable is
 (A) 1 (B) 2
 (C) 3 (D) none of these
75. $f(x)$ is a continuous function and takes only rational values. If $f(0) = 3$, then $f(2)$ equals
 (A) 5 (B) 0
 (C) 1 (D) none of these
76. $\lim_{x \rightarrow 0} \frac{|x+1| + |x-1| - 2}{x}$ is equal to
 (A) 1 (B) -1 (C) 2 (D) 0
77. $\lim_{x \rightarrow 0} \frac{x \cdot 2^x - x}{1 - \cos x}$ is equal to
 (A) $\log 2$ (B) $\frac{1}{2} \log 2$ (C) $2 \log 2$ (D) none of these
78. If $f(x) = \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x$. Then $\lim_{x \rightarrow \infty} f(x)$ is
 (A) e^4 (B) e^3 (C) e^2 (D) None of these
79. $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$ is equal to
 (A) $1/2$ (B) 2 (C) $-1/2$ (D) None of these
80. $\lim_{x \rightarrow 0} \frac{(a+x)^2 \sin(a+x) - a^2 \sin a}{x}$ is equal to
 (A) $a^2 \cos a + a \sin a$ (B) $a^2 \cos a + 2a \sin a$ (C) $2a^2 \cos a + a \cos a$ (D) None of these
81. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function and $f(1) = 4$. Then the value of $\lim_{x \rightarrow 1} \int_4^{f(x)} \frac{2t}{x-1} dt$ is
 (A) $8 f'(1)$ (B) $4 f'(1)$ (C) $2 f'(1)$ (D) None of these
82. If $f(x) = \begin{cases} x-3 & x < 0 \\ x^2 - 3x + 2, & x \geq 0 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$, then $g(x)$ is continuous at
 (A) $\mathbb{R} - \{0\}$ (B) \mathbb{R}^+
 (C) $\mathbb{R} - \{1, 2\}$ (D) $\mathbb{R} - \{0, 1, 2\}$
83. The value the limit $\lim_{x \rightarrow 0} \left(\frac{e^{a/x} - e^{-a/x}}{e^{a/x} + e^{-a/x}} \right)$, $a > 0$ is
 (A) 0 (B) 1
 (C) infinity (D) does not exist

84. The number of points where $g(f(x))$ is discontinuous given that $g(x) = \frac{1}{x^2 + x - 1}$ and $f(x) = \frac{1}{x-3}$ is
 (A) 1 (B) 2
 (C) 3 (D) 4
85. The value of $\lim_{x \rightarrow 0} \left(\frac{1+5x^2}{1+3x^2} \right)^{1/x^2}$ is
 (A) e^2 (B) e^3
 (C) e^5 (D) none of these
86. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval $(0, 2)$ is
 (A) 1 (B) 2 (C) 3 (D) 4
87. Let $f(x + y) = f(x) f(y) \quad \forall x, y \in \mathbb{R}$. Suppose that $f(3) = 3$ and $f'(0) = 11$ then $f'(3)$ is given by
 (A) 22 (B) 44 (C) 28 (D) 33
88. The function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ then which of the following is not true
 (A) continuous at $x = 1$ (B) continuous at $x = 3$
 (C) differentiable at $x = 1$ (D) differentiable at $x = 3$
89. The function $f(x) = \max\{1 - x, 1 + x, 2\}$, $x \in (-\infty, \infty)$ is
 (A) differentiable at all points
 (B) differentiable at all points except at $x = 1$ and $x = -1$
 (C) continuous at all points except at $x = 1$ and $x = -1$, where it is discontinuous
 (D) None of these
90. Let $f(x) = [\tan^2 x]$ where $[.]$ is greatest integer function then
 (A) $\lim_{x \rightarrow 0} f(x)$ does not exist (B) $f(x)$ is continuous at $x = 0$
 (C) $f(x)$ is not differentiable at $x = 0$ (D) $f'(0) = 1$

LEVEL-III

1. The number of critical points of $f(x) = \max(\sin x, \cos x)$ for $x \in (0, 2\pi)$
 - (A) 2
 - (B) 5
 - (C) 3
 - (D) non

2. If $f(x) = \int_0^x (t+1)(e^t - 1)(t-2)(t+4) dt$ then $f(x)$ would assume the local minima at;
 - (A) $x = -4$
 - (B) $x = 0$
 - (C) $x = -1$
 - (D) $x = 2$.

3. Let $f(x) = [\cos x + \sin x]$, $0 < x < 2\pi$ where $[x]$ denotes the greatest integer less than or equal to x . The number of points of discontinuity of $f(x)$ is
 - (A) 6
 - (B) 5
 - (C) 4
 - (D) 3

4. $\lim_{x \rightarrow -1^+} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}}$

5. $f(x) = \frac{1}{\log|x|}$ is discontinuous at $x =$

6. The value of the limit

$$\lim_{x \rightarrow 0} \left(1^{1/\sin^2 x} + 2^{1/\sin^2 x} + \dots + n^{1/\sin^2 x} \right)^{\sin^2 x}$$
 - (A) ∞
 - (B) 0
 - (C) $\frac{n(n+1)}{2}$
 - (D) n

7. $\lim_{x \rightarrow 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to
 - (A) $1/5$
 - (B) $1/6$
 - (C) $1/4$
 - (D) $1/2$

8. If $\tan^{-1}(x+h) = \tan^{-1}(x) + (h \sin y)(\sin y) - (h \sin y)^2 \cdot \frac{\sin 2y}{2} + (h \sin y)^3 \cdot \frac{\sin 3y}{3} + \dots$, where $x \in (0, 1)$, $y \in (\pi/4, \pi/2)$, then
 - (A) $y = \tan^{-1}x$
 - (B) $y = \sin^{-1}x$
 - (C) $y = \cot^{-1}x$
 - (D) $y = \cos^{-1}x$

9. The value of $\lim_{|x| \rightarrow \infty} \cos[\tan^{-1}(\sin(\tan^{-1} x))]$ is equal to
 - (A) -1
 - (B) $\sqrt{2}$
 - (C) $-\frac{1}{\sqrt{2}}$
 - (D) $\frac{1}{\sqrt{2}}$

10. If $\lim_{x \rightarrow 0} \int_0^x \frac{t^2 dt}{(x - \sin x)\sqrt{a+t}} = 1$, then the value of a is
 - (A) 4
 - (B) 2
 - (C) 1
 - (D) none of these

- 12 For some g , let $f(x) = x(x+3)e^{g(x)}$ be a continuous function. If there exists only one point $x = d$ such that $f'(d) = 0$, then
 (A) $d < -3$ (B) $d > 0$ (C) $-3 \leq d \leq 0$ (D) $-3 < d < 0$
- 13 $\lim_{n \rightarrow \infty} \left[1 - \ln \left(1 + \frac{1}{n} \right)^{n-1} \right]$ is equal to
 (A) 0 (B) 1 (C) e (D) none of these
- 14 The value of $\lim_{x \rightarrow \infty} \frac{x^n + nx^{n-1} + 1}{e^{[x]}}$, $n \in \mathbb{I}$ is
 (A) 1 (B) 0 (C) n (D) $n(n-1)$
- 15 Given a function $f(x)$ continuous $\forall x \in \mathbb{R}$ such that $\lim_{x \rightarrow 0} \left[f(x) + \log \left(1 - \frac{1}{e^{f(x)}} \right) - \log(f(x)) \right] = 0$, then $f(0)$ is
 (A) 0 (B) 1 (C) 2 (D) 3
- 16 Let \mathbb{R} be the set of real numbers and $f : \mathbb{R} \rightarrow \mathbb{R}$ be such that for all x and y in \mathbb{R} $|f(x) - f(y)| \leq |x - y|^7$. Then $f(x)$ is.
 (A) linear (B) constant
 (C) quadratic (D) none of these.
17. Find the value of $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \cot^2 x \right)$
 (A) $2/5$ (B) $2/3$
 (C) $1/4$ (D) $1/5$.
- 18 $\lim_{x \rightarrow 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x}$ is
 (A) 1 (B) -1
 (C) 0 (D) doesn't exist
- 19 Given that $f(x)$ is a non-zero differentiable function such that $f(x+y) = f(x) \cdot f(y)$, $\forall x, y \in \mathbb{R}$, and $f'(0) = 1$ then $\ln f(1)$ is equal to
 (A) 0 (B) 1
 (C) e (D) none of these
- 20 The largest interval where the function $f(x) = \frac{x}{1 + |x|}$ is differentiable
 (A) $(-\infty, \infty)$ (B) $(0, \infty)$
 (C) $(-\infty, 0) \cup (0, \infty)$ (D) none of these
- 21 $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e + \frac{ex}{2}}{x^2}$ is equal to

- (A) $\frac{11e}{24}$ (B) $-\frac{11e}{24}$ (C) $\frac{e}{24}$ (D) None of these

- 22 The value of the limit $\lim_{x \rightarrow 0} \left(\frac{1 - 3^x - 4^x + 12^x}{\sqrt{2 \cos x + 7} - 3} \right)$ is
 (A) 0 (B) $-6(\log 3)(\log 4)$
 (C) 1 (D) none of these
- 23 Let $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$, for all $x, y \in \mathbb{R}$ and if $f(x)$ is differentiable, and $f'(0) = -1$, $f(0) = 1$ then the function $f(x)$ is
 (A) $-x + 1$ (B) $x + 1$
 (C) $x^2 - 1$ (D) $x - 1$
- 24 The points of discontinuity of the function \log where $g(x) = \frac{1}{x-1}$ and $f(x) = \frac{1}{x^2 + x - 2}$ are
 (A) $\frac{1}{2}, 2, 1$ (B) 2, 1 (C) 2, $\frac{1}{2}$ (D) none of these

ANSWERS

LEVEL -I

1. C 2. B 3. A 4. C

- | | | | |
|-----------|-----------|-------------------------------------|----------|
| 5. B | 6. C | 7. 1 | 8. e^2 |
| 9. -2, -1 | 10. C | 11. D | 12. B |
| 13. D | 14. D | 15. C | 16. A |
| 17. A | 18. D | 19. A | 20. B |
| 21. B | 22. B | 23. D | 24. A |
| 25. A | 26. B | 27. B | 28. C |
| 29. A | 30. B | 31. A | 32. C |
| 33. C | 34. A | 35. C | 36. B |
| 37. A | 38. D | 39. A | 40. A |
| 41. A | 42. B | 43. B | 44. D |
| 45. D | 46. -2, 1 | 47. $f(x) = x^3/3 + x^2/2 - 2x + 2$ | |
| 48. B | | | |
| 49. A | 50. B | 51. C | 52. A |

LEVEL -II

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|----------|-----------------------|---------|----------------------------|
| 1. D | 2. C | 3. A | 4. D |
| 5. A | 6. B | 7. A, C | 8. A |
| 9. D | 10. C | 11. A | 12. C |
| 13. B, D | 14. 0 | 15. 7 | 16. $(2a\alpha + b)^2 / 2$ |
| 17. -1 | 18. 0, π , 2π | 19. 6 | 20. B |
| 21. B | 22. C | 23. B | 24. B |
| 25. A | 26. B | 27. B | 28. A |
| 29. C | 30. B | 31. D | 32. C |
| 33. C | 34. C | 35. A | 36. C |
| 37. A | 38. C | 39. D | 40. C |
| 41. A | 42. D | 43. B | 44. A |
| 45. B | 46. C | 47. B | 48. D |
| 49. A | 50. B | 51. A | 52. B |
| 53. C | 54. A | 55. B | 56. D |
| 57. C | 58. B | 59. D | 60. B |
| 61. C | 62. A | 63. C | 64. D |
| 65. B | 66. D | 67. A | 68. A |
| 69. B | 70. B | 71. B | 72. C |
| 73. B | 74. B | 75. D | 76. D |
| 77. C | 78. A | 79. A | 80. B |
| 81. A | 82. A | 83. D | 84. C |
| 85. A | 86. C | 87. D | 88. D |
| 89. B | 90. B | | |

LEVEL -III

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|---------------|-------|-------|----------------------|
| 1. C | 2. C | 3. B | 4. $1 / \sqrt{2\pi}$ |
| 5. 0, ± 1 | 6. D | 7. D | 8. C |
| 9. D | 10. A | | 12. D |
| 13. A | 14. B | 15. A | 16. B |
| 17. B | 18. D | 19. B | 20. A |
| 21. A | 22. B | 23. A | 24. A |