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Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

UNITS AND DIMENSION

Units and Dimension

Syllabus of IITJEE and Maharashtra Board:
Scope and excitement of physics, Technology & society. Forces in nature, Conservation laws, Examples of gravitation, electromagnetic and nuclear forces from daily life experiences (qualitative description only). Need of measurement, Units of measurement, System of units, SI units, Fundamental and derived unit; Length, mass and time measurement, Accuracy and precision of measuring instrument, errors in measurement, significant figures. Dimensions of physical quantities, dimensional analysis and its application.

Have you ever observed the nature and the various spectacular events like formation of rainbow on any rainy day?
Whenever we observe nature keenly, we can easily understand that the various events in nature like blowing of wind, flow of water, motion of planets, formation of rainbow, different forms of energies, the function of human bodies, animals, etc. are happening or taking place according to some basic laws. The systematic study of these laws of nature governing the observed events is called science. For our convenience, clear understanding and systematic study of Science is classified into various branches. Among these branches Chemistry, Mathematics, Botany, Zoology, etc. are ancient branches and Bio–technology, Bio–chemistry, Bio–Physics, Computer science, Space Science, etc. are considered to be modern branches of science and engineering. One of such ancient and reputed branches of this science is physics.

SCOPE AND EXCITEMENT OF PHYSICS

The domain of physics consists of wide variety and large number of natural phenomena. Hence, the scope of physics is very vast and obviously the excitement that one gets from the careful study of physics has got no boundaries.

Scope of Physics

For example, when we study one of the basic physical quantities called mass, we come across the values ranging from minute masses like mass of an electron (of the order of $10^{-3\circ}$ kg) to heavy masses like mass of universe (10^{55} kg). Similarly, in case of other basic quantities like length and time also the range is very wide.

Hence, the scope of physics can be understood easily, only when we can classify the study of physics chiefly into three levels. They are:

(a) Macroscopic level study of physics,
(b) Mesoscopic level study of physics, and
(c) Microscopic level study of physics.

Macroscopic level study of physics: Macroscopic level study of physics mainly includes the study of basic laws of nature and several natural phenomena like gravitational force of attraction between any two bodies in the universe (in mechanics), variation of quantities like pressure, volume, temperature, etc. of gases on their thermal expansion or contraction (in thermodynamics), etc.

Microscopic level study of physics: The microscopic level study of physics deals with constitution and structure of matter at the level of atoms or nuclei. For example, interaction between elementary particles like electrons, protons and other particles , etc.

Mesoscopic level study of physics: The mesoscopic level study of physics deals with the intermediate domain of macroscopic and microscopic, where we study various physical phenomena of atoms in bulk.

So, the edifice of physics is beautiful and one can appreciate the subject as and when one pursues the same seriously.

Excitement of Physics

The study of physics is exciting in many ways as it explains us the reason behind several interesting features like (a) ***how day and nights are formed?*** (b) ***how different climatic conditions are formed in different seasons?*** (c) ***how satellite works and helps in using several devices like television, telephones, etc.?*** (d) ***how an astronaut travels to celestial space ?*** (e) ***how we can convert one form of energy to another ?*** (f) ***how different types of forces are governing different types of motion in universe ? etc***.

It is quite common and simple that every human being on the earth will be interested to know the answers for at least few of the above questions. As physics is the subject which answers them, naturally the study of physics will be exciting.

TECHNOLOGY AND SOCIETY

Physics is almost an integral part of upgradation of technology. Technology was also a branch of science where we study the application of principles of physics for practical purposes. Based on laws and principles of physics, technocrats along with scientists develop technically advanced equipment to help the society.

For example, from the principles of thermodynamics James watt invented steam engine which was responsible for a big industrial revolution in England in the 18^{th} century. Another recent example is invention of mobile phones which are creating revolution in wireless communication technology. Yet another important example is invention of micro–processors by using silicon chips which has replaced valve technology and brought the computers from the size of your study room to the size of your geometry box. These are few examples. There are many more areas where physics is involved in upgrading technology and thereby helping the society. The following table gives us a list of various branches of physics that helped the field of technology.

Technology	**Scientific principle(s)**
Steam engine	Laws of thermodynamics
Nuclear reactor	Nuclear fission
Radio and Television	Propagation of electromagnetic waves
Computers	Digital logic
Lasers	Light amplification by stimulated emission of radiation (population inversion)

Production of ultra–high magnetic fields	Superconductivity
Rocket propulsion	Newton's laws of motion
Electric generator	Faraday's laws of electromagnetic induction
Hydroelectric power	Conversion of gravitational potential energy into electric energy
Aeroplane	Bernoulli's principle in fluid dynamics
Particle accelerators	Motion of charged particles in electromagnetic fields
Air conditioners / Refrigerators	Laws of thermodynamics
Washing machines, centrifuge, etc.	Centrifugal force
Sonar	Reflection of ultrasonic waves

The following table lists the involvement of various renowned physicists all across the world, who helped the society with their noble inventions.

Name	**Major Contribution / Discovery**	**Country of origin**
Isaac Newton	Universal law of gravitation: Laws of motion; reflecting telescope.	**U. K.**
Galileo Galilei	Law of inertia	**Italy**
Archimedes	Principle of buoyancy; principle of the lever	**Greece**
James Clerk Maxwell	Electromagnetic theory; light an electromagnetic wave	**U. K.**
W. K. Roentgen	x– rays	**Germany**
Marie Sklodowska Curie	Discovery of radium and polonium; Studies on natural radioactivity	**Poland**
Albert Einstein	Law of photo–electricity; Theory of relativity	**Germany**
S. N. Bose	Quantum statistics	**India**
James Chadwick	Neutron	**U.K.**
Niels Bohr	Quantum model of hydrogen atom	**Denmark**
Ernest Rutherford	Nuclear model of atom	**New Zealand**
C.V. Raman	Inelastic scattering of light by molecules	**India**
Christiaan Huygens	Wave theory of light	**Holland**
Michael Faraday	Laws of electromagnetic induction	**U.K.**
Edwin Hubble	Expanding universe	**U.S.A.**
Homi Jehangir Bhabha	Cascade process in cosmic radiation	**India**
Abdus Salam	Unification of weak and electromagnetic interactions	**Pakistan**
R. A. Millikan	Measurement of electronic charge	**U.S.A**
Ernest Orlando Lawrence	Cyclotron	**U.S.A.**
Wolfgang Pauli	Quantum Exclusion Principle	**Austria**
Louis victor de Broglie	Wave nature of matter	**France**
J.J. Thomson	Electron	**U.K.**
S. Chandrasekhar	Chandrasekhar limit, structure and	**India**

	evolution of stars	
Lev Devidovich Landau	Theory of condensed matter; liquid helium	**Russia**
Heinrich Rudolf Hertz	Electromagnetic waves	**Germany**
Victor Francis Hess	Cosmic radiation	**Austria**
M. N. Saha	Thermal ionisation	**India**
G. N. Ramachandran	Triple helical structure of proteins	**India**
Thomas Alwa Edison	Electric bulb, Projector	**US**
Graham Bell	Telephone	**US**
Cavendish	Determination of ‘G’	**England**
Robert Boyle	Boyle’s law	**England**

So, to put it in a nut shell, science, technology and society are inseparable as they are deeply interwined.

FUNDAMENTAL FORCES IN NATURE

Force is a very common word which we normally come across in our daily life. We need force to push or pull or throw a body. Even we need it to deform or break the bodies. Sometimes, we experience force like when we are standing in a great storm, we experience the force exerted by wind. When we are sitting in a bus which is negotiating a turn, we experience an outward push. So, what is this force? Let us try to understand the concept of force in terms of physics.

At macroscopic level study of physics, we normally encounter different kinds of forces like gravitational force, muscular force, frictional force, contact force, spring force, buoyant force, viscous force, pressure force, force due to surface tension, electrostatic force, magnetic force, etc. whereas at microscopic level of study we come across nuclear forces, interatomic forces, intermolecular forces, weak forces, etc.

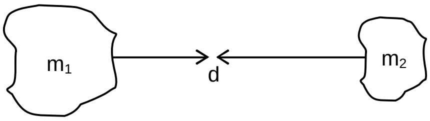
After analysing these various types of forces in nature, it was concluded that all the forces can be comfortably classified into four categories, which are known as fundamental forces in nature. They are

(1) Gravitational force (2) Electromagnetic force,
(3) Nuclear force, and (4) Weak force.

That means, any force other than the above four forces can be derived from these four basic forces. For example, elastic force or spring force arises due to the net attraction or repulsion between any two neighboring atoms of the spring. When it is elongated or compressed, attractive or repulsive forces produced between the atoms can be treated as the resultant of all electromagnetic forces between charged particles of an atom. Hence, this spring force is known as derived force and electromagnetic force which is the origin of this spring force is called fundamental force. Now, we will study about fundamental forces in brief.

Gravitational Force

Newton discovered that any two bodies in universe attract each other. This force of attraction exists by virtue of their masses, and is known as gravitational force of attraction. He found that the gravitational force is directly proportional to their masses and is inversely proportional to the square of the distance between them.



i.e. $F = G\dfrac{m_1m_2}{d^2}$ where ‘G’ is a Universal Gravitational Constant. This force is a universal force and is independent of any type of intervening medium between the two bodies. Though this is the weakest force in nature when compared to other types of fundamental forces, it plays vital role in governing the motion of planets around sun, natural satellites (like moon around earth), artificial satellites, etc.

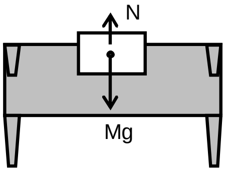
Electromagnetic Force:

The force of attraction or repulsion between any two charged particles is known as electrostatic force. If q_1 and q_2 charges are separated by a distance ‘d’ in air then the force of attraction or repulsion between them is given by $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{d^2}$. This is called Coulomb’s law of electric forces.

Charges in motion produce magnetic effects and a magnetic field gives rise to a force on a moving charge. In general electric and magnetic effects are inseparable and hence the name – electromagnetic force. This electromagnetic force between moving charged particles is comparatively more complicated and contains several other terms other than Coulomb’s force.

In atoms electromagnetic force between electrons and protons is responsible for several molecular and atomic phenomena. Apart from this it also plays vital role in the dynamics of chemical reactions, mechanical and thermal properties of materials, tension in ropes, friction, normal force, spring force, Vander Waals force .

Example: Let us consider a block which is placed on a horizontal surface of a table as shown in the figure. The table balances the weight (Mg) and exerts a force which comes from electromagnetic force between charged constituents of atoms or molecules of surface of block and that of the table. Thus a force called normal force acts on block.



This electromagnetic force is a strong force when compared to the gravitational force. The electromagnetic force between two protons is 10^{36} times the gravitational force between them for any fixed distance.

Nuclear Force

We know that, in general, nucleus of every atom consists of two elementary particles called protons and neutrons. As neutrons are uncharged and protons are charged, the electric force of repulsion between protons will cause nucleus to break into fragments. But this is not happening, and also we know that nucleus of a non–radioactive element is a stable one.

That means there must be some other attractive force which is dominating coulombic force of repulsion between protons and keeping all the particles in nucleus together in stable condition as gravitational force can’t dominate electric force. That new force existing between any two nucleons and which keeps all the particles in nucleus bound together is known as nuclear force. This force is stronger than electromagnetic force and is a charge independent force. Range of these forces is very small and will be of the order of nuclear size (10^{-21} th portion of size of an atom).

Latest developments in physics revealed that this strong nuclear force is also not a fundamental force as protons and neutrons consist of still elementary particles called quarks. And according to this latest development quark – quark force is fundamental force of nature and nuclear force is a derived force. However the study of quark – quark force is out of the scope of this book and our curriculum.

Weak Nuclear Force

This force appears only in certain nuclear processes. A neutron can change itself into a proton by emitting an electron and another elementary particle called antineutrino simultaneously. This process is called β^- decay. Similarly a proton can also change into neutron by emitting positron and a neutrino. This process is called β^+ decay. The forces which are responsible for these changes are known as weak forces. These forces are weak in nature when compared to nuclear and electromagnetic forces but stronger than gravitational forces. The range of these weak nuclear forces is exceedingly small, of the order of 10^{-15}m.

The following table gives us an overall idea about relative strengths and ranges of four fundamental forces.

Name	Relative strength	Range	Operates among
Gravitational force	10^{-38}	Infinite	**All objects in the universe**
Weak nuclear force	10^{-13}	Very short, within nuclear size ($\sim 10^{-15}$)	**Elementary particles**
Electromagnetic force	10^{-2}	Infinite	**Charged particles**
Strong nuclear force	1	Very short, within nuclear size ($\sim 10^{-15}$)	**Nucleons**

CONSERVATION LAWS

In any physical phenomena, few physical quantities associated with the phenomena may change with time and few physical quantities associated with it may not change. Those physical quantities which remain constant in time are known as conserved quantities.

For example, if a big liquid drop is sprayed into several small droplets the volume of liquid before spraying and after spraying remains same. Hence, we can say that a physical quantity called volume is conserved in this example. Similarly, we have several quantities which are conserved. Within the scope of our course, we can discuss the following conservation laws.

1. Law of conservation of linear momentum
2. Law of conservation of energy
3. Law of conservation of angular momentum
4. Law of conservation of charge.

Let us discuss them in brief.

Law of conservation of linear momentum

The linear momentum of a body is defined as the ability of a body by virtue of which it imparts its motion to other objects along a straight line. And mathematically it is equal to the product of mass of the body (m) and its velocity ($\vec{v}$) Mathematically, $\vec{P} = m.\vec{v}$.

According to this law, in absence of an external force, the total vector sum of linear momentum remains unchanged.

Example: When a bullet is fired with a gun, the total momentum vector of the system of bullet and gun is zero. After firing, bullet moves in forward direction with some momentum and gun recoils with the same amount of momentum in magnitude, but opposite in direction. Hence total vector sum of momentum after firing is also zero. Thus linear momentum of the system before and after firing is zero. Hence we can say that linear momentum is conserved.

Law of conservation of energy

According to this law the total energy of an isolated system is always constant and it never changes. But it can be transformed from one form to another. *For example* an electric cell in our daily life gives electrical energy by transforming chemical energy in it, electric motor converts electrical energy to mechanical energy, etc. However the total energy in these processes is conserved.

When an object is dropped from a certain height the total mechanical energy of the body is conserved. At its highest point all its mechanical energy will be in the form of potential energy and at its lowest point it will be in the form of kinetic energy, i.e. energy has transformed from one form into another, (i.e. potential to kinetic) but the total energy remains constant. Hence the total mechanical energy is conserved.

But this conservation of mechanical energy can't be applied in the presence of non – conservative force. For example in the above case if you consider air resistance on the freely falling body total mechanical energy does not remain constant. Here work done by air resistance gets converted into different forms of energy like heat energy. So such while applying nergy conservation principle heat energy should also be taken into consideration in such cases

Law of conservation of angular momentum

Angular momentum ($\vec{L}$) of a body about a point is defined as the cross product of its position vector about that point ($\vec{r}$) and its linear momentum at that instant ($\vec{p}$)

i.e. $\vec{L} = \vec{r} \times \vec{p}$
or $L = rp \sin\theta$ where θ is the angle between '$\vec{r}$' and '$\vec{p}$'.

According to this law the total angular momentum of the system remains conserved in absence of external torque.

Example: We know that planets revolves around sun in elliptical orbits. The angular momentum of a planet at any point during its motion in its path is conserved. We will study more clearly about this under rotatory motion concepts.
These are the few conservation laws in mechanics. Now let us discuss a conservation law in electrostatics.

Law of conservation of charge

This law states that the total electric charge of an isolated system is always conserved. Charge can neither be created nor destroyed, but it can be transferred or exchanged from one body to another.

Apart from these, there are several other physical quantities that are conserved in nature. During our further discussions in various chapters we will understand them.

MEASUREMENT AND UNITS

Physical quantity: Any meaningful term which can be measured is a physical quantity. For example length, velocity, time etc. are physical quantity. But handsomeness, beauty are not physical quantity.

Why measurement is needed?: Physics is an experimental science and experiments involve measurement of different physical quantities in which laws of physics are expressed. Without measuring results of experiments, it would not be possible for scientists to communicate their results to one another or to compare the results of experiments from different laboratories.

Units of measurement: To measure a physical quantity we need some standard unit of that quantity. For example, if a measurement of length is quoted as 5 meters, it means that the measured length is 5 times as long as the value accepted for a standard length defined to be **"one meter".**

Any set of standards of units must fulfill the following two conditions
(i) It must be accessible.
(ii) It must be invariable with the passage of time

Two more auxiliary conditions are:–
(i) It is necessary to have wide unlimited agreement about those standards.
(ii) It is inter convertible to different units of same quantity.

A measurement consists of two parts, one is numeric and the other is standard chosen. For example, 5 meter of length implies 5 times the **"standard meter"**. It is not necessary to establish a measurement standard for every physical quantity. Some quantities can be regarded as fundamental and the standard for other quantities can be derived from the fundamental ones. For example, in mechanics length, mass and time are regarded as fundamental quantities and the standard for speed (= length / time) can be derived from fundamental quantities length and time.

Quantity	SI Units	Symbols
Time	second	s
Length	meter	m
Mass	kilogram	kg
Amount of Substance	mole	mol
Thermodynamic Temp.	kelvin	K
Electric Current	ampere	A
Luminous Intensity	candela	Cd

And two supplementary units are

Plane Angle	Radian	rad

Solid Angle	Steradian	sr

Two other system of units compete with the international system. One is Gaussian System in terms of which much of the literature of physics is expressed. In India this system is not in use.
The other is the British system. This system is still in daily use in United states. But SI units are standard units worldwide.

C.G.S. Unit: In this system of unit, centimeter, gram and seconds are units of length, mass and time respectively.

Conversion of One System of Units to another System: The basic formula is $n_1u_1 = n_2u_2$ where n_1 and n_2 are numbers.

Illustration 1. *How many dyne–centimeter are equal to 1 N–m?*

Solution:
$$1\text{N}-\text{m} = (1\text{kg})(1\text{m})^2(1\text{s})^{-2}$$
$$1\text{dyne}-\text{centimeter} = (1\text{g})(1\text{cm})^2(1\text{ s})^{-2}$$
$$\therefore \frac{1\text{N}-\text{m}}{1\text{dyne}-\text{cm}} = \left(\frac{1000\text{g}}{1\text{g}}\right)\left(\frac{100\text{cm}}{1\text{cm}}\right)^2$$
$$= 1000 \times 10000$$
$$\therefore 1\text{N}-\text{m} = 10^7 \text{ dyne}-\text{cm}$$

Exercise: ***Calculate the value of 1 erg in SI system.***

Measurement of Length
Depending upon the range of length, there are three main methods for measuring length.
(i) Direct method using measuring instruments.
(ii) Indirect method or Mathematical method
(iii) Chemical method

(i) Direct method
The simplest method measuring the length of a straight line is by means of a meter scale. But there exist some limitations in the accuracy of the result:
(i) the dividing lines have a finite thickness.
(ii) naked eye cannot correctly estimate less than 0.5 mm

For greater accuracy devices like
(a) Vernier calliper (b) micrometer scale (screw gauge) are used .

Vernier calliper
It consists of a main scale graduated in cm/mm over which an auxiliary scale (or Vernier scale) can slide along the length. The division of the Vernier scale being slightly shorter than the divisions of the main scale.

Least count of Vernier Calliper

The least count or the Vernier constant (V.B.) is the minimum value of correct estimation of length without eye estimation. The difference between the values of one main scale division and one vernier scale division is known as vernier constant if N division of vernier scale coincides with (N–1) divisions of main scale, then vernier constant,

$$\text{n.V.S.D.} = (\text{n}-1)\ \text{M.S.D.}$$

$$1.\text{V.S.D.} = \left(\frac{\text{n}-1}{\text{n}}\right)\text{M.S.D., and}$$

$$1.\text{M.S.D.} - 1.\text{V.S.D.} = 1.\text{M.S.D.}\left(\frac{\text{n}-1}{\text{n}}\right)\text{M.S.D.}$$

$$= \frac{1}{\text{n}}\ \text{M.S.D.}$$

$$= \frac{\text{Smallest Division on main scale}}{\text{No. of divisions on vernier scale}}$$

Reading a Vernier scale

Let one main scale division be 1 mm and 10 vernier scale divisions coincide with 9 main scale divisions

$$\therefore\ 1.\text{V.S.D.} = \frac{9}{10}\ \text{M.S.D.}=0.9\ \text{mm}$$

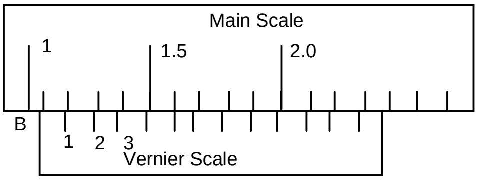
$$\therefore\ \text{Vernier constant} = 1.\text{M.S.D} - 1.\text{V.S.D.} = 1\ \text{mm} - 0.9\ \text{mm}$$
$$= 0.1\ \text{mm} = 0.01\ \text{cm}$$

The reading with vernier scale is read as given below :

(i) Firstly take the main scale reading (N) before on the left of the zero of the vernier scale.
(ii) Find the number (n) of vernier division which just coincides with any of the main scale division. Multiply this number (n) with vernier constant (V.C.)
(iii) Total reading = (N + n × V.C.)

Caution: The main scale reading with which the Vernier scale division coincides has no connection with reading
Suppose If we have to measure a length AB, the end A is coincided with the zero of the vernier scale as shown in fig. Its enlarged view is given in fig.

Length AB > 1.0 cm < 1.1. cm



Let 5th division of vernier scale coincide with 1.6 cm of main scale. From diagram it is clear that the distance between 4th division of vernier scale and 1.5 cm of main scale is equal to one V.C. and distance between zero mark of vernier scale and 1.0 cm mark on the main scale is equal to 5 times the vernier constant.

$$\therefore\quad AB = 1.0 + 5 \times \text{v.c.} = 1.0 + 5 \times 0.01 = 1.05\ \text{cm.}$$

Illustration 2. *In travelling microscope the vernier scale used has the following data.*
1 M.S.D. = 0.5 mm, 50 V.S.D. = 49 M.S.D.
and the actual reading for distance travelled by travelling microscope is 2.4 cm with 8th division coinciding with a main scale graduation . Estimate the distance travelled.

Solution : In this case vernier constant = 1.M.S.D. – 1.V.S.D.

$$= 1.\text{M.S.D.} - \frac{49}{50}\text{M.S.D.} = \frac{1}{50}\text{M.S.D} = \frac{1}{50}\times 0.5\,\text{mm}$$

$= \frac{5}{10} \times \frac{1}{50} = 0.01\text{mm} = 0.001\text{cm}$

$\therefore$ Distance travelled = 2.4 + 8 × 0.001 cm
= 2.408 cm

Illustration 3. *The Vernier scale used in Fortin's barometer has 20 divisions coinciding with the 19 main scale divisions. If the height of the mercury level measured is 5 mm and 15^{th} division of vernier scale is coinciding with the main scale division. Then calculate the exact height of the mercury level (given that 1.M.S.D. = 1mm)*

Solution: 20 V.S.D. = 19 M.S.D. (Given)

1.V.S.D. = $\frac{19}{20}$ M.S.D.

V.C. = 1. M.S.D. – 1.V.S.D $= \left(1 - \frac{19}{20}\right)$M.S.D.

$= \frac{1}{20}$ M.S.D.

$= \frac{1}{20} \times 1\text{mm} = 0.05\text{mm}$

= 0.005 cm

Height of mercury level = 5 + 0.05 × 15
= 5.75 mm

Exercise: ***The Vernier calliper is used to measure the length of an object. The least count of such a vernier calliper is 0.2 cm and scale reads its length to be 5.6 cm. 3^{rd} division of Vernier scale is coinciding main scale division Calculate the length of an object.***

Zero Error

If the zero marking of main scale and Vernier scale do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument. If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.

Illustration 4. *Consider the following data:*
10 main scale divisions = 1cm, 10 vernier division = 9 main scale divisions,
zero of Vernier scale is to the right of the zero marking of the main scale with 6^{th} Vernier division coinciding with a main scale division and the actual reading for length measurement is 4.3 cm with 2^{nd} Vernier divisions coinciding with a main scale graduation. Estimate the length.

Solution: In this case, vernier constant = $\frac{1\text{mm}}{10} = 0.1\text{mm}$

Zero error = 6 × 0.1 = + 0.6 mm

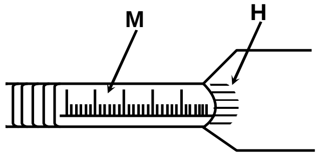
Correction = –0.6 mm

Actual length = (4.3 + 2 × 0.01) + correction
= 4.32 – 0.06 = 4.26 cm

Screw Gauge (or Micrometer Screw)

In general Vernier Callipers can measure accurately upto 0.02 mm and for greater accuracy micrometer screw devices, e.g. screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by turning it axially. The instrument is provided with two scales:

(i) The main scale or pitch scale M graduated along the axis of the screw.
(ii) The cap–scale or head scale H round the edge of the screw head.



Constants of the screw gauge:

(a) Pitch: The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is the distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus if 10 rotations of cap ≡5 mm, then pitch = 0.5 mm

In general, pitch $= \dfrac{\text{Distance travelled by screw on main scale}}{\text{No. of rotation taken by the cap to travel that much distance}}$

(b) Least count: In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the main scale which is equal to pitch divided by the total cap divisions. Thus in the aforesaid Illustration, if the total cap division is 100, then least count = 0.5 mm/100

= 0.005 mm
In general, In case of circular scale,

Least count $= \dfrac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

If pitch is 1 mm and there are 100 divisions on circular scale, then

Least count = $= \dfrac{1\text{mm}}{100}$ = 0.01 mm = 0.001 cm

= 0.00001 m = 10^{-5} m = 10 μm.

Since least count is of the order of 10 μm, So the screw is called a micrometer screw. Screw gauge and the spherometer which work on the principle of micrometer screw, consist essentially of the following two scales.

(i) Linear or Pitch scale: It is a scale running parallel to the axis of the screw.

(ii) Circular of Head scale: It is marked on the circumference of the circular disc or the cap attached to the screw.

Zero Error: In a perfect instrument the zero of the head scale coincides with the line of graduation along the screw axis with no zero–error, otherwise the instrument is said to heave zero–error which is equal to the cap reading with the gap closed. This error is positive when zero line or reference line of the cap lies below the line of graduation and vice–versa. The corresponding corrections will be just opposite.

Illustration 5. *A screw gauge has 100 divisions on its circular scale. Circular scale travels one division on linear scale in one rotation and 10 divisions on linear scale of screw gauge is equal to 5 mm. What is the least count of a screw gauge.*

Solution: Pitch $= \dfrac{1\,\text{division on linear scale}}{1\,\text{rotation}} = 1\,\text{div.}$

10 division = 5 mm

∴ 1 division = 0.5 mm

∴ pitch = 0.5 mm

$\text{least count} = \dfrac{\text{Pitch}}{\text{No. of divisions on circular scale}}$

$$= \frac{0.5\,\text{mm}}{100} = 0.005\,\text{mm}$$

Illustration 6. *The screw gauge mentioned in above illustration is used to measure thickness of a coin. The reading of the linear scale is 4^{th} div and 25^{th} division of circular scale is coinciding with it. What is the value of thickness of the coin..*

Solution: Reading = Linear scale Reading + Least count × circular scale reading
= 4^{th} division on linear scale + 0.005 mm × 2.5
= 4 × 0.5 mm + 0.125 mm
= 2 mm + 0.125 mm
= 2.125 mm

Illustration 7. *A spherometer has 250 equal divisions marked along the periphery of its disc and one full rotation of the disc advances it on the main scale by 0.0625 cm. The least count of the spherometer is*
(A) 2.5×10^{-2}cm *(B) 25×10^{-3}cm*
(C) 2.5×10^{-4} cm *(D) none of the above*

Solution: Least count = $\frac{0.0625}{250}$ cm = 2.5×10^{-4}cm
$\therefore$(C)

(ii) Indirect or Mathematical method

This method involves measurement of long distances. Main methods of this category are –

Reflection method: Suppose we want to measure the distance of a multi story building from a destination point P. If a shot be fired from P, the sound of shot travels a distance x towards the building, gets reflected from the building. The reflected sound travels the distance x to the point of P, when an echo of the shot is heard.

Let t = time interval between the firing of the shot and echo sound.
v = velocity of sound in air.

Distance = velocity x time

x + x = (v) (t)

⇒ x = (v) (t/2)

As v is known, x can be calculated by measuring the time t.

Illustration 8. *A rock is at the bottom of a very deep river. An ultrasonic signal is sent towards rock and received back after reflection from rock in 4 seconds. If the velocity of ultrasonic wave in water is 1.45 km/s, find the depth of river.*

Solution: Here x = ?

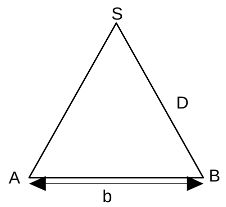
v = 1.45 km/s = 1450 m/sec.

t = 4 sec

so, x = v x t / 2 = 1450 x 4 / 2 = 2900 m.

Parallel method: This method is used for measuring distance of nearby stars.

Let we have to measure the distance D of a far away star S by this method. We observe this star from two different position A and B on the earth, separated by a distance AB = b at the same time as shown in figure. Let $\angle ASB = \theta$, the angle θ is called parallatic angle. As the star is very far away, b/D << 1 and θ is very small.



Here we can take AB as an arc of length b of a circle with centre at S and the distance D as the radius AS=BS so that AB = b = Dθ where θ is in radians.

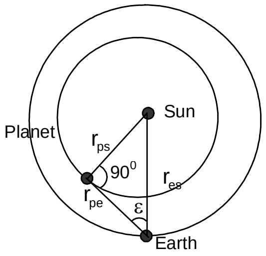
$D = b/\theta$

Knowing b and measuring θ, we can calculate D.

Copernicus method: This method is used to measure the relative distances of the planets from the Sun.

(a) For Interior Planets: The angle formed at earth between the earth–planet direction and the earth–sun direction is called the planet's elongation. This is the angular distance of the planet from the sun as observed from earth. When the elongation attains its maximum value ε as in the figure, the planet appears farthest from Sun.

$$r_{ps} = r_{es} \sin\varepsilon$$
$$= (\sin\varepsilon)\,AU \quad (AU = \text{Astronomical Unit})$$



(b) For Exterior Planets: This method is a consequence of Kepler's 3rd law of planetary motion. For two planets P_1 and P_2 we have,

$$\frac{a_2^3}{a_1^3} = \frac{T_2^2}{T_1^2}$$

where a_1 and a_2 are semi–major axes, of respective orbits. Period can be ascertained by direct observation. Therefore if a_1 is measured, a_2 can be calculated.

(iii) Chemical Method

This method is used to measure distance of the order of $10^{-1\circ}$ m. Let us calculate the size of an atom.

Let m = mass of substance,

V = volume occupied by substance &

ρ = density of the substance

$\therefore v = m/\rho$ (1)

Let M be the atomic weight of the substance and N be the Avogadro number.

$\therefore$ No. of atoms in mass m of the substance = Nm / M

If r = radius of each atom then V = volume of each atom = $\frac{4}{3}\pi r^3$

Volume of all the atoms in substance = $(\frac{4}{3}\pi r^3 \text{ x Nm})M$.

According to Avagordo's hypothesis,

Volume of all the atoms = (2/3) x volume of substance

$$\frac{4}{3}\pi r^3 \text{ x Nm/M} = (2/3)\, m/\rho$$

$$\therefore r = \left(\frac{M}{2\pi N\rho}\right)^{1/3}$$

MEASUREMENT OF MASS

Measurement of Inertial Mass

Inertial mass of a body is measured using a device which is known as inertial balance. It consists of a long metal strip. One end of the strip is clamped to a table such that its flat face is vertical, and it can easily vibrate horizontally. The other end of strip supports a pan in which the object whose inertial mass is to be found can be kept. It is found that the square of time period of vibration is directly proportional to total mass of the pan and the body placed in it.

$$t^2 \propto m$$

$$\therefore \qquad \frac{t_2^2}{t_1^2} = \frac{m_2}{m_1}$$

$$\Rightarrow \qquad m_2 = m_1 \frac{t_2^2}{t_1^2}$$

Measurement of Time: The following methods are used

(a) Quartz Crystal Clock
(b) Atomic Clock
(c) Radioactive dating

Significant figures:

Each measurement involves errors. The measure results has a number that includes all reliably known digits and first unknown digit. The combination of reliable digits and first uncertain digit are significant figures.

Example: If a length is measured as 2.43 cm then 2 and 4 are reliable while 3 is uncertain. Thus the measured value has three significant figures.

Common rules for counting significant figures

(1) All non zero digits are significant.
For example: 1745 has four significant digits.
(2) All zeros present between 2 non zero digits are significant, irrespective of the position of the decimal point.

Example: 208005 has 6 significant figures.
(3) If there is no decimal point, all zeros to the right of the right–most non zero digit are considered to be significant only if they come from a measurement.

Example: 41000 has only 2 significant digits while 41000 m has 5 significant digits.
(4) All zeros to the right of a decimal point but to the left of non–zero digits are considered to be non significant, provided there should be no non zero digit to the left of the decimal point.
Example: 0.00305 has 3 significant figures.
(5) All zeros are significant if they are placed to the right of a decimal point and to the right of a non zero digit.

Example: 0.04080 has 4 significant figures
50.000 has 5 significant figures

(6)The number of significant figures does not alter in different units.
If we want to write 450 m in different units, we can write it 4.50×10^{4} cm or 4.50×10^{5} mm etc. in which all of them are having 3 significant figures.

Illustration 9. *State the number of significant figures in the following –*
(a) 06500310
(b) 754400
(c) 15000 kg
(d) $8.314\times10^{+2}$ J
(e) 1.6×10^{-19} C
(f) 0.0065050

Solution: (a) 7 (b) 4 (c) 5 (d) 4 (e) 2 (f) 5

Exercise: ***The number of significant figures in 0.0160 is***
(a) 2 ***(b) 3*** ***(c) 4*** ***(d) 5***

Rounding off

(1) If all the digits to be discarded are such that the first discarded digit is less than 5, the remaining digits are left unchanged.

Example:

7.499498 can be written in 4 significant figures as 7.499

(2) If the digit to be discarded is 5 followed by digits other than zero, then the preceding digit should be raised by 1.

Example:

7.45001, on being rounded off to first decimal, became 7.5

(3) If the digits to be discarded is 5 or 5 followed by zero the preceding digit remains unchanged if it is even and the preceding digit is raised by 1 if it is odd.

Example:

3.6500 will become 3.6 and 4.7500 will become 4.8 in 2 significant figures.

Arithmetic operations with significant figures:

(1) Addition and subtraction In addition and subtraction, the number of decimal places in the result is the smallest number of decimal places of terms in the operation.

Let us consider the sum of following measurements.
3.45 kg., 7.6 kg. and 10.055 kg.

$$\begin{array}{r} 3.45 \\ 7.6 \\ 10.055 \\ \hline 21.105 \\ \hline \end{array}$$

So the sum will be 21.1 kg as 7.6 kg has only 1 digit after the decimal point while the others are having more than one digit.

Multiplication and Division:

In the result of multiplication or division, the number of significant figures is same as the smallest number of significant figures among the numbers.

Illustration 9: *Multiply 1.21 and 1.1.*

Solution: $1.21 \times 1.1 = 1.331$

So the result is 1.3 as there are only 2 significant digits in 1.1

The same procedure is followed for division.

Exercise: Value of 1.2 + 1.34 + 2.342 is
(a) 4.88 ***(b) 4.8*** ***(c) 4.90*** ***(d) 5***

Accuracy and Precision of measuring instrument: It is impossible to measure any physical quantity perfectly. It is due to imperfection in manufacturing and working of measuring instruments.

Accuracy: It is the degree of correctness of the measured quantity, i.e. how much close the result is to the true value of the physical quantity.

Precision: It is the degree of repeatability & refinement of a measurement.

ERRORS IN MEASUREMENT

In the experiment we may get some other value than that of the true value due to faulty equipment, carelessness or random causes. This will cause error in measurement.

There are 3 ways to express an error

(1) Absolute Error: It is the positive value of difference between the true value and measured value of the quantity. Since we don't know the correct value of quantity the best possible value can be given by mean value of all the measured value.

Arithmetic mean v, $A_m = \dfrac{A_1 + A_2 + ...A_n}{n} = \dfrac{1}{n}\sum_{i=1}^{n} A_i$

$\therefore$ The absolute error in the measurement can be given as.

$\Delta A_1 = |A_m - A_1|$ where A_m : Mean value of the measurements.

$\Delta A_2 = |A_m - A_2|$ where A_1, A_2 : Measured value of quantity.

$\Delta A_n = |A_m - A_n|$

Taking the arithmetic mean of all the absolute errors we get the mean absolute error ΔA_m.

$$\Delta A_m = \frac{\Delta A_1 + \Delta A_2 + ... + \Delta A_n}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} \Delta A_i$$

So the true value of A will be such that

$$\left(A_m - \Delta A_m\right) \le A \le \left(A_m + \Delta A_m\right)$$

(2) Relative Error: It is defined as the ratio of the mean absolute error to the mean value of the quantity being measured

Relative error $= \frac{\Delta A_m}{A_m}$

(3) Percentage Error: The relative error can be expressed in percentage error as

% error = Relative error $\times 100$

Propagation of Error: Any physical quantity depends on one or more than one physical quantities. So the error in any physical quantity will lead to error in the result.

(1) Error in result involving sum or difference of quantities

Let Z is defined as

$$Z = A + B - C$$

$\therefore \quad \Delta Z = \Delta A + \Delta B - \Delta C$

$\therefore$ Maximum possible error in Z is given by

$|\Delta Z|_{max} = \Delta A + \Delta B + \Delta C$ (Since ΔC can be positive or negative)

2. Error in the result having product or division of quantities:

$$Z = \frac{A^p B^q}{C^r}$$

$\Rightarrow \quad \ln z = p\ln A + q\ln B - r\ln c$

$\Rightarrow \quad \frac{dz}{z} = \frac{pdA}{A} + \frac{qdB}{B} + \frac{rdC}{C}$

For small change $dz \approx \Delta z. \Rightarrow \frac{\Delta z}{z} = P\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$

Illustration10. *Multiply 107.88 by 0.610 and express the result upto the correct number of significant figure.*

(A) 65.8068 *(B 65.807*

(C) 65.81 *(D) 65.8*

Solution: Number of significant figures in multiplication is three corresponding to the minimum number 107.88×0.610

= 65.8068 = 65.8

$\therefore$ (D)

Illustration11. *In measurement of the period of oscillation of a Helical spring, the readings comes out to be 2.15 sec, 2.25 sec, 2.36 sec, 2.45 sec and 2.54 sec, calculate the absolute errors, relative error or percentage error.*

Solution: The mean period of oscillation of the Helical spring is

$$T = \frac{2.15 + 2.25 + 2.36 + 2.45 + 2.54}{5}$$

= 2.35 sec

The absolute error in the measurements are

2.15 – 2.35 = – 0.20 sec

2.25 – 2.35 = – 0.10 sec

2.36 – 2.35 = 0.01 sec

2.45 – 2.35 = 0.10 sec

2.54 – 2.35 = 0.19 sec

The arithmetic mean of all the absolute errors (for arithmetic mean, we take only the magnitudes) is ΔT_{mean} = [(0.20 + 0.10 + 0.01 + 0.10 + 0.19)] /5

$= \frac{0.6}{5} = 0.12\,\text{sec}$

Period of oscillation of the simple pendulum is (2.35 ± 0.12) sec.
A more correct way to write its is
(2.4 ± 0.2) sec
The relative error or the percentage error is

$= \frac{0.2}{2.4} \times 100 = 8\%$

Combination of Errors

While doing an experiment we take several measurements, we must know how the errors in all the measurements combine.
To make such estimates, we should learn how errors combine in various mathematical operations. For this we use the following procedure

(I) Error of a sum or difference: Suppose two physical quantities A and B have measured values A ± ΔA, B ± Δ B respectively where ΔA and ΔB are their absolute errors.

(a) We wish to find the error Δz in the sum
z= A + B

We have by addition, z ± Δz

= (A ± ΔA) + (B ± Δ B)

The maximum possible error in z = Δ z = ΔA + ΔB

(b) For the difference z = A – B, we have

z ± Δz = (A ± Δ A) – (B ± Δ B)

= (A – B) ± Δ A ± Δ B

or, ± Δ z = ± Δ A ± Δ B

The maximum value of the error Δz is again ΔA + ΔB.

Hence the rule : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual.

Illustration 12. *The series combination of resistances is given by $R = R_1 + R_2$*
Suppose two resistances $R_1 = (50 \pm 4)\Omega$ and $R_2 = (100 \pm 3)\ \Omega$ are connected in series. Find equivalent resistance of the series combination.

Solution: $R_{eq} = R_1 + R_2$
= (50 ± 4) Ω + (100 + 3) Ω
= (150 ± 7) Ω

(II) Error in a product or a quotient

Suppose z = AB and the measured values of A and B are A ± ΔA and B ± ΔB. Then
z ± Δz = (A ± ΔA) (B ± ΔB)

= AB ± B ΔA ± A ΔB ± ΔA ΔB

Dividing L.H.S. by z and R.H.S. by AB, we have

$$1 \pm \frac{\Delta z}{z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \left(\frac{\Delta A}{A}\right)\left(\frac{\Delta B}{B}\right).$$

Since ΔA and ΔB are small we shall ignore their product.

Hence the maximum fractional error in Z

$$= \frac{\Delta z}{z} = \left(\frac{\Delta A}{A}\right) + \left(\frac{\Delta B}{B}\right)$$

Similarly, we can easily verify that this is true for division also.

So, when two or more quantities multiplied or divided, the fractional error in the result is the sum of the fractional errors in the multipliers.

(III) Error due to the power of a measured quantity.

Let $Z = X^2$

Then $\frac{\Delta Z}{Z} = \frac{\Delta X}{X} + \frac{\Delta X}{X} = \frac{2\Delta X}{X}$

Hence the fractional error in X^2 is two times the error in X.

In general if $Z = \frac{X^a Y^b}{Q^c}$

then $\frac{\Delta Z}{Z} = a\frac{\Delta X}{X} + b\frac{\Delta Y}{Y} + \left[\frac{\Delta Q}{Q} \times c\right]$

Illustration 13. *Find the fractional error in Z, if* $Z = \sqrt{\frac{XY}{M}}$

Solution: $\frac{\Delta Z}{Z} = \frac{1}{2}\frac{\Delta X}{X} + \frac{1}{2}\frac{\Delta Y}{Y} + \frac{1}{2}\frac{\Delta M}{M}$

Illustration 14. *Find maximum possible percentage error in* $x = \frac{a^{\ell} b^m}{y^p z^k}$

Solution: $\frac{\Delta X}{X} \times 100 = \left(\ell\frac{\Delta a}{a} + m\frac{\Delta b}{b} + p\frac{\Delta y}{y} + k\frac{\Delta z}{z}\right) \times 100$

Illustration 15. *In the relation $x = 3yz^2$, x, y and z represents various physical quantities, if the percentage error in measurement of y and z is 3% and 1% respectively, then final maximum possible percentage error in x.*

Solution: $\frac{\Delta x}{x} \times 100 = \left(\frac{\Delta y}{y} + 2\frac{\Delta z}{z}\right) \times 100$

$= 3\% + 2 \times 1\% = 5\%$

PHYSICAL QUANTITIES

All the physical quantities can be expressed in terms of some combination of seven base quantities: Length [L], mass [M], time [T], electric current [I], thermodynamic temperature [K], luminous intensity [cd] and amount of substance [mol]. These base quantities are considered as the seven dimensions of the physical world.

DIMENSIONS

The dimension of a physical quantity are the powers to which the fundamental (or base) quantities like mass, length and time etc. have to be raised to represent the quantity. Consider the physical quantity **"Force"**. The unit of force is Newton.

1 Newton = 1 kg m/sec^2

kg $\to M^1$ (Mass); m $\to L^1$ (Length); $s^{-2} \to T^{-2}$ (Time)

$\therefore$ Dimensions of force are $[M^1L^1T^{-2}]$

Dimensional formulae for some physical quantities

Physical quantity	Relation with other quantity	Dimensional formula
Area	Length × breadth	$L \times L = [L^2]$
Density	Mass/volume	$\frac{M}{L^3} = [ML^{-3}]$
Acceleration	$\frac{\Delta v}{\Delta t} = \frac{\text{change in velocity}}{\text{time}}$	$\frac{LT^{-1}}{T} = [LT^{-2}]$
Force	F = ma	$[MLT^{-2}]$
Linear momentum	P = mv	$[MLT^{-1}]$
Pressure	P = F/A	$[ML^{-1}T^{-2}]$
Universal gravitational constant	$G = \frac{Fr^2}{M_1M_2}$	$[M^{-1}L^3T^{-2}]$
Work	$W = F \times d$	$[ML^2T^{-2}]$
Energy (kinetic, potential and heat)	$\frac{1}{2}mv^2$	$[ML^2T^{-2}]$
Surface tension	$T = \frac{F}{\ell}$	$[ML^\circ T^{-2}]$
Strain	$e = \frac{\Delta \ell}{\ell}$	$[M^\circ L^\circ T^\circ]$
Modulus of elasticity	$E = \frac{\text{stress}}{\text{strain}}$	$[ML^{-1}T^{-2}]$
Angle	$\theta = \frac{\text{arc}}{\text{radius}}$	$[M^\circ L^\circ T^\circ]$
Coefficient of viscosity	$\eta = \frac{F \times r}{A \times v}$	$[M^1L^{-1}T^{-1}]$
Planck's constant	$h = mv\lambda$	$[ML^2T^{-1}]$
Thermal resistance	$\frac{\Delta\theta t}{Q}$	$[M^{-1}L^{-2}T^3\,\theta]$
Thermal conductivity	$K = \frac{H}{At(d\theta/dx)}$	$[MLT^{-3}\theta^{-1}]$
Boltzman's constant	k = R/N	$[ML^2T^{-2}\theta^{-1}]$
Universal gas constant	$R = \frac{PV}{T}$	$[ML^2T^{-2}\theta^{-1}]$
Mechanical equivalent of heat	J = W/H	$[M^\circ L^\circ T^\circ]$
Decay constant	$\lambda = \frac{0.693}{T_{1/2}}$	$[M^\circ L^\circ T^{-1}]$

Illustration 16. *Write the dimensions of: Impulse, Pressure, Work, Universal constant of Gravitation.*

Solution: (i) $[M^1L^1T^{-1}]$ (ii) $[M^1L^{-1}T^{-2}]$ (iii) $[M^1L^2T^{-2}]$ (iv) $[M^{-1}L^3T^{-2}]$

Four types of quantities

Dimensional constant: These are the quantities whose values are constant and they possess dimensions. For example, velocity of light in vacuum, universal gas constant etc.

Dimensional variables: These are the quantities whose values are variable, and they possess dimensions. For example, area, volume, density etc.

Dimensionless constants: These are the quantities whose values are constant, but they do not possess dimensions. For example, π, 1, 2, 3, ….. etc.

Dimensionless Variables: These are the quantities, whose values are variable, and they do not have dimensions, e.g., angle, strain, specific gravity etc.

Uses of dimensions: dimensional analysis

(1) Checking the correctness (dimensional consistency) of an equation: An equation contains several terms which are separated from each other by symbols of equality, plus or minus. The dimensions of all the terms in an equation must be identical. This means that we can not add velocity to force. This principle is called Principle of Homogeneity of dimensions.
Look at the equation : $v^2 = u^2 + 2as$
Dimensions of $v^2 : [L^2T^{-2}]$
Dimensions of $u^2 : [L^2T^{-2}]$
Dimensions of $2as : LT^{-2}][L] = [L^2T^{-2}]$
$\therefore$ The equation $v^2 = u^2 + 2as$ is dimensionally consistent, or dimensionally correct.

Note:
A dimensionally correct equation may not be actually correct. For example, the equation $v^2 = u^2 + 3as$ is also dimensionally correct but we know that it is not actually correct. However, all correct equations must necessarily be dimensionally correct.

Illustration 17. *Which of the following equations may be correct ?*

(i) $x = ut + \frac{1}{2}at^2$ *(ii)* $T = 2\pi\sqrt{\frac{L}{g}}$

(iii) $F = \frac{GM_1M_2}{r}$ *(iv)* $T^2 = \frac{4\pi^2R^3}{GM}$

(v) $V = \sqrt{GMR}$

Given: G = Gravitational constant, whose dimensions are $[M^{-1}L^3T^{-2}]$
M_1, M_2 *and M have dimensions of mass. L, x, r, R has dimenssions of length. And t has dimensions of Time. 'F' denotes Force and 'a' has dimensions of acceleration.*

Solution: (i) Yes (ii)Yes (iii) No (iv) Yes (v) No.

(2) Conversion of units: Dimensional methods are useful in finding the conversion factor for changing the units to a different set of base quantities. Let us consider one example, the SI unit of force is Newton. The CGS unit of force is dyne. How many dynes is equal to one newton. Now,

$$1 \text{ newton} = [F] = [M^1L^1T^{-2}] = (1\text{kg})^1)(1\text{meter})^1(1\text{s})^{-2}$$

$$1\text{dyne} = (1\text{g})(1\text{cm})(1\text{s})^{-2}$$

$$\therefore \frac{1\text{newton}}{1\text{dyne}} = \frac{(1\text{kg})^1(1\text{meter})^1(1\text{s})^{-2}}{(1\text{g})(1\text{cm})(1\text{s})^{-2}} = (10^3)(10^2) = 10^5$$

$$1 \text{ newton} = 10^5 \text{ dynes}$$

Thus knowing the conversion factors for the base quantities, one can work out the conversion factor of any derived quantity if the dimensional formula of the derived quantity is known.

Illustration 18. *Find the conversion factor for expressing universal gravitational constant from SI units to cgs units.*

Solution: $6.67\text{x}10^{-8}\ \text{cm}^3\text{s}^{-2}\text{g}^{-1}$

(3) Deducing relation among the physical quantities:

Suppose we have to find the relationship connecting a set of physical quantities as a product type of dependence. Then dimensional analysis can be used as a tool to find the required relation. Let us consider one example. Suppose we have to find the relationship between gravitational potential energy of a body in terms of its mass 'm', height 'h' from the earth's surface and acceleration due to gravity 'g', then,
Let us assume that: – Gravitational potential energy, U,

$$U = K[m]^a[g]^b[h]^c,$$

where K, a, b, and c are dimensionless constants.

Then $[ML^2T^{-2}] = [M]^a[LT^{-2}]^b[L]^c$

$$= [M^aL^{b+c}T^{-2b}]$$

$\therefore$ a =1, b + c = 2

–2b = –2

b = 1, c = 1.

$\therefore$ U = Kmgh, where K is a dimensionless constant.

Thus by dimensional analysis, we conclude that the gravitational potential energy of a body is directly proportional to its mass, acceleration due to gravity and its height from the surface of the earth.

Limitations of dimensional analysis:

This method does not give us any information about the dimensionless constants appearing in the derived formula, e.g. 1, 2, 3, .. . π etc.
We can't derive the formula having trigonometrical functions, exponential functions etc, which have no dimensions.

The method of dimensions cannot be used to derive an exact form of relation when it consists of more that one part on any side, e.g. the formula $v^2 = u^2 + 2as$ cannot be obtained.

If a quantity depends on more than three factors having dimensions the formula cannot be derived. This is because on equating powers of M, L and T on either side of the dimensional equation, we can obtain three equations from which only three exponents can be calculated.
It gives no information whether a physical quantity is a scalar or a vector.

Illustration 19. *Using the method of dimensions, find the acceleration of a particle moving with a constant speed v in a circle of radius r.*

Solution: Assuming that the aceeleration of a particle depends on v and r
$a \propto v^x r^y \Rightarrow a = k\, v^x r^y$
Now as we know dimensions of acceleration (a) = $M^\circ LT^{-2}$
and dimensions of velocity (v) = $M^\circ LT^{-1}$
dimension of radius (r) = $M^\circ LT^\circ$
Putting all thee dimensions in (1), we get
$M^\circ LT^{-2} = k\,(M^\circ LT^{-1})^x (M^\circ LT^\circ)^y$
$M^\circ LT^{-2} = k\, M^\circ L^{x+Y} T^{-x}$
Comparing the powers, we get
$x + y = 1$
$x = 2$
$\therefore\ y = 1-2 = -1$
$\therefore\ a = k\, v^2 r^{-1}$
$a = \dfrac{kv^2}{r}$

Illustration 20. *In the expression* $\left(P + \dfrac{a}{v^2}\right)(v - b) = RT$
P is pressure and v is the volume. Calculate the dimensions of a and b.

Solution: Only physical quantities having same dimensions are added or subtracted. So $\dfrac{a}{v^2}$ has the same dimensions as that of pressure.
As pressure = $\dfrac{\text{Force}}{\text{Area}}$
Dimensions of pressure $= \dfrac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$
$\therefore$ Dimensions of $\dfrac{a}{v^2} = ML^{-1}T^{-2}$
Dimensions of a $= ML^{-1}T^{-2}(V^3)^2$
$= (ML^{-1}T^{-2})(L^3)^2$
$= ML^{-1}T^{-2}L^6 = ML^5T^{-2}$
Similarly dimensions of b is same as that of volume.
Dimensions of $b = M^0L^3T^0$.

Illustration 21. *Does* $S_{nth} = u + \dfrac{a}{2}(L_n - 1)$ *dimensionally correct?*

Solution: Yes, this expression is dimensionally correct, yet it appears to be incorrect. As we are taking it to be for n^th^ second. Here one second is divided through the equation.

Illustration 22. *Find the dimensions of resistivity, thermal conductivity and coefficient of viscosity.*

Solution: (i) $R = \rho \frac{\ell}{A}$

$\rho = \frac{RA}{L} = [ML^3T^{-3}A^{-2}]$

(ii) Thermal conductivity, k

$\frac{d\theta}{dt} = \frac{k\ell}{A\Delta\theta} = \frac{ML^2T^{-3}L}{L^2K}$

$= MLT^{-3}k^{-1}$

(iii) Coefficient of viscosity

$\therefore\ F = \eta A \frac{dv}{dx}$

$\eta = \frac{Fdx}{Adv} = \frac{(MLT^{-2})(L)}{(L^2)(LT^{-1})} = ML^{-1}T^{-1}$

Illustration 23. *A displacement of a particle is given by equation y = A sin ωt, where y is in metres and A is also in metres, t is in seconds. What are the dimensions of ω.*

Solution: As the angles are always dimensionless, so

ωt = dimensionless quantity

Dimensions of $\omega t = M^\circ L^\circ T^\circ$

Dimensions of $\omega = M^\circ L^\circ T^{-1}$

Illustration 24. *If density ρ, acceleration due to gravity g and frequency f are the basic quantities, find the dimensions of force.*

Solution: We have $\rho = ML^{-3}$, $g = LT^{-2}$ $f = T^{-1}$

Solving for M, L and T in terms of ρ, g and f , we get

$M = \rho^2 g^3 f^{-6}$, $L = gf^{-2}$ & $T = f^{-1}$

Force = $[MLT^{-2}] = [\rho g^3 f^{-6}.gf^{-2}.\ f^2] = [\rho g^4 f^{-6}]$

Illustration 25. *An athlete's coach told his team that muscle times speed equals power. What dimensions does he view for "muscle"?*

(A) MLT^2 *(B) ML^2T^{-2}*

(C) MLT^{-2} *(D) L*

Solution: Power = force × velocity

= muscle times speed

$\therefore$ muscle represents force

muscle = $[MLT^{-2}]$

$\therefore$ (C)

Illustration 26. *If force, length and time would have been the fundamental units what would have been the dimensional formula for mass*

(A) $FL^{-1}T^{-2}$ *(B) $FL^{-1}T^2$*

(C) FLT^{-2} *(D) F*

Solution: Let $M = K\ F^aL^bT^c$

$= [MLT^{-2}]^a\ [L^b]\ T^c = [M^aL^{(a+b)}T^{(-2a+c)}]$

$a = 1,\ a + b = 0 \quad \& \ -2a + c = 0$
$\Rightarrow a = 1,\ b = -1,\ c = 2$
$\therefore$ (B)

Illustration 27. *The dimensions of the Rydberg constant are*
(A) $M^{\circ} L^{-1} T$ *(B) MLT^{-1}*
(C) $M^{\circ} L^{-1} T^{\circ}$ *(D) $ML^{\circ} T^{2}$*

Solution: From the relation $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$R = \frac{1}{L} = L^{-1} = M^{\circ}L^{-1}T^{\circ}$
$\therefore$ (C)

Illustration 28. *The error in the measurement of the radius of a sphere is 1%. Then error in the measurement of volume is*
(A) 1% *(B) 5%*
(C) 3% *(D) 8%*

Solution: $V = \frac{4}{3}\pi r^3$

$\frac{\Delta V}{V} \times 100 = 3\left(\frac{\Delta r}{r}\right) \times 100 = 3 \times 1 = 3\%$
$\therefore$ (C)

MISCELLANEOUS EXERCISE

1. The time period of a gas bubble formed under water oscillating with a time period depending on static pressure P, density of water ρ and ε total energy of explosion. Find the relationship between T, P, ρ and ε.

2. Name the three physical quantities having the same dimensions

3. A student measures the time period of a simple pendulum. If error in measurement of length is 2% and error in measurement of g is 2% calculate the error in measurements of Time period.

4. A physical quantity is given by x = a + bt, where x is in metres and t is in seconds. So calculate the dimensions of a and b.

5. Find the dimensional formulae of the following quantities
   (A) the universal gravitational constant (B) Surface tension
   (C) Potential energy (D) Surface energy

6. A Vernier calliper has 50 divisions on its Vernier scale. Minimum division in main scale is of 1 mm.
   (a) Find out the least count of the Vernier Calliper.
   (b) During a length measurement, zero of Vernier scale is between 12th and 13th divisions of the main scale and 26th division of the Vernier scale coincides with a main scale division. What is the length ?

7. Calculate the focal length of a spherical mirror if measured quantities u and v are as follows.
   $u = 50.1 \pm 0.5\,\text{cm}$
   $v = 20.1 \pm 0.2\ \text{cm}$

8. Young's modulus of steel is 19 x $10^{1\circ}$ N/m^2. Express it in dyne/cm^2 . Here dyne is the CGS unit of force.

9. Convert 1 joule into ergs.

10. A goldsmith puts some gold weighting 5.42 gm in a box weighing 1.2 kg. Find the total weight of the box to correct number of significant figures.

SOLUTION TO MISCELLANEOUS EXERCISE

1. $T \propto P^{-5/6}\rho^{1/2}E^{1/3}$
2. Work, energy and torque
3. 2%
4. x = a + bt
   Dimensions of a = $M^\circ L T^\circ$
   Dimensions of b = $M^\circ L T^{-1}$
5. (a) $M^{-1}L^2T^{-2}$ (b) MLT^{-2}
   (c) ML^2T^{-2} (d) ML^2T^{-2}
6. 0.02 mm, 12.52 mm
7. 14.3 ± 0.4 cm
8. 19×10^{11} dyne / cm^2
9. 1 joule = 10^7 ergs
10. 1.2 kg

SOLVED PROBLEMS

Subjective:

Prob 1. *The Bernoulli's equation is given by,*

$P + \frac{1}{2}\rho v^2 + \rho gh = constant$ *, where P is pressure. Compare the unit of the quantity* $\frac{1}{2}\rho v^2$ *with the unit of pressure.*

Sol. Only same quantities can be summed up or subtracted from each other. So $\frac{1}{2}\rho v^2$ has same unit as that of pressure.

Prob 2. *The relation between velocity and time of a moving body is given as,* $v = A + \frac{B}{t} + Ct^2$ *. Find the units of A, B and C.*

Sol. From the principle of homogeneity

v = A = m/sec

v = B/t $\Rightarrow$ B = m

v = (ct^2)

$\therefore$ c = m/sec^3

Prob 3. *The external and internal diameters of a hollow cylinder are measured to be (4.23 ±0.01) cm and (3.89 ± 0.01) cm. What is the thickness of the wall of the cylinder?*

Sol. Thickness = (4.23 – 3.89) cm = $\frac{0.34}{2}$ = 0.17 cm.

Error = ±(0.01 + 0.01) cm = ±0.02 cm

Prob 4. *A physical quantity x is calculated from the relation x =* $\frac{a^2 b^3}{c\sqrt{d}}$*. If percentage error in a, b, c and d are 2%, 1%, 3% and 4% respectively. What is the percentage error in x?*

Sol. As x = $\frac{a^2 \ b^3}{c\sqrt{d}}$

$$\frac{\Delta x}{x} = \pm\left[2\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}\right]$$

$$= \pm\ [2 \times 2\% + 3\times 1\% + 3\% + \frac{1}{2} \times 4\% = \pm\ 12\%$$

Prob 5. *A physical quantity A is defined as,* $A = \mathrm{pkx^2y/z}$*. The absolute errors in the measured of x, y, z are given as*

x = (0.26 ± 0.02)cm

y = (64 ± 2) Ω

z = (156.0 ± 0.1)cm

Find the percentage error in the quantity A.

Sol. $a = kx^2y / z$

$\Rightarrow \frac{\Delta A}{A} = 2\frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$

$= 2 \times \frac{0.02}{0.26} + \frac{2}{64} + \frac{0.1}{156}$

$\pm 0.186 = 18.6\ \%$

Prob 6. *10 rotations of the cap of a screw gauge is equivalent to 5 mm. The cap has 100 divisions. Find the least count. A reading taken for the diameter of wire with the screw gauge shows 4 complete rotations and 35 on the circular scale. Find the diameter of the wire.*

Sol. The least count = $\frac{5}{1000}$ = 0.005 mm

The diameter of the wire = $(4 \times 0.5 + 35 \times 0.005)$ mm = 2.175 mm

Prob 7. *The diameter of a sphere is 2.78 cm. Calculate its volume in proper significant figures.*

Sol. Volume = $\frac{4}{3}\pi(R)^3 = \frac{4}{3}\pi\left(\frac{2.78}{2}\right)^3$ cm^3 = 11.2437 cm^3

Hence the volume in proper significant figures is 11.2 cm^3

Prob 8. *Calculate the number of light years in one meter.*

Sol. We know 1 light year (ℓ_y) = 9.46×10^{15}m

or 9.46×10^{15} m = $1\ell y$

1 m = $1.057 \times 10^{-16} \ell y$

Prob 9. *Find the dimensions of a and b in the relation P =* $\frac{b - x^2}{at}$

where P is power, x is distance and t is time.

Sol. The given relation is, $P = \frac{b - x^2}{at}$

As x^2 is subtracted from b therefore the dimensions of b are of x^2

i.e. $b = L^2$

We can rewrite relation as

$P = \frac{\left[L^2\right]}{at} = \frac{L^2}{at}$

$a = \frac{L^2}{\left[ML^2T^{-3}\right]\ [T]} = M^{-1}L^{\circ}T^2$

Prob 10. *It is claimed that two cesium clocks if allowed to run for 100 years free from any disturbance may differ by only about 0.02 sec. What is the accuracy of the standard cesium clock in measuring a time interval of 1 sec?*

Sol. $\because$ t = 100 years = 100×365.25×86400s

$\Delta t = 0.02s$

Fractional error = $\dfrac{\Delta t}{t} = \dfrac{0.02}{100\times 365.25\times 86400}$

$= 0.63 \times 10^{-11}$

So, there is an accuracy of 10^{-11} Part in 1 or 1 sec in 10^{11} sec.

Prob11. *In screw gauge no. of division on circular scale is n and circular scale travels a distance of a units in one rotation. Calculate least count of the screw gauge.*

Sol. Pitch = a units

Least count $= \dfrac{\text{Pitch}}{\text{No. of divisions on circulat scale}}$

$= \dfrac{a}{n}$ units.

Prob 12. *The diameter of the spherical bob is measured by vernier Calipers (10 divisions of a Vernier scale coincide with a divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 12 mm and 7^th^ division of the main scale coincides with the main scale. Mass of the sphere is 4.532 g. Find the density of the sphere.*

Sol. Vernier constant = 1.M.S.D. – 1.V.S.D.

$= 1\text{ mm} - \dfrac{9}{10}\text{ mm}$

= 0.1 mm

Diameter of sphere = 12 mm + 0.1 × 7

= 12.7 mm

$\therefore$ Volume of sphere = $= \dfrac{4}{3}\pi\left(\dfrac{D}{2}\right)^3$

$= \dfrac{4}{3}\times 3.14\times\left(\dfrac{12.7}{2}\times 10^{-3}\right)^3$

Density $= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{4.532\times 3\times 8\times 10^{-3}}{4\times 3.14\times (12.7\times 10^{-3})^3}$

$= 4.227\text{ kg/m}^3$

$= 4.23\text{ kg/m}^3$ (in appropriate significant figures)

Prob 13. *A wire of length ℓ = 8 ± 0.02 cm and radius r = 0.2 ± 0.02 cm and mass m = 0.1 ± 0.001 gm. Calculate maximum percentage error in density*

Sol. $\rho = \dfrac{m}{\pi r^2 \ell}$

$\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta m}{m} + \dfrac{2\Delta r}{r} + \dfrac{\Delta\ell}{\ell}$

$\Delta\ell$ = 0.02 cm, ℓ = 8 cm

Δr = 0.02 cm, r = 0.02 cm

m = 0.1 gm, Δm = 0.001 gm

$\dfrac{\Delta\rho}{\rho} = \dfrac{0.001}{0.1} + 2\times\dfrac{0.02}{0.2} + \dfrac{0.001}{0.1}$

$$= \left(\frac{1\times10}{1000\times1} + \frac{2\times2}{100\times2}\times10 + \frac{1\times10}{1000\times1}\right)\times100$$

$$= \frac{(1+20+1)}{100}\times100$$

= 22%

Prob 14. *Planck's formula is given by*

$$u = \frac{\hbar\omega^3}{\pi^2 e^3}\times\frac{1}{e^{\hbar\omega/k^{-1}-1}}$$

where u is the energy radiated per unit area per unit time and h is Planck's constant. What will be the dimensions of k in the expression.

Sol. The power in exponential is always dimensionless. So,

$$\frac{\hbar\omega}{kT} = M^0L^0T^0$$

E = hv

$$\text{so, } h = \frac{E}{v} = \frac{ML^2T^{-2}}{M^0L^0T^{-1}}$$

$= ML^2T^{-1}$

$$\therefore \quad k = \frac{\hbar\omega}{T}$$

$$= \frac{ML^2T^{-1}T^{-1}}{T} = ML^2T^{-3}$$

Prob15. *According to Stoke's law the viscous force acting on a spherical body moving fluid depends on radius r of the body, co–efficient of viscosity η of the fluid and velocity f the body. Find the relation between F, η, r, v.*

Sol. Force acting on a spherical body depends on

$F \propto \eta^a r^b v^C$

$F = k\eta^a r^b v^c$

$(MLT^{-2}) = k\,(ML^{-1}T^{-1})^a\,(L)^b\,(LT^{-1})^c$

$MLT^{-2} = k\,(M)^a\,(L)^{-a+b+c}\,(T^{-a-c})$

a = 1

–a + b + c = 1

– a –c = –2

–1 –c = – 2

c = – 2 = 1

– a + b + c = 1

– 1 + b + 1 = 1

$\Rightarrow$ b = 1

F = k η r v

Objective:

Prob 1. *An experiment measures quantities a, b, c and x is calculated from $x = ab^2/c^3$. If the maximum percentage error in a, b and c are 1%, 3% and 2% respectively, the maximum percentage error in x will be*

(A) 13% *(B) 17%*

(C) 14% *(D) 11%*

Sol. (A) Maximum percentage error in x

As $x = \frac{ab^2}{c^3}$

$$\frac{\Delta x}{x} = \frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c}$$

$$\frac{\Delta x}{x} = 1\% + 2 \times 3\% + 3 \times 2\%$$

$= (1 + 6 + 6)\% = 13\%$

Prob 2. *If P represents radiation pressure, c represents speed of light and Q represents radiation energy striking a unit area per second, then non–zero integers x, y and z, such that $P^xQ^yc^z$ is dimensionless, may be*

(A) x = 1, y =1, z = 1. *(B) x = 1, y =–1, z = 1.*

(C) x = –1, y =1, z = 1. *(D) x = 1, y = 1, z = 1*

Sol. As $P^xQ^{y+}C^z$ is a dimensionless

$$\left(\frac{MLT^{-2}}{L^2}\right)^x\left(\frac{ML^2T^{-2}}{L^2T}\right)(LT^{-1})^2 = M^0L^0T^0$$

$$(M^1L^{-1}T^{-2})^x(ML^\circ T^{-3})^y(LT^{-1})^2 = (M^\circ L^\circ T^\circ)$$

Comparing powers, we get

$x + y = 0$ . . . (i)

$-x + z = 0$ . . . (ii)

$-2x - 5y - z = 0$ . . . (iii)

From (1) and (2),

$y = -x,\ z = x$

Substituting in (3), we get

If $x = k$

$y = -k,\ z = k$

$x = 1,\ y = -1,\ z = 1$

Prob 3. *The dimensional representation of Planck's constant is identical to that of*

(A) Torque *(B) Power*

(C) Linear momentum *(D) angular momentum*

Sol. (D) As Planck's constant has dimensions of $\frac{E}{\nu}$

$= \frac{ML^2T^{-2}}{T^{-1}}$

$= ML^2T^{-1}$

and Dimensions of angular momentum = r × p

$= (L \times MLT^{-1})$

$= ML^2T^{-1}$

Prob 4. *The parallel combination of two resistances is given by If the two resistances $R_1 = (2 \pm 0.2)\Omega$ and $R_2 = (1 \pm 0.1)\Omega$ are connected in parallel. Then the % error is given by*

(A) 0.1% *(B) 0.2%*

(C) 0.3% *(D) 0.4%*

Sol. (C) $R_p = \frac{R_1R_2}{R_1 + R_2}$

$$\frac{\Delta R_p}{R_p} = \left(\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}\right) \times 100$$

$$\frac{\Delta R_p}{R_p} = \left(\frac{0.2}{2} + \frac{0.1}{1} + \frac{0.2 + 0.1}{3}\right)$$

(0.1 + 0.1 + 0.1)

= 0.3%

Prob5. *If the units of M and L are quadrupled, then the units of torque becomes*

(A) 16 times *(B) 64 times*

(C) 8 times *(D) 4 times*

Sol. (B)

Dimensions of torque $= ML^2T^{-2}$

$= (4M)(4L)^2T^{-2}$

$= 64\, ML^2T^{-2}$

Prob6. *A radar signal is beamed towards a planet from earth and its echo is received seven minutes later. If distance between the planet and earth is $6.3 \times 10^{1\circ}$ m, then velocity of the signal will be*

(A) 3×10^8 m/s *(B) 2.97×10^6 m/s*

(C) 3.10×10^5 m/s *(D) 300 m/s*

Sol. (A).

Velocity of signal, $c = \frac{2x}{t} = \frac{2 \times 6.3 \times 10^{10}}{7 \times 60} = 3 \times 10^8$ m/s

Prob7. *If speed of light c, acceleration due to gravity g and pressure P are taken as fundamental units, then the dimensions of gravitational constant is*

(A) $[c^\circ g P^{-3}]$ *(B) $[c^2 g^3 P^{-2}]$*

(C) $[c^\circ g^2 P^{-1}]$ *(D) $[c^2 g^2 P^{-2}]$*

Sol. (C).

Let $G = c^x g^y P^z$

$\Rightarrow [M^{-1}L^3T^{-2}] = [LT^{-1}]^x [LT^{-2}]^y [ML^{-1}T^{-2}]^z$

$= [M^z L^{x+y} T^{-x-2y-2z}]$

Comparing powers of M, L and T on both sides, we get

$z = -1$, $x + y = 3$, $-x - 2y - 2z = -2$

On solving these equations for x, y and z, we get

$x = 0$, $y = 2$, $z = -1$

$\Rightarrow G = [c^\circ g^2 P^{-1}]$.

Prob 8. *The time dependence of a physical quantity P is given by $P = P_0 \exp(-\alpha t^2)$, where α is a constant and t is time. The constant α*

(A) is dimensionless *(B) has dimensions T^{-2}*

(C) has dimensions of P *(D) has dimensions T^2*

Sol. (B).

$P = P_0 [\exp(-\alpha t^2)]$.

Since αt^2 must be dimensionless, so $\alpha = \frac{1}{T^2} = T^{-2}$

Prob 9. *The displacement of a particle is given by $x = A^2 \sin^2 kt$, where t denotes time. The unit of k is*

(A) hertz *(B) metre*

(C) radian *(D) second*

Sol. (A).

Here, kt is dimensionless. Hence, $k = 1/t = \sec^{-1}$ = hertz

Prob10. *The parallel of a heavenly body measured from two points diametrically opposite on the equator of earth is 1.0 minute. If the radius of earth is 6400 km, find the distance of the heavenly body from the centre of earth in AU. Take 1 AU = 1.5×10^{11} m.*

(A) 0.293 AU *(B) 0.28 AU*

(C) 2.01 AU *(D) 3.97 AU*

Sol. (A).

Here, $\theta = 1' = \frac{1^\circ}{60} = \frac{1}{60} \times \frac{\pi}{180}$ rad

ℓ = diameter of earth = 2×6400 km

$= 1.28 \times 10^4$ km $= 1.28 \times 10^7$ m

Now, $\ell = r\theta$

$\Rightarrow r = \frac{1.28 \times 10^7}{(\pi/60) \times 180} = 4.4 \times 10^{10}$ m

$r = \frac{4.4 \times 10^{10}}{1.5 \times 10^{11}} = 0.293$ AU

Prob11. *Dimensions of ohm are same as (h is Planck's constant and e is charge)*

(A) $\frac{h}{e}$ (B) $\frac{h^2}{e}$

(C) $\frac{h}{e^2}$ (D) $\frac{h^2}{e^2}$

Sol. (C).

$$\frac{h}{e^2} = \frac{[ML^2T^{-1}]}{[AT]^2} = [ML^2T^{-3}A^{-2}] = \text{resistance}$$

Prob12. *Which of the following is a derived unit?*

(A) newton *(B) joule*

(C) pascal *(D) metre*

Sol. A, B, C.

Because, they are derived from the fundamental units, i.e. kg, m and sec.

Prob13. *Which of the following equations is dimensionally correct?*

(A) Pressure = energy per unit volume

(B) Pressure = energy per unit area

(C) Pressure = force per unit volume

(D) Pressure=momentum per unit volume

Sol. $\frac{\text{Energy}}{\text{Volume}} = \left[\frac{\frac{1}{2}\ mv^2}{\text{volume}}\right]$

$\Rightarrow \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$

$\therefore$ (A)

Prob 14. *Which of the following is/are dimensional constants is*

(A) Planck's constant *(B) dielectric constant*

(C) relative density *(D) gravitational constant*

Sol. A Planck's constant and gravitational constant G have constant values and dimensions

$\therefore$ A, D

Prob 15. *Which of the following is not a unit of time*

(A) solar year *(B) tropical year*

(C) leap year *(D) light year*

Sol. Tropical year is the year in which there is total eclipse.

Light year represents distance

$\therefore$ (D)

ASSIGNMENT PROBLEMS

Subjective:

Level– O

1. If force acting on a particle depends on the x–coordinates as $F = ax + bx^2$, find the dimensions of 'a' and 'b'.

2. If velocity, time and force are chosen as basic quantities, find the dimensions of mass.

3. Find the dimensional formula of
   (a) Charge Q
   (b) The potential V
   (c) The capacitance C,
   (d) The Resistance, R

4. Which of the following have same dimensions?
   (A) angular momentum and linear momentum
   (B) work and power
   (C) work and torque
   (D) Torque and Pressure

5. The Van der Waals interaction between two molecules separated by a distance r is given by the energy $E = -\frac{A}{r^6} + \frac{B}{r^{12}}$. Find the dimensions of A and B.

6. If error in measuring diameter of a circle is 4%, find the error in radius of circle.

7. Derive, by method of dimensions, an expression for the energy of a body executing S.H.M., assuming that this energy depends upon; the mass (m), the frequency (ν) and the amplitude of vibration (r)

8. Write the dimensions of $\frac{a}{b}$ in the relation
   $F = a\sqrt{x} + bt^2$, where F is force, x is distance and t is time.

9. Assuming that the mass of the largest stone that can be moved by a flowing river depends upon the velocity r, the density ρ and acceleration due to gravity, show that m varies with sixth power of the velocity of flow.

10. The density of a material in cgs system is 8 gcm^{-3}. In a system of units, in which unit of length is 5 cm and unit of mass is 20 g, what is the density of the material ?

11. To study the flow of a liquid through a narrow tube the following formula is used
    $\eta = \frac{\pi\rho r^4}{8v\ell}$ where the letters have their usual meanings. The values of ρ, r, v and ℓ are measured to be 76 cm of Hg, 0.28 cm, 1.2 cm^3s^{-1} and 18.2 cm respectively. If these quantities are measured to the accuracy of 0.5 cm of Hg, 0.01 cm, 0.1 $cm^3\ s^{-1}$ and 0.1 cm respectively, find the percentage error in the value of η.

12. The equation of a wave is given by $y = A\sin\omega\left(\frac{x}{v} - k\right)$, where ω is angular velocity and v is linear velocity. Find the dimension of k. Given that

13. The surface tension of a liquid is 70 dyne/cm. Express it in MKS system of units?

14. Name a physical quantity which has same unit as that of Torque.

15. If all measurements in an experiment are taken upto same number of significant figures then mention two possible reasons for maximum error.

Level – I

1. The mass of a block is 87.2 g and its volume is 25 cm^3. What is its density upto correct significant figures?

2. Suppose refractive index μ is given as $\mu = A + \frac{B}{\lambda^2}$, where A and B are constants, λ is wavelength. Then calculate the dimensions of A and B.

3. Suppose, the torque acting on a body, is given by $\tau = KL + \frac{MI}{\omega}$
   Where L = angular momentum, I = moment of inertia & ω = angular speed
   What is the dimensional formula for KM?

4. When a current of (2.5 ± 0.5). A flows through a wire it develops a potential difference of (20 ± 1) V. What is the resistance of wire?

5. Find out the result in proper significant figures, 291 x 0.03842 / 0.0080.

6. The radius of a sphere is (5.3 ± 0.1) cm. Find the percentage error in its volume.

7. If Planck's constant h; the velocity of light, c and Newton's gravitational constant G are taken as fundamental quantities, then express mass, length and time in terms of these quantities using dimensional notation.

8. What will be the unit of time in the system in which the unit of length is meter, unit of mass is kg and unit of force is kg. wt.?

9. Imagine a system of units in which the unit of mass is 10 kg, length is 1 km and time is 1 minute, then calculate the value of 1 J in this system.

10. A screw gauge of pitch 0.5 mm has a circular scale divided into 5 divisions. The screw gauge is used to measure the thickness of a coin. The main scale reading is 2 mm and 35^{th} circular division coincides with main scale with a positive zero error of divisions. Find the thickness of the coin

11. A Vernier Calliper is used to measure the thickness of the wall of cylinder by measuring its external and internal diameters. For external diameter, the zero if the Vernier scale coincides with the 5^{th} division of main scale and 6^{th} division of Vernier scale coincides with the main scale.
    For internal diameter, the zero of the Vernier scale coincides with the 3^{rd} division of mass scale and 2^{nd} division of Vernier scale coincides with 3^{rd} division of main scale and 2^{nd} division of Vernier scale coincides with main scale. Given that 1 main scale division is equal to 10 m
    1 V.S.D. = 0.09 cm.
    Calculate the thickness of the wall of a cylinder.

12. The time period of small oscillations of a spring mass system is given as $T = 2\pi\sqrt{\frac{m}{k}}$. What will be the accuracy in the determination of k if mass m is given as 10 kg with accuracy of 10 gm and time period is 0.5 sec measured for time of 100 oscillations with a watch of accuracy of 1 sec.

13. In a screw micrometer, main scale divisions are in mm. There are 100 cap divisions.
(a) Find out the least count of the micrometer.
(b) In fully closed condition, 4^{th} division of the cap scale coincides with the line of graduation along the screw axis. What is the zero error of the instrument ? Is it to be added or subtracted from the observed reading during a measurement ?
(c) In the above instrument, during a measurement, the cap is between 7^{th} and 8^{th} divisions of the main scale and 37^{th} division of cap scale coincides with the line of graduation of the main scale. What is the measurement corrected for zero error ?

14. The equation for energy (E) of a simple harmonic oscillator,
$$E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2x^2$$
is to be made "dimensionless" it with multiplying by a suitable factor, which may involve the constants, m(mass), ω(angular frequency) and h (Planck's constant). What will be the unit of momentum and Length ?

15. In in the equation $F = A\sin Bx^2 + \frac{C}{t}e^{Dt}$, F, x and t are force, position and time respectively, then give the dimensions of $\frac{A}{CB}$.

Objective:

1. Which of the following is a possible dimensionless quantity?
(A) Velocity gradient (B) Pressure gradient
(C) Displacement gradient (D) Force gradient

2. Dimensional formula of thermal conductivity is
(A) $ML^2T^{-3}\theta^{-1}$ (B) $ML^2T^{-2}\theta^{-4}$
(C) $ML^2T^{-2}\theta^{-1}$ (D) $MLT^{-3}\theta^{-1}$

3. The unit of power is
(A) kilowatt hour (B) joule
(C) dyne (D) kilo watt

4. The dimensional representation of Planck's constant is identical to that of
(A) torque. (B) power.
(C) linear momentum. (D) angular momentum.

5. Which of the following is a fundamental quantity?
(A) volume (B) velocity
(C) time (D) force

6. The displacement of a particle is given by $x = A^2 \sin^2 kt$, where t denotes time. The unit of k is
(A) hertz (B) metre
(C) radian (D) second

7. The dimensional representation of Planck's constant is identical to that of
(A) torque (B) work
(C) stress (D) angular momentum

8. A force F is given by $F = \frac{a}{t} + bt^2$, where t is time. The dimensions of a and b are
(A) $[MLT^{-3}]$ and $[MLT^{-4}]$ (B) $[MLT^{-4}]$ and $[MLT^{-3}]$
(C) $[MLT^{-1}]$ and $[MLT^{-4}]$ (D) $[MLT^{-2}]$ and $[MLT^{\circ}]$

9. A unit–less quantity
(A) may have non–zero dimensions (B) always has non–zero dimensions
(C) never has a non–zero dimensions (D) does not exist

10. Joule $\times$ sec is the unit of
(A) energy (B) momentum
(C) angular momentum (D) power

11. Given that v is speed, r is radius and g is gravitational acceleration, which of the following expression is dimensionless.
(A) $\frac{v^2}{gr}$ (B) $\frac{v^2 r}{g}$
(C) $\frac{v^2 g}{r}$ (D) $v^2 rg$

12. The dimensional formula for modulus of rigidity is
(A) $[ML^2T^{-2}]$
(B) $[ML^{-1}T^{-3}]$
(C) $[ML^{-2}T^{-2}]$
(D) $ML^{-1}T^{-2}]$

13. A highly rigid cubicle block A of small mass m and side L is rigidly fixed to an other similar cubical block of low modulus of rigidity η. Lower face of A completely covers the upper face of B. The lower face of B is rigidly held on horizontal surface. A small force T is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes simple harmonic motion, the time period of which is given by
(A) $2\pi\sqrt{m\eta L}$
(B) $2\pi\sqrt{m\eta / L}$
(C) $2\pi\sqrt{mL / \eta}$
(D) $2\pi\sqrt{m / \eta L}$

14. The time period of a soap bubble is $T \propto P^a d^b S^c$, where P is pressure, d is density and S is surface tension, then values of a, b and c, respectively, are
(A) –1, –2, 3
(B) –3/2, 1/2 1
(C) 1, –2, –3/2
(D) 1, 2, 1/3

15. The dimensional formula for specific resistance in term of M, L, T and Q is
(A) $[ML^3T^{-1}Q^{-2}]$
(B) $[ML^2T^{-2}Q^2]$
(C) $[MLT^{-2}Q^{-1}]$
(D) $[ML^2T^{-2}Q^{-2}]$

16. Which of the two have same dimensions?
(A) Force and strain
(B) Force and stress
(C) Angular velocity and frequency
(D) Energy and strain

17. The velocity of water waves depend on their wavelength λ, the density of water ρ and acceleration due to gravity g. The method of dimensions gives the relation between these quantities as
(A) $v^2 \propto g^{-1}\lambda^{-1}$ y
(B) $v^2 \propto g\lambda$ y
(C) $v^2 \propto g\lambda\rho$ y
(D) $v^2 \propto g^{-1}\lambda^{-3}$ y

18. L, C and R represent the physical quantities inductance, capacitance and resistance, respectively. The combination which have the dimensions of angle
(A) $\frac{1}{RC}$
(B) $\frac{R}{L}$
(C) $\frac{C}{L}$
(D) $\frac{R^2C}{L}$

19. The vernier of a circular scale is divided into 30 divisions, which coincides with 29 main scale divisions. If each main scale division is $(1/2)^o$, the least count by the instrument is
(A) 0.1′
(B) 1′
(C) 10′
(D) 30′

20. Dimensional analysis of the equation $(\text{velocity})^x = (\text{pressure difference})^{3/2} \times (\text{density})^{-3/2}$ gives the value of x as
(A) 1
(B) 2
(C) 3
(D) 4

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. $[a] = M^1L^0T^{-2}$, $[b] = M^1L^{-1}T^{-2}$
2. FTV^{-1}
3. (a) $[Q] = IT$ (b) $[V] = ML^2I^{-1}T^{-3}$ (c) $[C] = M^{-1}L^{-2}I^2T^4$ (d) $[R] = ML^2T^{-3}I^{-2}$
4. Work and torque
5. $[A] = ML^8T^{-2}$, $[B] = ML^{14}T^{-2}$
6. 4 %
7. $E = k\,mv^2\,r^2$
8. $M^\circ L^{-1/2}\,T^2$
10. 50 units
11. 23%
12 k= $M^\circ L^\circ T$
13. $7\text{x}10^{-2}$ N/m
14. Work
15. The maximum error will be due to (i) measurement, which is least accurate.
(ii) measurement of the quantity which has maximum power in formula's.

Level – I

1.	3.5 g/cc	2.	$M^\circ L^\circ T^\circ$, $M^\circ L^2 T^\circ$
3.	T^{-4}	4.	$(8 \pm 2)\ \Omega$
5.	1400	6.	5.7%
7.	$(hc)^{1/2}\,G^{-1/2}$, $(hG)^{1/2}c^{-3/2}$, $(hG)^{1/2}\,c^{-5/2}$	8.	$\frac{1}{\sqrt{9.8}}$ sec
9.	360	10.	2.25 mm
11.	1.02 cm	12.	$\pm$ 5%

13. (a) 0.01mm (b) + 0.04 mm, to be subtracted (c) 7.33 mm
14. $\frac{E}{\hbar\omega} = \frac{1}{2}\frac{mv^2}{\hbar\omega} + \frac{1}{2}\frac{\omega m x^2}{\hbar}$, $\sqrt{m\omega\hbar}$, $\sqrt{\frac{\hbar}{m\omega}}$
15. L^2T^{-1}

Objective:

1.	C	2.	D	3.	D	4.	D
5.	C	6	A	7.	D	8.	C
9.	C	10.	C	11	A	12.	D
13.	D	14.	B	15.	A	16.	C
17.	B	18.	D	19.	B	20.	C

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

VECTORS

Vectors

Syllabus for IITJEE and ISC Board:
Scalar and vector quantities; Position and displacement vectors, general vectors and notation; Equality of vectors, multiplication of vectors by a real number; Addition and subtraction of vectors; Unit vector, Resolution of a vector in a plane Rectangular components, Multiplication of vectors-scalar and vector products; vectors in three dimensions (elementary idea only).

VECTORS

Definition: The physical quantities specified completely by their magnitude as well as direction are called vector quantities. The magnitude and direction alone cannot decide whether a physical quantity is a vector. In addition to the above characteristics, a physical quantity, which is a vector, should follow law of vector addition. For example, electric current has magnitude as well as direction, but does not follow law of vector addition. Hence, it is not a vector.

A vector is represented by putting an arrow over it. The length of the line drawn in a convenient scale represents the magnitude of the vector. The direction of the vector quantity is depicted by placing an arrow at the end of the line.

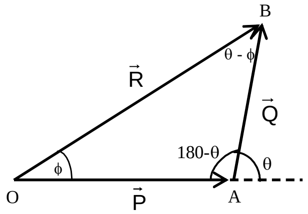
If two vectors have the same direction, they are parallel. Two vectors are said to be equal when their magnitudes and directions, both are same, e.g. if $\vec{a} = \vec{b}$ then $|\vec{a}| = |\vec{b}|$ and the directions of vectors are same. Thus, a vector is not altered by shifting it parallel to itself in the space.

The vector having same magnitude as of $\vec{a}$, but the opposite direction is defined as the negative or opposite of $\vec{a}$ and is denoted by $-\vec{a}$.

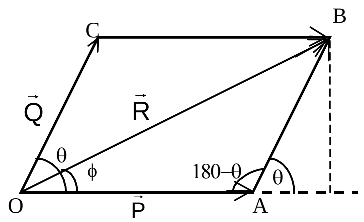
Laws of Addition of Vectors: Two or more vectors can be added to give another vector, which is called the resultant of the vectors. The resultant would produce the same effect as that of the original vectors together.

Triangle law of addition of vectors: If two vectors are represented in magnitude and direction by the two sides of a triangle taken in the same order, then their resultant will be represented in magnitude and direction by the third side of the triangle taken in reverse order.

To add the two vectors $\vec{P}$ and $\vec{Q}$, the vectors are drawn with the tail of $\vec{Q}$ coinciding with the terminus of $\vec{P}$. The vector sum i.e. the resultant vector $\vec{R}$ which completes the triangle drawn from the tail O of $\vec{P}$ to the terminus B of $\vec{Q}$ as shown in figure.



Parallelogram law of addition of vectors:
If two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant will be represented in magnitude and direction by the diagonal of the parallelogram drawn from that point.



We get , $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$$\phi = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

where ϕ is the angle that the resultant makes with $\vec{P}$

(i) $\theta = 0^o$

$\vec{P}$ and $\vec{Q}$ are in the same direction i.e. they are parallel $\cos 0^o = 1$

$\therefore |\vec{R}| = |\vec{P}| + |\vec{Q}|$ & $\phi = 0^o$

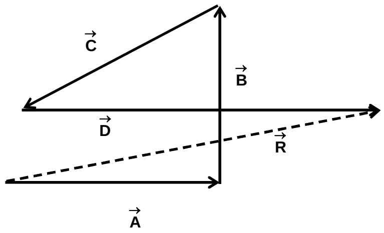
(ii) $\theta = 180^o$, $\vec{P}$ and $\vec{Q}$ are in opposite direction i.e. they are antiparallel $\cos 180^o = -1$

$\therefore |\vec{R}| = |\vec{P}| \sim |\vec{Q}|$ and $\vec{R}$ is in the direction of the larger vector.

(iii) $\theta = 90^o$, $\cos 90^0 = 0$

$\vec{P}$ and $\vec{Q}$ are perpendicular to each other

$\therefore |\vec{R}| = (|\vec{P}|^2 + |\vec{Q}|^2)^{1/2}$ & $\phi = \tan^{-1}(Q/P)$



Polygon law of addition of vectors

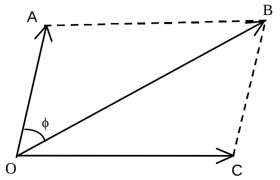
$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

Vectors obey commutative law

i.e. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Illustration 1. *Two forces of 60N and 80N acting at an angle of 60^0 with each other, pull an object. What single pull would replace the given forces?*

Solution: *Two forces are drawn from a common origin O, making an angle of 60^0. OA and OC represent the forces 60N and 80N respectively. The diagonal OB represents the resultant R.*



$\therefore R^2 = 60^2 + 80^2 + 2.60.80\cos 60^0$

$= 3600 + 6400 + 4800 = 14800$ $\therefore R = 121.7N$

Angle ϕ is given, $\tan\phi = \dfrac{80\sin 60^0}{60 + 80\cos 60^0}$

Which gives, $\phi = 34.7^0$

Exercise 1:

i) ***Is it possible that the resultant of two equal forces is equal to one of the forces?***

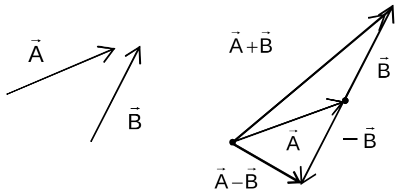
ii) ***If a vector has zero magnitude is it meaningful to call it a vector?***

iii) ***Can three vectors, not in one plane, give a zero resultant? Can four vectors do?***

Subtraction of Vectors:

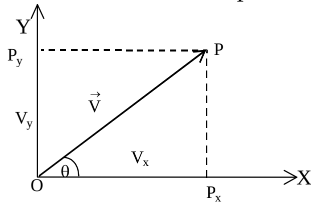
When a vector B is reversed in direction, then the reversed vector is written as - $\vec{B}$ then

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



Resolution of Vectors: If $\vec{P}+\vec{Q}=\vec{R}$, the resultant, then conversely $\vec{R}=\vec{P}+\vec{Q}$ i.e. the vector $\vec{R}$ can be split up so that the vector sum of the split parts equals the original vector $\vec{R}$. If the split parts are mutually perpendicular then they are known as components of $\vec{R}$ and this process is known as resolution. The orthogonal component of any vector along another direction which is at an angular separation θ is the product of the magnitude of the vector and cosine of the angle between them (θ). Therefore the component of $\vec{A}$ is A cosθ.

Note: In physics, resolution gives unique and mutually independent components only if the resolved components are mutually perpendicular to each other. Such a resolution is known as rectangular or orthogonal resolution and the components are called rectangular or orthogonal components.



O – the origin, OP – the given vector $\vec{V}$

PP_x – perpendicular to X axis.

PP_y – Perpendicular to Y axis.

$$\overrightarrow{OP_x} + \overrightarrow{P_xP} = \overrightarrow{OP} = \vec{V}$$

$$\vec{V} = \vec{V_x} + \vec{V_y}$$

$V_x = V\cos\theta$ & $V_y = V\sin\theta$

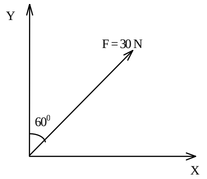
Illustration 2. *A force of 30 N is acting at an angle of 60^0 with the y-axis. Determine the components of the forces along x and y-axes.*

Solution :

$F_x = F\sin60^0$

$$= \frac{30 \text{ x } \sqrt{3}}{2} = 15\sqrt{3}\text{ N}$$

$F_y = F\cos60^0 = 30 \text{ x} \frac{1}{2} = 15\text{ N}$



Unit Vector: In order to make the algebraic operations with vectors simple, a given vector is expressed as a product of its magnitude and direction vector. Since the product should have the same magnitude, the direction vector having unit magnitude and is called unit vector.

A unit vector is not a physical quantity but represents only a given direction. Unit vector along the direction of $\vec{A}$ is $\hat{A}=\vec{A}/A$, Where A is magnitude of $\vec{A}$. $\hat{i}$, $\hat{j}$, $\hat{k}$, are the unit vectors along positive direction of X, Y and Z axis respectively, then the rectangular resolution of vector $\vec{A}$ can be represented.

$\vec{A} = A_x\hat{i}+A_Y\hat{j}+A_Z\hat{k}$ where A_X, A_Y, A_Z are the magnitudes of X, Y and Z components of $\vec{A}$. The magnitude of vector $\vec{A}$ is given by $|\vec{A}|=\sqrt{A_x^2+A_y^2+A_z^2}$.

Illustration 3. *Find the net displacement of a particle from its starting point if it undergoes three successive displacements given $\vec{S_1}$ = 20 m, 45^0 West of North, $\vec{S_2}$ = 15 m, 30^0 North of East; $\overrightarrow{S_3}$ =20m, due South.*

Solution: Let us set our axial system such that x-axis is along West-East and y-axis along South-North.

$\Rightarrow \ \vec{S_1} = 20\cos 45^0(-\hat{i}) + 20\sin 45^0(\hat{j})$

and $\vec{S_2} = 15\cos 30^0(\hat{i}) + 15\sin 30^0(\hat{j})$

$\vec{S_3} = 0(\hat{i}) + 20(-\hat{j})$

$\vec{S} = \vec{S_1} + \vec{S_2} + \vec{S_3}$

$= \left(-\frac{20}{\sqrt{2}} + \frac{15\sqrt{3}}{2} + 0\right)\hat{i} + \left(\frac{20}{\sqrt{2}} + \frac{15}{2} - 20\right)\hat{j}$

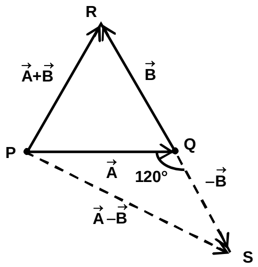
$= -1.15\hat{i} + 1.64\hat{j} = S_x\hat{i} + S_y\hat{j}$

$|\vec{S}| = \sqrt{S_x^2 + S_Y^2} = \sqrt{(-1.15)^2 + (1.64)^2} = 2\text{m}$

Direction $\theta = \tan^{-1}\frac{1.15}{1.64} = 35^0$ West of North

Illustration 4. *If the sum of two unit vectors $\vec{A}$ and $\vec{B}$ is also equal to a unit vector, find the magnitude of the vector $\vec{A} - \vec{B}$.*

Solution: Given that $|\vec{A}| = |\vec{B}| = |\vec{A} + \vec{B}| = 1$



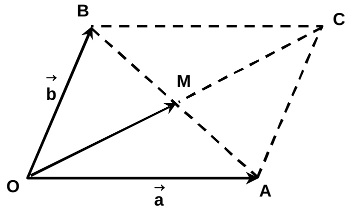
Hence the angle between $\vec{A}$ and $\vec{B}$ is 120°

Now $|\vec{PS}|^2 = |\vec{A}|^2 + |-\vec{B}|^2 + 2|\vec{A}||-\vec{B}|\cos 120°$

$= 1 + 1 + 2\times1\times(-1)\left(-\frac{1}{2}\right) = 3$

$\Rightarrow PS = \sqrt{3}$

Illustration 5. *If the position vector of point A and B are $\vec{a}$ and $\vec{b}$ respectively. Find the position vector of middle point of AB.*



Solution:

$\overline{OA} + \overline{OB} = \overline{OC}$

$\overline{OA} + \overline{OB} = 2\overline{OM}$

$\vec{a} + \vec{b} = 2\overline{OM}$

$\overline{OM} = \frac{1}{2}(\vec{a} + \vec{b})$

Multiplication of Vectors: Vector multiplications are of two types. One, in which we obtain a scalar quantity and the other in which we obtain a vector quantity on multiplication. The first one is called Dot Product and the other is called Cross Product.

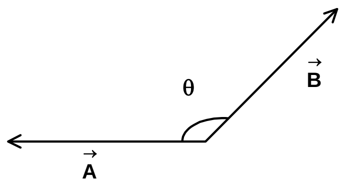
Scalar multiplication: $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$ where θ is the angle between the two vectors, when placed tail to tail.

For $\theta = 90^0$, $\cos\theta = 0$ then $\vec{A}.\vec{B} = 0$

Now for orthogonal unit vectors, $\hat{i} \ . \ \hat{j} = \hat{j} \ . \ \hat{k} = \hat{k} \ . \ \hat{i} = 0$

Again for $\theta = 0^0$, $\cos\theta = 1$ then $\vec{A}.\vec{B} = AB$

For orthogonal unit vectors, $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$



(in the same line)

Let there be two vectors given by

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A}.\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right).\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$= A_xB_x + A_yB_y + A_zB_z$$

Dot product is commutative. i.e. $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

Angle between two vectors:

As we know $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$

$$\Rightarrow \cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|}$$

$$= \frac{A_xB_x + A_yB_y + A_zB_z}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)\left(\sqrt{B_x^2 + B_Y^2 + B_Z^2}\right)}$$

Direction cosines:

If vector $\vec{A}$ makes angle α, β and γ with x, y and z axes respectively, then

$$\cos\alpha = \frac{A_x}{|\vec{A}|} = \frac{A_x}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)}$$

$$\cos\beta = \frac{A_y}{|\vec{A}|} = \frac{A_y}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)}$$

$$\cos\gamma = \frac{A_z}{|\vec{A}|} = \frac{A_z}{\left(\sqrt{A_x^2 + A_Y^2 + A_Z^2}\right)}$$

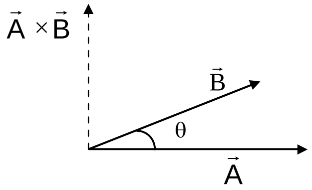
where cos α, cos β and cos γ are called direction cosines of the vector $\vec{A}$. The unit vector in the direction of $\vec{A}$ is

$$\hat{n} = \frac{\vec{A}}{|\vec{A}|} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{|\vec{A}|}$$

$\hat{n} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$. (Note: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$)

Cross product:

The cross product of two vectors $\vec{A}$ and $\vec{B}$ inclined to each other by an angle θ is given by $\vec{A} \times \vec{B} = \vec{C}$, a vector. where $\vec{C} = |\vec{A}|.|\vec{B}|\sin\theta\hat{n}$, where $\hat{n}$ is the unit vector along a direction which is perpendicular to plane containing $\vec{A}$ & $\vec{B}$. Its direction is given by the right hand thumb rule, or right hand screw rule.



If the vectors $\vec{A}$ and $\vec{B}$ lie in the x-y plane then the product is perpendicular to the plane i.e. is parallel to z-axis.

The vector product is not commutative i.e. $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ and $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

In terms of orthogonal vectors $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

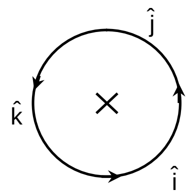
$$\hat{i} \times \hat{j} = \hat{k} = -(\hat{j} \times \hat{i}), \quad \hat{j} \times \hat{k} = \hat{i} = -\left(\hat{k} \times \hat{j}\right),$$

$$\left(\hat{k}\times\hat{i}\right) = \hat{j} = -\left(\hat{i}\times\hat{k}\right)$$

If $\vec{A} = A_X\hat{i} + A_Y\hat{j} + A_Z\hat{k}$, $\vec{B} = B_X\hat{i} + B_Y\hat{j} + B_Z\hat{k}$

Then, $\vec{A}\times\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right) \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$

$= \left(A_YB_Z - A_ZB_Y\right)\hat{i} + \left(A_ZB_X - A_XB_Z\right)\hat{j} + \left(A_XB_Y - A_YB_X\right)\hat{k}$



In determinant form we have,

Then, $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \end{vmatrix}$

Cross Product of two parallel or anti parallel vectors is zero.

Note: *Division of a vector by a vector is not defined.*

Exercise 2:

(i) ***Calculate the angle between a two dyne and a three dyne force so that their sum is four dyne.***

(ii) ***Resultant of two forces which have equal magnitudes and which act at right angles to each other is 1414 dyne. Calculate the magnitude of each forces.***

(iii) ***Find the direction cosines of*** $5\hat{i} + 2\hat{j} + 4\hat{k}$

(iv) ***Given:*** $\vec{a} = 3\hat{i} + 4\hat{j}$ ***and*** $\vec{b} = 3\hat{j} + 4\hat{k}$ ***, Calculate the magnitude of the resultant.***

(v) ***One of the rectangular component of an acceleration of 8 m/s²*** ***is 4 m/s², calculate the other component.***

(vi) ***Find the unit vector in the direction of*** $3\hat{i} + 4\hat{j} - \hat{k}$

Illustration 6. *If the magnitudes of the dot product and cross product of two vectors are equal, find the angle between the two vectors.*

Solution: $|\vec{A}\times\vec{B}| = |\vec{A}\cdot\vec{B}|$

A.B $\sin\theta$ = A.B $\cos\theta$ $\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = 45^0$.

Illustration 7. *If* $\vec{a} = 3\hat{i} + 4\hat{j}$ *and* $\vec{b} = 7\hat{i} + 24\hat{j}$, *find the vector having the same magnitude as* $\vec{b}$ *and parallel to* $\vec{a}$.

Solution: Magnitude of $\vec{a} = \left|\vec{a}\right| = \sqrt{3^2 + 4^2} = 5$

And magnitude of $\vec{b} = \left|\vec{b}\right| = \sqrt{7^2 + 24^2} = 25$

Now a unit vector parallel to $\vec{a} = \hat{a} = \dfrac{3\hat{i} + 4\hat{j}}{5}$

$\therefore$ The vector having the same magnitude as $\vec{b}$ and parallel to $\vec{a}$

$= 25\,\hat{a} = 15\hat{i} + 20\hat{j}$

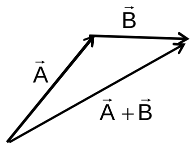
SUMMARY

1. Scalar quantities are quantities with magnitudes only and combine with the usual rules of arithmetic e.g. speed, mass and temperature.

2. Vector quantities have magnitude as well as direction and combine according to the rules of vector addition. e.g. velocity and acceleration.

3. $\vec{B} = \lambda\vec{A}$
Where λ is a real number. Magnitude of B is λ time the magnitude of A and direction is same as that of A. (If λ is positive).

4. Graphically, two vectors $\vec{A}$ and $\vec{B}$ are added by placing the tail of $\vec{B}$ at the head of $\vec{A}$. The vector sum $\vec{A}+\vec{B}$ then extends from the tail of $\vec{A}$ to the head of $\vec{B}$



5. Vector addition is
$\vec{A}+\vec{B} = \vec{B}+\vec{A}$ (Commutative)
$(\vec{A}+\vec{B})+\vec{C} = \vec{A}+(\vec{B}+\vec{C})$ (Associative law)

6. A vector with zero magnitude is called null vector and
$\vec{A}+\vec{0} = \vec{A}$
$\lambda\vec{0} = \vec{0}$

7. Subtraction of vectors
$\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$

8. Unit vectors describe directions in space. A unit vector has a magnitude of one, with no units.
The unit vectors $\hat{i}, \hat{j}, \hat{k}$ are vectors of unit magnitude and points in the direction of the x, y and z axes, respectively, in a right – handed coordinate system.

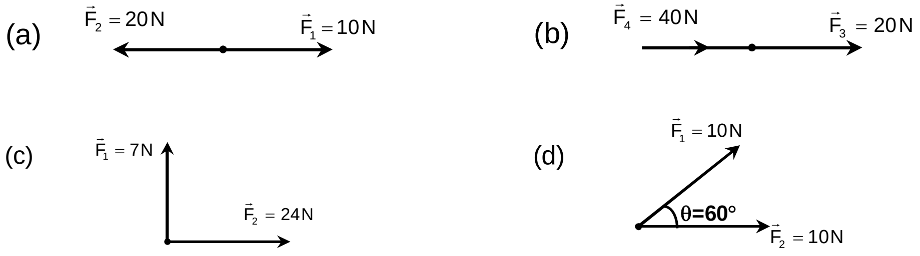
10. vector $\vec{A}$ can be expressed as $\vec{A} = A_X\hat{i} + A_y\hat{j}$ having magnitude $= \sqrt{A_x^2 + A_y^2}$ and angle θ with the x – axis = $\tan^{-1}\dfrac{A_y}{A_x}$.

11. Scalar product of two vectors, $C = \vec{A}\cdot\vec{B} = AB\cos\phi$, where ϕ is the angle between two vectors and scalar product of two vectors is a scalar quantity. Scalar products obey the commutative and the distributive laws.

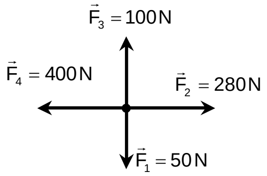
12. Cross – product of two vectors $\vec{A}$ and $\vec{B}$ is a vector quantity. $\vec{C} = \vec{A}\times\vec{B} = AB\sin\phi\ \hat{n}$ and its direction is given by right hand rule, $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ (non – commutative)

MISCELLANEOUS EXERCISE

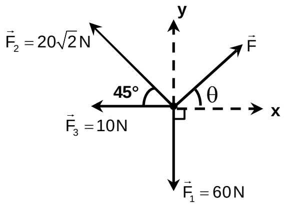
1. Find the magnitude and direction of resultant vectors as shown in the figures below.



2. (a) In the adjacent figure, find the magnitude and direction of the resultant vector.



(b) In the adjacent figure, find the value of F and θ so that the sum of the vectors will be zero.



3. Show that the vectors $\vec{A} = 12\hat{i} - 10\hat{j} + 2\hat{k}$ and $\vec{B} = 4\hat{i} + 8\hat{j} + 16\hat{k}$ are perpendicular to each other.

4. Resultant of two vectors of equal magnitude makes an angle 60° with one of the vectors. Find the angle between the vectors.

5. If $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = 4\hat{k}$ find the value of $\vec{a}\cdot\vec{b}$ and $\vec{a}\times\vec{b}$. Also find the angle between $\vec{a}$ and $\vec{b}$.

6. If $\vec{A} = 3\hat{i} - 2\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} + 2\hat{k}$, find $|\vec{A}|$, $|\vec{B}|$ and $|\vec{A}+\vec{B}|$. Also find the direction of $\vec{A}+\vec{B}$ with the x-axis. Check whether $|\vec{A}| + |\vec{B}|$ is equal to $|\vec{A}+\vec{B}|$.

7. Check whether the two vectors, $\vec{A} = -3\hat{i} - 7\hat{j} + 9\hat{k}$ and $\vec{B} = -2\hat{i} - 21\hat{j} + 6\hat{k}$ are parallel to each other.

8. Two vectors are given by $\vec{A} = 3\hat{i} + 2\hat{j} + 4\hat{k}$ and $\vec{B} = 4\hat{i} + 2\hat{j} + \hat{k}$. Find the magnitude and direction cosines of $\vec{A}+\vec{B}$.

9. The components of a vector $\vec{A}$ along x-axis and y-axis are 4 unit and 6 unit respectively. If the components of vector $\vec{A}+\vec{B}$ along x-axis and y-axis are 10 unit and 14 unit respectively, find the vector $\vec{B}$ and its direction with the x-axis.

10. (a) Find the unit vector which is parallel to the vector $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$.

    (b) Find the unit vector which is perpendicular to both of the vectors $\vec{A} = 2\hat{i}$ and $\vec{B} = 3\hat{i} + 4\hat{j} + 12\hat{k}$.

SOLUTION TO MISCELLANEOUS EXERCISE

1. (a) 10 N (towards $\vec{F}_2$)
   (b) 60 N (towards $\vec{F}_3$)
   (c) 25 N, at an angle $\tan^{-1}\left(\frac{7}{24}\right)$ from $\vec{F}_2$ towards $\vec{F}_1$
   (d) $10\sqrt{3}$ at an angle 30° from $\vec{F}_2$ towards F_1

2. (a) 130 N, $\tan^{-1}\left(\frac{5}{12}\right)$ (from $\vec{F}_4$ towards $\vec{F}_3$)
   (b) 50 N, $\theta = 53°$

4. 120^0

5. 0, $4\left(2\hat{i} - 3\hat{j}\right)$, 90°

6. $\sqrt{14}$, $2\sqrt{6}$ and $\sqrt{30}$ N, $\cos^{-1}\left(\sqrt{\frac{5}{6}}\right)$; No

7. No

8. $\sqrt{90}$, $\frac{7}{\sqrt{90}}$, $\frac{4}{\sqrt{90}}$, $\frac{5}{\sqrt{90}}$

9. $\vec{B} = 6\hat{i} + 8\hat{j}$ and $\theta = 53°$

10. (a) $\frac{2\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{14}}$ (b) $\frac{\left(-3\hat{j} + \hat{k}\right)}{\sqrt{10}}$

SOLVED PROBLEMS

Subjective:

Prob 1. *Find the component of a vector $\vec{A} = 3\hat{i} + 2\hat{j}$ along the direction of $(\hat{i} + \hat{j})$.*

Sol. Unit vector along $(\hat{i}+\hat{j})$ is $\hat{n} = \dfrac{\hat{i}+\hat{j}}{(1^2+1^2)^{1/2}} = \dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$

$\therefore$ The magnitude of the component of vector $\vec{A}$ along the $(\hat{i}+\hat{j})$ is

$$\vec{A}\cdot\hat{n} = (3\hat{i}+2\hat{j})\cdot\frac{\hat{i}+\hat{j}}{\sqrt{2}} = \frac{1}{\sqrt{2}}(3+2) = \frac{5}{\sqrt{2}}$$

$\therefore$ The component vector of $\vec{A}$ along the $(\hat{i}+\hat{j})$ is

$$\vec{A}_1 = \frac{5}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \frac{5}{2}\hat{i}+\frac{5}{2}\hat{j}$$

Prob 2. *At what angle do the forces $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$ act so that the magnitude of their resultant is $\sqrt{3A^2+B^2}$?*

Sol. $R^2 = 3A^2 + B^2 = (A + B)^2 + (A - B)^2 + 2(A^2 - B^2)\cos\theta$
where θ is the angle between (A + B) and (A–B)
or, $A^2 - B^2 = 2(A^2 - B^2)\cos\theta$
or, $\cos\theta = 1/2 \quad \Rightarrow \quad \theta = 60°$

Prob 3. *The resultant of two non-zero forces $\vec{P}$ and $\vec{Q}$ is of magnitude P. Prove that if $\vec{P}$ is doubled, the resultant force will be perpendicular to Q.*

Sol. $P^2 = P^2 + Q^2 + 2PQ\cos\theta \Rightarrow Q + 2P\cos\theta = 0$
Now, if $\vec{P}$ is doubled then

$\tan\alpha = \dfrac{2P\sin\theta}{Q+2P\cos\theta}$ [where α = angle made by the resultant with Q]

$= \dfrac{2P\sin\theta}{0} = \infty$ (undefined)

$\Rightarrow \alpha = 90°$

Prob 4. *$\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}.\vec{B} = 0$ and $\vec{A}.\vec{C} = 0$. To which vector, the vector $\vec{A}$ is parallel.*

Sol. $\vec{A}.\vec{B} = 0$ so $\vec{A}$ is perpendicular to $\vec{B}$
$\vec{A}.\vec{C} = 0$ so $\vec{A}$ is perpendicular to $\vec{C}$
But $(\vec{B}\times\vec{C})$ is a vector which is perpendicular to both $\vec{B}$ and $\vec{C}$.
So, $\vec{A}$ is parallel to $(\vec{B}\times\vec{C})$.

Prob 5. *Prove that* $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$.

Sol. $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}$

$= \vec{a}\times\vec{b}+\vec{a}\times\vec{b}$ $[\because \quad \vec{a}\times\vec{a}=\vec{b}\times\vec{b}=0 \text{ and } \vec{b}\times\vec{a}=-\vec{a}\times\vec{b}]$

$= 2(\vec{a}\times\vec{b})$

Prob 6. *Given* $\vec{A}.\vec{B}=\vec{A}.\vec{C}$. *Is it correct to conclude* $\vec{B}=\vec{C}$ *?*

Sol. $\vec{A}.\vec{B}=\vec{A}.\vec{C}$

i.e. $AB\cos\theta_1 = AC\cos\theta_2$

[where θ_1 and θ_2 are the angles formed by $\vec{B}$ and $\vec{C}$ respectively with $\vec{A}$.]

or, $B\cos\theta_1 = C\cos\theta_2$

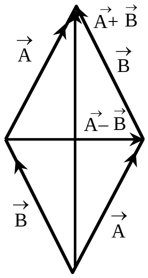
Now, if $\theta_1 = \theta_2$ then $\vec{B}=\vec{C}$

But if $\theta_1 \neq \theta_2$ then $\vec{B}\neq\vec{C}$

So, we can't conclusively say that $\vec{B}=\vec{C}$.

Prob 7. *Find the area of a parallelogram whose diagonals are represented by* $(3\hat{i}+\hat{j}+\hat{k})$ *and* $(\hat{i}-\hat{j}-\hat{k})$.

Sol. Let $\vec{A}$ and $\vec{B}$ be the two adjoining sides of the parallelogram drawn from a point. The area of the parallelogram = $\vec{A}\times\vec{B}$



Given that $\vec{A}+\vec{B}=3\hat{i}+\hat{j}+\hat{k}$

and $\vec{A}-\vec{B}=\hat{i}-\hat{j}-\hat{k}$

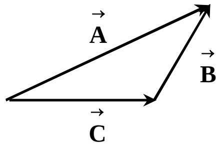
Now, $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})=-2(\vec{A}\times\vec{B})$

$\therefore$ $\vec{A}\times\vec{B}=-\frac{1}{2}[(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})]$

$$=-\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 1\\ 1 & -1 & -1\end{vmatrix}$$

$$=-\frac{1}{2}\left(0\hat{i}+4\hat{j}-4\hat{k}\right)=-2\hat{j}+2\hat{k}$$

Prob 8. *Prove that the vectors* $\vec{A}=3\hat{i}+\hat{j}-2\hat{k}$, $\vec{B}=-\hat{i}+3\hat{j}+4\hat{k}$ *and* $\vec{C}=4\hat{i}-2\hat{j}-6\hat{k}$, *can form a triangle.*



Sol. Let us add any two of the given vectors, say $\vec{C}$ and $\vec{B}$

$\vec{C}+\vec{B}=\left(4\hat{i}-2\hat{j}-6\hat{k}\right)+\left(-\hat{i}+3\hat{j}+4\hat{k}\right)=3\hat{i}+\hat{j}-2\hat{k}=\vec{A}$

As the sum of two vectors is equal to the third vector, the three vectors can form a triangle.

Prob 9. *If* $\vec{A}=3\hat{i}+7\hat{j}$ *and* $\vec{B}=2\hat{i}+5\hat{j}$, *find the angle between* $\vec{A}$ *and* $\vec{B}$.

Sol. $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$

or, $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|} = \dfrac{6+35}{\sqrt{58}\sqrt{29}} = 0.9997$ $[\because\ |\vec{A}| = \sqrt{9+49}\ \&\ |\vec{B}| = \sqrt{4+25}$

$\therefore\ \theta = 1°24'$

Prob 10. *If $\vec{A} = 3\hat{i}+2\hat{j}+4\hat{k}$ and $\vec{B} = 4\hat{i}+2\hat{j}+\hat{k}$, find the magnitude and direction cosines of $(\vec{A}+\vec{B})$.*

Sol. $\vec{A}+\vec{B} = \left(3\hat{i}+2\hat{j}+4\hat{k}\right)+\left(4\hat{i}+2\hat{j}+\hat{k}\right) = 7\hat{i}+4\hat{j}+5\hat{k}$

$\therefore |\vec{A}+\vec{B}| = \sqrt{7^2+4^2+5^2} = \sqrt{90}$

$\therefore$ The direction cosines of $\vec{A}+\vec{B}$ are

$\cos\alpha = \dfrac{\left|\vec{A}+\vec{B}\right|_x}{\left|\vec{A}+\vec{B}\right|} = \dfrac{7}{\sqrt{90}}$

$\cos\beta = \dfrac{\left|\vec{A}+\vec{B}\right|_y}{\left|\vec{A}+\vec{B}\right|} = \dfrac{4}{\sqrt{90}}$

$\cos\gamma = \dfrac{\left|\vec{A}+\vec{B}\right|_z}{\left|\vec{A}+\vec{B}\right|} = \dfrac{5}{\sqrt{90}}$

Prob 11. *Prove that the vectors $\vec{A} = 2\hat{i}-3\hat{j}-\hat{k}$ and $\vec{B} = -6\hat{i}+9\hat{j}+3\hat{k}$ are parallel.*

Sol. The given vectors will be parallel if their cross product is zero. Because if the two vectors are $\vec{A}$ and $\vec{B}$ then

$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n} = AB\sin 0°\ \hat{n}$ [if they are parallel then the angle between $\vec{A}$ and $\vec{B}$ is 0°]

$\therefore\ \vec{A}\times\vec{B} = 0$

Now, $\left(2\hat{i}-3\hat{j}-\hat{k}\right)\times\left(-6\hat{i}+9\hat{j}+3\hat{k}\right) = (9-9)\hat{i}+(6-6)\hat{j}+(18-18)\hat{k} = 0$

Hence, the two vectors $\vec{A}$ and $\vec{B}$ are parallel to each other.

$\vec{B} = -3\vec{A} \implies \vec{B} \parallel A$.

Prob12. *For a point P (2, 4, –5) in a three dimensional co-ordinate system, find*

(a) the position vector $\vec{r}$ of point P.

(b) $|\vec{r}|$

(c) the direction cosines of vector $\vec{r}$.

Sol. (a) $\vec{r} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

(b) $|\vec{r}| = \sqrt{r_x^2+r_y^2+r_z^2}$

$= \sqrt{(2)^2+(4)^2+(-5)^2} = \sqrt{45}$

(c) $\cos\alpha = \dfrac{r_x}{|\vec{r}|} = \dfrac{2}{\sqrt{45}}$

$\cos\beta = \dfrac{r_y}{|\vec{r}|} = \dfrac{4}{\sqrt{45}}$

and $\cos\gamma = \dfrac{r_z}{|\vec{r}|} = -\dfrac{5}{\sqrt{45}}$

Prob13. *Under the action of a force* $\left(10\hat{i} - 3\hat{j} + 6\hat{k}\right)$ *N, a body of mass 5 kg moves from position* $\left(6\hat{i} + 5\hat{j} - 3\hat{k}\right)$ *m to a position* $\left(10\hat{i} - 2\hat{j} + 7\hat{k}\right)$ *m. Deduce the work done.*

Sol. Displacement $\vec{r} = \vec{r}_2 - \vec{r}_1$

$$= \left(10\hat{i} - 2\hat{j} + 7\hat{k}\right) - \left(6\hat{i} + 5\hat{j} - 3\hat{k}\right)$$

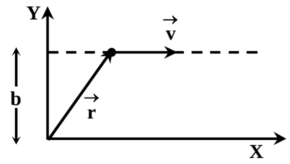
$$= \left(4\hat{i} - 7\hat{j} + 10\hat{k}\right) \text{ m}$$

Work $W = \vec{F} \cdot \vec{s}$

$$= \left(10\hat{i} - 3\hat{j} + 6\hat{k}\right) \cdot \left(4\hat{i} - 7\hat{j} + 10\hat{k}\right)$$

$$= 40 + 21 + 60 = 121 \text{ J}$$

Prob 14. *If a particle of mass m is moving with a constant velocity* $\vec{v}$ *parallel to the x-axis in x-y plane as shown in the figure, calculate its angular momentum w.r.t. origin at any time t.*



Sol. We know $\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$

As the motion is in x-y plane, $z = 0$ and $p_z = 0$

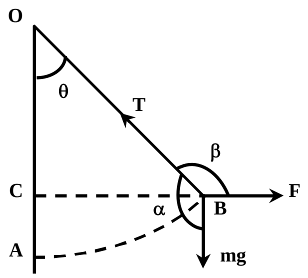
$\therefore$ $\vec{L} = \hat{k}\,(xp_y - yp_x)$

Hence $x = vt$, $y = b$, $p_x = mv$ and $p_y = 0$

$\therefore$ $\vec{L} = \hat{k}\,(vt \times 0 - bmv) = -mvb\,\hat{k}$

Prob15. *A bob weighing 50 gm hangs vertically at the end of a string 50 cm long. If 20 gm force is applied horizontally on the bob, by what distance is the bob pulled aside from its initial position when it reaches its equilibrium position?*

Sol. Let the bob be in equilibrium when it is pulled to B.



$$\frac{F}{\sin\alpha} = \frac{mg}{\sin\beta} = \frac{T}{\sin 90^\circ}$$

$$\frac{F}{\sin(\pi - \theta)} = \frac{mg}{\sin\left(\frac{\pi}{2} + \theta\right)} = T$$

or, $$\frac{F}{\sin\theta} = \frac{mg}{\cos\theta} = T$$

$\therefore$ $$\frac{F}{mg} = \tan\theta = \frac{20 \times 980}{50 \times 980} = 0.4 = \tan 21^\circ 48'$$

$\therefore$ $\theta = 21^\circ 48'$

$\therefore$ $CB = OB \sin\theta = OB \sin 21^\circ 48'$

$= 50 \times 0.3714 = 18.57$ cm

Prob16. *The x and y components of vector* $\vec{A}$ *are 4 m and 6 m, respectively. If the x and y components of vector* $(\vec{A} + \vec{B})$ *are 10 m and 9 m, respectively, calculate*

(a) x and y components of vector $\vec{B}$.

(b) its length and

(c) the angle made by vector $\vec{B}$ with the x-axis.

Sol. In terms of components

$\vec{A} + \vec{B} = (\hat{i}A_x + \hat{j}A_y) + (\hat{i}B_x + \hat{j}B_y)$

$\therefore\ \vec{A} + \vec{B} = \hat{i}(A_x + B_x) + \hat{j}(A_y + B_y)$

According to the given Problem

$A_x + B_x = 10$ m and $A_y + B_y = 9$ m

As $A_x = 4$ m and $A_y = 6$ m (given)

(a) $B_x = 6$ m and $B_y = 3$ m

(b) $B = \sqrt{B_x^2 + B_y^2} = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$ m

(c) $\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$

Objective:

Prob 1. *A force $\vec{F} = (4\hat{i} - 5\hat{j} + 3\hat{k})$N is acting at a point having a position vector $\vec{r}_1 = (\hat{i} + 2\hat{j} + 3\hat{k})$. The torque acting about a point having a position vector $\vec{r}_2 = (3\hat{i} - 2\hat{j} - 3\hat{k})$, is*

(A) $42\hat{i} + 30\hat{j} - 6\hat{k}$ *(B)* $42\hat{i} + 30\hat{j} + 6\hat{k}$
(C) $42\hat{i} - 30\hat{j} + 6\hat{k}$ *(D) zero.*

Sol. Calculate torque $(\vec{\tau}) = \vec{r} \times \vec{F}$
Where $\vec{r} = \vec{r}_1 - \vec{r}_2$
Hence (A) is correct.

Prob 2. *The area of a parallelogram formed by the vectors $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j} + \hat{k}$ as adjacent sides is*

(A) $8\sqrt{3}$ *units.* *(B) 64 units*
(C) 32 units. *(D)* $\sqrt{3}$ *units*

Sol. Calculate $\vec{A} \times \vec{B}$
and Area of a parallelogram $= |\vec{A} \times \vec{B}|$
Hence (A) is correct.

Prob 3. *If vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other, then which of the following statements is valid?*

(A) $\vec{A} \times \vec{B} = \vec{A} . \vec{B}$ *(B)* $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = 0$
(C) $\vec{A} . \vec{B} = 0$ *(D)* $\vec{A} . \vec{B} = |\vec{A}|\,|\vec{B}|$

Sol. $\vec{A} \perp \vec{B}$. Then.
$\vec{A} . \vec{B} = |A||B|\cos 90° = 0$
Hence (C) is correct.

Prob 4. *Which of the rectangular pair may be the components of a 13 N force?*

(A) 5 N, 12 N *(B) 10 N, 11 N*
(C) 6.5 N, 6.5 N *(D) 9 N, 12 N*

Sol. Rectangular components will follow

$$R = \sqrt{R_x^2 + R_y^2}$$

$\therefore$ $13^2 = 5^2 + 12^2$
Hence (A) is correct.

Prob 5. *If $\vec{A}$ and $\vec{B}$ are two mutually perpendicular vectors, where $\vec{A} = 5\vec{i} + 7\hat{j} + 3\hat{k}$ and $\vec{B} = 2\vec{i} + 2\hat{j} - a\hat{k}$, then the value of a is*

(A) –2 *(B) 8*
(C) –7 *(D) –8*

Sol. $\vec{A} \perp \vec{B}$

$\vec{A}\,.\,\vec{B} = 0 = (5\vec{i} + 7\hat{j} + 3\hat{k})(2\vec{i} + 2\hat{j} - a\hat{k}) = 10 + 14 - 3a$

$\therefore \quad 3a = 24 \Rightarrow a = 8$

Hence (B) is correct.

Prob 6. *The unit vector perpendicular to* $\vec{i} - 2\hat{j} + \hat{k}$ *and* $3\vec{i} + \hat{j} - 2\hat{k}$ *is*

(A) $\dfrac{5\vec{i} + 3\hat{j} + 7\hat{k}}{\sqrt{83}}$ *(B)* $\dfrac{3\vec{i} + 5\hat{j} + 7\hat{k}}{\sqrt{83}}$

(C) $\dfrac{5\vec{i} + 3\hat{j} - 7\hat{k}}{\sqrt{83}}$ *(D)* $\dfrac{3\vec{i} - 5\hat{j} + 7\hat{k}}{\sqrt{83}}$

Sol. $\vec{A} \times \vec{B}$ is a vector $\perp$ to both $\vec{A}$ and $\vec{B}$

Now, $\vec{A} \times \vec{B} = (\vec{i} - 2\hat{j} + \hat{k}) \times (3\vec{i} + \hat{j} - 2\hat{k}) = 3\vec{i} + 5\hat{j} + 7\hat{k}$

Now, $\hat{n} = \dfrac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

$= \dfrac{3\vec{i} + 5\hat{j} + 7\hat{k}}{\sqrt{3^2 + 5^2 + 7^2}} = \dfrac{3\vec{i} + 5\hat{j} + 7\hat{k}}{\sqrt{83}}$

Hence (B) is correct.

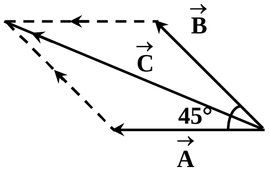
Prob 7. *For any two vectors* $\vec{A}$ *and* $\vec{B}$ *if* $\vec{A}\,.\,\vec{B} = |\vec{A} \times \vec{B}|$, *the magnitude of* $\vec{C} = \vec{A} + \vec{B}$ *is equal to*

(A) $\sqrt{A^2 + B^2}$ *(B) A + B*

(C) $\left[A^2 + B^2 + \dfrac{AB}{\sqrt{2}}\right]^{1/2}$ *(D)* $(A^2 + B^2 + \sqrt{2} \times AB)^{1/2}$

Sol. $\vec{A}\,.\,\vec{B} = AB\cos\theta$ . . . (1)

$|\vec{A} \times \vec{B}| = AB\sin\theta\,\hat{n}$ . . . (2)



$\therefore \quad AB\cos\theta = AB\sin\theta \Rightarrow \theta = 45°$

Again given $\vec{C} = \vec{A} + \vec{B}$

$\therefore \quad |\vec{C}| = (A^2 + B^2 + 2AB\cos 45°)^{1/2}$

$= (A^2 + B^2 + \sqrt{2}AB)^{1/2}$

Hence (D) is correct.

Prob 8. *What is the value of linear velocity, if* $\vec{\omega} = 3\vec{i} - 4\hat{j} + \hat{k}$ *and* $\vec{r} = 5\vec{i} - 6\hat{j} + 6\hat{k}$ *?*

(A) $6\vec{i} + 2\hat{j} - 3\hat{k}$ *(B)* $18\vec{i} + 13\hat{j} - \hat{k}$

(C) $4\vec{i} - 13\hat{j} + 6\hat{k}$ *(D)* $6\vec{i} - 2\hat{j} + 8\hat{k}$

Sol. $\vec{v}$ = tangential velocity $= \vec{r} \times \vec{\omega}$

$\vec{v} = (5\vec{i} - 6\hat{j} + 6\hat{k}) \times (3\vec{i} - 4\hat{j} + \hat{k})$

$= 18\vec{i} + 13\hat{j} - 2\hat{k}$

Hence (B) is correct.

Prob 9. *A particle moves in x-y plane under the action of a force $\vec{F}$ such that the value of its linear momentum $\vec{p}$ at any time t is $p_x = 2\cos t$, $p_y = 2\sin t$. The angle between $\vec{F}$ and $\vec{p}$ at given time t will be*

(A) $\theta = 0°$ *(B) $\theta = 30°$*
(C) $\theta = 90°$ *(D) $\theta = 180°$*

Sol. $P = \sqrt{P_x^2 + P_y^2} = 2\sqrt{\cos^2 t + \sin^2 t} = 2$, which is independent of t, which means the applied force is not changing the magnitude of velocity.

i.e. $\vec{F}$ is perpendicular to $\vec{p}$

Hence (C) is correct.

Prob10. *If $\hat{i}$ denotes a unit vector along an incident ray $\hat{r}$ the unit vector along the refracted ray in a medium of refractive index μ and $\hat{n}$ a unit vector normal to boundary of medium directed towards incident medium, law of refraction is*

(A) $\hat{i}.\hat{n} = \mu(\hat{r}.\hat{n})$ *(B) $\hat{i} \times \hat{n} = \mu(\hat{n} \times \hat{r})$*
(C) $\hat{i} \times \hat{n} = \mu(\hat{r} \times \hat{n})$ *(D) $\mu(\hat{i} \times \hat{n}) = \hat{r} \times \hat{n}$*

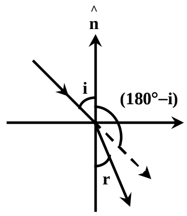
Sol. $|\hat{i} \times \hat{n}| = 1.1\sin(180° - i) = \sin i$

$|\hat{r} \times \hat{n}| = 1.1\sin(180 - r) = \sin r$

Now $\dfrac{|\hat{i} \times \hat{n}|}{|\hat{r} \times \hat{n}|} = \dfrac{\sin i}{\sin r} = \mu$

$\therefore$ $\hat{i} \times \hat{n} = \mu(\hat{r} \times \hat{n})$, as their directions are also same.

Hence (C) is correct.



ASSIGNMENT PROBLEMS

Subjective:

1. Two forces whose magnitudes are in ratio of 3 : 5 give a resultant of 35 N. If the angle of inclination be 60°, calculate the magnitude of each force.

2. Find the unit vector of $3\hat{i}+4\hat{j}-\hat{k}$

3. Given $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=\hat{i}+\hat{j}$. What is the vector component of $\vec{A}$ in the direction of $\vec{B}$?

4. Given $\vec{A}=\hat{i}-2\hat{j}-3\hat{k}$ and $\vec{B}=4\hat{i}-2\hat{j}+6\hat{k}$. Calcualte the angle made by $(\vec{A}+\vec{B})$ with x-axis.

5. Prove that the vectors $2\hat{i}-3\hat{j}-\hat{k}$ and $-6\hat{i}+9\hat{j}+3\hat{k}$ are parallel.

6. Calculate the area of the parallelogram when adjacent sides are given by the vectors $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{B}=2\hat{i}-3\hat{j}+\hat{k}$.

7. What is the angle between $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$?

8. Two vectors of magnitudes A and $\sqrt{3}$ A ar perpendicular to each other. What is the angle which their resultant makes with $\vec{A}$?

9. What should be the angle θ between two vectors $\vec{A}$ and $\vec{B}$ for their resultant $\vec{R}$ to be maximum ?

10. Find the direction cosines of $5\hat{i}+2\hat{j}+4\hat{k}$.

Objective:

1. Which of the following statement is correct?
(A) A vector having zero length can have a unique direction.
(B) If $\vec{A} \times \vec{B} = 0$, then either $\vec{A} = 0$ or $\vec{B} = 0$ or both $\vec{A}$ and $\vec{B}$ are zero.
(C) If $\vec{A} . \vec{B} = 0$, then either $\vec{A} = 0$ or $\vec{B} = 0$ or both $\vec{A}$ and $\vec{B}$ are zero.
(D) The vector $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$ are mutually perpendicular.

2. The magnitude of a given vector with end points (4, –4, 0) and (–2, –2, 0) must be
(A) 6 (B) $5\sqrt{6}$
(C) 4 (D) $2\sqrt{10}$

3. If the magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 12, 5 and 13 units, respectively, and $\vec{A} + \vec{B} = \vec{C}$, then the angle between $\vec{A}$ and $\vec{B}$ is
(A) zero. (B) π
(C) $\pi/2$ (D) $\pi/4$

4. If $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j}$, then their dot product will be
(A) 0 (B) 12
(C) 8 (D) 16

5. A particle is acted upon by two forces of 3 N and 4 N simultaneously. Which of the following is most correct?
(A) The resultant of these forces is 7 N.
(B) The resultant of these forces is 1 N.
(C) The resultant of these forces in 4 N.
(D) The resultant of these forces lies between 1 N and 7 N

6. Find the value of c if $\vec{A} = 0.4\hat{i} + 0.3\hat{j} + c\hat{k}$ is a unit vector.
(A) 0.5 (B) $\sqrt{0.75}$
(C) 1 (D) none of these.

7. Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A} . \vec{B} = 0$ and $\vec{A} . \vec{C} = 0$. The vector $\vec{A}$ is parallel to
(A) $\vec{B}$ (B) $\vec{C}$
(C) $\vec{B} . \vec{C}$ (D) $\vec{B} \times \vec{C}$

8. Angular momentum is
(A) scalar. (B) an axial vector.
(C) a polar vector. (D) a null vector.

9. Minimum number of forces having equal magnitudes, which can give a resultant zero, is
(A) 2 (B) 4
(C) 3 (D) 1

10. $\hat{i} \times (\hat{j} \times \hat{k})$ is
(A) $\hat{i} + \hat{j} + \hat{k}$
(B) $\hat{i} + \hat{j} + \hat{k}$
(C) zero vector
(D) unit vector.

11. If $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + 4\hat{j} + \hat{k}$, then the unit vector along $(\vec{A} + \vec{B})$ is
(A) $\frac{3\hat{i} + 7\hat{j} + \hat{k}}{\sqrt{59}}$
(B) $\frac{2\hat{i} + 3\hat{j}}{\sqrt{59}}$
(C) $\frac{\hat{i} + 4\hat{j} + \hat{k}}{\sqrt{18}}$
(D) $\frac{2\hat{i} + 3\hat{j}}{\sqrt{13}}$

12. A particle moves under a force $\vec{F} = 2\hat{i} + 4\hat{j}$ with a velocity $\vec{v} = 4\hat{i} - 2\hat{j}$, then the power delivered by the force is
(A) 16 W
(B) zero
(C) 8 W
(D) $8\sqrt{2}$ W

13. If $\vec{A} = \vec{B} + \vec{C}$, and the magnitude of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 5, 4 and 3 units respectively, the angle between $\vec{A}$ and $\vec{C}$ is
(A) $\cos^{-1}(3/5)$
(B) $\cos^{-1}(4/5)$
(C) $\pi/2$
(D) $\sin^{-1}(3/4)$

14. Given that $\vec{A} + \vec{B} + \vec{C} = 0$. Out of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ two are equal in magnitude and the magnitude of the third vector is $\sqrt{2}$ times that of either vector having equal magnitudes. Then, the angle between the vectors is
(A) 30°, 60°, 90°
(B) 45°, 45°, 90°
(C) 45°, 60°, 90°
(D) 90°, 135°, 135°

15. The vector sum of N coplanar forces, each of magnitude F, when each force is making an angle of $2\pi/N$ with the preceding one is
(A) $N\vec{F}$ (B) $N\vec{F}/2$
(C) $\vec{F}/2$
(D) zero

16. At what angle two forces 2F and $\sqrt{2}$ F should act, so that the resultant force is F $\sqrt{10}$?
(A) 45°
(B) 60°
(C) 120°
(D) 90°

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

1. 15 N, 25 N
2. $\frac{3}{\sqrt{26}}\hat{i}+\frac{4}{\sqrt{26}}\hat{j}-\frac{1}{\sqrt{26}}\hat{k}$
3. 2.5 ($(\hat{i}+\hat{j})$
4. 45°
6. 13.96 sq. units.
7. 90°
8. 60°
9. $\theta = 0°$
10. $\frac{5}{\sqrt{45}},\frac{2}{\sqrt{45}},\frac{4}{\sqrt{45}}$

Objective:

1.	D	2.	D
3.	C	4.	A
5.	D	6.	B
7.	D	8.	B
9.	A	10.	C
11.	A	12.	B
13.	A	14.	D
15.	D	16.	A

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Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

KINEMATICS

Kinematics

Syllabus:
Motion in a straight line, Position–time graph, speed & velocity, Uniform and non-uniform motion, Average speed & instantaneous velocity. Uniformly accelerated motion, velocity–time, position–time graphs, Relations for uniformly accelerated motion (graphical treatment), Elementary concepts of differentiation and integration for describing motion. Motion in a plane, cases of uniform velocity and uniform acceleration, Projectile motion, uniform circular motion.

KINEMATICS

Rest and Motion

When an observer says that a particle is in motion, it means that the particle is changing its position with respect to the observer as time passes, otherwise, it is said to be at rest. For an observer standing near a lamp-post, the moving car is in motion but the building appears to be at rest. Same statement is not true for an observer in a moving car. For this observer, the co-passengers are at relative rest whereas the buildings or lamp-posts appear to be in state of motion. Thus, the states of rest or motion are relative terms, relative to the state of the observer. Thus, motion is a combined property of the object under observation and the observer as well.

In order to define motion empirically, we locate the position of the particle with respect to the origin of a coordinate system (x-y-z axis) at different times. Such a system comprising x-y-z coordinate axes with a clock (to measure time interval) is called *frame of reference*.

If all the three coordinates (x, y, z) of a particle P remain unchanged as time passes, we say that P is at rest relative to the frame. However, if any one or more co-ordinates changed with time, the particle is said to be in motion with respect to the frame.

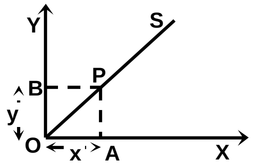
Motion in a Straight Line

When a particle moves along a straight line (assuming along the x-axis of the reference frame), we need only one coordinate (here the x-coordinate) to specify its position. This is also known as motion in one dimension or one-dimensional motion or rectilinear motion of the particle.

The remaining two coordinates (y and z) remain unchanged as time passes. Motion of a particle projected vertically upward is one-dimensional motion. It is appropriate to mention here that for a particle moving in a plane along a curved path, two coordinates are required (say, x and y) to specify the position. Such motions are called motion in a plane or motion in two dimensions. Examples of two dimensional motion are: (i) circular motion, (ii) projectile motion, (iii) motion of an insect on table top along a curved path, etc.

Similarly, we require all the three coordinates (x, y and z) to locate the position of a mosquito flying in space. Such motions are called three-dimensional motion or motion in three dimensions. In this chapter, we shall describe the simplest kind of motion, i.e. the motion in a straight line only.

Illustration 1. *A particle P is moving along a straight line OS. The coordinates x = OA and y = OB are required to describe the motion of the particle. Does it indicate motion in one dimension or in two dimensions?*



Solution: Yes, it is a one-dimensional motion.

Illustration 2. *Two particles A and B start together and are moving with speed 2 m/s and 3 m/s, respectively, in the same direction. Find how far will B be from A after 10 second.*

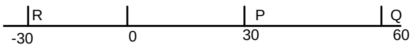
Solution: Distance travelled by A_1, $S_1 = 2 \times 10 = 20$ m,
Distance travelled by B, $S_2 = 3 \times 10 = 30$ m.
$\therefore$ Distance between them, $S = S_2 - S_1 = 30 - 20 = 10$ m.

In our discussions in the following sections, we shall treat the objects in motion as point objects or like a particle. This approximation is true in cases where the size of the object is much smaller than the distance it covers in a reasonable time interval. We consider moon as a particle during its orbital motion round the earth and even earth as a particle during its orbital motion round the sun.

POSITION, PATH LENGTH AND DISPLACEMENT

(i) When motion is along a straight line

Position: To locate the position of the particle at some time (instantaneous position) we choose an axis (*say x-axis*) with a fixed origin O in the given reference frame and find the distance from O. Thus, positions of P, Q and R are +30 cm, +60 cm and –30 cm, respectively. In vector notation, we represent the position vectors as $\overrightarrow{OP} = (+30\,\text{cm})\hat{i}$, $\overrightarrow{OQ} = +(60\,\text{cm})\hat{i}$ and $\overrightarrow{OR} = (-30\text{cm})\hat{i}$, respectively.



Path length: With reference to the above *figure*, let a particle starts moving from the origin O at time t = 0 and at subsequent times t_1, t_2 and t_3 ($t_3 > t_2 > t_1$) and it is at P, Q and R, respectively. Can you find the path length during the interval (i) t = 0 to t = t_1, (ii) t = 0 to t = t_3, (iii) t = t_1 to t = t_3?

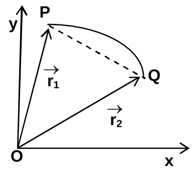
The path length is always equal to the total distance moved by the particle. Hence, the corresponding path lengths are 30 cm, 150 cm and 120 cm, respectively. Thus, path lengths add up like a scalar quantity, they have no direction but magnitude only.

Displacement: It is defined as the change in the position vectors in the given time interval. If x_i and x_f be the initial and final positions of the particle in the time interval ($t_2 - t_1$), then the displacement $\Delta x = x_2 - x_1$. In vector notation, $\vec{x}_i = x_1\hat{i}$, $\vec{x}_f = x_2\hat{i}$, hence $\Delta\vec{x} = (x_2 - x_1)\hat{i}$, since the motion is along the x-axis only. Can you find the displacement during the same time interval as done for calculating the path length in the previous section. The displacements are (i) (+30 cm)$\hat{i}$, (ii) (–30 cm)$\hat{i}$ and (iii) (–60 cm)$\hat{i}$. Note that no sign + or – has been mentioned in expressing the path length (since it is scalar) while ± sign has been mentioned in expressing the displacements (since displacement is a vector quantity). The magnitude of the displacement may or may not be equal to the path length traversed by the particle.

In general path length ≥ magnitude of displacement. If the initial and final positions of a particle in its motion be same, the displacement is zero but path length is not zero.

(ii) When the motion is along the curved path

Position: Initial position is at P and is represented by $\overrightarrow{OP} = \vec{r}_1$. Similarly, final position is at Q and is represented by $\overrightarrow{OQ} = \vec{r}_2$. In terms of coordinates of P and Q, $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$, $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$



Path length: Here, the path length is the length of the curve joining the initial and final positions (not the straight line joining P and Q as shown by dotted line) along which the particle has actually moved through.

Displacement: Magnitude of the displacement is the length of the straight line joining the initial and final positions and its direction is from the initial to the final position. We have already defined displacement as the change in position vector, hence displacement $\overrightarrow{PQ} = \vec{r}_2 - \vec{r}_1$,

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= \Delta x\,\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

Exercise 1: ***A person moves from A to B along the semicircular path. Compare the distance moved by him and the displacement.***

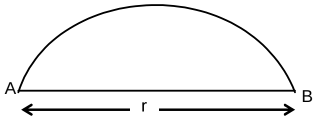
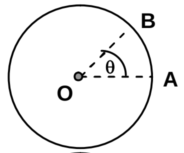


Illustration 3. *A boy travels from his house to a play ground along a straight path of length 'D' meter and return back to his house. Find the distance travelled and displacement of the boy.*

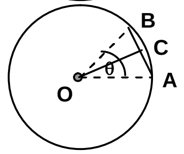
Solution: Distance = 2D meter, Displacement = Zero

Illustration 4. *A particle moves along a circle of radius R. Find the path length and magnitude of displacement from initial position A to final position B.*



Solution: Path length = Rθ

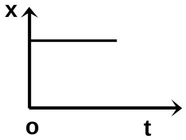
Displacement = AB = AC + BC = 2R sin (θ/2).



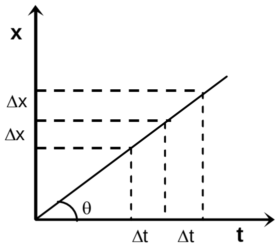
Position–Time Graph:

If we plot time t along the x-axis and the corresponding position (say x) from the origin O on the y-axis, we get a graph which is called the position–time graph. This graph is very convenient to analyse different aspects of motion of a particle. Let us consider the following case.

(i) In this case, position (x) remains constant but time changes. This indicates that the particle is stationary in the given reference frame. Hence, the straight line nature of position–time graph parallel to the time axis represents *the state of rest*. Note that its slope (tan θ) is zero.



(ii) When the x–t graph is a straight line inclined at some angle ($\theta \neq 0$) with the time axis, the particle traverses equal displacement Δx in equal interval of time Δt. The motion of the particle is said to be *uniform rectilinear motion.* The slope of the line measured by $\frac{\Delta x}{\Delta t} = \tan\theta$ represents the uniform velocity of the particle.



(iii) When the x–t graph is a curve, motion is not uniform. It either speeds up or slows down depending upon whether the slope ($\tan\theta$) successively increases or decreases with time. As shown in the figure, the motion speeds up from $t = 0$ to $t = t_1$ (since the slope $\tan\theta$ increases). From $t = t_1$ to $t = t_2$, AB represents a straight line indicating uniform motion. From $t = t_2$ to $t = t_3$, the motion slows down and for $t > t_3$ the particle remains at rest in the reference frame.

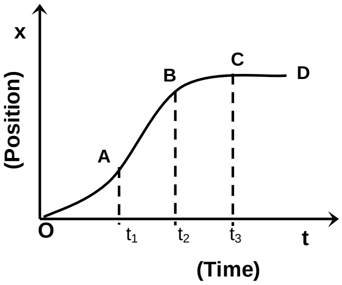
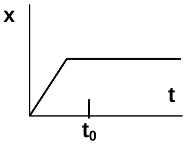


Illustration 5. *The adjacent figure shows the displacement–time graph of a particle moving on the x-axis. Choose the correct option given below.*



(A) The particle is continuously going in positive x direction.
(B) The particle is at rest.
(C) The particle moves at a constant velocity all time
(D) The particle moves at a constant velocity upto a time t_0, and then stops.

Solution: **(D).** *Upto time t_0,, particle is said to have uniform rectilinear motion and after that comes to rest as the slope is zero.*

Exercise 2:

(i) ***Distinguish between the distance covered by a body and its displacement. What are the characteristics of displacement?***

(ii) ***Under what condition will the distance and displacement of a moving object have the same magnitude.***

Speed

The term average indicates overall effect whereas instantaneous means the effect at a particular time. Hence, the average speed in a given time interval $(t_2 - t_1)$ is measured by the distance covered (path length s) divided by the time interval.

Thus, average speed $= \dfrac{s}{t_2 - t_1} = \dfrac{\text{path length}}{\text{time interval}}$

If the time interval $(t_2 - t_1)$ is divided into small segments Δt_1, Δt_2, ..., for which the corresponding path lengths be Δs_1, Δs_2, ..., then

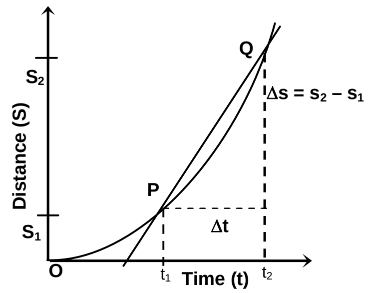
Average speed $= \dfrac{\Delta s_1 + \Delta s_2 + \Delta s_3 + ...}{\Delta t_1 + \Delta t_2 + \Delta t_3 + ...} = \dfrac{s}{t_2 - t_1}$

Hence, the average speed during one such time interval is equal to $\frac{\Delta s}{\Delta t}$. If Δt is infinitesimally small $|\Delta t \to 0|$, then we define the instantaneous speed at a time t as

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Speed is a scalar quantity. It has only the magnitude and no direction. For a particle in motion in a given reference frame the instantaneous or average speed during any time interval is always positive.

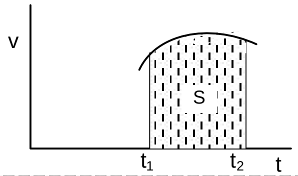
Consider the distance time graph as shown in the given figure. The average speed during the time interval Δt is $\frac{\Delta s}{\Delta t}$ which is the slope of the chord PQ. As $\Delta t \to 0$, the chord PQ becomes the tangent at P and the average speed becomes the instantaneous speed at P given by $\frac{ds}{dt} = \tan\theta$, which is the slope of the tangent at P.



Remember that the s–t graph (position–time graph) does not indicate the path of motion but represents increase in the path length as time increases, whether the particle does or does not retrace its path. Now, instantaneous speed $v = \frac{ds}{dt}$.

$\therefore$ $ds = v\,dt$ and $s = \int_{t_1}^{t_2} v\,dt$ = total distance travelled during the time interval $(t_2 - t_1)$

It is evident that the area under the speed time graph (shown by the shaded region) measures the total distance covered during the time interval $t_2 - t_1$.



Exercise 3:

i) ***Can a body moving with uniform speed have variable velocity?***

ii) ***Can a body moving with uniform velocity have variable speed?***

iii) ***Can average velocity ever become equal to instantaneous velocity ?***

Illustration 6. *A car covers the first half of the distance between two places at a speed of 40 km/hr and the second half at 60 km/hr. What is the average speed of the car?*

Solution : Let the distance between the two places be 2x km.

$\therefore$ Time taken by the car for the first half of the journey = $\frac{x}{40}$ hr

Also, the time taken for the second half = $\frac{x}{60}$ hr

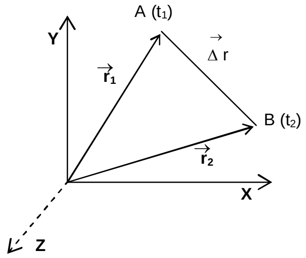
The total time of the journey = $\frac{x}{40} + \frac{x}{60} = \frac{5x}{120}$ hr

Average speed = $\frac{\text{distance}}{\text{time}} = \frac{2x}{5x/120}$ = 48 km/hr

Velocity

By definition,

Average velocity, $\vec{v}_{av} = \dfrac{\text{displacement vector}}{\text{time interval}} = \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$



Note that the positions in between the interval of time t_1 and t_2 are to be specified in finding the average velocity. If the particle takes different paths to travel from A to B in the same time interval, the average velocity will remain same but average speed will be different and greater than the magnitude of $\vec{v}_{av}$. In the special case when the points A to B is straight, the average speed is equal to the magnitude of average velocity.

If $\Delta t \to 0$, the path length Δs during the interval Δt is equal to the Δr.

Hence, the instantaneous velocity $\vec{v} = \lim \dfrac{\Delta\vec{r}}{dt} = \dfrac{d\vec{r}}{dt}$, and

magnitude of the velocity is $v = \left|\dfrac{\Delta\vec{r}}{dt}\right| = \dfrac{|d\vec{r}|}{dt} = \dfrac{ds}{dt}$.

Hence, the instantaneous speed at any time t is the magnitude of instantaneous velocity at that time.

On a graph of position as a function of time for straight line, the instantaneous velocity at any point is equal to the slope of the tangent to the curve at that point.

The figure depicts the motion of a particle.

	x, t group	Motion of particle
A	Positive slope, so $V_x > 0$	Moving in +ive x = direction
B	Larger positive slope, so $V_x > 0$	Moving in +x direction faster than at A
C	Zero slope, so $V_X = 0$	Instantaneously at rest
D	Negative slope, so $V_x < 0$	Moving in –x direction
E	Smaller negative slope, so $V_x < 0$	Moving in –ve x-direction more slowly than at D

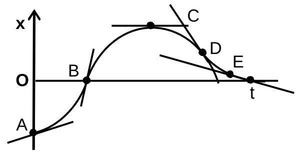
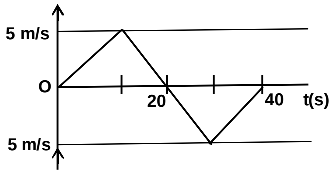


Illustration 7. *From the velocity– time plot shown in fig. Find*
(a) distance travelled by the particle during the first 40 seconds.
(b) displacement travelled by the particle during the first 40 seconds.
(c) Also find the average velocity during this period.



Solution: (a) Distance = area under the curve

$$= \frac{1}{2} \times 20 \times 5 + \frac{1}{2} \times 5 \times 20$$

= 50 + 50 = 100 m

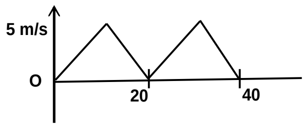


Fig. (a)

For distance measurement, the curve is plotted as in Fig. (a)

(b) Displacement = area under the curve in Fig. (b) = 0

(c) $V_{aV} = \dfrac{\text{Displacement}}{\text{time}}$

As displacement is zero,

$\therefore V_{av} = 0$

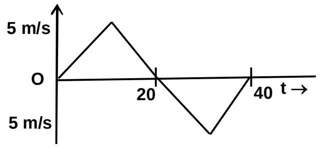


Fig. (b)

Illustration 8. *If velocity of a particle moving along a straight line changes with time as V(m/s) = 4 sin* $\left(\frac{\pi}{2}t\right)$, *its average velocity over time interval t = 0 to t = 2(2n – 1) sec, (n being any (+)ve integer) is*

(A) $\dfrac{8}{\pi(2n-1)}$ *m/s* *(B)* $\dfrac{4}{\pi(2n-1)}$ *m/s*

(C) zero (D) $\dfrac{16(2n-1)}{\pi}$ *m/s* *(D) none*

Solution: **(A).**

Displacement over the interval t = 0 to t = 2(2n –1) sec

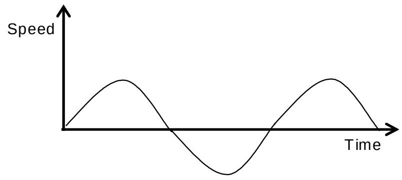
$$= 4\int_0^{2(2n-1)} \sin\left(\frac{\pi}{2}t\right)dt = -\left(\frac{8}{\pi}\right)\left|\cos\frac{\pi t}{2}\right|_0^{2(2n-1)}$$

$$= \frac{16}{\pi}\text{m}$$

$$\Rightarrow \text{Average velocity } = \frac{16}{2(2n-1)\pi} = \frac{8}{\pi(2n-1)} \text{ m/s}$$

Exercise 4:

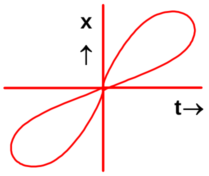
(i) ***Is the speed–time graph shown in figure is possible ?***



(ii) ***Under what circumstances, does the relationship Δx = v. Δt hold good.***

(iii) ***Show that average velocity of an object over an interval of time is either smaller than or equal to the average speed of the object over the same interval.***

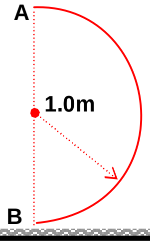
(iv) ***State with reasons why the graph in the adjacent figure cannot represent one-dimensional motion of a particle.***



(v) ***In 1.0 sec, a particle goes from point A to point B, moving in a semicircle of radius 1.0 m as shown in the adjacent figure. The magnitude of average velocity is***

(A) 3.14 m/s ***(B) 2.0 m/s***

(C) 1.0 m/s ***(D) Zero***



Acceleration

Motion of a particle moving with constant velocity along a straight line is said to be uniform motion because neither the speed nor the direction of motion changes with the passage of time. On the other hand, the motion is said to be accelerated, if either the speed or the direction or both continuously change with time.

Definition: *Acceleration is defined as the rate of change of velocity.* It is a vector quantity and has its direction along which velocity has changed.

Average acceleration, $\vec{a}_{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \dfrac{\Delta\vec{v}}{\Delta t}$

Note: When the velocity decreases, we say that the particle is decelerating. Deceleration is equivalent to negative acceleration.

It is also a vector quantity directed along the direction of the change $\Delta\vec{v}$ and independent of the intermediate values of velocities in between the interval $t_2 - t_1$.

Instantaneous acceleration at a time t is defined as

$$\vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{v}) = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

If the particle moves along x-axis,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i}) = \frac{dv_x}{dt}\hat{i} = \frac{d^2x}{dt^2}\hat{i}$$

If the particle moves in the xy plane,

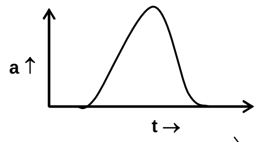
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j}) = \left(\frac{dv_x}{dt}\right)\hat{i} + \left(\frac{dv_y}{dt}\right)\hat{j} = a_x\hat{i} + a_y\hat{j} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

Exercise 5:

(i) ***Can a body have an acceleration with zero velocity?***

(ii) ***Can the direction of the velocity of a body change when its acceleration is constant ?***

(iii) ***A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time-interval. Suggest acceleration time graph for this situation.***



(iv) ***Figure shows the x–t graph of a particle moving along a straight line. What is the sign of the acceleration during the intervals OA, AB, BC and CD?***

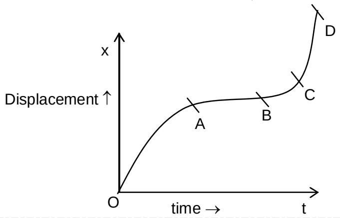


Illustration 9. *A body moving in a curved path possesses a velocity 3 m/s towards north at any instant of its motion. After 10s, the velocity of the body was found to be 4 m/s towards west. Calculate the average acceleration during this interval.*

Solution: To solve this problem the vector nature of velocity must be taken into account. In the figure, the initial velocity v_0 and the final velocity v are drawn from a common origin. The vector difference of them is found by the parallelogram method.

The magnitude of difference is

$|v - v_0| = OC = \sqrt{OA^2 + AC^2}$

$= \sqrt{4^2 + 3^2} = 5 \text{ m/s}$

The direction is given by

$\tan\theta = \frac{3}{4} = 0.75 \quad \therefore 37^0$

$\therefore$ Average acceleration

$= \frac{v - v_0}{t} = \frac{5}{10}$

$= 0.5 \text{ m/s}^2$ at 37° South of West.

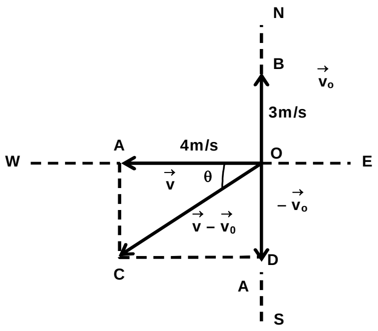


Illustration 10. *A car is moving eastwards with velocity 10 m/s. In 20 seconds, the velocity changes to 10 m/s northwards. Calculate the average acceleration in this time.*

Solution: The change in velocity

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

$= 10\hat{j} - 10\hat{i}$

$\Rightarrow \Delta\vec{v}$ has a magnitude of $10\sqrt{2}$ and is directed towards N-W.

So, $|\vec{a}| = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \frac{1}{\sqrt{2}}$ m/s² towards N-W.

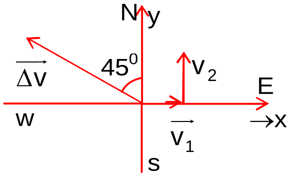


Illustration 11. *Which of the following statements is possible in a one/two-dimensional motion?*
(A) A body is having zero velocity and still accelerating.
(B) The velocity of an object reverses direction when acceleration is constant.
(C) An object increasing in speed as its acceleration decreases.
(D) None of these.

Solution: **(A), (B) and C**

(A) When a body is projected vertically upward, at the highest point its speed becomes zero while it is accelerating downwards with $g = 9.8 \text{ m/s}^2$.

(B) When a body is projected upward, the direction of its velocity reverses during its descent while the acceleration (g) remains constant.

(C) Steel ball falling through a viscous liquid before terminal velocity is attained.

Exercise 6:

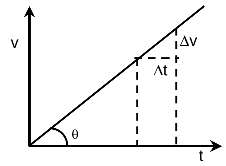
(i). ***A particle is moving along a circular path of radius 5 m with a uniform speed of 5 m/s. What will be the average acceleration when the particle completes half revolution?***
(A) Zero (B) 10 m/s²
(C) 10π m/s² (D) 10/π m/s²

(ii). ***Which of the following statement is incorrect?***
(A) Zero velocity of particle does not necessarily mean that its acceleration is zero.
(B) Zero acceleration of a particle does not necessarily mean that its velocity is zero.
(C) If speed of a particle is constant its acceleration must be zero.
(D) None of these.

Uniformly accelerated motion

Motion of a particle is said to be uniformly accelerated if acceleration (a vector quantity) remains constant in magnitude as well as in direction. Motion of a particle falling freely under gravity is an example of uniformly accelerated motion since the acceleration ($\vec{g}$) remains constant (assuming negligible air resistance).

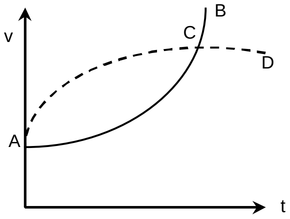
Velocity–time graph



When the v–t graph is a straight line, inclined at an angle θ with the time axis, the velocity increases equally in equal time interval. This indicates that the acceleration is uniform. Its magnitude is $a = \frac{\Delta v}{\Delta t} = \tan\theta$, the slope of v–t graph.

Let us find the area under the v–t graph:

$$A = \int dA = \int v dt = \int \frac{ds}{dt} dt = \int_i^f ds = \text{net displacement}$$



If the v–t graph is a curve, the slope continuously changes with time, which indicates that the magnitude of acceleration either increases with time (for curve AB) or it decreases with time (for curve ACD).

Note:

Features of v–t graphs:

(i) The slope of v–t graph gives the instantaneous acceleration.

(ii) The area under the v–t graph gives the net displacement (not distance) in the given time interval.

EQUATION OF MOTION IN A STRAIGHT LINE WITH UNIFORM ACCELERATION

Consider the motion of a particle moving along the x-axis with uniform acceleration.

Let u = Initial velocity (at time t = 0)

v = Final velocity (at time t), and

x = Net displacement in the time interval t = 0 to t = t.

The equations describing such uniformly accelerated motion are

$$v = u + at; \quad x = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2ax$$

Remember that these equations are valid or applicable if the acceleration remains constant both in magnitude and direction.

Derivation of Equations of Motion

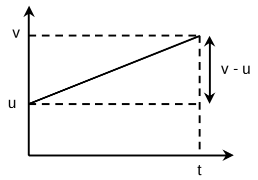
(i) v = u + at

Calculus method:

By definition $a = \frac{dv}{dt}$ or, $dv = a dt$

Integrating, $\int_u^v dv = a\int_0^t dt$ [$\because$ a = constant]

$v - u = at$, or $v = u + at$



Graphical Method:
By definition, slope of v–t graph gives the acceleration

$$\therefore \; a = \tan\theta = \frac{v-u}{t-0} = \frac{v-u}{t}$$

or $v - u = at$ $\therefore$ $v = u + at$

(ii) $\mathbf{x = ut + \frac{1}{2}at^2}$

Calculus method:

By definition, instantaneous velocity $v = \frac{dx}{dt}$

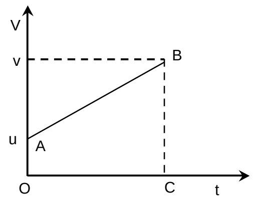
or, $dx = v\,dt$

$= (u + at)dt$ [$\because v = u + at$]

$= u\,dt + at\,dt$

Integrating, $\int_0^x dx = u\int_0^t dt + a\int_0^t t dt$ [$\because$ u and a are constant]

Hence, $x = ut + \frac{1}{2}at^2$



Graphical method:
We know that area under the velocity– time graph gives the net displacement during the given time interval. Hence, net displacement x = area OABC.

or $x = \frac{1}{2}(OA + BC) \times OC$

$= \frac{1}{2}(u+v).t = \frac{1}{2}[2u + \frac{v-u}{t}t]t$

$= \frac{1}{2}[2u + at]t$ [$\because a = \frac{v-u}{t}$] $= ut + \frac{1}{2}at^2$

(iii) $\mathbf{v^2 = u^2 + 2ax}$

Calculus method:

By definition, $a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt}$

or, $a = v.\frac{dv}{dx}$

or, $vdv = adx$

Integrating, $\int_u^v vdv = a\int_0^x dx$

or, $\frac{v^2 - u^2}{2} = ax$ $\therefore$ $v^2 = u^2 + 2ax$

Graphical method:
From the v–t graph, net displacement x = area under the v–t graph

or, $x = \frac{1}{2}(v + u)t$

Multyplying both sides by (v – u),

$$x\,(v - u) = \frac{1}{2}(v+u)(v-u)t$$

or, $\quad x\left(\dfrac{v-u}{t}\right)=\dfrac{v^2-u^2}{2}$

or, $\quad x \cdot a=\dfrac{v^2-u^2}{2} \quad \left[\because a=\dfrac{v-u}{t}\right]$

$\therefore \quad v^2 = u^2 + 2ax$

It is to be noted that u, v and a are vectors and may have positive or negative value depending on whether their directions are along the positive or negative directions of the x-axis.

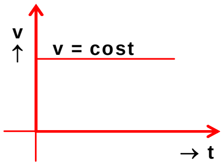
Graphs Representing Motion of a Particle

From the knowledge of calculus, we can say from Eq. (1) that:
- (i) Slope of s–t graph gives velocity;
- (ii) Slope of v–t graph gives acceleration;
- (iii) Area under v–t graph gives displacement; and
- (iv) Area under a–t graph gives change in velocity.

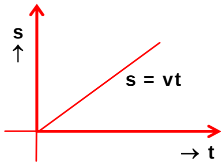
(a) **Uniform motion**

v = constant

$\Rightarrow a = \dfrac{dv}{dt} = 0$



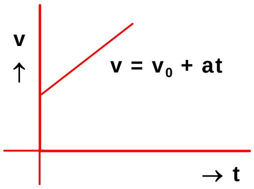
v–t graph
(a)



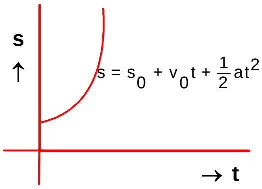
s–t graph
(b)

(b) **Uniformly accelerated motion**

a = constant

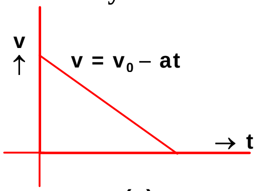


v–t graph
(a)

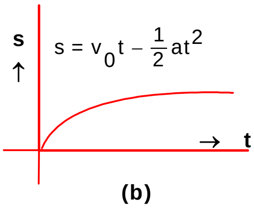


s–t graph
(b)

(c) **Uniformly retarded motion**



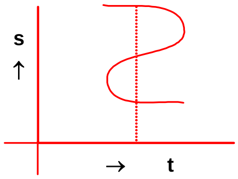
(a)



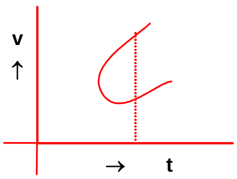
(b)

Some facts regarding graphs:

(i) Two or more values of velocity or displacement at any particular instant of time are not possible. So, the corresponding graphs should be rejected. For example, the following graphs are not possible.

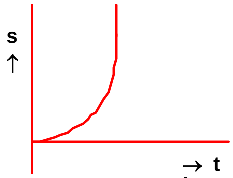


(a)

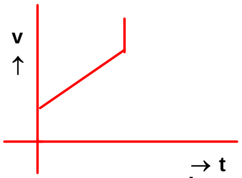


(b)

(ii) At any point, the slopes of s–t or v–t graph can never be infinite because infinite slope of s–t graph means infinite velocity and that of v–t graph means infinite acceleration, which are not possible. So, corresponding graphs are not acceptable. For example, the following graphs are all not possible.



s–t graph
(a)



v–t graph
(b)

In general, when a particle is moving with a uniform acceleration, to its motion is described by the following equations.

$$\vec{r}(t) = \vec{r}_0 + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\cdot\vec{s}$$

Here, $\vec{r}(t) =$ represents position vector of a particle at an instant t.

$\vec{r}_0$ = position vector of a particle at t = 0.

$\vec{u}$ = initial velocity of a particle at t = 0.

$\vec{v}$ = velocity of a particle at an instant t.

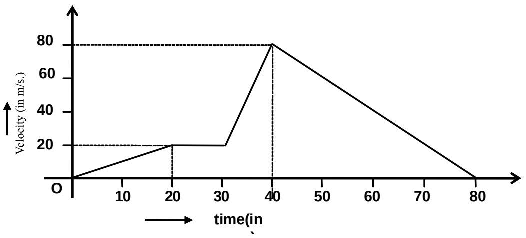
$\vec{a}$ = acceleration of the particle at an instant t.

Exercise 7:

(i) ***A boy sitting on a rail road car moving with a constant velocity tosses a coin up. Describe the path of the coin as seen by***
(a) the man on the train.
(b) the man standing on the ground near the rail.

(ii) ***A particle is moving along a straight path, draw its velocity– time graph for the following cases:***
(a) When the acceleration of the particle increases.
(b) When the displacement of the particle obeys the relation $s = 4 + 5t + 2t^2$
(c) When the acceleration of the particle is given by $a = 12 \cos 6t$

Illustration 12. *The velocity–time graph of a moving object is given in the figure. Find the maximum acceleration of the body and distance travelled by the body in the interval of time in which this acceleration exists.*



Solution: Acceleration is maximum when slope is maximum.

$$a_{max} = \frac{80-20}{40-30} = 6\text{m/s}^2$$

$$S = 20\text{m/s} \times 10\text{s} + \frac{1}{2} \times 6\text{m/s}^2 \times 100\text{s}^2 = 500\text{m}.$$

Illustration 13. *A particle having initial velocity is moving with a constant acceleration 'a' for a time t.*
(a) Find the displacement of the particle in the last 1 second.
(b) Evaluate it for u = 2 m/s, a = 1 m/s^2 and t = 5 sec.

Solution: (a) The displacement of a particle at time t is given by $s = ut + \frac{1}{2}at^2$

At time (t – 1), the displacement of a particle is given by

$$S' = u(t-1) + \frac{1}{2}a(t-1)^2$$

$\therefore$ Displacement in the last 1 second is

$$S_t = S - S'$$

$$= ut + \frac{1}{2}at^2 - \left[u(t-1) + \frac{1}{2}a(t-1)^2\right]$$

$$= ut + \frac{1}{2}at^2 - ut + u - \frac{1}{2}a(t-1)^2$$

$$= \frac{1}{2}at^2 + u - \frac{1}{2}a(t^2 + 1 - 2t) = \frac{1}{2}at^2 + u - \frac{1}{2}at^2 - \frac{a}{2} + at$$

$$S = u + \frac{a}{2}(2t-1)$$

(b) Putting the values of u = 2m/s, a = 1m/s^2 and t = 5 sec, we get

$$S = 2 + \frac{1}{2}(2\times 5 - 1) = 2 + \frac{1}{2}\times 9$$

$$= 2 + 4.5 = 6.5 \text{ m}$$

Illustration 14. *A car moving along a straight road with a speed of 72 km/h is brought to a stop with in a distance of 10m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?*

Solution: v = 72 km/h

$$= \frac{72\times 1000}{3600} = 20\,\text{m/s}$$

using equation

$$\vec{v}^2 = \vec{u}^2 + 2a(\vec{x} - \vec{x}_0)$$

$$= (20)^2 + 2 \times a \times 10$$

$$a = -\frac{20\times 20}{20} = -20\,\text{m/s}^2$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$0 = 20 - 20\,t$$

$$t = \frac{20}{20} = 1\text{sec}$$

So, it will take 1 sec for the car to stop.

Illustration 15. *Position of a particle moving along x-axis is given by x = 3t – 4t^2 + t^3, where x is in meters and t in seconds.*
(a) Find the position of the particle at t = 2s.
(b) Find the displacement of the particle in the time interval from t=0 to t=4s.
(c) Find the average velocity of the particle in the time interval from t=2s to t=4s.
(d) Find the velocity of the particle at t =2s.

***Solution*:** (a) $x_{(t)} = 3t - 4t^2 + t^3$

$\Rightarrow x_{(2)} = 3 \times 2 - 4 \times (2)^2 + (2)^3 = 6 - 4 \times 4 + 8 = -2$ m.

(b) $x_{(o)} = 0$

$x_{(4)} = 3 \times 4 - 4 \times (4)^2 + (4)^3 = 12$m.

Displacement $= x_{(4)} - x_{(0)} = 12$ m.

(c) $\langle v \rangle = \dfrac{x_{(4)} - x_{(2)}}{(4-2)} = \dfrac{12-(-2)}{2} \text{m/s} = 7\text{m/s}$

(d) $\dfrac{dx}{dt} = 3 - 8t + 3t^2$

$\Rightarrow v_{(2)} = \left(\dfrac{dx}{dt}\right)_{(2)} = 3 - 8 \times 2 + 3 \times (2)^2 = -1\text{m/s}$

Illustration 16. *An anti-aircraft shell is fired vertically upwards with a muzzle velocity of 294 m/s. Calculate (a) the maximum height reached by it, (b) time taken to reach this height, (c) the velocities at the ends of 20th and 40th second. (d) when will its height be 2450 m? Given g = 980 cm/s².*

Solution : (a) Here, the initial velocity u = 294 m/s and g = 9.8 m/s²

$\therefore$ The maximum height reached by the shell is,

$$H = \frac{u^2}{2g} = \frac{294^2}{2 \text{ x } 9.8} = 4410\text{m} = 4.41\text{km}$$

(b) The time taken to reach the height is,

$$T = \frac{u}{g} = \frac{294}{9.8} = 30\text{s}$$

(c) The velocity at the end of 20th second is given by,

v = u – gt = 294 – 9.8 x 20 = 98 m/s upward,

and the velocity at the end of 40th second is given by,

v = 294 – 9.8 x 40 = – 98 m/s

The negative sign implies that the shell is falling downward.

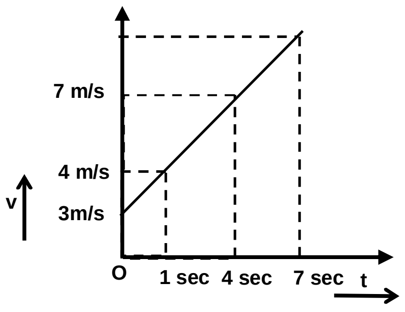
(d) From the equation

$h = ut + \frac{1}{2}gt^2$ or $2450 = 294\,t - \frac{1}{2} \times 9.8t^2$

or, $t^2 - 60\,t + 500 = 0$ $\therefore$ t = 10s and 50 s.

At t = 10 s the shell is at a height of 2450 m and is ascending, and at the end of 50 s it is at the same height, but is falling.

Illustration 17. *The velocity–time graph of a particle is given as shown in the figure. Find the distance travelled by the object in 7th second.*



Solution : $\dfrac{dv}{dt} = \dfrac{7-4}{4-1} = 1\text{m/s}^2$

$$\vec{s}_{t=7} = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

$$= 3 + \frac{1}{2}(2 \times 7 - 1) = 9.5\text{m}.$$

Exercise 8:

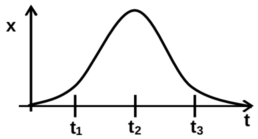
(i) ***A truck moving with constant acceleration covers the distance between two points 180 m apart in 6 seconds. Its speed as it passes the second point is 45 m/s. Find***
(a) its acceleration , and
(b) its speed when it was at the first point.

(ii) ***A body undergoing uniformly accelerated motion starts moving along +x-axis with a velocity of 5 m/s and after 5 seconds its velocity becomes 20 m/s in the same direction. What is the velocity of the body 10 seconds after the start of the motion ?***

(iii) ***What is the speed with which a stone is projected vertically upwards from the ground if it attains a maximum height of 20 m?***

(iv) ***A ball is thrown vertically upwards with a speed of 20 m/s from a hard floor. Draw a graph showing the velocity of the ball as a function of time if the ball suffers elastic collisions continuously.***

(v) ***The adjacent figure shows the x-coordinate of a particle as a function of time. Find the signs of v_x and a_x at $t = t_1$, $t = t_2$ and $t = t_3$.***



Solving problems in Kinematics using elementary concepts of differential and integral calculus

For the motion of a particle in a straight line, we always write instantaneous velocity $v = \frac{dx}{dt}$.

In case, the acceleration is non uniform and a function of displacement, we write,

$$a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx}$$

Let us solve some illustrative examples:

Illustration 18. *The instantaneous velocity of a particle moving along a straight line is given by $v = \alpha t^2$ whose α is a positive constant. Find the average speed during the interval $t = 0$ to $t = T$.*

Solution: By definition, average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{\int_0^T v\,dt}{\int_0^T dt}$

$$= \frac{1}{T}\int_0^T \alpha t^2 dt = \frac{\alpha}{T}\left[\frac{t^3}{3}\right]_0^T = \frac{\alpha T^2}{3}.$$

Illustration 19. *The displacement (x) of a particle moving in one dimension under the action of a constant force is related to the time t by the equation $t = \sqrt{x} + 3$, x in m and t in sec. Find the displacement of the particle when its velocity is zero.*

Solution: Here, $t = \sqrt{x} + 3$
or $x = t^2 - 6t + 9$

$\therefore v = \frac{dx}{dt} = 2t - 6$

When v = 0, 2t – 6 = 0 ⇒ t = 3 sec

At t = 3 sec, $x = t^2 - 6t + 9$

= 9 –6 (3) + 9 = 0

Hence, the displacement of particle is zero when its velocity is zero.

Motion Under Gravity (Free Fall)

When a body is dropped from some height (earth's radius = 6400 km), it falls freely under gravity with constant acceleration g (=9.8 m/s^2) provided the air resistance is negligible small. The same set of three equations of kinematics (where the acceleration $\vec{a}$ remains constant) are used in solving such motion.

Here, we replace $\vec{a}$ by $\vec{g}$ and choose the direction of y-axis conveniently. When the y-axis is chosen positive along vertically downward direction, we take $\vec{g}$ as positive and use the equations as

$$v = u + gh \qquad \Rightarrow \qquad v^2 = u^2 + 2gh$$

$$h = ut + \frac{1}{2}gt^2$$

where u is initial velocity of projection in the vertically downward direction.

However, if an object is projected vertically upward with initially velocity u, we can take y – axis positive in the vertically upward direction the set of equations reduces to

$$v = u - gt \qquad \Rightarrow v^2 = u^2 - 2gh$$

$$h = ut - \frac{1}{2}gt^2$$

In order to avoid confusion in selecting $\vec{g}$ as positive or negative, it is advisable to take the y-axis as positive along vertically upward direction and point of projection as the origin. We can now write the set of three equations in the vector form:

$$\vec{v} = \vec{u} + \vec{g}t \qquad \Rightarrow \qquad \vec{v}.\vec{v} = \vec{u}.\vec{u} + 2\vec{g}.\vec{h}$$

$$\vec{h} = \vec{u}t + \frac{1}{2}\vec{g}t^2$$

Exercise 9:

(i) ***A stone is thrown upwards with a speed v from the top of a tower. It reaches the ground with a velocity 3v, what is the height of the tower ?***

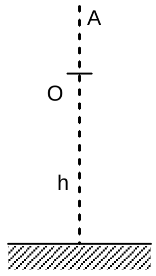
(ii) ***A stone is thrown vertically upwards with a velocity of 19.6 m/s. After 2 second, another stone is thrown upwards with a velocity of 9.8 m/s. When and where these stones will collide?***

Illustration 20. *A body is projected vertically upward, then find the velocity and acceleration of that body at it's highest point of motion?*

Solution: Velocity = 0 , acceleration = ± g

Illustration 21. *A ball is projected vertically upward with a speed of 4.0 m/s from a point 64 m above the ground. Find the time it takes to reach the ground. [g = 10 m/s^2]*

Solution: Before solving the problem, analyse the situation. As the ball will move up, it gradually slows down and attains the maximum height at A (where it comes to momentarily rest) and thereafter it retraces its path, attains the same speed at O but direction reversed down and finally it strikes the ground.



Choosing O (the point of projection) as the origin and positive y-axis as vertically upward, we collect the data in the vector notation which are given in the question.

Net displacement $\vec{y} = -h\ \hat{j} = (-64\text{ m})\,\hat{j}$

Constant acceleration $\vec{g} = -g\hat{j} = (-10\,\text{m/s}^2)\hat{j}$

Initial velocity $\vec{u} = +u\hat{j} = (+4.0\text{ m/s})\ \hat{j}$

Now, $\vec{h} = \vec{u}t + \frac{1}{2}\vec{g}t^2$

Writing the values with proper sign.

$$-64\hat{j} = 4t\hat{j} + \frac{1}{2}(-10\hat{j})t^2$$

This reduces to a simple quandratic equation, $5t^2 - 4t - 64 = 0$

The solution $t = -\frac{16}{5}$ s is not permissible. Hence, the required time= 4.0 seconds.

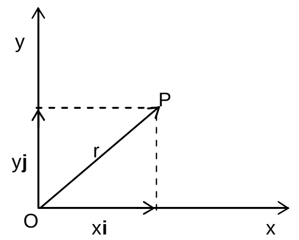
MOTION IN A PLANE

Position Vector and Displacement

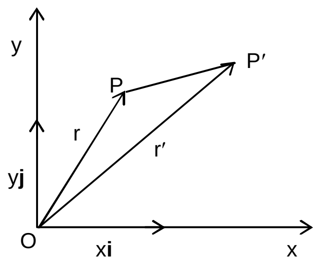
The position vector $\vec{r}$ of a particle located in a plane with reference to the origin of an x-y reference frame is

$$\vec{r} = x\hat{i} + y\hat{j}$$

where x, y are the coordinates of the object.



Let the particle be at a point P at any time t and at a point P′ at any time t′ as shown in figure.



$\therefore$ $\overrightarrow{OP} = \vec{r}(t)$ and $\overrightarrow{OP}' = \vec{r}'(t')$

$\therefore$ Displacement vector

$$\vec{PP'} = \Delta\vec{r} = \vec{r}'(t') - \vec{r}(t)$$

$$= (x'\hat{i} + y'\hat{j}) - (x\hat{i} + y\hat{j})$$

$$= \hat{i}\Delta x + \hat{j}\Delta y$$

where $\Delta x = x' - x$ and $\Delta y = y' - y$

Velocity

The average velocity $\vec{v}$ of an object = $\dfrac{\text{displacement}}{\text{corresponding time interval}}$

$\therefore$ $$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t} = \hat{i}\frac{\Delta x}{\Delta t} + \hat{j}\frac{\Delta y}{\Delta t}$$

or, $$\vec{v} = \vec{v}_x\hat{i} + \vec{v}_y\hat{j}$$

Direction of the average velocity is same as that of displacement.

The instantaneous velocity is the average velocity as the time interval approaches to zero.

$\therefore \qquad \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

The direction of the instantaneous velocity of an object at any point on the path is tangent to the path at that point and is in the direction of motion.

In component form,

$$\vec{v} = \hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt} = v_x\hat{i} + v_y\hat{j}$$

Magnitude of v,

$$v = \sqrt{v_x^2 + v_y^2}$$

and the direction of v,

$$\tan\theta = \frac{v_y}{v_x}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right).$$

Acceleration

Average acceleration = $\dfrac{\text{change in velocity}}{\text{time interval}}$

$$\vec{a}_{av} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} \qquad \text{or} \qquad \vec{a}_{av} = a_x\hat{i} + a_y\hat{j}$$

Instantaneous acceleration: It is the limiting value of the average acceleration as the time interval approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta \vec{v}}{\Delta t}$$

$$\therefore \quad \vec{a} = \hat{i}\lim_{\Delta t \to 0}\frac{\Delta v_x}{\Delta t} + \hat{j}\lim_{\Delta t \to 0}\frac{\Delta v_y}{\Delta t}$$

or, $\qquad \vec{a} = a_x\hat{i} + a_y\hat{j}$

where $a_x = \dfrac{dv_x}{dt}$, $a_y = \dfrac{dv_y}{dt}$

Motion in a plane with constant acceleration:

If an object is moving in x-y plane having constant acceleration a, then by the definition of average acceleration

$$a = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t}$$

where , v_0 = velocity of the object at time t = 0
and v = velocity of the object at time t.

or, $\qquad v = v_0 + at$

In terms of components,

$v_x = v_{0x} + a_x t$

$v_y = v_{0y} + a_y t$

Now, let r_0 and r be the position vectors of any particle at time zero and t and their velocities at these instant are v_0 and v, respectively. During the time interval t, $\left(\dfrac{v_0 + v}{2}\right)$ is the average velocity.

$\therefore$ Displacement,

$$r - r_0 = \left(\frac{v + v_0}{2}\right)t = \left(\frac{v_0 + at + v_0}{2}\right)t$$

$$r = r_0 + v_0 t + \frac{1}{2}at^2$$

Hence, in component form,

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

Therefore, a two dimensional motion can be treated as two separate simultaneous one-dimensional motions having constant acceleration along two perpendicular direction.

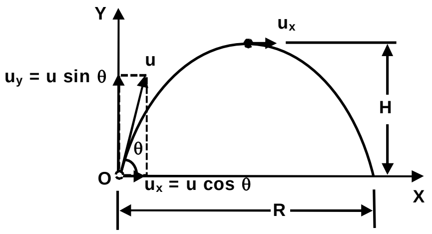
PROJECTILE

An object projected into space or air, such that it moves under the effect of gravity only, is called a projectile.

Projectile Motion

Motion in a vertical plane containing horizontal and vertical axes:

A particle when given a velocity at any arbitrary angle (other than 90°) made with the horizontal surface is known as a projectile.



If a particle is projected from point O, at any angle θ from the horizontal, with initial velocity $\vec{u}$, then the components of $\vec{u}$ in x and y directions are given as

$u_x = u\cos\theta$, $u_y = u\sin\theta$ where $\vec{u} = u_x\hat{i} + u_y\hat{j}$

$\Rightarrow \quad \vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

The X axis is parallel to the horizontal. Y axis is parallel to the vertical and the $\vec{u}$ lies in the X – Y plane.

The constant acceleration $\vec{a}$ is given as, $\quad \vec{a} = a_x\hat{i} + a_y\hat{j}$

where $a_x = 0$ [as there is no acceleration along the X-axis].

$a_y = -g$ [the acceleration is downward and equal to g].

Now, velocity after time t is given as.

$v_{tx} = u_x + a_x t = u\cos\theta \quad (\text{as } a_x = 0)$

$v_{ty} = u_y + a_y t = u\sin\theta - gt$

$\because \quad \vec{v} = v_x\hat{i} + v_y\hat{j} \quad \Rightarrow \quad \vec{v}_{(t)} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$

The direction of $\vec{v}$ with the x axis is given by $\tan^{-1}\left(\frac{v_y}{v_x}\right)$

Co-ordinates of the projectile after time t is given by

$$x = x_o + u_x t + \frac{1}{2}a_x t^2 \quad \Rightarrow \quad x = 0 + u\cos\theta\,.t + 0$$

$$\Rightarrow \quad x = u\cos\theta\, t \qquad \ldots (1)$$

and $\quad y = y_o + u_y t + \frac{1}{2}a_y t^2$

$$\Rightarrow \quad y = 0 + u\sin\theta\, t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad y = u\sin\theta\, t - \frac{1}{2}gt^2 \qquad \ldots (2)$$

Eliminating 't' from Eqs. (1) and (2), we get

$$y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$$

$$\Rightarrow \quad y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \qquad \ldots (3)$$

The above equation shows the relation between x and y and represents the path of the projectile known as trajectory. The inspection of eq. (3) shows that it is the equation of parabola of the form

$$y = bx + cx^2$$

where $b = \tan\theta$ = constant, and $c = -\frac{g}{2u^2\cos^2\theta}$ = constant

Time of flight: It is time interval during which the projectile remains in air.
Putting y = 0 in (2), we get

$$T = \frac{2u\sin\theta}{g},$$ where T = time of flight.

Range: The horizontal range R of the projectile is the horizontal distance between the initial point and the point where the projectile is again at same horizontal level.

If R be the horizontal range then $R = u\cos\theta \times \frac{2u\sin\theta.}{g} = \frac{u^2\sin 2\theta}{g}$

Note (i): Since $\sin 2\theta = \sin(\pi - 2\theta) = \sin 2\left(\frac{\pi}{2} - \theta\right)$

Let $(\pi/2 - \theta) = \beta \quad \Rightarrow \sin 2\theta = \sin 2\beta$

Hence, range is same for two angles of projection provided angles are complimentary.

Note (ii): For a given velocity of projection R is maximum for $\theta = 45^o$.

$$\Rightarrow \quad R_{max} = \frac{u^2}{g}$$

Maximum height: The maximum height attained by the projectile is given by

$$\because \quad v_y^2 = u_y^2 + 2a_y y \quad \text{at} \quad y = y_{max},\ v_y = 0$$

$$\Rightarrow \quad 0 = u^2\sin^2\theta - 2\,g\,y_{max} \quad \Rightarrow \quad y_{max} = \frac{u^2\sin^2\theta}{2g}$$

Exercise 10:

(i) ***A projectile is thrown horizontally from the top of a tower and strikes the ground after 3 second at an angle of 45° with the horizontal. Find the height of the tower and speed with which the body was projected. Given g = 9.8 m/s^2***

(ii) ***A ball is thrown with an initial velocity of 100 m/s at an angle of 30° above the horizontal. How far from the throwing point will the ball attain its original level? Solve the problem without using formula for horizontal range.***

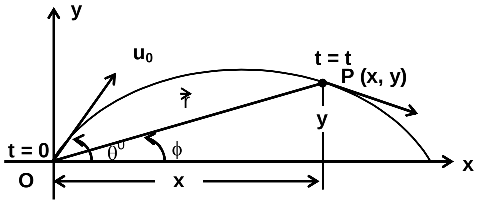
(iii) ***A bullet P is fired from a gun when the angle of elevation of the gun is 30°, another bullet Q is fired from the gun when the angle of elevation is 60° which of the two bullets would have a greater horizontal range and why ?***

(iv) ***Name the two quantities which would be reduced if air resistance is taken into account in the study of motion of oblique projectile.***

Illustration 22. *A boy throws a stone with an speed V_0 = 10 m/sec at an angle $\theta_0 = 30^o$ to the horizontal. Find the position of the stone w.r.t. the point of projection just after a time t = 1/2 sec.*

***Solution*:** The position of the stone is given by

$\vec{r} = xi + yj$



where $x = (v_0 \cos \theta_0)t$

$= (10\ \cos 30)\left(\frac{1}{2}\right) = 4.33$ m.

and $y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$

$= (10\ \sin 30)\left(\frac{1}{2}\right) - \frac{1}{2}\times 10\times\left(\frac{1}{2}\right)^2 = 1.25$ m

$\Rightarrow \vec{r} = (4.33i\ +\ 1.25j)$ m.

Illustration 23. *A particle is projected with velocity v_o = 100 m/s at an angle $\theta = 30^o$ with the horizontal. Find*

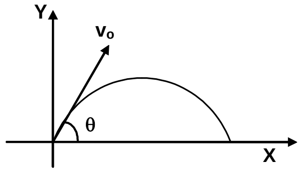
(a) velocity of the particle after 2 s.

(b) angle between initial velocity and the velocity after 2s

(c) the maximum height reached by the projectile

(d) horizontal range of the projectile.

(e) the total time of flight



Solution: (a) $\vec{v}_{(t)} = \vec{v}_{x(t)}\hat{i} + \vec{v}_{y(t)}\hat{j}$

where $\hat{i}$ and $\hat{j}$ are the unit vectors along +ve x and +ve y axis, respectively.

$\Rightarrow\ \vec{v}_{(t)} = (u_x + a_x t)\hat{i} + (u_y + a_y t)\hat{j}$

Here, $u_x = v_o \cos\theta = 50\sqrt{3}$ m/s

$a_x = 0$

$u_y = v_o \sin\theta = 50$m/s

$a_y = -g$ ($\because$ g acts downward)

$\Rightarrow\ \vec{v}_{(t)} = v_o \cos\theta\hat{i} + (v_o \sin\theta - gt)\hat{j}$

$\vec{v}_{(2)} = 50\sqrt{3}\hat{i} + (50 - 10\times 2)\hat{j} = 50\sqrt{3}\hat{i} + 30\hat{j}$ m/s

$\Rightarrow\ |\vec{v}_2| = \sqrt{v_x^2 + v_y^2} = 91.65$ m/s

(b) $\vec{v}_o = 50\sqrt{3}\hat{i} + 50\hat{j}$, and

$\vec{v}_{(t=2\sec)} = 50\sqrt{3}\hat{i} + 30\hat{j}$

$\Rightarrow\ \vec{v}_o.\vec{v}_{(2)} = 7500 + 1500 = 9000$

If α is the angle between $\vec{v}_o$ and $\vec{v}_{(2s)}$.

Then, $\cos\alpha = \frac{\vec{v}_o.\vec{v}_{(2s)}}{|\vec{v}_o|\times|\vec{v}_{(2s)}|} = \frac{9000}{1000\times 91.65}$

or $\alpha = \cos^{-1}(0.98) = 10.8^o$.

(c) $v_y^2 - u_y^2 = 2a_y y$

At $y = y_{max}$, $v_y = 0$

$\Rightarrow\ 0 - v_0^2 \sin^2\theta = 2(-g)y_{max}$

$\Rightarrow\ y_{max} = \dfrac{v_0^2 \sin^2\theta}{2g} = 125\text{m}$

(d) $R = \dfrac{u^2 \sin 2\theta}{g} = 866$ m

(e) $T = \dfrac{2v_0 \sin\theta}{g} = 50$ sec.

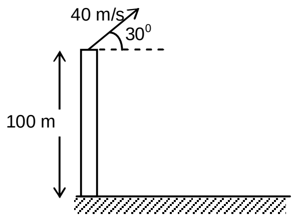
Illustration 24. *A ball is thrown at a speed of 50 m/s at an angle of 60° with the horizontal. Find*
(a) the maximum height reached, and.
(b) the range of ball. (Take g = 10 m/s²)

Solution: (a) Maximum height, $H = \dfrac{u^2 \sin^2\theta}{2g}$

$$= \frac{(50)^2}{2\times 10}\times\left(\frac{\sqrt{3}}{2}\right)^2 \text{ m } = 93.75 \text{ m}$$

(b) Range, $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{(50)^2 \times \sin 120}{10} = 216.5$ m

Illustration 25. *A stone is projected with a speed of 40 m/s at an angle of 30° with the horizontal from a tower of height 100 m above ground. Find*
(a) the maximum height attained by the stone, and
(b) the horizontal distance from the tower where it hits the ground.



Solution: (a) Maximum height above the tower, using $v^2 = u^2 + 2as$ in vertical direction.

$(u \sin 30°)^2 = 2gh$ As $u = 40$ m/s, $\theta = 30°$

$\dfrac{40\times 40\times 1}{4} = 2 \times 10 \times h \quad \Rightarrow h = \dfrac{1600}{80} = 20\text{m}$

$\therefore$ Height above ground $= 100 + 20 = 120$m.

(b) Range, time of flight = t, $H = u \sin\theta t - \dfrac{1}{2}gt^2$, $H = -100$ m,

$-100 = (40 \times \frac{1}{2})t - \frac{1}{2}\times 10\times t^2$

$-100 = 20t - 5t^2$

$t^2 - 4t - 20 = 0$, $t = 6.9$ sec.

$R = u\cos\theta \times t$, $R \rightarrow$ distance from tower

$R = 40 \times \dfrac{\sqrt{3}}{2}\times 6.9 = 238.9$ m.

Illustration 26. *The position of a particle at time t = 0 is P = (–1, 2, –1). It starts moving with an initial velocity* $\vec{u} = 3\hat{i} + 4\hat{j}$ *and with uniform acceleration* $-4\hat{i} + 4\hat{j}$. *Find the final position and the magnitude of displacement after 4 sec.*

Solution: Initial position vector of the particle $= \left(-\hat{i} + 2\hat{j} - \hat{k}\right)$

Final position of the particle after 4 seconds

$$S_f = S_i + \vec{u}t + \frac{1}{2}\vec{a}t^2$$

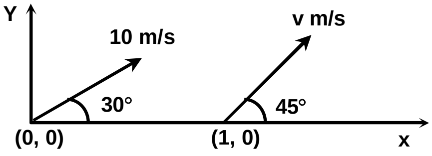
$$= \left(-\hat{i} + 2\hat{j} - \hat{k}\right) + \left(3\hat{i} + 4\hat{j}\right) \times 4 + \frac{1}{2} \times \left(-4\hat{i} + 3\hat{j}\right) \times 16$$

Final position $= -21\hat{i} + 42\hat{j} - \hat{k}$,

Displacement $= -20\hat{i} + 40\hat{j}$.

Magnitude of displacement = $\sqrt{(20)^2 + (40)^2} = 20\sqrt{5}$ m

Illustration 27. *Two particles projected vertically upward from points (0, 0) and (1, 0) with uniform velocity 10 m/s and v m/s, respectively, as shown in the figure. It is found that they collide after time t in space. Find v and t.*



Solution:

$x_1 = 10\cos 30°t$

$x_2 = v\cos 45°t$

$y_1 = 10\sin 30°t - \frac{1}{2}gt^2$

$y_2 = v\sin 45°t - \frac{1}{2}gt^2$

For collision:

$y_1 = y_2$

$10 \times \frac{1}{2} = \frac{v}{\sqrt{2}}$

$\Rightarrow v = 5\sqrt{2}$ m/s

$\Rightarrow x_1 = x_2 + 1$

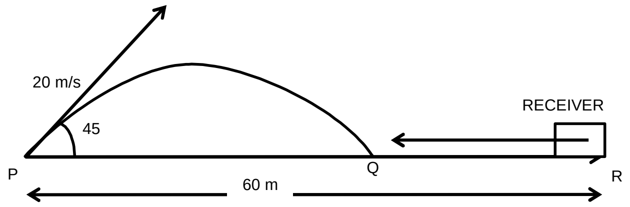
$10\cos 30°t = 5\sqrt{2}\cos 45°t + 1$

$t\left(5\sqrt{3} - 5\right) = 1$

and $t = \dfrac{1}{5\left(\sqrt{3} - 1\right)}$ sec

Illustration 28. *A football is kicked off with an initial speed of 20m/s at an angle of projection of 45°. A receiver on the goal line at a distance of 60 m away in the direction of the kick starts running to meet the ball at that instant. What must be his speed if he is to catch the ball before it hits the ground? [Take g = 10m/s²]*

Solution:



Let u = 20 m/s, $\theta = 45^\circ$ and v = speed of the receiver.
The ball is projected from Pand the receiver starts running from R to receive the ball at Q.
Let t be the time after which they meet.
So t is the time taken by the ball to go from P to Qin which the receiver goes from R to Q.

$\therefore PQ = \frac{u^2}{g}\sin 2\theta$ and QR = vt

$PR = 60 \Rightarrow \frac{u^2}{g}\sin 2\theta + vt = 60 \ldots(i)$

Putting the value of t (i.e. the time of flight) = $\frac{2u\sin\theta}{g}$

in equation (I) we get,

$$\frac{u^2}{g}\sin 2\theta + v\left(\frac{2u\sin\theta}{g}\right) = 60$$

$$\Rightarrow \quad v = \frac{60g - u^2\sin 2\theta}{2u \ \sin\theta}$$

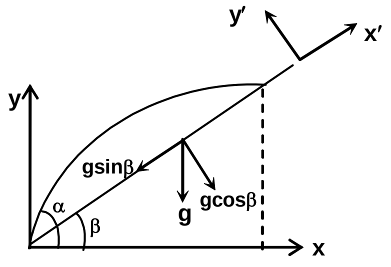
$$= \frac{600-400}{2(20)}\sqrt{2}$$

$$= 5\sqrt{2}\ \text{m/s}$$

The Projectile on an Inclined Plane

In case the projection is from an inclined plane, we consider two axes x′ and y′, along and perpendicular to the inclined plane.

Motion up the plane



In x′-y′ plane,

$u_{x'} = v_0\cos(\alpha - \beta)$, $u_{y'} = v_0\sin(\alpha - \beta)$

$a_{x'} = -g\sin\beta$, $a_{y'} = -g\cos\beta$

Since $y' = v_o\sin(\alpha - \beta)t - \frac{1}{2}g\cos\beta t^2$

at t = T, y′ = 0, where T = time of flight.

$\Rightarrow \quad T = \frac{2v_o\sin(\alpha-\beta)}{g\cos\beta}$ again $x = (v_0\cos\alpha).T$

$x = v_o\cos\alpha\ \frac{2v_o\sin(\alpha-\beta)}{g\cos\beta}$

So range along inclined plane (R) = x′ = $x/\cos\beta$

$\therefore \quad x' = \frac{2v_0^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$ [Apply formula 2cosA sin B = sin(A + B) – sin(A–B)]

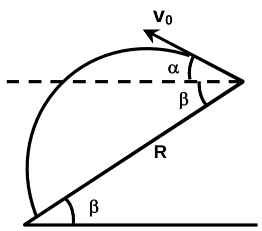
$x' = R = \frac{v_0^2\left[\sin(2\alpha-\beta)-\sin\beta\right]}{g\cos^2\beta}$

Now, R will be maximum when $\sin(2\alpha - \beta)$ is maximum, i.e. $\sin(2\alpha - \beta) = 1$.

$$\Rightarrow \quad R_{max} = \frac{v_0^2[1-\sin\beta]}{g(1-\sin^2\beta)} \Rightarrow \quad R_{max} = \frac{v_0^2}{g(1+\sin\beta)} \text{ up the plane}$$

Motion down the plane

Let the particle be thrown at a velocity v_0 at an angle 'α' with the horizontal as shown in figure.



$$V_0 \sin(\alpha+\beta)T - \frac{1}{2}\cos\beta T^2 = 0 \qquad [\text{for } y' = 0]$$

$$\Rightarrow \quad T = \frac{2v_o\sin(\alpha+\beta)}{g\cos\beta}$$

$$R = v_o\cos(\alpha+\beta)T + \frac{1}{2}g\sin\beta T^2 = \frac{v_0^2}{g}\left[\frac{\sin(2\alpha+\beta)+\sin\beta}{1-\sin^2\beta}\right]$$

For R to be maximum;

$$\sin(2\alpha+\beta) = 1$$

and $$R_{max} = \frac{v_0^2}{g}\left[\frac{1+\sin\beta}{1-\sin^2\beta}\right]$$

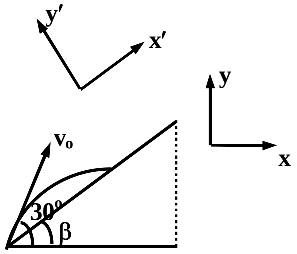
$$= \frac{v_0^2}{g(1-\sin\beta)} \text{ down the plane.}$$

Illustration 29. *Name a quantity which remains unchanged during the flight of projectile on an inclined plane.*

Solution: Horizontal component of velocity.

Illustration 30. *From the foot of an inclined plane, whose rise is 7 in 25, a shot is projected with a velocity of 196 m/s at an angle of 30^o with the horizontal (a) up the plane (b) down the plane. Find the range in each case.*

Solution : Let β be the inclination of the plane.



Hence, $\sin\beta = \frac{7}{25}$, and $\cos\beta = \frac{24}{25}$

(a) $v_{ox'} = v_o\cos(30^o - \beta)$

and $a_x' = -g\sin\beta$

$v_{oy'} = v_o\sin(30^o - \beta)$

and $a_y' = -g\cos\beta$

$\because \quad y' = v_{oy}'t + \frac{1}{2}a_y't^2$

If T = time of flight, then at $t = T$, $y' = 0$

$$\Rightarrow \quad 0 = v_o\sin(30^o - \beta)T - \frac{1}{2}g\cos\beta T^2$$

$$\Rightarrow \quad T = \frac{2v_o\sin(30^o-\beta)}{g\cos\beta}$$

If R_1 be the range then $R_1\cos\beta = x = v_o\cos30^o.\,T$

$$\Rightarrow \quad R_1\cos\beta = v_o\cos30.\,\frac{2v_o\sin(30°-\beta)}{g\cos\beta}$$

$$\Rightarrow \quad R_1 = \frac{2v_0^2 \cos 30^o \sin(30-\beta)}{g\cos^2\beta}$$

Solving we get, R_1 = 1749.8 m

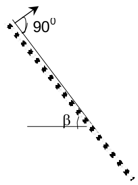
(b) For down the plane,

$$T = \frac{2v_o \sin(30^o + \beta)}{g\cos\beta}$$

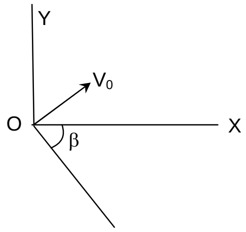
Hence $\quad R_2 \cos\beta = v_o \cos 30^o .T$

$$\Rightarrow \quad R_2 = \frac{2v_0^2 \cos 30^o \sin(30^o+\beta)}{g\cos^2\beta}.$$

Illustration 31. *A projectile is launched from an inclined plane with an initial velocity v_0 as shown in the figure. Find the time after which the projectile hits the plane for the first time.*



Solution: Let the projectile hit the plane after time t.



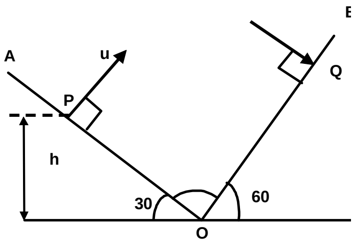
The horizontal displacement x = (v_0 sin β) t

The vertical displacement y = (v_0 cos β) t $-\frac{1}{2}gt^2$

y = –(tan β)x for the plane

$$\therefore \quad t = \frac{2v_0}{g\cos\beta}$$

Illustration 32. *Two inclined planes of inclinations 30° and 60°, respectively, meet at 90° as shown in figure. A particle is projected from point P on the first inclined plane with a velocity u = 10$\sqrt{3}$ m/s in a direction perpendicular to the inclined plane and it is observed to hit the other inclined plane at 90°.*
Find (a) the height of point P from ground,
(b) the length of $\overline{PQ}$.



Solution: (a) We observe the motion of projectile fixing y-axis with OP and x-axis with OQ. Hence, velocity at any instant t along x-axis:

$v_x = 10\sqrt{3} - (g \sin 60°)t$

$v_y = 0 - (g \cos 60°)t$

As v_x = 0 at the time of hitting,

Time of flight = T = 2 sec.

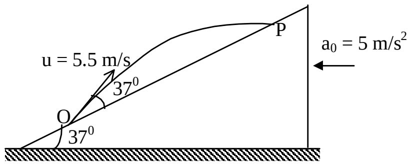
Displacement OP during this time = $\frac{1}{2}(g \cos 60°)t^2 = \frac{1}{2} \times 10 \times \frac{1}{2} \times 4$ = 10 m

Hence, h = OP sin 30° = 10 × $\frac{1}{2}$ = 5m

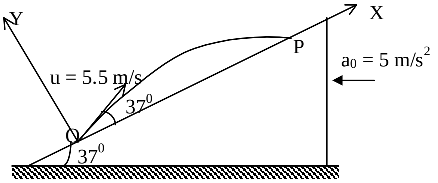
(b) Similarly, displacement OQ = $(10\sqrt{3})(2) - \frac{1}{2} \times 10 \times \frac{\sqrt{3}}{2} \times 4 = 10\sqrt{3}$ m

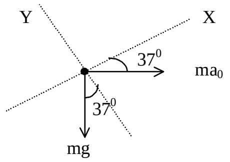
Hence, PQ = $\sqrt{OP^2 + OQ^2}$ = 20 m.

Illustration 33. *A particle is projected up a large inclined plane from a point O on it as shown in the figure. The projection velocity has a magnitude of 5.5 m/s and its direction makes an angle of 37° with the inclined plane. The inclination of the plane is also 37°. The inclined plane starts moving towards left with an acceleration $a_0 = 5$ m/s^2 at the moment the particle is projected. The particle strikes the inclined plane at a point P. Find the time taken by the particle to move from O to P. Also find the magnitude of displacement along the inclined plane as it moves from O to P. (Take sin 37° = 3/5)*



Solution: Let us take x and y axes as shown in the figure. The magnitude of pseudo force acting on the particle has a magnitude of ma_0 and its direction will be towards right as shown in the free body diagram.





F.B.D. of the particle

The components of the acceleration of the particle are

$$a_x = \frac{ma_o \cos 37^\circ - mg \sin 37^\circ}{m}$$

$$= 5 \times \frac{4}{5} - 10 \times \frac{3}{5} = -2 m/s^2$$

$$a_y = \frac{-(mg\cos 37^o + ma_o \sin 37^o)}{m}$$

$$= -\left(10 \times \frac{4}{5} + 5 \times \frac{3}{5}\right) = -11 m/s^2$$

$u_x = u\cos 37° = 5.5 \times \frac{4}{5} = 4.4$ m/s.

$u_y = u\sin 37° = 5.5 \times \frac{3}{5} = 3.3$ m/s

Displacement of the particle along y – axis

$y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow y = 3.3t - \frac{1}{2} \times 11t^2$

When the particle strikes the plane y = 0

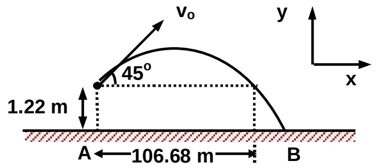
$\Rightarrow t = \frac{2u_y}{-a_y} = \frac{2 \times 3.3}{11}$ = 0.6 sec.

OP = x = $u_x t + \frac{1}{2}a_x t^2$

$= 4.4 \times 0.6 - \frac{1}{2} \times 2 \times (0.6)^2 = 2.28$ m.

Illustration 34. *A batsman hits a ball at a height of 1.22 m above the ground so that ball leaves the bat at an angle of 45° with the horizontal. A 7.31 m high wall is situated at a distance of 97.53 m from the position of the batsman. Will the ball clear the wall if its range is 106.68m? Take g = 10 m/s².*

Solution : $R(\text{range}) = \dfrac{v_0^2 \sin 2\theta}{g}$



$\Rightarrow \quad v_0^2 = \dfrac{Rg}{\sin 2\theta} = Rg \quad \text{as} \quad \theta = 45^o.$

$\Rightarrow \quad v_o = \sqrt{Rg} \qquad \ldots (1)$

Equation of trajectory

$$y = x \tan 45^o - \frac{gx^2}{2v_0^2 \cos^2 45^o} = x - \frac{gx^2}{2Rg\frac{1}{2}} \quad [\text{using (1)}]$$

Putting x = 97.53, we get

$$y = 97.53 - \frac{10 \times (97.53)^2}{106.68 \times 10} = 8.35$$

Hence, height of the ball from the ground level is

h= 8.35 + 1.22 = 9.577m.

As height of the wall is 7.31m, so the ball will clear the wall.

Illustration 35. *A particle is projected with velocity u and angle θ with the horizontal. Find the time after which the velocity will be perpendicular to the initial velocity.*

Solution : $\bar{v} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$

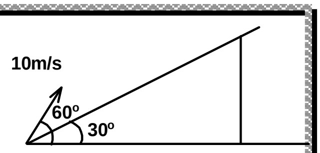
$\bar{u} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$

$\bar{u} \cdot \bar{v} = 0 = u^2\cos^2\theta + u^2\sin^2\theta - (u\sin\theta)gt$

$\therefore \; t = \dfrac{u}{g\sin\theta}$

Exercise 11:

(i) ***A particle is thrown at time t = 0, with a velocity of 10 m/s at an angle of 60° with the horizontal, from a point on an incline plane, making an angle of 30° with the horizontal. The time when the velocity of the projectile becomes parallel to the incline is***



(A) $\frac{2}{\sqrt{3}}$ sec (B) $\frac{1}{\sqrt{3}}$ sec

(C) $\sqrt{3}$ sec (D) $\frac{1}{2\sqrt{3}}$ sec

(ii) ***An object projected with the same speed at two different angles covers the same horizontal range R. If the two times of flight be t_1 and t_2, prove that R = $\frac{1}{2}gt_1t_2$.***

(iii). ***Is it important in the long jump that how much height you take for jumping?***

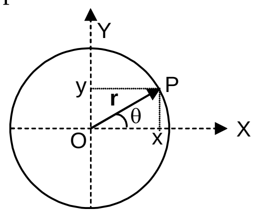
UNIFORM CIRCULAR MOTION

As another small illustration of motion of a particle in two dimensions let's analyse the uniform circular motion of a particle.

In uniform circular motion, the particle moves in a circular path with constant speed.

Let's choose the centre of the circular path as the origin of the reference frame. Point 'P' is an arbitrary point on the path whose position vector

$\vec{r} = x\hat{i} + y\hat{j}$.



where r, the radius of the circular path, is related to x and y by following equations

$x = r\cos\theta,\ \ y = r\sin\theta$ and $x^2 + y^2 = r^2$

$\Rightarrow \quad \vec{r} = r\cos\theta\hat{i} + r\sin\theta\hat{j}$

Now, the velocity of particle 'P' is given as

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \frac{d(r\cos\theta)}{dt}\hat{i} + \frac{d(r\sin\theta)}{dt}\hat{j}$$

$$\Rightarrow \quad \vec{v} = -r\sin\theta\frac{d\theta}{dt}\hat{i} + r\cos\theta\frac{d\theta}{dt}\hat{j}$$

but $\frac{d\theta}{dt} = \omega = \text{const.}$ [for uniform circular motion]

Thus, $\vec{v} = \omega r(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

Now, $\vec{v}\cdot\vec{r} = \omega r(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})(r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}) = \omega r^2(-\sin\theta\cos\theta + \cos\theta\sin\theta) = 0$

$\Rightarrow \quad \vec{v}$ is perpendicular to $\vec{r}$

$|\vec{v}| = \omega r\sqrt{\sin^2\theta + \cos^2\theta} = \omega r$

Now, acceleration $\vec{a}$ is given as

$$\vec{a} = \frac{d\vec{v}}{dt} = \omega r\left(-\cos\theta.\frac{d\theta}{dt}\hat{i} - \sin\theta\frac{d\theta}{dt}\hat{j}\right) \quad \Rightarrow \quad \vec{a} = -\omega^2 r\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$\Rightarrow \quad \vec{a} = \omega^2(-\vec{r})$$

which shows that $\vec{a}$ is directed in the opposite direction of '$\vec{r}$'. Thus, $\vec{a}$ is always directed towards the centre.

Magnitude of $\vec{a}$, $|\vec{a}| = \omega^2 r\sqrt{\cos^2\theta + \sin^2\theta} = \omega^2 r$

Note: If the circular motion is non-uniform, then tangential acceleration $a_t = dv/dt$ exists apart from normal acceleration $\omega^2 r$.

Illustration 36. *Does velocity remain constant in uniform circular motion ?*

Solution: No, magnitude remains constant but direction keep on changing.

Illustration 37. *Find the magnitude of average acceleration of the tip of the second hand of length 10 cm during 10 seconds.*

Solution: Average acceleration has the magnitude

$a = \Delta v/\Delta t$, where $\Delta v = 2v\sin\theta/2$

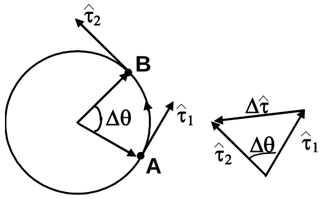
$$\Rightarrow \quad a = \frac{2V\sin\theta/2}{\Delta t}$$

Putting v = $\pi/300$ m/sec (obtained earlier), Δt = 10 seconds and $\theta = 60°$, we obtain

$$a = \frac{2(\pi/300)\sin 30°}{10}$$

$$\Rightarrow \quad a = \frac{\pi}{3000} \text{ m/sec}^2.$$

CIRCULAR MOTION



$|\vec{v}| = r\omega$ (variable)

Let $\hat{\tau}$ = unit vector along the tangent. and

$\hat{u}$ = unit vector along radius (outwards)

Since the velocity of the particle describing circular motion is along the tangent, hence it can be given by the expression.

$\vec{v} = v\hat{\tau}$ where v = magnitude of the velocity

$$\Rightarrow \quad \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{\tau} + v\frac{d\hat{\tau}}{dt}$$

Take A and B two positions of the particle.

Change in $\hat{\tau} = \Delta\hat{\tau} = \Delta\theta(-\hat{u})$

Negative sign shows that it is towards the centre as (S = R□)

$$\Rightarrow \quad \frac{d\hat{\tau}}{dt} = \frac{d\theta}{dt}(-\hat{u}) \qquad \Rightarrow \quad \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{\tau} + v\frac{d\theta}{dt}(-\hat{u})$$

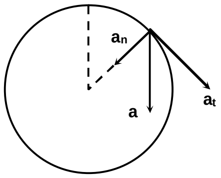
$$\Rightarrow \quad \vec{a} = \vec{a}_{tangential} + \vec{a}_{radial}, \qquad \text{where} \quad \vec{a}_{tangential} = \frac{dv}{dt} = r\frac{d\omega}{dt}$$

If 'v' is a constant then $\vec{a}_{tangential} = 0$ and $\vec{a}_{radial} = v\frac{d\theta}{dt} = v\omega = \frac{v^2}{r} = \omega^2 r$

$$|\vec{a}| = \sqrt{a_r^2 + a_T^2}$$

Illustration 38. *A point moves along a circle with velocity v = at where a = 0.5m/s^2. Find the total acceleration of the point at the moment when it covered (1/10)th of the circle after beginning of motion.*

Solution :



We know $S = ut + \frac{1}{2}at^2$

Here, $S = \frac{2\pi r}{10} = \frac{\pi r}{5}$, $a_t = 0.5$ m/s^2 and u = 0

$$\therefore \quad \frac{\pi r}{5} = 0 + \frac{1}{2}0.5t^2, \; t = \sqrt{\frac{4\pi r}{5}}$$

$$\therefore \quad v = at = 0.5\sqrt{\frac{4\pi r}{5}} = \sqrt{\frac{\pi r}{5}}$$

$$\therefore \quad a_n = \frac{v^2}{r} = \frac{\pi r}{5}\frac{1}{r} = \frac{\pi}{5}$$

$$\therefore \quad a = \sqrt{a_n^2 + a_t^2} = \sqrt{\left(\frac{\pi}{5}\right)^2 + 0.5^2} = \sqrt{\frac{\pi^2}{25} + \frac{1}{4}} = 0.8 \text{ m/s}^2$$

Exercise 12:

(i) ***By using vector method, show that direction of acceleration vector $\vec{a}$ is towards the centre of the circle in which body is revolving.***

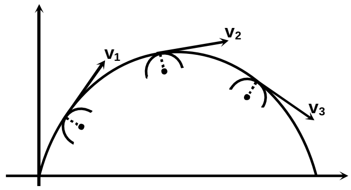
(ii) ***What is the direction of velocity vector of a particle in circular motion ?***

(iii) ***What is the acceleration associated with a body having variable speed in a circular path ?***

(iv) ***Two cars having masses m_1 and m_2 move in circles of radii r_1 and r_2, respectively. If they complete the circles in equal time, find the ratio of the their angular speeds $\frac{\omega_1}{\omega_2}$.***

RADIUS OF CURVATURE

In a curvilinear motion, every small path may be assumed to be an arc of a circular path, and here the radius of curvature will be different at different points. So if a particle moves on a curved path then radius of curvature is given by $R = \frac{v^2}{a_R}$.



where v = instantaneous velocity at any time at that point and a_R = acceleration acting normal to the path towards the centre.

Illustration 39. *Where is radius of curvature maximum at the highest point or at the point of projection?*

Solution: At the point of projection

Illustration 40. *Find the ratio of radius of curvature at the highest point of projectile to that just after its projection if the angle of projection is 30°.*

Solution : If $\vec{v}_0$ is the initial velocity

$v_p = v_0 \cos\theta$

Normal acceleration at $0 = g\cos\theta$

Normal acceleration at P = g

Hence, if r_0 and r_p be radii of curvature at O and P, respectively.

$$r_0 = \frac{v_0^2}{g\cos\theta} \quad \text{and} \quad r_p = \frac{v_0^2\cos^2\theta}{g}$$

Hence, the required ratio = $\frac{r_p}{r_0} = \cos^3\theta = \frac{3\sqrt{3}}{8}$.

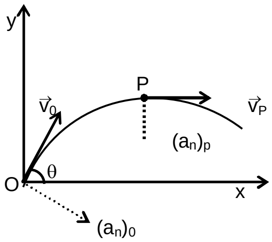
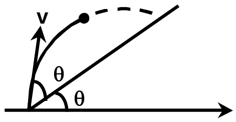


Illustration 41. *A particle is projected with a velocity u at an angle θ with an inclined plane which makes an angle θ < 45° with the horizontal. Calculate the radius of curvature of the path of projectile when velocity of projectile becomes parallel to the plane.*



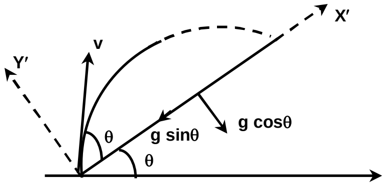
Solution : Along y′,

$$u\sin\theta - g\cos\theta t = 0$$

$$\Rightarrow \quad t = \frac{u}{g}\tan\theta$$

Along x′,

$$v'_x = u\cos\theta - g\sin\theta . t = u\cos\theta - g\sin\theta . \frac{u}{g}\tan\theta$$



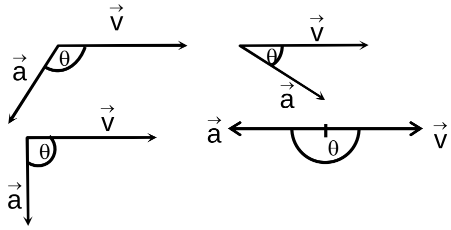
$$= \frac{u\cos^2\theta - u\sin^2\theta}{\cos\theta} = \frac{u\cos 2\theta}{\cos\theta}$$

$$r = \frac{(v_x')^2}{a_n} = \frac{u^2\cos^2 2\theta}{g\cos^3\theta}$$

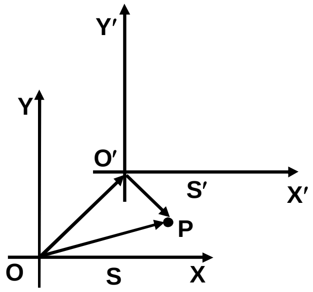
Exercise 13:

i) ***The tangential acceleration change the speed of the particle whereas the normal acceleration changes its direction. State whether the statement true or false?***

ii) ***At a certain moment, the angle between velocity vector $\vec{v}$ and the acceleration vector $\vec{a}$ of the particle is θ. What will be the motion of the particle at this moment for different θ's: rectilinear or curvilinear, accelerated or decelerated?***



RELATIVE VELOCITY



The position, velocity and acceleration of a particle depend on the reference frame chosen.

A particle P is moving and is observed from two frames 'S' and 'S′'. The frame S is stationary and the frame S′ is in motion.

Let at any time position vector of the particle P with respect to S is

$\overrightarrow{OP} = \vec{r}_{p,s}$ and with respect to S′ is $\overrightarrow{O'P} = \vec{r}_{p,s'}$

Position vector of the origin of S′ with respect to origin of S is

$\overrightarrow{OO'} = \vec{r}_{s',s}$

From vector triangle OO′P, we get

$\overrightarrow{O'P} = \overrightarrow{OP} - \overrightarrow{OO'} \Rightarrow \vec{r}_{p,s'} = \vec{r}_{p,s} - \vec{r}_{s',s}$

$\Rightarrow \quad \frac{d}{dt}\left(\vec{r}_{p,s'}\right) = \frac{d}{dt}\left(\vec{r}_{p,s}\right) - \frac{d}{dt}\left(\vec{r}_{s',s}\right)$

$\Rightarrow \quad \vec{v}_{p,s'} = \vec{v}_{p,s} - \vec{v}_{s',s}$

$\Rightarrow \quad \vec{v}_{p,s'} = \vec{v}_{p(\text{absolute})} - \vec{v}_{s'\,(\text{absolute})}$

If $\vec{v}_r$ and $\vec{v}_m$ are the absolute velocities of the rain and the man, respectively, then the relative velocity of rain w.r.t. (as seen by) the man is $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$.

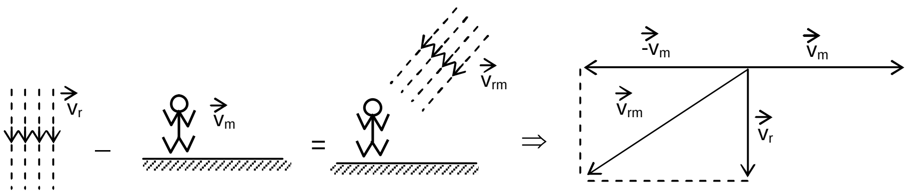


Illustration 42. *Two men are moving with same velocity in the same direction. What is there relative velocity?*

Solution: zero

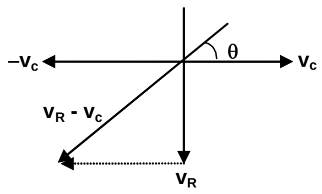
Illustration 43. *A stationary person observes that rain is falling vertically down at 30 km/hr. A cyclist is moving on the level road at 10 km/hr. In which direction the cyclist should hold his umbrella to prevent himself from rain?*

Solution: Relative to stationary frame, velocity of rain is 30 km/hr downward. Take horizontal axis as x-axis and vertical axis as y-axis and $\hat{i}, \hat{j}$ are the unit vectors along X- and Y-axes, respectively.

$\vec{v}_R = 0 - 30\hat{j}$, $\vec{v}_c = 10\hat{i}$

$\because$ $\vec{v}_{R,c} = \vec{v}_R - \vec{v}_c$

$= -30\hat{j} - 10\hat{i} = -10\hat{i} - 30\hat{j}$



If angle between horizontal and the $\vec{v}_{R,c}$ is θ, then

$$\tan\theta = \frac{-30}{-10} = 3$$

$\Rightarrow$ $\theta = \tan^{-1}3$ $\Rightarrow$ $\theta = 72^o$.

Therefore, to prevent himself from rain the cyclist should hold the umbrella at angle of 72^o from horizontal.

Illustration 44. *A man walking eastward at 5 m/s observes that wind is blowing from the north. On doubling his speed eastward, he observes that wind is blowing from north-east. Find the velocity of the wind.*

Solution: Let velocity of the wind be

$\vec{v}_w = v_1\hat{i} + v_2\hat{j}$ m/s

And velocity of the man is

$\vec{v}_m = 5\hat{i}$

$\therefore$ $\vec{v}_{wm} = \vec{v}_w - \vec{v}_m = (v_1 - 5)\hat{i} + v_2\hat{j}$

In first case,

$v_1 - 5 = 0$

$\Rightarrow$ $v_1 = 5$ m/s.

In the second case, $\tan 45° = \dfrac{v_2}{v_1 - 10}$

$\Rightarrow$ $v_2 = v_1 - 10 = -5$ m/s.

$\Rightarrow$ $\vec{v}_w = 5\hat{i} - 5\hat{j}$ m/s.

Illustration 45. *From a lift moving upward with a uniform acceleration 'a', a man throws a ball vertically upwards with a velocity v relative to the lift. The ball comes back to the man after a time t. Show that a + g = 2 v/t.*

Solution: Let us consider all the motion from lift frame. Then, the acceleration, displacement and velocity everything will be considered from the lift frame itself.

As the ball comes again to the man, therefore displacement from the lift frame is zero.

Again, the velocity with respect to the lift frame is v.

Similarly, the acceleration with respect the lift frame is

$g - (-a) = a + g$ (downwards)

Now, $s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad 0 = vt - \frac{1}{2}(a + g)t^2$

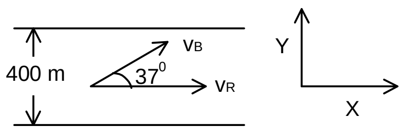
or $\quad a + g = 2\frac{v}{t}$.

Illustration 46. *A river 400 m wide is flowing at a rate of 4 m/s. A boat is sailing at a velocity of 20 m/s with respect to the still water in a direction making an angle of 37° with the direction of river flow.*

(a) Find time taken by the boat to reach the opposite bank.

(b) How far from the starting point does the boat reach on the opposite bank?

Solution: (a) Resultant velocity of the boat is



$v = (v_R + v_B \cos 37^\circ)\, i + v_B \sin 37^\circ\, j$

$= 4i + 20 \times \frac{4}{5} i + 20 \times \frac{3}{5} j$

$= 20\, i + 12\, j$ m/s

Time taken by boat to cross the river

$= \frac{\text{distance travelled in y-direction}}{\text{velocity in y-direction}} = \frac{400}{12} = \frac{100}{3}$ sec.

(b) Displacement along x $= v\,t$

$= 20 \times \frac{100}{3} = \frac{2000}{3}$ m

Distance from starting point $= \sqrt{(400)^2 + \left(\frac{2000}{3}\right)^2} = \frac{400}{3}\sqrt{34}$ m.

Illustration 47. *A stone is projected from a balloon which is ascending with a velocity 2 m/s. The velocity of the stone w.r.t. balloon is $\sqrt{2}$ m/s at an angle of 45°. Find the velocity of the stone with respect to ground.*

Solution : $\vec{v}_{SB} = v\cos 45^0\,\hat{i} + v\sin 45^0\,\hat{j}$

$= \sqrt{2} \times \frac{1}{\sqrt{2}}\hat{i} + \sqrt{2} \times \frac{1}{\sqrt{2}}\hat{j} = (\hat{i} + \hat{j})\,m/s$

$\vec{v}_{BG} = 2\hat{j}\,m/s$

Thus, $\vec{v}_{S,G} = \vec{v}_{SB} + \vec{v}_{BG}$

$= 2\hat{j} + (\hat{i} + \hat{j}) = ((\hat{i} + 3\hat{j})$

$v = \sqrt{1^2 + 3^2} = \sqrt{10}$ m/s and $\tan\theta = \dfrac{3}{1}$

$\theta = \tan^{-1}(3)$.

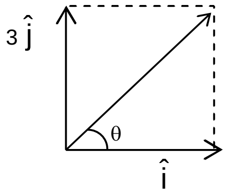
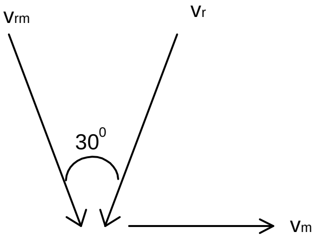


Illustration 48. *A man standing on a road has to hold his umberella at 30° with the vertical to keep the rain away. He throws the umbrella and runs at 10 kmph. He finds that rain drops are hitting his head vertically. find the speed of raindrops with respect to*
(a) the road
(b) the moving man.



Solution : Velocity ofrain w.r.t. road is $\vec{v}_r$ and velocity of rain w.r.t. moving man is $\vec{v}_{rm}$ but

$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$

$= -v_r \sin 30\ \hat{i} - v_r \cos 30\ \hat{j} - 10\ \hat{i}$

$= (-v_r \sin 30 - 10)\ \hat{i} - v_r \cos 30\ \hat{j}$

But $-v_r \sin 30 - 10 = 0$ $\therefore$ $v_r \sin 30 = -10$

$v_r = \dfrac{-10}{\sin 30} = -20$ m/s.

But v_r is not negative

$\therefore$ $\vec{v}_m = -10\hat{i}$

and $\vec{v}_{rm} = -[20\cos 30]$

$= 20 \cos 30\ \hat{j} = 10\sqrt{3}\,\hat{j}$.

Illustration 49. *A particle A is moving along a straight line with velocity 3 m/s and another particle B has a velocity 5 m/s at an angle of 30° to the path of A.. Find the velocity of B relative to A.*

Solution: $|v_B - v_A| = \sqrt{v_B^2 + v_A^2 - 2v_A v_B \cos 30^0}$

$= \sqrt{5^2 + 3^2 - 2\times 5\times 3\times(\sqrt{3/2})}$

$= \sqrt{8.02} = 2.832$ m/sec.

Using sine rule, $\dfrac{3}{\sin\theta} = \dfrac{2.832}{\sin 30^0}$

$\Rightarrow \theta = 32°$

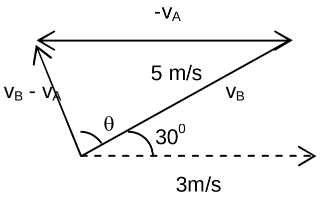
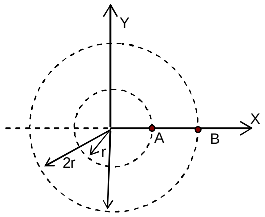


Illustration 50. *Two particles A and B are moving in a horizontal plane anticlockwise on two different concentric circles with different constant angular velocities 2ω and ω, respectively.*
(a) Find the relative velocity of B w.r.t. A after time $t = \pi/\omega$.
(Initial position of particles A and B are shown in figure.)
(b) Also find the relative position vector of B w.r.t. A.



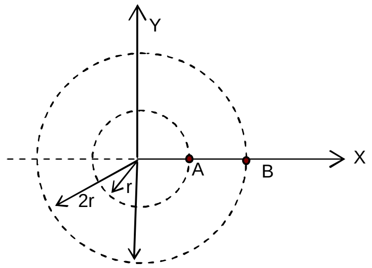
Solution:

$\theta_A = 2\omega\dfrac{\pi}{\omega} = 2\pi$, $v_A = 2\omega r\,\hat{j}$

$\theta_B = \omega\dfrac{\pi}{\omega} = \pi$, $v_B = 2\omega r(-\hat{j})$

(a) $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 2\omega r(-\hat{j}) - 2\omega r(\hat{j}) = -4\omega r\,\hat{j}$

(b) $\vec{r}_{BA} = \vec{r}_B - \vec{r}_A = 2r\,(-\hat{i}) - (r)\,\hat{i} = -3r\,\hat{i}$.



Exercise 14:

(i) ***How long will a boy sitting near the window of a train travelling at 54 km/h see a train passing by in the opposite direction with a speed of 36 km/hr? The length of the slow moving train is 100 m.***

(ii) ***Two particles A and B are moving with speeds of 2 km/hr and 3 km/hr, respectively, in the same direction. Find how far will B be from A after 1 hour ?***

(iii) ***Two cars are moving in the same direction with the same speed 30 km/hr. They are separated by a distance of 5 km. What is the speed of a car moving in the opposite direction if it meets these two cars at an internal of 4 minutes?***

(iv) ***A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/hr. He finds raindrops are hitting his head vertically. Find the speed of raindrops with respect to:***
(a) the road, and (b) the moving man.

(v) ***A boat travels downstream from point A to point B in two hours and upstream in four hours. Find the time taken by a log of wood to cover the distance from point A to point B.***

SUMMARY

Motion in a straight line:

1. Distance is the total length of the path traversed by an object.

2. Displacement is the change in position $\Delta x = x_2 - x_1$

3. Average speed $= \dfrac{\text{distance traversed}}{\text{time interval}}$

4. Speed: The speed of an object is equal to the distance traversed by it in a very short time interval divided by time interval.

5. Instantaneous velocity: It is defined as the limit of average velocity as the time interval Δt becomes infinitesimally small,
$v = \lim\limits_{\Delta t \to 0} \bar{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$
Slope of the tangent drawn on position–time graph at any instant gives the velocity at that particular time.

6. Average acceleration = $\dfrac{\text{change in velocity}}{\text{time interval}}$

7. Instantaneous acceleration: It is defined as the limit of the average acceleration as the time interval Δt goes to zero
i.e. $a = \lim\limits_{\Delta t \to 0} \bar{a} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$
At any particular instant, the slope of the velocity–time graph gives the acceleration of an object at that particular instant.

8. (i) The area under the speed–time graph gives the distance traversed by the object in the corresponding time interval.
(ii) The area under a velocity– time graph gives the displacement of the object.

9. For uniformly accelerated rectilinear motion, three equations of motion are
$v = u + at$
$x = ut + \dfrac{1}{2}at^2$
$v^2 = u^2 + 2ax$
where
u = initial velocity
v = final velocity
t = time taken
a = acceleration

Motion in a plane:

1. The position vector $\vec{r}$ of a point P in space is the vector from the origin to P. Its components are the coordinates x, y and z.
$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

2. $\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$

$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{r}}{dt}$

3. $\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$

$\vec{a}_{av} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

4. In projectile motion with no air resistance, $a_x = 0$ and $a_y = -g$. The coordinates and velocity component are simple functions of time, and the shape of the path is always a parabola.

5. When a particle moves in a circular path of radius R, its acceleration $\vec{a}$ is directed towards the centre of the circle and perpendicular to $\vec{v}$.

$a_{radial} = \frac{v^2}{R}$

where $v = \frac{2\pi R}{T}$

6. If the speed is not constant in circular motion, there is still a radial component of $\vec{a}$ but there is also a component of $\vec{a}$ parallel to the path.

$a_{rad} = \frac{v^2}{R}$

$a_{tangential} = \frac{dv}{dt}$

7. When two frames of reference A and B are moving relative to each other at constant velocity, the velocity of a particle P as measured by an observer in frame A usually differs from that measured from frame B.

$\therefore \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$

where $\vec{v}_{BA}$ is the velocity of B with respect to A.

Both observers measure the same acceleration for the particle; that is

$\vec{a}_{PA} = \vec{a}_{PB}$

MISCELLANEOUS EXERCISE

1. If a particle is accelerating, it is either speeding up or speeding down. Do you agree with this statement?

2. Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed. Do you agree with this statement?

3. Is the vertical height taken by a long jumper important while taking the jump?

4. A woman standing on the edge of a cliff throws a ball straight up with a speed of 8 km/h and then throws another ball straight down with a speed of 8 km/h from the same position. What is the ratio of the speeds with which the balls hit the ground ?

5. Find the average velocity during the time of flight, if a particle is projected with v at an angle θ with horizontal plane.

6. Establish the relation $x(t) = v(0)t + \frac{1}{2}at^2$ by calculus method.

7. Derive the velocity–time relationship by (i) calculus method, (ii) graphical method.

8. A stone is thrown upwards from the top of a tower 85 m high. It reaches the ground in 5 second. Calculate (i) the greatest height above the ground, (ii) the velocity with which it reaches the ground and (iii) the time taken to reach the maximum height.
Given: $g = 10\ m/s^2$.

9. A car starts from rest and accelerates uniformly for 10 s to a velocity of 8 m/s. It then runs at a constant velocity and is finally brought to rest in 64 m with a constant retardation. The total distance covered by the car is 584 m. Find the values of acceleration, retardation and total time taken.

10. Prove that there are two times for which a projectile travels the same vertical distance. Also prove that the sum of the two times is equal to the time of flight.

SOLUTION TO MISCELLANEOUS EXERCISE

1. No, not always. In case of uniform circular motion, the particle is accelerating but its speed is neither decreasing nor increasing, only direction of velocity changes.

2. This is true as motion under gravity is independent of mass of the body and so the body comes back to the point of projection with the same speed.

3. Yes, because for the longest jump the player should throw himself at an angle of 45° wrt horizontal. The vertical height required for this purpose should be $\frac{u^2}{4g}$, where u is velocity of

throw. If the vertical height is different from $\frac{u^2}{4g}$ then the angle will be different from 45° and the horizontal distance covered also will be less.

4. 1 : 1, both the balls will hit the ground with the same speed.

5. $v \cos \theta$

8. $h = 3.2$ m, $v = 42$ m/s, $t = 0.8$ sec.

9. 0.8 m/s^2, 0.5 m/s^2 and 86 sec.

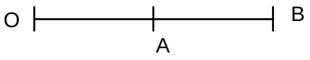
SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob 1. *A car starts from rest and moves with a constant acceleration of 2.0 m/s^2 for 30 seconds. The brakes are then applied and the car comes to rest in another 60 seconds. Find*
(a) total distance covered by the car.
(b) Maximum speed attained by the car
(c) Find shortest distance from initial point to the point when its speed is half of maximum speed.

Sol. Final velocity at A $v_A = 2 \times t_1 = 2 \times 30 = 60$ m/sec.
For AB, Let the retardation be 'b'



$\therefore\ 0 = v_A + bt^2$

$\therefore\ b = -\frac{v_A}{t} = -\frac{60}{60} = -1 \text{ m/s}^2$

(a) Total distance = OA + AB

$$OB = \frac{1}{2}at_1^2 + (v_A t_2 - \frac{1}{2}bt_2^2)$$
$$= (\frac{1}{2}\times 2\times 30\times 30 + 60\times 60\times 60 - \frac{1}{2}\times 1\times 60\times 60)$$
$$= 900 + 3600 - 1800 = 2700 \text{ m.}$$

(b) Maximum speed $v_A = 60$ m/s.

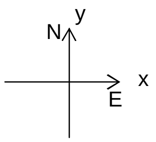
(c) $v^2 = 2 \times a \times s$

$$s = \frac{(v_A/2)^2}{2\times a} = \frac{30\times 30}{2\times 2} = 225 \text{ m.}$$

Prob 2. *A farmer has to go 500 m due north, 400 m due east and 200 m due south to reach his field. If he takes 20 minutes to reach the field,*
(a) what distance he has to walk to reach the field ?
(b) what is his displacement from his house to the field ?
(c) what is the average speed of farmer during the walk ?
(d) what is the average velocity of farmer during the walk ?

Sol. (a) Distance = 500 + 400 + 200 = 1100m

(b) Displacement = $500(\vec{j}) + 400(\hat{i}) + 200(-\hat{j}) = 300\vec{j} + 400\hat{i}$



Magnitude of displacement = $\sqrt{(400)^2 + (300)^2} = 500$m

(c) Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{1100}{20\times 60} = \frac{11}{12}$ m/s

(d) Average velocity = $\frac{\text{Displacement}}{\text{Time}} = \frac{500}{20\times 60} = \frac{5}{12}$ m/s

$\theta = \tan^{-1}\left(\frac{300}{400}\right) = 37^\circ$ due North of East.

Prob 3. *A body is projected up such that its position vector varies with time as $\vec{r} = 6t\hat{i} + (8t - 5t^2)\hat{j}$. Find the (a) initial velocity (b) time of flight*

Sol. (a) The position of the body at any time t is given as $\vec{r} = 6t\,\hat{i} + (8t-5t^2)\,\hat{j}$. When t = 0, r = 0. That means the body is projected from the origin of the coordinate system. Differentiating both sides w.r.t. time 't', we obtain

$$\frac{d\vec{r}}{dt} = 6\,\hat{i} + (8-10t)\,j \quad \Rightarrow \vec{v} = 6\,\hat{i} + (8-10t)\,j.$$

Putting t = 0, we obtain the initial velocity (velocity of projection) given as

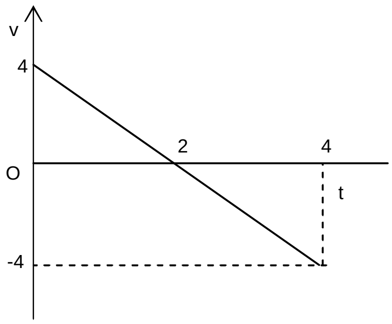
$$(\vec{v})_{t=0} = \vec{v}_0 = 6\,\hat{i} + 8\,\hat{j} \quad \Rightarrow v_0 = 10 \text{ m/sec};$$

(b) The time of flight $T = \dfrac{2v_0 \sin\theta_0}{g}$

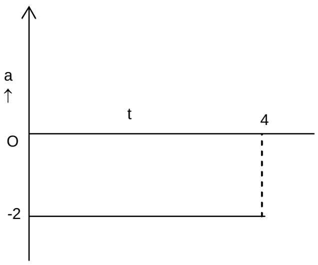
$$\Rightarrow T = \frac{2(v_y)_0}{g} \text{ where } (v_y)_0 = 8$$

$$\Rightarrow T = \frac{2\times 8}{10} = 1.6 \text{ sec.}$$

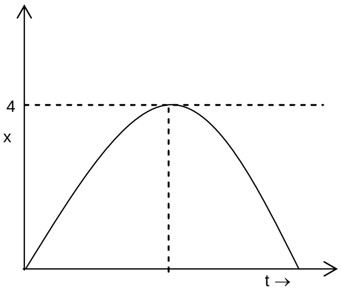
Prob 4. *A particle starts from origin at t = 0 along +ve x axis. It's velocity–time graph is shown in the figure. Draw*
(i) a, t graph
(ii) x, t graph.



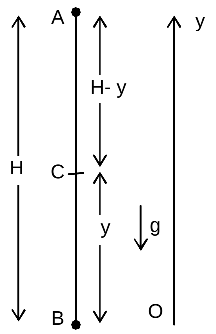
Sol. (i) Velocity is decreasing
so, a = –4/2 = –2



(ii)



Prob 5. *A stone 'A' is dropped from the top of a tower 20 m high. Simultaneously another stone 'B' is thrown up from the bottom of the tower so that it can reach just on the top of the tower. What is the distance of the stones from the ground while they pass each other?*



Sol. Let t be the time when they pass one another

For stone B, $y = v_B t + \frac{1}{2}(-g)t^2$ … (i)

Ffor stone A, $H - y = \frac{1}{2}gt^2$ … (ii)

From (i) and (ii),

$H = v_B t$ … (iii)

Stone B can reach just one the top of tower. We can calculate the velocity of stone B,

$v_f^2 = v_i^2 + 2a_y y$

$u_f = 0$, for $y_{max} = H = 20$ m

$v_i = v_B$; $a_y = -g$; $v_B = 20$ m/s

From (iii) $t = \frac{20m}{20m/s} = 1sec.$

From equation (i), the required distance (BC) from ground = $20 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 15$ m

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Prob 6. *A rocket is fired vertically and ascends with constant vertical acceleration of $20m/s^2$ for 1 minute. Its fuel is then all used and it continues as a free particle. Find the*
(a) maximum height reached by the rocket
(b) total time elapsed from the take off till the rocket strikes the earth.($g=10m/s^2$).

Sol. (a) For the time interval from 0 to 60 seconds rocket accelerates and thereafter it moves under gravity. Distance moved by it in 60 seconds is given by

$$S_1 = \frac{1}{2} \times \frac{20m}{s^2} \times (60s)^2 = 36000m.$$

$$v_{(60s)} = \frac{20m}{s^2} \times 60s = 1200m/s$$

If H be the maximum height reached.

Then, $0 = \left(1200\frac{m}{s}\right)^2 - 2g(H - 36000)$, $(v^2 = u^2 + 2as)$

$\Rightarrow H = 36000 + \frac{1200 \times 1200}{2 \times 10} m$

$\Rightarrow H = 108000m$

(b) Time taken to ascend is

$$t_1 = 60s + \frac{1200}{10}s = 180\text{ s}, \quad [t = t_1 + \frac{u}{a}]$$

Let time taken to descend is t_2 then

$$108000 = \frac{1}{2}gt_2^2$$

$\Rightarrow t_2 = \sqrt{\frac{2 \times 108000}{10}} = 146.96s$

Total time $T = t_1 + t_2 = 180 + 146.96 = 326.96$ s.

Prob 7. *The position of a particle moving along the x-axis depends on the time as $x = at^2 - bt^3$ where $a = 3.0\ m/s^2$ and $b = 1.0\ m/s^3$ respectively.*
(a) At what time does the particle reach its maximum positive x-position ?
(b) What total path length does the particle cover in the first 2.0 sec?
(c) Does the particle cover equal path length in the opposite direction in the subsequent 2.0 sec. ? If not, explain why?
(d) Find the total path length covered in the first 4.0 sec.
(e) Find the displacement during the first 4.0 sec.
(f) What is the particles speed and acceleration at the end of first 3.0 sec.?

Sol. Here, the position (x) is time dependent as $x = at^2 - bt^3 = 3t^2 - t^3$

Instantaneous velocity $v = \frac{dx}{dt} = 6t - 3t^2$ . . . (i)

And acceleration $a = \frac{dv}{dt} = 6 - 6t$ . . . (ii)

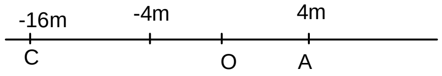
Note that the acceleration is not uniform (like gravity) but time dependent.

(a) At the maximum positive x-position the particle comes to momentary rest (v = 0) and then moves in the negative x-direction with non uniform acceleration.
From equation (i), $v = 0 = 6t - 3t^2$
$\therefore$ Required time = 2.0 seconds.

(b) The x-coordinate of the particle increases from zero to $(x)_{max}$ during the first 2 seconds.
$\therefore$ path length $= (x)_{t=2s} = (3t^2 - t^3)_{t=2s} = 4.0$ m

(c) For t > 2s, the particle moves in the backward direction with time dependent acceleration. Hence, subsequent motion is not repeated (as we have seen in free fall) where the acceleration $|\vec{g}|$ remains constant. Hence, the path length for the subsequent 2 seconds will be different.

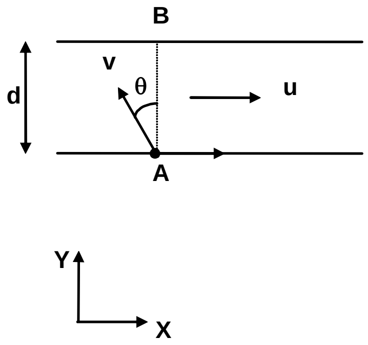
(d) Position at t = 2 sec. is
$(x)_{t=2} = 4.0$ m and
$(x)_{t=4} = (3t^2 - t^3)_{t=4} = 48 - 64$
$= -16$ m.
Hence, the path length during the first 4.0 is
OA + AO + OC
= 4 + 4 + 16 = 24.0 m



(e) Displacement during the first 4.0 sec is – 16.0 m.
(f) Speed at the end of 3.0 sec is
$(v)_{t=3} = (6t - 3t^2)_{t=3s} = -9.0$ m/s.
Negative sign indicates that motion is along the negative x-direction.
Acceleration $(a)_{t=3s} = (6 - 6t)_{t=3s} = -12\ m/s^2$.

Prob 8. *A man can row a boat with a speed of 4 km/hr in still water. He is crossing a river where the speed of current is 2 km/hr.*
(a) In what direction will his boat be headed if he wants to reach a point on the other bank, directly opposite to starting point?
(b) If width of the river is 4 km how long will it take him to cross the river, with the condition in part 'a' ?
(c) In what direction should he head the boat if he wants to cross the river in shortest time?
(d) How long will it take him to row 2 kms up the stream and then back to his starting point?

Sol. B is a point directly opposite to the starting point A.



Let the man heads the boat in a direction making an angle θ with the line AB.

Here $\vec{v}_w = 2\hat{i}$

$\vec{v}_{bw} = -4\sin\theta\hat{i} + 4\cos\theta\hat{j}$

$\because \quad \vec{v}_{(absolute)} = \vec{v}_{bw} + \vec{v}_w$

$= (2 - 4\sin\theta)\hat{i} + 4\cos\theta\hat{j}$

$\Rightarrow v_{bx} = 2 - 4\sin\theta$ and $v_{by} = 4\cos\theta$

(a) For directly opposite point $v_{bx} = 0$

$\Rightarrow \quad \sin\theta = \frac{1}{2} = \sin 30^o \Rightarrow \theta = 30^o$

Hence, to reach the point directly opposite to starting point he should head the boat an angle $\beta = (90^o + 30^o) = 120^o$ with the river flow.

(b) $t = \frac{y}{v_{by}} = \frac{d}{4\cos\theta} = \frac{4}{4\cos 30^o} = \frac{2}{\sqrt{3}}$ hr.

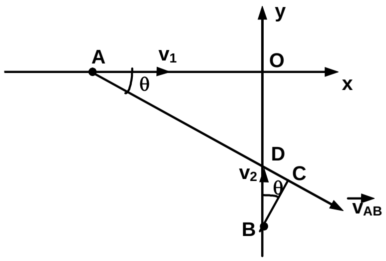
(c) For t to be minimum $\cos\theta = 1 \Rightarrow \theta = 0^o$

$\Rightarrow \quad t_{min} = \frac{4}{4\cos 0} = 1$ hr.

(d) $T = \frac{2}{(4-2)}\text{hr} + \frac{2}{(4+2)}\text{hr} = \left(1 + \frac{1}{3}\right)\text{hr} = \frac{4}{3}\text{hr}.$

Prob 9. *Two particles A and B move with constant velocities v_1 and v_2 along two mutually perpendicular straight lines towards the intersection point O. At moment t = 0, the particles were located at distances l_1 and l_2 from O, respectively. Find the time, when they are nearest and also this shortest distance.*

Sol. $\because \quad \vec{v}_{AB} = \vec{v}_A - \vec{v}_B = v_1\hat{i} - v_2\hat{j}$



Minimum distance is the length of the perpendicular to $\vec{v}_{AB}$ from B.

If θ is the angle between the x-axis and $\vec{v}_{AB}$, then

$\tan\theta = \left|-\frac{v_2}{v_1}\right| = \frac{v_2}{v_1}$

In ΔAOD, $OD = OA\tan\theta = \frac{v_2}{v_1} l_1$

Therefore, $BD = l_2 - OD = \frac{v_1 l_2 - v_2 l_1}{v_1}$

In ΔBCD, $\cos\theta = \frac{BC}{BD}$

$\Rightarrow BC = BD\cos\theta = \frac{v_1 l_2 - v_2 l_1}{v_1} \times \frac{v_1}{\sqrt{v_1^2 + v_2^2}}$

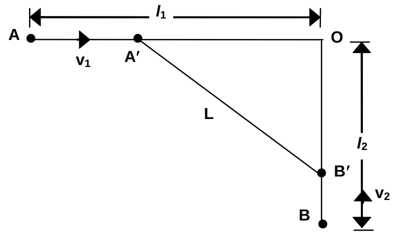
$\Rightarrow \quad BC = \frac{|v_1 l_2 - v_2 l_1|}{\sqrt{v_1^2 + v_2^2}}$

The required time $t = \dfrac{AC}{|\vec{v}_{AB}|} = \dfrac{AD + DC}{|\vec{v}_{AB}|}$

$$\Rightarrow \quad \frac{\ell_1 \sec\theta + BC\tan\theta}{\sqrt{v_1^2+v_2^2}} = \frac{\dfrac{\ell_1}{v_1}\sqrt{v_1^2+v_2^2} + \dfrac{v_1\ell_2 - v_2\ell_1}{\sqrt{v_1^2+v_2^2}}\dfrac{v_2}{v_1}}{\sqrt{v_1^2+v_2^2}}$$

$$= \frac{\ell_1 v_1 + \ell_2 v_2}{v_1^2 + v_2^2}$$

Alternatively



After time 't', the position of the point A and B are $(\ell_1 - v_1 t)$ and $(\ell_2 - v_2 t)$, respectively.

The distance L between the points A′ and B′ are

$L^2 = (\ell_1 - v_1 t)^2 + (\ell_2 - v_2 t)^2$ . . . (i)

Differentiating with respect to time,

$2L\dfrac{dL}{dt} = 2(\ell_1 - v_1 t)(-v_1) + 2(\ell_2 - v_2 t)(-v_2)$ For minimum value of L, $\dfrac{dL}{dt} = 0$

$(v_1^2 + v_2^2)t = \ell_1 v_1 + \ell_2 v_2$

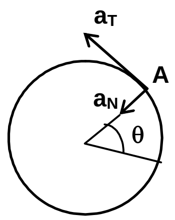
or $\quad t = \dfrac{\ell_1 v_1 + \ell_2 v_2}{v_1^2 + v_2^2}$

Putting the value of t in equation (i)

$$L_{min} = \frac{|\ell_1 v_2 - \ell_2 v_1|}{\sqrt{v_1^2 + v_2^2}}.$$

Prob 10. *A wheel rotates around a stationary axis so that rotation angle θ varies as $\theta = Pt^2$, where $P = 0.20\ rad/s^2$. Find the total acceleration a of the point A at the rim at the moment $t = 2.55$ sec, if the linear velocity of the point A at this moment is $v = 0.65$ m/s.*

Sol. Total acceleration of a body moving in a circular path



$\vec{a} = \vec{a}_R + \vec{a}_t$

$|\vec{a}| = \sqrt{|\vec{a}_R^2| + |\vec{a}_t^2|}$

The radial acceleration a_R is the centripetal acceleration

$a_R = \dfrac{v^2}{R} = \omega^2 R \qquad = \left(\dfrac{d\theta}{dt}\right)^2 R$

$= \left\{\dfrac{d}{dt}(Pt^2)\right\}^2 R = 4P^2t^2R$ . . . (i)

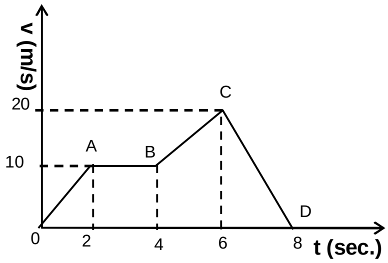
Tangential acceleration $a_T = \dfrac{d}{dt}(v)$

$= \dfrac{d}{dt}(\omega R) = \dfrac{R\ d^2(\theta)}{dt^2} = 2PR$ . . . (ii)

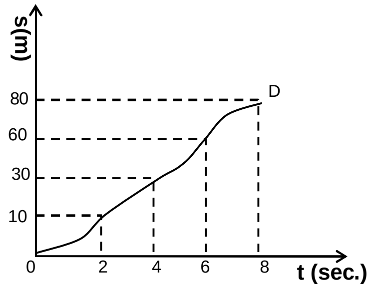
$\therefore \quad a = \sqrt{\left(\dfrac{v^2}{R}\right)^2 + (2PR)^2}$

$= \sqrt{(4P^2t^2R)^2 + (2PR)^2} = 2PR\sqrt{1 + 4P^2t^4} = \dfrac{v}{t}\sqrt{1+4P^2t^4} = 0.7\ m/s^2.$

Prob 11. *Velocity–time graph of a particle moving in a straight line is shown in figure. Plot the corresponding displacement–time graph of a particle if at t = 0 displacement s = 0.*



Sol.



Prob 12. *A particle moves in a circle of radius 20 cm at a speed given by $v = 1 + t + t^2$ m/s where t is time in s. Find (a) the initial tangential and normal acceleration. (b) the angle covered by the radius in first 2 s.*

Sol. (a) Tangent acceleration $a_t = \frac{dv}{dt} = 2t + 1$

Normal acceleration $a_n = \frac{v^2}{R}$ $\quad\therefore (a_t)_{t=0} = 1\ m/s^2$

$(a_n)_{t=0} = \frac{v_0^{\,2}}{R} = \frac{1}{(0.2)} = 5\ m/s^2$

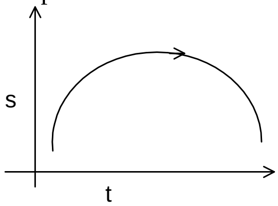
(b) $v = R\frac{d\theta}{dt}$

$R\,d\theta = (1+t+t^2)dt$ $\quad\therefore R\int_0^{\theta'} d\theta = \int_0^2 \left(1+t+t^2\right)dt$

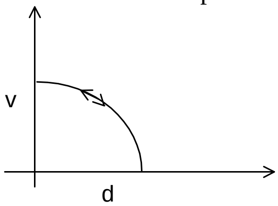
$\theta = 33.3$ rad

Prob 13. *A body of mass m is projected vertically upwards with a speed v_0. It goes up and comes back to the same point. For this motion draw displacement–time, velocity–time, acceleration–time and speed–displacement graphs.*

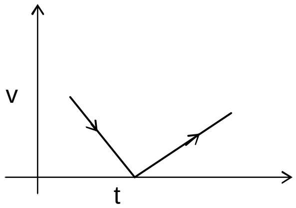
Sol. Displacement–time



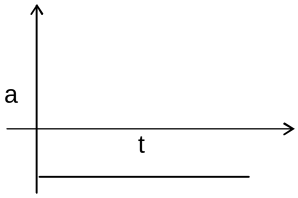
Speed–displacement



Speed time



Acceleration time



Prob 14. *A car starts moving from rest with an acceleration whose value linearly increases with time from zero to 6 m/s² in 6 sec after which it moves with constant velocity. Find the time taken by the car to travel first 72 m from starting point.*

Sol. Since acceleration varies linearly with time therefore

$a \propto t$

$\Rightarrow a = kt \quad \Rightarrow \int_0^6 da = k\int_0^6 dt \qquad \Rightarrow k = 1$

then, $\frac{dv}{dt} = t \quad \Rightarrow \; v = \frac{t^2}{2}$ m/s

then, $\frac{ds}{dt} = \frac{t^2}{2} \quad$ or, $s = \frac{t^3}{6}$

at the end of t = 6 sec. Acceleration becomes zero.

Distance moved by car at t = 6 sec is

$S_1 = \frac{6\times6\times6}{6} = 36$ m

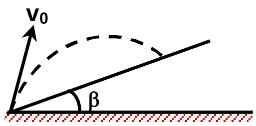
Speed of the car $= \frac{6\times6}{2} = 18$ m/s

Remaining distance = 72 – 36 = 36 m.

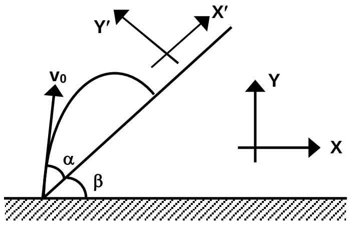
so time taken to cover this distance = $t_2 = \frac{36}{18}\text{sec.} = 2\,\text{sec.}$

Total time = 6+2 = 8 sec.

Prob 15. *A particle projected with velocity v_0 from an inclined plane whose angle of inclination with the horizontal is β. If afterwards the projectile strikes the inclined plane perpendicular to it. Find the height of the point struck, from horizontal plane through the point of projection.*



Sol. Let α be the angle between the velocity of projection and the inclined plane.



$v_{0x'} = v_0\cos\alpha,\; v_{0y'} = v_0\sin\alpha$

$a_{x'} = -g\sin\beta \qquad a_{y'} = -g\cos\beta$

$\Rightarrow \quad v_{x'}(t) = v_0\cos\alpha - g\sin\beta t$

At the point of impact $v_{x'} = 0$

$$\Rightarrow \quad t = \frac{v_0\cos\alpha}{g\sin\beta} \qquad ..(1)$$

Also y′ at the point is zero.

$$\Rightarrow \quad v_0\sin\alpha t - \frac{1}{2}g\cos\beta t^2 = 0$$

$$\Rightarrow \quad t = \frac{2v_0\sin\alpha}{g\cos\beta} \qquad ..(2)$$

From (1) and (2) $\qquad \tan\alpha = \frac{\cot\beta}{2} \qquad ..(3)$

$$x = v_0\cos(\alpha+\beta)t$$

$$= v_0\left[\cos\alpha\cos\beta - \sin\alpha\sin\beta\right].\frac{v_0\cos\alpha}{g\sin\beta}$$

$$= \frac{v_0^2}{g}\left[\cos^2\alpha\cot\beta - \sin\alpha\cos\alpha\right]$$

$$= \frac{v_0^2}{g}\left[\left(\frac{2}{\sqrt{4+\cot^2\beta}}\right)^2 \cot\beta - \frac{\cot\beta}{\sqrt{4+\cot^2\beta}}.\frac{2}{\sqrt{4+\cot^2\beta}}\right]$$

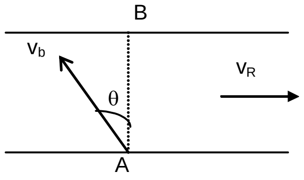
$$= \frac{v_0^2}{g}\frac{2\cot\beta}{4+\cot^2\beta}$$

$\therefore$ $$y = x\tan\beta = \frac{v_0^2}{g}.\frac{2\cot\beta}{4+\cot^2\beta}\tan\beta$$

$\Rightarrow$ $$y = \frac{2v_0^2}{g\left(4+\cot^2\beta\right)}$$

Prob 16. *The velocity of a boat in still water is n times less than the velocity of flow of a river. At what angle to the stream direction must the boat move so that drift is minimised ? If n = 2, show that the angle $\theta = 120°$.*

Sol. Given $v_b = \frac{v_R}{n}$



$\vec{v}_b = (-v_b \sin\theta)\hat{i} + (v_b\cos\theta)\hat{j}$

Resultant velocity of boat

$= \vec{v}_b + \vec{v}_R$

$= (v_R - v_b \sin\theta)\ \hat{i}\ + (v_b \cos\theta)\ \hat{j}$

If w = width of the river, time for crossing is

$$T = \frac{W}{v_b\cos\theta}$$

Drift during time T is $(v_R - v_b \sin\theta)$ T

$\Rightarrow$ Drift $x = v_b(n - \sin\theta)\frac{w}{v_b\cos\theta} = w(n \sec\theta - \tan\theta)$

For x to be minimum, $\frac{dx}{d\theta} = 0$ lead to $\theta = \sin^{-1}(1/n)$

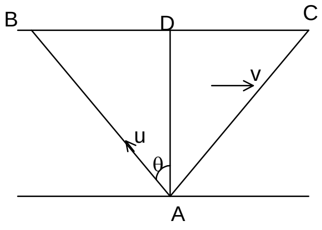
Direction of boat w.r.t. stream is

$90° + \theta = 90° + \sin^{-1}(1/n)$

For n = 1/2, the required angle $= 90° + 30° = 120°$

Prob 17. *A man can row a boat in still water at 3 km/h He can walk at a speed of 5 km/h on the shore. The water in the river flows at 2 km/h. If the man rows across the river and walks along the shore to reach the opposite point on the river bank find the direction in which he should row the boat so that he could reach the opposite shore in the least possible time. The width of the river is 500 m.*

Sol. Let the points towards B and reches at C



t_1 : the time taken by the boat to reach C

$t_1 = \frac{AD}{u\cos\theta}$ $\quad CD = (v - u\sin\theta)t_1$

$t_1 = \frac{500\times10^{-3}}{3\cos\theta}$ hr $= \frac{1}{6\cos\theta}$

$CD = (-3\sin\theta + 2)\frac{1}{6\cos\theta}$

$= -0.5\tan\theta + \frac{1}{3\cos\theta}$

t_2 : time taken by the man from C to D

$t_2 = \frac{CD}{v_s} = -\frac{0.5\tan\theta}{5} + \frac{1}{3\cos\theta \times 5} = \frac{1}{10}\tan\theta + \frac{1}{15\cos\theta}$

$= -\frac{\sin\theta}{10\cos\theta} + \frac{1}{15\cos\theta}$

$= \frac{(-3\sin\theta + 2)}{30\cos\theta}$

Total time $t = t_1 + t_2 = \frac{1}{6\cos\theta} + \frac{-3\sin\theta + 2}{30\cos\theta} = \frac{7 - 3\sin\theta}{30\cos\theta}$

$= \frac{7}{30}\sec\theta - \frac{1}{10}\tan\theta$

For minimum t

$\frac{dt}{d\theta} = 0 \quad \Rightarrow \quad \frac{7}{30}\sec\theta\tan\theta - \frac{1}{10}\sec^2\theta = 0$

$\Rightarrow \frac{1}{10}\sec\theta\left(\frac{7}{3}\tan\theta - \sec\theta\right) = 0 \Rightarrow \quad \frac{7}{3}\tan\theta - \sec\theta = 0$

$\frac{7\sin\theta - 3}{3\cos\theta} = 0 \quad \Rightarrow \quad \theta = \sin^{-1}(3/7)$

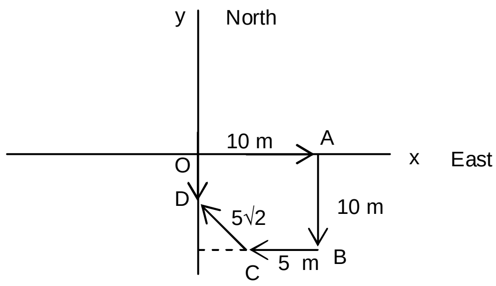
Prob 18. *A cyclist moves with constant speed 5 m/s along eastward for 2 seconds, and along southward for 2 seconds. Then, he moves along west for one second and finally along north-west for $\sqrt{2}$ seconds. Find*

(a) Distance and displacement of cyclist for whole journey.

(b) Average speed and average velocity for whole journey

(c) Average acceleration of cyclist for whole journey.

Sol. (a) In figure, shown final displacement



$\vec{OD} = -5\hat{j}\,m$

Distance = OA + AB + BC + CD

$= (25 + 5\sqrt{2})$ m

(b) Average speed $= \frac{\text{total distance}}{\text{total time}}$

$= \frac{25 + 5\sqrt{2}}{5 + \sqrt{2}} = 5$ m/s.

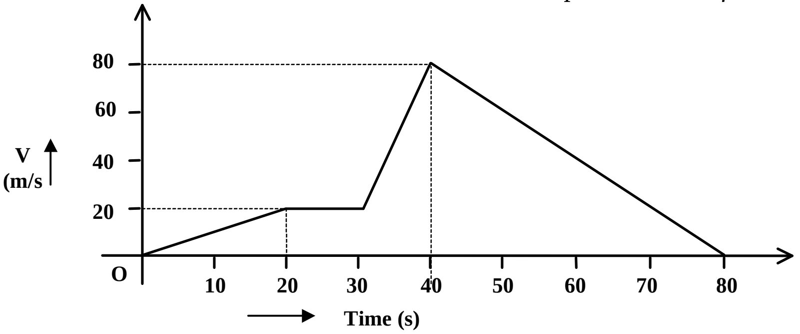
Average velocity $= \frac{-5\hat{j}}{(5 + \sqrt{2})}$ m/s

(c) For average acceleration $= \frac{\bar{v}_f - \bar{v}_i}{\Delta t}$

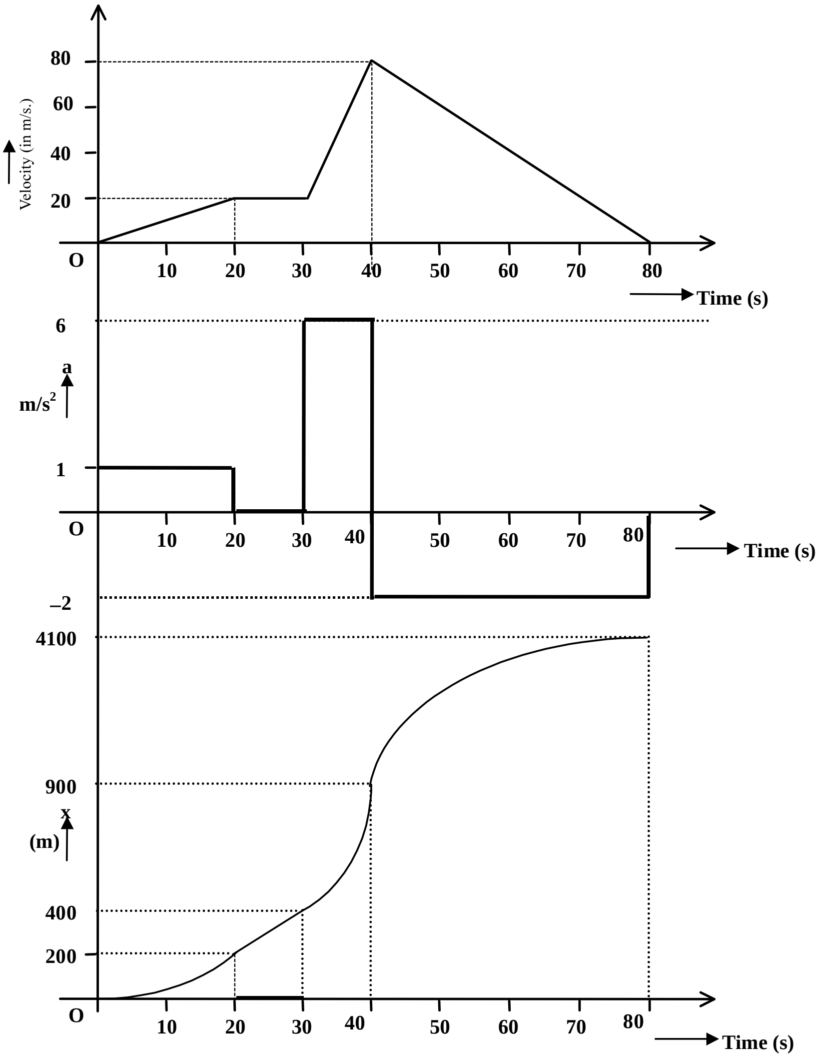
$\bar{v}_f = -5\hat{i} + 5\hat{j}$, $v_i = 5\hat{i}$

average $= \frac{-5\hat{i} + 5\hat{j} - 5\hat{j}}{(5 + \sqrt{2})} = \frac{-10\hat{i} + 5\hat{j}}{5 + \sqrt{2}}$ m/s^2

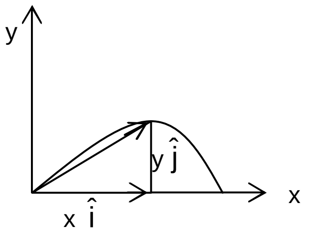
Prob 19. *The velocity-time graph of moving object is given in the figure. Draw the acceleration versus time and displacement versus time graph. Find the distance travelled during the time interval when the acceleration is maximum. Assume that the particle starts from origin.*



Sol.



Prob 20. *A projectile is fired with speed v_0 at an angle θ with the horizontal on a horizontal plane, Find*
(a) the average velocity of projectile in half of time of flight.
(b) the time in which the speed of projectile becomes perpendicular to its initial velocity.
(c) the radius of curvature of projectile at the instant when it is at its maximum height.



Sol. (a) $\vec{v}_{av} = \dfrac{\text{displacement}}{\text{time}} = \dfrac{x\hat{i} + y\hat{j}}{\frac{v_0 \sin\theta}{2g}}$

x = half of range $= \dfrac{v_0^2 \sin 2\theta}{2g}$

y = Max. height $= \dfrac{v_0^2 \sin^2\theta}{2g}$

$$\bar{v}_{av} = \frac{\frac{v_0^2 \sin 2\theta}{2g}\hat{i} + \frac{v_0^2 \sin^2\theta}{2g}\hat{j}}{\frac{v_0 \sin\theta}{2g}} = \frac{v_0 \sin 2\theta}{\sin\theta}\hat{i} + \frac{v_0 \sin^2\theta}{\sin\theta}\hat{j}$$

$= 2v_0 \cos\theta\ \hat{i} + v_0 \sin\theta\ \hat{j}$

(b) Let the perpendicular velocity be $\vec{v}$

$\bar{v}\cdot\bar{v} = 0 \quad \therefore\ \bar{v}\cdot\bar{v} = (v_0 \cos\theta\ \hat{i} + v_0 \sin\theta\ \hat{j}).(v_0 \cos\theta\ \hat{i} + (v_0 \sin\theta - gt)\hat{j}$

$\therefore\ v_0^2 \cos^2\theta + v_0 \sin\theta(v_0 \sin\theta - gt) = 0$

$v_0^2 - v_0 \sin\theta gt = 0 \ \therefore\ t = \dfrac{v_0}{g\sin\theta}$

(c) Radius of curvature $= \dfrac{v^2}{g} = \dfrac{v_0^2 \cos^2\theta}{g}$

Prob 21. *An elevator car whose floor to ceiling distance is equal to 2.7 m starts ascending with a constant acceleration of 1.2 m/s^2. Two seconds after it starts, a bolt begins to fall from the ceiling of the elevator. Find*

(a) the bolt's free fall time,

(b) the displacement and the distance covered by the bolt during the fall in the reference frame fixed to the ground. (Use g = 9.8 m/s^2.)

Sol. (a) Since a = 1.2 m/sec^2 is the constant acceleration of the elevator car while ascending and h = 2.7 m is the separation between the floor and the ceiling, therefore, the free fall time is given by

$\Rightarrow h = \dfrac{1}{2}(g + a)t^2 \ \Rightarrow t = \sqrt{\dfrac{2h}{g+a}} = 0.7$ sec

(b) Velocity of elevator at t = 2 sec is v = (1.2 m/s^2) (2 s) = 2.4 m/sec.

Thus, with respect to the reference frame fixed to the ground i.e. with respect to a stationary observer, the displacement in the course of free fall is

$y = (-2.4 \text{ m/s})(0.7 \text{ s}) + \dfrac{1}{2}(9.8 \text{ m/s}^2)(0.7 \text{ s})^2 = 0.72$ m

Total distance covered w.r.t. the ground during the free fall times is

s = y + 2h

$= 0.72 + 2 \times \dfrac{(2.4)^2}{2\times 9.8} = 1.31$ m.

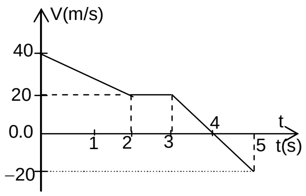
$v^2 = u^2 + 2gh$

$0 = (2.4 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)h$

$\Rightarrow \quad h = \dfrac{(2.4\,\text{m/s})^2}{2\times(9.8\,\text{m/s}^2)}$

Objective:

Prob 1. *In the given v–t graph, the distance travelled by the body in 5 sec will be*
(A) 100 m
(B) 80 m
(C) 40 m
(D) 20 m



Sol. **A.**
Distance travelled = area under the v–t curve

$$= \frac{20\times 2}{2} + 20\times 2 + 20\times 1 + \frac{20\times 1}{2} + \frac{20\times 1}{2} = 100 \text{ m}$$

Prob 2. *In Question 1, the displacement of the body in 5 sec will be*
(A) 100 m *(B) 80 m*
(C) 40 m *(D) 20 m*

Sol. **B.**
Displacement is a vector and is equal to algebraic sum of area under the v–t graph.
= 20 + 40 + 20 + 10 – 10 = 80 m.

Prob 3. *In Question 1, the average velocity of the body in 5 seconds is*
(A) 20 m/s *(B) 16 m/s*
(C) 8 m/s *(D) 4 m/s*

Sol. **B.**
Average velocity $= \frac{\text{displacement}}{\text{time}} = \frac{80}{5} = 16\,\text{m/s}$

Prob 4. *In above Question, the average speed of the body during 5 sec is*
(A) 20 m/s *(B) 16 m/s*
(C) 8 m/s *(D) 4 m/s*

Sol. **A.**
Average speed $= \frac{\text{distance}}{\text{time}} = \frac{100}{5} = 20\,\text{m/s}$

Prob 5. *A body when projected vertically up, covers a total distance D during its time of flight. If there were no gravity, the distance covered by it during the same time is equal to*
(A) 0 *(B) D*
(C) 2D *(D) 4D*

Sol. **C.**
The displacement of the body during the time t as it reaches the point of projection

$\Rightarrow S = 0 \Rightarrow v_0 t - \frac{1}{2} gt^2 = 0 \qquad \Rightarrow \; t = \frac{2v_0}{g}$

During the same time t, the body moves in absence of gravity through a distance
D′ = v.t, because in absence of gravity g = 0

$\Rightarrow D' = v_0 \left(\frac{2v_0}{g}\right) = \frac{2v_0^2}{g}$ …(1)

In presence of gravity, the total distance covered is

$= D = 2H = 2\ \frac{v_0^2}{2g} = \frac{v_0^2}{g}$ …(2)

(1) ÷ (2) $\Rightarrow D' = 2D$.

Prob 6. *A particle is projected vertically upward with initial velocity 25 ms^{-1}. During third second of its motion, which of the following statement is correct?*
(A) displacement of the particle is 30 m
(B) distance covered by the particle is 30 m
(C) distance covered by the particle is 2.5 m
(D) none of these

Sol. **C.**

Displacement of the particle during third second of the motion (i.e. between t = 2s and t = 3s) is zero. Hence, t = 2.5 sec is the turning point of the motion.

For distance $S_{t=2} = 25 \times 2 - \frac{1}{2} \times 10 \times 2^2 = 30$ m

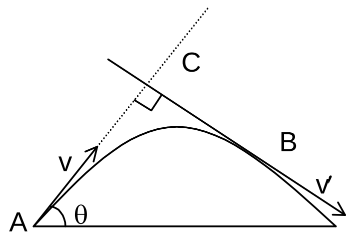
and $S_{t=2.5} = 25 \times 2.5 - \frac{1}{2} \times 10 \times 2.5^2 = 31.25$

Hence, distance covered by the particle during third second of motion

$= 2\,(31.25 - 30) = 2.5$ m.

Prob 7. *A particle is projected from a point A with a velocity v at an angle θ (upward) with the horizontal. At a certain point B, it moves at right angle to its initial direction. It follows that*
(A) velocity of the particle at B is v.
(B) velocity of the particle at B is v cos θ.
(C) velocity of the particle at B is v tan θ.
(D) the time of flight from A to B is $\frac{v}{g\sin\theta}$.

Sol. **D.**



$\vec{v} = \vec{u} + \vec{a}t$

Considering along the line AC

$0 = v - g\sin\theta t \Rightarrow t = \frac{v}{g\sin\theta}$

Now, consider along the line CB

$v' = 0 + g\cos\theta\ \frac{v}{g\sin\theta} = v\cot\theta$

Prob 8. *A particle is projected horizontally from the top of a cliff of height H with a speed $\sqrt{2gH}$. The radius of curvature of the trajectory at the instant of projection will be*
(A) H/2 *(B) H*
(C) 2H *(D) ∞*

Sol. **C.**

Since, $\vec{g} \perp \vec{v}$

Radial acceleration $a_r = g$

$\Rightarrow \dfrac{v_0^2}{r} = g$ where r is the radius of curvature.

$\Rightarrow \dfrac{2gH}{r} = g \quad (\because v = \sqrt{2gH})$

$\Rightarrow r = 2H$

Prob 9. *If a boat can have a speed of 4 km/hr in still water, for what values of speed of river flow, it can be managed to row boat right across the river, without any drift?*

(A) $\geq$ 4 km/hr *(B) greater than zero but less than 4 km/hr*

(C) only 4 km /hr *(D) none of these*

Sol. **B.**

Drift $(\Delta x) = (v_{b,x})\,\Delta t = (v_{br}\cos\theta + v_r)\Delta t$

where $v_{b,x}$ = velocity of boat w.r.t. ground

$v_{\perp, r}$ = velocity of boat w.r.t river

v_r = velocity of river w.r.t. ground

For $\Delta x = 0$, $v_r = -v_{br}\cos\theta$

$\Rightarrow (v_r)_{max} = v_{br}$

For, $v_r > v_{br}$ we can not have zero drift.

Prob 10. *A swimmer crosses a river of width d flowing at velocity v. While swimming, he keeps himself always at an angle of 120° with the river flow and on reaching the other end he finds a drift of d/2 in the direction of flow of river. The speed of the swimmer with respect to the river is*

(A) $(2-\sqrt{3})\,v$ *(B) $2(2-\sqrt{3})\,v$*

(C) $4(2-\sqrt{3})\,v$ *(D) $(2+\sqrt{3})\,v$*

Sol. **C.**

Drift = d/2 = $(V_r - V_s\sin30)d/V_s\cos30$

$\Rightarrow V_s = 4(2-\sqrt{3})V$

Prob 11. *A projectile is thrown into space so as to have the maximum possible horizontal range equal to 400 m. Taking the point of projection as the origin, the coordinates of the point where the velocity of the projectile is minimum, are*

(A) (400, 100) *(B) (200, 100)*

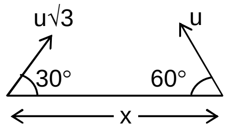
(C) (400, 200) *(D) (200, 200)*

Sol. **B.**

When the horizontal range is maximum, the maximum height attained is R/4 = 100 m. The velocity of the projectile is minimum at the highest point.

$\therefore$ Required point is (200, 100).

Prob 12. *Two particles are separated at a horizontal distance as shown in the adjacent figure. They are projected along the same line with different initial speeds. The time after which the horizontal distance between them becomes zero is*



(A) $\dfrac{x}{u}$ *(B)* $\dfrac{x}{2u}$

(C) $\dfrac{2u}{x}$ *(D) none of these*

Sol. **B.**

Both particles will collide at the highest point of their path. At highest point, only the horizontal component exists.

$$V_1 = u\sqrt{3}\cos 30° = \frac{3u}{2}$$

$$V_2 = u\cos 120° = -\frac{u}{2}$$

Relative velocity of the particle 1 w.r.t. particle 2 in x-direction = $\frac{3u}{2}+\frac{u}{2}=2u$

$\Rightarrow$ Required time $=\frac{x}{2u}$

Prob 13. *A particle is thrown vertically upward. Its velocity at half of the height is 10 m/s, then maximum height attained by it is: (g = 10 m/s²)*

(A) 8 m *(B) 20 m*
(C) 10 m *(D) 16 m*

Sol. **C.**

Suppose after travelling distance s, particle has the velocity 10 m/s.

So, $v^2 = u^2 - 2as$

$\Rightarrow$ $(10)^2 = u^2 - 2 \times 10s$ …(1)

At the maximum height, i.e. 2s, v = 0

$\Rightarrow$ $0 = u^2 - 2g(2s)$

$\Rightarrow$ $u^2 = 40s$ …(2)

From Eqs. (1) and (2), s = 5 m

$\Rightarrow$ 2s = 10 m

Prob 14. *A person walks up a stationary escalator in 90 sec. If the escalator moves with the person, first standing on it, it will take 1 minute to reach the top from ground. How much time it would take him to walk up the moving escalator?*

(A) 24 sec *(B) 48 sec*
(C) 36 sec *(D) 40 sec*

Sol. **C.**

Let L be the length of escalator.

$\therefore$ Relative speed $=\frac{L}{90}+\frac{L}{60}=\frac{L}{36}$

$\therefore$ Time taken to walk up the moving escalator $=\left(\frac{L}{L/36}\right)=36\,\text{sec}$

Prob 15. *A driver applies brakes on seeing a traffic signal 400 m ahead. At the time of applying the brakes the vehicle was moving with 15 m/s and retarding with 0.3 m/s². The distance of vehicle after 1 min from the traffic light is*

(A) 25 m *(B) 375 m*
(C) 360 m *(D) 40 m*

Sol. **A.**

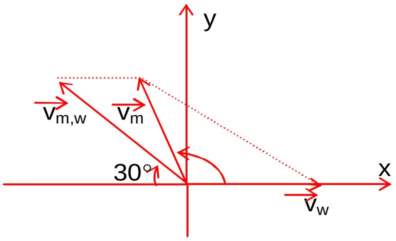
The maximum distance covered by the vehicle before coming to rest $=\frac{v^2}{2a}=\frac{(15)^2}{2(0.3)}=375$ m

The corresponding time = t = $\frac{v}{a}=\frac{15}{0.3}=50$ sec

$\therefore$ The distance of the vehicle from the traffic signal after one minute = 400 – 375 = 25 m

Prob 16. *A motorboat is to reach at a point 30° upstream on the other side of a river flowing with velocity 5 m/s. The velocity of the motorboat wrt water is $5\sqrt{3}$ m/s. The driver should steer the boat at an angle*
(A) 30° wrt the line of destination from starting point
(B) 60° wrt normal to the bank
(C) 120° wrt stream direction
(D) None of these

Sol. **C.**
The velocity of motorboat,



$$\vec{v}_m = \vec{v}_{mw} + \vec{v}_w$$
$$= -5\sqrt{3}\cos 30°\ \hat{i} + 5\sqrt{3}\sin 30°\,\hat{j} + 5\hat{i}$$
$$= -2.5\ \hat{i} + \frac{5\sqrt{3}}{2}\hat{j}$$
$$\phi = \tan^{-1}\left(-\frac{5\sqrt{3}}{2\times 2.5}\right) = \tan^{-1}\left(-\sqrt{3}\right) = 120^0$$

Prob 17. *The acceleration of a particle is increasing linearly with time t as bt. The particle starts from the origin with an initial velocity v_0. The distance travelled by the particle in time t will be*

(A) $v_0t + \frac{1}{6}bt^3$ *(B)* $v_0t + \frac{1}{3}bt^3$

(C) $v_0t + \frac{1}{3}bt^2$ *(D)* $v_0t + \frac{1}{2}bt^2$

Sol. **A.**
Given, acceleration a = bt
$$\Rightarrow \frac{dv}{dt} = bt \quad \Rightarrow v = \frac{bt^2}{2} + c$$
At t = 0, v = v_0 ⇒ c = v_0
$$\text{So, } v = \frac{bt^2}{2} + v_0$$
$$\Rightarrow \frac{ds}{dt} = \frac{bt^2}{2} + v_0$$
$$\Rightarrow s = \frac{bt^3}{6} + v_0t$$

Fill in the Blanks

Prob 1. The *position of a body w.r.t. time is given by x = $3t^3 - 6t^2 + 12t + 6$. At time t = 0, its acceleration is ____________.*

Sol.
$$\frac{dx}{dt} = 6t^2 - 12t + 12$$
$$\frac{d^2x}{dt^2} = 12t - 12$$
$$\left.\frac{d^2x}{dt^2}\right|_{t=0} = -12$$

Prob 2. *A body thrown up from the ground vertically passes the height of 10.2 m twice in an interval of 10 sec. Its initial velocity was ________ m/s and its time of journey upwards was ________sec (g = 10 m/s²).*

Sol. It takes 5 sec from its maximum height to the height of 10.2 m, travelling from rest at acceleration of 10 m/s^2. Hence, if this distance be s, then

$$s = \frac{1}{2} \times 10 \times 5^2 = 125 \text{ m}.$$

So, $u^2 = 2 \times 10 \times (125 + 10.2)$

$\Rightarrow$ $u = 52$ m/s

$\Rightarrow$ $t = \dfrac{52}{10} = 5.2$ sec.

Prob 3. *For a projectile projected at an angle ____________, the maximum height and horizontal range are equal.*

Sol. $\tan^{-1}(4)$

$$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$$

$\Rightarrow \sin 2\theta = \dfrac{\sin^2\theta}{2}$

$\Rightarrow \tan\theta = 4 \Rightarrow \theta = \tan^{-1} 4.$

Prob 4. *A car covers the first half of its distance between two places at a speed of 40 km/hr and the second half at 60 km/hr. The average speed of the car is ________ km/hr.*

Sol. 48 km/hr.

Let total distance be 2x km

$\therefore$ Total time taken $= \left(\dfrac{x}{40} + \dfrac{x}{60}\right)$hr $= \dfrac{x}{24}$hr

Therefore, average speed = $\dfrac{2x}{\left(\dfrac{x}{24}\right)} = 48$ km/hr.

Prob 5. *In a uniform motion, the particle travels in a ______________ and traces equal _________however ______________intervals of ________________ be taken.*

Sol. Straight line, displacements, small, time.

True or False Type Questions

Prob 1. *A particle in one-dimensional motion with positive value of acceleration must be speeding up.*

Sol. False.

If the velocity of the body is negative, then even in case of positive acceleration the body speeds down, e.g. a body projected up slows down even when acceleration is positive.

Prob 2. *A particle in one-dimensional motion with constant speed must have zero acceleration.*

Sol. True.

As the direction of motion remains unchanged, therefore, if the speed is zero the acceleration must also be zero.

Prob 3. *A particle moves with a uniform velocity in a straight line. If another particle moves such that it is always directed towards the first particle then the motion of the second particle is also along a straight line.*

Sol. False.
Because the second particle is always directed towards the first particle, the motion of the second particle can be straight line only in the special case when it follows the uniformly moving first particle along the same straight line.

Prob 4. *A particle in one-dimensional motion with zero speed at any instant may have non-zero acceleration at that instant.*

Sol. True.
When a body begins to fall freely under gravity, its speed is zero but it has non-zero acceleration of 9.8 m/s^2

Prob 5. *If a base ball player can throw a ball to a maximum distance d over the ground, then the maximum vertical height to which he can throw it will be equal to d/2. Assume that initial speed of the ball is same in both the cases.*

Sol. True.
$R_{max} = d = \frac{u^2}{g}$ and $H_{max} = \frac{u^2}{2g} = \frac{d}{2}$

Prob 6. *A bus moving towards north takes a turn and starts moving towards east with same speed. There will be no change in the velocity of the bus.*

Sol. False.
The direction will change.

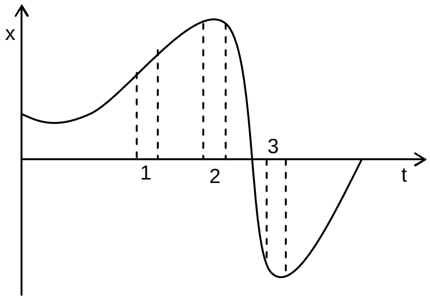
ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. A wooden block of mass 10 g is dropped from the top of a cliff 100 m high. Simultaneously, a bullet of mass 10 g is fired from the foot of the cliff upward with a velocity 100 m/s. After what time the bullet and the block meet ?

2. Figure shows the x–t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.



3. Two balls of different masses are thrown vertically upwards with the same speed. During their downward journey, they pass through the point of projection with the same speed. Neglect air resistance. Is this statement correct?

4. Galileo stated that "For elevations which exceed or fall short of 45° by equal amounts, the ranges are equal." Prove this statement.

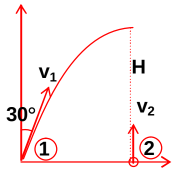
5. A block slides down a smooth inclined plane when released from the top while another falls freely from the same point. Which one of them will strike the ground earlier ?

6. A stone is thrown horizontally with a speed $\sqrt{2gh}$ from the top of a wall of height h. What is the distance from the wall when it reaches the ground?

7. What is the angle θ of projection with horizontal plane of a projectile if its range is $\frac{\sqrt{3}v^2}{2g}$, where v is velocity of projection?

8. Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km/h in the same direction, with A ahead of B. The driver of B desires to overtake A and accelerates by 1 m/s^2. If after 50 s, the guard of B just brushes past driver of A, calculate the original distance between the guard of B and the driver of A.

9. A particle 1 is projected with speed v_1 from a point A making an angle of 30° with the vertical. At the same instant, a second particle 2 is thrown vertically upwards from position B with velocity v_2. The two particles reach height H, the highest point on the parabolic path of particle 1 simultaneously. Calculate the ratio $\frac{v_1}{v_2}$.



10. A car is moving with a speed of 30 m/s on a circular path of radius 500 m. Its speed is increasing at a rate of 2 m/s^2. What is the acceleration of the car?

11. A particle is thrown vertically upward. Its velocity at half of the maximum height is 10 m/s, then calculate the maximum height attained by it. ($g=10\ m/s^2$)

12. A car moving with a speed of 40 km/hr can be stopped, by applying brakes, in 2 meters. If the same car is moving with a speed of 80 km/hr, what is the minimum stopping distance?

13. The position of a particle moving along x-axis is given by $x = a + bt^2$ where x is in meter and t in seconds. The constants a and b are 4.5 m and 3.5 m/s^2 respectively. Find
    (a) initial velocity
    (b) velocity at t = 3 seconds.
    (c) average velocity during the time interval t = 1 s to t = 3 s.

14. A body dropped from a height h, with initial velocity zero, strikes the ground with velocity 3 m/s. Another body of the same mass is dropped from the height h with an initial velocity of 4 m/s. Find the final velocity with which it strikes the ground.

15. The velocity of a train increases uniformly from 20 km/hr to 60 km/hr in 4 hours. Find the distance travelled by the train during this period.

16. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β and comes to rest. If the total time elapsed is t, then find the maximum velocity acquired by the car.

17. An aeroplane is flying horizontally with a velocity of 216 km/hr and at a height of 1960 m. When it is vertically above a point A on the ground, a bomb is released from it. The bomb strikes the ground at point B. Find the distance AB.

18. A body is projected horizontally with a speed of 20 m/s from the top of a tower. What will be its speed after nearly 5 sec? ($g = 10\ m/s^2$)

19. A bus moves a distance of 200 m. It covers the first half of the distance at speed 40 km/hr and the second half of the distance at speed v. The average speed is 48 km/hr. Find the value of v.

20. A ball is dropped from height of 90 m on a floor. The ball loses one tenth of its speed. Put the speed–time graph of its motion between t = 0 and 12 sec. ($g = 10\ m/s^2$)

Level - I

1. The position of a particle along the x-axis is given in centimeters by $x=9.75+1.50t^3$, where t is in seconds. Consider the time interval t = 2 s to t = 3 s and calculate
   (a) the average velocity
   (b) instantaneous velocity at t = 2 s;
   (c) the instantaneous velocity when t = 2.5 s;
   (d) the instantaneous velocity when the particle is mid way between its position at t = 2 s and t = 3 s.

2. A train started from rest and moved with constant acceleration. At one time it was travelling at 33.0 m/s and 160 m farther it was travelling at 54.0 m/s. Calculate
   (a) the acceleration.
   (b) the time required to travel the 160 m.
   (c) the time required to attain the speed of 33.0 m/s.
   (d) the distance moved from rest to the time the train had a speed of 33 m/s.

3. A body travelling in a straight line travels 2 m in the first two seconds and 2.2 m in the next four seconds with constant retardation. What will be its velocity at the end of the seventh second from the start?

4. A motorcyclist moving with uniform retardation takes 10 s and 20 s to travel successive quarter kilometer. How much further he will travel before coming to rest?

5. A car is moving on a straight road with a speed 20 m/s. At t = 0, the driver of the car applies the brakes after watching an obstacle 150 m ahead. After application of brakes the car retards with 2 m/s^2. Find the position of the car from the obstacle at t =15 s.

6. A ball is thrown with a velocity of 100 ms^{-1} at an angle of 30° to the horizontal and meets the same horizontal plane later. Find
   (a) its time of flight
   (b) the horizontal distance it travels
   (c) the velocity with which it strikes the ground at the end of its flight. [g = 9.8 ms^{-2}]

7. A projectile shot at an angle of 60° above the horizontal strikes a wall 30 m away at a point 15 m above the point of projection.
   (a) Find the speed of projection.
   (b) Find the magnitude of velocity of the projectile when it strikes the wall.

8. A ball is thrown vertically up with a certain velocity from the top of a tower of height 40 m. At 4.5 m above the top of the tower its speed is exactly half of that it will have at 4.5 m below the top of the tower. Find the maximum height reached by the ball above the ground?

9. A rotating fan completes 1200 revolutions every minute. Consider a point on the tip of the blade, which has a radius of 0.15 m
   (a) Through what distance does the point move in one revolution?
   (b) What is the speed of the point? (c) What is its acceleration?

10. A particle A is moving along a straight line with velocity 3 m/s and another particle B has a velocity 5 m/s at an angle 30° to the path of A. Find the velocity of B relative to A.

11. A ball dropped from some height covers half of its total height during the last second of its free fall. Find
(a) time of flight
(b) height of its fall
(c) speed with which it strikes the ground.

12. From the foot of an inclined plane, whose rise is 7 in 25, a shot is projected with a velocity of 196 m/s at an angle of 30^o with the horizontal up the plane. Find the range.

13. A man walking eastward at 5 m/s observes that wind is blowing from the north. On doubling his speed eastward, he observes that wind is blowing from north-east. Find the velocity of the wind.

14. The acceleration experienced by a moving boat after its engine is cut off, is given by $\frac{d\omega}{dt} = -kv^3$, where k is a constant if v_0 is the magnitude of the velocity at cutoff find the magnitude of the velocity at time t after the cut off.

15. A boy throws a ball vertically upward with an initial speed of 15.0 m/s. The ball was released when it was at 2.00 m above ground. The boy catches it at the same point as the point of projection.
(a) What is maximum height reached by the ball ?
(b) How long is the ball in the air?

Level- II

1. The equation of motion of a particle moving along a straight line is given as $x = \frac{1}{2}vt$ when x, v t, have usual meaning, prove that the acceleration is constant.

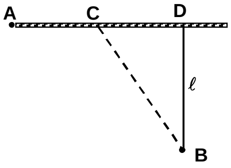
2. A point moving in a straight line traversed half the distance with a velocity v_o. The remaining part of the distance was covered with velocity v_1 for half the time, and with velocity v_2 for the other half of the time. Find the mean velocity of the point averaged over the whole time of motion.

3. At the instant the traffic light turns green, an automobile starts with a constant acceleration of 2.2 m/s^2. At the same instant a truck, travelling with a constant speed of 9.5 m/s, overtakes and passes the automobile.
   (a) How far beyond the starting point will the automobile overtake the truck?
   (b) How fast will the car be traveling at the instant?
   (It is instructive to plot a qualitative graph of 'x' versus t for each vehicle.)

4. A balloon is ascending vertically with an acceleration of 1 m/s^2. Two stones are dropped from it at an interval of 2 s. Find the distance between them 1.5 sec after the second stone is released.

5. Two particles move in a uniform gravitational field with an acceleration g. At the initial moment the particles were located at one point in space and moved with velocities v_1 = 3.0 m/s and v_2 = 4.0 m/s horizontally in opposite directions. Find the distance between the particles at the moment when their velocity vectors become mutually perpendicular.

6. From point A located on a highway (Fig.) one has to get by car as soon as possible to point B located in the field at a distance ℓ from the highway. It is known that the car moves in the field n times slower than the highway. At what distance from point D one must turn off the highway?



7. To a man walking at 7 km/hr due west the wind appears to blow from the north west, but when he walks at 3 km/hr due west the wind appears to blow from the north. What is the actual direction of the wind and what is its velocity?

8. A particle is projected with a velocity u at an angle θ with the horizontal. Find the radius of the curvature of the parabola traced out by the particle at the point where velocity makes an angle (θ/2) with the horizontal.

9. A ship A streams due north at 16 km/hr and a ship B due west at 12 km/hr. At a certain instant B is 10 km north-east of A. Find the velocity of A relative to B. Find also the nearest distance of approach of ships.

10. A particle moves in x-y plane with constant acceleration 'a' directed along the negative y-axis. The equation of motion of the particle has the form $y = px - qx^2$ where p and q are positive constants. Find the velocity of the particle at the origin.

11. The position vector of a particle varies with time t as x = kt(1– αt), where k is a constant vector and α is a positive factor. Find
    (a) the velocity v and the acceleration a of the particle as functions of time.
    (b) the time interval Δt taken by the particle to return to the initial points, and the distance covered during that time.

12. A balloon starts rising from the surface of the Earth. The ascent rate is constant and equal to v_o. Due to the wind the balloon gathers the horizontal velocity component v_x = ay, where a is a constant and y is the height of ascent. Find how the following quantities depend on the height of ascent.
    (a) The horizontal drift of the balloon x(y);
    (b) The total, tangential, and normal accelerations of the balloon.

13. Two boats A and B move away from buoy anchored at the middle of a river along mutually perpendicular straight lines, the boat A along the river and the boat B across the river. Having moved off an equal distance from the buoy the boat returned. Find the times of motion of boats t_A / t_B if the velocity of each boat with respect to water is n times greater than the stream velocity.

14. A ball starts failing with zero initial velocity on a smooth inclined plane forming an angle α with the horizontal. Having fallen the distance h, the ball rebounds elastically off the inclined plane. At what distance from the impact point will the ball rebound for the second time?

15. A man standing in an elevator observes a screw fall from the ceiling. The ceiling is 3m above the floor.
    (a) If the elevator is moving upward with a speed of 2.2 m/s, how long does it take for the screw to hit the floor.
    (b) How long is the screw in the air if the elevator starts from rest when the screw falls and moves upwards with a constant acceleration of a = 4.0 m/s^2.

Objective:

Level- I

1. A bus starts moving with an acceleration of 2 m/s^2. A cyclist 96 m behind the bus starts simultaneously towards the bus at 20 m/s. After what time will he be able to overtake the bus?
(A) 4 sec (B) 8 sec
(C) 12 sec (D) 16 sec

2. A body is projected at an angle of 30° to the horizontal with a speed of 30 m/s. What will be the angle with the horizontal after 1.5 seconds? [Take g = 10 m/s^2]
(A) 0° (B) 30°
(C) 60° (D) 90°

3. A ball rolls off the top of stairway with a horizontal velocity of magnitude 1.8 m/s. The steps are 0.20 m high and 0.2 m wide. Which step will the ball hit first?
(A) First (B) Second
(C) Third (D) Fourth

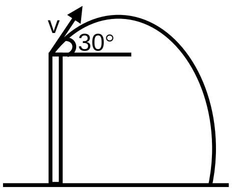
4. In a projectile motion, the velocity is
(A) never perpendicular to the acceleration
(B) always perpendicular to the acceleration
(C) perpendicular to acceleration at one instant only
(D) perpendicular to acceleration at two instants only

5. A boat is sent across a river with a velocity of 8 km/hr. If the resultant velocity of the boat is 10 km/hr, then velocity of the river is
(A) 12.8 km/hr (B) 6 km/hr
(C) 8 km/hr (D) 10 km/hr

6. The position vector of a particle is $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$. The velocity of the particle is
(A) parallel to position vector (B) perpendicular to position vector
(C) directed towards the origin (D) directed away from the origin

7. A particle starts from rest with constant acceleration. The ratio of space-average velocity to the time-average velocity is
(A) 1/2 (B) 3/4
(C) 4/3 (D) 3/2

8. A ball is projected with a speed of 20 m/s at an angle of 30° from a point on the top of a very high tower. The time after which its velocity becomes perpendicular to the velocity of projection (take g = 10 m/s^2) is
(A) 0.5 sec (B) 2 sec
(C) 4 sec (D) never

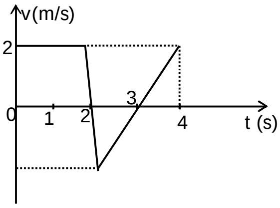


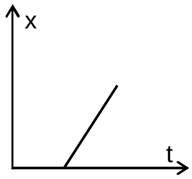
9. A particle is moving along a circular path of radius r with uniform speed v. Through what angle does its angular velocity change when it completes half of the circular path?
(A) 0° (B) 45°
(C) 180° (D) 360°

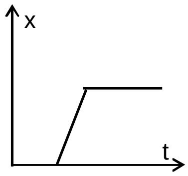
10. Three particles start moving simultaneously from a point on a horizontal smooth plane. First particle moves with speed v_1 towards east, second particle moves towards north with speed v_2 and third one moves towards north east. The velocity of the third particle, so that the three always lie on a straight line, is
(A) $\frac{v_1 + v_2}{2}$
(B) $\sqrt{v_1 v_2}$ s
(C) $\frac{v_1 v_2}{v_1 + v_2}$
(D) $\sqrt{2}\frac{v_1 v_2}{v_1 + v_2}$

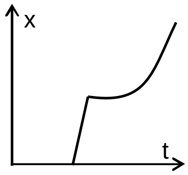
11. A particle is moving along a circular path of radius 5 m and with uniform speed 5 m/s. What will be the average acceleration when the particle completes half revolution?
(A) zero
(B) 10 m/s^2
(C) 10 π m/s^2
(D) 10/π m/s^2

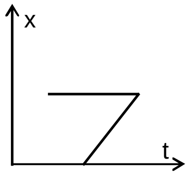
12. The velocity v versus t graph of a body in a straight line is as shown in the adjacent figure. The displacement of the body in 4 sec is
(A) 2 m
(B) 4 m
(C) 6 cm
(D) 8 m



13. Which of the following displacement–time graph is not possible?
(A)


(B)


(C)


(D)


14. A train of length 100 m travelling at 50 m/s overtakes another train of length 200 m moving at 30 m/s. The time taken by the first train to overtake the second is
(A) 5 sec
(B) 10 sec
(C) 15 sec
(D) 20 sec

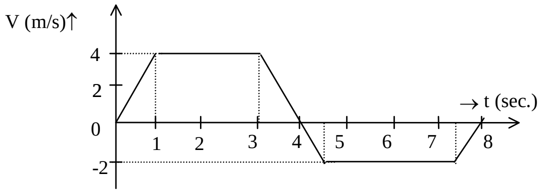
15. A balloon starts from the ground with an acceleration of 1.25 m/s^2. After 8 sec, a stone is released from the balloon. The stone will
(A) cover a distance of 40 m
(B) have a displacement of 50 m
(C) reach the ground in 4 sec
(D) begin to move down after being released.

16. A body moving with a uniform acceleration has velocities of u and v when passing through points A and B in its path. The velocity of the body midway between A and B is

(A) $\frac{u+v}{2}$ (B) $\sqrt{\frac{u^2+v^2}{2}}$

(C) $\sqrt{uv}$ (D) None of these

17. The velocity–time graph of a linear motion is shown in figure. The displacement from the origin after 8 sec. is



(A) 5 m
(B) 16 m
(C) 8 m
(D) 6 m

18. A ball is thrown up vertically with speed u. At the same instant another ball B is released from rest from a height h. At time t, the speed of A relative to B is

(A) u (B) u – 2gt

(C) $\sqrt{u^2 - 2gh}$ (D) u – gt

19. The greatest height to which a man can throw a stone is h. The greatest distance to which he can throw will be:

(A) h/2 (B) h

(C) 2 h (D) 4 h

20. A motor boat is to reach at a point 30° upstream on the other side of a river flowing with velocity 5 m/s. Velocity of motor boat with respect to water is $5\sqrt{3}$ m/sec. The driver should steer the boat an angle:

(A) 30° w.r.t. the line of destination from starting point
(B) 60° w.r.t.. normal to the bank
(C) 120° w.r.t. stream direction
(D) None of these

Fill in the Blanks

1. A particle moves in a circle of radius R. In half the period of revolution its displacement is _______________________ and distance covered is _______________________.
2. A particle is projected with an initial velocity of 200 m/s in a direction which makes an angle of 30° with the vertical, the horizontal distance travelled by the particle in 3 sec is ____________ m.
3. A stone is released from an elevator going up with an acceleration a. the acceleration of the stone after the release is _________________________.
4. For angles of projection which exceed or fall short of 45° by equal amounts, the ranges are ____________ .
5. The weight of a body in projectile motion is ______________________.

True or False Type Questions

1. If the displacement y of a particle is proportional to t^2, i.e. if $y \propto t^2$, then its initial velocity will be non-zero.

2. The instantaneous velocity vector is always in the direction of motion.

3. The magnitude of the sum of two displacement vectors must be greater than the magnitude of either displacement vectors.

4. A particle can move with constant velocity and constant acceleration simultanesouly.

5. The average velocity of a particle moving on a straight line is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval (infinite acceleration are not allowed).

Level - II

1. A stone is thrown vertically upward with an initial velocity v_0 . The distance travelled in time $4v_0/3g$ is

(A) $\frac{2v_0^2}{g}$ (B) $\frac{v_0^2}{2g}$

(C) $\frac{4v_0^2}{3g}$ (D) $\frac{4v_0^2}{9g}$

2. The motion of a body depends on time according to the equation $\frac{dv}{dt}$ = 6.0 – 3v, where v is speed in m/s and t is time in second. If the body was at rest at t = 0 which of the following statements is correct?
(A) The speed of the body approaches 2 m/s after long time
(B) The speed varies linearly with time
(C) The acceleration remains constant
(D) The initial acceleration is zero

3. If the angle (θ) between velocity vector and the acceleration vector is $90° < \theta < 180°$. The body is moving on a:
(A) Straight path with retardation (B) Straight path with acceleration
(C) Curvilinear path with acceleration (D) Curvilinear path with retardation

4. The relation between time t and distance x is $t = \alpha x^2 + \beta x$ where α and β are constants. The retardation is: (if v is velocity of the particle)

(A) $2\alpha v^3$ (B) $2\beta v^2$
(C) $2\alpha\beta v^2$ (D) $2\beta^2 v^3$

5. Two particles start moving along the same straight line starting at the same moment from the same point. The first moves with constant velocity u and the second with constant acceleration f. During the time that elapses before second catches the first, the greatest distance between the particles is

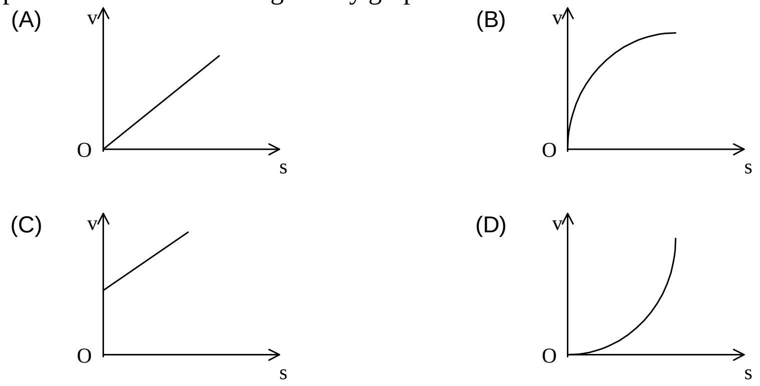
(A) $\frac{u}{f}$ (B) $\frac{u^2}{2f}$

(C) $\frac{f}{2u^2}$ (D) $\frac{u^2}{f}$

6. A particle is projected horizontally in air at a height of 25 m from the ground with a speed of 10 m/s. The speed of the particle after 2 seconds will be
(A) 10 m/s (B) 22.4 m/s
(C) 25 m/s (D) 28.4 m/s

7. A man can swim at a speed of 5 km/h w.r.t. water. He wants to cross a 1.5 km wide river flowing at 3 km/h. He keeps himself always at an angle of 60^o with the flow direction while swimming. The time taken by him to cross the river will be
(A) 0.25 hr. (B) 0.35 hr.
(C) 0.45 hr. (D) 0.55 hr.

8. A body starts from rest and moves along a straight line with constant acceleration. The variation of speed v with distance s is given by graph



9. The displacement of a particle in a straight line motion is given by $s = 1 + 10t - 5t^2$. The correct representation of the motion is

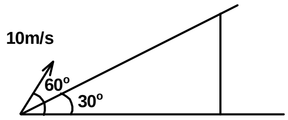
10. The position of a particle along x-axis at time t is given by $x = 1 + t - t^2$. The distance travelled by the particle in first 2 seconds is

(A) 1m (B) 2m
(C) 2.5 m (D) 3m

11. From the top of a tower, two particles A and B are projected simultaneously with speeds of 3 m/s and 4 m/s, respectively, in horizontally opposite directions at time t = 0. At time t = $\left(2\sqrt{3}/10\right)$ sec, the angle between their velocities is

(A) 60° (B) 45°
(C) 90° (D) 30°

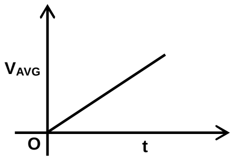
12. A particle is thrown at time t = 0, with a velocity of 10 m/s at an angle of 60° with the horizontal, from a point on an incline plane, making an angle of 30° with the horizontal. The time when the velocity of the projectile becomes parallel to the incline is



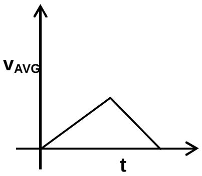
(A) $\frac{2}{\sqrt{3}}$sec (B) $\frac{1}{\sqrt{3}}$sec
(C) $\sqrt{3}$sec (D) $\frac{1}{2\sqrt{3}}$sec

13. A particle is moving along a straight line with a velocity of $\frac{1}{2}kt^2$, where k is a constant. Then, the average velocity of the particle as a function of time is best represented by

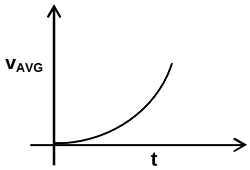
(A)



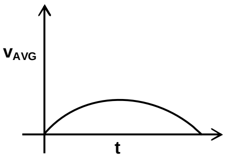
(B)



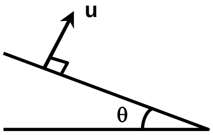
(C)



(D)



14. A particle is projected perpendicularly to an inclined plane as shown in the adjacent figure. If the initial velocity of the particle is u, calculate how far from the point of projection does it hit the plane again if the distance is measured along the plane?



(A) $\frac{2u^2}{g}$ (B) zero

(C) $\frac{2u^2}{g}\sin\theta$ (D) $\frac{2u^2}{g}\tan\theta\sec\theta$

15. A box is moving up on an inclined plane of inclination 30° with a constant acceleration of $5\,m/s^2$. A particle is projected with a velocity of $5\sqrt{3}$ m/s inside a box, at an angle of 30° with the base. Then, the time after which it again strikes the same base of the box is (assume during its flight, particle does not hit any other side of the box)

(A) 1 sec (B) 2 sec

(C) 1.5 sec (D) data insufficient

16. The height y and distance x along the horizontal for a body projected in the vertical plane are given by $y = 8t - 5t^2$ and $x = 6t$. The initial speed of projection is

(A) 8 m/s (B) 9 m/s

(C) 10 m/s (D) (10/3) m/s

More than one choice are correct:

17. Read and examine the following statements. Which of the following is /are correct/ true?
(A) $a_x \neq 0$, $a_y = 0$, $a_z = 0$ is necessarily a case of one dimensional motion.
(B) $v_x \neq 0$, $v_y = 0$, $v_z = 0$ is necessarily a case of one dimensional motion.
(C) If $v_x \neq 0$, $a_x \neq 0$; $v_y \neq 0$, $a_y \neq 0$; $v_z = 0$, $a_z = 0$ is necessarily a case of motion in one plane.
(D) If $a_x = a_y = a_z = 0$ is necessarily a case of one dimensional motion.

18. The rain is falling vertically downwards. A man walking on the road holds his umberalla tilted. Now, suddenly the rain stops and there is afternoon sun just above the head. In order to protect himself from sun-rays, he holds the umberalla vertical. The reason assigned can be
(A) The speed of light is much higher than that of speed of rain drops.
(B) The speed of light is much higher than that of speed of man.

(C) Actually, the angle of tilt of umberalla in sun-light is very small in comparison to the angle of tilt of umberalla in rain.
(D) Light is behaving as a wave, not as a particle here.

19. A swimmer swims in a flowing river.
$\vec{v}_{s,r}$ = velocity of swimmer with respect to (w.r.t.) river water.
$\vec{v}_{r,g}$ = velocity of river water w.r.t. ground,
$\vec{v}_{s,g}$ =velocity of swimmer w.r.t. ground.
The swimmer intends to reach at the opposite bank of the river. It is possible only when,
(A) $v_{r,g} > v_{s,r}$
(B) $v_{r,g} < v_{s,r}$
(C) $v_{s,g} < v_{s,r}$
(D) none of these.

True or False Type Questions

1. The instantaneous velocity of a body is equal to its average velocity when it is moving with uniform velocity.

2. The average velocity is always equal to the mean value of the initial and final velocities.

3. If the displacement y of a particle is proportional to time, i.e. if $y \propto t$, then the displacement of the particle will be non-zero.

4. Two bullets are fired simultaneously, horizontally and with different speeds from the same place. The two bullets will hit the ground simultaneously.

5. A man while walking observes that the rain is falling vertically downward, if he suddenly stops walking then the rain drops will strike him on his back.

Fill in the Blanks

1. If the velocity of a particle is given by $v = \sqrt{180 - 16x}$ m/s, its acceleration will be ___________.

2. A boat takes 2 hours to travel 8 km and back in a still water lake with water velocity 4 km/hr. The time taken for going up-stream 8 km and coming back is _______________ minutes.

3. The velocity of a particle moving with constant acceleration at an instant t is 10 m/s. After 5 sec the velocity is 20 m/s. The velocity at 3 sec before was ______________________.

4. A food packet is released from a helicopter which is rising steadily at 2 m/s. After 2 sec the velocity of the packet is ________________ ($g = 9.8$ m/s^2).

5. A horizontal stream of water leaves an opening in the side of a tank. If the opening is h metre above the ground and the stream hits the ground D meter away, and the acceleration due to gravity is ‘g’, the speed of water as it leaves the tank in terms of g, h and D is ________________.

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. $-x = -100t + \frac{1}{2}gt^2$; $100 - x = \frac{1}{2}gt^2$

   $-x = -100\,t + 100 - x \quad \Rightarrow t = 1$ s.

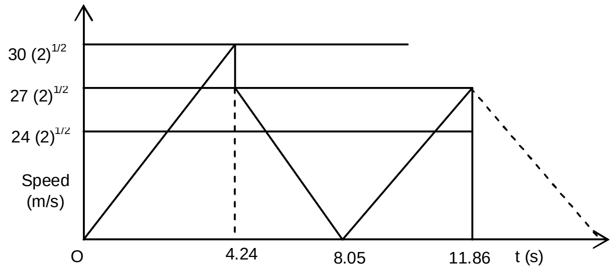
2. Greatest in 3, least in 2; v > 0 in 1 and 2, v < 0 in 3.

3. The given statement is true. Both the balls have equal acceleration due to gravity. Both the balls would attain the same height and would pass through the point of projection with the same speed.

4. The values of sin (90 + 2θ) and sin (90 – 2θ) are the same, equal to cos 2θ. Therefore, range are equal for elevation which exceed or falls short of 45° by equal amount θ.

5. The block will reach the ground earlier which falls freely.

6. x = 2h
7. $\theta = 30^\circ$
9. $\frac{2}{\sqrt{3}}$
10. 2.7 m/s^2
11. 10 m
12. 8m
13. (a) 0 m/s. (b) 21.0 m/s. (c) 14.0 m/s.
14. 5 m/s
15. 160 km
16. $\frac{\alpha\beta t}{\alpha + \beta}$
17. 1200 m
18. 54 m/s
19. 60 km/hr.
20.



Level - I

1. (a) 28.5 cm/s (b) 18.0 cm/s (c) 28.1cm/s (d) 30.4 cm/s
2. (a) 5.71m/s^2 (b) 3.68s (c) 5.78s (d) 95.4m
3. 0.1m/s
4. 10.42 m
5. 50 m
6. (a) 10.20s; (b)883.67 m; (c) 100 m/s
7. (a) 21.8 m/s, (b) 13.55 m/s
8. 47.5 m
9. (a) 94cm (b) 19m/s (c) 2400 m/s^2.
10. [2.832 m/s at an angle of 32^o with $\overrightarrow{V_B}$
11. (a) 3.41 seconds (b) 58.14 m(c) 34.10 m/s
12. 1749.8 m
13. $\left(5\hat{i}-5\hat{j}\right)$ m/s
14. $\dfrac{v_0}{\sqrt{1+2kv_0^2t}}$
15. (i) 13.5 m, (ii) 3.06 sec.

Level – II

2. $\dfrac{2v_o\left(v_1+v_2\right)}{2v_o+v_1+v_2}$
3. (A) 82 m; (B) 19 m/s
4. 55m
5. 2.5 m
6. $CD=\dfrac{\ell}{\sqrt{n^2-1}}$
7. 5 km/hr, 53^o North of East.
8. $\dfrac{u^2\cos^2\theta}{g\ \ \cos^3\dfrac{\theta}{2}}$
9. 20 km/hr at an angle $\tan^{-1}\dfrac{3}{4}$ or 37^o east of north, $\sqrt{2}$ km
10. $\sqrt{\dfrac{a\ \ \left(p^2+1\right)}{2q}}$
11. (a) $v = k(1-2\alpha t)$, $a = -2\alpha k$ (b) $\Delta t = 1/\alpha$, $s = k/2\alpha$
12. (a) $x=\left(\dfrac{a}{2v_o}\right)y^2$ (b) $\omega = av_o$, $\omega_t=\dfrac{a^2v_0y}{\sqrt{v_0^2+a^2y^2}}$, $\omega_n=\dfrac{av_0^2}{\sqrt{v_0^2+a^2y^2}}$
13. $\left[\dfrac{n}{\sqrt{n^2-1}}\right]$
14. $\ell = 8\,h\sin\alpha$
15. (a) 0.78 sec. (b) 0.66 sec.

Objective:

Level – I

1.	B	2.	A	3.	D
4.	C	5.	B	6.	B
7.	C	8.	C	9.	A
10.	D	11.	D	12.	B
13.	D	14.	C	15.	C
16.	B	17.	A	18.	A
19.	C	20.	A		

Fill In The Blanks

1.	2R and πR	2.	300 m
3.	downward	4.	equal
5.	zero		

True or False Type Questions

1.	False	2.	True
3.	False	4.	False
5.	False		

Level – II

1.	D	2.	A	3.	D
4.	A	5.	B	6.	B
7.	B	8.	D	9.	D
10.	C	11.	C	12.	B
13.	C	14.	D	15.	A
16.	C	17.	C, D	18.	B, C
19.	B, C				

True or False Type Questions

1.	True	2.	False
3.	True	4.	True
5.	True		

Fill In the Blanks

1.	$-8\ m/s^2$	2.	160
3.	4 m/s	4.	-17.6 m/s
5.	$D\sqrt{\frac{g}{2h}}$		

Learnaf
Educational Revolution

PHYSICS

FITJEE

PINNACLE

For – JEE (Main/Advanced)

LAWS OF MOTION

Laws of Motion

Syllabus:

Force and inertia, Newton's first law of motion; Momentum, Newton's second law of motion, Impulse; Newton's third law of motion; Law of conservation of linear momentum and its applications; Equilibrium of concurrent forces; Static and kinetic friction, laws of friction, rolling friction, lubrication; Examples of variable mass situation. Dynamics of uniform circular motion; Centripetal force, examples of circular motion (vehicle on level circular road, vehicle on banked road); Inertial and non- inertial frame (elementary idea).

Introduction:

In the introduction of the preceding chapter "Kinematics" we studied that mechanics can be broadly classified into two categories namely *"Kinematics and Dynamics"*.

In kinematics our prime concern was to define the physical quantities like position, velocity, acceleration and to establish the relations among them. But we never tried to answer the questions like "What causes the bodies to move from one place to another? What makes the body to gain or loose its speed? . . . etc.

Dynamics is that branch of mechanics which gives not only qualitative but quantitative description of the above quantities. This branch of mechanics also explains few basic laws which governs the motion of bodies.

Before you start studying this chapter try to analyse the answers for the following questions:

(i) Why to set a ball into motion in playground someone has to kick it or throw it?

(ii) Why a navigator has to row the boat in still water to move the boat ?

(iii) Why the branches of a tree swing when wind blows ?

In all these cases one can observe that some external agency (like hand or legs of player or wind) is coming into contact with the respective objects that are going into motion.

(iv) When a ball is released from the top of a building, the ball falls by itself, though no one is pushing it in downward direction. Why ?

(v) When a piece of iron is placed near a magnet, magnet attracts it from certain distance itself. How ?

In these cases we can observe that even though no external agency is physically coming into contact with the objects still the objects are moving.

Hence we can conclude that an animate or inanimate external agency is required to change the state of a body (i.e. from rest to motion or vice versa). To understand the logic behind this type of questions more clearly, we should know about two basic physical quantities namely *inertia and force.*

Inertia: It is a very common observation for all of us that any book kept on our study table will not move by itself, i.e. until and unless it is acted upon by any external force, it will not change its state of rest.

To explain the reason behind this type of questions Italian scientist Galileo has defined a new physical quantity known as *inertia*. Though it was introduced by Galileo the effective use of this term and its usage for explaining the motion of the bodies was done by another reputed physicist, Sir Isaac Newton.
"Inertia is an inherent property of a body by virtue of which it cannot change its state (i.e. rest or motion) by itself."

It says that every body in the universe does have a property which is hidden in itself and because of this property the body is unable to change its state by itself, i.e. from state of rest to state of motion or vice versa or even it's direction. This inertia is of 3 types, namely
(a) inertia of rest (b) inertia of motion and (c) inertia of direction

Inertia of rest: Inertia of rest is the inability of a body by virtue of which it can't change its state of rest to state of motion. That means any body which is at rest continues to be in the state of rest only and it can't go further into state of motion by itself.

Examples:
(i) *Passengers standing or sitting loosely in a bus experience jerk in the backward direction when the bus suddenly starts moving.* This is due to the fact that when the bus suddenly starts its motion, the lower parts of the human body shares the motion but the upper part tends to remain at rest due to inertia of rest.

(ii) *When a bullet is fired into a tightly- fitted glass pane from a reasonably close range, it makes a clear circular hole in the glass pane.* This is due to the fact that particles of glass around the hole tend to remain at rest due to inertia of rest. So they are unable to share the fast motion of the bullet.

Inertia of motion: Inertia of motion is the inability of a body by virtue of which it can not change its state of uniform motion along a straight line to state of rest. That means any body which is in uniform motion can't come to rest by itself until and unless some external force acts on it.

Examples:
(i) *A passenger standing in a moving bus falls forward when the bus stops suddenly*. This is due to fact that the lower part of the body comes to rest along with the bus but the upper part of the body remains in a state of motion on account of "inertia of motion".

(ii) *An athlete runs for some distance before taking a long jump.* In this way, the athlete gains momentum and this inertia of motion helps him in taking longer jump.

Inertia of direction: Inertia of direction is inability of a body by virtue of which it can't change its direction by itself. This means a body moving along a straight line can't change its direction by itself, until and unless it is acted upon by any external force.

Examples:
(i) *When a running car suddenly takes a turn, the passengers experience a jerk in the outward direction.* This is because the passengers tend to maintain their original direction of motion due to inertia of direction.

(ii) *A stone tied to one end of a string is whirled in a horizontal circle. When string breaks, the stone tends to fly off tangentially along a straight line.* This is due to inertia of direction.

Note: The mass of the body is the indirect measure of the inertia of that body.

Exercise 1.
(i) ***Why does the sparks coming out tangentially from the grid store when knife or any other such objects are sharpened ?***
(ii) ***A clothes line hangs between two poles. No matter how tightly the line is stretched, it always sags a little at the centre. Explain.***

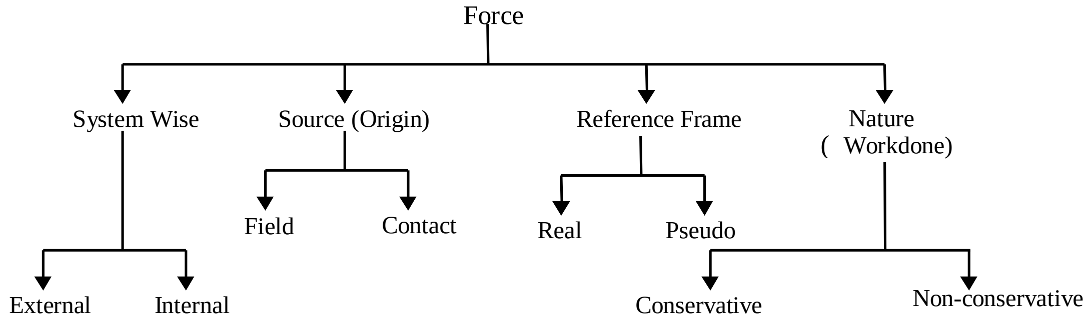
Now let us try to understand another physical quantity "*force*", with the help of which only the mechanical state of a body changes.

"Force is that which pushes or pulls the body or tends to change the state of rest or of uniform motion in a straight line."

(a) It produces or tries to produce motion in a body at rest.
(b) It stops or tries to stop a moving body.
(c) It changes or tries to change the direction of motion of body.
(d) It produces a change in the shape of the body.

CLASSIFICATION OF FORCES

There are different types of forces in our universe. Based on the nature of the interaction between two bodies, forces may be broadly classified as under



Since we are going to encounter these forces in our analysis we will briefly discuss each force and how it acts between two bodies, its nature etc and how we are going to take it into account.

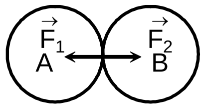
(a) Contact force: The force exerted by one surface over the surface of another body when they are physically in contact with each other is known as *contact force.*

If two surfaces that are coming into contact are perfectly smooth, then the entire contact force will act only perpendicularly (normal) to their surface of contact and it is known as "*Normal force or Normal reaction.*"

If two surfaces that are coming into contact are rough surfaces, then one component of this contact force acts perpendicular to their surface of contact and the other component of this force acts in tangential direction to their surface of contact and this component is known as "*force of friction*."

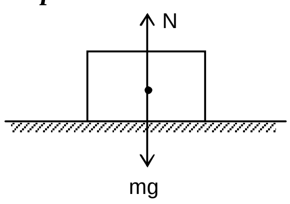
Normal Reaction, Tension, Friction, etc. are the examples of various contact forces.

Normal reaction: The forces $\vec{F}_1$, $\vec{F}_2$ shown in the diagram acting on A and B respectively act away from the surface of contact, and prevent the two bodies from " occupying the same space".

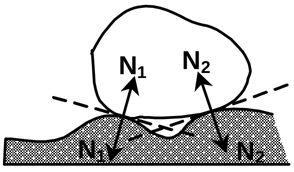


If $\vec{F}_1$ is the action, $\vec{F}_2$ is reaction: they are equal in magnitude but opposite in direction. Further, $\vec{F}_1$ and $\vec{F}_2$ are both perpendicular to the surfaces in contact and note that they act on two different bodies.

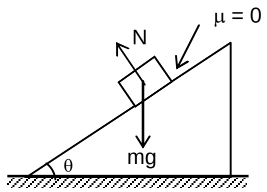
Examples:



N = mg

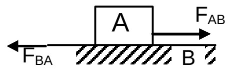


N is the normal reaction of the ground on the stone.



N = mg cos θ

Friction: It is a force that acts between bodies in contact with each other along the surface of contact and it opposes relative motion between the two bodies. The direction of frictional force on A is opposite to that of direction of frictional force on surface B and magnitude is same for both.

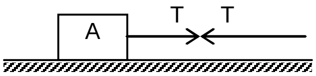


Tension (T): When a string, thread or wire is held taut, the ends of the string or thread (or wire) pull on whatever bodies are attached to them in the direction of the string. This force is known as Tension.

If the string is massless then the tension T has the same magnitude at all points throughout the string.

Examples:

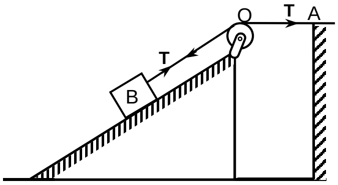
(i) Tension in a string: For a block A pulled by a string,



(ii) The direction of tension is always away from the point of attachment to the body. In the given figure two segments of tension act at O towards A and B.

For the wedge, there are two segments of thread at the point of attachment O to the body. Hence, two tensions act on the wedge; one along OB and the other along OA.

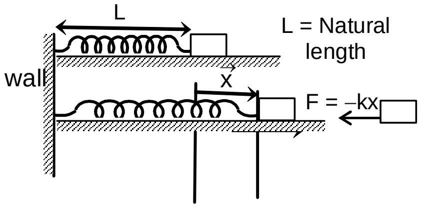
Hence a tension acts away from the point of attachment along BO.



Spring forces: Whenever a spring is compressed or extended, the elastic force developed in the spring which helps the spring to restore its original position is known as spring force.

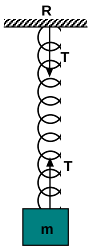
In an extended (or compressed) spring, force is proportional to the magnitude of extension (or compression).

$F \propto x$, in magnitude, but opposite in direction. So, $F = -kx$, where k is a positive constant, also known as the spring constant of the spring.
x is the compression or elongation from the natural length.



Example:

In case of the spring the tensions are oppositely directed on the block 'm' and on the roof R.



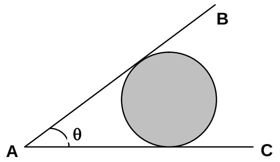
(b) Non-contact force: Bodies can exert forces on each other without actual physical contact. This is known as action at a distance. Such forces are known as *non-contact forces* (or) field forces, e.g. gravitational force, electrostatic forces, etc.

For the moment, we deal with actual forces. Suffice it to say that there exist pseudo-forces acting in a non-inertial frame of reference, which we will discuss later.

Forces may be conservative or non-conservative depending on whether work done against them by an external agent is recoverable or otherwise. This will be discussed in later chapters.

Exercise 2. ***(i). Is it necessary to have normal reaction whenever two surfaces are in contact with each other?***

(ii). Find the contact forces acting on a stationary sphere weighing 10 N is placed in a fixed frame BAC as shown in the diagram? (Assume θ is constant)



Now having understood about inertia and force, we can easily understand the three basic laws of motion which are known as Newton's laws of motion.

Newton's first law

Every body continues to be in the state of rest or of uniform motion in a straight line until and unless it is compelled to change the state of the body by an unbalanced force.

For better understanding we can divide this statement into two parts.

(i) "Every body continues to be in its state of rest until and unless some external force compels it to change the state of rest."

This part of the law is self explanatory and self evident as we come across several examples in our daily life like all inanimate objects will continue to be in the same place where they are put until they are disturbed by some external agents.

(ii) "Every body continues in its state of uniform motion in a straight line unless external force compels it to change that state."

The second part of the statement can't be readily understood as on the surface of the earth because of various types of frictional or resisting forces. For example when a ball is rolled on a horizontal surface the ball will come to halt after some time however smooth the surface may be, as we can't eliminate force of friction completely.

Momentum (Linear):

Till now we studied about inertia (translational) which is the inability of a body. Now we will study about another physical quantity called 'momentum' which is the ability of body.

Momentum is defined as the ability of a body by virtue of which it imparts or tends to impart its motion along a straight line.

Mathematically, momentum (p) is measured as the product of mass (m) and velocity (v) of the body. As velocity is a vector quantity, momentum is also a vector quantity.

$$\vec{p} = m\vec{v} \quad \text{or} \quad p = mv$$

Unit : Its unit is kg-m/s in SI system and gm-cm/s in CGS system.
Dimensions : Its dimensions are MLT^{-1}

Exercise 3.
(i) If a body is at rest, can we say that no force is acting on the body?
(ii) A car and a lorry are travelling with same velocity on a straight horizontal road. Which of the two has got greater momentum?

Illustration 1. *A block of mass 2 kg is moving with a velocity of* $2\hat{i} - \hat{j} + 3\hat{k}$ *m/s. Find the magnitude and direction of momentum of the block with the x-axis.*

Solution: The magnitude of momentum is $2\sqrt{14}$ kg m/s and the direction is at an angle of $\tan^{-1}\sqrt{\frac{2}{7}}$ with x-axis.

Illustration 2. *A block rests on air inclined plane with enough friction to prevent it from sliding down. To start the block moving, is it easier to push it up the plane, down the plane or sideways? Why?*

Solution: It is easier to push it down the plane because we have to apply the force only to overcome from the rest part of friction force.

Illustration 3. *When a force of constant magnitude always act perpendicular to the motion of a particle then :*

(A) Velocity is constant *(B) Acceleration is constant*
(C) KE is constant *(D) None of the above*

Solution : Force will provide centripetal acceleration, so it will move in a circular path therefore KE is constant because speed remains unchanged.
Option (C) is correct.

Newton's second law of motion

We have already studied the Newton's 1st law which has given us a qualitative idea about force. Now, we will study about Newton's IInd law which gives us a quantitative idea about force.

Whenever a cricketer catches a ball he allows a longer time for his hands to stop the ball. Otherwise the ball will hurt the cricketer. If you observe this incident carefully you can easily understand that cricketer is applying some force on ball in order to make the momentum of the body zero. And also we can understand that the magnitude of the retarding force that cricketer applies on the ball in order to stop depends on two factors.
(1) The momentum of the ball and
(2) Time for which he is applying the force

These type of observations lead Newton to state his second law of motion.

The rate of change of momentum of a body is directly proportional to the applied force and the change takes place in the direction of the force.

So for a body with constant mass,

$$\frac{d\vec{p}}{dt} \propto \vec{F} \qquad \text{or,} \qquad \frac{d}{dt}(m\vec{v}) \propto \vec{F}$$

$$\text{or,} \qquad m\frac{d\vec{v}}{dt} \propto \vec{F} \qquad ; \qquad \vec{F} = km\left(\frac{d\vec{v}}{dt}\right),$$

where k is a constant. With proper choice of units, k = 1. Thus,

$$\mathbf{\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}} \quad .$$

Unit of Force : Its unit is newton in SI system and dyne in CGS system.
Dimensions : $[MLT^{-2}]$

Note (1): The second law of motion is a vector law. These are actually three equations, one for each component of the vectors:

$$F_x = \frac{dp_x}{dt} = ma_x \; ; \; F_y = \frac{dp_y}{dt} = ma_y \; ; \quad F_z = \frac{dp_z}{dt} = ma_z$$

Note (2): The second law of motion is strictly applicable to a single point particle. The force F here stands for net external force on the particle.

Exercise 4. ***A body is acted upon by a number of external forces. Can it remain at rest ?***

Illustration 4. *A body of mass m = 1 kg falls from a height h = 20 m from the ground level*
(a) What is the magnitude of total change in momentum of the body before it strikes the ground?
(b) What is the corresponding average force experienced by it? (g = 10m/sec^2).

Solution: (a) Since the body falls from rest (u = 0) through a distance h before striking the ground, the speed v of the body is given by kinematics equation.

$v^2 = u^2 + 2\,as$; Putting a = g and s = h

we obtain $v = \sqrt{2gh}$

$\Rightarrow$ The magnitude of total change in momentum of the body

$= \Delta p = |\, mv - 0\,| = mv$, Where $v = \sqrt{2gh}$

$\Rightarrow \Delta p = m\sqrt{2gh} \;\; \Rightarrow \Delta p = (1)\sqrt{(2\times10\times20)}$ kg m/sec

$\Rightarrow \Delta p = 20$ kg m/sec.

(b) The average force experienced by the body $= F_{av} = \dfrac{\Delta p}{\Delta t}$

where Δt = time of motion of the body = t(say). We know Δp = 20 kg m/sec. Therefore we will have to find t using the given data. We know from kinematics that,

$S = ut + \dfrac{1}{2}at^2 \Rightarrow h = \dfrac{1}{2}g\,t^2$ (u = 0)

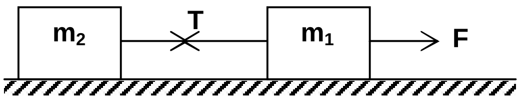
$\Rightarrow t = \sqrt{\dfrac{2h}{g}} \qquad \therefore \; F_{av} = \dfrac{\Delta p}{\Delta t} = \dfrac{\Delta p}{t}$

Putting the general values of ΔP and t we obtain

$F_{av} = \dfrac{m\sqrt{2gh}}{\sqrt{2h/g}} = mg \qquad \Rightarrow \vec{F}_{av} = m\vec{g}$.

Where mg is the weight (W) of the body and $\vec{g}$ is directed vertically downward. Therefore the body experiences a constant vertically downward force of magnitude mg.

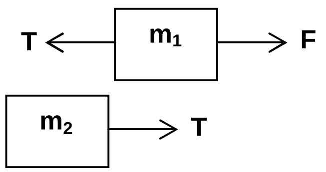
Illustration 5. *Two masses connected with a light string are placed on a horizontal frictionless surface. One mass is pulled by a constant force of F directed along the string. The acceleration of mass* m_1 *is*



(A) $\dfrac{F}{m_1}$ *(B)* $\dfrac{F}{m_2}$

(C) $\dfrac{F}{m_1 + m_2}$ *(D) zero*

Solution:



$$F - T = m_1 a$$
$$T = m_2 a$$
$$F = (m_1 + m_2) a$$
$$\Rightarrow a = \frac{F}{m_1 + m_2}$$

So option (C) is correct.

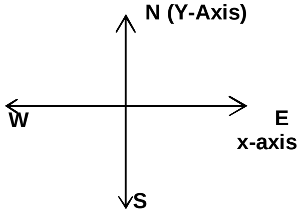
Illustration 6. *A body of mass 2 kg is moved towards east with a uniform speed of 2 m/s. A force of 3 N is applied to it towards north. Calculate the magnitude of the displacement of the body 2S after the application.*

Solution : According to the given reference frame:

$u_x = 2\,m/s$ and $a_x = 0$

$u_y = 0$ and $a_y = \frac{F_y}{m} = \frac{3}{2}$

$= 1.5\,m/s^2$



After 2 seconds, the displacement of the body along x-axis

$S_x = v_x t = 4m$

Along y-axis

$S_y = \frac{1}{2} a_y t^2 = \frac{1}{2} x 1.5 x 4 = 3m$

So magnitude of the displacement $= \sqrt{S_x^2 - S_y^2} = 5\,m$

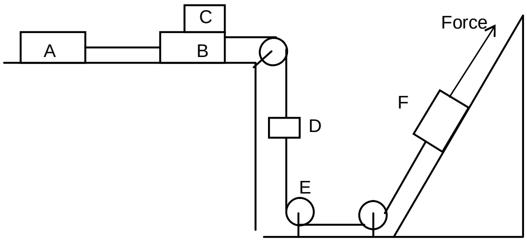
Working with Newton's first and second law

Before trying to write an equation from Newton's law, we should very clearly understand which particle we are considering. In any particle situation, we deal with extended bodies which are collection of a large number of particles. The laws stated a force ma be used even if the object under consideration is an extended body, provided each part of this body has the same acceleration (in magnitude and direction). A systematic algorithm for writing equations from Newton's law is as follow:

1^{st} step : Decide the system

The first step is to decide the system on which the laws of motion are to be applied. The system may be a single particle, a block, a combination of two blocks one kept over the other, two blocks connected by a string, a piece of string etc. The only restriction is that all parts of the system should have identical acceleration.

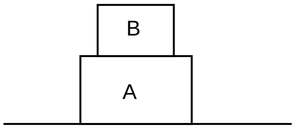
Here the distance covered by all the blocks is same but they all can not be taken as a system because even though magnitude of acceleration is same but the direction of acceleration in all the blocks is not same.



2nd step : Identify the forces

Once the system is decided, make a list of forces acting on the system due to all the objects other than system. Any force applied by the system should not be included in the list of the forces.

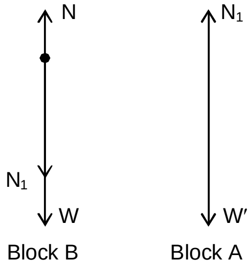
Consider the situation shown in the figure where small block of mass m is kept on bigger block of mass M. The load presses lower block, the lower pushes the upper block, the bigger block presses the floor downward, the floor pushes the block upward, the earth attracts the block.



3rd step: Make a free body diagram

Now, represent the system by a point in a separate diagram and draw vectors representing the forces acting on the system with this point as the common origin.

The forces may lie along a line, may be distributed in a plane (coplanar) or may be distributed in the space (non-planer). Indicate the magnitude and directions of the forces in the diagram. This is called a free body diagram.



4th step: Choose axes and write equations:

Any three mutually perpendicular directions may be chosen as the x-y-z axes. Some suggestion for choosing the axes to solve problems are,

(a) If the forces are coplaner, only two axes, say x and y, taken in the plane of forces are needed.
(b) Choose the x-axis along the direction in which the system is known to have or is likely to have the acceleration.
(c) If the system is in equilibrium, any mutually perpendicular directions in the plane of the diagram may be chosen as the axes.
(d) Write the components of all the forces along the x-axis and equate their sum to the product of the mass of the system and its acceleration. Write the components of the forces along the y-axis and equate the sum to zero.

Use mathematical techniques to get the unknown quantities out of these equations. This completes the algorithm.

Impulse: A large force acting for a short time to produce a finite change in momentum is called *impulse* and the force acted is called *impulsive force* or force of impulse.

Mathematically it is described as the product of force and time.

$\therefore$ Impulse (J) = F . t

$\therefore$ Impulse (J) = mv – mu and since force is variable, hence $J = \int_{t_1}^{t_2} F dt$

The area under F – t curve gives the magnitude of impulse.

Impulse is a vector quantity and its direction is same as the direction of $\vec{F}$.

Unit of Impulse **:** The unit in S.I. system is kgm/sec or newton -second.
Dimension **:** $M^1L^1T^{-1}$

Examples:

(i) *Automobiles are provided with spring shocker systems.* When the automobile bumps over an uneven road, it receives a jerk. The spring increases the time of the jerk, there by reducing the impulse of force. This minimises the damage to the vehicle.

(ii) *A man falling from a certain height receives more injuries when he falls on a marble floor than when he falls on a heap of sand.* This is because the marble floor does not yield under the weight of the man. The

man is stopped abruptly. A large change of momentum takes place in a very short interval of time. But when he falls on a heap of sand, the sand yields under the weight of the man and this increases the time interval. So it reduces the force exerted by sand on man.
(iii) *It is difficult to catch a cricket ball as compared to a tennis ball moving with the same velocity.* This is because cricket ball will have more momentum than tennis ball due to its heavier mass. The change in momentum in case of cricket ball is more. Hence more force is required to stop cricket ball than tennis ball.

Exercise 5. ***When a swimmer dives into water just before piercing himself into water, he stretches himself. Why ?***

Illustration 7. *A cricket ball of mass 200 gm moving with velocity 15 m/s is brought to rest by a player in 0.05 sec. What is the impulse of the ball and average force exerted by player ?*

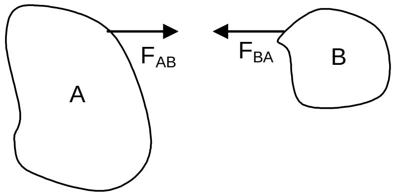
Solution: Impulse = change in momentum = m(v – u) = 0.2 (0 – 15) = – 3 Ns

Average force = Impulse / Time $= \dfrac{3}{0.05} = 60$ N

Newton's third law of motion

Now we have understood the qualitative and quantitative definitions of force from Newton's first and second laws. But how are the forces between two bodies related to each other if at all ? The answer is provided by the third law of motion.

Every action has an equal and opposite reaction, *which are equal in magnitude and opposite in direction.*



Consider two bodies A and B interacting with each other, by means of forces

$\vec{F}_{AB}$: the force exerted by body B on A

$\vec{F}_{BA}$: The force exerted by the body A on B.

According ot Newton's 3rd law : $\vec{F}_{AB} = -\vec{F}_{BA}$ (equal in magnitude & opposite in direction)

That may look fine, but it, apparently, raises a lot of questions. For example, if a horse pulls a cart and cart pulls the horse backward, how does the cart moves forward at all ?

If we observe we will find that the forces acting on the horse and the cart, though equal and opposite, they are acting not on the same body, rather, two bodies. It cannot produce equilibrium neither in horse nor in cart.

Exercise 6.

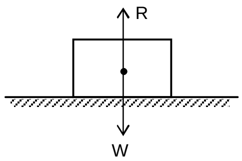
i) ***If action and reaction are equal and opposite to each other then how can a man move a box on a floor?***

ii) ***In a tug of war if the parties on both ends of the rope apply equal and opposite forces on each other then how can one party win?***

(iii) ***Suppose you are standing on a boat in the middle of a swimming pool. The boat is loaded with a bag of stones. Can you get to the shore using the stones? If yes, explain the concept behind it.***

Examples:

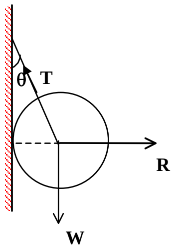
(i) Consider a body of weight W resting on a horizontal surface. The body exerts a force (action) equal to weight W on the surface. The surface exerts a reaction R on the body in the upward direction such that W = R or in vector notation, $\vec{W} = -\vec{R}$



(ii) In a lawn sprinkler, when water comes out of the curved nozzles, a backward force is experienced by the sprinkler. Consequently, the sprinkler starts rotating and sprinkles water in all directions.

(iii) In order to swim, a man pushes the water backwards with his hands. As a result of the reaction offered by water to the man, the man is pushed forward.

Illustration 8. *A metal of sphere is hung by by a string fixed to a wall. The force acting on the sphere are shown in figure. Which of the following statement is correct*



(A) $\vec{F}+\vec{T}+\vec{W} = 0$ *(B)* $T^2 = R^2 + W^2$
(C) $T = R + W$ *(D)* $R = W \tan\theta$

Solution : By the free body diagram

$T\cos\theta = w$

$T\sin\theta = R$

$T^2 = W^2 + R^2$

Option (B) is correct.

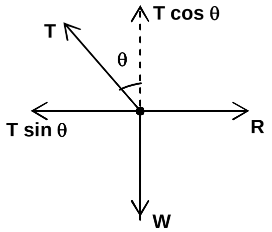
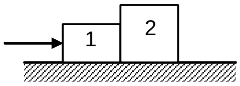
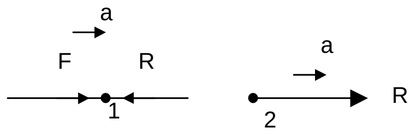


Illustration 9. *Two blocks of mass 2 kg and 4 kg are kept in contact with each other on a smooth horizontal surface. A horizontal force of 12 N is applied on the first block due to which they move with certain constant acceleration. Calculate the force between the blocks.*



Solution : Let a = common acceleration of blocks,
R = Force between the two blocks
From FBD of Ist block
$F - R = m_1 a$ . ... (i)
From FBD of block 2,
$R = m_2 a$ . . . .(ii)



F.B.D.

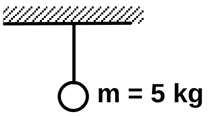
$\therefore$ From (i) & (ii), $F = (m_1 + m_2).\ a$

$$\therefore\ a = \frac{F}{m_1+m_2} = \frac{12}{6} = 2\ \text{m/sec}^2$$

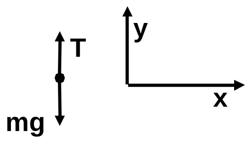
$\therefore$ Force between blocks $= R = m_2 a = (4 \times 2)$ N = 8 N.

Illustration 10. *A body of mass 5 kg is supported by a light cord. Find the tension in the cord.*

Solution:



(i) The body is isolated.
(ii) The two forces acting on it are :
Tension (T) in the cord (upwards)
Weight (mg) (downwards)
(iii) Equations of equilibrium :
$\Sigma F_x = 0$
$\Sigma F_y = 0 \Rightarrow T - mg = 0$
or $T = (5)(10) = 50$ N
(iv) Tension in the cord equals the weight of the body.



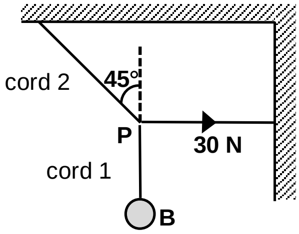
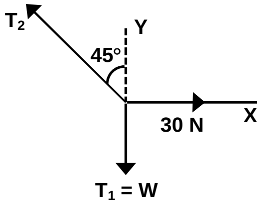


Illustration 11. *In the shown figure the tension in the horizontal cord is 30 N. Find the weight of the body B.*



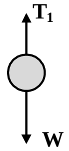
Solution:
(i) Isolate the point P
(ii) The forces acting on it are :
Unknown tension T_2 in cord 2
Unknown tension T_1 in cord1.
Known tension of 30N in the horizontal cord.
(iii) T_2 is resolved along x and y axes.
(iv) Condition of equilibrium:

$$\Sigma F_x = 0 \Rightarrow 30 - T_2 \sin 45° = 0 \qquad (1)$$

and $\Sigma F_y = 0 \Rightarrow T_2 \cos 45° - T_1 = 0 \qquad (2)$

Since body B is also in equilibrium,
Hence, $T_1 = W \qquad (3)$
(v) After solving these equations,
we get W = 30 N

For body *B*



FRAME OF REFERENCE

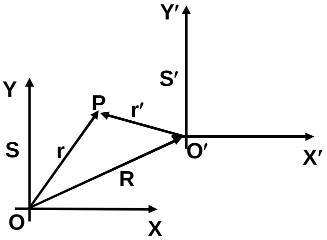
It is a conveniently chosen co-ordinate system, which describes the position and motion of a body in space.

Inertial frame of reference

A reference frame which is either at rest or in uniform motion along the straight line. Newton's laws are strictly valid only for inertial frame.

Non-inertial frame of reference

A reference frame which accelerates or rotates with respect to an inertial reference frame.
Motion of a particle (P) is studied from two frames of references S and S′. S is an inertial frame of reference and S′ is a non inertial frame of reference. At any time, position vectors of the particle with respect to those two frames are $\vec{r}$ and $\vec{r}\,'$ respectively. At the same moment position vector of the origin of S′ is $\vec{R}$ with respect to S as shown in the figure.



From the vector triangle OO′P , we get

$$\vec{r}\,' = \vec{r} - \vec{R}$$

Differentiating this equation twice with respect to time we get

$$\frac{d^2\vec{r}\,'}{dt^2} = \frac{d^2(\vec{r})}{dt^2} - \frac{d^2}{dt^2}(\vec{R})$$

$$\Rightarrow \vec{a}\,' = \vec{a} - \vec{A}$$

Here $\vec{a}\,'$ = acceleration of the particle P relative to S′
$\vec{a}$ = Acceleration of the particle relative to S
$\vec{A}$ = Acceleration of S′ relative to S.

Multiplying the above equation by m (mass of the particle) we get

$$m\vec{a}' = m\vec{a} - m\vec{A}$$

$$\Rightarrow \quad \vec{F}' = \vec{F}_{(real)} - m\vec{A}$$

$$\Rightarrow \quad \vec{F}' = \vec{F}_{(real)} + \left(-m\vec{A}\right).$$

In non-inertial frame of reference an extra force is taken into account in order to apply Newton's laws of motion. The magnitude of this force is equal to the product of the mass of the body and acceleration of the frame and it is always directed opposite to the acceleration of the frame. This force is known as ***Pseudo force***, because this force does not exist in the inertial frame of reference.

Exercise 7. *Apparent weight of a person, standing in an elevator which is going up or down with certain acceleration, changes. Why?*

Illustration 12. *A block slides down an inclined plane of slope angle θ with constant velocity. If it is then projected up the same plane with an initial speed v_0, the distance in which it will come to rest is*

(A) $\dfrac{v_0^2}{g\tan\theta}$ *(B)* $\dfrac{v_0^2}{4g\sin\theta}$

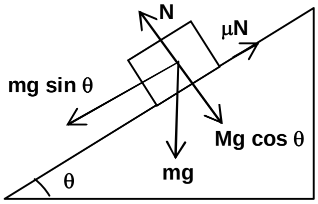
(C) $\dfrac{v_0^2}{2g}$ *(D)* $\dfrac{v_0^2}{2g\sin\theta}$

Solution : From the free body diagram

If the body move with constant velocity $mg\sin\theta = \mu N$

$N - mg\cos\theta = 0$

So, $\mu = \tan\theta$



Now in case (ii) if it is projected with velocity v_0 upward then net retardation against it is

$$a = \frac{mg\sin\theta + \mu\, mg\cos\theta}{m} = g\sin\theta + \mu g\cos\theta$$

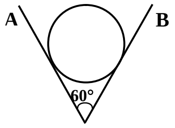
$$= g\sin\theta + g\sin\theta$$

$$= 2g\sin\theta$$

So distance travelled = $= \dfrac{{v_0}^2}{2a} = \dfrac{{v_0}^2}{4g\sin\theta}$

So option (B) is correct.

Illustration 13. *An iron sphere weights 10 N and rests in a V shaped trough whose sides form a angle 60°. The net force exerted by the walls on the sphere in case as shown in figure is*



(A) 0 N *(B) 10 N*

(C) $\dfrac{10}{\sqrt{3}}$ *N* *(D) 5 N*

Solution : By Symmetry $N_1 = N_2$

Therefore

$N_1 \sin 30^0 + N_2 \sin 30^0 = mg$

And $\frac{N_1 + N_2}{2} = mg$

So $N = mg = 10\,N$

So option (B) is correct.

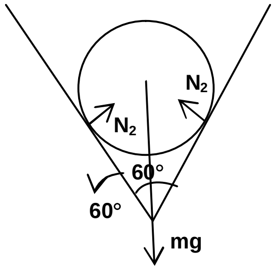


Illustration14. *A block of mass m is placed on an inclined plane. With what acceleration should the system move towards right on a horizontal surface so that m does not slide on the surface of inclined plane? Assume all surfaces are smooth.*

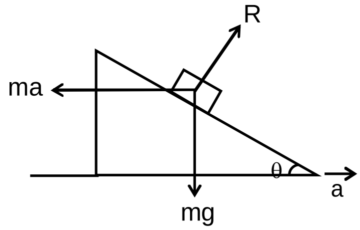
Solution : From ground frame of reference the forces acting on the block m are:

(i) its weight mg and

(ii) normal reaction R and its acceleration is rightward. If we analyse the motion of m relative to the inclined plane, its acceleration is zero and the forces acting are its weight, the normal reaction and a pseudo force of magnitude ma towards left.

$R\cos\theta = mg \therefore a = g\tan\theta$

$R\sin\theta = ma$



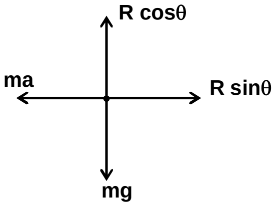
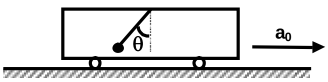


Illustration 15. *A pendulum of mass m is hanging from the ceiling of a car having an acceleration a_o with respect to the road in the direction shown. Find the angle made by the string with the vertical.*



Solution: Since bob of the pendulum is stationary relative to car

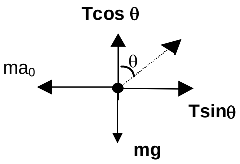
Hence

$T\sin\theta = ma_o$ (pseudo force) . . .(i)

$T\cos\theta = mg$ . . . (ii)

Dividing (i) by (ii), we get

$\tan\theta = \frac{a_o}{g} \Rightarrow \theta = \tan^{-1}\frac{a_o}{g}$



F.B.D. of pendulum relative to car

CONSTRAINT RELATIONS

The equations showing the relation of the motions of a system of bodies, in which motion of one body is constrained by motion of other bodies, are called the constraint relations.

Applying Newton's Laws alone is not sufficient in some cases where the number of equations is less than the number of unknowns.

In the given diagram for finding the acceleration of the masses there are three unknowns, tensions T, acceleration a_1 and a_2 of masses m_1 and m_2. However we will get only two equations. Clearly Newton's laws are not sufficient to solve the problem and constraint relations provide additional equations. When the motions of bodies in a system is constrained because of pulleys, strings, wedges or other factors, we use geometry to develop additional equations.

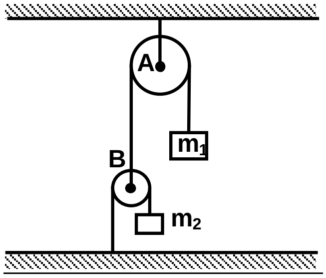
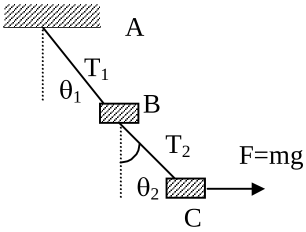


Illustration 16. *The blocks B and C in the figure have mass m each. The strings AB and BC are light, having tensions T_1 and T_2 respectively. The system is in equilibrium with a constant horizontal force mg acting on C.*

(A) $\tan\theta_1 = 1$ *(B) $\tan\theta_2 = 1/2$*
(C) $T_1 = \sqrt{5}$ mg *(D) $T_2 = \sqrt{5}$ mg*



Solution : From F.B.D.

$T_2\cos\theta_2 = mg$

$T_2\sin\theta_2 = mg$

$\Rightarrow T_2 = \sqrt{2}\,mg$

and $\theta_2 = 45^o$

Also $T_1\sin\theta_1 = T_2\sin\theta_2$

and $T_2\cos\theta_2 + mg = T_1\cos\theta_1$

$T_1\cos\theta_1 = mg + mg = 2mg$

and $T_1\sin\theta_1 = mg$

Therefore $T_1 = \sqrt{5}\,mg$

So option (C) is correct.

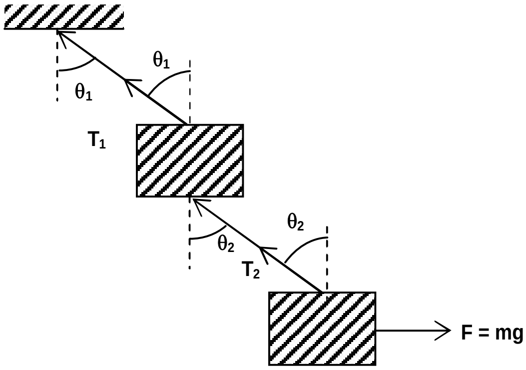


Illustration 17: *A rod is sliding along the wall as shown in figure. Find the ratio of velocity of its ends at the given instant.*

Solution : From the figure
The length of the rod should be constant, so

$v_A\cos\alpha = v_B\sin\alpha$

$\frac{v_A}{v} = \tan\alpha$

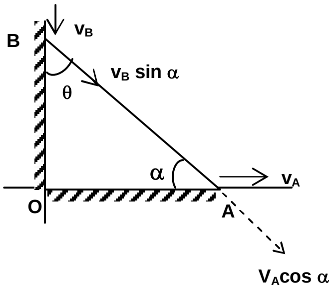
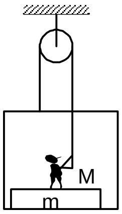
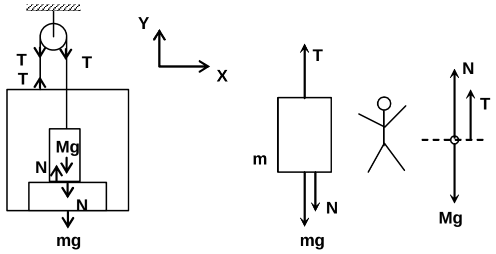


Illustration 18. *A man of mass M is standing on a plank kept in a box. The plank and box as a whole has mass m. A light string passing over a fixed smooth pulley connects the man and box. If the box remains stationary, find the tension in the string and the force exerted by the man on the plank.*



Solution: The fixed pulley is taken as frame of reference. The forces on man and box with plank are shown in figure.



The forces are as follows:

i. Weight of the man = Mg
ii. The tension in the string = T.
iii. The normal contact force between the man and the plank = N.
iv. The weight of the plank and box = mg

Referring to figure. The equation of motion of the man is given as

$T + N - Mg = Ma$

since M > m and the box remains at rest, the man will have to be at rest

$T + N - Mg = 0$ …(1)

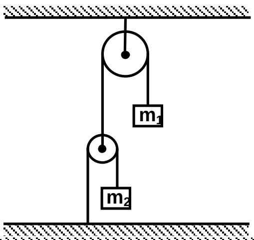
Similarly referring to figure,

$T - N - mg = ma = 0$ . . . (2)

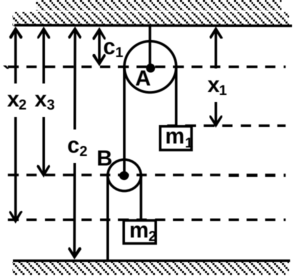
Solving (1) and (2) we obtain,

$T = \dfrac{(M+m)g}{2}$ and $N = \dfrac{(M-m)g}{2}$

Illustration 19. *Find the acceleration of the two blocks m_1 and m_2. Assume pulleys are massless and frictionless and the strings are inextensible.*



Solution: Let the acceleration of blocks m_1, m_2 and pulley B be a_1, a_2 and a_3 respectively.



Constraint relationship for the string attached to block of mass m_1:

$x_1 + x_3 - c_1 = \text{constant}$

Differentiating twice w.r.t. time we get,

$\dfrac{d^2x_1}{dt^2} + \dfrac{d^2x_3}{dt^2} = 0$

$\Rightarrow a_1 = -a_3$ . . . (i)

The minus sign signifies that acceleration of pulley B is opposite to that of block of mass m_1

Constraint relationship for the string attached to block of mass m_2:

$c_2 - x_3 + x_2 - x_3 = \text{constant}$

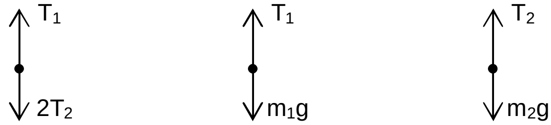
Differentiating twice w.r.t. time we get,

$$\frac{d^2x_3}{dt^2} + \frac{d^2x_2}{dt^2} = 0$$

$\Rightarrow a_2 = 2a_3$ . . . (ii)

$a_2 = 2a_1$ . . . (iii) (from equations (i) and (ii))

Taking the magnitudes only and ignoring the sign.



FBD of pulley B FBD of block m_1 FBD of block m_2

$T_1 = 2T_2$ . . . (iv)

$m_1g - T_1 = m_1a_1$ . . . (v)

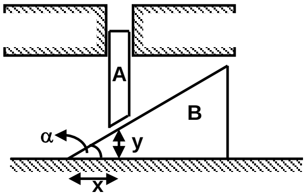
$T_2 - m_2 g = m_2a_2$ . . . .(vi)

Solving equations from (iii) to (vi) for a_1 and a_2 we get,

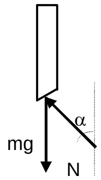
$$a_1 = \frac{m_1 - 2m_2}{m_1 + 4m_2}$$

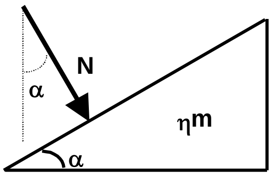
$$a_2 = 2a_1 = \frac{2(m_1 - 2m_2)}{m_1 + 4m_2}.$$

Illustration 20. *Find the accelerations of the rod A and the wedge B in the arrangement shown in the figure if the ratio of the mass of the wedge to that of the rod equals η and the friction between all surfaces is negligible.*



Solution: $y = x \tan\alpha \Rightarrow \dfrac{d^2y}{dt^2} = \dfrac{d^2x}{dt^2} \tan\alpha$





$a_{rod} = a_{wedge}.\tan\alpha$ (constraint equation)

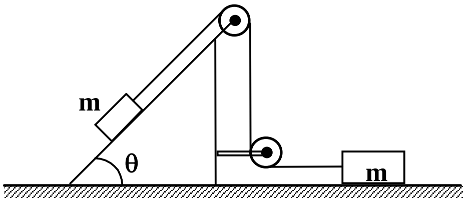
$mg - N\cos\alpha = ma_R$ & $N\sin\alpha = (\eta m).a_{wedge}$

$\Rightarrow mg - N\cos\alpha = m\, a_w \tan\alpha$

Solving we get,

$$a_{wedge} = \frac{g}{\tan\alpha + \eta\cot\alpha} \quad \text{and} \quad a_{Rod} = \frac{g}{1+\eta\cot^2\alpha}$$

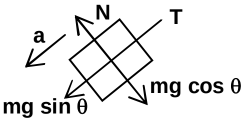
Illustration 21. *For the system shown in the figure, the pulleys are light and frictionless. The tension in the string will be*

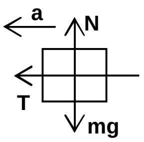


(A) $\frac{2}{3}mg\sin\theta$ *(B)* $\frac{3}{2}mg\sin\theta$

(C) $\frac{1}{2}mg\sin\theta$ *(D)* $2mg\sin\theta$

Solution: Form F.B.D.





$T = ma$

and $mg\sin\theta - T = ma$

So, $a = \dfrac{g\sin\theta}{2}$

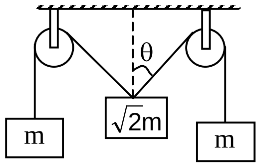
Therefore, $T = \dfrac{mg\sin\theta}{2}$

So, option (C) is correct.

Illustration 22. *The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be*

(A) 0° *(B) 30°*

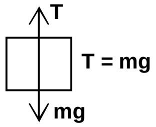
(C) 45° *(D) 60°*

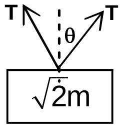


Solution:

$$\Rightarrow 2T\cos\theta = \sqrt{2}\,mg$$

$$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^{O}$$





So option (C) is correct.

Application of Newton's laws of motion: techniques and approach

A separate point diagram of the body is drawn showing the different forces exerted by the bodies in the environment, this is known as free body diagram.

Application of Newton's Laws to any system (consisting of one or more objects) can be done by following a systematic method. We recommend the following steps in the order given below –

(i) Draw the complete free body diagram (FBD), showing all the forces acting on each separate body.

(ii) Select proper coordinates for analysing the motion of each body.
Include any pseudo forces within the FBD if required.

(iii) If there are any constraints, write the proper constraint equations.

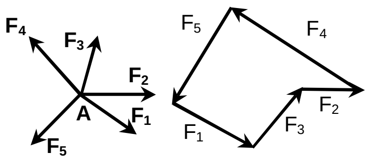
(iv) Apply Newton's 2nd law of motion : $\vec{F} = m\vec{a}$ for each body. This leads to a system of equations.

(v) Solve these equations.
(a) Identify the known and unknown quantities. Check that the number of equations equals the number of unknowns.
(b) Check the equations using dimensional analysis.
(c) After solving, check the final solution using back substitution.

(vi) If the velocity ($\vec{v}$) or position ($\vec{x}$) is required, proceed from a knowledge of acceleration ($\vec{a}$) as found from equations in step (v) and apply kinematics, e.g.
$\frac{d\vec{v}}{dt} = \vec{a}$ (known) and then integrate.

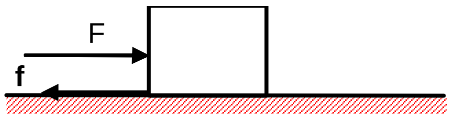
Equilibrium of concurrent forces

If the number of forces that are acting on a particle can be taken along the sides of any polygon both in direction as well as in magnitude, it will be in equilibrium.
Suppose that the force F_1, F_2, F_3, F_4 and F_5 are acting on the particle A and if they are in equilibrium then they will form a pentagon.

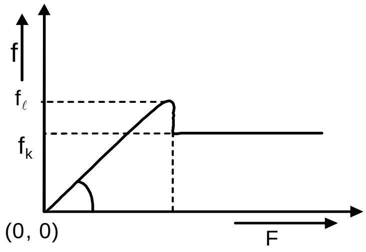


FRICTIONAL FORCE

Frictional force comes into play between two surfaces whenever there is relative motion or a tendency of relative motion between two surfaces in contact. Frictional force has the tendency to stop relative motion between the surfaces in contact.



Friction is a self-adjusting force. It changes its direction and magnitude according to the applied force or the force, which causes a tendency in the body to move. If the force increases then the opposing force also increases until the body moves beyond which it remains constant. If the applied force is plotted against the frictional force we obtain a graph as shown.



The graph shows that first frictional force increases to a certain maximum value f_ℓ with F and then suddenly decreases to a constant value f_k. For the range from 0 to f_ℓ frictional force is equal and opposite to F and hence block does not move. In this range, friction force is static.
Thus, friction can be classified as

(a) Static friction: It acts between surfaces in contact not in relative motion. It opposes the tendency of relative motion.

(b) Kinetic friction: It act acts between surfaces in contact which are in relative motion. It opposes the relative motion between the surfaces. Kinetic friction can be further classified as sliding friction and rolling friction.

Rolling friction: When a body rolls on a rough surface the frictional force developed is known as rolling friction. It is generally less than the kinetic friction or limiting friction.

Laws of static friction

Static Friction, acting between the surfaces in contact, (not in relative motion) opposes the tendency of relative motion between the surfaces.

The frictional force acts tangentially along the surfaces in contact, and the maximum value (or limiting value) of this force is proportional to the normal reaction between the two surfaces. The force of friction between two bodies is an *adjustable* force, only its maximum or limiting value is proportional to the normal reaction. Secondly, the direction of this force is determined by *all* other forces acting on the body that is by the forces that *tend* to cause relative motion. The force of static friction acts in a direction so as to *oppose* the other forces that tend to cause relative motion between the surfaces in contact.

Now, $f_{s(max)} \propto N$ where $f_\ell = f_{s(max)} \Rightarrow f_{s(max)} = \mu_s N$

Here μ_s = co-efficient of static friction.

N = normal reaction of the block from the surface.

$0 \le f_s \le \mu N$

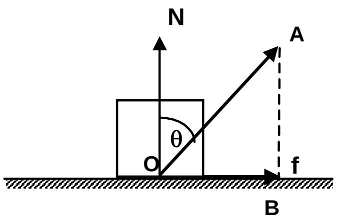
When F exceeds f_ℓ block starts moving and frictional force decreases to a constant value f_k. f_k is called kinetic friction and it has unique value which is given by

$f_k = \mu_k N$

Here μ_k = co-efficient of kinetic friction.

N = normal reaction.

Angle of friction: The angle made by the resultant reaction force with the vertical *(normal reaction)* is known as the angle of the friction.



Now, in the triangle OAB,

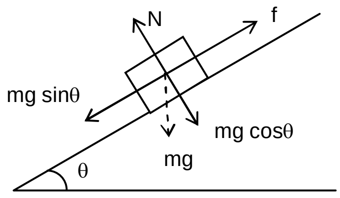
$$\frac{AB}{OB} = \cot\theta$$

$\Rightarrow OB = AB \tan\theta$

or, $\tan\theta = \dfrac{f}{N}$

Angle of Repose:

The angle of repose is defined as the angle of the inclined plane at which a body placed on it just begins to slide. Consider an inclined plane, whose inclination with horizontal is gradually increased, till the body placed on its surface just begins to slide down, then the angle made by the plane with horizontal is called angle of repose.



From the diagram:

$f = mg \sin\theta$ . . . (i)

$N = Mg \cos\theta$

Dividing (i) by (ii)

$$\frac{f}{N} = \frac{Mg\sin\theta}{Mg\cos\theta} = \tan\theta$$

Since $\dfrac{f}{N} = \mu$, $\tan\theta = \mu$

Therefore, coefficient of limiting friction is equal to the tangent to the angle of repose thus angle of repose is equal to the angle of friction.

Exercise 8.

(i) ***What causes the motion of a car on a road?***

(ii) ***Why the centrifugal force is never included in the free body diagram of circular motion. Explain why?***

Illustration 23. *A block weighing 2 kg rests on a horizontal surface. The coefficient of static friction between the block and surface is 0.40 and kinetic friction is 0.20.*

(a) How large is the friction force acting on the block ?

(b) How large will the friction force be if a horizontal force of 5N is applied on the block?

(c) What is the minimum force that will start the block in motion?

Solution: (a) As the block rests on the horizontal surface and no other force parallel to the surface is on the block, the friction force is zero.

(b) With the applied force parallel to the surfaces in contact 5 N, opposing friction becomes equal and opposite. Further the limiting friction is $\mu_s N = \mu_s Mg = 8$ N

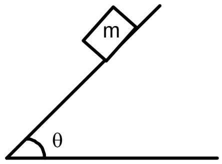
$\therefore$ Force of friction is 5N.

(c)The minimum force that can start motion is the limiting one. $\mu_s N = \mu_s mg = 8$ N

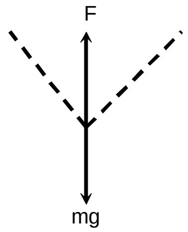
Illustration 24. *A block of mass m is at rest on a rough inclined plane of inclination θ as shown in the figure*

(a) Find the force exerted by the inclined plane on the block.

(b) What are the tangential and normal contact forces?



Solution: (a) The forces acting on the block are the field force $m\vec{g}$, vertically downward and total contact force $\vec{F}$ given by inclined plane on the block. As the block is at rest net force on the block is zero.



$\therefore \quad \vec{F} + mg = 0$ [as shown in Figure]

$\therefore \quad \vec{F} = -mg$

$\therefore$ The force exerted by the inclined plane on the block is $-\overrightarrow{mg}$ in vertically upward direction.

(b) The normal contact force N and tangential contact force f are shown in F.B.D. (Figure)

$f = mg \sin\theta$

$N = mg \cos\theta$

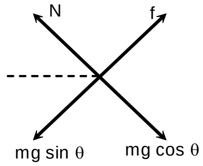
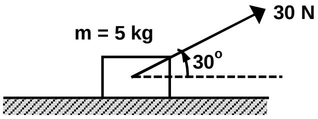


Illustration 25. *A 5 kg box is being moved across the floor at a constant velocity by a force of 30 N, as shown in the figure.*

(a) What is the force of friction acting on the box ?

(b) Find μ_k between the box and the floor.



Solution: The free body diagram of the box is shown in the figure.

Conditions of equilibrium :

$\Sigma F_y = 0$

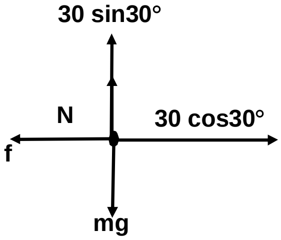
$\Rightarrow 30 \sin 30° + N - mg = 0$

or $N = (5)(10) - 30(1/2) = 35$ N

$\Sigma F_x = 0$

$\Rightarrow 30 \cos 30° - f = 0$

or $f = 30\left(\frac{\sqrt{3}}{2}\right) = 15\sqrt{3}$ N

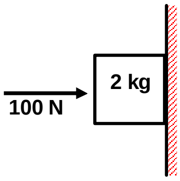


As there is a relative motion between the surfaces,

Hence $f = \mu_k N$, where μ_k = coefficient of kinetic friction.

$$\Rightarrow \mu_k = \frac{f}{N} = \frac{15\sqrt{3}}{35} = \frac{3\sqrt{3}}{7} = 0.74$$

Illustration 26. *A block of mass 2 kg is pressed against a rigid vertical wall by a horizontal force of 100 N. If co-efficient of static and kinetic friction are each equal to 0.3 then find the magnitude and direction of frictional force on the block.*

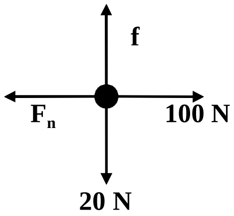


Solution: Since weight of block is 20 N which is acting downward, it has tendency to move the block downward. Hence, the frictional force will be upward. Maximum value of frictional force can be $f_{s(max)} = \mu F_n$

As block is in equilibrium along horizontal

$F_n = 100$ N

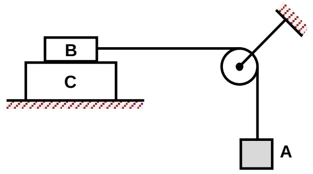
$\Rightarrow$ $f_{s(max)} = 0.3 \times 100$ N = 30 N



F.B.D. of the block

Since weight of the block is less than the limiting friction, it will not slide. Therefore, for vertical equilibrium f = 20 N.

Illustration 27. *In the figure shown co-efficient of friction between the block B and C is 0.4. There is no friction between the block C and the surface on which it is placed. The system of blocks is released from rest in the shown situation. Find the distance moved by the block C when block A descends through a distance 2 m. Given masses of the blocks are m_A = 3 kg, m_B = 5 kg and m_C = 10 kg.*



Solution: Let there is no relative motion between the blocks B and C

Hence

$T = (m_B + m_C)a$ . . . (1)

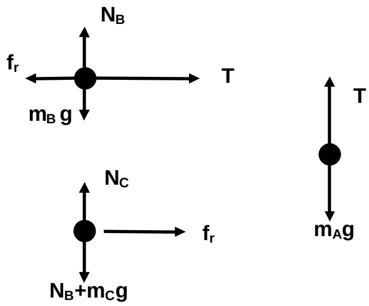
And

$m_A g - T = m_A a$ . . . (2)

From (1) and (2), we get

$$a = \frac{m_A g}{m_A + m_B + m_C} = \frac{30}{18} = \frac{5}{3}\ m/s^2$$

F.B.D. of the blocks



$\Rightarrow$ Net force on the block C is, $F = m_C a = 10 \times (5/3)$ N = 16.6 N

If maximum value of frictional force acting on block C is f_{max}, then

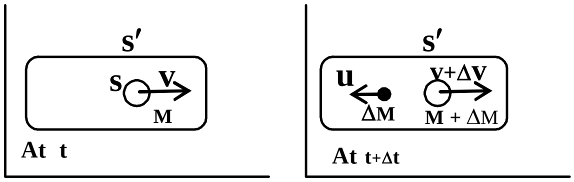
$f_{(max)} = \mu m_B g = 0.4 \times 5 \times 10 = 20$ N $\because F \le f_{max}$

Hence there is no relative motion between the block B and C. Therefore, distance moved by C is 2 m only.

Lubrication: In some cases friction acts as a hindrance when there are moving parts in contact. A great amount of energy is lost in such type of machines like an automobile or pump or any motor. This energy converted to heat can damage the machines. So friction is reduced by suitable lubricants like oil grease, graphite etc.

Variable Mass System: Till now, we were discussing mass which remains constant with time. But if the mass changes with time continuously, then can we apply the conservation of momentum as we discussed earlier.

Let a body move continuously either by ejecting mass or absorbing mass. At any instant let the mass of the body be M and its velocity be $\vec{v}$ from a given inertial frame of reference. Suppose that the mass increases by ΔM and the velocity



increases by Δv in a time Δt. As the mass increases by ΔM, the remainder of the system has a mass $-\Delta M$ and moves with a speed u, relative to the inertial frame.

The change in momentum of the system s′ will be:

$$\Delta p = (M + \Delta M)(v + \Delta v) + (-\Delta M)u - Mv$$

To generalize, let us imagine an external force F_{ext} acting on the sub-system M.

The external force acting on the mass is

$$\lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t} = \lim_{\Delta t \to 0} \frac{(M+\Delta M)\ (v+\Delta v) - \Delta M u - Mv}{\Delta t}$$

$$F_{ext} = M\frac{dv}{dt} + (v-u)\frac{dM}{dt} \quad \left[\because \frac{\Delta v \Delta M}{\Delta t} \text{ is very small}\right]$$

The relative velocity of the mass ΔM with respect to the sub-system is $(u - v) = v_{rel}$

$$\therefore F_{ext} = M\frac{Mdv}{dt} + v_{rel}\frac{dM}{dt}$$

Situations of variable mass

(i) Rocket Equation:

In distant space $F_{ext} = 0$ and $\frac{dv}{dt}$ is in positive direction, the direction of v_{rel} is negative. Now

$$M\frac{dv}{dt} = -v_{rel}\frac{dM}{dt}$$

$$\Rightarrow \quad dv = -v_{rel}\frac{dM}{M}$$

If the original mass of the rocket at t = 0 was M_0 and mass of the fuel burnt is m then

$$\int_{v_i}^{v_f} dv = -v_{rel}\int_{M_0}^{M_0 - m}\frac{dM}{M} \quad \Rightarrow \quad v_f - v_i = -v_{rel} \ln\frac{M_0 - m}{M_0}$$

$$M_f = M_0\, e^{-v_f / v_{rel}} \quad \text{where } M_f = M_0 - m \text{ and } v_i = 0$$

(ii) Rain drops accumulating in a moving rail road card is another example of variable mass situation.

(iii) A vertical chain falling on a fixed table also represents variable mass situation.

Exercise 9. ***A container filled with liquid has a hole on the side wall near bottom. Will momentum of the container and liquid system remain constant?***

Illustration 28. *A rocket of mass 40 kg has 360 kg of fuel. The exhaust velocity of the fuel is 2.0 km/sec. Calculate minimum rate of consumption of fuel so that the rocket may rise from the ground.*

Solution: We have relation $M\frac{dv}{dt} = F_{ext} + v_{rel}\frac{dM}{dt}$.

In the question, we have the force by the gravity as the external force. As the rocket is to lift from the ground the minimum acceleration of the rocket to do so is, o, beyond which, with a slight increase it will zoom into the sky.

Therefore, $\frac{dv}{dt} = 0$

Putting these into the equation, we obtain

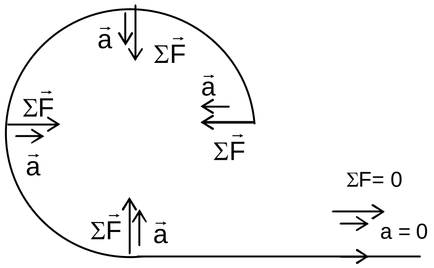
$$0 = Mg + v_{rel}\left(-\frac{dM}{dt}\right)$$

$\left(\frac{dM}{dt}\right)$ is the rate of decrease of mass hence taken negative. Mg is in the direction of the relative velocity of the fuel with respect to the rocket

Hence, $\frac{dM}{dt} = \frac{M}{v_{rel}}g = \frac{400\times 9.8}{2\times 1\left(10^3\right)} = 1.96\,\text{kg/sec}$

CIRCULAR MOTION

If a particle moves in a circular path at a constant speed, the velocity of the particle at any point of its path is directed along the tangent at that point. Due to continuous change in the direction of velocity, the particle has an acceleration. It is found that this acceleration is always directed radially inwards. It is known as radial or centripetal acceleration. So the particle is always acted on by a force directed radially inward, known as centripetal force.



Thus, centripetal force is defined as the force which acts towards the centre along the radius of a circular path on which a body is moving with a uniform speed.

Expression for the centripetal force:

(i) For uniform motion:

Acceleration towards centre

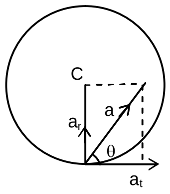
$a_r = r\omega^2 = \frac{v^2}{r}$ [$\because v = r\omega$]. Hence centripetal force, $F = \frac{mv^2}{r}$

(ii) For non uniform motion:

In this case a body has two accelerations

(a) Radial acceleration : $a_r = \frac{v^2}{r}$, it occurs due to change in direction of the body and

(b) Tangential acceleration, $a_t = \frac{dv}{dt}$, it occurs due to change in speed of the body.



Hence the total acceleration, $a = \sqrt{a_r^2 + a_t^2}$

$\tan\theta = \frac{a_r}{a_t}$

Exercise 10. ***For uniform circular motion does the direction of the centripetal force depend upon the sense of revolution?***

Illustration 29. *A stone, tied to the end of a string 80 cm long, is whirled in a horizontal circle with a constant speed. If the stone makes 5 revolutions in 10 s, what is the magnitude and direction of acceleration of the stone ?*

Solution: Acceleration $a = r\omega^2 = r\,(2\pi n)^2$

$$= 0.8 \times 4\times \pi^2 \times \left(\frac{5}{10}\right)^2 = 8\ \text{m/s}^2 \quad \text{(Towards centre)}$$

Vehicle moving round a circular path on horizontal road:

When a vehicle takes a turn on a road, it has a tendency to skid away from the centre of curvature of the road due to inertia. This tendency to skid brings into action a frictional force between the road and the tyres, directed towards the centre. This frictional force provides the necessary centripetal force.

If the maximum speed of a vehicle without skidding is v, then frictional force,

$$f = \mu mg = \frac{mv^2}{r}$$

$$\Rightarrow \quad v = \sqrt{\mu rg}$$

Banking of tracks:

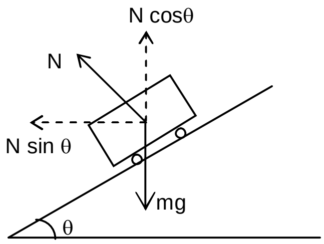
Friction is not always sufficient to provide the required centripetal force. Moreover, it causes damage of the tyres. Friction can be avoided by banking the road at a suitable angle θ to the horizontal.

The horizontal component of the normal reaction N provides the centripetal force, i.e.

$$N \sin\theta = \frac{mv^2}{r} \qquad \ldots \text{(i)}$$

And the vertical component of normal reaction balances the weight, i.e.

$$N \cos\theta = mg \qquad \ldots \text{(ii)}$$



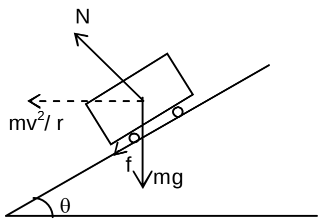
From (i) and (ii)

$$\tan\theta = \frac{v^2}{rg}$$

If the above relation is not satisfied, then frictional force will come into play. It may act down the plane or up the plane depending on whether the vehicle has a tendency to skid outward or inward.

If the vehicle has a tendency to skid outward, then the component of frictional force and the normal reaction in the horizontal direction produces necessary centripetal force,

$$N \sin\theta + f \cos\theta = \frac{mv^2}{r} \qquad \ldots \text{(i)}$$



And the component of frictional force and the normal reaction in upward direction balances the weight of the vehicle.

i.e. $N \cos\theta - f \sin\theta = mg \qquad \ldots$ (ii)

From (i) and (ii),

$$\frac{v^2}{rg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}$$

We know frictional force, $f = \mu N$.

Hence, $\dfrac{v^2}{rg} = \dfrac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$

$$\Rightarrow \frac{v^2}{rg} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}$$

$$\Rightarrow v = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

This is the maximum velocity of a vehicle without slipping. If $\mu = 0$, we get the optimum speed for least damage of tyres as

$$V = \sqrt{rg\tan\theta}$$

Illustration 30. *The road at a circular turn of radius 10 m is banked by an angle of 10^0. With what speed should a vehicle move on the turn so that the normal contact force is able to provide the necessary centripetal force? [tan 10° = 0.176]*

Solution: $\tan\theta = v^2 / rg$

or, $v = \sqrt{rg\tan\theta}$

$= \sqrt{(10)(9.8)\tan 10^0} = 4.2$ m/s

Illustration 31. *A cyclist is riding with a speed of 27 km/hr. As he approaches a circular turn on the road of radius 80 m. He applies brakes and reduces his speed at the constant rate of 0.5 m/s every second. What is the magnitude of the net acceleration of the cyclist?*

Solution: Tangential acceleration $= 0.5\,\text{m/s}^2$

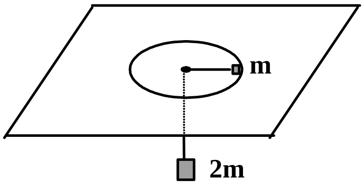
Centripetal acceleration $= \dfrac{v^2}{R} = \dfrac{\left(27\times\frac{5}{18}\right)^2}{80} = \dfrac{225}{320}$

$= 0.703\,\text{m/s}^2$

Net acceleration $\sqrt{a_T^2 + a_R^2}$

$= \sqrt{0.25+0.49} = \sqrt{0.74} = 0.86$ m/s^2

Illustration 32. *A mass m rotating freely in a horizontal circle of radius 1m on a frictionless smooth table supports a mass 2m in equilibrium attached to the other end of the string hanging vertically. If the instantaneous acceleration of the mass 2m is g m/s^2 vertically upward then angular velocity of rotation is*



(A) 5.78 rad/s *(B) 6.32 rad/s*
(C) 5.94 rad/s *(D) 6.11 rad/s*

Solution: Mass 2m is moving upward with an acceleration g

So, T – 2 mg = (2 m)g

$\Rightarrow$ T = 4 mg

Now for mass m, Tension will provide the necessary centripetal acceleration

4 mg = $mR\omega^2$

$\omega = 2\sqrt{g} = 6.32$ rad/sec.

So option (B) is correct.

SUMMARY

When a body is in equilibrium in an inertial frame of reference, the vector sum of forces acting on it must be zero. Free body diagrams are essential in identifying the forces that act on the body being considered.

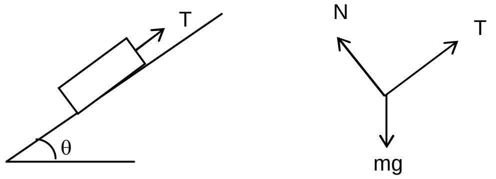
Newton's third law is also frequently needed in equilibrium problems. The forces in an action –reaction pair never act on the same body.

Vector form

$$\Sigma\vec{F} = 0$$

Component form

$$\Sigma F_x = 0,\ \ \Sigma F_y = 0$$

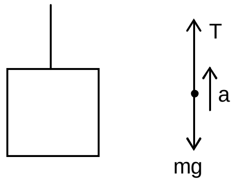


If the vector sum of forces on a body in not zero, the body accelerates. Its acceleration is given by Newton's 2^{nd} law.

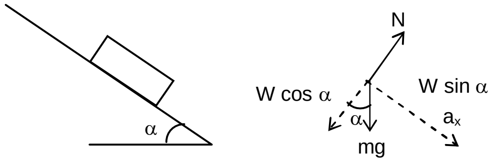
As they are for equilibrium problems, free-body diagrams are essential for solving problems involving Newton's second law.

Vector form: $\Sigma\vec{F} = m\vec{a}$

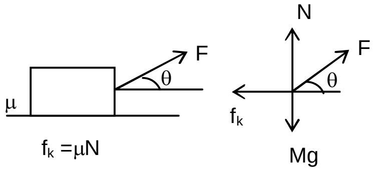
component form $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$



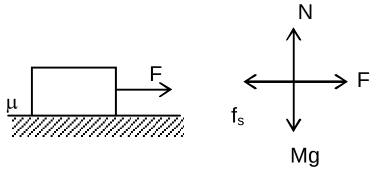
The contact force between two bodies can always be represented in terms of a normal force $\vec{n}$ perpendicular to the surface of contact and a friction force f parallel to the surface. The normal force exerted on a body by a surface is not always equal to the body's weight.



When a body is sliding over the surface, the friction force is called kinetic friction. Its magnitude f_k is approximately equal to the normal force magnitude N multiplied by the coefficient of kinetic friction μ_k.



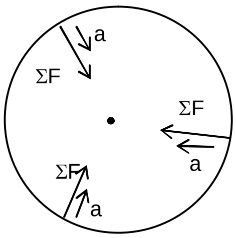
When a body is not moving relative to the surface, the friction force is called static friction. The maximum possible static friction force is approximately equal to the magnitude N of the normal force multiplied by the coefficient of static friction.



The actual static force may be anything from zero to this maximum value, depending on the situation usually μ_s is greater than μ_k for a given pair of surface in contact.

In uniform circular motion, the acceleration vector is directed towards the centre of the circle. Just as in any other dynamic problem, the motion is governed by Newton's second law $\Sigma\vec{F} = m\vec{a}$.

$a_{rad} = v^2/R$.



MISCELLANEOUS EXERCISE

1. If the net force acting on a body be zero, then will the body remain necessarily in rest position?

2. Can a body remain in rest position when external forces are acting on it?

3. The two ends of a spring-balance are pulled each by a force of 10 kg-wt. What will be the reading of the balance?

4. A force of 5 N changes the velocity of a body from 10 m/s to 20 m/s in 5 sec. How much force is required to bring out the same change in 2 sec.

5. An impulsive force of 100 N acts on a body for 1 s. What is the change in its linear momentum.

6. Two bodies of masses M and m are allowed to fall freely from the same height. If air resistance for each body is same, then will both the bodies reach the earth simultaneously?

7. When a ball is thrown upward, its momentum first decreases then increases. Is conservation of linear momentum violated in this process?

8. Four blocks of same mass m connected by cords are pulled by a force F on a smooth horizsontal surface, as shown in figure. Determine the tensions T_1, T_2 and T_3 in the cords.

SOLUTIONS TO MISCELLANEOUS EXERCISE

1. No, the body may be moving uniformly along a straight line.
2. Yes, if vector sum of all the forces acting on the body is zero.
3. The reading of balance will be 10 kg-wt.
4. From $F_1 = \frac{dp_1}{dt_1}$ and $F_2 = \frac{dp_2}{dt_2}$ $\Rightarrow \frac{F_2}{F_1} = \frac{dt_1}{dt_2} \times \frac{dp_2}{dp_1}$

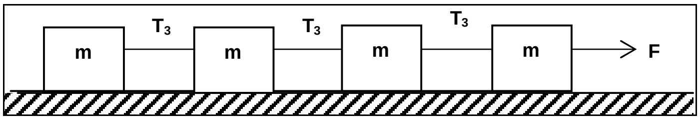
   Now, $dp_1 = dp_2$

   So, $\frac{F_2}{F_1} = \frac{dt_1}{dt_2} = \frac{5}{2} \Rightarrow F_2 = \frac{5}{2}F_1 = \frac{5\times5}{2} = 12.5\,\text{N}$
5. Change in linear momentum = Impulse = F× t = 100 N-s.
6. No, the net force on the body of mass M is (Mg — F). Therefore, its acceleration,

   $a = \frac{Mg - F}{M} = \left(g - \frac{F}{M}\right)$

   Thus, acceleration, a of a body of larger mass will be greater and it will appear lighter than before.
7. No, the momentum conservation principle is not violated. This is because vector sum of linear momentum of the ball and the earth remain constant.

8. Let a be the common acceleration of the whole system



   $\therefore$ F = 4 ma $\Rightarrow$ a = $\frac{F}{4m}$

   Applying Newton's 2^{nd} law for each blocks :

   $F - T_1 = ma \Rightarrow T_1 = \frac{3}{4}F$

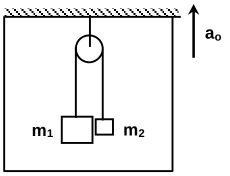
   $T_1 - T_2 = ma \Rightarrow T_2 = \frac{F}{2}$ $\Rightarrow T_2 - T_3 = ma \Rightarrow T_3 = \frac{F}{4}$

SOLVED PROBLEMS

Subjective:

BOARD TYPE

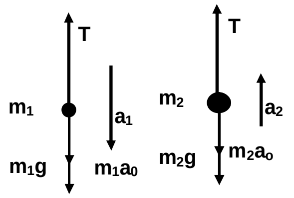
Prob 1. *A pulley fixed to the ceiling of an elevator car carries a thread whose ends are attached to the masses m_1 and m_2. The car starts going up with an acceleration a_o. Assuming the masses of the pulley and the thread as well as the friction to be negligible, find :*



(a) the acceleration of the load m_1 relative to the elevator shaft and relative to the car.
(b) the force exerted by the pulley on the ceiling of the car. Given, $m_1 > m_2$.

Sol. The elevator is an accelerated frame which is non-inertial, pseudo force $m_1a_0 < m_2a_0$ have been taken into consideration for m_1 and m_2 respectively.

F.B.D. of the blocks relative to elevator.



$m_1g + m_1a_o - T = m_1a_1$ . . . (i)

$T - m_2g - m_2a_o = m_2a_2$ . . . (ii)

$a_1 = a_2$ . . . (iii)

Solving the above equations , we get,

$$a_1 = \frac{(m_1 - m_2).(a_o + g)}{(m_1 + m_2)}$$

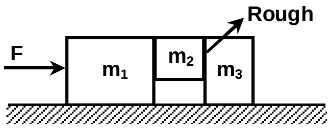
This is the acceleration of m_1 w.r.t car. Acceleration of the mass m_1 w.r.t the elevator shaft

$$= a_o - a_1 = \frac{2a_o m_2 - g(m_1 - m_2)}{(m_1 + m_2)}$$

Force exerted by the pulley on the ceiling of the car

$$= 2.T = (m_1a_o + m_1g - m_1a_1) \text{ x } 2 = \frac{4m_1.m_2(a_o + g)}{(m_1 + m_2)}.$$

Prob 2. *In the arrangement shown in the figure, the system of masses m_1, m_2 and m_3 is being pushed by a force F applied on m_1 horizontally, in order to prevent the downward slipping of m_2 between m_1 and m_3. If coefficient of friction between m_2 and m_3 is μ and all the other surfaces are smooth, What is the minimum value of force F?*

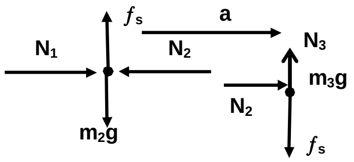


Sol. $f_s = m_2g$; $f_s \le \mu N_2$; $N_2 = m_3a$

$\therefore m_2g \le \mu m_3a$

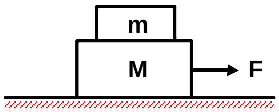
$$\Rightarrow a \ge \left(\frac{m_2g}{\mu m_3}\right)$$

$$F \ge (m_1 + m_2 + m_3)\frac{m_2g}{\mu \ m_3}.$$



F.B.D of m_2 F.B.D of m_3

Prob 3. *A block of mass m is placed on another block of mass M lying on a smooth horizontal surface as shown in the figure. The co-efficient of friction between the blocks is μ. What maximum horizontal force F can be applied to the block M so that the blocks move together ?*



Sol. If there is no relative motion between the blocks then acceleration of the blocks is

$$a = \frac{F}{M+m}$$

For vertical equilibrium $N = mg$

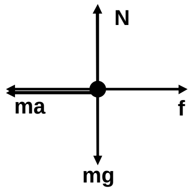
For horizontal equilibrium $f - ma = 0$

$$f \le \mu N$$

$\Rightarrow$ $\mu mg \ge m \dfrac{F}{(M+m)}$

$\Rightarrow$ $F \le (M + m)\mu g$

$\Rightarrow$ $F_{max} = (M + m)\,\mu g$



F.B.D. of m relative to M

Prob 4. *A mass of 200 kg. is placed on a rough inclined plane of angle 30°. If coefficient of limiting friction is $\frac{1}{\sqrt{3}}$, Find the greatest and the least forces in Newton, acting parallel to the plane to keep the mass in equilibrium .*

Sol. From the figure shown

$R = mg\cos\theta$

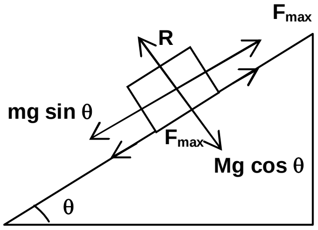
$F_{max} = \mu R = \mu\, mg\cos\theta$

So, the greatest force required to kept the body in equilibrium

$F_{max} = mg\sin\theta + f_{max}$

$= mg\sin\theta + \mu\, mg\cos\theta$

$= 200\times 9.8\left[\frac{1}{\sqrt{3}}\cos 30^\circ\right] = 1960$ N



From the figure (b)

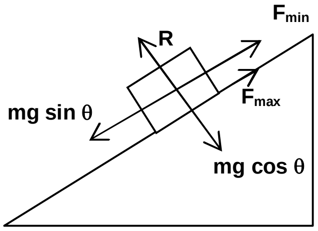
$F_{min} = mg\sin\theta - f_{max}$

$= mg\sin\theta - \mu\, mg\cos\theta$

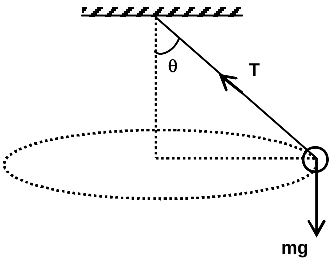
$= mg[\sin\theta - \mu\cos\theta]$

$= 200\times 9.8\left[\sin 30^{o} - \frac{1}{\sqrt{3}}\cos 30^{o}\right]$

= zero



Prob 5. *A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev/min in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can with stand a maximum tension of 200 N?*



Sol. From figure

$T\cos\theta = mg$

$T\sin\theta = mr\,\omega^2$

So, $T = m\sqrt{g^2 + r^2\omega^4}$

Now $\omega = 40\,\text{rev/min} = 2\pi\times\frac{40}{60}\,\text{rad/sec}$

$= \frac{4\pi}{3}\,\text{rad/sec}$

So, Tension (T) $= \sqrt{g^2 + (r\omega^2)^2})\times 0.25$

$= \left(\sqrt{100 + \left(1.5\times\frac{16\pi^2}{9}\right)^2}\right)^{0.25}$

$= \left(\sqrt{100 + \left(\frac{64}{9}\right)\pi^4}\right)\times 0.25 = 7.0\,\text{N}$

So, maximum speed, by the given data

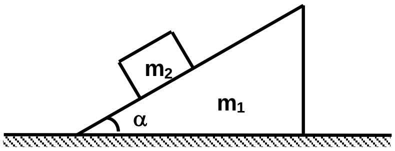
$T_{max} = 200$ N

So $200 = \sqrt{g^2 + r^2\omega_{max}^4})0.25 = 7.0\,\text{N}$

$\omega_{max} = 23.1$ rad/sec

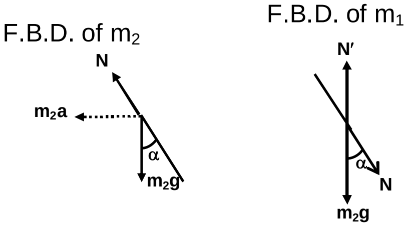
IITJEE TYPE

Prob 6. *A wedge 1 of mass m_1 and with angle α rests on a horizontal surface. Block 2 of mass m_2 is placed on the wedge. Assuming the friction to be negligible, find the acceleration of the wedge.*



Sol. Let us solve this problem by considering the motion of m_2 in non-inertial frame of wedge. In that frame the block is at rest along the normal to the inclined plane. Hence it is under equilibrium along the normal to the plane.

Due to the acceleration of the frame towards right pseudo force acts on the block towards left. As shown in the F.B.D.



$m_2 a\sin\alpha + N = m_2 g\cos\alpha$ ...(1)

and for m_1, $N\sin\alpha = m_1 a$ ...(2)

Multiplying (1) by $\sin\alpha$ and substituting (2) in it,

$m_2\sin^2\alpha + m_1 a = m_2 g\sin\alpha\cos\alpha$

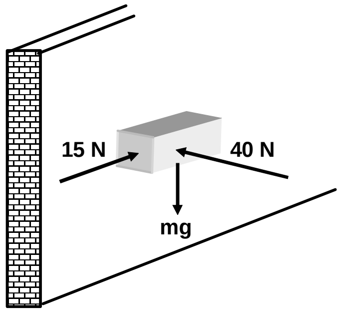
$\Rightarrow \quad a = \frac{g\sin\alpha.\cos\alpha}{\sin^2\alpha + (m_1/m_2)}$.

Prob 7. *A block of mass 2 kg is pushed normally against a rough vertical wall with a force of 40 N, coefficient of static friction being 0.5. Another horizontal force of 15 N, is applied on the block in a direction parallel to the wall. Will the block move? If yes, in which direction and with what minimum acceleration ? If no, find the frictional force exerted by the wall on the block.*

Sol. The force which may cause the tendency of motion or motion in the body is its own weight and the applied horizontal force of 15 N. The resultant of the forces

$$F = \sqrt{20^2 + 15^2} = 25N$$

In a direction $\tan^{-1}\left(\frac{15}{20}\right) = 37°$ with the vertical.



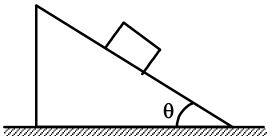
The friction will, by its very virtue of opposing the tendency of relative motion will act in a direction opposite to the resultant force. Now, for the acceleration to be minimum.

The minimum force required = $F - \mu N$ (as, μN is the maximum frictional force)

$$= 25 - 0.5 \times 40 = 5 \text{ N}$$

$\therefore$ Minimum acceleration is $\frac{5}{2} = 2.5 \text{ m/s}^2$.

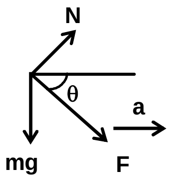
Prob 8. *A small block is resting on an inclined plane (coefficient of friction $\mu > \tan\theta$) as shown in the figure. The inclined plane is given a constant horizontal acceleration 'a' towards right.*



(a) Find the range of 'a' such that the block does not slide on the plane.

(b) Find the value of 'a' such that the friction force between the block and the plane is zero.

Sol. (a) As $\mu > \tan\theta$ the block does not slide when a = 0 which is the lower limit.



Newton's 2nd law

$$N - mg\cos\theta = ma\sin\theta \quad \ldots (1)$$

$$F + mg\sin\theta = ma\cos\theta \quad \ldots (2)$$

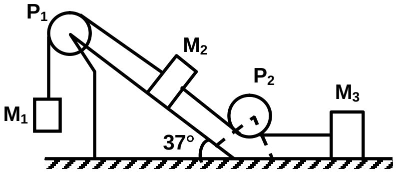
Force of friction $F \le \mu N$

$\therefore$ $a \le \frac{\mu + \tan\theta}{1 - \mu\tan\theta}.g$ (no slide condition)

$\therefore$ the range of a is O to $\frac{\mu + \tan\theta}{1 - \mu\tan\theta}.g$

(b) setting F = 0 we get $a = g\tan\theta$

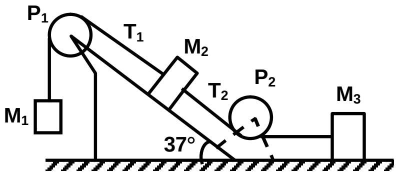
Prob 9. *Masses M_1, M_2 and M_3 are connected by strings of negligible mass which pass over massless and frictionless pulleys P_1 and P_2 as shown in the figure. The masses move such that the portion of the string between P_1 and P_2 is parallel to the inclined plane and the portion of the string between P_2 and M_3 is horizontal.*



The masses M_2 and M_3 are 4.0 kg each and the coefficient of kinetic friction between the masses and the surfaces is 0.25. The inclined plane makes an angle of 37° with the horizontal. If the mass M_1 moves downwards with a uniform velocity find the

(a) mass of M_1

(b) tension in the horizontal portion of the string. ($g = 9.8 \text{ m/s}^2$ and $\sin 37° \approx 3/5$)



Sol. Let T_1 be the tension in the string connecting M_1 and M_2 and T_2 be the tension in the string connecting M_2 and M_3. From the figure.

$M_1g = T_1$

$T_2 = \mu M_3 g = (0.25)4g$

or, $T_2 = g = 9.8$ N

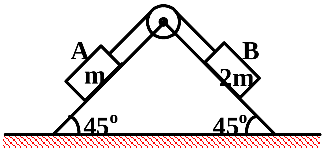
Also, $T_1 = T_2 + (0.25) \times 4g\cos 37° + 4g\sin 37°$

$= g\left(1 + \frac{4}{5} + \frac{4\times 3}{5}\right)$

$T_1 = \frac{21}{5}g$

or, $M_1 g = \frac{21}{5}g$

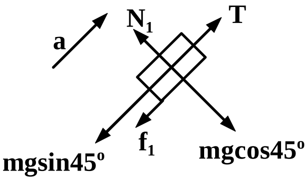
$\therefore$ $M_1 = \frac{21}{5} = 4.2$ kg



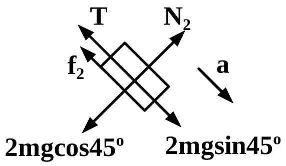
Prob10. *Block A of mass m and block B of mass 2m are placed on a fixed triangular wedge by means of a light and inextensible string and a frictionless pulley as shown in the figure. The wedge is inclined at 45° to the horizontal on both sides. The coefficient of friction between the block A and the wedge is 2/3 and that between the block B and the wedge is 1/3. If the system of A and B is released from rest, then find*
(a) the acceleration of A
(b) tension in the string
(c) the magnitude and direction of the frictional force acting on A.

Sol. (a) In the absence of friction the block B will move down the plane and the block A will move up the plane. Frictional force opposes this motion.

F.B.D. of the blocks.



(A)



(B)

$\Rightarrow$ $T - mg\sin 45° - f_1 = ma$ . . . (1)

and, $2mg\sin 45° - f_2 - T = 2ma$ . . . (2)

Adding (1) and (2), we get

$mg\sin 45° - (f_1 + f_2) = 3ma$

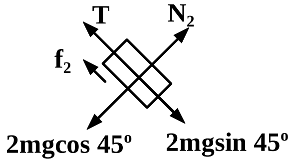
For a to be non-zero $mg\sin 45°$ must be greater than the maximum value of $(f_1 + f_2)$

$\because$ $(f_1 + f_2)_{max} = (\mu_1 m_1 + \mu_2 m_2)g\cos 45° = \frac{4}{3}mg\cos 45°$

$\Rightarrow$ $mg\sin 45° < (f_1 + f_2)_{max}$

Hence blocks will remain stationary

(b) F.B.D. of the block B



$f_{(2)max} = \frac{1}{3} 2mg\cos 45^o = \frac{2}{3\sqrt{2}} mg$ &

$2\,mg\sin 45° = \frac{2mg}{\sqrt{2}}$

$\because$ $2\,mg\sin 45° > f_{2(max)}$, therefore block B has tendency to slide down the plane.

For block B to be at rest

$T + f_{2(max)} = 2\,mg\sin 45°$

$\Rightarrow T = \frac{mg}{\sqrt{2}}\left(2 - \frac{2}{3}\right) = \frac{4mg}{3\sqrt{2}}$

$\Rightarrow T = \frac{2\sqrt{2}}{3} mg$

(c) $mg\cos 45° = \frac{mg}{\sqrt{2}}$

$\because$ T(tension) is greater than mg cos 45°.

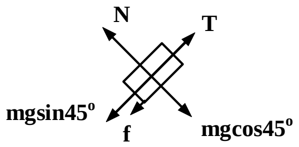
Hence block A has tendency to move up the plane, therefore frictional force on the block A will be down the plane.

For A to be at rest

$mg\sin 45^o + f = T$

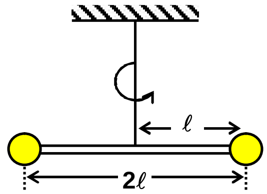
$\Rightarrow f = T - mg\sin 45^o$

$= \frac{2\sqrt{2}mg}{3} - \frac{mg}{\sqrt{2}} \Rightarrow f = \frac{mg}{3\sqrt{2}}$



F.B.D. of A

Prob 11. *Consider two small balls connected to a rigid rod of length '2ℓ' which in turn is suspended by a thread & rotated about the thread at angular velocity ω. What would be the magnitude & direction of total force exerted by rod on one of the balls?*



Sol. Consider the ball in ground's reference frame. It is observed to be doing uniform circular motion for which each ball must experience a resultant force of magnitude $m\omega^2\ell$ directed towards centre of its circular path.

$\Rightarrow \quad \vec{R} + m\vec{g} = m\omega^2\ell\,\hat{n}$ Where $\vec{R}$ is the force exerted by rod on one of the two balls.

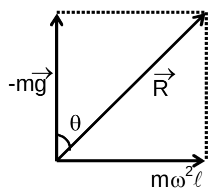
$\Rightarrow \quad \vec{R} = m\omega^2\ell\,\hat{n} - m\vec{g}$

As $\hat{n}$ and $\vec{g}$ are mutually perpendicular

$$|\vec{R}| = \sqrt{(m\omega^2\ell)^2 + (mg)^2} = m\sqrt{(\omega^2\ell)^2 + g^2}$$

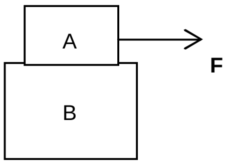
To decide direction of $\vec{R}$, consider the following vector diagram.

Thus, the angle that $\vec{R}$ makes with vertical to equals $\tan^{-1}\left(\frac{\omega^2\ell}{g}\right)$.



Prob 12. *A block A of mass 2 kg is placed on another blocks of mass 5 kg and a horizontal force F is applied on the block A . If co-efficient of friction between block A and B is 0.3 and between block B and the floor is frictionless, then what is the maximum value of F so both blocks will move together and what is the value of this acceleration?*

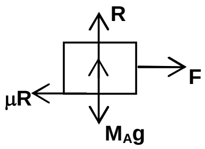
Sol. Suppose both blocks will move with common acceleration a, then



$F = (m_A + m_B)\, a$

$a = \frac{F}{2+5} = \frac{F}{7}$

Now for F.B.D. of block A.



$F - \mu R = m_A\, a$

$\Rightarrow \mu R = F - m_A\, a = F - \frac{2F}{7}$

$\Rightarrow \mu\, m_A g = \frac{5F}{7}$

$\Rightarrow \; = \frac{7 \times 0.3 \times 2 \times g}{5}$

$= 4.2 \times 2 = 8.4$ N and

$a = \frac{F}{7} = \frac{8.4}{7} = 1.2 \text{ m/s}^2$

Prob 13. *A car start from rest and accelerates uniformly with 2 m/s^2. At t = 10 s, a stone is dropped out of the window (1 m high) of the car. What are the (a) velocity and (b) acceleration of the stone at t = 10.1 sec? Neglect air resistance and take g = 9.8 m/s^2.*

Sol. At t = 10 s

Velocity of car = at = 20 m/s

(a) Horizontal component of stone V_s=20 m/s.

Now in vertical direction acceleration = g m/s^2

So at t = 10.1 sec.

$v_y = gt = 9.8 \times (10.1 - 10)$

$= 0.98$ m/s

So, resultant velocity of stone = $\sqrt{v_x^2 + v_y^2}$ = 20.02 m/s and

the angle of resultant velocity with horizontal direction is

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{0.98}{20}\right)$

$\theta = 2.8°$

(b) The moment of the stone is dropped out from the car, horizontal force on the stone =0. The only acceleration is due to gravity $a_y = g = 9.8$ m/s^2 (downwards)

Objective:

Prob 1. *An iron nail is dropped from a height h from the level of a sand bed. If it penetrates through a distance x in the sand before coming to rest, the average force exerted by the sand on the nail is,*

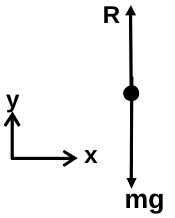
(A) $mg\left(\frac{h}{x}+1\right)$ (B) $mg\left(\frac{x}{h}+1\right)$

(C) $mg\left(\frac{h}{x}-1\right)$ (D) $mg\left(\frac{x}{h}-1\right)$

Sol. The nail hits the sand with a speed v_o after falling through a height h

$\Rightarrow v_0^2 = 2gh \Rightarrow v_0 = \sqrt{2gh}$ …(1)

The nail stops after sometime say t, penetrating through a distance, x into the sand. Since its velocity decreases gradually the sand exerts a retarding upward force, R (say). The net force acting on the nail is given as



$\Sigma F_y = R - mg = ma$

$\Rightarrow R = m(g + a)$ …(2)

Where a = deceleration of the nail. Since the nail penetrates a distance x

$0 - V^2_0 = - 2a\,x$ …(3)

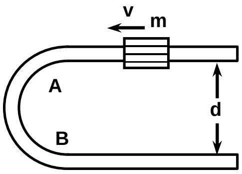
Putting V_0 from (1) and 'a' from (2) in (3), we obtain

$$2gh = 2\left(\frac{R - mg}{m}\right) x$$

$$\Rightarrow \quad R = \frac{mg(h+x)}{x}$$

$\Rightarrow \quad R = mg\left(\frac{h}{x}+1\right)$, Hence **(A)** is the correct choice.

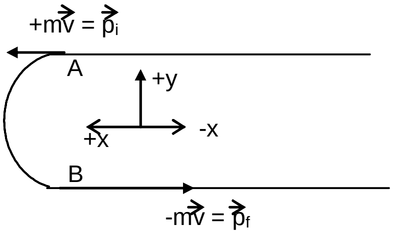
Prob 2. *A U shaped smooth wire has a semi-circular bending between A and B as shown in the figure. A bead of mass 'm' moving with uniform speed v through the wire enters the semicircular bend at A and leaves at B. The average force exerted by the bead on the part AB of the wire is,*



(A) 0 (B) $\frac{4mv^2}{\pi d}$

(C) $\frac{2mv^2}{\pi d}$ (D) none of these.

Sol. Choosing the positive x-y axis as shown in the figure, the momentum of the bead at A is $\vec{p}_i = +m\vec{v}$. The momentum of the bead at B is $\vec{p}_f = -m\vec{v}$.



Therefore, the magnitude of the change in momentum between A and B is

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -2m\vec{v}$$

i.e. $\Delta p = 2mv$ along positive x-axis.

The time interval taken by the bead to reach from A to B is

$$\Delta t = \frac{\pi . d/2}{v} = \frac{\pi d}{2v}.$$

Therefore, the average force exerted by the bead on the wire is

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{\pi d}{2v}} = \frac{4mv^2}{\pi d}$$

Hence the correct choice is **(B)**.

Note:

1. By mistake, if the change in the magnitude of the momentum is considered, average force will be equal to zero.
2. If someone accounts carelessly d = r instead r = d/2 then he will lead to wrong choice (c).

Prob 3. *A man holds a ball of mass (1/2) kg in his hand. He throws it vertically upward. During this process his hand moves up by 40 cm and the ball leaves his hand with an upward velocity of 4 ms^{-1}. The constant force with which the man pushes the ball is*

(A) 2 N *(B) 10 N*
(C) 15 N *(D) 7 N*

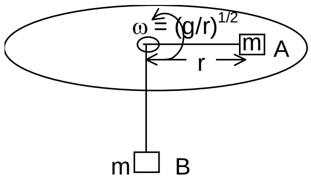
Sol. Acceleration of the ball

$$a = \frac{v^2}{2s} = \frac{4^2}{2\times 0.4} = 20 \text{ m/s}^2$$

Hence force applied by the man $= m(a+g) = \frac{1}{2}(20+10) = 15$ N

Hence the correct choice is **(C)**.

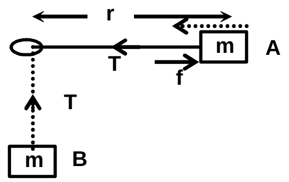
Prob 4. *Two particles A and B, each of mass m, are inter-connected by an inextensible string such that the particle B hangs below a table as shown in the figure and particle A is on a rough rotating disc at a distance r from the axis of rotation of the disc.*



If the angular speed of the disc and the block is $\omega = \sqrt{g/r}$, the frictional force developed at the interface of the particle & the disc is equal to

(A) mg/2 *(B) < mg/2*
(C) mg/√2 *(D) zero*

Sol. The particle of mass m experiences two forces (i) tension T (ii) frictional force f.



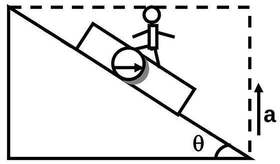
Since the particle A is rotating in a circular path of radius r, its centripetal acceleration,

$$\Rightarrow \quad r\,\omega^2 = \frac{T-f}{m}$$

Putting T = mg for equilibrium of the mass B & $\omega^2 = g/r$

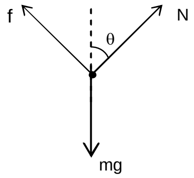
we obtain f = mg – mr g/r = 0

Prob 5. *A man of mass m = 60 kg is standing on weighing machine fixed on a triangular wedge of angle $\theta = 60°$ with horizontal as shown in the figure. The wedge is moving up with an upward acceleration $a = 2\ m/s^2$. The weight registered by machine is*



(A) 600 N *(B) 1440 N*
(C) 360 N *(D) 240 N*

Sol. $N - mg\cos\theta = ma\cos\theta$
$N = m(g + a)\cos\theta = 60\ (10 + 2)\cos 60°$
$= 360$ N
Hence the correct choice is **(C).**



Prob 6. *A massive platform of mass M is moving with speed $v=6\ ms^{-1}$. At t=0 a body of mass m ($m << M$) is gently placed on the platform. If coefficient of friction between body and platform is $\mu = 0.3$ and $g = 10\ m/s^2$, then*
(A) the body covers a distance 3 m on the platform in the direction of motion of the platform.
(B) the body covers a distance 3 m on the platform opposite to the direction of motion of platform before coming to rest.
(C) the body covers a distance of 6 m on the platform in the direction of motion of the platform.
(D) the body covers a distance of 6 m on the platform opposite to the direction of motion of platform before coming to rest.

Sol. Since M>>m, the velocity of M remains unchanged after m is placed on to it.

Acceleration of m, $a = \dfrac{\mu mg}{m} = \mu g$

$a_{mM} = a - 0 = a$ and initial relative velocity $v_{mM} = 0 - v = -v$

Hence $s = \dfrac{v^2}{2\mu g} = \dfrac{6^2}{2\times 0.3\times 10} = 6$ m. Hence the correct choice is **(D).**

Prob 7. *A body of mass m is kept on a rough horizontal surface of friction coefficient μ. A force is applied horizontally, but the body is not moving. The net force 'F' by the surface on the body will be*
(A) $F \le \mu\, mg$ *(B) $F = \mu\, mg$*
(C) $mg \le F \le mg\sqrt{1+\mu^2}$ *(D) $mg \ge F \ge mg\sqrt{(1-\mu^2)}$*

Sol. If the body is not moving, F = f, where f is the force of friction on the body and
$0 \le f \le \mu mg$ or $0 \le F \le \mu mg$ . . . (i)
The force by the surface on the body

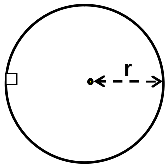
$R = \sqrt{f^2 + N^2} = \sqrt{F^2 + (mg)^2}$

or $R = mg\sqrt{\mu^2 + 1}$

$\therefore\ mg \le F \le mg\sqrt{1+\mu^2}$,

Hence the correct choice is **(C)**

Prob 8. *Consider a small cube of mass 'm' kept on a horizontal disc. If the disc is to rotate with uniform angular velocity, what could be its maximum value without causing any sliding between the cube and the disc? (Coefficient of static friction between cube & disc is μ).*



(A) $\sqrt{\frac{\mu g}{r}}$ (B) $\sqrt{\frac{2\mu g}{r}}$

(C) $\sqrt{\frac{\mu g}{2r}}$ (D) $2\sqrt{\frac{\mu g}{r}}$

Sol. In absence of any sliding, net force on the cube in the frame of reference rotating with disc will be zero. We find two forces in the plane of disc - frictional force and centrifugal force.

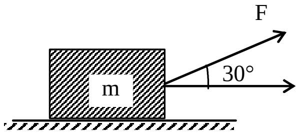
Hence, $m\omega^2 r = f$

but $f \le \mu mg$

Hence, $\omega \le \sqrt{\mu g / r} \Rightarrow \omega \le \sqrt{\mu g / r}$

$\Rightarrow \omega_{max} = \sqrt{\frac{\mu g}{r}}$, Hence **(A)** is the correct choice.

Prob 9. *A mass m rests on a horizontal surface. The coefficient of friction between the mass and the surface is μ. If the mass is pulled by a force F as shown in figure, the limiting friction between mass and the surface will be*



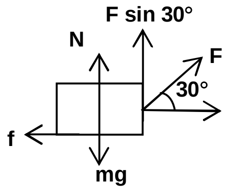
(A) μmg (B) $\mu(mg - \frac{\sqrt{3}}{2}F)$

(C) μ [mg – F/2] (D) μ (mg + F/2)

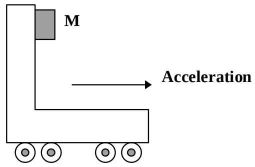
Sol. From F.B.D. N = mg — F sin 30°

So Limiting friction $= \mu N = \mu\left(mg - \frac{F}{2}\right)$

So, option (C) is correct.



Prob 10. *An accelerated system with a vertical wall has co-efficient of friction μ between block and walls as shown in the figure. A block M of mass 1 kg just remains in equilibrium with the vertical wall, when the system has an acceleration of 20 m/s^2. The co-efficient of friction has a value*



(A) 0.10 (B) 0.25

(C) 0.50 (D) 1

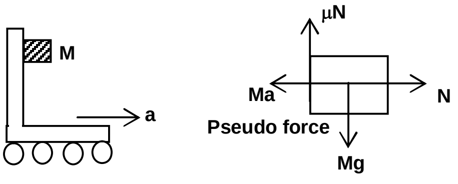
Sol. So, N = Ma

And if mass M is in equilibrium then

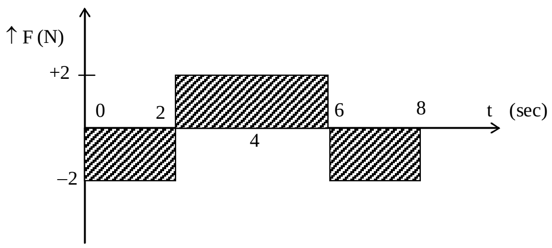
then, $\mu N - Mg = 0$

$$\mu = \frac{g}{a} = \frac{10}{20} = 0.5$$

So, option (C) is correct.

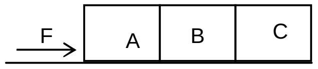


Prob 11. *Force time graph for the motion of a body of mass 2 kg is shown in figure. Change in velocity between 0 to 8 sec is*
(A) zero
(B) 4 m/s
(C) 8 m/s
(D) None of these



Sol. Area of graph = + $[2 \times (6-2)] - [2 \times 2] - [2 \times 2]$
Change in momentum = $\int F.dt = 8 - 4 - 4 = 0$
Thus there is no change in velocity between 0 to 8 sec.
So option (A) is correct.

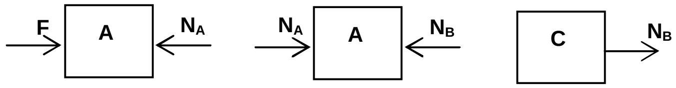
Prob 12. *A constant force F pushes three blocks A, B and C on a horizontal smooth surface. The masses of the blocks are m_A, m_B and m_C respectively. The normal reaction between the blocks B and C will be*



(A) $\frac{F(m_B + m_C)}{m_A + m_B + m_C}$ *(B)* $\frac{Fm_A}{(m_B + m_C)}$

(C) $\frac{Fm_C}{(m_B + m_C)}$ *(D)* $\frac{F.m_C}{(m_A + m_B + m_C)}$

Sol. F.B.D. of blocks



If the common acceleration of the block is a, then
F = $(m_A + m_B + m_C)$
So normal reaction between B and C is
$N_B = m_C a = \frac{F \cdot m_c}{(m_A + m_B + m_C)}$
So option (D) is correct.

ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. In general the normal force is not equal to the weight. Give an example where the two forces are equal in magnitude and at least two examples where they are not.

2. A car is driven up a steep hill at constant speed. Discuss all the forces acting on the car. What pushes it up the hill?

3. If there is a net force on a particle in uniform circular motion, why does the particle's speed not change.

4. A curve in a road has the banking angle calculated for 80 km/h. However, the road is covered with ice, and you plan to creep around the highest lane at 20 km/h. What may happen to your car ? Why ?

5. A reference frame attached to the earth cannot be an inertial frame. Explain.

6. A person is sitting on a moving train and is facing the engine. He tosses up a coin which falls behind him. Find out the reason.

7. Is a 'single isolated force' possible in nature ?

8. Which of Newton's laws of motion is involved in rocket propulsion?

9. Can a body in linear motion be in equilibrium position?

10. Friction is a self – adjusting force. Is this statement correct ? If yes then justify.
    What is limiting friction ? What are the laws of limiting friction ?

11. A body moving over the surface of another body suddenly comes to rest. What happens to friction between the two surfaces ?

12. When walking on ice, one should take short steps rather than long steps. Why

13. Why does a cyclist bend inwards from his vertical position while taking a turn?

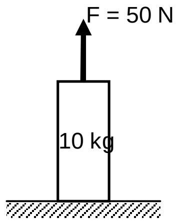
14. A stone tied to one end of a string is whirled in a circle. If the string breaks, the stone flies off tangentially. Explain.

15. Why is it easier to pull a body than to push it.

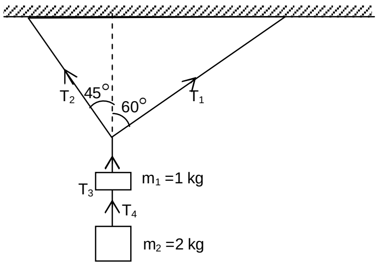
Level - I

1. A ball of mass 0.2 kg falls from a height of 45 m. On striking the ground, it rebounds in 0.1sec with two third of the velocity with which it struck the ground. Calculate
   (a) change in the momentum of the ball immediately after hitting the ground,
   (b) the average force on the ball due to the impact.

2. (a) Find the normal reaction between the block and the horizontal surface.

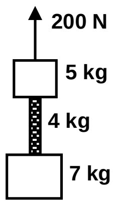


   (b) Find out the tensions T_1, T_2 , T_3 and T_4
   (Take g = 10 m/s^2)

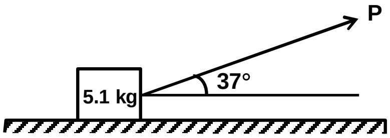


3. A body hangs from a spring balance supported from the roof of an elevator.
   (a) If the elevator has an upward acceleration of 2 m/s^2 and balance reads 240 N, what is the true weight of the body?
   (b) Under what circumstances will the balance read 160 N?
   (c) What will the balance read if the elevator cable breaks? (Take g = 10 ms^{-2})

4. Two blocks shown in figure are connected by a heavy uniform rope of mass 4 kg. An upward force of 200 N is applied as shown.
   (a) What is the acceleration of the system?
   (b) What is the tension at the top of heavy rope?
   (c) What is the tension at the mid-point of the rope?

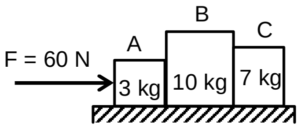


5. A 5.1 kg block is pulled along a frictionless floor by a cord that exerts a force P = 10 N at an angle θ = 37° above the horizontal, as shown in the figure.



   (a) What is the acceleration of the block?
   (b) The force P is slowly increased. What is the value of P just before the block breaks off the floor?
   (c) What is the acceleration of the block just before it is lifted off the floor?
   (d) Suppose the surfaces are rough with μ = 0.4, for what value of P the block just begins to move?

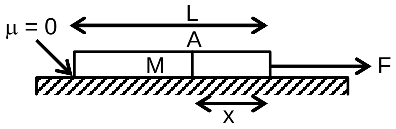
6. Find out the mutual contact forces between A and B and between blocks B and C.



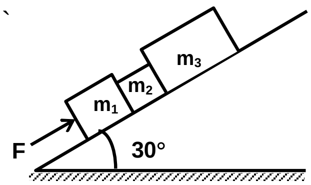
μ = 0 for all surfaces

7. A block weighing 100 kg is placed on an inclined plane of height 6 m and base 8 m. The co-efficient of friction is 0.3. ($g = 9.8$ m/s^2)
   (a) Would the block slide down the inclined plane due to its own weight? If so, how far it will move in 1s starting from rest?
   (b) What force parallel to the inclined plane must be applied to just support the block on the plane?
   (c) What force parallel to the inclined plane is required to keep the block moving up the plane at constant velocity?
   (d) If an upward force of 940 N parallel to the inclined plane is applied to the block what will be its acceleration?
   (e) How far will the block move in 1s starting from rest?
   (f) What will happen if an upward force of 500 N parallel to the inclined plane is applied?
   (g) If an upward force of 260 N parallel to the inclined plane is applied what will happen? How far will the block move in 1s starting from rest?

8. Find the tension in rope at section A, at a distance x from the right end.

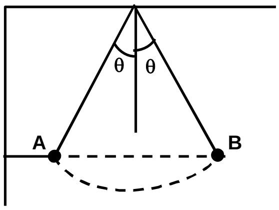


9. Three blocks $m_1 = 3$ kg, $m_2 = 2$ kg, $m_3 = 5$ kg, lie on an inclined frictionless surface as shown in the figure.
   (a) What force (F) parallel to the incline is needed to push the blocks up the plane with an acceleration $a = 2$ m/s^2?
   (b) Find the contact force between m_1 & m_2 and m_2 and m_3.



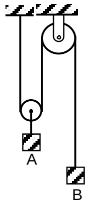
10. A child places a picnic basket on the outer rim of a merry go round that has a radius of 4.0 m and revolves once in every 24 s. What is the minimum co-efficient of static friction for the basket to stay on the merry go round?

11. A ball is held at rest in position A as shown in the figure by two light cords. The horizontal cord is cut and the ball swings as a pendulum. What is the ratio of the tensions in the supporting cord, in position A, to that in position B?

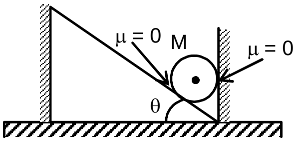


12. A balloon is descending with a constant acceleration 'a' less than the acceleration due to gravity. The mass of the balloon, with its basket and contents is M. What mass m, of ballast (Sand bags) should be released so that the balloon will begin to accelerate upward with constant acceleration 'a'? (Neglect air resistance)

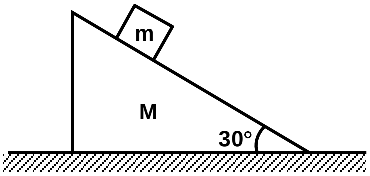
13. Two blocks 'A' and 'B' having masses m_A and m_B respectively are connected by an arrangement shown in the fig. Calculate the downward acceleration of the block B. Assume the pulleys to be massless. Under what condition the block A will have downward acceleration ?



14. Find the normal reaction forces acting on the vertical wall (say N_1) and the fixed incline surface (say N_2) respectively by the sphere of mass M.



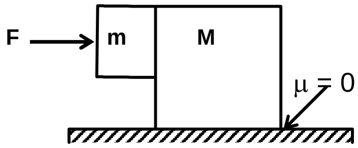
15. A block of mass m = 1 kg rests on a wedge of mass M = 9 kg, which in turn is placed on a table as shown in the figure. All the surfaces are smooth.
(a) What horizontal acceleration 'a' must M have relative to stationary table so that m remains stationary relative to the wedge?



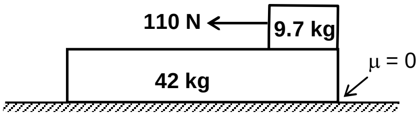
(b) Find the horizontal force required to maintain this acceleration.

Level- II

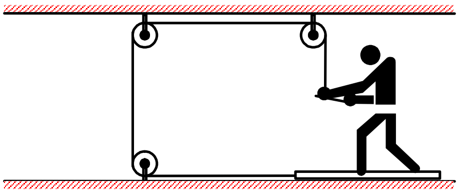
1. The two blocks m = 5 kg and M = 25 kg as shown in the figure are free to move. The coefficient of friction between the blocks is $\mu_s = 0.4$, but the coefficient of friction between ground and M is zero. What is the minimum horizontal force F required to hold m against M?



2. A 42 kg slab rests on a frictionless floor. A 9.7 kg block rests on the top of the slab as shown in the figure. The coefficient of static friction between the block and the slab is 0.53, while the coefficient of kinetic friction is 0.38. The 9.7 kg block is acted on by a horizontal force of 110 N. What are the resulting accelerations of
(a) the block? (b) the slab?

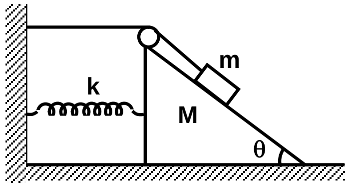


3. The friction coefficient between the board and the floor shown in figure is μ. Find the maximum force that the man can exert on the rope so that the board does not slip on the floor. The mass of man is M and the mass of plank is m.

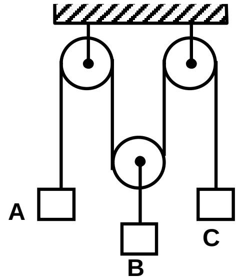


4. A car moves with constant tangential acceleration $a_T = 0.80$ m/s^2 along a horizontal surface circumscribing a circle of radius R = 40m. The coefficient of sliding friction between the wheels of the car and the surface is μ = 0.20. What distance will the car ride without sliding if its initial velocity is zero ?

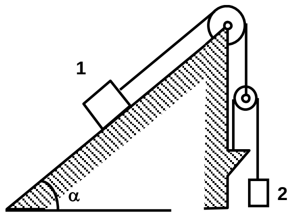
5. A wedge of mass 'M' and angle of inclination 'θ' and of mass 'm' is arranged in a manner shown in the figure. The spring of force constant 'k' attached to the wedge. Assuming the pulleys to be massless and all surfaces to be frictionless. Find the compression of the spring under equilibrium condition.



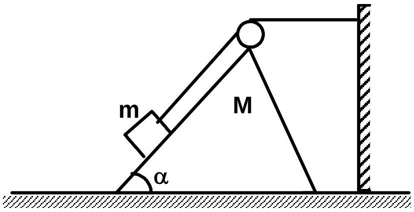
6. The pulley block system shown in the figure is released from rest. Assuming the pulleys to be light and frictionless and the string to be light and inextensible, find
(a) the acceleration of the blocks A, B and C.
(b) the tension in the string connecting the blocks. The masses of the block A, B and C are m, m and 2m respectively.



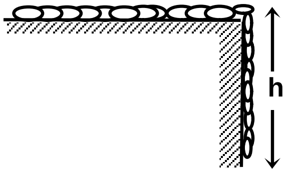
7. Find the acceleration a of body 2 in the arrangement shown in Fig. If its mass is n times as great as the mass of bar 1 and the angle that the inclined plane forms with the horizontal is equal to α. The masses of the pulleys and the threads, as well as the friction, are assumed to be negligible. Look into possible cases.



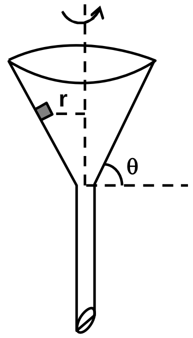
8. In the arrangement shown in the figure, the masses m of the bar and M of the wedge, as well as the wedge angle α, are known. The masses of the pulley and thread are negligible. Friction is absent. Find the acceleration of the wedge M.



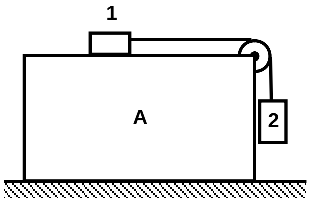
9. A uniform chain of length ℓ is released from rest on a smooth horizontal table with a portion h of the chain overhanging as shown in the figure. Find the time taken by the chain to slip off the table.



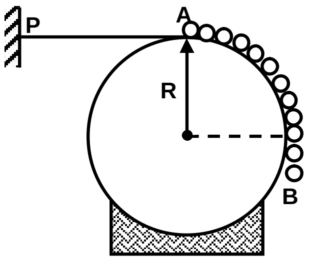
10. A small cube of mass m is placed on the inside of a funnel rotating about a vertical axis at a constant rate of f revolutions per second. The wall of the funnel makes an angle θ with the horizontal. The coefficient of static friction between the cube and the funnel is μ and the centre of the cube is at a distance r from the axis of rotation find the
(a) largest and
(b) smallest value of f for which the cube will not move with respect to the funnel.



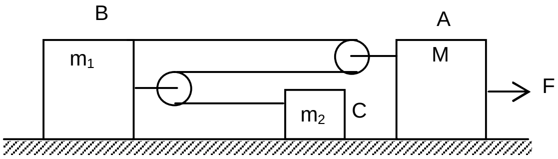
11. What is the minimum and maximum acceleration with which bar A (Fig.) should be shifted horizontally to keep bodies 1 and 2 stationary relative to the bar? The masses of the bodies are equal , and the co-efficient of friction between the bar and the bodies is equal to k. The masses of the pulley and the threads are negligible, the friction in the pulley is absent.



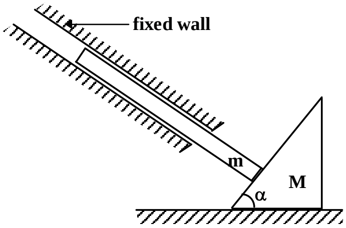
12. A uniform flexible chain of length 1.50 m rest on a fixed smooth sphere of radius $R = 2/\pi$ m such that one end A of chain is at top of the sphere while the other end B is hanging freely. Chain is held stationary by a horizontal thread PA as shown in figure.
Calculate the acceleration of the chain when the thread is burnt.



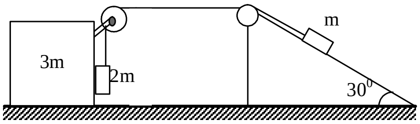
13. In the shown figure, the blocks and pulley are ideal and force of friction is absent. External horizontal force F is applied as shown in the figure. Find the acceleration of block C.



14. In the arrangement shown in the figure, the rod of mass m held by two smooth walls, always remains perpendicular to the surface of the wedge of mass M. Assuming that all the surfaces are frictionless, find the acceleration of the rod and that of the wedge.



15. Neglect friction. Find accelerations of m, 2m and 3m as shown in the figure. The wedge is fixed.



Objective:

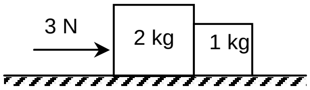
Level- I

1. When a bird of weight W sits on a stretched wire, the tension T in the wire is
   (A) > W/2 (B) = W
   (C) < W (D) None of these

2. A machine gun fires n bullets per second and the mass of each bullet is m. If v is the speed of each bullet then the force exerted on the machine gun is
   (A) mng (B) mnv
   (C) mnvg (D) mnv/g

3. A particle of mass 'm' moving with a velocity $\vec{v}$ strikes a horizontal surface at angle θ with the vertical and rebounds with the same magnitude and at the same angle with the vertical. The magnitude of change in momentum is
   (A) 2 mv cos θ (B) 2 m v sin θ
   (C) 2 mv (D) 0

4. Two blocks of masses 2 kg and 1 kg are in contact with each other on a horizontal frictionless table. When a horizontal force of 3.0 N is applied to the block of mass 2 kg, the value of the force of contact between the two blocks is:



   (A) 4 N (B) 3 N
   (C) 2 N (D) 1 N

5. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force F is applied at one end of the rope, the force which the rope exerts on the block is:
   (A) $\frac{FM}{m+M}$ (B) $\frac{Fm}{m+M}$
   (C) $\frac{FM}{M-m}$ (D) $\frac{Fm}{M-m}$

6. Two masses m and m' are tied with a thread passing over a pulley, m' is on a frictionless horizontal surface and m is hanging freely. If acceleration due to gravity is g, the acceleration of m' in this arrangement will be
   (A) g (B) $\frac{mg}{(m+m')}$
   (C) $\frac{m'g}{(m+m')}$ (D) $\frac{g(m-m')}{(m+m')}$

7. Which of the following statements is true in a tug of war.
   (A) The team which applies a greater force on the rope than the other wins.
   (B) The team which applies a smaller force on the other wins.
   (C) The team which pushes harder against the ground wins.
   (D) none of these

8. Two students of equal weight try to break a rope which can break if the tension in it is equal to the sum of their weights
   (A) They should pull against each other applying a force equal to their weight on the rope.
   (B) They should hang the rope over a pulley and pull on either side of the pulley downwards.
   (C) They should climb on either side of the rope suspended over the pulley.
   (D) They should tie one-end of the rope to the ceiling and pull the other end together.

9. A massless rope passes over a frictionless pulley. A monkey holds one end of the rope and a mirror, having the same weight as the monkey, is attached to the other end of the rope at the same level. The monkey gets scared by its own image. It can get away from its image by
   (A) climbing up the rope. (B) moving down the rope.
   (C) by releasing the rope. (D) None of the above.

10. A chain of length L and mass M is hanging by fixing its upper end to a rigid support. The tension in the chain at a distance *x* from the rigid support is:
   (A) Zero (B) $\frac{MgL}{(L-x)}$
   (C) $Mg\frac{(L-x)}{L}$ (D) Mgx / L

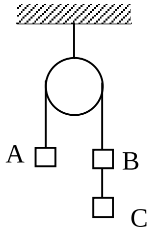
11. When a force of constant magnitude always act perpendicular to the motion of a particle then:
   (A) Velocity is constant (B) Acceleration is constant
   (C) KE is constant (D) None of these

12. A given object takes n times as much time to slide down a 45^o rough incline as it takes to slide down a perfectly smooth 45^o incline. The coefficient of kinetic friction between the object and the incline is given by
   (A) $\mu_k = \frac{1}{(1-n^2)}$ (B) $\mu_k = 1-\frac{1}{n^2}$
   (C) $\mu_k = \sqrt{\frac{1}{(1-n^2)}}$ (D) $\sqrt{1-\frac{1}{n^2}}$

13. Sand drops are falling gently at the rate of 2 kg/sec. on a conveyor belt moving horizontally with a velocity of 5 m/s. The extra force and extra power required to keep the belt moving will be respectively,
   (A) 10 N force and 20 watts (B) 10 N force and 50 watts.
   (C) 20 N force and 20 watts (D) 50 N force and 50 watts

14. A body of mass 60 kg is dragged with just enough force to start moving on a rough surface with coefficients of static and kinetic frictions 0.5 and 0.4 respectively. On applying the same force what is the acceleration ?
   (A) 0.98 m/s^2 (B) 9.8 m/s^2
   (C) 0.54 m/s^2 (D) 5.292 m/s^2

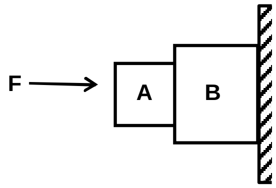
15. Three equal weights A, B, C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in figure. The tension in the string connecting weights B and C is



(A) Zero
(B) 13 N
(C) 3.3 N
(D) 19.6 N

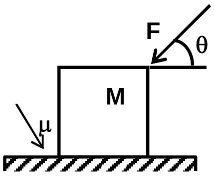
Level- II

1. Two blocks A and B are pushed against the wall with the force F. The wall is smooth but the surfaces in contact of A and B are rough. Which of the following is true for the system of the blocks?



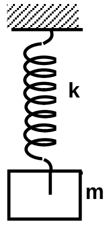
(A) F should be equal to weight of A and B.
(B) F should be less than the weight of A and B.
(C) F should be more than the weight of A and B
(D) The system cannot be in equilibrium (at rest).

2. A block of mass M lies on a rough surface of coefficient of friction μ. A force is applied on it at an angle θ to the horizontal as shown, and the block is at rest. The frictional force acting on the block will be



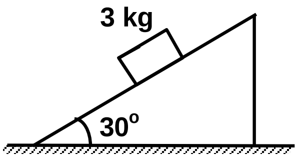
(A) $F \cos\theta$ (B) $\mu(mg + F\sin\theta)$
(C) $\mu(mg - F\sin\theta)$ (D) μmg

3. The spring mass system shown in the figure is in equilibrium. If the mass m is pulled down by a distance mg/3k and released, its instantaneous acceleration will be



(A) g/3 upward (B) 2g/3 downward
(C) g/3 downward (D) 2g/3 upward

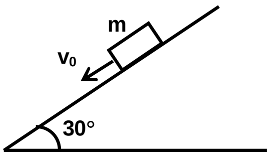
4. A block of mass 3 kg is at rest on a rough inclined plane as shown in the figure. The magnitude of net force exerted by the surface on the block will be



(A) 26 N (B) 19.5 N
(C) 10 N (D) 30 N

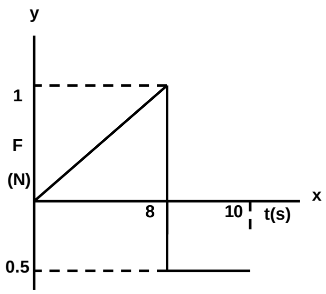
5. A body of mass 60 kg is dragged along a horizontal surface by a horizontal force which is just sufficient to start the motion of the body from rest. If the coefficients of static and kinetic friction are 0.5 and 0.4 respectively, the acceleration of the body is
(A) $0.98\ m/s^2$ (B) $9.8\ m/s^2$
(C) $0.54\ m/s^2$ (D) $5.292\ m/s^2$

6. A block of mass m is pushed down on a rough inclined plane (coefficient of friction is 0.25) with a velocity v_0 as shown in the figure. Then the block will

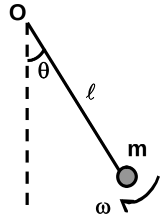


(A) decelerate and come to rest (B) accelerate downward
(C) move downward with velocity v_0 (D) first accelerate and then decelerate

7. A force time graph for the motion of a body is shown in the figure. The change in the momentum of the body between zero and 10 sec. is
(A) zero
(B) 4 kg m/s
(C) 5 kg m/s
(D) 3 kg m/s

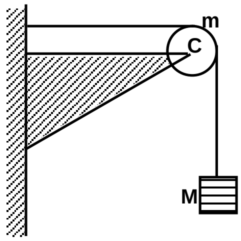


8. A simple pendulum swings in a vertical plane about the point of suspension O. In the position shown in the figure, the string has an angular velocity ω radian/second. The instantaneous tension in the string is



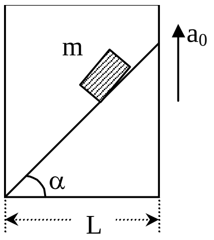
(A) $mg\cos\theta$
(B) $mg\left(\cos\theta + \frac{\ell\omega^2}{g}\right)$
(C) $mg\left(\cos\theta - \frac{\ell\omega^2}{g}\right)$
(D) $m\ell\omega^2$

9. A string of negligible mass, going over a clamped pulley of mass m, supports a block of mass M as shown in the figure. The force on the pulley by the clamp C is given by
(A) $\sqrt{2}Mg$
(B) $\sqrt{2}mg$
(C) $g\sqrt{(M+m)^2+m^2}$
(D) $g\sqrt{(M+m)^2+M^2}$

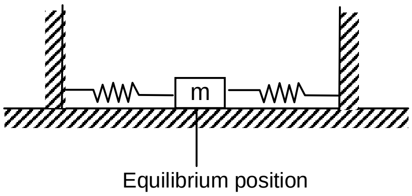


10. A long horizontal rod has a bead which can slide along its length, The bead is initially placed at a distance L from one end A of the rod. The rod is set in angular motion in the horizontal plane about the end A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping on the rod is
(A) $\sqrt{\frac{\mu}{\alpha}}$
(B) $\frac{\mu}{\sqrt{\alpha}}$
(C) $\frac{1}{\sqrt{\mu\alpha}}$
(D) infinite

11. A particle slides down a smooth inclined plane of elevation α fixed in the elevator going up with an acceleration a_0 as shown in figure. The base of the incline has a length L. The time taken by the particle to reach the bottom is
(A) $\left[\frac{2L}{(g+a_0)\sin\alpha\cos\alpha}\right]^{1/2}$
(B) $\left[\frac{2L}{g\sin\alpha\cos\alpha}\right]^{1/2}$
(C) $\left[\frac{g\sin\alpha\cos\alpha}{2L}\right]^{1/2}$
(D) $\left[\frac{2L}{a_0\sin\alpha\cos\alpha}\right]^{1/2}$



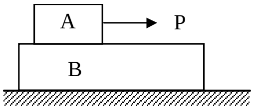
12. A body of mass 'm' is connected to two springs of spring constants K_1 and K_2 and is in equilibrium on a smooth horizontal surface as shown. If the body is displaced to the left by a small distance 'x' from the position shown, what is the velocity of the body as it passes through this position again? (springs are massless)

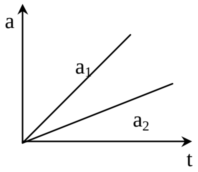


(A) 0
(B) $x\sqrt{\frac{K_1 + K_2}{m}}$
(C) $(K_1 + K_2)x$
(D) can't say

13. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1m. The angle made by the rod with the track is
(A) Zero
(B) 30°
(C) 45°
(D) 60°

14. Block A is placed on block B, whose mass is greater than that of A. There is friction between the blocks, while the ground is smooth. A horizontal force P, linearly increasing with time, begins to act on A. The acceleration a_1 and a_2 of A and B respectively are plotted against time t. Choose the correct graph.



(A)


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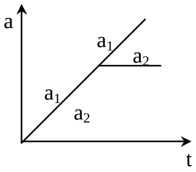
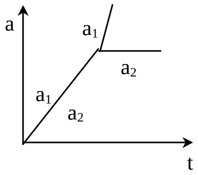
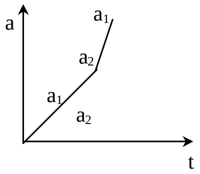


(C)


(D)


15. A block of mass M rest on a rough horizontal surface. The coefficient of friction between the block and the surface is μ. A force F = Mg acting at an angle θ with the vertical side of the block pulls it. In which of the following cases, the block can be pulled along the surface?
(A) $\tan \theta \geq \mu$
(B) $\cot \theta \geq \mu$
(C) $\tan \theta/2 \geq \mu$
(D) $\cot \theta/2 \geq \mu$

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – I

1. (a) 10 Ns (b) 100 N
2. (a) 50 N (b) $T_1 = \frac{60\,N}{1+\sqrt{3}}$, $T_2 = \frac{30\sqrt{6}}{1+\sqrt{3}}$ N, T_3 = 30 N, T_4 = 20 N
3. (a) 200 N (b) When the elevator as a downward acceleration of 2 m/s^2(c) zero
4. 2.5 m/s^2, 137.5 N, 112.5 N
5. (a) 1.56 m/s^2 (b) 83.3 N (c) 13 m/s^2 (d) 19.6 N
6. F_{AB} = 51 N, F_{BC} = 21 N
7. (a) Yes, 176.4 cm; (b) 36 kg wt; (c) 84 kg wt; (d) 116.8 cm/s^2 upwards; (e) 58.4 cm; (f) remains at rest, (g) slides down with acceleration 92.8 cm/s^2, 46.4 cm
8. $F\left(\frac{L-x}{L}\right)$
9. (a) 70 N (b) 49 N, 35 N
10. 0.032
11. $\frac{T_A}{T_B} = \sec^2\theta$
12. $m = \frac{2\ \ Ma}{g+a}$
13. 2g ($2m_B - m_A$) / ($4m_B + m_A$) ; when $m_A > 2m_B$
14. $N_1 = Mg\tan\theta$, $N_2 = Mg\sec\theta$
15. (a) 5.658 m/s^2 rigtward (b) 56.58 N

Level – II

1. 150 N
2. (a) 7.6 m/s^2 leftward (b) 0.86 m/s^2 leftward
3. $\frac{\mu(M+m)g}{(1+\mu)}$
4. 45.82 m
5. $\frac{mg\sin\theta}{k}$
6. (a) $a_A = \frac{3g}{11}$ (downward), $a_B = \frac{5g}{11}$ (upward), $a_C = \frac{7g}{11}$ (downward)

   (b) $T_A = T_C = \frac{8mg}{11}$, $T_B = \frac{16mg}{11}$
7. $\frac{2g(2n-\sin\alpha)}{(4n+1)}$

8. $\dfrac{mg \sin\alpha}{M+2m\left(1-\cos\alpha\right)}$

9. $\left\{\sqrt{\dfrac{\ell}{g}}\ln\left[\dfrac{\ell}{h}+\sqrt{\left(\dfrac{\ell}{h}\right)^2-1}\right]\right\}$ sec.

10. (a) $\dfrac{1}{2\pi}\sqrt{\dfrac{g\left(\tan\theta+\mu\right)}{r\left(1-\mu\tan\theta\right)}}$ (b) $\dfrac{1}{2\pi}\sqrt{\dfrac{g\left(\tan\theta-\mu\right)}{r\left(1+\mu\tan\theta\right)}}$

11. $a_{min} = g(1-k)/(1+k)$, $a_{max} = \left(\dfrac{1+k}{1-k}\right)g$

12. $\dfrac{4+\pi}{3\pi}g$

13. $\dfrac{2m_1F}{m(m_1+9m_2)+4m_1m_2}$

14. $\dfrac{mg\cos\alpha\sin\alpha}{\left(m\sin\alpha+\dfrac{M}{\sin\alpha}\right)}$, $\dfrac{mg\cos\alpha}{\left(m\sin\alpha+\dfrac{M}{\sin\alpha}\right)}$

15. $\dfrac{13g}{34}$, $\sqrt{\dfrac{397}{34}}g$, $\dfrac{3g}{17}$

Objective:

Level - I

1.	A	2.	B	3.	A
4.	D	5.	A	6.	B
7.	C	8.	D	9.	D
10.	C	11.	C	12.	B
13.	B	14.	A	15.	D

Level - II

1.	D	2.	A	3.	A
4.	D	5.	A	6.	B
7.	D	8.	B	9.	D
10.	A	11.	A	12.	B
13.	C	14.	C	15.	D

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

WORK, ENERGY & POWER

Work, Energy and Power

Syllabus for IITJEE and Maharashtra Board:

Work done by constant and variable forces, Conservative and non–conservative forces, Potential and Kinetic energy, Conservation of energy, Work energy theorem, Power.

WORK

Work is said to be done by a force when the point of application is displaced under the influence of the force. Work is a scalar quantity and it is measured by the product of the magnitude of force and the component of displacement along the direction of force. In fact, work is the scalar product (dot product) of the force vector and the displacement vector.

Thus, $W = \vec{F}\cdot\vec{S} = FS\cos\theta$, where F and S are the magnitudes of force and displacement vectors and θ is the angle between them.
For $0 \le \theta \le \pi/2$, work done is positive.
For $\theta = \pi/2$, work done is zero
For $\pi/2 < \theta < 3\pi/2$, work done is negative.

For example,
(a) When a person lifts a body from the ground, the work done by the lifting force is positive but the work done by the gravitational force is negative.
(b) When a body slides on a fixed rough surface, work done by the pulling force is positive while work done by the force of friction is negative. The work done by normal reaction is zero.

Note:
1. Work done by a constant force is path independent, i.e. it depends on initial and final positions only.
2. Work done depends on the frame of reference. For example, if a person is pushing a box inside a moving train, the work done in the frame of reference of train will be $\vec{F}\cdot\vec{S}$. While work done in the frame of earth will be $\vec{F}\cdot\left(\vec{S}+\vec{S}_0\right)$, where $\vec{S}_0$ is the displacement of the train relative to the ground.
3. Work done by friction may be zero, positive or negative depending upon the situation. When force applied on a body is insufficient to overcome the friction, work done by the frictional force is zero. When this force is large enough to overcome the friction, then work done by the frictional force is negative. When force is applied on a body, which is placed above another body, the work done by the frictional force on the lower body may be positive.

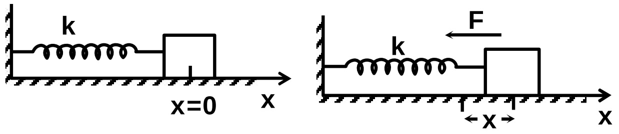
Work Done By a Variable Force

The equation $W = \vec{F}\cdot\vec{S} = FS\cos\theta$ is applicable when $\vec{F}$ remains constant, but when the force is variable work is obtained by integrating $\vec{F}.\overrightarrow{dS}$
Thus, $W = \int \vec{F}\cdot\overrightarrow{dS}$
An example of a variable force is the spring force in which force depends on the extension x,
i.e. $F \propto x$
When the force is time dependent, we have,

$$W = \int \vec{F}.\overrightarrow{dx} = \int \vec{F}\cdot\left(\frac{d\vec{x}}{dt}\right)dt = \int \vec{F}.\vec{v}\ dt$$

where $\vec{F}$ and $\vec{v}$ are force and velocity vectors at an instant.

Geometrically, the work done is equal to the area between the F(x) curve and the x–axis, between the limits x_i and x_f, e.g. Consider spring force:

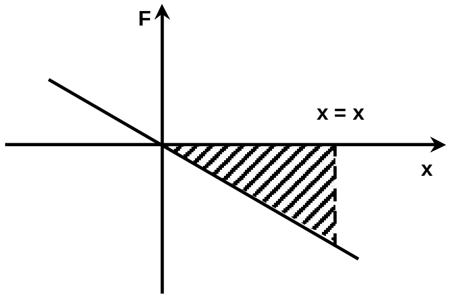


In the given figure, one end of a spring is attached to a fixed vertical support and other end to a block which can move on a horizontal frictionless table.

At x = 0, the spring is in its natural length. When the block is displaced by an amount x, a restoring force (F) due to elasticity is applied by the spring on the block.

i.e. $F = -kx$ . . . (1)

where k is the force constant of the spring which depends on the nature of the spring.



From equation (1), we can observe that

(i) F is a variable force

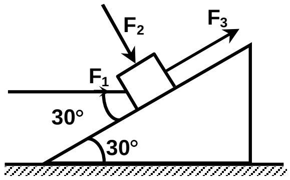
(ii) F–x graph is a straight line which passes through origin with slope –k.

(iii) Work done by the force F when block is displaced from x = 0 to x is

$$W = \int_0^x F dx = \int_0^x -kx dx = -\frac{1}{2}kx^2 = \text{area under F – x–graph.}$$

Since force (F) and displacement are opposite in direction, hence work done by restoring force is negative, but work done by the external agent is equal to the potential energy stored in the system.

Illustration 1. *A block moves up a 30° incline under the action of certain forces which are shown in the figure. F_1 is horizontal and of magnitude 40 N. F_2 is normal to the plane and of magnitude 20 N. F_3 is parallel to the plane and of magnitude 30 N. Determine the work done by each force as the block (and point of application of each force) moves 80 cm up the incline plane.*



Solution: Component of F_1 along the direction of displacement

$$= F_1 \cos 30° = 40 \times \frac{\sqrt{3}}{2} = 34.6 \text{ N}$$

Hence, the work done by $F_1 = F_1 s = 34.6 \times 0.8 = 28$ J

Work done by F_2 is zero because it has no component in the direction of the displacement.

Component of F_3 in the direction of displacement $= 30$ N

Hence, work done by $F_3 = 30 \times 0.80 = 24$ N

Illustration 2. *A block of mass 10 kg slides down on an incline 5 m long and 3 m high. A man pushes up on the ice block parallel to the incline so that it slides down at constant speed. The coefficient of friction between the ice and the incline is 0.1. Find:*

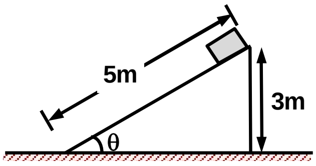
(a) the work done by the man on the block.

(b) the work done by gravity on the block.

(c) the work done by the surface on the block.

(d) the work done by the resultant forces on the block.

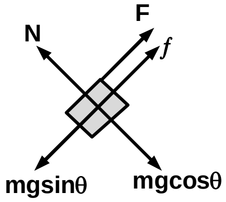
Solution: From the figure, $\sin\theta = 3/5$
and $\cos\theta = 4/5$



F = force by the man
f = frictional force
N = normal reaction of the surface,
mg = force of gravity

Since block slides with constant speed, Hence,

F.B.D. of the block



$mg\sin\theta = F + f$

$\Rightarrow$ $F = mg\sin\theta - f$

$= 10 \times 10 \times \frac{3}{5} - 0.1 \times 10 \times 10 \times \frac{4}{5} = 52 \text{ N}$,

as $f = \mu\, mg\cos\theta$

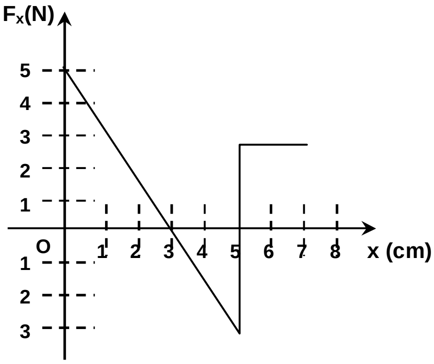
(a) $W_{man} = \vec{F}.\vec{S} = FS\cos 180° = -FS$
Here F = 52 N and S = 5 m.
$\Rightarrow$ $W_{man} = -52 \times 5 \text{ J} = -260 \text{ J}$

(b) $W_{gravity} = mg\, S\sin\theta = 10 \times 10 \times 5 \times (3/5) \text{ J} = 300 \text{ J}$

(c) $W_{surface} = W_N + W_{friction} = 0 + fS\cos 180^o = 0 - \mu mg\cos\theta\ S$
$= -0.1 \times 10 \times 10 \times (4/5) \times 5 \text{ J} = -40 \text{ J}$

(d) Work done by the resultant force is given by
$W = W_m + W_g + W_N + W_f = -260 \text{ J} + 300 \text{ J} + 0 - 40 \text{ J} = 0.$

Illustration 3. *The force acting along x–axis on an object as a function of x is shown in the figure. Find the work done by the force in the interval.*
(a) $0 \le x \le 3$ *cm*,
(b) $3 \le x \le 5$ *cm*,
(c) $0 \le x \le 6$ *cm*.



Solution: We know that, work done = Area under the curve

(a) For the interval $0 \le x \le 3$ cm

$W = \frac{1}{2} \times (0.03\,\text{m}) \times 5\text{N} = 0.075\text{J}$

(b) For the interval $3 \le x \le 5$ cm

$W = \frac{1}{2} \times (0.02\,\text{m}) \times 3\,\text{N} = -0.03\,\text{J}$

(c) For the interval, $0 \le x \le 6$ cm
Work done between 5 and 6 cm = 0.01 ×3 = 0.030 J
Adding this and (a) & (b), we will get total work done in the 6 cm interval
= 0.075 – 0.030 + 0.030) = 0.075 J.

Illustration 4. *A particle is moved by a force* $\vec{F} = (3\hat{i} + 4\hat{j})$ *N from point (2m, 3m) to (3m, 0m) in x–y plane. Find the work done by the force on the particle.*

Solution : Displacement of the particle is

$$\vec{S} = \left[(3-2)\hat{i} + (0-3)\hat{j}\right]\text{m} = \left(\hat{i} - 3\hat{j}\right)\text{m}$$

$$\therefore \quad W = \vec{F}.\vec{S} = \left(3\hat{i} + 4\hat{j}\right).\left(\hat{i} - 3\hat{j}\right) = -9 \text{ J}$$

Illustration 5. *A force of magnitude 26 N, along the direction of a vector* $5\hat{i} + 12\hat{j}$ *displaces a body of mass 2 kg from a point (2, 3, 4) to a point (4, 3, 2). What is the work performed by the force ?*

Solution :

$$W = \vec{F}\cdot\vec{S} = 26\times(5i + 12\,j)\cdot\left\{\left(4\hat{i} + 3\hat{j} + 2\hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)\right\}$$

$$= 2\left(5\hat{i} + 12\hat{j}\right)\cdot\left(2\hat{i} - 2\hat{k}\right) = 20 \text{ J}$$

Illustration 6. *A force F = kx acting on a particle moves it from x = 0 to x =* x_1*. The work done in the process is*

(A) kx_1^2 *(B)* $\frac{1}{2}kx_1^2$

(C) zero *(D)* kx_1^3

Solution: **(B).**

$$W = \int_0^{x_1} F.dx = \int_0^{x_1} kxdx = \frac{kx^2}{2}\Big|_0^{x_1} = \frac{kx_1^2}{2}$$

Illustration 7. *A force F = 9 – 4x + 6x*^{2} *N acts on a body of mass 5 kg and displaces it from x = 1 m to x = 3 m. What is the work done by the force ?*

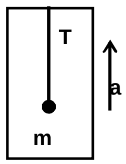
Solution:

$$W = \int F.dx = \int_1^3 (9 - 4x + 6x^2)dx = 9x - \frac{4x^2}{2} + \frac{6x^3}{3}\Big|_1^3$$

$$= 9x - 2x^2 + 2x^3\Big|_0^3 = (27 - 18 + 54) - (9 - 2 + 2)$$

= 54 J.

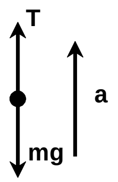
Illustration 8. *A block of mass m is suspended by a light thread from an elevator. The elevator is accelerating upward with uniform acceleration a. Find the work done during the first 't' seconds by the tension in the thread.*



Solution: Let the block moves up with an acceleration a

$\Rightarrow$ $F_{net} = T - mg = ma$

$\Rightarrow$ $T = m(g + a)$ …(1)



Now the work done W by the tension T in displacing the block through a distance x is given as**,**

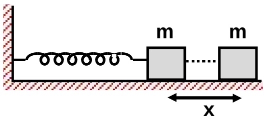
$W = T.x$ …(2)

where $x = \frac{1}{2}at^2$ …(3)

Putting x and T from (1) and (3) in (2), we obtain

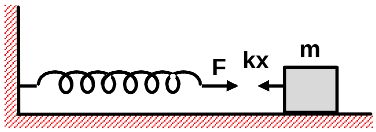
$W = m(g + a)\,(\frac{1}{2}at^2) \Rightarrow W = \frac{m}{2}(g + a)\,at^2.$

Illustration 9. *A block of mass m is attached rigidly with a light spring of force constant k. The other end of the spring is fixed to a wall. If block is displaced by a distance x, find the work done on the block by the spring for this range.*



(The spring force is given by F= –kx, where k is spring constant and x is displacement of the block from its free length.)

Solution: Since F = –kx, Therefore, force varies with displacement. This force has tendency to bring the block to its equilibrium point Hence, it is opposite to the displacement.



For infinitesimal displacement (dx) this force is supposed to be constant. Therefore, Work done by this force for the displacement dx is given by

$dW = \vec{F}.d\vec{x} = kxdx \cos \pi$

$\Rightarrow \quad W = \int dW = -\int_0^x kxdx \;\Rightarrow\; W = -\frac{1}{2}kx^2.$

Illustration 10. *A body is subjected to a constant force F in Newton given by $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z axes respectively of a co-ordinate system..*

(i) What is the work done by this force in moving the body through a distance of
(a) 4 m along the z–axis, and (b) 3 m along the y–axis?

(ii) What is the total work done by the force in moving the body through a distance of 4 m along the z–axis and then 3 m along the y–axis?

Solution : (i) The force is given by

$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$

(a) Displacement along z–axis $S = 4\hat{k}$ metres, Therefore, work done is

$W_1 = \vec{F}.\vec{S} = \left(-\hat{i} + 2\hat{j} + 3\hat{k}\right)\cdot\left(4\hat{k}\right) = -4\hat{i}\cdot\hat{k} + 8\hat{j}\cdot\hat{k} + 12\hat{k}\cdot\hat{k}$

Now, $\hat{i}\cdot\hat{k} = 0$ and $\hat{j}\cdot\hat{k} = 0$ because $\hat{i}$ and $\hat{j}$ are perpendicular to $\hat{k}$, but $\hat{k}\,.\,\hat{k} = 1$

Hence, $W_1 = 12$ J.

(b) Displacement along y–axis is $S = 3\hat{j}$ metres. Hence, work done in this case is

$W_2 = F\cdot S = \left(-66\hat{i} + 2\hat{j} + 3\hat{k}\right)\cdot\left(3\hat{j}\right) = 6\,J$

(ii) Since work is a scalar quantity, the total work done is the algebraic sum of W_1 and W_2,

i.e $W = W_1 + W_2 = 12 + 6 = 18$ J

Illustration 11. *The displacement of a particle of mass 1 kg on a horizontal smooth surface is a function of time is given by* $x = \frac{1}{3}t^3$. *The work done by external agent for first one second is*

(A) 0.5 J *(B) 2 J*
(C) 0.60 J *(D) none of these*

Solution: **(B).**

$$x = \frac{1}{3}t^3 \Rightarrow v = \frac{dx}{dt} = t^2$$

Acceleration, $a = \frac{dv}{dt} = 2t$

f = ma = 2mt

dW = Fdx = 2mt × t^2dt

$$\int_0^W dW = \int_0^1 2mt^3 dt$$

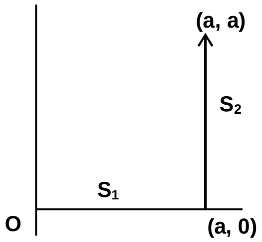
$$= \left.\frac{2m \times t^4}{4}\right|_0^1 = \frac{m \times 1}{2} = 0.5\,J$$

Illustration 12. *A force* $\vec{F} = -k(x\hat{i} + y\hat{j})$, *where k is a positive constant, acts on a particle moving in the x–y plane. Starting from the origin, the particle is taken along the positive x–axis to the point (a, 0) and then parallel to the y–axis to the point (a, a). Total work done by the force* $\vec{F}$ *on the particle is*

(A) $-2ka^2$ *(B)* $-ka^2$
(C) ka^2 *(D)* $2ka^2$

Solution : **(B).**

The displacement occurs in two slabs $\vec{S}_1$ and $\vec{S}_2$, the corresponding amount of work done W_1 and W_2, are given by



$$W_1 = -\int_{x=0}^{x=a} k\, x dx = -\frac{ka^2}{2}$$

$$W_2 = -k\int_{y=0}^{y=a} y\, dy = -\frac{ka^2}{2}$$

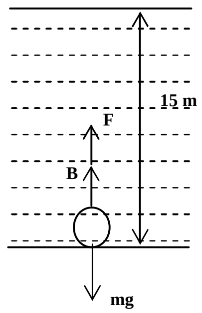
$W = W_1 + W_2 = -ka^2$

Illustration 13. *A stone of mass 10 kg and specific gravity 2.5 lies at the bed of a lake 15 m deep. The work required to be done to bring it to the top of the lake is (consider g = 10 m/s²)*

(A) 1000 J *(B) 1200 J*
(C) 750 J *(D) 900 J*

Solution : **(D).**

Forces acting on the stone are shown in the figure. To move the stone up without acceleration



F + B = mg

F = mg – B

W = Fh = (mg – ρwVg)h

$$= \left(mg - \frac{\rho Vg}{2.5}\right)h$$

$$= mgh\left\{1-\frac{1}{2.5}\right\} = mgh\left(\frac{1.5}{2\cdot 5}\right)$$
$$= \frac{10\times10\times15\times15}{25} = 900 \text{ J}$$

Exercise 1:

(i) ***Springs A and B are identical except that A is stiffer than B, i.e., force constant $k_A > k_B$. On which spring, more work will be done, if***
(a) they are stretched by same amount ?
(b) they are stretched by the same force ?

(ii) ***The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative.***
(a) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.
(b) Work done by the gravitational force in the above case.
(c) Work done by friction on a body sliding down an inclined plane.
(d) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.
(e) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Work depends on the frame of reference

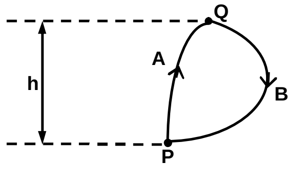
For example, if a person is pushing a box inside a moving train, the work done in the frame of train will be $\vec{F}.\vec{s}$. while work done in the frame of earth will be $\vec{F}.\left(\vec{s}+\vec{s}_0\right)$, where $\vec{s}_0$ is the displacement of the train relative to the ground and $\vec{s}$ is the displacement of box relative to the train.
Work done by friction may be zero, positive or negative depending upon the situation. When force applied on a body is insufficient to overcome the friction, work done by the friction force is zero. When this force is large enough to overcome the friction then work done by the friction force is negative. When force is applied on a body, which is, placed above another body the work done by the friction force on the lower body is positive.
Let us consider the situation in which a horizontal rough trolley, on which a block and a man is standing, is accelerating along the horizontal direction. The block is not slipping on the trolley. The following conclusions can be drawn from above:

(i) In this case work done by friction (between trolley and the block) is zero as observed by the man on trolley.
(ii) Work done by friction (between trolley and the block) is positive as observed by an observer on the ground.
(iii) Work done by friction is negative as observed by observer who is moving along the direction of motion of trolley with higher speed.

CONSERVATIVE AND NON-CONSERVATIVE FORCES

A force is said to be conservative if the work done by the force along a closed path is zero. Work done by the conservative forces depends only upon the initial and final positions and is path independent.
A force is said to be non-conservative if the work done by the force along a closed path is not zero.
Conservative forces are non-dissipative whereas non-conservative forces are dissipative.
Examples of conservative forces are gravitational force, electrostatic force, etc.
Examples of non-conservative forces are frictional force, viscous force, etc.

Illustration 14. *A particle is taken from point P to point Q via the path PAQ and then placed back to point P via the path QBP. Find the work done by gravity on the body over this closed path. The motion of the particle is in the vertical plane.*



Solution: Here, displacement of the particle is $\overrightarrow{PQ}$, gravity is acting vertically downward. The vertical component of $\overrightarrow{PQ}$ is h upward. Hence,

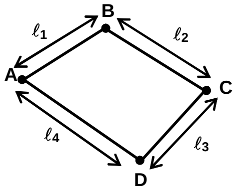
$$W_{(PAQ)} = -mgh \quad \ldots(1)$$

For the path QBP, component of the displacement along vertical is h(downward)

In this case, $W_{(QBP)} = mgh$

$\because$ Total work done = $W_{PAQ} + W_{QBP} = 0$.

Illustration 15. *A particle of mass m is moved on a rough horizontal surface on a closed path as shown in the figure. If co–efficient of friction between the particle and the surface is μ then find the work done by frictional force on the particle over closed path ABCDA.*



Solution: Since friction force is always opposite to the motion, hence work done by it is given by

$$W = -\oint f d\ell = -\mu mg \oint d\ell$$

–ve sign indicates that force is opposite to displacement

$$\Rightarrow \quad W = -\mu mg\left[\int_A^B d\ell + \int_B^C d\ell + \int_C^D d\ell + \int_D^A d\ell\right]$$

$$= -\mu mg\left[\ell_1 + \ell_2 + \ell_3 + \ell_4\right]$$

Mechanical Energy [Kinetic Energy + Potential Energy]

It is the capability of doing mechanical work.

Mechanical energy possessed by a body is of two types, kinetic and potential

Kinetic Energy

The capacity of a body to do work by virtue of its motion is known as kinetic energy of the body. Kinetic energy is equivalent to work done by an external force on a body of mass ‘m’ to bring the body from rest upto its velocity v in absence of dissipative forces.

Mathematical expression

Consider a body of mass m initially at rest. Let us consider that an external constant force F acts on the body to bring its velocity to v. If s be the displacement, then

$$v^2 = 2aS \text{ and } F = ma$$

Now, work done by the constant force, W = FS

$$= (ma)\left(\frac{v^2}{2a}\right) = \frac{1}{2}mv^2$$

Therefore, according to the definition

$$K.E. = \frac{1}{2}mv^2.$$

Note:

1. Both m and v^2 are always positive, Therefore, K.E. is also always positive.

2. Like work-done, K.E. is also frame dependent. For example, the kinetic energy of a person of mass m in a frame moving with velocity v is zero in the same frame but it is $\frac{1}{2}mv^2$ in a stationary frame.

Exercise 2:

(i) ***Is work done by a non- conservative force always negative.***

(ii) ***A man in an open car moving with high speed, throws a ball with his full capacity along the direction of the motion of the car. Now the same man throws the same ball when the car is not moving. In which case the ball possesses more kinetic energy in***
(a) ground frame,
(b) in car frame.

Potential Energy:

The energy possessed by a body by virtue of its position is called its potential energy.

The change in potential energy produced by a conservative force is defined as the negative of the work done by the conservative force.

$\therefore\ U_f - U_i = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{F}.d\vec{r}$, where U_i = Potential energy at the initial reference position,

U_f = Potential energy at the final position.

Usually, the initial reference position is taken as infinity and the potential energy at infinity is assumed to be zero.

Then, we get the potential energy of a body as, $U = -\int_{\vec{r}=\infty}^{\vec{r}} \vec{F}.d\vec{r}$.

The negative derivative of the potential energy function with respect to the position gives the conservative force acting on the particle. Mathematically, $F = -\frac{dU}{dr}$.

Illustration 16. *A 150 g mass has a velocity* $\vec{v} = \left(2\hat{i} + 6\hat{j}\right) m/s$ *at a certain instant. What is its kinetic energy?*

Solution: We know that, $K = \frac{1}{2}mv^2$

or, $K = \frac{1}{2}m\left(\vec{v}.\vec{v}\right)$ $[\because \vec{v}\,.\,\vec{v} = v^2]$

$= \frac{1}{2}\times\left(0.150\,kg\right)\left(2^2 + 6^2\right) = 3.0\ J$

Illustration 17. *The potential energy of a spring when stretched through a distance S is 10 J. The amount of work (in J) that must be done on this spring to stretch it through an additional distance S will be*

(A) 30 J *(B) 40 J*
(C) 10 J *(D) 20 J*

Solution: **(A).** $u_1 = \frac{1}{2}kS^2$; $u_2 = \frac{1}{2}k(2S)^2 = 4\left\{\frac{1}{2}kS^2\right\} = 4u_1$

$\Delta u = u_2 - u_1 = 4u_1 - u_1 = 3u_1 = 30\ J$

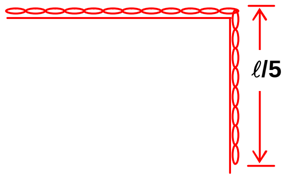
Illustration 18. *The momentum of a body is increased by 20%. The percentage increase in its kinetic energy is*

(A) 36% *(B) 44%*
(C) 20% *(D) 50%*

Solution: **(B).**

$$mv = mu + \frac{20}{100}mu = 1.2\,mu$$
$$\Rightarrow v = 1.2\,u$$
$$\Rightarrow v^2 = 1.44u^2$$
$$\frac{1}{2}mv^2 = \frac{1}{2}m(1.44\,u^2)$$
$$\therefore \frac{1}{2}mv^2 = 1.44(\frac{1}{2}mu^2)$$
$\therefore$ KE $_{final}$ = 1.44 KE$_{initial}$

Illustration 19. *A uniform chain is held on a frictionless table with one–fifth of its length hanging over the edge. If the chain has a length ℓ and a mass m, how much work is required to pull the hanging part back on the table?*



Solution: Mass of hanging part = m/5.

The weight $\frac{mg}{5}$ acts at the centre of gravity of the hanging chain, i.e. at a distance ℓ/10 below the surface of the table. The gain in potential energy in pulling the hanging part on the table.

$$W = \frac{mg}{5} \times \frac{\ell}{10} = \frac{mg\ell}{50}$$

$\therefore$ Work done = W = mgℓ/50

Illustration 20. *The kinetic energy of a particle moving along a circle of radius R depends upon the distance s as K = as^2. The force acting on the particle is*

(A) $\frac{2as^2}{s}$ *(B)* $2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$
(C) 2 as *(D) 2 a*

Solution: **(B).**

It is given that, K = as^2

$$\Rightarrow \frac{1}{2}mv^2 = as^2$$

Differentiating w.r.t time

$$\Rightarrow \frac{1}{2}m \times 2v\frac{dv}{dt} = 2a\,s\frac{ds}{dt}$$

But $\frac{ds}{dt} = v$

$$\Rightarrow m\frac{dv}{dt} = 2a\,s$$

The tangential force, $F_t = m\dfrac{dv}{dt} = 2as$

The radial or centripetal force, $F_r = \dfrac{mv^2}{R} = \dfrac{2as^2}{R}$

$$F = \sqrt{F_t^2 + F_r^2} = \sqrt{(2as)^2 + \left(\frac{2as^2}{R}\right)^2}$$

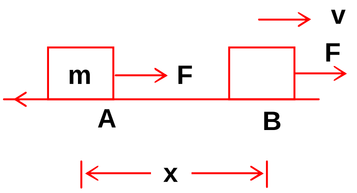
$$= 2as\sqrt{\left(1+\frac{s^2}{R^2}\right)} = 2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$$

Work-Energy Theorem

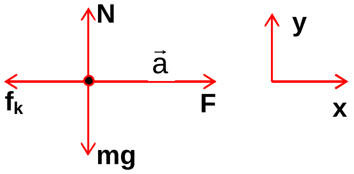
This theorem states that work done by all the forces acting on a particle or body is equal to the change in its kinetic energy.

Let us take an example shown in Figure (a), in which a block of mass m kept on a rough horizontal surface is acted upon by a constant force $\vec{F}$ parallel to the surface. The corresponding F.B.D. is shown in Fig.(b) which gives $\vec{F} + \vec{f}_k = m\vec{a}$ ...(1)

and $N = mg$ ...(2)



(a)



(b)

Initially, while the force $\vec{F}$ is just applied, the block is at the position A and has a velocity v_0. The force acts on it for some interval of time 't' so that the block reaches to position B at a distance x from A.

Now, the work done by the net external force is maximum along the surface and is given by

$W = \left(\vec{F} + \vec{f}_k\right)\cdot\vec{x}$ . . . (3)

Since cos θ = cos 0^0 = 1, θ being the angle between $\vec{a}$ and $\vec{x}$.

Therefore, $W = m\times a\times x$ . . . (4)

Again from the kinematics equation for the velocities at A and B, we have,

$$v^2 = v_0^2 + 2ax.$$

where v is the velocity of the block at position B.

Thus, $ax = \dfrac{v^2 - v_0^2}{2}$ ...(5)

Putting the value of 'ax' from equation (4) in equation (3), we have,

$$W = m\left(\frac{v^2 - v_0^2}{2}\right)$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \quad ...(6)$$

The work done by the other two forces in F.B.D., for the displacement $\vec{x}$, are zero because $\vec{N}\cdot\vec{x} = 0$ and also $m\vec{g}\cdot\vec{x} = 0$.

Considering $\dfrac{1}{2}mv_0^2 = k_i$ (initial kinetic energy) and $\frac{1}{2}mv^2 = k_f$ (final kinetic energy)

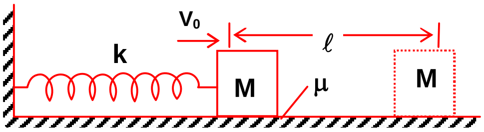
Equation (6) becomes

$$W = k_f - k_i \qquad \ldots(7)$$

Now, equation (7) can be explained as: the net work done by all the forces on a system gives the change in kinetic energy of the system. This is known as work–energy theorem.

Thus, the change in kinetic energy of the body equals the total work done by all the forces (conservative and non conservative).

Illustration 21. *The block of mass M shown in the figure initially has a velocity V_0 to the right and its position is such that the spring exerts no force on it, i.e. the spring is not stretched or compressed. The block moves to the right a distance ℓ before stopping in the dotted position shown. The spring constant is k and the coefficient of friction between block and table is μ. As the block moves the distance ℓ,*



(a) What is the work done on it by the frictional force?
(b) What is the work done on it by the spring force?
(c) Are there other forces acting on the block, and if so, what is the work done by these force on the block?
(d) What is the total work done on the block?

Solution: (a) Work done by friction = – $\mu Mg\ell$.

(b) Work done by spring force $= -\frac{1}{2}kt^2$

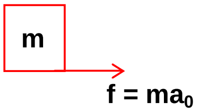
(c) Gravitational force and normal reaction of the table do not work as they act in direction perpendicular to displacement.

(d) Total work done on block = $-(\mu mg\ell + \frac{1}{2}k\ell^2)$

Illustration 22. *A truck is moving with constant acceleration a_0. A block of mass m is kept on the rough trolley of the truck and is observed stationary w.r.t. truck. Using work energy theorem, find the velocity of block: (a) relative to ground, and (b) relative to truck, when truck moves a distance x from the starting point.*

Solution: (a) From the ground frame of reference

Since there is no relative motion between the block and the truck, therefore, the force of friction on the block is f = ma in forward direction as shown in the figure



If the truck moves a distance x on the ground, the block will also move the same distance x as there is no slipping between the two. Hence, work done by friction on the block (w.r.t. ground) is $W_f = f \cdot x = m_{a_0} x$

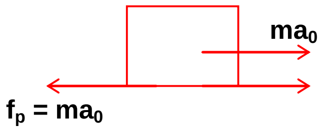
From work–energy theorem,

Δ K.E. = W_f

$\frac{1}{2}mv^2 = m_{a_0} x$ or v = $\sqrt{2_{a_0} x} = -\frac{1}{2}Mv_0^2$

where v is the velocity of block w.r.t. ground.

(b) Relative to the truck accelerating (non–inertial) frame
F.B.D. in truck frame: Here f_P = pseudo force = ma_0
Work done by all forces



$$W = W_f + W_{fp} = 0$$

From work energy theorem,

$$\frac{1}{2}mv_r^2 = W = 0 \text{ or } v_r = 0$$

where v_r is relative velocity of block w.r.t. truck

Illustration 23. *The displacement of a body in metre is a function of time according to* $x = 2t^4 + 5$. *Mass of the body is 2 kg. What is the increase in its kinetic energy one second after the start of motion?*

(A) 8 J *(B) 16 J*
(C) 32 J *(D) 64 J*

Solution: **(D).**

$x = 2t^4 + 5$

$$\Rightarrow v = \frac{dx}{dt} = 8t^3$$

$$a = \frac{dv}{dt} = 24t^2$$

$$\Rightarrow F = ma = 48t^2$$

$$dW = Fdx = 48t^2 \times 8t^3 dt = 48 \times 8t^5 dt$$

Increase in the kinetic energy results from the work done by the applied force

$$\Delta KE = \int_0^1 48 \times 8t^5\ dt = \frac{48 \times 8}{6} t^6 \Big|_0^1 = \frac{48 \times 8}{6} = 64\text{ J}$$

Illustration 24. *A bullet having a speed of 153 m/s crushes through a plank of wood. After passing through the plank, its speed is 130 m/s. Another bullet, of the same mass and size but travelling at 92 m/s is fired at the plank. What will be the second bullet's speed after tunneling through? Assume that the resistance of the plank is independent of the speed of the bullet.*

Solution: Since plank does the same amount of work on the two bullets, therefore, decreases their kinetic energies equally

$$\therefore\ \frac{1}{2}m(153)^2 - \frac{1}{2}m(130)^2 = \frac{1}{2}m(92)^2 - \frac{1}{2}mv^2$$

or, $v^2 = 1955$

$\Rightarrow$ $v = 44.2$ m/s

Illustration 25. *A ball of mass m is thrown in air with speed v_1 from a height h_1 and it is caught at a height $h_2 > h_1$ when its speed becomes v_2. Find the work done on the ball by the air resistance.*

Solution: Work done on the ball by gravity is

$$W_g = -mg(h_2 - h_1)$$

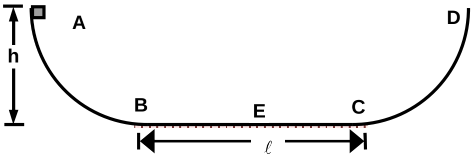
Work done on the ball by air resistance is W_{air} = ?

As $W_g + W_{air} = \Delta K.E.$

$\Rightarrow$ $-mg(h_2 - h_1) + W_{air} = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$

$\Rightarrow$ $W_{air} = mg(h_2 - h_1) + \frac{1}{2}m\left(v_2^2 - v_1^2\right)$

Illustration 26. *A particle slides along a track with elevated ends and a flat central part as shown in the figure. The flat part has a length ℓ = 3m. The curved portions of the track are frictionless.*



For the flat part the coefficient of kinetic friction is μ_K = 0.2. The particle is released at point A which is at height h = 1.5m above the flat part of the track. Where does the particle finally come to rest?

Solution: The particle will finally come to rest on the flat part. Hence, displacement of the particle along vertical is h. If W_g be the work done on the particle by the gravity then

$W_g = mgh$ where m = mass of the particle … (1)

If distance travelled by the particle on the flat part is x, the work done on the particle by the friction is

$W_f = -\mu mgx$ … (2)

Since initially particle was at rest and finally it comes to rest again. Hence change in its K.E. is zero.

From work energy theorem

$W_g + W_f = \Delta K.E.$

$\Rightarrow$ $mgh - \mu mgx = 0$

$\Rightarrow$ $x = \frac{h}{\mu} = \frac{1.5}{0.2}$ m $\Rightarrow x = 7.5$ m

Since x > ℓ, the particle will reach C and then will rise up till the remaining KE at C is converted into potential energy. It will then again descend to C and it will have the same kinetic energy as it had when ascending but now will move from C to B; At B , the same will be repeated (because 7.5 > 2ℓ) and finally, the particle will stop at E such that

BC + CB + BE = 7.5

$\Rightarrow$ BE = 7.5 – 6 = 1.5 m

Conservation of Energy and Conservation of Mechanical Energy

Conservation of energy means conservation of all forms of energy together. Accounting all forms of energy within an isolated system, the total energy remains constant.

The mechanical energy accounts for only two forms of energy, namely kinetic energy, K and potential energy, U.

If only conservative forces acts on a system then total mechanical energy of the system remains constant.

i.e. K + U = const. …(1)

Therefore,

$\Delta K + \Delta U = 0$ …(2)

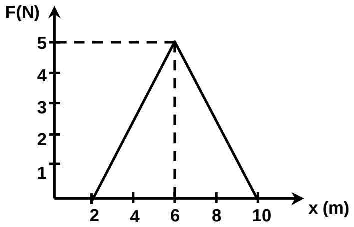
or, $\Delta K = -\Delta U$ …(3)

Note:
1. In case of any conservative force, the potential energy is a function of its position. i.e. U = U(x)
2. For conservative force field, the negative of differentiation of U(x) with respect to x, gives the force acting on the system, i.e. F(x) $= -\frac{dU(x)}{dx}$

Illustration 27. *An object is acted upon by the forces $\vec{F}_1 = 4\hat{i}$ N and $\vec{F}_2 = (\hat{i} - \hat{j})$ N. If the displacement of the object is ($\vec{i} + 6\vec{j} - 6\vec{k}$) metre, the kinetic energy of the object*
(A) remains constant *(B) increases by 1 J*
(C) decreases by 1 J *(D) decreases by 2 J*

Solution : **(C).**
$$\vec{F} = \vec{F}_1 + \vec{F}_2 = \left(4\hat{i} + \hat{i} - \hat{j}\right)N = \left(5\hat{i} - \hat{j}\right)N$$
$$W = \vec{F}\cdot\vec{S} = \left(5\hat{i} - \hat{j}\right)\cdot\left(\hat{i} + 6\hat{j} - 6\hat{k}\right) = 5 - 6 = -1\,J$$
Work done = change in KE = – 1 J

Illustration 28. *A 2 kg block is placed on a frictionless horizontal surface. A force shown in the F–x graph is applied to the block horizontally. The change in kinetic energy is*
(A) 15 J *(B) 20 J*
(C) 25 J *(D) 30 J*



Solution : **(B).**
Work done = Area under F–x graph
W = ½ × (10 – 2) × 5 = 20 J
Work done = change in kinetic energy = 20 J

Illustration 29. *Just before striking the ground, a 2.0 kg mass has 400 J of kinetic energy. If friction can be ignored, from what height was it dropped?(g = 9.8 m/s²)*

Solution: By conservation of mechanical energy,
$$U_f + K_f = U_i + K_i$$
or, $0 + K_f = mgh + 0$
$$\Rightarrow \quad h = \frac{K_f}{mg} = \frac{400}{2\times 9.8} = 20.4\,m$$

Illustration 30. *A body dropped from height h acquires momentum p just as it strikes the ground. What is the mass of the body ?*

Solution: By conservation of energy, mgh = $\frac{1}{2}mV^2$
$$\Rightarrow 2m^2gh = p^2$$
$$\Rightarrow m = \sqrt{\frac{p^2}{2gh}} = \frac{p}{\sqrt{2gh}}$$

Illustration 31. *A 40 g body starting from rest falls through a vertical distance of 25 cm before it strikes to the ground. What is the*
(a) kinetic energy of the body just before it hits the ground?
(b) velocity of the body just before it hits the ground? (g = 9.8 m/s²)

Solution: (a) As the body falls, its gravitational potential energy is converted in kinetic energy

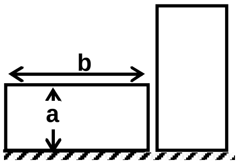
$\therefore$ K.E. = P.E. = mgh = (0.040 kg)(9.8 m/s^2) × 0.25 m = 0.098 J

(b) As K.E. = $\frac{1}{2}mv^2$

$\therefore \quad v^2 = \frac{2\times 0.098}{0.040}$

$\Rightarrow$ v = 2.21 m/s

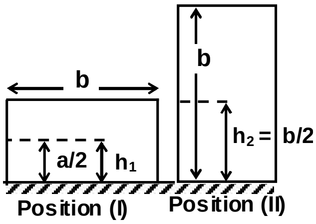
Illustration 32. *A plate of mass m, length b and breadth 'a' is initially lying on a horizontal floor with length parallel to the floor and breadth perpendicular to the floor. Find the work done to erect it on its breadth.*



Solution: Gravitational potential energy of the block at the position 1,

$U_1 = mgh_1$ …(1)

Gravitational potential energy of the block at the position 2 is given as



$U_2 = mgh_2$ …(2)

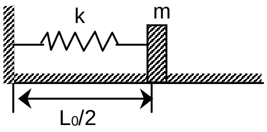
$\Rightarrow$ Work done by the external agent = W = ΔU = $(U_2 - U_1)$

$= mgh_2 - mgh_1$

where $h_2 = b/2$ and $h_1 = a/2$

$\Rightarrow \quad W = mg\left(\frac{b}{2}-\frac{a}{2}\right) = \frac{mg(b-a)}{2}$.

Illustration 33. *A block of mass 'm' is pushed against a spring of spring constant k fixed at other end to a wall. The block can slide on a frictionless table as shown. The natural length of the spring is L_0. It is compressed to half its natural length and released. Find the velocity of the block as a function of its distance from the wall.*



Solution: Here initial compression = $L_0/2$

$\therefore$ Energy stored in the spring = $\frac{1}{2}k\left(\frac{L_0}{2}\right)^2 = \frac{1}{8}kL_0^2$

As it moves a small distance x, say the velocity it has is v and so

$$\frac{1}{2}k\left[\frac{L_0}{2}-x\right]^2 + \frac{1}{2}mv^2 = \frac{1}{8}kL_0^2$$

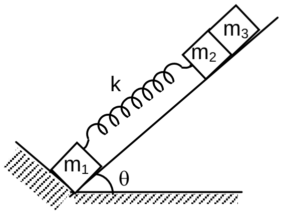
$\therefore \quad \frac{1}{2}kx^2 - \frac{1}{2}k(L_0\, x) + \frac{1}{2}mv^2 = 0$

$\therefore \quad kx^2 - kL_0x + mv^2 = 0$

$v^2 = \frac{[L_0x - x^2]k}{m}$

or $\quad v = \sqrt{\frac{k(L_0x - x^2)}{m}}$

Illustration 34. *Two blocks of masses m_1 and m_2 are connected by an ideal spring of spring constant k. This system is placed on a smooth inclined surface of inclination θ. Initially when the system is released, block of mass m_3 is in contact with m_2 and the spring is in its natural length. Then the maximum compression in the spring is*



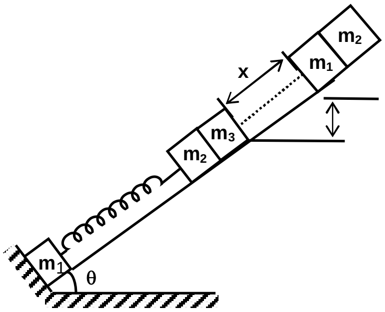
(A) $\frac{m_1 m_2}{m_1 + m_2}\frac{g\sin\theta}{k}$ (B) $\frac{m_2 g\sin\theta}{k}$

(C) $\frac{2(m_2 + m_3)g\sin\theta}{k}$ (D) $\frac{(m_2 + m_3)g}{k}$

Solution: **(C).**

At the position of maximum compression x, the masses m_1 and m_2 are displaced vertically by x sin θ.

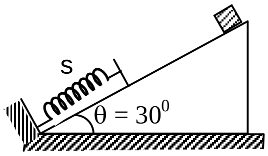
Loss of PE by the mass m_2 and m_3 = gain in KE of the spring



$$\frac{1}{2}kx^2 = \left(m_2 + m_3\right)gx\sin\theta$$

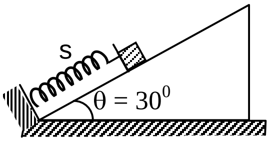
$$\Rightarrow\ x = \frac{2\left(m_2 + m_3\right)g\sin\theta}{k}$$

Illustration 35. *An ideal massless spring 'S' can be compressed 1.0 m by a 100 N force. It is placed as shown at the bottom of a frictionless inclined plane which makes an angle of $\theta = 30^0$ with the horizontal. A 10 kg block is released from rest from the top of the incline and is brought to rest momentarily after compressing the spring 2.0 m. Through what distance does the mass slide before coming to rest ?*



Solution: Spring constant = 100 N/m

Conserving energy,



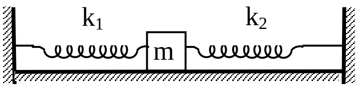
$$mg\ (x + 2)\sin 30° = \frac{1}{2}\ K.\ \left(2\right)^2$$

$$10\times10(x + 2) \times \frac{1}{2} = \frac{1}{2} \times 100 \times 4$$

x + 2 = 4

x = 2 m along the inclined surface.

Illustration 36. *A block of mass m is attached to two unstretched springs of spring constants k_1 and k_2 as shown in figure. The block is displaced towards right through a distance 'x' and is released. Find the speed of the block as it passes through a distance x/4 from its mean position.*

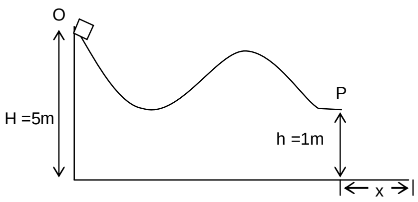


Solution: Applying conservation of energy

$$\frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2 = \frac{1}{2}mv^2 + \frac{1}{2}k_1(x/4)^2 + \frac{1}{2}k_2(x/4)^2$$

$$v = \frac{x}{4m}\sqrt{15(k_1 + k_2)}\ .$$

Illustration 37. *A body starts moving from the highest point of the smooth curved surface horizontal at the end as shown in figure without losing contact. Find out the horizontal distance moved by the body after breaking off at point P from the curved surface.*



Solution:

$0 + mgH = \frac{1}{2}mv^2 + mgh$

$mg(H - h) = \frac{1}{2}mv^2$

$\Rightarrow v^2 = 2g(H - h)$

$\therefore\ v = \sqrt{2\times10(5-1)} = \sqrt{80}$ m/s

For time:

$h = 0 + \frac{1}{2}gt^2$

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times1}{10}} = \sqrt{\frac{1}{5}}$ sec

Now $x = v.\ t$

$x = \sqrt{80}\ .\ \sqrt{\frac{1}{5}} = \sqrt{16}$

$x = 4$ m.

Exercise 3.

(i) ***A lorry and a car moving with the same kinetic energy are brought to rest by the application of brakes, which provide equal retarding forces. Which of them will come to rest in a shorter distance?***

(ii) ***Can a body have energy without having momentum? Can a body have momentum without having energy?***

(iii) ***In a ballistics demonstration, a police officer fires a bullet of mass 50 gm with speed 20 m/s on soft plywood of thickness 2 cm. The bullet emerges with only 10 % of its initial K.E. What is the emergent speed of the bullet ?***

(iv) ***A body of mass 5 kg initially at rest is moved by a horizontal force of 20 N on a frictionless table. Calculate the work done by the force in 10 second and prove that this equals to the change in kinetic energy.***

(v) ***A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Calculate the***
(a) work done by the applied force in 10 sec.
(b) work done by friction in 10 sec.
(c) work done by the net force on the body in 10 sec.
(d) change in K.E. of the body in 10 sec.

POWER

Power is defined as the rate of doing work.

Mathematically, $P = \frac{dW}{dt}$ …(1)

As $dW = \vec{F}\cdot d\vec{x}$

Therefore, $P = \frac{dW}{dt} = \vec{F}\cdot\frac{d\vec{x}}{dt}$

$P = \vec{F}\cdot\vec{v}$ …(2)

If the force is variable, we calculate the average power as

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{\int_0^t P\,dt}{\int_0^t dt} \quad \text{…(3)}$$

Power can also be expressed as the rate of change of kinetic energy.
Let a body of mass m moves with a velocity v. Then, its kinetic energy is,

$$K = \frac{1}{2}mv^2$$

Now, $\frac{dK}{dt} = \frac{1}{2}\frac{d}{dt}\left(mv^2\right)$

$= m\vec{v}\cdot\left(\frac{d\vec{v}}{dt}\right)$

$= \vec{F}_{ext}\cdot\vec{v}$

Therefore, $P = \frac{dK}{dt}$

Illustration 38. *An advertisement claims that a certain 1200 kg car can accelerate from rest to a speed of 25 m/s in a time of 8s. What average power must the motor produce to cause this acceleration ? (Ignore friction losses)*

Solution: The work done in accelerating the car is given by

$$W = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}(1200)[(25)^2 - 0] = 375\text{ kJ}$$

Power = $\frac{W}{t} = \frac{375}{8} = 46.9\,\text{kW}$

Illustration 39. *A hoist powered by a 15 kW motor is used to raise a 500 kg bucket to a height of 80 m. If the efficiency is 80%, find the time required.*

Solution: Upward force needed = bucket's weight (mg)

= 500 × 9.8 = 4900 N

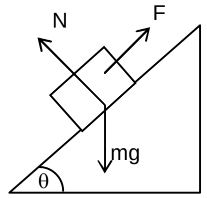
Power available (P) = $0.80 \times 15 \times 10^3$ W = 1.2×10^4 W

Since, P = W/t = Fs/t ,

$\therefore\quad t = \frac{4900\times 80}{1.2\times 10^4} = 32.7$ sec.

Illustration 40. *A block of mass m is pulled up on a smooth incline of angle θ with the horizontal. If the block moves with an acceleration of a m/s², find the power delivered by the pulling force at a time t after the motion starts. What is the average power delivered during the t seconds after the motion starts ?*

Solution: The forces acting on the blocks are shown in figure. Resolving the forces parallel to the incline, we get



$$F - mg\sin\theta = ma$$

$$F = ma + mg\sin\theta$$

The velocity at t sec is

$$v = at$$

The power elivered by the force at t is

$$P = \vec{F}.\vec{v}$$

$$= (ma + mg\sin\theta)\, v = mat\,(a + g\sin\theta)$$

The displacement during the first t seconds is

$$x = \frac{1}{2}at^2$$

The work done in these t seconds is Therefore,

$$W = \vec{F}\cdot\vec{x}$$

$$= \frac{1}{2}ma(a + g\sin\theta)\,t^2$$

The average power delivered $= \dfrac{1}{2}\dfrac{ma(a+g\sin\theta)t^2}{t}$

$$= \frac{1}{2}ma(a + g\sin\theta)t$$

Illustration 41. *A particle of mass m at rest is acted upon by a force P for a time t. Its kinetic energy after an interval t is*

(A) $\dfrac{P^2t^2}{m}$ *(B)* $\dfrac{P^2t^2}{2m}$

(C) $\dfrac{P^2t^2}{3m}$ *(D)* $\dfrac{Pt}{2m}$

Solution: **(B).** $S = ut + \dfrac{1}{2}at^2$

$$= 0 + \frac{1}{2}\left(\frac{P}{m}\right)t^2$$

$$\Delta KE = F.S = \frac{P^2t^2}{2m}$$

Illustration 42. *A particle of mass m is acted upon by a constant power P. The distance travelled by the particle when its velocity increases from v_1 to v_2 is*

(A) $\dfrac{m}{3P}\left(v_2^2 - v_1^2\right)$ *(B)* $\dfrac{2m}{3P}\left(v_2 - v_1\right)$

(C) $\dfrac{3P}{m}\left(v_2^2 - v_1^2\right)$ *(D)* $\dfrac{m}{3P}\left(v_2^3 - v_1^3\right)$

Solution: **(D)**

P = F v = mav

$\Rightarrow a = \dfrac{P}{mv}$

$v\dfrac{dv}{ds} = \dfrac{P}{mv}$

$\Rightarrow v^2 dv = \dfrac{P}{m} ds$

$\Rightarrow \dfrac{P}{m}\int_0^s ds = \int_{v_1}^{v_2} v^2 dv$

$\Rightarrow \dfrac{P}{m}s = \dfrac{1}{3}\left(v_2^3 - v_1^3\right)$

$s = \dfrac{m}{3P}\left(v_2^3 - v_1^3\right)$

Exercise 4:

(i) ***An elevator having mass 1200 kg with a passenger of mass 50 kg is moving up with an acceleration of 2 m/s^2. A friction force of 2000 N opposes its motion. Determine the minimum power delivered by the motor to the elevator.***

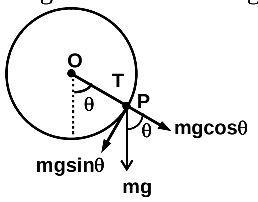
(ii) ***A car of mass 2000 kg is lifted up a distance of 30 m by a crane in 1 min. A second crane does the same job in 2 min. Do the cranes consume the same or different amounts of fuel ? What is the power supplied by each crane. Neglect power dissipation against friction.***

(iii) ***A pump on ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground and the efficiency of the pump is 30%, how much electric power is consumed by the pump.***

(iv) ***An elevator weighing 500 kg is to be lifted at a constant velocity of 0.4 m/s. What should be the minimum horse power of motor to be used ? Take g = 10 m/s^2.***

Motion in a vertical circle

A particle of mass m is attached to a light and inextensible string. The other end of the string is fixed at O and the particle moves in vertical circle of radius r equal to the length of the string as shown in the figure.



Consider the particle when it is at the point P and the string makes an angle θ with vertical. Forces acting on the particle are:

T = tension in the string along its length,

mg = weight of the particle vertically downward.

Hence, net radial force on the particle is $F_R = T - mg\cos\theta$

Since $F_R = \dfrac{mv^2}{R}$, where v = speed of the particle at P, and

R = radius of the circle, Here, R = ℓ(length of the string)

$\Rightarrow$ $T - mg\cos\theta = \dfrac{mv^2}{R}$ $\Rightarrow T = \dfrac{mv^2}{R} + mg\cos\theta$

Since speed of the particle decreases with height, hence, tension is maximum at the bottom, where $\cos\theta = 1$ (as $\theta = 0$)

$\Rightarrow$ $T_{max} = \dfrac{mv^2}{R} + mg$; $T_{min} = \dfrac{mv'^2}{R} - mg$ at the top

Here v' = speed of the particle at the top.

Critical Velocity

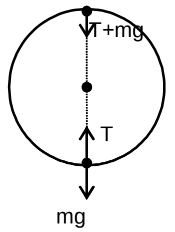
It is the minimum velocity given to the particle at the lowest point to complete the circle. The tendency of the string to become slack is maximum when the particle is at the topmost point of the circle.

At the top, tension is given by $T = \dfrac{mv_T^2}{R} - mg$; where v_T = speed of the particle at the top.

$\Rightarrow$ $\dfrac{mv_T^2}{R} = T + mg$

For v_T to be minimum, $T \approx 0$ $\Rightarrow$ $v_T = \sqrt{gR}$

If v_B be the critical velocity of the particle at the bottom, then from conservation of energy



$Mg(2R) + \dfrac{1}{2}mv_T^2 = 0 + \dfrac{1}{2}mv_B^2$

As $v_T = \sqrt{gR}$ $\Rightarrow$ $2mgR + \dfrac{1}{2}mgR = \dfrac{1}{2}mv_B^2$

$\Rightarrow$ $v_B = \sqrt{5gR}$

Note: In case the particle is attached with a light rod of length ℓ, at the highest point its minimum velocity may be zero. Then the critical velocity is $2\sqrt{g\ell}$

Illustration 43. *A pendulum bob has a speed u m/s while passing through its lowest position. What is its speed when it makes an angle of θ with the vertical? The length of the pendulum is ℓ.*

Solution: As Tension $\perp$ velocity

$\therefore$ Work done by tension = 0

$\therefore$ Total mechanical energy will remain conserved.

Increase in P.E. $= mg\ell(1 - \cos\theta)$

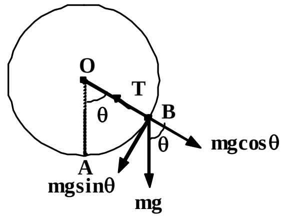
Increase in P.E. = decrease in K.E.

$mg\ell(1 - \cos\theta) = \dfrac{1}{2}mu^2 - \dfrac{1}{2}mv^2$

$v = \sqrt{u^2 - 2g\ell(1 - \cos\theta)}$

Illustration 44. *A heavy particle hanging from a fixed point by a light inextensible string of length ℓ is projected horizontally with speed $\sqrt{(g\ell)}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle.*

Solution: Let tension in the string becomes equal to the weight of the particle when particle reaches the point B and deflection of the string from vertical is θ. Resolving mg along the string and perpendicular to the string, we get net radial force on the particle at B, i.e.



$$F_R = T - mg\cos\theta \qquad \text{(i)}$$

If v_B be the speed of the particle at B, then

$$F_R = \frac{mv_B^2}{\ell} \qquad \text{(ii)}$$

From (i) and (ii), we get,

$$T - mg\cos\theta = \frac{mv_B^2}{\ell} \qquad \text{(iii)}$$

Since at B, T = mg

$$\Rightarrow \quad mg(1-\cos\theta) = \frac{mv_B^2}{\ell}$$

$$\Rightarrow \quad v_B^2 = g\ell\,(1-\cos\theta) \qquad \text{(iv)}$$

Applying conservation of mechanical energy of the particle at points A and B, we have

$$\frac{1}{2}mv_A{}^2 = mg\ell(1-\cos\theta) + \frac{1}{2}mv_B^2\,;$$

where $v_A = \sqrt{g\ell}$ and $v_B = \sqrt{g\ell(1-\cos\theta)}$

$$\Rightarrow \quad g\ell = 2g\ell(1-\cos\theta) + g\ell\,(1-\cos\theta)$$

$$\Rightarrow \quad \cos\theta = \frac{2}{3} \qquad \text{(v)}$$

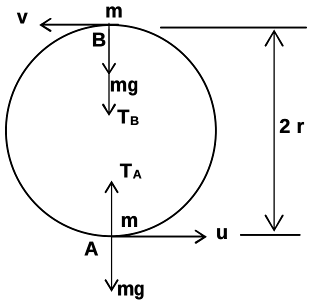
Putting the value of cos θ in equation (iv), we get : $v = \sqrt{\dfrac{g\ell}{3}}$

Illustration 45. *A mass 'm' is revolving in a vertical circle at the end of a string of length 20 cm. By how much does the tension of the string at the lowest point exceed the tension at the topmost point?*

(A) 2 mg *(B) 4 mg*
(C) 6 mg *(D) 8 mg*

Solution: **(C).**

At the lowest point A



$$T_A - mg = \frac{mu^2}{r} \qquad \ldots(1)$$

At the highest point B

$$T_B + mg = \frac{mv^2}{r} \qquad \ldots(2)$$

Gain in PE from A to B = 2mgr = loss in KE

$\Rightarrow$ 2 mgr = (1/2) m($u^2 - v^2$)

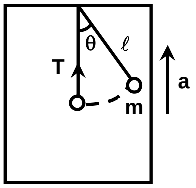
$\therefore$ $u^2 - v^2 = 4gr$ …(3)

From (1), (2) and (3)

$$T_A - T_B - 2mg = \frac{m}{r}\left(u^2 - v^2\right)$$

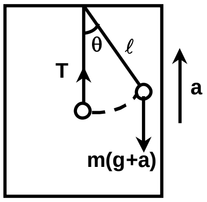
$$T_A - T_B = 2mg + \frac{m}{r}\times 4gr = 6\ mg$$

Illustration 46. *A particle of mass m is attached to the ceiling of a cabin with an inextensible light string of length ℓ. The cabin is moving upward with an acceleration a. The particle is taken to a position such that string makes an angle θ with vertical. When string becomes vertical, find the tension in the string.*



Solution: In a frame associated with cabin:

Work done on the particle when it comes in the vertical position = $mg\ell(1 - \cos\theta) + ma\ell(1 - \cos\theta)$



By work energy theorem,

$$\frac{mv^2}{2} = (mg\ell + ma\ell)(1 - \cos\theta)$$

$$\frac{v^2}{2} = (g+a)\ell(1 - \cos\theta)$$

At vertical position,

$$T - (mg + ma) = \frac{mv^2}{\ell}$$

$$T = (mg + ma) + 2m(g + a)(1 - \cos\theta)$$

Equilibrium

As we have studied in the chapter of LOM, a body is said to be in translatory equilibrium if net force acting on the body is zero, i.e.

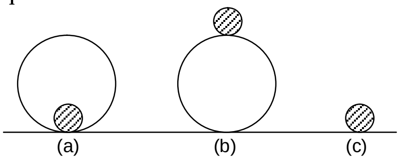
$$\vec{F}_{net} = 0$$

If the forces are conservative,

$$F = -\frac{dU}{dr}$$

and for equilibrium F = 0 $\Rightarrow \frac{dU}{dr} = 0$

i.e. at equilibrium position slope of U – r graph is zero or the potential energy is optimum (maximum or minimum or constant). Equilibria are of three types, i.e., the situation where F = 0 and $\frac{dU}{dr} = 0$ can be obtained under three conditions. These are stable equilibrium, unstable equilibrium and neutral equilibrium. These three types of equilibria can be better understood from the following three figures.



Three identical balls are placed in equilibrium position as shown in figures (a), (b) and (c), respectively. In figure (a), ball is placed inside a fixed smooth spherical shell. The ball is in stable equilibrium position. In figure (b), the ball is placed over a fixed smooth sphere. This is unstable equilibrium. In figure (c), the ball is placed on a smooth horizontal ground. This ball is in neutral equilibrium position.

Table: Types of equilibria

S. No.	Stable equilibrium	Unstable equilibrium	Neutral equilibrium
1.	$F_{net} = 0$	$F_{net} = 0$	$F_{net} = 0$
2.	$\frac{dU}{dr} = 0$ or slope of U– r graph is zero.	$\frac{dU}{dr} = 0$ or slope of U–r graph is zero.	$\frac{dU}{dr} = 0$ or slope of U–r graph is zero.
3.	When displaced from its equilibrium position, a net restoring force starts acting on the body which has a tendency to bring body back to its equilibrium position.	When displaced from its equilibrium position a net force starts acting on the body in the direction of displacement or away from the equilibrium position.	When displaced from its equilibrium position the body has neither the tendency to come back nor to move away from the original position.
4.	Potential energy in equilibrium position is minimum as compared to its neighbouring points or $\frac{d^2U}{dr^2} = +ve$	Potential energy in equilibrium position is maximum as compared to its neighbouring points or $\frac{d^2U}{dr^2} = -ve$	Potential energy remains constant even if the body is displaced from its. equilibrium position or $\frac{d^2U}{dr^2} = 0$
5.	When displaced from equilibrium position the centre of gravity of the body goes up.	When displaced from equilibrium position the centre of gravity of the body comes down.	When displaced from equilibrium position the centre of gravity of the body remains at the same level.

Illustration 47. *The potential energy function of a diatomic molecule is given by U =* $\frac{a}{r^{12}} - \frac{b}{r^6}$*, where a and b are constants. The equilibrium point for the potential field is at*

(A) $r = \left(\frac{2a}{b}\right)^{1/6}$ *(B)* $r = \left(\frac{a}{b}\right)^{1/6}$

(C) $r = \left(\frac{a}{2b}\right)^{1/6}$ *(D)* $r = (ab)^{1/6}$

Solution: **(A)**

$$U = \frac{a}{r^{12}} - \frac{b}{r^6}$$

For equilibrium

$$\frac{dU}{dr} = -12\frac{a}{r^{13}} + \frac{6b}{r^7} = 0 \Rightarrow \frac{12a}{r^{13}} = \frac{6b}{r^7}$$

$$r^6 = \frac{12a}{6b}$$

$$\Rightarrow r = \left(\frac{2a}{b}\right)^{\frac{1}{6}}$$

Different forms of energy

Some other forms of energies are also present in nature.

(a) ***Internal Energy:***
Internal energy of a body is possessed because of its temperature. A body can be supposed to be made of molecules. The sum of the kinetic and potential energies of all the molecules constituting

the body is called the internal energy. If the temperature of a body increases, this change cause increase in the kinetic and potential energy and Hence, in the internal energy.

(b) Heat Energy:

Due to the disordered motion of molecules of a body, it possesses heat energy.

(c) Chemical Energy:

Due to the chemical bonding of its atom, a body possesses chemical energy.

(e) Electrical Energy:

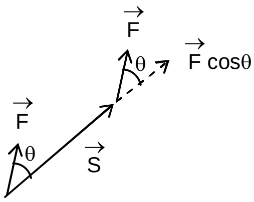
In order to move an electric charge from one point to the other in an electric field or for the transverse motion of a current carrying conductor in a magnetic field, work has to be done. This work done appears as the electrical energy of the system.

(f) Nuclear Energy

When a heavy nucleus breaks up into lighter nuclei on being bombarded by a neutron, a large amount of energy is released. This energy is called nuclear energy.

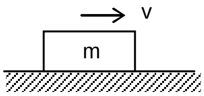
SUMMARY

1. When a constant force $\vec{F}$ acts on a particle that undergoes a displacement $\vec{S}$, the work done by the force on the particle is defined as the scalar product of $\vec{F}$ and $\vec{S}$. The unit of work in SI units is Joule = Newton–meter (1J = 1 Nm). Work is a scalar quantity; It has an algebraic sign (positive or negative) but no direction in space.



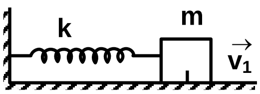
   $W = \vec{F}.\vec{S} = F\,S\cos\theta$, where θ = Angle between $\vec{F}$ and $\vec{S}$.

2. The kinetic energy of a particle equals the amount of work required to accelerate the particle from rest to speed v. It is also equal to the amount of work a particle can do in the process of being brought to rest. Kinetic energy is a scalar quantity that has no direction in space, it is always positive or zero. Its unit is the same as the unit of work.



   $K = \frac{1}{2}mv^2$

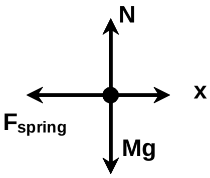
3. When forces act on a particle while it undergoes a displacement, the particle's kinetic energy changes by an amount equal to the total work done on the particle by all the forces. This relation, called the work energy theorem, is valid whether the forces are constant or varying and whether the particles move along a straight line or curved path.



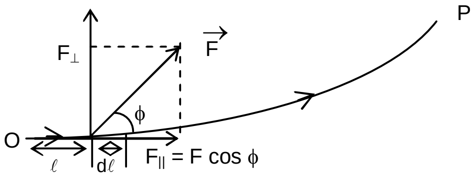
   $W_{tot} = K_2 - K_1 = \Delta K$

4. When a force varies during a straight line motion, the work done by the force is given by an integral:

   $$W = \int_{x_1}^{x_2} F_x dx$$



5. If the particle follows a curved path, then the work done by a force $\vec{F}$ is given by an integral that involves the angle ϕ between the force and the displacement. This expression is valid even if the force magnitude and the angle ϕ vary during the displacement.



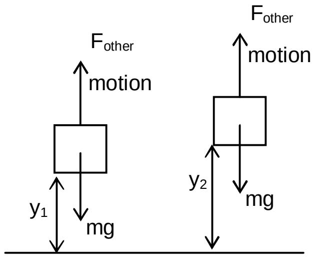
   $$W = \int F\cos\phi\, d\ell$$

6. The work done on a particle by a constant gravitational force can be represented as a change in the gravitational potential energy.

   U = mgy. This energy is a shared property of particle and earth.

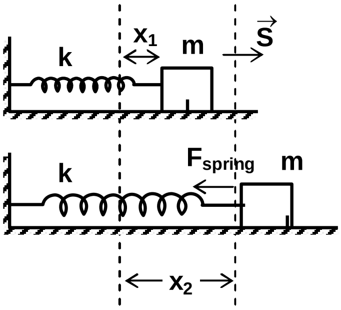
   $W_{gravity} = mgy_1 - mgy_2$

   $U_1 - U_2 = -\Delta U$



7. An ideal stretched or compressed spring exerts an elastic force $F_x = -kx$ on a particle, where x is the amount of stretch or compression. The work done by this force can be represented as a change in the elastic potential energy of the spring,

$$U = \frac{1}{2}kx^2$$



8. The total potential energy is the sum of gravitational and elastic potential energy. If no forces other than the gravitational and elastic force do work on a particle, the sum of kinetic and potential energy is conserved.

$K_1 + U_1 = K_2 + U_2$

9. When forces other than the gravitational and elastic forces do work on a particle, the work W_{other} done by these other forces equals the change in total mechanical energy, considering that the work done by gravitational and elastic forces has already been taken into account as P.E. of the system.

$K_1 + U_1 + W_{other} = K_2 + U_2$

10. All forces are either conservative or non–conservative. A conservative force is one for which the work–energy theorem relation is completely reversible. The work of a conservative force can always be represented by a potential energy function, but the work of a non–conservative force cannot.

11. The work done by non-conservative forces manifests itself as changes in internal energy of bodies. The sum of kinetic, potential and internal energy is always conserved.

$$\Delta K + \Delta U + \Delta U_{int} = 0$$

12. For motion along a straight line, a conservative force $F_x(x)$ is the negative derivative of its associated potential–energy function U. In three dimensions, the component of a conservative force is negative partial derivative of U.

$$F_x(x) = -\frac{dU(x)}{dx},\ F_x = -\frac{\partial U}{\partial x},\ F_y = -\frac{\partial U}{\partial y} \text{ and } F_z = -\frac{\partial U}{\partial z}$$

$$\therefore\quad \vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)$$

13. Power is the time rate of doing work. The average power (P_{avg}) is the amount of work ΔW done in time Δt divided by that time. The instantaneous power (P) is the limit of average power as Δt goes to zero. When a force $\vec{F}$ acts on a particle moving with velocity $\vec{v}$, the instantaneous power is the scaler product of $\vec{F}$ and $\vec{v}$. Like work and kinetic energy, power is also a scalar quantity.

$$P_{avg} = \frac{\Delta W}{\Delta t}$$

$$P = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \text{ and } P = \vec{F}.\vec{v}$$

MISCELLANEOUS EXERCISE

1. A force $F = 9 - 4x + 6x^2$ N acts on a body of mass 5 kg and displaces it from x = 1 m to x = 3 m. Find the work done by the force ?

2 An inelastic ball is dropped from a height of 100 cm. Due to collision with the ground, 20% of its energy is lost. To what height will the ball rise?

3. A body of 3 kg initially at rest is subjected to a force of 15 N. Find the kinetic energy acquired by the body at the end of 10 sec.

4. An object of mass 10 kg falls from rest through a vertical distance of 10 m and acquires a velocity of 10 m/s. Find the work done by the push of air on the object?

5. Water is falling at the rate of 100 kg/sec on the blades of a turbine from a height of 100 m. Find the power delivered to the turbine.

6. A constant force of 2.50 N accelerates a stationary particle of mass 15 g through a displacement of 2.50 m. Find the work done and the average power delivered.

7. A car of mass 1200 kg going with a speed of 30 m/s applies its brakes and skids to rest. If the frictional force between the sliding tyres and the road is 6000 N, how far does the car skid before coming to rest?

8. A block of mass 2kg initially at rest is subjected to a force of 20 N. Calculate the kinetic energy acquired by the body at the end of 10 sec. Assume gravity free space.

9. A driver of a 1200 kg car notices that the car slows down from 20 m/s to 15 m/s as it moves a distance of 130 m along the level ground. How large a force opposes the motion?

10. A motor having an efficiency of 90% operates a crane having an efficiency of 40%. With what constant speed does the crane lift a 500 kg weight if the power supplied to the motor is 5 kW?

ANSWERS TO MISCELLANEOUS EXERCISE

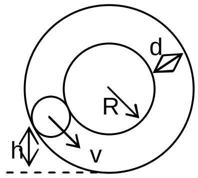
1.	54 J	2.	80 cm
3.	3750 J	4.	– 500 J
5.	100 kW	6.	6.25 J, 36.1 W
7.	90 m	8.	10000 J
9.	807.7 N	10.	0.36 m/s

SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob 1. *With what minimum speed v must a small ball should be pushed inside a smooth vertical fixed tube from a height h so that it may reach the top of the tube? Radius of the tube is R. (d <<R)*



Sol. Applying conservation of energy

$$\frac{1}{2}mv^2 + mgh = mg \,.\, 2R$$

$$v = \sqrt{2g(2R-h)}$$

Prob 2. *A projectile is fired from the top of a tower 40 meter high with an initial speed of 50 m/s at an unknown angle. Find its speed when it hits the ground.*

Sol. Initial K.E. = $\frac{1}{2}m.u^2 = \frac{1}{2}m.50^2$

Final K.E. = $\frac{1}{2}mv^2$

Work done by gravity = +mgh = mg. 40

From w~E principle

$mg \,.\, 40 = k_f - k_I = \frac{1}{2}m(v^2 - 50^2)$

$\Rightarrow v = 57.4$ m/s.

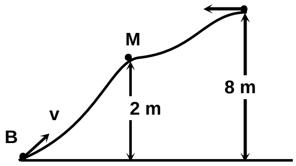
Prob 3. *A body of mass m is thrown vertically upward into air with initial velocity* v_0. *A constant force F due to air resistance acts on the body opposite to the direction of motion of the body. What is the velocity of the body at a height h above the surface during ascent?*

Sol. Using COE,

$$\frac{1}{2}mv_0^2 = mgh + F.h + \frac{1}{2}mv^2 \;\Rightarrow\; v_0^2 - v^2 = 2gh + \frac{2Fh}{m}$$

$$\Rightarrow\quad v = \sqrt{v_0^2 - \left(2gh + \frac{2Fh}{m}\right)}$$

Prob 4. *Two bodies A and B having masses 100 gm each are allowed to move on a frictionless path as shown in the figure. What is the initial velocity given to B such that each body have same kinetic energy at M. Body A starts from rest?*



Sol. K.E. of A at M = decrease in Potential energy = mg × (8 –2)

$= (100 \times 10^{-3}\text{ kg})(10\text{ m/s}^2) \times (8-2) = 6$ J

K.E. of B at M = 6 J

Potential of B at M = $\frac{1}{10}\times 10\times 2 = 2\,\text{J}$

By energy conservation at B and M

$$\frac{1}{2}mv_B^2 = (6+2) \Rightarrow v_B^2 = 160$$

$\Rightarrow$ $v_B = 4\sqrt{10}$ m/s

Prob 5. *A bob of mass 'm' is suspended by a light inextensible string of length 'l' from a fixed point. The bob is given a speed of $\sqrt{6gl}$. Find the tension in the string when string deflects through an angle 120^0 from the vertical.*

Sol. By C.O.E .theorem

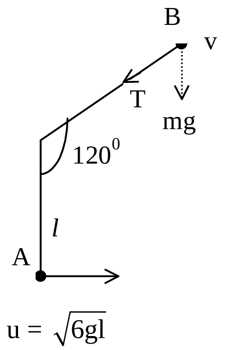
$$\frac{1}{2}mu^2 = mgl(1-\cos 120) + \frac{1}{2}mv^2$$

$\Rightarrow v = \sqrt{3gl}$

At point B,

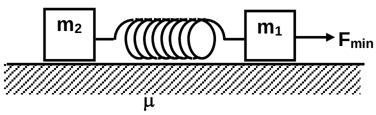
$$T + mg\cos 60^0 = \frac{mv^2}{l}$$

By putting $v = \sqrt{3gl}$ we get, $T = \frac{5}{2}mg$.



IITJEE TYPE

Prob 6. *Two bodies m_1 and m_2 are kept on a table with coefficient of friction 'μ' and are joined by a spring. Initially, the spring is in its relaxed state. Find the minimum force F which will make the other block m_2 move. (k is the spring constant).*



Sol. Motion of m_2 starts when, $kx = \mu.m_2.g$, where x = elongation in the spring

$$x = \frac{\mu m_2 g}{k}$$

The minimum force will be such that m_1 has no kinetic energy.

Applying work energy principle for m_1

$$\int_0^x (F - \mu m_1 g - kx)\,dx = 0$$

$$Fx - \mu m_1 gx - \frac{1}{2}kx^2 = 0$$

$$\Rightarrow \quad F = \left[\mu m_1 g + \frac{1}{2}kx\right] = \left[\mu m_1 g + \frac{\mu m_2 g}{2}\right]$$

$$\Rightarrow \quad F_{min} = \mu m_1 g + \frac{\mu m_2 g}{2}$$

Prob 7. *A small metallic sphere is suspended by a light spring of force constant k from the ceiling of a cage, which is accelerating uniformly by a force F. The ratio of mass of the cage to that of the sphere is 'n'. Find the potential energy stored in the spring.*

Sol. The force equation for the cage and the sphere are

$F - (T' + Mg) = Ma$ …(1)

And, $T' - mg = ma \Rightarrow T' = m(g + a)$ …(2)

Adding (1) and (2) $\Rightarrow (M + m)a = F - (M + m)g$

$\Rightarrow F = (M + m)(g + a)$ …(3)

dividing (2) by (3)

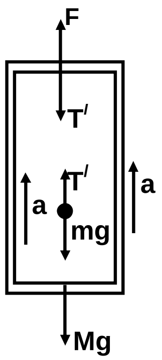
$\Rightarrow T' = [m(g + a) / (M + m)(g + a)]\,F = \dfrac{mF}{(M+m)}$

where $T' = kx$, Therefore, $x = \dfrac{mF}{(M+m)k}$.

Therefore, P.E. stored in the spring $= \dfrac{1}{2}kx^2 = \dfrac{m^2F^2}{2(M+m)^2k}$

As $M/m = n$,

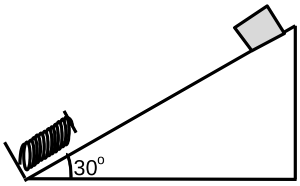
$\text{P.E.} = \dfrac{F^2}{2(n+1)^2k}$.



Prob 8. *An ideal massless spring can be compressed 1 m by a force of 100 N. This same spring is placed at the bottom of a frictionless inclined plane which makes an angle $\theta = 30^o$ with the horizontal. A 10 kg mass is released from rest at the top of the incline and is brought to rest momentarily after compressing the spring 2 meters.*

(a) Through what distance does the mass slide before coming to rest?

(b) What is the speed of the mass just before it reaches the spring?



Sol. (a) Let total distance moved by the block is

$S = (\ell + 2)$m, where ℓ is the distance moved by the block before touching the spring.

Now, work done by gravity on the block is

$W_g = mg\,S\sin\theta = 10 \times 10 \times S\sin 30$ J

$\Rightarrow W_g = 50\,S$ J . . . (1)

Work done by spring on the block is

$W_S = -\dfrac{1}{2}kx^2$

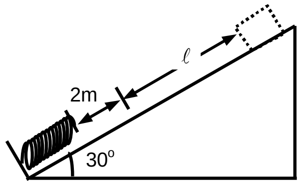
Here, $k = 100$ N/m and $x = 2$ m

$\Rightarrow W_S = -\dfrac{1}{2}\times 100 \times 4$ J $= -200$ J . . . (2)

Total work done $W = W_g + W_s = (50\,S - 200)$ J

Since change in K.E. of the block is zero as $W = \Delta$K.E.

$\Rightarrow 50\,S - 200 = 0 \quad \Rightarrow \quad S = 4$ m



(b) As $S = \ell + 2 \Rightarrow \ell = S - 2 = 2$ m

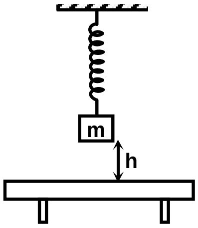
Work done by gravity over this path length is $W_g = mg \times 2\sin\theta$

As $W_g = \Delta$ K.E. $\Rightarrow 100 = \dfrac{1}{2}mv^2 - 0$

$\Rightarrow \ v^2 = \dfrac{100 \times 2}{10} = 20$

$\Rightarrow \ v = 2\sqrt{5}$ m/s.

Prob 9. *A body of mass 100 g is attached to a hanging spring whose force constant is 10 N/m. The body is lifted until the spring is in its unstretched state. The body is then released. Calculate the speed of the body when it strikes a table 15 cm below the release point.*



Sol. By conservation of mechanical energy,

$$K_f + U_f = K_i + U_i$$

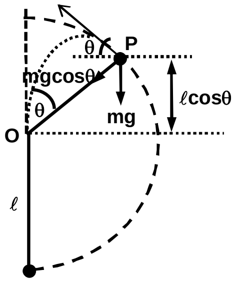
$$\frac{1}{2}mv_f^2 + \frac{1}{2}kh_i^2 = 0 + mgh$$

Since $v_i = 0$

$$\therefore v_f = \sqrt{2gh - \frac{kh^2}{m}} = \sqrt{2(9.80)(0.15) - \frac{(10)(0.15)^2}{0.1}} = 0.831 \text{ m/s}$$

Prob 10. *A heavy particle is suspended by a string of length ℓ from a fixed point O. The particle is given a horizontal velocity v_o. The string slacks at some angle and the particle proceeds on a projectile path. Find the value of v_o, if the particle passes through the point of suspension.*

Sol. Let at P, the string slacks when it makes an angle θ with the vertical, Hence, at the point P the centripetal force is only due to the component of the gravitational force.



$mg\cos\theta = \dfrac{mv^2}{\ell}$, where v = velocity of the particle at P.

$$v^2 = g\ell.\cos\theta \qquad \text{. . . (i)}$$

Conserving energy at initial point and at P , we get

$$\frac{1}{2}mv_o^2 = \frac{1}{2}mv^2 + mg\ell(1+\cos\theta) \qquad \text{. . . (ii)}$$

From (i) and (ii), we get, $\dfrac{1}{2}mv_o^2 = \dfrac{1}{2}mg\ell\cos\theta + mg\ell(1+\cos\theta)$

$$v_o^2 = g\ell\,[2 + 3\cos\theta] \qquad \text{. . . (iii)}$$

Now particle will pass through the point of suspension, if

$$\ell\sin\theta = (v\cos\theta)t \qquad \text{. . . (iv)}$$

And $-\ell\cos\theta = (v\sin\theta)t - \dfrac{1}{2}gt^2$ . . . (v)

Eliminating t from (iv) and (v) we get .

$$-\ell\cos\theta = (v\sin\theta)\left[\frac{\ell\sin\theta}{v\cos\theta}\right] - \frac{1}{2}g\left[\frac{\ell\sin\theta}{v\cos\theta}\right]^2$$

Substituting $v^2 = g\ell\cos\theta$, and simplifying $\tan\theta = \sqrt{2}$

Therefore, $\cos\theta = \dfrac{1}{\sqrt{3}}$.

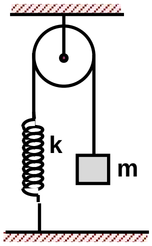
Substituting the value of cosθ in equation (iii), we get

$v = [g\ell\,(2 + \sqrt{3}\,)]^{1/2}$.

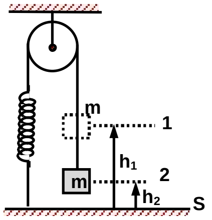
Probs 11. *In the figure shown, stiffness of the spring is k and mass of the block is m. The pulley is fixed. Initially, the block m is held such that, the elongation in the spring is zero and then released from rest. Find :*

(a) the maximum elongation in the spring,

(b) the maximum speed of the block m. Neglect the mass of the spring and that of the string and the friction.



Sol. Let the maximum elongation in the spring be x, when the block is at position 2.



(a) The displacement of the block m is also x.

If E_1 and E_2 are the energies of the system when the block is at position 1 and 2, respectively, then

$E_1 = U_{1g} + U_{1s} + T_1$

where U_{1g} = gravitational P.E. with respect to surface S,

U_{1s} = P.E. stored in the spring, and T_1 = initial K.E. of the block.

$\Rightarrow\ E_1 = mgh_1 + 0 + 0 = mgh_1$ . . . (1)

And $E_2 = U_{2g} + U_{2s} + T_2 = mgh_2 + \frac{1}{2}kx^2 + 0$ . . . (2)

From conservation of energy, $E_1 = E_2$

$\Rightarrow\ mgh_1 = mgh_2 + \frac{1}{2}kx^2 \ \Rightarrow\ \frac{1}{2}kx^2 = mg\left(h_1 - h_2\right) = mgx$

$\Rightarrow\ x = 2mg/k$

(b) From work energy theorem:

$$mgx - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

For maximum v,

$$\frac{dv}{dx} = 0 \Rightarrow x = \frac{mg}{k}$$

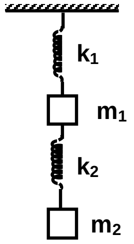
So, $mg\left(\frac{mg}{k}\right) - \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{1}{2}mv_{max}^2$

$\Rightarrow\ v_{max} = \left(\sqrt{\frac{m}{k}}\right)g$

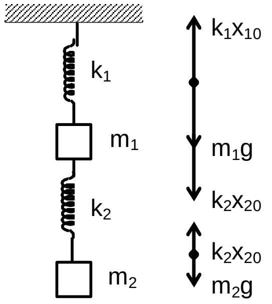
Prob 12. *Given k_1 = 1500 N/m, k_2 = 500 N/m, m_1 = 2 kg, m_2 = 1 kg. Find*

(a) potential energy stored in the springs in equilibrium.

(b) work done in slowly pulling down m_2 by 8 cm.



Sol. Let the initial extension in the springs of force constants k_1 and k_2, at equilibrium position, be x_{20} and x_{10}. Then



$x_{20} = \frac{m_2 g}{k_2}$, $x_{10} = \frac{(m_1+m_2)g}{k_1}$

(a) Potential energy stored in the springs in equilibrium position is

$$U_1 = \frac{1}{2}k_1 x_{10}^2 + \frac{1}{2}k_2 x_{20}^2$$

Putting values of x_{10}, x_{20} from above we get $U_1 = 0.4$ J

(b) Let Δx_1 and Δx_2 be additional elongations due to pulling m_2 by $\ell = 8$ cm. Additional forces on m_1 are equal and in opposite direction.

$\Rightarrow$ $k_1\Delta x_1 = k_2\Delta x_2$ . . . (i)

Also $\Delta x_1 + \Delta x_2 = \ell$ . . . (ii)

Δx_1 , Δx_2 can be found from (i) and (ii)

$w_g + w_P + w_s = 0$ (where w_p is the work done by the pulling force)

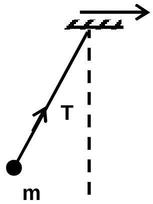
$\Rightarrow$ $w_p = -w_s - w_g = (U_2 - U_1) - [m_1 g\Delta x_1 + m_2 g(\Delta x_1 + \Delta x_2)]$

where $U_2 = \frac{1}{2}k_1(x_{10} + \Delta x_1)^2 + \frac{1}{2}k_2(x_{20} + \Delta x_2)^2$

Putting the values

$\Rightarrow$ $w_p = 1$ J

Prob 13. *A pendulum with a bob of mass m is suspended from a horizontal platform. The platform is given a horizontal uniform acceleration. The breaking tension in the light string of the pendulum is $\frac{2}{\sqrt{3}}$mg. Find the work done by the extreme tension T on the bob in the first one sec.*

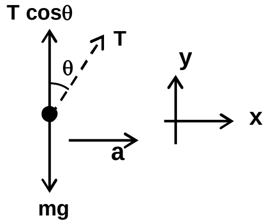


Sol. Work done, W = (F) (S).

Force F and displacement $\vec{S}$ are parallel. Here, the bob does not move in y axis.

$\Rightarrow$ The work done by the tension T along y axis is zero.

Since the bob moves horizontally (along x – axis) with an acceleration, a (say)



$\therefore$ $\Sigma F_x = ma$

$\Rightarrow$ $T\sin\theta = ma$

$\Rightarrow$ $a = \frac{T\sin\theta}{m}$ …(1)

For equilibrium of the bob along Y–axis

$\Sigma F_y = 0 \Rightarrow T\cos\theta = mg$ …(2)

From (1) and (2), $a = \left(\frac{mg}{\cos\theta}\right)\left(\frac{\sin\theta}{m}\right) = g\tan\theta$

$\Rightarrow$ $a = g\tan\theta$ …(3)

Since the breaking strength,

$T = \frac{2mg}{\sqrt{3}} \Rightarrow \frac{mg}{\cos\theta} = \frac{2mg}{\sqrt{3}}$

$\Rightarrow \theta = 30°$, putting $\theta = 30°$ in (3).

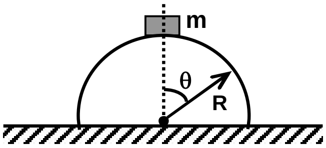
We obtain $a = g\tan 30° = \frac{g}{\sqrt{3}}$.

Hence, required work done = W = FS= (T sinθ) (1/2) at^2

where $T = \frac{2}{\sqrt{3}}mg$, θ = 30°, $a = \frac{g}{\sqrt{3}}$, t = 1 sec.

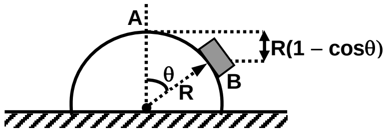
$\Rightarrow$ $W = \left(\frac{2}{\sqrt{3}}mg\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{g}{\sqrt{3}}\right)(1)^2 = \frac{mg^2}{6}$.

Prob 14. *A point mass m starts from rest and slides down the surface of a fixed frictionless solid hemisphere of radius R as shown in figure. Measure angles from the vertical and potential energy from the top. Find*



(a) the change in potential energy of the mass with angle
(b) the kinetic energy as a function of angle,
(c) the radial and tangential acceleration as a function of angle,
(d) the angle at which the mass flies off the sphere.

Sol. (a) Consider the mass when it is at the point B.



U_A (P.E. at A) = 0
U_B(P.E. at B)= –mgR (1 – cosθ)
$\Rightarrow \Delta U = U_B - U_A$
$\Rightarrow \Delta U = -mgR(1 - \cos\theta)$
Negative sign indicates that P.E. decreases as particle slides down.

(b) Conserving energy at points A and B.
$U_A + T_A = U_B + T_B$
where U_A = P.E. at A, U_B = P.E. at B
T_A = K.E. at A, T_B = K.E. at B
$\Rightarrow$ $0 + 0 = -mgR(1 - \cos\theta) + T_B$
$\Rightarrow$ $T = mgR(1 - \cos\theta)$

(c) Since $T = \frac{1}{2}mv^2$

$\Rightarrow$ $\frac{1}{2}mv^2 = mgR(1 - \cos\theta) = 2mgR\sin^2(\theta/2)$

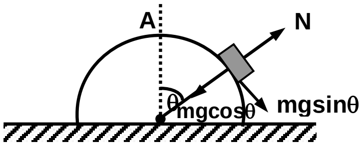
$\Rightarrow$ $v = 2\sqrt{gR}\ \sin(\theta/2)$ $\Rightarrow$ $a_r = v^2/R$

$\Rightarrow$ $a_r = 4g\sin^2(\theta/2)$ As $a_t = \frac{dv}{dt}$

$\Rightarrow$ $a_t = \sqrt{(gR)}\cos(\theta/2)\frac{d\theta}{dt} = \sqrt{(gR)}\ \omega\cos(\theta/2)$

$\Rightarrow$ $a_t = \sqrt{(g/R)}\ v\cos(\theta/2)$, as $\omega R = v$

(d) For circular motion



$mg\cos\theta - N = \frac{mv^2}{R}$

at the moment when the particle breaks off the sphere N = 0.

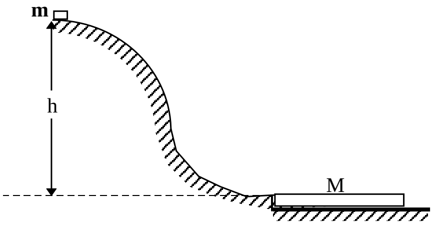
$\Rightarrow$ $mg\cos\theta = \frac{mv^2}{R}$ $\Rightarrow$ $g\cos\theta = \frac{v^2}{R}$

As $v = 2\sqrt{gR}\sin(\frac{\theta}{2})$

$\therefore$ g cosθ = 4g sin^2(θ/2) = 2g(1 − cosθ)

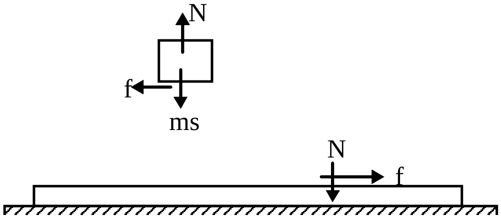
$\Rightarrow$ $\cos\theta = \frac{2}{3}$ $\Rightarrow$ $\theta = \cos^{-1}(\frac{2}{3})$

Prob 15. *A small disc of mass m slides down a smooth hill of height h without initial velocity and gets onto a plank of mass M lying on the horizontal plane at the base of the hill, as shown in the figure. Due to friction between the disc and the plank, the disc slows down and ultimately, both move together.*



(a) Find the common velocity of the disc and the plank.
(b) Find the work done by the friction
(c) Find the distance moved by the disc with respect to the planck before they start moving together.

Sol. (a) Velocity of the disc at the foot of the hill is $v_0 = \sqrt{2gh}$. When it slides on the plank, friction opposes its motion and favours the motion of the plank as shown in the figure.



Let their common velocity be V, then

for m, $V = v_0 - \mu g t$

for M, $V = \frac{\mu m g t}{M}$

Eliminating t from the above two equations,

$$V = \frac{m v_0}{m + M}$$

(b) Using Work – Energy theorem

$$W_f = K_f - K_i$$

$$W_f = \frac{1}{2}(m + M)V^2 - \frac{1}{2}mv_0^2$$

$$= -\frac{1}{2}\frac{mMv_0^2}{m+M} \qquad = -\frac{mMgh}{m+M}$$

(c) Acceleration of the disc w.r.t the plank is

$$a_{rel} = \mu g + \mu\frac{m}{M}g = \mu g\left(\frac{m+M}{M}\right)$$

$$x_{rel} = \frac{u_{rel}^2}{2a_{rel}} = \frac{Mv_0^2}{2\mu g(m+M)} = \frac{Mh}{\mu(m+M)}$$

Objective:

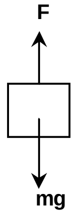
Prob 1. *A force of magnitude 2mg (1 – ay) starts acting in the vertically upward direction on a body of mass m placed on earth's surface, where y is the height of the object above the ground during ascent and 'a' is a positive constant. Total height through which the body ascends is*

(A) 1/a *(B) 1/2a*

(C) 2/3a *(D) 2/a*

Sol. Initial and final kinetic energy of the body is zero.

Applying work energy principle



$$\int_0^h \vec{F}_{net}.d\vec{y} = 0 \qquad \text{where } F_{net} = F - mg$$

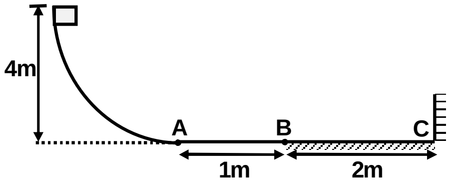
or $$\int_0^h (mg - 2mgay)dy = 0,$$

$$\int_0^h (1-2ay)dy = 0 \Rightarrow \left[y - ay^2\right]_0^h = 0$$

$$\Rightarrow h = \frac{1}{a}$$

Hence, (A) is correct.

Prob 2. *A block of mass m = 0.1 kg is released from a height of 4 m on a curved smooth surface. On the horizontal surface path AB is smooth and path BC offers coefficient of friction μ = 0.1.*



If the impact of block with the vertical wall at C be perfectly elastic, the total distance covered by the block on the horizontal surface before coming to rest will be (take g = 10ms^{-2})

(A) 29 m *(B) 49 m*

(C) 59 m *(D) 109 m*

Sol. KE attained by block at B = mgh = 4J

Work done by friction force on path BC = (–μmg)(BC) = –0.2 J.

On the other horizontal surface block can make total forward and backward

$\left(\frac{4}{0.2}\right)$ = 20 trips. but initially it will stop at B.

$\therefore$ Distance covered = 20 × (AB + BC) – AB = 59 m.

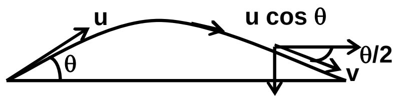
Hence, (C) is correct.

Prob 3. *A particle of mass m is projected with velocity u at an angle θ with horizontal. During the period when the particle descends from highest point to the position where its velocity vector makes an angle $\theta/2$ with horizontal, the work done by the gravity force is*

(A) $\frac{1}{2}mu^2\tan^2\theta/2$ *(B)* $\frac{1}{2}mu^2\tan^2\theta$

(C) $\frac{1}{2}mu^2\cos^2\theta\tan^2\theta/2$ *(D)* $\frac{1}{2}mu^2\cos^2\theta/2\ \sin^2\theta$

Sol. As horizontal component of velocity does not change,



$$v\cos\theta/2 = u\cos\theta$$

$$v = \frac{u\cos\theta}{\cos\theta/2}$$

$$W_{gravity} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}m(u\cos\theta)^2$$

$$= \frac{1}{2}mu^2\cos^2\theta\tan^2\theta/2$$

Hence, (C) is correct.

Prob 4. *A body of mass 1 kg thrown upwards with a velocity of 10 m/s comes to rest (momentarily) after moving up 4 m. The work done by air drag in this process is (Take g = 10 m/s²)*

(A) –20 J *(B) –10 J*
(C) –30 J *(D) 0 J*

Sol. According to work energy theorem

$W_{\text{gravitational force}} + W_{\text{air drag}}$ = change in KE.

$\Rightarrow$ $m\vec{g}.\Delta\vec{r} + W_{\text{air drag}}$ = change in KE.

$\Rightarrow$ $-mgh + W_{\text{air drag}}$ = change in KE

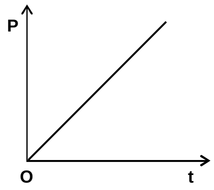
$\Rightarrow$ $W_{\text{air drag}}$ = change in KE + mgh = $[0 - \frac{1}{2}mu^2]$ + mgh

= –50 + 40 = –10 Joules.

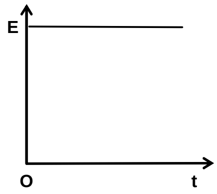
Hence, (B) is correct.

Prob 5. *A vehicle is driven along a straight horizontal track by a motor, which exerts a constant driving force. The vehicle starts from rest at t = 0 and the effects of friction and air resistance are negligible. If kinetic energy of vehicle at time t is E and power developed by the motor is P, which of the following graph is/ are correct?*

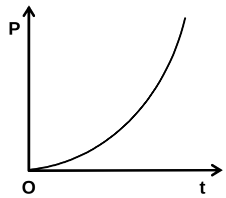
(A)



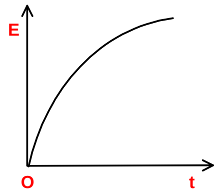
(B)



(C)



(D)



Sol. Since force on the vehicle is constant, therefore, it will move with a constant acceleration. Let this acceleration be 'a'.

Then at time t, its velocity will be equal to v = a.t

Hence, at time t, the kinetic energy, $E = \frac{1}{2}mv^2 = \frac{1}{2}ma^2t^2$

The power associated with the force is equal P = F.v = Fat. Hence, the graph between power and time will be a straight line passing through the origin.
Hence, (A) is correct.

Prob 6. *A block is suspended by an ideal spring of force constant K. If the block is pulled down by applying a constant force F and if maximum displacement of block from its initial position of rest is δ, then*

(A) $\frac{F}{K} < \delta < \frac{2F}{K}$

(B) $\delta = \frac{2F}{K}$

(C) $\delta = F/K$

(D) Increase in potential energy of the spring is $\frac{1}{2}K\delta^2$

Sol. Let mass of the block hanging from the spring be m. Then, initial elongation of the spring will be equal to mg/K. When the force F is applied to pull the block down, then work done by F and further loss of gravitational potential energy of the block is used to increase the potential energy of this spring.

Hence, $(F.\delta + mg.\delta)$

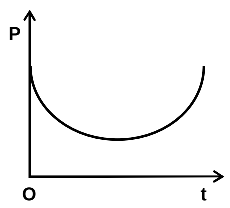
$$= \frac{1}{2}K\left(\frac{mg}{K} + \delta\right)^2 - \frac{m^2g^2}{2\ K}$$

From this equation, $\delta = \frac{2F}{K}$,

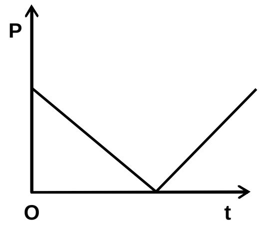
Hence, (B) is correct.

Prob 7. *A stone is projected at time t = 0 with a speed V_0 at an angle θ with the horizontal in a uniform gravitational field. The rate of work done (P) by the gravitational force plotted against time (t) will be as*

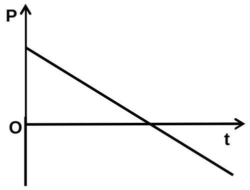
(A)



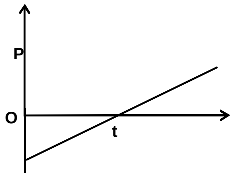
(B)



(C)



(D)



Sol. Rate of work done is the power associated with the force. It means rate of work done by the gravitational force is the power associated with the gravitational force.

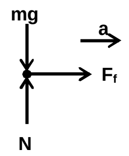
Gravitational force acting on the block is equal to its weight mg which acts vertically downwards.
Velocity of the particle (at time t) has two components,
(i) a horizontal component $v_0 \cos\theta$, (ii) a vertically upward component $(v_0 \sin\theta - gt)$
Hence, the power associated with the weight mg will be equal to

$$P = m\vec{g}.\vec{v} = -mg\left(v_0 \sin\theta - gt\right)$$

This shows that the curve between power and time will be a straight line having positive slope but negative intercept on Y–axis.
Hence, (D) is correct.

Prob 8. *A 5 kg block is kept on a horizontal platform at rest. At time t = 0, the platform starts moving with a constant acceleration of 1 m/s². The coefficient of friction μ between the block and the platform is 0.2. The work done by the force of friction on the block in the fixed reference frame in 10 s is*
(A) +250 J *(B) –250 J*
(C) +500 J *(D) –500 J*

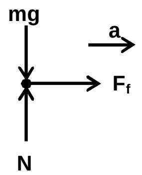
Sol. Assuming that the block does not slide on the platform



$F_F = ma = 5(1) = 5N$; $N - mg = 0$
$\Rightarrow$ $N = mg = 50$ N, As $\mu N = 10$ N
$F_f < \mu N$
The block will remain at rest relative to the platform.
Displacement D relative to the ground = $\frac{1}{2}(1)(10)^2 = 50$ m
$\therefore$ Work done by friction = $F_f D \cos 0 = +250$ J,
Hence, (A) is correct.

Prob 9. *In the previous Problem, if $\mu = 0.02$, the work done by the force of friction on the block in the fixed reference frame in 10 seconds is*
(A) +10 J *(B) –10 J*
(C) +250 J *(D) –250 J*

Sol. Limiting force of friction = $\mu N = 0.02\ (50) = 1N$



$\therefore$ The block will slide on the platform.
$F_f = ma = 1$; $a = \frac{1}{5}$ m/s²
$\therefore$ Displacement D = $\frac{1}{2}\left(\frac{1}{5}\right)(10)^2 = 10$m
Work done by F_f = 1N (10 m) = +10 J
Hence, (A) is correct.

Prob 10. *A particle of mass m starts from rest and moves in a circular path of radius R with a uniform angular acceleration α. The kinetic energy of the particle after n revolutions is*
(A) $n\alpha mR^2$ *(B) $2\pi n\alpha mR^2$*
(C) $(1/2)\, mn\alpha R^2$ *(D) $mn\alpha R^2$*

Sol. For uniform angular acceleration in a circular path, the angular speed is given by

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad = 0 + 2\alpha\,(2\pi n)$$

Kinetic energy of the particle

$$KE = \frac{1}{2}m(R\omega)^2 = \frac{1}{2}mR^2(4\alpha\pi n) = 2\pi nm\,\alpha R^2$$

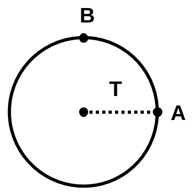
Hence, (B) is correct.

Prob 11. *A particle of mass m attached to an inextensible light string is moving in a vertical circle of radius r. The critical velocity at the highest point is v_0 to complete the vertical circle. The tension in the string when it becomes horizontal is*

(A) $\frac{3mv_0^2}{r}$ *(B)* $\frac{9mv_0^2}{r}$

(C) 3mg *(D) both (A) and (C) are correct.*

Sol. At point A, in the horizontal direction



$$T = \frac{mv_A^2}{r} \qquad \ldots (1)$$

At the highest point B,

$$mg = \frac{mv_0^2}{r} \qquad \ldots (2)$$

By conserving energy at points A and B,

$$\frac{1}{2}mv_0^2 + mg(2r) = \frac{1}{2}mv_A^2 + mgr \qquad \ldots (3)$$

From equations (2) and (3)

$$v_A = \sqrt{3gr}$$

Hence, the tension in the string at the point A,

$$T = \frac{m(3gr)}{r}$$

or, T = 3 mg

Hence, (C) is correct.

Prob 12. *A block of mass 10 kg accelerates uniformly from rest to a speed of 2 m/s in 20 sec. The average power developed in time interval of 0 to 20 sec is*

(A) 10 W *(B) 1 W*

(C) 20 W *(D) 2 W*

Sol. Average power

$$P_{av} = \frac{\text{Net work done}}{\text{Total time taken}}$$

Net work done = change in kinetic energy

$$= \frac{1}{2}\times 10\times 2^2 = 20J \qquad \ldots (1)$$

Average power = $\frac{20}{20}$ = 1 watt

Hence, (B) is correct.

Prob 13. *A pumping machine pumps water at a rate of 60 cc per sec at a pressure of 1.5 atm. The power delivered by the machine is*

(A) 9 watt *(B) 6 watt*

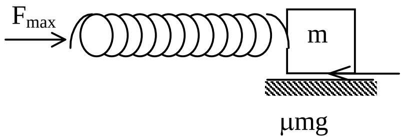
(C) 9 kW *(D) None of these*

Sol. Power = F.v Where F = force imparted by the machine , v = velocity of the liquid
P = p.A.v, Where p = pressure & A = effective area
$\therefore$ P = p $\frac{dV}{dt} = \left(\frac{3}{2}\times10^5\right)(60\times10^{-6})$ = 9 watt. ($\because$ 1 atm $\approx 10^5$ N/m^2)
Hence, (A) is correct.

Prob 14. *A spring, placed horizontally on a rough surface is compressed by a block of mass m, placed on the same surface so as to store maximum energy in the spring. If the coefficient of friction between the block and the surface is μ, the potential energy stored in the spring is*

(A) $\frac{\mu^2m^2g^2}{2k}$ (B) $\frac{2\mu m^2g^2}{k}$
(C) $\frac{\mu^2m^2g}{2k}$ (D) $\frac{3\mu^2mg^2}{k}$

Sol. For equilibrium of the block
$F_{max} - \mu mg = 0 \Rightarrow F_{max} = \mu mg$
$\therefore$ U = $\frac{F_{max}^2}{2k} = \frac{\mu^2m^2g^2}{2k}$
Hence, (A) is correct.



FILL IN THE BLANKS IN THE FOLLOWING QUESTIONS.

Prob 1. *A stone is projected up with a velocity u. It reaches to a maximum height of h. When it is at a height of 3h/4 from the ground, the ratio of K.E. and P.E. at that point is ……..*

Sol. 1 : 3.

Prob 2. *The negative of the work done by the conservative forces on a system equals the change in …….. energy.*

Sol. Potential

Prob 3. *The work done by all the forces (external and internal) on a system equals the change in ……………energy.*

Sol. Kinetic

Prob 4. *In the stable equilibrium position, a body has …….. potential energy.*

Sol. Minimum.

Prob 5. *A vehicle of mass m is moving on a rough horizontal road with momentum P. If the coefficient of friction between the tyres and the road be μ, then the stopping distance is ……..*

Sol. $\frac{P^2}{2\mu m^2g}$

STATE WHETHER THE FOLLOWING QUESTIONS ARE TRUE OR FALSE.

Prob 1. *Kinetic energy of an object can have negative value.*

Sol. **False.** K.E. $=\frac{1}{2}mv^2, v^2$ and m are always positive and hence KE is always positive

Prob 2. *A block starting from rest reaches the bottom of a rough inclined plane with velocity u. Now if velocity u is given at the bottom, up the incline plane, the block will reach the top.*

Sol. **False.** Some energy will be spent against frictional force

Prob 3. *When a body moves in a circular path with uniform speed, no work is done by the force.*

Sol. **True**. This is because force and displacement are always perpendicular to each other.

Prob 4. *Power is a vector quantity,*
False. It is a dot product of two vector quantities, force and velocity.

Prob 5. *Work done by frictional force is always negative because the force always acts opposite to displacement.*

Sol. **False.**

ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. How much work is done on a body of mass M in moving one round on a horizontal circle of radius r with constant speed ? Give reason.

2. A lorry and a car moving with the same kinetic energy are brought to rest by the application of brakes, which provide equal retarding forces. Which of them will come to rest in a shorter distance?

3. A car of mass m starts with acceleration 'a' along a straight level road against a constant external resistive force R. When the velocity of the car is v, what is the rate at which the engine of the car will be doing work ?

4. If a body of mass m accelerates uniformly from rest to the velocity v_1 in time t_1, then find out the instantaneous power delivered to the body as a function of time t.

5. A block of mass m is pulled along a horizontal surface by applying a force at an angle θ with the horizontal. If the block travels with a uniform velocity and has a displacement d and the coefficient of friction is F, then find the work done by the applied force.

6. A coin of mass m slides a distance D along a tabletop. If the coefficient of friction between the coin and table is μ, find the work done on the coin by friction.

7. A ball is thrown at an angle of 60° with the horizontal with initial kinetic energy K. What is the kinetic energy at the highest point of its flight ?
8. A bird is flying at a speed of 10 m/s. The kinetic energy of the bird is $\frac{1}{10}$ times its potential energy (with respect to the ground). What is the height above the ground at which the bird is flying ?

9. Explain why friction is a non–conservative force.

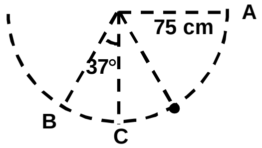
10. Prove the work–energy principle for a particle moving with constant acceleration (under a constant force) along a straight line.
11. State and explain the law of conservation of energy. Show that the energy in case of free fall of a body is conserved.

12. A bucket tied to a string is lowered at a constant acceleration of g/4. If the mass of the bucket is m and it is lowered by a distance ℓ, then find the work done by the string on the bucket.

13. Can you associate potential energy with a non–conservative force?

14. In a tug of war, one team is slowly giving way to the other. What work is being done and by whom?

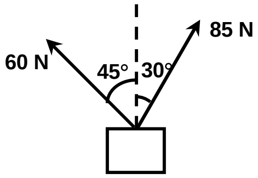
15. If the simple pendulum shown in the figure is released from point A, what will be the speed of the ball as it passes through point C?



16. A spring which stretches 10 cm under a load of 200 g requires how much work to stretch it 5 cm from its equilibrium position?

17. When a ball is thrown up, the magnitude of its momentum first decreases and then increases. Does this violate the conservation of momentum principle?

18. Top views of two horizontal forces pulling a box along the floor is shown in the figure.
    (a) How much work does each forces do as the box is displaced 70 cm along the broken line?
    (b) Calculate the total work done by the two forces in pulling the box through this distance.



19. A thin uniform rectangular slab of area (3.4×2.0) m^2 has a mass of 180 kg. It is lying on the flat ground. Calculate the minimum amount of work needed to stand it over its larger edge.

20. A man weighing 60 kg climbs up a stair case carrying a 20 kg load on his head. The stair case has 20 steps and each step has a height of 20 m. If he takes 10 sec to climb, calculate the power.

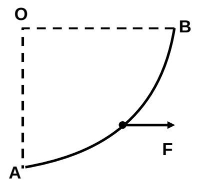
Level - I

1. A particle moves from a point $\vec{r}_1 = (2m)\hat{i} + (3m)\hat{j}$ to another point $\vec{r}_2 = (3m)\hat{i} + (2m)\hat{j}$ during which a certain force $\vec{F} = (5N)\hat{i} + (5N)\hat{j}$ acts on it. Find the work done by the force on the particle during the displacement.

2. A force F = a + bx acts on a particle in the x direction, where a and b are constants. Find the work done by this force during a displacement from x = 0 to x = d.

3. A small block of mass m is kept on a rough inclined plane of inclination θ fixed in an elevator going up with uniform velocity v and the block does not slide on the wedge. Find the work done by the force of friction on the block in time t.

4. The figure shows a smooth circular path of radius R in the vertical plane which subtends an angle $\frac{\pi}{2}$ at O. A block of mass m is taken from position A to B under the action of a constant horizontal force F.



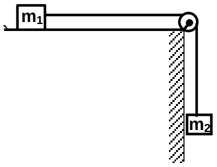
   (a) Find the work done by this force.
   (b) In part (a), if the block is being pulled by a force F which is always tangential to the surface, find the work done by the force F between A and B.

5. A block of mass M is kept on a platform which is accelerating upward from rest with a constant acceleration a, during the time interval from t = 0 to t = t_0. Find
   (a) the work done by gravity, (b) the work done by normal reaction.

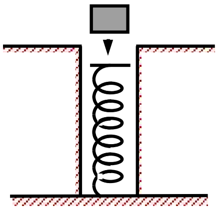
6. Two cylindrical vessels of equal cross–sectional area A contain water upto heights h_1 and h_2. The vessels are interconnected so that the levels in them become equal. Calculate the work done by the force of gravity during the process. The density of water is ρ.

7. A body of mass m is thrown at an angle θ to the horizontal with a velocity u. Find the mean power developed by gravity over the whole time of motion and the instantaneous power of gravity as a function of time.

8. Two blocks are connected by a string as shown in the figure. They are released from rest. If the co–efficient of friction between m_1 and the surface is μ, find the common speed of the blocks at the instant when they have moved a distance ℓ.



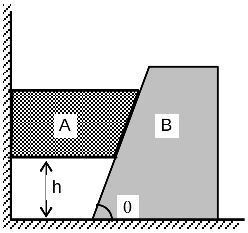
9. A 263 gm block is dropped onto a vertical spring with force constant k = 2.52N/cm. The block sticks to the spring, and the spring compresses 11.8 cm before coming momentarily to rest. While the spring is being compressed, how much work is done
   (a) by the force of gravity, and
   (b) by the spring?
   (c) What was the speed of the block just before it hits the spring?



(d) If this initial speed of the block is doubled, what is the maximum compression of the spring? Ignore friction.

10. The displacement x of a particle moving in one dimension under the action of a constant force is related to time 't' by the equation $t = \sqrt{x} + 3$, where x is in meter and t in sec. Calculate
(a) the displacement of the particle when its velocity is zero.
(b) the work done by the force in the first 6 sec.

11. Two smooth wedges of equal mass m are placed as shown in figure. All surfaces are smooth. Find the velocities of A & B when A hits the ground.

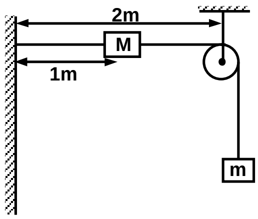


Level - II

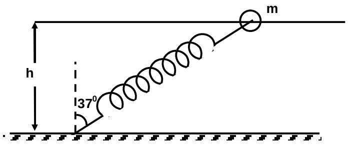
1. Two blocks of different masses are hanging on two ends of a string passing over a frictionless pulley. The heavier block has mass twice that of the lighter one. The tension in the string is 60 N. Find the decrease in potential energy during the first second after the system is released.

2. A block of mass 2.0 kg is pulled up on a smooth incline of angle 30^o with the horizontal. If the block moves with an acceleration of 1.0 m/s^2, find the power delivered by the pulling force at a time 4.0 s after the motion starts. What is the average power delivered during 4.0 s after the motion starts?

3. A string with one end fixed on a rigid wall passing over a fixed frictionless pulley at a distance of 2 m from the wall has a point mass M = 2 kg attached to it at a distance of 1 m, from the wall. A mass m = 0.5 kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley, and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when mass m is released?



4. One end of a spring of natural length h and spring constant k is fixed at the ground and the other is fitted with a smooth ring of mass m which is allowed to slide on a horizontal rod fixed at height h as shown in the figure. Initially, the spring makes an angle of 37^0 with the vertical when the system is released from rest. Find the speed of the ring when the spring becomes vertical.



5. An automobile of mass 'm' accelerates starting from rest, while the engine supplies constant power P. show that:
(a) the velocity is given as a function of time by $v = (2Pt/m)^{1/2}$
(b) the position is given as a function of time by $s = (8P/9m)^{1/2}t^{3/2}$.

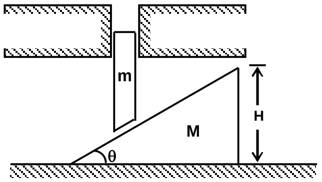
6. A stone with weight W is thrown vertically upwards into the air with initial speed v_0. If a constant force f due to air drag acts on the stone throughout its flight,
(a) show that the maximum height reached by the stone is $h = \dfrac{v_0^2}{2g\left[1+(f/W)\right]}$
(b) show that the speed of the stone upon impact with the ground is $v = v_0\left(\dfrac{W-f}{W+f}\right)^{1/2}$

7. A particle of mass m moves along a circle of radius R with a normal acceleration varying with time as $a_n = at^2$, where 'a' is a constant. Find the time dependence of the power developed by all the forces acting on the particle, and the mean value of this power averaged over the first t seconds after the beginning of motion.

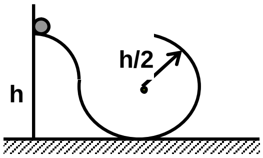
8. A rigid body of mass 400 kg is pulled vertically up by a light inextensible chain. Initially, the body is at rest and the pull on the chain is 60 kN. The pull gets smaller at the rate of 3.60 kN each meter through which it is raised. Find the speed of the body when it has been raised by 10 m.

9. A particle is hanging from a fixed point O by means of a string of length a. There is a small nail 'Q' in the same horizontal line with O at a distance b (b = a/3) from O. Find the minimum velocity with which the particle should be projected so that it may make a complete revolution around the nail without being slackened.

10. A rod of mass m and length ℓ is kept on a smooth wedge of mass M as shown in the figure. If the system is released when the rod is at the top of the wedge, find the speed of the wedge when the rod hits the ground level. Neglect friction between all surfaces in contact.

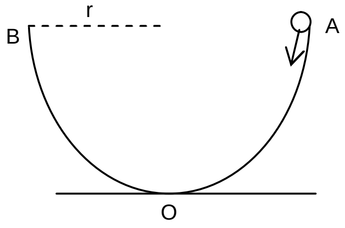


11. A small body starts sliding from the height 'h' down an inclined groove passing into a half circle of radius h/2. Find the speed of the body when it reaches the highest point.



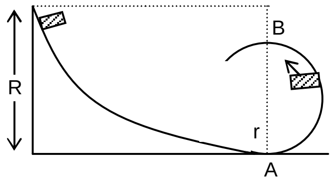
12. A smooth sphere of radius R is made to translate in a straight line with constant acceleration a. A particle kept on the top of sphere is released from there at zero velocity w.r.t. the sphere. Find the speed of the particle with respect to sphere as a function of angle θ as its slides on the spherical surface.

13 AOB is a smooth semicircular track of radius r. A block of mass m is given a velocity $\sqrt{2rg}$ parallel to track at point A. Calculate normal reaction between block and track when block reaches at point O.



14. An object dropped from a height h strikes the ground with a speed of k$\sqrt{gh}$ where k < $\sqrt{2}$. Calculate the work done by air–friction.

15. A small block of mass m slides along a frictionless loop inside loop track as shown in figure. Find the minimum ratio R/r so that the block may not lose contact at the highest point of inner loop.



Objective:

Level - I

1. Two bodies of masses m_1 and m_2 have equal momenta. Their kinetic energies E_1 and E_2 are in the ratio:
(A) $\sqrt{m_1} : \sqrt{m_2}$ (B) $m_1 : m_2$
(C) $m_2 : m_1$ (D) $m_1^2 : m_2^2$

2. A motor boat is travelling with a speed of 3.0 m/sec. If the force on it due to water flow is 500 N, the power of the boat is:
(A) 150 kW (B) 15 kW
(C) 1.5 kW (D) 150 W

3. A chain of mass m and length ℓ is placed on a table with one–sixth of it hanging freely from the table edge. The amount of work done to pull the chain on the table is:
(A) $mg\ell/4$ (B) $mg\ell/6$
(C) $mg\ell/72$ (D) $mg\ell/36$

4. A ball looses 15 % of its kinetic energy after it bounces back from a concrete slab. The speed with which one must throw it vertically down from a height of 12.4 m to have it bounce back to the same height is
(A) 2.5 m/s (B) 4.38 m/s
(C) 6.55 m/s (D) 8.25 m/s

5. A body is dropped from a certain height. When it has lost an amount of potential energy 'U', it subsequently acquires a velocity 'v'. The mass of the body is:
(A) $\frac{2U}{v^2}$ (B) $\frac{U}{2v^2}$
(C) $\frac{2v}{U}$ (D) $\frac{U^2}{2v}$

6. A man of mass 80 kg runs up a staircase in 15 s. Another man of same mass runs up the staircase in 20 s. The ratio of the power developed by them will be:
(A) 1 (B) 4/3
(C) 16/9 (D) None of the above

7. A particle moves with a velocity $\vec{v} = \left(5\hat{i} - 3\hat{j} + 6\hat{k}\right)$ m/s under the influence of a constant force $\vec{F} = \left(10\hat{i} + 10\hat{j} + 20\hat{k}\right)$ N . The instantaneous power delivered to the particle is:
(A) 200 J/s (B) 40 J/s
(C) 140 J/s (D) 170 J/s

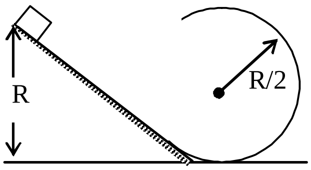
8. If v, P and E denote the velocity, momentum and kinetic energy of the particle, then:
(A) P = dE/dv (B) P = dE/dt
(C) P = dv/dt (D) none of these

9. Can a body have energy without having momentum?
(A) the body which has no momentum will have no energy
(B) the body which has no momentum, will have potential energy by virtue of its position.
(C) the body which has no momentum, may have no kinetic as well as potential energy.
(D) nothing can be said

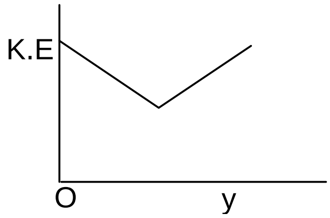
10. When a ball is thrown up, the magnitude of its momentum decreases and then increases. Does this violate the conservation of momentum? Answer by selecting the correct statement.
(A) The conservation of momentum is violated.
(B) The conservation of momentum is not violated. Here the system includes (ball + earth + air molecules) and the net momentum of the system remains constant.
(C) It depends upon how high the ball is thrown.
(D) Conservation of momentum is sometimes violated and sometimes not.

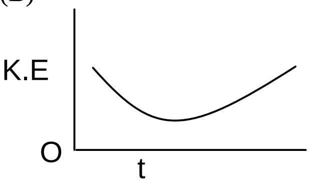
Multiple choice questions (More than one correct option)

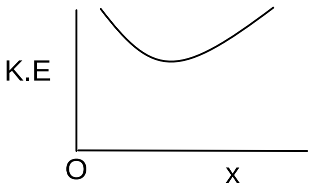
11. A block of mass m is released on the frictionless incline as shown from a height R. The incline turns into a circle of radius R/2. The maximum height attained by the block is
(A) less than R/2 (B) greater than R/2
(C) equal to R (D) less than R

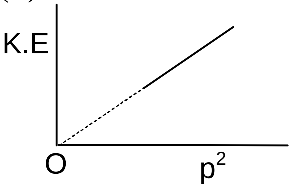


12. A heavy particle is projected up from a point at an angle with the horizontal. At any instant 't', if p = linear momentum, y = vertical displacement, x = horizontal displacement, then the kinetic energy of the particle plotted against these parameters can be

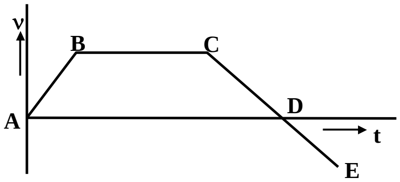
(A)


(B)


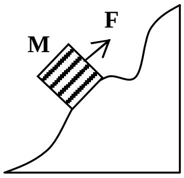
(C)


(D)


13. The plot of velocity versus time is shown in the figure. A varying force acts on the body. The correct statement(s) among the following is (are)
(A) In moving from A to B, work done on the body is negative.
(B) In moving from B to C no work is done on the body.
(C) In moving from C to D, work done by the force on the body is positive.
(D) In moving from D to E, work done by the force on the body is positive.



14. A body of mass M was slowly hauled up the rough hill by a force F which at each point was directed along tangent to the hill. Work done by the force:



(A) is independent of shape of trajectory
(B) depends upon vertical component of displacement but independent of the horizontal component
(C) depends upon both the components of displacement horizontal as well vertical.
(D) does not depend upon coefficient of friction

15. The potential energy U (in joule) of a particle of mass 1 kg moving in x–y plane obeys the law U = 3x + 4y, where (x, y) are the co–ordinates of the particle in metre. If the particle is at rest at (6, 4) at time t = 0, then:
(A) the particle has constant acceleration
(B) the particle has zero acceleration
(C) the speed of particle when it crosses the y–axis is 10 m/s
(D) coordinates of the particle at t = 1 sec are (4.5, 2)

True or False Type Questions

1. Total energy of a system is always conserved, no matter what internal and external forces on the body are present.
2. Work done in the motion of a body over a closed loop is zero for any force.
3. Work done in moving a body does not depend on how fast or slow the body is moved.
4. Work done by an individual force is zero if the work done by the resultant force is also zero.
5. Kinetic energy is an absolute value and it does not depend upon the frame of reference.

Fill in the Blanks

1. A ball whose K.E. is E, is thrown at an angle of 45° with the horizontal; its K.E. at the highest point of its flight will be …………
2. A mass M lowered with the help of a string by a distance x at a constant acceleration g/2. The work done by string is …………
3. Two bodies with masses M_1 and M_2 have equal kinetic energies. If P_1 and P_2 are their respective momenta, then P_1/P_2 is equal to …………
4. Work is said to be negative when angle between force and displacement lies between ………… and …………
5. Area under a ………… curve (taking due care of algebraic sign) gives work done by the force.

Level - II

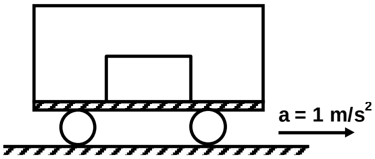
1. How much work is done in raising a stone of mass 5 kg and relative density 3 lying at the bed of a lake through a height of 3 metre? (Take g = 10 ms^{-2})
   (A) 25 J (B) 100 J
   (C) 75 J (D) None of the above

2. A body is acted upon by a force which is proportional to the distance covered. If distance covered is represented by s, then work done by the force will be proportional to
   (A) s (B) s^2
   (C) $\sqrt{s}$ (D) None of the above

3. A car is moving along a straight horizontal road with a speed v_0. If the coefficient of friction between the tyres and the road is μ, the shortest distance in which the car can be stopped is
   (A) $\frac{v_0^2}{2\mu g}$ (B) $\frac{v_0^2}{\mu g}$
   (C) $\frac{2v_0^2}{\mu g}$ (D) none of these

4. A block of mass m is released on top of a wedge, which is free to move on a horizontal surface. Neglecting friction between the surfaces in contact, which of the following statement is true ?
   (A) The kinetic energy of the block when it reaches the bottom of the wedge is mgH.
   (B) The kinetic energy of the wedge when the block reaches the bottom is mgH.
   (C) The work of normal reaction on the block in the ground reference is not zero.
   (D) The potential energy of the wedge continuously changes.

5. A rail road car is moving with a constant acceleration of 1 m/s^2. A block of 5 kg is put on a horizontal rough floor in the car. At time t = 0, velocity of the car is 5 m/s. Considering friction is sufficient and block is not slipping on the floor, the work done on the block by friction force during t = 0 to t = 2 sec will be (coefficient of friction is μ_s)



   (A) 60 J (B) –60J
   (C) 10 J (D) 600 μ_sJ

6. A particle of mass m moves in a conservative force field where the potential energy U varies with position coordinate x as $U = U_0(1 - \cos ax)$, U_0 and a are positive constants. Which of the following statement is true regarding its motion $(-\pi \le x \le \pi)$?
   (A) The acceleration is constant.
   (B) The kinetic energy is constant.
   (C) The acceleration is directed along the position vector.
   (D) The acceleration is directed opposite to the position vector.

7. A particle of mass m moves under the influence of the force $\vec{F} = a\,(\sin \omega t\,\hat{i} + \cos \omega t\,\hat{j})$, where a, ω are constants and t is time. The particle is initially at rest at the origin. The instantaneous power given to the particle is
   (A) zero
   (B) $a^2 \sin \omega t/ m\omega$
   (C) $a^2 \cos \omega t/ m\omega$
   (D) $a^2(\sin \omega t + \cos \omega t)/m\omega$

8. A horizontal massive platform is moving with a constant velocity v_0. At time t = 0, a small block of mass m is gently placed on the platform. If the coefficient of friction between the block and the platform is μ, the work done by the force of friction on the block in the fixed ground reference frame (from t = 0 to a sufficiently long time) is
   (A) $+\frac{1}{2}mv_0^2$
   (B) $-\frac{1}{2}mv_0^2$
   (C) $+\frac{1}{2}\mu mv_0^2$
   (D) zero

9. A block of mass m is moving with a constant acceleration 'a' on a rough horizontal plane. If the coefficient of friction between the block and ground is μ, the power delivered by the external agent after a time t from the beginning is equal to:
   (A) 2mat
   (B) μmgat
   (C) μm(a+μg)gt
   (D) m(a+μg)at

10. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to
   (A) $t^{1/2}$
   (B) $t^{3/4}$
   (C) $t^{3/2}$
   (D) t^2

More than one correct option.

11. Select the correct alternative(s).
   (A) Work done by the static friction is always zero.
   (B) Work done by kinetic friction can be positive also.
   (C) Kinetic energy of a system is always constant by applying external force on the system.
   (D) Work–energy theorem is valid in a non–inertial frame also.

12. A particle moves in a straight line with constant acceleration under a constant force F. Select the correct alternative (s).
   (A) Power developed by this force varies linearly with time.
   (B) Power developed by this force varies parabolically with time.
   (C) Power developed by this force varies linearly with displacement.
   (D) Power developed by this force varies parabolically with displacement.

13. In projectile motion, power of the gravitational force
   (A) is constant throughout
   (B) is negative for first half, zero at topmost point and positive for rest half
   (C) varies linearly with time
   (D) is positive for complete path

14. Kinetic energy of a particle moving in a straight line is proportional to the time t. The magnitude of the force acting on the particle is
    (A) directly proportional to the speed of the particle
    (B) inversely proportional to $\sqrt{t}$
    (C) inversely proportional to the speed of the particle
    (D) directly proportional to $\sqrt{t}$

15. A particle is taken from point A to point B under the influence of a force field. Now, it is taken back from B to A and it is observed that the work done in taking the particle from A to B is not equal to the work done in taking it from B to A. If W_{nc} and W_c are the work done by non–conservative and conservative forces, respectively, present in the system, ΔU is the change in potential energy and Δk is the change in kinetic energy, then
    (A) $W_{nc} - \Delta U = \Delta k$
    (B) $W_c = -\Delta U$
    (C) $W_{nc} + W_c = \Delta k$
    (D) $W_{nc} - \Delta U = -\Delta k$

True or False Type Questions

1. In circular motion, work done on a body is always zero.

2. Work done by frictional force may be positive, zero or negative.

3. Potential energy can be positive on negative but kinetic energy is never negative.

4. Potential energy of a spring given by relation $\frac{1}{2}kx^2$ is positive.

5. Area under the power–time graph gives the work done and the slope of the work–time curve represents the power at that instant.

Fill in the blanks

1. Kinetic energy ………… if work is negative and ………… if work is positive.

2. Work done by a constant force is ………… of the actual path followed.

3. Potential energy exist for………… forces and not for………… forces.

4. If different bodies have same linear momentum, the ………… body will have the maximum kinetic energy.

5. If the particle moves opposite to the conservative field, work done by the field will be …………and so change in potential energy will be …………, i.e. potential energy will…………

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. Zero, because the centripetal force needed to revolve the body is always perpendicular to the circular path.
2. Both will travel the same distance.
3. (R + ma) v
4. $\frac{mv_1^2}{t_1^2}t$
5. $\frac{\mu Mgd\cos\theta}{(\cos\theta+\mu\sin\theta)}$
6. $-\mu mgD$
7. $\frac{1}{4}K$
8. 50 m
9. Work done against friction along any closed path is non–zero.
10. $W_{AB}=K_B-K_A$
12. $-\frac{3}{4}mg\ell$
13. No, we can associate potential energy only with a conservative force.
14. The winning team is performing over loosing team.
15. 3.83 m/s
16. 0.025 J
17. The momentum of the system remains constant. Here, the system includes ball, earth and air molecules.
18. (a) 51.5 J, 29.7 J ; (b) 81.2 J
19. 3000 J
20. 32 watt

Level - I

1. Zero
2. (a + bd/2)d
3. mgvt $\sin^2\theta$
4. (a) FR (b) $\frac{\pi}{2}$ FR
5. (a) $-\frac{1}{2}\text{Mg a}\,t_0^2$ (b) $\frac{1}{2}\text{M(g + a)a}\,t_0^2$
6. $\frac{\rho A}{4}(h_1 - h_2)^2 g$
7. 0, mg(gt – usinθ)
8. $v = \sqrt{\frac{2(m_2 - \mu m_1)g\ell}{(m_1 + m_2)}}$
9. (a) 304 mJ, (b) –1.75 J, (c) 3.32 m/s, (d) 22.5 cm
10. (a) zero (b) zero
11. $v_B = \sqrt{2gh}\,\cos\theta$, $v_A = \sqrt{2gh}\,\sin\theta$

Level – II

1. 75 J
2. 48 W, 24 W
3. v = 1.84 m/s.
4. $\frac{h}{4}\sqrt{\frac{k}{m}}$
7. P = mRat , <P> = mRat/2
8. 43.58 m/s
9. $2\sqrt{ga}$
10. $\sqrt{\frac{2mgH}{M + m\tan^2\theta}}$
11. $\frac{2}{3}\sqrt{\frac{gh}{3}}$
12. $[2R(a\sin\theta + g - g\cos\theta)]^{1/2}$
13. 5mg
14. $-\left[1 - \frac{k^2}{2}\right]mgh$
15. $\frac{R}{r} = \frac{5}{2}$

Objective:

Level - I

1.	**C**	2.	**C**
3.	**C**	4.	**C**
5.	**A**	6.	**B**
7.	**C**	8.	**A**
9.	**B**	10	**B**
11.	**B, D**	12.	**B, C**
13.	**B, D**	14.	**A, C**
15.	**A, D**		

True or False Type Questions

1.	False	2.	False
3.	True	4.	False
5.	False		

Fill in the blanks

1.	E/2	2	$\frac{Mgx}{2}$
3.	$\sqrt{M_1};\sqrt{M_2}$	4.	$\pi/2, \pi$
5.	force displacement		

Level - II

1.	**B**	2.	**B**
3.	**A**	4.	**C**
5.	**A**	6.	**D**
7.	**A**	8.	**A**
9.	**D**	10.	**C**
11.	**B, D**	12.	**A, D**
13.	**B, C**	14.	**B, C**
15.	**A, B, C**		

True or False Type Questions

1.	False	2.	True
3.	True	4.	True
5.	True		

Fill in the blanks

1.	decreases, increases	2.	independent
3.	conservative, non conservative	4.	lightest.
5.	negative, positive, increase.		

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

CONSERVATION OF MOMENTUM

Conservation of Momentum

Syllabus for IITJEE and CBSE Board:
System of particles; Center of mass and its motion; Impulse; Elastic and Inelastic collisions; Variable mass system

CENTRE OF MASS

The centre of mass of an object is the single point that moves in the same way as a point mass having mass equal to the object would move when subjected to the same external forces that act on the object. That is, if the resultant force acting on an object (or system of objects) of mass m is $\vec{F}$, then the acceleration of the centre of mass of the object (or system) is given by $\vec{a}_{cm} = \frac{\vec{F}}{m}$.

If the system is considered to be composed of tiny masses m_1, m_2, m_3 and so on, at co-ordinates (x_1, y_1, z_1), (x_2, y_2, z_2) and so on, then the co-ordinates of the centre of mass are given by

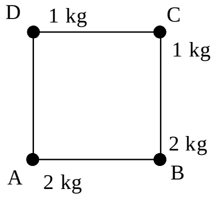
$$x_{cm} = \frac{\sum x_i m_i}{\sum m_i}, \quad y_{cm} = \frac{\sum y_i m_i}{\sum m_i}, \quad z_{cm} = \frac{\sum z_i m_i}{\sum m_i}$$

where the summations extend over all masses composing the object. In a uniform gravitational field, the centre of mass and the centre of gravity coincide.

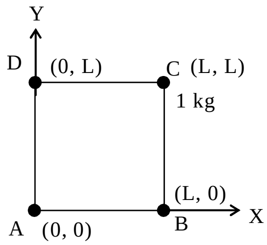
The position vector $\vec{r}_{cm}$ of the centre of mass can be expressed in terms of the position vectors $\vec{r}_1$, $\vec{r}_2$, . . of the particles as $\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...}{m_1 + m_2 + ...} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$

In statistical language, the centre of mass is a mass–weighted average of the particles.

Illustration 1. *Particles of masses 2 kg, 2 kg, 1 kg and 1 kg are placed at the corners A, B, C and D of a square of side L as shown in figure. Find the centre of mass of the system.*



Solution: If A is taken as origin, then



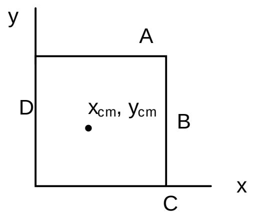
$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{2.0 + 2.L + 1.L + 1.0}{6} = \frac{1}{2}L$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{2.0 + 2.0 + 1.L + 1.L}{6} = \frac{L}{3}$$

Illustration 2. *There are four rods A, B, C and D of same length ℓ but different linear mass densities d, 2d, 3d & 4d, respectively. These are joined to form a square frame with sides C & D along x & y axes, respectively. Find coordinate of centre of mass of the structure.*



Solution: Let the coordinates of centre of mass of the structure be (x_{cm}, y_{cm})

$$x_{cm} = \frac{\ell d\ell/2 + \ell 2d\ell + \ell 3d\ell/2 + 0}{10\ell d} = \frac{4\ell}{10}$$

Similarly, $y_{cm} = \dfrac{\ell d\ell + \ell 2d\ell/2 + 0 + \ell 4d\ell/2}{10\ell d}$

$$y_{cm} = \frac{4\ell}{10}$$

Coordinates of centre of mass: $\left(\dfrac{4\ell}{10}, \dfrac{4\ell}{10}\right)$

Exercise 1.
(i) Must there necessarily be any mass at the centre of mass of a system?
(ii) Can a high jumper clear a height even if his center of mass does not clear the height?

The centre of mass after removal of a part of a body

If a portion of a body is taken out, the remaining portion may be considered as, original mass (M) – mass of the removed part (m)

= {original mass (M)} + {– mass of the removed part (m)}

The formula changes to:

$$X_{cm} = \frac{Mx - mx'}{M - m} \text{ and } Y_{cm} = \frac{My - my'}{M - m},$$

where primed ones represent the coordinates of the c.m. of the removed part.

Illustration 3. *Centre of mass is at point X (1, 1, 1) when system consists of particles of masses 2, 3, 4 and 5 kg. If the centre of mass shifts to point Y (2,2,2) on removal of the mass of 5 kg. what was its position.*

Solution: Let $\vec{r}_1, \vec{r}_2, \vec{r}_3$ and $\vec{r}_4$ are the position vectors of the particles of mass m_1 = 2 kg, m_2 = 3 kg, m_3 = 4 kg and m_4 = 5 kg respectively.

Initially, the centre of mass was at the point X(1,1,1), then

$$\hat{i} + \hat{j} + \hat{k} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4}{m_1 + m_2 + m_3 + m_4}$$

$$\Rightarrow \frac{2\vec{r}_1 + 3\vec{r}_2 + 4\vec{r}_3 + 5\vec{r}_4}{2+3+4+5} = \hat{i} + \hat{j} + \hat{k}$$

$$\Rightarrow 2\vec{r}_1 + 3\vec{r}_2 + 4\vec{r}_3 + 5\vec{r}_4 = 14\hat{i} + 14\hat{j} + 14\hat{k} \qquad \ldots (1)$$

On removing the mass m_4 = 5 kg, the centre of mass shifts to the point Y (2, 2, 2). Then

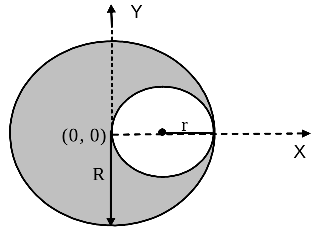
$$2\hat{i} + 2\hat{j} + 2\hat{k} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

$$\Rightarrow 2\vec{r}_1 + 3\vec{r}_2 + 4\vec{r}_3 = 18\hat{i} + 18\hat{j} + 18\hat{k} \qquad \ldots (2)$$

By subtracting the equation (2) from (1),

$$\vec{r}_4 = -\frac{4}{5}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

Illustration 4. *A thin homogeneous lamina is in the form of a circular disc of radius R. .From it a circular hole(exactly half the radius of the lamina and touching the lamina's circumference) is cut off. Find the centre of mass of the remaining part.*



Solution: Let the centre of the lamina be the origin. Due to symmetry, the c.m. will lie on the x–axis. Let M be the mass of the circular lamina. Then, mass m′ of the removed circular hole is

$$m' = \frac{M}{\pi R^2}\left(\pi r^2\right) = \frac{M}{R^2}\left(\frac{R}{2}\right)^2 = \frac{M}{4}$$

$$\therefore \qquad x_{cm} = \frac{M.0 - m'.r}{M - m'}$$

The negative sign of m′ denotes that it has been removed.

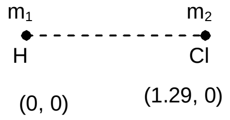
Thus, $x_{cm} = \dfrac{-\dfrac{M}{4}\dfrac{R}{2}}{M - \dfrac{M}{4}} = -\dfrac{R}{6}$ (the center of mass of the remaining part lies at a distance $\dfrac{R}{6}$, towards left of the origin i.e. the initial center of mass of the disc)

Illustration 5. *In the $HC\ell$ molecule, the separation between the nuclei of the two atoms is about 1.29Å. Find the approximate location of the centre of mass of the molecule, given that chlorine atoms is 35.5 times as massive as hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.*

Solution:

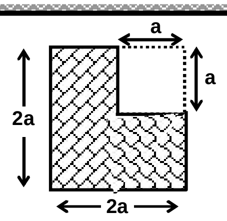
$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$x_{cm} = \frac{35.5 \times 1.29}{36.5} = 1.254\,\text{Å} \text{ from hydrogen atom.}$$



Note: **(i)** In the absence of external force, the position of the c.m. does not change in case of a stationary system.
(ii) In a particular coordinate system, if there is no net external force along a particular coordinate, then the centre of mass of the system will move with constant initial velocity along that direction.

Exercise 2. ***Find the centre of mass of the section. Consider the mass of the lamina to be uniformly distributed.***



Center of mass of a body having continuous distribution of mass

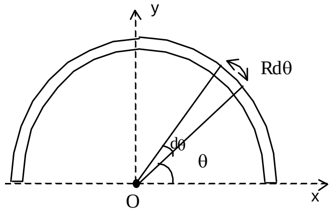
If the bodies given are not discrete and their distances are not specific, the centre of mass can be found out by taking an infinitesimal part of mass dm at a distance x, y and z from the origin of the chosen co–ordinate system.

$$X_{cm} = \frac{\int x\ dm}{\int dm}\ ; \quad Y_{cm} = \frac{\int y\ dm}{\int dm}\ ; \quad Z_{cm} = \frac{\int z\ dm}{\int dm}$$

In vector form, $\vec{r}_{cm} = \frac{\int \vec{r}\ dm}{\int dm}$.

Illustration 6. *Find the centre of mass of a uniform semi–circular ring of radius R and mass M.*

Solution: Consider the centre of the ring as origin. Consider a differential element of length $d\ell$ of the ring whose radius vector makes an angle θ with the x–axis. If the angle subtended by the length $d\ell$ is dθ at the centre, then $d\ell = R d\theta$.



Let λ be the mass per unit length.
Then, mass of this element is dm = λR dθ

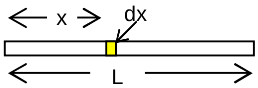
$$\therefore\ X_{cm} = \frac{1}{m}\int x\,dm = \frac{1}{m}\int_0^{\pi} R\cos\theta\lambda R\,d\theta = 0$$

and $Y_{cm} = \frac{1}{m}\int_o^{\pi}(R\sin\theta).\lambda R\,d\theta$

$$= \frac{\lambda R^2}{m}\int_o^{\pi}\sin\theta\,d\theta = \frac{\lambda R^2}{\lambda\pi R}\left[-\cos\theta\right]_o^{\pi} = \frac{2R}{\pi}$$

Illustration 7. *If the linear mass density of a rod of length L lying along x–axis and origin at one end varies as λ = A + Bx, where A and B are constants, find the coordinates of the centre of mass.*

Solution : As rod is kept along x–axis
Hence, $Y_{cm} = 0$ and $Z_{cm} = 0$
For x–coordinate



$$x_{cm} = \frac{\int_0^L x dm}{\int_0^L dm}$$

dm = λ. dx = (A + Bx) dx

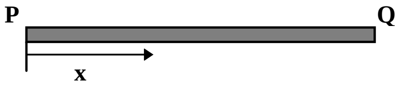
$$x_{cm} = \frac{\int_0^L x(A+Bx)dx}{\int_0^L (A+Bx)dx} = \frac{\frac{AL^2}{2}+\frac{BL^3}{3}}{AL+\frac{BL^2}{2}}$$

$$x_{cm} = \frac{L(3A+2BL)}{3(2A+BL)}$$

Hence, coordinates of the centre of mass are $\left(\frac{L(3A+2BL}{3(2A+BL)},\ 0,\ 0\right)$.

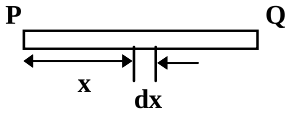
Illustration 8. *Linear mass density of a rod PQ of length l and mass m is varying with the distance x (from P), as* $\lambda = \frac{m}{2l}(1+ax)$

(i) Determine the value of a
(ii) also determine the distance of c.m. from the end P.



Solution: $dm = \lambda dx$

$$m = \int_0^l \frac{m}{2l}(1+ax)\,dx \quad \Rightarrow \quad a = 2/l$$



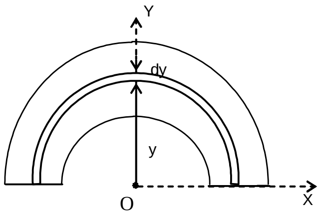
From P

$$x_{cm} = \frac{\int x\,dm}{m} = \frac{\int_0^l x\frac{m}{2l}\left(1+\frac{2x}{l}\right)dx}{m} = \frac{7l}{12}$$

Illustration 9. *Find the centre of mass of a thin, uniform disc of radius R from which a small concentric disc of radius r is cut.*

Solution: X_{cm} of the object is zero by symmetry. Y_{cm} can be calculated by considering a small element of mass dm and thickness dy at a distance of y from origin.

Then $\frac{M}{A}dA = \frac{2M}{\pi(R^2 - r^2)}\pi y\,dy$



$$\therefore\ Y_{cm} = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_r^R \frac{2y}{\pi}.\frac{2M}{\pi(R^2-r^2)}\pi\,y\,dy$$

On solving, we get $Y_{cm} = \frac{4}{3\pi}\left[\frac{R^3 - r^3}{R^2 - r^2}\right] = \frac{4(R^2+Rr+r^2)}{3\pi(R+r)}$.

Note: (i) If $R \to r$, then it becomes a wire, $Y_{cm} = \frac{2R}{\pi}$

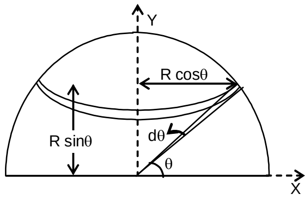
(ii) If r = 0, then it becomes a semicircular disc, $Y_{cm} = \frac{4R}{3\pi}$

Illustration 10. Find the centre of mass of a thin uniform hemispherical shell.

Solution: $X_{cm} = 0$ (by symmetry)

Y_{cm} can be calculated by considering a small element (taken by drawing two circles on its surface at angles θ and θ + dθ as shown).

$$\sigma = \frac{M}{2\pi R^2} = \frac{dm}{dA}$$



$$dm = \frac{M}{2\pi R^2} 2\pi R \cos\theta \, R \, d\theta = M \cos\theta \, d\theta$$

$$Y_{cm} = \frac{1}{M}\int y \, dm = \frac{1}{M}\int y M \cos\theta \, d\theta$$

$$= \int_0^{\pi/2} R \sin\theta \cos\theta \, d\theta \qquad (\because \quad y = R \sin\theta)$$

On solving, we get

$$Y_{CM} = \frac{R}{2}$$

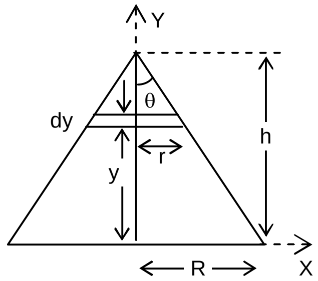
$\therefore$ C.M. of thin uniform hemispherical shell is $(0, \frac{R}{2})$

Illustration 11. *Find the centre of mass of a uniform solid cone.*

Solution: Let us consider a uniform solid cone of mass M, radius R and height h.

$X_{cm} = 0$ (by symmetry)

Let us consider a small element (disc) of mass dm, radius r and thickness dy at a distance y the from base as shown.



Then, $\rho = \frac{3M}{\pi R^2 h} = \frac{dm}{\pi r^2 dy}$

$$dm = \frac{3Mr^2}{R^2 h} dy$$

$$Y_{CM} = \frac{1}{M}\int y \, dm = \frac{1}{M}\int y \frac{3Mr^2}{R^2 h} dy = \frac{3}{R^2 h}\int y r^2 dy$$

$$= \frac{3}{h}\int_0^h y\left(1 - \frac{y}{h}\right)^2 dy \qquad \left[\because \frac{h-y}{r} = \frac{h}{R} \Rightarrow r = \left(1 - \frac{y}{h}\right) R\right]$$

On solving, we get $Y_{CM} = \frac{h}{4}$

$\therefore$ C.M. of a uniform solid cone is $(0, \frac{h}{4})$ from the centre of base.

Velocity of the centre of mass of a system of particles

Position vector of the centre of mass of a system of particle is given by

$$\vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n}$$

Differentiating both sides with respect to time, we obtain

$$\frac{d\vec{R}_{cm}}{dt} = \frac{m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + ... + m_n\frac{d\vec{r}_n}{dt}}{m_1 + m_2 + m_3 + ... + m_n}$$

$\therefore$ The velocity of the center of mass of the system is given by,

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n}{m_1 + m_2 + ... + m_n}$$

Differentiating both sides with respect to time, we obtain

$$\frac{d\vec{v}_{cm}}{dt} = \frac{m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} + ... + m_n \frac{d\vec{v}_n}{dt}}{m_1 + m_2 + ... + m_n}$$

$\therefore$ Acceleration of the c.m. of the system of particles,

$$\Rightarrow \vec{a}_{cm} = \frac{\vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n}{M} = \frac{\sum \vec{F}_{ext}}{M} \quad \Rightarrow \quad \vec{a}_{c.m.} = \frac{\sum \vec{F}_{ext}}{M}$$

If $\sum \vec{F}_{ext} = 0$, $\vec{a}_{cm} = 0$

Note: **(i)** In the absence of external force, the position of the c.m. does not change in case of a stationary system.
(ii) In a particular coordinate system, if there is no net external force along a particular coordinate then the centre of mass of system will move with constant initial velocity along that direction.

Exercise 3. ***Can a sailboat be propelled by air blown at the sails from a fan attached to the boat ?***

Illustration 12. *A dog of mass 10 kg chases a rabbit (running with a speed of 7 km/hr and having mass 2 kg) with a speed of 13 km/hr along a straight line. Find the speed of the centre of mass of the two.*

Solution: Velocity of centre of mass, $v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

$$= \frac{10 \times 13 + 2 \times 7}{10 + 2} = 12 \text{ km/hr}$$

CONSERVATION OF LINEAR MOMENTUM OF THE SYSTEM OF PARTICLES

Since $m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} + ... + m_n \frac{d\vec{v}_n}{dt} = M\vec{a}_{cm}$

$$\Rightarrow \frac{d}{dt}\left(m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n\right) = \sum \vec{F}_{ext}$$

$$\Rightarrow \frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2 + ... + \vec{p}_n\right) = \sum \vec{F}_{ext}$$

where $\vec{p}_1, \vec{p}_2, ..., \vec{p}_n$ are the linear momenta of the particles.

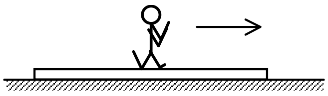
If $\sum \vec{F}_{ext} = 0$,

$\vec{p}_1 + \vec{p}_2 + ... + \vec{p}_n$ = constant.

Hence, in the absence of any external force, the total momentum of a system of particles remains constant. This is valid along a particular direction also.

Exercise 4.
(i) ***You are marooned on a big frictionless surface with your school bag in you hand. How will you get out of it?***
(ii) ***When a ball is thrown up, the magnitude of its momentum first decreases and then increases. Does this violate conservation of momentum principle?***

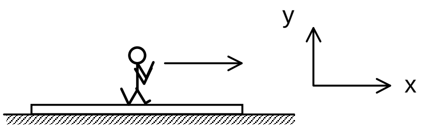
Illustration 13. *A man of mass m is standing on a stationary wooden board of mass M kept on smooth ice. The man starts running on the board and acquires a speed u relative to the board. Find the speed of the man relative to the stationary observer. The board is long enough.*



Solution: The system is initially at rest. Hence, its momentum is zero. Since no net external force is acting on the system along the direction in which the man is running, i.e. x–axis. Therefore, momentum of the system will remain zero.

Given velocity of the man with respect to board, $\vec{v}_{mb} = u\hat{i}$

Let velocity of the board, $\vec{v}_b = v_b\hat{i}$



Hence, velocity of the man,

$\vec{v}_m = \vec{v}_{mb} + \vec{v}_b$

$= u\hat{i} + v_b\hat{i} = (u + v_b)\hat{i}$

$M\vec{v}_b + m\vec{v}_m = 0$

$M v_b\hat{i} + m(u + v_b)\hat{i} = 0 \quad \Rightarrow \quad v_b = -\dfrac{mu}{m+M}$

Velocity of the man, $\vec{v}_m = \left(u - \dfrac{mu}{m+M}\right)\hat{i}$, $\quad \vec{v}_m = \dfrac{Mu}{m+M}\hat{i}$

Illustration14. *A gun is mounted on a stationary rail road car. The mass of the car, the gun, the shells and the operator is 50m, where m is the mass of one shell. Two shells are fired one after the other along same horizontal line in same direction. If the muzzle velocity (velocity with respect to gun) of the shells is u m/s, then find the speed of the car after second shot.*

Solution: Let $\vec{u}$ be the muzzle velocity of the shell and $\vec{v}$ be the velocity of the car after first shot.

$p_i = 0$ (1) [initial momentum of the system]

$p_f = 49m\vec{v} + m(\vec{u} + \vec{v})$ (2) [final momentum of the system]

$\because \; p_i = p_f \Rightarrow \quad 50m\vec{v} + m\vec{u} = 0$

$\Rightarrow \quad \vec{v} = -\dfrac{\vec{u}}{50}$ (3)

Negative sign shows that $\vec{u}$ and $\vec{v}$ are oppositely directed.

For the second shot: Before second shot, momentum of the system is

$p'_i = 49\,m\vec{v}$ (4)

If $\vec{v}$ be the velocity of the car after second shot, then

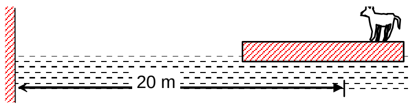
$p'_f = 48m\vec{v}' + m(\vec{u} + \vec{v}')$ (5)

From COM, we get $\quad 49\vec{v}' + \vec{u} = 49\vec{v}$

$\Rightarrow \quad 49\vec{v}' = 49\vec{v} - \vec{u} = -\dfrac{49}{50}\vec{u} - \vec{u} \quad$ as $\quad \vec{v} = -\dfrac{\vec{u}}{50}$

$\Rightarrow \quad \vec{v}' = -\vec{u}\left(\dfrac{1}{50} + \dfrac{1}{49}\right)$

Illustration 15. *A dog of mass 10 kg is standing on a flat boat so that it is 20 meters from the shore. It walks 8 m on the boat towards the shore and then stops. The mass of the boat is 40 kg and friction between the boat and the water surface is negligible. How far is the dog from the shore now?*



Solution: Take boat and dog as a system. Initially, centre of mass of the system is at rest. Since no external force is acting on the system, hence centre of mass of the system will remain stationary.

Let initial distance of the centre of mass of the boat from the shore be x m. Hence,

$$x_{1c.m.} = \frac{40\times x + 10\times 20}{40+10}\,m \qquad . . . (i)$$

Here $x_{1.cm.}$ = distance of the c.m. of the system from the shore. Since dog moves towards the shore and centre of mass of the system is to be at rest, therefore boat has to move away from the shore. Let distance moved by the boat be x′, then

$$x_{2c.m.} = \frac{40(x+x')+10(20-8+x')}{40+10} \qquad \because \qquad x_{1c.m.} = x_{2c.m.}$$

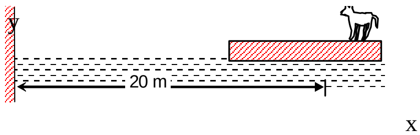
$$\Rightarrow \quad \frac{40x+200}{50} = \frac{40(x+x')+10(12+x')}{50}$$

$\Rightarrow$ 50x′ = 80 $\Rightarrow$ x′ = 1.6 m.

Hence distance of the dog from the shore is (20 – 8 + 1.6) m = 13.6 m

Alternatively

As there is no external force acting on the system (dog + boat) along the x–axis and initially centre of mass of the system is at rest, it means that the motion of the dog will not change.



x–coordinate of the centre of mass of the system

i.e. $$\Delta\vec{X}_{cm} = \frac{m_d\Delta\vec{x}_d + m_b\Delta\vec{x}_b}{m_b+m_d} = 0 \qquad . . . (i)$$

Let the displacement of boat is $s\hat{i}$.

The displacement of the dog with respect to boat, $\Delta\vec{r}_{d,\,b} = -\ell\hat{i}$

The displacement of the dog,

$$\Delta\vec{r}_d = \Delta\vec{r}_{d,\,b} + \Delta\vec{r}_b = -\ell\hat{i} + s i = (-\ell + s)\hat{i}$$

From (i), $m_d(-\ell + s)\hat{i} + m_b s\hat{i} = 0$

$$\Rightarrow \quad s = \frac{m_d\ell}{(m_d+m_b)} = \frac{10\times 8}{(10+40)} = 1.6\text{ m}$$

Hence, position of dog from the shore

$$\vec{L}' = \vec{L} + \Delta\vec{r}_d = 20\,\hat{i} + (-8+1.6)\hat{i} = 13.6\,\hat{i}.$$

COLLISION

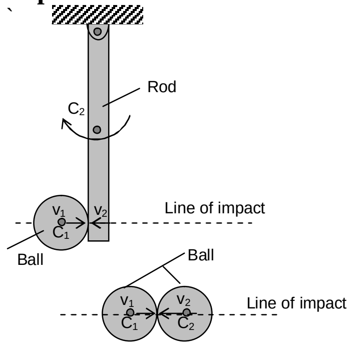
Collision is said to occur if two bodies interact strongly for very short interval of time.

Types of Collision:

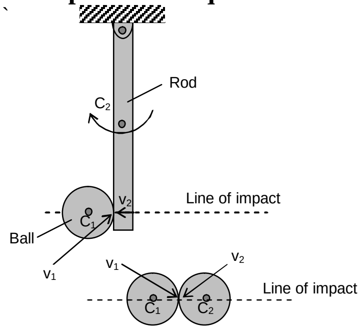
1. ***Direct collision:*** When the velocities at the point of contact coincide with the common normal of the two surfaces at point of contact (line of impact).

2. ***Oblique collision:*** When the velocities at the point of contact do not coincide with the line of impact.

Examples of Direct Collision



Examples of Oblique Collision



Newton's Experimental law:

Coefficient of restitution 'e'

$$= -\frac{\text{relative velocity of points of contact along the common normal after impact}}{\text{relative velocity of points of contact along the common normal before impact}}$$

Collisions are divided into two types:

Elastic collision

$e = 1$ $\Rightarrow$ The kinetic energy of the system is conserved.

Perfectly Inelastic collision

$e = 0$ $\Rightarrow$ Relative velocity of the points of contact along the line of impact (common normal) after impact $= 0$

$0 < e < 1$ (for partially inelastic collision).

Velocities of colliding bodies after collision

Let there be two bodies with masses m_1 and m_2 moving with velocities u_1 and u_2. They collide at an instant and acquire velocities v_1 and v_2 after collision. Let the coefficient of restitution of the colliding bodies be e. Then, applying Newton's experimental law and the law of conservation of momentum, we can find the value of velocities v_1 and v_2.

Conserving momentum of the colliding bodies before and after the collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{. . . (i)}$$

Applying Newton's experimental law

We have $\frac{v_2 - v_1}{u_2 - u_1} = -e$

$$v_2 = v_1 - e\,(u_2 - u_1) \quad \text{. . . . (ii)}$$

Putting (ii) in (i), we obtain

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2\,\{v_1 - e\,(u_2 - u_1)\}$$

$$v_1 = u_1\frac{(m_1 - em_2)}{m_1+m_2} + u_2\frac{m_2(1+e)}{m_1+m_2} \qquad \ldots \text{(iii)}$$

From (ii) $v_2 = v_1 - e\,(u_2 - u_1)$

$$= u_1\frac{m_1(1+e)}{(m_1+m_2)} + u_2\frac{(m_2 - m_1 e)}{m_1+m_2} \qquad \ldots \text{(iv)}$$

When the collision is elastic; e = 1.

Finally, $v_1 = \left(\frac{m_1 - m_2}{m_1+m_2}\right)u_1 + \left(\frac{2m_2}{m_1+m_2}\right)u_2$

and $v_2 = \left(\frac{2m_1}{m_1+m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1+m_2}\right)u_2$

Cases:

(i) **If $m_1 = m_2$**

$v_1 = u_2$ and $v_2 = u_1$

When the two bodies of equal mass collide head-on elastically, their velocities are mutually exchanged.

(ii) **If $m_1 = m_2$ and $u_2 = 0$, then**

$v_1 = 0$, $v_2 = u_1$

(iii) **If target particle is massive;**

(a) $m_2 \gg m_1$ and $u_2 = 0$

$v_1 = -u_1$ and $v_2 = 0$

The light particle recoils with same speed while the heavy target remains practically at rest.

(b) $m_2 \gg m_1$ and $u_2 \neq 0$

$v_1 \approx -u_1 + 2u_2$ and $v_2 \approx u_2$

(iv) **If particle is massive : $m_1 \gg m_2$**

$v_1 = u_1$ and $v_2 = 2u_1 - u_2$

If the target is initially at rest, $u_2 = 0$

$v_1 = u_1$ and $v_2 = 2u_1$

The motion of heavy particle is unaffected, while the light target moves apart at a speed twice that of the particle.

(v) **When collision is perfectly inelastic, e = 0**

$$v_1 = v_2 = \frac{u_1\, m_1}{m_1+m_2} + \frac{u_2\, m_2}{m_1+m_2} = \frac{m_1u_1 + m_2u_2}{m_1+m_2}$$

Exercise 5.

(i) ***Is it possible to have a collision in which all the kinetic energy is lost? If so, cite an example.***

(ii) ***Two objects collide and one is initially at rest. (a) Is it possible for both to be at rest after collision? (b) Is it possible for one of them to be at rest after collision? Explain.***

(iii) ***A ball of mass m_1 experienced a perfectly elastic collision with a stationary ball of mass m_2. What fraction of the K.E. does the striking ball lose, if***

(a) it recoils at right angle to the original direction of motion, and

(b) the collision is head - on one.

(iv) ***Two particles A and B, with different but unknown masses, collide. A is initially at rest while B has speed v. After collision, B has a speed v/2 and moves at right angles to its original direction of motion. Find the direction in which A moves after collision. Can you determine the speed of A from the information given ? Explain.***

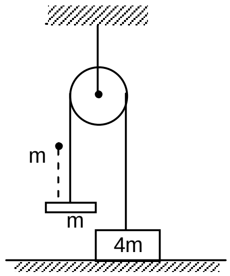
IMPULSE

If a very large force acts on a body for a very short interval of time, then product of the force and the time is known as impulse.

$$\vec{I} = \int \vec{F}dt = \int \frac{d\vec{P}}{dt}dt = \Delta\vec{P}$$

Hence, impulse is equal to the change in linear momentum $\vec{P}$.

Illustration 16. *An inextensible string is passing over a light frictionless pulley. One end of the string is connected to a block of mass 4m which is resting on a horizontal surface. To another end a plate of mass m is attached and is hanging in the air. A particle of mass m strikes from above with vertical velocity v_0 and sticks to the plate. Calculate maximum height attained by block of mass 4m.*



Solution: For 4m,

$\int Tdt = 4mv$ … (i)

For plate + particle

$-\int Tdt = 2mv - mv_0$ … (ii)

from (i) and (ii)

$v = \frac{v_0}{6}$

After collision,

For acceleration

$T - 2mg = 2ma$

$4mg - T = 4ma \quad \Rightarrow 2mg = 6ma$

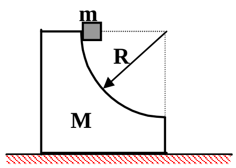
$\Rightarrow a = g / 6.$

At maximum height velocity is zero.

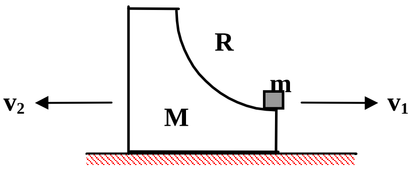
$\Rightarrow 0 - \left(\frac{v_0}{6}\right)^2 = -2(a/6)s$

$\Rightarrow S = \frac{v_0^2}{12g}.$

Illustration 17. *A small cube of mass m slides down a circular path of radius R cut into a larger block of mass M, as shown in the figure. M rests on a table, and both blocks move without friction. The blocks are initially at rest, and m starts from the top of the path. Find the velocity v of the cube as it leaves the block.*



Solution: Let v_1 and v_2 be the final velocities of the small and large blocks with respect to the ground.



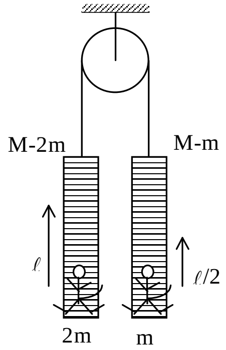
From momentum conservation: $mv_1 = Mv_2$

From energy conservation:

$$mgR = \frac{1}{2}Mv_2^2 + \frac{1}{2}mv_1^2$$

After solving, $v_1 = \sqrt{\frac{2MgR}{M+m}}$

Illustration 18. *Two ladders are hanging from ends of a light rope passing over a light and smooth pulley. A monkey of mass 2m hangs near the bottom of one ladder whose mass is M–2m. Another monkey of mass m hangs near the bottom of the other ladder whose mass is M–m. The monkey of mass 2m moves up a distance ℓ with respect to the ladder. The monkey of mass m moves up a distance $\ell/2$ with respect to the ladder.*
Does the centre of mass of the system change. If so, then calculate its value.



Solution:

$$y_{cm\,1} = \frac{2m.y_1 + (M-2m)y_2 + my_3 + (M-m)y_4}{2M}$$

Let the left ladder moves downward a distance y_0 with respect to ground. Then

$$y_{cm\,2} = \frac{2m.(y_1 - y_0 + \ell) + (M-2m)(y_2 - y_0) + m(y_3 + y_0 + \ell/2) + (M-m)(y_4 + y_0)}{2M}$$

$$\Delta y_{cm} = y_{cm2} - y_{cm1} = \frac{2m\ell + m\ell/2}{2M} = \frac{5m\ell}{4M}.$$

Illustration 19. *A shell of mass 3m is moving horizontally through the air with a velocity 'u'. An internal explosion causes it to separate into two parts of masses m and 2m which continue to move horizontally in the same vertical plane. If the explosion generates an additional energy of 12 mu^2, prove that the two fragments separate with the relative speed 6u.*

Solution: Let u_1 and u_2 be the velocities of m and 2m in the direction of u

By conservation of momentum

$3mu = mu_1 + 2mu_2$

or $3u = u_1 + 2u_2$ . . . (i)

Additional energy generated = $k_f - k_i = \frac{1}{2}mu_1^2 + \frac{1}{2}2m\,u_2^2 - \frac{1}{2} \times 3mu^2$

$= 12\ mu^2$ (given)

or $u_1^2 + 2u_2^2 = 27u^2$ . . . (ii)

$(3u - 2u_2)^2 + 2\,u_2^2 = 27u^2$

$u_2^2 - 2uu_2 - 3u^2 = 0$

$(u_2 - u)^2 = 4u^2$ or $u_2 = u \pm 2u = 3u$ or $-u$

Substituting the value of u_2 in (i)

$u_1 = -3u$ or $5u$

The values $u_1 = -3u$ and $u_2 = +3u$ are not acceptable because the two masses being unequal cannot have equal velocities.

$\therefore$ $u_1 = 5u$ and $u_2 = -u$

Relative velocity $= 5u - (-u) = 6u$.

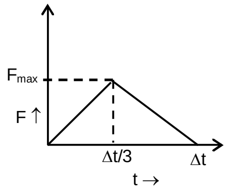
Illustration 20. *A 2 kg ball drops on floor from a height of 20 m and rebounds with 20% of the initial speed. Find the impulse received by the ball.*

Solution: $\vec{J} + \vec{P}_i = \vec{P}_f \qquad \Rightarrow \vec{J} = \vec{P}_f - \vec{P}_i$

$\vec{J} = m\left[v_2 - (-v_1)\right] = m[2v_1 + v_1]$

$\vec{J} = 2 \times 0.2\sqrt{2 \times 20 \times 10} = 48$ kg m/s

Illustration 21. *A body of mass m moving with speed v makes an elastic one dimensional collision with a stationary body of same mass. They are in contact for very small time interval Δt. The contact force between them varies as shown in graph. Find the magnitude of F_{max}.*



Solution: Impulse = area under f–t graph

Also, impulse = change in momentum

$\Rightarrow mv = \dfrac{F_{max}\Delta t}{2}$

$\Rightarrow \quad F_{max} = \dfrac{2mv}{\Delta t}$

Illustration 22. *A ball of mass m is initially moving with speed 5 m/s towards a ball of mass 4 m which is initially at rest. Initially, the distance between the balls is 2.5 m. Find the average impulsive force acting for initial 4 m of journey, considering collision to be completely inelastic.*

Solution: Using conservation of momentum,

5m = mv + 4mv

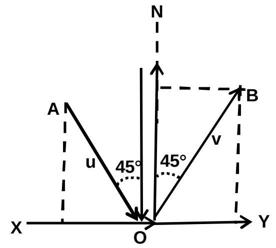
v = 1 m/s

Total time of journey $= \dfrac{2.5}{5} + \dfrac{1.5}{1}$

$F_{av} = \dfrac{\Delta P}{\Delta t} = \dfrac{4}{2.} = 2$ newton

Illustration 23. *A batsman deflects a ball by an angle of 90° without changing its initial speed, which is equal to 54 km/hr. What is the impulse to the ball? Mass of the ball is 0.15 kg.*

Solution: Let the point O represent the position of the bat. Draw a line XY through the point O and another line ON normal to the line XY. The ball of mass M initially moving along path AO with speed u is deflected by the batsman along OB (without change in speed of the ball), such that AOB = 90° the ∠AON = ∠BON = 45°.



Then initial momentum of the ball can be written as,

(i) Mu cos 45° along NO and

(ii) Mu sin 45° along XY

Also, the final momentum of the ball can be resolved into the following components

(i) Mv cos 45° along ON and

(ii) Mv sin 45° along XY

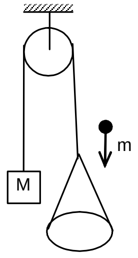
The component of the momentum of the ball along XY remains unchanged (both in magnitude and direction). However, the components of the momentum of the ball along normal are equal in magnitude but opposite in direction. Since the impulse imparted by the batsman to the ball is equal to the change in momentum of the ball along normal.

Normal Impulse $= Mv \cos 45° - (-Mu \cos 45°)$

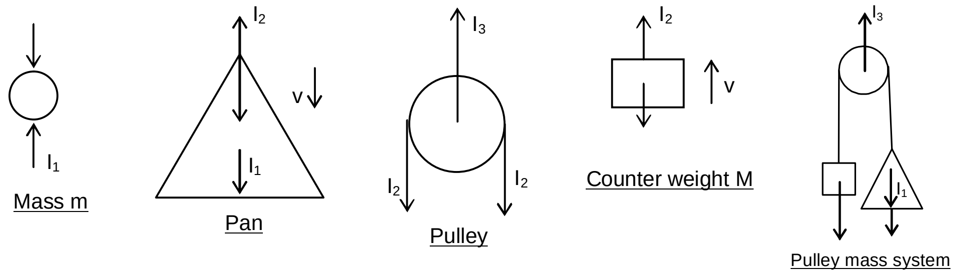
$= 2Mu \cos 45°$

Therefore, Impulse $= 2 \times 0.15 \times 15 \times \cos 45° = 3.18$ kgm/s

Illustration 24. *A pan being connected by a string passing over a pulley counter balances a block of mass M. A mass m falls on the pan at rest from a height h from it and sticks to it. What is the speed of the block and the pan soon after the mass hits the pan? Find the impulsive tension in the string:*
(A) connecting the pan and M.
(B) connecting the pulley and the ceiling.



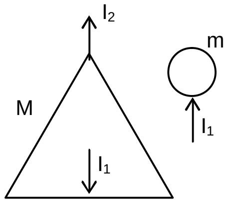
Solution: During the impact, the impulses of the external forces are shown in the respective FBD.



- For each FBD, the impulse momentum theorem says
  Σ (Impulses) = (Change in momentum)
- During the impact, weights of the objects are negligible as compared to the impulsive forces.
- Linear momentum cannot be conserved just before and just after the collision because an external impulse acts on the pulley and mass system.

Consider the FBD of particle of mass m:

$mv_0 - I_1 = mv$ . . . (i)



As the particle has stuck to the pan, thus velocity of particle and pan will be same.

$I_1 - I_2 = MV$ . . . (ii)

and from the constraints of the string

$I_2 = MV$ . . . (iii)

$v_0 = \sqrt{2gh}$ . . . (iv)

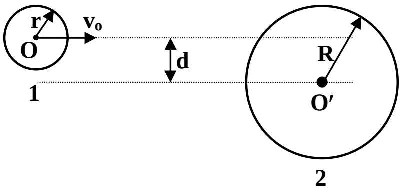
By solving (i), (ii), (iii) and (iv)

$$V = \frac{mv_0}{(m+2M)} = \frac{m\sqrt{2gh}}{m+2m}$$

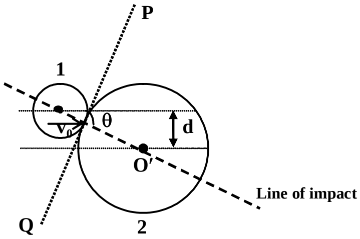
Impulsive tension in the string connecting the pulley and the ceiling

$$= I_3 = 2I_2 = \frac{2Mm\sqrt{2gh}}{m+2M}.$$

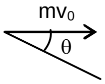
Illustration 25. *A disc of mass m is moving with constant speed v_0 on a smooth horizontal table. Another disc of mass M is placed on the table at rest as shown in the figure .If the collision is elastic, find the velocity of the discs after collision. Both discs lie on the horizontal plane of the table.*

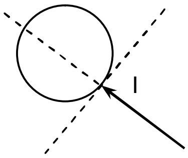


Solution: $\sin\theta = \dfrac{d}{r+R}$

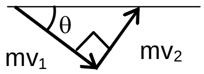


For mass m:
Momentum before impact

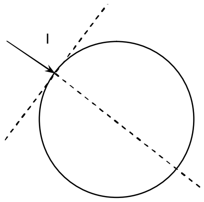


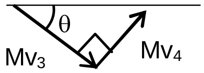


Momentum after impact



For mass M: Momentum before impact = 0
Momentum after impact





Net impulse on the system = 0
$\therefore$ Momentum of the system is conserved.

$mv_0\cos\theta + 0 = mv_1 + Mv_3$ . . . (i)

$mv_0\sin\theta + 0 = mv_2 + Mv_4$ . . . (ii)

Individually for mass m and M, the impulse along the tangent = 0

$\therefore$ For mass m, $mv_0\sin\theta = mv_2$ . . . (iii)

For mass M, $0 = Mv_4$ . . . (iv)

As the collision is elastic

$$e = -\frac{v_3 - v_1}{0 - v_0\cos\theta} = 1$$

$\therefore$ $v_3 - v_1 = v_0\cos\theta$ . . . (v)

Solving equations (i) through (v)

$v_2 = v_0\sin\theta$, $v_4 = 0$

$$v_3 = \frac{2mv_0\cos\theta}{m+M},\quad v_1 = \left(\frac{m-M}{m+M}\right)v_0\cos\theta$$

It can be verified that, $\frac{1}{2}mv_0^2 = \frac{1}{2}m(v_1^2+v_2^2) + \frac{1}{2}M(v_3^2+v_4^2)$

$\Rightarrow$ The kinetic energy is conserved in an elastic impact.

Particular case: If m = M

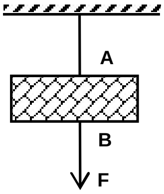
$$v_1 = 0,\ v_2 = v_0 \sin\theta$$

$$v_3 = v_0 \cos\theta,\ \ v_4 = 0$$

Note: The impulse along the common tangent during an impact may not be always negligible. For example, when normal reaction is impulsive and the effect of friction is to be considered.

Exercise 6.

(i) ***A heavy book is suspended with a length of thread. Another piece of same thread is tied to the lower end of the book as shown in the figure.***



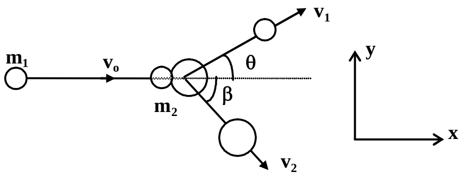
(a) The lower thread is pulled gradually, harder and harder in downward direction, so as to apply force on the upper thread. Which of the threads will break? Explain.

(b) In case the lower thread is pulled with a sharp jerk, will the same thread break as in case (a). Explain.

(ii) ***Two men, each of mass m, stand on the edge of a stationary car and jump off with a horizontal velocity u relative to the car, first simultaneously and then one after the other. If friction be negligible, in which case will they impart greater velocity to the car?***

Elastic Collision In Two Dimensions

A particle of mass m_1 moving along x–axis with speed v_o makes elastic collision with another stationary particle of mass m_2. After collision, the particles move in the directions shown in the figure with speeds v_1 and v_2.



Initial momentum of the system is, $\vec{P}_i = m_1 v_o \hat{i}$

After collision, momentum of the system is,

$$\vec{P}_f = \left(m_1 v_1 \cos\theta + m_2 v_2 \cos\beta\right)\hat{i} + \left(m_1 v_1 \sin\theta - m_2 v_2 \sin\beta\right)\hat{j}$$

Conserving momentum we get,

$$m_1 v_o = m_1 v_1 \cos\theta + m_2 v_2 \cos\beta \qquad \ldots (1)$$

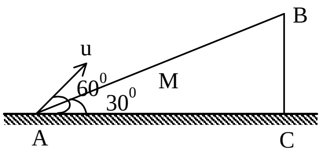
And $$m_1 v_1 \sin\theta - m_2 v_2 \sin\beta = 0 \qquad \ldots (2)$$

From conservation of K.E.

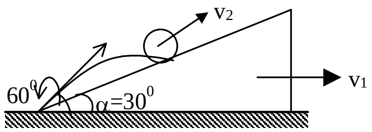
$$\frac{1}{2} m v_0^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \qquad \ldots (3)$$

Knowing the value of θ or β, we can get the values of v_1 and v_2 by solving the above equations.

Illustration 26. *A wedge of mass M = 9 kg is situated on a smooth horizontal plane. At t = 0 a shell of mass 4 kg is projected with velocity $10\sqrt{3}\ ms^{-1}$ from the point A situated at ground near foot of the wedge, at an angle 60^0 with horizontal (figure). At t = 1.5 sec the shell collide inelastically with the inclined surface of the wedge and slides up along the plane. Find the velocity of the wedge just after collision.*



Solution: At t = 1.5 sec vertical component of velocity of the shell $u'_y = u_y - gt = 10\sqrt{3} \times \frac{\sqrt{3}}{2} - 10 \times 1.5 = 0$



Hence the shell collides with the wedge while it is moving horizontally. Let velocity of wedge be v_1 and velocity of ball after collision be v_2 (with respect to wedge).

Since there is no horizontal external force acting on the (wedge + shell) system, conserving momentum of the system in horizontal direction.

$Mv_1 + m(v_1 + v_2 \cos\alpha) = mu'$ . .. .(i)

where u′ is the velocity with which the shell collides with the wedge.

Since the force applied by the wedge on the shell during collision is acting along the normal to the inclined plane, therefore momentum of the shell along the inclined plane will be conserved.

$m(v_2 + v_1 \cos\alpha) = mu' \cos\alpha$

$v_2 = (u' - v_1)\cos\alpha$ . . . (ii)

Substituting the value of v_2 from (ii) in (i), we get

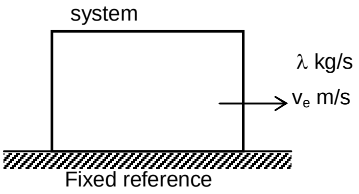
$$v_1 = \frac{mu'\sin^2\alpha}{M + m\sin^2\alpha}$$

$u' = u\cos\theta = 10\sqrt{3}\cos 60^0 = 5\sqrt{3}$ m/s

$$v_1 = \frac{4 \times 5\sqrt{3} \times \sin^2 30^0}{9 + 4\sin^2 30^0} = \frac{\sqrt{3}}{2}\ \text{ms}^{-1}$$

Momentum principle for variable mass stationary system

Consider a stationary system from which mass is being ejected at the rate of λ kg/second with a velocity of v_e m/s **relative to the system.**



The element of mass ejected in time dt = λ dt kg.

Momentum of this mass before ejection = 0

Momentum after ejection = $(\lambda dt)v_e \rightarrow$

∴ Change in momentum = $(\lambda dt)v_e \rightarrow$

∴ Rate of change of momentum $= \frac{(\lambda dt)v_e}{dt} = \lambda v_e \rightarrow$

From Newton's second law, the force exerted on the ejected mass $= \lambda v_e \rightarrow$

∴ The force exerted **by** the ejected mass **on** the system = $\lambda v_e \leftarrow$ $\left(\frac{kg}{s} \times \frac{m}{s} = \text{Newton}\right)$

If m is the instantaneous mass of the system, then $\frac{dm}{dt} = -\lambda$

Rocket equations

At any instant of time, momentum of the mass (λdt) relative to the ground before ejection = $\lambda dt\, V\hat{j}$

After ejection = $(\lambda dt)(-v_e\hat{j} + V\hat{j})$

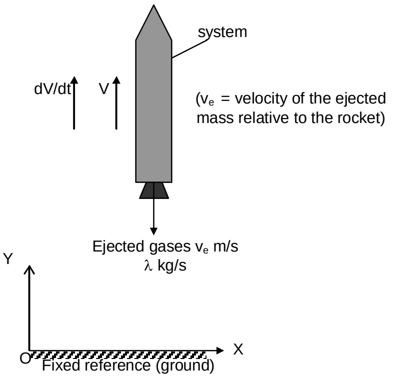
$\therefore$ Change in momentum = $-(\lambda dt)v_e\hat{j}$

$\therefore$ Rate of change of momentum of the ejected mass = $-\lambda v_e\hat{j}$

$\therefore$ Force exerted on the ejected mass = $-\lambda v_e\hat{j}$

Force exerted on the system (Rocket), i.e.

Thrust $F = \lambda v_e\hat{j}$



Newton's Second Law for the Rocket:

$$F - mg = m\frac{dV}{dt}$$

$$\lambda v_e - mg = m\frac{dV}{dt}$$

$$dV = \frac{\lambda v_e}{m}dt - gdt$$

Let m be the instantaneous mass and m_0 be the initial mass of the rocket

$\Rightarrow$ $m = m_0 - \lambda t$

$\Rightarrow$ $\therefore \int_0^V dV = \lambda v_e \int_0^t \frac{dt}{m_0 - \lambda t} - \int_0^t g dt$

Assuming g to be uniform

$$V = ve \ln\frac{m_0}{m_0 - \lambda t} - gt$$

$$= v_e \ln\frac{m_0}{m} - gt$$

Ignoring the effect of gravity, the instantaneous velocity and acceleration of the rocket (attributed only to the thrust by the ejected mass) are

$$V = v_e \ln\frac{m_0}{m} \quad \ldots \text{(i)}$$

$$\frac{dV}{dt} = \frac{\lambda v_e}{m} \quad \ldots \text{(ii)}$$

where $m = m_0 - \lambda t$ . . . (iii)

$\therefore$ Thrust $F = \lambda v_e$ . . . (iv)

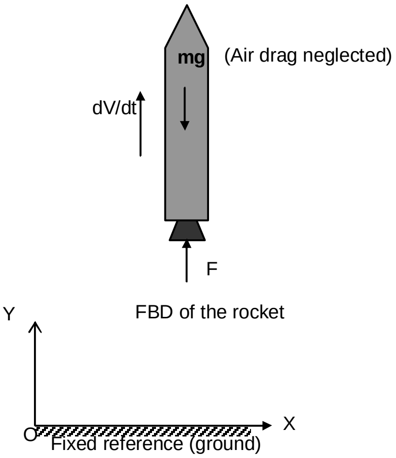
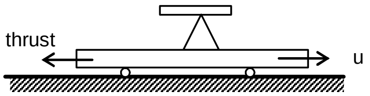


Illustration 27. *A machine gun is mounted on a car on a smooth horizontal surface. The mass of the system (car + gun) is M. The gun starts firing bullets of mass m along horizontal with speed u relative to stationary observer at the rate of μ/s. Find the acceleration with which it starts moving. (M $\gg$ m)*

Solution: Rate of mass ejected

$\lambda = m\mu$ kg/s

$v_e = u$ m/s



$\therefore$ Thrust on the machine gun in the horizontal direction

$= \lambda v_e = m\mu u$

$\therefore$ Instantaneous acceleration $= \dfrac{m\mu u}{M} \leftarrow$.

Illustration 28. *A rocket is in outer space, far from any planet. When the rocket engine is turned on, in the first second of firing, the rocket ejects (1/120) of its mass with a relative speed of 2400 m/s. What is the rocket's initial acceleration?*

Solution: m_0 = initial mass of the rocket

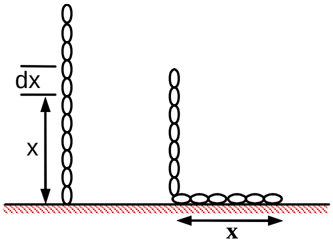
Rate of ejection $= \lambda = \dfrac{(m_0/120)\text{kg}}{1 \ \text{s}} = \dfrac{m_0}{120}$ kg/s

$v_e = 2400$ m/s

Initial acceleration $= \dfrac{\lambda v_e}{m_0} = \dfrac{(m_0/120)(2400)}{m_0} = 20$ m/s^2.

Illustration 29. *A uniform chain of mass m and length ℓ hangs by a thread and just touches the surface of a table by its lower end. Find the force exerted by the chain on the surface when half of its length has fallen on the table. Assume that the fallen part does not form heap and immediate comes to rest after collision.*

Solution: Consider an element of the chain of length dx at a height x from the surface. This element will hit the surface with speed $v = \sqrt{2gx}$



Mass of the element is

$dm = \lambda dx$, where $\lambda = m/\ell$

Hence, momentum of this element before collision is

$p_i = dm.v$ (downward)

Final momentum of the element is $p_f = 0$.

Therefore, change in momentum is given by

$$dp = p_f - p_i = -\,dm.v = -\lambda\, v dx \ \Rightarrow \frac{dp}{dt} = -\lambda v\frac{dx}{dt} = -\lambda v^2 \left(\text{as } \frac{dx}{dt} = v\right)$$

or $F = \lambda v^2 = 2\lambda gx$

F is the upward force exerted by the surface on the chain.

Hence, total force on the surface by the chain is given by

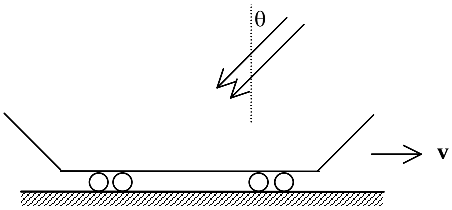
F' = F + weight of the fallen part

$= 2\lambda xg + \lambda\, xg \quad \Rightarrow \quad 3\lambda gx = 3mgx/\ell$.

When $x = \dfrac{\ell}{2}$,

$$F' = \frac{3mg}{\ell}.\frac{\ell}{2} = \frac{3mg}{2}$$

Illustration 30. *A freight car is moving on a smooth horizontal track. Rain is falling with a velocity u m/s at an angle θ with the vertical. Rain drops are collected in the car at the rate of μ kg/s. If the initial mass of the car is m_0 and velocity v_0, then find its velocity after time t.*



Solution: Instantaneous mass of the car at time t,

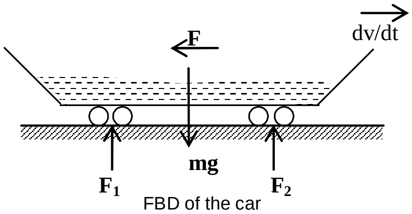
$m = m_0 + \mu t$.

Let v be the instantaneous velocity of the car.

Change in horizontal component of velocity of the rain drops

$= v - (-u \sin \theta)$

$= v + u \sin \theta \rightarrow$



FBD of the car

The rate of change of momentum (horizontal) of rain drops

$= (v + u \sin \theta)$ m/s $\times \mu$ kg/s $\rightarrow$

Force exerted by the rain drops on the car

$F = \mu(v + u \sin \theta)$ N $\leftarrow$

Newton's Second law in horizontal direction

$$-F = m\frac{dv}{dt} = (m_0 + \mu t)\frac{dv}{dt}$$

$$\int_{v_0}^{v} \frac{dv}{(v + u \sin \theta)} = -\int_0^t \frac{\mu dt}{m_0 + \mu t}$$

$$v = \frac{m_0(v_0 + u \sin \theta)}{m_0 + \mu t} - u \sin \theta$$

MISCELLANEOUS EXERCISE

1. Is linear momentum of a system always conserved ?

2. "The collision between two hydrogen atoms is perfectly elastic, so the momentum is conserved". Do you agree with this statement.

3. A ball of mass m moving with velocity v strikes head on elastically with a number of balls of the same mass in a line. Only one ball from other side moves with the same velocity. Explain why not two balls move simultaneously each with velocity v/2.

4. Is it possible to have a collision in which the whole of kinetic energy is lost.

5. If two objects collide and one is initially at rest. Is it possible for any one to be at rest after collision ?

6. A shot fired from a cannon explodes in air. What will be the momentum and the kinetic energy ?

7. Can kinetic energy of a system be changed without changing its momentum ?

8. Can momentum of a system be changed without changing its kinetic energy ?

9. Can a body have momentum when its energy is negative ?

10. A moving ball of mass m undergoes a head on collision with another stationary ball of mass 2m. What will be the fraction of total energy lost during collision ?

ANSWERS TO MISCELLANEOUS EXERCISE

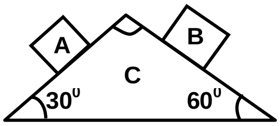
1. No, only when system is isolated.
2. No, because momentum is conserved even when the collision is inelastic.
3. Because momentum is conserved but kinetic energy not conserved.
4. Yes
5. Yes
6. Linear momentum is conserved while kinetic energy increases.
7. Yes
8. Yes
9. Yes, when KE < U
10. 8/9

SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob 1. *Two blocks A and B of equal mass are released on two sides of a fixed wedge C as shown in the figure. Find out the acceleration of centre of mass.*



Sol. Acceleration of block A = g sin 30° = g/2

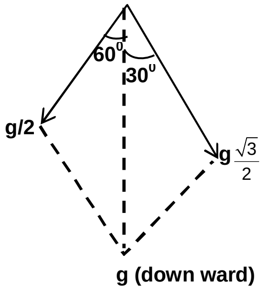
Block B = g sin 60° = $g\sqrt{3}/2$

Angle between $\vec{a}_1$ and $\vec{a}_2 = 90^0$

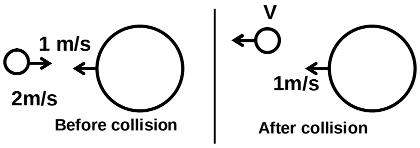
$$\vec{a}_{cm} = \frac{m_A\vec{a}_1 + m_B\vec{a}_2}{m_A + m_B}$$

Here, $m_A = m_B$

$$\therefore \quad \vec{a}_{cm} = \frac{\vec{a}_1 + \vec{a}_2}{2} = g/2 \text{ downward}$$



Prob 2. *A light ball is moving with a speed of 2 m/s toward the heavy ball which is moving with speed 1 m/s. Assuming collision to be perfectly elastic, find the speed of the lighter ball immediately after collision*



Sol. Separation speed =e approach speed

$v – 1 = 1(2+1)$

$v = 4$m/s

Prob 3. *A truck weighing 8000 kg is moving along a track with negligible friction at 1.8 m/s with the engine turn off when it begins to rain hard. The raindrops fall vertically with respect to the ground. Calculate the speed of the truck when it has collected 1000 kg of rain.*

Sol. As the system (truck + raindrops) is free from external horizontal force, so the momentum along the horizontal is conserved.

Therefore, $8000 \times 1.8 = 9000\,v'$

$\Rightarrow \quad v' = 1.6\,m/s$

Prob 4. *A 100 km/h wind blows normally against one wall of a house with an area of 50 m^2. Calculate the force exerted on the wall if the air moves parallel to the wall after striking it and has a density of 1.134 kg/m^3.*

Sol. Change in velocity of air along the normal to the wall

$\Rightarrow \quad v – u = v$

Amount of air striking the wall per second = $A \times v$, where A = area of wall

Mass of air striking the wall per second $= A \times v \times \rho$

where ρ = density of air

∴Change in momentum per second = (A vρ)v $= \rho A v^2$

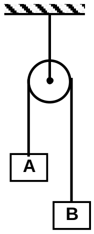
∴ Force exerted by the air on the wall

$$= \rho A v^2$$

$$= 1.134 \times 50 \times \left(\frac{100 \times 1000}{3600}\right)^2$$

$$= 4.375 \times 10^4 \text{ N}$$

Prob 5. *In the arrangement shown in the figure, m_A = 1 kg and m_B = 2kg. the string is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.*



Sol. Net pulling force on the system $= (m_B - m_A)g = (2-1)g = g$

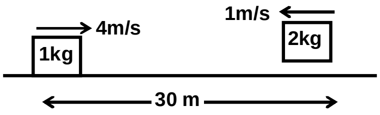
Total mass $m_A + m_B = 2 + 1 = 3$ kg

$$a = \frac{\text{Net pull force}}{\text{Total mass}} = \frac{g}{3}$$

Now, $\vec{a}_{com} = \frac{m_A \vec{a}_A + m_B \vec{a}_B}{1+2} = \frac{g}{9}$ (downward)

IITJEE TYPE

Prob 6. *Two particles of masses 1 kg and 2 kg respectively are 30 m apart. At time t = 0, they start moving towards each other with speeds of 4 m/s and 1 m/s. Find the displacement of their centre of mass between time t = 0 and t = 1.5 s.*

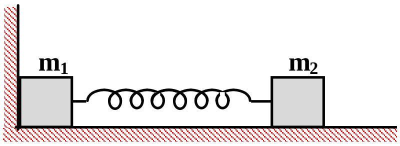


Sol. $V_{cm} = m_1V_1 + m_2V_2 / (m_1 + m_2) = (2/3)$ m/s

Displacement of C.M. = $V_{C.M} \times t$

$= \frac{2}{3} \times 1.5 = 1$m

Prob 7. *Two blocks of masses m_1 and m_2 connected by a light spring of stiffness k rest on a smooth horizontal plane, such that the block m_1 touches a vertical wall as shown in the figure. The block m_2 is shifted a small distance x towards the wall and then released. Find the velocity of the centre of mass of the system after block m_1 breaks off the wall.*



Sol. External force acting on the system is the normal reaction of the wall until the spring attains its natural length. Since work done by the normal reaction is zero hence total energy of the system will be conserved. At the moment when spring acquires its natural length velocity of the block m_1 is zero and velocity of the block m_2 is v_2 (say).

Hence conserving energy we get

$$\frac{1}{2}kx^2 = 0 + \frac{1}{2}m_2v_2^2 \qquad \Rightarrow v_2 = \sqrt{\frac{k}{m_2}}\,x$$

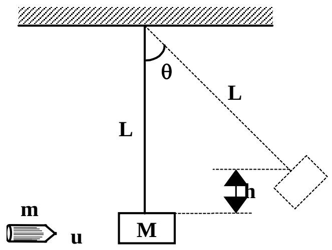
$\therefore \quad v_{c.m.} = \dfrac{m_1v_1 + m_2v_2}{m_1+m_2} = \dfrac{m_1 \times 0 + m_2\sqrt{k/m_2}\,x}{m_1+m_2} = \dfrac{\sqrt{km_2}\;x}{m_1+m_2}$

Prob 8. *A bullet of mass m moving with a horizontal velocity u strikes a stationary block of mass M suspended by a string of length L. The bullet gets embedded in the block. What is the maximum angle made by the string after impact.*

Sol. Let V be the combined velocity of the (bullet + block) system just after collision. Then, by conservation of linear momentum.

$mu = (m + M)V \quad (1)$

After collision, the KE of the (bullet + block) system gets converted into potential energy of the system.



$\therefore \quad \frac{1}{2}(m+M)V^2 = (m+M)gh$

Putting this value in equation (1), $mu = (m+M)\sqrt{2gh}$

or $\quad h = \dfrac{u^2}{2g}\left[\dfrac{m}{m+M}\right]^2$

From the figure, $\cos\theta = \dfrac{L-h}{L} = 1 - \dfrac{h}{L}$

$\therefore \quad \theta = \cos^{-1}\left[1 - \dfrac{u^2}{2gL}\left(\dfrac{m}{m+M}\right)^2\right]$

Prob 9. *A particle of mass 4m, which is at rest explodes into three fragments. Two of the fragments each of mass m are found to move with a speed v each in mutually perpendicular directions. Calculate*
(a) momentum of the third fragment of mass 2m after explosion.
(b) the energy released in the process of explosion.

Sol. The force of explosion is internal and system is initially at rest.

(a) Let the velocities be of the first two fragments be $v\,\hat{i}$ and $v\,\hat{j}$ and that of the fragment having mass 2m be $v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$

Hence,

$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$ where $\vec{p}_1, \vec{p}_2, \vec{p}_3$ are momenta

$\Rightarrow \quad mv\hat{i} + mv\hat{j} + 2m\left(v_1\hat{i} + v_2\hat{j} + v_3\hat{k}\right) = 0$

$\Rightarrow \quad \left(v + 2v_1\right)\hat{i} + \left(v + 2v_2\right)\hat{j} + v_3\hat{k} = 0$

$\Rightarrow \quad v_1 = -v/2, \quad v_2 = -v/2 \qquad$ and $v_3 = 0$

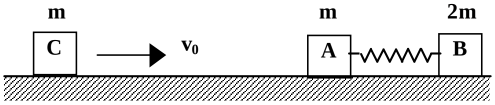
$\Rightarrow \quad \vec{p}_3 = -2m\left(\dfrac{v}{2}\hat{i} + \dfrac{v}{2}\hat{j}\right) = -\,(mv\hat{i} + mv\hat{j})$

(b) $K_f = \dfrac{1}{2}mv^2 + \dfrac{1}{2}mv^2 + \dfrac{1}{2}2m(v_1^2 + v_2^2 + v_3^2) = \dfrac{3}{2}mv^2$

Energy released in the explosion is

$\Delta E = K_f - K_i = = \dfrac{3}{2}mv^2$

Prob 10. *Two bodies A and B of masses m and 2m, respectively are placed on a smooth floor. They are connected by a light spring of stiffness k and are initially at rest. A third body C of mass m moving with velocity v_0 along the line joining A and B collides elastically with A. If l_o be the natural length of the spring, then find the minimum separation between the blocks A and B.*



Sol. Initially, there will be collision between C and A, which is elastic. Therefore, by using momentum conservation

$$mv_0 = mv_A + mv_C \quad \text{or} \quad v_0 = v_A + v_C$$

Since e = 1, $v_0 = v_A - v_C$

Solving the above two equations, $v_A = v_0$ and $v_C = 0$.

Now, A will move and compress the spring which in turn will accelerate B and retard A. Finally both A and B will compress the spring and at maximum compression, their relative velocity will be zero.

Since net external force is zero, therefore momentum of the system (A and B) is conserved.

Hence, $mv_0 = (m + 2m)v \Rightarrow v = v_0/3$

If x_0 is the maximum compression, then using energy conservation,

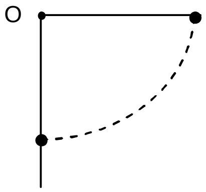
$$\frac{1}{2}mv_0^2 = \frac{1}{2}(m+2m)v^2 + \frac{1}{2}kx_0^2$$

$$\Rightarrow \quad \frac{1}{2}mv_0^2 = \frac{1}{2}(3m)\frac{v_0^2}{9} + \frac{1}{2}kx_0^2$$

$$\Rightarrow \quad x_o = v_0\sqrt{\frac{2m}{3k}}$$

Hence minimum distance $D = \ell_0 - x_0 = \ell_0 - v_0\sqrt{\frac{2m}{3k}}$.

Prob 11. *A small steel ball is suspended by a light inextensible string of length ℓ from a fixed point O. When the ball is in equilibrium, it just touches a vertical wall as shown in the figure. The ball is first taken aside such that the string becomes horizontal and then released from rest. If coefficient of restitution is e, then find the maximum deflection of the string after n^{th} collision.*



Sol. Let v_0 be the speed of the ball just before first collision and v_1 be the speed just after first collision. From the definition of the coefficient of restitution,

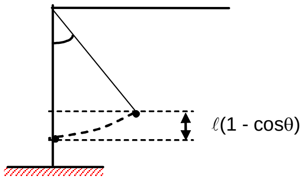
$$v_1 = ev_0 \qquad \text{(i)}$$

Just before second collision, speed of the ball will be v_1 only. If v_2 be the speed just after second collision, then

$$v_2 = ev_1 = e^2v_0 \qquad \text{(ii)}$$

Similarly, $V_n = e^n v_0$ (iii)

If after n^{th} collision maximum deflection of the string from vertical is θ, then from COE, we get

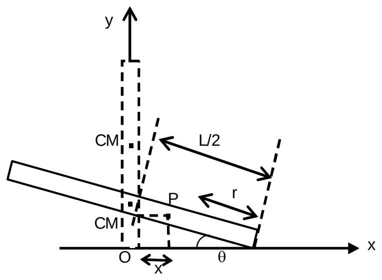


$$\frac{1}{2}mv_n^2 = mgl(1-\cos\theta)$$

$$\Rightarrow \quad e^{2n}v_0^2 = 2gl(1-\cos\theta) \qquad \text{(iv)}$$

Also, from COE, $v_o^2 = 2gl$ (v)
From (iv) and (v), we get $\theta = \cos^{-1}(1 - e^{2n})$

Prob 12. *A uniform thin rod of mass M and length L is standing vertically along the y–axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance causes the lower end to slip on the smooth surface along the positive x–axis, and the rod starts falling.*
(a) What is the path followed by the centre of mass of the rod during its fall.
(b) Find the equation of trajectory of a point on the rod located at a distance r from the lower end. What is the shape of the path of this point?



Sol. (a) As the floor is frictionless, no horizontal force acts on the rod. Thus centre of mass will have vertical acceleration. As it is initially at rest, it will move vertically downwards along y-axis, i.e. x = 0.

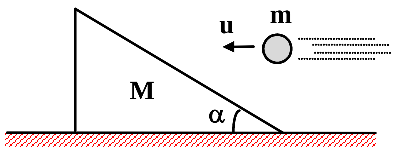
(b) At any instant, for a point P on the rod as shown in the figure

$x = [(L/2) - r]\cos\theta$ and $y = r\sin\theta$

So, eliminating θ between these we get trajectory of the point P as

$$\frac{x^2}{\left[(L/2)-r\right]^2} + \frac{y^2}{r^2} = 1$$. which is an ellipse.

Prob 13. *A wedge of mass M rests on a horizontal surface. The inclination of the wedge is α. A ball of mass m moving horizontally with speed u hits the inclined face of the wedge inelastically and after hitting slides up the inclined face of the wedge. Find the velocity of the wedge just after collision. Neglect any friction.*

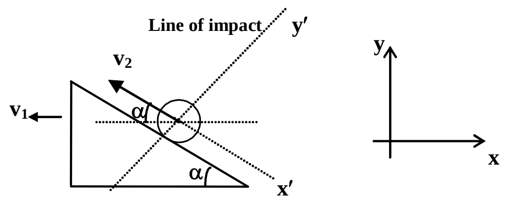


Sol. Let velocity of the block after collision be $\vec{v}_1$ and that of the ball $\vec{v}_2$ with respect to the wedge in directions as shown in the figure. As the net impulse on the system in the horizontal direction = 0.

Conserving momentum along horizontal, we get

$mu = m[v_2\cos\alpha + v_1] + Mv_1$

$\Rightarrow mu = mv_2\cos\alpha + (M + m)v_1$ . . . (i)



Since common normal is along y′, therefore momentum of the ball remains constant along the incline, i.e. along X′ ($\because$ $\vec{F}_{x'} = 0$ for the ball).

$\Rightarrow \quad u \cos\alpha = v_2 + v_1 \cos\alpha$

$\Rightarrow \quad v_2 = (u - v_1)\cos\alpha \qquad \ldots \text{(ii)}$

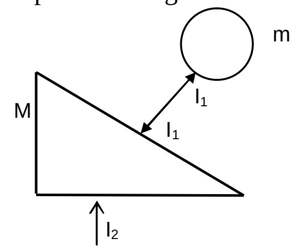
From equations (i) and (ii), we get

$$mu = mu\cos^2\alpha - mv_1\cos^2\alpha + (M + m)v_1$$

$$\Rightarrow \quad v_1 = \frac{mu\sin^2\alpha}{M + m\sin^2\alpha}.$$

Note: e = 0 is satisfied for perfectly inelastic collision.

Impulses during the collision



Prob 14. *Two identical buggies move one after the other due to inertia (without friction) with the same velocity $\vec{v}_0$. A man of mass m rides the rear buggy. At a certain moment, the man jumps into the front buggy with a velocity $\vec{u}$ relative to his buggy. Knowing that the mass of each buggy is equal to M, find the velocity with which the buggies will move after the jump.*

Sol. Conserving momentum for the rear buggy

$$(M + m)\,\vec{v}_0 = m[\vec{u} + \vec{v}_r] + M\vec{v}_r \qquad \ldots(1)$$

$\vec{v}_r$ = velocity of rear buggy finally. The velocity of the man with respect to ground,

$$\vec{v}_{\text{man, ground}} = \vec{v}_{\text{man, buggy}} + \vec{v}_{\text{buggy, ground}}$$

$$\Rightarrow \quad \vec{v}_{\text{man, ground}} = \vec{u} + \vec{v}_r \quad \Rightarrow \quad \vec{v}_r = \vec{v}_0 - \frac{m\vec{u}}{M+m}$$

Conserving momentum for front buggy and solving (1), we get,

$$M\vec{v}_o + m\left[\vec{u} + \vec{v}_r\right] = (M + m)\,\vec{v}_f$$

Here $\vec{v}_f$ = velocity of front buggy. Finally,

$$\vec{v}_r = \vec{v}_0 - \frac{m\vec{u}}{(M+m)} \Rightarrow M\vec{v}_0 + m\vec{u} + m\vec{v}_0 - \frac{m^2\vec{u}}{M+m} = (M+m)\vec{v}_f$$

$$\Rightarrow \quad (M+m)\vec{v}_0 + \frac{Mm\vec{u}}{M+m} = (M+m)\vec{v}_f$$

$$\Rightarrow \quad \vec{v}_f = \vec{v}_0 + \frac{Mm\vec{u}}{(M+m)^2}$$

Prob 15. *A cart is moving along +x direction with a velocity of 4 m/s. A person on the cart throws a stone with a velocity of 6 m/s relative to himself. In the frame of reference of the cart, the stone is thrown in y–z plane making an angle of 30° with vertical z-axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from the branch of tree by means of a string of length L. A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine:*

(a) the speed of the combined mass immediately after the collision with respect to an observer on the ground,

(b) the length L of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass.

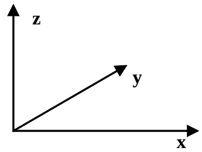
Sol. $V_{cx} = 4$ m/s; $V_{scy} = 3$ m/s; $V_{scz} = 3\sqrt{3}$ m/s; $V_{scx} = 0$

$V_{sy} = V_{scy} + V_{cy} = 3$ m/s

$V_{sz} = V_{scz} + V_{sz} = 3\sqrt{3}$ m/s

$V_{sx} = V_{scx} + V_{cx} = 0 + 4 = 4$ m/s.

$\Rightarrow$Velocity at the highest point of trajectory $= \sqrt{V_{sy}^2 + V_{sx}^2} = 5\ \text{m/s}$



Common velocity of stone and block just after the collision

$= \frac{mv}{2m} = 2.5\ m/s = V'$ (say)

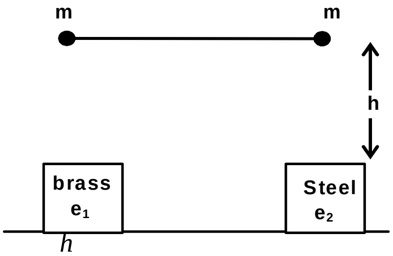
For the tension to be zero when string becomes horizontal,

$$V' = \sqrt{2gL} \Rightarrow V'^2 = 2gL$$

$$\Rightarrow \quad L = \frac{V'^2}{2g} = \frac{(2.5)^2}{19.6} = \frac{6.25}{19.6} = 0.3189\,m$$

Objective:

Prob 1. *A dumbell consisting of two spherical masses, each of mass m, which are connected by a light rigid rod of length ℓ, falls through a height h on two pads of equal height, one steel and the other brass. The co–efficients of restitution are e_1 and e_2 ($e_1 < e_2$). To what maximum height will the centre of mass of the dumbell rise after bouncing off the pads?*



(A) $\dfrac{h}{e_1+e_2}$ (B) $(e_1+e_2)^2\dfrac{h}{4}$

(C) $\dfrac{e_1^2+e_2^2}{4}$ (D) $\dfrac{4h}{e_1^2+e_2^2}$

Sol. The centre of mass of the system after collision will move up with velocity

$$=\frac{(e_1+e_2)\ \sqrt{2gh}}{2}$$

$$\Rightarrow 0=\left[\frac{(e_1+e_2)}{2}\sqrt{2\ \ gh}\right]^2-2gh'$$

$$h'=\frac{(e_1+e_2)^2\ \ h}{4}$$, Hence, (B) is the correct choice

Prob 2. *Two blocks of masses m_1 and m_2 interconnected with a spring of stiffness K are kept on a smooth horizontal surface. Which of the following ratios is/are correct, when the spring is extended and released?*

(A) $\dfrac{F_1}{F_2}=1$ and $\dfrac{X_1}{X_2}=1$ (B) $\dfrac{v_1}{v_2}=\dfrac{m_2}{m_1}$ and $\dfrac{KE_1}{KE_2}=\dfrac{m_2}{m_1}$

(C) $\dfrac{p_1}{p_2}=1$ and $\dfrac{KE_1}{KE_2}=\dfrac{m_1}{m_2}$ (D) $\dfrac{X_1}{X_2}=\dfrac{m_2}{m_1}$ and $\dfrac{a_1}{a_2}=\dfrac{m_2}{m_1}$

where X = displacement, v = speed, p = momentum, a = acceleration (all relative to the surface), KE = kinetic energy, F = force and suffixes 1 and 2 are used for bodies of masses m_1 and m_2, respectively.

Sol. $(F_{ext})_{system}=0\Rightarrow \vec{p}_1+\vec{p}_2=0\Rightarrow p_1=p_2$.

$$\Rightarrow\qquad m_1\vec{v}_1+m_2\vec{v}_2=0\quad\Rightarrow\quad\frac{v_1}{v_2}=\frac{m_2}{m_1}$$

$$\frac{KE_1}{KE_2}=\frac{p_1^2/2m_1}{p_2^2/2m_2}=\frac{m_2}{m_1}\ \text{(since } p_1=p_2\text{)}$$

$$\frac{F_1}{F_2}=\frac{kx}{kx},\quad \text{where x = deformation of the spring}$$

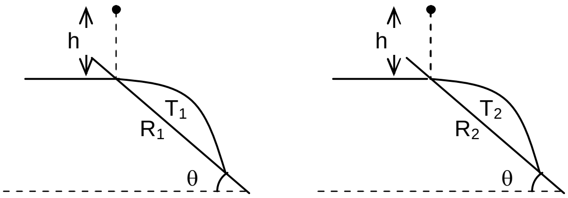
$$\Rightarrow\qquad\frac{F_1}{F_2}=1\Rightarrow\frac{m_1a_1}{m_2a_2}=1$$

$$\Rightarrow\qquad\frac{a_1}{a_2}=\frac{m_2}{m_1}$$. Hence, the correct choice is (B), (D)

Prob 3. *A ball strikes a wall with a velocity $\vec{u}$ at an angle θ with the normal to the wall surface and rebounds at an angle β with the surface of the wall. Then:*
(A) $(\theta + \beta) > 90°$, if wall is smooth
(B) if the wall is smooth, coefficient of restitution $= \frac{\tan\beta}{\cot\theta}$
(C) if the wall is smooth, coefficient of restitution $< \frac{\tan\beta}{\cot\theta}$
(D) none of these

Sol. Velocity $v \cos \beta = u \sin \theta$ . . . (i)
$v \sin \beta = e.\ u \cos \theta$ . . . (ii)
From the above equation, $\tan \beta = e \cot \theta$
or $\tan \beta = e \tan (90° - \theta)$.
But $\beta \not> (90 - \theta)$
Or $(\beta + \theta) \not> 90°$
From equation (ii)
$e = \frac{\tan\beta}{\cot\theta}$,
Hence, (B) is the correct choice.

Prob 4. *Two balls are dropped from same height h, one on a smooth plane and the other on a rough plane having same inclination θ with the horizontal as shown in figure.*



Both the planes have same coefficient of restitution. If range and time of flight of first and second balls are R_1, T_1 and R_2, T_2, respectively, then:
(A) $T_1 = T_2, R_1 = R_2$ *(B) $T_1 < T_2, R_1 < R_2$*
(C) $T_1 = T_2, R_1 > R_2$ *(D) $T_1 > T_2, R_1 > R_2$*

Sol. The velocity of the ball just before striking the plane $= \sqrt{2gh}$
Velocity resolved into two components:
(i) $\sqrt{2gh} \cos \theta$, normal to the plane, and
(ii) $\sqrt{2gh} \sin \theta$, parallel to the plane.
The normal component of velocity of both the balls will be same after collision as they have same coefficient of restitution. Hence time of flight is same for the two balls i.e.
$T_1 = T_2$. But, the component of velocity along the plane will decrease for the second ball after collision (because of frictional impulse) and therefore $R_1 > R_2$.
Hence, **(C)** is the correct choice.

Prob 5. *Two particles A and B, initially at rest, move towards each other under a mutual force of attraction. At the instant when the speed of A is v and the speed of B is 2v, the speed of centre of mass is*
(A) zero *(B) v*
(C) 1.5 v *(D) 3 v*

Sol. Since there is no external force acting on the system $\vec{v}_{cm}$ will be constant. Initially,
$\vec{v}_{cm} = 0$, hence finally also

$\vec{v}_{cm} = 0$

Hence, **(A)** is the correct choice.

Prob 6. *After perfectly inelastic collision between two identical particles moving with same speed in different directions, the speed of the particles becomes (2/3)rd of the initial speed. The angle between the two particles before collision is*

(A) $\cos^{-1}(-1/3)$ *(B) $\sin^{-1}(-1/3)$*
(C) $\cos^{-1}(-1/9)$ *(D) $\sin^{-1}(-1/9)$*

Sol. Let θ be the desired angle. Linear momentum of the system will remain conserved. Hence

$$P^2 = P_1^2 + P_2^2 + 2P_1P_2\cos\theta$$

$$= \left[2m\left(\frac{2}{3}\right)v\right]^2 = (mv)^2 + (mv)^2 + 2(mv)(mv)\cos\theta$$

$$\Rightarrow \frac{16}{9} = 2 + 2\cos\theta \qquad \Rightarrow 2\cos\theta = \frac{-2}{9}$$

$$\Rightarrow \cos\theta = -\frac{1}{9} \qquad \Rightarrow \theta = 96.37^0$$

Hence, **(C)** is the correct choice.

FILL IN THE BLANKS IN THE FOLLOWING QUESTIONS.

[

Prob 7. *A bullet of mass m leaves a gun of mass M kept on a smooth horizontal surface. If the speed of the bullet relative to the gun is v, the recoil speed of the gun will be__________________________.*

Sol. No external force acts on the system (gun + bullet) during their impact (till the bullet leaves the gun). Therefore the momentum of the system remains constant. Before the impact the system (gun + bullet) was at rest. Hence its initial momentum is zero. Therefore just after the impact, its momentum of the system (gun + bullet) will be zero.

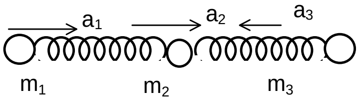
$$\Rightarrow \quad m\vec{v}_b + M\vec{v}_g = 0 \qquad \Rightarrow m\left[\vec{v}_{bg} + \vec{v}_g\right] + M\vec{v}_g = 0$$

$$\Rightarrow \quad \vec{v}_g = -\frac{m\vec{v}_{bg}}{M+m}, \quad \text{where } \left|\vec{v}_{bg}\right| = \text{velocity of bullet relative to the gun} = v$$

$$\Rightarrow \quad v_g = \frac{mv}{M+m}. \text{ (opposite to the direction of v.)}$$

Prob 8. *Shown in the figure is a system of three particles having masses m_1 =1 kg, m_2 = 2 kg and m_3 = 4 kg connected by two springs. The acceleration of A, B and C at any instant are 1 m/sec^2, 2m/sec^2 and (1/2) m/sec^2, respectively, directed as shown in the figure. The external force acting on the system is_________________.*

Sol. The acceleration of center of mass of the system



$$= \vec{a}_{cm} = \frac{\left|m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3\right|}{m_1 + m_2 + m_3}$$

$$\Rightarrow \quad \text{The net force acting on the system} = (m_1 + \vec{m}_2 + m_3)\ \vec{a}_{cm}$$

$$\Rightarrow \quad F_{net} = (m_1 a_1 + m_2 a_2 - m_3 a_3)$$

$$= \left[(1)(1) + (2)(2) - \frac{1}{2}\ (4)\right] \text{N} = 3 \text{ N}.$$

Prob 9. *A particle of mass m collides with another stationary particle of mass M. If the particle m stops just after the collision, the coefficient of restitution of collision is equal to* _________________

Sol. The net horizontal force acting on the system (M + m) is zero. Therefore the momentum of the system just before and after the collision remains constant. Let M move with the velocity v'.

$$\Rightarrow \quad m\vec{v} + 0 = m(0) + M\vec{v}\,'$$

$$\Rightarrow \quad \frac{v'}{v} = \frac{m}{M} \qquad \ldots(1)$$

Newton's experimental formula $e = -\frac{(v' - 0)}{0 - v}$

$$\Rightarrow \quad e = \frac{v'}{v} \qquad \ldots(2)$$

Equating (1) and (2), we find e = (m/M).

Prob 10. *A ping–pong ball of mass m is floating in air by a jet of water emerging out of a nozzle. If the water strikes the ping–pong ball with a speed v and just after collision water falls dead, the rate of mass flow of water in the nozzle is equal to*________________________.

Sol. The impact force F = (Δp/Δt), where Δp = change of momentum of water of mass Δm striking the ball with a speed v during time Δt.
Since water falls dead after collision with the ping–pong ball,

$$\Delta p = \Delta m\, v \quad \Rightarrow F = v\frac{\Delta m}{\Delta t} \text{ (upward on the ball)}$$

where $\frac{\Delta m}{\Delta t}$ = rate of flow of water in the nozzle.
Since the ball is in equilibrium,

$$F - mg = 0 \quad \Rightarrow F = mg$$

$$\Rightarrow \quad \frac{v\Delta m}{\Delta t} = mg \quad \Rightarrow \quad \frac{\Delta m}{\Delta t} = \frac{mg}{v}.$$

Prob 11. *A bomb of mass 9 kg explodes into two fragments of masses 3 kg and 6 kg. The velocity of mass 3 kg is 16 m/sec. The energy of explosion is equal to*_____________________.

Sol. Conservation of momentum just before and after the impact yields

$$P_i = P_f$$

$$\Rightarrow \quad 0 = \left|m_1\vec{v}_1 + m_2\vec{v}_2\right| \quad \Rightarrow m_1 v_1 = m_2 v_2 = p \text{ (say)}$$

$$m_1v_1 = m_2v_2 = p$$

$$\Rightarrow \quad \text{Energy of explosion} = \Delta\, KE_{system}$$

$$= \frac{p^2}{2m_1} + \frac{p^2}{2m_2} = \frac{p^2}{2m_1}\left[1 + \frac{m_1}{m_2}\right]$$

Putting p = m_1 v_1, where m_1 = 3 kg, m_2 = 6 kg and v = 16 m/sec we obtain,

$$E = \frac{(3\times16)^2}{2\times\ 3}\left[1 + \frac{3}{6}\right]$$

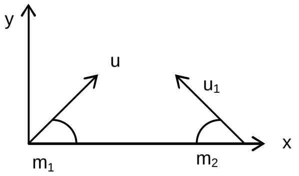
$\Rightarrow$ E = 576 J . Hence, correct choice is **(B)**

STATE WHETHER THE FOLLOWING QUESTIONS ARE TRUE OR FALSE.

Prob 12. *During the explosion, the total energy of the system remains conserved.*

Sol. The total mechanical energy will not be conserved but the total energy has to remain conserved.
$\Rightarrow$ True

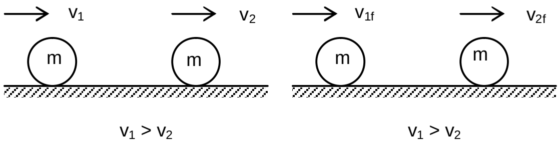
Prob 13. *Two heavy particles of masses m_1 and m_2 collide at an intermediate point in air. After collision, the total momentum of the two particle system will remain conserved.*



Sol. Momentum will be conserved along x–axis as there is an external force acting along x–axis. But there is an external force acting along y–axis, so the momentum along y–axis will not be conserved.
$\Rightarrow$ The total momentum of the two particle system is not conserved.
$\Rightarrow$ False

Prob 14. *Two blocks of equal masses have an elastic head-on collision. Then the velocities of the two blocks will be interchanged if the collision is head on.*

Sol. Momentum conservation



$\Rightarrow$ $mv_1 + mv_2 = mv_{1f} + mv_{2f}$ . . . (i)

$$e = -\frac{(v_{2f} - v_{1f})}{v_2 - v_1} = 1 \quad . . . (ii)$$

Solving equations (i) and (ii),
$\Rightarrow v_{1f} = v_2$ and $v_{2f} = v_1$
$\Rightarrow$ True.

Prob 15. *If there is no external force acting on the system, then the position of centre of mass of the system always remains the same.*

Sol. Position of centre of mass will remain the same only if
$\vec{F}_{ext} = 0$ and $\vec{v}_{cm} = 0$,
$\Rightarrow$ False

ASSIGNMENT PROBLEMS

Subjective:

Level– O

1. From a uniform circular disc of radius R, a circular disc of radius R/6 and having centre at a distance R/2 from the centre of the disc is removed. Determine the centre of mass of the remaining portion of the disc.

2. What do you mean by centre of mass of a system? Obtain an expression defining the centre of mass of a system of two particles. Prove that the centre of mass of a system moves with constant velocity in the absence of any external force on the system.

3. A gun weighing 15 kg fires a bullet of 150 gm with a velocity of 500 m/s. With what velocity does the gun recoil? What is the resultant momentum of the gun and the bullet system before and after firing?

4. A shell of mass 5 kg moving with a velocity of 20 m/s explodes into two particles of 3 kg and 2 kg respectively. If the former is just brought to rest after the explosion, find the velocity of the later.

5. Derive an expression for the velocity of a rocket at any time, after being fired. What is meant by burnt out velocity of the rocket?

6. A rocket consumes 24 kg of fuel per second. The burnt gases escape the rocket at a speed of 6.4 km/s relative to the nozzle. Calculate the upthrust received by the rocket. Also calculate the velocity acquired by the rocket, when its mass reduces to 1/100 of initial mass.

7. A glass marble, whose mass is 100 gm, falls from a height of 20 m and rebounds to a height of 5 m. Find the impulse and average force between the marble and the floor, if the time during which they are in contact is 0.01 sec.

8. Two billiard balls each of mass 0.05 kg, moving in opposite directions with equal speed of 16 m/s collide and rebound with same speed. What is the impulse imparted to each ball due to other.

9. A ball of mass 0.1 kg makes an elastic head-on collision with a ball of unknown mass, initially at rest. If the 0.1 kg ball rebounds at one-third of its original speed, what is the mass of the other ball.

10. A railway carriage of mass 10,000 kg moving with a speed of 54 km/hr strikes a stationary carriage of the same mass. After the collision, the carriages get coupled and move together. What is their common speed after the collision ?

11. A rocket is out in free space shooting out a steam of exhaust gases and picking up speed in the opposite direction. What happens to the centre of mass of all the matter, which is ejected and which is left in the rocket ?

12. What is the difference between centre of mass and centre of gravity?

13. The centre of mass of a rigid body always lies inside the body. Is this statement true or false?

14. All the particles of a body are situated at distance R from the origin. The distance of the centre of mass of the body from the origin is also R. Is this statement true or false?

15. A bomb, in a projectile motion, explodes in the air. The path of the centre of mass remains unchanged. Explain.

Level – I

1. A uniform steel rod 1 m in length is bent in a 90° angle at its mid point. Determine the position of its centre of mass.

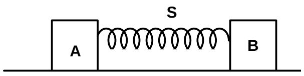
2. A body of mass 3 kg collides elastically with another body at rest and then continues to move in the original direction with one-half of its original speed. What is the mass of the target body?

3. A 1000 kg automobile is moving along a straight highway at 10m/s. Another car, with mass 2000 kg and speed 20 m/s at 30 m ahead of the first
(a) find the total momentum from the above data.
(b) find the velocity of the centre of mass.
(c) find the total momentum, using the velocity of the centre of mass. Compare your result with that of part (a).

4. A railroad hand car is moving along a straight frictionless track. In each of the following cases, the car initially has a total mass (car and contents) of 200 kg and is travelling with a velocity of 4 m/s. Find the final velocity of the car in each of the three cases.
(a) A 20 kg mass is thrown sideways out of the car with a velocity of 2 m/s relative to the car.
(b) A 20 kg mass is thrown backward out of the car with a velocity of 4 m/s relative to the car.
(c) A 20 kg mass is thrown into the car with a velocity of 6 m/s relative to the ground and opposite in direction to the velocity of the car.

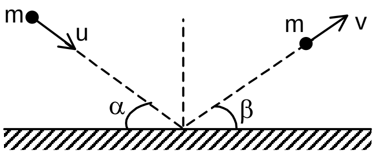
5. An acrobat of mass m clings to a rope ladder hanging below a balloon of mass M. The balloon is stationary with respect to ground.
(a) If the acrobat begins to climb the ladder at a speed v (with respect to the ladder), in what direction and with what speed (with respect to the Earth) will the balloon move ?
(b) If the acrobat stops climbing, what will be the velocity of the balloon?

6. Block A in the figure has a mass of 1 kg and block B has a mass of 2 kg. The blocks are forced together compressing a spring between them and the system is released from rest on a level frictionless surface. The spring is not fastened to either of the blocks and drops to the surface after it has expanded. Block B acquires a speed of 0.5 m/s. How much potential energy was stored in the compressed spring?



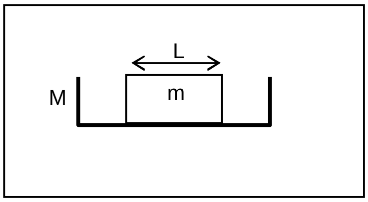
7. A projectile of mass m is fired with a velocity v_0 at an angle θ with the horizontal. At the highest position in its fight, it explodes into two fragments of masses $m_1 = \frac{m}{3}$ and $m_2 = \frac{2m}{3}$. The fragment of mass m_1 falls vertically with zero initial speed.
(a) Find the initial velocity of the fragment m_2.
(b) Find the distance d at which the fragment m_2 will land with respect to m_1.

8. A particle of mass m moving with a speed u strikes a smooth horizontal surface at an angle α. The particle rebounds at an angle β with a speed v. Determine the expression for v and β if the coefficient of restitution is e.



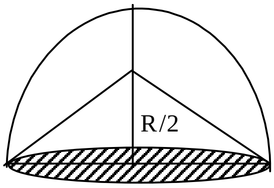
9. Find the ratio of the linear momenta of two particles of masses 1.0 kg and 4.0 kg if their kinetic energies are equal.

10. Consider a gravity-free hall in which a tray of mass M, carrying a cubical block of ice of mass m and edge L, is at rest in the middle. If the ice melts, by what distance does the center of mass of 'the tray plus the ice' system descend?



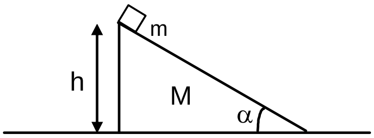
11. Find the centre of mass of (a) a hemisphere, (b) a solid cone.

12. From the base of a hemisphere, a right cone of height R/2 and same base has been scooped out. Find the centre of mass of the remaining part.

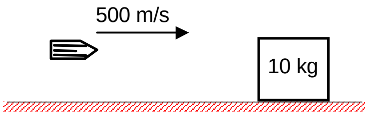


13. A pulley fixed to a rigid support carries a rope whose one end is tied to a ladder with a man and the other end to the counterweight of mass M. The man of mass m climbs up a distance h with respect to the ladder and then stops. If the mass of the rope and the friction in the pulley axle are negligible, find the displacement of the center of mass of this system.

14. A cubical block of mass m is released from rest at a height h on a frictionless surface of a movable wedge of mass M, which in turn is placed on a horizontal frictionless surface as shown in the figure. Find the velocity of the triangular block when the smaller block reaches the bottom.

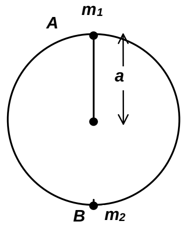


15. From a pistol a bullet of mass 20 gm is fired such that it passes horizontally with a speed of 500 m/s through a wooden block of mass 10 kg with a speed of 100 m/s and the block slides 20 cm on the surface before coming to rest. Find the coefficient of friction between the block and the surface.

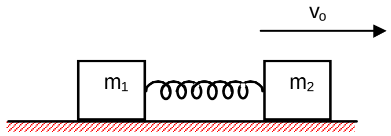


Level – II

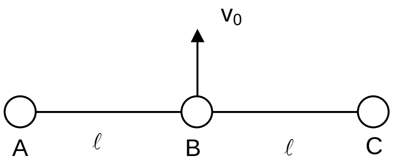
1. Two beads of masses m_1 and m_2 are threaded on a smooth circular wire of radius 'a' fixed in a vertical plane. B is stationary at the lowest point when A is gently dislodged from rest at the highest point. A collides with B at the lowest point. The impulse given to B due to collision is just great enough to carry it to the level of the centre of the circle while A is immediately brought to rest by the impact. Find $m_1 : m_2$.



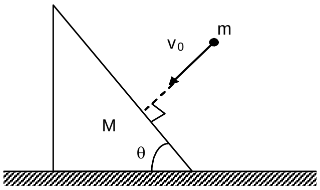
2. A spring of spring constant k connects two bocks of masses m_1 and m_2 as shown in the figure. The block of mass m_2 is given a sharp impulse so that it acquires a velocity V_0 towards right . Find
(a) the velocity of the centre of mass
(b) the maximum elongation of the spring.



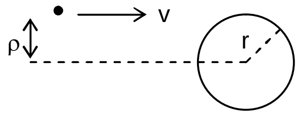
3. Three identical balls are connected by light inextensible strings with each other as shown and rest over a smooth horizontal table. At moment t = 0, ball B is imparted a velocity v_0. Calculate the velocity of A when it collides with ball C.



4. In the figure shown, a ball of mass m collides perpendicularly with a smooth stationary wedge of mass M. If the coefficient of the restitution of collision is e, then determine the velocity of the wedge after collision.

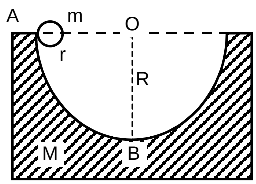


5. A small particle travelling with a velocity v collides elastically with a spherical body of equal mass and of radius r initially kept at rest. The center of this spherical body is located a distance $\rho(<r)$ away from the direction of motion of the particle. Find the final velocities of the two particles and the spherical body.

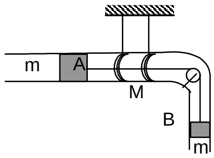


6. From the bottom of a cone of height h, a cone of same base is scooped out so that the centre of mass of the remaining part remains at the tip of the hollow cone. Find the height of the hollow cone.

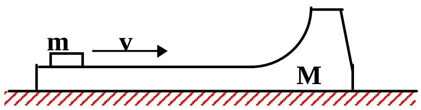
7. A block of mass M with a semicircular track of radius R rest on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A as shown in the figure. The cylinder slips in the semicircular frictionless track,
(a) How far does the block move when the cylinder reaches the bottom (point B) of the track?
(b) What is the velocity of the block when the cylinder reaches the bottom of track.



8. In the figure shown the hollow tube of mass M is free to move without friction in the horizontal direction supported by two fixed vertical lugs attached to the roof. The system is released from rest. Find the velocity of the tube, when the block B has fallen through a height h. Neglect any friction and mass of the string.

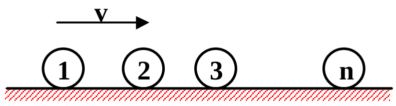


9. A body of mass M, with a small disc of mass m placced on it rests on a smooth horizontal plane. The disc is set in motion in the horizontal direction with velocity v. To what height (relative to the intial level) will the disc rise after breaking off the body M? The friction is assumed to be absent.

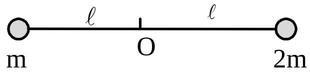


10. Two masses m_1 and m_2 connected by a light spring of natural length ℓ_0 is compressed completely and tied by a string. This system while moving with a velocity v_0 along +ve x-axis passes through the origin at t = 0. At this position the string snaps. Position of mass m_1 at time t is given by the equation $x_1(t) = v_0t - A(1 - \cos\omega t)$. Calculate
(a) position of the particle m_2 as a function of time.
(b) ℓ_0 in terms of A.

11. In figure a series of n identical balls is shown on a smooth horizontal surface. If number 1 moves horizontally with speed v into number 2 which in turn collides with number 3, etc. and if the coefficient of restitution for each impact is e, determine the speed of the nth ball.



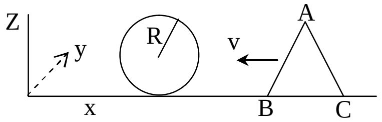
12. Two balls having masses m and 2m are fastened to two light strings of same length *l*. The other ends of the strings are fixed at O. The strings are kept in same horizontal line and the system is released from rest. The collision between the balls is elastic.
(a) Find the velocities of the balls after their collision.
(b) How high will the balls rise after the collision ?



13. Two equal spheres of mass m are suspended by vertical strings so that they are in contact with their centres at the same level. A third ball of mass 2m and same size falls vertically from height h and strikes the two spheres simultaneously so that their centres at the instant of impart form an equilateral triangle in a vertical plane. (Coefficient of restitution is e)
(a) Find the velocities of the spheres just after impact.
(b) Find the impulsive tension of the strings.

14. A sphere of mass m falls vertically on a smooth hemisphere of mass M resting on a smooth horizontal surface with a velocity u so that the point of impact makes an angle α with the horizontal at the centre of the hemisphere. Find the velocity of the hemisphere if the coefficient of restitution is e.

15. A wedge of mass m and triangular cross-section (AB = BC = CA = 2R) is moving with a constant velocity -vi towards a sphere of radius R fixed on a smooth horizontal table as shown in the figure.



The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time Δt, during which the sphere exerts a constant force F on the wedge. Find the force F and also the normal force N exerted by the table on the wedge during the time Δt.

Objective:

Level– I

1. The centre of mass of a body:
   (A) always lies at the geometrical center
   (B) always lies inside the body
   (C) always lies outside the body
   (D) lies within or outside the body

2. A bomb travelling in a parabolic path under gravity, explodes in mid air. The centre of mass of the fragments will:
   (A) move vertically upwards and then downwards
   (B) move vertically downwards
   (C) move in irregular path
   (D) move in the same parabolic path as the unexploded bomb would have travelled

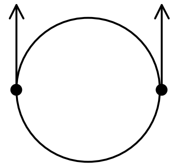
3. A bullet in motion hits and gets embedded in a solid block resting on a frictionless table. What is conserved for the bullet-block system ?
   (A) momentum and KE
   (B) kinetic energy alone
   (C) neither KE nor momentum
   (D) momentum alone

4. If a ball is thrown upwards from the surface of earth:
   (A) the earth remains stationary while the ball moves upwards
   (B) the ball remains stationary while the earth moves downwards
   (C) the ball and earth both move towards each other
   (D) the ball and earth both move away from each other

5. Consider a system of two identical particles. One of the particles is at rest and the other has an acceleration $\vec{a}$. The centre of mass has acceleration
   (A) zero
   (B) $\frac{1}{2}\vec{a}$
   (C) $\vec{a}$
   (D) $2\vec{a}$

6. If a shell, fired from a cannon, explodes in mid air, its total
   (A) momentum increases.
   (B) momentum decreases.
   (C) kinetic energy increases.
   (D) kinetic energy remains constant.

7. On a smooth horizontal surface, a ring of mass M lies with two insects of mass m, each on its diametrically opposite points. The insects start moving with velocity v in same direction relative to the ring. The velocity of the ring as the insects meet is (Considering that initially the system was at rest)



   (A) $\frac{Mv}{2m}$
   (B) $\frac{M-m}{M+m}$
   (C) zero
   (D) $\frac{2m}{M}v$

8. A bag of mass M hangs by a long thread and a bullet (mass m) comes horizontally with velocity v and gets caught in the bag. Then, for the combined system (bag + bullet):
(A) momentum is mMv/(M + m) (B) KE is (1/2) Mv^2
(C) momentum is Mv (D) KE is $m^2v^2/2(M + m)$

9. A bullet weighing 10 gm and moving at 300 m/s strikes a 5 kg ice block and drops dead. The ice block is resting on a frictionless level surface. The speed of the block after the collision, is
(A) 6 cm/sec (B) 6 m/sec
(C) 60 cm/sec (D) 60 m/sec.

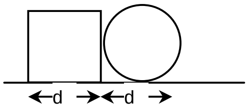
10. A body of mass 1 kg, initially at rest, explodes and breaks into three fragments of masses in the ratio 1:1:3. The two pieces of equal mass fly off perpendicular to each other with a speed of 15 m/s each. The speed of the heavier fragment is:
(A) $5\sqrt{2}$ m/s (B) 45 m/s
(C) 5 m/s (D) 156 m/s

11. Two putty balls A and B of masses 2 kg and 3 kg, respectively, collide plastically at O with velocities $v_1 = 11\hat{i} + 6\hat{j} + 5\hat{k}$ m/s and $v_2 = 6\hat{i} + 6\hat{j}$ m/s.
The velocity of the new ball is
(A) $8\hat{i} + 5\hat{j} + 2\hat{k}$ (B) $8\hat{i} + 6\hat{j} + 2\hat{k}$
(C) $8\hat{i} + 5\hat{j} + 5\hat{k}$ (D) $7\hat{i} + 5\hat{j} + 2\hat{k}$

12. A particle of mass m is fired with a velocity v, making an angle of 45° with the horizontal. The magnitude of the change in momentum, when the particle of mass m lands on the ground is
(A) Zero (B) $\sqrt{2mv}$
(C) $mv\sqrt{2}$ (D) $\sqrt{mv}$

13. A shell is fired from a canon with velocity v m/s at an angle θ with the horizontal direction. At the highest point in its path the shell explodes into two pieces of equal mass. One of the pieces retraces its path to the canon and the speed (in m/s) of the other piece immediately after the explosion is
(A) 3v cos θ (B) 2v cos θ
(C) 3/2 v cos θ (D) $\frac{\sqrt{3}}{2}$ v cos θ

14. A circular plate of diameter 'd' is kept in contact with a square plate of edge 'd' as shown in figure. The density of material and thickness are same everywhere. The center of mass of the composite system will be



(A) inside the circular plate (B) inside the square plate
(C) at the point of contact (D) outside the system

15. A ball is dropped from a height h on the ground. If the coefficient of restitution is e, find the height to which the ball will go up after it rebounds for the n^{th} time.
(A) e . h^n (B) h. e^n
(C) e. h^{2n} (D) h. e^{2n}

Level – II

1. A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless horizontal surface. The man walks to the other end of the plank. If the mass of the plank is (M/3), the distance that the man moves relative to ground is

(A) L
(B) $\frac{L}{4}$
(C) $\frac{3L}{4}$
(D) $\frac{L}{3}$

2. A particle of mass m is made to move with uniform speed v_0 along the perimeter of a regular hexagon. The magnitude of impulse applied at each corner of the hexagon is

(A) $2mv_0 \sin\frac{\pi}{6}$
(B) $mv_0 \sin\frac{\pi}{6}$
(C) $mv_0 \sin\frac{\pi}{3}$
(D) $2mv_0 \sin\frac{\pi}{3}$

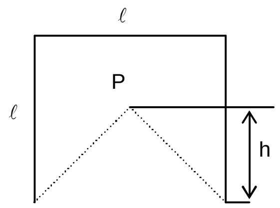
3. Two identical billiard balls are in contact on a table. A third identical ball strikes them symmetrically and remains at rest after impact. The coefficient of restitution is

(A) $\frac{2}{3}$
(B) $\frac{1}{3}$
(C) $\frac{1}{6}$
(D) $\sqrt{\frac{3}{2}}$

4. A small spherical ball strikes a frictionless horizontal plane with a velocity v making an angle θ to the normal at the surface. If the coefficient of restitution is e, the particle will again strike the surface after time

(A) $\frac{2v\sin\theta}{g}$
(B) $\frac{2ev\cos\theta}{g}$
(C) $\frac{2ev\sin\theta}{g}$
(D) $\frac{2v\cos\theta}{g}$

5. An isosceles triangle is to be cut from one edge of a square lamina (as shown in the figure) such that the remaining portion when suspended from the apex P of the cut will remain in equilibrium in any position. The value of h is



(A) $\frac{(3-\sqrt{3})\ell}{2}$
(B) $\frac{(3+\sqrt{3})\ell}{2}$
(C) $\frac{(2-\sqrt{2})\ell}{2}$
(D) $\frac{(2+\sqrt{2})\ell}{2}$

6. A Neutron travelling with a velocity v and kinetic energy E collides elastically head on with the nucleus of an atom of mass number A at rest. The fraction of total energy retained by the neutron is:

(A) $\left(\frac{A-1}{A+1}\right)^2$ (B) $\left(\frac{A+1}{A-1}\right)^2$

(C) $\left(\frac{A-1}{A}\right)^2$ (D) $\left(\frac{A+1}{A}\right)^2$

7. A sphere of mass m moving with a constant velocity u hits another stationary sphere of the same mass. If e is the coefficient of restitution, then ratio of velocities of the two spheres after collision will be:

(A) $\left(\frac{1-e}{1+e}\right)$ (B) $\left(\frac{1+e}{1-e}\right)$

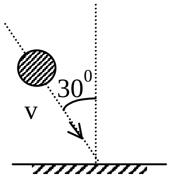
(C) $\left(\frac{e+1}{e+1}\right)$ (D) $\left(\frac{e-1}{e+1}\right)$

8. A particle strikes a horizontal frictionless floor with a speed u at an angle θ with the vertical, and rebounds with speed v at an angle ϕ with vertical. The coefficient of restitution between the particle and the floor is e. The magnitude of v is

(A) e u (B) (1 – e)u

(C) $u\sqrt{\sin^2\theta + e^2\cos^2\theta}$ (D) $u\sqrt{e^2\sin^2\theta + \cos^2\theta}$

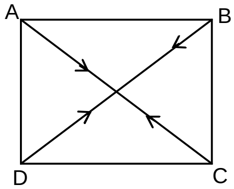
9. A smooth ball of mass m strikes a horizontal surface with a velocity v in a direction making an angle 30° with the normal to the surface as shown in the figure. If the coefficient of restitution for the collision between the ball and the surface is e and the ball was in contact with the surface for a small time 'Δt' the average force acting on the ball during collision is



(A) mg (B) $\frac{mv(1+e)}{2\Delta t}$

(C) $\frac{\sqrt{3}\,mv(1-e)}{2\Delta t}$ (D) $\frac{\sqrt{3}\,mv(1+e)}{2\Delta t}$

10. Four particles A, B, C and D of equal mass move with equal speed v along the diagonals of a square in a horizontal plane as shown in the figure. After the collision A comes to rest, while B and C retrace their paths. Then, the particle D will



(A) continue to move along the same line with speed v

(B) retrace its path with speed 2v

(C) come to rest

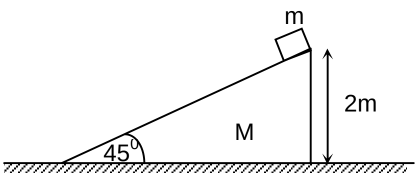
(D) move with speed $v\sqrt{2}$ along a line parallel to CD

11. A ball is projected from a flat horizontal floor vertically with speed v. If the coefficient of restitution for every collision between the ball and floor be 'e', the ball would finally come to rest after a time

(A) 2v/g(1 – e) (B) 2v/g(1 + e)

(C) v/g(1 – e) (D) $2v/g(1 - e^2)$

12. A wedge of mass M and a cube of mass m are shown in figure. The system is released. Considering no frictional force between any two surfaces, the distance moved by the wedge, when the cube just reaches on the ground is



(A) $\frac{m2\sqrt{2}}{(m+M)}$ (B) 2m

(C) $2\sqrt{2}$ m (D) $\frac{2m}{m+M}$

13. A bomb of mass 7m explodes into two fragments of masses 4m and 3m. If the momentum of the lighter fragment is 'p,' then the energy released in the explosion is

(A) $\frac{7P^2}{24m}$ (B) $\frac{9P^2}{16m}$

(C) $\frac{11P^2}{24m}$ (D) $\frac{5P^2}{14m}$

14. Two particles of masses M and 2M are at a distance D apart. Under their mutual gravitational force, they start moving towards each other. The acceleration of their centre of mass when they are distance D/2 apart is

(A) $\frac{2GM}{D^2}$ (B) $\frac{4GM}{D^2}$

(C) $\frac{8GM}{D^2}$ (D) zero

15. A particle moves in the x–y plane under the action of a force $\vec{F}$. At any time t, its linear momentum is given by $P_x = 2\cos(kt)$ and $P_y = 2\cos(kt)$.

The angle between $\vec{F}$ and $\vec{P}$ at time t is

(A) 0° (B) 30°

(C) 90° (D) 180°

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. [$\Delta x = -R/70$]
3. [5 m/s, 0 kg–m/s, 0 kg–m/s]
4. [50 m/s]
7. 3 N-sec, 300 N
8. [–1.6 kgm/s] opposite to the kg m/s $\propto$ N/s direction in which the ball was initially mixing.
9. [0.2 kg]
10. 27 km/hr
11. Taking rocket and exhaust gases as one system, there is no external force acting on the system. Thus acceleration of centre of mass must be zero.
12. The mass of two boats floating on still water constitute a single system and forces applied by them on each other are identical forces.

Level – I

1. $\frac{1}{4\sqrt{2}}$ m from the corner along 45° bisector line and inside the right angle
2. 1 kg
3. (a) 50,000 kg m/s (b) $\frac{50}{3}$ m/s (c) 50,000 kgm/s
4. (a) 4 m/s in the same direction, (b) 4.4 m/s in the same direction, (c) 3.1 m/s in the same direction
5. (a) down , $\frac{mv}{M+m}$; (b) balloon remains stationary
6. 0.75 J
7. (a) $\frac{3}{2}v_0\cos\theta$ (b) $\frac{3v_0^2\sin\theta\cos\theta}{2g}$
8. $v = u\sqrt{e^2\sin^2\alpha+\cos^2\alpha}$, $\tan\beta = e\tan\alpha$
9. 1 : 2
10. zero
11. (a) $\frac{3R}{8}$ from base, (b) $\frac{H}{4}$ from base
12. $\frac{11R}{24}$ from the circular base
13. $\frac{mh}{2M}$
14. $\left[\frac{2m^2gh\cos^2\alpha}{(M+m)(M+m\sin^2\alpha)}\right]^{1/2}$
15. 0.16

Level – II

1. $\frac{1}{\sqrt{2}}$

2. (a) $\frac{m_2 v_0}{m_1 + m_2}$ (b) $v_0\sqrt{\frac{m_1 m_2}{(m_1+m_2)k}}$

3. $\frac{2v_0}{3}$

4. $\frac{(1+e)mv_0 \sin\theta}{M + m\sin^2\theta}$

5. (a) $\frac{v\rho}{r}$ along common tangent (b) $\frac{v}{r}\sqrt{r^2-\rho^2}$ along common normal

6. $\frac{h}{3}$

7. (a) $-\frac{m\left(R - r\right)}{m + M}\hat{i}$ (b) $m\sqrt{\frac{2g\left(R - r\right)}{M\left(m + M\right)}}$ (to the left)

8. $\sqrt{\frac{2m^2gh}{(2m+M)(3m+2M)}}$ leftward

9. $\frac{Mv^2}{2g(m+M)}$.

10. (a) $v_0t + \frac{m_1}{m_2}A(1-\cos\omega t)$, (b) $\left(\frac{m_1}{m_2}+1\right)A$

11. $\frac{(1+e)^{n-1}}{2^{n-1}}v$

12. (a) $v_m = \frac{5}{3}\sqrt{2g\ell}$ leftward, $v_{2m} = \frac{1}{3}\sqrt{2g\ell}$ rightward ; (b) Ball m: 2ℓ, Ball 2m: $\frac{\ell}{9}$

13. (a) $v_{2m} = \frac{(3e-1)\sqrt{2gh}}{4}$ upward , $v_m = \frac{(e+1)\sqrt{6gh}}{4}$ horizontal; (b) $\frac{3m(e+1)\sqrt{2gh}}{4}$

14. $\frac{mu\sin\alpha\cos\alpha(1+e)}{M+m\cos^2\alpha}$

15. $N = \left[\frac{2mv}{3\Delta t} + Mg\right]\hat{k}$

Objective:

Level – I

1.	**D**	2.	**D**
3.	**D**	4.	**D**
5.	**B**	6.	**C**
7.	**C**	8.	**D**
9.	**C**	10.	**A**
11.	**B**	12.	**C**
13.	**A**	14.	**B**
15.	**D**		

Level –II

1.	**B**	2.	**A**
3.	**A**	4.	**B**
5.	**A**	6.	**A**
7.	**A**	8.	**C**
9.	**D**	10.	**D**
11.	**A**	12.	**D**
13.	**A**	14.	**D**
15.	**D**		

Learnaf
Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

ROTATIONAL MECHANICS

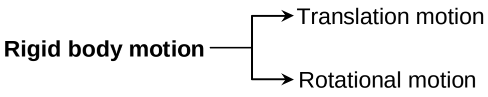
Rotational Mechanics

Syllabus for IITJEE and Maharashtra Board:
Moment of a force, Torque, angular momentum, Physical meaning of angular momentum, conservation of angular momentum with some examples (Planetary motion)

Equilibrium of rigid bodies, rigid body rotation and equation of rotational motion, comparison of linear and rotational motion, collision of point masses with rigid bodies, moment of inertia and its physical significance, radius of gyration, parallel and perpendicular axis theorems; moment of inertia of circular ring, disc and cylinder. Rolling without slipping of rings cylinders and spheres. Examples of Binary Systems in nature (Binary Stars, Earth–moon system, diatomic molecules)

Rigid Body

A body is said to be rigid if the distance between any two particles of the body remains invariant under the action of any force.
The plane motion of a rigid body is represented as the combination of **translational motion** and **rotational motion**.



Translational motion:

A rigid body is said to undergo translation if it moves such that it always remains parallel to itself, this means that a line connecting any two particles of the rigid body always remains parallel to itself throughout its motion. Any two particles of the rigid body have identical velocities in this case.

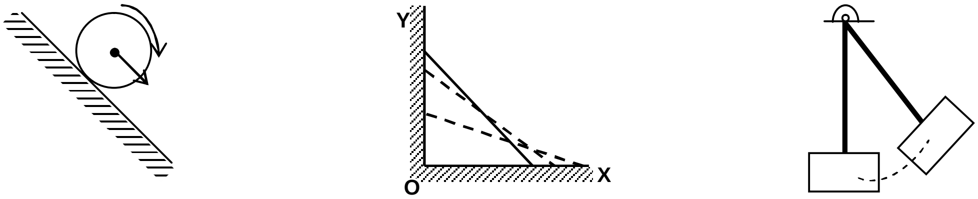
Rotational motion:

A rigid body is said to undergo rotation about an axis if there exists a straight line such that the motion of any particle of the rigid body takes place on an arc of a circle whose centre lies on this straight line and the planes of all such circles being perpendicular to this line.
This straight line is known as the axis of rotation. The rigid body undergoes rotation relative to this axis.

Plane motion:

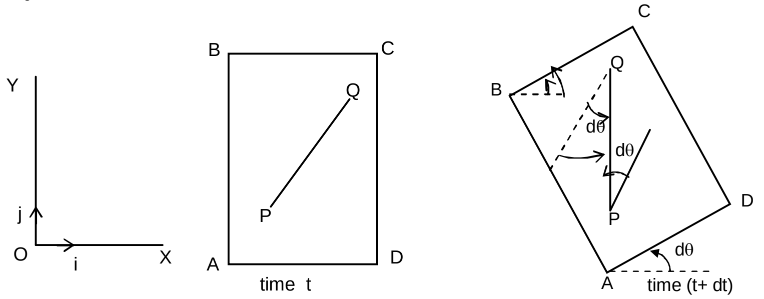
The motion of a rigid body is said to be plane motion if the motion of any particle of the rigid body is confined to a plane with the additional condition that for any two particles of the rigid body, their planes of motion are either completely identical or do not intersect at all (i.e. are parallel). During plane motion, the axis of rotation retains its orientation in space.

Examples of plane motion



Kinematics of plane motion

Consider a rigid body ABCD at times t and t + dt.



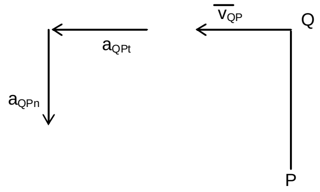
Any line on the rigid body rotates through the same angle dθ anticlockwise in the plane XOY in a small interval of time dt. A line perpendicular to the plane XOY and passing through any point on the rigid body is called an axis of rotation.

The angular velocity vector of the body (about any axis) is defined as

$$\vec{\omega} = \frac{d\theta}{dt}\hat{k} \qquad \text{and} \qquad \text{angular acceleration,}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\theta}{dt^2}\hat{k}$$

As length PQ remains constant, any point Q can have only circular motion relative to another point P.



(i) $\vec{v}_{QP} = \vec{\omega} \times \overrightarrow{PQ}$

$|\vec{v}_{QP}| = (PQ)\omega \sin 90^\circ$

(ii) a_{QPt} = tangential acceleration of Q with respect to P =(PQ)α

(iii) a_{QPn} = centripetal acceleration of Q with respect to P
= $(PQ)\omega^2$, is directed along the line QP towards the point P.

(iv) $\vec{a}_{QP} = \vec{a}_{QPt} + \vec{a}_{QPn}$

(v) For constant angular acceleration, the kinematics equations are

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Illustration 1. *A body executes 300 revolutions in 6 s. Find its (i) frequency (ii) angular speed (iii) time period and (iv) time required to turn through 60°.*

Solution : (i) Frequency, $f = \frac{300}{6} = 50$ per second

(ii) Angular speed, $\omega = 2\pi f = 2 \times \pi \times 50 = 100\,\pi$ rad/s

(iii) Time period, $T = \frac{1}{f} = \frac{1}{50}$ s

(iv) $\because 180° = \pi$ radian, $\therefore 60° = \frac{\pi}{3}$ rad

Hence, the time required to turn through 60° = $\frac{\pi/3}{100\pi} = \frac{1}{300}$s

Illustration 2. *A wheel rotates with an angular acceleration given by $\alpha = 4at^3 - 3bt^2$, where t is the time and a and b are constants. If the wheel has initial angular speed ω_0, write the equations for the (a) angular speed (b) angular displacement.*

Solution: (a) $\alpha = \frac{d\omega}{dt} \quad \Rightarrow \quad d\omega = \alpha\, dt$

$$\Rightarrow \int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha\, dt = \int_0^t (4at^3 - 3bt^2)\, dt$$

$$\Rightarrow \omega = \omega_0 + at^4 - bt^3$$

(b) Further,

$$\omega = \frac{d\theta}{dt} \Rightarrow d\theta = \omega\, dt$$

$$\Rightarrow \int_0^{\theta} d\theta = \int_0^t \omega\, dt = \int_o^t (\omega_0 + at^4 - bt^3)\, dt$$

$$\Rightarrow \theta = \omega_0 t + \frac{at^5}{5} - \frac{bt^4}{4}$$

Illustration 3. *An astronaut is uniformly rotating in a rotor of radius 4 m. If he can withstand acceleration upto 10 g, then what is the maximum number of permissible revolutions ? ($g = 10\ m/s^2$)*

Solution : In case of uniform circular motion

$$a_r = (v^2/r) = r\omega^2$$

$$a_r = (2\pi f)^2\, r$$

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{a_r}{r}\right)}$$

here $a_r < 10$ g, so

$$f < \frac{1}{2\pi}\sqrt{\frac{(10\times10)}{4}}, \quad \text{i.e.,} \quad f_{max} = \left[\frac{5}{2\pi}\right] \text{rev/sec}$$

Illustration 4. *A uniformly accelerated wheel reaches the angular velocity 20 rad/sec in 10 revolution after rotation begins. The angular acceleration of wheel is*

(A) 2π rad/s^2 *(B) π rad/sec^2*

(C) $\frac{10}{\pi}$ rad/s^2 *(D) $\frac{20}{\pi}$ rad/sec^2*

Solution: Given $\omega_0 = 0$, $\omega_f = 20$ rad/sec

and $\theta = 10$ rev. or $20\,\pi$ rad

So by the equation

$$\omega_f = \omega_0^2 + 2\alpha\theta$$

$$\alpha = \frac{\omega_f^2}{2\theta} = \frac{400}{2\times 20\pi} = \frac{10}{\pi}\text{rad/sec}^2$$

So option (C) is correct.

Illustration 5. *A pulley one meter in diameter rotating at 600 revolutions a minute is brought to rest in 20 s by a constant force of friction on its shaft. How many revolutions does it make before coming rest?*
(A) 200 *(B) 300*
(C) 400 *(D) 100*

Solution: Given D = 1 m, ω_0 = 600 rev/min = $\frac{600\times 2\pi}{60} = 20\pi \text{rad/sec}$

$\omega_f = 0$ in 20 sec
therefore, $\omega_f = \omega_0 - \alpha t$

$\Rightarrow \alpha = \frac{\omega_0}{t} = \frac{20\pi}{20} = \pi \text{ rad/sec}^2$

so the no. of revolution before coming to rest is

$$\omega_f^2 = \omega_0^2 - 2\alpha\theta$$

$$\theta = \frac{\omega_0^2}{2\alpha} = \frac{400\pi^2}{2\times\pi} = 200\,\pi \text{ rad} = 100 \text{ rev}$$

So option (D) is correct.

Comparison of linear and rotational motions

There are similarities in linear and rotational motions. The laws of rotational motion are analogous to linear motion and are given below.

First law: A body at rest or in uniform rotational motion will continue in the same state unless an external torque acts on it to change its state.

Second law: Time rate of change of angular momentum of a body is directly proportional to the external torque applied on it. The direction of change is in the direction of torque applied.

Third law: If a body A applies torque on body B, body B applies a torque of equal magnitude on body A in opposite direction.

The following chart gives corresponding equivalent terms in linear and rotational motion.

S. No.	Linear motion	Rotational motion
1.	Linear displacement (S)	Rotational displacement(θ)
2.	Linear velocity (v) = dS/dt	Angular velocity (ω) = dθ/dt
3.	Linear acceleration (a) = dv/dt	Angular acceleration (α) = dω/dt
4.	Mass (m)	Moment of Inertia (I)
5.	Linear momentum p = mv	Angular momentum L = Iω
6.	Force F = ma	Torque τ = Iα
7.	Work done by a force W = $\int$ FdS	Work done by a torque W = $\int \tau d\theta$
8.	K.E. = $\frac{1}{2}mv^2$	K.E. = $\frac{1}{2}I\omega^2$
9.	Instantaneous power P = Fv	Instantaneous power = $\tau\omega$

Corresponding equations of motion are

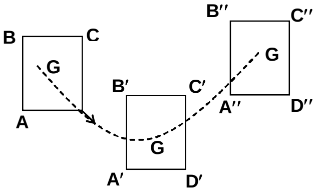
Linear motion	Rotational motion
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$

$S = v_0 t + \frac{1}{2}at^2$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$v^2 = v_0^2 + 2aS$ $\omega^2 = \omega_0^2 + 2\alpha\theta$

Special Types of Plane Motion

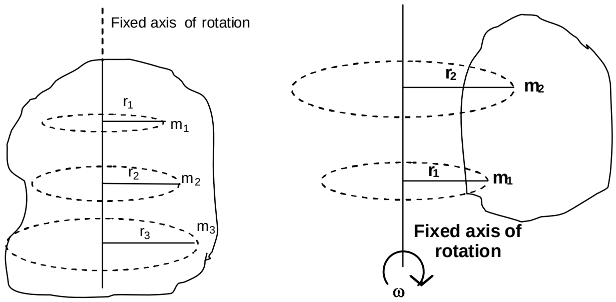
Translational motion:



1. Any line remains parallel to itself throughout the motion
   $\Rightarrow$ $\omega = 0$
2. All points of the body have the same displacement, the same velocity and same acceleration at a given instant.

Pure rotation (fixed axis rotation) – Kinetic energy and rotational inertia

The trajectory of each point on the rigid body must be an arc of a circle with its centre on the axis of rotation (a fixed line perpendicular to the plane of motion). The fixed axis of rotation may be inside or outside the body.



If $m_1, m_2, \ldots\ldots m_n$ are the masses of the constituent particles moving on circular paths with radii $r_1, r_2 \ldots r_n$ with velocities $v_1, v_2 \ldots\ldots v_n$, then the kinetic energy of the body is given by

$$KE = \sum_1^n \frac{1}{2}m_i v_i^2 = \sum_1^n \frac{1}{2}m_i r_i^2\omega^2 = \frac{1}{2}\omega^2\sum_1^n m_i r_i^2 = \frac{1}{2}I\omega^2$$

The terms $\sum_1^n m_i r_i^2$ is called rotational inertia or moment of inertia of the system of particles. For a continuous distribution of mass, $I = \int_{body} r^2 dm$ where dm is the mass of a particle at a distance r from the axis of rotation.

Moment of inertia of a body depends upon

(i) axis of rotation

(ii) mass of the system

(iii) distribution of mass about the axis. Its SI unit is kg-m^2

Physical significance of moment of inertia

Moment of inertia plays same role in rotational motion as mass does in translational motion. It is a measure of rotational inertia.

RADIUS OF GYRATION

If the moment of inertia (I) of a body of mass m about an axis be written in the form: $I = mk^2$

then, the quantity k, so defined, has the dimension of length and is known as the radius of gyration of the body about the given axis. It represents the radial distance from the given axis of rotation where the entire mass of the body can be assumed to be concentrated so that its rotational inertia remains unchanged.

Theorems on moment of inertia

(i) The parallel - Axis theorem:

The theorem states that moment of inertia of any distribution of matter about an arbitrary axis is equal to the moment of inertia of that matter about an axis through its centre of mass and parallel to the arbitrary axis plus the product of the mass of the matter and square of the distance between these two axes.

$I = I_{cm} + md^2$, where d is the perpendicular distance between two parallel axes.

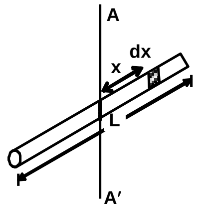
(ii) The perpendicular - Axes theorem:

The sum of moment of inertia of a plane lamina about two mutually perpendicular axes lying in its plane is equal to its moment of inertia about an axis perpendicular to the plane of lamina and passing through the point of intersection of these two axes.

Mathematically, $I_z = I_x + I_y$

Moment of Inertia of Some Common Bodies

(a) Thin Uniform Rod about a Perpendicular Bi-Sector: Consider a uniform rod of mass M and length L. Let us take a differential element of length dx, at a distance x from the middle of the rod which is assumed to be the origin.



Mass of the element = $\frac{M}{L}dx$

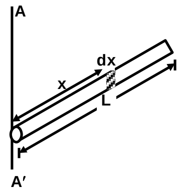
∴ Moment of inertia of this element about AA′ is

$$dI = \frac{M}{L}x^2dx$$

∴ Moment of inertia of the entire rod about AA′ is

$$I = \int_{-L/2}^{L/2}\frac{M}{L}x^2dx = \left[\frac{M}{L}\frac{x^3}{3}\right]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{ML^2}{12}$$

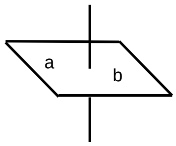
(b) Thin Uniform Rod about a Perpendicular axis passing through one of its ends:



If the axis AA′ passes through one of the ends then

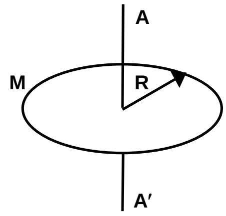
$$I = \int_0^L \frac{M}{L}x^2dx = \left[\frac{M}{L}\frac{x^3}{3}\right]_0^L = \frac{ML^2}{3}$$

(c) Rectangular lamina: For a rectangular lamina of sides of length ‘a’ and ‘b’ about a line perpendicular to its plane, passing through the intersection of the diagonals.



$$I = \frac{M\left(a^2+b^2\right)}{12}$$

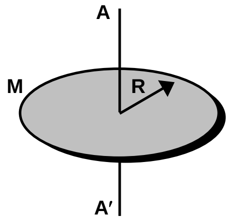
(d) Uniform circular ring about an axis



Let us consider the mass of the ring as M and radius R and all the elements of the ring are at the same perpendicular distance from the axis which passes through the centre and perpendicular to plane.

$\therefore \quad I = \int r^2 dm = \int R^2 dm = R^2 \int dm = MR^2$

(e) Uniform circular disc about an axis:



If the mass of the disc is M and its radius is R then mass per unit area $= \dfrac{M}{\pi R^2}$

The disc may be considered as combination of large number of elementary rings.

Let us consider one such elementary ring of radius x and width dx.

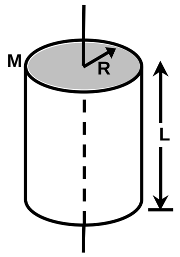
$\therefore$ Mass of the ring $= \dfrac{M}{\pi R^2}.2\pi x dx = \dfrac{2Mxdx}{R^2}$

The moment of inertia of the elementary ring about AA′, which passes through centre of disc and perpendicular to the plane is

$$dI = \left[\frac{2Mxdx}{R^2}\right]x^2$$

$\therefore$ Moment of inertia of the disc

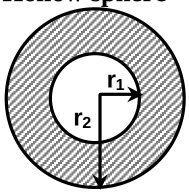
$$I = \int_0^R \frac{2M}{R^2} x^3 dx = \frac{MR^2}{2}$$



(f) Uniform cylinder about its axis:

Moment of inertia of a cylindrical shell and a solid cylinder about the given axis are respectively MR^2 and $(MR^2/2)$

Hollow sphere

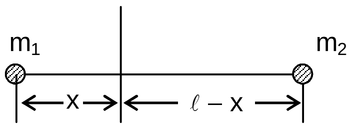


(g) Spherical Shell about a diameter:

Moment of inertia of a uniform thin spherical shell about a diameter is $(2/3)MR^2$ and that of a uniform solid sphere is $(2/5)MR^2$ where M is the corresponding mass and R be the radius. The moment of inertia of a hollow sphere with inner and outer radii r_1 and r_2 respectively, is given by $\dfrac{2}{5}M\dfrac{(r_2^5 - r_1^5)}{(r_2^3 - r_1^3)}$

Illustration 6. *What is the minimum moment of inertia of a system of particles of masses m_1 and m_2 being interconnected by a rigid light bar of length ℓ, about an axis perpendicular to the rod and passing through it.*

Solution: The M.I. $= I = m_1x^2 + m_2(\ell - x)^2$



$\Rightarrow \dfrac{dI}{dx} = 2m_1x + 2m_2(\ell - x)(-1) = 0,$

I is minimum for $x = \dfrac{m_2\ell}{m_1 + m_2}$

$\therefore \quad I_{min} = m_1\left(\frac{m_2\ell}{m_1+m_2}\right)^2 + m_2\left(\ell - \frac{m_2\ell}{m_1+m_2}\right)^2$

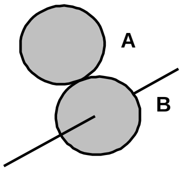
$\Rightarrow \quad I_{min} = \frac{m_1m_2\ell^2}{m_1+m_2}$

The axis must pass through the c.m. of the system.

Illustration 7. *Two thin discs each of mass 4.0 kg and radius 0.4 m are attached as shown in figure to form a rigid body. What is the rotational inertia of this body about an axis perpendicular to the plane of disc B and passing through its centre ?*

Solution : Moment of inertia of each disc A and B about the axis through their centre of mass and perpendicular to the plane will be

$I_{AA} = I_{BB} = (1/2)Mr^2$



Now moment of inertia of disc A about an axis through B by theorem of parallel axes will be

$I_{AB} = I_{AA} + M(2r)^2 = (9/2)Mr^2$

So, $I = I_{BB} + I_{AB} = \frac{1}{2}Mr^2 + \frac{9}{2}Mr^2$

$I = 5Mr^2 = 5 \times 4 \times (0.4)^2$ kg–m^2

$= 3.2$ kg–m^2

Illustration 8. *Find the moment of inertia of a square plate about a diagonal and show that it is equal to the rotational inertia about a median line.*

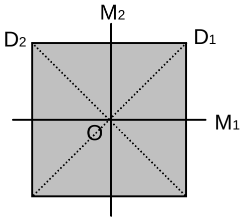
Solution : By symmetry $I_{D1} = I_{D2} = I_D$. The two diagonals of a square intersect at right angles, so using theorem of perpendicular axes

$I_z = 2I_D$

But for a square a = b = L

So, $I_z = M\frac{(L^2+L^2)}{12} = \frac{ML^2}{6}$

$\therefore \quad I_D = \frac{1}{2}I_Z = ML^2/12$



Again by symmetry $I_{M1} = I_{M2} = I_M$

Now as the two medians also intersect at right angles, by theorem of perpendicular axes

$I_z = 2I_M = 2I_D$

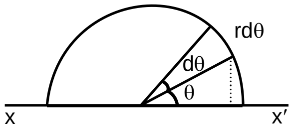
Thus, $I_M = I_D$

Illustration 9. *The moment of inertia of a uniform semicircular wire of mass M and radius r about a line passing through its ends is*

(A) Mr^2 *(B)* $\frac{1}{2}Mr^2$

(C) $\frac{1}{4}Mr^2$ *(D)* $\frac{2}{5}Mr^2$

Solution: Mass per unit length $= \dfrac{M}{\pi r}$



Mass of small element $= \dfrac{M}{\pi r} \times r\, d\theta \; = \dfrac{M\, d\theta}{\pi}$

The distance of the small element from the axis xx′

$(r') = r \sin\theta$

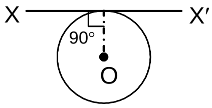
So moment of Inertia = $\int r^2\, dm$

$$= \int_0^{\pi} (r \sin\theta)^2 \cdot \frac{M\, d\theta}{\pi}$$

So, option (B) is correct.

$$= \frac{Mr^2}{\pi} \int_0^{\pi} r \sin^2 d\theta = \frac{M r^2}{2}$$

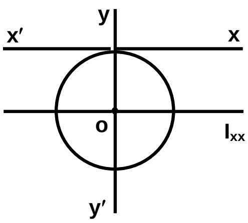
Illustration10. *A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown in figure. The moment of inertia of the loop about the axis XX′ is*



(A) $\dfrac{\rho L^3}{8\pi^2}$ *(B)* $\dfrac{\rho L^3}{16\pi^2}$

(C) $\dfrac{5\rho L^3}{16\pi^2}$ *(D)* $\dfrac{3\rho L^3}{8\pi^2}$

Solution: We know that moment of inertia of the ring about the line passing through centre and in the plane of the ring is



$$I_{xx} = \frac{MR^2}{2} = \frac{(\rho L)\left(\frac{L}{2\pi}\right)^2}{2}$$

$$= \frac{\rho L^3}{8\pi^2}$$

So by parallel axis theorem $I_{xx'} = I_{xx} + Md^2$

$$= \frac{\rho L^3}{8\pi^2} + \rho L \times R^2 = \frac{\rho L^3}{8\pi^2} + \rho L \times \left(\frac{L}{2\pi}\right)^2$$

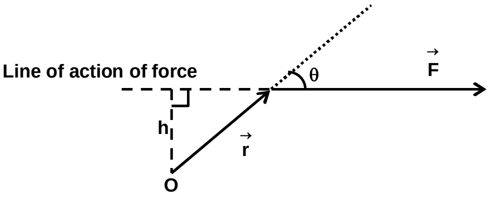
$$= \frac{\rho L^3}{8\pi^2} + \frac{\rho L^3}{4\pi^2} = \frac{3\rho L^3}{8\pi^2}$$

So option D is correct.

ROTATIONAL DYNAMICS

TORQUE (Moment of a force)

Torque or moment of force is the turning action of the force about a given point.



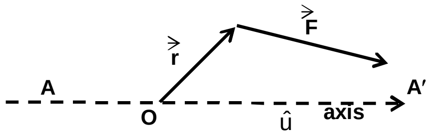
The torque vector $\vec{\tau}$ of a force $\vec{F}$ about the given point O is defined as the vector product

$\vec{\tau} = \vec{r} \times \vec{F}$

$= rF \sin\theta\ \hat{n} = Fh\,\hat{n}$

where $\hat{n}$ is the unit vector obtained by the vector product rule.

Torque about an axis



Let 'O' be any point on the axis AA′. The torque of the force $\vec{F}$ about the axis AA′ is defined as

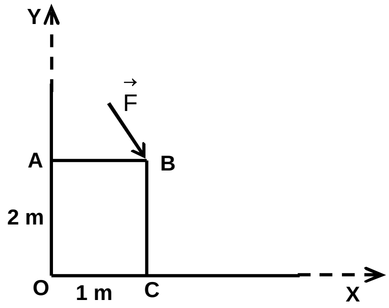
$$\tau_{AA'} = (\vec{r} \times \vec{F}).\hat{u}$$

where $\hat{u}$ is the unit vector along the axis.

The Torque Equation

If the net torque acting on a body about any inertial axis is $\vec{\tau}$ and the moment of inertia about that axis is I then the angular acceleration $\vec{\alpha}$ of the body is given by the relation: $\vec{\tau} = I\vec{\alpha}$

Illustration 11. *A rectangular plate in the x–y plane is shown in the figure. A force $\vec{F} = 2\hat{i} - 3\hat{j}$ N is applied at point B. Find the moment of $\vec{F}$ about*
(a) the origin O (b) the point C
(c) X–axis, Y–axis and Z–axis.



Solution: (a) Torque of $\vec{F}$ about O: $\vec{r} = \vec{OB} = \hat{i} + 2\hat{j}$

$$\vec{\tau}_0 = \vec{r} \times \vec{F} = (\hat{i} + 2\hat{j}) \times (2\hat{i} - 3\hat{j}) = -7\hat{k} \text{ N-m}$$

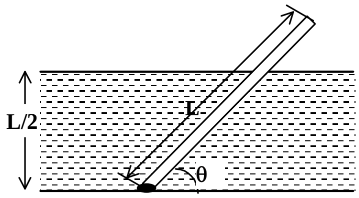
(b) $\vec{\tau}_C = \overrightarrow{CB} \times \vec{F} = 2\hat{j} \times (2\hat{i} - 3\hat{j}) = -4\hat{k}$ N-m

(c) $\tau_{XX'} = \vec{\tau}_0.\hat{i} = (-7\hat{k}).\hat{i} = 0$

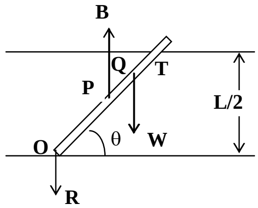
$\tau_{YY} = \vec{\tau}_0.\hat{j} = (-7\hat{k}).\hat{j} = 0$

$\tau_{ZZ'} = \vec{\tau}_0.\hat{k} = (-7\hat{k}).\hat{k} = -7$ N-m.

Illustration 12. *A uniform rod of length L pivoted at the bottom of pond of depth L/2 stays in stable equilibrium as shown in figure. Find*
(a) the angle θ if the density of the material of rod is half of the density of water.
(b) the force acting at the pivoted end of the rod in terms of mass m of the rod.



Solution: (a) Weight of the rod w = LAρg
(A = area of cross section,
ρ = density of material)
submerged length of the rod



$$OT = \frac{L}{2\sin\theta}$$

Buoyant force, $B = \left(\frac{L}{2\sin\theta}\right)A\rho_w g$

(from Archimedies principle) (ρ_w = density of water)

This acts at P, where $OP = \frac{OT}{2} = \frac{\ell}{4\sin\theta}$

Balancing torque about O

W. OQ cos θ = B. OP cos θ

$$(L\,A\,\rho g)\,L/2 = \left(\frac{L}{2\sin\theta}\right)A\rho_w g\left(\frac{L}{4\sin\theta}\right)$$

$$\sin^2\theta = \frac{\rho_w}{4\rho}$$

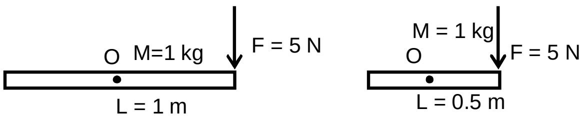
$\Rightarrow \sin^2\theta = 1/2 \Rightarrow \theta = 45°$
(b) Also, balancing forces
R + W = B
R = B – W

$= \dfrac{\rho A\rho_w g}{2\sin\theta} - LA\rho g$

$= L\,A\rho\,g\left[\dfrac{1}{\sin\theta} - 1\right]$ $(\because \rho_w = 2\rho)$

$= (\sqrt{2} - 1)\,mg$

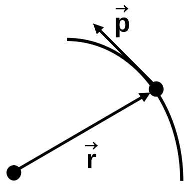
Exercise 1. ***(a) Which is easier to rotate about pivot at O***
(b) Can a single force rotate a body?



Angular momentum or moment of momentum

Angular momentum of a particle about a given point is defined as

$\vec{L} = \vec{r}\times\vec{p}$



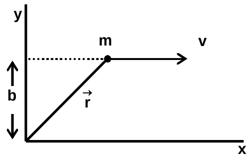
Here $\vec{r}$ = position vector of the particle with respect to the given point.
$\vec{p}$ = linear momentum of the particle relative to the point.

Direction of $\vec{L}$ can be given by the rules of vector product.

Magnitude of the angular momentum of a particle about a fixed point is equal to the product of its linear momentum and length of the perpendicular to the line of linear momentum from the fixed point.

A body, which translates, can also have angular momentum. It is not necessary for a body to rotate to have angular momentum. However, angular momentum depends on the point about which it is taken.

Illustration 13. *A particle of mass m is moving with constant velocity v parallel to the x–axis in x–y plane as shown in figure. Calculate its angular momentum with respect to origin at any time t.*



Solution : At any time t coordinates of the particle will be

$x = vt, \quad y = b, \quad z = 0$

and velocity components will

$v_x = v, \quad v_y = 0, \quad v_z = 0$

(The particle is moving parallel to x–axis)

$$\vec{L} = \vec{r}\times\vec{p} = m\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ vt & b & 0\\ v & 0 & 0\end{vmatrix}$$

i.e. $\vec{L} = m\hat{k}\left[vt\times 0 - vb\right] = -\,mvb\,\hat{k}$

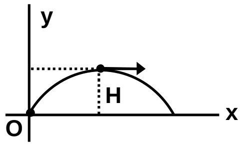
Thus, angular momentum has magnitude mvb and is directed along negative z–axis.

Illustration 14. *A particle of mass m is projected with velocity v at an angle θ with the horizontal. Find its angular momentum about the point of projection when it is at the highest point of its trajectory.*

Solution : At the highest point it has only horizontal velocity

$v_x = v\cos\theta$

Length of the perpendicular to the horizontal velocity from 'O' is the maximum height,

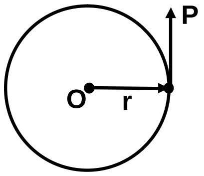


$$H_{max} = \frac{v^2 \sin^2\theta}{2g}$$

$\Rightarrow$ The required angular momentum

$$L = \frac{mv^3 \sin^2\theta\cos\theta}{2g}$$ and is directed along the negative z–axis.

Angular momentum of a particle describing circular motion:

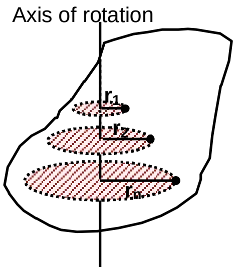


$\vec{L} = \vec{r}\times\vec{p}$; as linear momentum $(\vec{p})$ is along the tangent, hence $\vec{r}\times\vec{p} = rp\hat{n}$, where $\hat{n}$ is the unit vector perpendicular to the plane of the circle.

$\Rightarrow$ $|\vec{L}| = mvr = m\omega r^2$, where $|\vec{p}| = mv = m\omega r$

Angular momentum of a rigid body in a fixed axis rotation

In a fixed axis rotation, all the constituent particles describe circular motion. Hence, angular momentum of the particles about corresponding centres are



$L_1 = m_1\omega r_1^2,\ L_2 = m_2\omega r_2^2$

and similarly $L_n = m_n\omega\, r_n^2$ where ω is the angular speed of the body.

Since angular momentum of all the particles have same direction therefore angular momentum of the whole body is given by

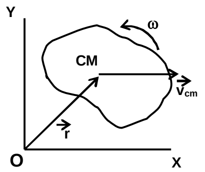
$L = L_1 + L_2 + \ldots\ldots\ldots.. + L_n$

$= \left(m_1r_1^2 + m_2r_2^2 + \ldots\ldots\ldots. + m_nr_n^2\right)\ \omega$

$\Rightarrow L = I\omega$

Angular momentum of a body describing general plane motion

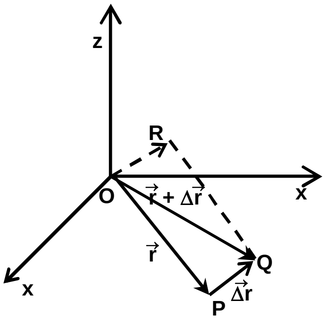
The angular momentum of a body about a point 'O' in the lab frame XOY is given by



$\vec{L}_O = I_{cm}\vec{\omega} + \vec{r}\times m\vec{v}_{cm}$

The angular momentum of a body point 'P' on the body is given by $\vec{L} = I_p\vec{\omega}$

Physical meaning of angular momentum



Consider a particle rotating about origin in x–y plane. Its position vector at time t is $\vec{r}$. At time (t + Δt), position vector is $\left(\vec{r}+\Delta\vec{r}\right)$. If $\vec{v}$ is velocity at time t,

$$\vec{v}\Delta t = (\vec{r}+\Delta\vec{r}) - \vec{r} = \Delta\vec{r}$$

Draw $\overrightarrow{OR}$ parallel to $\overrightarrow{PQ}$ and complete the parallelogram. Then, area of parallelogram,

$$\Delta\vec{A} = \vec{r}\times\Delta\vec{r}$$

Area of triangle OPQ, $\Delta\vec{A} = \frac{1}{2}(\vec{r}\times\Delta\vec{r}) = \frac{1}{2}(\vec{r}\times\vec{v}\Delta t)$

$$\frac{\Delta\vec{A}}{\Delta t} = \frac{1}{2}\vec{r}\times\vec{v} = \frac{1}{2}\vec{r}\times\frac{\vec{p}}{m} = \frac{1}{2m}\vec{r}\times\vec{p}$$

$\therefore$ $\frac{\Delta\vec{A}}{\Delta t} = \frac{1}{2m}\vec{L}$; where $\vec{L}$ is angular momentum

Hence, angular momentum = 2 × mass × Areal velocity of the particle.
Here areal velocity is rate of area swept per unit time.
If during the motion, angular momentum is constant, areal velocity of the particle remains constant.

Relation between Torque and Angular Momentum:

We know that $\vec{L} = \vec{r}\times\vec{p}$

$$\Rightarrow \quad \frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt} = \vec{v}\times\left(m\vec{v}\right) + \vec{r}\times\vec{F}_{ext}$$

$$= 0 + \vec{r}\times\vec{F}_{ext} = \vec{\tau}_{ext}$$

$$\Rightarrow \quad \frac{d\vec{L}}{dt} = \vec{\tau}_{ext}$$

Conservation of angular momentum

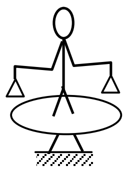
Since $\frac{d\vec{L}}{dt} = \vec{\tau}_{ext}$, then if $\vec{\tau}_{ext} = 0$, $\vec{L}$ must be constant.

Hence, "In absence of any net external torque about an axis, the total angular momentum of the system about that axis remains constant".

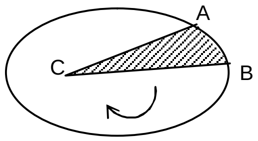
Examples of conservation of angular momentum:

1. A diver dives with his hands stretched out. In the mid air, he squeezes his hands and legs. His body rotates at a high angular speed. Before hitting the water he stretches his hands and legs which reduces his angular speed.
   By squeezing the hands and legs, momentum of inertia of the diver's body decreases resulting in corresponding increase in angular speed (L = Iω is constant)

2. Consider a man standing on a rotating platform with some weights in his hands. If he withdraws his hands close to his chest, his angular speed increase as his momentum of inertia is decreased.

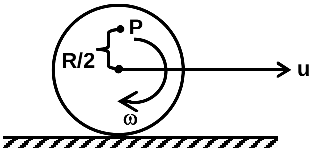


3. A planet is revolving around the sun in an elliptical orbit. When it goes near the sun, its speed increases and when it goes farther from the sun, its speed decreases. It happens because angular momentum is conserved and hence time rate of area swept by the planet is constant.



4. A planet revolves around the sun in an orbit in a fixed plane only. This is due to conservation of angular momentum.

Illustration 15. *Find the angular momentum of a cylinder of radius R rolling on a horizontal floor about a point P as shown in figure, if point 'P' is*
(i) On the ground
(ii) On the cylinder



Solution: For pure rolling, v = Rω and

MI of cylinder $= \dfrac{MR^2}{2}$

(i) If point P is on the ground then

$$\vec{L}_{LG} = I_{cm}\vec{\omega} + (\vec{r} \times \vec{v}_{cm})$$

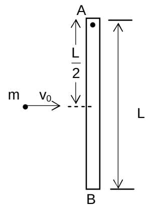
$$= \left[\frac{MR^2}{2}\omega M\left(-\frac{R}{2}\times v\right)\right](-\hat{k})$$

$$= -\left[\frac{MR^2\omega}{2} - \frac{MR^2\omega}{2}\right]\hat{k} = \text{zero}$$

(ii) If point P is on the cylinder, then

$$\left|\vec{L}_{PC}\right| = I\rho\omega = \left(I_{cm} + \frac{MR^2}{4}\right)\omega = \frac{3MR^2\omega}{4}$$

Illustration 16. *A uniform rod AB which is free to swing in the vertical plane about a horizontal axis through A, is hanging freely. A particle of equal mass strikes the rod with a velocity v_0 and gets stuck to it. Find the angular velocity of the combination immediately after the collision.*



Solution: As the net torque about A during the collision is zero, the angular momentum of the system about A is conserved.

$$\therefore\ mv_0\left(\frac{L}{2}\right) = \left(m\left(\frac{L}{2}\right)^2 + \frac{mL^2}{3}\right)\omega$$

$$\omega = \frac{6v_0}{7L} \text{ (anticlockwise)}$$

Illustration 17. *A particle performs uniform circular motion with an angular momentum L. If the frequency of particle's motion is doubled and its kinetic energy is halved, the angular momentum becomes:*

(A) 2L *(B) 4L*
(C) L/2 *(D) L/4*

Solution: Since $(KE)_f = \frac{1}{2}(KE)_i$ (By keeping the mass constant)

$\Rightarrow v_f = \frac{v_i}{\sqrt{2}}$

Also $(\text{frequency})_f = 2\ (f_i)$

So, $\omega_f = 2\ \omega_I$

Therefore $r_f = \left(\frac{v_f}{\omega_f}\right) = \frac{v_i/\sqrt{2}}{2\omega i} = \frac{r_i}{2\sqrt{2}}$

Therefore, $L_f = m \times r \times v_f = m \times \frac{r_i}{2\sqrt{2}} \times \frac{v_i}{\sqrt{2}} = \frac{L}{4}$

So option (D) is correct.

Exercise 2. ***(a) When ballet dancers dance on an ice floor, they suddenly seem to increase their angular velocity. How is it possible?***
(b) Why does a rolling thin disc not topple whereas a standing one topples?

Angular impulse (Moment of Impulse)

Angular impulse of a torque in a given time is equal to the change in angular momentum. If angular momentum of a body is changed by a torque $\vec{\tau}$ in time dt then

$$\vec{J} = \int_{t_1}^{t_2} \vec{\tau}\ dt, \text{ where } \vec{J} = \text{angular impulse.}$$

$$\Rightarrow \quad \vec{J} = \int_{t_1}^{t_2} \frac{d\vec{L}}{dt} dt = \int_{L_1}^{L_2} d\vec{L}$$

$$\Rightarrow \quad \vec{J} = \vec{L}_2 - \vec{L}_1 = \Delta\vec{L}$$

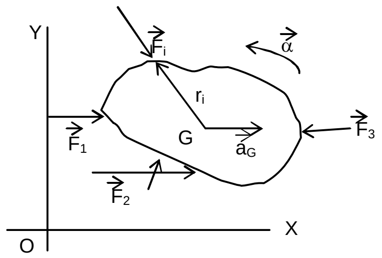
Force and torque equations for a rigid body in plane motion

(i) Σ (External forces) = mass of the body × acceleration of the centre of mass

$$\Sigma(\vec{F}_i) = m\vec{a}_G$$

(ii) Σ (moments of external forces about the centroidal axis) = moment of inertia of the body about the centroidal axis × angular acceleration

$$\Rightarrow \Sigma(\vec{r}_i \times \vec{F}_i).\hat{k} = \vec{a}_G I_G$$



Instantaneous axis of rotation in plane motion

Instantaneous axis of rotation is perpendicular to the plane of motion and passes through the point which is momentarily at rest.

About the instantaneous axis of rotation, the plane motion can be treated instantaneously as fixed axis rotation. (Only as far as velocities of the constituent particles are concerned). The acceleration of the particles, however, can not be found by treating the body as having fixed axis rotation about the instantaneous centre.

Let P be the instantaneous centre so that the instantaneous velocity of P, $\vec{v}_p = 0$

Then the following equations can be written which simplify the solution of the problems where the instantaneous centre is obvious, for example – fixed axis rotation and pure rolling motion.

1. The angular momentum of the body about the instantaneous axis of rotation

$$\vec{L}_P = I_P\vec{\omega}$$

2. The kinetic energy of the body KE = $\frac{1}{2} I_P\, \omega^2$.
3. Σ(moments of forces about the instantaneous axis) = $I_P\, \alpha$

Illustration 18. *A 10 m long ladder rests against a vertical wall and makes an angle of 60° with the horizontal floor. If it starts to slip, then find the position of its instantaneous axis of rotation.*

Solution : Consider the ends A and B of the ladder. Velocity of end A is vertically downward and velocity of end B is along the horizontal floor. As perpendiculars to the velocities meet at point O. Therefore, axis of rotation will pass through this point and will be perpendicular to the plane of ABC.

Taking C as origin co–ordinates of the point O are 10 cos60° m and 10sin60° m. Hence radius vector of the point O w.r.t. point C is $\vec{R} = 5\hat{i} + 5\sqrt{3}\hat{j}$ where $\hat{i}$ and $\hat{j}$ are the unit vectors along horizontal and vertical.

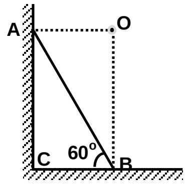
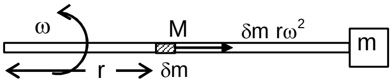


Illustration 19. *A rod of mass M, length ℓ is attached with a small bob of mass m at its end is freely rotating about a vertical axis passing through its other end with a constant angular speed ω. Find the force exerted by the rod and bob on the pivot. Assume that gravity is absent.*

Solution : We select a co–rotating frame of reference with respect to the rod. The centrifugal force experienced by the elementary segment of length dr & mass δm at a distance r from the axis is given by the expression



$dF = (\delta m)\, r\omega^2 \Rightarrow$ The total centrifugal force experienced by the rod

$$= F = \omega^2 \int r\,(dm) = \omega^2 \int_0^{\ell} r(M/\ell).dr \quad \Rightarrow \quad F = \frac{M\omega^2}{\ell}\left(\frac{r^2}{2}\right)\Bigg|_0^{\ell} = \frac{M\omega^2\ell}{2}$$

The centrifugal force exerted on the bob, $F' = m\,\ell\omega^2$

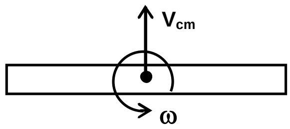
$\therefore$ The total centrifugal force = $F_t = F + F' = \frac{M\omega^2\ell}{2} + m\,\omega^2\,\ell$

$$F_t = \left(\frac{M}{2} + m\right)\omega^2\ell$$

Centrifugal force is a pseudo force being experienced in an non–inertial plane.

Hence the required force $= \left[\frac{M}{2} + m\right]\omega^2\,\ell$.

Illustration 20. *As shown in figure, an uniform rod has linear velocity v, and angular velocity about its centre is $\frac{6v}{5L}$ where L is the length of the rod, the instantaneous axis of rotation passing through the point at a distance from the centre of the rod is*



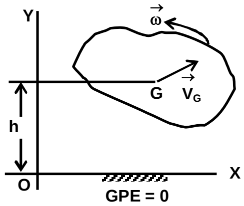
(A) $\frac{5L}{3}$ *towards left* *(B)* $\frac{5L}{6}$ *towards right*
(C) $\frac{5L}{6}$ *towards left* *(D) none of these*

Solution: For instantaneous axis of rotation, $V - R\omega = 0$

$$R = \frac{V}{\omega} = \frac{5L}{6} \text{ towards left}$$

So, option (c) is correct.

Total mechanical energy in plane motion



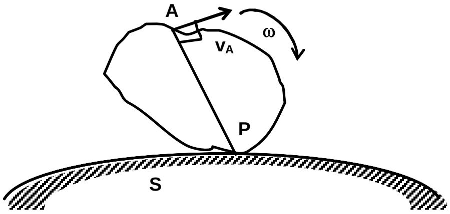
Gravitational potential energy = mgh

Kinetic energy = $\frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Total mechanical energy=Gravitational potential energy+ Kinetic energy

Exercise 3. ***Is it possible that angular momentum in a certain situation is conserved but linear momentum is not conserved?***

Rolling Motion



When a rigid body moves on a surface S at rest such that the velocity of point of contact P relative to the surface is zero, it is said to be rolling on surface 'S'

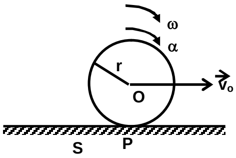
$$\vec{v}_{PS} = 0$$

i.e. P is the instantaneous centre of rotation

$$\therefore \quad \vec{v}_{AS} = \vec{v}_{AP} + \vec{v}_{PS} = \vec{v}_{AP} + 0 = \vec{v}_{AP} = \vec{\omega} \times \overrightarrow{PA}$$

$$v_{AS} = (PA)\omega$$

Rolling of disc /ring/sphere on a flat surface:

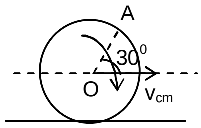


Velocity of the geometric centre O

$$v_{OS} = r\omega \rightarrow$$

$$a_{OS} = r\alpha \rightarrow$$

Illustrtion 21. *A disc of mass m is rolling on a rough surface. The velocity of centre of mass of disc is 10 m/s. Find the velocity of point A which makes an angle of 30° from horizontal anticlockwise as shown in figure. (There is no slipping at point of contact.)*



Solution: As disc is in pure rolling state

$v_{cm} = r\omega = 10$ m/s.

$\bar{v}_A = 10\hat{i} + 10\cos 60\hat{i} - 10\sin 60\hat{j}$

$\bar{v}_A = 15\hat{i} - 5\sqrt{3}\,\hat{j}$

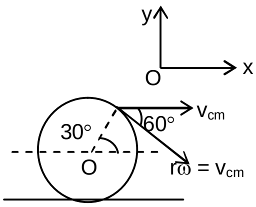
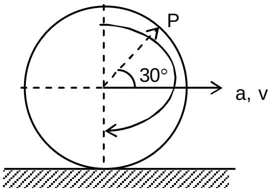


Illustration 22. *A disc of radius R is rolling on a horizontal surface. If the acceleration of centre of mass of the disc is a and velocity is v. Find the acceleration of point P at the instant represented in the figure.*



Solution: $a_P = \left(\dfrac{3a}{2} - \dfrac{v^2\sqrt{3}}{2R}\right)\hat{i} - \left(\dfrac{a\sqrt{3}}{2} + \dfrac{v^2}{2R}\right)\hat{j}$

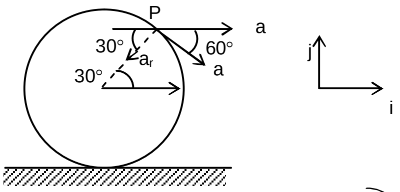
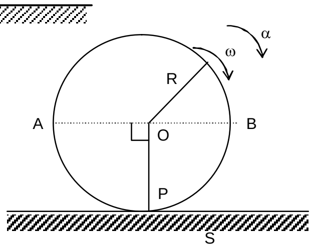


Illustration 23. *A disc of radius R is rolling on a straight surface S without slipping (velocity of the point of contact P relative to S = 0). The instantaneous angular velocity ω and angular acceleration α are as shown in the figure. Find*



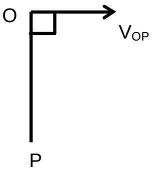
(a) the velocity and acceleration of point O, A and P on the disc relative to the surface S.
(b) the velocity and acceleration of point A relative to point B.

Solution : Geometric centre O :This point moves in a straight line parallel to the surface S

(OP = R at any time)

$\dfrac{F}{2} + \dfrac{mR\alpha}{2} = 2ma_2 = \vec{v}_{OP}$

= (OP) ω = R ω directed as shown.

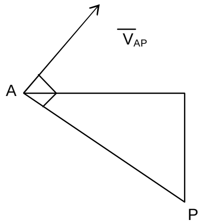


∴ Acceleration of O $\vec{a}_{OS} = \dfrac{d\vec{v}_0}{dt} = R\dfrac{d\omega}{dt} = R\alpha$

Point A :

$\dfrac{m_2}{m_1 + m_2}$

= (PA) ω = $\sqrt{2}$ Rω

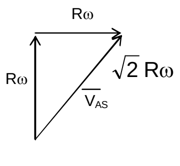


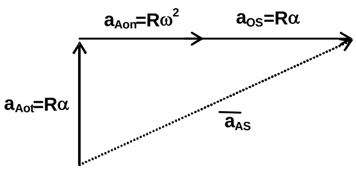
Alternatively:

$\vec{v}_{AS} = \vec{v}_{AO} + \vec{v}_{OS}$

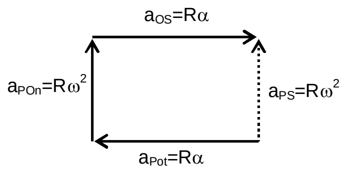
$\vec{a}_{AS} = \vec{a}_{AO} + \vec{a}_{OS}$

$= \vec{a}_{Aot} + \vec{a}_{Aon} + \vec{a}_{OS}$





Point P : $\frac{p^2}{2M}+\frac{p^2}{2m_e}$ (Given as a condition for rolling without slipping)



$\vec{a}_{PS}=\vec{a}_{PO}+\vec{a}_{OS}=\vec{a}_{POt}+\vec{a}_{POn}+\vec{a}_{OS}$

Note: The following argument is wrong:

$\vec{v}_{PS}=0 \quad \Rightarrow \quad \frac{d\vec{v}_{PS}}{dt}=0 \quad \Rightarrow \quad \vec{a}_{PS}=0$

(b) $\vec{v}_{AB}=(AB)\omega=2R\omega$

$\vec{a}_{AB}=\vec{a}_{ABt}+\vec{a}_{ABn}$

Thus, the velocity and acceleration of any point on the disc can be determined.

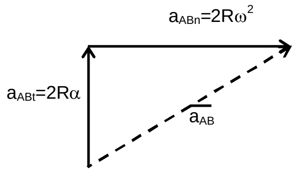
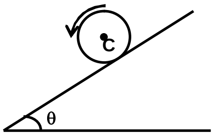


Illustration 24. *A uniform disc of mass m and radius R is rolling down a fixed inclined plane without slipping, under the action of gravitational force and the ground contact force as shown in the figure. Find*
(a) the acceleration of the centre C.
(b) the angular acceleration.
(c) the force of friction and the normal reaction.



Solution: For pure rolling motion

$a_c = R\alpha$ . . . (i)

As the centre of mass coincides with the geometric centre of the disc,

$a_G = R\alpha$ . . . (ii)

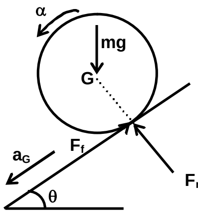
Force equations :

$F_n - mg\cos\theta = 0$ . . . (iii)

$-F_f + mg\sin\theta = ma_G$ . . . (iv)

Torque equation : $F_fR = I_G\,\alpha$ . .. (v)

For a uniform disc $I_G=\frac{mR^2}{2}$ . . . (vi)



FBD of the disc

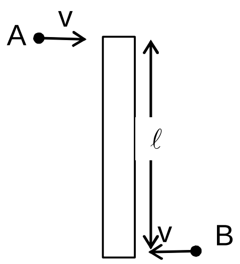
Solving the equation (ii) through (vi)

$a_G=\frac{2g\sin\theta}{3}$; $\alpha=\frac{2g\sin\theta}{3R}$

$F_f=\frac{mg\sin\theta}{3}$; $F_n = mg\cos\theta$

Pure rolling will prevail if $F_f \le \mu F_n$, i.e. $\mu \ge \frac{F_f}{F_n} \Rightarrow \mu \ge \frac{1}{3}\tan\theta$.

Illustration 25. *Two particles A & B, each of mass m and moving with velocity v, hit the ends of a rigid bar of mass M and length ℓ simultaneously and stick to the bar. The bar is kept on a smooth horizontal plane (as shown). Find the linear and angular speed of the system (bar + particle) after the collision.*



Solution: Conservation of linear momentum :

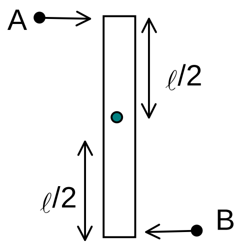
$P_i = mv - mv = 0$

$\Rightarrow \quad v_{cm} = 0$

Conservation of angular momentum

$\Rightarrow \quad L_{initial} = L_{final}$

where $L_{initial} = \left|\frac{m\ell}{2}v + \frac{m\ell}{2}v\right| = mv\ell$



Let the system rotate about its c.m. 'O' with an angular speed ω

$\Rightarrow \quad L_{final} = (I_{system})\,\omega$

where $I_{system} = \frac{M\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 + m\left(\frac{\ell}{2}\right)^2 = \left(\frac{M+6m}{12}\right)\ell^2$

$\therefore \quad \left(\frac{M+6m}{12}\right)\omega\ell^2 = mv\ell$

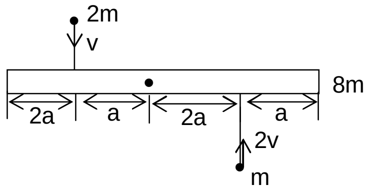
$\Rightarrow \quad \omega = \frac{12\ mv}{(M+6m)\ell}$.

Illustration 26.. *A uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two point masses 2m and m moving in the same horizontal plane with speed v and 2v respectively, strike the bar as shown in figure and stick to the bar after collision. Calculate*

(a) the velocity of the centre of mass.

(b) angular velocity about centre of mass.

(c) total kinetic energy just after collision.



Solution: (i) $F_{ext} = 0$, by applying COM

$-2m \times v + m \times 2v + 0 = (2m + m + 8m)v$

$v = 0$

(ii) $\tau_{ext} = 0$, by applying COAM

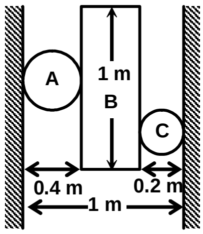
$2mva + m(2v)(2a) = [2m(a)^2 + m(2a)^2 + 8m \times (6a)^2/12]\omega$

$\omega = (v/5a)$

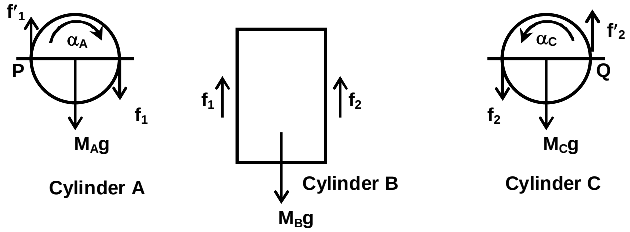
(iii) from part(i) and (ii), the system has no translating motion but only rotating motion

$E = \frac{1}{2}I\omega^2 = \frac{1}{2}(30ma^2)\left[\frac{v}{5a}\right] = \frac{3}{5}mv^2$

Illustration 27. *Three cylinders A, B and C are held between two walls at a separation of 1m (see in figure). The length of cylinder B is 1m and mass 2.5 kg while the masses and diameters of A and C are 2 kg, 0.4m and 1 kg and 0.2 m respectively. The system is in a vertical plane and is released from rest. The cylinders A and C roll without slipping along the vertical wall and cylinder B. Find the acceleration of the cylinder B which is constrained to move in a vertical direction.*



Solution: Free body diagrams :



For the cylinder B

$M_Bg - (f_1 + f_2) = M_B\, a_B$

$f_1 + f_2 = 25 - 2.5a_B$ . . . (i)

Taking torque about point P for cylinder A

$\tau_P = I_P\, \alpha_A = f_1 \times 2r_1 + M_Ag \times r$

$[\frac{1}{2}\times 2\times (0.2)^2 + 2 \times (0.2)^2\,]\ \alpha_A = f_1 \times 0.4 + 2 \times 10 \times 0.2$

$0.1\,2\ \alpha_A = 0.4\ f_1 + 4$ . . . (ii)

Taking torque about point Q for cylinder C

$\tau_Q = I_Q\ \alpha_C = f_2 \times 2r_2 + M_c.g \times r_2$

$[\frac{1}{2}\times 1\times (0.1)^2 + 1\times (0.1)^2]\alpha_c = f_2 \times 2\times 0.1 + 1\times 10\times 0.1\,]$

$0.015\ \alpha_c = 0.2\ f_2 + 1$ . . . (iii)

Also, $a_B = \alpha_A \times 0.4$

$\alpha_A = \frac{1}{0.4}a_B = 2.5a_B$

and $a_B = \alpha_C \times 0.2$

$\alpha_C = 5\ a_B$

Putting values of $\alpha_A \propto \alpha_C$ in (ii) and (iii) ,

$0.12 \times 2.5\ a_B = 0.4\ f_1 + 4$

$\Rightarrow$ $0.3\ a_B = 0.4\ f_1 + 4$ . . . (iv)

$0.015 \times 5a_B = 0.2\ f_2 + 1$

$\Rightarrow$ $0.15\ a_B = 0.4f_2 + 2$ . . . (v)

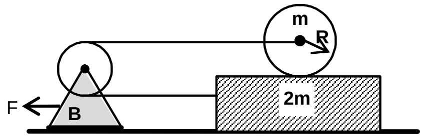
From (iv) and (v)

$(0.3 + 0.15)a_B = 0.4(f_1 + f_2) + 6$

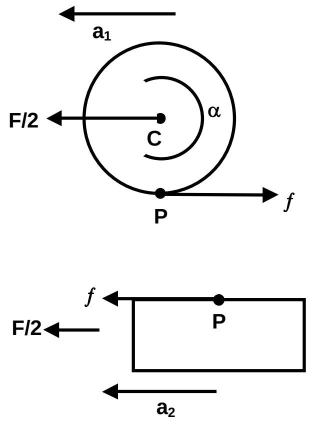
$0.45\ a_B = 0.4\ (25 - 2.5\ a_B) + 6$

$a_B = \frac{16}{1.45} = 11.03\ \text{m/s}^2$

Illustration 28. *A solid cylinder of mass m and radius R rest on a plank of mass 2m lying on a smooth horizontal surface. String connecting cylinder to the plank is passing over a massless pulley mounted on a movable light block B and the friction between the cylinder and the plank is sufficient to prevent slipping.*



If the block B is pulled with a constant force F, find the acceleration of the cylinder and that of the plank.



Solution: For the cylinder, $\frac{F}{2} - f = ma_1$ . . . (1)

Taking moment of forces about C, $fR = I\alpha$

$\Rightarrow \quad f = \frac{mR\alpha}{2}$ . . . (2)

$\because \quad I = mR^2/2$

For the plank, $\frac{F}{2} + f = (2m)a_2$ . . . (3)

And at the point of contact P, the acceleration of the two bodies must be same

$a_1 - R\alpha = a_2$ . . . (4)

From (1) and (3) , (2) and (4)

$F = (ma_1 + 2ma_2)$

and $\frac{F}{2} + \frac{mR\alpha}{2} = 2ma_2$

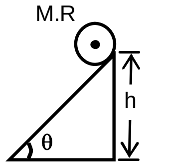
Substituting value of α and solving for a_1 and a_2 gives,

Acceleration of plank $a_2 = \frac{2F}{7m}$, Acceleration of cylinder $a_1 = \frac{3F}{7m}$

Illustration 29.. *A solid cylinder of mass M and radius R rolls down an inclined plane with height h without slipping. The speed V_A of its centre of mass when it reaches its bottom is*

(A) $\sqrt{2gh}$ *(B)* $\sqrt{\frac{4}{3}gh}$

(C) $\sqrt{\frac{3}{4}gh}$ *(D)* $\sqrt{\frac{4g}{h}}$



Solution: If it undergoes pure rolling than at any instant of time $v = R\omega$ and there is no loss of energy against friction, therefore by conservation of energy

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2} \times \frac{MR^2}{2} \times \left(\frac{v}{R}\right)^2$$

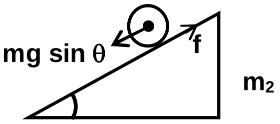
$$gh = \frac{3}{4}v^2; \quad v = \sqrt{\frac{4gh}{3}}$$

So option (B) is correct

Illustration 30. *A solid sphere rolls down two different inclined planes of the same height but of different inclinations*

(A) In both cases the speeds and time of descend will be same
(B) The speeds will be same but time of descend will be different
(C) The speeds will be different but time of descend will be same
(D) Speeds and time of descend both will be different

Solution: By law conservation of energy, velocities of the two spheres are same



$\text{Mg} \sin \theta - f = \text{Ma}$

$f \times R = I\alpha$

$\Rightarrow \quad f = \dfrac{Ia}{R^2}$

So, $\quad a = \dfrac{Mg\sin\theta}{\left(M + I/R^2\right)}$

$\Rightarrow \quad a \propto \theta$

$\Rightarrow$ time will be different.

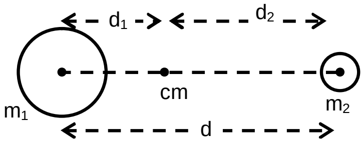
Hence, option (B) is correct.

Exercise 4. ***Without weighing how can you distinguish between hollow and solid spheres of same shape, size and mass ?***

Binary Systems in Nature

In a binary system two bodies revolve about their common centre of mass due to their mutual attractive forces i.e. no external force acts on them.

Let there be two celestial bodies of mass m_1 and m_2 revolving about their common centre of mass.



By definition of centre of mass

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

Taking centre of mass m_1, as origin,

$$d_1 = \frac{m_2}{m_1 + m_2} d$$

and $d_2 = d - d_1$ where d_1, d_2 and d are as shown in figure.

Gravitational force provides the required centripetal force, hence,

$$\frac{Gm_1m_2}{d^2} = \frac{m_1v_1^2}{d_1}$$

$$\frac{Gm_2}{d^2} = \frac{v_1^2}{d_1} = \left(\frac{2\pi d_1}{T}\right)^2 \frac{1}{d_1} \quad \left(\because \frac{2\pi d_1}{v_1} = T\right)$$

where T is time period of one revolution.

$$= \frac{4\pi^2 d_1}{T^2} = \frac{4\pi^2}{T^2}\left(\frac{m_2}{m_1 + m_2}\right)d$$

$$\therefore \qquad m_1 + m_2 = \frac{4\pi^2}{T^2G}d^3$$

If one of the masses is known, the other mass can be calculated knowing T and d.

Typical examples of such systems are earth–moon system, Binary stars.

Reduced Mass

Consider an isolated hydrogen atom. It is a binary system with a proton and an electron revolving about each other under mutual Coulomb's force of attraction.

In Bohr's model, nucleus is considered to be stationary which means nucleus mass is taken to be infinite compared to mass of an electron.

Taking M as mass of nucleus and m_e as mass of electron and assuming zero momentum of the atom,

Momentum of proton = momentum of electron = p (say).

Then, K. E. of the system $= \frac{p^2}{2M} + \frac{p^2}{2m_e} = \frac{p^2}{2\mu}$, where $\mu = \frac{Mm_e}{M + m_e}$

This μ is said to be the reduced mass.

By the above, a two particle system is reduced to an equivalent one particle system under the action of the interactive force.

Another example of a binary system is diatomic molecule where two atoms (similar or dissimilar) revolve about their common centre of mass. It can also be considered equivalent to a one particle system with reduced mass.

If one mass is much larger than the other, i.e. $m_1 >> m_2$, then

$$\mu = \frac{m_2}{1 + \frac{m_2}{m_1}} = m_2$$

The reduced mass becomes mass of the smaller mass. This is applicable to all binary systems where one mass is much smaller than the other one.

MISCELLANEOUS EXERCISE

1. A disc of metal is melted and a solid sphere is formed. What will happen to the moment of inertia about a vertical axis passing through the centre ?

2. What are the unit and dimensions of moment of inertia ? Is it a vector ?

3. Is radius of gyration of the body a constant quantity?

4. What is the rotational analogue of force and mass.

5. Can a body in translatory motion have angular momentum ?

6. Why spin angular velocity of star is greatly enhanced when it collapses under gravitational pull and becomes a neutron star.

7. Why there are two propellers in a helicopter.

8. The moment of inertia and the angular momentum of two bodies A and B are equal then which has a greater kinetic energy ?

9. A canon ball and marble ball roll from rest down and inclined which goes to the bottom first.

10. If the earth were to shrink suddenly what will happen to the length of the day ?

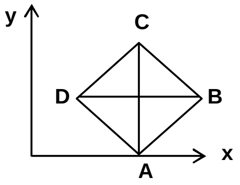
ANSWERS TO MISCELLANEOUS EXERCISE

1. Decrease
2. kg m^2, [$M^1L^2T^\circ$], No.
3. No.
4. Torque and moment of Inertia.
5. Yes
6. Since angular momentum is conserved therefore, as I decrease ω increase.
7. For balancing itself
8. $(K.E.)_B > (K.E.)_A$
9. Both will reach simultaneously.
10. Decrease

SOLVED PROBLEMS

Subjective:

Prob 1. *Four particles, each of mass m are on the vertices of square of side a as shown in the figure. Find out moment of inertia of the system about the x–axis.*



Sol. Perpendicular distance of A from x–axis = 0

Perpendicular distance of B from x–axis = $\frac{a}{\sqrt{2}}$

Perpendicular distance of C from x–axis = $a\sqrt{2}$

Perpendicular distance of D from x–axis = $\frac{a}{\sqrt{2}}$

$\therefore$ Moment of inertia = $0 + m\frac{a^2}{2} + m.2a^2 + m\frac{a^2}{2}$

$I = ma^2\left[\frac{1}{2}+2+\frac{1}{2}\right] = 3ma^2$

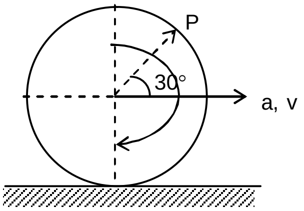
Prob 2. *If the radius of the earth were to contract to half its present value, find the new period. The angular velocity of earth about its own axis is $\frac{2\pi}{24}$ rad/hr*

Sol. $I_1\,\omega_1 = I_2\,\omega_2$

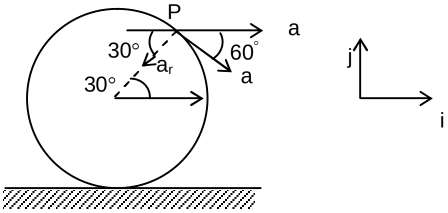
$\Rightarrow \frac{2}{5}mR^2.\frac{2\pi}{24} = \frac{2}{5}m\left(\frac{R}{2}\right)^2 \times \frac{2\pi}{T_2}$

$\Rightarrow T_2 = 6$ hrs.

Prob 3. *A disc of radius R is rolling on a horizontal surface. If the acceleration of centre of mass of the disc is a and velocity is v. Find the acceleration of point P at the instant represented in the figure.*



Sol. $a_P = \left(\frac{3a}{2} - \frac{v^2\sqrt{3}}{2R}\right)\hat{i} - \left(\frac{a\sqrt{3}}{2} + \frac{v^2}{2R}\right)\hat{j}$



Prob 4. *Earth spins with a uniform speed and completes one revolution about its own axis in 24 hours. What is*

(a) angular speed of earth?

(b) surface speed of earth (Radius of earth = 6400 km)

Sol. (a) Angular speed, $\omega = \frac{2\pi}{24\times60\times60}$ rad/s = 7.27×10^{-5} rad/s

(b) Surface speed $= \omega r = 7.27 \times 10^{-5}$ rad/ s $\times 6.4 \times 10^{6}$ m
$= 465.3$ m/s.

Prob 5. *A wheel rotates with a constant angular acceleration $\alpha = 0.5$ rad/s^2. It has an angular velocity*
$\omega_0 = -0.75$ rad/s of time $t = 0$. Consider angular position at $t = 0$ as $\theta_0 = 0$. Find out
(a) angular speed at $t = 5$ s.
(b) It's angular position at time $t = 5$s.
(c) how much time will it take to reach an angular position of $\theta = 10°$ radian?

Sol. (a) $\omega_{(t = 5s)} = \omega_0 + \alpha t = -0.75 + 0.5 \times 5 = 1.75$ rad/s

(b) $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = 0 - 0.75 \times 5 + \frac{1}{2} \times 0.5 \times 25 = 2.5$ rad

(c) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\Rightarrow \; 10 = -0.75\, t + \frac{1}{2} \times 0.5 t^2$

$\Rightarrow \; t^2 - 3t - 40 = 0$

$\Rightarrow \; t = 8$ sec. or -5 sec.

Since $t > 0$, time taken $= 8$ sec.

Solving this we get,

$t = 17.3$ s.

Prob 6. *A motor shaft rotating at 150 rpm comes to rest under uniform deceleration in 20 s. Find out no. of revolutions made by it before coming to rest.*

Sol. $\omega_0 = 150 \text{ rpm} = \frac{150 \times 2\pi}{60} = 5\pi \text{rad/s}$

$\omega = \omega_0 + \alpha t$

$0 = 5\pi - \alpha\, 20$

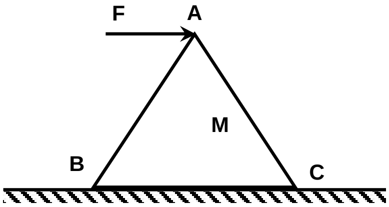
$\Rightarrow \alpha = \frac{5\pi}{20} = 0.25\pi$

$\theta = 5\pi \times 20 + \frac{1}{2}(-0.25\pi) \times 20^2$

$\Rightarrow \theta = 50\pi$

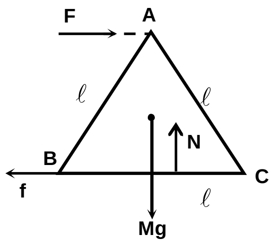
Number of revolutions $= \frac{50\pi}{2\pi} = 25$

Prob 7. *A constant horizontal force (F) is applied at the vertex A of an equilateral triangular wedge ABC of mass M and side length AB=BC=CA=ℓ, placed on a rough horizontal surface as shown in the figure. Even after the application of force F the wedge remains stationary. Calculate the torque of normal reaction acting between the wedge and the floor about the vertex B.*



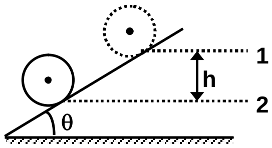
Sol. Torque of normal reaction about B on the wedge is τ_N = Torque due to frictional force (f) + torque due to applied force (F) + torque due to weight of the wedge (Mg)

$$\tau_N = 0 + F\left(\frac{\sqrt{3}}{2}\ell\right) + Mg\left(\frac{\ell}{2}\right) = \frac{(Mg+\sqrt{3}F)\ell}{2}$$



Prob 8. *A solid cylinder of mass m and radius r starts rolling down an inclined plane of inclination θ. Friction is enough to prevent slipping. Find the speed of its centre of mass when, its centre of mass has fallen a height h.*

Sol. Consider the positions 1 and 2 of the cylinder. As it does not slip, the total mechanical energy is conserved.



Energy at position 1 is $E_1 = mgh$

Energy at position 2 is $E_2 = \frac{1}{2}mv_{c.m.}^2 + \frac{1}{2}I_{c.m.}\omega^2$

$\because \quad \frac{v_{c.m.}}{r} = \omega$ and $I_{c.m.} = \frac{mr^2}{2}$

$\Rightarrow E_2 = \frac{3}{4}mv_{c.m.}^2$

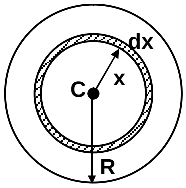
From COE, $E_1 = E_2$

$\Rightarrow \quad v_{c.m.} = \sqrt{\frac{4}{3}gh}$

Prob 9. *A uniform disc of radius R and mass M is spun to an angular velocity ω_0 in its plane about its centre and then placed on a rough horizontal surface such that the plane of the disc is parallel to the horizontal plane. If the co–efficient of friction between the disc and the surface is μ, how long will it take for the disc to come to stop?*

Sol. Consider a differential circular strip of the disc of radius x and thickness dx. Mass of this strip is

$dm = \rho\, 2\pi x dx$; where $\rho = \frac{M}{\pi R^2}$.



Frictional force on this strip is along the tangent and is equal to

$dF = \mu\rho 2\pi\, x dx\, g$

Torque on the strip due to frictional force is equal to $d\tau = \mu\rho g 2\pi x^2 dx$

The disc is a combination of a number of such strips. The torque on the disc is given by

$$\tau = \int d\tau = \mu g\rho\, 2\pi \int_0^R x dx = \frac{2}{3}\mu MgR$$

$\Rightarrow \quad \tau = \mu Mg\,(2/3)R$

$$\Rightarrow \quad \alpha = 2\mu MgR \Big/ 3\left(\frac{MR^2}{2}\right) = \frac{4}{3}\frac{\mu g}{R}$$

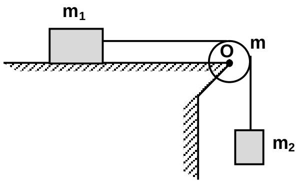
As the angular acceleration α is uniform and opposite to ω

$\because \quad \vec{\omega}_{(t)} = \vec{\omega}_o + \vec{\alpha}t$

$\Rightarrow \quad 0 = \omega_0 - \frac{4\mu g}{3R}t$

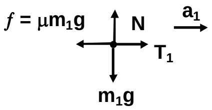
$\Rightarrow \quad t = \frac{3\omega_0 R}{4\mu g}$

Prob 10. *In the system shown in the figure, masses of the blocks are m_1 and m_2 and that of the pulley (a uniform disc free to rotate about the axle O) is m. The co–efficient of friction between the block of mass m_1 and the plane is μ. At t= 0 block m_2 starts descending. The string does not slip on the pulley. Find the work done by the friction acting on the block m_1 over the first t seconds. Neglect the mass of the string and friction in the axle of the pulley.*

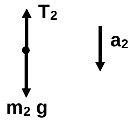


Sol. Tensions in the horizontal and vertical parts of the string are different because of friction, say T_1 and T_2.

F. B. D. of m_1



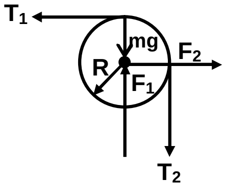
F. B. D. of m_2



$\Rightarrow\ T_1 - \mu m_1 g = m_1 a_1$ . . . (i)

$m_2 g - T_2 = m_2 a_2$ . . . (ii)

F.B.D. of the pulley



$\Rightarrow\ (T_2 R - T_1 R) = I_{c.m.}\alpha$

$\Rightarrow\ (T_2 - T_1)R = \dfrac{mR^2}{2}\alpha$

$\Rightarrow\ (T_2 - T_1) = \dfrac{m\alpha R}{2}$ . . . (iii)

Adding (i) and (ii) we get

$\Rightarrow\ (T_1 - T_2) - \mu m_1 g + m_2 g = m_1 a_1 + m_2 a_2$

$\Rightarrow$ Since the string does not slip, hence, $a_1 = a_2 = \alpha R$

$\Rightarrow\ (T_1 - T_2) - \mu m_1 g + m_2 g = (m_1 + m_2)\alpha R$ . . . (iv)

Adding (iii) and (iv) , we get

$$\alpha = \frac{2(m_2 - \mu m_1)\,g}{(2m_1 + 2m_2 + m)\,R}$$

$$\Rightarrow\ a_1 = \alpha R = \frac{2(m_2 - \mu m_1)\,g}{(2m_1 + 2m_2 + m)}$$

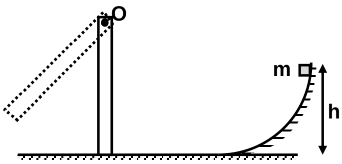
Distance moved by m_1 in time t is equal to $S = \dfrac{(m_2 - \mu m_1)\,g}{(2m_1 + 2m_2 + m)}t^2$

Using $S = \dfrac{1}{2}at^2$

Work done by friction over this distance is equal to, $W = -fS$

$$\Rightarrow\ W = -\frac{\mu m_1 (m_2 - \mu m_1) g^2 t^2}{(2m_1 + 2m_2 + m)}.$$

Prob 11. *In the shown figure, a particle of mass m slides down the frictionless surface from height h and collides with the uniform vertical rod of length L and mass M. After the collision, mass m sticks to the rod. The rod is free to rotate in a vertical plane about a fixed axis through O. Find the maximum angular deflection of the rod from its initial position.*



Sol. Just before collision velocity of the mass m is along the horizontal and is equal to $v_o = \sqrt{2gh}$.

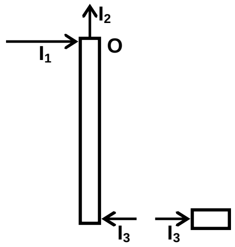
From the FBD during the collision, we note that net torque of impulses about O = 0

$\therefore$ The angular momentum of the system about O is conserved.

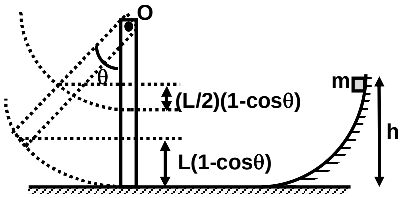
If L_1 and L_2 are the angular momentum of the system just before and just after the collision, then

$L_1 = mv_0L$

FBD during collision



and $L_2 = I\omega = \left(\frac{ML^2}{3} + mL^2\right)\omega$



From COAM

$$\left(\frac{M}{3} + m\right)L^2\omega = mv_0L$$

$$\Rightarrow \quad \omega = \frac{mv_0}{\left(\frac{M}{3} + m\right)L} \qquad \ldots \text{(i)}$$

Let the rod deflects through an angle θ

Initial energy of rod and mass system = $\frac{1}{2}I\omega^2$; where $I = \left(\frac{ML^2}{3} + mL^2\right)$

Gain in potential energy of the system

$$= mgL[1 - \cos\theta] + Mg\,\frac{L}{2}[1 - \cos\theta] = \left(m + \frac{M}{2}\right)gL(1 - \cos\theta)$$

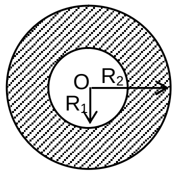
$\because$ From conservation of energy

$$\frac{1}{2}I\omega^2 = \left(m + \frac{M}{2}\right)gL(1 - \cos\theta) \qquad \ldots \text{(ii)}$$

Putting ω from (i) in (ii) we get,

$$\cos\theta = 1 - \frac{1}{2}\frac{m^2v_0^2}{\left[\frac{M}{3} + m\right]\left[\frac{M}{2} + m\right]gL}$$

Prob 12. *A uniform disc of radius R_2 has a concentric round cut of radius R_1 as shown in the figure. The mass of the remaining (shaded) portion of the disc is M. Find the moment of inertia of such a disc relative to the axis passing through its centre and perpendicular to the plane of the disc.*



Sol. Mass per unit area, $\sigma = \frac{M}{\pi(R_2^2 - R_1^2)}$

$I_0 = (I_{complete})_0 - (I_{removed})_0$

$$= \frac{1}{2}(\sigma\pi R_2^2)R_2^2 - \frac{1}{2}(\sigma\pi R_1^2)R_1^2$$

$$= \frac{1}{2}\sigma\pi(R_2^4 - R_1^4) = \frac{1}{2}M(R_2^2 + R_1^2)$$

Prob 13. *A cart with mass M has four wheels (idealized as uniform discs), each of radius r and mass m. Find the acceleration of the cart when a horizontal force F is applied on it. There is no slipping between the wheels and the horizontal road.*

Sol. For rolling motion

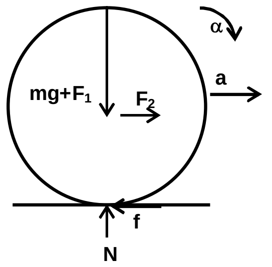
$a = r\alpha$

$fr = \left(\frac{mr^2}{2}\right)\alpha$

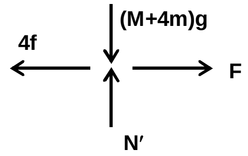
$f = \frac{mr\alpha}{2} = \frac{ma}{2}$

$F - 4f = (M + 4m)a$

$\therefore\ a = \frac{F}{M + 6m}$

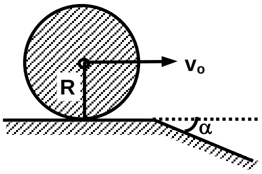


FBD of the wheel (F_1 and F_2 are the reactions of the axle)



FBD of the cart (with wheels)

Prob 14. *A uniform solid cylinder of radius R = 15 cm rolls over a horizontal plane passing into an inclined plane forming an angle $\alpha = 30^o$ with the horizontal. Find the maximum value of the velocity v_o which still permits the cylinder to roll onto the inclined plane section without a jump. The sliding is assumed to be absent.*



Sol. Initial energy

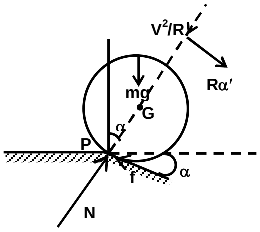
$$E_1 = \frac{1}{2}mv_0^2 + \frac{1}{2}I_{c.m.}\omega^2 + mgR$$

For rolling, $\frac{v_0}{R} = \omega$

$$\Rightarrow\ E_1 = \frac{1}{2}mv_0^2 + \frac{1}{2}.\frac{1}{2}mR^2\frac{v_0^2}{R^2} + mgR$$

$$= \frac{3}{4}mv_0^2 + mgR$$

$$E_2 = \frac{1}{2}mv^2 + \frac{1}{2}I_{c.m.}\omega'^2 + mgR\cos\alpha = \frac{3}{4}mv^2 + mgR\cos\alpha$$



From law of COE,

$$\frac{3}{4}mv^2 + mgR\cos\alpha = \frac{3}{4}mv_0^2 + mgR$$

$$\Rightarrow\ mv^2 = mv_0^2 + \frac{4}{3}mgR(1 - \cos\alpha) \qquad \ldots (i)$$

Consider the F.B.D. of the cylinder when it is at the edge. Centre of mass of the cylinder describes circular motion about P.

Hence, mg cos α – N = mv^2/R

$\Rightarrow$ N = mg cos α – mv^2/R

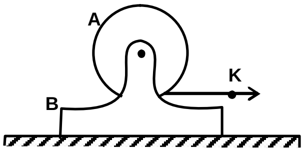
$$= mg\cos\alpha - \frac{mv_0^2}{R} - \frac{4}{3}mg + \frac{4}{3}mg\cos\alpha, \text{ from (i)}$$

For no jumping, $N \geq 0$

$$\Rightarrow\ \frac{7}{3}mg\cos\alpha - \frac{4}{3}mg - \frac{mv_0^2}{R} \geq 0$$

$$\Rightarrow\ v_0 \leq \sqrt{\frac{7gR}{3}\cos\alpha - \frac{4}{3}g\,R}.$$

Prob 15. *A uniform solid cylinder A of mass m_1 can freely rotate about a horizontal axis fixed to a mount of mass m_2. A constant horizontal force F is applied to the end K of a light thread tightly wound on the cylinder. The friction between the mount and the supporting horizontal plane is assumed to be absent. Find*



(a) the acceleration of the point K.
(b) the kinetic energy of the system t seconds after beginning of motion.

Sol. (a) The acceleration of the whole system, $a_1 = \dfrac{F}{m_1 + m_2}$

The acceleration of the of the point K w.r.t. the axis of the cylinder

$a_2 = \alpha R$ where, α is given by

$FR = I\alpha$

$$\Rightarrow \alpha = \frac{FR}{m_1R^2/2} = \frac{2F}{m_1R}$$

$$\Rightarrow a_2 = \frac{2F}{m_1}$$

$\therefore$ The acceleration of the point K w.r.t. ground

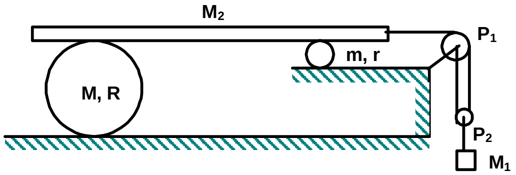
$$= a_1 + a_2 = \frac{F}{m_1+m_2} + \frac{2F}{m_1}$$

$$= F\frac{3m_1+2m_2}{m_1(m_1+m_2)}$$

(b) Total K.E. of system

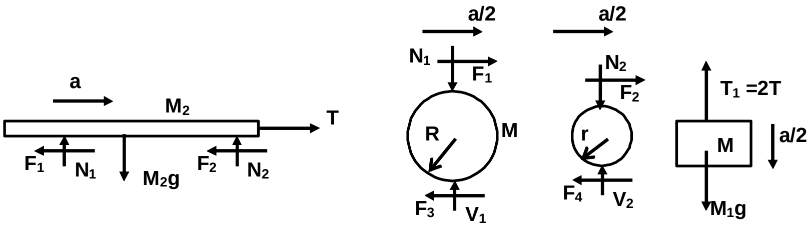
$$k = \frac{F^2t^2}{2(m_1+m_2)} + \frac{F^2t^2}{m_1} = \frac{F^2t^2(3m_1+2m_2)}{2m_1(m_1+m_2)}.$$

Prob 16. *A plank of mass M_2=1 kg is kept on two rollers of masses M = 5 kg and m = 2.5 kg respectively and radius R = 20 cm and r = 10 cm respectively. One end of the plank is attached with a string which passes through two light pulleys P_1 and P_2. A block M_1 = 1 kg is suspended through the pulley P_2 as shown in figure.*



Friction is sufficient to ensure pure rolling of both rollers. If the system is released from rest, then find the
(a) acceleration of the block M_1
(b) magnitude and direction of force of friction acting on bigger roller
(c) acceleration of smaller roller.

Sol.



Pure rolling is assumed for both rollers

$T - F_1 - F_2 = M_2a$ . . . (i)

$\alpha_1 = \frac{a}{2R}$, $\alpha_2 = \frac{a}{2r}$

$M_1g - 2T = M_1(a/2)$ . . . (ii)

From (i) and (ii)

$M_1g - 2(F_1 + F_2) = a(\frac{M_1}{2} + 2M_2)$

$a = \frac{2M_1g - 4(F_1 + F_2)}{(M_1 + 4M_2)}$ . . . (iii)

Taking torque about lower contact point of M

$F_1 \times 2R = I_1\,\alpha_1 = \left(\frac{MR^2}{2} + MR^2\right)\alpha_1 = \frac{3}{2}MR^2\frac{a}{2R} = \frac{3MRa}{4}$

$F_1 = \frac{3}{8}Ma$ . . . (iv)

Taking torque about lower contact point of m

$F_2 \times 2r = I_2\alpha_2 = \left(\frac{mr^2}{2} + mr^2\right)\alpha_2 = \frac{3}{2}mr^2.\frac{a}{2r} = \frac{3}{4}mr$

$F_2 = \frac{3}{8}ma$ . . . (v)

From (iv) and (v),

$F_1 + F_2 = \frac{3}{8}a(M + m) = \frac{3}{8}a(5 + 2.5) = \frac{3\times 75}{80}a = \frac{45}{16}a$

Putting this in (iii)

$a = \frac{2\times 1\times 10 - 4\times(45/16)a}{1+4}$

$\therefore\ a = \frac{16}{13}$ m/s^2

(a) Acceleration of block $M_1 = \frac{a}{2} = \frac{8}{13}$ m/s^2

(b) From FBD of M,

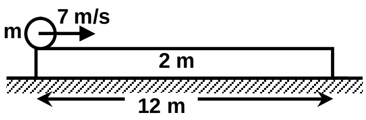
$F_1 - F_3 = M(a/2)$

$F_1 = \frac{3}{8}Ma = \frac{3}{8}\times 5\times\frac{16}{13} = \frac{30}{13}$ N (Rightward)

$F_3 = -\frac{1}{8}Ma = \frac{10}{13}$ N (rightward)

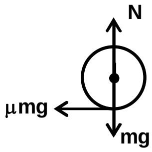
(c) Acceleration of smaller roller $= \frac{a}{2} = \frac{8}{13}$ m/s^2.

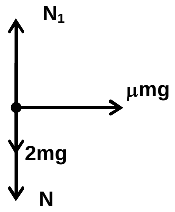
Prob 17. *A cylinder of mass m is kept on the edge of a plank of mass 2m and length 12 m, which in turn is kept on smooth ground. Coefficient of friction between the plank and the cylinder is 0.1. The cylinder is given an impulse, which imparts it a velocity of 7 m/s but no angular velocity. Find the time after which the cylinder falls off the plank.*



Sol. Initially, the cylinder will slip on the plank, therefore kinetic friction will act between the cylinder and the plank

F.B.D. of cylinder F.B.D. of plank





$a_c = -\frac{\mu mg}{m} = -\mu g$; $a_P = -\frac{\mu mg}{2m} = -\frac{\mu g}{2}$

$\alpha_c = \frac{2\mu mgR}{mR^2} = \frac{2\mu g}{R}$; $v_c = v_0 - \mu gt$

$v_p = \frac{\mu g}{2}t$, $\omega_c = \frac{2\mu gt}{R}$

Also $s_c = v_0t - (1/2)\mu\, gt^2$; $s_p = (1/2)\frac{\mu g}{2}t^2$

For pure rolling $v_c - R\omega_c = v_p$

$v_0 - \mu gt - 2\,\mu gt = \mu gt/2 \Rightarrow t = \frac{2v_0}{7\mu g} = 2s$

Putting the value of t we get, $s_c = \frac{12v_0^2}{49\mu g}$ $s_p = \frac{v_0^2}{4\mu g}$

$s_c - s_p = \frac{11v_0^2}{49\mu g} = 11\text{ m}$

Hence remaining distance = 12 m – 11 m = 1 m

Also $v_c = \frac{5}{7}v_0$, $v_p = \frac{v_0}{7}$

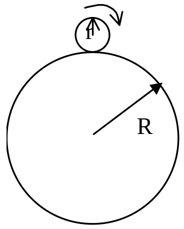
$\Rightarrow v_c - v_p = \frac{4v_0}{7} = 4\text{ m/s}$

After the start of pure rolling the velocities become constant because friction vanishes.

The time after the start of pure rolling, when the cylinder falls = $\frac{1\text{m}}{4\text{m/s}} = 0.25\text{ s}$

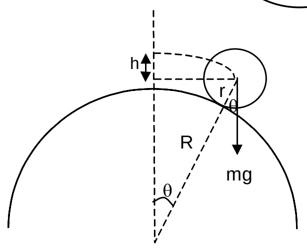
$\therefore$ Total time = 2 + 0.25 = 2.25 s

Prob 18. *A uniform sphere of radius r starts rolling down without slipping from the top of another fixed sphere of radius R shown in the figure. Find the angular velocity of sphere of radius r at the instant when it leaves contact with the surface of the fixed sphere.*



Sol. Let us assume that at the instant when the sphere of radius r breaks-off from the another sphere it has turned through angle θ w.r.t. its initial position. Since at this instant normal force between them is zero.

Centripetal force, $F = mg\cos\theta = \frac{mv^2}{R+r}$



$\Rightarrow \cos\theta = \dfrac{v^2}{g(R+r)}$ ...(i)

From conservation of mechanical energy,

$mgh = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$

Since there is no slipping

$v = \omega r$

Also, $I = \dfrac{2}{5}mr^2$

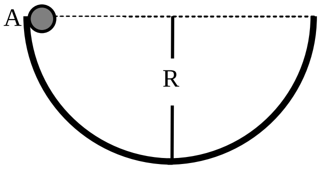
$h = (R + r)(1 - \cos\theta)$

$\Rightarrow mg(R + r)(1 - \cos\theta) = \dfrac{1}{2}m\omega^2r^2 + \dfrac{1}{2}\times\dfrac{2}{5}mr^2\omega^2$ . . . (ii)

Substituting the value of $\cos\theta$ from equation (i) in equation (ii) and solving we get

$\omega = \sqrt{\dfrac{10g(R+r)}{17r^2}}$.

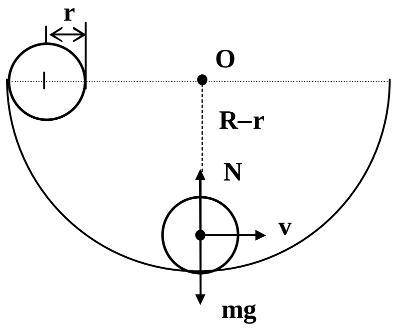
Prob 19. *A small sphere of radius r is released from point 'A' inside the fixed large hemispherical bowl of radius R as shown in figure. If the friction between the sphere and the bowl is sufficient enough to prevent any slipping then find*



(a) what fractions of the total energy are translational and rotational, when the small sphere reaches the bottom of the hemisphere.

(b) and also the normal force exerted by the small sphere on the hemisphere when it is at the bottom of the hemisphere.

Sol. $K_{trans} = \dfrac{1}{2}mv^2$



$K_{rot} = \dfrac{1}{2}I\omega^2 = \dfrac{1}{5}mv^2$

$K = K_{trans} + K_{rot} = \dfrac{7}{10}mv^2$

$\therefore \dfrac{K_{trans}}{K} = \dfrac{5}{7}$

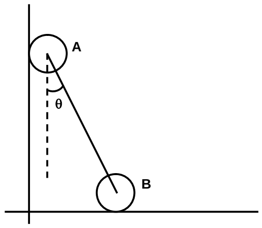
$\dfrac{K_{rot}}{K} = \dfrac{2}{7}$

$mg(R - r) = \dfrac{7}{10}mv^2$

$\left(\dfrac{mv^2}{R-r}\right) = \dfrac{10}{7}mg$

$N = mg + \left(\dfrac{mv^2}{(R-r)}\right) = \dfrac{17}{7}mg$

Prob 20. *Two ends of a light rigid rod having length $\ell = 60$ cm are connected to two identical uniform discs A & B. The wall in the shown diagram is smooth and the floor is sufficiently rough to ensure pure rolling. The system starts from the position $\theta = 0$. Find the velocity of the mid point of the rod when $\theta = 60^\circ$.*



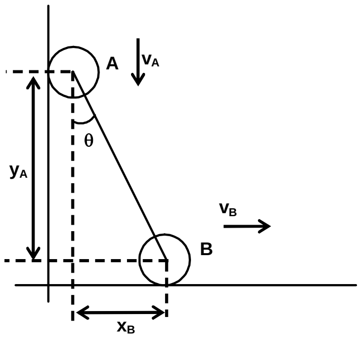
Sol. Apply law of COE

$$mg\ell\,(1-\cos\theta) = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}\left(m\frac{R}{2}\right)^2\left(\frac{v_B}{R}\right)^2$$

$$2g\ell\,(1-\cos\theta) = v_A^2 + \frac{3}{2}v_B^2 \qquad \ldots(1)$$

Also from constraint relation

$$y_A^2 + x_B^2 = \ell^2 \qquad (\ell = \text{length of the rod})$$



$$2y_A\frac{dy}{dt} + 2x_A\frac{dx_B}{dt} = 0$$

$$v_A = \frac{x_B}{y_A}v_B \qquad \left(\because\ v_A = \frac{-dy_A}{dt}\ \&\ v_B = \frac{dx_A}{dt}\right)$$

$$\Rightarrow\ v_A = v_B\tan\theta \qquad \ldots(2)$$

From (1) and (2)

$$v_A = \sqrt{\frac{2g\ell\,(1-\cos\theta)}{1+\frac{3}{2}\cot^2\theta}} \quad \&\ v_B = \sqrt{\frac{2g\ell\,(1-\cos\theta)}{\tan^2\theta+\frac{3}{2}}}$$

Velocity of mid-point of the rod, $v = \frac{1}{2}\left(v_B\hat{i} - v_A\hat{j}\right)$

Putting the values of ℓ and θ we get, $v = \left(\frac{1}{\sqrt{3}}\hat{i} - \hat{j}\right)$ m/s

Objective:

Prob 1. *A boy is standing on a rotating table with heavy spheres in his extended hands. If he suddenly withdraws his hands towards his chest, the angular velocity of the table:*

(A) becomes zero *(B) increases*
(C) decreases *(D) remains unchanged*

Sol. Since $I\omega$ = constant
So by withdrawing his hands towards the chest, he reduces the moment of inertia of the system so ω increases.
Hence (B) is correct.

Prob 2. *A uniform rod of mass m and length ℓ is rotating about one end on a hinge on a smooth horizontal table with angular speed ω. Force exerted by the hinge on the rod is*

(A) mg *(B) $m\omega^2\ell$*
(C) zero *(D) $\frac{1}{2}m\omega^2\ell$*

Sol. Force by hinge provides centripetal force for rotation.

$$F = \frac{mv^2}{R} = m\left(\frac{\ell}{2}\omega\right)^2 \frac{1}{\ell/2} = \frac{1}{2}m\omega^2\ell$$

Hence (D) is correct.

Prob 3. *Moment of inertia of a circular wire about an axis passing through its centre and perpendicular to its plane is*

(A) $\frac{mr^2}{12}$ *(B) $\frac{mr^2}{2}$*
(C) $\frac{2}{5}mr^2$ *(D) mr^2*

Sol. Moment of inertia = $\int_0^m dm.r^2 = mr^2$

Hence (D) is correct.

Prob 4. *A disc of moment of inertia I_1 is rotating freely with angular speed ω_1. Another disc of moment of inertia I_2 is dropped on it. The angular speed of the system will be*

(A) ω_1 *(B) $\frac{I_1\omega_1}{I_2}$*
(C) $\frac{I_1\omega_1}{I_1+I_2}$ *(D) $\frac{I_2\omega_1}{I_1}$*

Sol. L = constant

$$\therefore\ I_1\omega_1 = (I_1 + I_2)\omega'$$

$$\omega' = \frac{I_1\omega_1}{I_1 + I_2}$$

Hence (C) is correct.

Prob 5. *Moment of inertia of a system having three masses m connected by light rods and forming an equilateral triangle of side a about an axis passing through one of the masses and perpendicular to its plane is*

(A) ma^2 (B) $2ma^2$

(C) $3ma^2$ (D) $\frac{2}{3}ma^2$

Sol. Moment of inertia = $ma^2 + ma^2 = 2ma^2$

Hence (B) is correct.

Prob 6. *Particle of mass m is projected with a velocity v_0 making an angle of 45° with horizontal. The magnitude of angular momentum of the projectile about the point of projection at its maximum height is*

(A) Zero (B) $mv^3/\sqrt{2}\,g$

(C) $mv_0^2/4\sqrt{2}\,g$ (D) $mv_0^3/4\sqrt{2}g$

Sol. Speed of the particle at the top = horizontal component of the speed of projection

$\Rightarrow$ $v = v_0 \cos\theta_0 = v_0 \cos 45° = \frac{v_0}{\sqrt{2}}$

The angular momentum the particle about O

$L = mvr \sin\theta$ where θ = angle between $\vec{v}$ & $\vec{r}$

$\Rightarrow L = mv\,h$ ($\because r\sin\theta = h$)

Putting $h = \frac{v_0^2\sin^2\theta}{2g} = \frac{v_0^2\sin^2 45^0}{2g} = \frac{v_0^2}{4g}$ we obtain

$$L = \frac{mv_0^3}{4\sqrt{2}g}$$

Hence (D) is correct.

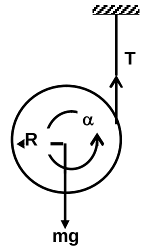
Prob 7. *A string is wrapped several times around a solid cylinder and then the end of the string is held stationary while the cylinder is released from rest with an initial motion. The acceleration of the cylinder and tension in the string will be*

(A) $\frac{2g}{3}$ and $\frac{mg}{3}$ (B) g and $\frac{mg}{2}$

(C) $\frac{g}{3}$ and $\frac{mg}{2}$ (D) $\frac{g}{2}$ and $\frac{mg}{3}$

Sol. Let a be the acceleration of the cylinder. Then

$m\,g - T = ma$ . . . (1)



If α be the angular acceleration, then

$T.\,R. = I\,\alpha = \left(\frac{mR^2}{2}\right)\alpha$ . . . (2)

As the string unwinds without slipping

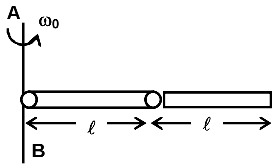
$a = R\,\alpha$ . . . (3)

Solving these equations, we have

$$a = \frac{2}{3}g \quad \text{and} \quad T = \frac{mg}{3}$$

Hence (A) is correct.

Prob 8. *A rod of mass m and length ℓ fits into a hollow tube of same length and mass. The tube is rotated with an initial angular velocity ω_0 and the rod slips through the rough hollow surface. The angular velocity of the rod as it slips out of the tube is*



(A) $\frac{\omega_0}{2}$ (B) $\frac{\omega_0}{4}$

(C) $\frac{\omega_0}{16}$ (D) $\frac{\omega_0}{7}$

Sol. There is no external torque acting on the system. Hence angular momentum remains conserved.

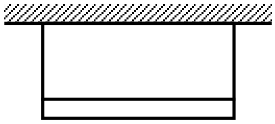
$I_1\omega_1 = I_2\omega_2$

$I_1 = 2M\ell^2/3$; $I_2 = [2M(2\ell)^2/3]$

$\Rightarrow \omega_2 = \omega_0/4$

Hence (B) is correct

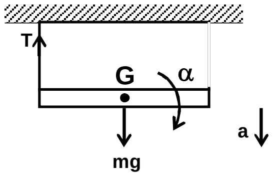
Prob 9. *A uniform rod of length ℓ and mass m is suspended on two vertical inextensible string as shown in the figure. Calculate tension T in left string at the instant, when right string snaps.*



(A) mg /2 (B) mg

(C) $\frac{mg}{4}$ (D) $\frac{mg}{8}$

Sol. Let the tension in the rope be T and the acceleration of the centre of mass of the rod downwards be a.



Then $mg - T = ma$ . . . (1)

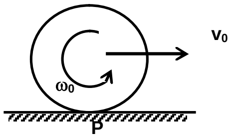
Again, $\frac{mg\ell}{2} = I\alpha = \frac{m\ell^2}{3}\times\frac{2\alpha}{\ell}$

$\Rightarrow a = \frac{3g}{4}$ . . . (2)

$\therefore T = \frac{mg}{4}$

Hence, (C) is correct.

Prob 10. *A uniform circular disc of radius r is placed on a rough horizontal surface and given a linear velocity v_0 and angular velocity ω_0 as shown. The disc comes to rest after moving some distance to the right. It follows that*



(A) $3 v_0 = 2\omega_0 r$ (B) $2 v_0 = \omega_0 r$

(C) $v_0 = \omega_0 r$ (D) $2 v_0 = 3 \omega_0 \mathbf{r}$

Sol. Since the disc comes to rest, it stops rotating & translating simultaneously

$\Rightarrow$ v = 0 & ω = 0. That means, the angular momentum about the instantaneous point of contact just after time of stopping is zero we know that, the angular momentum of the disc about P remains constant because frictional force f. N & mg pass through the point P, thus produce no torque about this point

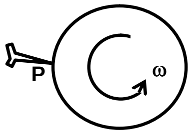
$\Rightarrow$ $L_{initial} = L_{final}$ $\Rightarrow$ $mv_0 r - I_0 \omega_0 = 0$

$\Rightarrow$ $m v_0 r = I_0 \omega_0$ $\Rightarrow$ $m v_0 r = \frac{1}{2} m r^2 \omega_0$.

$\Rightarrow$ $2 v_0 = \omega_0 r$.

Hence (B) is correct.

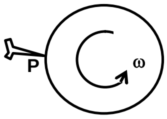
Prob 11. *A disc is freely rotating with an angular speed ω on a smooth horizontal plane. It is hooked at a rigid peg P & rotates about P without bouncing. Its angular speed after the impact will be equal to*



(A) ω (B) $\frac{\omega}{3}$

(C) $\frac{\omega}{2}$ (D) *none of these*

Sol. During the impact, the impact forces passes through the point P. Therefore, the torque produced by it about P is equal to zero.



Consequently, the angular momentum of the disc about P, just before & after the impact remains the same

$\Rightarrow$ $L_2 = L_1$ . . . (A)

Where L_1 = Angular momentum of the disc about P just before the impact

$= I_0\omega = \frac{1}{2}mr^2 \omega$.

$\Rightarrow$ L_2 = Angular momentum of the disc about P just after the impact

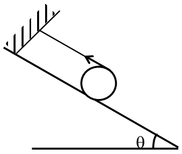
$= I_0 \omega = (\frac{1}{2}mr^2 + mr^2)\omega = \frac{3}{2}mr^2\omega'$

$\therefore$ $\frac{3}{2}mr^2\omega' = \frac{mr^2}{2}\omega$

$\Rightarrow$ $\omega' = \frac{1}{3}\omega$.

Hence (B) is correct.

Prob 12. *A sphere of radius r kept on a rough inclined plane is in equilibrium by a string wrapped over it. If the angle of inclination is θ, the tension in the string will be equal to*

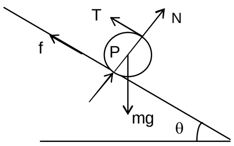


(A) $mg \sin \theta$ (B) $\frac{2mg}{5}$

(C) $\frac{mg \sin\theta}{2}$ (D) *none of these*

Sol. Since f & N passes through the point P, their torque about P will be zero.

For the equilibrium of the sphere the torques of tension T & weight mg about P will be zero.



$\Rightarrow$ $mg \sin \theta \times r - T(2r) = 0$

$\Rightarrow$ $2 Tr = mgr \sin \theta$

$\Rightarrow$ $T = \frac{mg\sin\theta}{2}$

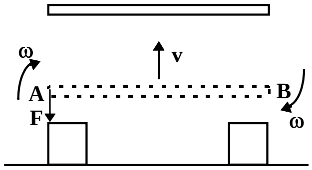
Hence (C) is correct.

Prob 13. *A rod of length ℓ falls on two metal pads of same height from a height h. The coefficients of restitution of the metal pads are e_1 and e_2 ($e_1 > e_2$). The angular velocity of the rod after it recoils, is*

(A) $\frac{e_1}{e_2}\ell\sqrt{2gh}$ (B) $\left(\frac{e_1+1}{e_2+1}\right)\sqrt{2gh}$

(C) $\frac{e_1-e_2}{\ell}\sqrt{2gh}$ (D) $\left(\frac{e_1+1}{e_2-1}\right)\sqrt{2gh}$

Sol. After collision, the end A moves with a linear velocity of $e_1\, e_1\sqrt{2gh}$, whereas end B moves with a velocity of $e_2\sqrt{2gh}$.



$\therefore$ The relative velocity between the ends is

$= e_1\sqrt{2gh} - e_2\sqrt{2gh}$

$\Rightarrow$ The angular velocity $= \frac{(e_1-e_2)\sqrt{2gh}}{\ell}$

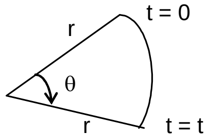
Hence (C) is correct.

Prob 14. *A particle of mass m is revolving in a horizontal circle of radius r with a constant angular speed ω. The areal velocity of the particle is*

(A) $r^2\,\omega$ (B) $r^2\,\theta$

(C) $\frac{r^2}{2}\omega$ (D) $\frac{r\,\omega^2}{2}$

Sol. Areal velocity $= \frac{dA}{dt}$ where A = area of the sector $= \frac{r^2\theta}{2}$



$\therefore \frac{dA}{dt} = \frac{d}{dt}\left(\frac{r^2\theta}{2}\right) = \frac{r^2}{2}\frac{d\theta}{dt} = \frac{r^2}{2}\omega$.

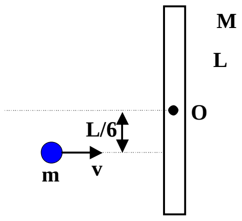
Hence (C) is correct.

Alternative method.

$$\frac{dA}{dt} = \frac{l}{2m} = \frac{I\Omega}{2m}$$

$$\frac{dA}{dt} = \frac{mr^2.\omega}{2m} = \frac{r^2}{2}\omega$$

Prob 15. *A uniform rod of mass M and length L lies on a smooth horizontal surface. A particle of mass m moving at a speed v perpendicular to the length of the rod strikes it at a distance $\frac{L}{6}$ from the centre and comes to rest, after collision. The angular velocity of the rod about its centre just after the collision is*



(A) $\frac{mv}{ML}$ (B) $\frac{2mv}{ML}$

(C) $\frac{3mv}{ML}$ (D) $\frac{mv}{2ML}$

Sol. For system (Rod + particle), $\tau_{ext} = 0$

So $\vec{L} = \text{constant}$

Now angular momentum about O (before collision)

$L_i = mv \times L/6$

After collision

$L_f = I\omega = \frac{ML^2}{12}\omega$

Therefore $L_i = L_f$

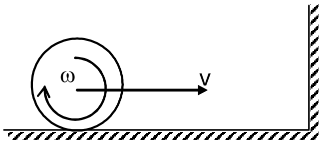
$mv\frac{L}{6} = \frac{ML^2}{12}\omega$

$\omega = \frac{2mv}{ML}$

Hence (B) is correct.

STATE WHETHER THE FOLLOWING QUESTIONS ARE TRUE OR FALSE.

A sphere rolling on a horizontal surface collides elastically with a smooth vertical wall as shown in figure.



*Select the following statement as **True** and **False.***

Prob 16. *After collision, the velocity of centre of mass gets reversed.*

Sol. **True**

Prob 17. *Angular momentum of the sphere about the point of contact with the wall is conserved.*

Sol. **True**

Prob 18. *Angular momentum of the sphere about a stationary point on the horizontal surface is conserved.*

Sol. **False**

Prob 19. *After collision the frictional force acts on the sphere such that it decreases the linear speed and increases the angular speed.*

Sol. **False**

Prob 20. *Finally, when the sphere starts rolling it is moving away from the wall.*

Sol. **True**

ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. Starting with $\omega = \frac{d\theta}{dt}$ and $\alpha = \frac{d\omega}{dt}$, derive equations for angular velocity vs time, angular position vs time and angular velocity vs angular position in terms of ω_0 and α.

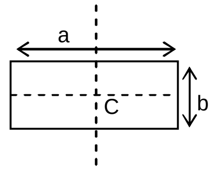
2. Define torque of a force about an axis of rotation. If force $\vec{F}$ intersects the axis what will be the value of torque ? If $\vec{F}$ is perpendicular to the axis but does not intersect it, what will he the value of torque?

3. Define angular momentum. Derive expression of angular momentum in terms of moment of inertia and angular velocity.

4. Derive the relationship between torque and angular momentum. Extend the concept to the law of conservation of angular momentum.

5. Derive an expression for moment of inertia of a uniform circular plate about its axis perpendicular to its plane.

6. Derive an expression for moment of inertia of a rectangular plate of sides a and b about an axis parallel to width b and passing through the centre. What will be the moment of inertia about an axis perpendicular to the plane of the plate and passing through its centre C?



7. Derive an expression of rotational kinetic energy of a body rotating with a constant angular speed ω.

8. A flywheel starts from rest. It attains an angular speed of 100 rev/sec in 5 sec under uniform angular acceleration. What are net angular acceleration and the angle by which it rotates during 5 sec?

9. A washing machine is switched off when it was rotating at an angular speed of 50 rev/sec. It slows down under uniform acceleration and stops after completing 150 revolutions. What was the value of deceleration and how much time it took to come to rest?

10. A disc is rotating about its axis with a constant angular acceleration 6 rad/s^2. Find radial and tangential acceleration of a point at a distance of 5 cm from the axis at the end of 1.5 seconds after starting of the rotation.

11. An electron of mass 9.1×10^{-31} kg is revolving around a nucleus with a speed of 2.2×10^6 m/s. Radius of the circular orbit is 0.53 A°. Find out angular momentum of the electron.

12. Three particles of mass 100 g each are kept on corners of an equilateral triangle of side 20 cm. Find moment of inertia of the system about an axis.
(a) passing through one of the particles and perpendicular to the plane of the triangle.
(b) passing through two particles.

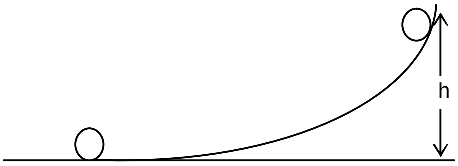
13. Derive an expression for moment of inertia of a thin rod of mass m and length ℓ about an axis perpendicular to it and passing through one of its ends.

14. A vehicle moving with speed of 36 km/hr is brought to rest by application of brakes in 5 sec. Moment of inertia of the wheel is 2.5 kg m^2. If the radius of a wheel is 40 cm, what is the average torque applied on the wheel.

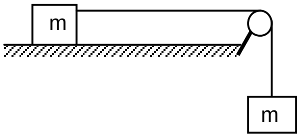
15. A sphere is rolling on a rough surface without slipping. Find out ratio of translational energy to total energy.

16. A solid sphere lying on a smooth horizontal surface is pulled by a force acting on its top most point. What will be the distance traveled by the sphere during the time it makes one complete revolution?

17. A hollow sphere is set rolling with a speed v on a rough surface as shown in the figure. Assuming no slipping find out the height h where it will come to instantaneous rest.



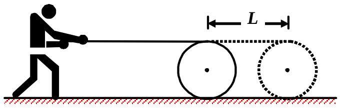
18. Two blocks of mass m each one connected by a string passing over a pulley as shown in the figure. The table is smooth. Moment of inertia of pulley is I. Find out acceleration of the hanging mass.



Level- I

1. A bicycle wheel of radius 0.3 m has a rim of mass 1.0 kg and 50 spokes, each of mass 0.01 kg. What is its moment of inertia about its axis of rotation?

2. A cylindrical drum, pushed along by a board, rolls forward on the ground. There is no slipping at any contact. Find the distance moved by the man who is pushing the board, when axis of the cylinder covers a distance L.

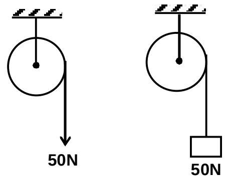


3. A cubical block of mass m and edge a slides down a rough inclined plane of inclination θ with a uniform speed. Find the torque of the normal force acting on the block about its centre.

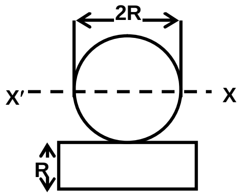
4. A fly wheel of gasoline engine is required to give up 300 J of kinetic energy while its angular velocity decreases from 600 rev/min to 540 rev/min. What moment of inertia is required ?

5. A metre stick is held vertically with one end on a rough horizontal floor. It is gently allowed to fall on the floor. Assuming that the end of the floor does not slip, find the angular speed of the rod when it hits the floor.

6. A cord is wrapped around the rim of a flywheel 0.5 m in radius and a steady pull of 50 N is on the cord as shown in the figure. The wheel is mounted on a frictionless bearing on the horizontal shaft through its centre. The moment of inertia of the wheel is 4 kg m^2.
(a) Compute angular acceleration of the wheel.
(b) If a mass having a weight 50 N hangs from the cord as shown in the figure. Compute the angular acceleration of the wheel. Why is this not the same as in part (a)?

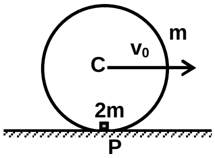


7. A disc of mass m and radius 'R' is attached to a rectangular plate of the same mass, breadth R and length 2R as shown in the figure. Find the moment of inertia of this system about the axis XX′ passing through the centre of the disc and along the plane.



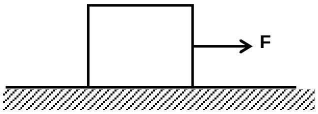
8. A spherical shell of mass m and radius R is released on a inclined plane of inclination θ. What should be the minimum coefficient of friction between the shell and the plane to prevent sliding?

9. A uniform ring of mass m and radius R has a particle of mass 2m rigidly attached to it on the inside surface. The system rolls on a horizontal surface without slipping. In the position shown at a certain instant, its centre C has a velocity v_0. Find the kinetic energy of the system.

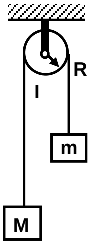


10. A tall chimney cracks near its base and falls over. Express
(i) the radial and (ii) the tangential acceleration of the top of chimney as a function of the angle θ made by the chimney with the vertical.

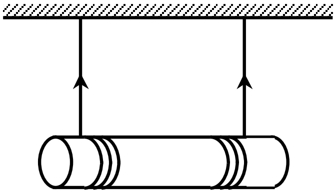
11. A uniform cube of side ℓ and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the center of the face at a height $3\ell/4$ above the base. Assuming that the cube may tip about an edge without slipping, find the angular acceleration of the cube when F is equal to (a) mg/3 (b) mg.



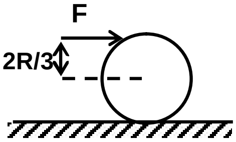
12. The pulley shown in figure has a moment of inertia I about its axis and its radius is R. Find the accelerations of the two blocks. Assume that the string is light and does not slip on the pulley.



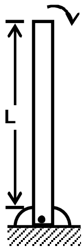
13. A cylinder of mass m is suspended through two strings wrapped around it as shown in figure. If cylinder is released from rest then find
(a) the tension T in the string and
(b) the speed of the cylinder as it falls through a distance h.



14. A cylinder of mass 6kg lies on a rough horizontal surface. The coefficient of friction between the cylinder and the surface is $\mu = 0.2$. A constant force acts horizontally on the cylinder. The line of action of the force F is at a height $\frac{2}{3}R$ above the centre of cylinder. Find the maximum value of F if the cylinder rolls without slipping.

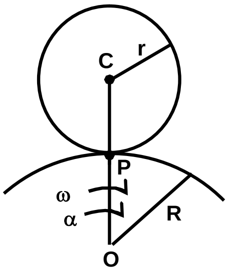


15. A uniform rod of mass m and length L is free to rotate in the vertical plane about a horizontal axis passing through its end. The rod is released from rest in the position shown by slightly displacing it clockwise. Find the hinge reaction at the axis of rotation at the instant the rod turns through
(a) 90° (b) 180°.



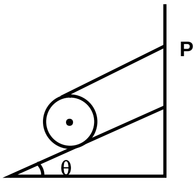
Level - II

1. A uniform cylinder of mass m of radius r rolls on a fixed cylindrical surface of radius R. At a certain instant, the line OC has an angular velocity ω and angular acceleration α. Find
(a) the acceleration of point of contact P of the cylinder with respect to the surface.
(b) the kinetic energy of the cylinder.

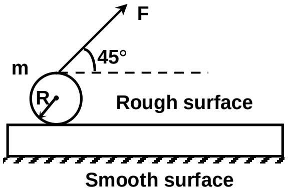


2. A student throws a stick of length L up in the air. At the moment the stick leaves his hand, the speed of the stick's end is zero. The stick completes N turns just as its is caught by the student at the initial release point. Show that the height h to which the centre of mass of the stick rose is $h = \pi NL/4$.

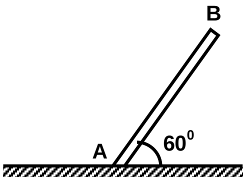
3. A flexible tape of length L is tightly wound on a cylinder. The tape is allowed to unwind as the cylinder slips down a frictionless inclined plane. The upper end of the tape is fixed to a point P as shown in the figure. Find the time taken by the tape to unwind completely.



4. A plank of mass m is placed on a smooth surface. Now a uniform solid sphere of equal mass m and radius R is placed on the plank as shown in the figure. A force F is applied at topmost point of the sphere at an angle of 45° to the horizontal. Surface between the plank and the sphere is extremely rough so that there is no slip between the plank and the sphere. Find the force of friction acting between the plank and the sphere.

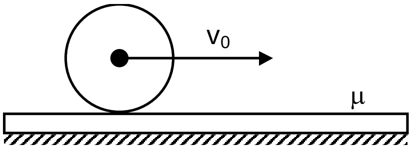


5. A uniform slender rod AB is released from rest on a horizontal surface as shown in the figure. Find the minimum static coefficient of friction between the rod and the surface so that the rod does not slip at the instant it is released. What is the instantaneous acceleration of (a) the centre of mass (b) end B under the no slip condition in the given situation ?



6. A solid sphere of mass M is released from the top of an incline plane of inclination θ.
(a) What should be the minimum coefficient of friction between the sphere and the plane to prevent sliding ?
(b) Find the kinetic energy of the ball when it moves down a length L on the inclined plane, if the coefficient of friction is half the value calculated in the part (a). Find the loss of the total mechanical energy, if any.

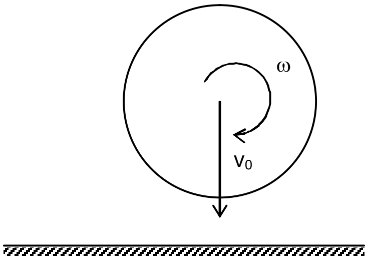
7. A solid sphere of mass m and radius R is placed on a plank of equal mass, which lies on a smooth horizontal surface. The sphere is given a sharp impulse in the horizontal direction so that it starts sliding with a speed of v_0. Find the time taken by the sphere to start pure rolling on the plank. The coefficient of friction between plank and sphere is μ.



8. A rod AB of mass M and length L is lying on a horizonal frictionless surface. A particle of mass m travelling along the surface hits the end 'A' of the rod with a velocity v_0 in a direction perpendicular to AB and comes to rest. The collision is completely elastic.
   (a) Find the ratio m/M
   (b) A point P on the rod is at rest immediately after the collision. Find the distance AP.
   (c) Find the linear speed of the point P at a time $\pi L/(3v_0)$ after the collision.

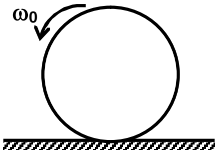
9. A solid sphere rolling on a rough horizontal surface with a linear speed v collides elastically with a fixed, smooth, vertical wall. Find the speed of the sphere after it has started pure rolling in the backward direction.

10. A rotating ball hits a rough horizontal plane with a vertical velocity v and angular velocity ω. Given that the coefficient of friction is μ and the vertical component of the velocity after the collision is v/2, find (a) the angular velocity after the collision (b) the impulsive ground reaction during the collision.

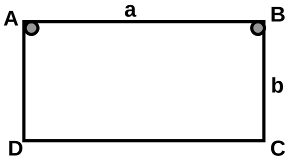


11. Two small balls A and B each of mass m, are connected by a light rod of length L. The system is lying on a frictionless horizontal surface. A particle of mass m collides with the rod horizontally with the velocity v_0 perpendicular to the rod and gets stuck to it as shown in the figure. Find
   (a) the velocities of A and B immediately after the collision.
   (b) the angular velocity of the system after the collision.
   (c) the velocity of the rod when the rod rotates through 90° after the collision.

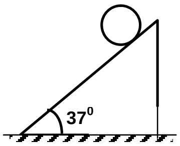
12. A uniform disc of mass m and radius R is gently kept on a horizontal rough surface (coefficient of friction μ) with an initial angular velocity ω_0 keeping its plane vertical. Find (a) the final velocity of its centre and the angular velocity after sufficiently long time (b) the dissipation of total mechanical energy for a long time because of friction.



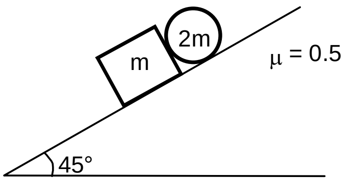
13. A rectangular plate ABCD of mass m and dimensions a × b is supported in a vertical plane by two hinges at A and B as shown in the figure. Find the instantaneous reaction of the hinge at A immediately after B is removed.



14. A solid metallic sphere of mass m and radius R is free to roll (without sliding) over inclined surface of wooden wedge of mass m. Wedge lies on a smooth horizontal floor. When the system is released from rest. Find frictional force between sphere and wedge.



15. A block of mass m and a cylinder of mass 2m are released on a rough inclined plane of inclination 45°. Coefficient of friction for all the surfaces of contact is 0.5. Find the accelerations of the block and the cylinder.



Objective:

Level- I

1. A wheel of mass 2 kg having practically all the mass concentrated along the circumference of a circle of radius 20 cm, is rotating on its axis with an angular velocity of 100 rad/s. The rotational kinetic energy of the wheel is
(A) 4J (B) 70J
(C) 400 J (D) 800 J

2. A rod of length L is hinged from one end. It is brought to the horizontal position and released. The angular velocity of the rod when it is in the vertical position is
(A) $\sqrt{\frac{2g}{L}}$ (B) $\sqrt{\frac{3g}{L}}$
(C) $\sqrt{\frac{g}{2L}}$ (D) $\sqrt{\frac{g}{L}}$

3. If a solid sphere, disc and cylinder are allowed to roll down an inclined plane from the same height
(A) cylinder will reach the bottom first
(B) disc will reach the bottom first
(C) sphere will reach the bottom first
(D) all will reach the bottom at the same time

4. A mass M is moving with a constant velocity parallel to the x axis. Its angular momentum w.r.t. the origin.
(A) is zero (B) remains constant
(C) goes on increasing (D) goes on decreasing

5. A solid homogeneous sphere is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of this sphere.
(A) total kinetic energy is conserved
(B) angular momentum of the sphere about the point of contact with the plane is conserved
(C) only the rotational kinetic energy about the centre of mass is conserved.
(D) angular momentum about the centre of mass is conserved.

6. A point moves along a circle of radius 20 cm with a constant tangential acceleration 5 m/s^2. How much time is required after motion begins for normal acceleration to become equal to tangential acceleration?
(A) 0.25 s (B) 0.025 s
(C) 0.2 s (D) 0.5 s

7. The moment of inertia of a cylinder of radius a, mass M and height h about an axis parallel to the axis of the cylinder and distant b from its centre is
(A) $\frac{3}{4}Ma^2$ (B) $\frac{2}{3}Ma^2$
(C) $\frac{3}{2}Ma^2$ (D) none of these

8. A thin bar of mass M and length L is free to rotate about a fixed horizontal axis through a point at its end. The bar is brought to a horizontal position and then released. The following statement is true regarding its motion
(A) The angular acceleration of the rod is constant.
(B) The angular momentum about the axis of rotation is constant.
(C) The force exerted by the rod on the hinge at the axis of rotation continuously changes.
(D) The acceleration of the centre of the mass is equal to g.

9. The ratio of the radii of gyration of a circular disc and a circular ring of the same mass and radius about a tangential axis parallel to the their planes is
(A) 6 : 5 (B) 1 : 2
(C) 5 : 6 (D) none of these

10. Assuming the mass distribution of the earth to be spherically symmetric, if the radius of the earth contracts to half of its present value, the duration of the day will be:
(A) 48 hrs (B) 24 hrs
(C) 12 hrs (D) 6 hrs

11. The centre of a wheel is rolling on a plane surface moves with a speed v_o. A particle on the rim of the wheel at the same level as the centre will be moving at speed
(A) zero (B) v_0
(C) $\sqrt{2}\,v_0$ (D) v_0

12. A particle moves on a straight line with a uniform velocity. The angular momentum of the particle is
(A) always zero (B) zero about a point on the straight line
(C) zero about a point away from the straight line. (D) none of the above.

13. A string is wrapped several times around a solid cylinder and then the end of the string is held stationary while the cylinder is released from rest with no initial motion. The acceleration of the cylinder and tension in the string will be
(A) g, mg/2 (B) $\frac{2g}{3}, \frac{mg}{3}$
(C) g, mg (D) 2g, $\frac{mg}{2}$

14. A particle performs uniform circular motion with an angular momentum L. If the frequency of particle's motion is doubled and its kinetic energy is halved, the angular momentum becomes:
(A) 2L (B) 4L
(C) L/2 (D) L/4

15. A uniform sphere is placed on a smooth horizontal surface and a horizontal force F is applied on it at a distance 'h' above the surface. The acceleration of the centre
(A) is maximum when h = 0 (B) is maximum when h = $\frac{2R}{5}$
(C) is maximum when h = $\frac{7}{5}R$ (D) is independent of h

STATE WHETHER THE FOLLOWING QUESTIONS ARE TRUE OR FALSE

16. A point on the rim of a flywheel has a peripheral speed of 10 m/s at an instant when it is decreasing at the rate of 60 m/s^2. If the magnitude of the total acceleration of the point at this instant is 100m/s^2, the radius of the flywheel is________________________

17. A boy is standing on a rotating table with heavy spheres in his extended hands. If he suddenly withdraws his hands to his chest, the angular velocity of the table is ________________________

18. A sphere cannot roll on a _________________inclined surface.

19. A uniform circular wheel is acted upon by a constant torque and its angular momentum changes from A_0 to $2A_0$ is 12 secs. The magnitude of this torque is ______________________

20. A thick walled hollow sphere has outer radius R. It rolls down an incline without slipping and its speed at bottom is v_o. Now the incline is waxed so that the friction is absent. The sphere is observed to slide down without rolling and the speed is now $\left(\frac{5v_o}{4}\right)$. The radius gyration of the hollow sphere about the axis through its centre is________________________

Level - II

1. Two uniform solid spheres having unequal masses and unequal radii are released from rest from the same height on a rough incline. If the spheres roll without slipping,

(A) the heavier sphere reaches the bottom first.

(B) the bigger sphere reaches the bottom first.

(C) the two spheres reach the bottom together.

(D) lighter sphere reaches the bottom first.

2. A thin bar of mass M and length L is free to rotate about a fixed horizontal axis through a point at its end. The bar is brought to the horizontal position and then released. The angular velocity when it reaches the vertical position is

(A) directly proportional to its length and inversely proportional to its mass.

(B) independent of mass and inversely proportional to the square root of its length.

(C) dependent only upon the acceleration due to gravity and the length of the bar

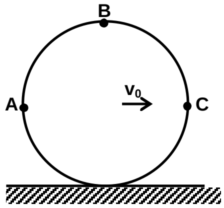
(D) directly proportional to its length and inversely proportional to the acceleration due to gravity.

3. The ratio of the radii of gyration of a spherical shell and a solid sphere of the same mass and radius about a tangential axis is

(A) $\sqrt{3}:\sqrt{7}$ (B) $\sqrt{5}:\sqrt{6}$

(C) $\sqrt{25}:\sqrt{21}$ (D) $\sqrt{21}:\sqrt{25}$

4. Three mass particle A, B and C having masses m, 2m and 3m respectively are rigidly attached to a ring of mass m and radius R which rolls on a horizontal surface without slipping. At a certain instant the velocity of the centre of the ring is v_0 as shown in the figure. The kinetic energy of the system is



(A) $7m\,v_0^2$ (B) $9m\,v_0^2$

(C) $m\,v_0^2$ (D) $\frac{7}{2}m\,v_0^2$

5. In the previous problem, the velocity of the centre of the mass of the system is

(A) v_0 (B) $7v_0$

(C) $\frac{\sqrt{85}}{7}v_0$ (D) $\frac{7}{\sqrt{85}}v_0$

6. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are:

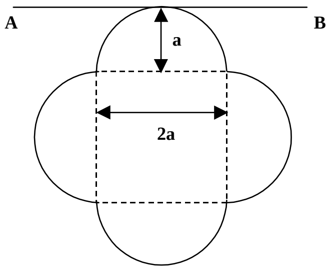
(A) up the incline while ascending and down the incline descending.

(B) up the incline while ascending as well as descending.

(C) down the incline while ascending and up the incline while descending.

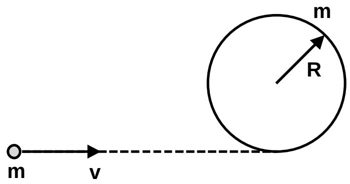
(D) down the incline while ascending as well as descending.

7. A symmetric lamina of mass m consists of a square shape with a semicircular section over each of the edge of the square as shown in the figure. The side of the square is 2a. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is 1.6 ma^2. The moment of inertia of the lamina about the tangent AB in the plane of the lamina is



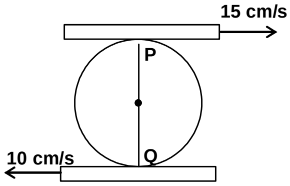
(A) 4.8 ma^2 (B) 2.4 ma^2
(C) 1.5 ma^2 (D) none of these

8. A circular hoop of mass m and radius R rests flat on a horizontal frictionless surface. A bullet, also of mass m, and moving with a velocity v, strikes the hoop and gets embedded in it. The thickness of the hoop is much smaller than R. The angular velocity with which the system rotates after the bullet strikes the hoop is



(A) $\frac{V}{4R}$ (B) $\frac{V}{3R}$
(C) $\frac{2V}{3R}$ (D) $\frac{3V}{4R}$

9. A cylinder of radius 10cm rides between two horizontal bars moving in opposite direction as shown in the figure. The location of the instantaneous axis of rotation and the angular velocity of the roller are respectively (There is no slipping at P or Q)



(A) 8 cm from Q, 1.25 rad/s (B) 8 cm from Q, 2.50 rad/s
(C)15 cm from Q 1.25 rad/s (D) 15 cm from Q, 2.50 rad/s

10. A uniform rod AB of mass m and length ℓ is at rest on a smooth horizontal surface. A horizontal impulse P is applied to the end B perpendicular to the rod. The time taken by the rod to turn through a right angle is
(A) $\pi m\ell/12P$ (B) $\pi m\ell/6P$
(C) $m\ell/6P$ (D) none of these

11. A thick walled hollow sphere has outer radius R. It rolls down an incline without slipping and its speed at bottom is v_o. Now the incline is waxed so that the friction is absent. The sphere is observed to slide down without rolling and the speed is now $\left(\frac{5v_o}{4}\right)$. The radius gyration of the hollow sphere about the axis through its centre is
(A) $\frac{3}{4}R$ (B) $\frac{\sqrt{3}}{4}R$
(C) $\frac{2}{3}R$ (D) None of these

12. A solid sphere rolls down a rough inclined plane of length ℓ and inclination θ without slipping. Find the speed of sphere when it reaches to bottom

(A) $\sqrt{\frac{10}{7}g\ell\sin\theta}$ (B) $\sqrt{5\,\frac{g\ell\sin\theta}{7}}$

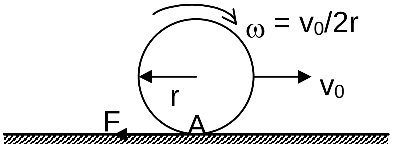
(C) $\sqrt{\frac{6}{5}g\ell\sin\theta}$ (D) none of these

13. A disc of radius R has a concentric hole of radius r (<R). Its mass is m. Its moment of inertia about an axis through its centre and perpendicular to its plane is

(A) $\frac{1}{2}m(R^2 - r^2)$ (B) $\frac{1}{2}m(R^2 + r^2)$

(C) $\frac{1}{2}m\,(R - r)^2$ (D) none of these

14. A sphere of mass M and radius r slips on a rough horizontal plane. At some instant it has translational velocity v_0 and rotational velocity about the centre $v_0/2r$. The translational velocity after the sphere starts pure rolling



(A) $6v_0/7$ in forward direction (B) $6v_0/7$ in backward direction

(C) $7v_0/6$ in forward direction (D) $6v_0/6$ in backward direction

15. A uniform rod is kept vertically on a horizontal smooth surface at a point O. If it is rotated slightly and released, it falls down on the horizontal surface. The lower end will be

(A) at O (B) at a distance less than $l/2$ from O

(C) at a distance $l/2$ from O (D) at distance larger than $l/2$ from O.

MORE THAN ONE CORRECT CHOICE

16. Four identical rods, each of mass m and length ℓ, are joined to form a rigid square frame. The frame lies in the xy plane, with its centre at the origin and the sides parallel to the x and y axes. Its moment of inertia about

(A) the x–axis is $(2/3)m\ell^2$

(B) the z–axis is $(4/3)m\ell^2$

(C) an axis parallel to the z–axis and passing through a corner is $(10/3)m\ell^2$

(D) one side is $(5/2)m\ell^2$

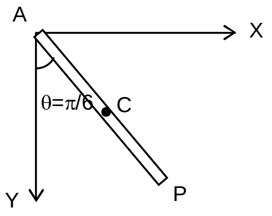
17. When slightly different weights are placed on the two pans of a beam balance, the beam comes to rest at an angle with the horizontal. The beam is supported at a single point P by a pivot. Now, select the correct statement from the following.

(A) The net torque about P due to the two weights is non–zero at the equilibrium position.

(B) The whole system does not continue to rotate about P because it has a large moment of inertia.

(C) The centre of mass of the system lies below P.

(D) The centre of mass of the system lies above P.



18. A uniform rod of length 1 m and mass 4 kg can rotate freely in a vertical plane about its end A. The rod is initially held in a horizontal position and then released. At the time the rod makes an angle 30° with the vertical, calculate
   (A) its angular acceleration is 7.5 rad/s
   (B) its angular velocity is 5.1 rad/s
   (C) its total acceleration vector of tip P is $[\omega^2\ell(\sin\theta) + \alpha\ell\cos\theta.]\left(-\hat{i}\right) + [\omega^2\ell(\cos\theta) + \alpha\ell\sin\theta.]\left(-\hat{j}\right)$
   (D) None Of These

19. A constant external torque τ acts for a very brief period Δt on a rotating system having moment of inertia I. Then
   (A) The angular momentum of the system will change by $\tau\,\Delta t$.
   (B) the angular velocity of the system will change by $\frac{(\tau\Delta t)}{I}$
   (C) If the system was initially at rest, it will acquire rotational kinetic energy $\frac{(\tau\Delta t)^2}{2I}$
   (D) The kinetic energy of the system will change by $\frac{(\tau\Delta t)^2}{I}$

20. A sphere is rolled on a rough horizontal surface. It gradually slows down and stops. The force of friction tries to
   (A) decrease the linear velocity
   (B) increase the angular velocity
   (C) increase the linear velocity
   (D) decrease the angular velocity

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level - O

8. $40\,\pi$ rad./ s^2, $500\,\pi$
9. $\frac{50\pi}{3}\text{rad/s}^2$, 6 sec
10. 405 cm/s^2, 30 cm/s^2
11. 1.06×10^{-34} kg–m^2/s
12. (a) 8×10^{-3} kg–m^2 (b) 3×10^{-3} kg m^2
13. $\frac{m\ell^2}{3}$
14. 12.5 Nm
15. $\frac{5}{7}$
16. $\frac{4\pi R}{5}$
17. $\frac{5v^2}{6g}$
18. $\frac{mg}{2m + I/r^2}$

Level - I

1. 0.105 kg–m^2
2. 2L
3. $\frac{1}{2}mga\sin\theta$
4. 0.80 kg–m^2
5. 5.4 rad/s
6. (a) 6.25 rad/s^2 (b) 4.74, rad/s^2,
7. $\frac{31}{12}mR^2$
8. $\frac{2}{5}\tan\theta$
9. $m\,v_0^2$
10. $a_n = 3g(1-\cos\theta)$, $a_t = \frac{3g}{2}\sin\theta$
11. (a) zero (b) $\frac{3g}{2}$
12. (a) $a_M = \frac{(M-m)g}{M+m+I/R^2}\downarrow$ (b) $a_m = \frac{(M-m)g}{M+m+I/R^2}\uparrow$
13. (a) $\frac{mg}{6}$, (b) $\sqrt{\frac{4gh}{3}}$
14. 108N
15. (a) Horizontal component $= \frac{3mg}{2}\leftarrow$, vertical component $= \frac{mg}{4}\uparrow$

    (b) Horizontal component = zero, vertical component $= 4mg\uparrow$

Level - II

1. (a) $\frac{R(R+r)\omega^2}{r}$ radially outward (b) $\frac{3}{4}m(r+R)^2\omega^2$

3. $\sqrt{\frac{3L}{g\sin\theta}}$

4. $-\frac{F}{3\sqrt{2}}$

6. (a) $\frac{2}{7}\tan\theta$ (b) $\frac{11}{12}mg\ell\sin\theta$, loss of energy $=\frac{mg\ell\sin\theta}{12}$

7. $\frac{2v_0}{9\mu g}$

8. (a) $\frac{1}{4}$ (b) $\frac{2L}{3}$ (c) $\frac{v_0}{2\sqrt{2}}$

9. $\frac{3v}{7}$

10. (a) $\omega - \frac{15\mu v}{4R}$ (b) $\frac{3mv}{2}\sqrt{1+\mu^2}$

11. (a) $v_A = 0$, $v_B = v_0/2 \uparrow$ (b) $v_0 / 2L$ (c) $v_0/3$

12. $\frac{R\omega_0}{3}$, $\frac{\omega_0}{3}$, loss of total mechanical energy is $\frac{mR^2\omega_0^2}{6}$

13. Hinge reaction on the plate:
Vertical component $\frac{(a^2+4b^2)mg}{4(a^2+b^2)} \uparrow$
Horizontal component $\frac{3ab}{4(a^2+b^2)}mg \leftarrow$

14. f = 2/9 Mg

15. $\frac{3g}{5\sqrt{2}}$.

Objective:

Level - I

1.	C	2.	B
3.	C	4.	B
5.	B	6.	C
7.	D	8.	C
9.	D	10.	D
11.	C	12.	B
13.	B	14.	D
15.	C	16.	1.25 m
17.	increases	18.	smooth
19.	$A_0 / 12$	20	$\frac{3}{4}R$

Level - II

1.	C	2.	B
3.	C	4.	B
5.	C	6.	B
7.	A	8	B
9.	A	10.	A
11.	A	12.	A
13.	B	14.	A
15.	C	16.	A, B, C, D
17.	A, C	18.	A, B, C
19.	A, B, C	20.	A, B.

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

ELASTICITY & SHM

Elasticity and SHM

Syllabus:
States of matter, Inter–atomic and Inter–molecular force, Elasticity, Hooke's Law, Young's Modulus, Shear And Bulk Modulii of Elasticity, Poisson's ratio Linear And Angular Simple Harmonic Motion, Energy Of A Body Executing S.H.M. & Simple Pendulum, Types of oscillations.

STATE OF MATTER

The molecules of a substance are the structural units of states in which substance may exist. There are three such states; gaseous, liquid and solid. The state in which a substance exists is in large measure determined by its temperature, which in turn, is a measure of the kinetic energy of its molecules. The higher the kinetic energy, i.e. higher the temperature, the less is the tendency of the molecules to pack themselves close to each other.

There exist, between molecules, forces of attraction, called weak forces which tend to oppose the effect of molecular kinetic energy. The strength of these forces depends not only on the distance between the molecules but also on the kinds of molecules between which they are acting. Thus they are stronger between the molecules of water than between those of hydrogen or oxygen. Between sugar and any solid molecules, they are still stronger. At a given temperature the average kinetic energy of all the molecules is same. Due to the difference in the characteristic strength of Vander–Waals forces, Oxygen is a gas, water is a liquid and sugar is a solid at the same temperature.

Gaseous state

In a gas, the energy of molecular motion is predominant. So, a given sample has neither a definite shape nor a characteristic volume. It expands to fill uniformly the volume of any vessel in which it is placed.

At atmospheric pressure and room temperature, the average distance between molecules is ten times greater than their diameter. As the gas is cooled, average K.E. decreases in turn the average distance between the molecules decreases. The average kinetic energy of a molecule is less than the average attractive potential energy and the gas condenses into a liquid.

Liquid state

A given mass of liquid has definite volume and shows a free surface in a container of greater volume. The Vandar Waals force in a liquid exerts a greater influence then the gas, especially since the molecules are close together. They have the same average kinetic energy as those in a gas at the same temperature. But forces of attraction are greater between molecules of a liquid than between those of a gas.

Solid state

A solid has not only a definite volume at a given temperature but also a regular geometric shape. In other words, a solid is made up of crystals. As the form of a crystal indicates, the particles that make up its structure are arranged in space, in a regular pattern, or crystal lattice. Many solids are made up of molecules also. Such substances are the elements iodine & sulphur and the compounds like sugar are examples. Each particle is a structure of their crystals in a cluster of a relatively small number of atoms, and each of these molecules bears no charge of electricity. On the other hand, there are substances, all of which are solids at room temperature, are composed of crystals in which electrically charged atoms or groups of atoms are the structural units. Such charged particles are called ions. Thus crystals of sodium chloride consists of equal number of positive and negative charge ions i.e. Na^+ and Cl^- ions. These are arranged alternatively in a cubic space pattern. Between the neutral molecules in crystals of iodine or sugar, Vandar–Waals force of attraction operates, but between the oppositely charged ions in the crystals of a salt, relatively stronger electrostatic forces hold the ions together. It should not be assumed that molecules or ions in a solid are at rest. But each must occupy a fixed position in the regular pattern of the crystal structure; otherwise fixed shape of the solid will be lost. Their motion is one of the vibrations about fixed points in the crystal structure. The kinetic energy due to this vibratory motion becomes greater as the temperature is raised and the molecules or ions requires greater space. The fact that solids expand on heating is the evidence.

SOLIDS
Solids forms one of the three phases of matter. At ordinary temperatures the solids possess a definite shape and volume. A solid is that state of matter in which its atoms and molecules are strongly bound so as to preserve their shape and volume. Solid are of two types; crystalline and amorphous.

Crystalline solid
It has a symmetrical shape and flat faces which meet at particular angles to each other.
A crystalline solid is one which has regular and periodic arrangement of atoms or molecules in the three dimensions.

The structure of a variety of crystalline solids has been studied with the help of X–ray diffraction techniques. Some crystalline solid do not appear as such to the naked eye, but under a powerful microscope, they appear to have minute crystalline structures known as crystallites or grains of that solids. The size may vary from several angstrom to macroscopic dimensions. The arrangement of atoms or molecules in a crystal is termed as crystal structure.

In single crystals, the periodicity of the atomic pattern is extended throughout the volume of the solids. In polycrystalline crystals, the periodicity of the pattern is interrupted at grain boundaries.

In a crystalline solid, all the bonds have the same bond strength. So, crystalline solids (both organic and inorganic) melts at a given fixed temperature. The melting point is commonly used for identifying the organic chemical.

When a liquid crystallizes into a solid, a definite amount of heat energy is released in solidification at a fixed temperature and the atoms get set into an orderly system of crystalline structure. Examples are diamonds, rock salts, mica, sugar etc.

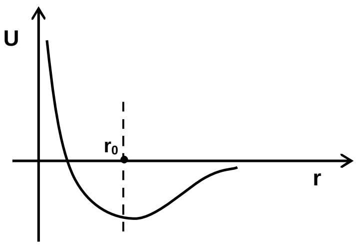
Amorphous solids or Glassy solids
An amorphous solid is one which does not have a periodic arrangement of atoms.

Just like liquids, amorphous solids have disordered arrangement of atoms or molecules. The essential difference between a liquid and an amorphous solid is that while the molecules of a liquid are free to move about within the volume of the liquid, the atoms in amorphous solids are fixed rigidly at their respective positive positions. When a melt or a solution is cooled quickly, we get an amorphous solid. The best example of an amorphous solid is glass.

In an amorphous solid, all the bonds are not equally strong. When an amorphous solid is heated, the weak bonds are the first to be broken. So, the amorphous solid gradually softens into a liquid and there is no fixed melting point. Likewise, there is no fixed freezing point. The molten liquid gradually hardens into a solid at a varying temperature. The liquid does not freeze at a fixed temperature because of high viscosity of the substance or the fast rate of cooling. Hence the disordered form is retained even in the solid state of an amorphous body.

MOLECULAR STRUCTURE AND INTERMOLECULAR FORCES
Matter consists of atoms and molecules. An atom has a positively charged nucleus with negatively charged electrons moving round the nucleus. Due to charged character of particles electromagnetic forces act between them.

The force between two molecules can be deduced from the potential energy vs intermolecular separation curve as shown in the figure. The potential energy is zero, when the separation is large. With decreasing separation, the potential energy also decreases, becoming negative. For a certain value r_0, the potential energy becomes minimum. At this separation, the force is zero and the atoms can stay in equilibrium. With further decrease in separation the potential energy increases. This means that the character of the force is repulsive at small separations.



BONDS

The atoms form molecules primarily due to electrostatic interaction between the electrons in one atom and the nucleus of another atom. These interactions can lead to different kinds of bonds.

Ionic bond

Ionic bonds are formed when atoms that have low ionization energies, (and hence lose electrons readily), interact with other atoms that tend to acquire excess electrons. The former atoms give up electrons to the latter. Thus the atoms become positive and negative ions respectively. These ions then come together into an equilibrium configuration in which attractive forces between positive and negative ions predominate over the repulsive forces between similar ions.

Covalent bond

In a covalent bond, atoms are held together by the sharing of electrons. Each atom participating in a covalent bond contributes an electron to the bond. These electrons are shared by both atoms rather than being the virtually exclusive property of one of them as in an ionic bond. Diamond is an example of a crystal whose atoms are linked by covalent bonds.

Metallic bond

In metallic crystals, a metallic bond is formed when all the atoms share all of the valence electrons in the lattice. Electrons of the atoms comprising a metal are common to the entire lattice, so that a kind of "gas" of free electrons pervades it. The crystal is held together by the electrostatic attraction between the negative electron gas and the positive metal ions.

Molecular bond

Neutral atoms with closed electron shells are bound together weakly by the Van der waal's forces. The Van–der Waal’s attraction was first explained for electrically neutral gas molecules by Debye. He assumed that neighbouring molecules induced dipoles in each other because of their own changing electric fields. This interaction produces an attractive force that is inversely proportional to the seventh power of the separation. The molecules are located at the crystal lattice points and the bonds between them are developed by Van der Waal's forces. Solid argon and solid methane (CH_4) are examples of molecular crystals.

ELASTICITY

The property of a body by virtue of which the body regains its original configuration (length, volume or shape) when external deforming forces are removed is called elasticity.

Deforming force

If a force applied to a body causes a change in the normal positions of the molecules of the body, resulting in a change in the configuration of the body either in length, volume or shape, then the force applied is deforming in nature.

Cause of elasticity

Molecules in a body are bound to each other by bonds. When elongated the bonds behave like springs due to inter molecular interaction. When subjected to compression beyond a certain distance or less than a certain inter molecular separation, the molecules, instead of attracting each other, repel.

Perfectly elastic body

A body which regains its original configuration immediately and completely after the removal of deforming force from it, is called perfectly elastic body. Quartz and phosphor bronze are the examples of nearly perfectly elastic bodies.

Perfectly plastic body

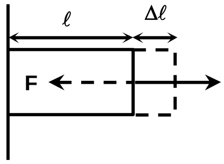
A body which does not regain its original configuration at all on the removal of deforming force, however small the deforming force may be, is a perfectly plastic body.

STRESS

When a deforming force is applied on a body, it changes the configuration of the body by changing the normal positions of the molecules or atoms of the body. As a result, an internal restoring force comes into play, which tends to bring the body back to its original configuration. This internal restoring force acting per unit area of a deformed body is called stress

i.e. Stress = Restoring force / area

(a) Tensile Stress



It is the restoring force developed per unit cross–sectional area of a body when the length of the body increases in the direction of the deforming force. It is also known as longitudinal stress.

Consider a body fixed to a rigid support at one end. Let a force F be applied normally outwards at the free end. Let $\Delta\ell$ be the increase in the length of the body in the direction of the force, then the tensile stress T = F/A, where A is the cross–sectional area of the body.

(b) Compresstional stress

It is the restoring force developed per unit cross–sectional area of body when the body is compressed i.e. when its length decreases under the action of the deforming force.

(c) Tangential or shearing stress

When the deforming force acts tangentially over an area, then the body gets sheared through a certain angle. Such a stress is called tangential stress.

STRAIN

When a deforming force is applied on a body, there is a change in the configuration of the body. The body is said to be strained or deformed. The ratio of change in configuration to the original configuration is called strain. i.e.

i.e. $\text{Strain} = \dfrac{\text{Change in configuration}}{\text{Original configuration}}$

Strain, being the ratio of two like quantities, has no units and dimensions.

Following are the three types of strains:

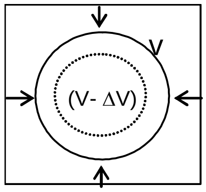
(a) Tensile or longitudinal strain

When the deforming forces produces a change in length, the strain is called longitudinal strain. Within elastic limit, it is the ratio of change in length to original length.
If the original length be ℓ and $\Delta\ell$ be the change in length then

$$\text{Longitudinal strain} = \frac{\Delta\ell}{\ell}$$

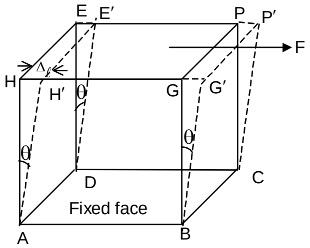
(b) Bulk or volumetric strain

If the deforming forces produces a change in volume, then the strain produced is called volumetric strain. Within elastic limit, it is the ratio of change in volume to the original volume. If the original volume be v and Δv be the change in volume, then



$$\text{Volume strain} = \frac{\Delta V}{V}$$

(c) Shear strain



If the deforming forces produce a change in the shape of the body, then the strain produced is called shear strain. Within elastic limit, it is measured by the ratio of the relative displacement of one plane to its distance from the fixed plane. It can also be measured by the angle through which a line originally perpendicular to the fixed plane is turned. This angle is called the angle of shear.

Shear strain = $\theta \approx \tan\theta$ (within elastic limit, θ is small)

$$= \frac{\Delta\ell}{L}$$

Elastic limit

Elastic limit is the upper limit of deforming force up to which, if deforming force is removed, the body regains its original form completely and beyond which if deforming force is increased, the body loses its property of elasticity and gets permanently deformed.

HOOKE'S LAW

Hooke's law states that the extension produced in the wire is directly proportional to the load applied within proportional limit.

Later on it was found that this law is applicable to all types of deformations such as compression, bending, twisting etc. and thus a modified form of Hooke's law was given as stated below.

"With–in proportional limit, the stress developed is directly proportional to the strain produced in a body"

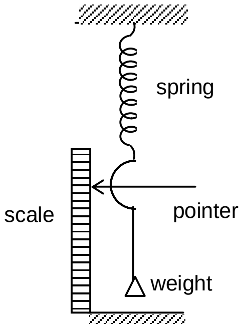
i.e. stress $\propto$ strain

or stress $= E \times$ strain

where E is a constant known as **modulus of elasticity**, which depends upon the nature of the material of the body and the manner in which the body is deformed.

Experimental verification of Hook's law

Suspend a spring from a rigid support. Attach a scale pan and a light point at its lowest end. The pointer can slide over a vertical scale graduated in centimetre.



Note the reading on the scale. Add a 5 gm weight in the pan and note reading. The difference of the two readings gives the extension produced. The same procedure should be repeated with every additional 5g weight.

It will be observed that the extension is directly proportional to load. This verifies Hook's law.

Table: Approximate value of Y

S. No.	Material	Young's modulus of elasticity Y ($\times 10^{11}$ N/m^2)
1.	Aluminium	0.70
2.	Brass	0.91
3.	Copper	1.1
4.	Glass	0.55
5.	Iron	1.9
6.	Lead	0.16
7.	Nickel	2.1
8.	Steel	2.0
9.	Tungston	3.6

Exercise 1.
(i). ***An elastic wire is cut to half its original length. How would it affect the maximum load that the wire can support ?***

(ii). ***Why is the water more elastic than air ?***

TYPES OF MODULUS OF ELASTICITY

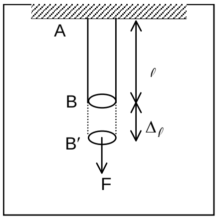
Corresponding to three types of strain, there are three types of modulus of elasticity, as described below:

Young's modulus of elasticity(Y).

It is defined as the ratio of longitudinal stress to the longitudinal strain within the elastic limit. Thus,

$$Y = \frac{\text{normal longitudinal stress}}{\text{longitudinal strain}}$$

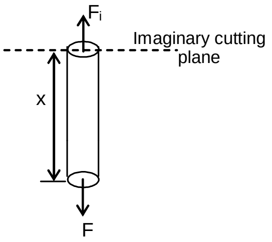
Consider a metal wire AB of length ℓ, radius r and of uniform area of cross–section a. Let it be suspended from a rigid support at A, as shown in figure.



Let a normal force F be applied at its free end B and let its length increase by $\Delta\ell$ (= BB′)

Then, longitudinal strain = $\frac{\Delta\ell}{\ell}$

Normal stress = $\frac{F_i}{a} = \frac{F_i}{\pi r^2}$ $\quad (\because a = \pi r^2)$



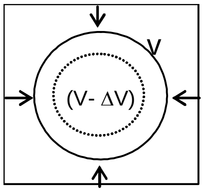
If the weight of the rod is negligible, the internal restoring force F_i at any cross–section is equal to F

$$\therefore \quad Y = \frac{F_i/\pi r^2}{\Delta\ell/\ell} = \frac{F_i\ell}{\pi r^2 \Delta\ell}$$

Bulk modulus of elasticity (k)

It is defined as the ratio of normal stress to the volumetric strain, within the elastic limit. Thus

$$K = \frac{\text{normal stress}}{\text{volumetric strain}}$$



Consider a spherical solid body of volume V and surface area a. In order to compress the body, let a force F be applied normally on the entire surface F and its volume decrease by ΔV as shown in figure. Then, volumetric strain $= -\frac{\Delta V}{V}$

Here, the negative sign shows that volume is decreasing when external force is applied.

Normal stress = F/a.

$$\therefore \quad K = \frac{F/a}{\Delta V/V} = -\frac{FV}{a\Delta V}$$

If Δp is the increase in pressure applied on the spherical body then F/a = Δp

$$k = -\frac{\Delta p}{(\Delta V/V)} = -V\frac{dp}{dv} = \rho\frac{dp}{d\rho} \quad \text{where } \rho \text{ is the density of the body.}$$

Compressibility The reciprocal of the bulk modulus of a material is called its compressibility.

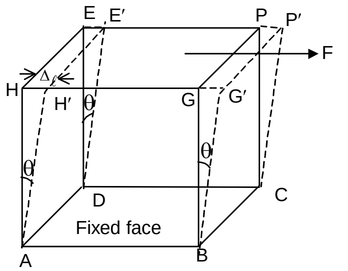
Compressibility = $\frac{1}{k}$

Modulus of rigidity (η)

It is defined as the ratio of tangential stress to the shearing strain, within the elastic limit. It is also called shear modulus of rigidity. Thus

$$\eta = \frac{\text{tangential or shearing stress}}{\text{shearing strain}}$$

$$\therefore \quad \eta = \frac{F/a_s}{\theta}$$

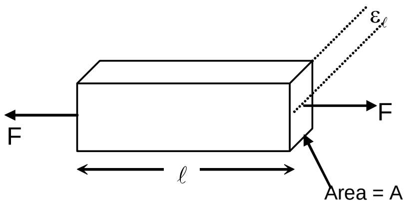


where a_s = Area of surface where tangential force is applied. (area HGPE)

Poisson's Ratio

It is observed that when a rod (see figure) is stretched by a force F (a pair actually as otherwise the rod would accelerate) there is longitudinal stress in the rod.

$$\eta_\ell = \frac{F}{A}$$



and consequently, a corresponding strain, ε_ℓ, is produced

$\varepsilon_\ell = \frac{\eta_\ell}{Y} = \frac{F}{AY}$, where Y is the Young's modulus of elasticity of the material.

In an actual experiment, the rod contracts in the lateral direction in an effort to maintain its volume. For a given material, the lateral strain is found to be proportional to the longitudinal strain (ε_ℓ), and their ratio

$$\frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \sigma,$$

Poisson's ratio, which is a constant for a given material. (In the limit of elasticity)

The value of Poisson's ratio lies between –1 to + 0.5 i.e. ($-1 < \sigma < +0.5$)

Relationship between Elastic constants:

$$\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K} \quad \text{. . . (i)}$$

$$Y = 3K(1 - 2\sigma) \quad \text{. . . (ii)}$$

$$Y = 2n(1 + \sigma) \quad \text{. . . (iii)}$$

Illustration 1. *A steel wire of length 4m and diameter 5 mm is stretched by 5 kg–wt. find the increase in its length, if the Young's modulus of steel of wire is 2.4×10^{12} dyne/cm^2.*

Solution: Here, ℓ = 4m = 400 cm ; 2r = 5 mm or r = 2.5 mm = 0.25 cm

f = 5 kg– wt = 5000 g – wt = 5000 × 980 dyne

$\Delta\ell$ = ? ; Y = 2.4×10^{12} dyne / cm^2

As $Y = \frac{F}{\pi r^2} \times \frac{\ell}{\Delta\ell}$

or $\Delta\ell = \frac{F\ell}{\pi r^2 Y} = \frac{(5000 \times 980) \times 400}{(22/7) \times (0.25)^2 \times 2.4 \times 10^{12}}$ = 0.0041 cm.

Illustration 2. *A cable is replaced by another one of same length and material but twice the diameter. How will this affect the elongation under a given load? How does this affect the maximum load it can support without exceeding the elastic limit?*

Solution : Young's Modulus $Y = \frac{Mg\ell}{\pi r^2.\Delta\ell} = \frac{Mg\ell}{\pi\left(\frac{D}{2}\right)^2.\Delta\ell} = \frac{4Mg\ell}{\pi D^2.\Delta\ell}$

Where D is the diameter of the wire.

$\therefore$ Elongation, $\Delta\ell = \frac{4Mg\ell}{\pi \; D^2.Y}$ i.e., $\Delta\ell \propto \frac{1}{D^2}$

Clearly, if the diameter is doubled, the elongation will become one–fourth.

Also load, $Mg = \frac{\pi D^2 Y \Delta\ell}{4\ell}$

i.e., $Mg \propto D^2$.

Clearly, if the diameter is doubled, the wire can support 4 times the original load.

Illustration 3. *The Bulk Modulus of water is 2.3 x 10^9 N/m^2.*
(a) Find its compressibility
(b) How much pressure in atmospheres is needed to compress a sample of water by 0.1%?

Solution : Here k = 2.3 x 10^9 N/m^2 = $\frac{2.3\times10^9}{1.01\times10^5} = 2.27\times10^4$ atm

(a) Compressibility = $\frac{1}{k} = \frac{1}{2.27\times10^4}$ = 4.4 x 10^{-5} atm^{-1}

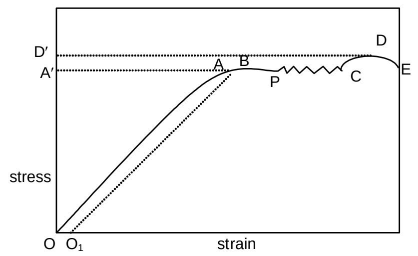
(b) Here $\frac{\Delta V}{V}$ = –0.1% = –0.001

Required increase in pressure, $\Delta p = k \times \left(-\frac{\Delta V}{V}\right)$

= 2.27 x $10^4 \times 0.001$ = 22.7 atm.

Stress strain relationship in a wire

If we plot a graph between stress and strain for the stretched wire, we shall get a curve of the type shown in figure. From this curve we note the following.
AO = elastic range, P = Yield point,
OD′ = breaking stress or tensile stress,
E = breaking point, OO_1= permanent set.

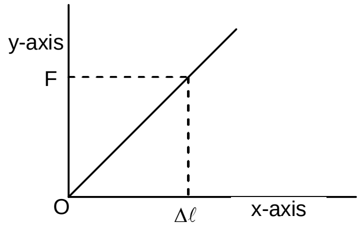


- The portion OA of the graph is a straight line showing that stress is directly proportional to strain up to a stage represented by point A on the graph. Thus Hooke's law is fully obeyed in the region OA. From A to B, the graph is in elastic range, but Hooke's law is not valid in this region.
- In the portion BP of the graph, Hooke's law fails and the wire is partly elastic and partly plastic in behavior.
- Beyond point P, the graph is represented by irregular wavy line from P to C. Here, the wire shows increase in strain without any increase in stress. It means the wire begins to flow down. After point P, without any increase in stress and it continue up to point C. The point P at which the wire yields to the applied stress and begins to flow down is called yield point. The behavior of the wire corresponding to the portion of graph from P to C is perfectly plastic.
- Beyond point C, the graph is a curved line CDE. It shows that if the wire is loaded beyond P, the thinning of the wire starts and the necks and waists (i.e. constrictions) are developed at few weaker portions in the wire and finally the wire breaks there which is shown by point E. The stress corresponding to point D is called breaking stress or ultimate stress or tensile stress of the wire.

Elastic potential energy in a stretched wire:

When a wire is stretched, some work is done against the internal restoring forces acting between particles of the wire. This work done appears as elastic potential energy in the wire.

Consider a wire of length ℓ and area of cross section a. Let F be the stretching forces applied on the wire and $\Delta\ell$ be the increase in length of the wire.
Hence, work done on the wire,



w = average force × increase in length $= \frac{F}{2} \times \Delta\ell$

This is stored as elastic potential energy U in the wire.

∴ $U = \frac{1}{2} F \times \Delta\ell = \frac{1}{2} \frac{F}{a} \times \frac{\Delta\ell}{\ell} \times a\ell$

$= \frac{1}{2}$ (stress) × (strain) × volume of the wire

∴ elastic potential energy per unit volume of the wire

$u = \frac{U}{a\ell} = \frac{1}{2}$ (stress) × (strain)

$= \frac{1}{2}$ (Young's modulus × strain) × strain

(∵ Young's modulus = stress / strain)

∴ $u = \frac{1}{2}$(young's modulus) × $(\text{strain})^2$

Illustration 4. *A steel wire of 4.0 m in length is stretched through 2.0 mm. The cross–sectional area of the wire is 2.0 mm^2. If Young's modulus of steel is 2.0×10^{11} N/m^2 find*
(a) the energy density of wire
(b) the elastic potential energy stored in the wire.

Solution: Here, $\ell = 4.0$ m ;
$\Delta\ell = 2 \times 10^{-3}$ m ; $a = 2.0 \times 10^{-6}$ m^2
$Y = 2.0 \times 10^{11}$ N/m^2.

(a) The energy density of stretched wire

$u = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$

$= \frac{1}{2} \times 2.0 \times 10^{11} \times (2 \times 10^{-3}) / 4)^2$

$= 0.25 \times 10^5 = 2.5 \times 10^4$ J/m^3.

(b) Elastic potential energy = energy density × volume

$= 2.5 \times 10^4 \times (8.0 \times 10^{-6}) \times 4.0$ J

$= 20 \times 10^{-2} = 0.20$ J.

Illustration 5. *Calculate the work done in stretching a uniform metal wire (area of cross–section $10^{-6}m^2$ and length 1.5 m) through 4×10^{-3}m. The Young's modulus for the wire is $2\times 10^{11} N/m^2$.*

Solution: Consider a wire of length L and area of cross–section A. Let ℓ be the increase in the length when a stretching force F is applied. The work done dW in increasing the length $d\ell$ is given by

$$dW = F \times d\ell$$

The total work done in increasing the length ℓ is given by

$$W = \int_0^{\ell} F \times d\ell \qquad \text{. . . (i)}$$

We know that $Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{F/A}{\ell/L}$

$$\therefore \qquad F = \frac{YA\ell}{L} \qquad \text{. . . (ii)}$$

Substituting the value of F from equation (ii) in equation (i), we get

$$W = \int_0^{\ell} \frac{YA\ell}{L} \times d\ell = \frac{YA}{L}\left(\frac{\ell^2}{2}\right)_0^{\ell} = \frac{YA\ell^2}{2\ L}$$

Substituting the given values, we have

$$W = \frac{2 \times 10^{11} \times 10^{-6} \times \left(4\times10^{-3}\right)^2}{2 \times 1.5} = 1.066\text{J}$$

Exercise 2. ***When a compressed spring is dissolved in acid, what happens to elastic potential energy of the spring?***

SIMPLE HARMONIC MOTION AND HARMONIC OSCILLATOR

PERIODIC MOTION

If a motion repeats itself after equal intervals of time, it is said to be periodic motion. The displacement of a particle in such motion can always be expressed in terms of sine and cosine function therefore periodic motion is often called harmonic motion.

OSCILLATORY MOTION

When a particle executes to and fro motion about some fixed point, its motion is said to be oscillatory or vibratory. The motion of the bob of a pendulum, mass attached to a spring, Violin strings, atoms in a solid lattice and air molecules as a sound wave passes by, are the examples of oscillatory motion.

An essential concept in oscillation is that of position of equilibrium. If a system is disturbed from its position of equilibrium and left to itself, it begins to oscillate about mean position.

An oscillatory motion is always periodic. A periodic motion may or may not be oscillatory. So, oscillatory motion is merely a special case of periodic motion such as the motion of the planets around the sun is periodic but not oscillatory.

SIMPLE HARMONIC MOTION

Important one among all oscillatory motion is the simple harmonic motion. A particle executing simple harmonic motion oscillates in straight line periodically in such a way that the acceleration is proportional to its displacement from a fixed point (called equilibrium point), and is always directed towards that point.

A system executing simple harmonic motion is called a harmonic oscillator.
Harmonic oscillation is that oscillation which can be expressed in terms of single harmonic function (sine function or cosine function).

In the absence of frictional forces, the harmonic oscillation possesses constant amplitude. A harmonic oscillation of constant amplitude and single frequency is called simple harmonic motion.

The displacement x (at any time t) of a particle executing simple harmonic oscillation may be given by any of the following two equation

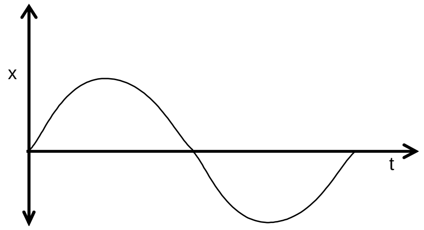
$$x = a \sin\frac{2\pi}{T}t \qquad \text{. . . (i)}$$

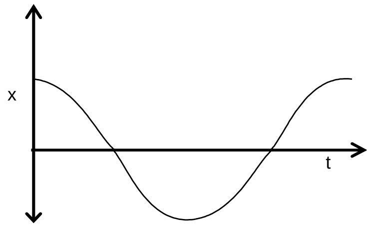
$$x = a \cos\frac{2\pi}{T}t \qquad \text{. . . (ii)}$$

Here T represents the time period and 'a' represents the maximum displacement i.e. displacement amplitude of the particle.

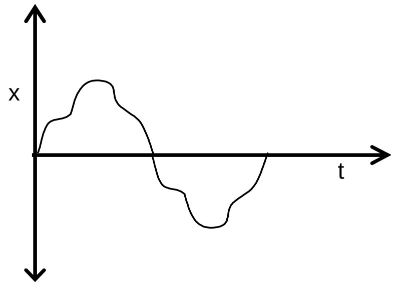
equation (i) can also be written as $x = a \cos\left(\frac{2\pi t}{T} - \frac{\pi}{2}\right)$

Thus, in general, a harmonic oscillation may be represented by $x = a \cos\left(\frac{2\pi t}{T} + \phi\right)$





Non harmonic oscillation is that which cannot be represented by a single harmonic function. A non–harmonic oscillation can be resolved into two or more than two simple harmonic oscillations.



Simple harmonic motion is a motion which is necessarily periodic and oscillatory about a fixed mean position. A particle executing such a motion is always in stable equilibrium about its mean position, so, if a particle is disturbed slightly from its mean position, the force which tends to bring the particle back to the mean position is called the restoring force. The greater is the displacement of the particle from the mean position, greater is the restoring force. Further, the restoring force is always directed towards the mean position.

Thus, simple harmonic motion is defined as such oscillatory motion about a fixed point (mean position) in which the restoring force is always proportional to the displacement from the point and is always directed towards that point.

Importance of the study of simple harmonic motion Any periodic motion can be expressed as the resultant of the two or more simple harmonic motions. Simple harmonic motion is the simplest and most fundamental of all types of periodic motions. example

(i) In mechanical wave motion, the particles of the medium execute either simple harmonic motion or combination of simple harmonic motions.

(ii) The motion of the molecules of a solid is nearly simple harmonic. This fact helps to explain certain characteristics of solids.

(iii) The vibrations of the air columns and string of musicals instruments are either simple harmonic or a superposition of simple harmonic motion.

(iv) The prongs of a vibrating tuning fork oscillate simple harmonically.

Note: The necessary and sufficient condition for a motion of a particle to be simple harmonic motion that, force acting on it must be directed towards a fixed point called equilibrium point and magnitude of the force must be proportional to the displacement of the particle from the equilibrium position.

In case of angular oscillation, the torque acting on the body must have tendency to bring the body to the equilibrium position.

Differential equation of simple harmonic motion is

$$a = -\omega^2 y \qquad \text{i.e.} \quad \frac{d^2y}{dt^2} = -\omega^2 y$$

Solution of the equation of simple harmonic motion:
The necessary and sufficient condition for a motion to be simple harmonic is that the net restoring force (or torque) must be linear i.e.

$$F = ma = -kx \qquad \text{(Where k is a constant)}$$

$$\Rightarrow \qquad a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

where x is the instantaneous displacement from the mean position i.e. x = 0. Multiplying both sides by $\frac{dx}{dt}$ and integrating with respect to t,

We get $\frac{1}{2}\left(\frac{dx}{dt}\right)^2 = -\frac{k}{m}\frac{x^2}{2} + C$ . . . . (i)

Where C is a constant of integration. Now when x is maximum $\frac{dx}{dt}$ will be zero. The maximum displacement x_{max} of the particle from the mean position is called **amplitude** and is represented by A, then the value of C comes out to be

$$C = \frac{k}{m}\ \frac{A^2}{2}$$

Hence $\frac{1}{2}\left(\frac{dx}{dt}\right)^2 = \frac{k}{2m}\left(A^2 - x^2\right)$

Putting $\frac{k}{m} = \omega^2$ we get

$$\frac{dx}{dt} = \omega\sqrt{A^2 - x^2} \qquad . . . \text{(ii)}$$

This equation gives the velocity of the particle in simple harmonic motion.

Again $\frac{dx}{\sqrt{A^2 - x^2}} = \omega dt$ . . . .(iii)

Integrating this equation with respect to t. we get

$$\sin^{-1}\frac{x}{A} = \omega t + \phi$$

Where ϕ is another constant of integration which depends on initial conditions.

Thus $x = A \sin(\omega t + \phi)$ . . . . (iv)

With the change of time t, sin ($\omega t + \phi$) varies between 1 and –1 and corresponding displacement x varies between A and –A. Thus the range of vibration is 2A. Here ω is called **angular frequency** and ϕ is called **phase constant,** whose value depends upon initial conditions. If the time is recorded from the instant when x is zero and is increasing then according to equation (iv) ϕ must be equal to zero.

Now, the periodic time or time required to complete one vibration or the time to increase the phase angle of the particle by 2π is given by $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ …(v)

T is called the **time period** of oscillation.

The **frequency f**, the number of complete vibration per second is given by

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{…(vi)}$$

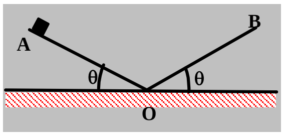
If we put $\pi/2 + \phi$ in place of ϕ then

$$x = A\cos(\omega t + \phi) \qquad \text{…(vii)}$$

Thus simple harmonic motion may also be represented by a cosine function but initial phase is different in two cases.

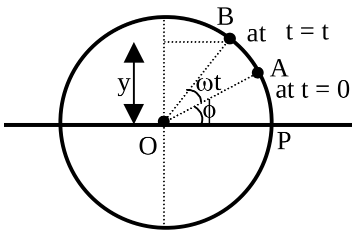
Exercise 3.

(i) ***A particle oscillates between A and B through O as shown. Will its motion be simple harmonic?***



(ii) ***A highly (super) elastic ball bouncing on a hard floor has motion that is approximately periodic. In what ways it is similar to S.H.M. and in what way it is different?***

Projection of uniform circular motion on a diameter



Let at t = 0 the particle is at the point A and after time t it is at B. The foot of the perpendicular from the point on the diameter oscillates about O which is simple harmonic motion in nature.

The displacement of the projection from centre is given by

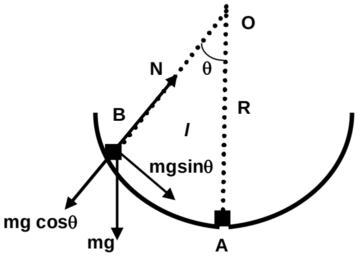
$$y = R\sin(\omega t + \phi)$$

The maximum displacement of the projection from the centre is called amplitude. If amplitude is denoted by A, then R = A.

$\Rightarrow$ $y = A\sin(\omega t + \phi)$ …(ii)

Illustration 6. *A small particle is kept on a fixed smooth concave glass surface of radius of curvature R placed in the vertical plane. Show that if the particle is displaced from the equilibrium position and left, it executes simple harmonic motion, and find its period. (displacement being very small compared to R).*

Solution: O is the centre of curvature of the glass. The equilibrium position of the particle is at A with OA as the vertical. If displaced, it will oscillate to and fro about the equilibrium position.



Let B be the position of the particle at any time, so that AB = x and ∠BOA = θ. The forces acting on the particle will be

(a) its weight mg acting vertically downwards
(b) the normal reaction N along BO

Resolving the weight mg into components along the normal to the surface and perpendicular to it, the component mg sin θ directed towards the mean position provides the restoring force.

∴ Restoring force = F = mg sinθ

∴ Acceleration = a = –gsinθ (–ve sign to indicate that acceleration is opposite in direction to the displacement).

$= -g\theta$ ($\because$ $\theta = x/R$ is very small)

$= -g(x/R)$

∴ The motion of the particle is a simple harmonic motion

∴ Acceleration $= -\omega^2 x = -\dfrac{g}{R}x$

$\omega^2 = g/R$, $\qquad \omega = \sqrt{\dfrac{g}{R}}$

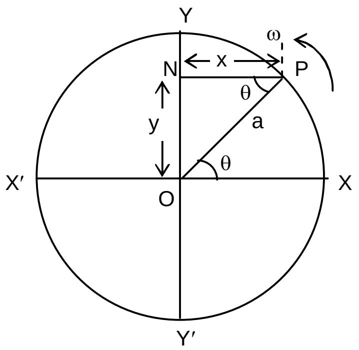
∴ Period of oscillation $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{R}{g}}$

CHARACTERISTICS OF SIMPLE HARMONIC MOTION

Displacement (x)

If time is measured from the equilibrium position, displacement x from equilibrium point at any instant of time t is given by x = A sinωt

In this figure, the magnitude of the displacement of P from the mean position at any instant is given by



$$x = a\cos\theta$$

Where a is the radius of the reference circle and θ is the angle covered by the reference particle in time t. If ω be the uniform angular velocity of the reference particle, then

$$x = a\cos\omega t \qquad [\because \omega = \theta/t]$$

If the projection N of the reference particle is taken on the diameter YOY′,

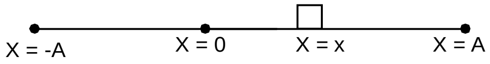
then $y = a\sin\theta$

$\Rightarrow$ $y = a\sin\omega t$

Velocity (v) :

$$v = \frac{dx}{dt} = \frac{d}{dt}(A \sin\omega t) = \omega A \cos\omega t$$

or $$v = \omega A\sqrt{1-\sin^2\omega t} = \omega\sqrt{A^2 - x^2}$$



(i) Velocity is minimum at extreme positions and is zero.

at $x = A$, v_{min} = zero.

(ii) Velocity is maximum at the equilibrium position and is ωA.

at $x = 0$, $v_{max} = \omega A$

(iii) Direction of velocity is either towards or away from the equilibrium position.

Acceleration (a) : $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx} = -\omega^2 A \sin\omega t = -\omega^2 x$

(i) The minimum value of acceleration is zero and it occurs at equilibrium, $a_{min} = 0$.

(ii) The maximum value of acceleration is $\omega^2 A$ and it occurs at extreme positions, $a_{max} = \omega^2 A$

(iii) Acceleration is always directed towards the equilibrium position and so is always opposite to displacement.

Amplitude: The amplitude of vibrating particle is its maximum displacement form the mean position to one extreme position.

Frequency: It is the number of oscillations (or vibrations) completed per unit time. It is denoted by ν or n. Since, In the time T seconds, 1 vibration is completed.

Thus in 1 sec, $\frac{1}{T}$ vibration is completed.

or $\nu = \frac{1}{T}$ or $\nu T = 1$

Angular frequency: It is frequency ν multiplied by a numerical quantity 2π. It is denoted by ω so that

$$\omega = 2\pi\nu = \frac{2\pi}{T}$$

thus, $x = a\cos\left(\frac{2\pi}{T} + \phi_0\right)$, can be written as $x = a\cos(\omega t + \phi_0)$

Phase: Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.

The augment of the cosine equations $x = a\cos(\omega t + \phi_0)$ gives the phase of oscillation at time t. It is denoted by ϕ

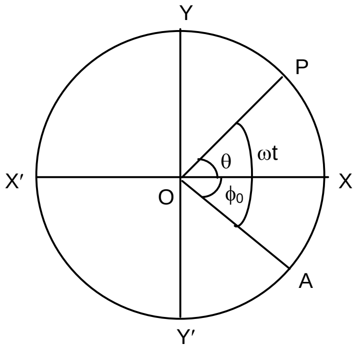
$$\phi = \frac{2\pi}{T}t + \phi_0, \quad \text{or} \quad \phi = \omega t + \phi_0$$

It is clear that phase ϕ is a function of time t. The phase of a vibrating particle can be expressed in terms of the fraction of the time period that has elapsed, since the vibrating particle last left its mean position in the positive direction.

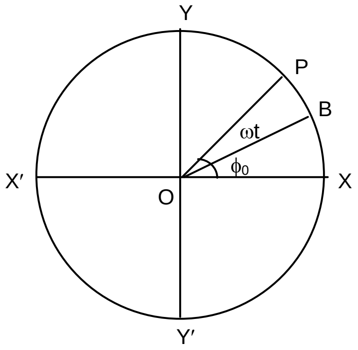
Again $\phi - \phi_0 = \omega t = \frac{2\pi}{T}t$

So the phase change in time t is $\frac{2\pi}{T}t$

Epoch: It is the initial phase of the vibrating particle i.e. phase at t = 0.
At t = 0, $\phi = \phi_0$ $[\because \ \phi = \omega t + \phi_0]$



(Epoch $-\phi_0$)



(Epoch $+\phi_0$)

If the reference particle starts from its standard position X, then the displacement of the projections on X–axis, at any instant, is given by $x = a \cos \theta$
for projection on y axis $y = a \sin\theta$

If instead of counting time from the instant when the reference particle crosses x–axis, it is counted from the instant, when the reference particle is at A, then

$\theta = \omega t - \phi_0$
$x = a \cos (\omega t - \phi_0)$
and $y = a \sin (\omega t - \phi_0)$

The angle $(\omega t - \phi_0)$ is called the phase of the vibrating particle at time t.

when t = 0, phase $= -\phi_0$ which is epoch

If a time is counted from the instant the reference particle is at B, then
$x = a \cos (\omega t + \phi_0)$
and $y = a \sin (\omega t + \phi_0)$
In this case, epoch $= +\phi_0$.

Graphical representation of particle displacement, particle velocity and particle acceleration:

Let us consider equation $x = a \sin (\omega t + \phi_0)$
If epoch ϕ_0 is assumed to be zero, then displacement $x = a \sin \omega t$
velocity $dx/dt = a\omega \cos \omega t$

$= a\omega \sin (\omega t + \pi/2)$

acceleration $d^2x/dt^2 = -a\omega^2 \sin \omega t$

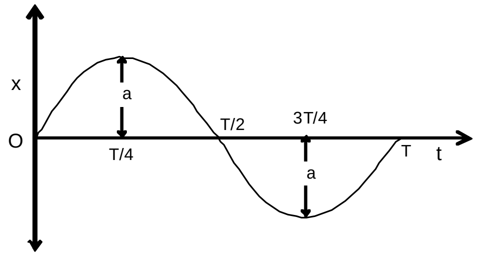
$= -\omega^2 x$

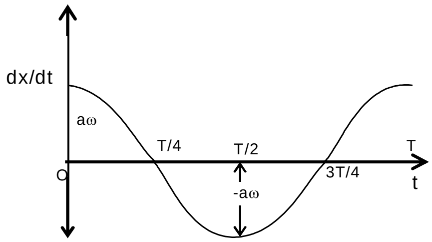
Using these relations, we can calculate the values of displacement, velocity and acceleration for different values of t. These values for one complete vibration are given in the following table, for displacement x = a sin ωt

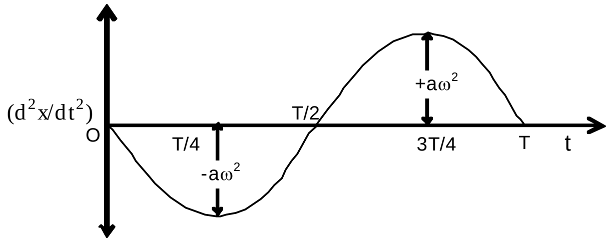
Time	0	T/4	T/2	3T/4	T
Displacement (x)	0 (min)	+a (max)	0 (min)	–a (max)	0(min)
Velocity (dx/dt)	Aω (max)	0 (min)	–aω (max)	0 (min)	aω (max)
Acceleration (d^2x/dt^2)	0 (min)	$-a\omega^2$ (max)	0 (min)	$a\omega^2$ (max)	0 (min)

These results lead us to the following conclusions

(i) All the three quantities (x, dx/dt, d^2x/dt^2) vary harmonically with time t.
(ii) The velocity amplitude is ω times the displacement amplitude.
(iii) The acceleration amplitude is ω^2 times the displacement amplitude.
(iv) the velocity is $\pi/2$ ahead of the displacement in phase.
(v) The acceleration is ahead of the velocity in phase by $\pi/2$ or π ahead of displacement.







Expression for time period and frequency in SHM

We know $\omega^2 = k / m$ or $\omega = \sqrt{k/m}$

But $\omega = \dfrac{2\pi}{T} = \sqrt{\dfrac{k}{m}}$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Depending on what kind of oscillation we discuss, the quantities corresponding to m and k will go on taking different forms and names. As a general nomenclature, k is called the spring factor and m is called the inertia factor.

In general, $T = 2\pi\sqrt{\dfrac{\text{Inertia factor}}{\text{spring factor}}}$

frequency $\nu = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{\text{spring factor}}{\text{inertia factor}}}$

Again, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\text{force/displacement}}}$

$$T = 2\pi\sqrt{\frac{m\times\text{displacement}}{\text{force}}} = 2\pi\sqrt{\frac{m\times\text{displacement}}{m\times\text{acceleration}}}$$

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$\text{Frequency } \nu = \frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}}.$$

Illustration 7. *A harmonic oscillation is represented by x = 0.4 cos (3000 t + 0.77) where x and t are in mm and sec respectively. Deduce (i) Amplitude (ii) frequency (iii) Angular frequency (iv) Period and (v) initial phase.*

Solution: $x = 0.4\cos(3000t + 0.77)$
comparing with general equation
$x = a\cos(\omega t + \phi_0)$
we get, a = 0.4 nm, ω = 3000 rod/s, ϕ_0 = 0.77 rad.
(i) Amplitude a = 0.4 mm = 0.04 cm.
(ii) Frequency $\nu = \frac{\omega}{2\pi} = \frac{300}{2\pi}$ Hz = 471.27 Hz.
(iii) Angular frequency, ω = 3000 rad/sec.
(iv) Period $T = \frac{2\pi}{\omega} = 0.002\,\text{sec}$.
(v) Initial phase ϕ_0 = 0.77 rad.

Illustration 8. *Obtain equation for simple harmonic motion of a particle whose amplitude is 0.5 m and frequency is 60 Hz. Initial phase is π/3.*

Solution: $x = a\sin(\omega t + \phi)$
Given a = 0.5 m ν = 60 Hz, $\phi_0 = \pi/3$
$\therefore\ x = 0.5\sin(2\pi\times 60\,t + \frac{\pi}{3})$
$x = 0.5\sin(120\,\pi t + \frac{\pi}{3})$

Illustration 9. *A particle executes simple harmonic motion about the point x = 0. At time t = 0 it has displacement x = 2 cm and zero velocity. If the frequency of motion is 0.25 sec^{-1}, find (a) The period, (b) angular frequency, (c) the amplitude, (d) maximum speed, (e) the displacement at t = 3 sec and (f) the velocity at t = 3 sec*

Solution : (a) Period $T = \frac{1}{n} = \frac{1}{0.25\ \text{sec}^{-1}} = 4\ \text{sec}$.
(b) Angular frequency $\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}\ \text{rad/sec}$ = 1.57 rad/s
(c) Amplitude is the maximum displacement from mean position.
Hence, A = 2 – 0 = 2 cm
(d) Maximum speed $V_{max} = A\,\omega = 2.\,\pi/2 = \pi$ cm/s = 3.14 cm/s
(e) The displacement is given by
$$x = A\sin(\omega t + \phi)$$
Initially at t = 0, x = 2cm, then

$2 = 2\sin\phi$
or $\sin\phi = 1 = \sin 90°$
or $\phi = 90°$
Now at t = 3 sec

$$x = 2\sin\left(\frac{\pi}{2} \times 3 + \frac{\pi}{2}\right) = 0$$

(f) Velocity at x = 0 is v_{max} i.e. 3.14 cm/sec

Illustration 10. *If a S.H.M. is represented by the equation $x = 10 \sin[\pi t + \pi/6]$ in S.I. units, determine its amplitude, time period and maximum velocity v_{max}.*

Solution : Comparing the above equation with
$x = A\sin(\omega t + \phi_o)$, we get,
$A = 10$ m
$\omega = \pi s^{-1}$ and $\phi_0 = \pi/6$
$\because$ $T = 2\pi/\omega$
$\Rightarrow$ $T = 2s$
$v_{max} = \omega A = 10\,\pi$ m/s.

Illustration 11. *A particle executes S. H. M. with a time period of 4s. Find the time taken by the particle to go directly from its mean position to half of its amplitude.*

Solution: $x = A\sin(\omega t + \phi_o)$
At t = 0, x = 0 $\Rightarrow$ $A\sin\phi_o = 0$ or $\phi_o = 0$
Hence, $x = A\sin(\omega t)$
or $A/2 = A\sin(\omega t)$
or $½ = \sin(\omega t)$

$$\omega t = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$t = \frac{\pi}{6\omega} = \frac{\pi.T}{6(2\pi)}$$

as $\omega = 2\pi/T$
$\Rightarrow$ $t = T/12 = 1/3$ sec

Illustration 12. *Two particles move parallel to x–axis about the origin with the same amplitude and frequency. At a certain instant they are found at distance (A/3) from the origin on opposite sides but their velocities are found to be in the same direction. What is the phase difference between the two?*

Solution : Let equations of two S.H.M. be
$x_1 = A\sin\omega t$ . . . (i)
$x_2 = A\sin(\omega t + \phi)$ . . . (ii)
Given that $\frac{A}{3} = A\sin\omega t$ and $-\frac{A}{3} = A\sin(\omega t + \phi)$
Which gives

$$\sin\omega t = \frac{1}{3} \quad \text{. . . (iii)}$$

$$\sin(\omega t + \phi) = -\frac{1}{3} \qquad \ldots \text{(iv)}$$

from (iv)

$$\sin\omega t \cos\phi + \cos\omega t \sin\phi = -\frac{1}{3}$$

$$\Rightarrow \frac{1}{3}\cos\phi + \sqrt{1-\frac{1}{9}}\sin\phi = -\frac{1}{3}$$

or $9\cos^2\phi + 2\cos\phi - 7 = 0$

or $\cos\phi = -1, 7/9$

$$\Rightarrow \phi = \pi \text{ or } \cos^{-1}\left(\frac{7}{9}\right)$$

differentiating (I) and (ii) we obtain

$v_1 = A\omega \cos\omega t$ and $v_2 = A\omega\cos(\omega t + \phi)$

If we put $\phi = \pi$, we find v_1 and v_2 are of opposite signs. Hence $\phi = \pi$ is not acceptable

$$\therefore \phi = \cos^{-1}\left(\frac{7}{9}\right)$$

Exercise 4.

(i). Determine whether or not the following quantities can be in the same direction for a simple harmonic motion. Find
(a) Displacement & Velocity, (b) Velocity & Acceleration,
(c) Displacement & Acceleration.

(ii). A body executes SHM with a period of 11/7 second and an amplitude of 0.025 m. What is the maximum value of acceleration ?

(iii). Imagine a situation where the motion is not simple harmonic but the particle has maximum velocity in mean position and zero velocity oat extreme position.

ENERGY OF A BODY EXECUTING SIMPLE HARMONIC MOTION

In general the total energy of a harmonic oscillator consists of two parts, potential energy (P.E) and Kinetic Energy (K.E.), the former being due to its displacement from the mean position and later due to its velocity. Since the position and velocity of the harmonic oscillator are continuously changing, P.E. and K.E. also change but their sum i.e., the total energy (T.E) must have the same value at all times.

During an oscillatory motion, when the displacement is maximum, the restoring force is maximum and thus P.E. is maximum. At this position velocity of the particle and hence the kinetic energy is zero. Also when the particle is just crossing the mean position in either direction, the potential energy is zero. At the mean position, the velocity and hence the kinetic energy is maximum. In between mean and extreme positions, the energy of particle is partly kinetic and partly potential. However the total energy remains constant.

Expression for potential energy (E_p)

The linear restoring force acting on the harmonic oscillator is given by

$$F = m\frac{d^2x}{dt^2} = -kx$$

Now if the oscillator is displaced through a further displacement dx against the restoring force, work done in displacing the particle is given by

$$dW = kx\,dx$$

Hence the total work done in displacing the particle from mean position (x = 0) to (x = x) is given by

$$W = \int_0^x kxdx = \frac{1}{2}kx^2$$

As P.E. at the mean position is taken zero, the above equation gives the values of P.E. of harmonic oscillator at a displacement x from the mean position i.e.,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 \qquad (1)$$

This shows the P.E. is proportional to the square of the displacement and graph showing the variation of potential energy with the displacement will be a parabola given by continuous line in the figure. P.E. is maximum at maximum displacement and is given by

$$U_{max} = \frac{1}{2}m\omega^2A^2$$

Exercise 5.

(i) ***Prove that the elastic potential energy density of a stretched wire is equal to half the product of stress and strain.***

(ii) ***What will happen to the potential energy if a wire is (a) compressed (b) stretched ?***

Expression for Kinetic energy (E_k)

Velocity of harmonic oscillator is given by equation as

$$v = \frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$$

Hence Kinetic energy KE of the oscillator is given by

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) \qquad (2)$$

The graph showing the variation of K.E. with x is shown in figure by dotted line.
The kinetic energy is maximum when x = 0. Thus

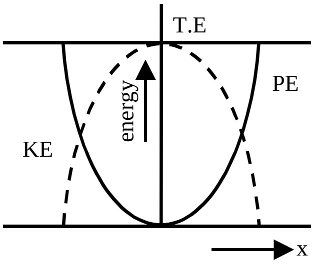
$$KE_{max} = \frac{1}{2}m\omega^2A^2$$

Now total energy E of the oscillator for displacement x is given by

$$E = P.E. + K.E. = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

$$= \frac{1}{2}m\omega^2A^2 \qquad (3)$$

Thus total energy is independent of the displacement, it remains constant throughout the motion of the oscillator. It is being represented by a straight line parallel to displacement axis in figure. Also the total energy is equal to maximum value of either P.E. or K.E.



Graphical Representation

Displacement (x)	**0**	**a/2**	**a**	**–a/2**	**–a**
Kinetic energy (E_k)	E	3E/4	0	3E/4	0
Potential energy (E_P)	0	E/4	E	E/4	E

Illustration 13. *A particle of mass 1kg is executing SHM of amplitude 0.1 m. When the particle passes through the mean position, its kinetic energy 32 × 10^2 J. Obtain the equation if this particle if the initial phase of oscillation is 60°.*

Solution: $x = a \sin(\omega t + \phi_0)$

Given $a = 0.1$ m, $\phi_0 = 60° = \dfrac{\pi}{3}$

Now, $\dfrac{1}{2}m\omega^2 a^2 = 32 \times 10^{-2}$

$\omega^2 = \dfrac{32 \times 10^{-2} \times 2}{1 \times 0.1 \times 0.1}$

$\omega = 8$ rad/s.

thus $x = 0.1 \sin(8t + \pi/3)$

Illustration 14. *A body of mass 10^{-4} kg has a velocity of 4 × 10^{2}m/s after one second of its starting from mean position. If time period is 6 sec., determine the kinetic energy, potential energy, total energy, maximum kinetic energy, and maximum potential energy.*

Solution: $m = 10^{-4}$ kg, $v = 4 \times 10^2$ m/s, $t = 1$ sec. $T = 6$ sec.

As $v = a\omega \cos \omega t$

$4 \times 10^2 = a \times \dfrac{2\pi}{6}\cos\left(\dfrac{2\pi}{6} \times 1\right)$

or $a = \dfrac{24}{\pi} \times 10^2$ m

kinetic energy $= \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 10^{-4} \times (4 \times 10^2)^2 = 8$ J

Total energy $= \dfrac{1}{2}ma^2\omega^2 = \dfrac{1}{2} \times 10^{-4} \times \left(\dfrac{24}{\pi} \times 10^2\right)^2 \times \left(\dfrac{2\pi}{6}\right)^2 = 32$ J

Potential energy (32 –8) J = 24 J.

Maximum KE = Max. potential energy = total energy = 32 J.

Average value of P.E. and K.E.:

By equation (1) P.E. of a particle when it is at distance x from the mean position is given by

$$U = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

The average value of P.E. for complete vibration is given by

$$U_{average} = \frac{1}{T}\int_0^T U dt = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)\, dt$$

$$= \frac{m\omega^2 A^2}{4T}\int_0^T 2\sin^2(\omega t + \phi) dt = \frac{1}{4}m\omega^2 A^2$$

because the average value of square of sine or of cosine function for the complete cycle is equal to 1/2.

Now K.E. at a distance x is given by

$$+ KE = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\left[\frac{d}{dt}\{A\sin(\omega t + \phi)\}\right]^2$$

$$= \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi)$$

The average value of K.E. for complete cycle

$$KE_{average} = \frac{1}{T}\int_0^T \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \phi)dt$$

$$= \frac{m\omega^2 A^2}{4T}\int_0^T \{1+\cos 2(\omega t + \phi)\}dt$$

$$= \frac{m\omega^2 A^2}{4T}.T = \frac{1}{4} m\omega^2 A^2$$

Thus average values of K.E. and P.E. of harmonic oscillator are equal and each is equal to half of the total energy

Exercise 6. ***What is the frequency of the K.E. of a body in S.H.M. if the time period of S.H.M. is T?***

Illustration 15. *A particle of mass 4 gm. lies in a potential field is given by $V = 200x^2 + 150$ ergs/gm. Deduce the frequency of vibration.*

Solution: The potential energy of the 4gm mass

$U = mV = 4(200x^2+150) = 800x^2 + 600)$ ergs

The force F acting on the particle is given by

$$F = -\frac{dU}{dx} = \frac{d}{dx}(800x^2 + 600) \text{ dynes} = -1600x$$

Then the equation of motion of the particle is given by

$$m\frac{d^2x}{dt^2} = -1600x$$

$$\frac{d^2x}{dt^2} = -\frac{1600}{4}x = -400x$$

hence frequency of oscillation $n = \frac{1}{2\pi}\sqrt{400} = \frac{10}{\pi} = 3.2 \text{ sec}^{-1}$

Illustration 16. *A particle executes S.H.M.*

(a) What fraction of total energy is kinetic and what fraction is potential when displacement is one half of the amplitude?

(b) At what value of displacement are the kinetic and potential energies equal?

Solution: We know that,

$$E_{total} = \frac{1}{2}m\omega^2A^2$$

$$K.E. = \frac{1}{2}m\omega^2(A^2 - x^2) \text{ and } U = \frac{1}{2}m\omega^2x^2$$

(a) when $x = \frac{A}{2}$

$$K.E. = \frac{1}{2}m\omega^2\frac{3A^2}{4} \Rightarrow \frac{K.E.}{E_{total}} = \frac{3}{4}$$

At $x = A/2$, $U = \frac{1}{2}m\omega^2\frac{A^2}{4}$

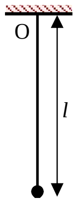
$$\Rightarrow \quad \frac{P.E.}{E_{total}} = \frac{1}{4}$$

(b) Since K = U

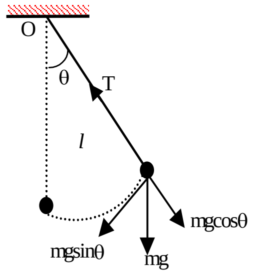
$$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}m\omega^2 x^2$$

or $2x^2 = A^2$ or $x = \frac{A}{\sqrt{2}} = 0.707A$

SIMPLE PENDULUM



A heavy small mass (bob) suspended by a light, long and inextensible string forms simple pendulum.
Length of the simple pendulum is the distance between the point of suspension and the centre of mass of the suspended mass.
Consider the bob when string deflects through a small angle θ.



Force acting on the bob are tension T in the string and weight mg of the bob.
Torque on the bob about point O is

$$\tau = \tau_{mg} + \tau_T = mg\,\ell\sin\theta + 0$$

$\Rightarrow$ $\tau = mg\,\ell\,\theta$ as θ is very small. . . . (1)

$\because$ M.I. of the bob about the point O is

$$I = m\ell^2$$

Hence $\tau = m\ell^2\frac{d^2\theta}{dt^2}$ . . . (2)

As torque τ and θ are oppositely directed hence from (1) and (2), we get

$$m\ell^2\frac{d^2\theta}{dt^2} = -mg\,\ell\theta \quad \Rightarrow \quad \frac{d^2\theta}{dt^2} = -(g/\ell)\,\theta$$

Comparing with the equation $\frac{d^2x}{dt^2} = -\omega^2 x$, we get $\omega = \sqrt{\frac{g}{\ell}}$

Since $T = 2\pi/\omega \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g}}$

If a simple pendulum is made to oscillate in a non inertial frame of reference, The torque due to pseudo force must be taken in account.

Exercise 7.

(i) ***At what point in the motion of a simple pendulum is the string tension greatest? Least?***

(ii) ***A pendulum – controlled clock is transferred from earth to moon. What would be the effect on the clock.***

(iii) ***The bob of a simple pendulum is a hollow sphere filled with water. How will the period of oscillation change if the water begins to drain out of the hollow sphere ?***

Illustration 17. *Show that the period of oscillation of simple pendulum at depth h below earth's surface is inversely proportional to $\sqrt{R-h}$ where R is the radius of earth. Find out the time period of a second pendulum at a depth R/2 from the earth's surface?*

Solution: At earth's surface the va $\sqrt{R-h}$ lue of time period is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Where L is the effective length of the simple pendulum and g is the acceleration due to gravity and is equal to $\frac{GM}{R} = \frac{4}{3}\pi RG\rho$. At depth h if the period is T_h and acceleration due to gravity is g_h,

Then $T_h = 2\pi\sqrt{\frac{L}{g_h}}$

Hence $\frac{T_h}{T} = \sqrt{\frac{g}{g_h}} = \sqrt{\frac{R}{R-h}}$

or $T_h = \frac{T\sqrt{R}}{\sqrt{R-h}}$

Thus T_h is inversely proportional to

Time period at h = R/2 is given by

$$T_{R/2} = 2\sqrt{\frac{R}{R-R/2}} = 2\sqrt{2} = 2.8\,\text{sec}$$

Illustration 18. *A cabin is moving in a gravity free space along y–axis with an acceleration a. What is the time period of oscillation of a particle of mass m attached with an inextensible string of length ℓ, in this cabin*

(A) $2\pi\sqrt{\frac{\ell}{g}}$ (B) $2\pi\sqrt{\frac{\ell}{a}}$

(C) $2\pi\sqrt{\frac{\ell}{a+g}}$ (D) $2\pi\sqrt{\frac{\ell}{g-a}}$

Solution: **B**

In gravity free space g = 0.

Tension in the string at equilibrium position = ma

Effective g = a

And thus, $T = 2\pi\sqrt{\frac{\ell}{a}}$

Illustration 19. *The total energy of a simple pendulum is 2 J when the length of the pendulum is 3 m and its amplitude is 1 cm. If the amplitude is made larger by 2 cm then taking length of the pendulum unchanged the energy becomes*

(A) 6 J *(B) 9 J*

(C) 18 J *(D) none of these.*

Solution: **C**

Energy E $\propto \text{Amp}^2$

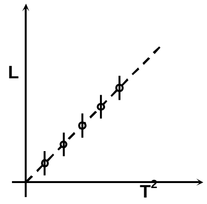
$$\frac{E_1}{E_2} = \left(\frac{A_1}{A_2}\right)^2$$

$$\Rightarrow \frac{2}{E_2} = \left(\frac{1}{3}\right)^2 \Rightarrow E_2 = 18\text{ J}$$

Measurement of g using a simple pendulum

A small spherical bob is attached to a cotton thread and the combination is suspended from a point A. The length of the thread (L) is read off a meter scale. A correction is added to L to include the finite size of the bob and the hook. The corrected value of L is used for further calculation. The bob is displaced slightly to one side and is allowed to oscillate, and the total time taken for 50 complete oscillations is noted on a stop–watch. The time period (T) of a single oscillation is now calculated by division.

Observations are now taken by using different lengths for the cotton thread (L) and, pairs of values of L and T are taken. A plot of L vs T^2, on a graph, is linear.The value of g is given by $g = 4\pi^2 \frac{L}{T^2}$



The major errors in this experiment are:

(a) Systematic: Error due to finite amplitude of the pendulum (as the motion is not exactly SHM). This may be corrected by using the correct numerical estimate for the time period. However, the practice is to ensure that the amplitude is small.

(b) Statistical: Errors arising from measurement of length and time are known as statistical errors

$$\frac{\delta g}{g} = \frac{\delta L}{L} + 2\left(\frac{\delta T}{T}\right)$$

The contributions to δL, δT are both statistical and systematic. These are reduced by the process of averaging.

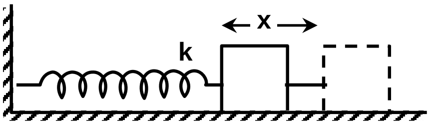
The systematic error in L can be reduced by plotting several values of L vs T^2 and fitting to a straight line. The slope of this fit gives the correct value of L/T^2.

Exercise 8. ***From the following measurements of L and T calculate the value of g***

L(cm)	*80.4*	*36.0*	*30.0*	*50.0 cm*
T(sec)	*90*	*60*	*55*	*71*

The time was measured with a stop watch for 50 oscillations. g is calculated by taking the mean of (L/T^2) from all the measurements. Estimate error in g as well.

Oscillation of a mass attached to a spring
Mass spring system when spring is horizontal:



If x is a small extension or compression in the spring from the equilibrium state the restoring force produced is given by

$F = -kx$

Where k is called force constant or spring factor.

$\therefore$ Equation of motion of the mass M is given by

$$M\frac{d^2x}{dt^2} = -kx \quad \text{or} \quad \frac{d^2x}{dt^2} = -\frac{k}{M}x$$

This represents a S.H.M. its angular frequency ω is

$$\omega = \sqrt{\frac{k}{M}}$$

and the period of oscillation is

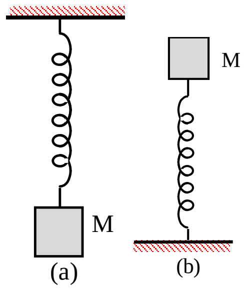
$$T = 2\pi\sqrt{M/k}$$

Mass spring system when spring is vertical

If y is a small extension or compression in the spring from the equilibrium state the restoring force produced is given by

$$F = -ky$$

Where k is called force constant or spring factor. Then equation of motion of the mass M is given by



$$M\frac{d^2y}{dt^2} = -ky$$

or $$\frac{d^2y}{dt^2} = -\frac{k}{M}y$$

This represents a S.H.M. of angular frequency ω given by $\omega = \sqrt{\frac{k}{M}}$

and the period of oscillation is $T = 2\pi\sqrt{\frac{M}{k}}$

Note that the gravity has no effect on the oscillations. The weight mg of the body produces an initial elongation such that $Mg - ky_o = 0$. If y is the displacement from this equilibrium position the total restoring force will be $Mg - k(y_0 + y) = -ky$

The displacement of the mass at any instant from position of rest is given by

$$y = A\sin(\omega t + \phi)$$

Where A and ϕ are amplitude and phase constant to be known from initial conditions of motion. If time is measured from the instant when the mass passes from its equilibrium position towards the direction in which y is measured positive

Then $y = 0$ at $t = 0$ or $\phi = 0$

Hence $y = A\sin\omega t$

But if at $t = 0$, the end of the spring is at its maximum displacement, then $\phi = \pi/2$

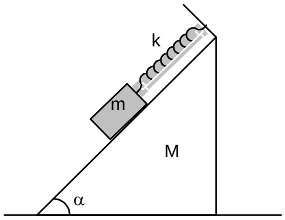
And $y = A\cos\omega t$

Exercise 9.

(i) ***A spring mass system oscillating vertically has a time period T. What shall be the time period if oscillated horizontally ?***

(ii) ***A spring is cut into two equal parts. What will be the difference in time period of the spring pendulum thus formed from the original spring pendulum?***

(iii) ***A system consisting of a smooth movable wedge of angle α and a block A of mass m are connected together with a massless spring of spring constant k, as shown in the figure. The system is kept on a frictionless horizontal plane. If the block is displaced slightly from equilibrium and left to oscillate, find the frequency of small oscillations.***



Simple harmonic motion of a massive particle under uniform spring of mass m

If L is the length of spring, then mass per unit length is given by m/L.

Consider an element of length dx at a distance x from the fixed end of the spring.

The mass of the element is $\frac{m}{L}dx$

The instantaneous velocity of the free end of the spring is v which is equal to the velocity of mass M. As velocity of spring at fixed end is always zero,

$\therefore$ The velocity of the element at a distance x is given by $\frac{vx}{L}$

Then K.E. of the element $= \frac{1}{2}\ \frac{m}{L}\ dx\ \left(\frac{vx}{L}\right)^2$

K.E. of the spring at that instant

$$= \int_0^L \frac{1}{2}\ \frac{mv^2}{L^3}\ x^2\ dx = \frac{1}{2}\ \frac{mv^2}{L^3}.\ \frac{L^3}{3} = \frac{mv^2}{6}$$

K.E. of mass M = $\frac{1}{2}\ Mv^2$

Total K.E. of the system

$$T = \frac{1}{2}\ Mv^2 + \frac{mv^2}{6} = \frac{1}{2}\left(M + \frac{m}{3}\right)\ v^2$$

If y is the displacement of the mass M at that instant from its equilibrium position, then

$$v = dx/dt$$

and restoring force produced is, F = – kx

Then P.E. $U = \int_0^y kx\ \ dx = \frac{Kx^2}{2}$

Hence total energy E of the system

$$E = T + U = \frac{1}{2}\ \left(M + \frac{m}{3}\right)\ \left(\frac{dx}{dt}\right)^2 + \frac{kx^2}{2}$$

Now as the total energy of the system is conserved dE/dt = 0

or $$\frac{d}{dt}\left[\frac{1}{2}\ \left(M + \frac{m}{3}\right)\ \left(\frac{dx}{dt}\right)^2 + \frac{kx^2}{2}\right] = 0$$

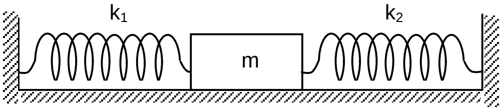
or $$\left(M + \frac{m}{3}\right)\ \frac{d^2x}{dt^2} + kx = 0$$

This represents a simple harmonic motion. Its angular frequency is,

$$\omega = \sqrt{\frac{k}{M + m/3}}$$

Hence, frequency (n) = $\frac{1}{2\pi}\ \sqrt{\frac{k}{M + m/3}}$

Illustration 20. *A body of mass 16 kg is oscillating on a spring of force constant 100 N/m. Deduce the angular frequency.*



Solution: m = 16 kg, k = 100 N/m, ω = ?,

$\omega = \sqrt{k/m}$ where $k = k_1 + k_2$

$= \sqrt{\frac{200}{16}}$ rad/s.

$= 5\sqrt{2}$ rad/s.

Illustration 21. *A spring has a load of 1.0 kg attached to its one end. A weight of 6 kg extends the spring by 24 cm. This 6.0 kg– wt. is removed and the block is allowed to execute harmonic vibration with the 0.1 kg load. Calculate its time period. (Given g = 9.8 m/s^2.)*

Solution: Since 6 kg wt. extends the spring by 24 cm i.e. 0.24 m.

$\therefore$ force constant $k = \frac{6\times 9.8}{0.24} = 245$ N/m.

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{245}} = 0.4$ sec.

Illustration 22. *A mass M attached to a spring oscillates every 2 sec. If the mass is increased by 2 kg, the period increases by 1 sec. Find the initial mass M, assuming that Hooke's law is obeyed.*

Solution: Period of oscillation $T = 2\pi\sqrt{\frac{M}{k}}$

Where M is the suspended mass and k is the spring constant of the spring.

In the first case, $2 = 2\pi\sqrt{\frac{M}{k}}$

Second case, $3 = 2\pi\sqrt{\frac{M+2}{k}}$

Squaring and dividing $\frac{9}{4} = \frac{M+2}{M}$

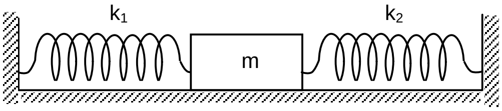
M = 1.6 kg.

Illustration 23. *A block of mass m is placed on a frictionless surface and is connected to two springs of force constant k_1 and k_2 as shown. Calculate the time period of oscillation of the block.*

Solution: Suppose block is displaced to the left through a small distance x. Thus left spring shall get compressed and the right spring shall elongate. Let F_1 and F_2 be the restoring force in the springs of force constant k_1 and k_2,

Then, $F_1 = -kx$

and $F_2 = -kx$



total restoring force

$F = F_1 + F_2$

$F = -k_1x - k_2x$

$F = -(k_1 + k_2)x$

$\Rightarrow$ $ma = -(k_1 + k_2)x$

$a = -\left(\frac{k_1 + k_2}{m}\right)x$

Time period, $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

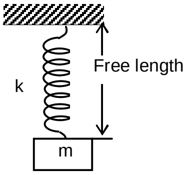
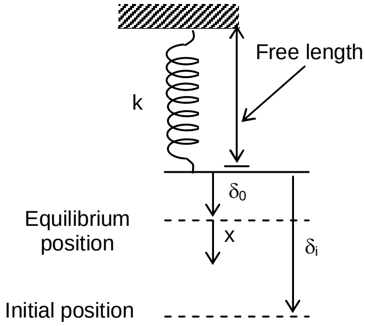


Illustration 24. *The spring mass system is shown in the figure. The spring stretches 2 cm from its free length when a force of 10 N is applied. This spring is stretched 10 cm from its free length, a body of mass m = 2kg attached to it and released from rest at time t = 0. Find*

(a) the force constant of the spring.
(b) the time period and frequency of vibration
(c) the amplitude of vibration.
(d) the initial velocity and acceleration
(e) the maximum velocity and acceleration
(f) the spring force at the two extreme position of the body.
(g) the time taken by the body to move half way towards the equilibrium position from its initial position. Write the equation of motion of the body in the form $x = A \sin(\omega t + \phi)$ where x is the displacement from the equilibrium position. Express the spring force as a function of time.



Solution : (a) $k = \dfrac{10\text{N}}{0.02\text{m}} = 500$ N/m

(b) time period $T = 2\pi\sqrt{\dfrac{m}{k}}$

$= 2\pi\sqrt{\dfrac{2}{500}} = 0.397$ s

frequency $f = \dfrac{1}{T} = 2.51$ Hz

angular frequency =15.8 rad/s

$\omega = 2\pi f$

(c) In equilibrium position acceleration = 0

$k\delta_0 - mg = 0$

$\delta_0 = \dfrac{mg}{k} = \dfrac{2(10)}{500} = 0.04$ m

amplitude = maximum displacement from the equilibrium position

$A = \delta_I - \delta_0 = 0.10 - 0.04 = 0.06$ m.

(d) Initial velocity = 0 (given)

Initial acceleration $= \dfrac{k\delta_i - mg}{m} = \dfrac{500(0.1) - 2(10)}{2}$

$= 15$ m/s^2 upward.

(e) Maximum velocity $= A\omega$

$= 0.06 \times 15.8 = 0.95$ m/s

Maximum acceleration $= A\omega^2 = 15$ m/s^2

(f) At one extreme position

spring force $F_s = k\delta_I = 500\,(0.1) = 50$ N

At the other extreme position

$F_s = k(A - \delta_0) = 500\ (0.06 - 0.04) = 10$ N compression.

The equation of motion in SHM is

$x = A \sin (\omega t + \phi)$

$\therefore$ $dx/dt = A\ \omega \cos (\omega t + \phi)$

initial condition t = 0, x = 10 – 4 = 6 cm = 0.06 m.

$$\frac{dx}{dt} = 0$$

$A \sin \phi = 0.06$

And $A\ \omega \cos \phi = 0$

$\therefore$ $\phi = \frac{\pi}{2}$ and $A = 0.06$ m

$\therefore$ $x = 00.06 \sin (\omega t + \pi/2)$

(g) At the given position x = A/2 = 0.03 m

$\therefore$ $0.03 = 0.06 \sin (\omega t + \pi/2)$

$\omega t = \pi/3 \quad \Rightarrow \quad t = \pi/3\omega$

$= 0.066$ s

The instantaneous velocity

$v = \omega\sqrt{A^2 - x^2} = \omega\sqrt{A^2 - A^2/4}$

$= \omega A\frac{\sqrt{3}}{2}$

= 0.82 m/s upward

[or $v = A\omega \cos (\omega t + \pi/2)$

$= A\ \omega \cos (\pi/3 + \pi/2)$

$= A\ \omega \cos (5\pi/3) = -A\omega\ \frac{\sqrt{3}}{2} = 0.82$ m/s upward]

Instantaneous acceleration

$a = -\omega^2 x$

$= -(A/2)\ \omega^2 = -7.5\ \ m/s^2$ i.e. $7.5\ m/s^2$ upwards

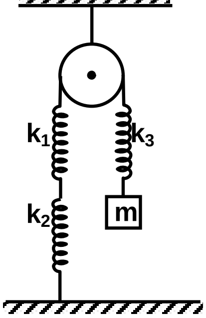
spring force $F_s = k\ (\delta_0 + x)$

$= mg + kA \sin (\omega t + \phi)$

$= 20 + 30 \sin (\omega t + \pi/2)$

$= 20 + 30 \cos \omega t$ N.

Illustration 25. *A block of mass m is connected by 3 identical spring of force constant k_1, k_2 and k_3 as shown in the figure. The block is given a small vertical displacement from its equilibrium position, the time period of oscillation of the block is*



(A) $2\pi\sqrt{\frac{m}{k_1}}$ (B) $2\pi\sqrt{\frac{m}{k_1}}$

(C) $2\pi\sqrt{\frac{m}{k_2}}$ (D) $2\pi\sqrt{m\left[\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}\right]}$

Solution: **(D).**

Restoring force on the block of mass m is given by

$= (k_1^{-1} + k_2^{-1} + k_3^{-1})x$

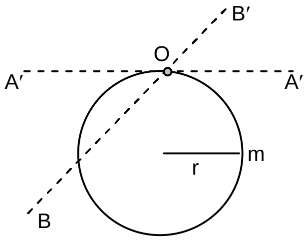
$$F = \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)x$$

$$\Rightarrow k_{eff} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

Therefore, the time period is given by

$$T = 2\pi\sqrt{m\left[\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right]}$$

Illustration 26. *A uniform ring of mass m, radius r is suspended as shown in the figure. There are two horizontal axes A–A′ (in the plane of ring) and B–B′ (perpendicular to the plane of ring)*



Assertion (A) : Time period of small oscillations of ring about A–A′ and B – B′ is same.

Reason (R) : $T = \sqrt{\frac{I}{mgr}}$ *where I is moment of inertia of ring.*

So time period is same for both cases.

(A) **(A)** is correct & **(R)** is correct explanation of **(A)**.
(B) Both are correct. But **(R) is not** correct explanation of **(A)**.
(C) **(A)** is incorrect & **(R)** is correct.
(D) Both are incorrect.

Solution: **D**

$$T = \sqrt{\frac{I}{mgr}}$$

$$T_{AA'} = \sqrt{\frac{I_{AA'}}{mgr}}$$

$$T_{BB'} = \sqrt{\frac{I_{BB'}}{mgr}}$$

$\Rightarrow I_{AA'} \neq I_{BB'}$

$\Rightarrow T_{AA'} \neq T_{BB'}$

Hence both A & R are wrong.

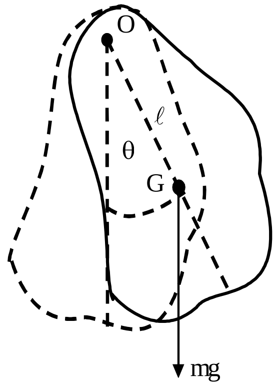
Exercise 10. ***Out of two clocks one based on simple pendulum and the other on spring pendulum, which one will show correct time on moon as written on earth ?***

PHYSICAL PENDULUM

Any rigid body suspended from a fixed support constitutes a physical pendulum. Consider the situation when the body is displaced through a small angle θ. Torque on the body about O is given by

$$\tau = mg\ell \sin\theta \qquad (1)$$

where ℓ = distance between point of suspension and centre of mass of the body.



If I be the M.I. of the body about O.

Then $\tau = I\alpha \qquad (2)$

From (1) and (2), we get

$I\frac{d^2\theta}{dt^2} = -mg\ell \sin\theta$ as θ and $\frac{d^2\theta}{dt^2}$ are oppositely directed.

$\Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{mg\ell}{I}\theta$ since θ is very small.

Comparing with the equation $\frac{d^2\theta}{dt^2} = -\omega^2\theta$, we get

$$\omega = \sqrt{\frac{mg\ell}{I}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{mg\ell}}$$

Exercise 11. ***What is the equivalent length of a simple pendulum for the physical pendulum given above?***

Illustration 27. *A uniform square lamina of side 2a is hung up by one corner and oscillates in its own plane which is vertical. Find the length of the equivalent simple pendulum.*

Solution: When the lamina ABCD is at rest, its centre of gravity G must lie vertically below the corner A by which it is hung.

By geometry, the distance between centre of gravity and centre of suspension i.e., $AG = \sqrt{2}\ a = L$

M.I. of lamina about horizontal axis passing through C.G. and perpendicular to its plane is given by

$$I_g = m\,\frac{(2a)^2 + (2a)^2}{12} = \frac{2}{3}\,ma^2$$

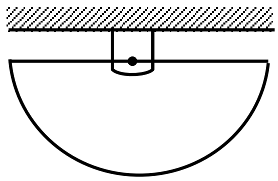
Radius of gyration about this axis is given by

$$K^2 = \frac{2}{3}\,a^2$$

Then length of the equivalent simple pendulum

$$= L + \frac{K^2}{L} = \sqrt{2}\ a + \frac{\frac{2}{3}a^2}{\sqrt{2}\ a} = \frac{4\sqrt{2}}{3}a$$

Illustration 28. *A uniform semi–circular disc of mass m and radius r is suspended as shown in the figure. If T is the time period of small oscillations about an axis passing through point of suspension and perpendicular to plane of disk. Then T is equal to*



(A) $2\pi\sqrt{\frac{3r}{8g}}$ (B) $2\pi\sqrt{\frac{3r\pi}{8g}}$

(C) $2\pi\sqrt{\frac{2r\pi}{9g}}$ (D) $2\pi\sqrt{\frac{2r}{9g}}$

Solution: **(B)**

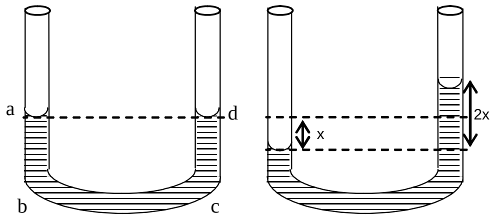
$$I = \frac{mr^2}{2}$$

$$d = \frac{4r}{3\pi}$$

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{3r}{8g}}$$

Oscillation of liquid column in U – tube

Consider a liquid column abcd of length L in a U–tube of uniform cross–sectional area A. In first figure, the static equilibrium position of liquid column is shown. Let the liquid level in the left limb be depressed through a distance x. The liquid level in the right limit will go up by the same amount x. Thus restoring force will be equal to the weight of liquid column of length 2x. As soon as the external force ceases to act, the liquid column in the U–tube will begin to oscillate up and down about its mean position.



Restoring force, F = – weight of height '2x' of liquid column

$$F = -(2xA)\rho g = -2A\rho g x$$

where ρ is the density of the liquid and 'g' is the value of acceleration due to gravity.

Now, $F = -kx$

where $k = 2A\rho g$ is a force constant of the system.

Mass of oscillating liquid column, $m = (LA)\rho$

Acceleration $\frac{d^2x}{dt^2} = \frac{F}{m} = \frac{kx}{m}$

$$\frac{d^2x}{dt^2} = -\frac{2A\rho g}{LA\rho}x$$

$\Rightarrow \quad \frac{d^2x}{dt^2} = -\frac{2g}{L}x$

put, $\quad \omega^2 = \frac{2g}{L}$.

Thus $\quad \frac{d^2x}{dt^2} = -\omega^2 x$

The acceleration is directly proportional to displacement and is directed towards the mean position. Thus, the motion of the oscillating liquid is simple harmonic motion,

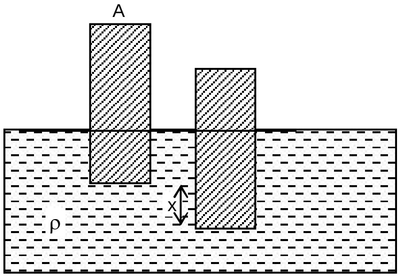
Time period $\quad T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{spring factor}}}$

$$T = 2\pi\sqrt{\frac{m}{k}} = \sqrt{\frac{LAg}{2A\rho g}}$$

$$T = 2\pi\sqrt{\frac{L}{2g}}.$$

The time period T is independent of the mass of the liquid column, density of the liquid and the cross–sectional area of the tube. However, it depends upon the length of the liquid column and also on the value of acceleration due to gravity.

Floating Cylinder



Consider a cylinder of mass m and area of cross–section A floating vertically in a liquid of density ρ. In equilibrium position, according to Archimedes principle, the weight of the cylinder is balanced by the upthrust of the liquid displaced by the immersed part of the cylinder.

Let the cylinder be now pushed down through a small distance x. A restoring force will come into play which will be equal to the increase in upthrust due to the additional immersion of x length of the cylinder in the liquid.

Restoring force, $F = -(Ax)\rho g = -A\rho g x$

$$F = -kx \text{ where } k = A\rho g$$

When the cylinder is released, this restoring force will make cylinder oscillate up and down with acceleration $\frac{d^2x}{dt^2}$.

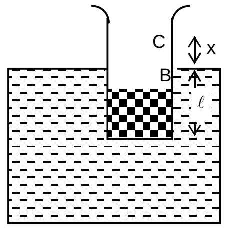
$\therefore \quad m\frac{d^2x}{dt^2} = -kx$

$\Rightarrow \quad \frac{d^2x}{dt^2} = -\frac{k}{m}$

Hence motion is SHM.

Now time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{A\rho g}}$.

Illustration 29. *A weighted glass tube is floating in a liquid with 20 cm of its length immersed. It is pushed down a little and released, calculate the time period of oscillations.*



Solution: Let B be the point upto which the weighted glass tube is immersed in the liquid such that

ℓ = 20 cm = 0.20 m.

Applying the law of floating,
weight of tube = weight of the liquid displaced = upthrust

$\therefore$ mg = (Aℓ)ρg

where m is the mass of the tube, A is cross–sectional area of the tube and ρ is the density of liquid.

$$m = A\ell\rho = A \times 0.20 \times \rho$$

Let the tube be pushed into the liquid upto point C such that

BC = x

Restoring force = weight of the extra liquid displaced
= Additional upthrust
= – Axρg = – Aρgx

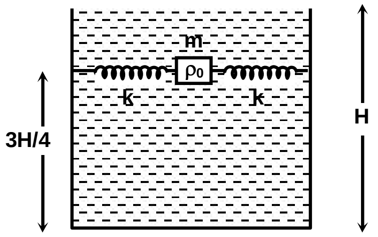
or – kx = – Aρgx

$\therefore$ k = Aρg

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{A \times 0.20 \times \rho}{A\rho g}}$$

$$T = 2\pi\sqrt{\frac{0.20}{9.8}} = 0.898 \text{ sec.}$$

Illustration 30. *The density of the block shown in the figure is ρ_0. It is tied by two massless springs, each of spring constant k. The mass of the block is m. The liquid in the vessel is non–viscous but the density of liquid varies as $\rho = \rho_0\left(\frac{5}{4} - \frac{h}{H}\right)$, where h is the depth below the free surface. The spring mass assembly is at a height $\left(\frac{3H}{4}\right)$ from the base. Find the time period of small oscillations of the mass m if it is slightly displaced in the direction of one of the springs.*



(A) $T = 2\pi\sqrt{\frac{m}{k}}$ *(B)* $T = 2\pi\sqrt{\frac{m}{2k}}$

(C) $T = 2\pi\sqrt{\frac{2m}{k}}$ *(D) cannot be determined.*

Solution: (A).

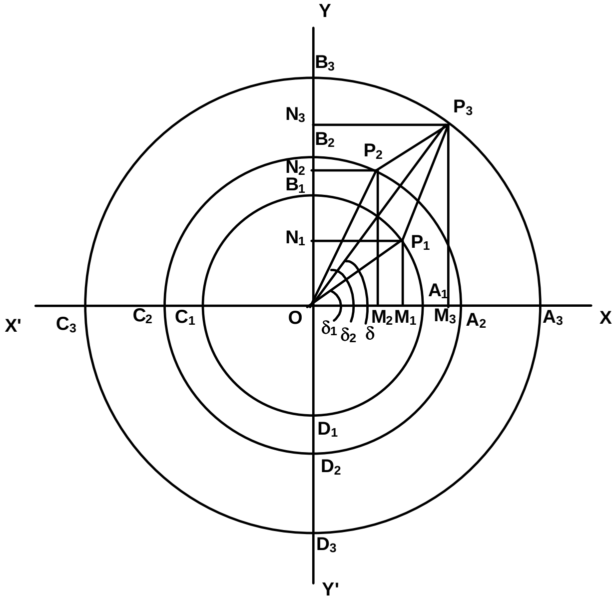
As water drains out the effective length of the pendulum increases, hence time period will also increases. When water is completely drained out, the C.G. will point again at the same geometric centre as before. Hence, final time period will be same.

Exercise 12. ***A simple pendulum vibrates in a non–viscous liquid. The density of the liquid is $\rho / 8$, where ρ is the density of the material of the bob. What is the time period if the time period of the pendulum in air is T.***

COMBINATION OF SIMPLE HARMONIC MOTION

We have previously learnt about the simple harmonic motion of a body. A body may, sometimes, possess two or more simple harmonic motions simultaneously. Take the case of a simple pendulum suspended from a stand placed in a boat. If the bob of the pendulum is set into vibrations when the boat is tossing sideways, the bob is having its own S.H.M. and is also sharing the S.H.M. of the boat and moves at any moment in the direction of the resultant of these two motions. We shall now discuss the methods of finding the resultant of such simultaneous motions. Let us first take the case of two simple harmonic motions in the same direction when they have the same periodic time. Their resultant can be found either graphically or analytically.

Graphic method of finding the resultant of two S.H.M. in the same direction when they have the same periodic time.



Suppose that a particle N possesses simultaneously two S.H. motions about O in the direction YOY′ or the Y axis and the periodic time of each equals T. Let the amplitudes of the two S.H. motions be a_1 and a_2 and their epoch angles δ_1 and δ_2 respectively. With O as centre and radii = a_1 and a_2 draw circles $A_1B_1C_1D_1$ and $A_2B_2C_2D_2$ cutting the axes of X and Y at the points shown in figure. Take the points P_1 and P_2 on the circumferences of the two circles such that $\angle P_1OX = \delta_1$ and $\angle P_2OX = \delta_2$. From P_1 and P_2 draw P_1N_1 and P_2N_2 perpendiculars on YOY′ and P_1M_1 and P_2M_2 perpendiculars on XOX′. Then ON_1 and ON_2 denote the component displacements y_1 and y_2 of the particle N at the start. With OP_1 and OP_2 as sides, complete the parallelogram $OP_1P_3P_2$ and join OP_3.

Draw P_3N_3 and P_3M_3 perpendiculars on YOY′ and XOX′ respectively. The resultant displacement of the particle N on Y–axis at the start is given by ON_3. Let this be denoted by y.

Since $N_2N_3 = ON_1 = y_1$

$y = ON_3 = ON_2 + N_2N_3 = y_2 + y_1$

With O as centre and radius = OP_3, draw the circle $A_3B_3C_3D_3$. Because the component simple harmonic vibrations have the same periodic time T, the angular velocities OP_1 and OP_2 will be same $\left(= \dfrac{2\pi}{T}\right)$ and the resultant OP_3 will also rotate with the same angular velocity as OP_1 or OP_2. Thus P_3 will always lie on the circumference $A_3B_3C_3D_3$. The resultant motion of the particle N being the projection of uniform circular motion of P_3 will therefore, be simple harmonic motion about O along YOY′ with periodic time =

T = periodic time of each component. The amplitude a of the resultant is given by the diagonal of the parallelogram,

$$a = OP_3^2 = OP_1^2 + OP_2^2 + 2OP_1OP_2 \cos\angle P_2OP_1$$
$$= a_1^2 + a_2^2 + 2a_1a_2 \cos(\delta_2 - \delta_1)$$

The epoch angle of the resultant = $\delta = P_3OX$ is given by

$$\tan\delta = \frac{P_3M_3}{OM_3} = \frac{ON_3}{OM_1 + M_1M_3} = \frac{y_1 + y_2}{OM_1 + OM_2}$$
$$= \frac{a_1 \sin\delta_1 + a_2 \sin\delta_2}{a_1 \cos\delta_1 + a_2 \cos\delta_2}$$

We, therefore, find that the resultant of two S.H. motions of equal period in the same direction is another simple harmonic motion of the same period, but of an amplitude given by compounding the component amplitudes according to the Parallelogram law.

Special Cases

(i) When $\delta_1 = \delta_2 = \phi$, i.e. the two S.H. vibrations are in the same phases $\delta_2 - \delta_1 = 0$ and

$$a^2 = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$$

or $a = a_1 + a_2$

and $$\tan\delta = \frac{(a_1 + a_2)\sin\phi}{(a_1 + a_2)\cos\phi} = \tan\phi$$

$$\delta = \phi$$

The resultant vibration has, therefore, in this case, an amplitude equal to the sum of the amplitudes of component vibrations and is in phase with each of them.

(ii) When $\delta_2 - \delta_1 = 180°$or component vibrations are opposite in phase then

$$a^2 = a_1^2 + a_2^2 + 2a_1a_2 \times (-1)$$
$$= a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$$

or $a = a_1 - a_2$

and $$\tan\delta = \frac{a_1 \sin\delta_1 + a_2 \sin(180 + \delta_1)}{a_1 \cos\delta_1 + a_2 \cos(180 + \delta_1)}$$

$$= \frac{a_1 \sin\delta_1 - a_2 \sin\delta_1}{a_1 \cos\delta_1 - a_2 \cos\delta_1} = \frac{(a_1 - a_2)\sin\delta_1}{(a_1 - a_2)\cos\delta_1} = \tan\delta_1$$

or $\delta = \delta_1$ or $180° + \delta_1$.

The resultant vibration has thus an amplitude equal to the difference of amplitudes of component vibrations and is in phase with one of the component vibrations – the one with the bigger amplitude.

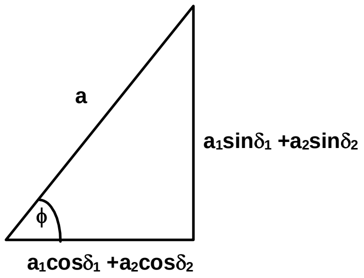
Analytical method of finding the resultant of two Simple Harmomic Motions in the same direction when they have the same periodic time.

The two S.H.Ms having the same periodic time can be represented by the equations

$$y_1 = a_1 \sin(\omega t - \delta_1)$$

or $$y_2 = a_2 \sin(\omega t - \delta_2)$$

Since the displacement y_1 and y_2 are in the same direction, the resultant displacement y at any time t is obtained by their algebraic addition, or



$$y = y_1 + y_2$$
$$= a_1 \sin(\omega t - \delta_1) + a_2 \sin(\omega t - \delta_2)$$
$$= a_1\sin\omega t \cos\delta_1 - a_1\cos\omega t \sin\delta_1)$$
$$+ a_2\sin\omega t \cos\delta_2 - a_2\cos\omega t \sin\delta_2)$$
$$= \sin\omega t(a_1\cos\delta_1 + a_2\cos \delta_2)$$
$$- a_2 \cos\omega t (a_1\sin\delta_1 + a_2\sin\delta_2)$$

Now take an angle ϕ as shown in figure such that a

$$a\cos\phi = a_1\cos \delta_1 + a_2\cos\delta_2$$

and $$a \sin\phi = a_1\sin\delta_1 + a_2\sin \delta_2$$

Then, $$y = a\sin\omega t \cos\phi - a \cos\omega t \sin\phi$$
$$= a\sin(\omega t - \phi),$$

This represents a S.H. motion of the same periodic time as the component motions and having an amplitude = a and epoch angle = ϕ. From figure a is given by the relation

$$a^2 = (a_1\sin\delta_1 + a_2\sin \delta_2)^2 + a_2^2 (a_1\cos \delta_1 + a_2\cos\delta_2)^2$$

$$= a_1^2 (\sin^2\delta_1 + \cos^2\delta_1) + (\sin^2\delta_2 + \cos^2\delta_2) + 2a_1a_2(\sin\delta_1 \sin\delta_2 + \cos\delta_1\cos\delta_2)$$

$$= a_1^2 + a_2^2 + 2a_1a_2 \cos(\delta_1 - \delta_2)$$

And ϕ is given by the relation

$$\tan\phi = \frac{a_1 \sin\delta_1 + a_2 \sin\delta_2}{a_1 \cos\delta_1 + a_2 \cos\delta_2}$$

The results are the same as those obtained graphically in the previous article.

Proceeding in a similar manner as above, it can be shown that, for a number of simple harmonic motions acting on a point in the same direction when their periodic times are equal, amplitude of the resultant motion is given by

$$a^2 = (\sum a\sin\delta)^2 + (\sum a\cos\delta)^2,$$

where $\sum a \sin\delta$ and $\sum a \cos\delta$ are the sums of the y and x components of amplitudes respectively.

The epoch of the resultant is given by

$$\tan\phi = \frac{\sum a \sin\delta}{\sum a \cos\delta}$$

TYPES OF OSCILLATION

(a) Free Vibrations

When a body oscillates with its own natural frequency, it is said to execute free vibrations. The frequency of vibration and the time period depends only on the dimensions (size, shape) of the body and the force constant i.e. the inertia factor and the spring factor. The frequency of free vibrations (or natural vibration is given by)

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Example:
(i) When a stretched spring is plucked, it executes free vibrations.
(ii) When air is blown gently across the mouth of a test tube or a bottle, the free vibrations are produced.
(iii) When a tunning fork is struck against a rubber pad, the prongs begin to execute free vibrations.

(b) Forced vibrations

When a body is maintained in a state of vibration by a strong periodic force of frequency other than the natural frequency of the body, the vibrations are called forced vibrations.

The frequency of forced vibrations is different from the natural frequency of the body. It is equal to the frequency of the applied force.

Let an external periodic force of frequency ν be applied to a body A of natural frequency ν_0. The body A will start vibrating with frequency ν and not ν_0. the external force is called the driver while the body A is called the driven oscillator.

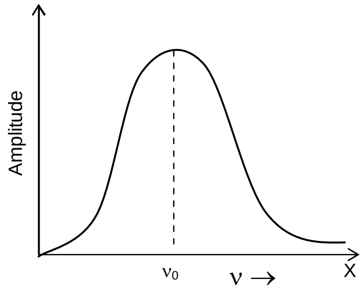
The amplitude of the forced vibrations is determined by the difference between the frequency of the applied force and the natural frequency. If the difference in frequencies is large, then the amplitude will be small. It is possible to obtain forced vibrations of larger amplitude by applying a series of small forces at proper intervals.

Example: (i) Press the stem of a vibrating tunning fork against the top of a table. The table will suffer forced vibration.
(ii) Hold the bob of a simple pendulum and give it any number of vibrations in unit time.
(iii) The sound boards of stringed musical instruments suffer forced vibrations.

(c) Resonant Vibrations

When a body is maintained in a state of vibrations by periodic force having the same or their internal multiples natural frequency as that of the body, the vibrations are called resonant or sympathetic vibrations. Resonant vibrations are merely a special case of forced vibrations. The phenomenon of producing resonant vibrations is termed as resonance.

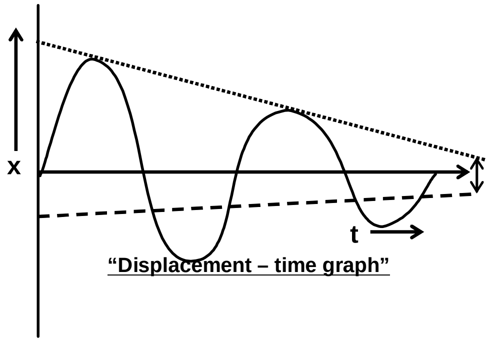
The amplitude of the forced vibrations depends on the difference between the natural frequency ν_0 of the body and the frequency ν of the applied force. The amplitude becomes larger as the difference between the frequencies decreases. So, the amplitude becomes maximum when the two frequencies are exactly equal to each other.



Example:
(i) Soldiers are asked to break step while crossing a bridge. If the soldiers march in step, there is possibility that the frequency of the foot steps may match the natural frequency of the bridge. Due to resonance, the bridge may start oscillating violently, thereby damaging itself.

(d) Damped oscillations

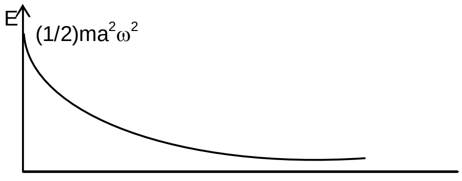
Most of the oscillations occur in viscous media like air, water etc. Therefore part of the energy is dissipated in overcoming resistive forces. Consequently, the amplitude of oscillations goes on decreasing exponentially with time and finally the oscillations die out. Such oscillations are called damped oscillations.



"Displacement – time graph"

(e) Maintained oscillations

If energy is supplied to the oscillator at the same rate at which it is dissipated, the amplitude of oscillations remains unchanged. Such oscillations are called maintained oscillations.



"Energy time graph)"

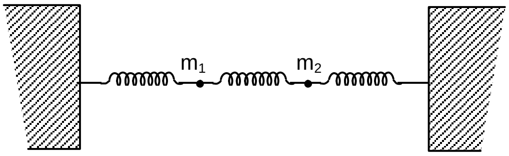
(f) Coupled oscillations

The vibrating systems (simple pendulum, block – spring etc.) are all single or isolated vibrating systems. It was assumed that the flow of energy was only from the driving system to drivers system. In simple words, the energy flow was assumed to be unidirectional. However it is more appropriate to also take into account the feed back of energy from the drivers system to the driving system. This consideration leads us to the concepts of coupled system and coupled oscillations.

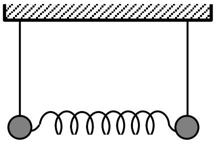
A system of two or more oscillations linked together in such a way that there is a mutual exchange of energy between them is called a coupled system. The oscillations of such a system are called coupled oscillations.

Example:

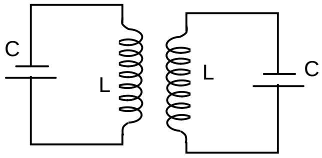
(i) Two masses attached to each other by three springs between two rigid supports. The middle spring can be driven system and driving system.



(ii) Two simple pendulums hanging from the same rigid support with their bobs attached to each other by a spring.



(iii) The two LC circuits placed close to each other as shown. The two circuits are coupled to each other by magnetic lines of force.



MISCELLANEOUS EXERCISE

1. The breaking stress of aluminium is 7.5×10^8 dynes/cm^2. Find the greatest length of aluminium wire that can hang vertically without breaking. Density of aluminium is 2.7 g/cm^3. Given g = 980 cm/s^2.

2. A material has normal density ρ and bulk modulus k. Find the increase in the density fo the material when it is subjected to an external pressure P from all sides.

3. Some work has to be does in stretching a wire where is this work stored ?

4. What is the bulk modulus of a perfectly rigid body ?

5. A particle of mass 0.1 kg executing HSM of amplitudes 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.

6. What is the length of a simple pendulum, which ticks seconds?

7. Can a simple pendulum be used in an artificial satellite ?

8. A girl is swinging a swing in the sitting position. What shall be the effect on eth frequency of oscillation if she stands up ?

9. What is the acceleration of a particle in simple harmonic motion ?

10. If the magnitude of displacement is numerically equal to that of acceleration, then what is the time period?

SOLUTION TO MISCELLANEOUS EXERCISE

1. 2.834 km
2. $\frac{\rho P}{k}$
3. This is stored as potential energy in the wire.
4. Infinity
5. y = 0.1 sin (4t + π/4)
6. 0.99 m
7. No. this because there exists a state of weightlessness in a satellite.
8. frequency of oscillation is increased.
9. Maximum at the extreme position.
10. 2π second.

SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob 1. *What is the density of ocean water at a depth, where the excess pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kg/m^3 ? Compressibility of water $= 45.8 \times 10^{-11}$ Pa^{-1}. Given 1 atm = 1.013×10^5 Pa.*

Sol. Here P = 80.0 atm.

$= 80.0 \times 1.013 \times \frac{1}{k} = 45.8\times10^{-11}\,10^5$ Pa

Compressibility, Pa^{-1}

Density of water at surface

$\rho = 1.03 \times 10^3$ kg/m^3.

Let ρ' be the density of water at the given depth. If V and V′ are volumes of certain mass M of ocean water at surface and at a given depth, then

$V = M/\rho$ and $V' = M/\rho'$

Change in volume

$$\Delta V = V - V' = M\left(\frac{1}{\rho} - \frac{1}{\rho'}\right)$$

Volumetric strain $\frac{\Delta V}{V} = M\left(\frac{1}{\rho} - \frac{1}{\rho'}\right) \times \frac{\rho}{M} = 1 - \frac{\rho}{\rho'}$

or, $\frac{\Delta V}{V} = 1 - \frac{0.03\times10^3}{\rho'}$ . . . (i)

As, Bulk modulus $k = \frac{\rho V}{\Delta V}$

or, $\frac{\Delta V}{V} = \frac{P}{k}$

$\therefore \quad \frac{\Delta V}{V} = (80.0\times1.013\times10^5) \times 45.8 \times 10^{-11}$

$= 3.712 \times 10^{-3}$

putting this value in (i) we get

$$\frac{1 - 1.03\times10^3}{\rho'} = 3.712\times10^{-3}$$

$$\rho' = \frac{1.03\times10^3}{1 - 3.712\times10^{-3}} = 1.034 \times 10^3 \text{ kg / m}^3.$$

Prob 2. *A body of mass 12 kg is suspended by a coil spring of natural length 50 cm and force constant 2.0×10^3 N/m. What is the stretched length of the spring? If the body is pulled down further stretching the spring to a length of 5.9 cm and then released. What is the frequency of oscillation of the suspended mass? (Neglect the mass of the spring).*

Sol. Here, m = 12 kg, original length ℓ = 50 cm

$K = 2.0 \times 10^3$ N/m

As $F = ky$

$\therefore \quad y = F/k = \dfrac{mg}{k} = \dfrac{12 \times 9.8}{2 \times 10^3}$

$= 5.9 \times 10^{-2}$ m = 5.9 cm

$\therefore$ Stretched length of the spring $= \ell + y$

= 50 + 5.9 = 55.9 cm

Frequency of oscillations $\nu = \dfrac{1}{T}$

$$= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2 \times 3.14}\sqrt{\frac{2 \times 10^3}{12}} = 2.06 \text{ s}^{-1}.$$

Prob 3. *If the displacement of a particle in SHM is given by y = (1/a)sinωt – (1/b) cosωt, where a and b are constant, ω is angular frequency and t represents time, find amplitude of motion.*

Sol.

$$y = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}\left\{\frac{\frac{1}{a}}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\sin\omega t - \frac{\frac{1}{b}}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\cos\omega t\right\}$$

$$= \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}\left\{\frac{\frac{1}{a}}{\frac{\sqrt{a^2 + b^2}}{ab}}\sin\omega t - \frac{\frac{1}{b}}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\cos\omega t\right\}$$

$$= \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}\left\{\frac{b}{\sqrt{a^2 + b^2}}\sin\omega t - \frac{a}{\sqrt{a^2 + b^2}}\cos\omega t\right\}$$

$$= \frac{\sqrt{a^2 + b^2}}{ab}[\sin\omega t.\cos\alpha - \cos\omega t.\sin\alpha], \quad \text{where } \sin\alpha = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow y = \frac{\sqrt{a^2 + b^2}}{ab}\sin[\omega t - \alpha]$$

$\therefore$ Amplitude of motion is $\dfrac{\sqrt{a^2 + b^2}}{ab}$.

Prob 4. *A particle in S.H.M. has time period 6 sec. After one second of its starting from mean position, its speed is 12 × 10^{-2} m/s. Find the ratio of kinetic energy and potential energy at this moment.*

Sol. K.E. $= \dfrac{1}{2}mv^2 = \dfrac{1}{2}m(12 \times 10^{-2})^2$

Total energy $= \dfrac{1}{2}m\omega^2a^2$

$\therefore \quad$ P.E. $= \dfrac{1}{2}m\omega^2a^2 - \dfrac{1}{2}mv^2$

$v = a\omega\cos\omega t$

$\Rightarrow \quad 12 \times 10^{-2} = a\omega\cos\left(\frac{2\pi}{T}\times 1\right) = a\omega\cos(60°) = a\omega/2$

$\Rightarrow \quad a = \frac{24\times10^{-2}}{\omega} = \frac{24\times10^{-2}\times 6}{2\pi} = 22.92$ cm

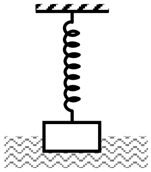
$\therefore \quad$ P.E. $= \frac{1}{2}m.\frac{4\pi^2}{T^2}.a^2 - \frac{1}{2}m\left(12\times10^{-2}\right)^2$

$= \frac{1}{2}m\frac{(22.92)^2\times10^{-4}}{36}\left(4\pi^2\right) - \frac{1}{2}m\left(12\times10^{-2}\right)^2$

$= \frac{1}{2}m\left[432.08\right]10^{-4}$

Hence required ratio = 0.33

Prob 5. *A cube of side a and mass m is held vertically by a spring of spring constant k and buoyant force by water such that at rest its half part is in air. [ρ = density of water]*
Find the time period of small oscillation.



Sol. In equilibrium, elongation of the spring = x_0 (let)

$\therefore \quad mg = kx_0 + B = kx_0 + \rho\frac{a^3}{2}g \quad \quad ...(1)$

When in motion,

net force $= mg - k(x_0 + x) - \rho a^2\left(\frac{a}{2} + x\right)g \quad \quad ...(2)$

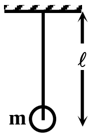
$= -kx - \rho a^2 xg$ from equation (1)

$\therefore \quad$ Acceleration $= \frac{-\left(k+\rho a^2 g\right)x}{m}$

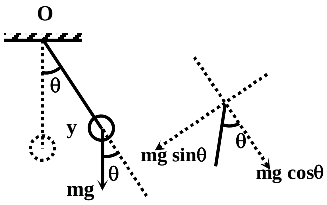
$\therefore \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k+\rho a^2 g}}$

IITJEE TYPE

Prob 6. *A bob of mass m is connected to a massless string of length as shown. The bob is slightly displaced towards right. Find the magnitude of restoring force acting on the bob as a function of angle θ made by string with the vertical. How does the time period change if mass is doubled?*

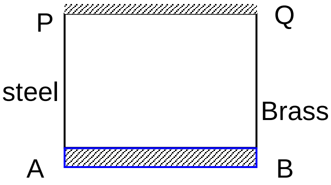


Sol. a) The component mg sinθ acts as restoring force to bring the bob to equilibrium condition. Thus, magnitude of restoring force = mg sinθ



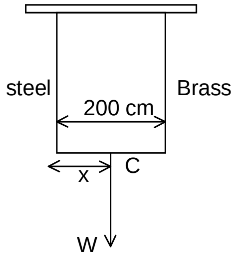
b) Time period $T = 2\pi\sqrt{\frac{\ell}{g}}$ As the time period is independent of mass, hence changing the mass has no effect on time period.

Prob 7. *A light rod of length 200 cm is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and is of cross–sectional area 0.1 cm^2 and the other is of brass of cross–sectional area 0.2 cm^2. Find out the position along the rod at which weights may be hung to produce*



(a) equal stress in both wires
(b) equal strains in both wires. $Y_{steel} = 20 \times 10^{11}$ dynes/cm^2 and $Y_{Brass} = 10 \times 10^{11}$ dynes/cm^2.

Sol. (a) Let a weight W be suspended from a point C on the rod such that it is x cm from the steel wire. Let the forces in steel & brass wires be F_1 and F_2 respectively.



$\therefore \quad F_1(x) = F_2(200 - x)$

$\Rightarrow \quad \frac{F_1}{F_2} = \frac{200 - x}{x}$

Stress developed in steel wire = $F_1/0.1$
= 10 F_1 dynes/cm^2

Stress developed in brass wire = $F_2/0.2$ = $5F_2$ dynes/cm^2

Now if stresses are equal, then

$10 F_1 = 5F_2 \Rightarrow \frac{F_1}{F_2} = \frac{1}{2}$ . . . (ii)

From (i) and (ii), $x = 133\frac{1}{3}$ cm.

(b) Let the weight w be hung at a point y cm away from the steel wire and forces developed in the steel & brass wires be F_3 and F_4 respectively.

$\therefore \quad F_3y = F_4(200 - y) \quad \Rightarrow \quad F_3/F_4 = \frac{200 - y}{y}$ . . . (iii)

Strain in steel wire = strain in brass wire

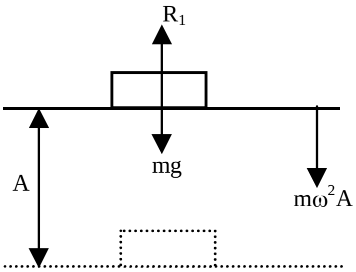
or $\frac{F_3}{0.1Y_{steel}} = \frac{F_4}{0.2Y_{brass}}$

$\Rightarrow \quad \frac{F_3}{F_4} = 1$ . . . (iv)

From (iii) and (iv), y = 100 cm.

Prob 8. *A horizontal platform vibrate up and down with a simple harmonic motion of amplitude 20 cm. At what frequency will an object kept on the platform just lose contact with the platform? [Take g = 9.81 m/s^2.]*

Sol. Let m be the mass of the object and A be the amplitude of motion of platform. If R_1 is the normal reaction of the platform at the highest point then



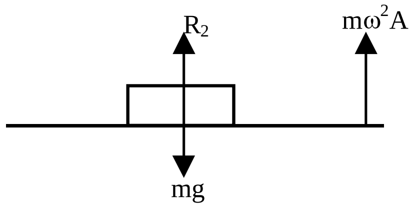
$(mg - R_1) = m\omega^2 A$

$\therefore \quad R_1 = (mg - m\omega^2 A)$ (1)

At the lowest point, if R_2 is the normal reaction of the platform, then

$(R_2 - mg) = m\omega^2 A$

$\therefore$ $R_2 = (mg + m\omega^2 A)$ (2)



When the object is about to leave the platform the normal reaction of the surface is zero. From (1) and (2), it can be seen that the object can leave the platform only at the highest point.

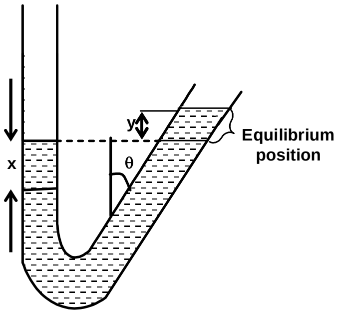
Putting $R_1 = 0$, we get , $m\omega^2 A = mg$

$\therefore$ $\omega^2 = g/A$

$\therefore$ $\omega = \sqrt{g/A}$

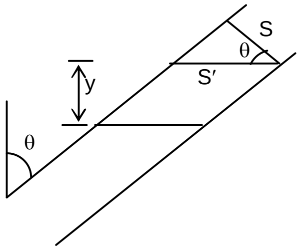
$\therefore$ Frequency = f = $\omega/2\pi$ = $\dfrac{1}{2\pi}\sqrt{\dfrac{9.8}{0.2}}$ = $\dfrac{7}{2\pi}$ Hz

Prob 9. *Determine the period of oscillation of mercury of mass m = 200g poured into a bent tube whose right arm forms an angle θ = 30° with the vertical. The cross–sectional area of the tube S = 0.50 cm². The viscosity of mercury is to be neglected.*



Sol. Volume $Sx = S'y$

$(S/\cos\theta)y$ $\therefore$ $x = y/\cos\theta$



The restoring force which causes the acceleration of the mercury column

= weight of the portion x + component of the weight of position y along the tube.

$= Sx\rho g + (S'y)\rho g \, . \cos\theta$

$= Sx\,\rho g\,(1 + \cos\theta)$

$\therefore$ from Newton's IInd law of motion

$$Sx\rho g\,(1 + \cos\theta) = -m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{S\rho g(1+\cos\theta)}{m}x = -\omega^2 x \qquad \text{. . . (i)} \quad \text{where } \omega^2 = \frac{S\rho g(1+\cos\theta)}{m}$$

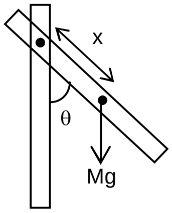
the equation (i) represents SHM

$\therefore$ Time period of oscillation

$$T = 2\pi/\omega = 2\pi\sqrt{\frac{m}{\rho S g(1+\cos\theta)}}$$

$$= 2\pi\sqrt{\frac{0.2}{13600(0.5)\times10^{-4}\times10(1+\cos30^0)}} = 0.8 \text{ s.}$$

Prob 10. *Find the distance form the top end of a uniform bar of length 24 cm is to be mounted on wall about an axis perpendicular to its length, so that its time period of oscillation will be minimum.*



Sol. Let axis is at a distance of 'x' from centre of mass.

$\therefore$ restoring force $\tau = (Mg)\, x \sin\theta \approx Mg\, x\, \theta$

$\therefore$ torque constant $C = Mgx$ … (i)

and moment of inertia of bar about axis of rotation

$= \frac{M\ell^2}{12} + Mx^2 = M\left(\frac{\ell^2}{1} + x^2\right)$ … (ii)

from (i) and (ii) $T = 2\pi\sqrt{\frac{I}{C}}$

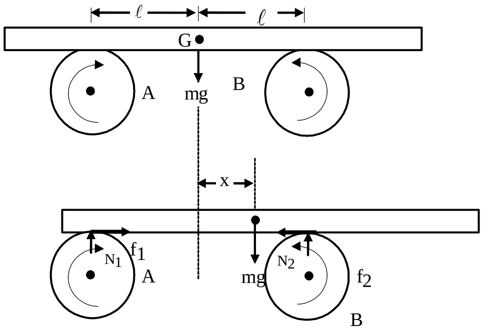
$T = 2\pi\sqrt{\frac{\ell^2 + 12x^2}{12gx}}$

For T to be minimum $\frac{dT}{dx} = 0 \Rightarrow \frac{d}{dx}\left(\frac{\ell^2}{12} + x\right) = 0$

$x = \frac{\ell}{\sqrt{12}} = \frac{24}{\sqrt{12}} = 2\sqrt{12}$ cm.

$\therefore$ from the top end $= 12 - 2\sqrt{12} = 5.08$ cm.

Prob 11. *A uniform bar with mass m lies symmetrically across two rapidly rotating fixed rollers, A and B, with distance 'L' between the bars centre of mass and each roller. The rollers whose direction of rotation are shown in the figure slip against the bar with coefficient of friction μ. Suppose the bar is displaced horizontally by a distance 'x' and then released, find the time–period of oscillation.*



Sol. Let N_1 and N_2 be the normal reactions at the rollers, Then

$N_1 + N_2 = mg$ & $L = 2\ell$

As the bar is in rotational equilibrium

Σ (moments about G)

$= I_G\,(O) = 0$

$N_1(\ell + x) - N_2(\ell - x) = 0$

$\Rightarrow \quad N_1 = \frac{mg(\ell - x)}{2\ell}, \quad N_2 = \frac{mg(\ell + x)}{2\ell}$

$F = f_1 - f_2 = \mu\left[N_1 - N_2\right] = \frac{\mu mg}{2\ell}\left[(\ell - x) - (\ell + x)\right] \quad = -\left(\frac{\mu mg}{\ell}\right).x$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\ell}{\mu.g}} = 2\pi\sqrt{\frac{L}{2\mu g}}$

Prob 12. *A horizontal spring block system of mass M executes simple harmonic motion. When the block is passing through its equilibrium position, an object mass m is put on it and the two move together. Find the new amplitude and frequency of vibration.*

Sol. Let A and ω be the initial amplitude and angular frequency of the spring block system. Then

$\omega = \sqrt{\frac{K}{M}}$, where K = spring constant.

When an object of mass m is placed on the block, the new angular frequency ω′ is given by

$$\omega' = \sqrt{\frac{K}{M+m}} = \omega\sqrt{\frac{M}{M+m}} \quad \ldots (1)$$

Let v and v′ be the velocities of the system before and after putting the object, then

$$Mv = (M+m)v' \quad \ldots (2)$$

$$\frac{1}{2}Mv^2 = \frac{1}{2}Mv^2 \quad \ldots (3)$$

$$\frac{1}{2}(M+m)v'^2 = \frac{1}{2}KA'^2 \quad \ldots (4)$$

From eq. (2) $\frac{v}{v'} = \frac{M+m}{M}$

Dividing equation (3) by equation (4), we get

$$\frac{M}{M+m} \cdot \frac{v^2}{v'^2} = \frac{A^2}{A'^2} \quad \text{or} \quad \frac{A'}{A} = \frac{v'}{v}\left[\frac{M+m}{M}\right]^{\frac{1}{2}}$$

$$A' = \frac{M}{M+m} \cdot \frac{v^2}{v'^2} = \frac{A^2}{A'^2} \quad \text{or} \quad \frac{A'}{A} = \frac{v'}{v}\left[\frac{M+m}{M}\right]^{\frac{1}{2}}$$

$$= A\left(\frac{M}{M+m}\right)\left(\frac{M+m}{M}\right)^{\frac{1}{2}}$$

$$A' = \frac{v}{v'} = \frac{M+m}{M}$$

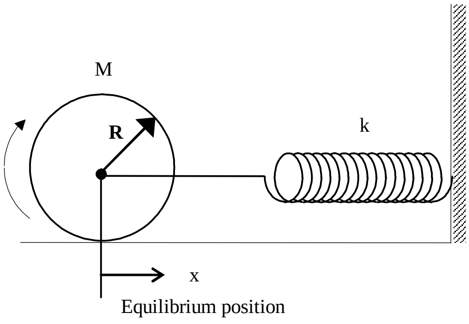
Prob 13. *A solid cylinder attached to a horizontal massless spring can roll without slipping along a horizontal surface. Show that if the cylinder is displaced and released, it executes S. H. M. Find the corresponding time period.*

Sol. At any instant of rolling, the cylinder has rotational and tranilational kinetic energies.

$$K(\text{rotational}) = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{1}{2}MR^2\right]\left[\frac{v^2}{R^2}\right] = \frac{1}{4}Mv^2 \quad \text{where } v = \frac{dx}{dt}$$

$$K(\text{translational}) = \frac{1}{2}Mv^2$$

$$U(\text{potential energy of system}) = \frac{1}{2}kx^2$$



$$E = \frac{3}{4}Mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

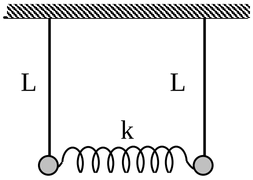
$$\frac{dE}{dt} = \frac{3}{4}M(2v)\frac{dv}{dt} + \frac{1}{2}k(2x)\frac{dx}{dt} = 0$$

$$\Rightarrow \quad \frac{dv}{dt} = -\left(\frac{2k}{3M}\right).x$$

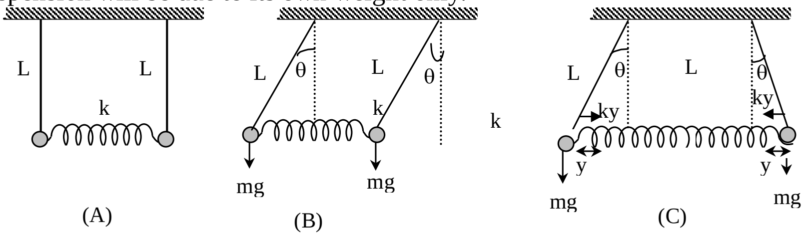
$\Rightarrow$ Comparing with $a = -\omega^2.x$

$$\omega = \sqrt{\frac{2k}{3M}} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{3M}{2k}}$$

Prob 14. *Two identical simple pendulums each of length L are connected by a weightless spring as shown in figure. The force constant of the spring is k. In equilibrium, the pendulums are vertical and the spring is horizontal and un–deformed. Find the time of small oscillations of the linked pendulums, when they are deflected from their equilibrium positions through equal displacements in the same vertical plane (a) in the same direction, (b) in opposite direction and released.*



Sol. (a) When both the pendulums are displaced in the same direction by same amount, the spring will neither compress nor stretch, so the restoring torque on each pendulum about the point of suspension will be due to its own weight only.



i.e., $\tau = -\,mgl\sin\theta = -mgL\theta$ [as for small θ, $\sin\theta = \theta$]

But as by definition $\tau = I\alpha = mL^2\dfrac{d^2\theta}{dt^2}$ [as $I = mL^2$]

$$\text{so } mL^2\ \frac{d^2\theta}{dt^2} = -\,\omega^2\theta$$

Therefore $\omega^2 = \dfrac{g}{L}$

This is the standard equation of angular SHM with time period $T = (2\pi/\omega)$.

So here $T_1 = 2\pi\sqrt{\dfrac{L}{g}}$, $f_1 = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$

(b) When both the pendulums are displaced in opposite directions by equal amount (say y), the restoring torque on each pendulum will be due to its own weight and also by elastic force of the spring which is stretched by 2y (=2L sinθ). So the restoring torque on a pendulum about the point of suspension will be

$$\tau = -[mgL\sin\theta + k(2L\sin\theta)L] = -\,[mgL + 2k\,L^2]\,\theta$$

But by definition $\tau = I\alpha = mL^2\,(d^2\theta/dt^2)$

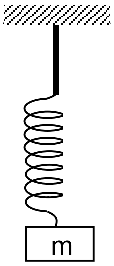
$$\text{so}\quad \frac{d^2\theta}{dt^2} = -\left(\frac{g}{L}+\frac{2k}{m}\right)\theta$$

$$\text{or}\quad \frac{d^2\theta}{dt^2} = -\,\omega^2\theta \ \text{ with } \omega^2 = \left(\frac{g}{L}+\frac{2k}{m}\right)$$

$$\text{so}\quad f_2 = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{g}{L}+\frac{2k}{m}\right)}\ \ (> f_1).$$

Prob 15. *One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross–section and the Young's modules of the wire are A and Y respectively. Find the time period with which mass m will oscillate if it is slightly pulled down and released.*

Sol. As there is no mass between the spring and wire, the restoring forces in the spring and the wire will be the same, i.e. $F_1 = F_2 = F$.



If mass m is pulled by y.

$\therefore\ y = y_1 + y_2$ …(i)

Now, By Hook's law

i.e. F = ky

$y_1 = (F_1 / k)$ …(ii)

and for a wire by Young's modulus

i.e. $y_2 = \dfrac{F_2 L}{AY}$ …(iii)

So, $y = \dfrac{F_1}{k} + \dfrac{F_2 L}{AY} = F\left[\dfrac{AY + kL}{AYk}\right]$

[as, $F_1 = F_2 = F$]

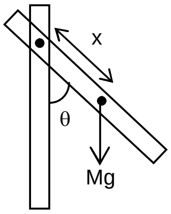
$\Rightarrow\ m\dfrac{d^2y}{dt^2} = -\left[\dfrac{AYk}{m(Ay + kL)}\right]$

This is standard equation of SHM with time period $T = (2\pi/\omega)$

$\therefore\ T = 2\pi\sqrt{\dfrac{m(AY + kL)}{AYk}}$.

Prob16. *Find the distance form the top end of a uniform bar of length 24 cm is to be mounted on wall about an axis perpendicular to its length, so that its time period of oscillation will be minimum.*

Sol. Let axis is at a distance of 'x' from centre of mass.



$\therefore$ restoring force $\tau = (Mg)\, x \sin\theta \approx Mg\, x\, \theta$

$\therefore$ torque constant $C = Mgx$ … (i)

and moment of inertia of bar about axis of rotation

$= \dfrac{M\ell^2}{12} + Mx^2 = M\left(\dfrac{\ell^2}{1} + x^2\right)$ … (ii)

from (i) and (ii) $T = 2\pi\sqrt{\dfrac{I}{C}}$

$T = 2\pi\sqrt{\dfrac{\ell^2 + 12x^2}{12gx}}$

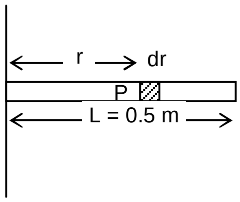
For T to be minimum $\dfrac{dT}{dx} = 0 \ \Rightarrow\ \dfrac{d}{dx}\left(\dfrac{\ell^2}{12} + x\right) = 0$

$x = \dfrac{\ell}{\sqrt{12}} = \dfrac{24}{\sqrt{12}} = 2\sqrt{12}$ cm.

$\therefore$ from the top end $= 12 - 2\sqrt{12} = 5.08$ cm.

Prob 17. *A thin uniform metallic rod of length 0.5 m and radius 0.1cm rotates with an angular velocity 400 radian/second in horizontal plane about a vertical axis passing through one of its ends. Calculate elongation of the rod. The density of the material of the rod is 10^4 kg/m^3 and Young's modulus is 2×10^{11} N/m^2.*

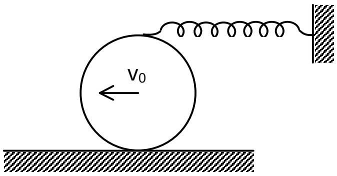
Sol. Consider a small element of the rod. Tension developed in this element at any distance r from axis of rotation is



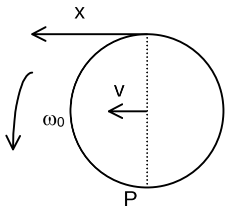
$$T = \frac{\rho A\omega^2}{2}(L^2 - r^2) \quad \Rightarrow \int_0^{\Delta\ell} d\,(\Delta\ell) = \int_0^L \frac{Tdr}{Ay}$$

$$\Rightarrow \Delta\ell = \frac{\rho\omega^2 L^3}{3} \qquad = 3.33 \times 10^{-4}\ \text{m.}$$

Prob 18. *A uniform disc of mass m and radius r is free to roll on a horizontal surface as shown in the figure. If the disc is given an initial velocity v_0 in the equilibrium position, find the time period and amplitude of small oscillations of the centre of the disc.*



Sol. During oscillations about the equilibrium position PE $= \frac{1}{2}kx^2$



kinetic energy KE $= \frac{1}{2}I_p\omega^2 = \frac{3}{4}mv^2$

PE + KE = constant

$$\frac{1}{2}kx^2 + \frac{3}{4}mv^2 = \text{constant}$$

differentiating w.r.t time

$$kx\,\frac{dx}{dt} + \frac{3}{4}m\left(2v\frac{dv}{dt}\right) = 0 \qquad \text{. . . (i)}$$

from kinematic constraints for rolling motion

$$\frac{dx}{dt} = 2v \qquad \Rightarrow \qquad \frac{d^2x}{dt^2} = 2\frac{dv}{dt} \qquad \text{. . . (ii)}$$

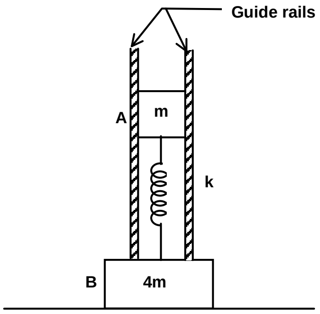
From (i) and (ii)

$$\therefore \frac{d^2x}{dt^2} = -\frac{8k}{3m}x \qquad \Rightarrow \qquad \text{time period } T = 2\pi\sqrt{\frac{3m}{8k}}$$

$\left(\frac{dx}{dt}\right)_{max} = \omega A$ where ω = circular frequency $\sqrt{8k/3m}$ and A is amplitude of point of attachment of spring.

∴ Amplitude of the centre of the disc = A/2 = $v_0/\omega = v_0\sqrt{\frac{3m}{8k}}$.

Prob 19. *A block A of mass m is attached by means of a spring to another block B of mass 4m and A is allowed to oscillate vertically on a pair of guide rails. If the upper block A is pressed down and released, then for what value of the compression would the lower block B just lose contact with the ground?*

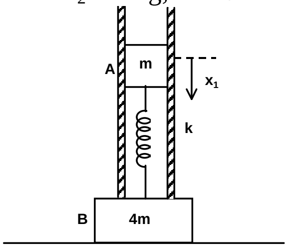


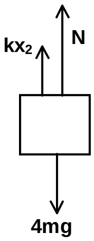
Sol. Suppose that the block of mass m is depressed by x_1 and left so that the entire system just rebounds & loses contact with the ground.

Initially, there is a compression x_o in the spring so that, $kx_o = mg$

If the block B just loses contact, then the normal reaction becomes 0.

$N + kx_2 = 4mg$; $N = 0$





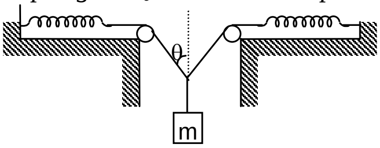
$X_2 = \dfrac{4mg}{k}$

Then, using the equation of energy conservation, we get,

$$\frac{1}{2}k\left(x_1 + \frac{mg}{k}\right)^2 = mg\,(x_o + x_1 + x_2) + \frac{1}{2}kx_2^2$$

where from, $x_1 = \dfrac{5mg}{k}$

Prob 20. *Consider a block symmetrically attached to two identical massless springs by means of strings passing over light frictionless pulleys as shown. In the equilibrium position extension in the springs is x_0. Find the time period of small vertical oscillations of the block.*



Sol. For equilibrium of the block

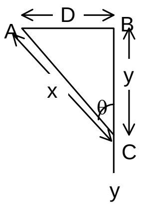
$mg = 2k\,x_0 \cos\theta$ . . . (I)

In ΔABC

$x^2 = y^2 + D^2$

$2x\,dx = 2y\,dy$

$dx = \cos\theta\,dy$



If the block is displaced by a small vertical distance dy, the elongation in the springs increases by $\cos\theta\,dy$. Hence increase in the tensions in the strings attached to the springs $= k\cos\theta\,dy$

Restoring force $= F = -\,2k\cos\theta\,dy\cos\theta$

$= -2k\cos^2\theta\,dy$

$\Rightarrow$ time period $T = 2\pi\sqrt{\dfrac{m}{2k\cos^2\theta}} = 2\pi\sqrt{\dfrac{x_0}{g\cos\theta}}$.

Objective:

Prob 1. *The frequency (f) and time period (T) of a particle in SHM are related as*
(A) f.T = 1 *(B) f = 1/T^3*
(C) f.T^2 = 1 *(D) T.f^2 = 1*

Sol. Frequency = $\frac{\omega}{2\pi}$ and time period = $\frac{2\pi}{\omega}$
$\therefore$ (A).

Prob 2. *The displacement of a particle in SHM is always measured from*
(A) mean position.
(B) extreme position.
(C) mid point of mean & extreme position.
(D) none of the above.

Sol. $\therefore$ (A).

Prob 3. *For a spring block system in SHM, during one time period the kinetic energy of the block (starting from mean position)*
(A) is always constant
(B) initially increases and then decreases.
(C) first decreases and the increases.
(D) keeps on changing

Sol. As speed keeps on changing, so is the kinetic energy.
$\therefore$ (D).

Prob 4. *A steel wire of uniform cross–section of 1 m^2 is heated to 70°C and stretched by tying its two ends rigidly. What is the change in the tension of the wire when the temperature falls from 70°C to 35°C? Coefficient of linear expansion of steel is 1.1×10^{-5} /°C and Young's modulus is 2.0×10^{11}N/m^2.*
(A) 250N *(B) 116N*
(C) 77N *(D) 28N*

Sol. The tension produced in a stretched wire due to fall in temperature by Δt is.
$F = YA\,\alpha\Delta t$
Where Y and α are the Young's modulus and coefficient of linear expansion of the material of the wire and A is the cross–sectional area.
Here $Y = 2.0\times10^{11} N/m^2$
$A = 1.0\ mm^2 = 1.0\times10^{-6} m^2$
$\alpha = 1.1\times10^{-5}/°C$
$\Delta t = (70 - 35) = 35°C$
$\therefore$ $F = 2.0\times10^{11} \times (1.0 \times 10^{-6}) \times (1.1\times10^{-6}) \times 35 = 77N$

Prob 5. *A metallic wire is stretched by suspending a weight to it. If β is the stress produced in the wire and Y its Young's modulus of elasticity, the potential energy stored per unit volume in the wire is*

(A) $\beta^2 Y/2$ *(B) $\beta Y/2$*
(C) $\beta/2Y$ *(D) $\beta^2/2Y$*

Sol. Elastic energy per unit volume = (1/2)stress × strain, Here, Strain = β

And $Y = \frac{\text{Stress}}{\text{Strain}}$ $\therefore$ Strain = stress/Y = β/Y

$\therefore$ Strain = Y/ β

Elastic energy per unit volume = $\frac{\beta}{2}\times\frac{\beta}{Y}=\frac{\beta^2}{2Y}$

Prob 6. *Which of the following functions represent S.H.M.*

(A) $t + \sin\omega t$ *(B) $\sin^2\omega t$*
(C) $\sin\omega t + 2\cos\omega t$ *(D) $\sin\omega t + \cos 2\omega t$*

Sol. The motion will be SHM only if acceleration (or $\frac{d^2y}{d\ \ t^2}$) α – y

This is true for (C).

Prob 7. *The potential energy U(x) of a particle executing SHM is given by*

(A) $U(x) = k(x-a)^2/2$ *(B) $U(x) = k_1x + k_2x^2 + k_3x^3$*
(C) $U(x) = A\,e^{-bx}$ *(D) $U(x)$ =constant*

Sol. $F = -\frac{dU}{dx}$

For $U(x) = \frac{k}{2}(x-a)^2$

$\frac{dU}{dx} = k(x-a) \qquad \Rightarrow F = -k(x-a)$

This is the condition for SHM about point x =+ a

Other functions do not satisfy this condition.

$\therefore$ (A)

Prob 8. *When a mass under goes SHM there is always a constant ratio between its displacement and*

(A) period *(B) acceleration*
(C) mass *(D) velocity*

Sol. $T = 2\pi\sqrt{\left|\frac{\text{displacement}}{\text{acceleration}}\right|}$ & T in SHM is constant

Therefore ratio (displacement / acceleration) is constant

$\therefore$(B)

Prob 9. *Two SHM are given by $y_1 = a\sin[(\pi/2)t + Q]$ and $y_2 = b\sin[2\pi t/3 + Q]$.*
The phase difference between these after 1 sec. is

(A) π *(B) π/2*
(C) π/4 *(D) π/6*

Sol. t = 1 sec.
$y_1 = a \sin(\pi/2 + Q)$ & $y_2 = b \sin(2\pi/3 + Q)$
phase difference is $\left(\frac{2\pi}{3} + Q\right) - (\pi/2 + Q) = \pi/6$
$\therefore$ (D)

Prob 10 . *A simple harmonic motion has an amplitude A & time period T. The time required by it to travel from x =A to x = A/2 is –*
(A) T/6 *(B) T/4*
(C) T/3 *(D) T/2*

Sol. Time required to travel from x = A to x = 0 is T/4 & time required to travel from x = A/2 to x = 0 is equal to the time required to travel from x = 0 tot x = A/2 which is equal to
$y = A \sin(2\pi/T)t$
$A/2 = A \sin(2\pi/T)t$
$\Rightarrow \sin(\pi/6) = \sin(2\pi/T)t$
$\Rightarrow t = T/12.$
$\Rightarrow$Time required to travel from x = A to x = A/2 will be
$T/4 - T/12 = T/6$
$\therefore$ (A)

Prob 11. *A particle executes SHM with a frequency f. The frequency with which its K.E. oscillates is*
(A) f/2 *(B) f*
(C) 2f *(D) 4f*

Sol. As we have seen in theory, the KE oscillates with double the frequency i.e. with 2f.
$\therefore$ (C)

Prob 12. *A simple pendulum with a solid metal bob has a period T. The metal bob is now immersed in a liquid of density one tenth that of the bob. The liquid is non viscous. Now the period of the same pendulum with its bob remaining all the time in the liquid wil be*
(A) t *(B) (9/10)T*
(C) $\sqrt{10/9}\,T$ *(D) $\sqrt{9/10}\,T$*

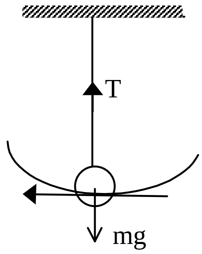
Sol. Apparent acceleration due to gravity because of force of buoyancy
$g' = g\left(1 - \frac{\rho_i}{\rho_s}\right)$ $\rho_i = \rho_s/10$
$\Rightarrow g' = g\left(1 - \frac{1}{10}\right)$
$= \frac{9}{10} g$
$T \propto \frac{1}{\sqrt{g}} \Rightarrow T' = \sqrt{\frac{10}{9}}\,T$
$\therefore$ (C)

Prob 13. *A simple pendulum with angular frequency ω oscillates simple harmonically. The tension in the string at lowest point is T. The total acceleration of the bob at its lowest position is*

(A) (T/m – g) *(B) Zero*

(C) g – T/m *(D) T + g/m*

Sol. At the lowest point, the tangential acceleration = 0, but since the bob is moving in a circle, it will have centripetal acceleration.

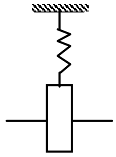


Centripetal force = T – mg

$\Rightarrow$ centripetal acceleration $= \dfrac{T - mg}{m} = \dfrac{T}{m} - g$

$\therefore$ (A)

Prob 14. *The time period of a spring–mass system is T in air when the mass is partially immersed in water the time period of oscillation is*



(A) T *(B) < T*

(C) >T *(D) ≤T*

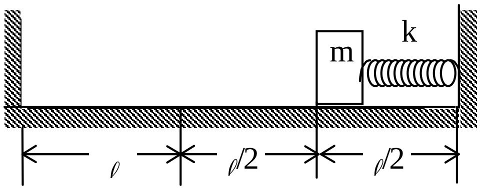
Sol. The time period of the spring mass system in air $= T = 2\pi\sqrt{\dfrac{m}{k}}$. When the mass is immersed in water partially, the buoyent force = ρAxg & the spring force = k.x. when the body is displaced through a small distance x.

The restoring force = kx + ρAgx

$$\omega' = \sqrt{\frac{k + \rho Ag}{m}} \quad \Rightarrow \quad T' = 2\pi\sqrt{\frac{m}{k + \rho Ag}}$$

$\therefore$ T′ < T $\therefore$ (B).

Prob 15. *A block of mass m compresses a spring of stiffness k through a distance (ℓ/2) as shown in the figure. If the block is not connected with the spring and the impact of the block with the vertical wall is elastic, the period of motion of the block is*



(A) $2\pi\sqrt{\dfrac{m}{k}}$ *(B)* $(\pi + 4)\sqrt{\dfrac{m}{k}}$

(C) $(1 + \pi)\sqrt{\dfrac{m}{k}}$ *(D) none of these.*

Sol. The period of oscillation $= 2\pi\sqrt{\dfrac{m}{k}}$

$\Rightarrow$ The period of motion till the block is in contact with the spring

$t_1 = \pi\sqrt{\dfrac{m}{k}}$ then it leaves the spring with a speed v = ωA

$$v = \left(\sqrt{\frac{k}{m}}\right)\left(\frac{\ell}{2}\right)$$

Then it moves with constant velocity v for a distance $D = \ell + \ell = 2\ell$

$\Rightarrow$ The corresponding time of motion = $t_2 = 2\ell/v$

$\Rightarrow$ $t_2 = \dfrac{2\ell}{\frac{\ell}{2}\sqrt{\frac{k}{m}}} = 4\sqrt{\dfrac{m}{k}}$

$\therefore$ The time period of motion $= t = t_1 + t_2$

$= \pi\sqrt{\dfrac{m}{k}} + 4\sqrt{\dfrac{m}{k}} = \sqrt{\dfrac{m}{k}}\,[\pi + 4]$.

$\therefore$ (B).

FILL IN THE BLANKS IN THE FOLLOWING QUESTIONS.

Prob 16. *Two bodies M and N of equal masses are suspended from two separate massless springs of constants k_1 and k_2 respectively. If the two bodies oscillates vertically such that their maximum velocities are equal, the ratio of amplitude of vibration of M to that of N is* __________________

Sol. $T_1 = 2\pi\sqrt{\dfrac{m}{k_1}}$, $T_2 = 2\pi\sqrt{\dfrac{m}{k_2}}$

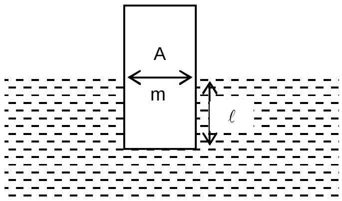
$\therefore$ $\dfrac{T_1}{T_2} = \left(\dfrac{k_2}{k_1}\right)^{1/2}$

Given $a_1\omega_1 = a_2\omega_2$

or, $\dfrac{a_1}{a_2} = \dfrac{T_1}{T_2} = \sqrt{\dfrac{k_2}{k_1}}$

Prob 17. *A rectangular block of mass m and area of cross–section A floats in a liquid density ρ. If it is given a small vertical displacement from equilibrium, it undergoes oscillation with a time period T, the T^2 is proportional to* _______________

Sol. In equilibrium position, let ℓ be the length of the block immersed in the liquid as shown in figure.



Then, $mg = A\ell\rho g$

Or $m = A\ell\rho$

If the block is given a further downward displacement y, then effective upward force (i.e. restoring force) on block is

$F = -\,[A(\ell + y)\rho g - mg]$

$= -[A(\ell + y)\rho g - A\ell\rho g]$

$= -A\rho g y$

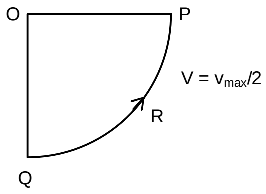
it means $F \propto y$ and it is directed towards equilibrium position. Therefore, if the block is left free, it will executes SHM mass of block = m = inertia factor

spring factor = $A\rho g$

$\therefore$ Time period, $T = 2\pi\sqrt{\dfrac{m}{A\rho g}}$

i.e. $T^2 \propto \dfrac{m}{A\rho}$

Prob 18. *A particle starts SHM from the mean position as shown in figure. Its amplitude is A and its time period is T. At one time its speed is half that of the maximum speed. This displacement is ________________________*



Sol. Let the particle be at R, when its velocity $v = \dfrac{v_{max}}{2} = \dfrac{a\omega}{2}$ and its displacement from the mean position Q be y

As $v = \omega\sqrt{A^2 - y^2}$

So, $y = \sqrt{A^2 - v^2/\omega^2}$

Given $v = A\omega/2$

Then, $y = \sqrt{A^2 - \dfrac{A^2\omega^2}{4\omega^2}} = \dfrac{\sqrt{3}}{2}A$

STATE WHETHER THE FOLLOWING QUESTIONS ARE TRUE OR FALSE.

Prob 19. The spring constant of a spring is k. When it is divided into n equal parts, the spring constant of one piece so obtained is nk.

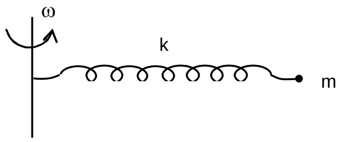
Sol. **True**

When a spring is cut into n equal parts, then spring constant k of each part is given by

$\dfrac{1}{k} = \dfrac{1}{k} + \dfrac{1}{k} + ...n \text{ terms} = \dfrac{n}{k}$

$\Rightarrow k = n\,k$

Prob 20. *A particle of mass m is fixed to one end of a light spring of force constant k and unstretched length ℓ. The system is rotated about the other end of the spring with an angular velocity ω, in gravity free space. The increase in length of the spring will be* $\dfrac{m\omega^2\ell}{k + m\omega^2}$



Sol. **False**

Let $d\ell$ be the increases in length of spring. It means the particle will move on a circular path of radius $(\ell + d\ell)$ and the restoring force due to spring $(=kd\ell)$ will provide the required centripetal force.

$\therefore m\omega^2(\ell + d\ell) = kd\ell$

or, $d\ell = \dfrac{m\omega^2\ell}{k - m\omega^2}$

ASSIGNMENT PROBLEMS

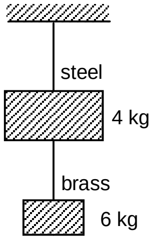
Subjective:

Level – O

1. What mass must be suspended from a steel wire 2m long and 1 mm in diameter to stretch it by 1mm ? Given Young's modulus of steels 2×10^{12} dyne/cm^2 , g = 981 cm/s^2.

2. How much will a 3.0 m long copper wire elongate if a weight of 10 kg is suspended from one end and the other end is fixed. The diameter of the wire is 0.4 mm.
Given $Y_c = 10^{11}$ N/m^2 g = 9.8 m/s^2

3. Compute the elongation of the steel and brass wire in the figure. Unloaded length of steel wire = 1.5m, unloaded length of brass wire = 10 m, diameter of each wire 0.25 cm. Young's modulus of steel = 2×10^{11} N/m^2 and that of brass is 0.91×10^{11} N/m^2.



4. The diameter of brass rod is 4 mm. Young's modulus of brass is $Y = 9.2 \times 10^{10}$ N/m^2.
(i) Calculate in SI units the stress and strain when it is stretched by 0.25 % of its length.
(ii) What is the force exerted.

5. Find the greatest length of a steel wire that can hang vertically without breaking. The breaking stress for steel is 7.9×10^9 dyne/cm^2 and density = 8 gm/cm^3.

6. Two parallel wires A and B are fixed to a rigid support at the upper ends and subjected to the same load at the lower ends. The lengths of the wire are in the ratio 4 : 3. The increase in length of the wire A is 1mm. Calculate the increase in the length of the wire B.

7. Calculate the bulk modulus of water from the following data: Initial volume = 100.0 litre. Pressure increase = 100.0 atm. Final volume 100.5 litre (1 atm = 1.013×10^5 N/m^2). Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large. Given bulk modulus of air = 0.10×10^7 N/m^2

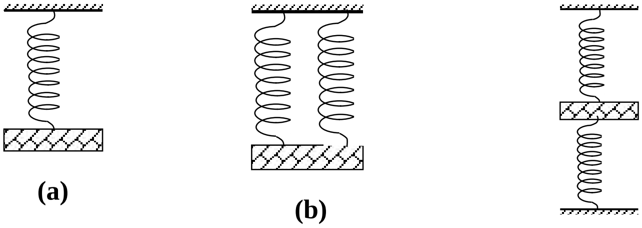
8. A square lead slab of side 50 cm and thickness 5.0 cm is subjected to a shearing force (on its narrow face) of magnitude 9.0×10^4 N. The lower edge is riveted to the floor. How much is the upper edge displaced if the shear modulus of lead is 5.6×10^9 N/m^2.

9. Two wires of same length and material but different radii are suspended from a rigid support. Both carry the same load at the lower end. In the two wires, (i) stress (ii) strain and (iii) extension be same or different ? Why ?

10. A particle executes SHM of amplitude A. At what distance from the mean position is its kinetic energy (i) equal to potential energy (ii) half of potential energy and (iii) At what point is its speed half the maximum speed.

11. A metal cylinder of mass 6 kg vibrates harmonically with an amplitude of 0.5 m and period of 2 sec. Calculate (i) the frequency (ii) its maximum speed (iii) speed when x = 0.15 m (iv) maximum acceleration (v) acceleration when x = 0.15 m (vi) maximum restoring force (vii) maximum restoring force when x = 0.15 m (viii) total energy.

12. Two identical springs, each of spring factor k, may be connected in different ways as shown. Deduce the equivalent spring factor of the oscillation of the body in each case. Also find the frequency in each case.



13. A vertical U–tube of uniform cross–section contains water up to a height of 2.45 cm. When the water on one side is depressed and then released, its up and down motion in the tube is simple harmonic motion. Show that the time period is equal to $\frac{\pi}{10}$ seconds. Given g = 9.8 m/s^2.

14. One end of a U–tube mercury is connected to a pump and the other end is open to the atmosphere. A small pressure difference is maintained between the two columns. Show that when the suction pump is removed, the liquid column in the U–tube executes simple harmonic motion.

15. A person normally weight 100 kg stands on a platform which oscillates vertically harmonically at frequency of 2.0 per sec. and an amplitude of 0.05m. If a machine on a platform gives the person's weight against time, deduce the maximum and minimum reading it will show. Given g = 10 m/s^2.

16. What will be the effect of the time period, if the amplitude of a simple pendulum increases ?

17. A mass m is dropped in a tunnel along the diameter of earth from a height h (<<R) above the surface of earth. Find the time period of motion. Is the motion simple harmonic ?

18. A simple pendulum of length ℓ and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period.

19. A ball of radius r is made to oscillate in a bowl of radius R, find its time period of oscillation.

20. A body of mass 1 kg is executing SHM which is given by x = 6 cos (100 t + π/4) cm. What is the
(i) amplitude of displacement
(ii) angular frequency
(iii) initial phase
(iv) velocity
(v) acceleration
(vi) maximum kinetic energy ?

Level – I

1. What is the density of ocean water at a depth, where the pressure is 80 atm, given that its density at the surface is 1.03 x 10^3 Kg m^{-3}? Compressibility of the water = 45.8 x10^{-11} Pa^{-1}.
Given 1 atm = 1.013 x 10^5 Pa

2. A particle moving with simple harmonic motion has speeds 3 cm/s and 4 cm/s at displacements 8cm and 6 cm respectively from the equilibrium position. Find
(a) the period of oscillation
(b) the amplitude of oscillation.

3. A particle moves with simple harmonic motion along x–axis. At times t and 2t, its positions are given by x = a and x = b respectively. Find the time period of oscillation.

4. A uniform rod of mass m and length l is free to rotate about a fixed horizontal axis through its end and perpendicular to its length. Find the period of small oscillations of the rod.

5. A particle oscillating harmonically with an amplitude of 1.5 cm, has a maximum energy of 0.25 μJ. At what displacement from the equilibrium position will the particle be acted upon by a force of 2.5×10^{-5} N?

6. A small bob of mass 50 g oscillates as a simple pendulum, with an amplitude 5 cm and period 2 s. Find the velocity of the bob and the tension in the supporting thread, when the velocity of the bob is maximum. [Take g = 10 m/s^2]

7. A horizontal platform vibrates up and down with a simple harmonic motion of frequency 2/π Hz. Find the maximum permissible amplitude so that an object kept on the platform remains in contact with the platform.

8. A particle of mass m is moving in a field of conservative force whose potential is given by u = a+ bx^2 J/kg. Show that the motion of the particle is simple harmonic. Find the frequency in Hz.

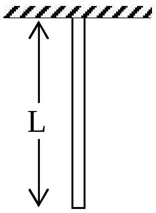
9. A meter stick swinging from one end oscillates with a frequency f_o. What would be the frequency, in terms of f_o, if the bottom third of the stick were cut off ?

10. A uniform rod having density σ, length L, cross–sectional area A is rotating with a uniform angular velocity ω about a fixed vertical axis through its end. Neglecting the effect of gravity, find the normal stress developed in the rod at its mid point.

11. (a) A copper wire of length 8 m and a steel wire of length 4 m, each of cross section 0.5 cm^2 are joined end to end and stretched with a tension of 500 N. If $y_{copper} = 1 \times 10^{11}$ N/m^2, $y_{steel} = 2 \times 10^{11}$ N/m^2. Find the elastic potential energy stored in the system ?
(b) At what depth a rubber ball shall be kept in a lake so that its volume decrease by 0.1 %? (Given Bulk modulus of rubber = 10×10^8 N/m^2)

12. If two SHM's are represented by equations $y_1 = 10 \sin [3\pi t + \pi/4]$ and $y_2 = 5[\sin 3\pi t + \sqrt{3} \cos 3\pi t]$, find the ratio of their amplitudes.

13. A uniform heavy rod of weight W, cross sectional area A and length L is hanging from a fixed support. Young's modules of the material of the rod is Y; find the elongation of the rod due to its own weight, Neglect the lateral contraction.



14. If a particle of mass m moves in a potential energy field $v = u_0 - ax + bx^2$ where u_0, a and b are positive constant. Calculate the force constant, frequency of the particle and point of equilibrium.

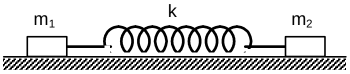
15. A particle executes S. H. M. with a time period of 4s. Find the time taken by the particle to go directly from its mean position to half of its amplitude.

Level – II

1. A copper rod of length L and radius r is suspended from the celling by one of its ends. Find
(a) the elongation of the rod due to its own weight if ρ and Y are the density and Young's Modulus of copper respectively,
(b) The elastic potential energy stored in the rod due to its own weight.

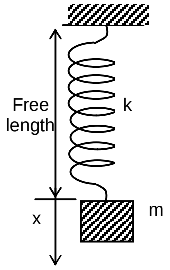
2. A particle in S.H.M. crosses its equilibrium position at time t =2 sec. . When the motion advances by one second more, its velocity is found to be $\pi/3$ m/s. If the frequency of the motion is 1/6 s^{-1}, find:
(a) the amplitude of the motion
(b) the velocity of the particle at t = 4s.

3. A spring mass system is lying on a horizontal frictionless surface. The spring is stretched by x_0 from its natural length and the system is released from rest. Find



(a) the variation of the elongation δ in the spring with respect to time and hence the time period of oscillations.
(b) is the motion of m_1 and m_2 with respect to ground simple harmonic ? If yes, find the amplitude of motion of m_1 and m_2 as seen by the ground.

4. A block of mass m is gently attached to the spring and released at time t = 0 when the spring has its free length as shown in the figure. During subsequent motion of the block, find the variation of x with respect to time.

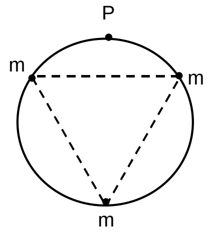


5. The average speed over the period of a complete oscillation of a particle performing rectilinear SHM is 2 cm/s. If the particle attains this speed when it is at a point P whose distance from the centre O is 4 cm, determine
(a) the amplitute
(b) the periodic time of the motion expressing the answers in terms of π. Show also that the least time taken by the particle to travel from O to P is given by

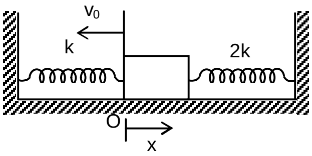
$$\frac{4}{\sqrt{(\pi^2-4)}}\sin^{-1}\left(\frac{\sqrt{(\pi^2-4)}}{\pi}\right)$$

6. A pendulum is formed by pivoting a long thin rod of length L and mass m about a point on the rod which is a distance d above the center of the rod.
(a) Find the small–amplitude period of this pendulum in terms of d, L, m and g.
(b) Show that the period has a minimum value when $d = L/\sqrt{12} = 0.289$ L.

7. Three particles of the same mass m are fixed to a uniform circular hoop of mass m and radius R at the corners of an equilateral triangle. The hoop is free to rotate in a vertical plane about the point on the circumference opposite to one of the masses. Find the equivalent length of a simple pendulum.



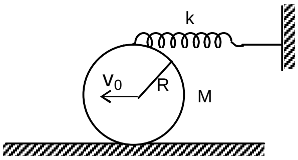
8. A spring mass system is shown in the figure. At t = 0, the mass is given an initial velocity v_0 towards left. Denoting by O the equilibrium position,



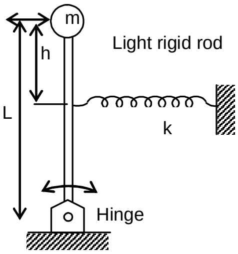
(i) find the time period of oscillations.

(ii) derive an expression for x in terms of x_0, m, k and time t.

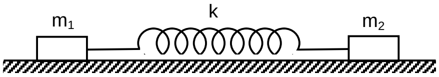
9. A uniform disc of mass M and radius R, which is free to roll on a horizontal surface, is connected by a spring k as shown in the figure. If the disc is given an initial velocity v_0 in the equilibrium position, find the time period and amplitude of small oscillations of the centre of the disc.



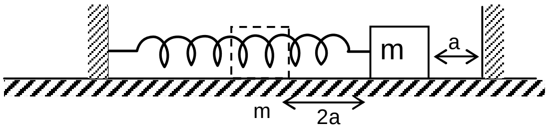
10. Find the time period of small oscillations of the spring – mass system shown in figure. Find the range of values of h and k so that the system has simple harmonic motion.



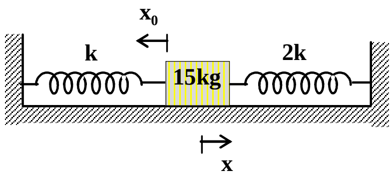
11. In the figure shown m_1 and m_2 are rigidly connected to a light spring of stiffness K. the system is placed on a smooth horizontal surface. The masses are slightly pulled apart and released. Find the period of oscillation of the masses.



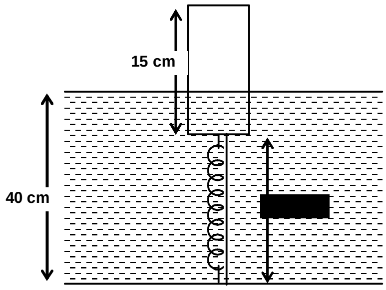
12. A block of mass 'm' connected by a spring is kept on a horizontal frictionless floor. When spring is at its natural length 'ℓ_0' the block is at a distance 'a' from the wall. Now the block is moved by a distance of '2a' away from the wall and released. If the collision between the block and wall is elastic, determine the time period of oscillation ?



13. A spring mass system is shown in the figure. The mass is displaced towards left by x_0 and released.
(a) find the time period of oscillations
(b) derive an expression for x as a function of x_0, t, k and m.



14. The figure shows a cylinder having length 15cm, cross sectional area $100 cm^2$ and density $500 kg/m^3$. The base of the cylinder is connected with a thread of length 35cm and a spring of natural length 40cm. (The spring constant of spring is 150 N/m) The arrangement is kept in a large water tank (The tank has depth 40cm and the density of water is $1000 kg/m^3$). Find
(a) tension in the thread.
Now, if the thread snaps at t = 0. Find
(b) maximum length of the cylinder out of the water.
(c) the value of t at which the maximum length of the cylinder will be out of water for the first time.



15. ABCD is a horizontal square plate of diagonal 2b. A particle of mass m rests on the plate and is joined to each corner by elastic strings of natural length ℓ and elastic constant (Youngs modulus Y) which remain in tension throughout the motion. The particle is initially at the centre of square and is then drawn aside along the diagonal CA towards A by a small distance, and then released. If the area of cross–section of the wire is A, find the time period of oscillations of the particle.

Objective:

Level – I

1. A particle moves in x–y plane according to the equation $\vec{r} = (\hat{i} + 2\hat{j})A\cos\omega t$. The motion of the particle is

(A) simple harmonic (B) uniformly accelerated
(C) circular motion (D) projectile motion

2. Which of the following quantities are always positive in a simple harmonic motion

(A) $\vec{F}.\ \vec{a}$ (B) $\vec{v}.\ \vec{r}$
(C) $\vec{a}.\ \vec{r}$ (D) $\vec{F}.\ \vec{r}$

3. The magnitude of average acceleration in half time period in a simple harmonic motion is

(A) $\frac{2A\omega^2}{\pi}$ (B) $\frac{A\omega^2}{2\pi}$
(C) $\frac{A\omega^2}{\sqrt{2}\pi}$ (D) Zero

4. A particle performs S.H.M. with time period T. The time taken by the particle to move from half the amplitude to the maximum displacement is

(A) $\frac{T}{2}$ (B) $\frac{T}{4}$
(C) $\frac{T}{6}$ (D) $\frac{T}{8}$

5. Two pendulums begin to swing simultaneously. During first fifteen oscillations of the first pendulum, the other pendulum makes only ten swings. The ratio between the lengths of these pendulums is

(A) $\frac{4}{9}$ (B) $\frac{2}{3}$
(C) $\frac{5}{9}$ (D) $\frac{2}{15}$

6. A small block oscillates back and forth on a smooth concave surface of radius R. The time period of small oscillations is

(A) $T = 2\pi\sqrt{\frac{R}{g}}$ (B) $T = 2\pi\sqrt{\frac{2R}{g}}$
(C) $T = 2\pi\sqrt{\frac{R}{2g}}$ (D) None of these

7. A simple pendulum with a brass bob has a time period T. The bob is now immersed in a non–viscos liquid and made to oscillate. The density of the liquid is (1/8)th that of the brass. The time period of the pendulum will be

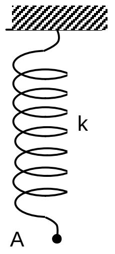
(A) $\sqrt{\frac{8}{7}}T$ (B) $\frac{8}{7}T$
(C) $\frac{8^2}{7^2}T$ (D) T

8. A particle of mass m executing SHM makes f oscillation per second. The difference of its kinetic energy when at the centre, and when at a distance x from the centre is

(A) $\pi^2 f^2 x^2$ m (B) $2\pi^2 f^2 x^2$ m

(C) $\frac{1}{2}\pi^2 f^2 x^2$ m (D) $f^2 x^2$ m

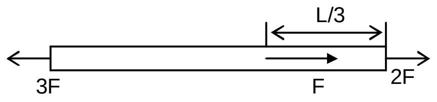
9. A light spring of stiffness k is hanging from the ceiling. It is stretched by a distance δ, then a mass m attached to the end A, and then released. The amplitude of the oscillations will be



(A) mg/k (B) 2mg/k

(C) δ (D) |(mg/k) – δ|

10. A uniform slender rod of length L, cross–sectional area A and Young's modulus Y is acted upon by the forces shown in the figure. The elongation of the rod is



(A) $\frac{3FL}{5AY}$ (B) $\frac{2FL}{5AY}$

(C) $\frac{3FL}{8AY}$ (D) $\frac{8FL}{3AY}$

11. Three masses 700 gm, 500 gm, 400 gm are suspended at the end of a spring and in equilibrium. When 700 gm mass is removed, the system oscillates with a period of 3 seconds, when 500 gm is also removed, it will now oscillates with

(A) 1 sec. (B) 2 sec.

(C) 3 sec. (D) $\sqrt{\frac{12}{5}}$ sec.

12. A particle of mass m is executing oscillations about the origin on the x – axis. Its potential energy is $V(x) = k|x|^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is

(A) proportional to $\frac{1}{\sqrt{a}}$ (B) independent of a

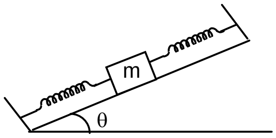
(C) proportional to $\sqrt{a}$ (D) proportional to $a^{3/2}$

13. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross section and the Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

(A) $2\pi\sqrt{(m/k)}$ (B) $2\pi\sqrt{m(YA+kL)/(YAk)}$

(C) $2\pi\sqrt{(mYA/kL)}$ (D) $2\pi\sqrt{(mL/YA)}$

14. A uniform spring has certain mass suspended from it and its period for vertical oscillation is T_1. The spring is now cut into two parts having the lengths in the ratio of 1 : 2 and same mass is suspended with those two spring as shown in figure. Now time period is T_2. The ratio T_1 / T_2 is



(A) 1 (B) sin θ
(C) $\sqrt{\frac{2}{9}}$ (D) $\sqrt{\frac{9}{2}}$

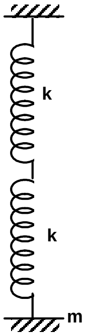
15. A particle starts SHM from the mean position. Its amplitude is A and total energy E. At one instant its kinetic energy 3E/4. Its displacement at that instant is
(A) $A/\sqrt{2}$ (B) $A/\sqrt{\frac{3}{2}}$
(C) A/2 (D) A

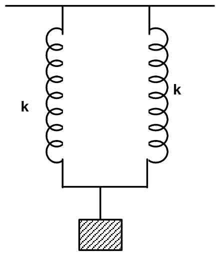
16. For a simple pendulum, graph between L and T will be a
(A) hyperbola (B) parabola
(C) straight line (D) a curve line

17. A clock 'S' is based on the oscillation of a spring and a clock 'P' is based on motion of the pendulum. Both clocks run at the same rate on the earth. On a planet having the same density as earth, but twice the radius
(A) S will run faster than P
(B) P will run faster than S
(C) they will both run at the same rate, as on the earth.
(D) none of these

18. A simple pendulum is setup in trolley which moves to the right with an acceleration a on a horizontal plane. At that moment the thread of the pendulum is at the mean position and makes an angle θ with the vertical
(A) $\tan^{-1}$ (a/g) in the forward direction
(B) $\tan^{-1}$ (a/g) in the backward direction
(C) $\tan^{-1}$ (g/a) in the backward direction
(D) $\tan^{-1}$ (g/a) in the forward direction

19. Two identical springs of constant k are connected in series and parallel as shown in figure. A mass m is suspended from them. The ratio of their frequencies of vertical oscillations will be





(A) 1:1 (B) 2:1
(C) 1:2 (D) 4:1

20. **FILL IN THE BLANKS**

(i). A wire of length L and cross–sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is ____________________.

(ii). A particle executes simple harmonic motion between x = –A and x = + A. The time taken for it to go from 0 to A/2 is T_1 and to go from A/2 to A is T_2. Then _________________

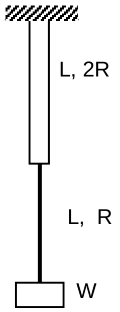
(iii). Work done by simple pendulum in one complete oscillation is _______________

(iv). A simple pendulum with length ℓ and bob of mass m is executing SHM of small amplitude a. The maximum tension in the string will be ___________________–

Level – II

1. A particle of mass m executes SHM according to equation x = A cos ωt. The average velocity and average kinetic energy over a time interval 0 to T/2
(T = Time period) are, respectively
(A) $0, \frac{mA^2\omega^2}{2}$
(B) $\frac{A\omega}{\pi}, \frac{mA^2\omega^2}{4}$
(C) Aω, 0
(D) $\frac{2A\omega}{\pi}, \frac{mA^2\omega^2}{4}$

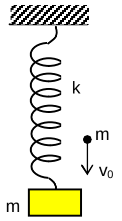
2. Two wires of the same material (Young's modulus Y) and same length L but radii R and 2R respectively are joined end to end and a weight W is suspended from the combination as shown in the figure. The elastic potential energy in the system is



(A) $\frac{3W^2L}{4\pi R^2Y}$
(B) $\frac{3W^2L}{8\pi R^2Y}$
(C) $\frac{5W^2L}{8\pi R^2Y}$
(D) $\frac{W^2L}{\pi R^2Y}$

3. A particle executes SHM along the line AB. If C divided AB in the ratio 3 : 1, the ratio of times taken to travel AC and CB is
(A) 3 : 1
(B) 1 : 1
(C) 2 : 1
(D) 4 : 1

4. A block of mass m is hanging freely from a spring of stiffness k. A particle of mass m falls on the block with a velocity v_0 and gets stuck to it. The amplitude and angular frequency of oscillations during subsequent motion are, respectively



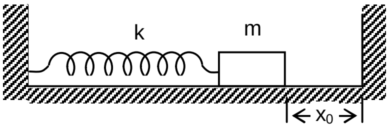
(A) $\sqrt{\frac{k}{2m}}$ and $v_0\sqrt{\frac{2m}{k}}$
(B) $\sqrt{\frac{2k}{m}}$ and $v_0\sqrt{\frac{m}{2k}}$
(C) $\sqrt{\frac{k}{2m}}$ and $v_0\sqrt{\frac{m}{2k}}$
(D) $\sqrt{\frac{k}{m}}$ and $v_0\sqrt{\frac{m}{k}}$

5. A particle executing SHM has velocities v_1 , v_2 and acceleration f_1 , f_2 in two of its positions A and B on the same side of the equilibrium position. Then the length AB is equal to
(A) $\left|\frac{v_1^2+v_2^2}{f_1+f_2}\right|$
(B) $\left|\frac{v_1^2-v_2^2}{f_1-f_2}\right|$
(C) $\left|\frac{v_1^2-v_2^2}{f_1+f_2}\right|$
(D) $\left|\frac{v_1^2+v_2^2}{f_1-f_2}\right|$

6. A particle moves on the x–axis according to equation $x = x_0 \sin^2 \omega t$. The motion is
(A) periodic but not simple harmonic
(B) simple harmonic with a periodic $2\pi/\omega$ and amplitude x_0
(C) simple harmonic with a period π/ω and amplitude $2x_0$
(D) simple harmonic with a period π/ω and amplitude $x_0/2$.

7. Two bodies A and B of equal mass are suspended from two separate springs of spring constants k_1 and k_2 respectively. Both oscillate vertically such that their maximum velocities are equal. The ratio of the amplitude of A to that of B is
(A) k_2 / k_1 (B) k_1 / k_2
(C) $\sqrt{\frac{k_2}{k_1}}$ (D) $\sqrt{\frac{k_1}{k_2}}$

8. One end of a spring of force constant k is fixed to a vertical wall and the other to a body of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the body. The spring is compressed by $2x_0$ and released. The time taken by the block to strike the wall is

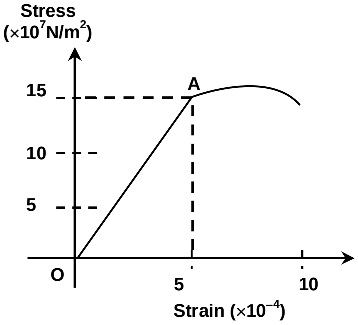


(A) $\frac{\pi}{6}\sqrt{\frac{m}{k}}$ (B) $\frac{\pi}{3}\sqrt{\frac{m}{k}}$
(C) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$ (D) $\frac{4\pi}{3}\sqrt{\frac{m}{k}}$

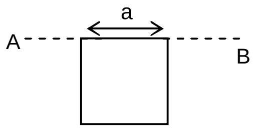
9. A disc of radius R and mass M is pivoted at the rim and is set for small oscillations. If a simple pendulum has to have the same period as that of the disc, the length of the simple pendulum should be
(A) $\frac{5}{4}R$ (B) $\frac{2}{3}R$
(C) $\frac{3}{4}R$ (D) $\frac{3}{2}R$

10. A particle executes SHM along x–axis about the centre x = – a with a frequency f Hz. Initially the particle at rest at the origin. Its equation of motion will be
(A) $x = 2a(1 - \cos 2\pi ft)$ (B) $x = a \cos 2\pi ft$
(C) $x = a(\cos 2\pi ft - 1)$ (D) $x = a(1 - \cos 2\pi ft)$

11. A graph plotted between longitudinal stress and longitudinal strain subjected on a unfirom elastic wire is as given in the diagram. Then the Young's modulus of elasticity of that wire is
(A) $5\times10^{11}\ N/m^2$
(B) $10\times10^{10}\ N/m^2$
(C) $15\times10^{11}\ N/m^2$
(D) $3\times10^{11}\ N/m^2$



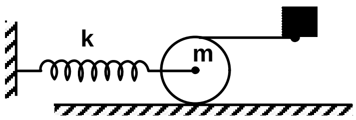
12. A thin square plate of mass m and side a is hinged about AB so as to execute small oscillations about AB. The time period of oscillations is



(A) $2\pi\sqrt{\frac{a}{g}}$ (B) $2\pi\sqrt{\frac{2a}{3g}}$

(C) $2\pi\sqrt{\frac{a}{6g}}$ (D) none of the above

13. One end of spring of spring constant k is attached to the centre of a disc of mass m and radius R and the other end of the spring connected to a rigid wall. A string is wrapped on the disc and the end A of the string (as shown in the figure) is pulled through a distance a and then released. The disc is placed on a horizontal rough surface and there is no slipping at any contact point. What is the amplitude of the oscillation of the centre of the disc?



(A) a (B) 2a
(C) a/2 (D) none of these

14. If a particle performs SHM with a frequency ν, then its K.E. will oscillate with a frequency
(A) ν/2 (B) ν
(C) 2ν (D) zero

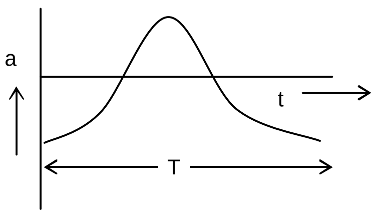
15. Two pendulums oscillate with a constant phase difference of 90^o and of the same amplitude. The maximum velocity of one is v. The maximum velocity of other will be
(A) v (B) $\sqrt{2}$ v
(C) 2v/3 (D) v$\sqrt{2}$

16. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant k, a mass m hangs freely from the free end of the spring. The area of cross–section and Young's modulus of the wire are A and y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

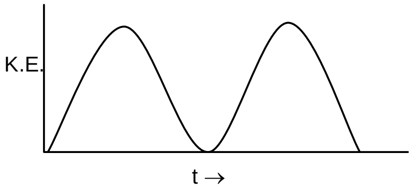
(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{mL}{yA}}$

(C) $2\pi\sqrt{\frac{myA}{kL}}$ (D) $2\pi\sqrt{\frac{m(yA+kL)}{yAk}}$

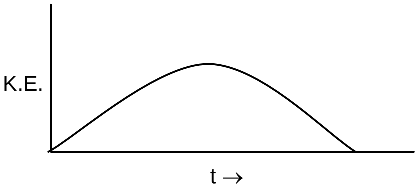
17. Acceleration 'a' and time period T of a body in SHM is given by a curve shown. The shape of the graph between KE and time t will be like



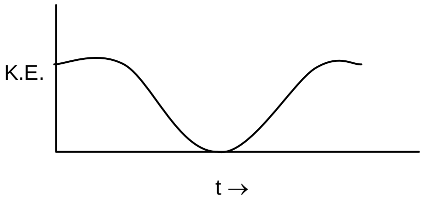
(A)



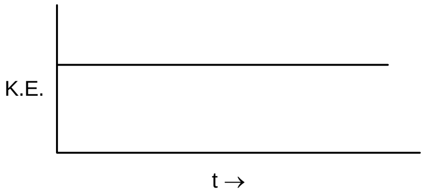
(B)



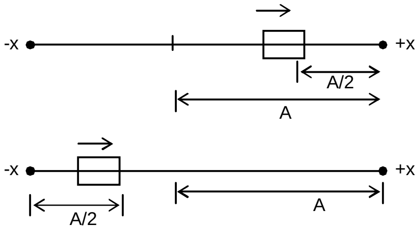
(C)



(D)



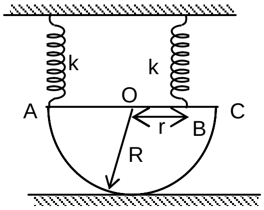
18. Two bodies executing SHM have same amplitude and frequency. Their position at a certain instant are as shown in the figure. The phase difference between them



(A) $11\pi/6$ (B) π

(C) $5\pi/3$ (D) $3\pi/5$

19. A half cylinder of mass m kg radius R m and length ℓ m is placed on a rough horizontal surface as shown. Two springs of force constant k N/m each, are connected to the cylinder along the same diametric line on the flat surface which divides the axis into two equal parts, as shown in the figure. If the corner C is pushed downward through a very small distance, then the time period of the resulting motion will be (Assume gravity free space & no slip of the cylinder on the surface).



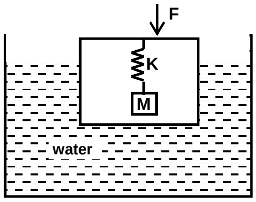
(A) $T = 2\pi\sqrt{\dfrac{m}{2k}}$

(B) $T = 2\pi R\sqrt{\dfrac{m}{2k(R^2 + r^2)}}$

(C) $T = 2\pi R\sqrt{\dfrac{m}{2(R^2 - r^2)}}$

(D) none of the above

20. Suppose there is an empty box in which a block of mass 'm' is hanging in equilibrium with the help of a vertical spring of stiffness k. Now the box is moved downwards in water ($\rho_\omega =$ density of water) with a constant acceleration 'a' by applying a vertically downward force F as shown in the figure. The volume of the box is V. The time period of oscillation of the block in the frame of the box is



(A) $2\pi\sqrt{\dfrac{KF}{M\rho_\omega Vg}}$

(B) $2\pi\sqrt{\dfrac{K\rho_\omega Vg}{MF}}$

(C) $2\pi\sqrt{\dfrac{K\left(F+Mg\right)}{M\left(\rho_\omega Vg+Mg\right)}}$

(D) none of these

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. 8.009 kg.
2. 2.339 cm
3. 1.316×10^{-4} m
4. (i) 2.3×10^{8} N/m^{-2} (ii) 2.89×10^{3} N
5. 10.1 km
6. 2.22 mm
7. 2.026×10^{9} Pa, 20260
8. 3.214×10^{-4} m
9. all different
10. (i) $x = \pm\frac{A}{\sqrt{2}}$, (ii) $x = \pm\sqrt{\frac{2}{3}}A$ (iii) $x = \pm\frac{\sqrt{3}}{2}$
11. (i) 0.5 Hz (ii) 0.25π m/s (iii) $-0.25\pi^2$ m/s^2, $-0.15\pi^2$ m/s^2
(iv) $-\pi^2$ N, $-0.6\pi^2$ N (v) $0.125\pi^2$ J
12. (a) $\nu = \frac{1}{4\pi}\sqrt{\frac{2k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{2m}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$
16. Motion will be oscillatory but not simple harmonic.
17. $T = 2\pi\sqrt{R/g} + 4\sqrt{\frac{2h}{g}}$
18. $T = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2 + v^4/R^2}}}$
19. $T = 2\pi\sqrt{\frac{R-r}{g}}$
20. (i) 6 cm. (ii) 100/sec. (iii) $\pi/4$ (iv) $-100\sqrt{\left[6^2 - x^2\right]}$ (v) $-10^4 x$ (vi) 18 J

Level – I

1. 1.034×10^{3} kg/m^3
2. (a) 12.56s, (b) 10 cm
3. $\frac{2\pi t}{\cos^{-1}\left(\frac{b}{2a}\right)}$
4. $2\pi\sqrt{\frac{2l}{3g}}$
5. 1.125 cm
6. 0.157 m/s, 0.5 N
7. 5/8 m
8. $\frac{1}{\pi}\sqrt{\frac{b}{2}}$
9. $1.225\, f_0$
10. $\frac{3}{8}\sigma L^2\omega^2$
11. (a) 0.25 J. (b) 100 m
12. 1
13. $\frac{W}{LAY}\left(Lx - \frac{x^2}{2}\right)_0^L = \frac{WL}{2AY}$.
14. $x = \frac{a}{2b}$.
15. t = 1/3 s

Level – II

1. (a) $\frac{\rho g L^2}{2Y}$ (b) $\frac{1}{6}\frac{\pi r^2 \rho^2 g^2 L^3}{Y}$

2. 2m, $-\pi/3$ m/s

3. (a) $\delta = x_0 \cos \omega t$ where $\omega = \sqrt{k\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}$, time period $= 2\pi / \omega$

   (b) Yes, $A_1 = \frac{x_0 m_2}{m_1 + m_2}$, $A_2 = \frac{x_0 m_1}{m_1 + m_2}$

4. $x = \frac{mg(1-\cos\omega t)}{k}$ where $\omega = \sqrt{\frac{k}{m}}$

5. (a) $4\pi/(\pi^2 - 4)^{1/2}$ cm (b) $8\pi/(\pi^2 - 4)^{1/2}$ s

6. $2\pi\sqrt{\left(L^2 + 12d^2\right)/12gd}$

7. 2R

8. (a) $2\pi\sqrt{\frac{3k}{m}}$ (b) $x = -v_0\sqrt{\frac{m}{3k}}\sin\left(\sqrt{\frac{3k}{m}}\ t\right)$

9. $2\pi\sqrt{\frac{3M}{8k}}$, $v_0\sqrt{\frac{3M}{8k}}$

10. $T = \frac{2\pi}{\sqrt{\frac{k}{m}\left(\left(1-\frac{h}{L}\right)^2 - \frac{g}{L}\right)}}$, $k > \frac{mg}{L}$ and $h < L - \sqrt{\frac{mgL}{k}}$

11. $2\pi\sqrt{\frac{m_1 m_2}{K\ (m_1 + m_2)}}$

12. $\frac{4\pi}{3}\sqrt{\frac{m}{k}}$ sec.

13. (a) $\sqrt{\frac{k}{5}}$ (b) $x = x_0 \sin\left(\sqrt{\frac{k}{5}}t - \frac{\pi}{2}\right)$.

14. (a) 5N (b) 14cm (c) 0.17 sec

15. $2\pi\sqrt{\frac{b\ \ell\ m}{2\ Y\ A(2b - \ell)}}$

Objective:

Level – I

1.	**A**	2.	**A**
3.	**A**	4.	**C**
5.	**A**	6.	**A**
7.	**A**	8.	**B**
9.	**D**	10.	**D**
11.	**B**	12.	**B**
13.	**B**	14.	**B**
15.	**D**	16.	**C**
17.	**B**	18.	**A**
19.	**A**		
20.	(i). $\frac{YAx^2}{2L}$	(ii).	$T_2 = 2T_1$, $T_1 < T_2$
	(iii). zero	(v).	$mg\left[1+\frac{a^2}{\ell^2}\right]$

Level – II

1.	**D**	2.	**C**
3.	**C**	4.	**C**
5.	**C**	6.	**D**
7.	**C**	8.	**C**
9.	**D**	10.	**C**
11.	**D**	12.	**B**
13.	**C**	14.	**C**
15.	**A**	16.	**A**
17.	**D**	18.	**C**
19.	**B**	20.	**D**

Learnaf
Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

GRAVITATION AND FLUIDS

Gravitation and Fluids

Syllabus :

The universal law of gravitation, Gravitational constant; Acceleration due to gravity and its variation with the altitude, latitude, depth and rotation of the earth. Gravitational potential energy near the surface of the earth, gravitational potential and field; Escape velocity, orbital velocity of satellite, Weightlessness, motion of satellite, geostationary and polar satellites; Statement of Kepler's laws of planetary motion; proof of second and third law (circular orbits); Inertial and Gravitational mass.

Fluids: *Pressure due to fluid column, Pascal's law and its applications (hydraulic lift and hydraulic brakes), Effect of gravity on fluid pressure, Buoyancy, floatation and Archimedes' principle; Viscosity, Stoke's law, Terminal velocity, Streamline flow, Reynold's number, Bernoulli's theorem and its applications.*

Surface energy and surface tension, angle of contact, applications of surface tension in (i) formation of drops and bubbles (ii) capillary rise.

NEWTON'S LAW OF GRAVITATION

Newton's Law of Gravitation states that "Every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them".

Consider two bodies A and B of masses m_A and m_B, attracting each other with forces $\vec{F}_{AB}$ (force on A due to B) and $\vec{F}_{BA}$ (force on B due to A) respectively.

Then, $\vec{F}_{AB} = -\vec{F}_{BA}$ and,

$F_{AB} = |\vec{F}_{AB}|$ = magnitude of the attractive force

$= G.\dfrac{m_A m_B}{d_{AB}^2}$ (Valid only for point masses having mass m_A and m_B)

where d_{AB} is the distance between them and G is a universal constant known as Newton's Gravitational constant. Its value was first measured by Cavendish (1798) and is now known to be:

- $G = 6.6726 \times 10^{-11}\ Nm^2/kg^2$

The net gravitational force on a particle A due to particles P_1, P_2 , ……, P_n is given by:

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n$ (vector sum)

where $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$ … are the gravitational forces acting on the particle A due to particles P_1, P_2, P_3, …respectively.

Important points:

- The gravitational force is an attractive force.
- The gravitational force between two particles does not depend on the medium.
- The gravitational force between two particles is along the straight line joining the particles (called line of centres).

Exercise1.

(i). ***Newton's apple fell towards the earth; why not the earth towards the apple?***

(ii). ***A mass M is broken into two parts; m and (M–m). How could m be related to M so that the gravitational force between two parts is maximum?***

Illustration 1. *Calculate the force between the sun and Jupiter. Assume that the mass of the sun = 2×10^{30} kg, the mass of Jupiter = 1.89×10^{27} kg and the radius of Jupiter's orbit = 7.73×10^{11} m.*

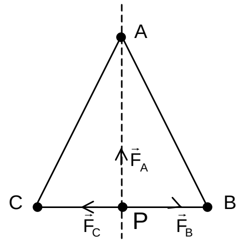
Solution: $F = \frac{6.67\times10^{-11}\times2\times10^{30}\times1.89\times10^{27}}{5.98\times10^{23}}\,N = 4.22 \times 10^{23}$ N

Thus, enormous force is required to hold Jupiter in its orbit round the sun.

Illustration 2. *Three identical particles each of mass m are placed at the three corners of an equilateral triangle of side a. Find the force exerted by this system on another particle of mass m placed at*
(a) the mid–point of a side,
(b) the centre of the triangle.

Solution : (a) Using the principle of superposition

When the particle is placed at the mid point of a side (at P)



$\vec{F}_C = -\vec{F}_B$, Hence cancel each other

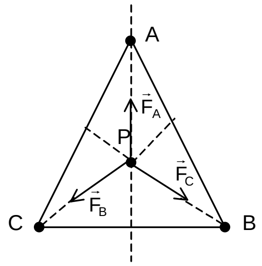
$\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C$

Hence, force experienced by the particle $\vec{F} = \vec{F}_A$

$$|\vec{F}| = |\vec{F}_A| = \frac{Gm.m}{(AP)^2} = \frac{Gm^2}{(a\sin 60)^2}$$

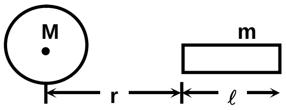
$$= \frac{4Gm^2}{3a^2} \text{along PA}$$

(b) If the particle is placed at the centre of the triangle. The net force on the particle at P, due to particles placed at the corners A, B and C will be zero



Hence, $\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C = 0$

Illustration 3. *The gravitational force of attraction between a uniform sphere of mass M and a uniform rod of length ℓ and mass m oriented as shown is*

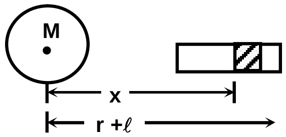


(A) $\frac{GMm}{r(r+l)}$ *(B) $\frac{GM}{r^2}$*

(C) $M\,mr^2 + \ell$ *(D) $(r^2 + \ell)\,m\,M$*

Solution: Mass per unit length $= \frac{m}{\ell}$

Mass of section of length $dx = \frac{m}{\ell}dx$



Let dF be the gravitational force of attraction between this section and sphere

Then $dF = \frac{GM\left(\frac{m}{\ell}dx\right)}{x^2}$

$$F = \int_r^{r+\ell} \frac{GMm}{\ell x^2} dx = \frac{GMm}{r(r+\ell)}$$

Hence, answer (A) is correct.

Exercise 2. ***A sphere of mass 40 kg is attracted by another sphere of mass 15 kg, with a force of $\frac{1}{40}$ mg wt. Find the value of gravitational constant if their centres are 0.40 m apart.***

GRAVITATIONAL FIELD AND INTENSITY

The space around a body where the gravitational force exerted by it can be experienced by any other particle is known as the gravitational field of the body. The strength of this gravitational field is referred to as intensity, and it varies from point to point.

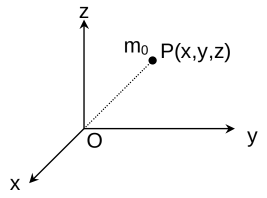
Consider the gravitational field of a particle of mass m located at the origin (O).

Suppose that a test particle of mass m_0 is placed at the point P(x, y, z). The force of gravitational attraction exerted on the test particle is given by,

$$\vec{F}_g = \frac{Gmm_0}{r^2}\hat{r}\ , \quad \text{where the position vector, } \overrightarrow{OP} = \vec{r}\ ,$$

$$r = OP = |\overrightarrow{OP}| = |\vec{r}| \text{ and the unit vector, } \hat{r} = \frac{\vec{r}}{r}$$

The intensity of this gravitational field at a point (P) is given by the force per unit mass on a test particle kept at P:



$$\vec{E} = \frac{\vec{F}_g}{m_0}$$

where $\vec{E}$ is the gravitational intensity, $\vec{F}_g$ is gravitational force acting on the mass m_0. The gravitational field is, therefore, a vector field.

The gravitational field at P of a particle of mass m kept at the point O (origin) is given by

$$\vec{E} = \frac{\vec{F}_g}{m_0} = \left\{-\frac{Gmm_0}{r^2}\hat{r}\right\} \times \frac{1}{m_0} = -\frac{G\ m}{r^2}\ \hat{r}$$

where $\vec{r} \equiv x\hat{i} + y\hat{j} + z\hat{k}$, represents the position vector of the point P with respect to the source O and $\hat{r} = \frac{\vec{r}}{r}$ represents the unit vector along the radial direction.

The superposition principle extends to gravitational fields (intensities) as well:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \ ..... + \vec{E}_n$$

where $\vec{E}_1$, $\vec{E}_2$ . . . . . $\vec{E}_n$ are the gravitational field intensities at a point due to particles 1, 2 . . .n respectively.

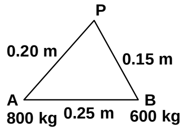
For a distributed mass the formula changes to $\vec{E} = \int d\vec{E}$, where $d\vec{E}$ = gravitational field intensity due to an elementary mass dm.

Illustration 4. *Two masses 800 kg and 600 kg are at a distance 0.25 m apart. Calculate the magnitude of the gravitational field intensity at a point distant 0.20 m from the 800 kg mass and 0.15 m from the 600 kg mass. Given G = 6.66 $\times 10^{-11}$ Nm^2 kg^{-2}.*

Solution: Let E_A be the gravitational field intensity at P due to 800 kg mass at A.

Then, $E_A = \dfrac{G\,800}{(0.2)^2} = 2\times10^4$ G along PA

Let E_B be the gravitational field intensity at P due to 600 kg mass at B.



Then, $E_B = G\dfrac{600}{(0.15)^2}$ (along PB) = $\dfrac{80000}{3}$ G (along PB)

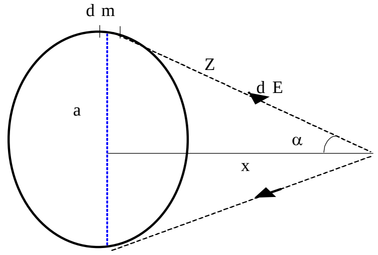
The angle between $\vec{E}_A$ and $\vec{E}_B$ is 90°

If E be the magnitude of resultant intensity, then

$$E = \sqrt{E_A^2 + E_B^2} = 2.22\times10^{-6}\ \text{N kg}^{-1}$$

The gravitational field of a ring on its axis

Let us consider a ring of mass M. We want to find the gravitational field on its axis at a distance x.



Consider a differential length of the ring of mass dm.

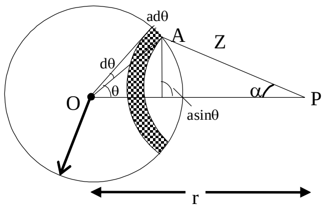
$$dE = \frac{Gdm}{Z^2}$$

The Y components of the fields due to diametrically opposite elements cancel each other. Thus, the X components add up.

$$E = \int\frac{Gdm}{z^2}\cos\alpha = \frac{GM\cos\alpha}{z^2} = \frac{GM.x}{(a^2+x^2)^{3/2}} \leftarrow$$

Field due to a uniform thin spherical shell

Consider a thin spherical shell of radius a, mass M and of negligible thickness. Out of the spherical shell, we consider a small ring of thickness a dθ. The shaded ring has mass dm = (M/2) sin θ dθ.



The field at P due to this ring is

$$dE = \frac{G.dm}{z^2}\cos\alpha = \frac{GM}{2}\frac{\sin\theta\, d\theta.\cos\alpha}{z^2}$$

From ΔOAP,

$z^2 = a^2 + r^2 - 2ar\cos\theta$

or $2zdz = 2ar\sin\theta\, d\theta$

or $\sin\theta\, d\theta = zdz/a.r$

Also, from ΔOAP,

$a^2 = z^2 + r^2 - 2zr.\cos\alpha$

$$\cos\alpha = \frac{z^2+r^2-a^2}{2.z.r}$$

Thus, $$dE = \frac{GM}{4ar^2}\left[1-\frac{a^2-r^2}{z^2}\right]dz$$

or $$\int dE = \frac{GM}{4ar^2}\left[z+\frac{a^2-r^2}{z}\right]$$

Case I (P is outside the shell, r > a)

$$E = \frac{GM}{4ar^2}\left[z+\frac{a^2-r^2}{z}\right]_{r-a}^{r+a} = \frac{GM}{r^2}$$

We see that the shell may be treated as a point particle of the same mass placed at its centre to calculate the gravitational field at an external point.

Case II (Inside the shell, r < a).

$$E = \frac{GM}{4ar^2}\left[z + \frac{a^2 - r^2}{z}\right]_{a-r}^{a+r} = 0$$

We see that the field inside a uniform spherical shell is zero.

Gravitational field outside a solid sphere

The sphere can be thought of as composed of many shells from radius = 0 to radius = a.

The centers of all these shells are at a distance 'r' from the point P.

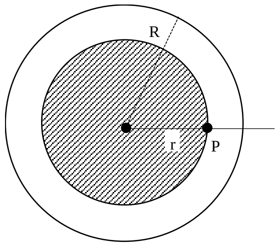
$\Rightarrow \quad E = \frac{G}{r^2}[\Delta M_1 + \Delta M_2 ...]$

or $\quad E = \frac{GM}{r^2}$

Gravitational field inside a uniform solid sphere of radius 'r'

To find the field at a point P inside the sphere, at a distance r < R from the centre. Consider a sphere of radius r. Now consider a point P on the surface of the shaded sphere. Since this point is inside the shells having radii larger than R, do not contribute to the field at P.

Shells with less than radii 'r', contribute to the gravitational field at P.



The mass of the sphere of radius r is

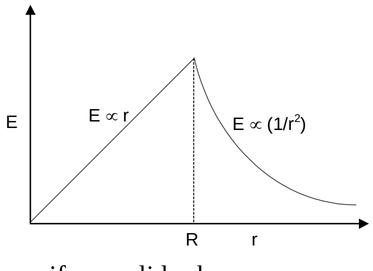
$$M' = \frac{M.\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{M.r^3}{R^3} \qquad \left(\because E_P = \frac{GM'}{r^2}\right)$$

$\Rightarrow \quad E_P = \frac{GM\ r}{R^3}$

(i) The graph shows the variation of E due to solid sphere of radius R with the distance r from its centre.

$E = \frac{GM}{r^2}, \qquad r \geq R$

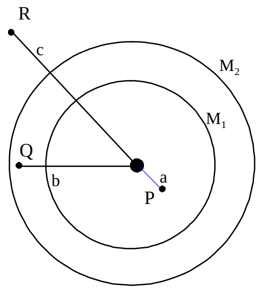
$E = \frac{GM}{R^3}.r, \qquad r < R$



- This result holds good for the earth if it is assumed to be a uniform solid sphere.

- As by definition $g = \frac{F_g}{m}$ and also $E = \frac{F_g}{m}$, so g = E, i.e. acceleration due to gravity and gravitational intensity E at a point are synonymous.

Illustration 5. *Two concentric shells of masses M_1 and M_2 are present. Calculate the gravitational force on 'm' due to M_1 and M_2 at points P, Q and R .*



Solution: Field at P, $E_P = 0 \Rightarrow F = 0$

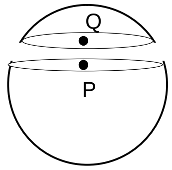
Field at Q, due to M_2 will be zero but there will be field due to M_1, $E_Q = \frac{GM_1}{b^2}$ $\Rightarrow F = \frac{GM_1 m}{b^2}$

Field at R, is the sum of fields due to M_1 and M_2, $E_R = \frac{G(M_1 + M_2)}{c^2}$

$\Rightarrow F_R = \frac{G(M_1 + M_2).m}{c^2}$

Illustration 6. *A spherical shell is cut into two pieces along a chord as shown in figure. For points P and Q [I = magnitude of gravitational field intensity]*

(A) $I_P > I_Q$ *(B) $I_P < I_Q$*
(C) $I_P = I_Q = 0$ *(D) $I_P = I_Q \neq 0$*



Solution: **(D)** At the point P we have $I_p - I_Q = 0$

Because the gravitational field inside a shall is zero, hence $I_p = I_Q \neq 0$

Acceleration due to gravity

Consider a body of mass m kept free near earth's surface. Force of attraction between the body and earth will be

$$F = \frac{GMm}{R^2}$$

where G is gravitational constant, M is mass of earth and R is the radius of earth. Height of the body (x) above earth's surface, being very small compared to the radius of earth ($x << R$), has been neglected.

The body, when left free in the gravitational field of earth, accelerates towards centre of earth with acceleration,

$$g = \frac{F}{m} = \frac{GM}{R^2}$$

This is called acceleration due to gravity. It results from the gravitational pull of the earth and does not depend on the mass of the body. Its value near the surface of the earth is 9.8 m/s^2. Value of g is affected by

(i) altitude above the earth's surface
(ii) depth of body inside the earth,
(iii) rotational motion of the earth ,
(iv) shape of the earth.

Variation of acceleration due to gravity 'g'

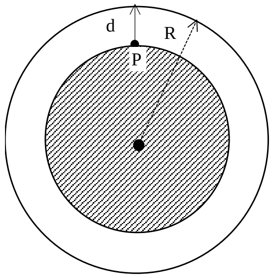
(i) Due to altitude: Consider a mass m at a height h from the surface of the earth. Now, the force acting on the mass due to gravity is $F = G\frac{Mm}{(R+h)^2}$, where M is the mass of the earth and R is the radius of the earth.

If the acceleration due to gravity at the given height is g', then

$$mg' = G\frac{Mm}{(R+h)^2},$$

$$\Rightarrow \quad g' = G\frac{M}{R^2\left(1+h/R\right)^2} = \frac{GM}{R^2}\times\left(1+\frac{h}{R}\right)^{-2} = g\,(1 - 2h/R)$$

(expanding binomially and neglecting the higher order terms)

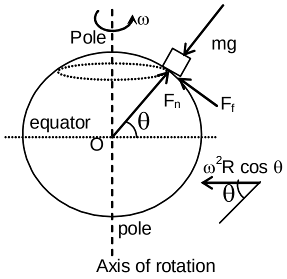


(ii) Due to depth: If we are going to find g at a depth d, we may consider a mass kept at that depth. A sphere of radius (R – d) will exert gravitational force on the mass. Now the mass of the sphere is $\frac{M}{R^3}\left(R-d\right)^3$

Then, the gravitational force exerted on the mass m is

$$\frac{GM'm}{\left(R-d\right)^2} = \frac{GM\left(R-d\right)m}{R^3}$$

$$g'' = \frac{GM}{R^3}R\left(1-\frac{d}{R}\right) = g\,(1 - \frac{d}{R})$$



(iii) Due to rotation of the earth: Consider a body at a point with latitude θ, on the surface of the earth.

R = radius of the earth

ω = angular velocity of the earth about its own axis.

F.B.D. of the body with respect to earth is shown in the figure.

Acceleration of the body with respect to the earth's centre O is (Rcos θ) ω^2 directed towards the axis of rotation (i.e. the centripetal acceleration)

From Newton's second law in the radial direction

$$mg - F_n = m(R\cos\theta)\,\omega^2\cos\theta$$

$$F_n = m[g - R\omega^2\cos^2\theta]$$

$$F_n/m = g''' = g - R\omega^2\cos^2\theta$$

where g''' is the apparent value of the acceleration due to gravity at the latitude θ.

At poles, $\theta = 90° \Rightarrow g''' = g$

At the equator, $\theta = 0^0 \quad \Rightarrow g''' = g - R\omega^2$.

(iv) Due to shape of the earth: Earth is not exactly spherical. Its radius near equator is more then its radius near poles. Since acceleration due to gravity is inversely proportional to the square of the radius of the earth, it varies with latitude due to shape of the earth.

$$\frac{g_e}{g_p} = \frac{R_p^2}{R_e^2}$$

where g_e & g_p are accelerations due to gravity at equator and pole, respectively, and R_P, R_e are radii of earth near pole and at equator, respectively.

MASS OF THE EARTH

Mass of the earth can be calculated with the help of the equation,

$$g = \frac{GM}{R^2}; \qquad M = \frac{gR^2}{G}$$

Putting the values of g, R and G in this equation, mass of earth comes out to be

$$M_e = 6\times10^{24}\text{ kg}$$

Illustration 7: *The mass of a planet is $\frac{1}{40}$ th the mass of the earth and its radius is $\frac{1}{4}$ th the radius of the earth.*
(a) Find acceleration due to gravity at the planet's surface.
(b) at what height above the surface of planet does the acceleration due to gravity reduce by 36 % of its value on the surface of planet ?
(c) at what depth inside the planet does the acceleration due to gravity reduce by 36 % of its value on the surface ?

Solution: (a) As $g \propto \frac{M}{R^2}$

$\therefore \frac{g_P}{g_E} = \frac{1}{40} \times (4)^2 = 0.4$

$\Rightarrow g_P = 0.4$ g

(b) $g_P \times 0.64 = g_p'$

$\therefore \frac{g'_P}{g_P} = \left(\frac{R_P}{R_P + h}\right)^2 = \left(\frac{8}{10}\right)^2$

$\Rightarrow h = \frac{R_e}{16} = 400\,\text{km}$

(c) Acceleration due to gravity inside the planet is given by

$g' = g_P\,(1 - d/R_P)$

$\frac{64}{100} g_P = g_P\,(1 - d/R_P)$

$\Rightarrow \quad d = \frac{36}{100} R_P$

Illustration 8: *If the earth were a perfect sphere of radius 6400 km rotating about its axis with a period of one day, then*
(a) find the weight of a body of mass 10 kg at the poles, equator and at a latitude of 60°
(b) how much faster than its present value should the earth rotate about its axis so that the weight of a body at the equator becomes zero ?
(c) calculate the length of the new day.
(d) what would happen if the rotation becomes still faster ?

Solution: (a) At the pole $\theta = 90°$

$g' = g - R\omega^2 \cos 90°$
$= g$

(i) Weight of the body at the pole is = 10 g
= 98.1 N

(ii) At the equator $\theta = 0°$

$g' = g - R\omega^2 \cos 0°$

$\Rightarrow g' = g - \frac{6400 \times 10^3 \times 4 \times (3.14)^2}{(86400)^2} = 9.77\ \text{m/s}^2 \quad \left[\because \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 60 \times 60}\,\text{rad/s.}\right]$

mg′ = weight of the body at the equator = 97.76 N

(iii) At latitude $\theta = 60°$

$g' = g - R\omega^2 \cos^2 60°$

$= 9.81 - R\,\omega^2\,(1/4)$

$= 9.81 - 6400 \times 10^3 \times \frac{4\pi^2}{T^2}\frac{1}{4}$

$\Rightarrow \quad g' = 9.80\ \text{m/s}^2$

Weight of the body = 9.80 × 10 = 98.0 N

(b) Weight of the body at the equator (θ = 0°) will be zero only when g = 0

$\Rightarrow 0 = g - R\omega^2 \cos^2 0°$

$$\omega^2 = \sqrt{\frac{9.81}{6400 \times 10^3}}$$

$\therefore$ ω= 1.23×10^{-3} rad/sec.

(c) New time period will be

$$T' = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{1.23 \times 10^{-3}} \text{ sec} = 1.41 \text{ hours.}$$

(d) If the angular velocity of the earth further increases, then a net outward force will act and all objects simply placed at the equator will start leaving the earths surface.

Illustration 9. *The value of 'g' at a certain height h above the free surface of earth is* $\frac{x}{4}$*, where x is the value of 'g' at the surface of earth. The height h is*

(A) R *(B) 2 R*

(C) 3 R *(D) 4 R*

Solution:

$$x = \frac{GM}{R^2}$$

$$\text{Again } \frac{x}{4} = \frac{GM}{(R+h)^2}$$

$$\text{or } x = GM\left(\frac{2}{R+h}\right)^2$$

$$\therefore \frac{1}{R^2} = \left(\frac{2}{R+h}\right)^2$$

$$\text{or } \frac{1}{R} = \frac{2}{R+h}$$

or, R + h = 2R

or, h = R

Exercise 3.

(ii) ***If the earth shrinks, what will be the effect on the value of acceleration due to gravity?***

(iii) ***The radius of the earth is 6400 km and g is 10 m/s^2. What should be the angular speed of earth so that a 5 kg body placed at the equator of earth weighs zero?***

(iii) ***Somebody says "I weigh less than what I exactly weigh" . Is he right ? Is there any place on earth where he can weigh exactly?***

ORBITAL VELOCITY

When a satellite revolves in an orbit around a planet, it requires centripetal force to do so. This centripetal force is provided by the gravitational force between the planet and the satellite. If the mass of the satellite is m and that of the planet is M and satellite is at a height h from the surface of the planet, then the gravitational force between them is

$$F = \frac{GMm}{(R+h)^2},$$ where R is the radius of the earth.

If the speed of the satellite in its orbit is v, then the required centripetal force is $\frac{mv^2}{(R+h)}$.

Then $\frac{mv^2}{(R+h)} = \frac{GMm}{(R+h)^2}$

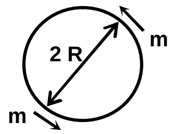
$$\Rightarrow \quad v = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{GM}{R\left(1+\frac{h}{R}\right)}} = \sqrt{\frac{gR}{\left(1+\frac{h}{R}\right)}}$$

If the height is very small compared to the radius of the earth, then

$$v = \sqrt{gR}\,.$$

Illustration 10. *Two particles of equal mass m go round a circle of radius R under the action of their mutual gravitational attraction. What is the speed of each particle?*

Solution: The gravitational force of attraction provides the necessary centripetal force



$$\frac{mv^2}{R} = \frac{G(m)(m)}{(2R)^2}$$

$$\Rightarrow v^2 = \frac{GM}{4R}$$

$$v = \frac{1}{2}\sqrt{\frac{GM}{R}}$$

Exercise 4.

(i) ***Two satellites are at different heights. Which would have greater velocity?***

(ii) ***The acceleration due to gravity on a planet is 1.96 m/s^2. If it is safe to jump from a height of 2 m on the earth, what will be the corresponding safe height on the planet?***

(iii) ***Two artificial satellites of different masses are moving in the same orbit around the earth. Can they have the same speed?***

GEOSTATIONARY AND POLAR SATELLITES

Geostationary Satellite: If the time period of a satellite is 24 hrs, rotating in the same sense as the rotation of the earth and if the plane of the orbit is at right angle to the polar axis of the planet (earth) then the satellite will always be above a certain place of the earth.

This kind of a satellite is called geostationary satellite (GSAT). Its orbit is above the equator of the earth. These satellites can be used for communication purposes.

Polar Satellite: A polar satellite revolves around a polar orbit, i.e. an orbit passing over the north and south poles of earth. These satellites are close to earth and, due to rotation of earth, pass through different parts of the earth in each revolution.

These satellites can scan all parts of earth in a few revolutions. Their time periods are also small and a few revolutions are made in one day. These can be used for weather monitoring, military spying purposes and monitoring health of crops.

Exercise 5. ***Can the GSAT launched by India be placed above Delhi in space?***

Illustration 11. *An artificial satellite of mass 100 kg is in circular orbit at 500 km above the earth's surface. Take the radius of the earth as 6.5×10^6 m.*
(a) Find the acceleration due to gravity at any point along the satellite path.
(b) What is the centripetal acceleration of the satellite?

Solution: Here, $h = 500 \text{ km} = 0.5 \times 10^6$ m

$R = 6.5 \times 10^6$ m

$r = R + h = 6.5 \times 10^6 + 0.5 \times 10^6 = 7.0 \times 10^6$ m

(a) Now, $g' = g\left(\frac{R}{R+h}\right)^2 = 9.8\left(\frac{6.5\text{x}10^6}{7.0\text{x}10^6}\right)^2 = 8.45 \text{ m/s}^2$

(b) Centripetal acceleration of the satellite,

$a = \frac{mg'}{m} = g' = 8.45 \text{ m/s}^2$

GRAVITATIONAL POTENTIAL

Gravitational field around a material body can be described not only by gravitational intensity vector $\vec{E}$, but also by a scalar function, the gravitational potential V. The gravitational potential at any point may be defined as the potential energy per unit mass of a test mass placed at that point.

$V = \frac{U}{m}$ (where U is the gravitational potential energy of the test mass m.)

Thus, if the reference point is taken at infinite distance, the potential at a point in the gravitational field is equal to the amount of work done against the gravitational force per unit mass in bringing a test mass from infinite distance to that point. The expression for the potential is given by

$$V = -\int_{\infty}^{P} \vec{E}.d\vec{s}$$

With the above definition the gravitational potential due to a point mass M at a distance r from it is

$$V = -\int_{\infty}^{r} \frac{GM}{r^2}\hat{r}.d\vec{s} = \int_{\infty}^{r} \frac{GM}{r^2}dr = -\frac{GM}{r}$$

Potential is a scalar quantity. Therefore, at a point in the gravitational field of a number of material particles, the resultant potential is the arithmetic sum of the potentials due to all the particles at that point. If masses m_1, m_2 ……. m_n are at distances r_1, r_2, r_3 ….. r_n, then potential at the given point is

$$V = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3}........\right)$$

The field and the potential are related as

$$E_r = -\frac{\partial V}{\partial r}$$

where E_r is the Gravitational field in the direction of $\vec{r}$.

.

Gravitational potential due to a shell

(i) Outside the shell is : $-\frac{GM}{r}$ (r > R)

(ii) On the surface of the shell is: $-\frac{GM}{R}$

(iii) Inside the shell is: $-\frac{GM}{R}$

Gravitational Potential due to a Solid Sphere

(i) Outside of the sphere at a distance r from the center is: $-\dfrac{GM}{r}$

(ii) Inside the sphere at a distance r from the center is : $-\dfrac{3GM}{R^3}\left(\dfrac{R^2}{2}-\dfrac{r^2}{6}\right)$

Illustration 12. *Two heavy spheres each of mass 100 kg and radius 0.1 m are placed 1.0 m apart on a horizontal table. What is the gravitational field and potential at the mid-point of the line joining the centres of the spheres?*
Take G = 6.67 × 10^{-11} m^2 kg^{-2}

Solution: Here, G = 6.67 × 10^{-11} N m^2 kg^{-2}
M = 100 kg; R = 0.1 m
Distance between the two spheres, d = 1.0 m. Suppose that the distance of the either sphere from the mid-point of the line joining their centres is r. Then

$$r = \frac{d}{2} = 0.5\,m$$

The gravitational field at the mid-point due to the two spheres will be equal and opposite.
Hence, the resultant gravitational field at the mid point = 0
The gravitational potential at the mid point

$$= \left(\frac{-GM}{r}\right)\times 2 = \frac{-6.67\times10^{-11}\times100\times2}{0.5}$$

= –2.668 × 10^{-8} J kg^{-1}

Illustration 13. *At a point above the surface of earth, the gravitational potential is –5.12 × 10^7 J/kg and the acceleration due to gravity is 6.4 m/s^2. Assuming the mean radius of the earth to be 6400 km, calculate the height of this point above the earth's surface.*

Solution: Let r be the distance of the given point from the centre of the earth. Then

Gravitational potential = $-\dfrac{GM}{r} = -5.12\times10^7$ ….(i)

and acceleration due to gravity, $g = \dfrac{GM}{r^2} = 6.4\ m/s^2$ . . . (ii)

Dividing (i) by (ii), we get

$$R = \frac{5.12\times10^7}{6.4} = 8\times10^6\,m = 8000\,km$$

∴ Height of the point from earth's surface
= r – R = 8000 – 6400 = 1600 km

BINDING ENERGY

Binding energy of system of two bodies is the amount of minimum energy needed to separate the body to a large distance.

If two particles of masses m_1 and m_2 are separated by a distance r, the gravitational potential energy of the system is given by

$$U = -\frac{Gm_1m_2}{r} \quad \ldots (i)$$

Let T amount of energy is given to the system to separate the body by a large distance. When particles are separated by a large distance gravitational potential energy of the system is zero. For optimum T conserving energy for initial and final position,

$$T + U = 0 \;\Rightarrow\; T - \frac{Gm_1m_2}{r} = 0 \;\Rightarrow T = \frac{Gm_1m_2}{r}$$

Hence, Binding energy of the system of two particles separated by a distance r is equal to $T = \frac{Gm_1m_2}{r}$, where m_1 and m_2 are the masses of the particles.

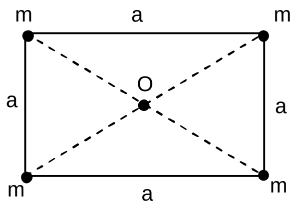
Illustration 14. *The energy required to remove a body of mass m from earth's surface is equal to*
(A)2mgR *(B) mgR*
(C)–mgR *(D) zero*

Solution: **(B).**
The potential energy of the body on the surface of earth, $U_1 = -\frac{GmM}{R}$.
The potential energy of the body at infinity, $U_2 = 0$.

$$\Rightarrow \Delta U = U_2 - U_1 = \frac{GmM}{R} = mgR \qquad \left(\because g = \frac{GM}{R^2}\right)$$

Illustration 15. *Find the potential energy of a system of four particles each of mass m placed at the vertices of a square of side a. Also find gravitational potential at the center of square.*

Solution: (i) Potential energy



$$= -\frac{Gmm}{a} - \frac{Gmm}{a} - \frac{Gmm}{a} - \frac{Gmm}{a} - \frac{Gmm}{\sqrt{2}a} - \frac{Gmm}{\sqrt{2}a}$$

$$= -\frac{4Gm^2}{a} - \sqrt{2}\frac{Gm^2}{a}$$

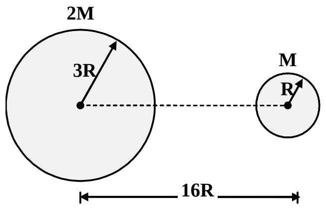
$$= -\frac{Gm^2}{a}[4 + \sqrt{2}]$$

(ii) Gravitational potential

$$V = -\frac{Gm}{a/\sqrt{2}} - \frac{Gm}{a/\sqrt{2}} - \frac{Gm}{a/\sqrt{2}} - \frac{Gm}{a/\sqrt{2}}$$

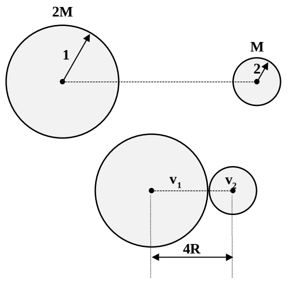
$$= \frac{4\sqrt{2}Gm}{a}.$$

Illustration 16. *Two spherical bodies of masses 2M and M and of radii 3R and R, respectively, are held at a distance 16R from each other in free space. When they are released, they start approaching each other due to the gravitational force of attraction. Tthen, find*



(a) the ratio of their accelerations during their motion
(b) their velocities at the time of impact.

Solution : Taking both the bodies as a system, from conserving momentum of the system



$m_1v_1 - m_2v_2 = 0 \Rightarrow \frac{m_1}{m_2} = \frac{v_2}{v_1} = 2$

Again, if the accelerations are a_1 and a_2, the net external force on the system = 0

$\Rightarrow \bar{a}_{CM} = 0 \qquad \Rightarrow m_1a_1 - m_2a_2 = 0$

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = 2$$

Now conserving the total mechanical energy, we have

$$-\frac{G(2M)M}{16R} = -\frac{G(2M)M}{4R} + \frac{1}{2}(2M)v_1^2 + \frac{1}{2}(M)v_2^2$$

$$v_1 = \sqrt{\frac{GM}{8R}}, \quad v_2 = 2\sqrt{\frac{GM}{8R}}$$

Note: The velocities and accelerations are w.r.t. the inertial reference frame (i.e. the centre of mass of the system).

Illustration 17. *The masses and radii of two fixed bodies are 2M, R and M, 2R . Their centres are at a distance 10 R apart. What is the minimum speed with which a particle of mass M/10 should be projected from a point midway between the two so as to escape to infinitey..*

Solution : Potential energy midway between 2M and M

$$= -\frac{2GMM}{5R(10)} + \frac{(-GM)}{5R}\frac{M}{10}$$

Potential energy at infinity = 0

For minimum speed K.E. at infinity should be zero.

By COE, $-\frac{3GM^2}{50R} + \frac{1}{2}\left(\frac{M}{10}\right)v^2 = 0$

$$\Rightarrow v = \sqrt{\frac{6GM}{5R}}.$$

ESCAPE VELOCITY

Escape velocity on the surface of earth is the minimum velocity given to a body to make it free from the gravitational field, i.e. it can reach an infinite distance from the earth.

Let v_e be the escape velocity of the body on the surface of earth and the mass of the body to be projected be m. Now conserving energy at the surface of the earth and infinity,

$$\frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0 \qquad \Rightarrow \qquad v_e = \sqrt{\frac{2GM}{R}}.$$

Illustration 18. *The mass of Jupiter is 318 times that of earth, and its radius is 11.2 times the earth's radius. Estimate the escape velocity of a body from Jupiter's surface. [Given that the escape velocity from the earth's surface is 11.2 km/s.]*

Solution: Hence, $M_J = 318\ M_e$; $R_J = 11.2\ R_e$; $v_e = 11.2$ km/s

We know, $v_J = \sqrt{\dfrac{2\ GM_J}{R_J}}$

and $v_e = \sqrt{\dfrac{2\ GM_e}{R_e}}$

$\therefore \quad \dfrac{v_J}{v_e} = \sqrt{\dfrac{M_J}{M_e} \times \dfrac{R_e}{R_J}}$

$\Rightarrow \quad v_J = v_e\sqrt{\dfrac{M_J}{M_e} \times \dfrac{R_e}{R_J}}$

$$v_J = 11.2 \left\{\frac{(318\ M_e)}{M_e} \times \frac{R_e}{(11.2\ R_e)}\right\}^{\frac{1}{2}}$$

$$= 11.2\left(\frac{318}{11.2}\right)^{\frac{1}{2}} = 59.7 \ \text{km/s}$$

Illustration 19. *Find the escape velocity at the surface of a planet whose mass is one- fourth the mass of the earth and radius is 1/16th of the earthes radius.*

Solution: $v_{ep} \rightarrow$ escape velocity at planet

$v_{ep} = \sqrt{\dfrac{2GM_P}{R_P}}$, ($M_p$ is the mass of the planet)

$= \sqrt{\dfrac{2G.\frac{M}{4}}{R/16}} = \sqrt{\dfrac{2GM}{R}.4}$

$= 2\sqrt{\dfrac{2GM}{R}} = 2 \times 11.2$ km/sec.

$= 22.4$ km/sec.

Illustration 20. *A spaceship is launched into a circular orbit close to earth's surface. What additional velocity has now to be imparted to the spaceship in the orbit to overcome the gravitational pull? Given radius of earth = 6400 km and g = 9.8 m/s^2.*

Solution: $\dfrac{mv^2}{R} = mg$ (h has been neglected)

$v = \sqrt{gR}$

$\therefore v_e = \sqrt{2g\ R}$

Additional velocity $= \sqrt{gR}\left(\sqrt{2} - 1\right)$

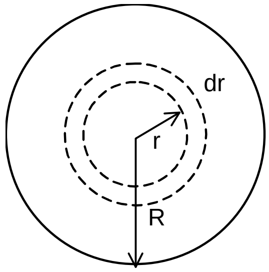
Exercise 6.

(i) ***What will be the escape velocity of a body if it is projected at 45° to the horizontal?***

(ii) ***Why the lighter gases are rare on the surface of the earth?***

(iii) ***If a projectile is fired straight up from the earth's surface, what will happen if the total mechanical energy is (a) less then zero? (b) greater than zero? Ignore the air resistance and effects of other heavenly bodies.***

GRAVITATIONAL SELF ENERGY OF A UNIFORM SPHERE



Consider a sphere of radius R and mass M uniformly distributed. Consider a stage of formation at which the radius of the spherical core is r. Its mass will be $\frac{4}{3}\pi r^3\rho$, where ρ is the density of the sphere.

Let, through an additional mass, the radius of the core be increased by dr in the form of a spherical layer over the core. The mass of this layer will be $4\pi r^2 dr.\rho$

The mutual gravitational potential energy of the above mass and the spherical core of radius r

$$dU = \frac{-G\left(\frac{4}{3}\pi r^3\rho\right)\left(4\pi r^2 dr.\rho\right)}{r} = -\frac{16}{3}\pi^2\rho^2 G r^4 dr$$

Hence, total energy evolved in the formation of the spherical body of radius R i.e. self energy

$$U = -\int_0^R \frac{16}{3}\pi^2\rho^2 G r^4 dr = -\frac{16}{15}\pi^2\rho^2 G R^5 = -\frac{3}{5}\frac{GM^2}{R}$$

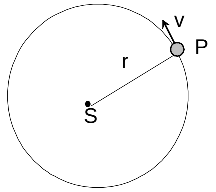
MOTION OF PLANETS AND SATELLITES

One of the greatest ideas proposed in human history is the fact that the earth is a planet, among the other planets, that orbits the sun. The precise determination of these planetary orbits was carried out by Johannes Kepler, using the data compiled by his teacher, the astronomer Tycho Brache. Johannes Kepler discovered three empirical laws by using the data on planetary motion:

1. Each planet moves in an elliptical orbit, with the sun at one focus of the ellipse.
2. A line from the sun to a given planet sweeps out equal areas in equal times.
3. The square of the periods of the planets are proportional to cubes of their mean distances from the sun.

These laws go by the name Kepler's laws of planetary motion. It was in order to explain the origin of these laws, among other phenomena, that Newton proposed the theory of gravitation.

In our discussion, we are not going to derive the complete laws of planetary motion from Newton's law of gravitation. Since most of the planets actually revolve in near circular orbits, we are going to assume that the planets revolve in circular orbits.



Consider a planet of mass m, rotating around the sun (mass M, M>>m) in a circular orbit of radius r with velocity v.

Then, by applying Newton's law of gravitation and the second law of motion, we can write

Gravitational force = mass × centripetal acceleration

or, $$\frac{GMm}{r^2} = m\left(\frac{v^2}{r}\right) \qquad \ldots(1)$$

or, $$v^2 = \frac{GM}{r} \qquad \ldots(2)$$

As the moment of the gravitational force about S is zero, the angular momentum of the planet about the sun remains constant. This is the meaning of Kepler's 2nd law of motion, as will be shown later.

The time period of rotation, T, of the planet around the sun is given by,

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{GM/r}} = \frac{2\pi}{\sqrt{GM}}\ r^{3/2} \qquad \ldots(3)$$

Squaring $\quad T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \qquad \ldots(4)$

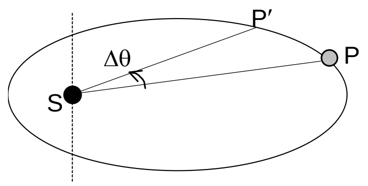
Which is Kepler's 3rd law of motion

Note: That the constant of proportionality in the above equation depends only on the mass of the sun (M) but not on the mass of the planet.

Kepler's Laws are also valid for the motion of satellites around the earth.

Exercise 7. ***Is it necessary for the plane of a satellite motion to pass through the centre of the earth?***

KEPLER'S 2ND LAW AND ITS MEANING



Consider a planet P that moves in an elliptical orbit around the sun, and let P and P′ be the positions of the planet at times t and t + Δt (where Δt is a very small time interval). If the angular displacement of the planet is Δθ, then the area swept out by the line joining the planet SP in time Δt is:

ΔA = area of the section SPP′

$= \frac{1}{2} r^2.\Delta\theta$ (where r = the length SP.)

The areal velocity

$$v_A = \frac{\Delta A}{\Delta t} = \frac{\frac{1}{2}\ r^2\ \Delta\theta}{\Delta t} = \frac{1}{2}\ r^2\omega = \text{constant} \qquad \ldots(5)$$

In other words,

$m \times (2v_A) = \text{constant}$ (m = mass of the planet)

Arial velocity $\frac{dA}{dt} = \frac{L}{2m} \qquad \ldots(6)$

This is the expression for the angular momentum of the planet, $L = I\omega = mr^2\,\omega$

$= mr^2\left(\frac{d\theta}{dt}\right)$ perpendicular to the plane of its orbit.

The gravitational force $\vec{F} = -\frac{G\ M\ m}{r^2}\ \hat{r}$ is centripetal, and the torque on the planet is zero:

$$\vec{r} \times \vec{F} = \vec{r} \times \left(-\frac{G\ M\ m}{r^2}\hat{r}\right) = 0 \qquad \ldots(7)$$

Hence, the angular momentum of the planet does not vary i.e. the areal velocity of the planet remains constant. At its aphelion (farthest point from the sun, r is large) the planet moves slowly, and at its perihelion (nearest point from the sun, r is small) the planet moves fastest.

Illustration 21. *Two satellites A and B revolve around a planet in coplanar circular orbits in the same direction with periods of revolution 1 hour and 8 hours respectively. The radius of satellite A is 10^4 km. Find the angular speed of 'B' with respect to A.*

Solution : According to Kepler's III law, $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$

$\Rightarrow$ $R_B = \left(\frac{8}{1}\right)^{2/3}(10^4) = 4\times10^4$ km

$v_A = \frac{2\pi R_A}{T_A} = \frac{2\pi\times10^4}{1} = 2\pi\times10^4$ km/h

$v_B = \frac{2\pi R_B}{T_B} = \frac{2\pi\times4\times10^4}{8} = \pi\times10^4$ km/h

$\therefore$ Relative velocity of 'B' w.r.t. A is $[v_B - v_A] = -\pi\times10^4$ km/h.

$\therefore$ Relative angular speed of 'B' w.r.t. A is $= \left|\frac{v_B - v_A}{R_B - R_A}\right|$

$= \frac{\pi\times10^4}{4\times10^4 - 1\times10^4} = \frac{\pi}{3}$ rad/hour

Illustration 22. *A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.5 $\times 10^8$ km away from the sun?*

Solution : It is given that

$T_S = 29.5\ T_e$; $R_e = 1.5 \times 10^{11}$ m

Now, according to Kepler's third law

$\frac{T_S^2}{T_e^2} = \frac{R_S^3}{R_e^3}$

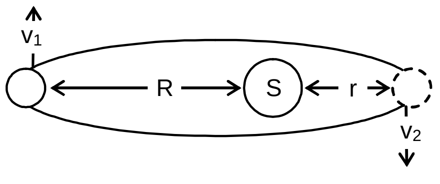
$\Rightarrow$ $R_S = R_e\left(\frac{T_S}{T_e}\right)^{\frac{2}{3}}$

$= 1.5\times10^{11}\left(\frac{29.5\ T_e}{T_e}\right)^{\frac{2}{3}} = 1.43\times10^{12}$ m

$= 1.43\times10^9$ km

Illustration 23. *A planet of mass m moves along an ellipse around the sun so that its maximum and minimum distances from the sun are equal to R and r respectively. Find the angular momentum of this planet relative to the centre of the sun.*

Solution : According to Kepler's Second Law, the angular momentum of the planet is constant. We have



$mv_1R = mv_2r$, $v_1R = v_2r$

If the mass of the Sun is M, conserving total mechanical energy of the system at two given positions we have,

$-\frac{GMm}{R} + \frac{1}{2}mv_1^2 = -\frac{GMm}{r} + \frac{1}{2}mv_2^2$

$\therefore$ $GM\left[\frac{1}{R} - \frac{1}{r}\right] = \frac{v_1^2}{2} - \frac{v_2^2}{2}$

or $GM\left[\frac{r-R}{Rr}\right] = \frac{v_1^2}{2} - \frac{v_1^2R^2}{2r^2}$

$$\therefore \qquad v_1^2 = \frac{2GM\left(R-r\right)\ r^2}{Rr\left(R^2-r^2\right)} = \frac{2GMr}{R\left(R+r\right)}$$

Now angular momentum = $mv_1R = m\sqrt{\frac{2GMRr}{\left(R+r\right)}}$

WEIGHTLESSNESS

Weight of a body is the gravitational force due to earth acting on it towards centre of the earth. When measured by a machine, the weight will be measured by the reaction by the machine opposing the force applied by the body.

In certain situations, the force of gravitation is nullified and weightlessness is felt. It may happen in following situations.

1. Free fall – During free fall of a body, it is in a state of weightlessness.

2. In a satellite revolving around the earth – Force of gravitation provides the centripetal force. Hence, if a body is kept in a satellite, there is no reaction from the point touching it (N = 0). The body is in a state of weightlessness.

3. At null point:
   If a body moves from the earth to moon, its gravitational pull by earth goes on decreasing whereas the gravitational pull by the moon keeps on increasing. At a point called null point, the gravitational pull by the earth and by the moon will be equal and opposite. The body will be in the state of weightlessness at this point.

Exercise 8. ***A thief with a box in his hand jumps from the top of a building. What will be the load experienced by him during the state of free fall?***

INERTIAL AND GRAVITATIONAL MASS

Mass of a body B can be established by comparing it with the mass of another body A in two ways.

1. Apply equal force F to the two bodies and measure the acceleration.

   $F = m_A a_A = m_B a_B \qquad \therefore\ m_B = \frac{a_A}{a_B} m_A$

   This is called inertial mass of the body.

2. Measure the gravitational pull of earth on both the bodies.

   $F_A = \frac{GMm_A}{R^2}$

   $F_B = \frac{GMm_B}{R^2}$

   $\therefore\ m_B = \frac{F_B}{F_A}.m_A$

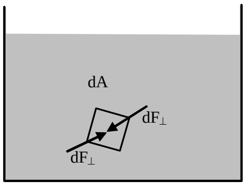
Mass measured by this method is called gravitational mass of the body.

The mass of a body measured by the above two methods, in various experiments, has been found to be identical.

FLUID

A fluid is a substance that can flow, so the term fluid includes both liquids and gases. Fluids differ from solids in being unable to support a shear stress.

Fluid Statics



Fluid pressure: Consider an elemental area dA inside a fluid, the fluid on one side of area presses the fluid on the other side and vice-versa. We define the pressure p at that point as the normal force per unit area.

$$p = \frac{dF_\perp}{dA}$$

If the pressure is same at all the points of a finite plane surface with area A, then $p = \frac{F_\perp}{A}$; where $F_\perp$ is the normal force on one side of the surface.

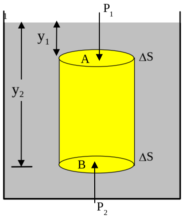
The SI unit of pressure is the 'Pascal 'where 1 Pascal = 1 Pa = 1 N/m^2

Atmospheric Pressure: It is the pressure of the earth's atmosphere. Normal atmospheric pressure at sea level (an average value) is 1 atmosphere (atm) that is equal to 1.013×10^5 Pa.

- Fluid force acts perpendicular to any surface in the fluid, no matter how that surface is oriented. Hence, pressure, has no intrinsic direction of its own, it is a scalar.

- The excess pressure above atmospheric pressure is called gauge pressure, and total pressure is called absolute pressure.

Variation of Pressure with Depth:



(i) Let pressure at A is P_1 and pressure at B is P_2

Then, $p_2 \Delta S = p_1 \Delta S + \rho g \Delta S (y_2 - y_1)$

or $p_2 = p_1 + \rho g(y_2 - y_1)$

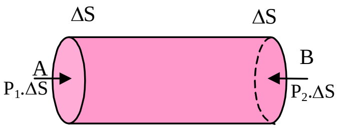
Pressure increases with depth.

i.e. $\frac{dp}{dy} = \rho g$, where ρ = density of the fluid

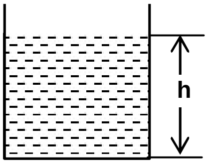
(ii) Pressure is same at two points in the same horizontal level.

As body is in equilibrium, $P_1 \Delta S = P_2 \Delta S$

or $P_1 = P_2$



Pressure exerted by a liquid (effect of gravity)



Consider a liquid of density ρ contained in a cylinder of cross-sectional area A. Let h be the height of the liquid column. The weight of the liquid column exerts a downward thrust and hence a downward pressure.

Weight of liquid column

= Volume of liquid column × density of liquid × g

= Ah ρ g

$$\text{Pressure} = \frac{\text{weight of liquid column}}{\text{cross sectional area}} = \frac{Ah\rho g}{A} = h\rho g$$

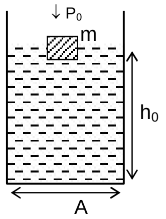
So, the pressure exerted by a liquid column at rest is proportional to

(i) height of liquid column, and (ii) density of liquid.

Illustration 24. *Water is filled upto a height of 20 cm. The bottom of the flask is circular and has an area of 1 m^2. If the atmospheric pressure is 1.01×10^5 Pa, then what force is exerted by water on the bottom? (Take g = 10 m/s^2 and density of water = 1000 kg m^{-3})*

Solution:
$$P = P_0 + \rho gh$$
$$= 1.01 \times 10^5 + 0.20 \times 1000 \times 10$$
$$= 1.03 \times 10^5 \text{ Pa}$$
$$\therefore \text{ Force} = P \times A = 1.03 \times 10^5 \text{ N}$$

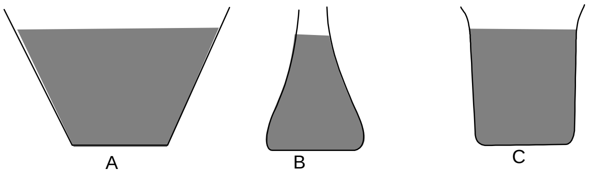
Illustration 25. *The height of a liquid of density 'ρ' in a beaker of area of cross–section 'A' is 'h_0'. Now a block of mass m is floating on the surface of the liquid. Determine the pressure on the bottom of the beaker, if atmospheric pressure is P_0.*



Solution: Net pressure $= P_0 + h_0\rho g + \dfrac{mg}{A}$

Exercise 9.

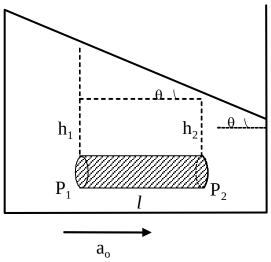
(i). ***Water is poured to same level in each of the vessels shown, all having the same base area as shown in figure. In which vessel the force experienced by the base is maximum?***



(ii) ***Why the blood pressure in humans is greater at the feet than at the brain.***

Illustration 26. *A vessel containing liquid accelerates to the right, what is the angle made by the free surface of the liquid with the horizontal?*

Solution: The shaded element accelerates to the right.



$\Rightarrow P_1 \Delta S - P_2 \Delta S = ma_o = (\rho \ell \Delta S)\, a_o$ … (1)

$P_1 = h_1 \rho g + p_o$; $P_2 = h_2 \rho g + P_o$

or $P_1 - P_2 = (h_1 - h_2)\rho g$

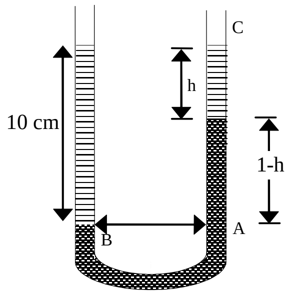
$\tan\theta = \dfrac{h_1 - h_2}{\ell}$ …(2)

From (1), $(P_1 - P_2) = \rho\, \ell\, a_o$

or $(h_1 - h_2)\rho g = \rho \ell a_o$ or $\dfrac{h_1 - h_2}{\ell} = \dfrac{a_o}{g}$

From (2), $\tan\theta = (a_o/g)$.

Illustration 27. *A vertical U–tube of uniform cross-section contains mercury in both arms. A glycerine (relative density 1.3) column of length 10 cm is introduced into one of the arms. Oil of density 800 kg m^{-3} is poured into the other arm until the upper surface of the oil and* *glycerine are at the same horizontal level. Find the length of the oil column. Density of mercury is $13.6 \times 10^3\ kgm^{-3}$.*



Solution : Pressures at A and B must be same.

Pressure at, $B = p_0 + 0.1 \times (1.3 \times 1000) \times g$

p_0 = atmospheric pressure

Pressure at A,

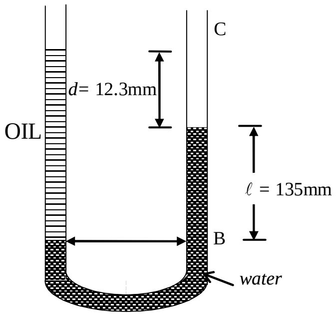
$= p_0 + h \times 800 \times g + (0.1 - h) \times 13.6 \times 1000\ g$

$\therefore \quad p_0 + 0.1 \times 1300 \times g$

$= p_0 + 800\ gh + 1360\ g - 13600 \times g \times h$

$\Rightarrow h = 9.6\ cm$

Illustration 28. *For the arrangement shown in the figure, what is the density of oil?*



Solution :

$p_{surface} = P_o + \rho_w . g\ell$

$p_{surface} = P_o + \rho_{oil}\ (\ell + d)g$

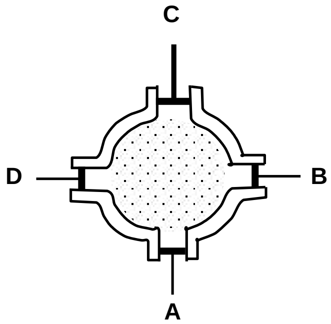
$$\Rightarrow \rho_{oil} = \frac{\rho_\omega . \ell}{(\ell + d)} = \frac{1000.(135)}{(135 + 12.3)} = 916.5\ kg/m^3$$

PASCAL'S LAW

A change in the pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.

Experimental Proof of Pascal's law

Consider a vessel filled with water and fitted with pistons in different positions as shown in figure below. The four pistons A, B, C and D have different cross-sectional areas a_1, a_2, a_3, and a_4 respectively. Suppose the Piston A is pushed with a force F_1.

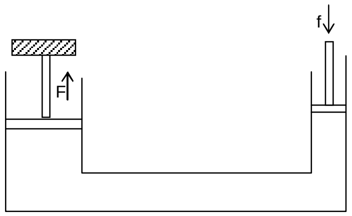


The pressure on the piston is $\frac{F_1}{a_1}$. It will be observed that the piston B, C and D can be held in their positions only if we apply forces F_2, F_3 and F_4 respectively such that

$$\frac{F_1}{a_2} = \frac{F_3}{a_3} = \frac{F_4}{a_4}$$

This experiment demonstrates that the pressure is transmitted undiminished in all directions as required by Pascal's law.

HYDRAULIC LIFT



A hydraulic press is used to magnify force applied on a piston. It is based on Pascal's law. As shown in figure, a hydraulic press consists of two cylinders of different cross–sectional areas connected together.

A small force f is applied to the piston of smaller cross–sectional area 'a' and a magnified force F is produced by fluid pressure on the piston of larger cross–sectional area (A).

By Pascal's law, $\frac{f}{a} = \frac{F}{A}$

$\therefore\ F = \frac{A}{a}f$

Hence, force F is magnified by $\frac{A}{a}$.

Illustration 29. *The diameter of the piston P_2 is 50 cm and that of the piston P_1 is 10 cm. What is the force exerted on P_2 when a force of 1 N is applied on P_1?*

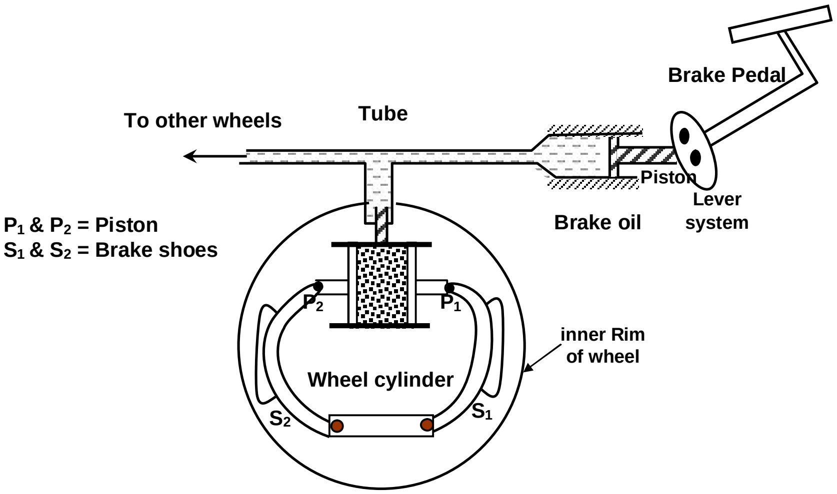
Solution: $r_2 = 25$ cm, $r_1 = 5$ cm, $F_2 = ?$

$F_1 = 1$ N

$\because\ \frac{F_2}{\pi r_2^2} = \frac{F_1}{\pi r_1^2}$

or $F_2 = F_1\left(\frac{r_2}{r_1}\right)^2 = 25$ N

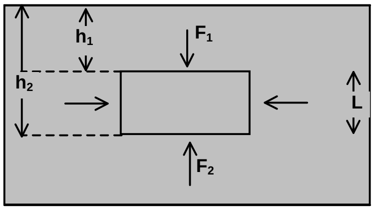
Hydraulic Brakes: Pascal's law is used in application of hydraulic brakes also.



A small force is applied on the brake pedal with small area. It is magnified to a large force acting on the brakes due to larger area of the piston in brake shoes.

ARCHIMEDES' PRINCIPLE

When a body is partially or fully dipped into a fluid, the fluid exerts force on the body. At any small portion of the surface of the body, the force by the fluid is perpendicular to the surface and is equal to the product of the pressure at the surface and area. The resultant of all these contact forces is called buoyant force (upthrust).



Let a body is immersed in a fluid of density σ as shown in figure.

$F_1 = A\,(h_1\sigma g + p_0)$

And $F_2 = A\,(h_2\sigma g + p_0)$

As $h_2 > h_1$, hence $F_2 > F_1$, so the body will experience a net upward force

$F = F_1 - F_2 = A\sigma g\,(h_2 - h_1) = A\,g\sigma L$

$= (AL)\,g\sigma = V\sigma g$ (V = volume of the body),

Hence, F = weight of fluid displaced by the body. This force is called buoyant force and acts vertically upwards (opposite to the weight of the body) through the centre of gravity of the displaced fluid.
Apparent decrease in weight of body = upthrust = weight of liquid displaced by the body.

Illustration 30. *A cubical block of steel of side 'ℓ' is floating in mercury in a vessel. The densities of steel and mercury are ρ_s and ρ_m. The height of block above the mercury level is given by*

(A) $\ell\left(1+\frac{\rho_s}{\rho_m}\right)$ (B) $\ell\left(1-\frac{\rho_s}{\rho_m}\right)$

(C) $\ell\left(1+\frac{\rho_m}{\rho_s}\right)$ (D) $\ell\left(1-\frac{\rho_m}{\rho_s}\right)$

Solution: **B.**

Volume of block = ℓ^3. Let 'h' be the height of block above mercury surface, then

volume of mercury displaced = $(\ell - h)\ell^2$, and

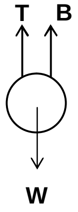
weight of mercury displaced = $(\ell - h)\ell^2 \rho_m g$

$(\ell - h)\ell^2\rho_m g = \ell^3\rho_s g$.

$$h = \ell\left(1-\frac{\rho_s}{\rho_m}\right)$$

Illustration 31. *A 0.5 kg block of brass (density 8×10^2 kg/m^3) is suspended from a string. What is the tension in the string if the block is completely immersed in water? ($g = 10$ m/s^2)*

Solution: Volume of block $= \frac{0.5}{8\times10^3}$ m^3



Volume of displaced water $= \frac{0.5}{8\times10^3}$ m^3

Weight of displaced water $= \frac{0.5}{8\times10^3}10^3\times10$ N $= \frac{5}{8}$N

This is equal to upward buoyant force.
Since, T + B = W
$\therefore$ T = W – B

$= 5-\frac{5}{8} = 4.375$ N

Illustration 32. *A pipe of copper having an internal cavity weighs 264 gm in air and 221 gm in water. Find the volume of the cavity. Density of copper is 8.8 gm/cc.*

Solution : The buoyant force on the copper piece

F = Vρg

Hence volume of the copper piece V = $\frac{F}{\sigma\ \ g}$

$= \frac{(264-221)g}{1\times g} = 43$ cc

The volume of the material of the copper piece

$V_0 = \frac{\text{mass of copper piece}}{\text{density of material}} = \frac{264}{8.8} = 30\,\text{cc}$

Hence volume of the cavity = $V - V_0$

= 43 – 30= 13 cc.

Illustration 33. *A piece of brass (alloy of copper and zinc) weighs 12.9 gm in air. When completely immersed in water it weighs 11.3 gm. What is the mass of copper contained in the alloy? Specific gravities of copper and zinc are 8.9 and 7.1, respectively.*

Solution : Let the mass of copper in alloy = x gm.

Hence, amount of zinc = (12.9 – x) gm

Volume of copper $V_{Cu} = \dfrac{x}{\rho_{Cu}} = \dfrac{x}{8.9}$

And volume of zinc $V_{Zn} = \dfrac{(12.9 - x)}{7.1}$

Hence total volume of the alloy $V = V_{Cu} + V_{Zn}$

$$V = \frac{(12.9 - x)}{7.1} + \frac{x}{8.9} \qquad \ldots \text{(i)}$$

Buoyant force F = Vρ g = loss of weight (P is the density of water)

= (12.9 – 11.3)g = 1.6 g

Substituting the value of V in equation (i), we get x = 7.6 gm

Floatation: A body floats in a liquid if the average density of the body is less than that of the liquid.

There will be equilibrium of floating bodies if the following conditions are fulfilled:

(a) The weight of the liquid displaced by the immersed part of body must be equal to the weight of the body.

(b) The centre of gravity of the body and centre of buoyancy must be along the same vertical line.

Illustration 34. *A solid floats in water with $(3/4)^{th}$ of its volume below the surface of water. What is the density of the solid?*

Solution: Let the volume and density of the solid be V and ρ, respectively.

Weight of the solid = vρg

Volume of block under water $= \dfrac{3V}{4} m^3$ = volume of water displaced

Weight of water displaced = $\left(\dfrac{3V}{4} \times 1000 \times g\right)$ newton

Since weight of body = weight of water displaced by the body

$\therefore\ V\rho g = \dfrac{3V}{4} \times 1000 \times g$

or $\rho = 750\ \text{kg/m}^3$

Illustration 35. *A cube of wood supporting 200 gm mass just floats in water. When the mass is removed, the cube rises by 2 cm. What is the size of the cube?*

Solution : Let the edge of the cube be x. As the cube rises 2 cm on removing the mass, hence the weight of the body must be equal to the thrust provided by 2 cm height of cube of base area x^2

i.e. $200\ g = (2\ x^2)\ 1g \quad \Rightarrow \quad x = 10$ cm

Illustration 36. *A cubical block of iron edge 5 cm is floating on mercury in a vessel.*
(a) What is the height of the block above mercury level?
(b) Water is poured into the vessel so that it just covers the iron block. What is the height of the water column?
[Relative density of Hg = 13.6 and that of Fe = 7.2]

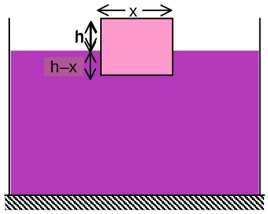
Solution: (a) Let h be the height of the iron block above mercury.

In case of flotation

Weight of the block = Buoyant force

$x^3 \rho g = [(x - h) \sigma g] x^2$

$\Rightarrow \; h = x\left(1 - \frac{\rho}{\sigma}\right) = 5\left(1 - \frac{7.2}{13.6}\right) = 2.35$ cm.



(b) Let y be the height of the water level.

For equilibrium of the block

$x^3 \rho g = [\sigma_w g y + \sigma_{Hg} g(x - y)]x^2$

$x\rho = (x - y)\sigma_{Hg} + y\sigma_w$

$\Rightarrow y = x\left(\frac{\sigma_{Hg} - \rho}{\sigma_{Hg} - \sigma_w}\right)$

$= 5\left(\frac{13.6 - 7.2}{13.6 - 1}\right) = 2.54$ cm

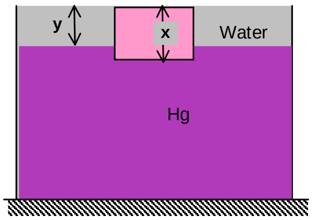


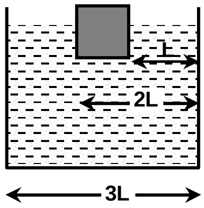
Illustration 37. *The 'tip of the iceberg' in popular speech has come to mean a small visible fraction of something that is mostly hidden. For real icebergs, what is this fraction? (ρ_{ice} = 917 kg/m^3, $\rho_{sea\ water}$ = 1024 kg/m^3)*

Solution: $W_{ice} = \rho_i V_i g, \quad W_{sea\ water} = \rho_w . V_w . g$

For floatation, $\rho_i V_i g = \rho_w . V_w . g$

$$\text{Fraction} = \frac{V_i - V_w}{V_i} = 1 - \frac{917}{1024} = \frac{107}{1024} = 10.45\ \%$$

Illustration 38. *A cube of specific gravity 0.6 and side L floats in a rectangular tank containing water with square base of side 3L. Assume the cube remains vertical.*



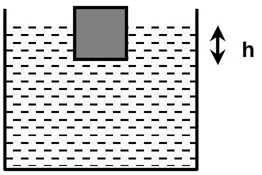
(a) Find the ratio of area of the vertical face of cube, which is inside water to that of vertical face of cube, which is outside water.

(b) Now the tank is accelerated horizontally with acceleration of g m/s^2. Find angle made by water surface with horizontal.

Solution: (a) Wt. of cube = buoyant force

$\Rightarrow L^3(0.6)g = L^2 h(1)g \Rightarrow \frac{h}{L} = 0.6$

$\Rightarrow \frac{h}{L - h} = \frac{3}{2}$



(b) $\tan\theta = \frac{a}{g} = \frac{g}{g} = 1 \Rightarrow \theta = 45°$

Exercise 10.

(i). ***A tank containg water is placed on a spring scale which registers a total weight W. A stone of weight ω is hung from a string and is lowered into water without touching the sides or bottom of the tank. What will be the reading on the spring scale?***

(ii). ***It is easier to swim in sea water than in river water. Why?***

(iii) ***Why a soft plastic bag weighs the same when empty as when filled with air at atmospheric pressure?***

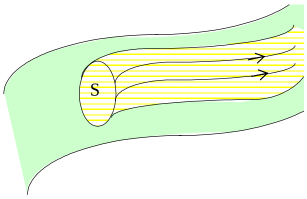
(iv) ***Explain why a balloon filled with helium does not rise in air indefinitely but halts after a certain height. (Neglect winds)***

FLUID DYNAMICS

Steady Flow (Streamline Flow): Wherein the velocity of fluid particles reaching a particular point is the same at all times. Thus, each particle takes the same path as taken by a previous particle through that point.

Line of flow: It is the path taken by a particle in flowing liquid. In case of a steady flow, it is also called a streamline.

Tube of flow: Consider an area S in a fluid in steady flow. Draw streamlines from all the points of the periphery of S. These streamlines enclose a tube, of which S is a cross–section. No fluid enters or leaves across the surface of this tube.
It is called a tube of flow.



Reynolds Number: Flow of a liquid turns from laminar to turbulent when its velocity exceeds a particular value. It is called critical velocity. Reynolds established a number which determines the nature of flow, i.e. laminar or turbulent.

The number $N = \frac{\rho D v}{\eta}$ is called Reynolds number.

Where ρ is the density of liquid, v is its velocity, η is viscosity of the liquid and D is the diameter of the tube in which the liquid is flowing.

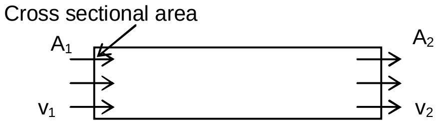
If N < 2000, the flow is laminar. If N is in between 2000 and 3000, the flow is unsteady and may change from laminar to turbulent and vice–versa. If N > 3000, the flow is turbulent.

Equation of continuity: In a time Δt, the volume of liquid entering the tube of flow in a steady flow is $A_1 V_1 \Delta t$. The same volume must flow out as the liquid is incompressible. The volume flowing out in Δt is $A_2 V_2 \Delta t$.

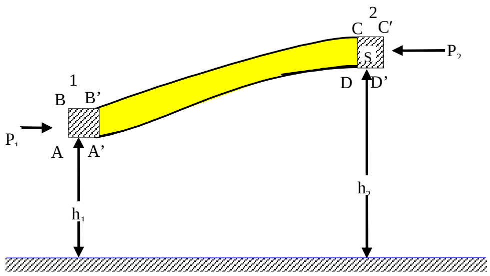
$\Rightarrow \quad A_1 V_1 = A_2 V_2$

Mass flow rate = ρAV

where ρ is the density of the liquid.



Bernoulli's Theorem



Consider a tube of flow, ABCD. In a time Δt, liquid moves and the liquid element becomes A'B'C'D'. In other words, we can also interpret that ABB'A' has gone to DCC'D'.

$$\Delta m = \rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

Work done by fluid pressure at 1

$$= (P_1A_1)\, v_1 \Delta t = P_1 \Delta m/\rho$$

Work done by fluid pressure at 2 $= -(P_2A_2)\, v_2 \Delta t = -P_2 \Delta m/\rho$

Work done by gravity $= -(\Delta m).g.\,(h_2 - h_1)$

Change in kinetic energy $= ½\, \Delta m\, [v_2^2 - v_1^2]$

Using, work–energy theorem, $(W = \Delta K)$,

$$P_1 \frac{\Delta m}{\rho} - P_2 \frac{\Delta m}{\rho} - \Delta mg\,(h_2 - h_1) = \frac{1}{2}\Delta m\,[v_2^2 - v_1^2]$$

$$\frac{P_1}{\rho} + gh_1 + \frac{v_1^2}{2} = \frac{P_2}{\rho} + gh_2 + \frac{v_2^2}{2}$$

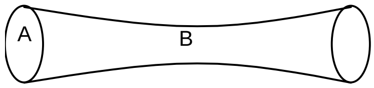
$$\Rightarrow \quad P_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g h_2 + \frac{\rho v_2^2}{2}$$

$$P + \rho g h + \frac{\rho v^2}{2} = \text{Constant}$$

In a stream-line flow of an ideal fluid, the sum of pressure energy per unit volume, potential energy per unit volume and kinetic energy per unit volume is always constant at all cross sections of the liquid.

➢ Bernoulli's equation is valid only for incompressible steady flow of a fluid with no viscosity.

Illustration 39. *Water flows in a horizontal tube as shown in figure. The pressure of water changes by 600 N/m² between A and B where the areas of cross–section are 30 cm² and 15 cm² respectively. Find the rate of flow of water through the tube.*



Solution: $\Delta P + \Delta \rho g h + \Delta \frac{1}{2}\rho v^2 = 0$ $A_Av_A = A_Bv_B$

$\Rightarrow \quad v_B = 2v_A$

$$-600 + 0 + \frac{1}{2}\rho(2v_A)^2 - \frac{1}{2}\rho(v_A)^2 = 0$$

$\Rightarrow \quad v_A = 0.63$ m/s

Rate of flow = (30 cm²) (0.63 m/s) = 1890 cm³ /sec.

Illustration 40. *Water enters a house through a pipe with inlet diameter of 2.0 cm at an absolute pressure of 4.0×10^5 Pa (about 4 atm). A 1.0 cm diameter pipe leads to the second floor bathroom 5.0 m above. When flow speed at the inlet pipe is 1.5 m/s, find the flow speed, pressure and volume flow rate in the bathroom.*

Solution: Let points 1 and 2 be at the inlet pipe and at the bathroom. Flow speed at point 2 is obtained from continuity equation

$a_1 v_1 = a_2 v_2 \Rightarrow v_2 = 6.0$ m/s

Now applying Bernoulli's equation at the inlet (y=0) and at the bathroom (y_2=5.0 m)

$$p_2 = 4 \times 10^5 - \frac{1}{2}P(v_2^2 - v_1^2) - \rho g(y_2 - y_1)$$

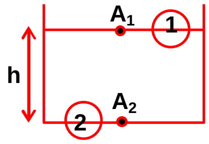
Which gives $p_2 = 3.3 \times 10^5$ pa

The volume flow rate = $A_2V_2 = A_1V_1$

$$= \frac{\pi}{4}(0.01)^2 6 = 4.7\times10^{-4}\ \text{m}^3/\text{s}.$$

Application of Bernoulli's Equation

1. Speed of efflux



Consider a tank of cross sectional area A_1 having a hole of area A_2 (<<A_1) at its bottom. Let us calculate the speed of water coming out from the hole.

Applying Bernoulli's equation between (1) and (2), we get

$$\frac{\rho v_1^2}{2} + \rho gh + P_0 = \frac{\rho v_2^2}{2} + 0 + P_0 \qquad (1)$$

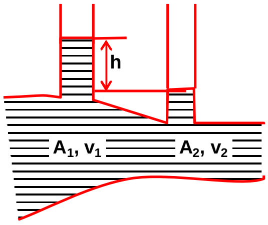
By continuity equation,

$$A_1 v_1 = A_2 v_2 \qquad (2)$$

On solving Eqs. (1) and (2), we get

$$v_2 = \sqrt{\frac{2gh}{1-\left(\frac{A_2}{A_1}\right)^2}}$$

2. Venturi tube



It is used to find the speed of flow of a liquid at different sections of a pipe of varying cross section.

By continuity equation, $A_1v_1 = A_2v_2$ (1)

By Bernoulli's equation,

$$P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho v_2^2}{2}$$

or $\quad P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$ (2)

Applying Pascal's law in the tube,

$$P_1 - P_2 = \rho gh \qquad (3)$$

From Eqs. (2) and (3), we have

$$2gh = v_2^2 - v_1^2$$

For $A_2 >> A_1$,

$$\Rightarrow v_2 = \sqrt{2gh}$$

Exercise 11.

(i) ***To keep a piece of paper horizontal you should blow over, not under it.***

(ii) ***Two row boats moving parallel to each other and nearby are pulled towards each other. Explain.***

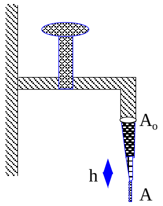


Illustration 41. *Figure shows how the stream of water emerging from a faucet necks down as it falls. The area changes from A_o to A through a fall of h. At what rate does the water flow from the tap?*

Solution : Equation of continuity :

$A_0V_0 = AV$

Bernoulli Equation :

$$P_0 + \rho gh + \frac{1}{2}\rho v_0^2 = P_0 + \rho g(0) + \frac{1}{2}\rho v^2$$

Solving for v_0,

$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$$

$$\text{Flow rate } R = A_0v_0 = \sqrt{\frac{2ghA^2A_0^2}{A_0^2 - A^2}} = \frac{AA_0\sqrt{2gh}}{\sqrt{A_0^2 - A^2}}.$$

Note: As the jet is going into the atmosphere, the pressure at A and A_0 are equal to the atmospheric pressure.

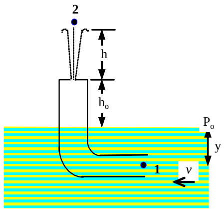


Illustration 42. *A bent tube is lowered into the stream as shown. The velocity of the stream relative to the tube is equal to V. The closed upper end of the tube is located at the height h_o. To what height h will the water jet spurt?*

Solution : Let tube's entrance be at depth 'y' below the surface. Take point 1 at entry, 2 at the maximum height of the fountain. This is a tube of flow, let's apply Bernoulli's theorem,

$$P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$$

Taking, $h_1 = 0$, $h_2 = (y + h_o + h)$, $V_1 = V$, $V_2 = 0$,

$P_1 = P_o + \rho gy$, $P_2 = P_o$,

Substituting, $P_o+\rho gy + \rho g.o + \frac{1}{2}\rho V^2 = P_o+\rho g\,(y + h_o+h)+ \frac{1}{2}\rho 0^2$

$$\Rightarrow \frac{1}{2}\rho V^2 = \rho g\,(h_o + h) \quad \Rightarrow \quad h = \left(\frac{V^2}{2g} - h_o\right)$$

Surface Tension

The free surface of a liquid contracts so that its exposed surface area is a minimum, i.e. it behaves as if it were under tension, somewhat like a stretched elastic membrane. This property is known as surface tension. The surface tension of a liquid varies with temperature as well as dissolved impurities, etc. When is soap mixed with water, the surface tension of water decreases.

Surface tension of a liquid is measured by the normal force acting per unit length on either side of an imaginary line drawn on the free surface of the liquid. The direction of this force is perpendicular to the line and tangential to the free surface of liquid.

$$T = \frac{F}{L}.$$

Consider a wire frame as shown in figure equipped with a sliding wire AB. It is dipped in soapy water. A film of liquid is formed. A force F has to be applied to hold the wire in place. Since, the soap film has two surfaces attached to the wire, the total length of the film in contact with the wire is 2L.

T (surface tension) = $\frac{F}{2L}$.

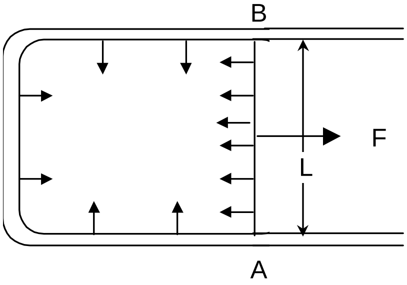


Illustration 43. *A soap film is formed on a rectangular frame of length 0.03 m dipped in a soap solution. The frame hangs from the arm of a balance. An extra mass of 2.20×10^{-4} kg must be placed in the other pan to balance the pull. Calculate the surface tension of the soap solution.*

Solution: Force acting on the frame due to surface tension $F = \sigma \times \ell$

Where ℓ is the length of the frame in contact with the liquid.

Since the soap film has two surfaces

$\therefore\ \ell = 2 \times 0.03 \text{ m} = 0.06 \text{ m}$

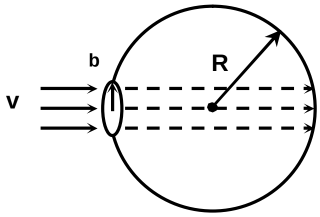
$\therefore\ F = 0.06\ \sigma$ Newton

This must be equal to the extra weight.

$\therefore\ 0.06\ \sigma = 2.20 \times 10^{-4} \times 9.81$

or $\sigma = 0.036$ N/m

Illustration 44. *A bubble having surface tension T and radius R is formed on a ring of radius b (b << R). Air is blown inside the tube with velocity v as shown. The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring.*



Solution: $2\pi b \times 2T\sin\theta = \rho A v^2$

$\Rightarrow\quad 4\pi bT \times \frac{b}{R} = \rho\pi b^2 v^2$

$\Rightarrow\quad R = \frac{4T}{\rho v^2}$

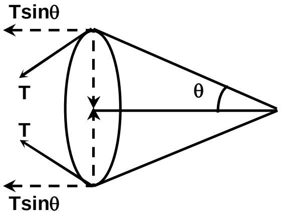


Illustration 45. *Find the work done to break a drop of water of radius 0.5 cm into identical drops of radius 1 mm. ($T_{water} = 7 \times 10^{-2}$ N/m).*

Solution: No. of drops $= \frac{4/3\pi(0.5)^3}{(4/3)\pi(0.1)^3} = 125$

Surface area of big drop $= 4\pi\,(0.5)^2 \times 10^{-4} = \pi \times 10^{-4}\ \text{m}^2$

Total surface area of small drops $= 125 \times 4\pi\,(0.1)^2 \times 10^{-4} = 5\,\pi \times 10^{-4}\ \text{m}^2$

Total increase in surface area $= 4\pi \times 10^{-4}\ \text{m}^2$

$\therefore$ Work done $= T \times A = 7 \times 10^{-2}(4\pi \times 10^{-4}) = 8.8 \times 10^{-5}$ J.

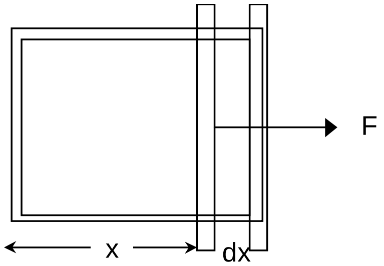
Exercise 12. ***How do detergents clean dirty clothes?***

Properties of surface Tension

- Scalar quantity.
- Temperature sensitive.

- Impurity sensitive.
- Depends only on the nature of the liquid.
- Unit of surface tension, N/m.
- Dimension of surface tension, ML^0T^{-2}.

Surface Energy



If the area of the liquid surface has to be increased work has to be done against the force of surface tension. The work done to form a film is stored as potential energy in the surface and the amount of this energy per unit area of this surface under isothermal condition is the "intrinsic surface energy" or free surface energy density.

Work done in small displacement dx

$dW = F \times dx = 2TL\,dx$

$W = \int_0^x 2TL\,dx = 2TLx = TA$

As A= 2Lx (area of both sides)

W/A = T (intrinsic surface energy)

Illustration 46. *What is the surface energy of a soap bubble of radius r?*

Solution: $E = TA = T \times 4\pi r^2 \times 2$ (as it has two surfaces)

$= 8\pi r^2\,T$.

Illustration 47. *What is the surface energy of an air bubble inside a soap solution?*

Solution: $E = T \times A = 4\pi r^2 T$, as it has only one surface

Illustration 48. *A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Given surface tension of liquid = σ*

Solution: If r is the radius of small drop then

$27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left[\frac{D}{2}\right]^3 \quad \therefore r = \frac{D}{6}$

Initial surface area $= 4\pi\left[\frac{D}{2}\right]^2 = \pi D^2$

Final surface area = $27 \times 4\pi r^2 = 27 \times 4\pi\left[\frac{D}{6}\right]^2$

Increase in surface area $= 3\pi D^2 - \pi D^2$

$= 2\pi D^2$

Increase in energy $= 2\pi D^2\sigma$

Illustration 49. *Calculate the amount of energy evolved when eight droplets of mercury (surface tension 0.55 N/m) of radius 1 mm each combine into one.*

Solution: Energy $= 4\pi(r_1^2 - r_2^2)\,T$

$\frac{4}{3}\pi r_2^3 = 8\frac{4}{3}\pi r_1^3$

$r_2 = 2r_1$

$E = 27.65 \times 10^{-6}$ J

Exercise 13. ***Several small drops of liquid are added together to make a big drop. How is the temperature of the drops affected?***

Excess Pressure

The pressure inside a soap bubble and outside it, are not identical due to surface tension of the soap bubble. To calculate this pressure difference, let's first consider an air bubble inside a liquid. If the pressure difference is Δp, then the work done to increase the radius of bubble from r to $(r+\Delta r)$ is given by:

$$W = F\,\Delta r = 4\,\pi r^2\,\Delta p\,\Delta r$$

while change in area

$$\Delta S = 4\pi\,(r + \Delta r)^2 - 4\pi r^2 = 8\,\pi r\,\Delta r$$

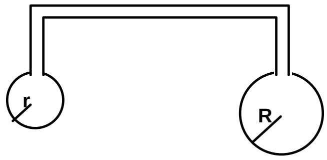
From the definition of surface tension

$$T = W/\Delta S = \frac{4\pi r^2 \Delta p \Delta r}{8\pi\ \ r\Delta r} \Rightarrow \qquad \Delta p = \frac{2T}{r}$$

For a soap bubble in air, there are two surfaces, and so,

$$\Delta p = 2 \times 2T/\,r = \frac{4T}{r}$$

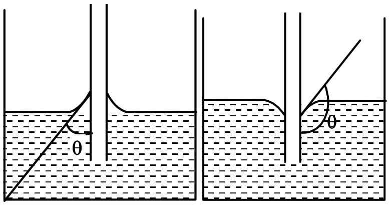
Exercise 14. ***Two soap bubbles of different radii (r, R : r < R) are connected by means of a tube. What will happen to the larger bubble?***



Angle of Contact

1. Angle of contact, for a solid and a liquid is defined as the angle between tangent to the liquid surface drawn at the point of contact and the solid surface inside the liquid.

2. The angle of contact of a liquid surface on a solid surface depends on the nature of the liquid and the solid.



(a) (b)

Case I: When $\theta < 90°$:

The liquid surface curves up towards the solid. This happens when the force of cohesion between two liquid molecules is less than force of adhesion between the liquid and the solid. If such a liquid is poured into a solid tube, it will have a concave meniscus. For example, a glass rod dipped in water, or water inside a glass tube.

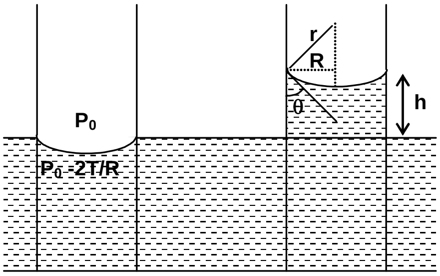
Case II: When $\theta > 90°$:

The liquids surfaces get curved downward in contact with a solid. In this case the force of cohesion is greater than the force of adhesion. In such cases, solids do not get "wet". When such liquids are put into a solid tube, a convex meniscus is obtained.

For example, a glass rod dipped in mercury or mercury within a solid glass tube.

Capillarity

When a piece of chalk is dipped into water, it is observed that water rises through the pores of chalk and wets it.



Consider a glass capillary of radius R dipped in water as shown in the figure. The pressure below the meniscus will be $p_0 - \frac{2T}{r}$. To compensate for this pressure difference, water in the capillary rises so that

$$\frac{2T}{r} = \rho gh \quad \Rightarrow \quad h = \frac{2T}{rg\rho}$$

where r is the radius of meniscus,

and $r = \frac{R}{\cos\theta}$, where θ is the angle of contact and thus $h = \frac{2T\cos\theta}{R\rho g}$

If $\theta < 90°$, the meniscus will be concave, for Illustration: at a water–glass interface.

If $\theta > 90°$, the meniscus will be convex, for Illustration: at a mercury–glass interface.

Illustration 50. *A U-tube is supported with its limbs vertical and is partly filled with water. If internal diameters of the limbs are 1×10^{-2} m and 1×10^{-4} m respectively, what will be the difference in heights of water in the two limbs? Surface tension of water is 0.07 N/m.*

Solution: Surface tension, T = 0.07 N/m

Density $\rho = 1000\ kg/m^3$, $g = 9.8\ m/s^2$

Angle of contact $\theta = 0°$

radius $r_1 = 0.5 \times 10^{-2}$ m, and

radius $r_2 = 0.5 \times 10^{-4}$ m

Let h_1 be the height of water in the limbs of radius r_1

Then, $h_1 = \frac{2T\cos\theta}{r_1\rho g} = 2.86 \times 10^{-3}$ m

Let h_2 be the height of water in the limbs of radius r_2

Then $h_2 = \frac{2T\cos\theta}{r_2\rho g} = 2.86 \times 10^{-1}$ m

Difference in heights $h_2 - h_1$

$= 2.86 \times 10^{-1}\ m - 2.86 \times 10^{-3}\ m$

$= (0.286 - 0.00286)m = 0.283\ m$

Illustration 51. *A meniscus drop of radius 1 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expended if surface tension of mercury is 435×10^{-3}N/m.*

Solution : Energy expended will be the work done against the increase in surface area

i.e. $n(4\pi r^2) - 4\pi R^2$

$E = W = T\Delta S$

$= T.\ 4\pi\ (nr^2 - R^2)$

But the total volume remains constant.

i.e. $\frac{4}{3}\pi R^3 = n\ \frac{4}{3}\ \pi r^3$

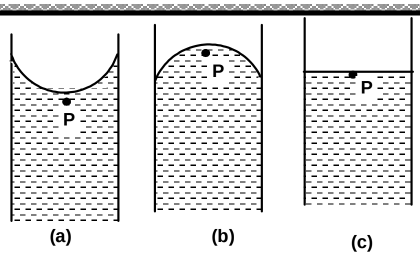
or $r = \frac{R}{(n)^{1/3}}$ and hence, $E = 4\ \pi R^2 T\ (n^{1/3} - 1)$

$= 4 \times 3.14 \times (1 \times 10^{-2})^2 \times 435 \times 10^{-3}\ (10^2 - 1)$

$= 54.1 \times 10^{-3}$ J.

Exercise 15.

(i) ***Compare the pressure at the point P in the three tubes shown in the figure.***

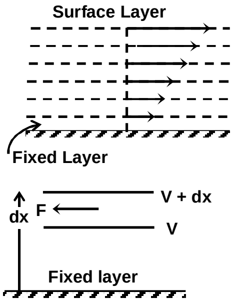


(ii) ***Why the pressure of water decreases when it flows from a broader pipe to a narrower pipe?***

Viscosity
Viscosity is the property of a fluid (liquid or gas) by virtue of which it opposes the relative motion between its different layers.

Cause of viscosity:
Considering two neighbouring liquid layers A and B. Suppose A moves faster than B. B would tend to retard the motion of A. On the other hand A would try to accelerate B. Due to these two different tendencies a backward tangential force is set up. This force tends to destroy the relative motion between the two layers. This accounts for the viscous behaviour of both liquids and gases.

Coefficient of viscosity
Consider a liquid flowing steadily over a solid horizontal surface. The layer of the liquid in contact with the solid horizontal surface is at rest. So, this layer is a fixed layer. The velocities of other layers increase uniformly with the increase in distance from the fixed layer as shown in figure.



Consider two liquid layers P and Q at distances x and x + dx respectively from the fixed layers. Let v and v + dv be their respective velocities.

The velocity gradient $\frac{dv}{dx}$ is in a direction perpendicular to the direction of flow of the liquid.

Viscous forces F acting tangentially on a layer of the liquid is proportional to (i) the area A of the layer and (ii) the velocity gradient $\frac{dv}{dx}$

$$\therefore F \propto A\frac{dv}{dx}$$

or $$F = -\eta A\frac{dv}{dx}$$

where η is the coefficient of viscosity of the liquid. It depends upon the nature of the liquid. The negative sign shows that the viscous force acts in a direction opposite to the direction of the motion of the liquid

If A = 1 and $\frac{dv}{dx} = 1$,

then η = F (numerically)

The coefficient of viscosity of a liquid is the viscous force acting tangentially per unit area of a liquid layer having a per unit velocity gradient in a direction perpendicular to the direction of flow of the liquid.
The CGS unit of η is poise.
The SI unit of viscosity is equal to poise. It dimension is $ML^{-1}T^{-1}$.

Illustration 52. *A metal plate 0.04 m^2 in area is lying on a liquid layer of thickness 10^{-3}m and coefficient of viscosity 140 poise. Calculate the horizontal force needed to move the plate with a speed of 0.040 m/s.*

Solution : Area of the plate A = 0.04 m^2

Thickness $\Delta x = 10^{-3}$m

Δx is the distance of the free surface with respect to the fixed surface.

Velocity gradient, $\frac{\Delta v}{\Delta x} = \frac{0.04\ \ m/s}{10^{-3}} = 40 s^{-1}$

Coefficient of viscosity, $\eta = 14 kg/ms^{-1}$

Let F be the required force,

Then, $F = \eta A \frac{\Delta v}{\Delta x} = 22.4$ N

FLOW OF VISCOUS LIQUID THROUGH A CAPILLARY TUBE (POISEUILLE'S FORMULA)

The velocity v at a distance y from the capillary axis for a flow of liquid of viscosity η in a capillary tube of length L and radius r under a pressure difference p across it is given by

$$v = \frac{P}{4\eta L}(r^2 - y^2)$$

and the volume of liquid flowing per second is given by

$$\left(\frac{dQ}{dt}\right) = \frac{\pi P r^4}{8\eta L}.$$

Illustration 53. *A liquid flows through a pipe of 1.5 mm radius and 15 cm length under a pressure of 15,000 dyne/cm^2. Calculate the rate of flow and the speed of the liquid coming out of the pipe. The coefficient of viscosity of the liquid is 1.40 centipoise.*

Solution: Radius r = 1.5 mm = 0.15 cm

Length L = 15 cm

Pressure difference, $p = 15\times10^3$ dyne/cm^2

Coefficient of viscosity, η = 1.40 centipoise = 0.0140 poise

Rate of flow, $V = \frac{\pi\ \ pr^4}{8\eta\ell}$

$= \frac{3.14\times15\times10^3\times(0.15)^4}{8\times0.0140\times15}$ cm^3/s

$= 14.19 cm^3/s$

Velocity, $v = \frac{V}{\text{cross sectional area}}$

$= \frac{14.19}{3.14\ \times\ (0.15)^2}$

= 200.84 cm/s

Illustration 54. *Liquid flows through two capillary tubes connected in series. Their lengths are L and 2L and radii r and 2r, respectively. The pressure difference across the first and second tubes are in the ratio.*

Solution:

$$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2$$

$$\frac{\pi p_1 r_1^4}{8\eta L_1} = \frac{\pi p_2 (2r_1)^4}{8\eta 2L_2}$$

$$\frac{p_1}{p_2} = 8$$

STOKE'S LAW AND TERMINAL VELOCITY

When a sphere of radius r moves with a velocity v through a fluid of viscosity η, the viscous force opposing the motion of the sphere is

$$F = 6\pi\eta\, rv$$

If for a sphere viscous force becomes equal to the net weight acting downward, the velocity of the body becomes constant and is known as terminal velocity.

$$6\pi\eta\, rv_T = \frac{4}{3}\pi r^3 (\rho - \sigma)g$$

where ρ and σ are densities of the sphere and the fluid, respectively.

$$\Rightarrow \quad v_T = \frac{2}{9} r^2 \left\{\frac{\rho - \sigma}{\eta}\right\} g\,.$$

Illustration 55. *Find the viscous force on a steel ball of 2 mm radius (density 8 g/cc) that acquires a terminal velocity of 4 cm/s in falling freely in a tank of glycerine (density of glycerine 1.3 g/cc).*

Solution:

$$mg = 6\pi\eta rv + \frac{4}{3}\pi r^3 \sigma g$$

$$\frac{4}{3}\pi r^3 \rho_{steel} g - \frac{4}{3}\pi r^3 \sigma g = \text{viscous force}$$

$$= 980 \times \frac{4}{3} \times \frac{22}{7} \times \left(\frac{2}{10}\right)^3 [8 - 1.3]$$

$$= 220.12 \text{ dynes.}$$

MISCELLANEOUS EXERCISE

1. A sphere of mass 40 kg is attracted by another sphere of mass 15 kg with a force of $\frac{1}{40}$mg . Find the value of gravitational constant, if their centres are 0.40 m apart.

2. If the diameter of earth becomes two times of its present value, its mass remains unchanged. Then, how would the weight of an object on the surface of earth be affected ?

3. At what height from the surface of earth will the value of acceleration due to gravity be reduced by 36 % from the value at the surface? Given, radius of earth = 6400 km

4. An artificial satellite circulated around the earth at a height of 3400 km of revolution. Find the acceleration and time period of satellite. Radius of earth = 6400 km and g = 9.8 m/s^2.

5. The escape velocity of a projectile on the earth's surface is 11.2 km/s. A body is projected out with thrice this speed. What is the speed of the body far away from the earth ? Ignore the presence of sun and other planet.

6. Atmospheric pressure is nearly 100 kPa. How large the force does the air in a room exert on the inside of a window pane that is 40 cm × 80 cm ?

7. A 0.5 kg block of brass (density : 8 × 10^3 kg/m^3) is suspended from a string. What is the tension in the string if the block is completely immersed in water ? (g = 10 m/s^2)

8. A cubical block of wood 10 cm on a side floats at the interface between oil and water with its lower face 2 cm below the interface. The density of oil is 0.8 is 0.8 g /cm^3. What is the mass of the block ?

9. A wooden rod of uniform cross–section and of length 120 cm is hinged at the bottom of a tank which is filled with water to a height of 40 cm. In the equilibrium position, the rod makes an angle of 60° with the vertical. Calculate the distance of the centre of buoyancy from the hinge.

10. A 700 g solid cube having an edge of length 10 cm floats in water. What volume of the cube is outside water ?

ANSWERS TO MISCELLANEOUS EXERCISE

1. 6.54×10^{-11} Nm^2 kg^{-2}
3. 1600 km
4. 6400 m/s^2, 9.6×10^3 s
5. 31.68 km/s
6. 32 kN
7. 4.375 N
8. 840 gram
9. 40 cm
10. 300 cm^3

SOLVED PROBLEMS

Subjective:

Prob 1. *Acceleration due to gravity at moon's surface is g/6, where g is acceleration due to gravity on earth. If moon's radius is 1.74×10^6 m, find out the mass of the moon. ($G = 6.67 \times 10^{-11}$ Nm^2/kg^2).*

Sol. On moon, $m\frac{g}{6} = \frac{GMm}{R_m^2}$ (R_m = radius of the moon, M = mass of the moon)

$\therefore$ $M = \frac{g.R_m^2}{6G} = \frac{9.8 \times (1.74 \times 10^6)^2}{6 \times 6.67 \times 10^{-11}}$

Mass of moon = 7.41×10^{22} kg.

Prob 2. *What will be the value of g at the bottom of sea 8.5 km deep ? (Take radius of earth as 6400 km.)*

Sol. Taking g′ as value of g at the bottom of sea with 8.5 km depth,

$g' = g(1 - \frac{d}{R})$

$= 9.8\ (1 - \frac{8.5}{6400})$

$g' = 9.787\ m/s^2$.

Prob 3. *Escape velocity of a particle on earth's surface is 11.2 km/s. A body is projected out with twice this velocity. What is the speed of the body far away from the earth, i.e. at infinity? Ignore presence of sun and other planets.*

Sol. By conservation of energy near earth's surface and at infinity,

$\frac{1}{2}m\left(\frac{8GM}{R}\right) - \frac{GMm}{R} = \frac{1}{2}mv'^2$

where v′ is the speed at infinity

$\Rightarrow 8\ GM - 2GM = v'^2R$

$v' = \sqrt{\frac{6GM}{R}} = \sqrt{3} \times \sqrt{\frac{2GM}{R}} = \sqrt{3} \times 11.2\ km/s$

v′ = 19.4 km/s.

Prob 4. *The distances of two planets from the sun are 10^{13} m and 10^{12}m respectively. Calculate the ratio of time periods and the speeds of the two planets.*

Sol. We know that, from Kepler's third law,

$T^2 \propto R^3$

$$\therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \quad \ldots (i)$$

$$= \left(\frac{10^{13}}{10^{12}}\right)^3 = 10^3$$

$$\frac{T_1}{T_2} = (10)^{3/2} = 10\sqrt{10}$$

Now, since $v_1 = \frac{2\pi R_1}{T_1}$

and $v_2 = \frac{2\pi R_2}{T_2}$

$$\Rightarrow \frac{v_1}{v_2} = \left(\frac{2\pi R_1}{T_1}\right)\times\left(\frac{T_2}{2\pi R_2}\right)$$

$$= \frac{R_1}{R_2}\times\frac{T_2}{T_1} = \left(\frac{R_1}{R_2}\right)\left(\frac{R_2}{R_1}\right)^{3/2} \quad \text{[from equation (i)]}$$

$$= \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{10^{12}}{10^{13}}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{1}{10}}.$$

Prob 5. *A particle is thrown up from the surface of the earth with velocity v in vertically upward direction. Neglecting effect of air friction, find out the maximum height H attained by the particle.*

Sol. By conservation of energy on the surface of the earth and at the maximum height (H),

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+H)}$$

$$v^2 = \frac{2GM}{R} - \frac{2GM}{R+H}$$

$$= \frac{2GMH}{R(R+H)} = \frac{2gRH}{R+H} = \frac{2gH}{\left(1+\frac{H}{R}\right)}$$

$$\therefore\ H = \frac{v^2R}{2gR - v^2}$$

Prob 6. *A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the relative density of spirit ?*

Sol. Since the mercury columns in the two arms are at the same level.

$\therefore$ Pressure due to water column

= pressure due to spirit column

$h_\omega\rho_\omega g = h_s\rho_s g$

or $h_\omega\rho_\omega = h_s\,\rho_s$

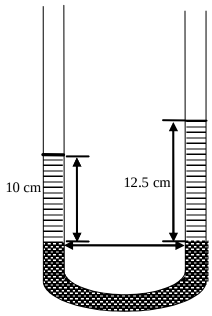
But $h_\omega = 10$ cm, $\rho_\omega = 1$ g cm^{-3},

$h_s = 12.5$ cm.

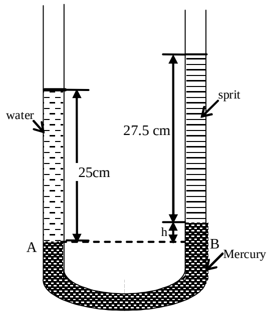
$\therefore\ 10 \times 1 = 12.5 \times \rho_s$

or $\rho_s = \frac{10}{12.5}\,\text{gcm}^{-3} = 0.8\ \text{gcm}^{-3}$

$\therefore$ relative density of spirit = 0.8.



Prob 7. *In the above given example if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of the mercury of the two arms? (Relative density of mercury = 13.6)*



Sol. Let two points A and B be in the same horizontal plane.
Applying Pascal's law, pressure at A = pressure at B
$\therefore\ p_0 + h_\omega\rho_\omega g = p_0 + h_s\rho_s g + h_m\rho_m g$
where p_0 is the atmospheric pressure
Now, $h_\omega\rho_\omega = h_s\rho_s + h_m\rho_m$
$25 \times 1 = 27.5 \times 0.8 + h \times 13.6$
or, $h = 0.22$ cm

Prob 8. *An air bubble of 0.01 m radius is rising at a steady rate of 5×10^{-3} m/s through a liquid of density 800 kg m^{-3}. Calculate the coefficient of viscosity of the liquid. Neglect the density of air.*

Sol. Radius r = 0.01 m, Density of air = 0
Density of liquid (ρ') = 800 kg m^{-3}
Terminal velocity (v_0) = -5×10^{-3} m/s
–ve sign indicates that the bubble is moving upwards

$$v_0 = \frac{2}{9}r^2\frac{(\rho-\rho')g}{\eta}$$

$$\text{or}\quad \eta = \frac{2}{9}\frac{r^2(\rho-\rho')g}{v_0}$$

$$= \frac{2}{9}\frac{(0.01)^2(0-800)9.81}{-5\times10^{-3}}\ \text{decapoise.}$$

$\eta = 34.88$ decapoise.

Prob 9. *Water is flowing through a horizontal pipe of varying cross-section. If the pressure of water equals 2×10^{-2} m of mercury where velocity of flow is 32×10^{-2} m/s, what is the pressure of another point, where the velocity of flow is 40×10^{-2} m/s ?*

Sol. $P_1 = 2 \times 10^{-2}$ m of mercury
$= 2 \times 10^{-2} \times 13600 \times 9.8$ N/m^2
$v_1 = 32 \times 10^{-2}$ m/s.
$v_2 = 40 \times 10^{-2}$ m/s, P_2 = ?
From Bernoulli's theorem for horizontal flow,

$$P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2$$

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$= P_1 - \frac{1}{2}\rho(v_2^2 - v_1^2)$$

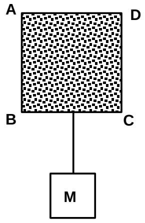
$$= 2 \times 10^{-2} \times 13600 \times 9.8 - \frac{1}{2}\times1000\times[(40\times10^{-2})^2 - (32\times10^{-2})^2]$$

$= 2636.8\ N/m^2$

$= \dfrac{2636.8}{13600 \times 9.8}$ m of Hg

$= 1.98$ cm of Hg.

Prob 10. *As shown in the figure, a rectangular wire frame ABCD in which side BC is movable is suspended vertically. To side BC a mass M is suspended through strings attached to BC. If shaded portion contains a liquid film and BC is in equilibrium, then find the surface tension of the liquid if BC = ℓ.*



Sol. Let surface tension be S.

Force upward on wire BC

$F_w = 2BC \times S = 2\ell S$ . . . (1)

Force downward due to tension of strings

$F_D = 2T$ . . . (2)

Equilibrium of mass m gives

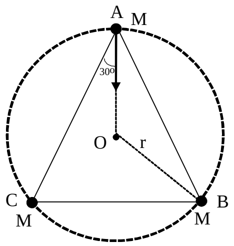
$2T = Mg$ . . . (3)

Equating (1), (2) and (3) we have

as $F_w = F_D = Mg$

$2\ell S = Mg \Rightarrow S = \dfrac{Mg}{2\ell}$

Prob 11. *Three particles, each of mass M, are located at the vertices of an equilateral triangle of side 'a'. At what speed must they move if they all revolve under the influence of their gravitational force of attraction in a circular orbit circumscribing the triangle while still preserving the equilateral triangle?*



Sol. $\vec{F}_A = \vec{F}_{AB} + \vec{F}_{AC} = 2\left[\dfrac{GM^2}{a^2}\right]\cos 30^o = \left[\dfrac{GM^2}{a^2}.\sqrt{3}\right]$

$r = \dfrac{a}{\sqrt{3}}$,

Now, $\dfrac{Mv^2}{r} = F$

or $\dfrac{Mv^2.\sqrt{3}}{a} = \dfrac{GM^2}{a^2}.\sqrt{3}$

$\therefore$ $v = \sqrt{\dfrac{GM}{a}}$

Prob 12. *An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth.*

(a) Determine the height of the satellite above the earth's surface.

(b) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth. ($g = 9.8$ m/s and $R_E = 6400$ km).

Sol. (a) We know that for satellite motion

$$v_o = \sqrt{\frac{GM}{r}} = R\sqrt{\frac{g}{(R+h)}} \qquad \left[\text{as } g = \frac{GM}{R^2} \text{ and } r = R + h\right]$$

In this Problem, $v_o = \dfrac{1}{2}v_e = \dfrac{1}{2}\sqrt{2gR}$

$$\text{So } \frac{R^2 g}{R+h} = \frac{1}{2}gR, \text{ i.e. } 2R = h + R \quad \text{or} \quad h = R = 6400 \text{ km}$$

(b) By conservation of ME

$$0 + \left(-\frac{GMm}{r}\right) = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$$

$$\text{or} \quad v^2 = 2GM\left[\frac{1}{R} - \frac{1}{2R}\right] \qquad [\text{as } r = R + h = R + R = 2R]$$

$$\text{or} \quad v = \sqrt{\frac{GM}{R}} = \sqrt{gR} = \sqrt{10 \times 6.4 \times 10^6} = 8 \text{ km/s}.$$

Prob 13. *A capillary tube of inner radius 0.5mm is dipped in water for which surface tension is 72 dynes/cm. To what height is the water raised by the capillary action above the normal water level? Also calculate the weight of the water raised. Given that the angle of contact = 0°.*

Sol. Given that r = 0.5mm, T = 75 dyne/cm, angle of contact, $\theta = 0°$.
We know that,

$$h = \frac{2T \cos\theta}{r\rho g} - \frac{r}{3}$$

$$\text{or} \quad h = \left(\frac{2\times 75 \cos 0°}{0.05\times1\times980} - \frac{0.05}{3}\right) \text{cm} = 3.041 \text{ cm}.$$

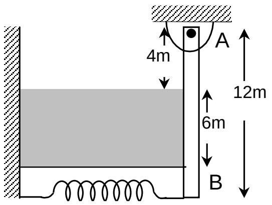
Weight of water raised, W = mg

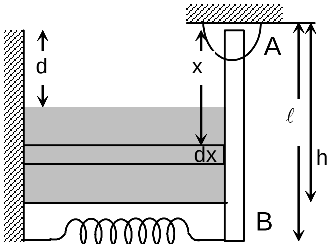
$$\text{or} \quad W = \text{volume} \times \text{density} \times g = \pi r^2\left(h + \frac{r}{3}\right)\rho g$$

$$= 3.142\times(0.05)^2 \times \left(3.041 + \frac{0.05}{3}\right) 981 \text{ dyne}$$

$$= 23.56 \text{ dyne} \quad \text{or} \quad \frac{23.56}{981} \text{ gmf, i.e. } 0.024 \text{ gmf}.$$

Prob 14. *A barrier AB of length 12 m is hinged at A. At the lower end a horizontal spring keeps the barrier closed. The height of the water is 6 m and the width of the barrier is 5 m. Water level is 4 m below the hinge A. If minimum elongation of spring to keep barrier closed is 1 m, find the spring constant. Neglect atmospheric pressure?*





Sol. Torque applied by pressure force about A

$$\tau = \int_d^h (x-d)\rho g w dx . x = \rho g w\left[\frac{h^3}{3} - \frac{dh^2}{2} + \frac{d^3}{6}\right]$$

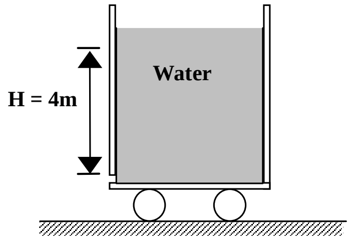
Putting values = $\tau = 72 \times 10^5$ Nm.

Taking moments about point A,

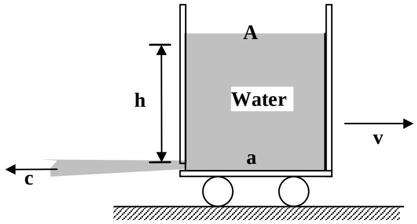
$72 \times 10^5 = (k \times 1)(12)$

$\Rightarrow$ k = 600 kN/m.

Prob 15. *A tank, initially at rest, is filled with water to a height H = 4 m. A small orifice is made at the bottom of the wall. Find the velocity attained by the tank when it becomes completely empty. Assume mass of the tank to be negligible. Friction is negligible.*



Sol. Let v be the instantaneous velocity of the tank and c be the instantaneous velocity of efflux with respect to the tank.



Thrust exerted on the tank is

$$F = \rho a c^2$$

where 'a' is the cross–sectional area of the orifice.

$$c = \sqrt{2gh}$$

where h is the instantaneous height of water in the tank.

Mass of the tank at any time t is, $m = \rho A h$

A = cross–sectional area of the tank.

Using Newton's second law,

$$F = m\frac{dv}{dt} = \rho A h\frac{dv}{dt} \qquad \therefore\ \rho A h\frac{dv}{dt} = \rho a c^2 = 2\rho g a h$$

or $\quad \dfrac{dv}{dt} = 2g\left(\dfrac{a}{A}\right) \qquad (1)$

In a time dt if the water level falls by dh, then according to the conservation of mass.

$$-\rho A dh = \rho a c dt \quad \text{or} \quad \frac{dh}{dt} = -\frac{ac}{A}$$

Equation (1) can be written as

$$\frac{dv}{dh}\frac{dh}{dt} = 2g\left(\frac{a}{A}\right) \quad \text{or} \quad \frac{dv}{dh}\left(-\frac{ac}{A}\right) = 2g\left(\frac{a}{A}\right)$$

or $\quad \dfrac{dv}{dh} = -\dfrac{2g}{c} = -\dfrac{2g}{\sqrt{2gh}} = -\sqrt{\dfrac{2g}{h}}$

On integrating

$$\int_0^v dv = -\sqrt{2g}\int_H^0 \frac{dh}{\sqrt{h}}, \qquad v = 2\sqrt{2gH}$$

Since H = 4 m, therefore $v = 2\sqrt{2(10)(4)} = 17.9\,m/s$

Prob 16. *What is the excess pressure inside a bubble of soap solutions of radius 5.00 mm? Given that the surface tension of soap solution at the temperature (20°C) is 2.50×10^{-2} N/m. If an air bubble of the same dimension were formed at a depth of 4.0 cm inside a container containing soap solution (relative density 1.20), what would be the pressure inside the bubble? (1atm = 1.01×10^5 Pa)*

Sol. $r = 5.00\times10^{-3}$m, $T = 2.50\times10^{-2}$ N/m, h = 40 cm = 0.4 m, $\rho = 1.20\times10^3$kg/m^3 and $P_0 = 1.01\times10^5$ Pascal.

Excess pressure inside a bubble of soap solution

$$\frac{4T}{r} = \frac{4\times2.50\times10^{-2}}{5.00\times10^{-3}} = 20 \text{ Pa}$$

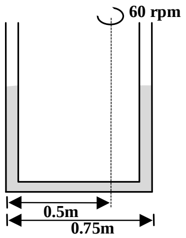
Excess pressure inside an air bubble in soap solution

$$\frac{2T}{r} = \frac{2\times2.50\times10^{-2}}{5.00\times10^{-3}} = 10 \text{ Pa}$$

Total pressure inside the air bubble = $P_0 + h\rho g + \frac{2T}{r}$

$= 1.01\times10^5 + 0.04\times1.2\times10^3\times9.8 + 10 = 1.015\times10^5$ Pa

Prob 17. *A vertical U-tube with the two limbs 0.75 m apart is filled with water and rotated about a vertical axis 0.5 m from the left limb, as shown in the figure. Determine the difference in elevation of the water levels in the two limbs when the speed of rotation is 60 rpm.*

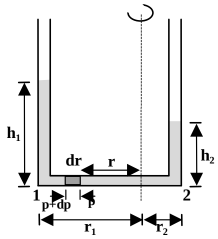


Sol. Consider a small element of length dr at a distance r from the axis of rotation. Considering the equilibrium of this element,

$$(p + dp) - p = \rho\omega^2 r\, dr$$

or $dp = \rho\omega^2 r\, dr$

On integrating between 1 and 2 ,



$$p_1 - p_2 = \rho\omega^2 \int_{-r_2}^{r_1} r dr$$

$$p_1 - p_2 = \frac{\rho\omega^2}{2}\left(r_1^2 - r_2^2\right)$$

or $h_1 - h_2 = \frac{\omega^2}{2g}\left[r_1^2 - r_2^2\right] = \frac{(2\pi)^2}{2(10)}[(0.5)^2 - [0.25)^2] = 0.37$ m.

Prob 18. *Two long capillary tubes of diameter 5.0 mm and 4.0 mm are held vertically inside water one by one. How much high the water will rise in each tube? ($g = 10$m/s^2, surface tension of water = 7.0×10^{-2}N/m.*

Sol. Height of water column in a capillary tube of radius r is given by

$$h = \frac{2T\ \cos\theta}{r\rho g} \qquad \text{. . . (1)}$$

where T is surface tension, ρ is density and θ is angle of contact of water–glass which can be assumed zero.

For the first tube, r = 2.5 mm = 2.5×10^{-3} m

$$\therefore \quad h = \frac{2\times\left(7.0\times10^{-2}\ \ N/m\right)}{\left(2.5\times10^{-3}m\right)\ \times\ \left(1\times10^{3}kg/m^{3}\right)\times\left(10N/kg\right)} = 5.6\text{ mm}$$

According to equation (1), for the same liquid, we have

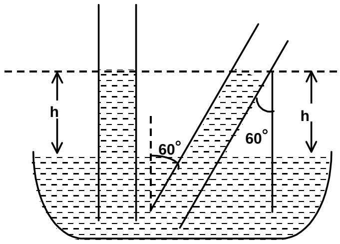
$$\therefore \quad hr = \frac{2T\ \ \cos\theta}{\rho g} = \text{constant}$$

If a liquid rises to a height h_1 in a capillary tube of radius r_1 and to a height h_2 in a capillary tube of radii r_2, then

$$h_1r_1 = h_2r_2$$

$$\text{or}\ \ h_2 = \frac{h_1r_1}{r_2} = \frac{5.6\times2.5}{2.0} = 7.0\text{ mm}$$

Prob 19. *Water rises in a capillary tube to a height 2.0 cm. In another capillary whose radius is one–third of it, how much the water will rise? If the first capillary is inclined at an angle of 60° with the vertical, then what will be the length of water column within the tube?*



Sol. Height of water column in a capillary tube of radius r is, in its vertical position, given by

$$h = \frac{2T\ \ \cos\theta}{r\rho g} \qquad \text{or}\ \ hr = \frac{2T\ \ \cos\theta}{\rho g}$$

or $\quad hr = \text{constant}\ (\therefore\ T, \theta, \rho, g \text{ are constant})$

In a capillary tube of radius one-third (r/3) that of the first tube, water will rise more. Suppose it rises to a height

$$h' = \frac{3hr}{r} = 3h = 3\times2.0 = 6.0\text{ cm}$$

When the first capillary is inclined at an angle of 60° to the vertical, the vertical height h (= 2.0) of the liquid will remain the same. Thus, if the length of water column in the capillary be h′ cm, then from figure, we get

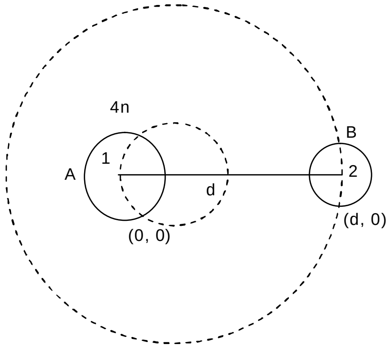
$$h' = \frac{h}{\cos60^\circ} = \frac{2.0}{1/2} = 4.0\text{ cm}$$

Prob 20. *Two masses 4m and m are separated by a distance 'd' from their centres are revolving about their centre of mass due to mutual force of attraction. Then determine*
(a) ratio of their time periods?
(b) ratio of their kinetic energies ?
(c) ratio of their velocities ?

Sol.

$$x_{cm} = \frac{4m(0)+m(d)}{5m} = \frac{d}{5} \quad\Rightarrow\ R_1 = d/5$$

$$R_2 = 4d/5$$



$$\text{(a)}\quad mr\omega^2 = \frac{Gm_Am_B}{d^2}$$

$$\Rightarrow\ \omega = \sqrt{\frac{Gm_Am_B}{m_0rd^2}} \quad = \frac{\omega_1}{\omega_2} = \sqrt{\frac{(m_0r)_2}{(m_0r)_1}}$$

$$\Rightarrow\ \frac{\omega_1}{\omega_2} = \sqrt{\frac{m(4d/5)}{(4m)d/5}} = 1:1$$

$$\Rightarrow T_1 : T_2 = \omega_2 :\ \omega_1 = 1:1$$

(b) $\frac{KE_1}{KE_2} = \frac{1/2(I_1\omega_1^2)}{1/2(I_2\omega_2^2)} = \frac{(1/2)(4m)(d/5)^2(T_2)^2}{(1/2)(m)(4d/5)^2(T_1)^2} = \frac{4(1)^2}{16(1)} = 1:4$

(c) $\frac{v_1}{v_2} = \frac{r_1\omega_1}{r_2\omega_2} = \frac{r_1T_2}{r_1T_1} = \frac{(d/5)(1)}{(4d/5)1} = 1:4$

Objective:

Prob 1. *The period of rotation of the earth so as to make any object weight–less on its equator is*
(A) 84 min (B) 74 minutes
(C) 64 minutes (D) 54 minutes
where M and r are the mass and radius of earth.

Sol. Put $g_e = 0$, in the expression

$g_e = g_0 - r\omega^2 \Rightarrow \quad \omega = \sqrt{\frac{g_0}{r}}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{r}{g_0}}$,

putting $r = 6.4 \times 10^6$ m and $g_0 = 9.8$ m/sec^2, we obtain, T = 84 min
Therefore (**A**)

Prob 2. *The gravitational field due to a mass distribution is given by $I = (A/x^3)$ in X–direction. The gravitational potential at a distance x is equal to*
(A) $-\frac{A}{x^3}$ (B) $-\frac{A}{2x^2}$
(C) $-\frac{A}{x^4}$ (D) $\frac{A}{2x^2}$.

Sol. The potential at a distance x is

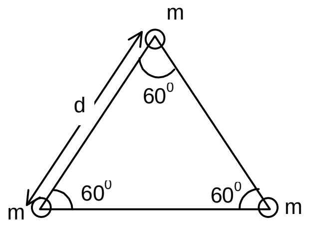
$V(x) = -\int_{\infty}^{x} I \, dx = -\int_{\infty}^{x} \frac{A}{x^3} dx$

$V(x) = \left[\frac{A}{2x^2}\right]_{\infty}^{x} = \frac{A}{2x^2}$

Therefore (**D**).

Prob 3. *Three particles each of mass m are placed at the corners of an equilateral triangle of side d. The potential energy of the systems is*
(A) $\frac{3Gm^2}{d}$ (B) $\frac{Gm^2}{d}$
(C) $\frac{-3Gm^2}{d}$ (D) none of these

Sol. For the system of two particles, gravitational energy is given as



$U = -Gm_1m_2/r$
$U_A = U_{12} + U_{23} + U_{31}$
$U_A = -3\frac{GMm}{d} = -\frac{3\ Gm^2}{d}$
Therefore (**C**).

Prob 4. *A body of mass m is approaching towards the centre of a hypothetical hollow planet of mass M and radius R. The speed of the body when it passes the centre of the planet through its diametrical chute is*

(A) $\sqrt{\frac{GM}{R}}$ (B) $\sqrt{\frac{2GM}{R}}$

(C) Zero (D) none of these.

Sol. At infinity, the total energy of the body is zero. Therefore the total energy of the body just before hitting the planet P will be zero according to the conservation of energy

$\Rightarrow$ $F_p = E_\infty = 0$ $\Rightarrow$ $U_p + K_p = 0$

$\Rightarrow$ $-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0$

$\Rightarrow$ $v = \sqrt{\frac{2GM}{R}}$.

Since the force imparted on a particle inside spherical shell is zero, therefore the velocity of the particle inside the spherical shell remain constant. Therefore, the body passes the centre of the planet with same speed $v = \sqrt{\frac{2GM}{R}}$.

Therefore (**B**).

Prob 5. *A planet has twice the density of earth but the acceleration due to gravity on its surface is exactly the same as that on the surface of earth. Its radius in terms of earth R will be*

(A) R/4 (B) R/2

(C) R/3 (D) R/8.

Sol. We know that

$g = \frac{GM}{R^2} = \frac{G}{R^2}\left[\frac{4}{3}\pi R^3 d\right]$

where d = mean density of earth.

For planet

$g' = \frac{G}{(R')^2} \times \left[\frac{4}{3}\pi R'^3 (2d)\right]$

Given that g = g′

Therefore, $= \frac{G}{R^2}\left[\frac{4}{3}\pi R^3 d\right] = \frac{G}{(R')^2} \times \frac{4}{3}\pi R'^3 (2d)$

Solving we get, R′ = (R/2).

Therefore (**B**).

Prob 6. *The energy required to remove a body of mass m from earth's surface is equal to*

(A) 2mgR (B) mgR

(C) –mgR (D) zero

Sol. The potential energy of the body on the surface of earth = $U_1 = -\frac{GMm}{R}$. The potential energy of the body at infinity $U_2 = 0$

$\Rightarrow \Delta U = U_2 - U_1 = \frac{GMm}{R} = mgR \quad \left(\because g = \frac{GMm}{R^2}\right)$

Therefore (**B**).

Prob 7. *A body weighs W in air and it loses its weight by 25% in water. The relative density of the body is:*

(A) 4 *(B) 1/4*
(C) 3/4 *(D) 4/3*

Sol. Relative density = Specific gravity

$$\text{R.D} = \frac{\text{Weight of the body in air}}{\text{Weight of the body in air} - \text{weight of the body in water}}$$

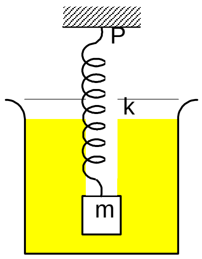
$$= \frac{W_a}{W_a - W_w} = \frac{W_a}{\Delta W} = \frac{W_a}{W_a/4}$$

[since weight of the body decreases by 25 % of its original weight W_a (weight in air)].

$\Rightarrow$ R.D. = 4.

Therefore **(A)**.

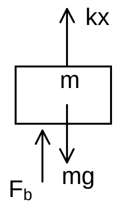
Prob 8. *A cube of mass m and density D is suspended from the point P by a spring of stiffness k. The system is kept inside a beaker filled with a liquid of density d. The elongation in the spring, assuming D > d, is:*



(A) $\frac{mg}{k}\left(1-\frac{d}{D}\right)$ *(B)* $\frac{mg}{k}\left(1-\frac{D}{d}\right)$

(C) $\frac{mg}{k}\left(1+\frac{d}{D}\right)$ *(D) None of these.*

Sol. The cube is in equilibrium under three forces as,



(a) spring force kx, where x = elongation of the spring,
(b) gravitational force w, weight of the cube = mg
(c) buoyant force F_b (or upward thrust) imparted by the liquid on the cube given as F_b = Vdg where V = volume of the immersed portion of the cube. For complete immersion, V = volume of the cube. For equilibrium of the cube,

kx + F_b = mg

$\Rightarrow$ $x = \frac{mg - F_b}{k} = \frac{mg - Vdg}{k}$ where V = (m/D)

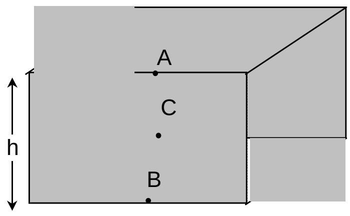
$\Rightarrow$ $x = \frac{mg}{k}\left(1-\frac{d}{D}\right)$

Therefore **(A)**.

Prob 9. *A cubical water tank is completely filled with water of mass m = 1000 kg. The force exerted by the water on the side wall of the tank, is:*

(A) 1.062 × 10^5 N *(B) 1.62 × 10^5 N*
(C) 0.049 × 10^5 N *(D) None of these.*

Sol. The cubical tank contains water of mass,



M = 1000 kg

$\Rightarrow$ Volume of water $V = \frac{m}{\rho_w}$

$= \frac{1000 \text{ kg}}{1000 \text{ kg/m}^3} = 1\text{m}^3$.

$\Rightarrow$ Volume of the tank = V = 1m^3.

$\Rightarrow$ Side of the tank = b = 1m.

The pressure at A = $P_A = P_{atm}$.

The pressure at B = $P_B = P_{atm} + \rho gh = P_{atm} + \rho g\ell$

$\Rightarrow$ The pressure at = $P_C = \dfrac{P_A + P_B}{2}$ = average pressure over the side wall

$= P_{atm} + \dfrac{\rho g\ell}{2}$

The force exerted on the side wall is

$$F = \left(P_{atm} + \frac{\rho gh}{2}\right)\left(\ell^2\right)$$

$$= \left(1.013\times10^5 + \frac{10^3\times9.8\times1}{2}\right)(1)^2\ \text{N}$$

$= (1.013 + 0.049) \Rightarrow \times 10^5$ N

$= 1.062 \times 10^5$ N

Therefore **(A)**.

Prob 10. *The piston of a syringe pushes a liquid with a speed of 1 cm/sec. The radii of the syringe tube and the needle are R = 1cm and r = 0.5 mm respectively. The velocity of the liquid coming out of the needle is:*

(A) 4 cm/sec *(B) 400 cm/sec*
(C) 10 cm/sec *(D) None of these.*

Sol. According to the equation of continuity,

$Av = av'$

$\Rightarrow \quad v' = (A/a)v$

where $A = \pi R^2$ and $a = \pi r^2$.

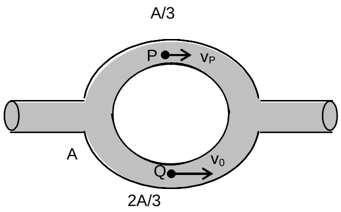
$$\Rightarrow \quad v' = \left(\frac{R}{r}\right)^2 v$$

$$= \left(\frac{1\ \text{cm}}{0.05\ \text{cm}}\right)^2 (1\ \text{cm/sec})$$

= 400 cm/sec.

Therefore **(B)**.

Prob 11. *In the figure shown, a liquid is flowing through a tube at the rate of 0.1 m^3/ sec. The tube is branched into two semi circular tubes of cross sectional areas A/3 and 2A/3. The velocity of liquid at Q is [the cross–section of the main tube (A) = 10^{-2} m^2 and V_P = 20 m/sec.]*



(A) 5 m/s *(B) 30m/s*
(C) 35 m/s *(D) None of these.*

Sol. Equation of continuity:

Rate = r = A/3 v_p + 2A/3v_Q

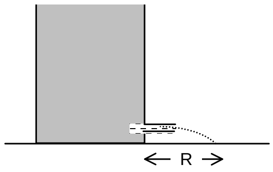
$$\Rightarrow \quad v_p + 2V_Q = \frac{3r}{A} \qquad \ldots(1)$$

$$\Rightarrow \quad v_p + 2v_Q = \frac{3\times0.1}{10^{-2}} = 30\ \text{m/sec.}$$

where v_p = 20 m/sec. $\Rightarrow v_Q$ = 5 m/sec.

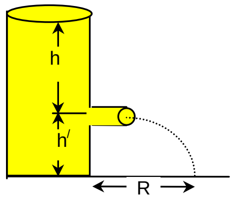
Therefore **(A)**.

Prob 12. *A small hole is made at a height of $h' = (1/\sqrt{2})$ m from the bottom of a cylindrical water tank and at a depth of $h = \sqrt{2}$ m from the upper level of water in the tank. The distance where the water emerging from the hole strikes the ground is:*



(A) $2\sqrt{2}$ m *(B) 1 m*
(C) 2 m *(D) None of these.*

Sol. Applying Bernoulli's Principle between the points 1 and 2



$\rho gh_1 + P_1 + (1/2)\,\rho v_1^2 = P_2 + (1/2)\,\rho v_2^2 + \rho gh_2$

Substituting $h_1 = h_2 = h'$, $v_1 = a/A\ v^2$ or, $v \approx$ as $a << A$

$v_{1\simeq 0}$, $P_1 = P_0 + \rho gh$, $P_2 = P_0$,

we obtain, $\rho gh = 1/2\,\rho v_2^2$ $v_2 = \sqrt{2gh}$ = v (say),

P_0 is the atmospheric pressure.

The range $R = v_2 \times t$, (where t = time of fall) can be given by

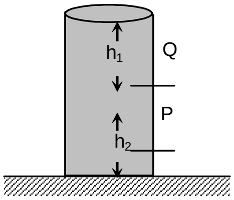
$h_1 = 1/2\ gt^2 \Rightarrow t = \sqrt{\dfrac{2h_1}{g}}$ $\Rightarrow R = v_2\sqrt{\dfrac{2h_1}{g}}$

Putting $v_2 = \sqrt{2gh}$, we obtain $R = 2\sqrt{h\ \ h'}$

Putting $h' = \dfrac{1}{\sqrt{2}}$ m and $h = \sqrt{2}$ m, we obtain R = 2 m

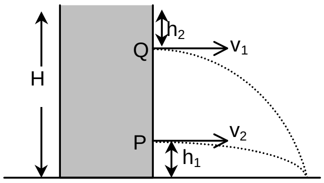
Therefore **(C)**.

Prob 13. *In a cylindrical water tank there are two small holes Q and P on the wall at a depth of h_1 from upper level of water and at a height of h_2 from the lower end of the tank respectively as shown in the figure. Water coming out from both the holes strike the ground at the same point. The ratio of h_1 and h_2 is:*



(A) 1 *(B) 2*
(C) >1 *(D) <1.*

Sol. The two streams strike at the same point on the ground,



$\Rightarrow$ $R_1 = R_2 = R$

$\Rightarrow$ $v_1 t_1 = v_2 t_2$ …(1)

where v_1 = velocity of efflux at Q = $\sqrt{2gh_1}$

and v_2 = velocity of the efflux at P = $\sqrt{2g(H-h_2)}$

t_1 = Time of fall of the water stream through P = $\sqrt{\dfrac{2(H-h_1)}{g}}$

t_2 = Time of fall of the water stream through Q = $\sqrt{\dfrac{2h_2}{g}}$

Putting these values in equation (1), we obtain,

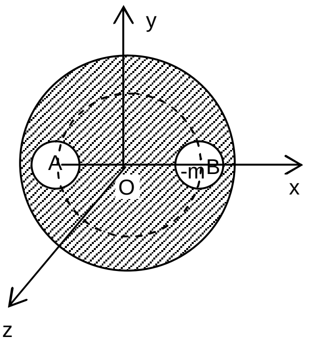
$(H - h_1)\,h_1 = (H - h_2)\,h_2 \Rightarrow [H-(h_1 + h_2)]\,[h_1 - h_2] = 0$

$\Rightarrow$ $H = h_1 + h_2$ is irrelevant because the holes are at two different heights, there fore $h_1 = h_2$

$\Rightarrow (h_1/h_2) = 1$

Therefore **(A)**.

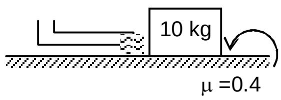
Prob 14. *A solid sphere of uniform density and radius 4 units is located with its center at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centers at A (–2, 0, 0) and B(2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in figure. Then*



(A) the gravitational force due to this object at the origin is zero.
(B) the gravitational force at the point B (2, 0, 0) is zero
(C) the gravitational potential is the same at all points of circle $y^2 + z^2 = 36$.
(D) the gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

Sol. The gravitational field is zero at the center of a solid sphere. The small spheres can be considered as negative mass m located at A and B. The gravitational force due to these masses on a mass at O is equal and opposite, hence the resultant force on mass at O is zero.
*(C and D) are correct because plane of these circles is y–*z .i.e. perpendicular to x–axis i.e. potential at any point on these two circles will be equal due to the positive mass M and negative masses –m and –m.

Prob 15. *A block of metal weighing 10 kg is resting on rough surface (μ = 0.4). It is struck by a jet releasing water at a rate of 2 kg/s and at a speed 8 m/s. The velocity of the block after 1 sec. is (Assume water comes to rest after striking the block.)*



(A) 0 *(B) 1*
(C) 2 *(D) 3*

Sol. $F = v\dfrac{dm}{dt} = 8 \times 2 = 16$ N
$f_{max} = 40$ N
So, a = 0
Velocity = 0
Hence, (A) is correct.

Prob 16. *Bhima was powerful enough to throw an elephant of mass m completely away from the earth. Assuming the mass of earth to be M and radius R, find the work done by the Bhima.*
(A) GM/R *(B) gMm/R*
(C) $mv^2/2$ *(D) GMm/R*

Sol. Work done = (change of P.E.)
Initial energy of the mass = –GMm/R
Final energy of the mass = 0
Work done = 0 – (–GMm/R) = GMm/R
Hence, (D) is correct.

Prob 17. The work done by gravity of earth in maintaining a satellite in its orbit is
(A) 0 *(B) >0*
(C) $\dfrac{\pi GMm}{r}$ *(D)* $-\dfrac{GMm}{2r}$
where M, m are masses of earth and satellite and r = radius of the orbit.

Sol. Since, the earth satellite is moving in circular orbit, the gravitational force of earth $\vec{F}_{gr}$ and the displacement $\vec{ds}$ of the satellite are mutually perpendicular.

$\Rightarrow$ Work done by gravity = $w_{gr} = \int \vec{F}_{gr} \cdot \vec{ds}$

$= \int F_{gr}\ ds\ \cos 90° = 0.$

Hence, (A) is correct.

18. STATE WHETHER THE FOLLOWING QUESTIONS ARE TRUE OR FALSE.

Prob 1. *Continuity equation is law of conservation of volume.*

Sol. **False.**
It is basically the law of conservation of mass.

Prob 2. *A body falling freely in a fluid cannot attain a velocity more than the terminal velocity.*

Sol. **True**
When a body starts falling in a fluid, its velocity increases. As the velocity increases, the viscous force on the body keeps on increasing. Thus, the body attains its making velocity as terminal velocity and then moves with that velocity only.

Prob 3. *The pressure of a fluid at a point depends upon the velocity of flow at that point.*

Sol. **True.**
Bernoulli's equation states that

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Thus, pressure depends upon the velocity of flow.

Prob 4. *The upthrust on a body which is floating in a fluid in a gravity free space is zero.*

Sol. ***True.***
The upthrust on the body is a property of weight of the liquid displaced. In a gravity–free space, the value of g = 0.
Thus, upthrust = 0.

Prob 5. *The water flowing through the same tube having two different cross section areas has different flow rates.*

Sol. **False.**
By equation of continuity, $a_1v_1 = a_1v_2$.

19. FILL IN THE BLANKS IN THE FOLLOWING QUESTIONS.

Prob 1. *A cube of wood supporting 200 gm mass just floats in water. When the mass is removed, the cube is raised by 2 cm. The size of the cube is __________________.*

Sol. **10 cm.**
We have

$0.2 \times 10 \times g = (2 \times \ell^2)\ g$

$\ell = 10$ cm

Prob 2. *A piece of cork is embedded inside an ice block which floats in water. When ice melts, the level of water will ________________.*

Sol. **Remain same.**

Prob 3. *A block of wood floats with (3/4)th of its volume submerged in water. The density of wood is ___________ .*

Sol. **0.75 g/cm^3**

We have, $\frac{3}{4} \times V \times 1 \times g = V \times \rho_w \times g$

$\rho_w = \frac{3}{4}$

Prob 4. *A body of density 2.5 g/cm^3 is let free in a liquid of density 1.25 g/cm^3. The downward acceleration of the body while sinking in the liquid is _____________ cm/s^2.*

Sol. **490 cm/s^2.**

We have,

$(V)(2.5) \times g - V(1.25)\, g = (V \times 2.5)\, a$

$a = g/2$

$a = \frac{980}{2} = 490 \text{ cm/s}^2$

Prob 5. *If the velocity of flow increases inside a tube, then at that point the pressure will _________.*

Sol. **decrease.**

According to Bernoulli's equation

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

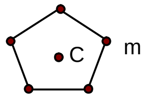
If $v_2 > v_1$, then $P_1 > P_2$.

ASSIGNMENT PROBLEMS

Subjective:

Level– O

1. Define gravitational field. What will be the gravitational field of 5 particles each of mass m kept at vertices of a pentagon on its centre?



2. Derive an expression for gravitational field due to a uniform thin spherical shell of radius R at a distance r from the centre of the shell when (a) $r > R$, (b) $r = R$, (c) $r < R$.

3. What will be the gravitational field due to a solid sphere of radius R at a point at a distance r from the centre if, (a) $r > R$, (b) $r = R$ (c) $r < R$?

4. If earth rotates with an angular speed ω and R is the radius of earth, find out acceleration due to gravity (g') at a point at latitude θ. What will be the value of g' at the equator ?

5. Define gravitational potential. Derive an expression for gravitational potential due to a solid sphere of radius R at a distance r from its centre when
(a) $r > R$, (b) $r = R$, (c) $r < R$.

6. What is binding energy ? Find out self energy of a sphere of radius R and mass M.

7. What are Kepler's laws of planetary motion ? Prove the second and third laws.

8. (a) State Archimedes principle and prove it with the help of principle of variation of pressure with height.
(b) What are the conditions of floating of a body in a liquid in equilibrium ?

9. State Bernoulli's theorem explaining the terms used.

10. (a) What are the properties of surface tension ?
(b) Define and derive an expression for surface energy of a liquid film.

11. What is a capillary tube? Define capillarity ?

12. What is Stoke's law ? Derive an expression for terminal velocity of a sphere of radius r falling vertically in a liquid with coefficient of viscosity η. Densities of material of sphere and liquid are ρ and σ, respectively.

13. Answer in one or two lines only.
(a) Can a geostationary satellite be placed above Chennai. Why ?
(b) For what purposes a geostationary satellite can be used ?
(c) For what purposes a polar satellite can be used ?
(d) Why doesn't moon fall down on earth due to gravitational force ?

(e) What is the time period of revolution of moon around the sun ?
(f) How is a satellite launched ?
(g) What is Reynold no. of a liquid ?
(h) Explain streamlined flow and turbulent flow.
(i) What are the conditional requirements of a liquid to follow Bernoulli's theorem ?
(j) Why a lifting force is produced on the wings of an aeroplane when air passes through them?
(k) What is the effect of temperature of a liquid on its surface tension ?
(l) Why are rain drops spherical ?
(m) Why does a chalk piece absorb ink after coming into its contact ?
(n) If air is blown into a soap bubble, what effect will it have on the pressure inside the bubble ?

14. A planet of mass 10×10^{23} kg is compressed into the shape of a sphere of radius R. If the escape velocity of a particle of mass 12.5 kg from its surface is 1.5×10^6 m/s, find the value of R.

15. A simple pendulum has time period of oscillation t at the equator. What will be its time period when checked at the South Pole? Take acceleration due to gravity, angular speed and radius of earth as g, ω and R, respectively.

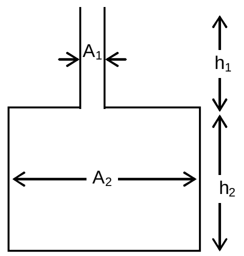
16. Mass and radius of planets A and B are m_1, r_1 and m_2, r_2 respectively. Distance between them is d. A particle of mass m is projected from midpoint of the line connecting centres of A and B so that it is just able to escape their gravitational forces. Find out the speed with which it is projected.

17. A body of mass 50 kg travelling in free space with speed 5 km/s falls on earth. What will be its velocity and energy when it touches the earth? (R_e = 6400 km, g = 9.8 m/s^2)

18. A spaceship is launched into a circular orbit close to the earth's surface. What additional velocity has now to be imparted to the spaceship in the orbit to overcome the gravitational pull?

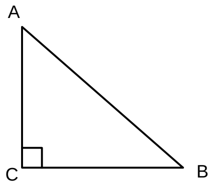
19. A liquid of density ρ is filled in a beaker of cross section A to a height H and then a cylinder of mass M and cross section 'a' is made to float in it. If the atmospheric pressure is P_0, find the pressure (a) at the top face B of the cylinder (b) at the bottom face C of the cylinder and (c) at the base D of the beaker. (d) Can ever there pressures be equal ?

20. The vessel shown in the given figure has two sections of areas of cross section A_1 and A_2. A liquid of density ρ fills in both the sections, upto a height h_1 and h_2 respectively. Neglect atmospheric pressure. Find
(a) the pressure exerted by the liquid on the base of the vessel.
(b) the force exerted by the liquid on the base of the vessel.
(c) the downward force exerted by the walls of the vessel at the level B.



Level – I

1. Two small particles of mass m each are placed at the vertices A and B of a right angle isosceles triangle. If AB = ℓ, find the gravitational field strength at C.



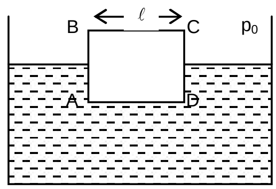
2. A body of mass m is taken to a height of kR from the surface of the earth very slowly, R being the radius of the earth. Find the change in gravitational potential energy in this process. (m_e is the mass of earth)

3. A space vehicle of mass m is in a circular orbit of radius $2R_e$ about the earth (m_e). What is the work done by an external agent to transfer it to an orbit of radius $4R_e$?

4. A body is thrown up (radially outward from the surface of the earth) with a velocity equal to one-fourth of the escape velocity. Find the maximum height reached from the surface of the earth. (Radius of earth is R_e)

5. Determine the binding energy of an earth–satellite system for an orbital radius r. Assume a circular orbit. (Given mass of satellite is m and mass of earth is m_e)

6. A cubical wooden block of specific gravity s_1 floats in a liquid of specific gravity s_2 as shown in the figure. The atmospheric pressure is P_0. Determine
(i) the force exerted on the face AD,
(ii) the force exerted on the face BC.
(Density of water is ρ_w).



7. (a) A hollow metal sphere of mass m has an instantaneous upward acceleration 'a' when released from rest fully submerged in a liquid of density d_1. If the density of the metal is d_2, find the volume of the cavity in the sphere.
(b) A wooden cube of side 10 cm and specific gravity 0.8 floats in water with its upper surface horizontal. What depth of the cube remains immersed ? What mass of aluminum of specific gravity 2.7 must be attached to
(i) the upper surface,
(ii) the lower surface so that the cube will be just immersed?

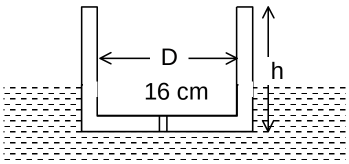
8. (a) Find the work done in increasing the radius of a soap bubble from initial radius r_1 to final radius r_2. Given T = surface tension of soap solution.
(b) Two soap bubbles of the same radius r coalesce isothermally to form a bigger bubble of radius R. If the atmospheric pressure is P_o, find the surface tension of the soap solution.

9. (a) If n identical water droplets falling under gravity with terminal velocity v coalesce to form a single drop which has the terminal velocity 4v, find the number n.

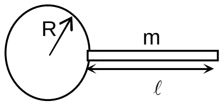
(b) If the velocity of water near the surface of a 6 m deep river is 6 m/s, assuming uniform velocity gradient along the depth, find the shear stress between the horizontal layers of water (coefficient of viscosity of water = 10^{-2} poise).

10. An incompressible non–viscous fluid (density δ) flows steadily through a cylindrical pipe which has radius 2R at point A and radius R at point B (at the same height as A) further along the flow direction. If the velocity and pressure at point A are v and P, respectively, find the pressure at B.

11. A cylindrical vessel of thickness 2 cm floats in a liquid as shown in figure. With a depth of 8 cm immersed, the vessel develops a leak in its bottom. What should be the minimum height of the vessel so that it may not sink.

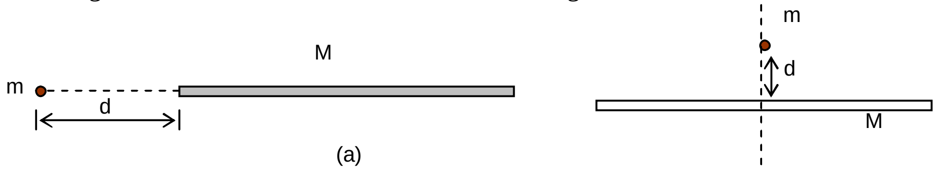


12. Find the potential energy of interaction between a solid sphere of mass M and radius R and a rod of mass m and length ℓ placed as shown in the figure.



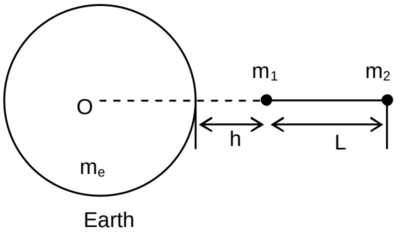
Level – II

1. Find the gravitational force of attraction between a mass particle m and a uniform slender rod of mass m and length L for the two orientations shown in the figure.



(a) (b)

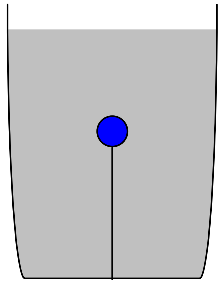
2. Two bodies of masses m_1 and m_2 are connected by a long inextensible cord of length L. The combination is allowed to fall freely towards to the earth (mass m_e), the direction of the cord being always radial as shown in the figure. Find (a) the tension in the cord, and (b) the accelerations of m_1 and m_2. Does the cord ever become slack ?



3. A missile is launched at an angle of 60° to the vertical with a velocity $\sqrt{\frac{0.75Gm_e}{R_e}}$ from the surface of the earth. Neglecting air resistance, find the maximum height reached by the missile from the surface of the earth. (m_e = mass of the earth, R_e = radius of the earth)

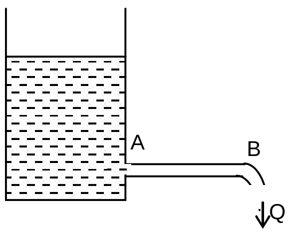
4. Distance between the centre of two stars is 10a. The masses of these start are M and 16M and their radii are a and 2a, respectively. A body of mass m is fired straight from the surface of larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G, M and a.

5. A solid sphere of mass m = 2 kg and specific gravity s = 0.5 is held stationary relative to a tank filled with water as shown in figure. The tank is accelerating vertically upward with acceleration $\alpha = 2\ ms^{-2}$.
   (a) Calculate the tension in the thread connecting the sphere and the bottom of the tank.
   (b) if the thread snaps, calculate the acceleration of the sphere with respect to the tank.
   (density of water is $\rho = 1000\ kg/m^3$ & $g = 10\ m/s^2$).

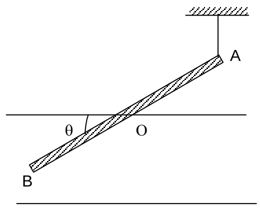


6. A small hollow vessel which has a small hole in it is immersed in water to a depth of 40 cm before water enters into the vessel. Calculate the radius of the hole. [Surface tension of water $= 70 \times 10^{-3}$ N/m, density of water $= 10^3\ kg/m^3$]

7. Water flows out of a big tank along a horizontal tube AB of length ℓ and radius r and bent at right angles at the other end as shown in the figure. The rate of flow is Q m^3/s. Calculate the moment of the force exerted by the water on the tube about the end A.

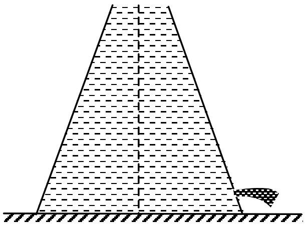


8. A uniform rod AB, 4 m long and weighing 12 kg, is supported at end A, with a 6 kg lead weight at B. The rod floats as shown in the figure with one-half of its length submerged. The buoyant force on the lead mass is negligible as it is of negligible volume. Find the tension in the cord and total volume of the rod.



9. Two soap bubbles of radii a and b combine under isothermal conditions to form a single bubble of radius c without any leakage of air. If P_o = atmospheric pressure and T = surface tension of soap solution, show that $P_o = \dfrac{4T\left(a^2+b^2-c^2\right)}{c^3-a^3-b^3}$.

10. The curved surface of a vessel has the shape of a truncated cone having semi–vertex angle $\alpha = 37°$. The top and bottom radii of the vessel are $r_1 = 3$ cm and $r_2 = 12$ cm, respectively, and height h=12 cm. The vessel is full of water (density $\rho = 1000$ kg/m^3) and is placed on a smooth horizontal plane. Calculate



   (a) mass of the liquid in the vessel,
   (b) force on the bottom of the vessel,
   (c) force on the curved wall.
   Now, hole having area S = 1.5cm^2 is made in curved wall near the bottom. Calculate
   (d) velocity of efflux,
   (e) horizontal force required to keep the vessel in static equilibrium.

11. A uniform ring of mass 'm' and radius 'a' is placed directly above a uniform sphere of mass M and of equal radius. The centre of the ring is at a distance $\sqrt{3}$ a from the centre of the sphere. Find the gravitational force exerted by the sphere on the ring.

12. A metal sphere of mass m having cavity inside has an instantaneous upward acceleration 'a' when released from rest fully submerged in a liquid of density d_1. Taking the density of the metal as d_2, find the volume of the cavity in the sphere.

Objective:

Level – I

1. The weight of a body at the centre of the earth is:
   (A) zero (B) infinite
   (C) same as on the surface of earth (D) none of the above.

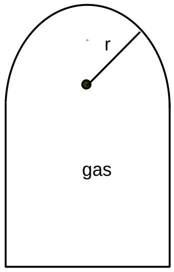
2. The earth revolves round the sun in an elliptical orbit. Its speed
   (A) goes on decreasing continuously
   (B) is greatest when it is closest to the sun
   (C) is greatest when it is farthest from the sun
   (D) is constant at all the points on the orbit.

3. Two satellites are orbiting around the earth in circular orbits of same radius. One of them is 10 times greater in mass than the other. Their periods of revolutions are in the ratio:
   (A) 100:1 (B) 1:100
   (C) 10:1 (D) 1:1.

4. An earth satellite is moving round the earth in a circular orbit. For such a satellite, which of the following statements is wrong ?
   (A) It is a freely falling body.
   (B) It is moving with a constant speed.
   (C) Its acceleration is zero.
   (D) Its angular momentum remains constant.

5. An earth satellite has a mass M and angular momentum L. Its areal velocity is given by
   (A) L/M m/s (B) 2L/M m^2/s
   (C) L/2M m^2/s (D) L/2M m/s

6. The pressure of the gas in a cylindrical chamber is P_0. The vertical force exerted by the gas on its hemispherical end is
   (A) P_0r^2 (B) $4P_0\pi r^2$
   (C) $2P_0\pi r^2$ (D) $P_0\pi r^2$



7. Let V and E be the gravitational potential and gravitational field at a distance r from the centre of a uniform spherical shell. Consider the following two statements:
   (a) The plot of V against r is discontinuous.
   (b) The plot of E against r is discontinuous:
   (A) Both (a) and (b) are correct (B) (a) is correct but (b) is wrong
   (C) (b) is correct but (a) is wrong (D) Both (a) and (b) are wrong.

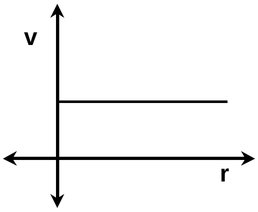
8. If a body is projected with a speed less than the escape velocity,
   (A) it must reach a certain height and may fall down following a straight path.
   (B) it must reach a certain height and may fall down following a parabolic path.
   (C) it may orbit the earth in a circular or elliptic orbit.
   (D) it must orbit the earth in a circular orbit.

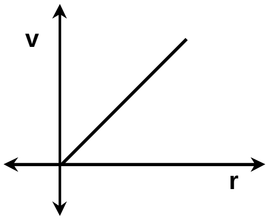
9. A planet has twice the density of earth but the acceleration due to gravity on its surface is exactly the same as on the surface of earth. Its radius in terms of radius of earth R will be:
   (A) R/4 (B) R/2
   (C) R/3 (D) R/8.

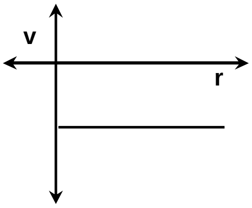
10. A mass particle of mass m_1 lies inside a spherical shell of mass m_2 and radius R at a distance r from the centre. The gravitational potential energy of the system is
   (A) $-\frac{Gm_1m_2}{r}$ (B) $-\frac{Gm_1m_2}{R}$
   (C) $+\frac{Gm_1m_2}{r}$ (D) $+\frac{Gm_1m_2}{R}$

11. A missile is launched with a velocity less than the escape velocity. The sum of its kinetic and potential energies is
   (A) positive (B) negative
   (C) zero
   (D) may be positive or negative depending upon its initial velocity

12. The variation of gravitational potential, inside a hollow sphere is given as

(A)


(B)


(C)


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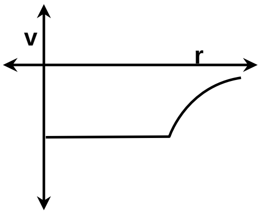


13. A spherical planet far out in space has a mass M_0 and diameter D_0. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity which is equal to
   (A) $\frac{GM_0}{D_0^2}$ (B) $\frac{4mGM_0}{D_0^2}$
   (C) $\frac{4GM_0}{D_0^2}$ (D) $\frac{GmM_0}{D_0^2}$

14. An earth satellite is moved from one stable circular orbit to a farther stable circular orbit. Which one of the following quantities increase?
   (A) gravitational force. (B) gravitational potential energy.
   (C) linear orbital speed. (D) centripetal acceleration.

15. A body of mass m is taken from earth's surface to a height equal to radius of earth. The change in potential energy will be
(A) mgR
(B) $\frac{1}{2}$mgR
(C) 2mgR
(D) $\frac{1}{4}$mgR

16. The weight of a body at earth's surface is W. At a depth half way to the centre of the earth, it will be (assuming uniform density in earth)
(A) W
(B) W/2
(C) W/4
(D) W/8

17. A mass of 1kg on earth weighs (1/6) kg on moon. The radius of the moon is 1.738×10^6m. The mass of the moon is ($g_e = 9.8$ m/s^2, $G = 6.67\times10^{-11}$ Nm^2/kg^2)
(A) 7.4×10^{22} kg
(B) 4.35×10^{16} kg
(C) 4.4×10^{23} kg
(D) 7.4×10^{20} kg

18. The escape velocity of an object on a planet, whose g value is 9 times on earth and whose radius is 4 times that of earth in km/s, is :
(A) 67.2
(B) 33.6
(C) 16.8
(D) 25.2

19. The escape velocity from earth is 11.2 km/s. If a body is to be projected in a direction making an angle of 45° to the vertical, then the escape velocity is
(A) 11.2×2 km/s
(B) 11.2 km/s
(C) $11.2 \times \frac{1}{\sqrt{2}}$ km/s
(D) $11.2\sqrt{2}$ km/s

20. The radius of the earth is 6400km and g = 10 m/s^2. In order that a body of 5kg weighs zero at the equator, the angular speed of the earth is
(A) 1/80 rad/s
(B) 1/400 rad/s
(C) 1/800 rad/s
(D) 1/1600 rad/s

21. Small droplets of a liquid are usually more spherical in shape than larger drops of the same liquid because
(A) force of surface tension is equal and opposite to the force of gravity
(B) force of surface tension predominates the force of gravity
(C) force of gravity predominates the force of surface tension
(D) force of gravity and force of surface tension act in the same direction and are equal

22. Water flows in a streamline manner through a capillary tube of radius ‘a’. The pressure difference is P and the rate of flow is Q. If the radius is reduced to a/2 and the pressure is increased to 2P, then the rate of flow becomes
(A) 4Q
(B) Q/2
(C) Q
(D) 2/8

23. When a venturimeter is used in an inclined position, it will show
(A) same reading
(B) higher reading
(C) lower reading
(D) depends on viscosity of liquid

24. A river is flowing at 5 m/s. The river is 10 m deep. What is the shearing stress between the horizontal layers of the river? Given that viscosity of water is 1 mPI.
(A) 2×10^{-3} N/m^2 (B) 10^{-3} N/m^2
(C) 0.5×10^{-3} N/m^2 (D) 0.1×10^{-3} N/m^2

25. A hollow glass stopper of relative density 2.5 just floats in water. The ratio of volume of cavity to that of stopper is
(A) 1 : 2 (B) 3 : 5
(C) 1 : 5 (D) 3 : 2

26. If a graph is drawn between the capillary height and radius of the capillary for a given liquid, which of the following will it be?
(A) Straight line through origin having positive slope.
(B) Straight line through origin having negative slope.
(C) Rectangular hyperbola.
(D) Parabola.

27. A wooden cylinder is floating on water in a beaker which is placed in a lift. When the lift is at rest, one–third of the volume of the wooden cylinder is exposed above water. The lift now moves up with an acceleration g/2. The fraction of the volume exposed now is
(A) 1/3 (B) 1/2
(C) 1/6 (D) 2/3

28. A boat having a length of 3 m and breadth 8 m is floating on water in a lake. The boat sinks by 1 cm when a person gets on it. The mass of the person is
(A) 60 kg (B) 62 kg
(C) 12 kg (D) 128 kg

29. A block of volume V and density ρ_b is placed in a liquid of density σ_ℓ ($\sigma_\ell > \sigma_b$), then the block is moved upwards due to buoyant force upto a height h. The increase in potential energy is
(A) $\sigma_b vgh$ (B) $(\sigma_b + \sigma_\ell)vgh$
(C) $(\sigma_b - \sigma_\ell)vgh$ (D) none of these

30. The height of mercury column in a simple barometer is h. As the tube is inclined to the vertical at an angle α, the length of mercury column along the length of the tube is ℓ. Thus,
(A) $\ell = \dfrac{h}{\cos\alpha}$ (B) $\ell = h \cos \alpha$
(C) $\ell = h$ (D) $h = \ell \cos \alpha$

FILL IN THE BLANKS IN THE FOLLOWING QUESTIONS.

31. The work done in taking a unit mass from one point to another on an equipotential surface is ____________________.

32. The velocity of the satellite in orbit relies upon ________________ of the radius and the acceleration due to gravity at that planet.

33. The escape velocity of a body is ______________ on different celestial bodies.

34. All the bodies fall in uniform gravitational field with the same ________.

35. If a planet gets inflated keeping its density constant, then the escape velocity will ______.

STATE WHETHER THE FOLLOWING QUESTIONS ARE TRUE OR FALSE.

36. The earth, moving around the sun in a circular orbit, is acted upon by a force and hence work must be done on earth by this force.

37. If a planet gets inflated, keeping its density constant, then g will increase.

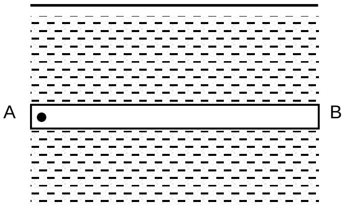
38. Meniscus of mercury in capillary is convex.

39. A liquid drop of radius 'R' is divided into eight identical droplets. If the surface tension is 'T', then the work done in this process is equal to $4\pi R^2 T$.

40. Density of ice is ρ and that of water is σ. The decrease in volume when a mass M of ice melts is $\left(\frac{M}{\sigma-\rho}\right)$.

Level – II

1. A uniform wooden rod of length L and specific gravity S is free to rotate in vertical plane about the end A. The rod is released from the horizontal position inside a liquid of specific gravity 1.5S as shown in the figure. The instantaneous acceleration of the end B is

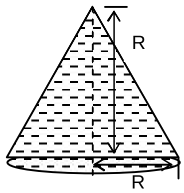


(A) g (B) g/4
(C) 3g/4 (D) 3g/8

2. The ratio of the radius of the earth to that of the moon is 10. The ratio of the acceleration due to gravity on the earth to that on the moon is 6. The ratio of the escape velocity from the earth's surface to that from the moon is
(A) 4 (B) 6
(C) 7.75 (D) 12

3. If the radius of the earth shrinks by 1%, its mass remaining same, the acceleration due to gravity on the surface of the earth will
(A) decrease by 2% (B) decrease by 0.5 %
(C) increase by 2% (D) increase by 0.5 %

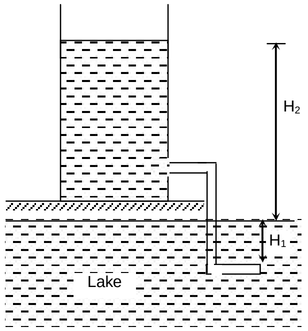
4. A conical vessel is completely filled with a liquid of density σ as shown in the figure. The force exerted by the liquid on the curved surface is



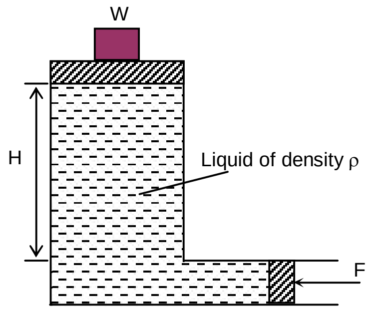
(A) $\frac{\pi R^3 \sigma g}{3} (\uparrow)$ (B) $\frac{\pi R^3 \sigma g}{3} (\downarrow)$
(C) $\frac{2\pi R^3 \sigma g}{3} (\uparrow)$ (D) $\frac{2\pi R^3 \sigma g}{3} (\downarrow)$

5. Air stream flows horizontally past an aeroplane wing of surface area 4 m^2. The speed of air over the top surface is 60 m/s and under the bottom surface is 40 m/s. The force of lift on the wing is (density of air =1 kg/m^3)
(A) 800 N (B) 1000 N
(C) 4000 N (D) 3200 N

6. Water flows from a large reservoir through a pipe of cross sectional area S into a big stagnant lake. The level of water in the lake is at a height H_1, above the end of the pipe as shown in the figure. The flow rate of water under steady state is
(A) $S\sqrt{2gH_1}$ (B) $S\sqrt{2gH_2}$
(C) $S\sqrt{2g(H_2 - H_1)}$ (D) $S\sqrt{2g(H_1 + H_2)}$



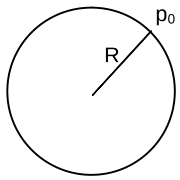
7. A heavy load W is supported on a platform of area S by applying a force F on a small piston of area S/10. The value of F for equilibrium is



(A) $\frac{W}{10}$ (B) $\frac{W+\rho gHS}{10}$
(C) $\frac{W-\rho gHS}{10}$ (D) 10 W

8. An artificial satellite moving in a circular orbit around the earth has total mechanical energy E_0. Its potential energy is
(A) $-2E_0$ (B) $1.5\ E_0$
(C) $2E_0$ (D) E_0

9. A spherical soap bubble of radius R has an internal pressure p in excess of the surrounding atmospheric pressure P_0. The force exerted by one half of the bubble on the other half is



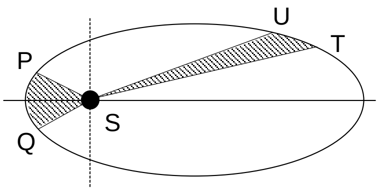
(A) p_0R^2 (B) $4\pi pR^2$
(C) πpR^2 (D) $p_0\pi R^2$

10. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then T is proportional to
(A) $R^{1.75}$ (B) $R^{2.25}$
(C) $R^{1.5}$ (D) R^3

11. The gravitational field due to a mass distribution is $E = K/x^3$ in the x–direction (K is constant). Taking the gravitational potential to be zero at infinity, its value at a distance x is:
(A) K/x (B) K/2x
(C) K/x^2 (D) $K/2x^2$

12. If velocity V be given to an object on surface of earth, then the object's total mechanical energy becomes (mass of earth M, radius of earth R, mass of object M)
(A) $-\frac{GMm}{r}$ (B) $-\frac{GMm}{r}+\frac{1}{2}mV^2$
(C) $\frac{1}{2}mV^2$ (D) $-2\frac{GMm}{r}$

13. If figure shows the path of a planet about sun. The two shaded parts have areas PSQ = 4×10^{16} m^2 and TSU = 6×10^{16} m^2. If time taken to go from P to Q is t_1 and T that from to U is t_2, then



(A) $t_2 = t_1$ (B) $t_2 = (3/2)\ t_1$
(C) $t_2 = 2t_1$ (D) $t_2 = (2/3)\ t_1$

14. A body of mass m starts falling from rest from a point at a distance R from the centre of earth. If R_o and M be the radius and mass of earth, respectively, the velocity acquired by the body when it reaches the surface of the earth will be

(A) $\sqrt{2GM\left(\frac{1}{R}-\frac{1}{R_o}\right)}$ (B) $\sqrt{2GM\left(\frac{1}{R_o}-\frac{1}{R}\right)}$

(C) $\sqrt{2GM/R}$ (D) $\sqrt{2gR}$

15. Two satellites of masses m_1 and m_2 ($m_1 > m_2$) are revolving round the earth in circular orbits of radii r_1 and r_2 ($r_1 > r_2$), respectively. Which of the following statements is true regarding their velocities v_1 and v_2?

(A) $v_1 = v_2$ (B) $v_1 < v_2$

(C) $v_1 > v_2$ (D) $\frac{v_1}{r_1} = \frac{v_2}{r_2}$

16. A planet moves around the Sun. At a point P, it is closest from the Sun at a distance d_1 and has a speed v_1. At another point Q, when it is farthest from the sun at a distance d_2, its speed will be

(A) $\frac{d_1^2 v_1}{d_2^2}$ (B) $\frac{d_2 v_1}{d_1}$

(C) $\frac{d_1 v_1}{d_2}$ (D) $\frac{d_1^2 v_1}{d_1^2}$

17. Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(A) $v = \frac{1}{2R}\sqrt{\frac{1}{GM}}$ (B) $v = \sqrt{\frac{GM}{2R}}$

(C) $v = \frac{1}{2}\sqrt{\frac{GM}{R}}$ (D) $v = \sqrt{\frac{4GM}{R}}$

18. The distances of Neptune and Saturn from the sun are nearly 10^{13}m and 10^{12}m, respectively. Assuming that they move in circular orbits, their periodic times will be in the ratio

(A) 10 (B) 100

(C) $10\sqrt{10}$ (D) 1000

19. Gravitational field intensity on the surface of the earth is maximum at the equator due to

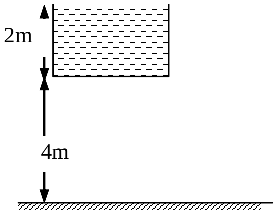
(i) rotation of the earth around its axis.
(ii) the shape of the earth.
(iii) temperature difference.
(iv) all of these

Select the correct option.

(A) (i) only. (B) (ii) only.

(C) (i) & (ii) only. (D) none of these.

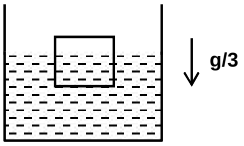
20. At what height from the ground a small hole should be cut in a tank to get a maximum range on horizontal ground.



(A) 3 m (B) 4 m
(C) 5 m (D) 6 m

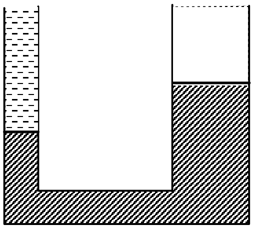
21. The amount of work done in increasing diameter of a soap bubble from 2 cm to 5 cm is (Surface tension of soap solution is $3.0\times10^{-2}\,N/m$)
(A) 3.2×10^{-4} J (B) 2.8×10^{-4} J
(C) 3.96×10^{-4} J (D) 2.42×10^{-4} J

22. A cubical block is floating in a liquid with half of its volume immersed in the liquid. When the whole system accelerates downward with an acceleration g/3, the fraction of volume immersed in liquid will be



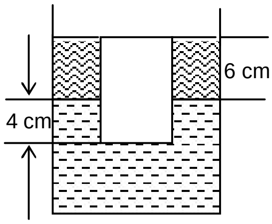
(A) 1/2 (B) 3/8
(C) 2/3 (D) 3/4

23. A U tube in which the cross sectional area of the limb on the left is one-third of the limb on the right contains mercury (density 13.6 gm/cm^3). The level of mercury in the narrow limb extends to a distance of 30 cm from the upper end of the tube. What will be the rise in the level of mercury in the right limb if the left limb is filled gradually to the top with water (Neglect surface tension effects)



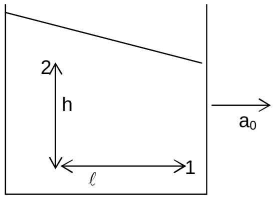
(A) 2.2 cm (B) 0.6 cm
(C) 1.8 cm (D) 1.2 cm

24. A 10 cm cubical block of wood floats with 4 cm inside the water and 6 cm inside the oil. The density of oil is 0.6 g/cm^3. The mass of the block is



(A) 607 g (B) 670 g
(C) 760 g (D) 706 g

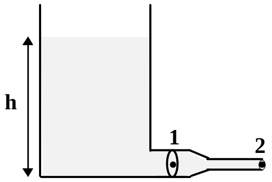
25. A beaker containing a liquid of density ρ is accelerated forward with constant acceleration = a_0. Then, the pressure difference between the two points 2 and 1 as shown in the figure is



(A) $\rho(a_0\ell - gh)$ (B) $\rho(a_0\ell + gh)$
(C) ρgh (D) $\rho g\sqrt{\ell^2 + h^2}$

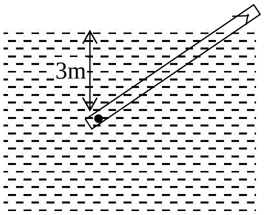
26. An incompressible non–viscous fluid (density δ) flows steadily through a cylindrical pipe which has radius 2R at point A and radius R at point B (at the same height as A) further along the flow direction. If the velocity and pressure at point A are v and P respectively, the pressure at B will be
(A) $P - \frac{1}{2}\delta V^2$ (B) $P - 8\delta V^2$
(C) $P + 2\delta V^2$ (D) $P - 7.5\,\delta V^2$

27. A tank of very large cross–sectional area is filled to a height h with a liquid of density ρ. If the atmospheric pressure is equal to P_0, and the cross–sectional area at 1 is twice of that at 2, then the pressure at the cross–section 1 will be equal to



(A) $P_0 - \frac{3}{4}\rho gh$ (B) $P_0 + \frac{1}{2}\rho gh$
(C) $P_0 + \frac{3}{4}\rho gh$ (D) $P_0 - \frac{1}{2}\rho gh$

28. A rod having specific gravity 0.5 and length 6 m has a mass of 12 kg. It is hinged at one end at a distance of 3 m below the water surface. The weight w attached to the other end so that 5 m rod be submerged is



(A) 25 kgf (B) 7/3 kgf
(C) 20 kgf (D) 15/4 kgf

29. There are three immiscible liquids of densities ρ_1, ρ_2 and ρ_3. If the first two liquids are taken in a U–tube, then they stand like Fig.(i). When all the three liquids are taken in a U – tube, they stand like Fig.(ii). The ratio of the densities of the three liquids are

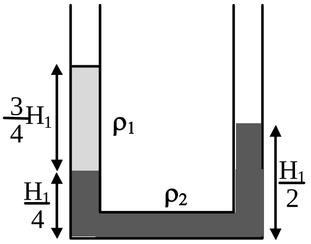


Fig.(i)

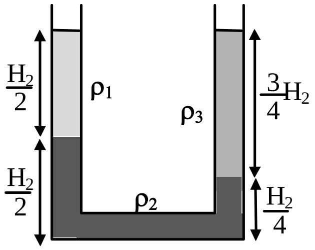


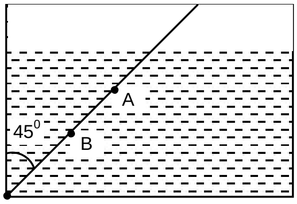
Fig.(ii)

(A) 3 : 9 : 5 (B) 1 : 3 : 5
(C) 1 : 5 : 3 (D) 3 : 5 : 9

30. Let v and E denote the gravitational potential and gravitational field at a point. It is possible to have
(A) v = 0 and E = 0 (B) v = 0 and E # 0
(C) v # 0 and E = 0 (D) v # 0 and E # 0

31. If r be the distance of a point from the center of a uniform solid sphere of mass M and radius R, then the gravitational field intensity I is proportional to
(A) r for r < R (B) $\frac{1}{r^2}$ for r > R
(C) both (A) and (B) (D) neither (A) nor (B)

32. A wooden plank of length 1m and uniform cross-section is hinged at one end to the bottom of a tank. The tank is filled with water upto a height of 0.5 m and the specific gravity of the plank is 0.5. The plank makes an angle of 45° with the vertical in the equilibrium position. A is the mid point of the plank and B is the middle point of the dipped part of the plank. Which of the following is correct?

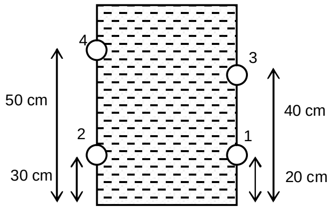


(A) The weight of the plank acts downwards at A. (B) The buoyant force acts through B.
(C) $OB = \frac{1}{\sqrt{2}}$ m (D) Both (B) and (C)

33. Which of the following is correct?
(A) If ρ is the mass density, then ρg is weight density.
(B) The SI unit of weight density is N/m^3
(C) Specific gravity is a dimensionless ratio which has the same value for all systems of units.
(D) Only (B) and (C) are correct.

34. A vertical glass capillary tube open at both ends, contains some water. Which of the following shapes may be taken by the water in the tube ?

35. Figure shows a cylindrical vessel of height 90 cm filled upto the brim, there are four holes in the vessel as shown. The liquid falling at maximum horizontal distance from the vessel would come from
(A) hole 1 (B) hole 2
(C) hole 3 (D) hole 4



36. The magnitudes of the gravitational fields at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2, respectively. Then,
(A) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$
(B) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 > R$ and $r_2 > R$
(C) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$
(D) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. Zero

2. (a) $\frac{GM}{r^2}$ (b) $\frac{GM}{R^2}$ (c) zero

3. (a) $\frac{GM}{r^2}$ (b) $\frac{GM}{R^2}$ (c) $\frac{GMr}{R^3}$

4. $g - R\omega^2\cos^2\theta$, $g - R\omega^2$

5. (a) $-\frac{GM}{r}$ (b) $-\frac{GM}{R}$ (c) $-\frac{3GM}{R^3}\left[\frac{R^2}{2}-\frac{r^2}{6}\right]$

6. $-\frac{3}{5}\frac{GM^2}{R}$

12. $\frac{2\pi r^2}{9\eta}(\rho-\sigma)g$

14. 59.3 m

15. $t\sqrt{1-\frac{\omega^2 R}{g}}$

16. $\sqrt{\frac{4(m_1+m_2)G}{d}}$

17. 12.26 km/s, 3.76×10^9 J

18. 3.28×10^3 m/s

20. (a) $2h\rho g$ (b) $2h\rho gA_2$ (c) $h\rho g(A_2 - h)$

Level – I

1. $\frac{2\sqrt{2}Gm}{\ell^2}$

2. $\frac{k}{k+1}\left(\frac{Gmm_e}{R}\right)$

3. $\frac{Gmm_e}{8R_e}$

4. $\frac{R_e}{15}$

5. $\frac{Gmm_e}{2r}$

6. (i) $P_0\ell^2 + \rho_w s_1 g\ell^3$ (ii) $P_0\ell^2$

7. (a) $\left[\frac{m}{d_1}\left(1+\frac{a}{g}-\frac{d_1}{d_2}\right)\right]$ (b) 8 cm (i) 200 gm (ii) 317.6 gm.

8. (a) $8\pi T(r_2^2 - r_1^2)$ (b) $\frac{P_o\left(R^3-2r^3\right)}{4\left(2r^2-R^2\right)}$

9. (a) 8 (b) 10^{-3} N/m^2

10. $P - \frac{15\delta v^2}{2}$

11. 18.66 cm.

12. $\frac{GMm}{\ell}\ln\left(\frac{R}{R+\ell}\right)$

Level – II

1. (a) $\frac{GmM}{d(d+L)} \rightarrow$ (b) $\frac{2GmM}{d\sqrt{L^2+4d^2}} \downarrow$

2. Tension in the cord $= \frac{Gm_e m_1 m_2}{m_1+m_2}\left[\frac{1}{(h+R)^2} - \frac{1}{(h+R+L)^2}\right]$

   Acceleration $a_1 = a_2$

   $= \frac{Gm_e}{m_1+m_2}\left[\frac{m_1}{(h+R)^2} + \frac{m_2}{(h+R+L)^2}\right]$

   towards the centre of the earth. The cord will always be in tension.

3. $0.238\ R_e$

4. $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

5. (a) 24 N, (b) 12 m/s^2 (upward)

6. 3.6×10^{-5} m

7. $\rho Q^2 \ell/\pi r^2$

8. 19.6 N, 32×10^3 cc.

10. (a) 0.756π kg (b) 17.28π N (c) 9.72π N ($\uparrow$) (d) $\sqrt{2.4}$ m/s (e) 0.288 N

11. $\frac{\sqrt{3}GMm}{8a^2}$

12. $\frac{m}{d_1}\left[1+\frac{a}{g}-\frac{d_1}{d_2}\right]$.

Objective:

Level – I

1.	A	2.	B	3.	D
4.	C	5.	C	6.	D
7.	C	8.	C	9.	B
10.	B	11.	B	12.	C
13.	C	14.	B	15.	B
16.	B	17.	A	18.	A
19.	B	20.	C	21.	B.
22.	B.	23.	A.	24.	C
25.	D	26.	C	27.	A
28.	A	29.	C	30.	A
31.	zero	32.	product	33.	Different
34.	acceleration	35.	increase	36.	False.
37.	True.	38.	True	39.	True
40.	False				

Level – II

1.	C	2.	C	3.	C
4.	C	5.	C	6.	B
7.	B	8.	C	9.	C
10.	A	11.	D	12.	B
13.	B	14.	B	15.	B
16.	C	17.	C	18.	C
19.	C	20.	B	21.	B.
22.	C	23.	A.	24.	B
25.	C	26.	A	27.	D
28.	B	29.	A	30.	A, B, C, D
31.	A, B & C	32.	A, B	33.	A, B, C
34.	B, D	35.	C, D	36.	A and B.

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Learnaf
Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

HEAT & THERMODYNAMICS

Heat & Thermodynamics

Syllabus:
Thermal expansion, calorimetry, Latent heat, heat conduction in one dimension, Elementary concept of conduction and radiation, Newton's law of cooling, Ideal gas laws, specific heat, Isothermal and adiabatic processes, First law of thermodynamics and its application, black body radiation, absorptive power and emissive power, kirchoff's law, Wein's displacement law, Stefan's law.

Thermal equilibrium

Two (or more than two) bodies are said to be in thermal equilibrium with each other if there is no flow of heat from one body to other when they are brought in contact (thermal contact) with each other.

Heat

It is "Energy in transition" due to temperature difference. When heat transfer takes place, the temperature of one body decreases and the temperature of the other body increases, except in a particular case where phase transition takes place.

Exercise 1. What is the unit of heat ?

Zeroth law of thermodynamics and temperature:

If a system A is in thermal equilibrium with system B and the system B is in thermal equilibrium with system C, then systems A and C are in thermal equilibrium with each other.

The common property of these systems in thermal equilibrium is the temperature. Thus, the temperature of a system is the property which determines whether or not a system is in thermal equilibrium with other systems.

Three different scales of temperatures

The Celsius Scale: The interval between the lower fixed point and the upper fixed point in Celsius scale is divided into 100 equal parts. Each division of the scale is called as one degree Celsius (1^0C). At normal pressure, the melting point of ice is 0^0C (lower fixed point) and the boiling point of water is 100^0C (upper fixed point) of the Celsius scale.

The Fahrenheit Scale: At normal pressure, the melting point of ice is regarded as 32^0 F and the boiling point of water as 212^0 F. The interval between the lower (32^0) and the upper (212^0F) fixed point is divided into equal 180 parts. Each division of this scale is called one degree Fahrenheit (1^0F).

The Reaumer Scale: At normal pressure, the melting point of ice is regarded as 0^0R and the boiling point of water as 80^0 R. The interval between the lower (0^0R) and the upper (80^0R) fixed points is divided into 80 equal parts. Each division is called 1^0R.

Conversion of temperature from one scale to another

To convert temperature from one scale to another following relation is used

$$\frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{R-0}{80-0}$$

$$\frac{C}{5} = \frac{F-32}{9} = \frac{R}{4}$$

Illustration 1. *The temperature of room is 77^0F. What would be the temperature on the Celcius scale?*

Solution: We know that

$$\frac{F-32}{9} = \frac{C}{5}$$

or, $C = \frac{5}{9}(F-32) = \frac{5}{9}(77-32)$

$C = 25^0C$.

Illustration 2. *What is the temperature at which we get the same reading on both the centigrade and Fahrenheit scale?*

Solution: If t is the required temperature,

then $\frac{t}{100} = \frac{t-32}{180}$

or $t = -40$

so the required temperature is -40^0C or -40^0F.

Absolute scale of temperature

It is possible to define an absolute temperature scale, which does not depend on any property of any substance.

The kelvin scale, the ideal gas scale, the absolute temperature scale and thermodynamic temperature scale- all refer to same temperature scale.

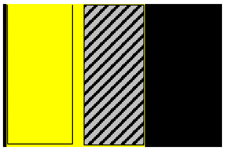
The Celsius temperature scale is defined such that 273.15 K equals 0^0C but the value of the intervals have the temperature (between two fixed temperatures) on the celsius scale is same as that on the kelvin scale i.e. the unit temperature intervals have same magnitude on the two scales. In other words $T(K) = t(^0C) + 273.15$

Thermal Expansion

If temperature of a body increases, the amplitude of molecular vibration increases which inturn increases intermolecular separation. So change in temperature produces change in the overall volume of body.

Thermal expansion of an isotropic object may be treated as a photographic enlargement. So if there is a hole in a plate (or cavity inside a body) the area of hole (or volume of cavity) will increase when a body expands on heating, just as if the hole (or cavity) were solid of the same material. Also expansion of a area (or volume) of a body will be independent of shape and size of hole (or cavity).

Exercise 2.



(i) ***Take a metallic cylindrical vessel of steel of internal volume V_o and a steel solid cylinder of similar shape and volume V_o. The temperature of both is raised up to ΔT, can we put the solid inside the vessel?***

(ii) ***A silver ring is tightly fitted on a silver rod. Can we remove the ring from the rod by raising or lowering the temperature ?***

When temperature is increased the size of the body also increases, in general. If dL be the linear expansion of the rod of length L, due to increase in temperature $d\theta$,

$dL \propto L$ and $dL \propto d\theta$ or $dL = \alpha L d\theta$ where α is the coefficient of linear expansion.

or $\displaystyle\int_{L_o}^{L} \frac{dL}{L} = \int_{\theta_0}^{\theta} \alpha d\theta$;

or $\ln\left[\frac{L}{L_o}\right] = \alpha\Delta\theta$ (where $\Delta\theta = \theta - \theta_0$, α is a constant)

or $\ln\left\{1 + \left[\frac{L}{L_o} - 1\right]\right\} = \alpha\Delta\theta$ or $\frac{L}{L_o} - 1 = \alpha\Delta\theta$

as ΔL is very small, we neglect the higher order terms.

$L = L_o(1 + \alpha\Delta\theta)$ or $\Delta L = L_o\ \alpha\Delta\theta$

$\therefore$ Coefficient of linear expansion $\alpha = \frac{\Delta L}{L_o\Delta\theta}$

Similarly, for expansion in area

$A = A_o[1 + \beta\Delta\theta]$ or $\Delta A = A_o\ \beta\Delta\theta$

$\therefore$ Co-efficient of area expansion $\beta = \frac{\Delta A}{A_o\Delta\theta}$

Similarly, for expansion in volume,

$V = V_o[1 + \gamma\Delta\theta]$ or $\Delta V = V_o\gamma\Delta\theta$

$\therefore$ Co-efficient of volume expansion $\gamma = \frac{\Delta V}{V_o\Delta\theta}$

Relation between α, β and γ

Assuming isotropic material, $\alpha : \beta : \gamma = 1 : 2 : 3$

(Isotropic material is the material which has uniform properties in all the three x, y and z directions.)

In case of liquid contained in a container, both the liquid as well as container expand.

$\therefore$ Relative expansion of liquid with respect to container is given by $\gamma_r = \gamma_l - \gamma_C$

Exercise 3.

A steel glass is half filled with a liquid. The coefficient of volume expansion of steel and liquid are same. When temperature is increased, will the volume of empty glass increases or decreases. If temperature is decreased, what will happen to the empty glass volume.

Illustration 3. *A rod 3m long is found to have expanded 0.091 in length for a temperature rise of 60^0K What is α for the material of the rod ?*

Solution: Given that,

$L_0 = 3m$, $\Delta L = 0.091$ cm. $\Delta T = 60^0C$

Since $\alpha = \frac{1}{L_0}\frac{\Delta L}{\Delta T}$

$\therefore\ \alpha = \dfrac{(0.091\times10^{-2})m}{3m\times60K}$

$\alpha = 5.1 \times 10^{-6}\ K^{-1}$.

Illustration 4. *An iron sphere has a radius of 10 cm at a temperature of 0^0C. Calculate the change in volume of the sphere if it is heated to 100^0C. Coefficient of linear expansion of iron = $11 \times 10^{-6}\ {}^0C^{-1}$*

Solution: volume at 0^0C,

$V_0 = \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\times\dfrac{22}{7}\times(10)^3\ cm^3$

$= 4190.\ 48\ cm^3$.

$\Delta V = \gamma V_0\ \Delta T$

$= 33 \times 10^{-6} \times 4190.48 \times 100\ cm^3$ ($\gamma = 3\alpha$)

$\Delta V = 13.83\ cm^3$.

Illustration 5. *A brass disc at 20^0C has a diameter of 30 cm and a hole cut in its centre is 10 cm in diameter. Calculate the diameter of the hole when the temperature of the disc is raised through 50^0C. Coefficient of linear expansion of brass = $1.8 \times 10^{-5}\ /{}^0C$.*

Solution: When the disc suffers thermal expansion, the circumference of the hole also increases.

Length of circumference of hole at 20^0C, $(L_1) = 10\ \pi$ cm.

Length of circumference of hole at 70^0C, $(L_2) = \pi D$ cm.

Rise of temperature = $t_2 - t_1 = (70 - 20)^0C = 50^0C$.

$L_2 = L_1\ \{1+ \alpha(t_2 - t_1)\}$

$\pi\ D = 10\pi\{1 + 1.8 \times 10^{-5} \times 50]$ cm.

$D = 10\ \{ 1 + 0.0009]$ cm = 10.009 cm.

Variation of density with temperature

For a given mass of a substance, mass = volume $\times$ density

$V_o\, \rho_o = V_t\, \rho_t$ = constant

or $\dfrac{\rho_t}{\rho_o} = \dfrac{V_o}{V_t} = \dfrac{1}{\left[1+\gamma\Delta\theta\right]}$; $\rho_t = \rho_o[1 + \gamma\Delta\theta]^{-1}$

$\rho_t = \rho_o\,[1 - \gamma\Delta\theta]$, (neglecting higher order terms)

Exercise 4. ***A sphere of silver is floating in a mercury bath, if temperature is increased, will sphere sink deeper or rise. It is given $\gamma_{Silver} > \gamma_{mercury}$.***

Illustration 6. *A sphere of diameter 7cm and mass 266.5gm floats in a bath of liquid. As the temperature is raised, the sphere just sinks at a temperature of 35°C. If the density of the liquid at 0°C is $1.527 gm/cm^3$, find the co-efficient of cubical expansion of the liquid. (γ of sphere is negligible)*

Solution: The sphere will sink in the liquid at 35°C , when its density becomes equal to the density of liquid at 35°C.

The density of sphere, $\rho_{35} = \dfrac{266.5}{\frac{4}{3}\times\left(\frac{22}{7}\right)\times\left(\frac{7}{2}\right)^3} = 1.483\ \text{gm/cm}^3$

Now $\rho_0 = \rho_{35}\,[1 + \gamma\Delta T)$

$1.527 = 1.483\,[1 + \gamma \times 35]$; $1.029 = 1 + \gamma \times 35$

$\gamma = \dfrac{1.029 - 1}{35} = 0.00083\,/^\circ\text{C}$

Thermal stress

When a rod is held between two rigid supports and the temperature is allowed to change, the rigid support prevents the rod from expanding or contracting. Then stress is produced in the rod which is called thermal stress.

Thermal expansion (if the rod were free), $\Delta L = \alpha . L . \Delta\theta$

$\therefore$ Thermal stress = strain $\times$ Y = $\left(\dfrac{\Delta L}{L}\right)Y = \alpha Y(\Delta\theta)$

Illustration 7. *A light steel wire of length ℓ and area of cross-section A is hanging vertically downward from a ceiling. It cools to the room temperature (30^0C) from the initial temperature of 100^0C. Calculate the weight which should be attached at its lower end such that its length remains same. Young's Modulus of steel is Y and coefficient of linear expansion is α.*

Solution: Stress due to temperature change = $\alpha\,Y(100 - 30)$

Stress due to weight = W/A

$\therefore\ W/A = \alpha\ Y.\ (100 - 30)$ W = 70 ($A\alpha Y$)

Kinetic Theory of Gases

Gases are made-up of tiny particles, consisting of molecules, atoms or even ions (sometimes) which retain the chemical properties of the sample of which they are composed.

The kinetic theory of gases develop a model of the molecular behavior which should result in the observed behavior of an ideal gas.

Ideal Gas

Any gas which obeys gas law PV = n RT is called as an ideal gas, where R = universal gas constant and R = 8.31 J/mol. K. An ideal gas is a simple theoretical model of a gas. No real gas is truly ideal. Real gas approaches the ideal gas behavior at low pressures and high temperatures.

Assumptions:

(i) Molecules (whose size is extremely small in comparison to the separation between them), moves randomly in all directions.

(ii) Molecules exert no appreciable force on one another or on the walls of the container except during collision.

(iii) All collisions between the molecules or with the wall of the container are perfectly elastic.

(iv) The duration of a collision is negligible in comparison to the time spent by a molecule between collisions.

(v) In steady state, the density and the distribution of molecules with different velocities are independent of position, direction and time. This assumption is justified if the number of molecules is very large.

(vi) The molecules obey Newton's Laws of motion. The assumptions of kinetic theory are close to the real situation at low densities.

Exercise 5. ***Can an ideal gas be liquified?***

Illustration 8. *An air bubble starts rising from the bottom of a lake. Its diameter is 3.6 mm at the bottom and 4 mm at the surface. The depth of the lake is 250 cm and the temperature at the surface is 40^oC. What is the temperature at the bottom of the lake ? Given atmospheric pressure = 76 cm of Hg and g = 980 cm/sec^2. (Specific gravity of mercury = 13.6.*

Solution: At the bottom of the lake, volume of the bubble,

$$V_1 = \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi(0.18)^3 \text{ cm}^3$$

Pressure on the bubble P_1 = Atmospheric pressure + Pressure due to a column of 250 cm of water

$$= 76 \times 13.6 \times 980 + 250 \times 1 \times 980 = (76 \times 13.6 + 250)\, 980 \text{ dyne/cm}^2$$

T_1 = Temperature at the bottom

At the surface of the lake, Volume of the bubble

$$V_2 = \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi(0.2)^3 \text{ cm}^3$$

Pressure on the bubble P_2 = atmospheric pressure

$$= 76 \times 13.6 \times 980 \text{ dyne/cm}^2$$

$$T_2 = 273 + 40^oC = 313^o \text{ K}$$

$$\text{Now } \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\text{or } \frac{(76\times 13.6 + 250)980\times(4/3)\pi(0.18)^3}{T_1} = \frac{(76\times 13.6)\times 980 \quad (4/3)\pi(0.2)^3}{313}$$

$$\text{or } \frac{1283.6\times(0.18)^3}{T_1} = \frac{1033.6(0.2)^3}{313}$$

$$\text{or } T_1 = \frac{1283.6\times(0.18)^3\times 313}{1033.6(0.2)^3} = 283.37\text{K}$$

$$\therefore \; T_1 = 283.37 - 273 = 10.37\ ^oC$$

Illustration 9. *Calculate the number of molecules in a gas of volume 1 litre and pressure 1.2×10^7 N/m^2 and temperature 400 K.*
(Given: Boltzman constant = 1.32×10^{-23} J/K)

Solution: Given that, $P = 1.2 \times 10^7$ N/m^{-2}, V = 1 litre, T = 400 K.

$$nN = \frac{PV}{KT} = \frac{(1.2\times10^{7}N/m^{2})(1litre)10^{-3}m^{3}}{(1.38\times10^{-23}J/K)(400K)}$$

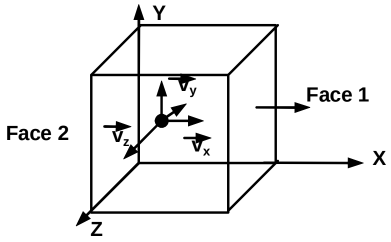
$$n_1 = nN = 2 \times 10^{24}.$$

Concept of gas pressure:

According to kinetic theory of gases of molecules of a gas move randomly in all directions. The molecules not only collide with one another but also with the wall of the containing vessel. A molecule imparts a certain momentum to the wall when it collides against the wall of the container. Since rate of change of momentum is proportional to the force. Therefore, in each collision the molecule exerts a force on the wall of the container. Thus the walls of the container experience a steady force. This force per unit area is called the pressure of the gas. i.e. the pressure of a gas is equal to the momentum imparted per second per unit area of the walls of the container by the molecules.

Pressure of an ideal gas:

Consider an ideal gas enclosed in a cubical vessel of edge L.



Considering a molecule moving with velocity

$$\vec{v} = \vec{v}_x + \vec{v}_y + \vec{v}_z\,; \qquad |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

where v_x, v_y and v_z are components of velocity along x, y and z axis respectively.

The change in momentum of a molecule on collision with face 1

$$\Delta P = (-mv_x) - (mv_x) = -2mv_x$$

$\therefore$ Momentum imparted to the wall = $2mv_x$

Time between two successive collisions on face 1, $\Delta t = \dfrac{2L}{v_x}$

Rate at which momentum is imparted to the wall, by a molecule $\Delta F = \Delta P/\Delta t = mv_x^2/L$

Total force on the face 1 due to all the molecules

$$F = \Sigma\, mv_x^2/L = m/L\Sigma v_x^2 \qquad \ldots(1)$$

As $\Sigma v_x^2 = \Sigma v_y^2 = \Sigma v_z^2$ ($\because$ molecules move randomly in all the directions)

$$\Sigma v_x^2 = \frac{1}{3}\Sigma\left(v_x^2 + v_y^2 + v_z^2\right) = \frac{1}{3}\Sigma v^2 \qquad \ldots(2)$$

$$F = \frac{1}{3}\frac{m}{L}\Sigma v^2 = \frac{1}{3}\frac{mN}{L}\frac{\Sigma v^2}{N}$$ were N = total number of molecules in the sample.

Now, pressure = $\dfrac{Force}{Area} = \dfrac{1}{3}\dfrac{mN}{L^3}\dfrac{\Sigma v^2}{N}$

or, $P = \dfrac{1}{3}\rho\dfrac{\Sigma v^2}{N}$, where $\rho = \dfrac{mN}{L^3}$

or, $P = \dfrac{1}{3}\rho\overline{v^2}$, $\overline{v^2} = \dfrac{\Sigma v^2}{N}$ is the mean square speed.

Mean speed or Average speed:

Mean or average speed is the arithmetic mean of the speeds of molecules in a gas at a given temperature. Consider a gas having n molecules. Let v_1, v_2, …………….v_n be the speeds of molecules. Then, the mean speed is given by $\bar{v} = \dfrac{v_1 + v_2 + - - + v_n}{n}$

From Maxwell Boltzman statistics,

$$\bar{v} = \sqrt{\frac{8KT}{\pi m}} \qquad \text{. . . (1)}$$

Where T is absolute temperature of the gas and m is the mass of gas molecule, k is Boltzman constant.

Illustration 10. *Four molecules of a gas have velocities 2, 4, 6 and 8 km/s, respectively. Calculate average speed.*

Solution: Average speed $= \dfrac{2+4+6+8}{4}$ = 5 km/s

RMS speed:

It is defined as $v_{rms} = \left[\dfrac{\int v^2 dN}{\int dN}\right]^{1/2} = \left[\dfrac{N_1 v_1^2 \;+\; N_2 v_2^2 \;+\; ....}{N_1 \;+\; N_2 + ...}\right]^{1/2}$,

here N_1 molecules have speed v_1, N_2 molecules have speed v_2 , ……

Since P = $\dfrac{1}{3}\dfrac{m}{V}v_{rms}^2$

If we take n moles of a gas, $PV = \dfrac{1}{3}n.M \;\; v_{rms}^2$ [$\because$ PV = nRT]

$\therefore$ $nRT = \dfrac{1}{3}n.M \;\; v_{rms}^2$ where R = universal gas constant = 8.314 J/mol-k.

or $$v_{rms} = \sqrt{\frac{3RT}{M}}. \qquad \text{. . . (2)}$$

Illustration 11. *Calculate the temperature at which rms velocity of SO_2 molecules is the same as that of O_2 molecules at 27^0C. Molecular weights of oxygen and sulphur dioxide are 32 and 64 respectively.*

Solution: For oxygen,

T = (27 + 273)K = 300 K.

Molecular weight of oxygen, M = 32

rms velocity of O_2 molecule at 27^0C.

$v_{rms} = \sqrt{\dfrac{3RT}{M}}$

$$= \sqrt{\frac{3 \times R \times 300}{32}} \qquad \text{. . . (i)}$$

Let t be the temperature at which the rms velocity of SO_2 is also v_{rms}. Let T be the corresponding absolute temperature.

For SO_2, M = 64

Then, $$v_{rms} = \sqrt{\frac{3 \times R \times T}{64}} \qquad \text{. . . (ii)}$$

Equating equation (i) and (ii) we get,

$T = 600\ k$

or $273 + t = 600$

or, $t = (600 - 273) = 327^0C$.

Most probable speed:

The most probable speed is that speed with which the maximum number of molecules move in a given gas at a given temperature. It is denoted by v_{mp}

From Maxwell Boltzman statistics,

$$v_{mp} = \sqrt{\frac{2KT}{m}} \quad \text{(iii)}$$

From the equation (i) (ii) & (iii), it follows that $v_{mp} < \bar{v} < v_{rms}$

Kinetic Energy and Temperature:

Consider one mole of a gas. If M and V be its mass and volume respectively, the pressure exerted by the gas is given by

$$P = \frac{1}{3}\frac{M}{V}v_{rms}^2$$

or, $PV = \frac{1}{3}Mv_{rms}^2$

or, $\frac{1}{3}Mv_{rms}^2 = RT$ $[\because PV = RT\ ,\ n = 1]$

or, $\frac{1}{2}Mv_{rms}^2 = \frac{3}{2}RT$ . . . (i)

i.e. The average translational kinetic energy per gram molecule of the gas is equal to $\frac{3}{2}RT$

Dividing both sides of equations by N, we get

$$\frac{1}{2}\frac{M}{N}v_{rms}^2 = \frac{3}{2}\frac{R}{N}T$$

where N = Avagadro's number

or, $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$ . . . (ii)

where $\frac{M}{N} = m$ (mass of one molecule of gas)

$\frac{R}{N} = k$ Boltzman constant

Thus, the average translational kinetic energy per molecules is equal to $\frac{3}{2}kT$

From equation (ii), it is clear that mean kinetic energy of a molecule is directly proportional to the absolutes temperature of a gas.

Kinetic interpretion of temperature:

Kinetic energy of random motion is the internal energy, given by

$$KE_{int} = \frac{1}{2}m.v_{rms}^2 = \frac{1}{2}n.M\frac{3RT}{M} = \frac{3}{2}.n.RT$$

$\therefore$ $KE_{int} \propto T$

where n = number of moles of a gas.

This means temperature of a body depends on its internal KE. At absolute zero temperature KE_{int} becomes zero.

Illustration 12. *Given : Avogadro's number N=6.02×10^{23} and Boltzmann's constant k=1.38×10^{-23} J/k. Calculate*

(i) The average kinetic energy of translation of the molecules of an ideal gas at 0^oC and at 100^oC

(ii) Also calculate the corresponding energies per mole of the gas.

Solution: (i) According to the kinetic theory, the average kinetic energy of translation per molecule of an ideal gas at kelvin temperature. T is (3/2)kT, where k is Boltzmann's constant.

At 0^oC (T = 273 K), the kinetic energy of translation = $\frac{3}{2}kT$

$$= \frac{3}{2}\times\left(1.38\times10^{-23}\right)\times273 = 5.65\times10^{-21}\,J/molecule$$

At 100^oC (T = 373 K), the energy is

$$= \frac{3}{2}\times\left(1.38\times10^{-23}\right)\times373 = 7.72\times10^{-21}\,J/molecule$$

(ii) 1 mole of gas contains N (=6.02×10^{23}) molecules. Therefore, at 0^oC, the kinetic energy of translation of 1 mole of the gas is
= $(5.65\times10^{-21})(6.02\times10^{23}) \approx 3401$ J/mole and at 100^oC.

The kinetic energy of translation of 1 mole of gas is

$$= (7.72\times10^{-21})(6.02\times10^{23}) \approx 4647 \text{ J/mole.}$$

Avogadro's Number

The number of molecules in one gram mole of a substance is called Avogadro's number or the number of atoms in 12 gms of C^{12} is called Avogadro number. It is denoted by N.
N = 6.0225×10^{23}.

Boyle's Law

At a given temperature, the pressure of a given mass of a gas is inversely proportional to its volume.
This is known as Boyle's Law.
i.e. PV = constant

Charles Law

At a given pressure the volume of given mass of gas is proportional to its absolute temperature. This is know as Charle's law.

i.e $\frac{V}{T}$ = constant

Gay Lussac's Law (Law of pressure)

At a given volume, the pressure of a given mass of a gas is proportional to its absolute temperature. This is called Law of pressure.

i.e. $\frac{P}{T} = \text{constant}$

Avogadro's law

Equal volumes of all gases at the same temperature and pressure contain equal number of molecules. Consider two gases having the same temperature pressure and volume. First gas contains n_1 molecules, each of mass m_1 and the second gas contain n_2 molecules, each of mass m_2. If c_1 and c_2 be the rms velocities of the two gases.

Then, $P = \frac{1}{3}\frac{m_1 n_1}{V} c_1^2$ . . . (1)

and $P = \frac{1}{3}\frac{m_2 n_2}{V} c_2^2$ . . . (2)

or $m_1 n_1 c_1^2 = m_2\, n_2\, c_2^2$ . . . (3) (from equation (1) & (2))

Since T is same for both the gases therefore average translation K.E. per molecule is same for each gas.

$\therefore \frac{1}{2} m_1 c_1^2 = \frac{1}{2} m_2 c_2^2$ . . . (4)

from equation (3) & (4)

$n_1 = n_2$

This proves Avogadro's law

Dalton's law of partial pressure

"Total pressure exerted by mixture of an ideal gas which do not interact with each other is equal to the sum of their individual pressures."

$\therefore P = p_1 + p_2 + - - - -$

Graham's Law of Diffusion

At any specified temperature and pressure, the relative rates of diffusion of two gases are inversely proportional to the square roots of their densities.

$$\frac{r_1}{r_2} = \frac{c_1}{c_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

(since rate of diffusion depend upon the velocity of the gas molecules)

Specific heats at constant volume and pressure (C_p and C_v)

The molar heat capacities of a gas are defined as the heat given per mole of the gas per unit rise in the temperature the molar heat capacity at constnt volume denoted by C_v, is

$$C_v = \left(\frac{\Delta Q}{n\Delta T}\right)_{\text{constant volume}}$$

And the molar heat capacity at constant pressure, denoted by C_p, is

$$C_P = \left(\frac{\Delta Q}{n\Delta T}\right)_{\text{constant pressure}}$$

Where n is the amount of the gas in number of moles. The unit of specific heat capacity *is J/kg - k whereas the molar heat capacity is J/mol – k.*

Illustration 13. *0.32 g of oxygen is kept in a rigid container and is heated. Find the amount of heat needed to rise the temperature from 25^0C to 35^0C. The molar heat capacity of oxygen at constant volume is 20 J/mol – k.*

Solution: The molecular weight of oxygen = 32 g/mol.
The amount of the gas in moles is

$$n = \frac{0.32}{32} = 0.01 \text{ mol.}$$

The amount of heat needed is

$$= nC_v\Delta T$$
$$= (0.01) \times (20) \times (10)$$
$$= 2.0 \text{ J}$$

Illustration 14. *A tank of volume 0.2 m^3 contains helium gas at a temperature of 300 k and pressure 1×10^5 N/m^2. Find the amount of heat required to raise the temperature to 400 k. The molar heat capacity of helium at constant volume is 3.0 cal/mol- k.*

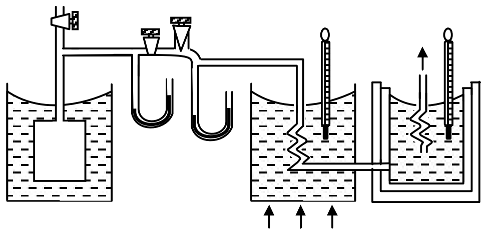
Solution: The amount of heat gas in moles is

$$n = \frac{PV}{RT} = \frac{(1\times10^5)\times(0.2)}{8.31\times300} = 8.0 \text{ mol.}$$

= The amount of heat required is

$$q = nC_v\Delta T$$
$$= 8.0 \times 3.0 \times 100$$
$$= 2400 \text{ cal.}$$

Figure shows a schematic diagram of the renault's experiment, the oil bath is heated with a burner to keep it at a high temperature which is measured by thermometer T_1. A measured mass m of water is taken in the calorimeter. The calorimeter should be almost full with water. So that the gas flowing the coil F gets maximum time to exchange heat with water. The initial temperature of the water is measured by T_2. The difference in heights of mercury in the two arms of the manometer M is noted. The valve 'v' is opened to allow the gas to how through the system. As the amount of the gas in the tank reduces, the pressure in the decreases and the rate of how tends to decrease. The screw value S is continuously adjusted to keep the rate of flow constant. This is decided by keeping the difference in the levels of mercury in manometer N constant. The pressure of the gas going in the coil then remains constant. The gas is allowed to how for some time and the final temperature of water and the final difference in the mercury levels in manometer M are noted.



Let the water equivalent of the calorimeter together with the coil D = W

Mass of the water = m
Temperature of the oil bath = Q_1
Initial Temperature of water= Q_2
Final temperature of water = Q_3
And the amount of the gas (in moles) passed through the water = n

The gas at temperature Q_1 enters the coil D. In the beginning of the experiment, the gas levels the coil D at temperature Q_2. This temperature gradually increases and a the end of experiment it becomes Q_3. The average temperature of the gas leaving the coil D is, therefore, $\frac{Q_2 + Q_3}{2}$.

The heat lost by the gas is

$$\Delta Q = nC_p\left[\theta_1 - \frac{\theta_2+\theta_3}{2}\right] \qquad \text{. . . (i)}$$

This heat is used to increase the temperature of the calorimeter, the water and the coil is from Q_2 and Q_3.

The heat received by them is

$$\Delta Q = (w+m)s\ (\theta_3 - \theta_2) \qquad \text{. . . (ii)}$$

where s is the specific heat capacity of water.
From (i) and (ii)

$$nC_p\left[\theta_1 - \frac{\theta_2+\theta_3}{2}\right] = (w+m)s\ (\theta_3 - \theta_2)$$

or, $$C_p = \frac{(w+m)s(\theta_3-\theta_2)}{n\left[\theta_1 - \frac{\theta_2+\theta_3}{2}\right]}$$

where $n = n_1 - n_2$

$$n = \frac{(P_1-P_2)V}{RT}$$

P_1 - initial pressure in M
P_2 – final pressure in M

Mayer's Relation

We know that
$dQ = dU + dW$ . . . (i)
Let the gas be heated at constant pressure P.
The amount of gas is n moles. The change in volume is dV and change in temperature is dT
Then $dW = P\,dV$ . . . (ii)
And $dQ = nC_p\,dT$ . . . (iii)
Putting (ii) and(iii) in (i), we get
$nC_p\,dT = dU + PdV$ . . . (iv)
Let heat dQ be supplied at constant volume
Then $dW = PdV = 0$
Because $dV = 0$
And $dQ = nC_v dT$ . . . (v)
From (i) and (v) we get
$nC_v\,dT = dU$ . . . (vi)
substracting from (iv) we get
$n(C_p - C_v)\,dT = PdV$
for ideal gas $PV = nRT$
which gives $PdV = nR\,dT$
$n(C_p - C_v)dT = nRdT$
$C_p - C_v = R$ . . . (v)
Relation (v) is called Mayer's formula.

Degrees of Freedom

The degrees of freedom of a particle is the number of independent motions, which the particle can undergo.

Law of equipartition of energy

For a system in equilibrium at absolute temperature T, the average energy per particle associated with each degree of freedom is $\frac{1}{2}kT$, where k is Boltzmann's constant.

The internal energy of an ideal gas is entirely the kinetic energy of its molecules.

$\therefore$ Internal energy of one mole of an ideal gas, having 'f' degrees of freedom.

$$U = N \times f \times \frac{1}{2}kT = \frac{1}{2}fRT, \; N = \text{Avogadro's Number}, \; R = kN$$

The specific heat of the gas at constant volume $C_v = \frac{dU}{dT} = \frac{fR}{2}$

$\therefore$ $C_P = C_v + R = \frac{f}{2}R + R = \left(\frac{f}{2}+1\right)R$

$\therefore$ Ratio of the two specific heats $\gamma = \frac{C_P}{C_V} = \frac{\left(\frac{f}{2}+1\right)R}{\frac{f}{2}R} = 1 + \frac{2}{f}$

- A molecule of monatomic gas has only 3(translational) degrees of freedom f = 3.

$\therefore$ $C_v = \frac{f}{2}R = \frac{3}{2}R$

$C_P = \left(\frac{f}{2}+1\right)R = \frac{5}{2}R$; $\gamma = 1 + \frac{2}{f} = 5/3 = 1.66$

- A molecule of diatomic gas has 5 degrees of freedom (3 - translational and 2 - rotational) at ordinary atmospheric temperatures (because the vibrational modes are not excited.) The moment of inertia about the line joining the two atoms is negligibly small. Hence the rotational energy about that axis is zero.

f = 5

$C_V = \frac{f}{2}R = \frac{5}{2}R$

$C_P = \left(\frac{f}{2}+1\right)R = \frac{7}{2}R$

$\gamma = 1 + \frac{2}{f} = \frac{7}{5} = 1.40$

Note: At very high temperatures, if vibrational mode is also considered then f = 7.

- A molecule of triatomic or polyatomic gas has 6 degrees of freedom (3 translational and 3 rotational)

f = 6

$C_V = \frac{f}{2}R = 3R$

$C_P = \left(\frac{f}{2}+1\right)R = 4R$

$\gamma = 1 + \frac{2}{f} = \frac{4}{3} = 1.33$

However if the atoms of the molecule are arranged in one line (like the molecule of CO_2) then the degrees of freedom are only 5 and its C_v, C_p & γ values will be similar to those of the diatomic gases.

Illustration 15. *Calculate the total number of degrees of freedom possessed by the molecules in one cm^3 of H_2 gas at NTP.*

Solution: 22400 cm^3 of every gas contains, 6.02×10^{23} molecules.

$\therefore$ No. of molecules in 1 cm^3 of H_2 gas

$$= \frac{6.02 \times 10^{23}}{22400} = 0.26875 \times 10^{20}.$$

Number of degrees of freedom of a H_2 gas molecule = 5

$\therefore$ Total number of degrees of freedom of 0.26875×10^{20} molecules

$= 0.26875 \times 10^{20} \times 5$

$= 1.34375 \times 10^{20}$

Illustration 16. *How many degrees of freedom have the gas molecules, if under standard conditions the gas density is 1.3 kg/m^3 and the velocity of sound in it is v = 330 m/s.*

Solution: $\because \quad v = \sqrt{\left(\frac{\gamma P}{\rho}\right)} \quad$ or $\quad \gamma = \frac{\rho v^2}{P}$

If f be the number of degree of freedom, then

$$f = \frac{2}{\gamma - 1} = \frac{2}{\left[\left(\rho v^2 / P\right) - 1\right]} = 5$$

($\because$ $P = 1.013 \times 10^5\ N/m^2$, $\rho = 1.3\ kg/m^3$, $v = 330$ m/s.)

Internal Energy

The internal energy of a system is the sum of kinetic and potential energies of the molecules of the system. It does not include the overall kinetic energy of the system.

Internal energy of ideal gas

In case of an ideal gas no internal force of interaction exists. Hence the molecular potential energy of the gas molecules is zero. So, the internal energy of the ideal gas is wholly kinetic in nature. According to kinetic interpretation, the kinetic energy of gas molecules depends only upon temperature. Hence, the internal energy of an ideal gas depends only upon the temperature of the gas i.e. U = f nRT/2

Where f is the number of degrees of freedom of gas molecule

Illustration 17. *One mole of an ideal monatomic gas is taken at a temperature of 300K. Its volume is doubled keeping its pressure constant. Find the change in internal energy.*

Solution: Since pressure constant

$\therefore V \propto T \qquad \therefore \frac{V_i}{T_i} = \frac{V_f}{T_f}$

$\therefore T_f = \frac{V_f}{V_i} T_i \quad \Rightarrow \quad 2T_i = 600K$

$\therefore \Delta U = \frac{f}{2} n.R\Delta T = \frac{3}{2} 1.R.(600 - 300) = 450\ R$

Concept of mean free path

Mean free path of a molecule in a gas is the average distance travelled by the molecule between two successive collisions. It is denoted by λ.

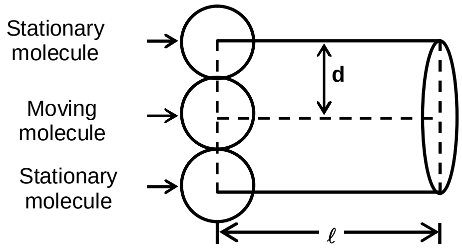
If λ_1, λ_2 - λ_N be the free paths travelled by the molecules in N successive collisions, then, mean free path

$$\lambda = \frac{\lambda_1 + \lambda_2 + ...\lambda_n}{N}$$

Expression for mean free path:

Let us assume that only one molecule is in motion and all other molecules are at rest. Let d be the diameter of each molecule.

If ℓ is the distance travelled by the moving molecule, the moving molecule shall make a collision with all those molecules whose centers be inside a volume $\pi d^2 \ell$.



Suppose, n is the number of molecules per unit volume in the gas.

Then number of collisions = number of molecules in volume $\pi d^2 \ell$ = n $\pi d^2 \ell$

Now $$\lambda = \frac{\text{distance travelled}}{\text{number of collisions}}$$ or, $$\lambda = \frac{\ell}{n\pi d^2 \ell}$$

$$\lambda = \frac{1}{n\pi d^2} \quad ...(1)$$

Since all the molecules are moving in random motion, so, the chances of a collision by a molecule is greater. Taking this into account, mean free path can be shown to be $\sqrt{2}$ times less than that given in equation (1).

$$\therefore \quad \lambda = \frac{1}{\sqrt{2} n\pi d^2} \quad ...(2)$$

or, $$\lambda = \frac{m}{\sqrt{2}\pi d^2 mn}$$

$$\lambda = \frac{m}{\sqrt{2}\pi d^2 \rho} \quad ...(3)$$

where mn = ρ = density of the gas.

Again since, PV = RT

$$\Rightarrow P = \frac{RT}{V} = \frac{N}{V} \times \frac{R}{N} \times T = nKT$$

$$\Rightarrow n = P/KT$$

Putting this value of n in equation (2)

$$\therefore \lambda = \frac{KT}{\sqrt{2}\pi d^2 P}$$

THERMODYNAMICS

System and its surroundings

SYSTEM
Any portion of universe, which is, considered as "separate", forms a "system".

There are three types of systems

OPEN
If exchange of matter and energy takes places with the surrounding.

CLOSED
If exchange of energy takes place but no exchange of matter.

ISOLATED
If no exchange of matter and energy takes place.

All those things which influence the behaviour of a **'system'** are known as its **"surroundings".**

Thermodynamic parameter:
The variables which describe the state of matter e.g. pressure, volume, temperature .

Thermodynamic equilibrium:
If the variable remains unchanged with time, then the system is said to be in equilibrium.

First law of thermodynamics

If ΔQ is heat given to the system and ΔW is work done by the system, then ΔU the change in internal energy can be written as

$$\Delta U = \Delta Q - \Delta W$$

This is the law of conservation of energy. It can be stated in differential form, $dQ - dW = dU$

And time rate form $\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dU}{dt}$

Note:
* Heat supplied to the system is taken as positive and heat rejected by the system is taken as negative.
* Work done by the system is positive and work done on they system is negative.
* Change in internal energy depends on initial and final states of the system.
* $\Delta U = nC_vdT$ for all processes.

Joule's law
It states that the work done by a thermodynamical system is directly proportional to the heat produced.
$W \propto Q$
$W = JQ$
where J is called mechanical equivalent of heat and J is equal to 4.18 4 J/cal

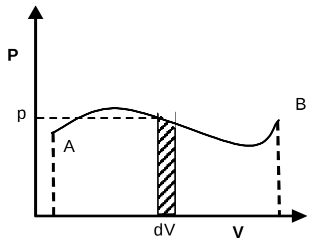
Work done:

Work done by the gas (or system) over the surroundings can be calculated as

$$dW = PdV$$

$$\therefore W = \int_{V_i}^{V_f} PdV$$

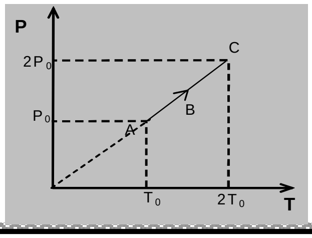
Here dW is elemental work done by pressure P, of the system, during elemental change in volume dV.



Work done in the process AB is equal to the area under the AB curve and V-axis.

Exercise 6. ***3 moles of a diatomic gas is taken through the process A-B-C as shown in the fig. Find work done by the system in the process A-B-C.***

(i)



(ii)

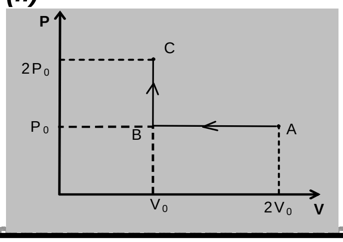


Illustration 18. *One mole of an ideal gas undergoes a cyclic change ABCD. Calculate the following from figure shown.*
(i) Work done along AB, BC, CD and DA.
(ii) Net work done in the processes.
(iii) Net change in the internal energy of the gas.
(Given 1 atm = 1.01×10^5 Nm^{-2})

Solution: (i) $W_{AB} = PdV = 5 \times 1.01 \times 10^5 \times 2 \times 10^{-3}$ J = 1010 J.
$W_{BC} = 0$
$W_{CD} = -2 \times 1.01 \times 10^5 \times 2 \times 10^{-3} = -404$ J.
$W_{DA} = 0$
(ii) Net work done, W = (1010 – 404) = 606 J.
(iii) Net change in internal energy = 0. Because given process is cyclic.

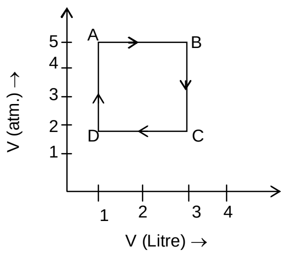


Illustration 19. *Two moles of a diatomic ideal gas is taken through the process PT = const. Its temperature is increased from T_0 K to $2T_0$K. Find the work done by the system?*

Solution: $W = \int Pdv$, Here $PT = P_1T_1 = P_2T_2 = c$ (Constant)

$$\therefore PT = c; \qquad \Rightarrow \quad P.\frac{PV}{nR} = c$$

$$\therefore P^2V = ncR \quad \Rightarrow \; P = \sqrt{\frac{ncR}{V}}$$

$$\therefore \int Pdv = \sqrt{ncR}\int_{V_1}^{V_2} \frac{1}{\sqrt{V}} dv = \sqrt{ncR}\left[2\left(\sqrt{V_2} - \sqrt{V_1}\right)\right]$$

$$= 2 \left[\sqrt{nR.P_2T_2V_2} - \sqrt{nRP_1T_1V_1}\right] = 2nR\,(T_2 - T_1) = 4RT_o$$

Calorimetry: Principle of Calorimatry

When two objects having different temperatures are brought in contact, heat flows from the hot object to the cold object. If the system is sufficiently thermally isolated from its surrounding.
Heat lost by the hot object = heat gained by the cold object.

Latent Heat (L)

Latent heat is defined as amount of heat absorbed or released by body during the change of state while its temperature remaining constant. Heat absorbed or released during change of state.
$Q = mL$ where m is mass of body and L is latent heat.

Latent heat of fusion

The heat supplied to a substance to change its state at constant temperature is called latent heat of fusion.

The latent heat of fusion of ice is 80 kcal/kg

Latent heat of vaporization:

The heat supplied to a substance to change it from liquid to vapor state at constant temperature is called latent heat of vaporization.

Latent heat of vaporization of water is 540 k cal/kg

Heat capacity If a quantity ΔQ of heat raises the temperature of a body through ΔT, then heat capacity is given by

$$H = \frac{\Delta Q}{\Delta T} \text{ J/K}$$

The heat capacity of a body is numerically equal to the quantity of heat required to raise its temperature by unity.

Specific heat It is the heat required to raise the temperature by 1^0C or 1^0K for a unit mass of the body.

$$C = \frac{\Delta Q}{m\Delta T}$$

In the problems related with heat and thermodynamics we usually work with ideal gases and require molar specific heat.

$\therefore$ Molar heat capacity $C = \dfrac{\Delta Q}{n\Delta T}$ where n = number of moles.

In differential form $C = \dfrac{1}{n}\left(\dfrac{dQ}{dT}\right) = \dfrac{1}{n}\left(\dfrac{dU}{dT}\right) + \dfrac{1}{n}\left(\dfrac{dW}{dT}\right)$

Since dW is process dependent, the molar heat capacity is process dependent.
If the volume of a gas of mass m is kept constant and heat ΔQ is given to it, and the temperature rises by ΔT, then, the heat capacity at constant volume is expressed as

$$C_v = \left(\frac{\Delta Q}{n\Delta T}\right)_{\Delta V = 0} = \frac{1}{n}\frac{dU}{dT}$$

If the pressure of a gas of mass m is kept constant and heat ΔQ is given to it, and temperature rises by ΔT, then molar heat capacity at constant pressure is expressed as

$$C_p = \left(\frac{\Delta Q}{n.\Delta T}\right)_{\Delta P = 0} = \frac{1}{n}\frac{dU}{dT} + \frac{1}{n}P\frac{dV}{dT} = C_v + R$$

Relation between C_P and C_V is, $C_p - C_v = R$

Exercise 7. ***What is the specific heat of pure boiling water at 100^0C?***

Illustration 20. *5 gm of water at 30^oC and 5 gm of ice at - 20^oC are mixed together in a calorimeter. Find the final temperature of mixture and also the final masses of ice and water. Water equivalent of calorimeter is negligible, specific heat of ice = 0.5 cal gm^oC and latent heat of ice = 80 cal/gm.*

Solution: In this case heat is given by water and taken by ice

Heat available with water to cool from 30^oC to 0^oC

$= ms\Delta\theta = 5\times1\times30 = 150$ cal.

Heat required by 5 gm ice at increase its temperature up to 0^oC $ms\Delta\theta = 5\times0.5\times20 = 50$cal

Out of 150 cal heat available, 50 cal is used for increasing temperature of ice from - 20^oC to 0^oC. The remaining heat 100 cal is used for melting the ice.

if mass of ice melted is m gm then

$m\times 80 = 100 \quad\Rightarrow\quad m = 125$ gm

Thus 1.25 gm ice out of 5 gm melts and the mixture of ice and water is at 0^oC.

Illustration 21. *What amount of heat must be supplied to 2.0×10^{-2} kg of Nitrogen at room temperature to raise its temperature by 45^0C at constant pressure? Given molecular mass of N_2 = 28, $R = 8.3\ J\ mol^{-1}\ K^{-1}$.*

Solution: Mass of gas, $m = 2.0 \times 10^{-2}$ kg = 20 g.

number of moles, n = $\frac{\text{mass in gram}}{\text{molecular mass}}$

$n = \frac{20}{28} = 0.714$.

increase in temperature, $\Delta T = 45^0C$

Molar specific heat at constant pressure

$c_p = \frac{7}{2}\times 8.3\ J\ mol^{-1}k^{-1}$

heat supplied , $Q = nC_p\ \Delta T$

$= 0.714 \times \frac{7}{2}\times 8.3\times 45$ Joule. = 933.4 Joule.

Illustration 22. *Find the molar heat capacity of the process P = a /T for a monatomic gas, 'a' being positive constant.*

Solution : We known that dQ = dU + dW

Specific heat $C = \frac{dQ}{ndT} = \frac{dU}{ndT} + \frac{dW}{ndT}$ …(i)

Since $dU = nC_v dT$ …(ii)

$C = C_v + \frac{dW}{ndT} = C_v + \frac{PdV}{ndT}$

$\therefore$ For the given process,

$$V = \frac{nRT}{P} = \frac{nRT^2}{a}; \quad \frac{dV}{dT} = \frac{2nRT}{a} \qquad (\because\ PV = nRT)$$

$$\therefore \quad C = C_v + \frac{P}{n}\left(\frac{2nRT}{a}\right) = C_v + 2R = \frac{3}{2}R + 2R = \frac{7}{2}R. \ (\because\ P = a/T).$$

Illustration 23. *A gaseous mixture enclosed in a vessel consists of one gm mole of a gas A with $\gamma = (5/3)$ and some amount of gas B with $\gamma = 7/5$ at a temperature T. The gases A and B do not react with each other and are assumed to be ideal. Find the number of gm moles of the gas B if γ for the gaseous mixture is (19/13).*

Solution : As for an ideal gas, $C_p - C_v = R$ and $\gamma = (C_p / C_v)$

So $C_v = \frac{R}{(\gamma - 1)} \quad \therefore \quad (C_v)_1 = \frac{R}{(5/3) - 1} = \frac{3}{2}R; \ (C_v)_2 = \frac{R}{(7/5) - 1} = \frac{5}{2}R$

and $(C_v)_{mix} = \frac{R}{(19/13) - 1} = \frac{13}{6}R$

Now from conservation of energy , i.e. $\Delta U = \Delta U_1 + \Delta U_2$

$$(\mu_1 + \mu_2)(C_v)_{mix}\,\Delta T = [\mu_1(C_v)_1 + \mu_2(C_v)_2]\,\Delta T$$

i.e. $(C_v)_{mix} = \frac{\mu_1(C_v)_1 + \mu_2(C_v)_2}{\mu_1 + \mu_2}$

We have, $\frac{13}{6}R = \frac{1 \times \frac{3}{2}R + \mu_2 \frac{5}{2}R}{1 + \mu_2} = \frac{(3 + 5\mu_2)R}{2(1 + \mu_2)}$

or $13 + 13\mu_2 = 9 + 15\mu_2$,

i.e., $\mu_2 = 2$ gm mole

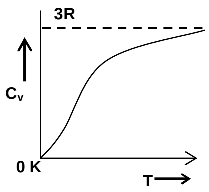
Dulong and Petit's Law

The specific heat per mole of a chemically pure crystalline solid is approximately 6 cal mole^{-1} K^{-1}. This is know as Dulong and Petit's law.

Variation of specific heat of solids with temperature:

According to Dulong and Petit's law, the molar specific heat of every solid must be equal to 6 cal. But the result is approximately equal to 6 cal.

At various temperatures, specific heat is not a constant quantity. Instead, it varies with temperature and only at a specific temperature depending upon the nature of the material, it approaches 6. If a graph is drawn between the temperature and the C_v for a solid, we actually get the result as shown in figure.



Variation of C_V with T

Equation of state

A relation between pressure, volume and temperature for a system is called its equation of state. The state of system is completely known in terms of its pressure, volume and temperature, e.g. for n mole of an ideal gas, the equation of state is

PV = nRT

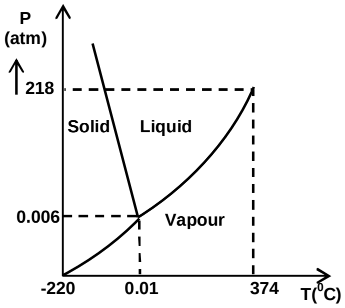
Vander Wall's Equation of state:

For a real gas the equation of state is different from an ideal gas, the size of a molecule is not negligible in comparison to the average separation between them. Also the molecular attraction is not negligible taking these two facts into account, Vander Waals. Derived for the following of state for a real gas

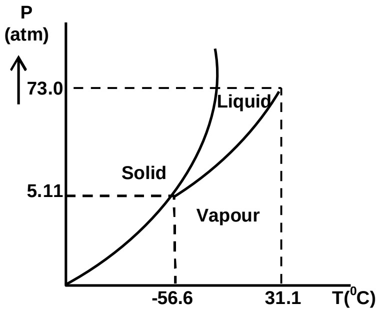
$$\left(P + \frac{an^2}{v^2}\right)(v - nb) = nRT$$

where a and b are small positive constant for of attraction between the molecules and b is related to the total volume of the molecules.

Phase: Real gases under appropriate conditions of temperature and pressure can liquefy or solidify. e.g. at 1 atm pressure, water exist in liquid form between 0 to 100^0 C, solid below 0^0C and vapour above 100^0. These different forms of water are called its phases. At 100^0 under 1 atm pressure, the system can be in equilibrium with a part of it liquid water and the rest vapour. Similarly, at 0^0C under 1 atm pressure ice and liquid water can co-exist in equilibrium. Thus, in equilibrium state a system can have two (or move) phases.



(a) (water)



(b) CO_2

Phase Diagrams: A phase diagram divides the P – T plane into a solid region, the vapour region and the liquid region. The regions are separated by the curves representing the solid-liquid equilibrium (melting), liquid – vapour equilibrium (boiling) and solid – vapour equilibrium (sublimation). The point where these curves meet is called the triple point (solid – liquid – vapour equilibrium). The P – T diagrams for water and CO_2 are shown in figures. (a) and (b) respectively.

Thermodynamic Processes

Quasi – Static process:

Quasi – static process is an infinitely slow process such that the system remains in equilibrium with the surroundings throughout. In this process, the pressure and temperature of the environment can differ from the system only infinitesimally.

Isobaric process

A process taking place at constant pressure is called an isobaric process. For example, the boiling of water to steam in an open pot or the freezing of water to ice taking place at a constant atmospheric pressure are isobaric processes. In this process, the work done is equal to PΔV.

Illustration 24. *At 1 atmospheric pressure, 1.000 gm of water having a volume of 1.000 cm^3 becomes 1.091 cm^3 of ice on freezing. The heat of fusion of water at 1 atmosphere is 80.0 cal/gm. What is the change in internal energy during the process?*

Solution: Heat given out during freezing

$$Q = -mL = -1 \times 80 = -80 \text{ Cal}$$

work done by the system

$W = p(V_{ice} - V_{water}) = 1.013 \times 10^5 \times (1.091 - 1.000) \times 10^{-6}$

$= 9.22 \times 10^{-3}$ J = $\frac{9.22 \times 10^{-3}}{4.18}$ Cal = 0.0022 cal

$\therefore$ Change in internal energy

$\Delta U = Q - W = -80 - 0.0022 = -80.0022$ Cal.

Isochoric process

A process taking place at constant volume is called an isochoric process, $\Delta v = 0$)
work done, $dW = p.dV = 0$
$\therefore$ Net change in internal energy, $\Delta U = \Delta Q$

Adiabatic process

When a system passes from an initial state i to a final state f through a process such that no heat flows into or out of the system, then the process is called adiabatic.

Such a process can occur when the system is thermally insulated from the surroundings, or when the process is very rapid so that there is little or no time for the heat to flow into or out of the system.

As heat supplied, Q = 0
$\therefore$ Net change in internal energy, $\Delta U = -\Delta W$
i.e. the system does work at the cost of its own internal energy in an adiabatic process.
For an adiabatic process dQ = 0

$\Rightarrow \quad dU + PdV = 0 \qquad \ldots (1)$

for an ideal gas, $dU = nC_v\, dT \qquad \ldots (2)$

also $PV = nRT \Rightarrow dT = \frac{PdV + VdP}{nR} \qquad \ldots (3)$

Substituting equation (2) and (3) in equation (1)

$$nC_v\left(\frac{PdV + VdP}{nR}\right) + PdV = 0 \quad \Rightarrow \left(\frac{C_v}{R} + 1\right)PdV + \frac{C_v}{R}VdP = 0$$

$$\Rightarrow \quad C_p\, PdV + C_v\, VdP = 0 \quad \Rightarrow \frac{C_p}{C_v}\left(\frac{dV}{V}\right) + \frac{dP}{P} = 0$$

$\Rightarrow \quad \gamma \ln V + \ln P = 0 \Rightarrow PV^\gamma = K$ (Constant)

If an ideal gas expands adiabatically from initial volume V_i to final volume V_f, then work done

$$W = \int_{V_i}^{V_f} pdV \qquad \because PV^\gamma = K \text{ (constant)}$$

$$= K\int_{V_i}^{V_f} V^{-\gamma} dV = K\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_i}^{V_f} = \frac{1}{\gamma-1}\left[\frac{K}{V_i^{\gamma-1}} - \frac{K}{V_f^{\gamma-1}}\right] = \frac{1}{\gamma-1}\left[\frac{P_iV_i^\gamma}{V_i^{\gamma-1}} - \frac{P_fV_f^\gamma}{V_f^{\gamma-1}}\right]$$

$$W = \frac{P_iV_i - P_fV_f}{\gamma - 1}$$

Since $P_iV_i = nRT_i$

$P_fV_f = nRT_f \qquad \therefore \qquad W = \frac{nR(T_i - T_f)}{\gamma - 1}$

Adiabatic relations

(i) PV^γ = constant $\Rightarrow \quad P_iV_i^\gamma = P_fV_f^\gamma$

(ii) PV^{γ}= constant as $P = \dfrac{nRT}{V}$

$\therefore \dfrac{nRT}{V}V^{\gamma} = \text{constant}$

or $TV^{\gamma-1}$ = constant $\Rightarrow T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$

(iii) PV^{γ} = constant as $V = \dfrac{nRT}{P}$ $\therefore$ $P\left(\dfrac{nRT}{P}\right)^{\gamma} = \text{constant}$

or $T^{\gamma}.P^{1-\gamma}$ = constant

$\Rightarrow T_iP_i^{(1-\gamma)/\gamma} = T_fP_f^{(1-\gamma)/\gamma}$

Illustration 25. *A certain gas at atmospheric pressure is compressed adiabatically so that its volume becomes half of its original volume. Calculate the resulting pressure in dyne cm^{-2}. Given γ = 1.4.*

Solution: Initial volume, V_1 = V (say)
Final volume V_2 = V/2
Initial pressure P_1 = 76 cm of mercury column.
$P_1 = 76 \times 13.6 \times 981$ dyne cm^{-2}.
Final pressure P_2 = ?
Since, $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

$P_2 = P_1\left(\dfrac{V_1}{V_2}\right)^{\gamma} = P_1\left(\dfrac{V}{V/2}\right)^{1.4}$

$P_2 = 2.7 \times 10^6$ dyne cm^{-2}.

Isothermal process

When a system undergoes a process under the condition that its temperature remains constant, then the process is said to be "isothermal".

Such a process can occur when the system is contained in a perfectly conducting chamber and the process is carried out very slowly, so that there is sufficient time for heat exchange.

As the temperature of the system remains constant (T = constant), i.e. PV = constant.

For an ideal gas, the internal energy depends only on the temperature

$\therefore$ dU = 0 $\Rightarrow$ dQ = dW.

If an ideal gas expands from initial volume V_i to final volume V_f at constant temperature, the work done

$$W = \int_{V_i}^{V_f} p\,dV = nRT\int_{V_i}^{V_f}\frac{dV}{V} \qquad (\because PV = nRT)$$

$$W = nRT\ln\left(\frac{V_f}{V_i}\right); \quad W = nRT\ln\left(\frac{P_i}{P_f}\right).$$

Illustration 26. *A gram molecule of a gas at 127^{0}C expands isothermally until its volume is doubled. Find the amount of work done.*

Solution: Temperature (T) = (127 + 273) k = 400 k
Molar gas constant
R = 8.3×10^7 erg/gram molecule / ^{0}C

Initial volume $V_1 = V$ (say)
Final volume $V_2 = 2V$
work done (W) = $RT \ln_e\left(\frac{V_2}{V_1}\right)$ ($\because$ n = 1)

$$= 2.3026 \times 8.3 \times 10^7 \times 400 \log_{10}\frac{2V}{V} = 2.301 \times 10^{10} \text{ erg.}$$

Free expansion:

If a system (a gas), expands in such a way that no heat enters or leaves the system (adiabatic process) and also no work is done by or on the system, then the expansion is called the free expansion.

Consider an adiabatic vessel with rigid walls divided into two parts. One containing a gas and the other evacuated. When the partition is suddenly broken, the gas rushes into the vacuum and expands freely.

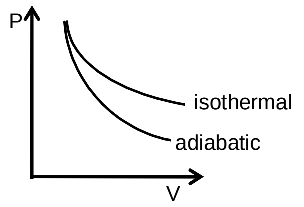
$\therefore$ Net change in internal energy

$U_f - U_i = \Delta Q - W$ as $\Delta Q = 0$ and $W = 0$

$\therefore$ $U_i = U_f$

The initial and final internal energies are equal in free expansion.

Slope of isothermal and adiabatic processes



For an isothermal process

PV = constant

Differentiating, PdV + VdP = 0

or $\left(\frac{dP}{dV}\right)_{\text{isothermal}} = -\frac{P}{V}$

For an adiabatic process

PV^γ = constant

Differentiating

$PV^{\gamma-1}\,dV + V^\gamma\,dP = 0$ or $\left(\frac{dP}{dV}\right)_{\text{adiabatic}} = -\gamma\frac{P}{V}$

The slope of the adiabatic for given pressure & volume is γ times the slope of the isothermal, both of them being negative.

Cyclic process:

When a particular system passes through various processes such that the initial and final states are the same, then the combination of such processes is called a cyclic process.

$\Delta U = 0$ for a cyclic process. Since,

$\Delta U = \Delta Q - W$ $\therefore$ $0 = \Delta Q - W$

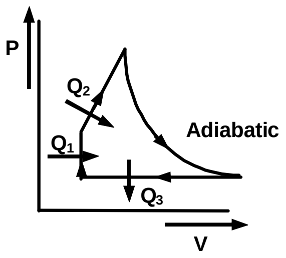
or $\Delta Q = W$, over a cyclic process.

Efficiency of a cyclic process:

Efficiency of a cycle is the ratio of the work done to the heat supplied

Efficiency of a cycle = $\frac{\text{Workdone}}{\text{Heat supplied}}$.

Let us consider an ideal gas going through the cycle as shown in figure.



If Q_1 and Q_2 be the heat supplied, while Q_3 is the heat rejected in the process.

$\therefore$ Efficiency = $\frac{\text{Work done}}{\text{Heat supplied}}$

$$= \frac{(Q_1 + Q_2) - Q_3}{(Q_1 + Q_2)} = 1 - \frac{Q_3}{Q_1 + Q_2}$$

$\because$ (Σ W = ΣQ) for a cyclic process.

Exercise 8.

(i) ***Is it possible that the temperature of a system may increase even when no heat is supplied to the system.***

(ii) ***A body is heated but there is no change of temperature. Is it possible?***

Illustration 27. *One mole of an ideal gas undergoes a cyclic change ABCD. Calculate the following from figure shown.*
(i) Work done along AB, BC, CD and DA.
(ii) Net work done in the process.
(iii) Efficiency of the process.
(iv) Net change in the internal energy of the gas.
(Given 1 atm = 1.01 $\times$ 10^5 Nm^{-2})

Solution: (i) $W_{AB} = PdV = 5 \times 1.01 \times 10^5 \times 2 \times 10^{-3}$ J
$= 1010$ J.
$W_{BC} = 0$
$W_{CD} = - 2 \times 1.01 \times 10^5 \times 2 \times 10^{-3} = - 404$ J.
(ii) Net work done, W = (1010 – 404) = 606 J.
(iii) Efficiency, $\eta = \frac{606}{1010} \times 100 = 60$ %.
(iv) Net change in internal energy = 0.
Because given process is cyclic.

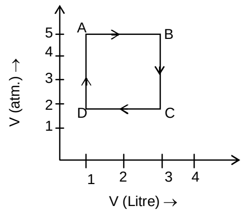


Illustration 28. *An ideal monoatomic gas at temperature 27^oC and pressure 10^6 N/m^2 occupies 10 litre volume. 10,000 cal of heat is added to the system without changing the volume. Calculate the final temperature of the gas. Given : R = 8.31 J/(mol-K) and J=4.18 J/Cal.*

Solution: For n mole of gas, we have PV = nRT
Here p = 10^6 N/m^2, V = 10 litre = $10^{-2} m^3$ and T = 27^oC = 300 K

$$\therefore \quad n = \frac{pV}{RT} = \frac{10^6 \times 10^{-2}}{8.31 \times 300} = 4.0$$

For "monatomic" gas, $C_v = \frac{3}{2}R$ (by kinetic theory and equipartition of energy).

Thus, $C_v = \frac{3}{2} \times 8.31$ J/mol-K $= \frac{3}{2} \times \frac{8.31}{4.18} \approx 3\text{Cal}/(\text{mole}-\text{K})$.

Let ΔT be the rise in temperature when n mole of the gas is given Q cal of heat at constant volume. Then,

$$Q = nC_v \Delta T$$

or $\Delta T = \frac{Q}{nC_v} = \frac{10,000 \text{ cal}}{4.0 \text{ mole} \times 3\text{cal}(\text{mole}-\text{K})} = 833$ K

$\therefore$ Final temperature of the gas is, $T + \Delta T = 300 + 833 = 1133$ K $= 860^oC$

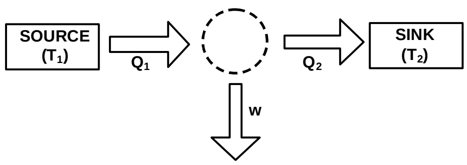
Heat Engine

Heat engine is a device used for converting heat energy into mechanical energy. A heat engine consists essentially of the following parts:

(i) Source or Heat reservoir: It is the supplier of heat energy.

(ii) Sink or Cold reservoir: That heat which has not been converted into work is rejected to the sink

(iii) Working substance: It absorbs a certain quantity of heat from the source, converts a part of it into work and rejects the remaining heat to the sink. It is taken through a cyclic operations



Block diagram of heat engine

Efficiency: It is defined as the ratio of the net external work done by the engine during one cycle to the heat absorbed from the source during that cycle It is denoted by η.

i.e. $\eta = \frac{W}{Q_1}$

Where W is the net external work done by the engine during one cycle and Q_1 is the heat absorbed from the source during that cycle.

Since the working substance returns to its initial state after completing one cycle therefore there is no change in internal energy.

Therefore, by applying first law of thermodynamics, we get

$$Q_1 - Q_2 = W$$

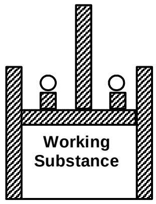
Where Q_2 is the amount of heat rejected to the sink,

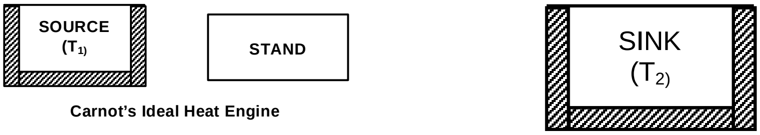
i.e. $\eta = \frac{Q_1 - Q_2}{Q_1}$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

For a given value of Q_1, the smaller the value of Q_2, the higher is the efficiency of the heat engine,

Carnot's Ideal Heat Engine: It is an ideal heat engine which is free from all the imperfections of an actual engine. It consists essentially of the following parts.





Carnot's Ideal Heat Engine

(i) Source: It serves as source of heat. It is maintained at a constant high temperature T_1K It has infinite thermal capacity.

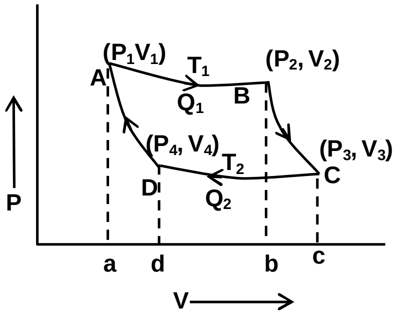
(ii) Sink: It is a cold body maintained at constant low temperature T_2K. It also has infinite thermal capacity.

(iii) Insulating stand: It is a perfectly non – conducting pad.

(iv) Cylinder it has perfectly non-conducting wall but bottom is perfectly conducting. It is fitted with perfectly non- conducting and frictionless piston over which some weights are placed. One mole of an ideal gas is enclosed in the cylinder. This ideal gas acts as the working substance,

The working substance is subjected to the following four successive reversible operations so as to complete a reversible cycle. This cycle is called Carnot's cycle.

Let the pressure, volume and temperature be P_1, V_1, and T_1 respectively. The state of the working substance is represented by the point A in the P – V diagram.



Operation I (Isothermal Expansion):

The cylinder containing the working substance is placed in contact with the heat source. So that the gas acquires the constant temperature T_1 of the source. Now, the gas is allowed to expand slowly. Therefore, it draws heat from the source through its conducting base and the piston slowly moves out of the cylinder. Piston moves slowly to maintain the temperature at T. Thus gas expands isothermally from the initial state A (P_1, V_1) to the final state B (P_2, V_2) along the isothermal AB at temperature T_1. If Q_1 be the amount of heat absorbed by the gas from the source and W_1 be the work done by it. Then

$$Q_1 = W_1 = \int_{v_1}^{v_2} PdV = RT\ell n_e \frac{V_2}{V_1} = \text{are a AB ba A} \quad \dots \text{(i)}$$

Operation II (Adiabatic expansion):

The cylinder is now removed from the source and is placed in contact with the insulating stand. The gas is completely thermally isolated from the surroundings. The gas expands adiabatically from volume V_2 to V_3 till its temperature falls to T_2K. The pressure falls from P_2 to P_3. This expansion is represented by the curve BC in the given figure.

The work done by the gas is

$$W_2 = \int_{V_2}^{V_3} PdV = \frac{R}{\gamma - 1}\left(T_1 - T_2\right) = \text{Area BC cb B} \quad \text{. . . (ii)}$$

Operation – III (Isothermal Compression):

Now the cylinder is placed on the sink and the gas compressed isothermally untill the pressure and volume because P_4 and V_4 respectively as shown in the PV diagram by the isothermal curve CD. The heat Q_2 developed in compression is absorbed by the sink. If W_3 be the work done on the gas then

$$Q_2 = W_3 = RT_2 \ell n_e \frac{V_3}{V_4} = \text{Area CcdDC} \qquad \text{. . . (iii)}$$

Operation IV (Adiabatic Compression):

Now, the cylinder is pleased on the insulating stand and the gas is compressed adiabatically till it attains pressure P_1, volume V_1 and temperature T_1 again. This compression is represented by the curve DA. Work done on the gas

$$W_4 = -\int_{V_2}^{V_1} PdV = \frac{R}{\gamma - 1}(T_1 - T_2) = \text{Area ADdaA} \qquad \text{. . . (iv)}$$

Net work done W by the working substance during one cycle.
W = work done by the gas – work done on the gas
$W = W_1 + W_2 - W_3 - W_4 = W_1 - W_3$
= area ABCDA

Efficiency of Carnot's cycle:

The efficiency of Carnot's heat engine is given by

$$\eta = 1 - \frac{Q_2}{Q_1}$$

from equation (1) & (3)

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \frac{\ell n_e \left(\frac{V_3}{V_4}\right)}{\ell n_e \left(\frac{V_2}{V_1}\right)} \qquad \text{. . . (v)}$$

The points B and C lie on the same adiabatic

$$\therefore T_1 V_1^{\gamma-1} = T_2 V_3^{\gamma-1} \qquad \text{. . . (vi)}$$

The points A and D lie on the same adiabatic

$$\therefore T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1} \qquad \text{. . . (vii)}$$

from equation (6) & (7)

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

from equation (5) $\dfrac{Q_2}{Q_1} = \dfrac{T_2}{T_1}$

$$\therefore \eta = 1 - \frac{T_2}{T_1}$$

Therefore,
(i) Efficiency is independent of the nature of the working substance.
(ii) It depends upon the temperatures of the heat source and sink only.

Illustration 29. *During isothermal expansion at 800 K, the working substance of a Carnot's engine extracts 480 cal of heat. If the sink be at 300K, calculate*

(i) the work done by the working substance during isothermal expansion.
(ii) the work done on the substance during isothermal comparison.
(iii) the efficiency of ideal engine.

Solution: Given that T_1 = 800 K, T_2 = 300 K, Q_1 = 480 cal. and we know that,

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

or $$Q_2 = \frac{T_2}{T_1} \times 480\text{cal.}$$

$$Q_2 = \frac{300}{800} \times 480 \text{ cal} = 180\text{cal.}$$

(i) Work done during isothermal expansion
$W_1 = Q_1 = 480$ cal $= 480 \times 4.2$ J
= 2016 J.

(ii) Work done during isothermal compression
$Q = 180$ cal $= 180 \times 4.2$ J $= 756$ J

(iii) Efficiency $\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100$

$$= \left(1 - \frac{180}{480}\right) \times 100 = 62.5\ \%$$

Illustration 30. *A Carnot engine operates between 27^0C and 227^0C. Calculate the work done if it takes 10^4 calories of heat from the source per cycle.*

Solution: Given that T_2 = (27 + 273)K = 300 K, T_1 = (227 + 273) = 500 K.

Efficiency $(\eta) = \frac{W}{Q} = 1 - \frac{T_2}{T_1}$

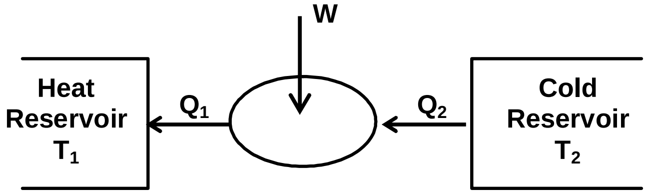
$$W = \left(1 - \frac{300}{500}\right) \times 10^4$$

$$= \left(1 - \frac{3}{5}\right) \times 10^4 = \frac{2}{5} \times 10^4 \times 4.18\text{ J}$$

$W = 1.68 \times 10^4$ J.

Refrigerators or heat pumps:

A refrigerator is reverse of a heat engine. In a refrigerator working substance extracts heat Q_2 from the cold reservoir at temperature T_2. Some external work W is done on it and heat Q_1 is released to the hot reservoir at temperature T_1. A heat pump is the same as a refrigerator.
As heat is to be removed from the sink at lower temperature, an amount of work equal to $Q_1 - Q_2$ is performed by the compressor of the refrigerator to remove heat from sink and then to reject the total heat $Q_1 = (Q_2 + Q_1 - Q_2)$ to the source through the radiator fixed at its back.



Coefficient of performance:

It is defined as the ratio of the quantity of heat removed per cycle from the contents of the refrigerator to the work done by the external agency to remove it. It is denoted by β.

Thus $\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$

$$\beta = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

For a carnat's cycle,

$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ $\therefore \beta = \frac{1}{\frac{T_1}{T_2} - 1}$

$$\beta = \frac{T_2}{T_1 - T_2}$$

Coefficient of performance cannot be infinite,

Reversible process:

It is that process which can be retraced in the opposite direction so that the system and the surroundings pass through exactly the same state at each stage as in the direct process.
e.g. If some work is done by the system in the direct process, the same amount of work is done on the system in the reverse process.
In order that a process may be reversible, it should satisfy the following conditions.
(i) The process should proceed at an extremely slow rate so that the following requirements are met.
(a) The system should remain in mechanical equilibrium,
(b) The system should remain in chemical equilibrium
(c) The system should remain in thermal equilibrium,
(ii) No dissipative forces should be present.

Irreversible process:

It is the process which is not exactly reversed, i.e. the system does not pass through the same intermediate state as in the direct process.
Every process in nature is an irreversible process. e.g.
(1) All chemical reactions are irreversible.
(2) Flow of current through a conductor is an irreversible process.

Second Law of Thermodynamics:

There are several statements of this law but the following two are the most significant.

(i) Kelvin - Plank statement:
It is impossible to construct an engine, operating in a cycle, which will produce no effect other than extracting heat from a reservoir and performing an equivalent amount of work.

(ii) Clausius Statement:

It is impossible to make heat flow from a body at a lower temperature to a body at a higher temperature without doing external work on the working substance.

These two statements are equivalent. The second law of thermodynamics is applicable only to a cyclic process in which the system returns to its original state after a complete cycle of changes.

The second law implies that no heat engine can have efficiency η equal to 1 or no refrigerator can have coefficient of performance α equal to infinity.

Carnot's Theorem:

No heat engine working between given temperatures can have efficiency greater than that of a reversible engine working between the same temperatures.

Illustration 31. *An ideal monoatomic gas at temperature 27^oC and pressure 10^6 N/m^2 occupies 10 litre volume. 10,000 cal of heat is added to the system without changing the volume. Calculate the final temperature of the gas. Given : R = 8.31 J/(mol-K) and J=4.18 J/Cal.*

Solution: For n mole of gas, we have PV = nRT

Here p = 10^6 N/m^2, V = 10 litre = 10^{-2}m^3 and T = 27^{o}C = 300 K

$$\therefore \quad n = \frac{pV}{RT} = \frac{10^6 \times 10^{-2}}{8.31 \times 300} = 4.0$$

For "monatomic" gas, $C_v = \frac{3}{2}R$ (by kinetic theory and equipartition of energy).

Thus, $C_v = \frac{3}{2} \times 8.31$ J/mol-K $= \frac{3}{2} \times \frac{8.31}{4.18} \approx 3\text{Cal}/(\text{mole} - \text{K})$.

Let ΔT be the rise in temperature when n mole of the gas is given Q cal of heat at constant volume. Then,

$$Q = nC_v \Delta T$$

$$\text{or} \quad \Delta T = \frac{Q}{nC_v} = \frac{10{,}000 \text{ cal}}{4.0 \text{ mole} \times 3\text{cal}(\text{mole} - \text{K})} = 833 \text{ K}$$

$\therefore$ Final temperature of the gas is, T + ΔT = 300 + 833 = 1133 K = 860^{o}C

Heat Transfer

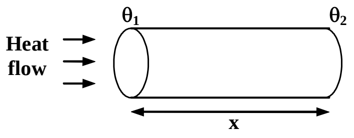
Heat Transfer — There are three modes of Heat Transfer:

CONDUCTION
Heat energy is transferred (usually through solids) from one part of the material medium to other without transferring the material particles.

CONVECTION
Heat energy is transferred (usually through liquids and gases) by bulk movement of the particles from one point to another. (Due to gravity & buoyant force).

RADIATION
Heat energy is transferred by electro-magnetic waves even in absence of medium.

Thermal conduction: Consider a slab of uniform cross-section A of length x, with lateral surface insulated. Let one face of the slab be maintained at temperature θ_1 and the other at θ_2.



In steady state, if ΔQ amount of heat crosses through any cross-section in time Δt, then

$$\frac{\Delta Q}{\Delta t} \propto \text{ A and } \frac{\theta_1 - \theta_2}{x} \quad \text{or} \quad \frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{X}$$

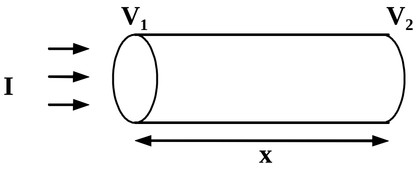
Where K is the co-efficient of thermal conductivity of the material of the slab.

For a small thickness dx along the direction of heat flow whose temperature difference is dQ.

$$\frac{dQ}{dt} = -KA\frac{d\theta}{dx}$$

Quantity $\frac{d\theta}{dx}$ is called the temperature gradient and minus sign indicates that $\frac{d\theta}{dx}$ is negative along the direction of heat flow.

From ohm's law, if current I flows through a conductor at potential difference (V_1 - V_2), then



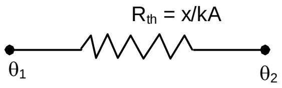
$V_1 - V_2 = I.R$

or $\quad I = \frac{V_1 - V_2}{R} \quad$ or $\frac{\Delta Q}{\Delta t} = \frac{(V_1 - V_2)}{\frac{1}{\sigma}\frac{x}{A}}$

where, R - resistance of conductor; σ - conductivity of the material; x - length of conductor

A - area of cross-section of the conductor; ΔQ - quantity of charge flowing through any cross-section in time Δt. Rewriting the equation for thermal conduction,

$$\frac{\Delta Q}{\Delta t} = \frac{(\theta_1 - \theta_2)}{\frac{1}{k}\frac{x}{A}}$$



$\rightarrow$ Heat transfer rate $\frac{dQ}{dt}$

The quantity $\frac{x}{KA}$ is called thermal resistance R_{th} .

Exercise 9.

(i) ***Pieces of copper and glass are heated to the same temperature. Why do the pieces of copper feel hotter on touching? At what common temperature the pieces of copper and glass feel equally hot when touched?***

(ii) ***Thermal conductivity of air is less than that of felt, but felt is a better heat insulator in comparison to air, why?***

Illustration 32. *The inside of the glass widow 2mm thick and one square metre in area is at a temperature of 15^0C and the temperature outside is -5^0C. Calculate the rate at which heat escaping from the room by condition through the glass. K of glass is 0.002 cal $cm^{-1}\ s^{-1}\ {}^0C^{-1}$.*

Solution: Given that, d = 2 mm = 0.2 cm

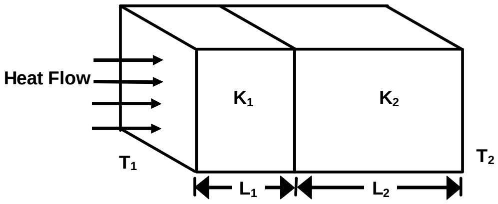
$A = 1\ m^2 = 10^4\ cm^2$

$T_1 = 15^0C$, $T_2 = -5^0C$

$k = 0.002$ cal $cm^{-1}\ s^{-1}\ {}^0C^{-1}$

Now $\frac{dQ}{dt} = \frac{kA(T_1 - T_2)}{d} = \frac{0.002 \times 10^4 \times [15 - (-5)]}{0.2}$ = 2000 cal s^{-1}.

Illustration 33. *Two plates each of area A, thickness L_1 and L_2 and thermal conductivities K_1 and K_2 respectively are joined to form a single plate of thickness (L_1 + L_2). If the temperatures of the free surfaces are T_1 and T_2. Calculate*



(a) Rate of flow of heat

(b) Temperature of interface and

(c) Equivalent thermal conductivity.

Solution: (a) If the thermal resistance of the two plates are R_1 and R_2 respectively then as plates are in series.

$$R_s = R_1 + R_2 = \frac{L_1}{AK_1} + \frac{L_2}{AK_2} \qquad \text{as} \quad R = \frac{L}{KA}$$

and so $$H = \frac{dQ}{dt} = \frac{\Delta\theta}{R} = \frac{(T_1 - T_2)}{(R_1 + R_2)} = \frac{A(T_1 - T_2)}{\left[\frac{L_1}{K_1} + \frac{L_2}{K_2}\right]}$$

(b) If T is the common temperature of interface then as in series rate of flow of heat remains same, i.e. $H = H_1(=H_2)$

$$\frac{T_1 - T_2}{R_1 + R_2} = \frac{T_1 - T}{R_1}, \qquad \text{i.e.} T = \frac{T_1R_2 + T_2R_1}{(R_1 + R_2)}$$

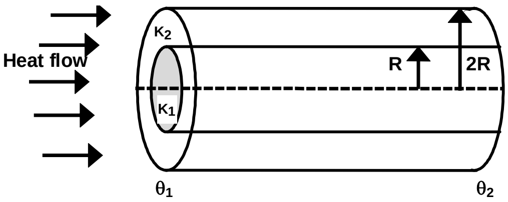
or $$T = \left[T_1\frac{L_2}{K_2} + T_2\frac{L_1}{K_1}\right] \Big/ \left[\frac{L_1}{K_1} + \frac{L_2}{K_2}\right] \qquad \text{as} \quad R = \frac{L}{KA}$$

(c) If K is the equivalent conductivity of composite slab, i.e., slab of thickness L_1 + L_2 and cross-sectional are A, then as in series

$$R_s = R_1 + R_2 \qquad \text{or} \qquad \frac{(L_1 + L_2)}{AK_{eq}} = R_1 + R_2$$

i.e. $$K_{eq} = \frac{L_1 + L_2}{A(R_1 + R_2)} = L_1 + L_2 \Big/ \left[\frac{L_1}{K_1} + \frac{L_2}{K_2}\right] \qquad \text{as} \quad R = \frac{L}{KA}.$$

Illustration 34. *A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by cylindrical shell of inner radius R and outer radius 2R made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and system is in steady state. What is the effective thermal conductivity of the system?*



Solution: In this situation a rod of length L and area of cross section πR^2 and another of same length L and area of cross-section $\pi[(2R)^2 - R^2] = 3\pi R^2$ will conduct heat simultaneously so total heat flowing per sec will be

$$\frac{dQ}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt} = \frac{k_1\pi R^2(\theta_1 - \theta_2)}{L} + \frac{k_2 3\pi R^2(\theta_1 - \theta_2)}{L} \quad \ldots(1)$$

Now if the equivalent conductivity is K.

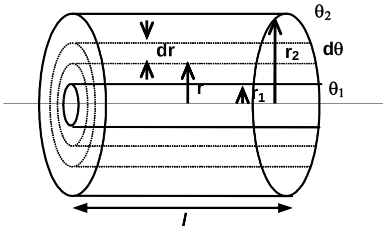
$$\frac{dQ}{dt} = K\frac{4\pi R^2(\theta_1 - \theta_2)}{L} \qquad [\text{as } A = \pi(2R)^2] \quad \ldots(2)$$

So from equations (1) and (2) we have

$4K = K_1 + 3K_2$ i.e. $K = (K_1 + 3K_2)/4]$

Illustration 35. *Find the rate of heat flow and temperature as a function of radius r in case of a cylindrical shell whose inner surface temperature and outer surface temperature are θ_1 and θ_2. The inner and the outer radii of the cylindrical shell are r_1 and r_2.*

Solution: Considering a cylindrical shell of length *l*, internal and external radii r_1 and r_2 respectively. Let its inner surface be maintained at a steady temperature θ_1 and the outer surface at a steady temperature θ_2 ($\theta_2 < \theta_1$).



Considering an elementary cylindrical shell of thickness dr, at temperature difference $d\theta$. Rate of radial flow of heat in steady state

$$H = \frac{dQ}{dt} = -K2\pi rl\,\frac{d\theta}{dr} \quad \Rightarrow \int_{r_1}^{r_2}\frac{dr}{r} = -\frac{K2\pi l}{H}(\theta_2 - \theta_1)$$

$$\ln(r_2/r_1) = \frac{K2\pi l(\theta_1 - \theta_2)}{H}$$

$$\text{Rate of heat flow, } H = \frac{dQ}{dt} = \frac{K2\pi l(\theta_1 - \theta_2)}{\ln(r_2/r_1)}$$

Considering the temperature of the layer θ at a distance r from the axis.

$$\int_{r_1}^{r}\frac{dr}{r} = -K\frac{2\pi l}{t_1}\int_{\theta_1}^{\theta} d\theta$$

$$\ln(r/r_1) = \frac{K2\pi l}{H}(\theta_1 - \theta) \quad \ldots(1)$$

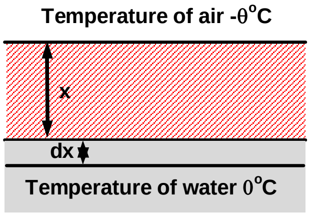
$$\text{and } \int_{r_1}^{r_2}\frac{dr}{r} = -K\frac{2\pi l}{H}\int_{\theta_1}^{\theta_2} d\theta$$

$$\ln(r_2/r_1) = K\frac{2\pi l}{H}(\theta_1 - \theta_2) \quad \ldots(2)$$

$$\text{Dividing, } \frac{\ln(r/r_1)}{\ln(r_2/r_1)} = \frac{\theta_1 - \theta}{\theta_1 - \theta_2}$$

$$\text{or } \theta = \theta_1 - (\theta_1 - \theta_2)\frac{\ln\ (r/r_1)}{\ln\ (r_2/r_1)}$$

Accretion of ice:



Consider a layer of ice of thickness x. The air temperature is -θ^oC and water temperature below the ice is 0^oC.

Considering unit cross-sectional area of ice, if a layer of thickness dx grows in time dt,

The heat given by this layer
= mass × latent heat = $1.dx.\rho.L$
where ρ = density of ice,
L = latent heat of fusion of ice.

If this quantity of heat is conducted upwards through the ice layer in time dt.

$$\therefore \quad dx.\rho.L = K\frac{0-(-\theta)}{x}dt, \quad \text{time taken } t = \frac{\rho L}{K\theta}\int_{x_1}^{x_2} x\,dx = \frac{\rho L}{2K.\theta}\left(x_2^2 - x_1^2\right)$$

Radiation

Coefficient Of Absorption, Reflection And Transmission

The coefficient of absorption or absorptive power 'a' of a body is the ratio of the quantity of heat radiation absorbed by the body Q_a to the quantity of heat radiation 'Q' incident on the body in the same time. $a = \frac{Q_a}{Q}$

The coefficient of reflection or reflective power 'r' of a body is defined as the ratio of the quantity of heat radiation reflected by the body 'Q_r' to the quantity of heat radiation 'Q' incident on the body in the same time. $r = \frac{Q_r}{Q}$

The coefficient of transmission 't' of a body is defined as the ratio of the quantity of heat transmitted by the body 'Q_t' to the quantity of heat radiation Q incident on the body in the same time.

$$t = \frac{Q_t}{Q}$$

By principle of conservation of energy
$Q = Q_a + Q_r + Q_t$
Dividing both sides by Q
$1 = a + r + t$

Emissive power E:

The emissive power denotes the energy radiated per unit time per unit area of the surface.

Emissivity ε:
Emissivity of a surface is the ratio of the emissive power of the surface to the emissive power of black body at the same temperature.
Emissivity ε = (Emissive power of the surface) ÷ (Emissive power of black body at the same temperature)

Black body:
A perfectly black body is one which absorbs completely all the radiation, of whatever wave-length, incident on it.

Exercise 10. ***On a winter night you feel warmer when clouds cover the sky than, when the sky is clear. Why?***

Kirchoff's law

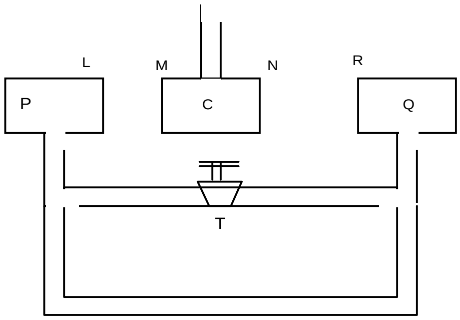
It states that the ratio of the emissive power to the absorptive power for radiation of a given wavelength is the same for all bodies at the same temperature, and is equal to the emissive power of a perfectly black body at that temperature.

$$\frac{e_\lambda}{a_\lambda} = E_\lambda \quad \text{(Constant)}$$

Exercise 11. ***A metallic ball has a black spot. The wall is heated to 1000^{0}C and is then taken in to a dark room. It is found that black sport looks brighter then rest of the ball. How can you understand this?***

RITCHIES EXPERIMENT

This experiment was conducted by Ritchie to verify Kirchoff's law. The experiment setup consists of two identical hollow cylinders P and Q connected by a U tube manometer particle filled with some coloured liquid.



A third cylinder C is mounted symmetrically between P and Q and all of them are made co-axial. All three cylinders have the same area of cross section and the cylinder C can be rotated about a vertical axis.

A horizontal tube with a stop case T, connects the vertical arms of the u tube. In the beginning the liquid levels in both the arms are made equal by opening the stopcock which is subsequently closed. Faces M and R are polished to the same extent. Hot water is poured in the cylinder C. It is observed that liquid levels is same in both arms. This shows the cylinder P and Q absorbs the same amount of heat exerting equal pressure on the liquid in the two arms.

Let E_b = emissive power of the block surface L and N.
E = emissive power of the polished surface M and R
a = coefficient of absorption of M and R.
A = Area of cross-section of each face.

Radiant energy emitted per second by N = A E_b, out of this only a fraction kAE_b is incident on the face R heat absorbed per second by R = $akAE_b$ . . . (i)
And it is used to heat the air in the cylinder Q.
Similarly, heat radiated per second by M = AE
Heat energy absorbed by L per second = kAE . . . (ii)

As L is blackened. As there is no change in the liquid levels it is airdent that L and R absorb heat at the same rate. Thus from (i) and (ii) we get $akAE_b = kAE \quad \Rightarrow a = E/E_b$

However by definition $e = \frac{E}{E_b} \quad \therefore \; a = e$

In the second part of the experiment, C is rotated through 180^0 so that the black force N faces the black force L and the polished faces M and R face each other. Now, during the same time interval N emits much more heat compared to M and L absorbs much more heat compared to R. The air in cylinder P expands more and pushes the liquid level downward as observed. This experiment shows that good emitter is a good absorber while bad emitter is a bad absorber.

Provost theory of exchange

Every body is continuously emitting radiant energy in all directions at a rate depending only on the nature of its surface and its temperature, and also it is absorbing radiant energy from all surrounding bodies at a rate depending on its surface and the temperature of the surrounding bodies. Thus there is continuous exchange of heat energy between a body and its surrounding. This is also known as Provost theory of exchanges.

Stefan's law of radiation

The total radiant energy emitted E per unit time by a black body of surface area A is proportional to the fourth power of its absolute temperature.

$E \propto T^4$

or $E = \sigma A T^4$ σ - Stefan's constant

For a body which is not a black body

$E = \varepsilon\sigma A T^4$ ε - emissivity of the body.

Using Krichoff's law

$$\frac{e_{body}}{a_{body}} = E_{black\ body}$$

or $$\frac{\varepsilon\sigma A T^4}{\sigma A T^4} = a$$

or $\varepsilon = a$

Emissivity and absorptive power have the same value.

Net loss of thermal energy

If a body of surface area A is kept at absolute temperature T in a surrounding of temperature T_o ($T_o < T$). Then the energy emitted by the body per unit time

$E = \varepsilon\,\sigma A T^4$

And energy absorbed per unit time by the body

$E_o = \varepsilon\,\sigma\ A T_o^4$

$\therefore$ Net, loss of thermal energy per unit time

$\Delta E = E - E_o = \varepsilon\sigma A(T^4 - T_o^4)$

Newton's law of cooling

For a small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference.

If a body of surface area A is kept at absolute temperature T in a surrounding of temperature T_o ($T_o < T$). Then net loss of thermal energy per unit time

$$\frac{dQ}{dt} = \varepsilon\sigma\ A\ \left(T^4 - T_0^4\right)$$

If the temperature difference is small

$\therefore \quad T = T_o + \Delta T \quad = \varepsilon\sigma A\{(T_0 + \Delta T)^4 - T_0^4\} \quad = \quad \varepsilon\sigma A\left\{T_0^4\left(1+\frac{\Delta T}{T_o}\right)^4 - T_0^4\right\}$

$= \varepsilon\sigma A{T_o}^4\left\{1+4\frac{\Delta T}{T_o} + \text{higher} \quad \text{powers} \quad \text{of} \quad \frac{\Delta T}{T_o} - 1\right\}$

$= 4\varepsilon\sigma A{T_o}^3\,\Delta T$ . . . (1)

Now, rate of loss of heat at temperature T

$\frac{dQ}{dt} = -\quad mc\frac{dT}{dt}$ . . . (2)

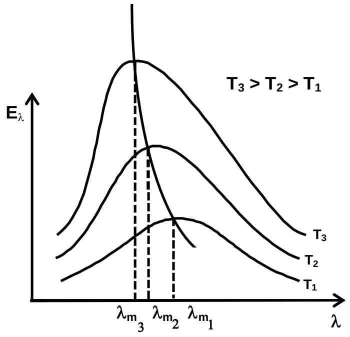
$\therefore \quad mc\frac{dT}{dt} = -4\,\varepsilon\sigma A{T_o}^3\,(T - T_o)$

$\frac{dT}{dt} = \frac{-4\varepsilon\quad\sigma AT_o^3}{mc}\left(T - T_o\right) \qquad \Rightarrow \quad \frac{dT}{dt} = -k(T - T_o)$ where $k = \frac{4\varepsilon\sigma\quad AT_o^3}{mc}$

$\frac{dT}{dt} \propto (T - T_o)\ ;\ -\int_{T_i}^{T_f}\frac{dT}{\left(T - T_o\right)} = \frac{4\varepsilon\sigma\quad AT_o^3}{mc}\int_0^t dt$

Spectral distribution of energy in black body radiation

The distribution of energy among the various wavelengths in black body radiation was studied by Lummer and Pringsheim, using electrically- heated chamber with a small aperture, acting as the black body. The information obtained is as such



(i) At a constant temperature T, when wavelength λ is increased, the energy emitted E_λ first increases, reaches a maximum and then decreases. i.e. at a particular temperature, the spectral radiancy E_λ is a maximum at a particular wavelength λ_m (say).

(ii) As the temperature increases, the maximum radiancy of energy occurs at shorter wavelength i.e. $\lambda_m.T = b$. where b is a constant. This is called Wein's displacement Law. b = 0.293 cm-kelvin

(iii) As the temperature rises, the area enclosed by the curve goes on increasing.

Simple Radiation Correction:

In experiments on calorimetry a solid is heated to a very high temperature and introduced in to a liquid in the calorimeter. The final temperature of the mixture is noted after stirring the liquid. Loss of heat due to conduction is minimized by keeping the calorimeter on pointed supports and surrounding it by non – conducting materials. Heat loss by radiation inside the calorimeter is minimized by the inner surface. Howerver, in spite of all the loss of heat due to radiation can not be stopped. Radiation correction is applied to the observed temperature for accurate results.

Radiation correction by half time method. In the experiment of finding the specific heat of a solid by method of mixture, the initial temperature θ of the liquid is measured. The hot solid is dropped in the mixture and the time 't' required by the mixture to attain the maximum temperature θ_2 is after time (t/2) is noted. The radiation correction is given by $\Delta\theta = \theta_2 - \theta_3$. The corrected final temperature of the mixture is given by $\theta_2' = \theta_2 + \Delta\theta$.

Illustration 36. *Due to change in the mains voltage, the temperature of an electric bulb rises from 3000 K to 4000 K. What is the percentage rise in electric power consumed?*

Solution: Electric power consumed in first case

$P_1 = \sigma T_1^4$

$= \sigma(3000)^4 = \sigma \times 4^4 \times 10^{12}$

Electric power consumed in second care $P_2 = \sigma\ T_2^4$

$= \sigma\ (4000)^4 = \sigma \times 4^4 \times 10^{12}$

$\therefore\ \frac{P_2}{P_1} = \frac{256}{81}$

Rise in electric power consumed = 256 – 81 = 175 units.

Percentage rise = $\frac{175}{81} \times 100 = 216$.

Wien's Displacement Law:

It states that the product of the wavelength of the maximum emission and the corresponding absolute temperature of the block body is always constant

i.e. $\lambda_m T = b$

Where b is constant and its value is 2.9×10^{-3} m^0K, in SI system and λ_{max} is the wavelength with maximum energy of radiation,

Illustration 37. *The temperature of an ordinary electric bulb is 3000 K. At what wavelength will it radiates maximum energy ? Will this wavelength be within visible region? Given Wien's constant b = 0.288 cmK*

Solution: Given T = 300 K, b = 0.288 cmK

Now, $\lambda_{max} = \frac{b}{T} = \frac{0.288\text{cm}}{3000} = 9600\ A^0$.

Illustration 38. *Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K calcualate*

(a) The temperature of B and (b) wavelength λ_B

Solution: (a) According to Stefan's law the power radiated by a body is given by

$P = e\sigma AT^4$

According to the given problem $P_A = P_B$ with $A_A = A_B$

so that $e_A T_A^4 = e_B T_B^4$, i.e. $0.01 \times (5802)^2 = 0.81(T_B)^4$

or $T_B = (1/3)(5802) = 1934$ K

(b) According to Wien's displacement law

$\lambda_A T_A = \lambda_B T_B$, i.e. $\lambda_B = (5802/1934)\lambda_A$

i.e. $\lambda_B = 3\lambda_A$ and also $\lambda_B - \lambda_A = 1\mu m$ (given)

So $\lambda_B - (1/3)\lambda_B = 1\mu m$, i.e. $\lambda_B = 1.5\ \mu m$.

Solar Constant (S):
It is the amount of heat energy received per second per unit area of a perfectly black surface placed at a mean distance of the Earth from the Sun, in the absence of Earth's atmosphere, the surface being held perpendicular to the direction of the sun's rays. The value of solar constant is 1388 Wm^{-2} or 2 cal cm^{-2} min^{-1}

Surface temperature of the Sun:
Energy falling in one second on the unit area of the Earth's surface held normal to Sun's rays is called solar constant S. Experimentally S is found to be equal to 1388 Wm^{-2}. Let R be the radius of the Sun and r be the radius of Earth's orbit around the Sun. Let E be the energy emitted by the Sun per second per unit area. Then, the total energy emitted by the Sun in one second $= 4\pi R^2 \times E$. This energy is falling on a sphere of radius equal to the radius of the Earth's orbit around the Sun i.e. on a sphere of surface area $4\pi r^2$.

The energy falling per unit area $= \dfrac{4\pi R^2 \times E}{4\pi r^2} = \dfrac{E \times R^2}{r^2}$

By definition, this is solar constant S

i.e. $S = \dfrac{ER^2}{r^2}$ But $E = \sigma T^4$

According to Stefan's law

$$S = \frac{\sigma T^4 R^2}{r^2} \quad \text{or} \quad T^4 = \frac{Sr^2}{\sigma R^2}$$

$$\text{or } T = \left[\frac{S \times r^2}{\sigma \times R^2}\right]^{1/4}$$

Now, $S = 1388\ wm^{-2}$, $R = 6.96 \times 10^8 m$,
$r = 1.496 \times 10^{11}$ m, $\sigma = 5.68\times10^{-8}$ SI units

Therefore, on substituting these values, temperature of the sun is
T = 5791K

Illustration 39. *A black body radiates heat energy at the rate of 2×10^5 joule/sec m^2 at a temperature of 127°C. The temperature of the black body at which the rate of heat radiation is 32×10^5 joule/sec m^2 is*
(A) 273°C *(B) 527°C*
(C) 873°C *(D) 927°C*

Solution: (B).
$E \propto T^4$ or $E_1/E_2 = (T_1/T_2)^4$

$$\therefore \frac{2\times10^5}{32\times10^5} = \left(\frac{400}{T}\right)^4$$

On solving, we get T = 527°C

Illustration 40. *A partition wall has two layers A and B in contact each made of a different material. They have the same thickness but the thermal conductivity of layer A is twice that of B. If the steady state temperature difference across layer B is 60 K, then the corresponding temperature difference across layer A is*
(A) 10 K *(B) 20 K*
(C) 30 K *(D) 40 K*

Solution: (C).
$k_1 \Delta T_1 \equiv k_2 \Delta T_2$

$2k\,\Delta T = k \times 60$
or $\Delta T = \Delta T_1 = 30$ K

Illustration 41. *A body cools in 7 minute from 60° to 40°C. How much time (in minute) does it take to cool from 40°C to 28°C if the surrounding temperature is 10°C? Assume Newton's law of cooling.*

(A) 10 *(B) 8*
(C) 7 *(D) 5*

Solution: (C).
For small temperature difference between a body and its surrounding, the rate of cooling is proportional to the temperature difference.

$$\therefore \frac{(60-40)/7}{(40-28)/t} = \frac{(50-10)}{(34-10)} = \frac{40}{24}$$

$$\Rightarrow t = \frac{40 \times 7 \times 12}{24 \times 20} = 7 \text{ min}$$

Illustration 42. *Two cylindrical rods of the same material have the same temperature difference between them ends. The ratio of the rates of flow of heat through them is 1:8. The radii of the rods are in the ratio 1:2. What is the ratio of their lengths?*

(A) 2:1 *(B) 4:1*
(C) 1:8 *(D) 1:32*

Solution: (A).

$$\frac{Q}{t} = \frac{K\pi r^2(\theta_1 - \theta_2)}{\ell_1} \quad ...(1)$$

$$\frac{8Q}{t} = \frac{K\pi (2r)^2(\theta_1 - \theta_2)}{\ell_2} \quad ...(2)$$

Dividing Eq. (2) by Eq. (1),

$$8 = \frac{\ell_1}{\ell_2} \times 4 \quad \text{or} \quad \frac{\ell_1}{\ell_2} = \frac{2}{1}$$

Illustration 43. *Hot water cools from 60°C to 50°C in the first 10 minute and to 42°C in the next 10 minute. The temperature of the surroundings is*

(A) 5°C *(B) 10°C*
(C) 15°C *(D) 20°C*

Solution: (B). From Newton's law of cooling

$$\frac{dQ}{dt} \propto (\theta - \theta_0)$$

$$\frac{60-50}{10} = K\left(\frac{60+50}{2} - \theta_0\right) \quad ...(1)$$

$$\frac{50-42}{10} = K\frac{50+42}{2} - \theta_0 \quad ...(2)$$

Dividing Eq. (1) by Eq. (2), we get

$$\frac{10}{8} = \frac{55-\theta_0}{46-\theta_0}$$

or $460 - 10\theta_0 = 440 - 8\theta_0$

or $2\theta_0 \equiv 20$

or $\theta_0 = 10^{\circ}C$

Illustration 44. *Ice starts forming in a lake with water at 0°C when the atmospheric temperature is –10°C. If the time taken for the first 1 cm of ice to be formed is 7 hour, then the time taken for the thickness of ice to change from 1 cm to 2 cm is*

(A) 7 hour *(B) 14 hour*

(C) 21 hour *(D) 3.5 hour*

Solution: (C).

Conduction formula is $Q = \dfrac{KA(\theta_1 - \theta_2)t}{d}$

Let A be the area of lake. Then, mass of ice formed in the first case, m = (A) $(\ell)\rho$,

Now, Q = mL, where L is the latent heat of ice

$$\therefore A \times \ell \times \rho L = \frac{KA[0-(-10)]t_1}{(0+1)/2} \quad \ldots(1)$$

Mean thickness, $d = \left(\dfrac{0+1}{2}\right)$ cm

In second case, mass of ice formed is $A \times (2-1) \times \rho$

$$\therefore A \times (2-1) \times \rho = \frac{KA[0-(-10)]t_2}{(1+2)/2} \quad \ldots(2)$$

Here, mean thickness, $d = \dfrac{1+2}{2}$

Dividing Eq. (2) by Eq. (1),

$$1 = \left(\frac{t_2}{3/2}\right) \Big/ \left(\frac{t_1}{1/2}\right) = \frac{t_2}{3t_1}$$

$\therefore t_2 = 3t_1 = 3 \times 7 = 21$ hour

Illustration 45. *Two insulating cylinders A and B fitted with pistons contain equal amount of an ideal diatomic gas at temperature 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in cylinder A is 30 K, then the rise in temperature of the gas B is*

(A) 30 K *(B) 18 K*

(C) 50 K *(D) 42 K*

Solution: (D) For cylinder A,

$dQ = nC_p\, dT_1$

$= n(C_v + R)\, dT_1$

For cylinder B,

$dQ = nC_v\, dT_2$

$\therefore \quad n(C_v + R)\, dT_1 = nCv\, dT_2$

$dT_2 = \left(\dfrac{C_v + R}{C_v}\right) dT_1$

For diatomic gas, $C_v = \dfrac{5}{2}R$

$\therefore \quad dT_2 = \dfrac{7}{5} \times dT_1 = \dfrac{7 \times 30}{5} = 42\text{ K}$

Illustration 46. *A thermodynamical system goes from state: (i) (P, V) to (2P, V) and (ii) (P, V) to (P, 2V), then the work done in the two cases is*

(A) (i) Zero, (ii) Zero *(B) (i) Zero, (ii) PV*

(C) (i) PV, (ii) Zero *(D) (i) PV, (ii) PV*

Solution: The correct answer is (B).

Illustration 47. *A sample of ideal gas (γ = 1.4) is heated at constant pressure. If an amount of 100 J heat is supplied to the gas, the work done by the gas is*

(A) 42.12 J *(B) 56.28 J*

(C) 28.57 J *(D) 36.23 J*

Solution: (C) $C_p = \left(\dfrac{\gamma}{\gamma - 1}\right)R$

$dw = dq - du = n(C_p - C_v)dT = nR\, dT.$

$dq = 100$ J

$n\, dT = 100/C_p$

$dw = R\left(\dfrac{100}{C_p}\right)$

$= R\left(\dfrac{\gamma - 1}{\gamma . R}\right) \times 100$

$= \dfrac{1.4 - 1}{1.4} \times 100 = \dfrac{40}{1.4}$

= 28.57 J.

Illustration 48. *One mole of an ideal gas undergoes a pressure change according to the equation* $P = \dfrac{P_0}{1 + \left(V_0/V\right)^2}$*, where P_0 and V_0 are constants. The temperature of the gas when the volume is changed from $V = V_0$ to $V = 2V_0$ is*

(A) $\dfrac{-2P_0V_0}{5R}$ *(B)* $\dfrac{11P_0V_0}{4R}$

(C) $\dfrac{-5P_0V_0}{2R}$ *(D) P_0V_0*

Solution: (B) At $V = V_0$, $P = P_0/2$ ($\because P = \dfrac{P_0}{1 + (V_0/V)^2}$)

$$\therefore\ T_i = \frac{PV}{nR} = \frac{\left(\frac{P_0}{2}\right)V_0}{R} = \frac{P_0V_0}{2R}$$

and at $V = 2V_0$; $P = \frac{4P_0}{5}$

$$\therefore\ T_f = \frac{PV}{nR} = \frac{(2V_0)\left(\frac{4P_0}{5}\right)}{R} = \frac{8P_0V_0}{5R}$$

$$\therefore\ \Delta T = T_f - T_i = \left(\frac{8}{5} - \frac{1}{2}\right)\frac{P_0V_0}{R} = \frac{11P_0V_0}{10R}$$

Illustration 49. *The volume of steam produced by 1 g of water at 100°C is 1650 cm^3. The change in internal energy during the change of state is Given J = 4.2×10^7 erg/cal, g =981 cm/s^2, latent heat of steam = 540 cal/g.*

(A) 21 × 10^9 ergs *(B) 12 × 10^9 erg*
(C) 21 × 10^2 ergs *(D) 12 × 10^2 ergs.*

Solution: (A) Here, mass of water = 1 g,
Volume of water =1 cm^3,
Volume of steam =1650 cm^3,
As the state of water is changing, so
$\Delta Q = mL = 1 \times 540$ cal.
$= 540 \times 4.2 \times 10^7$ ergs $= 2.268 \times 10^{10}$ ergs
Now, taking P = 1 atm = $76 \times 13.6 \times 981$ dyne/cm^2
$\therefore\ \Delta W = P\,\Delta V = 76 \times 13.6 \times 981 \times (1650 - 1)$
$= 1.67 \times 10^9$ ergs
By first law of thermodynamics,
$\Delta Q = \Delta U + \Delta W$
$\therefore\ \Delta U = \Delta Q - \Delta W$
$= (22.68 - 1.67) \times 10^9$ ergs
$= 21 \times 10^9$ ergs .

Illustration 50. *Three samples of same gas A, B and C (γ = 3/2) have initially equal volume. Now, the volume of each sample is doubled. The process is adiabatic for A, isobaric for B and isothermal for C. If the final pressure is equal for all three samples, the ratios of their initial pressure are*

(A) 2$^{3/2}$: 2 : 1 *(B) 2$^{3/2}$: 1 : 2*
(C) $\sqrt{2}$:1:2 *(D) 2 : 1 :$\sqrt{2}$*

Solution: (A) Let the initial pressures of three samples be P_A, P_B and P_C then

$P_A(V)^{\frac{3}{2}} = (2V)^{\frac{3}{2}}P$

$P_B = P$

$P_C(V) = P(2V)$

$\Rightarrow\ P_A : P_B : P_C = 2^{\frac{3}{2}} : 1 : 2$

Illustration 51. *An ideal gas is taken from the state A (pressure P, volume V) to a state B (pressure P/2, volume 2V) along a straight line path in the P-V diagram. Select the correct statement(s) from the following.*
(A) The work done by the gas in the process A to B exceeds the work that would be done by it if the system were taken from A to B along the isotherm
(B) In the T-V diagram, the path AB becomes a part of parabola.
(C) In the P-T diagram, the path AB becomes a part of the hyperbola.
(D) In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases.

Solution: (A) & (B)

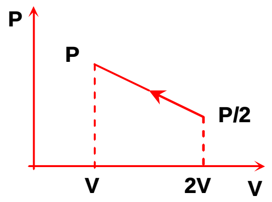


Fig. 1

Work done from A to B =area of P-V diagram

$$=\left(P+\frac{P}{2}\right)V=\frac{3PV}{2}=\frac{3}{2}RT$$

In case of isothermal process,

W = RT $\log_e(V_2/V_1)$ = RT × 2.303 log 2

= 0.693 RT {option (A)}

The equation of straight line is given by

$$\frac{P}{P_0}+\frac{V}{V_0}=1$$

Here, P_0, V_0 are the intercepts on P- and V-axes, respectively.

In T-V graph, P = $\frac{PT}{V}$ (assuming one mole of the gas)

$$\therefore \quad \frac{RT}{P_0V}+\frac{V}{V_0}=1$$

$$\text{or,}\quad T=\frac{P_0V}{R}\left(1-\frac{V}{V_0}\right)=\frac{P_0V}{R}-\frac{P_0V^2}{RV_0}$$

This equation is the equation of parabola, so {option (B)}.

Illustration 52. *A gas mixture consists of 2 moles of O_2 and 4 moles of Ar at a temperature T. Neglecting all vibrational modes, the total internal energy of the system is*

(A) 4RT	*(B) 15RT*
(C) 9RT	*(D) 11RT*

Solution: (D) For O_2 (diatomic), n_1 = 2 moles, $C_{v1}=\frac{5}{2}R$

So, $U_1 = n_1C_{v1}T$ = 5RT

For Ar (monoatomic), n_2 = 4 moles, $C_{v2}=\frac{3R}{2}$

So, $U_2 = n_2C_{v2}T = 4\times\frac{3R}{2}\times T$ = 6 RT

∴ Total internal energy

U = $U_1 + U_2$

= 5 RT + 6 RT = 11 RT.

SUMMARY

1. **Zeroth Law of thermodynamics**
If a body A is in thermal equilibrium with another body B and a body B is in thermal equilibrium with another body C, then A is also in thermal equilibrium with C.

2. Absolute Scale T(K) = T(°C) + 273

3. For small changes in temperature:
$L = L_0(1+\alpha\Delta T)$; $A = A_0(1+\beta\Delta T)$; $V = V_0(1+\gamma\Delta T)$
$\alpha : \beta : \gamma = 1 : 2 : 3$ (for isotropic substances only)

4. For a liquid:
$\gamma_a = \gamma_R - \gamma_c$
γ_R = coefficient of real expansion of liquid
γ_a = coefficient of apparent expansion of the liquid
γ_c = coefficient of cubical expansion of container

5. Variation of density with temperature, $\rho = \dfrac{\rho_0}{1+\gamma\Delta T}$

6. Thermal stress = $\dfrac{F}{A} = \alpha Y \Delta\theta$ (where $\Delta\theta$ is the increase in temperature)

7. **RMS Velocity of Molecules of an Ideal Gas**
$v_{rms} = \sqrt{\dfrac{v_1^2 + v_2^2 + \cdots v_n^2}{n}} = \sqrt{\dfrac{3RT}{M}}$
$PV = \dfrac{1}{3}mnv_{rms}^2$, where m = mass of molecule and n = number of molecules

8. Kinetic energy of random motion is the internal energy, given by
KE = $\dfrac{1}{2}m.v_{rms}^2 = \dfrac{1}{2}n.M\dfrac{3RT}{M} = \dfrac{3}{2}.n.RT$, where n is the number of moles of a gas in the system.

9. The number of degrees of freedom that a molecule can possess is equal to the number of independent motions that it can have and is denoted by the letter 'f'
For an ideal gas molecule,
$U = \dfrac{1}{2}fRT$, for one mole of the gas, R = kN; N = Avogadro's Number,

10. Ratio of the two specific heats, $\gamma = \dfrac{C_P}{C_V} = 1 + \dfrac{2}{f}$

11. A molecule of a monatomic gas has only three (translational) degrees of freedom,
i.e. f = 3 and γ = 1.66

12. A molecule of a diatomic gas has 5 degrees of freedom (3 - translational and 2- rotational) at ordinary temperatures (because the vibrational modes are not excited.) and $\gamma = 1.40$

13. A molecule of triatomic or polyatomic gas has 6 degrees of freedom (3 translational and 3 rotational) $f = 6$ and $\gamma = 1.33$

14. First Law of Thermodynamics
If ΔQ is heat given to a system, and ΔW is work done by the system, then ΔU, the change in its internal energy can be written as
$\Delta Q = \Delta U + \Delta W$, where $\Delta U = nC_v\Delta T$,
$\Delta W = \int_{v}^{v_f} PdV$ = Area enclosed under P-V graph.
$\Delta Q = nC\Delta T$, where C = specific heat capacity of a gas, for that process.
Note: For a cyclic process work done = area enclosed under P-V graph, and is positive if the cycle is clockwise

15. **Important Cases of the First Law of Thermodynamics**
(a) Isobaric process (Pressure = constant)
$$\Delta Q = \Delta U + \Delta W$$
$$nC_p\Delta T = nC_v\Delta T + nR\Delta T$$
$$C_p - C_v = R \text{ (Mayer's equation)}$$
(b) Isochoric process ($\Delta W = 0$, Volume = constant)
$$\Delta Q = \Delta U = nC_v\Delta T$$
(c) Isothermal process ($\Delta U = 0$; temperature = constant)
$$\Delta Q = \Delta W = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT\ln\left[\frac{V_2}{V_1}\right] = nRT\ln\left[\frac{P_1}{P_2}\right]$$
(d) Adiabatic process ($\Delta Q = 0$)
$TV^{\gamma-1}$ = constant
(e) Cyclic process
$\Delta U = 0$, since process returns to the same initial state
$\Delta U = \Delta Q - \Delta W = 0$
$\therefore \Delta Q = \Delta W$
(f) Free expansion
In free expansion (a gas) expands in such a way that no heat enters or leaves the system (adiabatic process) and also no work is done by or on the system, then the expansion is called the free expansion
$U_f - U_i = Q - W$, Now $Q = 0$, $W = 0$
$\therefore U_f = U_i$

16. For an isothermal process, $\left(\frac{dP}{dV}\right)_{\text{isothermal}} = -\frac{P}{V}$

17. For an adiabatic process, $\left(\frac{dP}{dV}\right)_{\text{adiabatic}} = -\gamma\frac{P}{V}$

18. Efficiency of a process $\eta = \frac{\Delta W}{\Delta Q} \times 100$

19. Thermal Conduction

$$\frac{\Delta Q}{\Delta t} = \frac{KA(T_1 - T_2)}{x}$$

where K is the co-efficient of thermal conductivity of the material of the slab.

Also R = thermal resistance = $\frac{x}{KA}$

In parallel connection, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

In series connection, $R_{eq} = R_1 + R_2 + \cdots + R_n$

20. Every body is continuously emitting radiant energy in all direction at a rate depending only on the nature of its surface and its temperature, and also it is absorbing radiant energy from all surrounding bodies at a rate depending on its surface and the temperature of the surrounding bodies. This is also known as Prevost's theory of exchanges.

21. **Stefan's Law of Radiation**

Energy emitted per unit time by a black body of surface area A is given by

or $\frac{d\theta}{dt} = \sigma A T^4$ [Stefan's constant $\sigma = 5.67 \times 10^{-8}$ Wm^{-2}-Kelvin^{-4}]

Energy emitted per unit time by a body other than a black body,

$\frac{d\theta}{dt} = \sigma A \varepsilon T^4$ [ε = emissivity]

22. Net Loss of Thermal Energy $\Delta E = \sigma\varepsilon A\,(T^4 - T_o^4)$

23. Absorptive power of a body is defined as the fraction of the incident radiation that is absorbed by the body.

Absorptive power (A) = $\frac{\text{Energy absorbed}}{\text{Energy incident}}$

24. The emissive power (E) denotes the energy radiated per unit time per unit area of the surface.

25. Emissivity **(**ε) of a surface is the ratio of the emissive power of the surface to the emissive power of black body at the same temperature.

Emissivity ε = (Emissive power of the surface) ÷ (Emissive power of black body at the same temperature)

26. A perfectly blackbody is one which absorbs completely all the radiation, of whatever wave-length, incident on it.

27. **Kirchhoff's Law**

It states that the ratio of the emissive power to the absorptive power for radiation of a given wavelength is the same for all bodies at the same temperature, and is equal to the emissive power of a perfectly blackbody at that temperature.

$\frac{e_\lambda}{a_\lambda} = E_\lambda$ (Constant)

28. Spectral Distribution of Energy in Black Body Radiation
Wien's displacement law is $\lambda_m T$ = constant
Where, λ_m = wavelength corresponding to maximum spectral intensity
T = absolute temperature of surface of the body.

29. **Newton's Law of Cooling**
For a small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference.
$\frac{dT}{dt} = -k(T - T_0)$ (rate of fall of temperature)
T = temperature of the body.
T_0 = temperature of the surroundings

MISCELLANEOUS EXERCISE

1. Prove that the slope of PV graph for an adiabatic process is γ times that of the isothermal process.

2. Prove for an adiabatic process. (i) $TV^{\gamma-1}$ = constant, (ii) $P^{1-\gamma}T^{\gamma}$ = constant.

3. Distinguish between an isothermal and adiabatic process.

4. Define C_P and $C_{V.}$

5. 200 joule of work is done on a gas to reduce its volume by compressing it. If the change is done under adiabatic conditions, find out the change in internal energy of the gas and also the amount of heat absorbed by the gas.

6. Differentiate between evaporation and boiling.

7. What is the value of γ - for a gas having 'n' degrees of freedom?

8. What are cyclic and non-cyclic processes?

9. Obtain an expression for work done by a gas in an isothermal expansion.

10. Prove that for an adiabatic process PV^{γ} = constant, where the symbols have their usual meanings.

SOLUTIONS OF MISCELLANEOUS EXERCISE

1. For isothermal process, PV = constant
Differentiating, VdP + PdV = 0

$$\therefore \quad \frac{dP}{dV} = -\frac{P}{V}$$

For adiabatic process, PV^{γ} = constant.
Differentiating, $V^{\gamma}dP + \gamma PV^{\gamma-1}dV = 0$

$$\therefore \quad \frac{dP}{dV} = -\frac{\gamma P}{V}$$

Comparing the two ratio's we can say, slope of adiabatic process = γ times of the slope of isothermal process.

2. (i) We know PV^{γ} = constant for adiabatic process.

Also, PV = nRT. $\therefore\ P = \frac{nRT}{V}$

Replacing P, we have $\frac{nRT}{T}\cdot V^{\gamma}$ = constant (or) $TV^{\gamma-1}$ = constant.

(ii) From PV = nRT, we have $V = \frac{nRT}{P}$

Replacing V, we have, $P\left(\frac{nRT}{P}\right)^{\gamma}$ = constant.

$\therefore\ T^{\gamma}P^{1\therefore\gamma}$ = constant.

3.

Isotherm process	Adiabatic process
1. Temperature remains same or $\Delta T = 0$. 2. Has to be a slow process. 3. Slope of PV graph is comparatively small. 4. PV = constant. 5. $W = 2.3026\ nRT \log\left(\frac{V_f}{V_i}\right)$	1. Heat energy exchange is zero. 2. Has to be a fast process. 3. Slope of PV graph is comparatively larger (γ times). 4. PV^γ = constant. 5. $W = -nC_v(T_f - T_i)$.

4. C_p id defined as the amount of heat required to raise the temperature on =e gram of a gas thr4ough 1°C keeping its pressure constant. It is measured in Cal $g^{-1}{}^\circ C^{-1}$.
C_v is defined as the amount of heat required to raise the temperature of one gram of a gas through 1°C keeping its volume constant. It is also measured in Cal $g^{-1}{}^\circ C^{-1}$.

5. In adiabatic changes, dQ = 0.
$\therefore$ From dQ = dU + dW = 0
dU = –dW = –(–200 J) = 200 J
Internal energy increases by 200 J. Heat absorbed is zero.

6. Evaporation is a slow process from the liquid to the gaseous state which takes place at the surface of a liquid and at all temperatures. Boiling is a rapid change of a substance from the liquid to the gaseous state which takes place throughout the mass of the liquid of a definite temperature.

7. For n degrees of freedom,

$C_v = (r/2)R,\ C_p = \left(\frac{n}{2}+1\right)R$

$y = \frac{C_P}{C_v} = \left(\frac{2+n}{n}\right)$

8. Cyclic processes are those in which the thermo dynamical co-ordinates remain the same before and after the process. The path followed may be the same or different. The area enclosed by the PV-graph will give the work done in the entire cycle. Since temperature is same there is no internal energy variation.
Non-cyclic processes are those in which the thermo dynamical co-ordinates are different before and after the process The area below PV-graph gives the work done. In this process, work is done and internal energy also changes.

9. For a small changes in volume, work done is given by dW = P dV.
We know, PV = nRT $\therefore\ P = \frac{nRT}{V}$ for T = constant, $dW = nRT\frac{dV}{V}$.
Net work doen under isothermal conditions to change the volume from V_i to V_f is,

$$W = \int_{V_i}^{V_f} dW = nRT\int_{V_i}^{V_f}\frac{dV}{V}$$

$$= nRT\left|\log_e V\right|_{V_i}^{V_f} = nRT\log_e\left(\frac{V_f}{V_i}\right)$$

$$\therefore\quad W = 2.3026\, nRT\ \log_{10}\left(\frac{V_f}{V_i}\right)$$

where n is the number of moles. If P_f and P_i are the pressures, we can also write,

$$W = 2.3026\ nRT \log_{10}\left(\frac{P_i}{P_f}\right)$$

10. For an adiabatic process, dQ = 0

$dU = nC_v dT$ for a process, where there is a temperature change by dT.

From gas equation, PV = nRT

Differentiation both sides, we have

$$PdV + VdP = nRdT$$

$$dT = \frac{PdV + VdP}{nR} \qquad ...(i)$$

From first law of thermodynamics, $0 = nC_V dT + PdV$ …(ii)

Putting dT from (i) in (ii), we have,

$$nC_v\left(\frac{PdV + VdP}{nR}\right) + PdV = 0 \text{ or } C_v(PdV + VdP) + RPdV = 0$$

or $C_v(PdV + VdP) + (C_p - C_v)PdV = 0$ $[\because R = C_p - C_v]$

$$C_v vdP + C_p PdV = 0 \text{ or } \frac{dP}{P} + \frac{dV}{V}y = 0 \qquad [\because C_p/C_v = \gamma]$$

Integrating, we get, $\int \frac{dP}{P} + \gamma \int \frac{dV}{V} = \text{constant}$

or $\log P + \gamma \log V$ = constant or $\log PV^{\gamma}$ = constant.

or PV^{γ} = constant.

This is the equation for an adiabatice change in an ideal gas.

SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob 1. *An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are Q_1 = 5960 J; Q_2 = –5585 J; Q_3 = –2980J; and Q_4 = 3645 J respectively. The corresponding works involved are W_1 = 2200 J; W_2 = -825 J; W_3 = -1100 J and W_4 respectively.*
(a) Find the value of W_4 (b) What is the efficiency of the cycle?

Sol. (a) According to the given Prob
$\Delta Q = Q_1 + Q_2 + Q_3 + Q_4 = 5960 - 5585 - 2980 + 3645$
$\Delta Q = 9605 - 8565 = 1040$ J.
$\Delta W = W_1 + W_2 + W_3 + W_4 = 2200 - 825 - 1100 + W_4 = 275 + W_4$
And as for cyclic process, $U_F = U_I$, $\Delta U = U_F - U_I = 0$
So from first law of thermodynamics , i.e., $\Delta Q = \Delta U + \Delta W$. we have
$1040 = (275 + W_4) + 0$, i..e. $W_4 = 765$ J

(b) As efficiency of a cycle is defined as

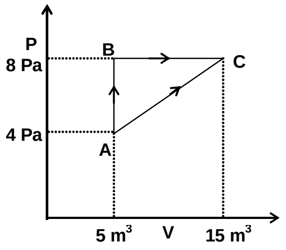
$$\eta = \frac{\text{Net work}}{\text{Input heat}} = \frac{\Delta W}{(Q_1 + Q_4)} = \frac{\Delta Q}{(Q_1 + Q_4)}$$

$$\eta = \frac{1040}{9605} = 0.1082 = 10.82\%$$

Prob 2. *In the given figure, an ideal gas changes its state from state A to state C by two paths ABC and AC.*
(i) Find the path along which work done is the least.
(ii) The internal energy of gas at A is 10 J and the amount of heat supplied to change its state to C through path AC is 200 J. Calculate the internal energy of gas at C.
(iii) The internal energy of gas at state B is 20 J. Find the amount of heat supplied to the gas to go from A to B.

Sol. (i) Minimum work done is along path AC
(ii) Process A to C

$$Q = 200 \text{ J}$$



Work done W_{AC} = area under AC

$$= (10 \times 4) + \left(\frac{10 \times 4}{2}\right) = 60 \text{ J}$$

From Ist law of thermodynamics.

$$\Delta U = Q - W_{Ac}$$
$$U_C - U_A = 200 - 60$$
$$\therefore U_C = U_A + 140 = 10 + 140 = 150\text{J}.$$

(iii) Path A to B

$U_B = 20\ J$

$\therefore \quad \Delta U = Q - W_{AB}$

$U_B - U_A = Q - 0$

$20 - 10 = Q$

$\therefore \quad Q = 10\ J$

Prob 3. *5 mole of O_2 is heated at constant volume from 10°C to 20°C. What is the change in its internal energy? Molar specific heat of O_2 at constant pressure is 8 cal/mole-°C and R = 8.36 J/mole-°C.*

Sol. R = 8.36 J/mol-°C = 2 cal/mole-°C.

Since, $C_P = C_v + R$

$\therefore \quad C_v = C_P - R = 8 - 2 = 6$

$\therefore \quad \Delta Q = nC_v\,\Delta T = 5\times6\times10 = 300$ cal

Since work done = $\int PdV = 0$

$\therefore \quad \Delta U = \Delta Q = 300$ cal.

Prob 4. *A slab of stone of area 0.36 m^2 and thickness 0.1 m is exposed on the lower surface to steam at 100^0C. A block of ice at 0^0C rests on the upper surface of the slab. In one hour, 4.8 kg of is melted. Calculate the thermal conductivity of stone.*

Sol. Given, A = 0.36 m^2, d = 0.1 m

$T_1 - T_2 = 100 - 0 = 100^0C$.

t = 1 h = 3600 s.

Mass of ice melted, M = 4.8 kg

Latent heat of ice $L = 336 \times 10^3\ J\ kg^{-1}$

Therefore, heat required to melt the ice

$Q = ML = 4.8 \times 336 \times 10^3$

$Q = 1.613 \times 10^6\ J$ . .. (i)

$$Q = \frac{kA(T_1 - T_2)}{d}t = \frac{k \times 0.36 \times 100 \times 3600}{0.1}$$

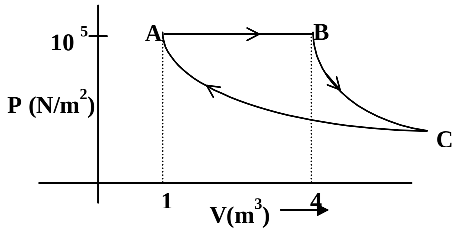
$Q = 1.296 \times 10^6\ k$ . . . (ii)

From equation (i) & (ii)

$1.296 \times 10^6\ k = 1.613 \times 10^6$

$k = 1.245\ Wm^{-1}\ {}^0C^{-1}$.

Prob 5. *A fixed mass of gas is taken through a process $A \to B \to C \to A$. Here $A \to B$ is isobaric. $B \to C$ is adiabatic and $C \to A$ is isothermal. Find*
(a) Pressure and volume at C
(b) work done in the process
(take $\gamma = 1.5$)



Sol. (a) For adiabatic process BC

$P_B V_B^{\gamma} = P_C V_C^{\gamma}$ . . . (1)

For isothermal process CA

$P_AV_A = P_CV_C$ . . . (2)

From (1) and (2)

$V_c = \left[\frac{V_B^\gamma}{V_A}\right]^{\frac{1}{\gamma-1}} = 64\ m^3$; $P_C = \frac{P_A V_A}{V_C} = \frac{10^5}{64}$ N/m^2

(b) Work done, $W = W_{AB} + W_{BC} + W_{CA}$

$= P(V_B - V_A) + \frac{1}{\gamma-1}[PV_B - P_CV_C] + PV_A \ell n\frac{V_A}{V_C}$

Putting the values

$W = 4.9 \times 10^5$ J

IITJEE TYPE

Prob 6. *A vertical hollow cylinder of height 1.52 m is fitted with a movable Piston of negligible mass and thickness. The lower half portion of the cylinder contains an ideal gas and the upper half is filled with mercury. The cylinder is initially at 300 K. When the temperature is raised, half of the mercury comes out of the cylinder. Find this temperature, assuming the thermal expansion of mercury to be negligible.*

Sol. Initial pressure of gas

$P_1 = P_0 + \frac{H}{2}dg$, P_0 is atmospheric pressure

Initial volume of gas $V_1 = \frac{V}{2}$

Initial temperature $T_1 = 300\,K$

When half of mercury comes out of the cylinder final pressure of gas $P_2 = P_0 + \frac{H}{4}dg$

and final volume of gas $V_2 = \frac{V}{2} + \frac{V}{4} = \frac{3V}{4}$

We have $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\therefore \quad T_2 = \frac{P_2V_2}{P_1V_1}T_1 = \left[\left\{\left(P_0 + \frac{H}{4}dg\right).\left(\frac{3V}{4}\right)\times 300\right\}\Big/\left\{\left(P_0 + \frac{H}{2}dg\right).\frac{V}{2}\right\}\right]$

Putting the valued $P_0 = 0.76\,dg$ (atmospheric Pressure) we get

$T_2 = 337.5\,K$.

Prob 7. *One mole of an ideal gas whose pressure changes with volume as $P = \alpha V$, where α is a constant, is expanded so that its volume increases η times. Find work done and change in internal energy adiabatic exponent = γ*

Sol. Let V be the initial volume of the gas. It is expanded to a volume ηV. The work done in this process is given by

$$W = \int_V^{\eta V} PdV = \int_V^{\eta V} \alpha VdV = \alpha\left[\frac{V^2}{2}\right]_V^{\eta V} = \frac{\alpha V^2}{2}\left[\eta^2 - 1\right]$$

The pressure of the gas varies with volume as $P = \alpha V$. So, the initial and final pressures will be αV and $\eta\alpha V$. The change in internal energy is given by

$$U = \frac{P_f V_f - P_i V_i}{\gamma - 1} = \frac{\eta^2 \alpha V^2 - \alpha V^2}{\gamma - 1} = \frac{\alpha V^2}{\gamma - 1}\left(\eta^2 - 1\right).$$

Prob 8. *At 27°C two moles of an ideal monoatomic gas occupy a volume V. The gas expands adiabatically to a volume 2 V. Calculate*
(a) final temperature of the gas
(b) change in its internal energy and
(c) the work done by the gas during the process. [R = 8.31 J/mol K]

Sol. (a) In case of adiabatic change

PV^γ = constant with $PV = \mu RT$

so that $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ [with $\gamma = (5/3)$]

i.e. $300 \times V^{2/3} = T(2V)^{2/3}$ or $T = 300/(2)^{2/3} = 189$ K

(b) As $\Delta U = \mu C_v \Delta T = \dfrac{\mu R \Delta T}{(\gamma - 1)}$ $\left[\text{as } C_v = \dfrac{R}{(\gamma - 1)}\right]$

$$\text{so } \Delta U = 2 \times \left(\frac{3}{2}\right) \times 8.31(189 - 300) = -2767.23\text{ J}$$

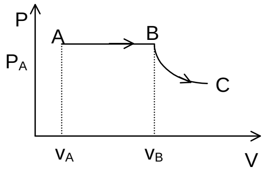
Negative sign means internal energy will decrease.

(c) According to first law of thermodynamics

$Q = \Delta U + \Delta W$

And as for adiabatic change $\Delta Q = 0$, $\Delta W = -\Delta U = 2767.23$ J

Prob 9. *Two moles of Helium gas (γ = 5/3) are initially at temperature 27°C and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value.*
(i) Sketch the process on P-V diagram.
(ii) What are the final volume and pressure of gas?
(iii) What is the work done by the gas? (Gas constant R = 8.3 J/mole K)



Sol. (i). From ideal gas equation

$PV = nRT$

initial pressure

$$P = \frac{nRT}{V} = \frac{2 \times 8.3 \times 300}{20 \times 10^{-3}}$$

$$= 2.49 \times 10^5\ \text{N/m}^2$$

When volume of gas is doubled at constant pressure, its temperature is also doubled. This process is shown on P-V curve by line AB. The gas then cools to temperature T adiabatically. This is shown by curve BC. The whole process is represented by curve ABC.

(ii) At point B, pressure $P_B' = P_A = 2.49 \times 10^5\ \text{N/m}^2$.

Volume $V_B = 2V_A = 40 \times 10^{-3}\ \text{m}^3$, Temperature $T_B = 600$ K.

Now from adiabatic equation $TV^{\gamma - 1}$ = constant

We have $T_A\ V_A^{(\gamma - 1)} = T_C\ V_C^{(\gamma - 1)}$

$\therefore \quad \left(\frac{V_C}{V_B}\right)^{\gamma-1} = \frac{T_B}{T_C} = \frac{600}{300} = 2$

$\therefore \quad \frac{V_C}{V_B} = 2^{1/(\gamma-1)} = 2^{3/2}$

Final volume

$\therefore \quad V_C = 2\sqrt{2}\,V_B$

$= 2 \times 1.414 \times 40 \times 10^{-3} = 113.13 \times 10^{-3}\ m^3$

and final pressure

$P_C = \frac{nRT_C}{V_C} = \frac{2 \times 8.3 \times 300}{113.13 \times 10^{-3}} = 0.44 \times 10^5\ N/m^2$

(iii) The work done by gas in isobaric process AB

$= 2.49 \times 10^5 \times (40 - 20) \times 10^{-3} = 4980\,J$

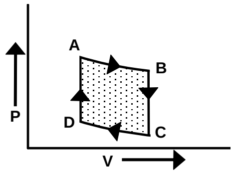
The work done by gas during adiabatic process BC

$W_2 = \frac{nR}{1-r}\left[T_2 - T_1\right] = \frac{2 \times 8.3}{1-\left(5/3\right)}\left[300 - 600\right] = 7470\ J.$

$\therefore$ Net work done by gas $W = W_1 + W_2$

$= 4980 + 7470 = 12450\ J.$

Prob 10. *One mole of a monotomic ideal gas is taken through the cycle shown in figure.*



$A \rightarrow B$ Adiabatic expansion
$B \rightarrow C$ Cooling at constant volume
$C \rightarrow D$ Adiabatic compression.
$D \rightarrow A$ Heating at constant volume

The pressure and temperature at A, B etc,. are denoted by P_A, T_A; P_B, T_B etc. respectively.

Given T_A = 1000K, $P_B = (2/3)P_A$ and $P_C = (1/3)P_A$. Calculate (a) The work done by the gas in the process $A \rightarrow B$ (b) the heat lost by the gas in the process $B \rightarrow C$ and (c) temperature T_D given $(2/3)^{2/5} = 0.85$ and R = 8.31 J/mol K.

Sol. (a) As for adiabatic change PV^γ = constant

i.e. $P\left(\frac{\mu RT}{P}\right)^\gamma = \text{constant}$ $\quad [\text{as}\ PV = \mu RT]$

i.e. $\frac{T^\gamma}{P^{\gamma-1}} = \text{constant}$ so $\left(\frac{T_B}{T_A}\right)^\gamma = \left(\frac{P_B}{P_A}\right)^{\gamma-1}$ where $\gamma = \frac{5}{3}$

i.e. $T_B = T_A\left(\frac{2}{3}\right)^{1-\frac{1}{\gamma}} = 1000\left(\frac{2}{3}\right)^{2/5} = 850K$

so $W_{AB} = \frac{\mu R[T_F - T_I]}{[1-\gamma]} = \frac{1 \times 8.31[1000 - 850]}{[(5/3) - 1]}$

i.e. $W_{AB} = (3/2) \times 8.31 \times 150 = 1869.75$ J

(b) For $B \rightarrow C$, V= constant so $\Delta W = 0$

So from first law of thermodynamics

$\Delta Q = \Delta U + \Delta W = \mu C_v \Delta T + 0$

or $\Delta Q = 1 \times \left(\frac{3}{2}R\right)(T_C - 850)$ as $Cv = \frac{3}{2}R$

Now along path BC, V = constant ; $P \propto T$

i.e. $\frac{P_C}{P_B} = \frac{T_C}{T_B}$, $T_C = \frac{(1/3)P_A}{(2/3)P_A} \times T_B = \frac{T_B}{2} = \frac{850}{2} = 425$ K …(2)

So $\Delta Q = 1 \times \frac{3}{2} \times 8.31(425 - 850) = -5297.625$J

[Negative heat means, heat is lost by the system]

(c) As A and D are on the same isochoric

$$\frac{P_D}{P_A} = \frac{T_D}{T_A}, \quad \text{i.e.,} \quad P_D = P_A\frac{T_D}{T_A}$$

But C and D are on the same adiabatic

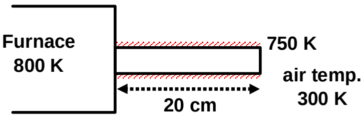
$$\left(\frac{T_D}{T_C}\right)^{\gamma} = \left(\frac{P_D}{P_C}\right)^{\gamma-1} = \left(\frac{P_A T_D}{P_C T_A}\right)^{\gamma-1}$$

or $(T_D)^{1/\gamma} = T_C\left[\frac{P_A}{P_C T_A}\right]^{1-\frac{1}{\gamma}}$, i.e. $T_D^{3/5} = \left(\frac{T_B}{2}\right)\left[\frac{P_A}{(1/3)P_A 1000}\right]^{2/5}$

i.e. $T_D^{3/5} = \left[\frac{1}{2}\left(\frac{2}{3}\right)^{2./5} \times 1000\right]\left[\frac{3}{1000}\right]^{2/5}$ i.e., $T_D = 500$K

Prob 11. *One end of a rod of length 20 cm is inserted in a furnace at 800 K. The sides of the rod are covered with an insulating material and the other end emits radiation like a blackbody. The temperature of this end is 750 K in the steady state. The temperature of the surrounding air is 300 K. Assuming radiation to be the only important mode of energy transfer between the surrounding and the open end of the rod, find the thermal conductivity of the rod. Stefan constant* $\sigma = 6.0 \times 10^{-8}$ *W/m*^{2}*-K*^{4} *.*

Sol. Quantity of heat flowing through the rod in steady state



$$\frac{dQ}{dt} = \frac{K.A.d\theta}{x} \quad …(1)$$

Quantity of heat radiated from the end of the rod in steady state

$$\frac{dQ}{dt} = A\sigma\left(T^4 - T_o^4\right) \quad … (2)$$

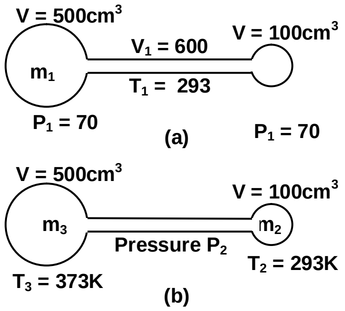
From (1) and (2), $\frac{K.d\theta}{x} = \sigma\left(T^4 - T_0^4\right)$

$$\frac{K \times 50}{0.2} = 6.0 \times 10^{-8}[(7.5)^4 - (3)^4] \times 10^8$$

or K = 74 W/m-K .

Prob 12. *Two glass bulbs of volumes 500cm*^{3} *and 100cm*^{3} *are connected by a narrow tube whose volume is negligible. When the apparatus is sealed off, the pressure of the air inside is 70 cm of mercury and its temperature 20*^{o}*C. What does the pressure become if the 100cm*^{3} *bulb is kept at 20*^{o}*C and the other is heated to 100*^{o}*C.*

Sol. The situation is as shown in figure.



Case (i)

Let m be the mass of air in both bulbs in first case. We know that

$P_1V_1 = n_1RT_1$

Where R is the gas constant of 1 gm of air.

$\therefore \quad n_1 = \frac{P_1V_1}{RT_1} = \frac{70C \times 600}{R \times 293}$ moles …(1) $(C = 10^{-2} \times 13.6 \times 10^3 \times 9.8)$

($\because$ $V_1 = 500 + 100 = 600$)

Case (ii)

After heating, let n_2 be no of moles of air in the bulb of volume 100cm^3 and n_3 be no of moles of air in bulb of volume 500cm^3. Let the pressure of the gas be P_2.

For the bulb of volume 100cm^3, applying the equation PV = nRT, we have

$P_2C \times 100 = n_2 .R. 293$

or $\quad n_2 = \frac{P_2C \times 100}{R \times 293}$ …(2)

For the bulb of volume 500 cm^3, $P_2 \times 500 = n_3.R.373$

$\Rightarrow \quad n_3 = \frac{P_2C \times 500}{R \times 373}$ …(3)

But $n_1 = n_2 + n_3$ …(4)

Substituting the values of n_1, n_2 and n_3 from equation (1), (2) and (3) in equation (4), we get

$$\frac{70C \times 600}{R \times 293} = \frac{P_2C \times 100}{R \times 293} + \frac{P_2C \times 500}{R \times 373}$$

or $\quad \frac{70 \times 600}{293} = P_2\left[\frac{100}{293} + \frac{500}{373}\right]$

or $\quad \frac{420}{293} = P_2\left[\frac{1}{293} + \frac{5}{373}\right]$

Solving we get P_2 = 85.23 cm of mercury.

Prob 13. *Suppose the pressure P and the density ρ of air are related as $P/\rho^n = C$ regardless of height (n & c are constant here). Find the corresponding temperature gradient.*

Sol. The temperature gradient is given by

$$\frac{dT}{dh} = \frac{dT}{d\rho}.\frac{d\rho}{dP}.\frac{dP}{dh} \quad …(1)$$

$\because \quad dP = -\rho g dh$

$$\therefore \quad \frac{dP}{dh} = -\rho g \quad …(2)$$

Given that $P/\rho^n = c$ (constant)

$\therefore \quad P = c\rho^n$

$$\frac{dp}{d\rho} = cn\rho^{n-1} \quad \Rightarrow \quad \frac{d\rho}{dp} = \frac{1}{cn\rho^{n-1}} \quad \ldots(3)$$

We know that $P = \rho\frac{R}{M}T$

or $C\rho^n = \rho\,(R/M)T \;\therefore \quad T = (M/R)C\rho^{n-1}$

or $\frac{dT}{d\rho} = \frac{M}{R}C(n-1)\rho^{n-2} \quad \ldots(4)$

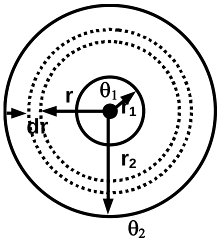
From (2), (3) and (4) substituting these values in (1), and solving we get,

$$\frac{dT}{dh} = -\frac{Mg(n-1)}{nR}$$

Prob 14. *Find temperature as a function of radius r in case of hollow sphere having inner and outer radii* r_1 and r_2, maintained at temperature θ_1 and θ_2 respectively.

Sol. Considering a spherical shell of inner radius r_1 and outer radius r_2, maintained at temperature θ_1 and θ_2 respectively ($\theta_2 < \theta_1$).

Considering an elementary spherical shell of thickness dr at a temperature difference $d\theta$. Rate of radial flow of heat in steady state



$$H = \frac{dQ}{dt} = -K\left(4\pi r^2\right)\frac{d\theta}{dr}$$

or $\int_{r_1}^{r_2}\frac{dr}{r^2} = -\frac{K4\pi}{H}\int_{\theta_1}^{\theta_2} d\theta\,; \quad \left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{K.4\pi}{H}\left[\theta_1 - \theta_2\right]$

Rate of flow,

$$H = \frac{dQ}{dt} = \frac{K4\pi\left(\theta_1 - \theta_2\right)}{\left[\frac{1}{r_1} - \frac{1}{r_2}\right]} = \frac{4\pi K r_1 r_2\left(\theta_1 - \theta_2\right)}{\left(r_2 - r_1\right)}$$

Considering the temperature of the layer as θ at a distance r from the centre.

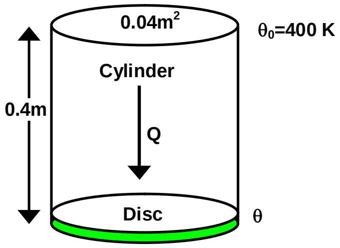
$$\int_{r_1}^{r}\frac{dr}{r^2} = -\frac{K4\pi}{H}\int_{\theta_1}^{\theta} d\theta \Rightarrow \quad \left[\frac{1}{r_1} - \frac{1}{r}\right] = \frac{K.4\pi}{H}\left[\theta_1 - \theta\right]$$

and $\int_{r_1}^{r_2}\frac{dr}{r^2} = -\frac{K.4\pi}{H}\int_{\theta_1}^{\theta_2} d\theta \quad \ldots(1)$

$\Rightarrow \quad \left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{K.4\pi}{H}\left[\theta_1 - \theta_2\right] \quad \ldots(2)$

Dividing, $\dfrac{\frac{1}{r_1} - \frac{1}{r}}{\frac{1}{r_1} - \frac{1}{r_2}} = \dfrac{\theta_1 - \theta}{\theta_1 - \theta_2}\,; \quad \theta = \theta_1 - \dfrac{\frac{1}{r_1} - \frac{1}{r}}{\frac{1}{r_1} - \frac{1}{r_2}}\left(\theta_1 - \theta_2\right)$

Prob 15. *A cylindrical block of length 0.4m and area of cross-section 0.04m² is placed coaxially on a thin metal disc of mass 0.4 kg and same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is 300K. If the thermal conductivity of the material of the cylinder is 10 w/mK and the specific heat of the material of the disc is 600 J/kg K, how long will it take for the temperature of the disc to increase to 350 K? Assume for purpose of calculation the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper and lower face of the cylinder.*



Sol. If the temperature of disc at time t is θ, the heat conducted by the cylinder to the disc in time dt

$$dQ = KA\frac{(\theta_0 - \theta)}{L}dt \qquad \ldots(1)$$

This heat increases the temperature of disc of specific heat c and mass m from θ to $\theta + d\theta$ so $dQ = mc\, d\theta$ …(2)

So from equation (1) and (2)

$$KA\frac{(\theta_0 - \theta)}{L}dt = mc\,d\theta \quad \text{i.e.} \quad dt = \frac{mcL}{KA}\frac{d\theta}{(\theta_0 - \theta)}$$

Integrating the above between limit, $\theta = \theta_1$ to $\theta = \theta_2$

$$t = \frac{mcL}{KA}\left[-\ln(\theta_0 - \theta)\right]_{\theta_1}^{\theta_2} = \frac{mcL}{KA}\ln\left[\frac{\theta_0 - \theta_1}{\theta_0 - \theta_2}\right]$$

Substituting the given data -

$$t = \frac{0.4 \times 600 \times 0.4}{10 \times 0.04}\ln\left[\frac{400 - 300}{400 - 350}\right] = 240\ \ln 2$$

$t = 240 \times 0.7 = 168$ sec = 2 min 48 sec [as $\ln 2 \approx 0.7$]

Objective:

Prob 1. *Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. The mass of the steam condensed in kg is*

(A) 0.130 *(B) 0.065*
(C) 0.260 *(D) 0.135*

Sol. Heat taken = Heat given

Let mass be m

$\therefore$ m $\times$ 540 + m(100 – 80) = (1.1 + 0.02)(80 – 15)

540 m + 20 m = 1.12 $\times$ 65 $\Rightarrow$ 560m = 1.12 $\times$ 65

$\Rightarrow$ m = $\frac{1.12 \times 65}{560}$ = 0.130

Here A is correct.

Prob 2. *The average translational kinetic energy and the rms speed of molecules in a sample of oxygen at 300K are 6.21 x 10^{-21}J and 484 m/s respectively. The corresponding values at 600K area nearly (assuming ideal gas behavior).*

(A) 12.42 x 10^{-21}J, 968 m/s. *(C) 6.21 x 10^{-21}J, 968 m/s*
(B) 8.78 x 10^{-21}J, 684m/s *(D) 12.42 x 10^{-21}J, 684 m/s*

Sol. $KE = \frac{3}{2}KT$, $v_{rms} = \sqrt{\frac{3RT}{M}}$

i.e. $\frac{KE_2}{KE_1} = \frac{T_2}{T_1} = 2 \quad \therefore KE_2 = 2KE_1 = 2 \text{ x } 6.21 \text{ x } 10^{-21}$

$\frac{V_{rms,2}}{V_{rms,1}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{2}$

$\therefore V_{rms,2} = \sqrt{2} \text{ x } V_{rms,1}$ = 684 m/s.

Hence (D) is correct.

Prob 3. *In a room where temperature is 30°C a body cools from 61°C to 59°C is 4 minutes. The time taken by the body to cool from 51°C to 49°C will be:*

(A) 4 minutes *(B) 6 minutes*
(C) 5 minutes *(D) 8 minutes*

Sol. Rate of cooling α difference in temperature

$-\frac{dT}{dt} \alpha \Delta\theta$; $\frac{dT}{dt} = -K\Delta\theta$

dT = – K$\Delta\theta$.dt

In First Case

dT = 61–59=2

$\Delta\theta$ = 60 – 30 = 30

dt = 4 minutes

For second case

dT = 2

$\Delta\theta$ = 50 – 30 = 20

$\therefore K = -\frac{dT}{\Delta\theta dt} = -\frac{2}{30 \times 4} = -\frac{1}{60}$ $\therefore dt = \frac{dT}{K\Delta\theta} = \frac{2}{\frac{1}{60} \times 20} = 6$ min.

Prob 4. *Gas at pressure P_o is contained in a vessel. If the masses of all the molecules are doubled and their speed is halved, the resulting pressure P will be equal to*

(A) $2P_o$ *(B) $P_o/4$*
(C) P_o *(D) $P_o/2$*

Sol.

$P_o = \frac{1}{3}\left(\frac{mn}{v}\right)v_{rms}^2$

$P = \frac{1}{3}\left(\frac{m'n}{v}\right)v_{rms}'^2$ where $m' = 2m$, $v'_{rms} = \frac{v_{rms}}{2}$

putting the value

$\therefore \frac{P}{P_o} = \frac{m'v_{rms}'^2}{mv_{rms}^2} = \frac{2m}{m}\frac{v_{rms}}{4 \times v_{rms}} = \frac{1}{2}$

$P = P_o/2$

Prob 5. *When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is:*

(A) 2/5 *(B) 3/5*
(C) 3/7 *(D) 5/7*

Sol.

$dQ = n\,C_P dT$
$= n\,(C_v + R)\,dT$
$dQ = n\,C_v dT + n\,RdT$
$n\,C_P dT = n\,C_v dT + n\,RdT$
$n\,C_v dT$ increases internal energy.

$\therefore$ Fraction of energy that increases internal energy is $\frac{C_v}{C_P} = \frac{5}{7}$

($\because$ diatomic gas)

Hence, D is correct Answer.

Prob 6. *Two insulting cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at temperature 300K. The piston A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30K. Then the rise in temperature of the gas in B is.*

(A) 30K *(B) 18K*
(C) 50K *(D) 42K*

Sol.

For cylinder A.
$dQ = nC_P dT_1$
$= n(C_V+R)\,dT_1$

For cylinder B
$dQ = nC_V dT_2$

$\therefore nC_V dT_2 = n(C_V+R)\,30$

$\therefore dT_2 = \frac{(C_v + R)30}{C_v}$

For diatomic gas $C_v = \frac{5}{2}R$

$\therefore dT_2 = 42K$. Hence (D) is correct.

Prob 7. *Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of gas contained in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The change in the pressure in A and B are found to be ΔP and 1.5 ΔP respectively. Then*

(A) $4m_A = 9m_B$ *(B) $2m_A = 3m_B$*
(C) $3m_A = 2m_B$ *(D) $9m_A = 4m_B$*

Sol. For gas in A, $P_1 = \left(\frac{m_A}{M}\right)\frac{RT}{V_1}$

$P_2 = \left(\frac{m_A}{M}\right)\frac{RT}{V_2} \therefore \Delta P = P_2 - P_1 = \left(\frac{RT}{M}\right) m_A \left(\frac{1}{V_1} - \frac{1}{V_2}\right)$

Putting $V_1 = V$ and $V_2 = 2V$

We get $\Delta P = \left(\frac{RT}{M}\right) \frac{m_A}{2V}$

Similarly for Gas in B, $1.5 \;\; \Delta P = \left(\frac{RT}{M}\right) \frac{m_B}{2V}$

From eq. (I) and (II) we get $2m_B = 3m_A$ Hence (C) is correct.

Prob 8. *The average translational energy and the rms speed of molecules in a sample of oxygen at 300K are 6.21 x 10^{-21}J and 484 m/s respectively. The corresponding values at 600K are nearly (assuming ideal gas behavior).*

(A) 12.42 x 10^{-21}J, 968 m/s. *(C) 6.21 x 10^{-21}J, 968 m/s*
(B) 8.78 x 10^{-21}J, 684m/s *(D) 12.42 x 10^{-21}J, 684 m/s*

Sol. $KE = \frac{3}{2}KT$, $V_{rms} = \sqrt{\frac{3RT}{M}}$

i.e. $\frac{KE_2}{KE_1} = \frac{T_2}{T_1} = 2 \;\; \therefore KE_2 = 2KE_1 = 2 \text{ x } 6.21 \text{ x } 10^{-21} = 12.42 \times 10^{-21}J$

$\frac{V_{rms,2}}{V_{rms,1}} = \sqrt{\frac{T_2}{T_1}} = \sqrt{2} \quad \therefore \quad V_{rms,2} = \sqrt{2} \text{ x } V_{rms,1}$ = 684 m/s. Hence (D) is correct.

Prob 9. *At room temperature the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s the gas is*

(A) H_2 *(B) F_2*
(C) O_2 *(D) Cl_2*

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}} \therefore M = \frac{3RT}{V_{rms}^2} = \frac{3 \text{ x } 8.31 \text{ x } 300}{(1.930)^2}$

$= 2.0078 \text{ x } 10^{-3}$ kg $= 2.00$ gm. Hence (A) is correct.

Prob 10. *In a room where the temperature is 30^oC, a body cools from 61^oC to 59^oC is 4 minutes. The time taken by the body to cool from 51^oC to 49^oC will be:*

(A) 4 minutes *(B) 6 minutes*

(C) 5 minutes *(D) 8 minutes*

Sol. Rate of cooling α difference in temperature

$$\frac{dT}{dt} \alpha\ \Delta\theta \quad \Rightarrow \frac{dT}{dt} = K\Delta\theta$$

in first case, $dT = 61 - 59 = 2$

$\Delta\theta = 60 - 30 = 30$

dt = 4 minute, $\therefore K = \frac{dT}{\Delta\theta dt} = \frac{2}{30\,x\,4} = \frac{1}{60}$

For second case, $dT = 2$

$\Delta\theta = 50 - 30 = 20$

$\therefore dt = \frac{dT}{K\Delta\theta} = \frac{2}{1/60\ x\ 20} = 6$ min. Hence (B) is correct.

Prob 11. *A monoatomic gas (r = 5/3) is suddenly compressed to $(1/8)^{th}$ its volume adiabatically. The pressure of the gas will changes to*

(A) 24/5 *(B) 8*

(C) 40/3 *(D) 32*

Sol. $P_1 V_1^r = P_2 V_2^r$; $P_2 = P_1\left(\frac{V_1}{V_2}\right)^r$

$= P(8)^r = P(8)^{5/3} = 32P$, Hence (D) is correct.

Prob 12. *Two rods of length ℓ_1 and ℓ_2 are made of materials whose coefficients of linear expansion are α_1 and α_2. If the difference between two lengths is independent of temperature then,*

(A) $\frac{\ell_1}{\ell_2} = \frac{\alpha_1}{\alpha_2}$ *(B)* $\frac{\ell_1}{\ell_2} = \frac{\alpha_2}{\alpha_1}$

(C) $\ell_2^2\alpha_1 = \ell_1^2\alpha_2$ *(D)* $\frac{\alpha_1^2}{\ell_1} = \frac{\alpha_2^2}{\ell_2}$

Sol. $L_1 = \ell_1 [1 + \alpha_1 \Delta T]$

$L_2 = \ell_2 [1 + \alpha_2 \Delta T]$

$L_1 - L_2 = (\ell_1 - \ell_2) + (\Delta T)(\ell_1 \alpha_1 - \ell_2 \alpha_2)$

If $L_1 - L_2$ is to be independent of ΔT

Then $\ell_1\alpha_1 - \ell_2\alpha_2 = 0 \quad \Rightarrow \frac{\ell_1}{\ell_2} = \frac{\alpha_2}{\alpha_1}$. Hence (B) is correct.

Prob 13 . *The volume of a gas at constant pressure of 10^3 N/m^2 expands by 0.25 m^3. The work done by gas is equal to*

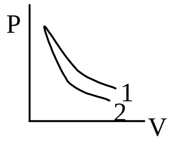
(A) 2.3 erg. *(B) 250J*

(C) 250W *(D) 250N*

Sol. Work done $= P\Delta V$

$= 10^3 \times 0.25 = 250J$, Hence (B) is the correct answer.

Prob 14. *P - V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to*

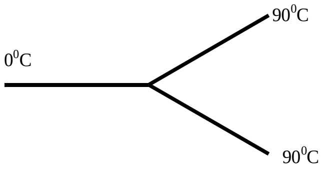


(A) He and O_2 *(B) O_2 and He*
(C) He and Ar *(D) O_2 and N_2*

Sol. Slope of the curve $= \frac{dP}{dV} = -\gamma \frac{P}{V}$.

As curve 2 is steeper, its γ is greater.
Also, γ = 5/3 for monoatomic gas
and γ = 7/5 for diatomic gas. Hence (B) is the correct.

Prob 15. *Three rods made of the same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0^0C and 90^0C respectively. The temperature of the junction of the three rods will be*



(A) 45^0C *(C) 60^0C*
(C) 30^0C` *(D) 20^0C*

Sol. T/R = 2(90 `– T)/R, T=60
where R is thermal resistance of one rod.
Hence (B) is correct.

Fill in the blanks:

Prob 16. *In an isothermal change of an ideal gas $\Delta V = 0$ hence ΔQ = ……………*

Sol. $\Delta W =$: $\Delta Q = \Delta U + \Delta W$, $\Delta u = 0$ $\therefore$ $\Delta Q = \Delta W$

Prob 17. *One mole of a mono-atomic gas is mixed with three moles of a diatomic gas. The molar specific heat of the mixture at constant volume is …………..*

Sol. $\frac{9R}{4}$

For mono-atomic gas $C_v = \frac{R}{\gamma - 1} = \frac{R}{\left(\frac{5}{3} - 1\right)} = \frac{3}{2}R$

For diatomic gas $C_v = \frac{R}{\left(\frac{7}{5} - 1\right)} = \frac{5}{2}R$

Specific heat of mixture $= \dfrac{1\times\frac{3}{2}R + 3\times\frac{5}{2}R}{1+3} = \dfrac{9R}{4}$

Prob 18. *Two rods A and B are of equal length each rod has its ends at temperature T_1 and T_2 then rate of flow of heat through the rods A and B is equal*

Sol. False.

The rate of flow is equal when $k_1A_1 = k_2A_2$. When k_1 and k_2 are the thermal conductivities and A_1 and A_2 are the area of cross section respectively.

Prob 19. *The root mean square speed of oxygen molecules at a certain temperature T is v. If dissociates into atomic oxygen when temperature is raised to 2T, the rms speed remains unchanged.*

Sol. False

$V_{rms} = \sqrt{\dfrac{3RT}{M}}$

When temperature is doubled the new temperature is 2T and as the molecules dissociates the new molecular mass is (M/2).

$\therefore$ New $v_{rms} = \sqrt{\dfrac{3R(2T)}{(M/2)}} = 2v$

ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. Define specific heat capacity at constant volume for a gas.

2. Is C_p greater than C_v?

3. Change in internal energy is independent of path followed. Is it true? Why?

4. How much will be the internal energy change in (i) Isothermal process (ii) Adiabattic process?

5. Define temperature.

6. Define specific heat of a body.

7. Calculate the total number of degrees of freedom for a mole of diatomic gas at NTP.

8. What is the value of specific heat of water in S.I. units? Does it vary with temperature?

9. Define latent heat. What are the S.I. units of latent heat?

10. Define internal energy of a system.

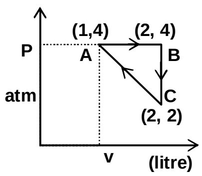
11. The volume of a thin brass vessel and the volume of a solid cube are both equal to 1 lt. What will be the change in volumes of vessel and the cube on being heated to 25°C? Given α for brass = 1.9×10^{-5}/°C.

12. One mole of a monoatomic gas is mixed with 3 moles of a diatomic gas. What is the molecular specific heat of the mixture at constant pressure?

13. At what temperature the rms velocity is equal to escape velocity from the surface of earth for hydrogen and for oxygen? Given radius of earth = 6.4×10^6 m, g = 9.8 m/s^2, k = 1.38×10^{-23} J/K.

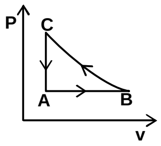
14. A cylinder containing one mole of a monoatomic gas at 127°C expand isothermally until its volume is doubled, and then compressed adiabatically until its temperature rose from 127°C to 227°C. Calculate total work done and heat absorbed.

15. A P–V diagram for a cyclic process is a triangle ABC. Calculate work done during the process AB, BC and CA. Also calculate the work done in the complete cycle.



16. 30 gm of ice of at –14°C is added to 200 gm of water at 25°C. Find the equilibrium temperature. [Take specific heat of ice = 0.5 cal/g. C°, specific heat of water = 1 cal/g. C°, and latent heat of ice = 80 cal/g.]

17. Consider the cyclic process ABCA on a sample of 2 mole of an ideal gas as shown in figure. The temperatures of the gas at A and B are 300 K and 500 K, respectively. A total of 1200 J heat is withdrawn from the sample in the process. Find the work done by the gas in part BC. [Take R = 8.3 J/mol-K]



18. A sphere, a cube and a thin circular plate all made of the same material and having the same mass are initially heated to 200°C. Which of these objects will cool faster and which one slowest when left in air at room temperature?

19. A body which has a surface area of 5 cm^2 and temperature of 727°C radiates 300 J of energy each minute. What is its emissivity? ($\sigma = 5.67 \times 10^{-8}$ W/m^2k^4)

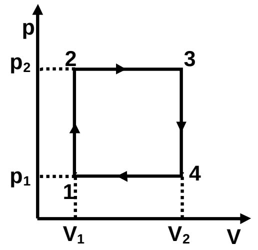
20. A body cools in 7 minutes from 60°C to 40°C. What will be its temperature after the next 7 minutes? The temperature of surrounding is 100°C.

Level- I

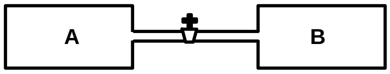
1. A brass disc at 20^0C has a diameter of 30cm and a hole cut inside the disc 10cm in diameter. Calculate the` diameter of hole when temperature of the disc is raised to 50^0C. The coefficient of linear expansion of brass = $0.000018/^0$C

2. A steel wire of cross section area 0.5 mm^2 is held between two fixed supports. The tension in the wire is negligible and it is just taut at a temperature of 20^0C. Determine the tension when the temperature of the wire falls to 0^0C. Assume that the distance between the supports remains constant. ($Y = 2.1 \times 10^{11} N/m^2$, $\alpha = 12 \times 10^{-6}/^0C$.)

3. A gas has been subjected to an isochoric-isobaric cycle 1 - 2 - 3 - 4 - 1. Plot the graph of this cycle in the P - ρ, V-T and P-T coordinates.
(ρ = density of the gas)



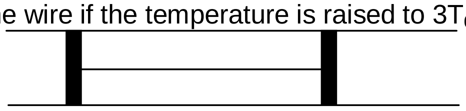
4. The figure shows two vessels A and B with rigid walls containing ideal gases. The pressure, temperature and the volume are P_A, T_A, V in the vessel A and P_B, T_B , V in the vessel B. The vessels are now connected through a small tube. Show that the pressure P and the temperature T satisfy $\frac{P}{T} = \frac{1}{2}\left[\frac{P_A}{T_A} + \frac{P_B}{T_B}\right]$ when equilibrium is achieved.



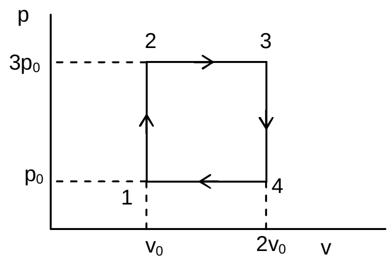
5. A vessel contains a mixture of 7 gm of Nitrogen and 11gm of Carbon dioxide at room temperature T = 290 K. If the pressure of the mixture P = 1atm calculate its density. (R = 8.31 J/ mole K)

6. A gas (γ=1.4) at 2 m^3 volume and pressure 4×10^5 N/m^2 is compressed adiabatically to a volume 0.5 m^3. Find its new pressure. Compare it with the pressure obtained if the compression were isothermal. Calculate the work done in each process.
Given : $(4)^{1.4} = 6.9$, $\log_{10} 4 = 0.6021$.

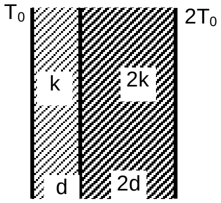
7. Figure shows a horizontal cylindrical tube of cross-sectional area A fitted with two frictionless pistons. The pistons are connected to each other by an inextensible wire. Initially the temperature of the gas is T_0 and its pressure is p_0 which equals the atmospheric pressure.
Find the tension in the wire if the temperature is raised to $3T_0$.



8. An ideal monoatomic gas undergoes a cyclic process as shown in the P-V diagram. For each process, find
(a) the work done
(b) the change in internal energy
(c) the heat transfer. Also, find the efficiency of the cycle.



9. Two slabs of thickness d and 2d, thermal conductivity k and 2k respectively, are joined face to face as shown in the figure. Under steady state condition, find (a) the rate of heat transfer per unit face area (b) the temperature at the interface.

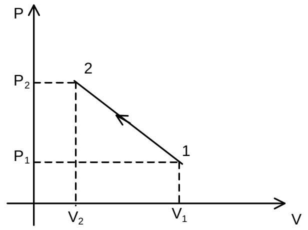


10. Find the average translational kinetic energy per molecule if one mole of a mono-atomic gas is contained in a volume 1.23×10^{-3} m^3 at a pressure 2×10^5 N/m^2. Avogadro's number is 6.02×10^{23} molecules/mole.

11. Find the amount of work done to increase the temperature of one mole of an ideal gas by 30^0C, if it is expanding according to $V \propto T^{2/3}$.

12. A monatomic gas undergoes a cycle consisting of two isothermal and two isobars. If the minimum and maximum temperature of the gas during the cycle is T_1 = 400K and T_2 = 800 K respectively and the ratio of maximum to minimum volume is 4. Calculate the efficiency of the cycle.

13. An ideal gas of mass m follows a process from state (P_1 V_1) to (P_2, V_2). What maximum/minimum temperature will the gas reach in this process, if it is depicted on the P-V diagram as a straight line as shown in figure. The molecular weight of the gas is M.



14. The volume of n moles of an ideal gas with the adiabatic exponent γ is varied according to the law $V = \frac{a}{T}$, where a is a constant. Find the amount of heat received by the gas in this process if the gas temperature is increased by ΔT.

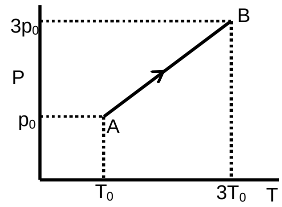
15. An ideal gas has $C_P = \frac{5R}{2}$. The gas is kept in a closed rigid vessel of volume v_0, temperature T_0 and pressure P_0. An amount of 10 P_0V_0 J of heat is supplied to the gas. Find the final pressure and temperature of the gas.

Level- II

1. Two moles of a monoatomic ideal gas undergoes a process AB as shown in the figure. Find

   (a) the work done

   (b) the heat transfer during the process.



2. A lead bullet at 100^0C strikes a block of wood, gets embedded in it and melts. Find the speed of the bullet if 50% of the heat produced is shared by the bullet. Specific heat of lead = 0.03 cal./g^0C, latent heat of lead = 5 cal/g and melting point of lead = 327^0C.

3. A copper and a tungsten plate having a thickness 2mm each are riveted together so that at 0^0C they form a flat bimetallic plate. Find the average radius of curvature of this plate at 200^0C. The coefficients of linear expansion for copper and tungsten are 1.7×10^{-5} k^{-1} and 0.4×10^{-5} k^{-1} respectively.

4. Ice of mass 2 kg at -20^0C and 5 kg water at 20^0 C are mixed. (specific heat of water = 1Cal/g ^{0}C, specific heat of ice = 0.5 Cal/g ^{0}C, find the net amount of water in the container.

5. The earth receives at its surface, the radiation from the sun at the rate of 1400 w/m^2. The distance between the sun and the earth is 1.5×10^{11}m and the radius of the sun is 7×10^8m. Treating the sun as a black body, find the surface temperature of the sun.

6. The temperature of an ideal gas in a tube of uniform cross section of length L, varies linearly from one end (T_o) to other end (T_L). calculate the total number of moles of gas present in the tube. The gas pressure is P_o and Volume V_o.

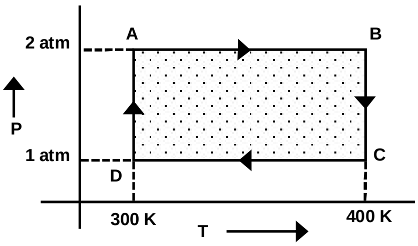
7. Two moles of helium gas undergoes a cyclic process as shown in the figure. Assuming the gas to be ideal, calculate the following quantities in the process

   (a) the net work done

   (b) the net change in internal energy

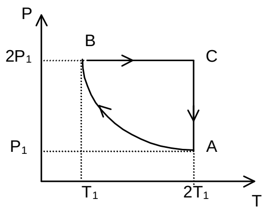
   (c) the net amount of heat received by the gas.

   [R = 8.32 J/mol K]



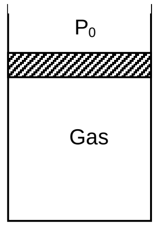
8. A hot body placed in air is cooled according to Newton's Law of Cooling, the rate of decrease of temperature being k times the temperature difference from the surrounding. Starting from t = 0, find the time in which the body will loose 90 % of the maximum heat it can loose.

9. Two moles of an ideal monatomic gas is taken through a cycle ABCA as shown in the P-T diagram. During the process AB, pressure and temperature of the gas vary such tht PT = constant. If T_1 = 300 K, calculate
(a) The work done on the gas in the process AB and
(b) The heat absorbed or released by the gas in each of the processes.
Give answers in terms of the gas constant R.

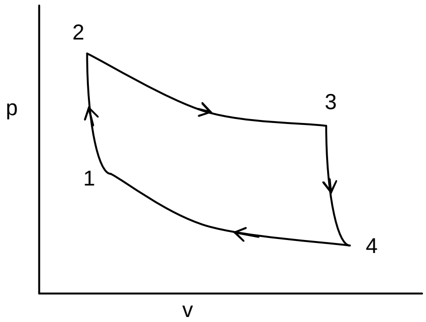


10. An ideal gas has a molar heat capacity C_V at constant volume. Find the molar heat capacity of this gas as a function of its volume V, if the gas undergoes the following process (a) $T = T_0e^{\alpha V}$ (b) $P = P_0e^{\alpha V}$

11. An ideal gas is contained in a piston cylinder arrangement as shown in the figure. The area of the piston is A and its mass is m. The surrounding atmospheric pressure is P_0. Under equilibrium condition, the volume of the gas is v_0. Find the angular frequency of small vertical oscillations of the piston assuming the expansion and compression of the gas to be adiabatic (adiabatic constant = γ).



12. An ideal gas undergoes a cycle consisting of two adiabatic and two isothermal processes as shown in the P-V diagram. The temperatures during the isothermal processes 2 – 3 and 4 – 1 are T_1 and T_2 respectively. Find the efficiency of the cycle.



13. A point source of power W watt is buried deep inside the earth. The thermal conductivity of the earth is k. Under steady state condition the temperature far away from the power source is found to be T_0. Show that the temperature T at a distance r from the source is given by $T = T_0 + \frac{W}{4\pi kr}$.

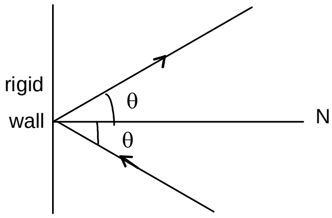
14. Suppose the molar specific heat at constant volume of an ideal gas varies according to law $C_V = a + bT$, where T is the absolute temperature of the gas, a and b are constants. Find the equation of the adiabatic process for this gas.

15. A tall vertical cylinder of height h and cross-sectional area A is in uniform gravitational field. The cylinder contains an ideal gas at uniform temperature and is closed at both ends. If the density of the gas at bottom and top of the cylinder is ρ_1 and ρ_2 respectively, calculate the mass of the gas contained in the cylinder.

Objective:

Level – I

1. The mass of hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike per second at 2 cm^2 area of a rigid wall at an angle of 45^0 from the normal and rebound back with a speed of 1000 m/s, then the pressure exerted on the wall is



(A) 2.34×10^3 Pascal
(B) 0.23×10^3 Pascal
(C) 0.23×10^3 Pascal
(D) 23.4×10^3 Pascal

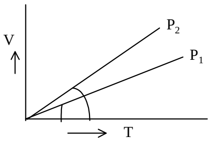
2. The number of molecules per unit volume of a gas given by
(A) $\frac{P}{kT}$
(B) $\frac{kT}{P}$
(C) $\frac{P}{RT}$
(D) $\frac{RT}{P}$
Where R & k are universal gas constant and Boltzman' constant respectively.

3. A gas is filled in a container at pressure P_0. If the mass of molecules is halved and their r.m.s. speed is doubled then the resultant pressure will be
(A) P_0
(B) 4 P_0
(C) 2 P_0
(D) $P_0/2$

4. When an ideal diatomic gas is heated at constant pressure then the fraction of heat energy which is used to increase the internal energy of gas system, is
(A) 3/7
(B) 3/5
(C) 5/7
(D) 2/5

5. The velocities of three molecules are 3v, 4v and 5v respectively. Their rms speed will be
(A) $\sqrt{\frac{50}{3}}\,v$
(B) $\sqrt{\frac{3}{50}}\,v$
(C) $\frac{50}{3}v$
(D) $\frac{3}{50}v$

6. In the following V-T diagram what is the relation between P_1 and P_2



(A) $P_2 = P_1$
(B) $P_2 > P_1$
(C) $P_2 < P_1$
(D) cannot be predicted

7. The average degree of freedom per molecule for a gas is 6. The gas performs 25J of work when it expends at constant pressure. The heat absorbed by the gas is
(A) 75J
(B) 100J

(C) 150 J (D) 125J

8. Equal masses of three liquids A, B and C have temperatures 10°C, 25°C and 40°C respectively. If A and B are mixed, the mixture has a temperature of 15°C. If B and C are mixed, the mixture has a temperature of 30°C. If A and C are mixed, the mixture has a temperature of

(A) 16°C (B) 20 °C
(C) 25°C (D) 29 °C

9. During an experiment, an ideal gas is found to obey the equation P^2V = constant. The gas is initially at temperature T and volume V. When it expands to a volume 2V, the temperature becomes

(A) $\sqrt{2}$ T (B) T/2
(C) $\sqrt{3}$ T (D) 2T

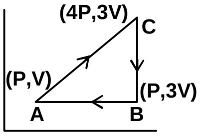
10. The bulk modulus of elasticity for a mono atomic ideal gas during an isothermal process is (P = pressure of the gas)

(A) P (B) $\frac{2P}{3}$
(C) $\frac{5P}{3}$ (D) $\frac{7P}{5}$

11. There are two spheres of same material and radius. One is solid and the other is hollow. If they are heated to the same temperature, the expansion of:

(A) solid sphere is more (B) hollow sphere is more
(C) solid and hollow spheres are equal (D) solid is outwards while that of hollow inwards

12. An ideal gas is taken through a series of changes represented in figure. The total work done by the gas at the end of the cycle is equal to



(A) zero (B) 2PV
(C) 3PV (D) 5PV

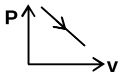
13. When a strip made of iron α_1 and copper α_2 ($>\alpha_1$) is heated

(A) its length does not change (B) it gets twisted
(C) it bends with iron on concave side (D) it bends with iron on convex side

14. A bucket full of hot water is kept in a room and it cools from 75°C to 70°C in T_1 minutes, from 70°C to 65°C in T_2 minutes and from 65°C to 60°C in T_3 minutes. Then,

(A) $T_1 = T_2 = T_3$ (B) $T_1 < T_2 < T_3$
(C) $T_1 > T_2 > T_3$ (D) $T_1 < T_2 > T_3$

15. If the indicator diagram for expansion of a gas is shown in figure, the gas



(A) is heated only (B) is cooled only
(C) is first heated & then cooled (D) is first cooled & then heated

16. For a certain gas, the ratio of specific heats is 3/2. What is the value of C_P?

(A) R (B) 2R
(C) 3R (D) 5R

17. If one mole of a monatomic gas is mixed with one mole of a diatomic gas, the value of γ for the mixture is
(A) 1.40 (B) 1.50
(C) 1.53 (D) 3.07

18. An iron ball of weight w is immersed in water at 4°C. Its weight in water is w_1. If the weight of ball in water at 54°C be w_2, then which of the following relation is correct?
(A) $w_1 < w_2$ (B) $w_1 > w_2$
(C) $w_1 = w_2$ (D) $w = \frac{w_1 + w_2}{2}$

19. The formation of ice is started in a lake with water at 0°C when the atmospheric temperature is –10°C. If time taken for 1 cm of ice to be formed is 7 hours, the time taken for the thickness of ice to increase from 1 cm to 2 cm is
(A) less than 7 hours (B) 7 hours
(C) more than 14 hours
(D) more than 7 hours but less than 14 hours

20. Two stars S_1 and S_2 radiate maximum energy at 360 nm and 480 nm, respectively. Ratio of their absolute temperatures is
(A) 3/4 (B) 4/3
(C) 9/16 (D) 16/9

Level – II

1. n_1 and n_2 moles of two ideal gases having adiabatic constants γ_1 and γ_2 respectively are mixed. C_v for the mixture is

(A) $\left(\frac{\gamma_1+\gamma_2}{2}\right)R$
(B) $\left(\frac{n_1\gamma_1+n_2\gamma_2}{n_1+n_2}\right)R$
(C) $\left(\frac{n_1\gamma_2+n_2\gamma_1}{n_1+n_2}\right)R$
(D) none of these

2. When the state of a system is changed from A to B adiabatically, the work done on the system is 322 Joule. Then the state of the same system is changed from A to B by another method which requires 100 J of heat. The work done on the system in this process will be

(A) 222Joule
(B) 100 Joule
(C) 422 Joule
(D) 322 Joule

3. A pendulum clock keeps correct time at 0^0 C. If the coefficient of linear expression is α, then what will be the loss in time per day, when the temperature rises by t^0C. ?

(A) $\frac{\alpha t}{2}$ day
(B) $\frac{\alpha t}{2}$ second
(C) $\left(1+\frac{\alpha t}{2}\right)$ day
(D) $\left(1+\frac{\alpha\ t}{2}\right)$ second

4. Two straight metallic strips each of thickness t and length ℓ are riveted together. Their coefficient of linear expansions are α_1 and α_2. If they are heated through temperature Δt, the bimetallic strip will bend to form an arc of radius

(A) $\frac{t}{(\alpha_1+\alpha_2)\Delta t}$
(B) $\frac{t}{\Delta t(\alpha_2-\alpha_1)}$
(C) $t(\alpha_1+\alpha_2)\Delta t$
(D) $t(\alpha_2-\alpha_1)\Delta t$.

5. The radius of a ring is R and its coefficient of linear expansion is α. If the temp of ring increases by θ then its circumference will increase by

(A) $\pi R\alpha\theta$
(B) $2\pi R\alpha\theta$
(C) $\pi R\alpha\theta/2$
(C) $\pi R\alpha\theta/4$

6. A gas mixture consists of 2 moles of O_2 and 4 moles of Ar at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(A) 4RT
(B) 15 RT
(C) 9 RT
(D) 11 RT

7. A monatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expend adiabatically to a temperature T_2 by

releasing the piston suddenly. If L_1 and L_2 are lengths of the gas column before and after expansion respectively, then T_1/T_2 is given by

(A) $\left(\frac{L_1}{L_2}\right)^{\frac{2}{3}}$ (B) $\left(\frac{L_1}{L_2}\right)$

(C) L_2/L_1 (D) $\left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$

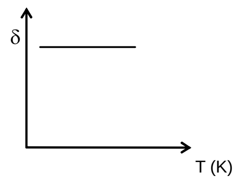
8. Starting with the same initial conditions, an ideal gas expends from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is isothermal, W_2 if isobaric and W_3 if adiabatic, then

(A) $W_2 > W_1 > W_3$ (B) $W_2 > W_3 > W_1$

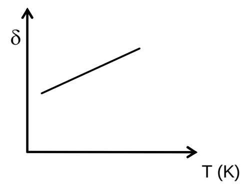
(C) $W_1 > W_2 > W_3$ (D) $W_1 > W_3 > W_2$

9. An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = (\Delta V/ V\Delta T)$ varies with temperature as

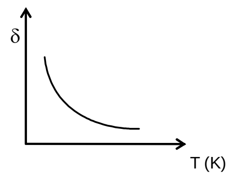
(A)



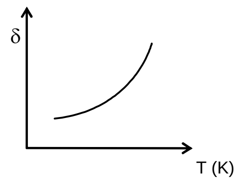
(B)



(C)

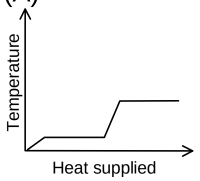


(D)

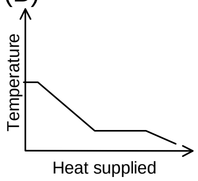


10. A block of ice at –10°C is slowly heated and converted to steam at 100°C. Which of the following curves represent the phenomenon qualitatively?

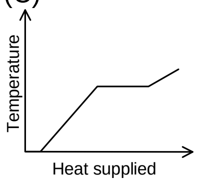
(A)



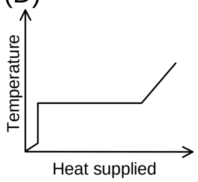
(B)



(C)



(D)



11. Two spheres of same material and of same emissivity have radii 1 m and 4 m and temperatures 4000 K and 1000 K, respectively. The ratio of radiation emitted per sec is

(A) 4:1 (B) 1:4

(C) 1:1 (D) 16:1

12. Two spheres of radii R_1 and R_2 have densities ρ_1 & ρ_2 and specific heats C_1 & C_2. If they are heated to the same temperature, the ratio of their rates of cooling will be

(A) $\frac{R_2\rho_2C_2}{R_1\rho_1C_1}$ (B) $\frac{R_1\rho_2C_2}{R_2\rho_1C_1}$

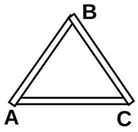
(C) $\frac{R_1\rho_1C_2}{R_1\rho_2C_1}$ (D) $\frac{R_2\rho_2C_1}{R_1\rho_1C_2}$

13. Liquid is filled in a vessel which is kept in a room with temperature 20°C. When the temperature of the liquid is 80°C, then it loses heat at the rate of 60 cal/s. What will be the rate of loss of heat when the temperature of the liquid is 40°C?

(A) 180 cal/s (B) 40 cal/s

(C) 30 cal/s (D) 20 cal/s

14. Three rods of same material AB, BC and CD have equal length and their areas of cross section are in ratio 1:2:4, respectively. If A and C are maintained at different temperatures, the ratio of heat transferred through AB, BC and AC in a given time is



(A) 1:2:4 (B) 1:1:6

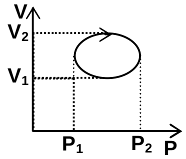
(C) 1:1:4 (D) 1:3:6

15. Consider a compound slab consisting of two different materials having equal thicknesses and thermal conductivities K and 2K. The equivalent thermal conductivity of the slab is

(A) $\sqrt{2}$ K (B) 3 K

(C) (4/3) K (D) (2/3) K

16. In the given figure, the magnitude of thermodynamic work is given by



(A) $\frac{\pi}{4}(P_2 - P_1)(V_2 - V_1)$ (B) $\pi(P_2 - P_1)(V_2 - V_1)$

(C) $\frac{\pi}{2}(P_2 - P_1)(V_2 - V_1)$ (D) $\frac{\pi}{3}(P_2 - P_1)(V_2 - V_1)$

17. A gas undergoes a process such that $P \propto \frac{1}{T}$. If the molar heat capacity for this process is 24.93 J/mol K, then what is the degree of freedom of the molecules of the gas?

(A) 8 (B) 4

(C) 2 (D) 6

18. Five mole of a diatomic gas is kept at temperature T. The volume of the gas varies according to the law $V = aT^{-2}$, where 'a' is positive constant. The final temperature of the gas is found to be 5T. What amount of heat is supplied to the gas?

(A) 5 RT (B) (5/2) RT

(C) (10/3) RT (D) 10 RT

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. The amount of energy required for 1 mole of a gas to change its temperature by unity at constant volume conditions is called specific heat capacity C_v.

2. Yes, $C_p - C_v = R$.

3. Yes, change in internal energy depends on the change in temperature alone. So it si independent of path.

4. (i) Zero, (ii) $nC_v dT$.

5. Temperature is a measurable magnification of the thermal energy of a body.

6. The specific heat of a substance is the amount of heat required to raise the temperature of a unit mass of the substance through unit degree.

7. No. of degrees of freedom of a diatomic gas molecule at 273 K = 5. One mole of gas contains 6.02×10^{-23} molecules at NTP.
Total no. of degrees of freedom of 1 mole of distance diatomic gas at NTP
$= 5 \times 6.02 \times 10^{23} = 30.1 \times 10^{23}$.

8. Specific heat of water is 4180 J $kg^{-1}K^{-1}$. Yes, it does vary little with temperature.

9. It is the amount of heat required to change the state of a unit mass of substance without any rise in the temperature Q = mL, where Q is heat, m is mass and L is latent heat S.I. unit of latent heat is J kg^{-1}.

10. The internal energy of a system is defined as the sum total of the kinetic energy possessed by all the particles comprising the system due to their vibratory or translatory motion. It is directly proportional to the absolute temperature of the system and is denoted by U.

11. 1.425 cm^3 in each case

12. 13/4 R

13. 1.01×10^4K, 16.1×10^4 K.

14. 1057.5 J, 2304 J

15. 1.01×10^2 J.

16. 10.39°C

17. –4500 J.

18. Disc will cool fastest while sphere slowest.

19. 0.18

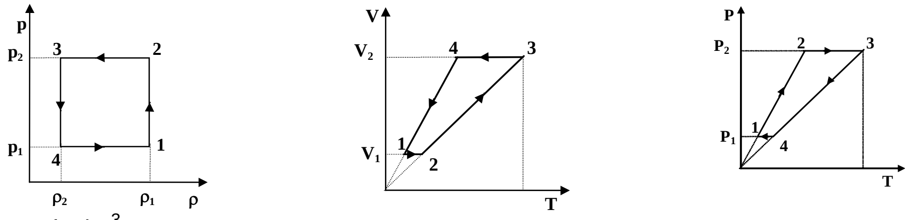
20. 28°C

Level - I

1. 10.0054 cm.

2. 25.2N

3.



5. 1.5 kg/m^3

6. $P = 2.79 \times 10^6$ N/m^2, $W = -1.48 \times 10^6$ J for adiabatic compression.

   $P = 1.6 \times 10^6$ N/m^2 $W = -1.109 \times 10^6$ J for isothermal compression.

7. $2p_0A$

8. (a) zero, $3P_0V_0$, zero, $-P_0V_0$

   (b) $3P_0V_0$, $\frac{9P_0V_0}{2}$, $-6P_0V_0$, $-\frac{-3P_0V_0}{2}$

   (c) $3P_0V_0$, $\frac{15P_0V_0}{2}$, $-6P_0V_0$, $-\frac{5P_0V_0}{2}$

   efficiency = 19 %

9. (a) $\frac{kT_0}{2d}$ (b) $\frac{3T_0}{2}$

10. 6.1×10^{-22} J

11. 167 J

12. $\frac{2\ln 2}{5+4\ \ln 2}$

13. $T_{max} = \frac{(P_2V_1 - P_1V_2)^2\ M}{4\ m\ R\ (V_1 - V_2)\ (P_2 - P_1)}$

14. $nR\Delta T\left[\frac{2-\gamma}{\gamma-1}\right]$

15. $\frac{23}{3}P_0$, $\frac{23}{3}T_0$

Level - II

1. (a) zero (b) $6RT_0$

2. 444.6 m/s

3. 0.769 m

4. 6 kg

5. 5795 K

6. $$\frac{P_oV_o}{R\dfrac{T_L - T_o}{\ln\left(T_L/T_o\right)}}$$

7. (a) $\Delta W = 1153$ J (b) $\Delta U = 0$; (c) Q = 1153 J

8. $\dfrac{\ln 10}{k}$

9. (a) 1200 R units (b) 2100R (released), 1500 R (absorbed, 831.6 R (absorbed)

10. (a) $C_V + \dfrac{R}{\alpha V}$ (b) $C_V + \dfrac{R}{1+\alpha V}$

11. $\sqrt{\dfrac{\gamma A}{v_0}(g + \dfrac{P_0A}{m})}$

12. $\left[1 - \dfrac{T_2}{T_1}\right]$

14. $V^R T^a e^{bT}$ = constant

15. $m = \dfrac{Ah\left(\rho_1 - \rho_2\right)}{\log\rho_1 - \log\rho_2}$

Objective:

Level – I

1.	**A**	2.	**A**
3.	**C**	4.	**C**
5.	**A**	6.	**C**
7.	**B**	8.	**A**
9.	**A**	10.	**A**
11.	**C**	12.	**C**
13.	**C**	14.	**B**
15.	**C**	16.	**C**
17.	**B**	18.	**A**
19.	**B**	20.	**B**

Level – II

1.	**D**	2.	**A**
3.	**B**	4.	**B**
5.	**B**	6.	**D**
7.	**D**	8.	**A**
9.	**C**	10.	**A**
11.	**D**	12.	**A**
13.	**B**	14.	**B**
15.	**C**	16.	**A**
17.	**C**	18.	**D**

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

WAVES AND SOUND

Waves & Sound

Syllabus:
Longitudinal and transverse waves, wave motion, speed of wave motion, Displacement relation for a progressive wave; Principle of superposition of waves, Reflections of waves, Refraction of waves, standing waves in strings and pipes, Resonance, fundamental mode and harmonics, Beats, speed of sound, Doppler effect.

Wave Motion

The vehicle, which is responsible for the transmission of energy from one place to another through a medium without any bulk motion of the medium in the direction of energy flow, is called a wave.

Types of waves:

(a) ***Mechanical waves:*** The wave, produced due to the vibration of material particles of an elastic medium e.g. sound wave, vibrating string.

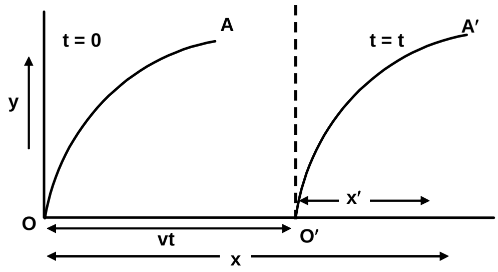
(b) ***Electro magnetic waves:*** Waves which are produced due to the periodic vibration of two mutually perpendicular electric and magnetic fields are Electro magnetic waves. It propagates in a direction perpendicular to both electric and magnetic field. e.g. light waves, X-ray, γ-ray etc.

Equation of wave motion

The common feature of all waves is the transmission of some sort of disturbance with a certain velocity. This disturbance may be the displacement or velocity of the particle of the medium or the magnitude of the fluctuating electric and magnetic field.
Suppose the disturbance is propagating along positive x-axis with a velocity v. The disturbance which is a function of x and t can be represented as :

$$y = f(x, t)$$

The function is called wave function.
Let us choose an origin
O such that at the instant t = 0, y = f(x)
i.e. y is a function of x-only.
At t = 0, the wave form is represented by OA



If this wave form advances unchanged, it is called plane wave of constant type. After time t the waveform takes the position O′A′ such that OO′ = vt. Now the distance x′ measured from O′,
$x' = x - vt$.
The wave form at that instant

$$y = f(x') = f(x - vt) \qquad \text{. . . (i)}$$

This is the equation for any disturbance which travels with a constant velocity v towards the positive direction of x. The function f determines the shape and size of the wave.

For the wave propagating along negative x-axis

$$y = f(x + vt) \quad \ldots (ii)$$

Wave function condition

Differentiating equation (i) twice with respect to time.

We obtain

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \quad \ldots. (iii)$$

Any finite function which satisfy (iii) is called a wave.

Exercise 1.

Which of the following functions represent wave?

(a) sin kx cosω t ***(b) $k^2x^2 - \omega^2t^2$,***

(c) $\cos^2(kx - \omega t)$ ***(d) $\cos(kx^2 - \omega^2x^2)$***

Equation of a simple harmonic plane wave

In case of harmonic wave the displacement of successive particles of the medium is given by a sine function or cosine function of position.

The displacement y at t = 0 is given by

$$y = A \sin kx \quad \ldots (iv)$$

Where A and k are constants.

Suppose this disturbance is propagating along positive x - direction then

$$y = A \sin k(x - vt) \quad \ldots (v)$$

Since the waveform represented by equation (iv) is based on sine function, it would repeat itself at regular distances. The first repetition would take place when

$$kx = 2\pi \quad \text{or} \quad x = 2\pi/k$$

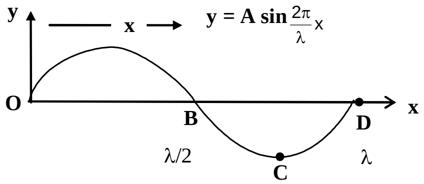
This distance after which the repetition takes place is called the wavelength and denoted by λ. Hence

$$\lambda = 2\pi/k \quad \text{or} \quad k = 2\pi/\lambda$$

This constant k is called propagation constant or wave number. Now equation (v) turns into

$$y = A \sin (x - vt) \quad \ldots (vi)$$

At $t = 0$ $y = A \sin \frac{2\pi}{\lambda} x$ . . (vii)



Relation between wavelength and velocity of propagation

Time taken for one complete cycle of wave to pass any point is the time period (T).
This is also the time taken by the disturbance in propagating a distance λ.

$v = \;= f\lambda$ where f = frequency (Hz)

$\omega = \dfrac{2\pi}{T} = 2\pi f$ = circular frequency (rad/s)

Different forms of simple harmonic wave equation:
The wave equation of a wave traveling in x-direction

$$y = A \sin \omega(t - x/v)$$

$$\Rightarrow \quad y = A \sin(\omega t - kx) = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$= A \sin k(vt - x)$$

Also, it is to be noted that we have made our particular choice of t=0 in writing equation y=A sin ωt. The origin of time is chosen at an instant when the left end x=0 is crossing its mean position y=0 and is going up. For a general choice of the origin of time, we have to add a phase constant so that equation will be

$$y = A \sin(\omega(t - x/v) + \phi)$$

The constant ϕ will be $\pi/2$ if we choose t=0 at an instant when the left end reaches its extreme position y=A. The equation will then be

$$y = A \cos \omega(t - x/v)$$

If t=0 is taken at the instant when the left end is crossing the mean position from upward to downward direction, ϕ will be π and the equation will be

$$y = A \sin \omega(x/v - t)$$

$$y = A \sin(kx - \omega t)$$

Illustration 1. *Consider the wave* $y = 5\sin(2x - 60t)\,\text{mm}$. *X is in centimetres and t is in second. Find (a) The Amplitude (b) The wave number (c) The wavelength (d) The frequency (e) The time period and (f) The wave velocity*

Solution: Comparing the given equation with $y = A \sin(kx - \omega t)$

(a) Amplitude A= 5mm
(b) Wave number K=2 /cm
(c) Wave length $\lambda = 2\pi/2 = \pi$ cm
(d) Frequency $\nu = \dfrac{\omega}{2\pi} = \dfrac{60}{2\pi} = \dfrac{30}{\pi}\text{Hz}$
(e) Time period $T = \dfrac{1}{\nu} = \dfrac{\pi}{30}\text{sec}$
(f) Wave velocity $v = \nu\lambda = 30$ cm.

Exercise 2. ***What is the nature of a wave on the surface of water?***

Illustration 2. *Equation of a transverse wave travelling in a rope is given by*
y = 5 sin (4.0t – 0.02 x)
Where y and x are expressed in cm and time in seconds. Calculate
(a) The amplitude, frequency, velocity and wavelength of the wave.
(b) The maximum transverse speed and acceleration of a particle in the rope.

Solution: (a) Comparing this with the standard equation of wave motion

$$y = A \sin\left(2\pi ft - \frac{2\pi}{\lambda}x\right)$$

Where A, f and λ are amplitude, frequency and wavelength respectively.
Thus amplitude A = 5cm
$2\pi f = 4$

$\Rightarrow$ Frequency $f = \frac{4}{2\pi} = 0.673$ cycle/s

Again $\frac{2\pi}{\lambda} = 0.02$ or Wavelength $\lambda = \frac{2\pi}{0.02} = 100\,\pi$ cm

Velocity of the wave $v = f\lambda = \frac{4}{2\pi}\ \frac{2\pi}{0.02} = 200$ cm/s

(b) Transverse velocity of the particle $u = \frac{dy}{dt} = 5 \times 4\cos(4.0\,t - 0.02x)$

$= 20\cos(4.0\,t - 0.02x)$
Maximum velocity of the particle = 20 cm/s

Particle acceleration $a = \frac{d^2y}{dt^2} = -20 \times 4 \sin(4.0\,t - 0.02\,t)$

Maximum particle acceleration = 80 cm/s^2

Illustration 3. *An observer standing at a sea-cost observes 54 waves reaching the coast per minute. If the wavelength of the wave is 10m, find the velocities of the waves.*

Solution: $f = \frac{54}{60}\,s^{-1}$

$\lambda = 10$ m

$v = f\lambda = \frac{54}{60} \times 10 = 9\,m/s$.

Illustration 4. *A progressive wave of frequency 550 Hz is travelling with a velocity of 360 m/s. How far apart are the two points 60^0 out of phase?*

Solution: f = 550 Hz, v = 360 Hz

$\lambda = \frac{v}{f} = \frac{360}{550} = \frac{36}{55}$ m

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$\Rightarrow \frac{\pi}{3} = \frac{2\pi \times 55\Delta x}{36}$

$\Rightarrow \Delta x = 0.109$ m.

Illustration 5. *Given: $y = 0.8 \sin 16\pi\left(t + \frac{x}{40}\right)$ meter. Calculate the wavelength and the velocity of the wave represented by this equation.*

Solution: $y = 0.8 \sin 2\pi\left(8t + \frac{8x}{40}\right)$

Comparing with y = a sin $2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$

$\frac{1}{T} = 8$, λ = 5m

Thus, v = $\frac{\lambda}{T}$ = 5 × 8 = 40 m/s.

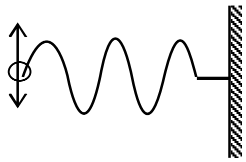
Longitudinal and transverse wave:

In a longitudinal wave, the particles of the medium carrying the mechanical wave move back and forth along the direction of propagation. Sound in air is a longitudinal wave.

In a transverse wave, the particles of the medium oscillate in the direction perpendicular to the direction of propagation of wave, for example the waves in a taut string.

Sine wave travelling on a string

When one end of tight string is fixed and other end is displaced slightly up and down continuously, then it keeps on doing work on the string and the energy is continues to vibrate up and down from the left once the first disturbance has reached it. It receives the energy from left, transmits it to the right and process continues. The nature of vibration of any particle similar to that of the left end, the only difference being that the motion is repeated after a time delay of x/v.



When the left end vibrates i.e. x=0 in an SHM, The equation of motion of this end may then be f(t) = A sinωt

Where A= amplitude

ω = the angular frequency.

The wave produced by such a vibrating source is called a sine wave or sinusoidal wave.

Since the displacement of the particle at x = 0 is

Given by y= A sin ωt, the displacement of the particle at x at time t will be

$$y = f(t - x/v)$$

$$y = A\sin\omega(t - x/v)$$

The reason is that the wave moves along the string with a constant speed v and the displacement of the particle at x at time t was original as at x=0 at time(t-x/v).

Next, velocity

$$\frac{dy}{dt} = A\omega\cos\omega(t - x/v)$$

Here variable x is taken constant .It is the velocity of the same particle whose displacement should be considered as a function of time.

Illustration 6. *A transverse sinusoidal wave is generated at one end of a long horizontal string by a bar that moves the end up and down through a distance of 1.50 cm. The motion is continuous and is repeated regularly 100 times per sec.*

(a) If the distance between the adjacent wave crests is observed to be15.0 cm, find the amplitude, frequency, speed and the wavelength of the wave motion.

(b) Assuming the wave moves in the +x direction and that at t = 0, the element of the string at x = 0 is at its equilibrium position y = 0 and moving downward, find the equation of the wave.

Solution: (a) As the bar moves a total of 1.50 cm, the end of the string moves

$\frac{1}{2}$(1.50 cm) = 0.75 cm. away from the equilibrium position, therefore the

amplitude A is 0.75 cm. The entire movement is repeated 100 times per second, thus frequency $f = 100$ Hz.
The distance between adjacent crest i.e. wavelength, $\lambda = 0.15$ m
wave speed $v = f\lambda$
$= 100 \times 0.15 = 15$ m/s.

(b) The general expression for a transverse sinusoidal wave moving in the +x direction is
$y = (x, t) = A \sin (kx - \omega t - \phi)$
At $y = 0$ $\frac{dy}{dt} < 0$ for x = 0 and t = 0 yields
$A \sin (-\phi) = 0$
and $- A\omega \cos (-\phi) < 0$,
Which means that the phase constant ϕ may be taken to be zero (or any integer multiple of 2π). Hence
$y (x, t) = A \sin (kx - \omega t)$
$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.15}$ rad/m.
$\omega = vk = 15 \times \frac{2\pi}{0.15} = 200\,\pi = 628$ rad/s.
Thus, $y (x, t) = 0.75 \times 10^{-2} \sin \left(\frac{2\pi}{0.15} x - 628t\right)$ m
Where x in meter and t in seconds.

Illustration 7. *As the wave passes along the string, each particle of the string moves up and down at right angles to the direction of the wave motion. Find the expression for the velocity and acceleration of a particle P located at $x_P = 0.30$ m, with reference to illustration 6.*

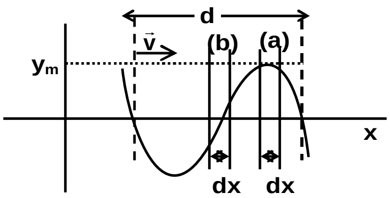
Solution: For a particle at $x_p = 0.30$ m in the above wave
$v_y(x_p, t) = - 0.75 \times 10^{-2} \times 628 \times \cos [\frac{2\pi}{0.15} \times 0.30 - 628t]$ m/s
$= - 4.71 \cos (4\pi - 628\, t)$ m/s.
Similarly acceleration,
$a_y(x_p, t) = - (0.75) \times 10^{-2} \times (628)^2 \times \sin (4\pi - 628\, t)$ m/s^2
$\approx 2.96 \times 10^3 \sin (4\pi - 628t)$ m/s^2

Energy and power of a traveling string wave:

When a wave is set up on stretched string, the energy is provided for the motion of the string. As the wave moves away, it transports that energy as both kinetic energy and elastic potential energy

Kinetic Energy
An element of the string of mass dm, oscillating transversely is SHM as the wave pass through it, has kinetic energy associated with its transverse velocity$= \frac{\partial y}{\partial t}$. When the element is passing through its y=0 position, its transverse velocity and thus kinetic energy is maximum. At the extreme position y=A, It transverse velocity and thus its kinetic energy is zero.

Elastic potential energy



To send a sinusoidal wave along the string, the wave must stretch the string. As a string element of length dx oscillates transversely, its length must increase and decrease in periodic way if the string element is to fit the sinusoidal wave form.
When the string element is at y=A position, its length has its normal undisturbed value dx, so its elastic potential energy is zero. At y=0 position, elastic potential energy is maximum.

Energy of a plane progressive wave:

The kinetic energy dE associated with a string element of mass dm is given by

$$dE = \frac{1}{2}mv^2$$

Where v is the transverse speed of the oscillating string element.

$$v = \frac{\partial y}{\partial t} = \omega A\cos(\omega t - kx)$$

Thus $dE = \frac{1}{2}(dm)\omega^2 A^2 \cos^2(\omega t - kx)$

$$= \frac{1}{2}(\mu dx)\omega^2 A^2 \cos^2(\omega t - kx)$$

Rate of energy transmitted

$$\frac{dE}{dt} = \frac{1}{2}\mu\left(\frac{dx}{dt}\right)\omega^2 A^2 \cos^2(\omega t - kx)$$

$$= \frac{1}{2}\mu v\omega^2 A^2 \cos^2(\omega t - kx)$$

The averages rate at which K.E is transmitted is

$$\left(\frac{dE}{dt}\right)_{avg} = \frac{1}{4}\mu v\omega^2 A^2$$

($\because$ Average of $\cos^2(\omega t - kx)$ over integer multiples of wavelengths crossed is ½)

Elastic potential energy is also carried along with the wave, and at the same average rate.
The average power, which is the average rate at which energy of both kinds is transmitted by the wave,

$$P_{avg} = 2\left(\frac{dE}{dt}\right)_{avg}$$

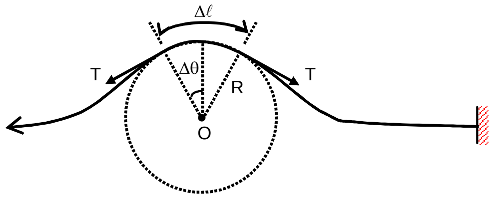
$$P_{avg} = \frac{1}{2}\mu v\omega^2 A^2$$

Wave Speed

The speed of any mechanical wave, transverse or longitudinal, depends on both an inertial property of the medium (to store kinetic energy) and an elastic property of the medium (to store potential energy).

Transverse wave in a stretched string

Consider a transverse pulse produced in a taut string of linear mass density μ. Consider a small segment of the pulse, of length Δl , forming an arc of a circle of radius R. A force equal in magnitude to the tension T pulls tangentially on this segment at each end.



Let us set an observer at the centre of the pulse which moves along with the pulse towards right. For the observer any small length $\Delta\ell$ of the string as shown will appear to move backward with a velocity v.

Now the small mass of the string is in a circular path of radius R moving with speed v. Therefore the required centripetal force is provided by the only force acting, (neglecting gravity) is the component of tension along the radius.

The net restoring force on the element is

$$F = 2T\sin(\Delta\theta) \approx T(2\Delta\theta) = T\frac{\Delta l}{R}$$

The mass of the segment is $\Delta m = \mu\Delta\ell$

The acceleration of this element toward the centre of the circle is

$a = \dfrac{v^2}{R}$, where v is the velocity of the pulse.

Using second law of motion,

$$T\frac{\Delta l}{R} = (\mu\Delta l)\left(\frac{v^2}{R}\right) \qquad \text{or} \qquad v = \sqrt{\frac{T}{\mu}}$$

Illustration 8. *A sonometre wire resonates with a given tuning fork forming standing waves with 5 antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, it resonates with the same tuning fork forming 3 antinodes for the same positions of bridges. The value of M is*

(A) 25 kg *(B) 5 kg*
(C) 12.5 kg *(D) 1/25 kg*

Solution : (A).

$$f = \frac{x}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}} = \frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}}$$

$\therefore 25 \times 9 = 9 \times m$

$\Rightarrow m = 25$ kg

Illustration 9. *A wire of uniform cross-section is stretched between two points 100 cm apart. The wire is fixed at one end and a weight is hung over a pulley at the other end. A weight of 9 kg produces a fundamental frequency of 750Hz.*

(a) What is the velocity of the wave in wire ?

(b) If the weight is reduced to 4 kg, what is the velocity of wave ?

What is the wavelength and frequency?

Solution : (a) L = 100 cm, f_1 = 750 Hz.

$v_1 = 2Lf_1 = 2 \times 100 \times 750 = 150000\ cms^{-1} = 1500\ ms^{-1}$

(b) $v_1 = \sqrt{\frac{T_1}{m}}$ and $v_2 = \sqrt{\frac{T_2}{m}}$

$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow \frac{v_2}{1500} = \sqrt{\frac{4}{9}}$

$\therefore\ v_2 = 1000\ m\ s^{-1}$

λ_2 = wave length = 2L = 200 cm = 2 m

$f_2 = \frac{v}{\lambda_2} = \frac{1000}{2} = 500$ Hz

Illustration 10. *The fundamental frequency of a sonometer wire increases by 5 hz if its tension increases by 21 %. How will the frequency be affected if its length is increased by 10 % ?*

Solution : $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$f + 5 = \frac{1}{2\ell}\sqrt{\frac{1.21T}{\mu}}$

by solving f = 50 Hz.

Next, $f' = \frac{1}{2(1.1)\ell}\sqrt{\frac{T}{\mu}}$

$= \frac{f}{1.1} = 45.45$ Hz.

Illustration 11. *Two perfectly identical wires are in unison. When the tension in one wire is increased by 1 %, then on sounding together, 3 beats are heard in 2 seconds. What is the initial frequency of each wire ?*

Solution : $\nu = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$

$\nu + \frac{3}{2} = \frac{1}{2\ell}\sqrt{\frac{1}{m}\left(T + \frac{T}{100}\right)}$

dividing $\frac{\nu + 1.5}{\nu} = \sqrt{\frac{101}{100}}$

on solving ν = 300 Hz.

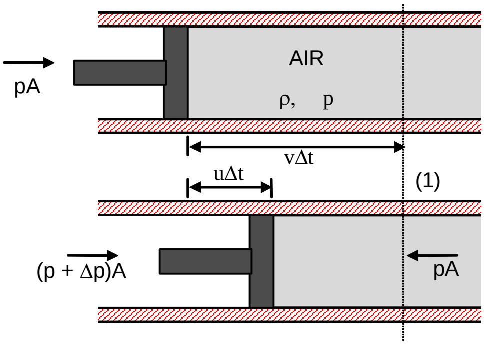
Longitudinal wave in fluids

Sound wave in air is a longitudinal wave. As a sound wave passes through air, potential energy is associated with periodic compressions and expansions of small volume elements of the air. The property that determines the extent to which an element of the medium changes its volume as the pressure applied to it is increased or decreased is the bulk modulus B.

$$B = \frac{-\Delta p}{\Delta V / V}$$

Where $\frac{\Delta V}{V}$ is the fractional change in volume produced by a change in pressure Δp.

Suppose air of density ρ is filled inside a tube of cross-sectional area A under a pressure p. Initially the air is at rest.



At t = 0, the piston at the left end of the tube(as shown in the figure) is set in motion towards right with a speed u. After a time interval Δt, all portions of the air at the left of section 1 are moving with speed u whereas all portions at the right of the section are still at rest. The boundary between the moving and the stationary portions travels to the right with v, the speed of the elastic wave (or sound wave). In the time interval Δt, the piston has moved $u\Delta t$ and the elastic disturbance has traveled a distance $v\Delta t$.

The mass of air that has attained a velocity u in a time Δt is $\rho(v\Delta t)A$. Therefore, the momentum imparted is $[\rho v\Delta t\, A]u$. And, the net impulse acting is $(\Delta p\, A)\Delta t$

Thus, impulse = change in momentum

$$(\Delta p\, A)\Delta t = [\rho v(\Delta t)\, A]u$$

or $\Delta p = \rho v u$ . . . (1)

Since $B = \dfrac{\Delta p}{\Delta V / V}$

$\therefore$ $\Delta p = B\left(\dfrac{\Delta V}{V}\right)$

Where $V = Av\,\Delta t$ and $\Delta V = Au\,\Delta t$

$\therefore$ $\dfrac{\Delta V}{V} = \dfrac{Au\Delta t}{Av\Delta t} = \dfrac{u}{v}$

Thus, $\Delta p = B\dfrac{u}{v}$ . . . (2)

From (1) and (2) $v = \sqrt{\dfrac{B}{\rho}}$.

Illustration 12. *Determine the speed of sound waves in water, and find the wavelength of a wave having a frequency of 242 Hz. Take $B_{water} = 2 \times 10^9$ Pa.*

Solution: Speed of sound wave, $v = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{\left(2\times 10^9\right)}{10^3}} = 1414\ m/s$

Wavelength $\lambda = \dfrac{v}{f} = \dfrac{1414}{242} = 5.84m$

Speed of sound in an ideal gas

Newton assumed that the motion of sound waves in air is isothermal. In the case of ideal gas relation between pressure and volume during the isothermal process is

$$PV = \text{constt.}$$

By differentiating,

$$pdv + vdp = 0$$

$$\frac{dp}{dv} = -\frac{p}{v} \qquad \dots (i)$$

Since $B = -v\frac{dp}{dv}$ … (ii)

From (i) and (ii)

$$B = -v\frac{dP}{dv} = -v \times -\frac{P}{v} = P \quad \text{… (iii)}$$

$$\therefore \quad v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{p}{\rho}} \quad \text{… (iv)}$$

Laplace correction: Since the gaseous medium is a bad conductor of heat, the heat produced at a compression or rarefaction can not be conducted with surrounding medium therefore, taking relation in p and v for adiabatic process,

pV^{γ} = constant.

Where γ is the ratio of the heat capacity at constant pressure to that at constant volume.

After differentiating , we get

$$\frac{dp}{dV}V^{\gamma} + \gamma p V^{\gamma-1} = 0 \qquad \Rightarrow \qquad \frac{dP}{dV} = -\frac{\gamma P}{v}$$

Since $B = -V\frac{dp}{dV} = \gamma p$

$\therefore \quad v = \sqrt{\frac{\gamma p}{\rho}}$

Using the gas equation $\frac{p}{\rho} = \frac{RT}{M}$ where M is the molar mass.

Thus, $v = \sqrt{\frac{\gamma RT}{M}}$ (T = Temperature in kelvin)

Illustration 13. *The velocity of sound through hydrogen is 1400 m/s. What will be the velocity of sound through a mixture of parts by two volume of hydrogen and one part of oxygen?*

Solution: If ρ be the density of the hydrogen, then the density of oxygen is 16 ρ. If x be the total volume, then volume of hydrogen in mixture$=\frac{2x}{3}$; volume of oxygen in mixture = x/3.

mass of hydrogen in mixture $= \frac{2x\rho}{3}$; mass of oxygen in mixture $= \frac{16}{3}x\rho$

Total mass $= \frac{2x\rho}{3} + \frac{16x\rho}{3} = 6x\rho$

Density of mixture $= \frac{6x\rho}{x} = 6\rho$

Now, $\frac{v_M}{v_{H_2}} = \sqrt{\frac{\rho_{H_2}}{\rho_m}}$

$v_m = v_{H_2}\sqrt{\frac{\rho_{H_2}}{\rho_M}} = 1400\sqrt{\frac{\rho}{6\rho}}$

$= \frac{1400}{2.45} = 571.4 m/s.$

Sound waves

From practical standpoint it is easier to measure pressure variations in a sound wave than the displacements, so it is worthwhile to develop a relation between the two. Let p be the instantaneous pressure fluctuation at any point, that is, the amount by which the pressure differs from normal atmospheric pressure. If the displacements of two neighbouring points x and x + Δx are the same, the gas between these points is neither compressed nor rarefied, there is no volume change, and consequently p =0. Only when y varies from one point to a neighbouring point there is a change of volume and therefore of pressure.

The fractional volume change $\Delta V/V$ in an element near point x turns out to be $\left(\frac{\partial y}{\partial x}\right)$, which is the rate of change of y and x as we go from one point to the neighbouring point. To see why this is so, we note that $\Delta V/V$ is proportional to change in length of an element which has length Δx when no wave disturbance is present, divided by Δx. The change in length is the value of y at the point x + Δx, minus the value at the point x. If Δx is very small, this is approximately multiplied by the derivative of y with respect to x. thus

$$y(x + \Delta x, t) - y(x, t) = \frac{\partial y}{\partial x}\Delta x$$

$$\frac{\Delta V}{V} = \frac{y(x+\Delta x,\ t) - y(x,\ t)}{\Delta x} = \frac{\partial y}{\partial x} \qquad . . . (i)$$

Now from the definition of the bulk modulus B,

$p = -B\frac{\Delta V}{V}$, and we find

$$p = -B\frac{\partial y}{\partial x}$$

Now $\quad y = A \sin(\omega t - kx) \Rightarrow p = BkA \cos(\omega t - kx)$

Maximum amount by which the pressure differs from atmospheric, that is, the maximum value of p, is called the pressure amplitude, denoted P.

$$\Rightarrow \qquad P = BkA$$

Intensity in terms of pressure variation

$I = 2\pi^2 f^2 A^2 \rho v$

$= P^2/2\rho v$ (Using P = Bkv, $k = 2\pi/\lambda$, $v = f\lambda$ and $v = \sqrt{B/\rho}$

Illustration 14. *The maximum pressure variations Δp_m that the ear can tolerate in loud sounds is about 28 Pa at 1000 Hz. The faintest sound that can be heard at 1000 Hz has a pressure amplitude of about 2.8×10^{-5} Pa. Find the corresponding density and the displacement amplitudes. The bulk modulus for air under standard condition is 1.4×10^5 Pa, and the speed of sound in air is 343 m/s at room temperature.*

Solution: The wave number is $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v}$

$$= \frac{2\pi \times 10^3}{343} = 18.3 \text{ rad/m}$$

The density of air under these conditions is 1.21 kg/m^3. Hence for $\Delta p_m = 28P_a$.

$\Delta\rho_m = \Delta p_m \frac{\rho_0}{B}$.

$= 28 \times \frac{1.21}{1.4 \times 10^5} = 2.4 \times 10^{-4}$ kg/m^3.

and $s_m = \frac{\Delta p_m}{k_B} = \frac{28}{18.3 \times 1.4 \times 10^5} = 1.1 \times 10^{-5}$ m.

The displacement amplitudes for the loudest sounds are about 10^{-5} m, a very small valve indeed. For the faintest sounds,

$\Delta\rho_m = 2.8 \times 10^{-5} \times = 2.4 \times 10^{-10}$ kg/m^2

and $\delta_m = \frac{2.8 \times 10^{-5}}{18.3 \times 1.4 \times 10^5} = 1.1 \times 10^{-11}$ m.

Illustration 15. *A stone dropped into a well of depth 300 m splashed into the water. When is the splash heard at the top? Velocity of sound = 340 m/s, g = 10 m/s^2. (Neglecting Byoyancy effect)*

Solution: Time after which the splash is heard is equal to the time t_1 taken by the stone to fall down and the time t_2 taken by the sound to travel from the bottom to the surface level.

Using $S = ut + \frac{1}{2}at^2$

$t_1 = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times 300}{10}} = \sqrt{60} = 7.74$ sec.

Again, $t_2 = \frac{\text{depth of well}}{\text{velocity of sound}} = \frac{300}{340} = 0.88\text{s}$

Total time, $t = t_1 + t_2 = 7.74 + 0.88 = 8.62$ sec.

Illustration 16. *The faintest sound the human ear can detect at frequency 1 kHz corresponds to an intensity of about 10^{-12} W /m^2. Determine the pressure amplitude and the maximum displacement associated with this sound assuming the density of the air = 1.3 kg/m^3 and velocity of sound in air = 332 m/s.*

Solution : $I = \frac{P^2}{2\rho v}$

$\Rightarrow \quad P = \sqrt{I \text{ x } 2\rho v}$

$= \sqrt{10^{-12}\text{x}2\text{x}1.3\text{x}332} = 2.94\text{x}10^{-5}\text{N/m}^2$

Again,

$P = \rho v \ \omega A$

$\Rightarrow A = \frac{P}{\rho v \omega} = \frac{2.94\text{x}10^{-5}}{1.3\text{x}332\text{x}2\pi\text{x}10^3} = 1.1\text{x}10^{-11}\text{m}$.

Illustration 17. *Measurement of sound waves show that the maximum pressure variations in the loudest sound that the ear can tolerate without pain are of the order of 30 Pa. Find the corresponding maximum displacement, if the frequency is 1000 Hz and v = 350 m/s. If the density of air is ρ = 1.22 kg/m^3, then find the intensity of a sound wave of the largest amplitude tolerable to the human ear.*

Solution : $k = \frac{\omega}{v} = \frac{2(3.14)(1000)}{350} = 18 m^{-1}$

$B = \gamma p = (1.4)(10^5)$

$A = \frac{P}{Bk} = \frac{30}{(1.4\times10^5)(18)} = 1.18\times10^{-5} m$

Using, $I = \frac{P_m^2}{2\rho v} = \frac{(30)^2}{2(1.22)(350)} = 1.05\ W/m^2$.

Exercise 3.

(i) ***Transverse waves are not produced in liquids and gases. Why ?***

(ii). ***What are the conditions for resonance of air column with a tuning fork ?***

Loudness

Human ear is sensitive for extremely large range of intensity. So a logarithmic rather than an arithmetic scale is convenient. Accordingly, intensity level β of a sound wave is defined by the equation.

Loudness, $\beta = 10 \log\left(\frac{I}{I_o}\right)$ decibel

Where $I_o = 10^{-12}$ W/m^2 is the reference intensity (threshold level for normal human ear) level to which any intensity I is compared.

Illustration 18. *. Spherical sound waves are emitted uniformely in all directions from a point source, the radiated power P being 25 P being 25 W. What are the intensity and sound level of sound waves a distance r = 2.5 m from the source ?*

Solution: All the radiated power P must pass thourgh a sphere of radius r centred on the source.

Thus, $I = \frac{P}{4\pi r^2}$

$\Rightarrow I = \frac{25}{4\pi(2.5)^2} = 0.32 W/m^2$,

SL = $10 \log \frac{I}{I_0} = 10 \log \frac{0.32}{10^{-12}} = 115$ dB.

Exercise 4. ***How can your mom distinguish between you and your sisters voice even if the loudness is same?***

Illustration 19. *A window whose area is 2m^2 opens on a street where the street noise results in an intensity level at the window of 60 dB. How much acoustic power enters the window through sound waves? Now, if a sound absorber is fitted at the window, how much energy from the street will it collect in a day ?*

Solution: By definition sound level = $10 \log \frac{I}{I_o} = 60$

or $I/I_0 = 10^6$

$I = 10^{-12} \times 10^6 = 1\mu\ W/m^2$

Power entering the room

$= 1 \times 10^{-6} \times 2 = 2\ \mu W$

Energy collected in a day = $2 \times 10^{-6} \times 86400$

$= 0.173$ J

Superposition of Waves

Two or more waves can traverse the same space independently of one another. Thus the displacement of any particle in the medium at any given time is simply the sum of displacements that the individual waves would give it. This process of the vector addition of the displacement of a particle is called superposition.

Illustration 20. *Standing waves are produced by superposition of two waves $y_1 = 0.05 \sin (3\pi t - 2x)$ and $y_2 = 0.05 \sin (3\pi t + 2x)$ where x and y are measured in metre and t in second. Find the amplitude of the particle at x = 0.5 m.*

Solution: Resultant displacement $y = y_1 + y_2$

$= 0.05 \sin (3\pi t - 2x) + 0.05 \sin (3\pi t + 2x)$

$= 0.1 \cos 2x \sin 3\pi t$

$= A \sin 3\pi t$ where $A = 0.1 \cos 2x$

For x = 0.5 m. we have

$A = 0.1 \cos (1 \text{ rad}) = 0.1 \cos (\pi/3.14)$

$= 0.054$ m.

Illustration 21. *Two waves are given as $y_1 = 3A \cos(\omega t - kx)$ and $y_2 = A \cos(3\omega t - 3kx)$. Amplitude of resultant wave will be*

(A) A *(B) 2A*
(C) 3A *(D) 4A*

Solution: (D).

Using trigonometric identity $\cos 3\theta = 4 \cos^3\theta - 3 \cos \theta$)

$y_R = y_1 + y_2 = 3A\cos(\omega t - kx) + A \cos(3\omega t - 3kx)$

$\Rightarrow y_R = 3A \cos (\omega t - kx) + A \{ 4 \cos^3 (\omega t - kx)) - 3 \cos (\omega t - kx)\}$

$= 4A \cos^3 (\omega t - kx)$

$\Rightarrow A_{Resultant} = 4$ A

Exercise 5. Is superposition principle applicable to electromagnetic waves ?

Interference

When two waves of the same frequency, superimpose each other, there occurs redistribution of energy in the medium which causes either a minimum intensity or maximum intensity which is more than the sum of the intensities of the individual sources. This phenomenon is called interference of waves. Let the two waves be

$y_1 = A_1 \sin(kx - \omega t)$, $y_2 = A_2 \sin(kx - \omega t + \delta)$

According to the principle of superposition

$y = y_1 + y_2$

$= A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \delta)$

$= A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t)\cos\delta + A_2 \cos(kx - \omega t)\sin\delta$

$= \sin(kx - \omega t)(A_1 + A_2\cos\delta) + \cos(kx - \omega t)(A_2 \sin\delta)$

$= R\sin(kx - \omega t + \phi)$

where, $A_1 + A_2\cos\delta = R\cos\phi$ and $A_2\sin\delta = R\sin\phi$

and $R^2 = (A_1 + A_2\cos\delta)^2 + (A_2\sin\delta)^2$

$= A_1^2 + A_2^2 + 2A_1A_2\cos\delta$

If I_1 and I_2 are intensities of the interfering waves and δ is the phase difference, then the resultant intensity is given by

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$

Now, $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ for $\delta = 2n\pi$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ for $\delta = (2n+1)\pi$

Illustration 22. *Two coherent sound sources are at distances $x_1 = 0.2$ m and $x_2 = 0.48$m from a point. Calculate the intensity of the resultant wave at that point if the frequency of each wave is f = 400 Hz and velocity of wave in the medium is v = 448 m/s. The intensity of each wave is $I_o = 60 W/m^2$.*

Solution : Path difference, $\Delta x = x_2 - x_1 = 0.48 - 0.2 = 0.28$ m

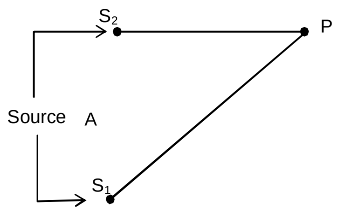
$$\phi = \frac{2\pi}{\lambda}p = \left(\frac{2\pi f}{v}\right)p = \frac{2\pi(400)(0.28)}{448} = \frac{\pi}{2}$$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

or $I = I_o + I_o + 2I_o\cos(\pi/2)$

$= 2I_o = 2(60) = 120\ W/m^2$.

Illustration 23. *A listener is seated at a point a distance of 1.2 m directly in front of one speaker. The two speakers, which are separate by a distance D of 2.3 m, emit pure tones of wavelength λ. The waves are in phase when they leave the speakers. For what wavelengths will the listener hear a minimum in the sound intensity?*



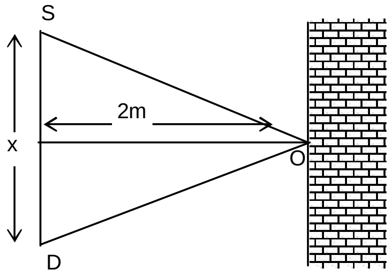
Solution : The minimum sound intensity occurs when the waves from the two speakers interfere destructively. If the listner is seated in front of speaker 2, then r_2 =1.2 m and r_1 can be found as

$r_1 = \sqrt{r_2^2 + D^2} = \sqrt{(1.2)^2 + (2.3)^2} = 2.6$ m

Thus, $r_1 - r_2 = 2.6 - 1.2 = 1.4$ m,

and accordingly 1.4 m $= \lambda/2,\ 3\lambda/2,\ 5\lambda/2$ . . . .
corresponding to λ = 2.8 m, 0.93 m, 0.56 m, ...

Illustration 24. *A source 'S' of sound emitting waves at 360 Hz is placed in front of a vertical wall, at a distance 2m as shown in figure. Detector D receives waves from two different paths SD and SOD. Find the minimum distance between the source and the detector for which the detector detects a maximum of sound. Take speed of sound in air = 360 m/s.*



Solution : SD = x, SOD $= 2\sqrt{4+\frac{x^2}{4}} = \sqrt{x^2+16}$

Path difference $\Delta x = \sqrt{x^2+16}$ - x
(For constrictive interference at D and minimum value of x)
$\Delta x = \lambda$

$\sqrt{x^2+16} - x = \frac{v}{f} = \frac{360}{360} = 1m.$

x = 7.5 m.

Illustration 25. *Two SHM's with same amplitude and time period, when acting together in perpendicular direction with a phase difference of $\pi/2$ gives rise to*

(A) elliptical motion *(B) circular motion*
(C) straight line motion *(D) none*

Solution : (B).
We have the equation of SHM's and adding together

$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy\cos\delta}{A_1A_2} = \sin^2\delta$

Given: $A_1 = A_2 = A$ and $\delta = \pi/2$
$\Rightarrow x^2 + y^2 = A^2$
This represents the general equation of a circle with its center at origin

Standing waves

A standing wave is formed when two identical waves travelling in the opposite directions along the same line, interfere.

On the path of the stationary wave, there are points where the amplitude is zero, they are known as NODES. On the other hand there are points where the amplitude is maximum, they are known as ANTINODES.

The distance between two consecutive nodes or two consecutive anitnodes is $\frac{\lambda}{2}$.

The distance between a node and the next antinode is $\frac{\lambda}{4}$.

Consider two waves of the same frequency, speed and amplitude, which are travelling in opposite directions along a string. Two such waves may be represented by the equations

y_1 = a sin (kx - wt) and

$y_2 = a \sin(kx + wt)$

Hence the resultant may be written as

$y = y_1 + y_2 = a \sin(kx - wt) + a \sin(kx + wt)$

$y = 2a \sin kx \cos wt$

This is the equation of a standing wave.

Note:

(i) In this equation, it is seen that a particle at any particular point 'x' executes simple harmonic motion and all particles vibrate with the same frequency.

(ii) The amplitude is not the same for different particles but varies with the location 'x' of the particle.

(iii) The points having maximum amplitudes are those for which 2asinkx, has a maximum value of 2a, these are at the positions,

$kx = \pi/2, 3\pi/2, 5\pi/2, \ldots\ldots\ldots\ldots$

or $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots\ldots\ldots\ldots$

These points are called **antinodes.**

(iv) The amplitude has minimumn value of zero at positions where

$kx = \pi, 2\pi, 3\pi, \ldots\ldots\ldots$

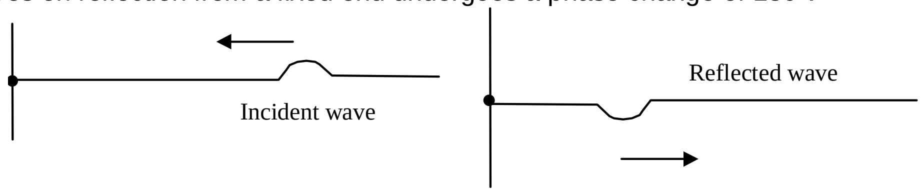
or $x = \lambda/2, \lambda, 3\lambda/2, 2\lambda \ldots\ldots\ldots$

These points are called **nodes.**

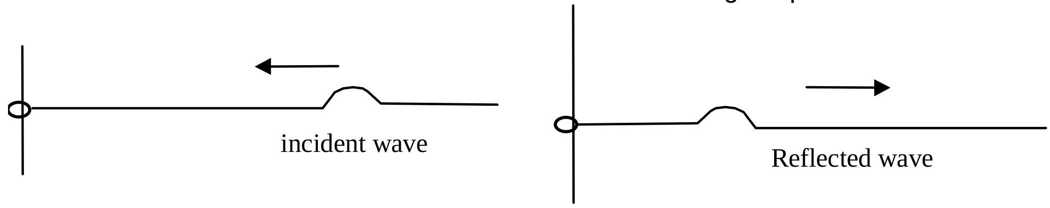
(v) Energy is not transported along the string to the right or to the left, because energy can not flow past the nodal points in the string which are permanently at rest.

Reflection of waves

(a) Waves on reflection from a fixed end undergoes a phase change of 180°.

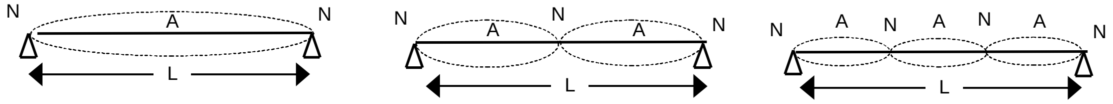


(b) While a wave reflected from a free end is reflected without a change in phase.



(c) In case of pressure wave there is no phase change when reflected from a denser medium or fixed end.

Stationary waves in strings



(a) (b) (c)

A string of length L is stretched between two points. When the string is set into vibrations, a transverse progressive wave begins to travel along the string. It is reflected at the other fixed end. The incident and the reflected waves interfere to produce a stationary transverse wave in which the ends are always nodes.

(a) In the simplest form, the string vibrates in one loop in which the ends are the nodes and the centre is the antinode. This mode of vibration is known as the fundamental mode and the frequency of vibration is known as the fundamental frequency or first harmonic.
Since the distance between consecutive nodes is $\lambda/2$

$L = \frac{\lambda_1}{2} \quad \therefore \ \lambda_1 = 2L$

If f_1 is the fundamental frequency of vibration, then the velocity of transverse waves is given as, $v = \lambda_1 f_1$ or $f_1 = v/2L$ $\Rightarrow$ $v = 2Lf_1$ . . . (1)

(b) The same string under the same conditions may also vibrate in two loops, such that the centre is also the node.

$\therefore \quad L = 2\frac{\lambda_2}{2} \ \therefore \quad \lambda_2 = L$

If f_2 is the frequency of vibrations, then the velocity of transverse waves is given as,
$v = \lambda_2 f_2 \quad \therefore \ v = L f_2$ or $f_2 = v/L$ . . . (2)
The frequency f_2 is known as second harmonic or first overtone.

(c) The same string under the same conditions may also vibrate in three segments.

$\therefore \quad L = 3\frac{\lambda_3}{2} \ \therefore \quad \lambda_3 = \frac{2}{3}L$

If f_3 is the frequency in this mode of vibration, then,

$v = \lambda_3 f_3 \ \therefore \ v = \frac{2}{3}Lf_3$ or $f_3 = 3v/2L$ (3)

The frequency f_3 is known as the third harmonic or second overtone. Thus a stretched string in addition to the fundamental mode, also vibrates with frequencies which are integral multiples of the fundamental frequencies. These frequencies are known as harmonics.

The velocity of transverse wave in a stretched string is given as

$v = \sqrt{\frac{T}{\mu}}$ where T = tension in the string.

μ = linear density or mass per unit length of string.
If the string fixed at two ends, vibrates in its fundamental mode, then

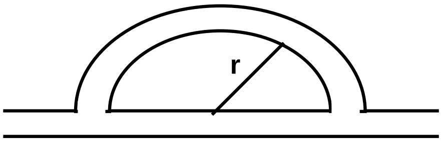
$v = 2Lf \ \therefore \quad f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

μ = volume of unit length $\times$ density

$= \pi r^2 \times 1 \times \rho = \pi \times \frac{D^2}{4} \times \rho$ where D = diameter of the wire, ρ = density.

Note : For a given fundamental frequency f, 2f is called upper octave of f and f/2 is called the lower octave of f.

Illustration 26. *A sound wave of 40 cm wavelength enters the tube as shown in the figure. What must be the smallest radius r such that a minimum will be heard at the deletion?*



Solution: Path difference is = $\pi r - 2r = r(\pi - 2)$

$$\frac{2\pi}{\lambda} r(\pi - 2) = (2n - 1)\pi$$

$$\frac{2\pi}{\lambda} r(\pi - 2) = \pi$$

$$r = \frac{\lambda}{2(\pi - 2)} = \frac{40}{2(3.14 - 2)} = \frac{20}{1.14} = 17.52 \text{ cm}$$

Illustration 27. *The vibrations of a string of length 60 cm fixed at both ends are represented by the equation $y = 4 \sin\left(\frac{\pi x}{15}\right) \cos(96\pi t)$ where x and y are in cm and t in seconds.*

(i) What is the maximum displacement of a point at x = 5 cm?
(ii) Where are the nodes located along the string ?
(iii) What is the velocity of the particle at x = 7.5 cm and at t = 0.25 s ?
(iv) Write down the equations of the component waves whose superposition give the above wave.

Solution: $y = 4 \sin(\pi x/15) \cos(96\pi t)$ can be broken up into

$$y = 2\left[\sin\left(\frac{\pi x + 1440\pi t}{15}\right) + \sin\left(\frac{\pi x - 1440\pi t}{15}\right)\right]$$

Thus the waves are of the same amplitude and frequency but travelling in opposite directions which thus, superimpose to give a standing wave,

(i) At x = 5 cm the standing wave equation gives

$y = 4 \sin(5\pi/15) \cos(96\pi t)$

$= 4 \sin \pi/3 \cos(96\pi t) = 4 \times \sqrt{3}/2 \cos(96\pi t)$

$\therefore$ Maximum displacement = $2\sqrt{3}$ cm.

(ii) The nodes are the points permanently at rest. Thus they are those points for which

$\sin(\pi x/15) = 0$

i.e. $\pi x/15 = n\pi$, n = 0, 1,2,3, 4 ......

$x = 15n$ i.e. at x = 0,15,30,45 and 60 cm

(iii) The particle velocity is equal to

$$\left(\frac{\partial y}{\partial t}\right) = 4\sin\left(\frac{\pi x}{15}\right)(96\pi)(-\sin 96\pi t)$$

$$= -384\pi\sin\left(\frac{\pi x}{15}\right)\sin(96\pi t)$$

at $x = 7.5$ and $t = 0.25$ we get

$$\left(\frac{\partial y}{\partial t}\right) = -384\pi\sin\left(\frac{\pi x}{15}\right)\sin(96\pi t) = -384\pi\sin\left(\frac{\pi}{2}\right)\sin(24\pi) = 0$$

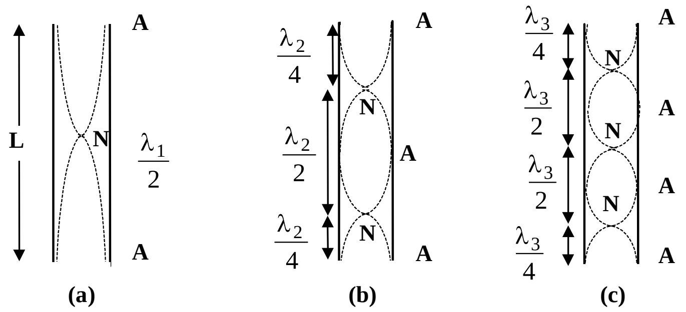
(iv) The equations of the component waves are

$$y_1 = 2\sin\left(\frac{\pi x}{15} + 96\pi t\right)$$

and $$y_2 = 2\sin\left(\frac{\pi x}{15} - 96\pi t\right)$$

Stationary waves in air column

Open pipe: If both ends of a pipe are open and a system of air is directed against an edge, standing longitudinal waves can be set up in the tube. The open end is a displacement antinode



(a) For fundamental mode of vibrations,

$L = \frac{\lambda_1}{2}$ $\therefore$ $\lambda_1 = 2L$

$v = \lambda_1 f_1$ $\therefore$ $v = 2Lf_1$ ...(1)

(b) For the second harmonic or first overtone,

$L = \lambda_2$ or $\lambda_2 = L$

$v = \lambda_2 f_2$ $\therefore$ $v = Lf_2$ ...(2)

(c) For the third harmonic or second overtone,

$L = 3 \times \frac{\lambda_3}{2}$ $\therefore$ $\lambda_3 = \frac{2}{3}L$

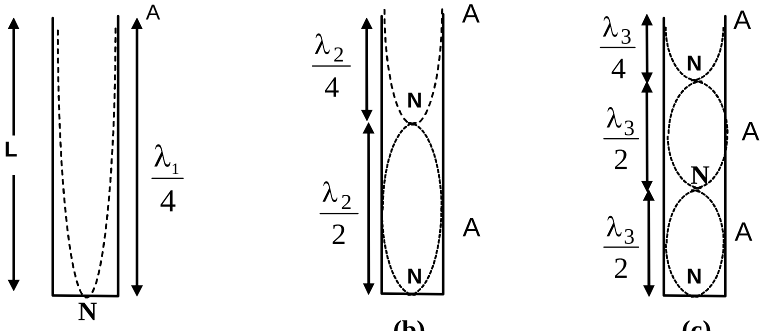
$v = \lambda_3 f_3$ $\therefore$ $v = \frac{2}{3}L\, f_3$

From (1) , (2) and (3) we get , $f_1 : f_2 : f_3 : ....= 1 : 2 : 3 : ....$
i.e. for a cylindrical tube, open at both ends, the harmonics excitable in the tube are all integral multiples of its fundamental.

$\therefore$ In the general case, $\lambda = \frac{2L}{n}$ where n = 1,2 ......

Frequency $= \frac{v}{\lambda} = \frac{nv}{2L}$ where $n = 1, 2.....$

Closed pipe: If one end of a pipe is closed the reflected wave is 180° out of phase with the incoming wave. Thus the displacement of the small volume elements at the closed end must always be zero. Hence the closed end must be a displacement node.



(a) This represents the fundamental mode of vibration.

$L = \frac{\lambda_1}{4}$ $\therefore$ $\lambda_1 = 4L$

If f_1 is the fundamental frequency, then the velocity of sound waves is given as,

$v = \lambda_1 f_1$ $\therefore v = 4Lf_1$ . . . (1)

(b) This is the third harmonic or first overtone.

$L = 3 \times \frac{\lambda_2}{4}$ $\therefore$ $\lambda_2 = \frac{4}{3}L$

$v = \lambda_2 f_2$ $\therefore$ $v = \frac{4}{3}L\,f_2$ . . . (2)

(c) This is the fifth harmonic or second overtone.

$L = 5 \times \frac{\lambda_3}{4}$ $\therefore$ $\lambda_3 = \frac{4}{5}L$

$v = \lambda_3 f_3$ $\therefore$ $v = \frac{4}{5}L\,f_3$ . . . (3)

From (1) , (2) and (3) we get, $f_1 : f_2 : f_3 : .... = 1 : 3 : 5 : ....$

In the general case, $\lambda = \frac{4l}{(2n+1)}$ where n = 0, 1, 2, ...

Velocity of sound = v

Frequency = $\frac{(2n+1)v}{4L}$ where $n = 0, 1, 2....$

Note : For more accurate results end correction must be taken in to account at open end of the pipe, as antinode will form at 0.6r (r = radius of pipe) out side the end of the open pipe.

Exercise 6.

(i) ***Why a flute produces a melodious sound where as a whistle produces a shrill noise?***

(ii) ***What are harmonics ?***

Illustration 28. *For a certain organ pipe, three successive resonance frequencies are observed at 425, 595 and 765 Hz respectively. Taking the speed of sound in air to be 340 m/s.*
(a) explain whether the pipe is closed at one end or open at both ends.
(b) determine the fundamental frequency and length of the pipe.

Solution: (a) The give frequencies are in the ratio 5 : 7 : 9. As the frequencies are i.e. notes are odd multiple of 85Hz the pipe must be closed at one end.
(b) Now the fundamental frequency is the lowest i.e. one time the found value 85Hz.

$$\therefore \quad 85 = \frac{v}{4\ell} \quad \Rightarrow \quad \ell = \frac{v}{4 \times 85} = 1 \text{ m}$$

Illustration 29. *Third overtone of a closed organ pipe is in unison with fourth harmonic of an open organ pipe. Find the ratio of the lengths of the pipes.*

Solution: Third overtone of closed pipe $\Rightarrow$ seventh harmonic
$(f_7)_{closed} = (f_4)_{open}$

$$7\left(\frac{v}{4\ell_c}\right) = 4\left(\frac{v}{2\ell_0}\right)$$

$$\frac{\ell_c}{\ell_0} = \frac{7}{8}.$$

Illustration 30. *An air column in a pipe which is closed at one end, is in resonance with a vibrating tuning fork of frequency 264 Hz. in its fundamental mode. Length of the column in cm is (v = 330 m/s)*
(A) 31.25 *(B) 62.5*
(C) 93.75 *(D) 125.75*

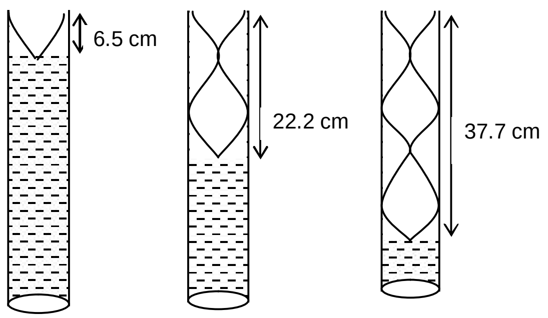
Solution: (A).

$$\ell = \frac{\lambda}{4} \Rightarrow \lambda = 4\ell,$$

$$n = \frac{v}{\ell} = \frac{v}{4\ell} \quad \Rightarrow 264 = \frac{330}{4\ell}$$

$\Rightarrow \ell = 0.3125$ m = 31.25 cm

Illustration 31. *A small speaker is held above a cylindrical resonance tube partially filled with water. In the experiment, the speaker is driven at a fixed frequency of 1080 Hz. and three resonances are observed when the water level is at distances of x_1 = 6.5 cm, x_2 = 22.2 cm and x_3 = 37.7 cm below the top of the tube. Find the speed of the sound.*



Solution: The air column acts like a tube of variable length closed at one end. The pattern of standing waves shows a pressure node near the speaker and a pressure antinode at the surface of the water. Here we donot know the end correction, we can not use the given data directely to find the speed of sound.

Howerver by the resonance pattern we can judge that
$\lambda/2 = x_2 - x_1 = 22.2 \text{ cm} - 6.5 \text{ cm} = 15.7 \text{ cm}$.
and similarly, from the second and third resonances.

$$\frac{\lambda}{2} = x_3 - x_2 = 37.7 \text{ cm} - 22.2 \text{ cm} = 15.5 \text{ cm}$$

Now avg. $(\lambda/2) = \dfrac{15.7 + 15.5}{2} = 15.6\text{cm}$

$\Rightarrow \lambda = 31.2$ cm
$= 0.312$ m
$v = f\lambda = 180 \times 0.312 = 337$ m/s.

Illustration 32. *An air column with a tuning fork of frequency 256 Hz, gives resonance at column length 33.4 cm and 101.8 cm. Deduce (i) the end correction. (ii) the speed of sound in air.*

Solution: (i) $e = \dfrac{\ell_2 - 3\ell_1}{2}$

$$= \frac{101.8 - 3 \times 33.4}{2}\text{cm} = 0.8\text{cm}.$$

(ii) $\dfrac{\lambda}{2} = \ell_2 - \ell_1$

$= 101.8 - 33.4 = 68.4$ cm $= 0.684$ m
$\lambda = 2 \times 0.684$
$= 1.368$ m
$v = f\lambda = 256 \times 1.368 = 350.2$ m/s.

Illustration 33. *A pipe of length 1.5 m closed at one end is filled with a gas and it resonates in its fundamental mode with a tunning fork. Another pipe of the same length but open at both ends is filled with air and it resonates in its fundamental mode with the same tuning fork. Calculate the velocity of sound at 0^0C in the gas, given that the velocity of sound in air is 360 m/sec at 30^0C, where the experiment is performed.*

Solution: Fundamental frequency for open pipe;

$$f_0 = \frac{V}{2L} = \frac{360}{2(1.5)} = 120 \text{ Hz}.$$

Let velocity of sound in the gas be v_1 at 30^0C
Fundamental frequency for closed pipe with gas inside

$$= \frac{v_1}{4\ell} = \frac{v_1}{4(1.5)} = \frac{v_1}{6}$$

given $120 = \dfrac{v_1}{6}$ $\therefore v_1 = 720$ m/s

Now relation between velocity of sound and temperature is $v \propto \sqrt{T}$

$$\therefore \frac{720}{v_2} = \sqrt{\frac{273 + 30}{273}} \qquad \Rightarrow v_2 = 683.4 \text{ m/s}.$$

Beats: When two interfering waves have slightly different frequencies the resultant disturbance at any point due to the superposition periodically fluctuates causing waxing and waning in the resultant intensity. The waxing and waning in the resultant intensity of two superposed waves of slightly different frequency are known as beats.

Let the displacement produced at a point by one wave be

$$y_1 = A \sin (2\pi f_1 t - \phi_1)$$

and the displacement produced at the point produced by the other wave of equal amplitude as

$$y_2 = A \sin (2\pi f_2 t - \phi_2)$$

By the principle of superposition, the resultant displacement is

$$y = y_1 + y_2 = A \sin (2\pi f_1 t - \phi_1) + A \sin (2\pi f_2 t - \phi_2)$$

$$y = 2A\sin\left\{2\pi\left(\frac{f_1+f_2}{2}\right)t - \left(\frac{\phi_1 - \phi_2}{2}\right)\right\} \cos 2\pi\left(\frac{f_1-f_2}{2}\right)t$$

$$Y = R\sin\left\{2\pi\left(\frac{f_1+f_2}{2}\right)t - \left(\frac{\phi_1 - \phi_2}{2}\right)\right\}$$

Where, $R = 2A \cos 2\pi\left(\frac{f_1-f_2}{2}\right) t$

The time for one beat is the time between consecutive maxima or minima.

First maxima would occur when

$$\cos 2\pi\left(\frac{f_1-f_2}{2}\right) t = +1$$

Then $2\pi\left(\frac{f_1-f_2}{2}\right) t = 0 \therefore t = 0$

For second maxima would occur when

$$\cos 2\pi\left(\frac{f_1-f_2}{2}\right) t = -1$$

Then $2\pi\left(\frac{f_1-f_2}{2}\right) t = \pi$

or $t = \dfrac{1}{f_1 - f_2}$

The time for one beat = $\left(\dfrac{1}{f_1-f_2} - 0\right) = \dfrac{1}{f_1-f_2}$

Similarly it may also be shown that time between two consecutive minima is $\dfrac{1}{f_1-f_2}$. Hence frequency of beat i.e. number of beats in one second

Beat frequency $= f_1 \sim f_2$

Illustration 34. *The first overtone of an open pipe and the fundamental note of a pipe closed at one end give 5 beats/sec when sounded together. If the length of the pipe closed at one end of 25cm, what are the possible lengths of the open pipe? (Neglect end corrections and take the velocity of sound in air to be 340 m/s).*

Solution: Let the fundamental frequency of the closed end pipe of length 25 cm be f_0. Then

$f_0 = \frac{V}{4\ell} = \frac{340 \times 100}{4 \times 25} = 340$ Hz

Possible frequencies of first overtone of the required open pipe are 340 $\pm$ 5, i.e. 345 or 335 Hz

For the first overtone of an open pipe, the length of the pipe ℓ equals the wavelength of the vibration

Hence $345 = \frac{V}{\ell}$

or $\ell = \frac{34000}{345} = 98.5$ cm

Other possible length ℓ' is given by

335 =

$\ell' = \frac{34000}{335} = 101.5\,\text{cm}$

Hence possible lengths of the open pipe are 98.5 and 101.5 cm.

Illustration 35. *Two tuning forks P and Q, when set vibrating, give 4 beats per second. If a prong of the fork P is filed, the beats are reduced to 2 per second. Determine the frequency of P, if that of Q is 250 Hz.*

Solution : There are four beats between P and Q , therefore the possible frequencies of P are 246 or 254 (that is 250 $\pm$ 4) Hz.

When the prong of P is filed, its frequency becomes greater than the original frequency.

If we assume that the original frequency of P is 254, then on filing its frequency will be greater than 254. The beats between P and Q will be more than 4. But it is given that the beats are reduced to 2, therefore, 254 is not possible.

Therefore, the required frequency must be 246 Hz.

(This is true, because on filing the frequency may increase to 248, giving 2 beats with Q of frequency 250 Hz.)

Illustration 36. *Two tuning forks A and B sounded together give 8 beats per second. With an air resonance tube closed at one end, the two forks give resonances when the two air columns are 32cm and 33cm respectively. Calculate the frequencies of forks.*

Solution: Let the frequency of the first fork be f_1 and that of second be f_2 .

We then have,

$f_1 = \frac{v}{4 \times 32}$ and $f_2 = \frac{v}{4 \times 33}$

We also see that $f_1 > f_2$

$\therefore \quad f_1 - f_2 = 8$ (1)

and $\frac{f_1}{f_2} = \frac{33}{32}$ (2)

Solving (1) and (2) , we get

$f_1 = 264$ Hz

and $f_2 = 256$ Hz

Illustration 37. *In an experiment, it is observed that a tuning fork and a sonometer wire gave 5 beats per second both when the length of wire was 1m and 1.05 m. Calculate the frequency of the fork.*

Solution : Let the frequency of the fork by ν. At the smaller length of the sonometer wire (ℓ_1 = 1m), the frequency of the wire must be higher and vice versa.

$\nu_1 = \nu + 5$

$\nu_2 = \nu - 5$

$$\frac{\nu_1}{\nu_2} = \frac{\ell_2}{\ell_1}$$

$$\frac{\nu + 5}{\nu - 5} = \frac{1.05}{1.00}$$

on solving, ν = 205 Hz.

Exercise 7. Can beats be observed in two light sources of nearly equal frequencies ?

Doppler Effect

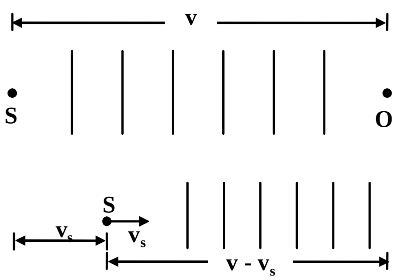
The apparent shift in frequency of the wave motion when the source of sound or light moves with respect to the observer, is called Doppler Effect.

Calculation of apparent frequency

Suppose v is the velocity of sound in air, v_s is the velocity of the source of sound(s), v_o is the velocity of the observer (O), and f is the frequency of the source.

(i) Source moves towards stationary observer.

If the source were stationary the f waves sent out in one second towards the observer O would occupy a distance v, and the wavelength would be v/f. If S moves with a velocity v_s towards O, the f waves sent out occupy a distance (v - v_s) because S has moved a distance v_s towards O in 1s. So the apparent wavelength would be

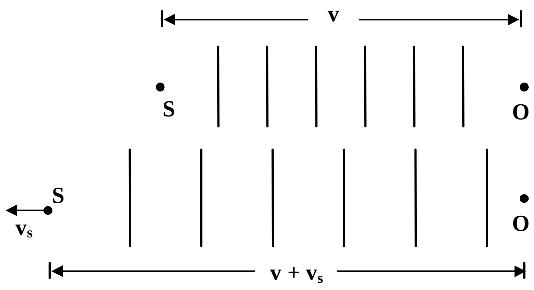


$$\lambda' = \frac{v - v_s}{f}$$

Thus, apparent frequency $f' = \dfrac{\text{Velocity of sound relative to O}}{\text{Wavelength of wave reaching O}}$

$$f' = \frac{v}{\lambda'} = f\left(\frac{v}{v - v_s}\right)$$

(ii) Source moves away from stationary observer.



Now, apparent wavelength

$$\lambda' = \frac{v + v_s}{f}$$

$\therefore$ Apparent frequency

$$f' = v/\lambda'$$

or $$f' = f\left(\frac{v}{v + v_s}\right)$$

(iii) Observer moves towards stationary source.

$$f' = \frac{\text{velocity of sound relative to O}}{\text{wavelength of wave reaching O}}$$

here, velocity of sound relative to O = $v + v_O$

and wavelength of waves reaching O = v/f

$\therefore$ $$f' = \frac{v + v_o}{v/f} = f\left(\frac{v + v_o}{v}\right)$$

(iv) Observer moves away from the stationary source.

$$f' = \frac{v - v_o}{v/f} = f\left(\frac{v - v_o}{v}\right)$$

(v) Source and observer both moves toward each other.

$$f' = \frac{v + v_o}{\frac{v - v_s}{f}} = f\left(\frac{v + v_o}{v - v_s}\right)$$

(vi) Both moves away from each other.

$$f' = f\left(\frac{v - v_o}{v + v_s}\right)$$

(vii) Source moves towards observer but observer moves away from source

$$f = f'\left(\frac{v - v_o}{v - v_s}\right)$$

(viii) Source moves away from observer but observer moves towards source

$$f = f'\left(\frac{v + v_o}{v + v_s}\right)$$

Note : The velocities are with respect to the medium.

Illustration 38. *The siren of a police can emit a pure note at a frequency of 1125 Hz. Find the frequency that one can perceive in the vehicle under the following circumstances*
(a) one's car is at rest; police car moving towards listener at 29 m/s.
(b) Police car at rest, listener's car is moving towards police at 29 m/s.
(c) Both police and the listener's are moving towards each other at 14.5 m/s.
(d) Listener's car moving at 9 m/s white police car is chasing with a speed of 38 m/s.

Solution: (a) Here, $v_0 = 0$ m/s, $v_s = 29$ m/s

$$f' = f\left(\frac{v}{v - v_s}\right) = 1125\left(\frac{343}{343 - 29}\right) = 1229\text{Hz}.$$

(b) $v_s = 0$ m/s, $v_0 = 29$ m/s

$$f' = f\left(\frac{v + v_0}{v}\right) = 1125 \times \left(\frac{343 + 29}{343}\right) = 1220\,\text{Hz}$$

(c) v_0 = 14.5 m/s, v_s = 14.5 m/s.

$$f' = f\left(\frac{v + v_0}{v - v_s}\right) = 1125 \times \left(\frac{343 + 14.5}{343 - 14.5}\right) = 1224\,\text{Hz}$$

(d) v_0 = 9 m/s, v_s = 38 m/s

$$f' = f\left(\frac{v - v_0}{v - v_s}\right) = 1125 \times \left(\frac{343 - 9}{343 - 38}\right) = 1232\,\text{Hz}.$$

Illustration 39. *A siren emitting a sound of frequency 1000 Hz moves away from you towards a cliff at a speed of 10 m/s.*
(a) What is the frequency of the sound you hear coming directly from the siren.
(b) What is the frequecny of sound you hear reflected off the cliff. Speed of sound in air is 330 m/s.

Solution: **(a)** Sound heard directly.

$$f_1 = f_0\left(\frac{v}{v + v_s}\right);\ V_s = 10 \text{ m/s.}$$

$$\therefore \quad f_1 = \left(\frac{330}{330 + 10}\right) \times 1000$$

$$= \frac{33}{34} \times 1000\text{Hz}$$

(b) The frequency of the reflected sound is given by

$$f_2 = f_0\left(\frac{v}{v - v_s}\right)$$

$$\therefore \quad f_2 = \left(\frac{330}{330 - 10}\right) \times 1000 = \frac{33}{32} \times 1000 \text{ Hz}$$

Illustration 40. *A stationary source emits a sound towards a wall moving towards it with a velocity u. Speed of sound in air = v. Find the fractional change in wavelength of the sound sent and the reflected sound.*

Solution : Let the frequency of the sound sent by the source be f_o.
Frequency of sound as observed by the wall is

$$\therefore \quad f' = \left(\frac{v + u}{v}\right) f_o$$

Frequency of the sound reflected by the wall is equal that received by it.
Now, the frequency f′′ received by the source is

$$f'' = \left(\frac{v}{v - u}\right) f'$$

$$f'' = \left(\frac{v}{v - u} \times \frac{v + u}{v}\right) f_o$$

$$f'' = \left(\frac{v + u}{v - u}\right) f_o$$

Initial wavelength $\lambda_i = \dfrac{v}{f_o}$

Final wavelength $\lambda_f = \dfrac{v}{f''} = \dfrac{v}{f_o}\left(\dfrac{v-u}{v+u}\right)$

Fractional change in wavelength = $\dfrac{\Delta\lambda}{\lambda_i} = \dfrac{2u}{v+u}$.

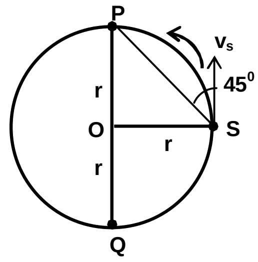
Illustration 41. *Two cars each moving with a velocity of 216 km/h, cross each other in the opposite directions. One of the cars gives a whistle whose frequency is 810 Hz. What will be the apparent frequency for passengers sitting in the other car before crossing? Given speed of sound = 330 m/s.*

Solution : v_s = 216 km/h = 216 × $\dfrac{5}{18}$ m/s = 60 m/s.

Similarly, v_0 = 216 km/hr = 60 m/s

Apparent frequency $\nu' = \left(\dfrac{v+v_0}{v-v_s}\right)\nu = \left(\dfrac{330+60}{330-60}\right)\times 810 = \dfrac{390}{270}\times 810$ = 1170 Hz.

Illustration 42. *A small sources of sound moves in a circle of radius r with constant speed v_s. Let the frequency of the source is 450 Hz. For the indicated positions P, O, Q of three different observers, find the frequency heard by the observers. Take the position of source as 'S' and velocity of source is $v_s = v/4\sqrt{2}$, where v is the velocity of sound of in air.*



Solution : $n' = n\left\{\dfrac{v}{v - v_s\cos\theta}\right\}$

(a) at P, $\theta = \pi/4$, $n_p = n\left[\dfrac{v}{v - v_s/\sqrt{2}}\right]$

$= \dfrac{450\times v}{v - v/8} \quad \Rightarrow \qquad 450 \times 8/7 = 514.28$ Hz.

(b) at 0, $\theta = \pi/2$, $n_0 = n = 450$ Hz

(c) at s, $\theta = 3\pi/4$, $n_Q = n\left[\dfrac{v}{v + (v_s/\sqrt{2})}\right]$

$= 450\left[\dfrac{v}{v + v/8}\right] = 450 \times \dfrac{8}{9}$

= 400 Hz.

Illustration 43. *A train approaching a hill at a speed of 40 km/hr sounds a whistle of frequency 580 Hz. When it is at a distance of 1 km from a hill, a wind with a speed of 40 km/hr is blowing in the direction of motion of the train find*
(a) the frequency of the whistle as heard by an observer on the hill.
(b) the distance from the hill at which the echo from the hill is heard by the driver and its frequency. *(Velocity of sound in air = 1200 km/hr.)*

Solution : $f' = \left(\frac{v+\omega}{v+\omega-v_s}\right)f$

$= \left(\frac{1200+40}{1200+40-40}\right)580$

$\approx$ 599 Hz.

For echo to be heard by the driver, the source is to be considered at the hill having frequency 599 Hz.

$f'' = \left(\frac{1200-40+40}{1200-40}\right)599$ = 619.65 Hz.

t_1 (wave to reach the hill) = $\frac{1}{1200+40} = \frac{1}{1240}$ hr.s

in the above duration train moves = $\frac{40}{1240} = \frac{1}{31}$ km.

Now the distance between train and hill = 1 - $\frac{1}{31}$ = $\frac{30}{31}$ km

After this instant echo will be heard

40 t + (1200 – 40)t = 30/31

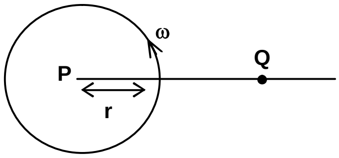
t = $\frac{1}{1240}$ hrs.

The distance travelled by the train in this time

= $40 \times \frac{1}{1240} = \frac{1}{31}$ km

Distance from the hill = $1 - \frac{2}{31}$ = 0.935 km.

Illustration 44. *A and B source is traveling on a circular path with angular speed ω as shown in figure. The source has a natural frequency to the maximum frequency observed by observer is: [v = speed of sound in air]*



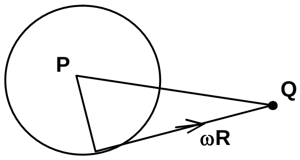
(A) $\frac{f_0 v}{V-\omega R}$ *(B)* $\frac{f_0 v}{v+\omega R}$

(C) $\frac{f_0(v+\omega R)}{v}$ *(D) None of these*

Solution : (D)

$f' = f_0\left(\frac{v \pm v_0}{v \pm v_S}\right)$

$\Rightarrow f' = f_0\left(\frac{v}{v-\omega R}\right)$



Exercise 8.

(i) ***When a source of sound passes us, wheather it be a car horn or a train whistle, the pitch we hear goes from high to low. Explain why.***

(ii) ***Is there a Dopplers effect for sound when the observer or the sources move at right angles to the line joining them.***

SUMMARY

1. Equation of Travelling Wave $y = f(x \pm vt)$

2. A harmonic wave can be represented as

   $y = A\sin\frac{2\pi}{\lambda}(x \pm vt)$ (Sinusoidal wave)

   or $y = A\sin(kx \pm \omega t)$

   The negative sign refers to the wave travelling along the positive x - axis, and versa.

   $k = \frac{2\pi}{\lambda}$ = angular wave number

   $\omega = \frac{2\pi}{T} = 2\pi f$ = angular frequency

   $v = \lambda f = \frac{\omega}{k}$ = wave velocity

3. In sound waves there is a phase gap of $\pi/2$ between the displacement and pressure waves, i.e at displacement minima there is pressure maxima and vice versa.

4. Transverse wave in a taut string

   $v = \sqrt{\frac{T}{\mu}}$ T = tension , μ = mass per unit length.

5. Longitudinal wave in a solid; $v = \sqrt{\frac{Y}{\rho}}$ Y = Young's Modulus , ρ= density

6. Longitudinal wave in a fluid; $v = \sqrt{\frac{K}{\rho}}$ [K = Bulk Modulus]

7. Energy density = $\frac{1}{2}(\rho S dx)(A\omega)^2 / S.dx = \frac{\rho}{2}(A\omega)^2 = 2\pi^2 f^2 A^2 \rho \left[J/m^3\right]$

8. Energy per unit length = $PS\left(2\pi^2 f^2 A^2 \rho\right)$

9. Power transmitted = $PS\left(2\pi^2 f^2 A^2 \rho\right)V$ (watt = J/s)

10. Intensity = $\frac{\text{Power}}{\text{unit area}} = 2\pi^2 f^2 A^2 \rho V$

11. A standing wave is produced by the superposition of two identical waves travelling in opposite directions viz., $y_1 = a \sin[kx + \omega t]$ and $y_2 = a\sin(\omega t - kx)$, gives the standing wave, $y = 2a \sin \omega t \cos kx$

12. The points having the maximum amplitude are those where 2a cos kx has a maximum value of 2a, these are at the position,

    $kx = 0, \pi, 2\pi, \ldots$

    i.e. $x = 0, \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots$

    These points are called antinodes

13. The amplitudes reaches a minimum value of zero at the positions

where $kx = \pi/2, 3\pi/2, 5\pi/2, \ldots$

or $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots$

These points are called nodes.

14. Energy is not transported along the string to the right or to the left, because energy cannot flow through the nodal points in the string which are permanently at rest.

15. If two or more waves of slightly different frequencies are superimposed, the intensity of the resulting wave has alternate maxima and minima. The number of maximas in one second is called beat frequency.
Beat frequency = $|f_1 - f_2|$

16. Doppler Effect: The apparent shift in frequency of the wave motion when the source of sound moves with respect to the observer, is called **Doppler Effect**.
Apparent frequency, $n' = \left(\frac{c + v_0}{c - v_s}\right) \times n$, where symbols represent their usual meaning.

MISCELLANEOUS EXERCISE

1. Speed of sound in air is 332 m/s at NTP what will be its value in hydrogen at NTP if density of hydrogen at NTP if density of hydrogen at NTP is $1/16^{th}$ that of air ?

2. A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far apart are two points 60^0 out of phase ?

3. Calculate the speed of propagation of a wave represented by y = 0.5 sin π (0.01 x – 3t), where y and x are in metre and t in sec.

4. The equation for the transverse wave on a string is $y = 4 \sin 2\pi \left(\frac{t}{0.05} - \frac{x}{50}\right)$ with length expressed in cm. and time in second. Calculate the wave velocity and maximum particle velocity.

5. A guitar string is 90 cm long and has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 186 Hz ?

6. A sound wave in fluid medium is reflected at a barrier so that a standing wave is formed. the distance between the nodes is 3.84 and the speed of propagation is 1520 m/s. Find the frequency.

7. A certain violin string is 30 cm long between its fixed ends and has a mass of 2.0 g. The string sounds an A note (440 Hz) when played without fingering. Where must one put one's finger to play a C (528Hz) ?

8. A tunning fork of unknown frequency makes three beats per sec with a standard fork of frequency 384 Hz. The beat frequency decreases when a small pieces of wax is put on a prong of the first fork. What is the frequency of this fork ?

9. One is given four tuning forks. The fork with the lowest frequency vibrates at 500 Hz. By using two tuning fork at a time, the following beat frequencies are heard : 1, 2,3,5, 7 and 8 Hz. What are the possible frequencies of the other three tuning forks ?

10. A whistle of frequency 538 Hz moves in a circle of radius 71.2 cm at an angular speed of 14.7 rad/s. What are (a) the lowest and (b) the highest frequencies heard by a listener a long distance away at rest with respect to the centre of the circle ?

ANSWERS TO MISCELLANEOUS EXERCISE

1. 1328 m/s.
2. 0.12
3. 300 m/s
4. 10 m/s. 5.03 m/s
5. 60 cm
6. 197.92 Hz
7. 5.0 cm from one end
8. 387 Hz.
9. 505, 507, 508 Hz or 501, 503, 508 Hz.
10. (a) 522 Hz. (b) 554 Hz.

SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob1. *A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 1275 Hz source ?*

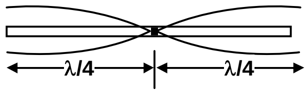
Sol. Here, L= 20 cm = 0.2 m
f_n = 430 Hz
v = 340 m/s.
frequency of nth normal mode of vibration of closed pipe is

$$f_n = (2n-1)\frac{v}{4L}$$

$$\Rightarrow 1275 = (2n-1)\frac{340}{4\times 0.2} \qquad \Rightarrow n = 2.$$

$\therefore$ third harmonic or first overtone.

Prob 2. *A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz. What is the speed of sound in steel?*



Sol. Here, $\frac{\lambda}{4}+\frac{\lambda}{4} = 100\text{ cm} = 1\text{ m}$

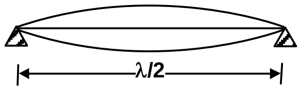
$\Rightarrow \frac{\lambda}{2} = 1\text{ m}$

$\Rightarrow \lambda = 2$ m

Here, ν = 2.53 kHz = 2530 Hz.

$\therefore$ Speed of sound in steel, v = $\nu\lambda$ = 2530 Hz $\times$ 2 m
= 5060 m/s

Prob 3. *A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5 $\times$ 10^{-2} kg and its linear mass density is 4.0 $\times$ 10^{-2} kgm^{-1}, What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?*



Sol.

$$L = \frac{m}{\mu} = \frac{3.5\times10^{-2}\text{kg}}{4.0\times10^{-2}\text{kg/m}} = \frac{3.5}{4}\text{m} = \frac{350}{4}\text{cm} = 87.5\text{ cm}$$

Here, $L = \frac{\lambda}{2} \Rightarrow \lambda$ = 2L = 175 cm = 1.75 m

(a) Speed of transverse wave on string, v = $\nu\lambda$
= (45 Hz) (1.75 m) = 78.75 m/s

(b) $\because$ Speed of transverse wave on string, $v = \sqrt{\dfrac{F}{\mu}}$

$\Rightarrow$ $F = \mu v^2$
$= (4.0 \times 10^{-2}$ kg/m$)$ $(78.75$ m/s$)^2$
$= 4 \times 78.75 \times 78.75$ N
$\approx$ 2.5×10^4 N
$\approx$ 25 kN.

Prob 4. *A hospital used an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 km s^{-1}? The operating frequency of the scanner is 4.2 MHz.*

Sol. Here frequency, ν = 4.2 MHz
Speed of sound, v = 1.7 kms^{-1}
$\because$ $v = \nu\lambda = \Rightarrow \lambda = \dfrac{v}{\nu}$.
$\therefore$ Wavelength of sound in tumour tissue, $\lambda = \dfrac{1.7 \times 10^3 \text{ms}^{-1}}{4.2 \times 10^6 \text{Hz}}$
$= \dfrac{1.7}{4.2} \times 10^{-3}\ \text{m} = 0.40\ \text{mm}$

Prob 5. *A sonar system fixed in a submarine operates at a frequency 40.0 kHz. An energy submarine moves towards the SONAR with a speed of 360 km/h. what is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 ms^{-1}.*

Sol. The frequency of sound reflected by the energy equals to the frequency of sound received by the same submarine.
The frequency of the sound wave received by the submarine, ν
$= \dfrac{v + u_0}{v} \nu_0$
$= \dfrac{1450 + 100}{1450} \times 40\ \text{kHz}$
= 42.75 kHz
$\left[\because\ u_0 = 360\ \text{km/h} = \dfrac{360 \times 1000}{60 \times 60} \text{m/s} = 100\ \text{m/s}\right]$

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Prob 6. *Show that if the room temperature changes by a small amount from T to T + ΔT, the fundamental frequency of an organ pipe changes from ν to $\nu + \Delta\nu$, where*
$\dfrac{\Delta\nu}{\nu} = \dfrac{1}{2}\dfrac{\Delta T}{T}$.

Sol. Frequency of an organ pipe,
For a closed organ pipe, $\nu = \dfrac{\left(n + \dfrac{1}{2}\right)v}{2L}$, $n \in \mathbf{I}$

and for a open organ pipe, $\nu = \frac{nv}{2L}$, $n \in I$

$\therefore$ ν = kv, where k is a constant.

= k′v, for the fundamental frequency, where k′ is a constant.

$$\nu = k'\sqrt{\frac{P\gamma}{\rho}}$$

$\because$ PV = nRT

$\Rightarrow$ $PV = \frac{m}{M}RT$

$\Rightarrow$ PM = ρRT

$\Rightarrow$ $\frac{P}{\rho} = \frac{RT}{M}$

Using in (ii);

$$\nu = k'\sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow \quad \nu = c\sqrt{T} \qquad \text{...(i)},$$

where c is another constant.

$$\therefore \quad \nu + \Delta\nu = c\sqrt{T + \Delta T}$$

$$= c(T + \Delta T)^{1/2} = c\sqrt{T}\left(1 + \frac{\Delta T}{T}\right)^{1/2}$$

$$\approx c\sqrt{T}\left(1 + \frac{1}{2}\frac{\Delta T}{T}\right) \qquad \text{...(ii)}$$

subtracting (i) from (ii);

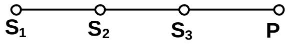
$$\Delta\nu = c\sqrt{T} \cdot \frac{\Delta T}{2T} = c\frac{\Delta T}{2\sqrt{T}} \qquad \text{...(iii)}$$

dividing (iii) by (i), we get;

$$\frac{\Delta\nu}{\nu} = \frac{c\frac{\Delta T}{2\sqrt{T}}}{c\sqrt{T}}$$

$$\Rightarrow \quad \frac{\Delta\nu}{\nu} = \frac{1}{2} \cdot \frac{\Delta T}{T}$$

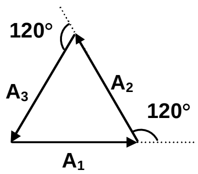
Prob 7. *Three sources of sound S_1, S_2 and S_3 of equal intensity are placed in a straight line with $S_1S_2 = S_2S_3$. At a point P, away from the sources, the wave coming from S_2 is 120° ahead in phase of that from S_1. Also, the wave coming from S_3 is 120° ahead of that from S_2. What would be the resultant intensity of sound at P? What is minimum value of S_1S_2 and S_2S_3 if the wavelength is λ?*



Sol.

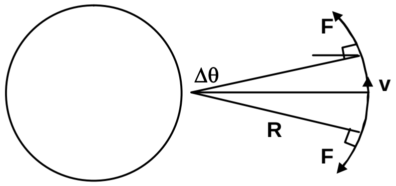
$$\frac{S_1S_2}{\lambda}2\pi = \frac{2\pi}{3} \Rightarrow S_1S_2 = \frac{\lambda}{3},$$

$$\frac{S_2S_3}{\lambda}2\pi = \frac{2\pi}{3} \Rightarrow S_2S_3 = \frac{\lambda}{3}.$$



The amplitude of the resulting sound wave is zero.

Prob 8. A circular loop of string rotates about its axis on a frictionless horizontal plane at a uniform rate, so that the tangential speed of any particle of the string is v. If a small transverse disturbance is produced at a point of the loop, with what speed (relative to the string) will this disturbance travel on the string?



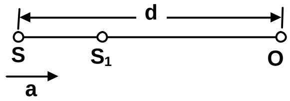
Sol. $2\left(F\sin\frac{\Delta\theta}{2}\right) = \mu(R\Delta\theta)\frac{v^2}{R}$

$\Rightarrow \quad 2F\frac{\Delta\theta}{2} = \mu\Delta\theta \cdot v^2$

$\Rightarrow \quad v = \sqrt{\frac{F}{\mu}}$ i.e. wave speed with respect to the string.

$\Rightarrow$ wave speed with respect to string is v.

Prob 9. *A source emitting sound of frequency ν is placed at a large distance from an observer. The source starts moving towards the observer with a uniform acceleration 'a'. Find the frequency heard by the observer corresponding to the wave emitted, just after the source starts. The speed of sound in the medium is v.*



Sol. Let a compression wave pulse is generated by source S at t = 0.
Now, the next compression wave pulse is generated by source at time t = T.
The distance travelled by the source in this time period

$$SS_1 = \frac{1}{2}aT^2$$

The time at which the first compression wave reaches to observer O,

$$t_1 = \frac{d}{v}.$$

The time at which the second compression wave reaches to observer O,

$$t_2 = T + \frac{d - SS_1}{v} = T + \frac{d - \frac{1}{2}aT^2}{v}$$

Thus, the apparent time-period to the observer O,

$$T' = t_2 - t_1$$

$$= \left(T + \frac{d - \frac{1}{2}aT^2}{v}\right) - \frac{d}{v} = T - \frac{aT^2}{2v} = \frac{2vT - aT^2}{2v}$$

Hence, the apparent frequency to the observer O,

$$\nu' = \frac{1}{T'} = \frac{2v}{2vT - aT^2} = \frac{2v}{2v\frac{1}{\nu} - a\frac{1}{\nu^2}} \qquad \left[\because \quad \nu = \frac{1}{T}\right]$$

$$= \frac{2v\nu^2}{2v\nu - a}$$

Prob10. *Wavelengths of two notes in air are* $\left(\frac{90}{175}\right)$*m and* $\left(\frac{90}{173}\right)$ *m . Each note produces four beats per second with a third note of a fixed frequency. Calculate the velocity of sound in air.*

Sol. Given $\lambda_1 = \frac{90}{175}$ m and $\lambda_2 = \frac{90}{173}$ m.

Let f_1 and f_2 be the corresponding frequencies and v be the velocity of sound in air.

$v = \lambda_1 f_1$ and $v = \lambda_2 f_2$

$\therefore \quad f_1 = \frac{v}{\lambda_1}$ and $f_2 = \frac{v}{\lambda_2}$

Since $\lambda_2 > \lambda_1 \;\therefore\; f_1 > f_2$

Let f be the frequency of the third note.

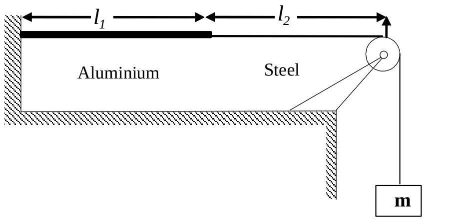
$\therefore \quad f_1 - f = 4$ and $f - f_2 = 4$

$\therefore \quad f_1 - f_2 = 8 \qquad \therefore \quad \frac{v}{\lambda_1} - \frac{v}{\lambda_2} = 8$

$v\left[\frac{175}{90} - \frac{173}{90}\right] = 8 \qquad \therefore \quad v\left[\frac{2}{90}\right] = 8$

$\therefore \quad v = 360\ ms^{-1}$

Prob 11. *An Aluminum thin wire of length* $l_1 = 60.0$ *cm and of cross-sectional area* 1.00×10^{-2} *cm*^{2} *is connected to a steel wire of the same cross sectional area. The compound wire , loaded with a block m of mass 10.0 kg, is arranged as shown, so that the distance* l_2 *from the joint to the supporting pulley is 86.6cm*



Transverse waves are set up in the wire by using an external source of variable frequency

(a) Find the lowest frequency of excitation for which standing waves are observed such that the joint in the wire is a node.

(b) What is total number of nodes observed at this frequency excluding the two at the ends of the wire ?

Density of aluminum is 2.6 g/cm^3 ; *Density of steel is 7.8 g/cm*^{3}

Sol. The distance between two nodes is equal to $\lambda/2$. The whole wire vibrates with the frequency of the turning fork. But as the velocity of the wave in each part of the wire is $\sqrt{T/\mu}$ and the mass per unit length in both the wires are different the velocity of the wave in the two parts of the string is different. Thus the wavelength is different in both parts of the string.

Let there be p_1 loops in the aluminium part of the wire and p_2 loops in the steel part of the wire, the joint being a node. In such a case the allowed frequencies of the Aluminium part of the wire is

$$f = \frac{p_1}{2l_1}\sqrt{\frac{T}{\mu_A}} \qquad (1)$$

and for steel,

$$f = \frac{p_2}{2l_2}\sqrt{\frac{T}{\mu_{steel}}} \qquad (2)$$

$l_1 = 60.0$ cm.

Area of cross section = 1.00×10^{-2} cm^2. Density of Aluminium=2.6 gm/cm^3

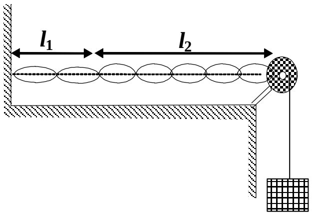
$\mu_{A1} = (2.6)(1 \times 10^{-2}) = 0.026$ g/cm

$\mu_{steel} = (7.8)(1 \times 10^{-2}) = 0.078$ g/cm.

Equating (1) and (2) we get,

$$\frac{p_1}{2\times 60}\sqrt{\frac{T}{0.026}} = \frac{p_2}{2\times 86.6}\sqrt{\frac{T}{0.078}}$$

$$\therefore \frac{p_1}{p_2} = \frac{60}{86.6}\sqrt{\frac{0.026}{0.078}} = \frac{2}{5}$$



$\therefore$ The lowest frequency of excitation causes $p_1 = 2$, $p_2 = 5$.

$\therefore$ The total number of loops = 5 + 2 = 7

and the frequency

$$\frac{2\sqrt{981\times 10^4}}{2\times 60\sqrt{0.026}} = 323.74\text{Hz}$$

and the wire looks like as shown in the figure.

Prob 12 . *Two wires are fixed on a sonometer. Their tensions are in the ratio 8 : 1, their lengths are in the ratio 36 : 35, the diameters are in the ratio 4 : 1 and densities are in the ratio 1 : 2. Find the frequencies of the beats produced if the note of the higher pitch has a frequency of 360 per second.*

Sol. Given: $\frac{T_1}{T_2} = \frac{8}{1}$, $\quad \frac{L_1}{L_2} = \frac{36}{35} \Rightarrow \quad \frac{D_1}{D_2} = \frac{4}{1}$, $\quad \frac{\rho_1}{\rho_2} = \frac{1}{2}$

Let μ_1 and μ_2 be the linear densities.

$\therefore \quad \mu_1 = \pi \times \frac{D_1^2}{4} \times \rho_1 \quad$ and $\quad \mu_2 = \pi \times \frac{D_2^2}{4} \times \rho_2$

$$\therefore \quad \frac{\mu_1}{\mu_2} = \left(\frac{D_1}{D_2}\right)^2 \times \frac{\rho_1}{\rho_2} = \left(\frac{4}{1}\right)^2 \times \frac{1}{2} = \frac{8}{1}$$

$$\therefore \quad \frac{f_1}{f_2} = \frac{L_2}{L_1} \times \sqrt{\frac{T_1}{T_2} \times \frac{\mu_2}{\mu_1}} = \frac{35}{36}\sqrt{\frac{8}{1} \times \frac{1}{8}} = \frac{35}{36}$$

$f_2 > f_1$, we have $f_2 = 360$ $\quad \therefore \quad f_1 = 350$

Beats = $f_2 - f_1 = 10$ per second.

Prob 13 . *A column of air and a tuning fork produces 4 beats per second. When sounding together, the tuning fork gives the lower note. The temperature of air is 15° C. When* *the temperature falls to 10°C, the two produce 3 beats per second. Find the frequency of the tuning fork.*

Sol. The frequency of the air column is given by

$f = v/\lambda$ Where v is the velocity of sound in air and λ is the wavelength.

But $v = \sqrt{\frac{\gamma RT}{M}}$, thus it is dependent of temperature.

Thus $f \propto \sqrt{T}$.

Let the frequency of the tuning fork be f.

Thus the frequency of the air column at 15° C = f + 4

Thus the frequency of the air column at 10° C = f + 3.

As the frequency decreases with temperature

$$\Rightarrow \quad \frac{f+4}{f+3} = \sqrt{\frac{288}{283}} \quad \Rightarrow \quad \frac{f+4}{f+3} = 1.00879$$

$$\Rightarrow \quad f + 4 = 1.00879f + 3.02638$$

$$\Rightarrow \quad 8.79 \times 10^{-3} f = 0.97362$$

$$f = 110.76\,\text{Hz}.$$

Prob 14. *An air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air 330 m/s. End corrections may be neglected. Let p_0 denote the mean pressure at any point in the pipe and ΔP_0 the maximum amplitudes of pressure variation*

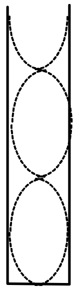
(a) Find the length L of the air column

(b) What is the amplitude of pressure variation at the middle of the column?

(c) What are the maximum and minimum pressures at the open end of at the pipe?

(d) What are the maximum and minimum pressures at the closed end of the pipe?

Sol. (a) Fundamental frequency of the closed organ pipe $= \frac{V}{4\ell}$



In closed organ pipe only odd harmonics are present

$\therefore$ Second over tone $= 5\frac{V}{4\ell}$

Given $5\frac{V}{4\ell} = 440$

$$\ell = \frac{5 \times 330}{4 \times 440} = \frac{15}{16}\text{m} = 93.75 \text{ cm}$$

(b) The equation of variation of pressure amplitude at any distance x from the node

$\Delta P = \Delta P_0 \cos kx$

Pressure variation is maximum at a node and minimum (zero) at antinode. Distance of the center from the close end

$$= \frac{\lambda}{2} + \frac{\lambda}{8} \Rightarrow \Delta P = \Delta P_0 \cos\frac{2\pi}{\lambda} \times \frac{5\lambda}{8} = \frac{\Delta P_0}{\sqrt{2}} \qquad \left(\because k = \frac{2\pi}{\lambda}\right)$$

(c) At antinode the pressure variation is minimum (zero), therefore antinode pressure remains equal to P_o (always)

$\therefore$ At antinode $P_{max} = P_{min} = P_0$

(d) At node, pressure variation is maximum equals to ΔP_0.

$P_{max} = P_0 + \Delta P_0$ and $P_{min} = P_0 - \Delta P_0$

Prob 15 . *Two gases of different densities but same atomicity are mixed in proportions V_1 and V_2 by volume. If v_1 and v_2 be the velocity of sound in them, respectively, find the velocity of sound in the mixture.*

Sol. For same atomicity $v_1 = \sqrt{\frac{\gamma p}{d_1}}$, $v_2 = \sqrt{\frac{\gamma p}{d_2}}$ . . . (i)

and speed in the mixture $v_m = \sqrt{\frac{\gamma p}{d_m}}$ . . . (ii)

where $d_m = \frac{v_1 d_1 + v_2 d_2}{v_1 + v_2}$ . . . (iii)

$$\frac{d_m}{d_1} = \frac{v_1 + v_2 d_2 / d_1}{v_1 + v_2}$$

from (i) $\frac{v_1}{v_2} = \sqrt{\frac{d_2}{d_1}} \Rightarrow \frac{d_2}{d_1} = \frac{v_1^2}{v_2^2}$ . . . (iv)

from (i) and (ii)

$$\frac{v_m}{v_1} = \sqrt{\frac{d_1}{d_m}} \Rightarrow \frac{v_m}{v_1} = \sqrt{\frac{v_1 + v_2}{v_1 + v_2(v_1^2 / v_2^2)}}$$

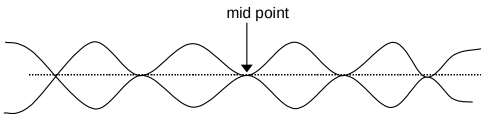
$\Rightarrow v_m = \sqrt{v_1 v_2}$.

Prob 16 . *A metallic rod of length 1m is rigidly clamped at its midpoint. Longitudinal stationary waves are setup in the rod in such a way that there are two nodes on either side of the mid point. The amplitude of antinodes is 2×10^{-6} m. Write the equation of motion at a point 2 cm from the mid point and those of constituent waves in the rod. ($y= 2 \times 10^{11}$ N/m^2), ρ=8000 kg/m^3)*

Sol. As the rod is clamped at mid point with two nodes on either side

$\Rightarrow \ell = 2 \times \lambda + \frac{\lambda}{2} = \frac{5\lambda}{2}$

$\Rightarrow \lambda = \frac{2\ell}{5} = 0.4$ m



Speed of the wave produced $= \sqrt{\frac{y}{d}} = \sqrt{\frac{2\times10^{11}}{8000}}$ = 5000 m/s . . . (i)

Frequency of the wave $= \frac{v}{\lambda} = \frac{5000}{0.4}$ = 125000 Hz.

Assuming left end of rod as origin, the equation of stationary waves

$$y = A\cos\frac{2\pi}{\lambda}x\sin 2\pi ft$$

The amplitude at any instant is given by

$$R = 2A\cos\frac{2\pi}{\lambda}x = 2A\cos\frac{2\pi x}{0.4} \quad \text{. . . (ii)}$$

At one end i.e. at x = 0 and t = 0

We have $2A = 2 \times 10^{-6}$ m

$\Rightarrow$ $y = 2 \times 10^{-6}\cos 5\pi x \sin 2500\,\pi t$ . . . (iii)

At a point 2cm from the mid point

$X = 0.5 + 0.02 = 0.52$ m

$\therefore$ $y = 2 \times 10^{-6}\cos 5\pi \times 0.52 \sin 25000\,t$

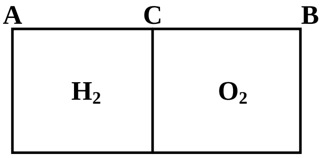
$= 2 \times 10^{-6}\cos 2.6\,\pi \sin 25000\,t$

Then constituent waves are

$y_1 = 1 \times 10^{-6}\sin(25000\,\pi t - 5\pi x)$

$y_2 = 1 \times 10^{-6}\sin(25000\,\pi t + 5\,\pi x)$

Prob 17. *AB is a cylinder of length 1.0 m filled with a thin flexible diaphragm C at the middle and two other thin flexible diaphragm A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of same frequency. What is the minimum frequency of these vibrations for which the diaphragm C is a node? Under the conditions of the experiment, the velocity of sound in hydrogen is 1100 m/s and in oxygen is 300m/s.*



Sol. Since C is node, the two parts AC and BC behave as closed pipes with the closed end of each at C, since diaphragms A and B are set into vibrations, the ends A and B must be antinodes.

The fundamental frequency of each pipe corresponds to just one node and one antinode. If n_1 and n_2 are the fundamental frequencies of gases in AC and BC respectively, we have

$$f_1 = \frac{v_1}{4L} = \frac{1100}{4 \times 0.5} = 550\,\text{Hz} \quad \text{and } f_2 = \frac{v_2}{4L} = \frac{300}{4 \times 0.5} = 150\text{ Hz}$$

Where v_1 and v_2 are the sound speeds in hydrogen and oxygen respectively.

These frequencies are not same. Hence the two gas columns are not vibrating in fundamental mode. We know that a closed pipe has only odd harmonics with frequencies, which are 3, 5, 7, 9, 11, etc times the fundamental frequency. To find out which harmonics of f_1 and f_2 have the same frequency . We notice that

$$\frac{f_1}{f_2} = \frac{550}{150} \quad \text{or} \quad 3f_1 = 11n_2$$

Thus, the third harmonic of f_1 and eleventh harmonic of f_2 have equal frequencies, similarly sixth harmonic of f_1 and 22^{nd} harmonic of f_2 have equal frequencies and so on.

$\therefore$ Common minimum frequency = $3n_1 = 3 \times 550 = 1650$ Hz.= $11n_2$

Prob 18. *The figure represents two snaps of a travelling wave on a string of mass per unit length μ = 0.25 kg/m. The first snap is taken at t = 0 and the second is taken at t =0.05 s. Determine the following*

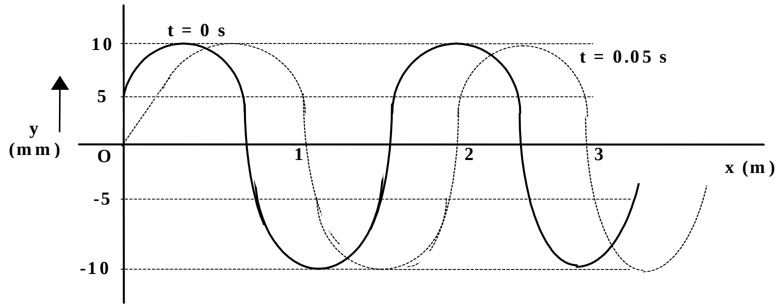
(a) the speed of the wave

(b) the wavelength and frequency of the wave

(c) the maximum speed of the particle

(d) the tension in the string

(e)the equation of the wave



Sol. The wave is travelling along the positive x -axis

$\therefore$ $y = A \sin[kx - \omega t + \phi]$

at $x = 0$, $y = A \sin[-\omega t + \phi]$

Also, at t = 0; $y = A\sin\phi = \frac{A}{2}$, or $\sin\phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{6}, \frac{5\pi}{6}, ...(1)$

and at t = 0.05 s; $y = A\sin\left(\frac{-\omega}{20} + \phi\right) = 0$

Since motion of the particle is downward, $\phi = 5\pi/6$

$\phi - \frac{\omega}{20} = 0 \Rightarrow \omega = \frac{50\pi}{3}$

$\therefore$ $f = \frac{\omega}{2\pi} = \frac{25}{3}$ Hz

Velocity of wave is, $V = \lambda f = 2\left(\frac{25}{3}\right) = \frac{50}{3}$ m/s

Maximum velocity of the particle is

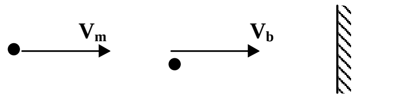
$V_{max} = \omega A = \left(\frac{50}{3}\pi\right)\left(10 \times 10^{-3}\right) = \frac{\pi}{6}$ m/s

Tension in the string is $T = \mu V^2 = (0.25)\left(\frac{50}{3}\right)^2 = \frac{625}{9}$ N

The equation of the wave is $y = 10 \sin\left[\pi x - \frac{50}{3}\pi t + \frac{5\pi}{6}\right]$ mm

Prob 19. *A band playing music at a frequency f is moving towards a wall at a speed v_b . A motorist is following the band with a speed v_m. If v is the speed of sound, obtain an expression for the beat frequency heard by the motorist.*

Sol.



Required beat frequency = $|f_1 - f_2|$

Where, f_1 = apparent frequency for the motorist corresponding to the signals directly coming to him from source, and f_2 = apparent frequency for the motorist corresponding to the signals coming to him after reflection.

Now, $f_1 = f\left[\dfrac{V+V_m}{V+V_b}\right]$ & $f_2 = f'\,\dfrac{V+V_m}{V}$

Where f ′ is the frequency at which signals from sources are incident on wall.

$$f' = f\left[\frac{V}{V-V_b}\right]$$

$$\Rightarrow\ f_2 = f\left[\frac{V}{V-V_b}\right]\left[\frac{V+V_m}{V}\right] = f\left[\frac{V+V_m}{V-V_b}\right]$$

Hence , the beat frequency = $|f_1 - f_2| = \dfrac{2V_b(V+V_m)f}{(V^2 - V_b^2)}$.

Objective:

Prob 1 . *An accurate and reliable audio oscillator is used to standardize a tuning fork marked as 512 Hz. When the oscillator reading is 514, two beats are heard per second. When the oscillator reading is 510, the beat frequency is 6 Hz. The frequency of the tuning fork is*

(A) 506 *(B)510*

(C) 516 *(D) 158*

Sol. It should be remembered that the oscillator reading is correct and the tuning fork frequency marked is wrong. When the oscillator reading is 514, two beats are heard. Hence the frequency of the tuning fork is $514 \pm 2 = 516$ or 512. When the oscillator reading is 510, the frequency of the tuning fork is $510 \pm 6 = 516$ or 504. The common value is 516. Hence the frequency is 516 Hz.

$\therefore$ (C)

Prob 2 . *A sound wave of wavelength λ travels towards the right horizontally with a velocity V. It strikes and reflects from a vertical plane surface, travelling at a speed v towards the left. The number of positive crests striking in a time interval of three seconds on the wall is*

(A) $3 (V + v) /\lambda$ *(B)$3(V - v)/\lambda$*

(C) $(V + v)/ 3\lambda$ *(D) $(V - v) /3\lambda$*

Sol. The relative velocity of sound waves with respect to the walls is V + v. Hence the apparent frequency of the waves striking the surface of the wall is $(V + v)/ \lambda$. The number of positive crests striking per second is the same as frequency. In three seconds the number is $[3 (V + v)]/ \lambda$.

$\therefore$ (A)

Prob 3 . *Two waves represented by $y_1 = 10 \text{ Sin } (2000 \pi t)$ and $y_2 = 10 \sin (2000 \pi t + \pi/2)$ are superposed at any point at a particular instant. The resultant amplitude is*

(A) 10 units *(B)20 units*

(C) 14.1 units *(D) zero*

Sol. The resultant amplitude A of two waves of amplitudes a_1 and a_2 and phase difference ϕ is $((a_1^2 + a_2^2 + 2a_1a_2 \cos\phi)^{1/2}$. Substituting $a_1 = 10$, $a_2 = 10$ and $\phi = 90^0$, we get A = 14.1.

$\therefore$ (C)

Prob 4. *When two simple harmonic motions of same periods, same amplitude, having phase of $3\pi/2$, and at right angles to each other are super imposed, the resultant wave form is a*

(A) circle *(B) parabola*

(C) ellipse *(D) figure of eight*

Sol. When two SHM of different amplitudes and same period are superposed, the resulting motion is in general elliptical. When the phase difference is 0 or 180^0, the motion will be a straight line. When the phase difference is 90 or 270^0, the motion will be an ellipse whose axes coincide with coordinate axes. If the amplitudes are the same and the phase difference is 90^0 or 270^0 , the motion will be circular.

$\therefore$ (A)

Prob 5 . *When two linear simple harmonic vibrations of the same frequency and amplitude are combined while acting on a particle at right angles, the resulting motion of the particle is circular when the phase difference between them is*

(A) π *(B) $\pi/4$*
(C) 2π *(D) $\pi/2$*

Sol. $\therefore$ (D)

Prob 6 . *A line source emits a cylindrical wave. If the medium absorbs no energy, the amplitude will vary with distance r from the source as proportional to*

(A) r^{-1} *(B) r^{-2}*
(C) $r^{-1/2}$ *(D) $r^{1/2}$*

Sol. The energy is inversely proportional to the square of distance. Hence the amplitude is inversely proportional to the distance.

$\therefore$ (C)

Prob 7 . *A transverse wave is described by the equation $y = y_0 \sin 2\pi (ft - x/a)$. The maximum particle velocity is equal to four times the wave velocity if a is equal to (a wavelength)*

(A) $\pi y_0 / 4$ *(B) $\pi y_0/2$*
(C) πy_0 *(D) $2\pi y_0$*

Sol. The maximum particle velocity of a SHM of amplitude y_0 and frequency f is $2\pi f Y_0$. The wave velocity is $f\lambda$. For $2\pi\, fy_0$ to be equal to $4f\lambda$, λ has to be $\pi y_0 / 2$

(Here $\lambda = a$).

$\therefore$ (B)

Prob 8 . *Inside a gas, sound transmission is possible for*

(A) longitudinal waves only
(B transverse waves only
(C) neither longitudinal waves nor transverse waves
(D) both longitudinal and transverse waves

Sol. Inside a gas, only the longitudinal mode of transmission is possible for sound waves.

$\therefore$ (A)

Prob 9. *A flat horizontal platform moves up and down in S.H.M. with an amplitude of 1 cm. A small object is placed on the platform. What is the maximum frequency the platform can have, if the object is not to separate from it during any part of the motion ?*

(A) $\frac{\sqrt{980}}{2\pi}$ *per second* *(B)* $\sqrt{980}/\sqrt{2\pi}$ *per second*

(C) 980 / 2π per second *(D) 2 π × 980 per second*

Sol. The maximum restoring force of S.H.M. is the weight of the object in the platform. If A is the amplitude, we have $m\omega^2A = mg$, where $\omega = 2\pi$ f. This solves to

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{a}} = \frac{1}{2\pi}\sqrt{980} \quad \therefore \text{(A)}$$

Prob 10. *The amplitude of a wave disturbance propagating in the positive x-direction is given by* $y = 1/(1+x^2)$ *at time t = 0 and by* $y = 1/[1+(x-1)^2$ *at t = 2 seconds, where x and y are in meters. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is*

(A) $1\ ms^{-1}$ *(B)* $0.5\ ms^{-1}$

(C) $1.5\ ms^{-1}$ *(D)* $2\ ms^{-1}$

Sol. Writing the general expression for y in terms of x as

$y = \frac{1}{1+(x-vt)^2}$ at t = 0, $y = 1/(1+x)^2$. At t = 2 s, $y = \frac{1}{1+[x-v(2)]^2}$

Comparing with the given equation we get 2v = 1 and v = 0.5 m/s.

$\therefore$ (B)

Prob 11. *Which of the following is not a wave equation?*

(A) $y = A\sin k(x^2 - vt^2)$ *(B)* $y = A\cos 2\pi(x/\lambda - t)$

(C) $y = A\sin 2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)$ *(D)* $y = A\sin kx\cos\omega t$

Sol. (A).

For $y = A\sin k(x^2 - vt^2)$

$$\frac{d^2y}{dx^2} \neq \frac{1}{v^2}\frac{d^2y}{dt^2}$$

$\therefore$ It is not the equation of a wave.

Prob 12. *Wave traveling in a stretched string is described by the equation* $y = a\sin(kx - \omega t)$*. The maximum velocity of the particle is*

(A) $A\omega$ *(B)* ω/k

(C) $x\omega$ *(D) x/t*

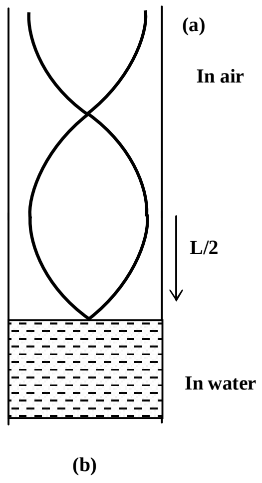
Sol. (A).

$$\frac{dy}{dt} = -A\omega\cos(kx-\omega t) = V_{particle}$$

$\Rightarrow V_{max} = A\omega$

Prob 13. *A cylindrical tube open at both ends has fundamental frequency f in air. Tube is dipped vertically in water so that half of it is in water. Fundamental frequency of air column is*
(A) f/2 *(B) 3f/4*
(C) f *(D) 2f*

Sol. (C).
Fundamental frequency in air,
$L = \lambda/2$
$\Rightarrow \lambda = 2L$
$\frac{V}{2L} = f_1$
When t is half dipped in water
$\frac{L}{2} = \frac{\lambda}{4}$
$\Rightarrow \lambda = 2L$
$v = \lambda f_2$
$\Rightarrow f_2 = v/2L$
$f_1 = f_2 = f$



Prob 14. *A complex wave $y = 3 \sin^2 t \cos 500t$ is formed by superposition of how many waves*
(A) 2 *(B) 3*
(C) 4 *(D) 5*

Sol. (B).
y = 3 sin2t cos500t
$$y = \frac{3}{2}(1 - \cos 2t)\cos 500t = \frac{3}{2}\cos 500t - \frac{3}{4}[2\cos 2t \cos 500t]$$
$$y = \frac{3}{2}\cos 500t - \frac{3}{4}\cos 502t - \frac{3}{2}\cos 498t$$
This implies, $y = 3\sin^2 t \cos 500t$ is the superposition of 3 waves

Prob 15. *A whistling train approaches a junction. An observer standing on platform observes the frequency to be 2.2 kHz and 1.8 KHz. When the train is approaching and receding him, respectively. The speed of the train is (speed of sound = 300 m/s)*
(A) 20 m/s *(B) 30 m/s*
(C) 40 m/s *(D) 50 m/s*

Sol. (B).
While approaching the junction, $f' = f_0\left(\frac{V}{V - V_S}\right)$
$$\Rightarrow 2200 = f_0\left(\frac{300}{300 - V_S}\right) \qquad \ldots \text{(i)}$$
While receding the junction, $f'' = f_0\left(\frac{V}{V + V_S}\right)$

$$\Rightarrow 1800 = f_0\left(\frac{300}{300+V_s}\right) \qquad \ldots\text{(ii)}$$

Solving (i) and (ii) gives v_s = 30 m/s.

Prob 16. *An air column in a pipe, which is closed at one end will be in resonance with a vibrating tuning fork of frequency 264 Hz, if the length of the pipe in cm is*

(A) 31.25 *(B) 62.5*

(C) 93.75 *(D) 125*

Speed of sound in air = 330 m/s

Sol. (A), (C).

$$(2n-1)\frac{v}{4\ell} = 264$$

$$\Rightarrow \ell = \frac{(2n-1)\times 330}{264\times 4}$$

$\Rightarrow \ell$ = 31.25 if n = 1

ℓ = 93.75 if n = 2

ℓ = 156.25 if n = 3

Prob 17. *The phase difference between waves y_1 = asin ωt and y_2 = a cos (ωt) is*

(A) π *(B) $\pi/2$*

(C) zero *(D) none of these*

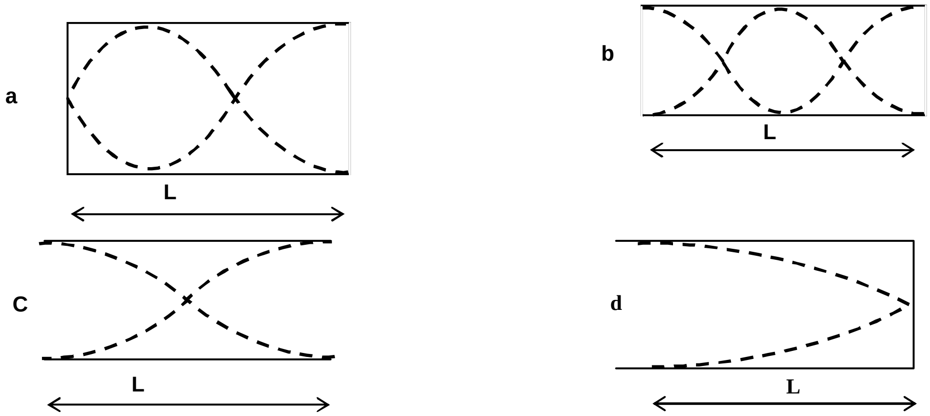
Sol. (B).

y_1 = asinωt = a cos ($\pi/2$ - ωt) = a cos (ωt - $\pi/2$)

Y_2 = a cos ωt

$\therefore \Delta\phi = \pi/2$

Prob 18. *The vibration of four air columns are shown in the figure. The ratio of frequencies fa:fb:fc:fd is*



(A) 6:8:3:1 *(B) 12:16:8:3*

(C) 3:4:2:1 *(D) 4:3:2:1*

Sol. (C).

$$\frac{3}{4}\lambda_a = L \;\; \therefore \lambda_a = \frac{4}{3}L \quad \therefore f_a = \frac{v}{4\ell/3} = \frac{3}{4}\frac{v}{L}$$

$$\lambda_b = L \quad \Rightarrow f_b = \frac{v}{L}$$

$$\frac{1}{2}\lambda_c = L \;\; \Rightarrow \lambda_c = 2L$$

$$\Rightarrow f_c = \frac{v}{2L} = \frac{1}{2}\frac{v}{L}$$

$$\frac{1}{4}\lambda_d = L \quad \Rightarrow \lambda_d = 4L$$

$$\Rightarrow f_d = \frac{v}{4L} = \frac{1}{4}\left(\frac{v}{L}\right)$$

$$\therefore f_a : f_b : f_c : f_d = \frac{3}{4} : 1 : \frac{1}{2} : \frac{1}{4} = 3 : 4 : 2 : 1$$

ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. Why the speed of sound in moist air is greater than in dry air?

2. An observer at a sea-cost observes waves reaching the coast. What type of waves does he observe? Justify your answer.

3. An organ pipe is in resonance with a tuning fork. If the length of the wire between the bridges is made twice even than it can resonate with the same fork. Why?

4. A tuning fork produces resonances with a pipe. If the temperature is increased, then how should the length of the pipe be changed for resonance?

5. Why two organ pipes of same length open at both ends produces sounds of different frequencies if their radii are different?

6. An observer is moving towards a wall at 2 m/s. He hears a sound from a source at some distance behind him directly as well as after reflection from the wall. Determine the beat frequency between these two sounds if the actual frequency of the source is 680 Hz. Given velocity of sound = 340 m/s.

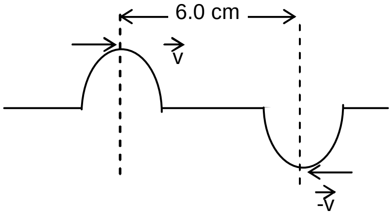
7. Consider the wave y = 100 (cm) sin [(1 cm^{-1})x - (50s^{-1})t]. Find
(a) the amplitude
(b) the wave number
(c) the wavelength
(d) the wave velocity.

8. A wave has a wave speed of 243 m/s and a wavelength of 3.27 cm. Calculate (a) the frequency (b) the period of the wave.

9. The speed of a wave on a string is 172 m/s when the tension is 123 N. Find tension in the string if the wave speed is 180 m/s.

10. An observer measures an intensity of 1.13 W/m^2 at an unknown distance from a source of spherical waves whose power output is unknown. The observer walks 5.30 m closer to the source and measures an intensity of 2.41 W/m^2 at this new location. Calculate the power output of the source.

11. Two pulses are travelling along a string in opposite directions. If the wave speed is 2.0 m/s and the pulses are 6.0 cm apart, sketch the pattern after 5.0, 10, 20 and 25 ms.

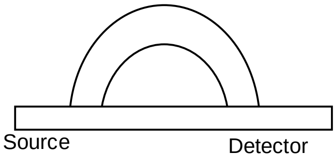


12. A string fixed at both ends is 2m long and has a mass of 122 gm. it is subjected to a tension of 61gm and set vibrating.
(a) what is the speed of the waves in the string ?
(b) What is the wavelength of the longest possible standing wave?

13. What are the three lowest frequencies for standing waves on a wire 9.88 m long having a mass of 0.107 kg, which is stretched under a tension of 236 N ?

14. Show that the sound wave intensity I can be written in terms of the frequency and displacement amplitude A in the form $I = 2\pi^2 \rho v f^2 A^2$

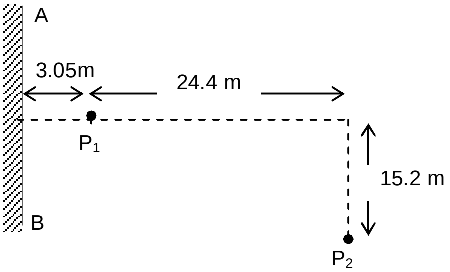
15. A sound wave of 42.0 cm wavelength enters the tube as shown. What must be the smallest radius r such that minimum will be heard at the detector ?



16. A sound wave of intensity 1.60 μW/cm^2 passes through a surface of area 4.70 cm^2. How much energy passes through the surface in 1h?

17. A source emits spherical waves with equal intensities in all directions. The intensity of the wave 42.5 m from the source is 197 μW/m^2. Find the power output of the source.

18. A spherical sound source is placed at P_1 near a reflecting wall AB and microphone is located at point P_2. The frequency of the sound source is variable. Find the two lowest frequencies for which the sound intensities, as observed at P_2, will be maximum. There is no phase change on reflection; the angle of incidence equals the angle of reflection.



19. A continuous sinsusoidal longitudinal wave is sent along a coiled spring from a vibrating source attached to it. The frequency of the source is 25 Hz, and the distance between the two successive rarefactions in the spring is 24 cm. (a) Find the wave speed. (b) If the maximum longitudinal displacement of a particle in the spring is 0.30 cm and wave moves in the -x direction. Write the equation for the wave. Let the source be at x = 0 and the displacement s = 0 at the time when t = 0.

20. Two submarines are on a head-on collision course during manoeuvres. The first sub is moving at 20.2 km/h and the second sub at 94.6 km/h. The first submarine sends out a sonar signal at 1030 Hz. Sonar waves travel at 5470 km/hr.
    (a) The second sub picks up the signal. What frequency does the second sonar detector hears?
    (b) The first sub picks up the reflected signals. What frequency does the first sonar detector hear? Consider the ocean is calm; assume no currents at all.

Level - I

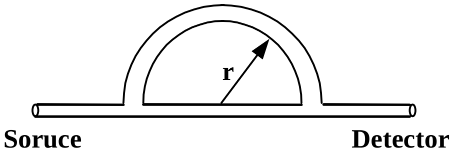
1. Given the equation for a wave in a string

   $y = 0.03 \sin(3x - 2t)$

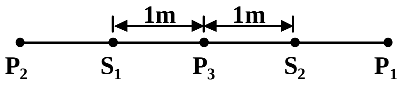
   Where y and x are in metre and t is in second, answer the following:
   (a) At t = 0, what is the displacement at x = 0 ?
   (b) At x = 0.1 m, what is the displacement at t = 0.2 s ?
   (c) What is the equation for the velocity of oscillation of the particles of the string ? What is the maximum velocity of oscillation?
   (d) What is the velocity of propagation of the wave?

2. A source of sound of frequency 550 Hz emits waves of wavelength 600 mm in air at 20^oC. What is the velocity of sound in air at this temperature ? What would be the wavelength at 0^oC ?

3. A sound wave of 40 cm wavelength enters the tube as shown in the figure. What must be the smallest radius r such that a minimum will be heard at the detector ?



4. Two sources S_1 and S_2, separated by 2.0 m, vibrate according to equation $y_1 = 0.03 \sin\pi t$ and $y_2 = 0.02 \sin\pi t$ where y_1, y_2 and t are in S.I. units. They send out waves of velocity 1.5 m/s. Calculate the amplitude of the resultant motion of the particle co-linear with S_1 and S_2 and located at a point



   (a) P_1 to the right of S_2
   (b) P_2 to the left of S_2 and
   (c) P_3 in the middle of S_1 and S_2 .

5. At a certain point in space, two waves produce pressure variations given by

   $\Delta p_1 = \Delta p_m \sin\omega t$

   $\Delta p_2 = \Delta p_m \sin(\omega t - \phi)$

   What is the pressure amplitude of the resultant wave at this point when $\phi = 0$ and $\phi = \pi/2$

6. To determine the velocity of sound propagation in air by acoustic resonance technique one can use a pipe with a piston and a sonic membrane closing one of its ends. Find the velocity of sound if the distance between the adjacent positions of the piston at which resonance is observed at a frequency f = 2000 Hz is equal to ℓ = 8.5cm ?

7. The water level in a vertical glass tube 1 m long can be adjusted to any position in the tube. A tuning fork vibrating at 686 Hz is held just over the open top end of the tube. At what positions of the water level will there be resonance? (velocity of sound = 330 m/s)

8. The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2Hz. The fundamental frequency of the closed organ pipe is 110Hz. Find the lengths of the pipes. Speed of sound in air is 330 m/s.

9. A simple harmonic wave has the equation $y = 0.5 \sin(314t - 1.57x)$, where y, x are in metres and t in seconds. Find
(a) the frequency and the wavelength of this wave.
(b) another wave has the equation $y' = 0.1 \sin(314t - 1.57x + 1.57)$. Deduce the phase difference and the ratio of their intensities.

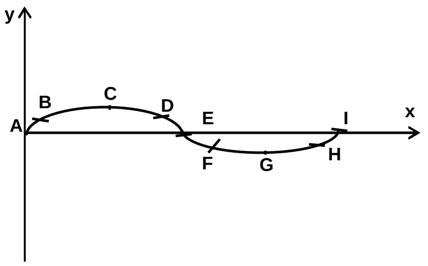
10. A stationary source of sound emits sound of frequency f_0. A screen mounted on a vehicle moving with velocity v_0 towards the source. An observer detects beat frequency f_1. If vehicle move away from the source with same speed v_0, observe beat frequency f_2. Assuming observers remains between the source and vehicle all the time and if the ratio of beat frequency $\frac{f_1}{f_2} = 2$, find the velocity v_0. Assume the velocity of sound is c.

Level - II

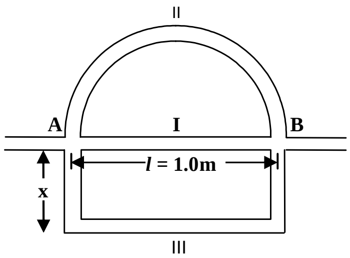
1. A receiver and a source of sonic oscillations of frequency 2000 Hz are located on the x - axis. The source swings harmonically along x - axis with circular frequency ω and amplitude 50 cm. At what value of ω will the frequency band width registered by stationary receiver be equal to 200 Hz ? The velocity of sound in air is equal to 340 m/s. (Assume that the velocity of source is small compared to the velocity of sound)

2. A column of air at 51^oC and a tuning fork produce 4 beats per second when sounded together. As the temperature of the air column is decreased, the number of beats per second tends to decrease and when the temperature is 16^oC, the two produce one beat per second. Find the frequency of the tuning fork.

3. A transverse wave is travelling along a string from left to right. Adjoining figure represents the snap shot at a given instant. At this instant
(a) which points have an upward velocity?
(b) which points will have downward velocity?
(c) which points have zero velocity?
(d) which have maximum magnitude of velocity ?



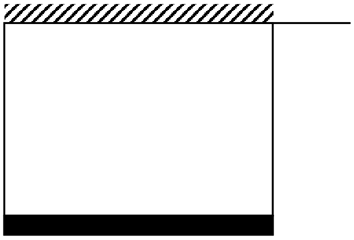
4. A sound of intensity 27 I_0 is produced at the end A. It travels down to the point B along three different paths I, II and III as shown in the figure. Path II is semi-circular. At B, no sound is heard.
(a) Find the maximum possible wavelength of sound
(b) Find the value of x.
(c) Find the equations of the three waves arriving at B.
Assume that sound energy is equally distributed among the three paths. The path length III is more than that of II.



5. Find the harmonic components in the complex wave $y = 3 \sin^2 t \cos (500t)$. What are their circular frequencies?

6. Two wires of radii r and 2r respectively are welded together end to end. This combination is used as a sonometer wire kept under tension T. The welded point is midway between the two bridges. What would be the ratio of the number of loops formed in the wires such that the joint is a node when stationary vibrations are set up in the wires.

7. A uniform horizontal rod of length 0.40 m and mass 1.2 kg is supported by two identical wires as shown in the figure. Where should a mass of 4.8 kg be placed on the rod so that the same tuning fork may excite the wire on the left to its fundamental vibration and that on the right to its first overtone. (take g = $10 m/s^2$)



8. A certain loudspeaker (assumed to be a point source) emits 30 W of sound power. A small microphone of effective cross-sectional area 0.75 cm^2 is located 200 m from the loudspeaker. Calculate
(a) the sound intensity at the microphone and
(b) the power received by the microphone.

9. A standing wave setup in a medium is given by

$$y = 4\cos\frac{\pi x}{3}\ \sin 40\pi t$$

Where x and y are in cm and t in seconds,
(a) What are the amplitude and velocity of the two component waves which produce this standing wave?
(b) What is the distance between the adjacent nodes?
(c) What is the velocity of the particle of the medium at x = 3cm at time $t = \frac{1}{8}$ sec?

10. A copper rod of length ℓ is clamped at its mid point. Find the expression for the n^{th} natural frequency of longitudinal oscillations in the rod. Young's modulus and density of the rod are Y and ρ respectively.

Objective:

Level – I

1. A sine wave is travelling in a medium. The minimum distance between the two particles, always having same speed is
   (A) $\lambda/4$ (B) $\lambda/3$
   (C) $\lambda/2$ (D) λ

2. A transverse wave travels along the z-axis. The particles of the medium must move
   (A) Along the Z-axis (B) Along the X-axis
   (C) Along the y-axis (D) in the X-Y plane.

3. A wire having a linear density 0.10 kg/m is kept under a tension 490 N it is observed that it resonates at a frequency of 400 Hz and the next frequency 450 Hz. The length of the wire is
   (A) 14 m (B) 0.7 m
   (C) 1. 6 m (D) 0.49 m

4. Two particles in a medium are vibrating in same phase with a path difference of 10 m and corresponding time difference is 0.1 s. The speed of wave in the medium is
   (A) 100 m/s (B) 300 m/s
   (C) 50 m/s (D) 200 m/s

5. A wire of length ℓ having tension T and radius r vibrates with a natural frequency f. Another wire of same metal with length 2ℓ having tension 2T and radius 2r will vibrate with natural frequency
   (A) f (B) 2f
   (C) $2\sqrt{2}f$ (D) $\frac{f}{2\sqrt{2}}$

6. In a resonance column experiment the first resonance is obtained when the level of the water in tube is 20 cm from the open end. Resonance will also be obtained when the water level is at a distance of
   (A) 40 cm from the open end (B) 50 cm from the open end
   (C) 80 cm from the open end (D) 100 cm from the open end

7. A policeman on duty detects a drop of 10% in the pitch of the horn of a motor car as it crosses him. If the velocity of sound is 330 m/s the speed of the car will be
   (A) 20 m/sec (B) 36.3 m/sec
   (C) 25 m/s (D) 27 m/sec

8. Two waves travelling in a medium in the x-direction are represented by $y_1 = A \sin(\alpha t - \beta x)$ and $y_2 = A \cos(\alpha t - \beta x - \pi/4)$, where y_1 and y_2 are the displacements of the particles of the medium, t is time and α and β are constants. The two waves have different
(A) Speeds (B) Direction of propagation
(C) Wavelengths (D) Frequencies

9. A wave travelling in a stretched string is described is described by the equation $y = A \sin(kx - \omega t)$. The maximum particle velocity is
(A) $A\omega$ (B) ω/k
(C) $x\omega$ (D) x/t

10. Two sounds waves of equal intensity I generates beats. The maximum intensity of sound produced in beats will be
(A) I (B) 4I
(C) 2I (D) I/2

11. Two trains, one coming towards and another going away from an observer both at 4 m/s whistle simultaneously, with a frequency of 300 Hz. Velocity of sound is 340 m/s. The number of beats produced are
(A) 7 (B) 6
(C) 5 (D) 12

12. Standing waves are produced in 10 m long stretched string. If the string vibrates in 5 segments and wave velocity is 20 m/s. Its frequency in Hz is
(A) 5 (B) 4
(C) 2 (D) 10

13. When the length of vibrating segment of a sonometer wire is increased by 1%, the percentage change in its frequency is
(A) $\frac{99}{100}$ (B) $\frac{100}{101}$
(C) 2 (D) 1

14. Doppler effect is valid for
(A) space waves (B) sound waves
(C) light waves (D) both B and C

15. Equation of progressive wave is given by $Y = a \sin \pi\left(\frac{t}{2} - \frac{x}{4}\right)$, where t is in second and x is in meter. Then the distance through which the wave moves in 8 seconds is
(A) 16 (B) 2
(C) 4 (D) 8

16. Two strings of the same material and same length have their tensions in the ratio of 4:1 and radii in the ratio 2:1. The ratio of their fundamental frequencies is
(A) 1:1 (B) 10:20
(C) 3:4 (D) 2:1

17. A resonance air column of length 20 cm resonates in fundamental mode with a tuning fork of frequency 450 Hz. Velocity of sound in air will be
(A) 920 m/s (B) 720 m/s
(C) 820 m/s (D) 360 m/s

18. A tuning fork produces 4 beats/sec, with another tuning fork B of frequency 320 cps. On filling fork A , 4 beats/second are again heard. Then the frequency of fork A before filling (in cps) is
(A) 316 (B) 328
(C) 320 (D) 324

19. What is the path difference for destructive interference?
(A) $n\lambda$ (B) $\frac{(n+1)\lambda}{2}$
(C) $n(\lambda+1)$ (D) $\frac{(2n+1)\lambda}{2}$

20. Resonance air column of length 20 cm resonates with a tuning fork of frequency 250 Hz. Speed of air in m/s is
(A) 250 (B) 200
(C) 150 (D) 300

Level – II

1. An organ pipe P_1 closed at one end vibrating in first harmonic and another pipe P_2 open at both ends vibrating in the third harmonic are in resonance with given turning fork. The ratio of length of P_1 to that of P_2 is
   (A) 8/3 (B) 3/8
   (C) 1/ 6 (D) 1/3

2. The temperature at which the speed of sound in air beocmes double of its value at 27^0C is
   (A) 54^0C (B) 327^0C
   (C) 927^0C (D) –123 ^{0}C

3. A cylindrical tube, open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of air column is now
   (A) f/2 (B) 3f/4
   (C) f (D) 2f

4. A wave is represented by the equation y = A sin ($10\pi x + 15\pi t + \pi/3$) where x is in meter's and t is in seconds. The expressions represents
   (A) A wave travelling in positive x direction with a velocity 1.5 m/s
   (B) A wave travelling in negative x-direction with a velocity 1.5 m/s
   (C) A wave travelling in the negative x-direction having a wave length 0.02 m.
   (D) A wave travelling in positive x-direction having a wave length 0.02 m.

5. In the interference of waves from two sources of intensities I_o and $4I_0$.. The intensity at a point where the phase difference is π is
   (A) I_0 (B) $2I_0$
   (C) $3I_0$ (D) $4I_0$

6. A string vibrates according to equation $y = 5 \sin \frac{\pi x}{3} \cos 40\pi t$. The potential energy of the string will be zero at times
   (A) $\frac{1}{20}$ s and $\frac{1}{40}$ s (B) $\frac{1}{40}$ s and $\frac{1}{80}$ s
   (C) $\frac{1}{80}$ s and $\frac{3}{80}$ s (D) $\frac{1}{40}$ s and $\frac{3}{40}$ s

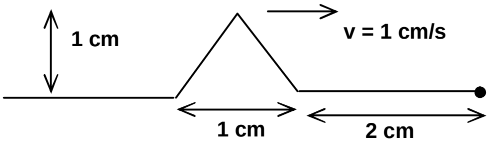
7. Which of the following is not a wave equation ?
   (A) $y = A \sin k (x^2 - vt^2)$ (B) $y = A \cos 2\pi (x/\lambda - t)$
   (C) $y = A \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right)$ (D) $y = A \sin kx \cos \omega t$

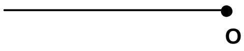
8. A source of sound of frequency 256 Hz is moving towards a wall with a velocity of 5 m/s. The number of beats per second heard by a stationary observer between the source and the wall is (velocity of sound = 330 m/s)
(A) 0 (B) 3
(C) 5 (D) 2

9. An open pipe has a fundamental frequency of 300 Hz. The first overtone of this pipe is the same as the first overtone of a closed pipe. The length of the closed pipe is (velocity of sound = 330 m/s)
(A) 82 cm (B) 90 cm
(C) 41 cm (D) 78 cm

10. An open pipe is suddenly closed at one end with the result that the frequency of the third harmonic of the closed pipe is found to be higher than its fundamentals frequency by 100 Hz. The fundamental frequency of the open pipe is
(A) 200 Hz. (B) 300 Hz.
(C) 100 Hz. (D) 50 Hz.

11. A wave pulse on a string has the dimension as shown is figure. The wave speed is v = 1 cm/s. If point O is a free end. The shape of wave at time t = 3s is



(A)


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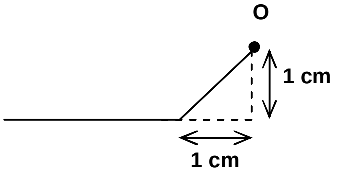

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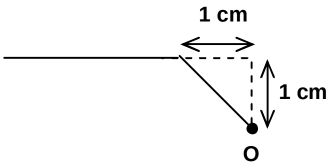
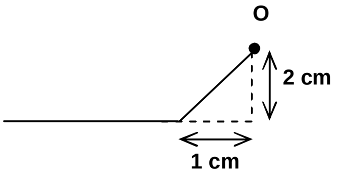
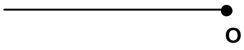
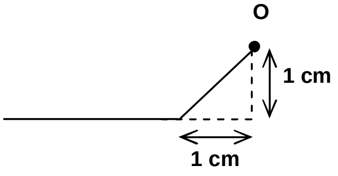
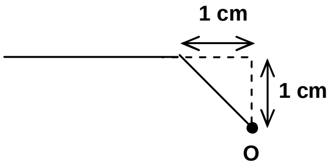


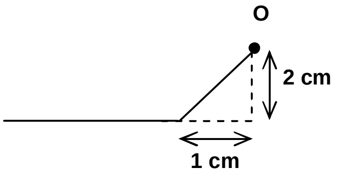
(D)


12. In the above problem, shape of the wave at time t = 3s, if O is the fixed end, will be

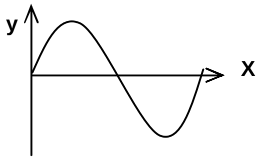
(A)


(B)


(C)


(D)


13. In a sine wave position of different particles at time t = 0 is shown in figure, the equation of this wave if it is traveling along +ve x-axis can be



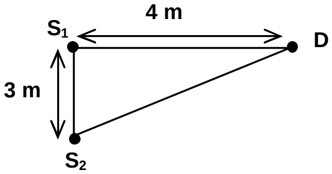
(A) $y = A\sin(\omega t - kx)$ (B) $y = A\sin(kx - \omega t)$
(C) $y = A\cos(\omega t - kx)$ (D) $y = A\cos(kx - \omega t)$

14. In a stationary wave that form as a result of reflection of waves from an obstacle, the ratio of the amplitude at an antinode to amplitude at node is n. The fraction of energy reflected is
(A) $\left(\frac{n-1}{n}\right)^2$ (B) $\left(\frac{n-1}{n+1}\right)^2$
(C) $\left(\frac{1}{n}\right)^2$ (D) $\left(\frac{n}{n+1}\right)^2$

15. In a certain organ pipe three successive resonance frequencies are observed to be 88, 104 and 120 we say that the pipe is
(A) definitely open (B) definitely closed
(C) may be open or closed (D) none of these

16. Which of the following is not the standard form of a sine wave
(A) $y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ (B) $y = A\sin(vt - kx)$
(C) $y = A\sin\omega\left(t - \frac{x}{v}\right)$ (D) $y = A\sin(k)(vt - x)$

17. In the given figure the intensity of waves arriving at D from two coherent sources S_1 and S_2 is I_0.Wavelength of wave is λ = 4 m. Resultant intensity at D will be

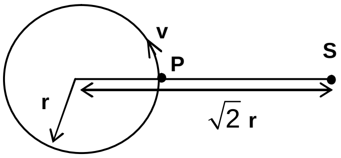


(A) $4I_0$ (B) I_0
(C) $2I_0$ (D) none of these

18. Two waves of intensities I and 2I and with opposite phase superpose. Intensity of superposed wave will be
(A) I (B) 3 I
(C) $I(\sqrt{2} - 1)^2$ (D) $I(\sqrt{2} + 1)^2$

19. If λ_1, λ_2 and λ_3 are the wavelength of the waves giving resonances with the fundamental, first and second overtones respectively in a closed organ pipe, then the ratio of $\lambda_1:\lambda_2:\lambda_3$ is
(A) 1:2:3 (B) 1:1/3:1/5
(C) 1:3:5 (D) 5:3:1

20. A stationary source emitting sound of frequency is kept as shown in figure. An observer is running on the circular track with constant speed v. What will be the minimum frequency recorded by observer. (c = speed of sound in air)



(A) $f_0\left(1 + \frac{v}{c}\right)$ (B) $f_0\left(1 - \frac{v}{C}\right)$
(C) f_0 (D) zero

ANSWERS

Subjective:

LEVEL - O

6. 8 Hz
7. (a) 100 cm (b) 1 rad/cm (c) 2π cm (d) 50 cm/s
8. (a) 7.43 kHz. (b) 135 μS
9. 135 N
10. 1420 W
12. (a) 3.13 m/s. (b) 4 m
13. 7.47 Hz, 14.9 Hz, 22.4 Hz.
15. 18.4 cm.
16. 0.0271 J
17. 4.47 W
18. 42.86 Hz. 85.73 Hz.
19. (a) 600 cm/s (b) y= 0.30 cm sin [(0.26 rad/cm)x + (157 rad/s)t]
20. (a) 1050 Hz (b) 1070 Hz.

LEVEL - I

1. 0 ; -3 mm; v = -0.06 cos(3x - 2t), 0.06 m/s; 0.667 m/s
2. 330 m/s, 579 mm
3. 17.52 cm
4. 2. 65 cm ; 2.65 cm, 5 cm
5. $2\Delta P_m$ when $\phi = 0$, $\sqrt{2}\,\Delta P_m$, when $\phi = \pi/2$
6. 340 m/s
7. 12 cm, 36 cm, 60 cm, 84 cm
8. open pipe : 0.99 m ; closed pipe : 0.75 m
9. (a) 50 Hz, 4 m (b) $\pi/2$; 25
10. c / 3

LEVEL - II

1. 34 rad/s

2. 50 Hz.

3. (a) D, E, F (b) A, B, H, I (c) C, G (d) A, E, I

4. (a) $3(\pi - 1)$m (b) $(\pi - 1)$m

(c) $y_1 = 3\sqrt{I_0}\,\sin(\omega t - kx)$, $y_2 = 3\sqrt{I_0}\,\sin(\omega t - kx + \frac{2\pi}{3})$, $y_3 = 3\sqrt{I_0}\,\sin(\omega t - kx + \frac{4\pi}{3})$

5. $y_1 = \frac{3}{2}\cos 500\,t$, $y_2 = -\frac{3}{4}\cos 502\,t$, $y_3 = -\frac{3}{4}\cos 498\,t$, 500 rad/s , 502 rad/s, 498 rad/s

6. $n_r : n_{2r} = 1 : 2$

7. 0.05 m from left end.

8. (a) 5.96×10^{-5} W/m^2 (b) 4.47×10^{-9} W

9. (a) 2 cm, 120 m/s (b) 3 cm (c) $160\,\pi$ cm/s

10. $(2n+1)\,\frac{1}{2\ell}\sqrt{\frac{y}{\rho}}$ (n = 0, 1, 2, 3 ,…..)

Objective:

LEVEL - I

1.	**C**	2.	**D**
3.	**B**	4.	**A**
5.	**D**	6.	**D**
7.	**B**	8.	**B**
9.	**A**	10.	**B**
11.	**A**	12.	**A**
13.	**B**	14.	**D**
15.	**A**	16.	**A**
17.	**D**	18.	**A**
19.	**D**	20.	**B**

LEVEL - II

1.	**C**	2.	**C**
3.	**C**	4.	**B**
5.	**A**	6.	**C**
7.	**A**	8.	**A**
9.	**C**	10.	**D**
11.	**D**	12.	**A**
13.	**B**	14.	**B**
15.	**B**	16.	**B**
17.	**C**	18.	**C**
19.	**B**	20.	**B**

Learnaf
Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

WAVE OPTICS

Wave Optics

Syllabus :
Newton's Corpuscular theory, Wave front and Huygen's principles; Reflection and refraction of plane wave at a plane surface using wave fronts (qualitative idea); Interference-Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light; Diffraction - diffraction due to a single slit, width of central maximum, difference between interference and diffraction; Resolving power of microscope and telescope; Polarisation, Plane polarised light, Brewester's law. Photometry.

Newton's Corpuscular theory

This theory was proposed by Sir Isacc Newton in 1678. According to this theory, known as corpuscular theory, light consists of stream of extremely light and tiny material particles known as corpuscles. These particles are shot out by every point of a source of light with a very high speed. When the corpuscles, which travel in straight lines, fall on the retina of the eye, they produce the sensation of vision. According to Newton, different colours of light were due to different sizes of corpuscles.

The theory could explain the propagation of light through vacuum and phenomenon of reflection and refraction. But the theory could not explain the phenomena of interference, diffraction and polarisation. It could not explain why velocity of light is lesser in a denser medium compared to vacuum.

Origin of the wave theory

As only few physical phenomenon could be explained by corpuscular theory, Christian Huygens proposed the wave aspect of light. According to which, a luminous body is a source of disturbance in a hypothetical medium called ether. The medium assumed to be spread in the entire space. The disturbance from the source is propagated in the form of waves through ether and the energy is distributed equally in all directions. Huygens assumed these waves to be longitudinal in which the vibration of particle of the medium is parallel to the direction of propagation of the wave.

Huygens successfully explained the linear propagation of light, phenomenon of reflection, refraction and double refraction. However, the phenomenon of polarisation discovered by him could not be explained.
Later, Fresnel and Young suggested that light waves are transverse.

Maxwell's electromagnetic theory
According to Maxwell, light is not a mechanical wave but electromagnetic in character, i.e. it propagates as transverse non-mechanical wave at speed c in free space given by

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3 \times 10^8 \text{ m/s.}$$

In it electric vector $\vec{E}$ and magnetic vector $\vec{B}$ are related to each other through the relation

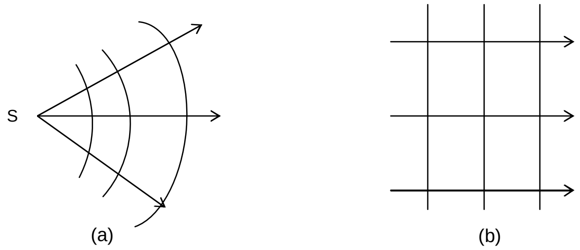
$$E/B = c$$

E and B are mutually perpendicular and transverse to the direction of propagation of light.

Wave front and wave normal:
A wave front can be defined as the locus of all the points of the medium to which the waves reach simultaneously, so that all the points vibrate in the same phase. If the distance of the source is small, the wave-front is spherical. For large distance, parallel beam of light gives rise to a plane wave front. A linear source, like an illuminated slit produces a cylindrical wavefront.

Wave normal:

A perpendicular drawn to the surface of a wave-front at any point, in the direction of propagation of light, is called a wave normal. A wave front carries light energy in a direction perpendicular to its surface. This direction is represented by a wave normal. The direction in which light travels is also called a ray of light. Thus, a wave normal is same as a ray of light.



(a) (b)

The successive positions of a spherical wave-front originating from a point source S and the corresponding wave normal are shown in fig. (a). It can be seen that the wave normal or rays are radial in the case of a spherical wave-front. In figure (b), the successive positions of a plane wave-front travel from left to right and the corresponding wave normal are shown. The wave normal or rays in this case are parallel to each other.

Exercise 1.

(i) ***Draw the type of wave-front that corresponds to a beam of white light coming from a very far off source.***

(ii) ***What is the relation of a wave-front with a ray of light ?***

Huygens' Principle:

1. Every point on a wave-front acts as a secondary source of light and sends out secondary wavelets in all directions. They are effective only in the forward sense. The waves travel with the speed of light in the medium.

2. The position of the wave-front at a later instant is given by the surface of tangency or envelope of all the secondary wavelets at that instant. (An envelope is a curve tangential to a family of surfaces).

Refer to fig (a). XY is a portion of sphere of radius vt. Here v is the velocity of propagation of light wave. XY is called the primary wavefront. Similar is the case in figure (b).

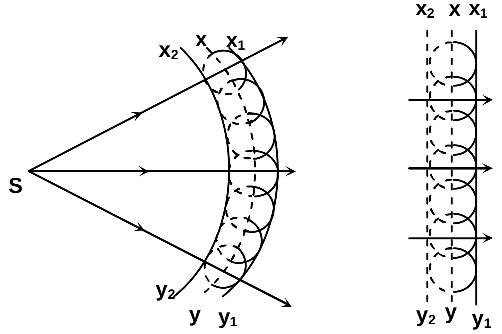


fig. (a) fig. (b)

Exercise 2.

What is the shape of a wavefront when

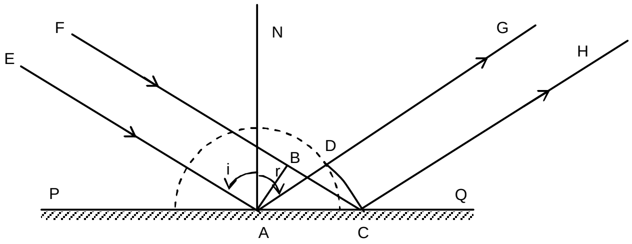
(a) light is emitted by a point source in an isotropic medium.

(b) light is emerging from a convex lens when a point source is placed at its focus.

(c) light of the sun is reaching the earth.

(d) light is diverging from a slit.

Reflection of a plane wave-front at a plane surface:



PQ is a plane reflecting surface (say a plane mirror). A plane wave-front AB bounded by two rays (wave-normals) EA and FB is approaching PQ obliquely. When AB touches the surface PQ at A, then according to Huygens' principle, A acts like a secondary source and sends out secondary wavelets traveling in the same medium only. As time progresses different points on AB will come in contact with PQ and secondary wavelets will start from these points. If the disturbance at B reaches point C in time t, then the distance BC = Vt, where V is the speed of light in the medium.
In the same time, the secondary waves starting from A, travel the distance Vt. With centre A and radius BC = Vt. Draw a hemispherical surface (semi circle in two dimensions) and draw a tangent CD to this surface.

The points C and D are in the same phase. Hence, CD represents the reflected wave-front at the time t and it moves parallel to itself. Join AD, AG and CH are the reflected rays. Draw AN $\perp$ PQ.

In triangles ABC and ADC
(1) AC is common (2), AD = BC = Vt (by construction), (3) $\angle ABC = \angle ADC = 90^0$.

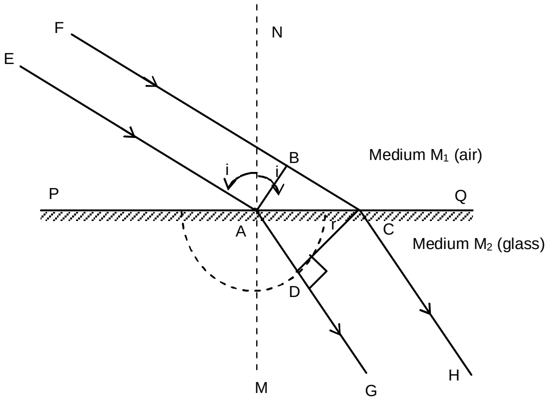
$\therefore$ The triangles are congruent $\therefore$ $\angle BAC = \angle ACD$.
From the figure,
$\angle EAN = i$, $\angle NAD = r$,
$\angle EAN = i = \angle BAC$ [$\because$ $\angle EAN + \angle NAB = \angle NAB + \angle BAC = 90^0$]
$\angle NAD = r = \angle ACD$. [$\because \angle NAD + \angle DAC = \angle DAC + \angle ACD = 90^0$]
$\therefore$ i = r

Thus, we get the following laws of reflection.
(i) The angle of incidence = angle of reflection.
(ii) Similarly, from the figure we find that the incident ray and the reflected ray lie on the opposite sides of the normal at the point of incidence and all three lie in the same plane (plane of the paper).

Thus, the laws of reflection are proved on the basis of Huygens' wave theory.

Refraction of a plane wave-front at a plane surface:

A plane wavefront AB bounded by the wave normals (rays) EA and FB is approaching obliquely the surface PQ. By Huygens' principle when the wavefront touches PQ at A, the point A becomes a secondary source and sends out secondary wavelets in all directions. But in the case of refraction of light, we consider the secondary wavelets traveling in the medium M_2 only. If the disturbance at B reaches C in time t then BC = V_1 t. In the same time, secondary wavelets from A travel a distance V_2t in M_2.



With centre A and radius V_2t, draw a hemi-spherical surface (semicircle in two dimensios) in M_2. Through C, draw the tangent CD to this wavefront. In time t, different points on AB come in contact at various points between A and C and they become the secondary sources. CD is tangential to all the secondary wavelets emitted by the secondary sources and it represents the refracted wavefront. It moves parallel to itself.

Join A and D then $AD = V_2t$
Draw NAM $\perp$ PQ. From the figure
$\angle EAN = i = \angle BAC$
$\angle MAD = r = \angle ACD$
In ΔABC: $\sin i = \frac{BC}{AC}$
In ΔADC ; $\sin r = \frac{AD}{AC}$

$\therefore \quad \frac{\sin i}{\sin r} = \frac{BC/AC}{AD/AC}$

$\therefore \quad \frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{V_1 t}{V_2 t} = \frac{V_1}{V_2}$

Since V_1 and V_2 are constants in the two media, $\frac{\sin i}{\sin r}$ = constant

This is Snell's law and the ratio $\frac{\sin i}{\sin r}$ is known as the refractive index of the second medium (M_2) w.r.t. the first medium (M_1). It is denoted by ${}_1n_2$.
Thus, ${}_1n_2 = \frac{\sin i}{\sin r} = \frac{V_1}{V_2}$
Thus Snell's law is proved.

Similarly, from the figure we find that the incident ray, the refracted ray and the normal to the refracting surface at the point of incidence lie in the same plane.
Note 1: By definition,
Absolute refractive index, $n = \frac{c}{V}$
Where c = speed of light (of a given frequency) in vacuum, and
V = speed of light (of the same frequency) in the medium.
$n_1 = \frac{c}{V_1}; n_2 = \frac{c}{V_2}$
${}_1n_2 = \frac{V_1}{V_2} = \frac{n_2}{n_1}$

Note 2: Frequency (ν) of radiation is determined by the source. On refraction, frequency of radiation does not change, but speed and wavelength are changed.
Thus $V_1 = \nu\lambda_1$ (in medium 1)
And $V_2 = \nu\,\lambda_2$ (in medium 2)

$$\frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$$

Thus, ${}_1n_2 = \frac{n_2}{n_1} = \frac{\sin i}{\sin r} = \frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$
For all practical purposes, speed of light in air (V_a) $\approx$ c.
Thus, the laws of refraction are proved.

Illustration 1. *Light is incident on a glass slab making an angle of 30^0 with the surface. If the speed of light in air is 3×10^8 m/s and the refractive index of glass is 1.5, find:*
(i) the angle of refraction.
(ii) the speed of light in glass.

Solution: Given that,
$V_{air} = 3 \times 10^8$ m/s
$n_g = 1.5$
$i = 90 - 30 = 60^0$
(i) $_an_g = \dfrac{\sin i}{\sin r}$
$\Rightarrow \sin r = \dfrac{\sin 60^0}{1.5} = \dfrac{\sqrt{3}/2}{3/2}$
$\Rightarrow r = 35^0\ 16$
(ii) $_an_g = \dfrac{V_{air}}{V_{glass}}$
or, $V_{glass} = \dfrac{V_{air}}{n_g} = 2 \times 10^8\ m/s$.

Interference: The phenomenon of interference of light is based on the principle of superposition of waves, which is stated as follows: "When two or more waves arrive at a point simultaneously, each wave produces its own displacement or effect at that point which does not depend upon other waves." The resultant displacement at that point is the vector sum of the instantaneous displacements due to individual waves meeting at that point.

The resultant displacement is maximum, if the displacements due to the waves are in the same phase.

The resultant displacement is minimum, if the two displacements are in opposite phase. The resultant displacement is zero if the amplitudes of the two waves are equal.

Therefore, the phenomenon of enhancement or cancellation of displacement produced due to the superposition of waves is called interference.

Condition for steady interference pattern:
(i) The two sources of light must be coherent.
(ii) The two sources of light must be monochromatic.
(iii) The two sources must be equally bright.
(iv) The sources should be narrow.
(v) The two sources should be close to each other.

Note:

- When two waves with amplitudes A_1 and A_2 superimpose at a point, the amplitude of resultant wave is given by $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

  where ϕ is the phase difference between the two waves at that point.
- Intensity $\propto A^2$. Hence, for I to be constant, ϕ must be constant.
- Intensity (I) = $I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$.
- When ϕ changes with time arbitrarily at a point, the intensity = $I_1 + I_2$.

- When ϕ does not change with time, we get an intensity pattern and the sources are said to be coherent. Coherent sources have a constant phase relationship in time.
- The intensity at a point becomes a maximum when $\phi = 2n\pi$ and there is constructive interference, where n = 0, 1, 2, ...
- If $\phi = (2n - 1)\pi$ there is destructive interference. (Hence n is a non-negative integer)

Illustration 2. *Four incoherent waves are expressed as $y_1 = a_1 \sin \omega t$, $y_2 = a_2 \sin 2\omega t$, $y_3 = a_3 \cos \omega t$ and $y_4 = a_4 \sin[\omega t + (\pi/3)]$. In which two waves, interference is possible.*

Solution: Out of all these four waves, no two can produce interference pattern. Here, the sources are not coherent.

Illustration 3. *If instead of using a single source of light, we use two different sources*
(A) interference pattern will never be obtained
(B) broad fringe width will be obtained
(C) very thin fringe pattern will be obtained
(D) overlapped fringes will be obtained

Solution : **(A)**

Interference pattern is obtained only if coherent sources of light are used, i.e. two sources having a constant phase difference, which is produced when two slits get light from a single source. Two different sources can never have a constant phase difference.

Exercise 3.

(i) ***What would happen if the path difference between the interfering beams, became very large?***

(ii) ***When two light waves interfere, then at some points, there is darkness. Where does the light energy of these points go? Does energy remains conserved in the phenomenon?***

(iii). ***No interference pattern is detected when two coherent sources are infinitely close to each other. Why ?***

Coherent sources

Two sources are said to be coherent if they emit light waves of the same frequency having the same wavelengths and always maintained with constant phase difference.
For experiments, two virtual sources are formed from a single source which can act as coherent sources.

Conditions for obtaining two coherent sources of light:

(i). Coherent sources of light should be obtained from a single source by same device.

(ii) The two sources should give monochromatic light.

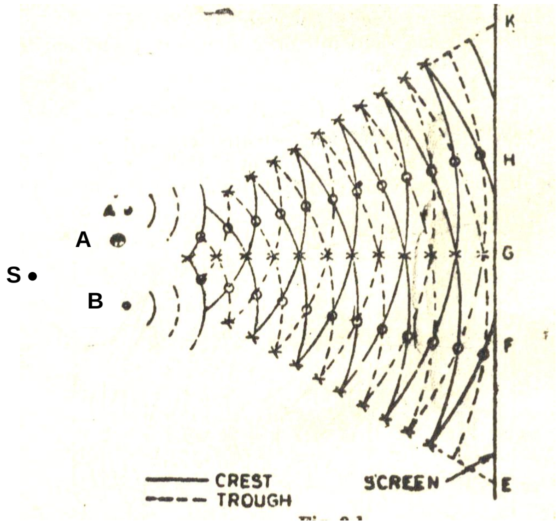
(iii) The path difference between light waves from two sources should be small.

Exercise 4.

Is it possible to have coherence between light sources emitting light of different wavelengths?

Young's double experiment

In the year 1802, Young demonstrated the experiment on the interference of light. He allowed sunlight to fall on a pinhole S and then at some distance away on two pinholes A and B.



A and B are equidistant from S and are close to each other. Spherical waves spread out from S. Spherical waves also spread out from A and B. These waves are of the same amplitude and wavelength. On the screen interference bands are produced which are alternatively dark and bright. The points such as E are bright because the crest due to one wave coincides with the crest due to the other and therefore they reinforce each other. The points, such as F are dark because the crest of one falls on the trough of the other and they neutralise the effect of each other.

Exercise 5.

(i) ***In Young's double slit experiment, if S_1 and S_2 are illuminated by two bulbs of same power, what will be observed on the screen ?***

(ii) ***The monochromatic source of light in Young's double slit experiment is replaced by another wavelength. What will be the effect.***

Phase difference (Δϕ) and path difference (Δϕ)

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

If the path difference between the two waves is λ, the phase difference $= 2\pi$ and time difference is T.

For a path difference x, the phase difference is δ

Then, $\delta = \dfrac{2\pi x}{\lambda}$

Determination of phase difference

The phase difference between two waves at a point will depends upon:

(a) the difference in path lengths of the two waves from their respective sources;
(b) the refractive index of the medium;
(c) initial phase difference, between the sources, if any;
(d) reflections, if any, in the path followed by the waves.

➢ In the case of light waves, the phase difference on account of path difference

$= \dfrac{\text{Optical path difference}}{\lambda} 2\pi$, where λ is the wavelength in free space.

$= \dfrac{\mu[\text{Geometrical path difference}]}{\lambda} 2\pi$

- In the case of reflection, the reflected disturbance differs in phase by π with respect to the incident one if the wave is incident on a denser medium from a rarer medium. No such change of phase occurs when the wave is reflected in going from a denser medium to a rarer medium.

Illustration 4. *Two light rays having the same wavelength λ in vacuum are in phase initially. Then, the first ray travels a path of length L_1 through a medium of refractive index n_1, while the second ray travels a path of length L_2 through a medium of refractive index n_2. The two waves are combined to observe interference. Then, what will be the phase difference between the two waves?*

Solution : Phase difference = $\frac{2\pi}{\lambda} \times$ path difference

Here, path difference = $(n_1L_1 - n_2L_2)$

$\therefore \quad \phi = \frac{2\pi}{\lambda}\left(n_1L_1 - n_2L_2\right)$

Exercise 6.

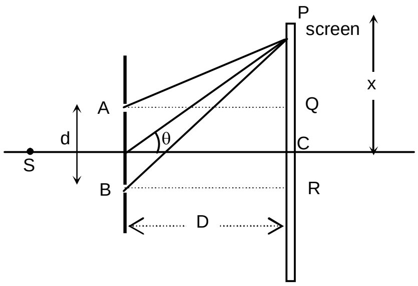
(i) ***When a wave undergoes reflection at a rarer medium, what happens to its phase ?***

(ii) ***When a wave undergoes reflection at a denser medium, what happens to its phase?***

(iii) ***What is the phase difference between any two points of a wavefront ?***

Analytical treatment of Interference:

Assume a monochromatic source of light S emits waves of wavelength λ and two narrow pinholes A and B are located at equidistant from S. A and B act as virtual coherent sources. Let the amplitude of waves be a. The phase difference between the two waves reaching the point P, at any instant, is δ.



If y_1 and y_2 are the displacements

$y_1 = a \sin \omega t$

$y_2 = a \sin (\omega t + \delta)$

$\therefore\ y = y_1 + y_2 = a \sin \omega t + a \sin (\omega t + \delta)$

$= a \sin \omega t + a \sin \omega t \cos\delta + a \cos \omega t \sin \delta$

$= a \sin \omega t\,(1 + \cos \delta) + a \cos \omega t \sin \delta$

Taking $a(1+ \cos \delta) = R \cos \theta$ and $a \sin \delta = R \sin \theta$

$\Rightarrow y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$ and

$= R \sin (\omega t + \theta)$

This represents the equation of simple harmonic vibration of amplitude R.

Next $R^2 \sin^2\theta + R^2 \cos^2\theta = a^2 \sin^2\delta + a^2 (1 + \cos\delta)^2$

$\Rightarrow R^2 = a^2 \sin^2\delta + a^2 + a^2 \cos^2\delta + 2a^2 \cos\delta$

$\Rightarrow R^2 = 2a^2 + 2a^2 \cos\delta$

$= 4a^2 \cos^2 (\delta/2)$

intensity at a point is given by the square of the amplitude

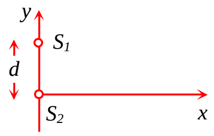
$\Rightarrow I = R^2$

$I = 4a^2 \cos^2 \delta/2$

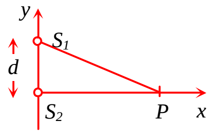
Special cases:

(i) When the phase difference $\delta = 0, 2\pi, 2(2\pi) \ldots\ldots n(2\pi)$
or $x = 0, \lambda, 2\lambda, \ldots\ldots n\lambda$
then $I = 4a^2$
(ii) When the phase difference $\delta = \pi, 3\pi \ldots\ldots (2n+1)\pi$
or $x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots\ldots (2n+1)\lambda/2$
then $I = 0$

Illustration 5. *There are two coherent point sources S_1 and S_2 placed on the y-axis as shown in the figure. The wavelength of the light emitted from the sources is λ. The distance between two sources is $d = 2\lambda$. Find the locations of all the minima on the positive x-axis.*



Solution: Consider a point P on the positive x-axis at a distance x from the origin, for minima,



$$(S_2P - S_1P) = \left(n + \frac{1}{2}\right)\lambda$$

$$\sqrt{x^2 + (2\lambda)^2} - x = \left(n + \frac{1}{2}\right)\lambda$$

$$\Rightarrow \sqrt{x^2 + 4\lambda^2} = x + \left(n + \frac{1}{2}\right)\lambda$$

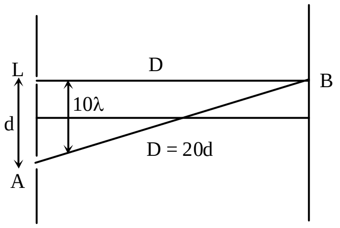
On squaring and solving, we get

$$x = \frac{\left[4 - \left(n + \frac{1}{2}\right)^2\right]\lambda}{(2n+1)}, \text{ where } n = 0, 1, 2, \ldots$$

For $n = 1$, $x = 7\lambda/12$
For $n = 0$, $x = 15\lambda/14$

Illustration 6. *The maximum intensity in Young's double slit experiment is I_0. Distance between the slits is $d = 10\lambda$, where λ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen at a distance $D = 20d$?*

Solution: Path difference = $\sqrt{D^2 + d^2} - D$



i.e. $\Delta x = D\left(1 + \frac{d^2}{2D^2}\right) - D$ ($D \gg d$ approximation)

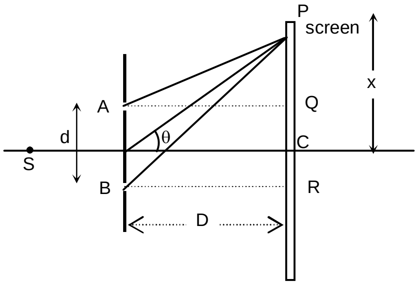
$$= \frac{d^2}{2D} = \frac{d^2}{2 \times 20d}$$

$$\therefore \quad \Delta x = \frac{d}{40} = \frac{10\lambda}{40} = \frac{\lambda}{4}$$

$$\therefore I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right) = I_0 \cos^2\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{4}\right) = I_0 \cos^2\left(\frac{\pi}{2}\right) = 0.$$

Fringe Width

Consider a narrow coherent source S and pinholes A and B, equidistant from S. A and B act as two coherent sources separated by a distance d. Let a screen the placed at a distance D from the coherent source. The point C on the screen is equidistant from points A and B, thus, the point C has maximum intensity.



Consider a point P at a distance x from C. The waves reach at the point P from A and B.

Here PQ = x – d/2

PR = x + d/2

$$(BP)^2 - (AP)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$\Rightarrow$ $(BP)^2 - (AP)^2 = 2xd$

$\Rightarrow$ $BP - AP = \dfrac{2xd}{BP + AP}$

But $BP \approx AP \approx D$

$\therefore$ Path difference $BP - AP = \dfrac{2xd}{2D} = \dfrac{xd}{D}$

Phase difference $= \dfrac{2\pi}{\lambda}\left(\dfrac{xd}{D}\right)$

(i) Bright fringe:

$$\frac{xd}{D} = n\lambda$$

where n = 0, 1, 2, 3, ...

$$x = \frac{n\lambda D}{d}$$

This equation gives the distance of the bright fringe from the point C.

When n = 1, $x_1 = \dfrac{\lambda D}{d}$

When n = 2, $x_2 = \dfrac{2\lambda D}{d}$

When n = n, $x_n = \dfrac{n\lambda D}{d}$

Therefore, the distance between any two consecutive bright fringes is

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{\lambda D}{d}$$

$\Rightarrow$ $\beta = \dfrac{\lambda D}{d}$

(which is fringe width.)

(ii) Dark fringes

$$\frac{xd}{D} = (2n-1)\frac{\lambda}{2}, \text{where } n = 1, 2, 3 \ldots.$$

or $x = \dfrac{(2n-1)\lambda D}{2d}$

where, n = 1, $x_1 = \dfrac{\lambda D}{2d}$

n = 2 $x_2 = \dfrac{3}{2}\dfrac{\lambda D}{d}$

n = 3 $x_3 = \frac{5}{2}\frac{\lambda D}{d}$

The distance between any two consecutive dark fringes,

$$x_3 - x_3 = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{\lambda D}{d} = \Rightarrow x_2 - x_1$$

Thus, $\beta = \frac{\lambda D}{d}$

Which is same for bright fringe also.

Illustration 7. *A monochromatic light of wavelength 5100 $\overset{0}{\mathrm{A}}$ from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 100 cm away is 1 cm, find the slit separation.*

Solution: $\beta = \frac{\lambda D}{d}$

$\lambda = 5100 \times 10^{-8}$ cm

D = 100 cm

Given 10 β = 1 cm , β = 0.1 cm

$$d = \frac{\lambda D}{\beta} = \frac{5100 \times 10^{-8} \times 100}{0.1} = 0.051 \text{ cm.}$$

Illustration 8. *Two coherent sources are 0.27 mm apart and the fringes are observed on a screen 120 cm away. It is found that with a certain monochromatic source of light, the fourth bright fringe is situated at a distance of 10.8 mm from the central fringe. Calculate the wavelength of the light.*

Solution: Here D = 120 cm , d = 0.27 mm = 0.027 cm

n = 4, x = 10.8 mm = 1.08 cm

λ = ?

$$x = \frac{nD\lambda}{d};\ \ \lambda = \frac{xd}{nD} = \frac{1.08 \times 0.027}{4 \times 120}$$

$\Rightarrow \lambda$ = 6075 Å

Illustration 9. *In Young's experiment, the width of the fringes obtained with light of wavelength 6000 $\overset{0}{\mathrm{A}}$ is 2.0 mm. What will be the fringe width if the entire apparatus is immersed in a liquid of refractive index 1.33?*

Solution: *As* $\beta = \frac{\lambda D}{d}$ and $\beta_1 = \frac{\lambda_1 D}{d}$

$$\therefore \quad \frac{\beta_1}{\beta} = \frac{\lambda_1 D/d}{\lambda D/d} = \frac{\lambda_1}{\lambda} = \frac{1}{\mu}$$

or, $\beta_1 = \frac{\beta}{\mu} = \frac{2.0}{1.33} = 1.5\,\text{mm}$

Illustration 10. *In Young's experiment, two slits are 0.2 mm apart. The interference fringes for light of wavelength 6000 $\overset{0}{\mathrm{A}}$ are formed on a screen 80 cm away from slits.*

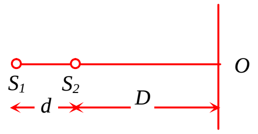
(a) How far is the second bright fringe from the central fringe ?

(b) How far is the second dark fringe from the central fringe ?

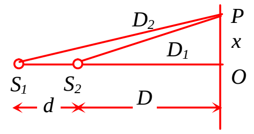
Solution: Here, d = 0.2 mm = 2×10^{-4} m
$\lambda = 6000\,\overset{0}{A} = 6 \times 10^{-7}$ m
D = 80 cm = 0.8 m.
(a) x = ?, n = 2 (for second bright fringe)
$$x = \frac{n\lambda D}{d} = \frac{2\times 6\times 10^{-7}\times 0.8}{2\times 10^{-4}}$$
$= 4.8 \times 10^{-3}$ m.

(b) x = ? n = 2 (for 2^{nd} dark fringe)
$$x = (2n-1)\left(\frac{\lambda}{2}\right)\frac{D}{d}$$
$$= \frac{3\times 6\times 10^{-7}\times 0.8}{2\times 2\times 10^{-4}}$$
$= 3.6 \times 10^{-3}$ m.

Illustration 11. *Two point coherent sources are on a straight line $d = n\lambda$ apart. The distance of a screen perpendicular to the line of the sources is D >> d from the nearest source. Calculate the distance of point on the screen where the first bright fringe is formed.*



Solution: Consider any point P on the screen at a distance x from O. Then

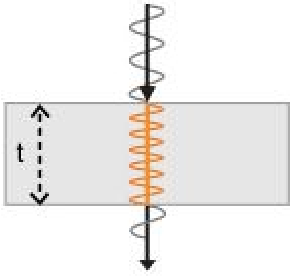


$D_1^2 = D^2 + x^2$
$$D_1 = D\left(1+\frac{x^2}{D^2}\right)^{1/2} = D\left(1+\frac{x^2}{2D^2}\right)$$
$$\text{Similarly, } D_2 = (D+d)\left\{1+\frac{x^2}{2(D+d)^2}\right\}$$
$$\therefore\ D_2 - D_1 = (D+d) + \frac{x^2}{2(D+d)} - D - \frac{x^2}{2D} = d + \frac{x^2}{2}\left(\frac{1}{(D+d)} - \frac{1}{D}\right)$$
$$= d - d\frac{x^2}{2D(D+d)}$$
For point O, $D_2 - D_1 = d = n\lambda$ (given)
Thus, there is brightness at O of n^{th} order. Since, the path difference decreases, the other fringes will be of lower order. The next bright fringe will be of (n – 1)th order. Hence, for next bright fringe
$D_2 - D_1 = (n-1)\lambda$
$$d - d\frac{x^2}{2D(D+d)} = (n-1)\lambda$$
$$n\lambda - n\lambda\frac{x^2}{2D(D+d)} = (n-1)\lambda$$
$$\Rightarrow x = \sqrt{\frac{2D(D+n\lambda)}{n}}$$

Optical path

When a ray of light travelling in air enters into a medium M, its speed changes. If c and v be the speeds in the two media, respectively, then the refractive index of medium M with respect to air is given by,

$${}_{Air}\mu_{medium} = c/v$$



Waves are longer in the medium where light travels faster

But the velocity of light in each medium is the product of the frequency and the wavelength in that medium. The frequency (colour) of light remains the same in the two media but the wavelength undergoes change. Thus,

$$\frac{c}{v} = \frac{f\lambda_{air}}{f\lambda_{medium}} = \frac{\lambda_{air}}{\lambda_{medium}}$$

$$\therefore \quad \lambda_{mediuim} = \frac{\lambda_{air}}{\mu}$$

If a ray of light is normally incident on a thickness t of glass of refractive index μ, then the number of waves contained in the glass slab would be

$$t/\lambda_g$$

Since each wave in air is of greater length λ_a (which is n times λ_g), the greater length of path required to contain the same number of waves would be given by,

$$\frac{t}{\lambda_g} = \frac{\ell}{\lambda_a} \qquad \therefore \qquad \ell = \frac{\lambda_a}{\lambda_g} t = {}_a\mu_g\, t$$

This length ℓ of path in air that contains the same number of waves as the path of length t in glass is the optical equivalent of the path t in glass.

For the path containing t/λ_g number of waves in air, the phase difference between its end points would still be

$$2\pi \frac{t}{\lambda_g}$$

Thus, the same phase difference is caused by a path of length t in glass and a path of length $\ell = \mu t$ in air. This length of path in air, that causes the same phase difference as a path of length t in glass is called the optical path or optical path equivalent for the path of length t in glass. A path of length t in a medium of refractive index μ is optically equivalent to a path of length μt in air.

Effects of changes made in Young's apparatus

In the Young's double slit experiment, therefore, the central bright fringe is formed at that point to which the optical paths from the source slit are equal.

If the Young's double slit experiment is conducted in a medium of refractive index μ, then the wavelength of light used changes from λ to λ/μ, thus making the fringe width smaller. But the position of the central bright fringe remains unchanged as the two paths remain equal.

But if one of the slits is covered by thickness t of transparent medium of refractive index μ, then the path from that slit to the centre of the fringe pattern becomes optically longer than the other.

Thus, the zeroth fringe shifts to a new position where the two optical paths are equal. The entire fringe pattern being grouped symmetrically around the zeroth fringe shifts along the screen in the direction of the covered slit. But the width of the fringes remains unchanged as the wavelength in air remains the same as before.

If the source slit is moved away from the double slits, then the fringe placement and width remain unaltered but fringes become less bright.

In the event that the source slit is moved parallel to the plane containing double slits, the entire fringe pattern shifts along the screen in the opposite direction.

If the slits are made wider, then the fringes become brighter but less sharply defined.

If the monochromatic source is replaced by a white light source, then only the zeroth fringe is observed (white fringe) with coloured edges, red being the outermost. The other fringes are a blur of overlapping colours.

Exercise 7.

What changes in interference pattern in Young's double slit experiment will be observed when

(i) distance between the slits is reduced.

(ii) the apparatus is immersed in water.

Intensity variation on screen

If A and I_o represent amplitude and intensity of each wave, then the resultant intensity at a point on the screen corresponding to the angular position θ as in figure below, is given by

$$I = 4I_o \cos^2\frac{\phi}{2} \quad \text{where } \phi = \frac{2\pi(d\sin\theta)}{\lambda}$$

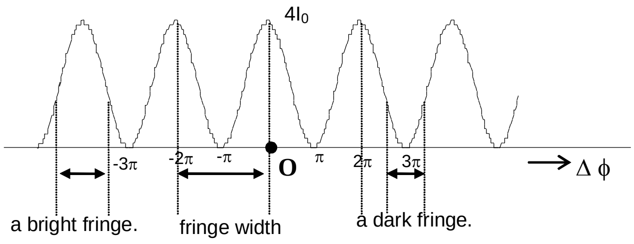


Illustration 12. *The intensity of the light coming from one of the slits in a Young's double slit experiment is double the intensity from the other slit. Find the ratio of the maximum intensity to the minimum intensity in the interference fringe pattern observed.*

Solution : .
$$\frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right]^2 \qquad \text{As} \quad I_1 = 2I_2$$

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{2}+1}{\sqrt{2}-1}\right]^2$$

Illustration 13. *The width of one of the two slits in a Young's double slit experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportional to the slit width, find the ratio of the maximum to minimum intensity in the interference pattern.*

Solution : Amplitudes from the slits are A and 2A.

$A_{max} = A + 2A = 3A$ and $A_{min} = 2A - A = A$

$$\frac{I_{max}}{I_{min}} = \frac{A^2_{max}}{A^2_{min}} = 9$$

Illustration 14. *In a double slit experiment, the separation between the slits is d = 0.25 cm and the distance of the screen D = 120 cm from the slits. If the wavelength of light used is λ = 6000 Å and I_0 is the intensity of the central bright fringe, what is the intensity at a distance $x = 4.8 \times 10^{-5}$ m from the central maximum?*

Solution: $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$

At central fringe, $\delta = 0$

$\therefore\ I_0 = I_1 + I_1 + 2I_1 = 4I_1$

At a distance x, path difference $= \frac{xd}{D}$.

The corresponding phase difference, $\delta = \left(\frac{2\pi}{\lambda}\right)\left(\frac{xd}{D}\right)$

$$\therefore\quad I' = \frac{I_0}{4} + \frac{I_0}{4} + 2\left(\frac{I_0}{4}\right)\cos\left(\frac{2\pi xd}{\lambda D}\right)$$

$$\Rightarrow\quad I' = I\cos^2\left(\frac{\pi \times 4.8 \times 10^{-5} \times 0.25 \times 10^{-2}}{6000 \times 10^{-10} \times 1.2}\right)$$

$$= I_0\cos^2\left(\frac{\pi}{6}\right) = \frac{3I_0}{4}.$$

Illustration 15. *Two waves $y_1 = A_1\sin(\omega t - \beta_1)$ and $y_2 = A_2\sin(\omega t - \beta_2)$ superimpose to produce a resultant wave. The amplitude of the resultant wave is*

(A) $A_1 + A_2$ *(B)* $|A_1 - A_2|$

(C) $\sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\beta_1 - \beta_2)}$ *(D)* $\sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\beta_1\beta_2)}$

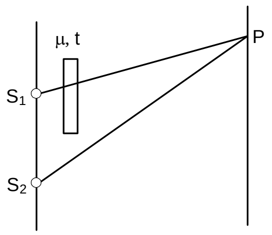
Solution: **(C)**

Angle between A_1 and A_2 is the phase difference $(\beta_1 - \beta_2)$ between them,

$$\therefore\quad R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\beta_1 - \beta_2)}$$

Displacement of fringes

When a film of thickness 't' and refractive index 'μ' is introduced in the path of one of the source of light, then fringe shift occurs as the optical path difference changes.



Optical path difference at

$P = S_2P - [S_1P + \mu t - t] = S_2P - S_1P - (\mu - 1)t$

$= y.d/D - (\mu - 1)t$

$\Rightarrow$ nth fringe is shifted by $\Delta y = \frac{D(\mu - 1)t}{d}$

As $w = \frac{D\lambda}{d}$, $\Delta y = \frac{w}{\lambda}(\mu - 1)t$

Illustration 16. *Interference pattern with Young's double slit 1.5 mm apart are formed on a screen at a distance of 1.5 m from the plane of slits. In the path of the beam from one of the slits, a transparent film of 10 micron thickness and of refractive index 1.6 is inserted. While in the path of the beam from the other slit, a transparent film of 15 micron thickness and of refractive index 1.2 is interposed. Find the displacement of the fringe pattern.*

Solution: Shift = $[(\mu_2 - 1)t_2 - (\mu_1 - 1)t_1]\frac{D}{d}$

$= [(1.2 - 1) \times 15\times10^{-6} - (1.6 - 1) \times 10 \times 10^{-6}]\frac{1.5}{1.5\times10^{-3}}$

$= 3 \times 10^{-3}$ m = 3 mm

Illustration 17. *S_1 and S_2 are two point sources of radiation emitting the waves in phase with each other of wavelength 400 nm. The sources are located on x-axis at x = 6.5 μm and x = –6.0 μm, respectively.*

(a) Determine the phase difference (in radians) at the origin between the radiations coming from S_1 and S_2.

(b) Suppose a slab of transparent material with thickness 1.5 μm and refractive index μ = 1.5 is placed between x = 0 and x = 1.5 μm. What then is the phase difference at the origin between the radiation from S_1 and the radiation from S_2.

Solution: (a) $\Delta x = 0.5\ \mu m = 5.0 \times 10^{-7}$ m

$$\phi = \frac{2\pi}{\lambda}\Delta x = \left(\frac{2\pi}{400\times10^{-9}}\right)\left(5\times10^{-7}\right) = \frac{5\pi}{2}$$

(b) $\Delta x = (\mu - 1)t = (1.5 - 1)(1.5) = 0.75\ \mu m$

$(\Delta x)_{net} = \Delta x + \Delta x = 1.25\ \mu m = 1.250 \times 10^{-6}$ m

$$\therefore\ \phi' = \left(\frac{2\pi}{400\times10^{-9}}\right)\left(1.25\times10^{-6}\right) = 6.25\ \pi$$

Illustration 18. *Sodium light (λ = 5890 Å) falls on a double slit whose separation is d = 0.20 mm. A thin lens (f = +1m) is placed near the slits. What is the linear fringe separation on the screen placed in the focal plane of the lens?*

Solution: $\beta = \frac{\lambda D}{d}$, if θ is the angular width of the fringe, $\beta = D\theta$

$\therefore\ D\theta = \frac{\lambda D}{d}$ or $\theta = \lambda/d$,

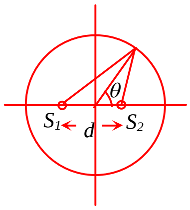
Let β' be the fringe width at the focal plane of the lens of focal length f. Then

$\theta = \frac{\beta'}{f}$

$\frac{\beta'}{f} = \frac{\lambda}{d} \Rightarrow \beta' = \frac{\lambda f}{d}$

$\therefore\ \beta' = \frac{5890\times10^{-1}\times1}{0.2\times10^{-3}}$m = 2.94 mm

Illustration 19. *Two coherent point sources S_1 and S_2 emitting light of wavelength λ are placed symmetrically about the centre of a circle of large radius, as shown in the figure. The distance between the two sources is $d = 2\lambda$. Find the position of the maxima on the circle in terms of the angle θ.*



Solution: For maxima, $d \sin\theta = \pm n\lambda$

$$\sin\theta = \pm\frac{n\lambda}{d}$$

$\sin\theta = 0, \pm ½, \pm 1$

$\theta = 0°, 30°, 90°, 150°, 180°, 210°, 270°, 330°$

illustration 20. *In a Young's double slit experiment, the fringes are displaced by a distance y when a glass plate of refractive index 1.5 is introduced in the path of one of the beams. When this plate is replaced by another plate of the same thickness, the shift of fringes is $\frac{3}{2}y$. The refractive index of the second plate is*

(A) 1.75 *(B) 1.50*
(C) 1.25 *(D) 1.00*

Solution: **(A)**

The path difference introduced by the plate of thickness t and refractive index μ is given by

$\Delta = (\mu - 1)t$.

Optical path difference (extra) shifts the fringe pattern by

$$y = \frac{(\mu-1)t\beta}{\lambda}.$$

For first plate, $y = \frac{(\mu_1-1)t\beta}{\lambda}$

For second plate, $\frac{3}{2}y = \frac{(\mu_2-1)t\beta}{\lambda}$

$$\therefore \quad \frac{(\mu_2-1)}{(\mu_1-1)} = \frac{3}{2} \Rightarrow \frac{\mu_2-1}{1.5-1} = \frac{3}{2} \Rightarrow \mu_2 = 1.75.$$

Illustration 21. *In a double slit experiment, when a glass plate of refractive index 1.5 and thickness t is introduced in the path of one of the interfering beams of wavelength λ, intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass plate is*

(A) 2λ *(B) $2\lambda/3$*
(C) $\lambda/3$ *(D) λ*

Solution: ***(A)***

For minimum thickness, $(\mu - 1)\,t = \lambda$

or $$t = \frac{\lambda}{\mu-1} = \frac{\lambda}{1.5-1}$$

$\Rightarrow$ $$t = \frac{\lambda}{0.5} = 2\lambda.$$

Exercise 8.

(i) ***In a Young's double slit experiment, the apparatus used is the source S and two slits A and B, with slit A above slit B. The fringes are observed on a vertical screen k. The optical path length from S to B is increased very slightly (by introducing a transparent material of higher refractive index) and optical path length from S to A is not changed. What is the effect of all this on the fringe system?***

(ii) ***In a Young's double slit experiment, monochromatic light is used to illuminate the two slits A and B. Interference fringes are observed on a screen placed in front of the slits. Now, if a thin glass plate is placed normally in the path of the beam coming from the slit A, then what will be the effect on fringes in the interference pattern?***

Illustration 22. *Monochromatic light of wavelength of 600 nm is used in a YDSE. One of the slits is covered by a transparent sheet of thickness 1.8 x 10^{-5} m made of a material of refractive index 1.6. How many fringes will shift due to the introduction of the sheet?*

Solution: As derived earlier, the total fringe shift = $\frac{\beta}{\lambda}(\mu-1)t$.

As each fringe width = β,

The number of fringes that will shift = $\frac{\text{total fringe shift}}{\text{fringe width}}$

$$=\frac{\frac{\beta}{\lambda}(\mu-1)t}{\beta}=\frac{(\mu-1)t}{\lambda}=\frac{(1.6-1)\,x1.8\,x10^{-5}\,m}{600\,x10^{-9}\,m}=18$$

Illustration 23. *A thin sheet of glass (μ = 1.520) is introduced normally in the path of one of the two interfering waves. The central bright fringe is observed to shift to the position originally occupied by the fifth bright fringe. If λ = 5890A°, find the thickness of the glass sheet.*

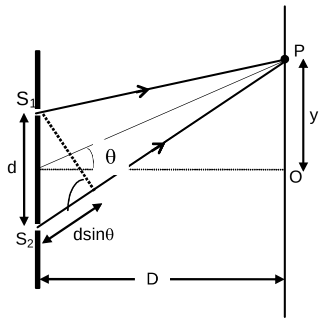
Solution: x_0 = displacement of images = 5w

$$\therefore\ \frac{D}{d}(\mu-1)t=5\times\frac{\lambda D}{d}$$

$$t=\frac{5\lambda}{2(\mu-1)}=\frac{5}{2}\frac{5890\times10^{-8}}{(1.52-1)}$$

t = 2.83 $\times$ 10^{-4} cm. or 0.283 mm.

Illustration 24. *In an interference arrangement similar to YDSE, slits S_1 and S_2 are illuminated with coherent microwave sources each of frequency 1MHz. The sources are synchronized to have zero phase difference. The slits are separated by distance of 150 m. The intensity is measured as a function of θ, θ is defined as shown in the figure. If I_0 is maximum intensity, calculate I_θ for (a) $\theta = 0^0$ (b) $\theta = 30^0$ and (c) $\theta = 90^0$*



Solution : For microwave, as c = fλ $\Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{10^6}$ = 300 m.

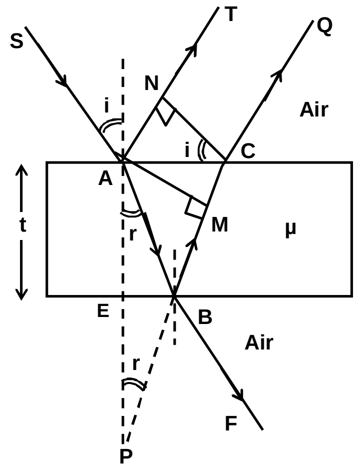
Phase difference $\phi = \frac{2\pi}{\lambda}(d\sin\theta) = \pi \sin\theta$

So, $I_\theta = I_0 \cos^2 (\phi/2) = I_0 \cos^2 \frac{\pi\sin\theta}{2}$

(a) $\theta = 0^0 \Rightarrow I = I_0$
(b) $\theta = 30^0 \Rightarrow I = I_0 / 2$
(c) $\theta = 90^0 \Rightarrow I = 0$

Interference due to reflected light (thin films)

Consider a transparent film of thickness t and refractive index μ.



A ray SA incident on the upper surface of the film is partly reflected along AT and partly refracted along AB. At B part of it is reflected along BC and finally emerges out along CQ. The difference in path between the two rays AT and CQ can be calculated. The angle of incidence is i and angle of refraction is r.

The optical path difference

x = μ(AB + BC) – AN

Here μ = sin i/sin r = AN/CM

$\therefore$ AN = μ CM

$\therefore$ x = μ (AB + BC) – μCM

= μ (AB + BC – CM)

= μ (PC – CM) = μ PM

In the Δ APM

cos r = PM/AP

PM = AP cos α = (AE + EP) cos r = 2t cos r

x = 2μt cos r … (i)

This equation in the case of reflected light does not represent the correct path difference but only the apparent. When light is reflected from the surface of an optically denser medium a phase change π, equivalent to a path difference $\lambda/2$, occurs

i. If $\Delta x = n\lambda$ (where n = 0, 1, 2, 3, … 4) then
   Constructive interference takes place and the film appears bright.
   $2\mu t \cos r - \lambda/2 = n\lambda$
   or $2\mu t \cos r = (2n + 1)\lambda/2$
ii. If the path difference $\Delta x = 2(2n - 1)\lambda/2$ where n = 1, 2, … etc.),
   Destructive interference takes place.
   $\therefore 2\mu t \cos r - \lambda/2 = (2n - 1)\lambda/2$
   $\Rightarrow 2\mu t \cos r = n\lambda$

Illustration 25. *A glass plate, having refractive index of 1.8, is coated with a thin layer of thickness d and refractive index 2.0. Light of wavelength λ, travelling through air, is incident normally on the layer. It is partly reflected at the upper and lower surfaces of the layer and the two reflected rays interfere. Obtain the least value of t for which the rays interfere constructively if λ = 600 nm.*

(A) 90 nm *(B) 60 nm*
(C) 75 nm *(D) 50 nm*

Solution : ***(C)***

There are two rays considered for interference. One ray is the result of reflection at denser medium, hence it suffers an additional path difference of $\lambda/2$ and phase difference of π. The other ray is produced after reflection at the lower surface. This reflection takes place at rarer medium, so net path difference is

$$2\mu t + \frac{\lambda}{2} = n\lambda \qquad \text{(this is the condition for constructive interference)}$$

or $$2\mu t = (2n-1)\frac{\lambda}{2},$$

It will be minimum for n = 1

i.e. $$t_{min} = \frac{\lambda}{4\mu} = \frac{600}{4\times 2} = 75\ \text{nm}.$$

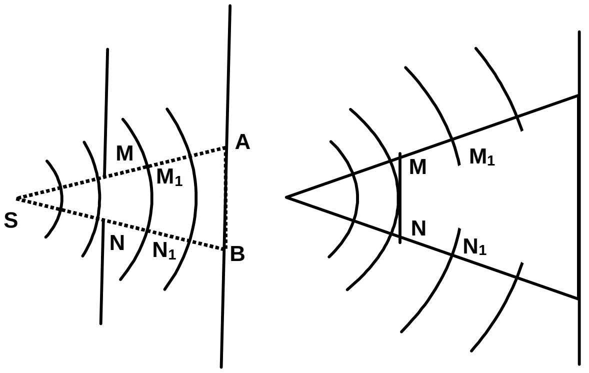
Exercise 9.

(i) White light is incident normally on a glass plate of thickness 0.50 x 10^{-6} m and index of refraction 1.50. Which wavelengths in the visible region (400 nm - 700 nm) are strongly reflected by the plate?

(ii) Why a thin film of oil on the surface of water appears coloured ?

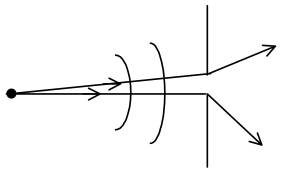
DIFFRACTION

It is a matter of common experience that the path of light entering a dark room through a hole illuminated by sunlight is straight. This phenomenon of straight line motion can be explained by Newton's corpuscular theory. But it has been observed that when a beam of light passed through a small opening, it spreads to some extent into the region of geometrical shadow also. If the light energy is propagated in the form of waves, then similar to sound waves one would expect bending of a beam of light round the edges of an opaque obstacle or illumination of the geometrical shadow.



(a) Diffraction is the bending or spreading of waves that encounter an object (a barrier or an opening) in their path.

(b) In Fresnel class of diffraction, the source and/or *screen are at a finite distance from the aperture.*

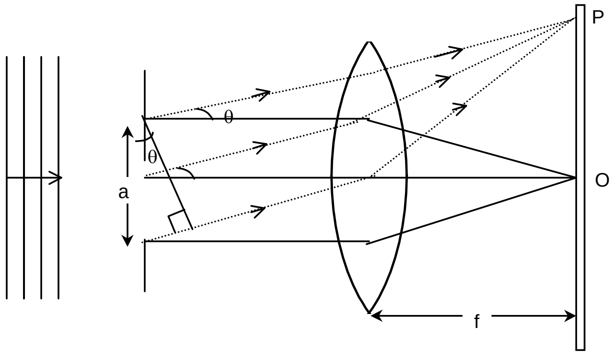


(c) In Fraunhoffer class of diffraction, the source and screen are at infinite distance from the diffracting aperture. Fraunhoffer is a special case of Fresnel diffraction.

Single Slit Fraunhoffer Diffraction

In order to find the intensity at point P on the screen as shown in the figure the slit of width 'a' is divided into N parallel strips of width Δx. Each strip then acts as a radiator of Huygen's wavelets and produces a characteristic wave disturbance at P, whose position on the screen for a particular arrangement of apparatus can be described by the angle θ.

The amplitudes ΔE_o of the wave disturbances at P from the various strips may be taken as equal if θ is not too large.



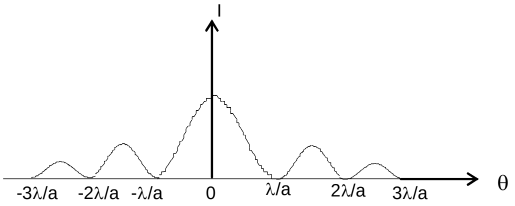
The intensity is proportional to the square of the amplitude. If I_m represents the intensity at O, its value at P is

$$I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 \;;$$

where $\alpha = \frac{\phi}{2} = \frac{\pi a \sin\theta}{\lambda}$

A minimum occurs when, $\sin\alpha = 0$ and $\alpha \neq 0$, so $\alpha = n\pi$, $n = 1, 2, 3...$

$$\Rightarrow \quad \frac{\pi a \sin\theta}{\lambda} = n\pi \quad \Rightarrow a\sin\theta = n\lambda$$



Angular width of central maxima of diffraction pattern = $2\theta_1 = 2\sin^{-1}(\lambda/a)$

[θ_1 gives the angular position of first minima]

The concept of diffraction is also useful in deciding the resolving power of optical instruments.

Illustration 26. *Light of wavelength 6×10^{-5}cm falls on a screen at a distance of 100 cm from a narrow slit. Find the width of the slit if the first minima lies 1mm on either side of the central maximum.*

Solution: Here n = 1, $\lambda = 6 \times 10^{-5}$ cm.

Distance of screen from slit = 100 cm.

Distance of first minimum from central maxima = 0.1 cm.

$$\sin\theta = \frac{\text{Distance of 1}^{\text{st}}\text{ minima from the central maxima}}{\text{Distance of the screen from the slit}}$$

$$\theta_1 = \frac{0.1}{100} = \frac{1}{1000}$$

We know that $a\sin\theta = n\lambda$

$$a = \frac{\lambda}{\theta_1} = 0.06 \text{ cm.}$$

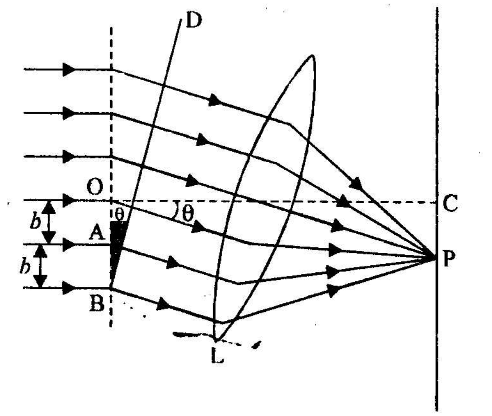
Exercise 10.

(i) ***Describe what happens to the Fraunhofer single slit diffraction pattern if the whole apparatus is immersed in water***

(ii) ***Light of wavelength 633 nm is incident on a narrow slit. The angle between the first minima on one side of the central maxima and the first minima on the other side is 1.97^0. Find the width of the slit.***

The diffraction grating:

It is glass plate upon which is ruled a large number of equally spaced opaque lines. These lines are generally several thousand per centimeter.



Consider the parallel rays making an angle θ with the normal (OC) to the grating. The rays are brought to focus on the screen at point P by a converging lens L.

If the ray AP travels a distance λ farther than ray BP, then waves from A and B will interfere constructively at P. The wave front BD makes an angle θ with the grating.
From the right angled triangle as shown in figure

$$\sin\theta = \frac{\lambda}{AB}$$

or, $\lambda = AB\sin\theta$
or, $\lambda = b\sin\theta$
This is the condition for reinforcement in the direction θ.

There will be other directions on each side of OC for which waves from adjacent slits differ in phase by 2λ, 3λ . . . .

In general, the grating equation may be written as

$$b\sin\theta_n = n\lambda$$

where b is the grating space and n is the order of the spectrum.

Rayleigh Criterion
According to Rayleigh criterion, when the central maximum in the diffraction pattern of one point source falls over the first minimum in the diffraction pattern of the other point source. Then the two point sources are said to have been resolved by the optical instrument.

Resolving power of microscope
The resolving power of microscope is its ability to form separate images of two point objects lying close together.

The resolving power of a microscope is defined as the reciprocal of the distance between two objects which can be just revolved when seen through the microscope.

∴ Resolving power of microscope $= \frac{1}{\Delta d} = \frac{2\mu\sin\theta}{\lambda}$

$$= \frac{1}{\Delta d} = \frac{2u\sin\theta}{\lambda}$$

Where
(i) μ is the refractive index of the medium between the object and the objective of microscope.
(ii) λ is the wavelength of the light.
(iii) The angle θ subtended by the radius of the objective on one of the objects.

Resolving power of a telescope

The resolving power of a telescope is the reciprocal of the smallest angular separation between two distinct objects whose images are separated in the telescope. This is given by

$$d\theta = \frac{1.22\lambda}{a}$$

Where $d\theta$ is the angle subtended by the point object at the objective. λ is the wavelength of light used and a is the diameter of the telescope objective.
Clearly, a telescope having larger aperture objective gives a high resolving power.

Illustration 27. *Assume that the mean wavelength of white light is 555 nm. Estimate the smallest angular separation of the two stars which can be just resolved by the telescope. Given the diameter of the objective of an astromical telescope is 25 cm.*

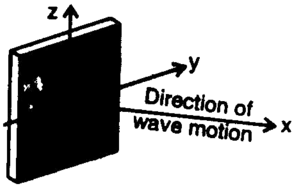
Solution: Since, $\theta = \frac{1.22\lambda}{a}$

$\therefore\ \theta = \frac{1.22 \times 555 \times 10^{-9}}{0.25}$ rad

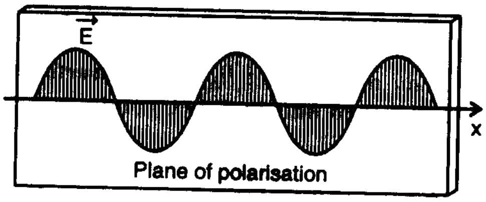
$\theta = 2.7 \times 10^{-6}$ rad.

Polarization

An ordinary beam of light consist of a large number of waves emitted by the atoms or molecules of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$. Since, all directions of vibrations of $\vec{E}$ are equally probably therefore resultant electromagnetic wave is called un-polarized light and it is symmetrical about the direction of wave propagation as shown in figure (A).



Unpolarised light
(A)

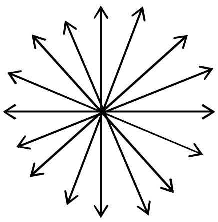


Plane polarised light
(B)

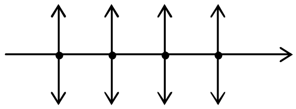
However, if by some means we confine the vibrations of electric vector in one direction perpendicular to the direction of wave motion, the light is said to be plane polarized or linearly polarized as shown in the figure (B).

Hence the phenomenon of confining the vibrations of a wave in a specific direction perpendicular to the direction of wave motion is called polarization. The plane perpendicular to the plane of polarization i.e. the plane in which no vibration occur is known as plane of vibration. The plane of polarization is that plane in which no vibrations occur.
For the shape of convenient representation, the vibrations may be assumed to be resolved into two rectangular components, in the planes of the paper and perpendicular to the plane of paper.

The vibrations in the plane of paper are represented by double arrow straight lines. While the vibrations perpendicular to the plane of paper are represented by dots as shown in figure.

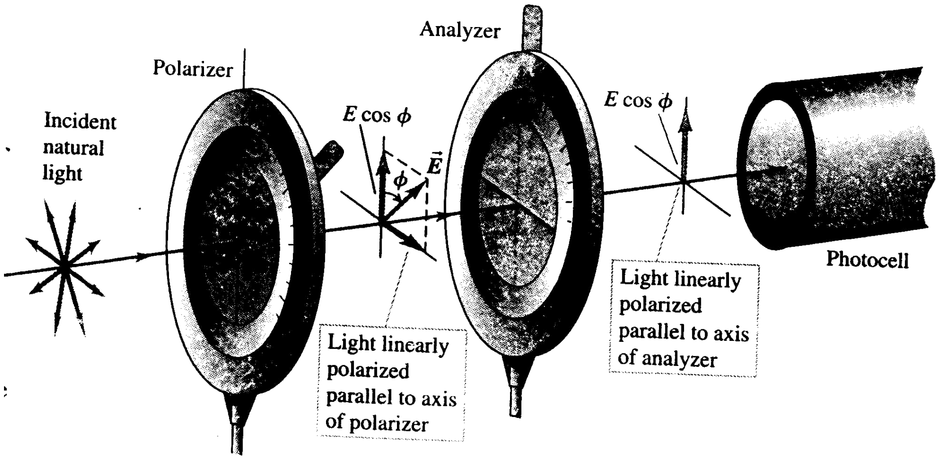


Natural light

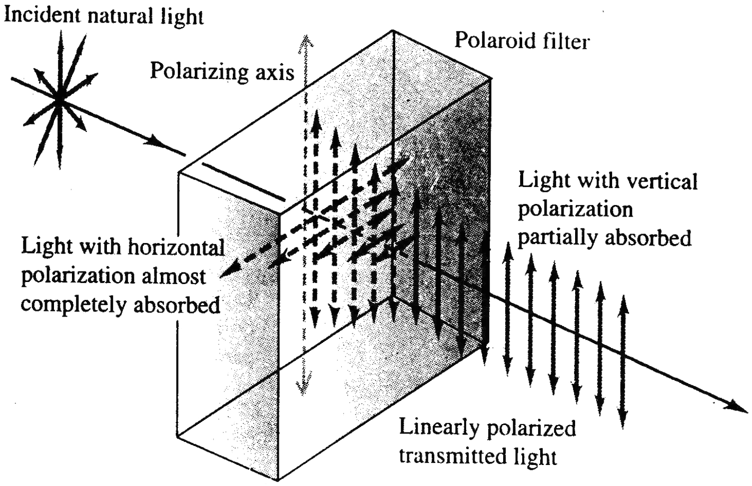


Representation of natural light

Polarizing filter: The emitted light is a random mixture of waves linearly polarized in all possible transverse directions. Such light is called unpolarised light or natural light. To create polarized light from unpolarised natural light requires a filter that is analogous to the slot for mechanical waves.



The most common polarizing filter for visible light is a material known by the trade name Polaroid, widely used for sunglasses and polarizing filter for camera lenses.

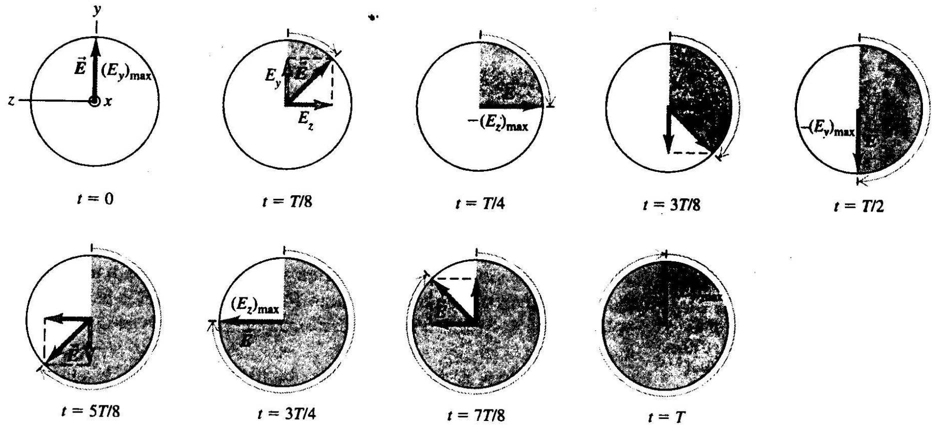


This material incorporates substances that have dichroism, a selective absorption in which one of the polarized component is absorbed much more strongly that the other. A Polaroid filter transmits 80 % or more of the intensity of a wave that is polarized parallel to a certain axis in the material, called the polarizing axis, but only 1 % or less for waves that are polarized perpendicular to the axis. In one type of Polaroid filter, long – chain molecules within the filter are oriented with their axis perpendicular to the polarizing axis. These molecules preferentially absort light that is polarized along their lengths.

An ideal polarizing filter passes 100 % of the incident light that it polarized in the direction of filter's polarizing axis but completely blocks all light that is polarized perpendicular to this axis.

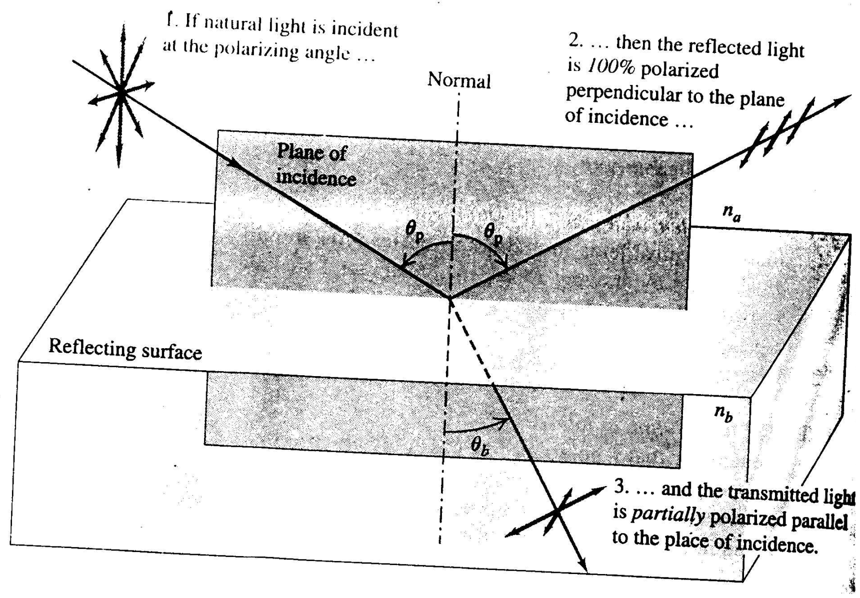
When unpolarised light is incident on an ideal polarizer, the intensity of the transmitted light is exactly half that of the incident unpolarised light, no matter how the polarizing axis is oriented. Here's why we can resolve the $\vec{E}$ field of the incident wave into a component parallel to the polarizing axis and a component perpendicular to it. Because the incident light is a random mixture of all sates of polarization, these two components are, on average, equal.

What happens when the linearly polarized light emerging from a polarizer passes through a second polarizer. Consider the general case in which the polarizing axis of the second polarizer or analyzer makes an angle ϕ with the polarizing axis of first polarizer.
Then $I = I_{max} \cos^2 \phi$



Since the intensity of an electromagnetic wave is proportional to the square of the amplitude of the wave, the ratio of transmitted to incident amplitude is cos ϕ, so the ratio of transmitted to incident intensity is $\cos^2 \phi$

Polarization by reflection: Unpolarized light can be polarized, partially or totally, by reflection. When unpolarised natural light is incident on a reflecting surface between two transparent optical materials, then for the most of the incident, waves, for which the electric – field vector $\vec{E}$ is perpendicular to the plane of incidence are refracted more strongly than those for which $\vec{E}$ lies in this plane. In this case the reflected light is practically polarized in the direction perpendicular to the plane of incidence. But at one particular angle of incidence called the polarizing angle θ_p, the light for which $\vec{E}$ lies in the plane of incidence is not reflected at all but is completely refracted. At this same angle of incidence the light for which $\vec{E}$ is perpendicular to the plane of incidence is partially reflected and partially reflected and partially refracted. The refracted light is therefore completely polarized perpendicular to the plane of incidence.



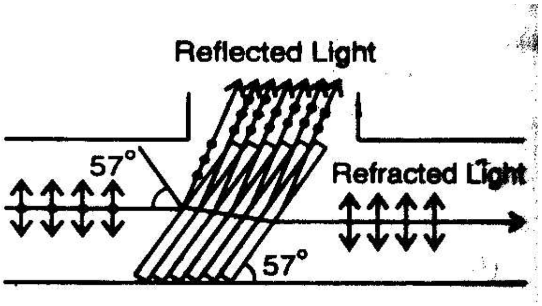
If natural light is incident at the polarizing angle.

Brewster's law: When the angle of incidence is equal to the polarizing angle i_p the reflected ray and refracted ray are perpendicular to each other. In this case the angle of refraction r_c becomes the complement of i_p, so $r_c = 90^0 - i_p$

$\Rightarrow \mu_1 \sin i_p = \mu_2 \sin (90^0 - i_p)$ $\Rightarrow \tan i_p = \mu_2 / \mu_1$

Polarization by Refraction

By refraction method, a pile of glass is formed by taking 20 to 30 microscope slides and light is made to be incident at polarizing angle (57^0). In accordance with Brewster law, the reflected light will be plane polarized with vibrations perpendicular to plane of incidence and the transmitted light will be partially polarized. Since in one reflection about 15 % of the light with vibration perpendicular to plane of paper is reflected, therefore after passing through a number of plates as shown in figure emerging light will become plane polarized with vibrations in the plane of paper.



Double Refraction: When a ray light is refracted by a crystal of calcite it gives two refracted rays. This phenomenon is called double refraction.

When a ray of light AB is incident on the calcite crystal making an angle of incidence = i, it is refracted along two paths inside the crystal as shown in figure.

(i) along BC making an angle of refraction = r_2 and
(ii) along BD making an angle of refraction = r_1. These two rays emerge out along DO and CE which are parallel. The ordinary ray (O-ray) has a refractive index $\mu_0 = \frac{\sin i}{\sin r_1}$ and the extraordinary ray (e – ray) has a refractive index $\mu_e = \frac{\sin i}{\sin r_2}$.

The e-ray and the e-ray travel with the same speed along a particular direction inside the crystal called the optic axis.

Dichroism

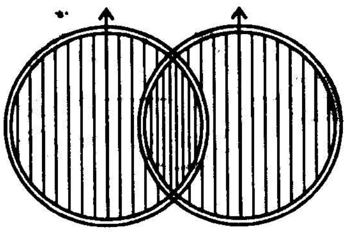
There are certain crystals and minerals which are doubly refracting and have the property of absorbing the ordinary and extraordinary rays unequally. In this way, plane polarized light is produced. The crystals showing this property are said to be dichroic and the phenomenon is known as dichroism. Tourmaline is a dichroic crystal and absorbs the ordinary ray completely. But these crystals are not stable and are affected by slightly strain. To remove this difficulty, a polarizer in the forms of large sheets is developed which is called Polaroid.

Polorids

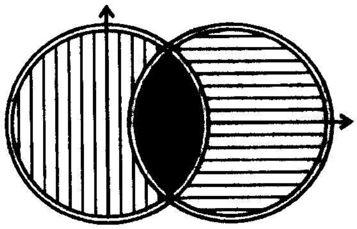
Herapathite crystals are embedded in a volatile viscous medium and the crystals are aligned with their optics axes parallel. The layer of crystals are mounted between glass sheets so that the crystals are not spoilt.

It is a sheet of polariser and also known as Polaroid.

Two Polaroid films mounted separately in rings between thin glass plates are used on the parallel position as shown in figure (a) light vibrating in the plane indicated by parallel lines is transmitted. In the crossed position as shown in figure (b). the axes of the polaroids are perpendicular to each other. So no light is transmitted.



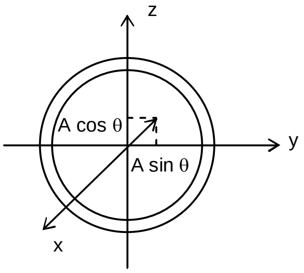
Parallel-polaroids
(A)



Crossed-polaroids
(B)

Intensity of light emerging from a Polaroid

If a plane polarized light of light intensity I_0 ($=kA^2$) is incident on a polaroid and its vibrations of amplitude A make an angle θ with the transmission axis, then the component of vibrations parallel to transmission axis, will be A $\cos\theta$ while perpendicular to it will be A $\sin\theta$



Now, Polaroid will pass only those vibrations which are parallel to its transmission axis. i.e. A $\cos\theta$, so the intensity of emergent light will be

$$I = k(A\cos\theta)^2 = kA^2\cos^2\theta$$

$$I = I_0\cos^2\theta \qquad [\because I_0 = kA^2]$$

This law is called Malus law.

Illustration 28. *Two polarizing sheets are placed with their planes parallel, so that light intensity transmitted is maximum. Through what angle must either sheet be turned so that light intensity drops to half the maximum value ?*

Solution: According to law of Malus

$I = I_0\cos^2\theta$

$\therefore \cos^2\theta = \frac{I}{I_0} = \frac{1}{2}$

$\therefore \quad \cos\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \pm 45^0$ or $\pm 135^0$

The effect will be same when any of the two sheets is turned through θ in any direction.

Uses of Polaroids

Polaroids are widely used as polarizing sun glass. Polaroid films are used to produce three – dimensional moving pictures. They are also used to eliminate the head light glare in motor cars.

SUMMARY

1. If two coherent waves with intensity I_1 and I_2 are superimposed with a phase difference of ϕ, the resulting wave intensity is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

2. When sources are in phase:
For maxima o.p.d. = $n\lambda$ (o.p.d. = Optical Path Difference)
For minima o.p.d. = $(n - \frac{1}{2})\lambda$
Phase difference $\phi = \frac{2\pi}{\lambda}$ (o.p.d.)

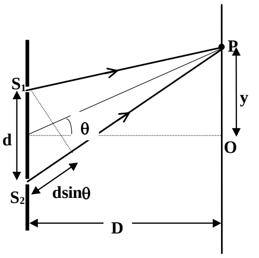
3. The phase difference between two waves at a point will depend upon
(a) the difference in path lengths of the two waves from their respective sources.
(b) the refractive index of the medium
(c) initial phase difference, if any.
(d) Reflections, if any, in the path followed by waves.
In the case of light waves, the phase difference on account of path difference

$$= \frac{\text{Optical path difference}}{\lambda} 2\pi \quad \text{where } \lambda \text{ is the wavelength in free space.}$$

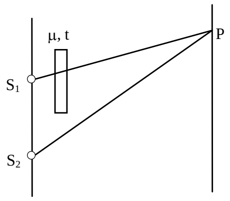
$$= \frac{\mu[\text{Geometrical path difference}]}{\lambda} 2\pi$$

4. In the case of reflection, the reflected disturbance differs in phase by π with the incident one if the incidence occurs in rarer medium. There would be no phase difference if incidence occurs in denser medium.

5. Young's Double Slit Experiment
(i) For maxima $d\sin\theta = n\lambda$
(ii) For minima $d\sin\theta = (2n + 1)\lambda/2$
(iii) Fringe width , $\omega = \lambda D/D$
(iv) Displacement of fringe pattern



6. When a film of thickness 't' and refractive index 'μ' is introduced in the path of one of the source's light, then fringe shift occurs as the optical path difference changes.
Optical path difference at
$P = S_2P - [S_1P + \mu t - t]$
$= S_2P - S_1P - (\mu - 1)t = y.d/D - (\mu - 1)t$



$\Rightarrow$ nth fringe shifted by $\Delta y = \frac{D(\mu - 1)t}{d}$. As $\omega = D\lambda/d$, also $\Delta y = \frac{\omega}{\lambda}(\mu - 1)t$

MISCELLANEOUS EXERCISE

1. In Young's double slit experiment, light of wavelength 5000 $\overset{0}{A}$ is used. The third bright band on the screen is formed at a distance of 0.01 m from the central bright band. If the screen is at a distance of 1.5 m from the centre of the two narrow slits, calculate the separation between the slits.

2. In a Young's double slit experiment, we observe the 10^{th} maximum for $\lambda = 7000\,\overset{0}{A}$. What order will be visible if the source of light is replaced by light of wavelength 5000 $\overset{0}{A}$?

3. State the principle of superposition of waves and conditions for obtaining a steady, sharp and clear interference pattern.

4. State the path difference between two waves for constructive interference in terms of λ.

5. What will be the effect on the fringes formed in Young's slit experiment if (i) the apparatus is immersed in water; (ii) white light is used instead of monochromatic light ?

6. What will happen to the interference pattern in Young's experiment, if the source is not exactly on the centre line between the slits ?

7. The two slits in Young's double slit experiment are separated by a distance of 0.03 cm. When light of wavelength 5000 A^0 falls on the slits, an interference pattern is produced on the screen 1.5 m away. Find the distance of 4^{th} bright fringe from the central maximum.

8. Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 8.3 mm. A second light produces an interference pattern in which the bright fringes are separated by 7.6 mm. Find the wavelength of the second light.

9. Find the minimum thickness of a film which will strongly reflect the light of wavelength 589 nm, when incident normally on it. The refractive index of the material of the film is 1.25.

10. What change is observed in interference pattern of Young's double slit experiment, if one of the two slits is pointed so that it transmits half the intensity of the other.

ANSWERS TO MISCELLANEOUS EXERCISE

1. 2.25×10^{-4} m
2. n = 14
4. path difference = 2n (λ/2) when n = 0, 1, 2, . . .
7. 1 cm
8. 576.87 nm
9. 117.8 nm

SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob 1. *Two coherent monochromatic light beams of intensities I and 4I are superposed. What would be the maximum and minimum possible intensities in the resulting beam?*

Sol. Here, $\frac{I_1}{I_2} = \frac{I}{4I} = \frac{a^2}{b^2}$

$\therefore \quad \frac{a}{b} = \frac{1}{2}$

or, b = 2a

Now, $\frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2} = \frac{(a+2a)^2}{(a-2a)^2} = \frac{9a^2}{a^2} = \frac{9I}{I}$

Prob 2. *A double slit is illuminated by light of λ = 6000 $\overset{0}{\text{A}}$. The slits are 0.1 cm apart and the screen is placed 1m away. Calculate (i) angular position of 10th maximum in radian, and (ii) separation of two adjacent minima.*

Sol. Here, λ = 6000 $\overset{0}{\text{A}}$ = 6×10^{-7} m

d = 0.1 cm = 10^{-3} m

D = 1 m, n = 10

$x = n\lambda \frac{D}{d}$

(i) If θ is the angle of diffraction. (angular position of 10th max),

then

$\sin\theta = \frac{x}{D} = \frac{n\lambda}{d} = \frac{10 \times 6 \times 10^{-7}}{10^{-3}}$

As $\sin\theta$ is small

$\theta \approx \sin\theta = 6 \times 10^{-3}$ rad.

(ii) Separation of two adjacent minima

$\beta = \frac{\lambda D}{d} = \frac{6 \times 10^{-7} \times 1}{10^{-3}} = 6 \times 10^{-4}$ m.

Prob 3. *The interference fringes for sodium light (λ = 5890 $\overset{0}{\text{A}}$) in a double slit experiment have an angular width of 0.2^0. For what wavelength will the width be 10 % greater.*

Sol. As angular width $\propto \lambda$

$\therefore \quad \frac{\lambda'}{\lambda} = \frac{\theta'}{\theta}$

Now, $\theta = 0.2^0 = 0.2 + \frac{10}{100}(0.2) = 0.22$

λ = 5890 $\overset{0}{\text{A}}$

$$\therefore \quad \lambda' = \frac{\theta'}{\theta}\lambda = 6479\,\overset{0}{A}$$

Prob 4. *A central fringe of interference pattern produced by light of wavelength 6000 $\overset{0}{A}$ is shifted to the position of 5th bright fringe by introducing a thin film of μ = 1.5. Calculate the thickness of the film.*

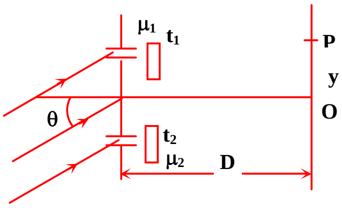
Sol. Here, $\lambda = 6000\ \overset{0}{A} = 6 \times 10^{-7}$ m
n = 5, μ = 1.5, t = ?
Since, $(\mu - 1)t = n\lambda$
$\Rightarrow t = \dfrac{n\lambda}{(\mu-1)} = 6 \times 10^{-6}$ m

Prob 5. *Find the thickness of a film which will strongly reflect the light of wavelength 589 nm. The refractive index of the material of the film is 1.25.*

Sol. For strong reflection, the least optical path difference introduced by the film should be $\lambda/2$
i.e. $2\mu t = \lambda/2$
$\therefore \quad t = \dfrac{\lambda}{4\mu} = \dfrac{589\mu m}{4 \times 1.25} = 118\,nm$

IITJEE TYPE

Prob 6. *In Young's double slit experiment two thin films are placed in front of two slits and the parallel beam of coherent light is incident at an angle θ with the normal to the slit plane, as shown in the figure. It is given that μ_1 = 1.7, μ_2 = 1.5, t_1 = 3 mm, t_2 = 2 mm, θ = 30°, d = 2 mm and D = 1 m. Find y-coordinate of the central maxima at point P. [t_1 and t_2 are thicknesses of the two plates].*



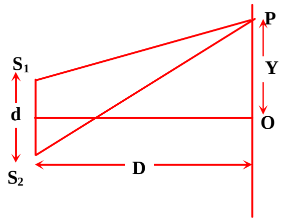
Sol. $\dfrac{yd}{D} = (\mu_1 - 1)t_1 - (\mu_2 - 1)t_2 + d \sin\theta$

$y = \dfrac{D}{d}\left[(\mu_1 - 1)t_1 - (\mu_2 - 1)t_2 + d\sin\theta\right]$

$\therefore$ Substituting values, we get
y = 105 cm

Prob 7. *Young's double slit experiment is carried out using microwaves of wavelength λ = 3 cm. Distance between the slits is d = 5 cm and the distance between the plane of slits and the screen is D = 100 cm. Find*
(a) the number of maxima.
(b) the position on the screen.

Sol. (a) The maximum path difference that can be produced = distance between the sources = 5 cm.
Thus, in this case we can have only three maxima, one central maxima and two on its either sides for path difference of λ or 3 cm.



(b) For maximum intensity at P

$S_2P - S_1P = \lambda$

or $\sqrt{\left(y+\frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2} = \lambda^2$

On substituting, we get,

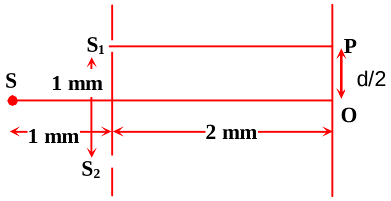
$d = 5$ cm, $D = 100$ cm

& $\lambda = 3$ cm and on solving, we get,

$y = \pm\, 75$ cm

Thus, there will be three maxima at $y = 0$ and $y = \pm 75$ cm

Prob 8. *In a Young's double slit experimental setup source S of wavelength 5000 Å illuminates two slits S_1 and S_2 which act as two coherent sources. The source S oscillates about its own position according to the equation $y = 0.5 \sin \pi t$, where y is in millimeter and t in seconds. Find*

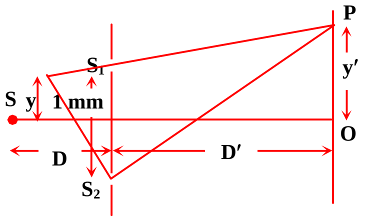


(a) position of the central maxima as a function of time.

(b) minimum value of t for which the intensity at point P on the screen exactly in front of the upper slit becomes maximum.

Sol. (a) Net path difference at O,

$\Delta x = \frac{yd}{D} + \frac{y'd}{D'}$



For central maximum, $\Delta x = 0$

$y' = -\frac{D'y}{D}$

$= -\left(\frac{2}{1}\right)(0.5 \sin \pi t) = -(\sin \pi t)$ mm

(b) $y' = d/2$, at point P exactly in front of S_1

$\Delta x = \frac{yd}{D} + \frac{d^2}{D'}$

For maximum intensity,

$\Delta x = n\lambda$

Putting, we get $0.5 \sin \pi t + 0.25 = 0.5\, n$

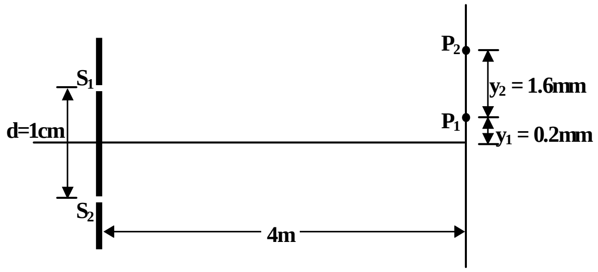
$\sin \pi t = \frac{0.5n - 0.25}{0.5}$

For minimum value of t, n = 1

$\therefore \sin \pi t = 0.5$

$\pi t = \pi/6 \Rightarrow t = 1/6 = 0.167$ sec

Prob 9. *In the YDSE conducted with white light (4000Å-7000Å), consider two points P_1 and P_2 on the screen at y_1 = 0.2 mm and y_2 = 1.6 mm, respectively. Determine the wavelengths which form maxima at these points*



Sol. The optical path difference at P_1 is

$p_1 = \dfrac{dy_1}{D} = \left(\dfrac{10}{4000}\right)(0.2) = 5\times10^{-4}\,\text{mm} = 5000\,\mathring{A}$

In the visible range 4000 - 7000Å

$n_1 = \dfrac{5000}{4000} = 1.25$ and $n_2 = \dfrac{5000}{7000} = 0.714$

The only integer between 0.714 and 1.25 is 1

$\therefore$ The wavelength which forms maxima at P is λ = 5000Å

For the point P_2, $p_2 = \dfrac{dy_2}{D} = \left(\dfrac{10}{4000}\right)(1.6) = 4\times10^{-3}\,\text{mm} = 40000\,\mathring{A}$

Here, $n_1 = \dfrac{40000}{4000} = 10$ and $n_2 = \dfrac{40000}{7000} = 5.71$

The integers between 5.71 and 10 are 6, 7, 8, 9 and 10

$\therefore$ The wavelengths which form maxima at P_2 are

λ_1 = 4000Å	for	n = 10
λ_2 = 4444Å	for	n = 9
λ_3 = 5000Å	for	n = 8
λ_4 = 5714Å	for	n = 7
λ_5 = 6666Å	for	n = 6

Prob 10. *In YDSE if the source consists of two wavelengths λ_1 = 4000Å and λ_2 = 4002Å, find the distance from the centre where the fringes disappear, if d = 1cm ; D = 1 m .*

Sol. The fringes disappear when the maxima of λ_1 fall over the minima of λ_2. That is

$\dfrac{p}{\lambda_1} - \dfrac{p}{\lambda_2} = \dfrac{1}{2}$, where p is the optical path difference at that point.

or $p = \dfrac{\lambda_1\lambda_2}{2(\lambda_2 - \lambda_1)}$

Here λ_1 = 4000Å, λ_2 = 4002Å

$\therefore$ p = 0.04 cm

In YDSE, p = dy/D

$\therefore$ $y = \dfrac{D}{d}p = \dfrac{(1000)}{10}(0.4) = 40\,\text{mm}$

Prob 11. *A beam of light consisting of two wavelengths 6500 Å and 5200 Å is used to obtain interference fringes in a Young's double slit experiment.*

(i) Find the distance of the third fringe on the screen from the central maximum for the wavelength 6500 Å.
(ii) What is the least distance from the central maximum where the bright fringes due to both wavelengths coincide?
(iii) The distance between the slits is 2mm and the distance between the plane of the slits and screen is 120cm. What is the fringe width for λ = 6500 Å?

Sol. (i) The width of the fringe $\dfrac{D\lambda}{d}$

Then distance of the third fringe

$3\omega = \dfrac{3D\lambda}{d} = \dfrac{3\times120\times6500\times10^{-8}}{0.2}$

= 0.117 cm

(ii) Let m^th and nth bright fringe of the wavelength coincide.
Now position of m^th bright fringe is

$y_m = m\lambda_1 \dfrac{D}{d}$

and $y_n = n\lambda_2\dfrac{D}{d} \Rightarrow \dfrac{m}{n} = \dfrac{\lambda_1}{\lambda_2} = \dfrac{5200}{6500} = \dfrac{4}{5}$

least distance = $n\lambda_1(D/d)$

$= \dfrac{5(5200\times10^{-10})120\times10^{-2}}{2\times10^{-3}}$

$= 1560 \times 10^{6}$ m.

(iii) Fringe width, $\omega = \dfrac{\lambda D}{d} = \dfrac{6500\times10^{-10}\times1.2}{2\times10^{-3}}$

$= 3900 \times 10^{-7}$m = 0.039 cm

Prob 12. *A transparent paper (μ = 1.45) of thickness 0.02 mm is pasted on one of the slits of a Young's double slit experiment which uses monochromatic light of wavelength 620 nm. How many fringes will cross through the centre if the paper is removed?*

Sol. Due to pasting the fringes shift which will restore position after removal.
Path difference will be

$S_2P - S_1P - t + \mu t = t(\mu - 1) + \dfrac{yd}{D}$

$\Rightarrow$ For a bright fringe

$\Delta x = n\lambda$

$t(\mu - 1) + yd/D = n\lambda.$

$y = \{n\lambda - t(\mu - 1)\}.\dfrac{D}{d}$

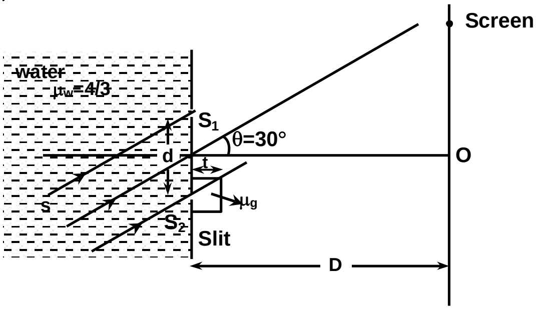
Again after removing

$y' = n\lambda\dfrac{D}{d} \Rightarrow y' - y = t(\mu - 1)\dfrac{D}{d}$

Number of fringes shifted will be

$$n = \frac{y'-y}{N} = t(\mu-1)\frac{D}{d}\Big/\frac{\lambda D}{d} = \frac{t}{\lambda}(\mu-1)$$
$$= \frac{0.02\times10^{-3}(0.45)}{620\times10^{-9}} = 14.5$$

Prob 13. *In a YDSE experiment the two slits are covered with a transparent membrane of negligible thickness which allows light to pass through it but does not allow water. A glass slab of thickness t = 0.41mm and refractive index μ_g=1.5 is placed infront of one of the slits as shown in the figure. The separation between the slits is d = 0.30 mm. The entire space to the left of the slits is filled with water of refractive index μ_w=4/3.*



A coherent light of intensity I and absolute wavelength λ = 5000A° is being incident on the slits making an angle of 30° with horizontal. If screen is placed at a distance D = 1 m from the slits, find
(a) the position of central maxima.
(b) the intensity at point O.

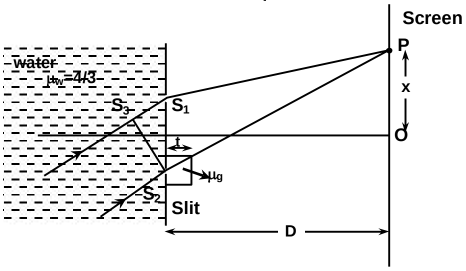
Sol. (a) Let central maxima lies at point P at a distance x from the central line. Then optical path difference Δp at point P is

$(S_2P - t) + t\mu_g - \mu_w d\sin\theta - S_1p$

$= S_2P - S_1P + (\mu_g-1)t - \mu_w d\sin\theta$

$= \frac{xd}{D} + (\mu_g-1)t - \mu_w d\sin\theta$

For central maxima, $\Delta p = 0$

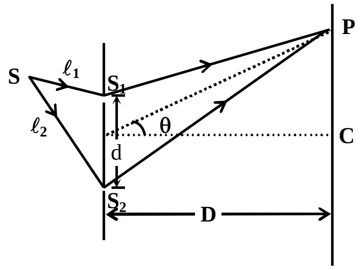


$$x = \frac{D}{d}\left[(\mu_w d\sin\theta) - (\mu_g - 1)t\right]$$
$$= \frac{1}{3\times10^{-4}}\left[\frac{4}{3}\times3\times10^{-4}\times\frac{1}{2} - 0.5\times0.41\times10^{-3}\right]$$
$$= \frac{1}{3\times10^{-4}}\left[2\times10^{-4} - 2.05\times10^{-4}\right] = -\frac{5\times10^{-6}}{3\times10^{-4}} = -1.66\times10^{-2}\,\text{m}$$

$x = -1.66$ cm
So, central maxima lies at a distance 1.66 cm below the central line

(b) At point O, optical path difference is
$(\mu_g - 1)t - \mu_w d\sin\theta$
$= -5 \times 10^{-6}$ m
So, intensity at O
$I_0 = I + I + 2\sqrt{II}\cos\frac{2\pi}{5\times10^{-7}}\left(-5\times10^{-6}\right)$
$I_0 = 4I$

Prob 14. *In a Young's double slit experiment the light source is at distance $\ell_1 = 20\ \mu m$ and $\ell_2 = 40\ \mu m$ from the slits. The light of wavelength $\lambda = 500$ nm is incident on slits separated at a distance $d = 10\ \mu m$. A screen is placed at a distance D = 2m away from the slits as shown in the figure.*



(a) Find the angles with central line where maxima will appear on the screen.
(b) How many maxima will appear on the screen?
(c) What should be the minimum thickness of a slab of refractive index 1.5 to be placed on the path of one of the ray so that minima occur at C?

Sol. (a) The optical path difference between two beams arriving at P,
$\Delta x = (\ell_2 - \ell_1) + d\sin\theta$
The condition for constructive interference
$\Delta x = n\lambda$, n = 0, 1, 2, 3 . . . . . .
Thus $\sin\theta = \frac{1}{d}\ [\Delta x - (\ell_2 - \ell_1)]$
$= \frac{1}{d}\ (n\lambda - (\ell_2 - \ell_1)]$
$= \frac{1}{10\times10^{-6}}\ [n \times 500 \times 10^{-9} - 20 \times 10^{-6}\ ]$
$\sin\theta = 2\left[\frac{n}{40} - 1\right]$ Hence, $\theta = \sin^{-1}\left[2\left(\frac{n}{40} - 1\right)\right]$

(b) $|\sin\theta| \le 1$
$-1 \le 2\left[\frac{n}{40} - 1\right] \le 1$
$-20 \le [n - 40] \le 20$
$20 \le n \le 60$
Hence, number of maxima = 60 – 20 = 40.

(c) At C, phase difference $\Delta\phi = k\Delta\ell$
$= \frac{2\pi\times20\times10^{-6}}{500\times10^{-9}} = 80\pi$
Hence, maxima will appear on C.

For minima at C, $(\mu - 1)t = \lambda/2$

Hence, $t = \dfrac{\lambda}{2(\mu-1)} = \dfrac{500\times10^{-9}}{2\times0.5}$ = 500 nm.

Prob 15. *In Young's double slit experiment one slit is covered by a glass plate of thickness 2t and refractive index 1.5. The second slit is covered by another glass plate of thickness 3t and refractive index 1.6. The point of central maxima on the screen before the plates were introduced is now occupied by previous 8^{th} bright fringe. Find the thickness of plates. The wavelength, λ, of light is 6000A°.*

Sol. The additional path difference introduced due to the glass plates is

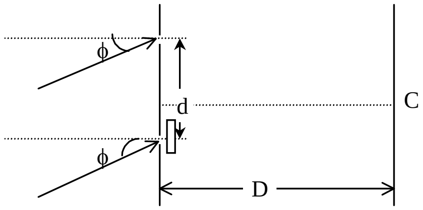
$(1.6 - 1)3t - (1.5 - 1)2t = 8\lambda$,

Since the 8^{th} fringe is now located at the position of the central maximum.

$\Rightarrow$ $0.8t = 8\lambda$

or, $t = 6 \times 10^{-6}$ m

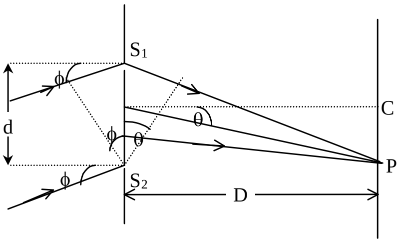
Prob 16. *Light of wavelength λ = 500 nm falls on two narrow slits placed a distance $d = 50 \times 10^{-4}$ cm apart, at an angle $\phi = 30^0$ relative to the slits shown in figure. On the lower slit a transparent slab of thickness 0.1 mm and refractive index 3/2 is placed. The interference pattern is observed on screen at a distance D = 2m from the slits. Then, calculate*



(a) position of the central maxima,

(b) the order of minima closest to centre C of screen.

(c) How many fringes will pass over C, if we remove the transparent slab from the lower slit ?



Sol. Phase difference ϕ = kd sin ϕ + kd sin θ – k b (n - 1)

where $k = \dfrac{2\pi}{\lambda}$

Central maxima is obtained when $\Delta\phi = 0$

$kd\sin\phi + kd\sin\theta - kb(n - 1) = 0$

$\sin\theta = \dfrac{b(n-1)}{d} - \sin\phi$

$= \dfrac{1}{2} \Rightarrow$ Therefore, $\theta = 30^0$

At C, $\theta = 0$

$\Delta\phi = kd\sin\phi - kb(n-1) = \frac{kd}{2}\left[1 - \frac{2b}{d}(n-1)\right]$

$= \frac{2\pi d}{2\lambda}\left[1 - \frac{2b}{d}(n-1)\right]$

Substituting the values, we get, $\Delta\phi = -100\pi = 2m\pi \Rightarrow m = -50$

As $I \propto \cos^2\left(\frac{\Delta\phi}{2}\right)$

Hence, at C there will be maxima.

For minima $\frac{\Delta\phi}{2} = (2m-1)\pi/2$

or $\Delta\phi = (2m-1)\pi$

For m = –50 and – 49 we get $\Delta\phi = -101\pi$ and -99π respectively

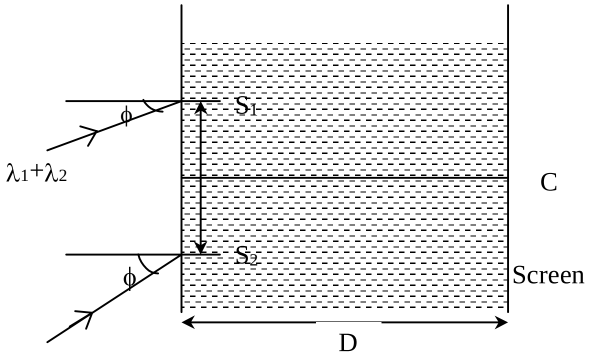
Hence the minima closest to $\theta = 0$ are of order m = –49 and –50 on both side

The phase difference at C when we remove the slab

$\Delta\phi = 100\pi$, i.e. m = 50

Hence, number of fringes passed over = 150

Prob 17. *In a Young's double slit experiment a parallel beam containing wavelengths λ_1 = 4000 Å and λ_2 = 5600 Å is incident at an angle $\phi = 30^0$ on a diaphram having narrow slits at a separation d = 2mm. The screen is placed at a distance D = 40cm from slits. A mica slab of thickness t =5 mm is placed infront of one of the slits and the whole apparatus is submerged in water. If the central bright fringe is observed at C, calculate*
(a) the refractive index of the slab.
(b) the distance of the first black line from C.



Sol. (a) Total phase difference at C, $\Delta\phi = kd\sin\phi - kt(\mu - 1)$

for central maxima at C,

$\Delta\phi = 0$

$t = \frac{d\sin\phi}{(\mu-1)} \Rightarrow \frac{2\times10^{-3}\times\sin30^0}{(\mu-1)} = 5\times10^{-3}$.

$\mu' = 1.2 \Rightarrow \mu = 1.2 \times (4/3) = 1.6$

Hence refractive index of mica slab = 1.6.

(b) A black line is formed at the position where dark fringes are formed for both the wavelengths.

The distance of first black line from centre bright line

$$y = \frac{(2n-1)\lambda D}{2d} \qquad \ldots . \text{(i)}$$

For black line ; $\frac{(2n_1-1)\lambda_1' D}{2d} = \frac{(2n_2-1)\lambda_2' D}{2d}$

$\frac{(2n_1-1)}{(2n_2-1)} = \frac{\lambda_2'}{\lambda_1'}$, where $\lambda_1' = \frac{\lambda_1}{\mu_\omega}$ and $\lambda_2' = \frac{\lambda_2}{\mu_\omega}$

$$\frac{(2n_1-1)}{(2n_2-1)} = \frac{7}{5}$$

For minimum value, $n_1 = 4$ and $n_2 = 3$.

Hence, distance of first black line

$$y = \frac{(2\times 4-1)4000\times 10^{-10}\times 40\times 10^{-2}\times 3}{2\times 2\times 10^{-3}\times 4}$$

$= 2.1 \times 10^{-4}$ m . $= 210\ \mu$m.

Prob 18. *The light of wavelength 600 nm is incident normally on a slit of width 3mm. Calculate the linear width of central maximum on a screen kept 3m away from the solid.*

Sol. Here, $\lambda = 600$ nm $= 6 \times 10^{-7}$ m

$a = 3$mm $= 3 \times 10^{-3}$ m,

D = 3m

$\therefore$ width of central maximum $= \frac{2D\lambda}{a}$

$$= \frac{2\times 3\times 6\times 10^{-7}}{3\times 10^{-3}}$$

$= 12 \times 10^{-4}$ m $= 1.2$ mn

Objective:

Prob 1. *In a Young's double slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelengths, λ_0 = 750 nm and λ = 900 nm. The minimum distance from the common central bright fringe on a screen 2 m away from the slits where a bright fringe from one interference pattern coincides with a bright fringe from the other is*

(A) 1.5 mm *(B) 3 mm*
(C) 4.5 mm *(D) 6 mm*

Sol. **(C)**

$y_n = n\left(\frac{D\lambda}{d}\right)$ and $y'_n = n'\left(\frac{D\lambda'}{d}\right)$

Equating y_n and y'_n, we get

$\frac{n}{n'} = \frac{\lambda'}{\lambda} = \frac{900}{750} = \frac{6}{5}$

Hence, the first position at which the overlapping occurs is

$y_6 = y'_5 = \frac{6\times 2\times 750\times 10^{-9}}{2\times 10^{-3}}$ = 4.5 mm

Prob 2. *In Young's interference experiment, if I_0 be the intensity at the central bright fringe and, β be the fringe width, then the intensity as a function of distance x from the central bright fringe is*

(A) $I = I_0\cos^2\left(\frac{x}{\beta}\right)$ *(B)* $I = I_0\cos^2\left(\frac{\beta}{x}\right)$

(C) $I = I_0\cos^2\left(\frac{\beta\pi}{x}\right)$ *(D)* $I = I_0\cos^2\left(\frac{\pi x}{\beta}\right)$

Sol. **(D)**. $\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}d\sin\theta$, As $I = I_0\cos^2(\Delta\phi/2)$

$I = I_0\cos^2\left(\frac{\pi}{\lambda}\times d\sin\theta\right)$

$\therefore I = I_0\cos^2\left(\frac{\pi}{\lambda}\frac{dx}{D}\right) = I_0\cos^2\left(\frac{\pi x}{\beta}\right)$, where $\beta = \lambda D/d$

Prob 3. *In Young's double slit experiment, the 8^{th} maximum with wavelength λ_1 is at a distance d_1 from central maximum and 6^{th} maximum with wavelength λ_2 is at a distance d_2. Then d_1/d_2 is*

(A) $\frac{4}{3}\left(\frac{\lambda_2}{\lambda_1}\right)$ *(B)* $\frac{4}{3}\left(\frac{\lambda_1}{\lambda_2}\right)$

(C) $\frac{3}{4}\left(\frac{\lambda_2}{\lambda_1}\right)$ *(D)* $\frac{3}{4}\left(\frac{\lambda_1}{\lambda_2}\right)$

Sol. **(B)**. Here, $d_1 = 8 \times \frac{\lambda_1 D}{d}$

$d_2 = 6 \times \frac{\lambda_2 D}{d}$

$\therefore \frac{d_1}{d_2} = \frac{4}{3}\left(\frac{\lambda_1}{\lambda_2}\right)$

Prob 4. *Two slits separated by a distance of 1 mm are illuminated with light of wavelength 6×10^{-7}m. The interference fringes are observed on a screen placed 1 m from the slits. The distance between third dark fringe and the fifth bright fringe is equal to*

(A) 0.60 mm *(B) 1.50 mm*
(C) 3.00 mm *(D) 4.50 mm*

Sol. **(B)**. $\beta = \lambda D/d$

$\beta = \frac{6.0 \times 10^{-7} \times 1}{1 \times 10^{-3}} = 0.6$ mm

The fifth bright fringe is separated from third fringe by = $2.5\beta = 2.5 \times 0.6 = 1.50$ mm

Prob 5. *Interference fringes were produced in Young's double slit experiment using light of wavelength 5000Å. When a film of material 2.5×10^{-3} cm thick was placed over one of the slits, the fringe pattern shifted by a distance equal to 20 fringe widths. The refractive index of the material of the film is*

(A) 1.25 *(B) 1.33*
(C) 1.4 *(D) 1.5*

Sol. **(C)**. $S = (\mu - 1)\frac{tD}{d}$

But, $\beta = \frac{\lambda D}{d}$, or $\frac{D}{d} = \frac{\beta}{\lambda}$

$\therefore S = (\mu - 1)\frac{t\beta}{\lambda}$

$20\beta = (\mu - 1)2.50 \times 10^{-3} \times \frac{\beta}{5000 \times 10^{-8}}$

$(\mu - 1) = \frac{20 \times 5000 \times 10^{-8}}{2.5 \times 10^{-3}} = 0.4$

$\Rightarrow \mu = 1.4$

Prob 6. *In Young's double slit experiment, the slits are 0.5 mm apart and interference is observed on a screen placed at a distance of 100 cm from the slits. It is found that 9th bright fringe is at a distance of 4.00 mm from the second dark fringe from the centre of the fringe pattern. The wavelength of light used is*

(A) 3000 Å *(B) 6000 Å*
(C) 9000 Å *(D) 12000 Å*

Sol. **(C)**. We know that, $x_n = \dfrac{n\lambda D}{d} = n\beta$, where d is distance between two slits and β is fringe,

For 9th fringe width

$x_9 = 9\beta$

The distance of nth dark fringe from central fringe is

$$x'_n = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d} = \left(n - \frac{1}{2}\right)\beta$$

For 2nd dark fringe, $x'_2 = \dfrac{3}{2}\beta$

The separation between 9th bright fringe and second dark fringe is, $x_9 - x'_2 = 9\beta - \dfrac{3}{2}\beta$

$\Rightarrow 9.0 = \dfrac{15}{2}\beta$ or $\beta = 1.2$ mm, $\beta = 1.2 \times 10^{-3}$ m

$\beta = \dfrac{\lambda D}{d} \Rightarrow 1.2 \times 10^{-3} = \dfrac{\lambda \times 1}{0.5 \times 10^{-3}}$

$\therefore \lambda = 6 \times 10^{-7}$ m = 6000Å

Prob 7. *In Young's double slits experiment, the distance between the slits S_1 and S_2 is 1.0 mm. What should be the width of each slit so as to obtain 10 maxima of the two slits interference pattern within the central maximum of the single slit diffraction pattern?*

(A) 0.1 mm *(B) 0.2 mm*
(C) 0.3 mm *(D) 0.4 mm*

Sol. **(B).** $x_n = \dfrac{n\lambda D}{d}$. As D >> d, angular separation between n bright fringes will be

$$\theta_n = \frac{x_n}{D} = \frac{n\lambda D}{Dd} = \frac{n\lambda}{d}$$

For 10 bright fringes

$$\theta_{10} = \frac{10\lambda}{d} \qquad \text{. . . (1)}$$

The angular width of principal maxima in diffraction pattern due to a slit of width 'a' is given by

$$\theta_{10} = \frac{2\lambda}{a} \qquad \text{. . . (2)}$$

$\therefore$ From (1) and (2)

$$\Rightarrow \frac{10\lambda}{d} = \frac{2\lambda}{a}$$

$$a = \frac{2d}{10} = \frac{2 \times 1.0}{5} = 0.2 \text{ mm}$$

Prob 8. *In an experiment similar to Young's experiment interference is observed using waves associated with electrons. The electrons are being produced in an electron gun. In order to increase the fringe width*

(A) electron gun voltage should be increased.
(B) electron gun voltage should be decreased.

(C) the slits should be moved away.
(D) the screen should be moved closer to interfering slits

Sol. **(B)**. $\beta = \lambda D/d$ and according to de-Broglie equation, $\lambda = h/p = \dfrac{h}{\sqrt{2meV}}$

As V decreases, λ increases and thus β increases

Prob 9. *Two waves originating from sources S_1 and S_2 have zero phase difference and common wavelength. The waves will show completely destructive interference at a point P, if $S_1P - S_2P$ is*

(A) 5λ *(B) $3\lambda/4$*
(C) 2λ *(D) $11\lambda/2$*

Sol. **(D).** For destructive interference:

Path difference = $(2n + 1)\dfrac{\lambda}{2}$

When n = 5, $S_2P - S_1P = \dfrac{11\lambda}{2}$

Prob 10. *In Young's double slit experiment, we get 60 fringes in the field of view of monochromatic light of wavelength 4000Å. If we use monochromatic light of wavelength 6000Å, then the number of fringes obtained in the same field of view is*

(A) 60 *(B) 90*
(C) 40 *(D) 1.5*

Sol. **(C).** Width of the field of view = $n_1\beta_1 = n_1\left(\dfrac{D\lambda_1}{d}\right)$

Hence, $n_1\left(\dfrac{D\lambda_1}{d}\right) = n_2\left(\dfrac{D\lambda_2}{d}\right)$

$\Rightarrow n_2 = \dfrac{n_1\lambda_1}{\lambda_2} = \dfrac{60 \times 4000}{6000} = 40$

Prob 11. *A double slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å. The fringe width is*

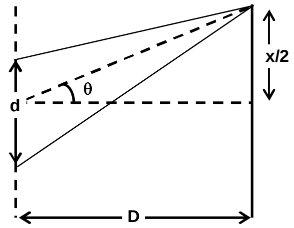
(A) 0.63 mm (B) 0.5 mm
(C) 0.32 mm (D) 0.8 mm

Sol. **(A)**. The fringe width in a medium of refractive index n is given by

$\beta_\ell = \beta_a/n$, where $\beta_a = \dfrac{D\lambda}{d}$ is the fringe width in air

Thus $\beta_\ell = \dfrac{D\lambda}{n_\ell d} = \dfrac{6300 \times 10^{-10} \times 1.33}{1.33 \times 1 \times 10^{-3}} = 0.63 \times 10^{-3}$ m = 0.63 mm

Prob 12. *A beam of light of wavelength 600 nm from a distant source falls on a single slit 1.00 mm wide and the resulting diffraction pattern is observed on a screen 2m away. The distance between the first dark fringes on either side of the central bright fringe is,*



(A) 1.2 cm *(B) 1.2 mm*
(C) 2.4 cm *(D) 2.4 mm*

Sol. **(D).** $\frac{x}{2} = \frac{\lambda D}{d}$

$x = \frac{2\lambda D}{d} = \frac{2 \times 600 \times 10^{-9} \times 2}{1 \times 10^{-3}}$

$x = 2.4 \times 10^{-3}$ m = 2.4 mm

Prob 13. *If the amplitude ratio of two sources producing interference is 3 : 5, the ratio of intensities at the maxima and the minima is*
(A) 16 : 25 *(B) 5 : 3*
(C) 16 : 1 *(D) 9 : 25*

Sol. **(C).** $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$I \propto A^2$

And for maximum intensity, $\cos\phi = +1$

For minimum intensity, $\cos\phi = -1$

$\therefore I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(A_1 + A_2\right)^2$

$\therefore I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \left(A_1 - A_2\right)^2$

$\therefore \frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left(\frac{3+5}{3-5}\right)^2 = \left(\frac{8}{-2}\right)^2 = \frac{16}{1}$

Prob 14. *White light is used to illuminate the two slits in a Young's double slit experiment, the separation between the slits is a and the screen is at a distance D (>>a) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing, some of these missing wavelengths are*

(A) $\lambda = \frac{a^2}{D}$ *(B)* $\lambda = \frac{2a^2}{D}$

(C) $\lambda = \frac{a^2}{3D}$ *(D)* $\lambda = \frac{2a^2}{3D}$

Sol. **(A) & (C).** According to theory of interference position y of a point on the screen is given by

y = $\frac{D}{d}(\Delta x)$ and as for missing wavelength intensity will be min (= 0)

$\Delta x = \frac{(2n-1)\lambda}{2}$

So, $y = \frac{D(2n-1)\lambda}{2d}$

Here, x = a and y = a/2

So, $\lambda = \frac{a^2}{(2n-1)D}$, with n = 1, 2, 3, ...

i.e. wavelengths, $\frac{a^2}{D}$, $\frac{a^2}{3D}$, $\frac{a^2}{5D}$, . . . etc will be absent (or missing) at point P.

Prob 15. *The ratio of the intensity at the centre of a bright fringe to the intensity at a point one-quarter of the distance between two fringes from the centre is*

(A) 2 *(B) 1/2*
(C) 4 *(D) 16*

Sol. **(A)** Two waves of a single source having an amplitude A interfere. The resulting amplitude

$A_r^2 = A_1^2 + A_2^2 + 2A_1A \cos\delta$

where $A_1 = A_2 = A$ and δ = phase difference between the waves

$\Rightarrow I_r = I_1 + I_2 + 2\sqrt{I_1I_2} \cos\delta$

When the maxima occurs at the center, $\delta = 0$

$\Rightarrow I_{r_1} = 4I$ …(1)

Since the phase difference between two successive fringes is 2π, the phase difference between two points separated by a distance equal to one-quarter of the distance between the two, successive fringes is equal to

$\delta = (2\pi)\left(\frac{1}{4}\right) = \frac{\pi}{2}$ radian

$\Rightarrow I_{r_2} = 4I \cos^2\left(\frac{\pi/2}{2}\right) = 2I$ …(2)

Using (1) and (2), $\frac{I_{r_1}}{I_{r_2}} = \frac{4\ I}{2\ I} = 2$

ASSIGNMENT PROBLEMS

Subjective:

Level- O

1. What is the effect on the interference fringes in a Young's double slit experiment due to each of the following operations?
   (a) The screen is moved away from the plane of the slits.
   (b) The monochromatic source is replaced by another monochromatic source of shorter wavelength.
   (c) The separation between the two slits is increased.
   (d) The monochromatic source is replaced by a source of white light.

2. Two slits in YDSE are illuminated by two different sodium lamps emitting light of same wavelength. Do you observe any interference pattern on the screen?

3. Explain how Newton's corpuscular theory predicts that the speed of light in a medium, say water is greater than the speed of light in vacuum. Is the prediction confirmed by the experimental determination of speed of light in water ? If not, which alternative picture of light is consistent with experiment ?

4. What are the coherent sources of light ? Why no interference pattern is observed when two coherent sources are (i) too close; (ii) very far apart ?

5. What is meant by wavefront ? Explain three types of wave-fronts.

6. State Huygen's principle and prove the laws of reflection on the basis of wave theory.

7. Prove Snell's laws of refraction on the basis of Huygen's principle.

8. What do you understand by fringe width? Derive an expression for fringe width in the diffraction pattern.

9. Explain what is meant by diffraction of light. Describe a simple experiment to demonstrate diffraction at a single slit.

10. In a single slit diffraction pattern, how is the angular width of central bright maximum changed when: (i) the slit width is decreased; (ii) the distance between the slit and screen is increased; (iii) light of smaller wavelength is used. Justify your answer.

11. What do you understand by polarisation of light? What are plane of polarization and plane of vibration.

12. State and explain Brewster's law.

13. At what angle should the axes of two polaroids be placed so as to reduce the intensity of incident unpolarised light to 1/3.

14. A parallel beam of monochromatic light of wavelength 450 nm passes through a long slit of width 0.2 mm. Find the angular divergence in which most of the light is diffracted.

15. For a given medium, the polarising angle is 60^0. What will be the critical angle for this medium?

Level- I

1. In a double slit interference experiment, the separation between the slits is 1.0 mm, the wavelength of light used is 5.0×10^{-7} m and the distance of the screen from the slits is 1.0 m.
   (a) Find the fringe width.
   (b) How many bright fringes are formed in one centimeter width on the screen?

2. A plate of thickness t made of a material of refractive index μ is placed in front of one of the slits in a double slit experiment.
   (a) Find the change in the optical path due to the introduction of the plate.
   (b) What should be the minimum thickness t which will make the intensity at the centre of the fringe pattern zero? Wavelength of the light used is λ. Neglect any absorption of light in the plate.

3. Two sources of intensities I and 4I are used in an interference experiment. Find the intensity at points where the waves from two sources superimpose with a phase difference
   (a) zero, (b) $\pi/2$, and (c) π.

4. In Young's double slit experiment fringe width is found to be 0.4 mm. If the whole apparatus is immersed in water of refractive index 4/3 without disturbing the geometrical arrangement, find the new fringe width.

5. Two beams of light having intensities I and 4I interfere to produce a fringe pattern on the screen. Phase difference between the beams is $\pi/2$ at point A and π at point B. Then, find the difference between resultant intensities at A and B.

6. Interference fringes are produced by a double slit arrangement and a piece of plane parallel glass of refractive index 1.5 is interposed in one of the interfering beam. If the fringes are displaced through 30 fringe widths for light of wavelength 6×10^{-5} cm, find the thickness of the plate.

7. In a young's double slit experiment, the path difference of waves for a point P on screen is one third of wavelength of the light. Find the ratio of intensity at P to that at maximum.

8. A light source emits light of two wavelengths λ_1 = 4300 $\AA$ and λ_2 = 5100 $\AA$. The source is used in a double slit interference experiment. The distance between the slit is 0.025 mm and between source and screen is 1.5 m. Calculate the separation between the third order bright fringes due to these two wavelengths.

9. A beam of light consisting of two wavelength 6500 $\AA$ and 5200 $\AA$ is used to obtain interference fringes in a Young's double slit experiment. What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?
   The distance between the slit is 2 mm and the distance between the plane of the slits and the screen is 120 cm.

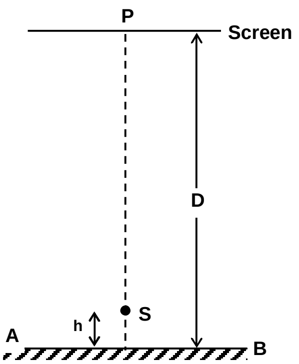
10 In a modified Young's double slit experiment (λ = 6000 $\AA$) the zero order maxima and tenth order maxima fall at 12.34 mm and 14.73 mm on a screen from a particular reference point. If λ is changed to 5000 $\AA$, find the new positions of the zero-order maxima and 10^{th} order maxima if other arrangements remaining unchanged.

11. A glass plate of refrective index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If λ = 648 nm, obtain the least value of t for which the rays interfere construcively.

Level- II

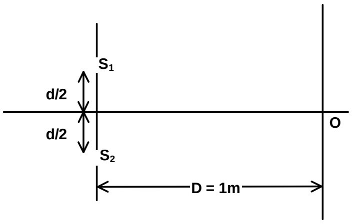
1. In Young's double slit experiment the slits are 0.5 mm apart and interference is observed on a screen placed at a distance of 100 cm from the slits. It is found that the 9^{th} bright fringe is at a distance of 8.835 mm from the 2^{nd} dark fringe from the centre of fringe pattern. Find the wavelength of light used.

2. A point source *S* emitting light of wavelength 600 nm is placed at a very small height *h* above a flat reflecting surface AB (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance *D* from it.



   (a) What is the shape of the interference fringes on the screen?
   (b) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).
   (c) If the intensity at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface *AB* should be shifted so that the intensity at P again becomes maximum.

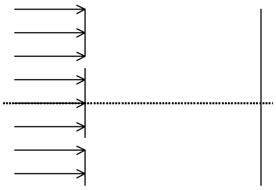
3. In a double slit experiment the separation between the slits is 1mm. Light rays fall normally on the plane of the slits and the interference pattern is observed on a screen placed at a distance of 1m from the plane of the slits. The arrangement is shown in the figure. When one of the slits is covered by a transparent strip of thickness 4μm, the central maximum is formed at a distance of 2mm from the point O. When the entire apparatus (one of the slits remaining covered) is immersed in a liquid, the distance between the central maximum and the point O is reduced to 0.5mm.



   (a) Find the refractive index of the material of the strip and the liquid.
   (b) If the wavelength of light used is 500 nm and the experiment is performed in the liquid, find distance of the nearest maximum from the point O.

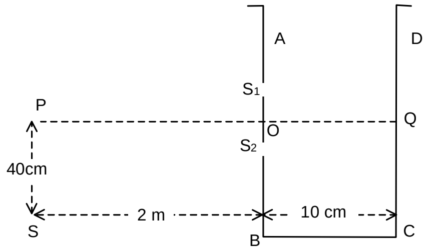
4. Fringes are produced using light of wavelength λ = 4800 Å in a double-slit experiment. One of the slit is covered by a thin plate of glass of refractive index 1.4 and other slit by another plate of glass of double thickness and of refractive index 1.7.
During this process the central bright fringe shifts to a position originally occupied by the fifth bright fringe from the centre. Find the thickness of each glass plate.

5. A parallel beam of light consisting of two wavelengths 14000 Å and 26000 Å coherent in themselves falls on a double slit apparatus. The separation between the two slits is 2 cm and that of between plane of the slits and screen is 1 meter. Find out
   (a) the location of central maximas for both the wavelengths.



   (b) the location of a point on the screen when the maximas of two waves coincide for the first time after central maxima.
   (c) the location where the minimas of the two waves coincide for the first time after central maxima.

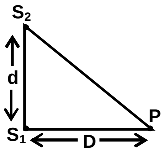
6. A vessel ABCD of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure below.



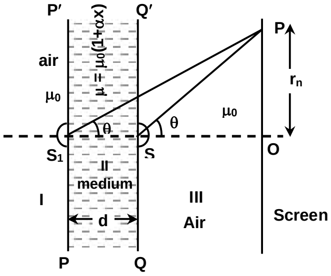
(a) Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ.
(b) Now, a liquid is poured into the vessel and filled up to OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid.

7. In Young's experiment, the source is red light of wavelength 7×10^{-7}m. When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beam, the central bright fringe shifts by 10^{-3}m to the position previously occupied by the 5^{th} bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength 5×10^{-7}m the central fringe shifts to a position initially occupied by the 6^{th} bright fringe due to red light. Find the refractive index of glass for the green light. Also estimate the change in fringe width due to the change in wavelength.

8. Two sources S_1 and S_2 emitting light of wavelengths 600 nm are placed at a distance of 1.0×10^{-2} cm. A detector can be moved on the line S_1P which is perpendicular to S_1S_2. Find out the position of first minimum detected.



9. Consider the situation of the interference experiment set up as shown in the figure. The S_1S_2 part of the set up is put in a medium whose refractive index varies as $\mu = \mu_0(1 + \alpha x)$ where x is the displacement from the line PP′. Find the [Take (D>>d, D>>r) and α = constant].
(a) nature of fringes obtained on the screen.



(b) distance of n^{th} bright fringe from the central fringe on the screen. [Take θ to be small]. Taking μ_0 as refractive index in medium I and III and also of S_1 and S_2 sources.

10. At a certain point on a screen the path difference for the two interfering rays is $(1/8)^{th}$ of a wavelength. Find the ratio of the intensity at this point to that at the centre of a bright fringe.

Objective:

Level- I

1. Two coherent monochromatic light beams of intensities 4I and 9I are superposed. The maximum and minimum intensities in the resulting beam are
(A) 5I and I (B) I and 25I
(C) 25I and I (D) 9I and 3I

2. In Young's double slit experiment, the fringe width is β. If the entire arrangement is now placed inside a liquid of refractive index μ, the fringe width will become
(A) $\mu\beta$ (B) β/μ
(C) $\frac{\beta}{\mu+1}$ (D) $\frac{\beta}{\mu-1}$

3. In a Young's double slit experiment, let S_1 and S_2 be the two slits, and C be the centre of the screen. If $\angle S_1CS_2=\theta$ and λ is the wavelength, the fringe width will be
(A) $\frac{\lambda}{\theta}$ (B) $\lambda\ \theta$
(C) $2\lambda/\theta$ (D) $\lambda/2\theta$

4. The path difference between two interfering waves at a point on a screen is 14.5 times the wavelength. The point is
(A) neither dark not bright (B) bright
(C) dark (D) data is inadequate

5. In the Young's double slit experiment, films of thicknesses t_A and t_B and refractive indices μ_A and μ_B are placed in front of A and B respectively. If $\mu_A t_A = \mu_B t_B$, the central maximum will
(A) not shift (B) shift towards A
(C) shift towards B (D) option (B), if $t_B>t_A$ and option(C) if $t_B<t_A$

6. In the Young's double slit experiment both the slits are similar. If the length of one of the slits is halved, which of the following is true?
(A) Bright fringes becomes narrower. (B) Bright fringes become wider.
(C) Dark fringes become darker. (D) Dark fringes become brighter.

7. Waves from two different sources overlap near a particular point. The amplitude and the frequency of the two waves are same. The ratio of the intensity when the two waves arrive in phase to that when they arrive 90^0 out phase is
(A) 1 : 1 (B) $\sqrt{2}$: 1
(C) 2 : 1 (D) 4 : 1

8. Instead of using two slits as in Young's experiment, if we use two separate but identical sodium lamps, which of the following occur?
(A) general illumination (B) widely separate interference
(C) very bright maximum (D) very dark minimum

9. For best contrast between maxima and minima in the interference pattern of Young's double slit experiment, the intensity of light emerging out of the two slits should be
(A) equal
(B) double
(C) small
(D) large

10. The path difference between two interfering waves at a point on a screen is 11.5 times the wavelength. The point is
(A) dark
(B) bright
(C) neither dark nor bright
(D) data is inadequate

11. Two points P and Q are situated at the same distance from a source of light but on opposite sides. The phase difference between the light waves passing through P and Q will be
(A) π
(B) 2π
(C) $\pi/2$
(D) zero.

12. In Young's double slits experiment, one of slits, say S_1, is covered with a glass plate that absorbs half the intensity of light from it. Then,
(A) the bright fringes becomes brighter and dark fringes become darker.
(B) the bright fringes becomes less bright and the dark fringes have a finite light intensity.
(C) the fringe width decreases.
(D) no fringes will be observed.

13. In Young's double slit experiment the 7th maximum with wavelength λ_1 is at a distance d_1 and that with wavelength λ_2 is at distance d_2. Then d_1/d_2 is
(A) λ_1/λ_2
(B) λ_2/λ_1
(C) λ_1^2/λ_2^2
(D) λ_2^2/λ_1^2

14. The distance between two slits is 0.03 cm. A screen is placed at a distance of 1.5 m where an interference pattern is observed in which fourth bright fringe is at 1 cm from the central maximum. The wavelength of light used will be
(A) 7000 Å
(B) 6000 Å
(C) 5000 Å
(D) 4500 Å

15. A transparent micasheet of refractive index 1.5 and uniform thickness 2λ is placed in front of both the slits in a Young's double slit experiment, λ is the wavelength of the monochromatic light used in the experiment. The fringe pattern on the screen will
(A) shift upwards by one fringe width.
(B) shift upwards by two fringe width.
(C) shift downwards by one fringe width.
(D) remain unaffected.

16. In a double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern the intensity of
(A) the maxima and the minima increases.
(B) the maxima increases and the minima has zero intensity.
(C) the intensity of maxima decreases and that of minima increases.
(D) the intensity of the maxima decreases and the minima has zero intensity.

17. Two monochromatic coherent point sources S_1 and S_2 are separated by a distance L. Each source emits light of wavelength λ (where $L >> \lambda$). The line S_1S_2 when extended meets a screen perpendicular to it at a point A. Then
    (A) The interference fringes on the screen are circular in shape.
    (B) The interference fringes on the screen are straight lines perpendicular to the line S_1S_2 A.
    (C) The point A is an intensity maxima if $L = n\lambda$.
    (D) The point A is always an intensity maxima for any separation L.

18. In a two slits experiment with white lights, a white fringe is observed on a screen kept behind the slits. When the screen is moved away by 0.05 m, this white fringe.
    (A) does not move at all (B) gets displaced from its earlier position.
    (C) becomes coloured. (D) disappears.

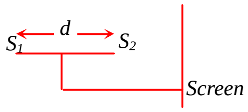
19. In a Young's double slit experiment, the distance between two sources is 0.1 mm. The distance of the screen from the source is 20 cm. Wavelength of light used is 5460Å. Then, the angular position of the first dark fringe is
    (A) 0.08° (B) 0.16°
    (C) 0.20° (D) 0.32°

20. The ratio of the intensity at the centre of a bright fringe to the intensity at a point one-quarter of the distance between two fringes from the centre is
    (A) 2 (B) 1/2
    (C) 4 (D) 16

FILL IN THE BLANKS IN THE FOLLOWING QUESTIONS.

21. A slit of width d is placed in front of a lens of focal length 0.5 m and is illuminated normally with light of wavelength 5.89×10^{-7} m. The first diffraction minima on either side of the diffraction pattern are separated by 2×10^{-3} m. The width d of the slits is ______________ m.

22. Consider a plane which is normal to the line joining two point coherent sources S_1 and S_2. The shape of fringes is _____________________.



23. If Young's double slit experiment is performed in water, then fringe width will ______________.

24. When light is refracted into a medium, ______________ remains unchanged, but ____________ decreases.

25. A laser beam of wavelength 6300Å is incident on a pair of slits and produces an interference pattern in which the bright fringes are separated by 8.3 mm. A second light produces an interference pattern of fringe separation 7.6 mm. The wavelength of the second light is _________________.

STATE WHETHER THE FOLLOWING STATEMENTS ARE TRUE OR FALSE.

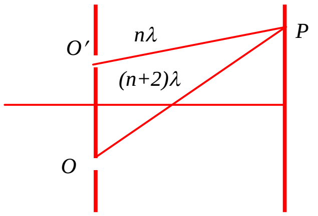
26. In an interference pattern the energy is destroyed at the minima.

27. The shape of interference fringes observed in Young's double slit experiment is straight lines parallel to slits.

28. When monochromatic light is incident on a surface separating two media, the reflected and refracted light both have the same frequency as the incident frequency.

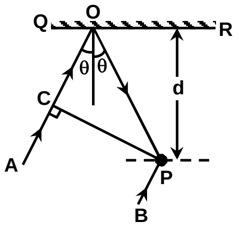
29. In Young's experiment β and β' denote the distance between any two consecutive bright and dark fringes respectively, then $\beta < \beta'$.

30. In a Young's double slit experiment, O and O′ are two slits. The path lengths OP and O′P are $n\lambda$ and $(n + 2)\lambda$ respectively, where n is a whole number and λ is the wavelength. Taking the central bright fringe as zero, the second bright fringe is formed at P.



Level - II

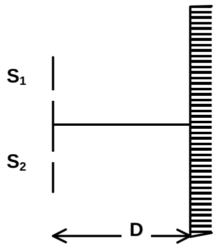
1. A plane wave of wavelength λ is incident at an angle θ on a plane mirror. Maximum intensity will be observed at P (See figure), when
   (A) $\cos\theta = 3\lambda/2d$
   (B) $\sec\theta - \cos\theta = 3\lambda/4d$
   (C) $\sec\theta - \cos\theta = \lambda/4d$
   (D) $\sec\theta - \cos\theta = \lambda/2d$



2. In a Young's double slit experiment, the position of first bright fringe coincides with S_1 and S_2 respectively on either side of central maxima. What is the wavelength of the light used? [Take D = 1m and d = 1.2 mm]
   (A) 3600 Å (B) 5400 Å
   (C) 7200 Å (D) none of these.

3. In a Young's double slit experiment, if the slits are of unequal width,
   (A) fringes will not be formed
   (B) the positions of minimum intensity will not be completely dark.
   (C) bright fringe will not be formed at the centre of the screen
   (D) distance between two consecutive bright fringes will not be equal to the distance between two consecutive dark fringes.

4. Two identical coherent sources of light S_1 and S_2 separated by a distance 'a' produce an interference pattern on the screen. The wave length of the monochromatic light emitted by the sources is λ. The maximum number of interference fringes that can be observed on the screen is nearly equal to



   (A) $\frac{2a}{\lambda}+1$ (B) $\frac{a-\lambda}{\lambda}$
   (C) $\frac{a+\lambda}{\lambda}$ (D) $\frac{\lambda}{a}+1$

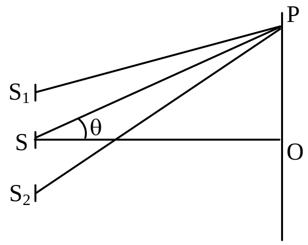
5. In Young's double slit experiment, we get 120 fringes in the field of view of monochromatic light of wavelength 4000 Å. If we use monochromatic light of wavelength 6000 Å, then the number of fringes obtained in the same field of view is
   (A) 60 (B) 90
   (C) 80 (D) 1.5

6. In Young's double slit experiment, the 7^{th} maximum with wavelength λ_1 is at a distance d_1 and that with wavelength λ_2 is at a distance d_2 . Then d_1/d_2 is
   (A) λ_1/λ_2 (B) λ_2 / λ_1
   (C) λ_1^2/λ_2^2 (D) λ_2^2/λ_1^2

7. In a two slit experiment with white light, a white fringe is observed on a screen kept behind the slits. When the screen is moved away by 0.05 m, this white fringe
(A) does not move at all
(B) gets displaced from its earlier position
(C) becomes coloured
(D) disappears

8. A source emits electromagnetic waves of wavelength 3m. One beam reaches the observer directly and other after reflection from a water surface, travelling 1.5m extra distance and with intensity reduced to 1/4 as compared to intensity due to the direct beam alone. The resultant intensity will be
(A) (1/4) fold (B) (3/4) fold
(C) (5/4) fold (D) (9/4) fold

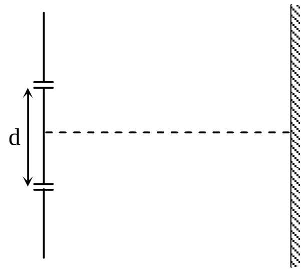
9. Two coherent sources emitting light of wavelength λ are a distance $\lambda/4$ apart. I_0 is the intensity due to either of the two sources. The intensity at a point in a direction making an angle θ, as shown in the adjacent figure, will be



(A) $4I_0 \cos^2(\theta/2)$ (B) $4I_0 \cos^2\theta$
(C) $4I_0 \cos^2\left(\frac{\pi}{2}\sin\theta\right)$ (D) $4I_0 \cos^2\left(\frac{\pi}{4}\sin\theta\right)$

10. In the Young's experiment with sodium light, the slits are 0.589 m apart. What is the angular width of the fourth maximum ? Given that λ = 589 nm.
(A) $\sin^{-1}(3 \times 10^{-6})$ (B) $\sin^{-1}(3 \times 10^{-8})$
(C) $\sin^{-1}(0.33 \times 10^{-6})$ (D) $\sin^{-1}(0.33 \times 10^{-8})$

11. In Young's double slit experiment, d is the distance between the two slits and intensity of the central maxima is I_1. If the system of double slits is displaced through d/2 along the plane of the slit, then intensity of central maxima become I_2. Therefore, I_1/I_2 is equal to



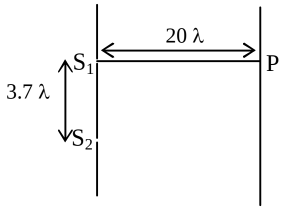
(A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 1 : 4

12. In Young's double-slit experiment, the separation between the slits is halved and the distance between the slits and screen is doubled: The fringe-width will
(A) remain unchanged (B) be halved
(C) be doubled (D) be four times.

13. Light originating from a coherent source, consisting of wavelengths λ_1 = 4500 Å and λ_2 = 6000 Å, illuminates Young's double slit apparatus simultaneously. Which of the following statements is true?
(A) No interference pattern will be formed.
(B) The third-order bright fringe of λ_1 will coincide with the fourth-order bright fringe of λ_2.
(C) The third-order bright fringe of λ_2 will coincide with the fourth-order bright fringe of λ_1.
(D) The fringes of wavelength λ_1 will be brighter than the fringes of wavelength λ_2.

14. In Young's double slit experiment, the fringe width is β. If the entire arrangement is placed in a liquid of refractive index n, the fringe width becomes
(A) $\frac{\beta}{n-1}$ (B) $\frac{\beta}{n}$
(C) $n\beta$ (D) $\frac{\beta}{n+1}$

15. Consider a Young's double slit experimental setup as shown in the adjacent figure. The light from S_1 and S_2 have equal intensities I. P is a point on the screen at the same horizontal level as of S_1. Which of the following statement is true?



(A) Intensity at point P must be equal to I.
(B) Intensity at point P must be 2I.
(C) Intensity at point P must be greater than I.
(D) P will always be a dark spot.

16. When one of the slits of Young's experiment is covered with a transparent sheet of thickness 4.8 mm, the central fringe shifts to a position originally occupied by the 30th bright fringe. What should be the thickness of the sheet if the central fringe has to shift to the position occupied by 20th bright fringe?
(A) 3.8 mm (B) 1.6 mm
(C) 7.6 mm (D) 3.2 mm

17. Light waves travel in vacuum along the y-axis. Which of the following may represent the wavefront?
(A) y = constant (B) x = constant
(C) z = constant (D) x + y + z = constant

18. Two identical sources each of intensity I_0 have a separation d = $\lambda/8$, where λ is the wavelength of the waves emitted by either source. The phase difference of the sources is $\pi/4$. The intensity distribution $I(\theta)$ in the radiation field as a function of θ, which specifies the direction from the sources to the distant observation point P, is given by
(A) $I(\theta) = I_0 \cos^2\theta$ (B) $I(\theta) = \frac{I_0}{4}\cos^2\left(\frac{\pi\theta}{8}\right)$
(C) $I(\theta) = 4I_0 \cos^2\left(\frac{\pi}{8}\right)(\sin\theta + I)$ (D) $I(\theta) = I_0 \sin^2\theta$

19. Two coherent sources are virtual in
(A) Fresnel's biprism experiment (B) Young's double slit experiment
(C) Lloyd's mirror experiment (D) all of these

Multiple choice questions (More than one choice is correct)

20. In which of the following experiments is the interference due to the division of wavefront ?
    (A) Young's double slit experiment
    (B) Fresnel's biprism experiment
    (C) Lloyd's mirror experiment
    (D) Experiment demonstrating colour thin films

21. Huygens' principle of secondary wavelets may be used to
    (A) find the velocity of light in vacuum
    (B) explain the particle behaviour of light
    (C) find the new position of a wavefront
    (D) explain Snell's law

22. If the Young's double slit experiment is to be done in a medium of refractive index 1.33, then
    (A) fringes will be hazy
    (B) fringes will come nearer
    (C) fringes will be wider
    (D) fringes will be sharpened

23. As a wave propagates
    (A) the wave intensity remains same for a plane wave
    (B) the wave intensity decreases as the inverse of the distance from the source for a spherical wave
    (C) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
    (D) total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times

24. In Young's double slit experiment, source emits light of wavelength 7000 Å. If distance between the two slits is 1 mm and distance between slits and screen is 1 m, then distance between the third dark and fifth bright fringe is
    (A) 1.75 mm
    (B) 1.85 mm
    (C) 0.875 mm
    (D) 1.75 cm

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level - O

13. 54.7^0

14. 4.5×10^{-3} rad.

15. $35^0\ 16'$

Level - I

1. (a) 0. 5 mm (b) 20

2. (a) $(\mu - 1)t$ (b) $\frac{\lambda}{2(\mu-1)}$

3. (a) 9I (b) 4I (c) I

4. 0.3 mm

5. 4I

6. 3.6×10^{-3} cm.

7. $I_P / I_{max} = 1/4$

8. 1.44 cm.

9. 1.56 mm

10. 14.33 mm

11. 90 nm.

Level - II

1. 5890 Å

2. (a) Fringes will be circular. (b) 1/16 (c) 300 nm

3. (a) 4/3 (b) 0.125 mm,

4. 4.8×10^{-6} m

5. (a) Central maximas of both the waves will be at the centre of the screen
(b) 9.1×10^{-4} m. (from centre on either side)
(c) 4.55×10^{-4} m

6. (a) 2 cm (b) 1.0016

7. 7×10^{-6} m, 1.6 and 5.7×10^{-5} m

8. 1.7 cm

9. (a) circular (b) $d' = \mu_0\left(d + \frac{\alpha d^2}{2}\right)$

10. 0.853

Objective:

Level - I

1.	C	2.	B	3.	A	4.	C
5.	D	6.	D	7.	C	8.	A
9.	A	10.	A	11.	D	12.	B
13.	A	14.	C	15.	D	16.	A
17.	A, C	18.	A	19.	B	20.	A

Fill in the blanks

21.	$\pi/2$	22.	Circular
23.	smaller	24.	Frequency, wavelength.
25.	7567 A^0		

True or False.

26.	False	27.	False
28.	True	29.	False
30.	True.		

Level - II

1.	C	2.	C	3.	B	4.	A
5.	C	6.	A	7.	A	8.	D
9.	D	10.	A	11.	A	12.	D
13.	C	14.	B	15.	C	16.	D
17.	A	18.	C	19.	A	20.	A, B, C
21.	C, D	22.	A, B	23.	A, C, D	24.	A

Learnaf

Educational Revolution

PHYSICS

FITJEE

PINNACLE

For – JEE (Main/Advanced)

GEOMETRICAL OPTICS

Geometrical Optics

Syllabus:

Reflection and refraction of light at plane and spherical surfaces, total internal reflection and its applications, spherical lenses, thin lens formula, lens maker's formula; magnification, Power of a lens, combination of mirrors and thin lenses; Refraction and dispersion of light due to a prism, Scattering of light.

Optical instruments-Compound microscope, astronomical telescope (refraction and reflection type) and their magnifying powers.

GEOMETRICAL OPTICS

In geometrical optics, we analyze the formation of images by assuming the light to travel in a straight line path or rays. For this reason, geometrical optics is also known as ray optics.

A ray of light gives the direction of propagation of light. When light meets a surface, which separates the two media, reflection and refraction take place. An image or an array of images may be formed due to this.

Beam of light

A beam of light is a bunch of light rays. It may be convergent, divergent or parallel.

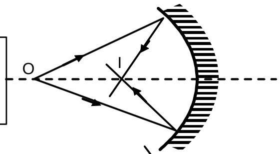
1. A convergent beam is that in which the rays are directed towards a point.
2. A divergent beam is that in which the light rays are directed away from a point.
3. In a parallel beam of light, all the rays are parallel to each other.

Object and image

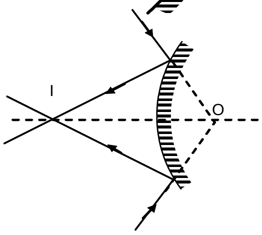
The point of intersection of the incident rays is called **object** and the point of intersection of the corresponding reflected / refracted rays is called **image**.

Object

- **REAL**
  If the incident rays diverge from an object, the object is a real object.

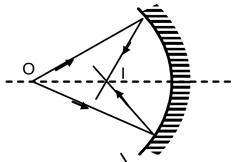


- **VIRTUAL**
  If incident rays converge and appear to intersect at a point behind the mirror (or lens), the point is a virtual object.

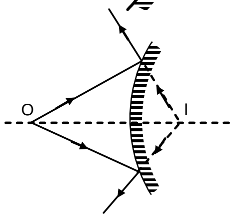


IMAGE

- **REAL**
  If the reflected (refracted) rays intersect, the point of intersection is the real image. An image, which is formed due to the actual intersection of light rays (can be held on a screen) is called a real image.



- **VIRTUAL**
  If the rays diverge after reflection (refraction) a virtual image is formed.



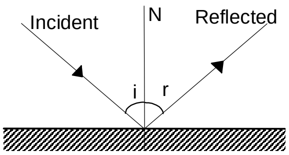
Note: In most of the problems in this section, our primary goal is to locate the final image formed by certain optical systems for a given object. The optical system may be just a mirror, or a lens or a combination of several reflecting and refracting surfaces. In such problems, first of all, we identify the sequence in which the reflection and refraction are taking place.

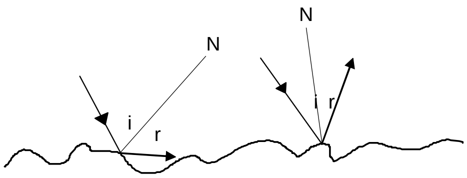
All cases of reflections and refractions are governed by laws of reflection and of refraction, respectively.

REFLECTION

When a light wave strikes a smooth interface separating two transparent materials, (such as air and glass or glass and water), the wave is in general partly reflected and partly refracted (transmitted) into the second material. For example when you look into a restaurant window from the street, you see a reflection of the street scene but a person inside the restaurant can look out through the window at the same scene as light reaches him by refraction.

The directions of the incident, reflected and refracted rays at a smooth surface between two optical materials is described in terms of two angles they make with the normal (perpendicular) to the surface at the point of incidence. If the interface is rough, both the transmitted and reflected light scatter in various directions, and there is no single angle of transmission or reflection. Reflection at a definite angle from a very smooth surface is called speculars reflection; scattered reflection from a rough surface is called diffuse reflection.





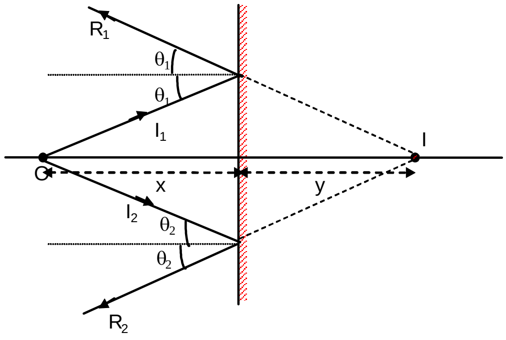
Laws of Reflection

Laws of reflection are obeyed at every reflecting surface, i.e.

1. Angle of incidence = Angle of reflection, $\angle i = \angle r$
2. The incident ray, reflected ray and the normal all lie in one plane.

Reflection at plane surface

The adjacent diagram illustrates the formation of image of a point object by a plane mirror



I_1 and I_2 are the two incident rays while R_1 and R_2 are the corresponding reflected rays. It is clear that rays R_1 and R_2 would not intersect in front of the mirror. However, for an observer in front of the mirror, reflected rays appear to be coming from point I and thus I is the virtual image of the real object O.

Using simple geometry, it can be shown that, object distance = image distance; where distances are measured from the mirror.

Other important points which should be remembered in this context are as follows.

- ☞ The image is laterally inverted.
- ☞ The linear magnification is unity.

- When the plane mirror is rotated through an angle θ, the reflected ray turns through double the angle, i.e. 2θ.

- When two plane mirrors are kept facing each other at an angle θ and an object is placed between them, multiple images of the object are formed as a result of successive reflection. The number of images, *n*, is given by,

Case 1: If $m = \frac{360}{\theta}$ is even integer, no. of images formed n = (m – 1) for all positions of object.

Case 2 : If $m = \frac{360}{\theta}$ is odd integer, no. of images formed n = m, for unsymmetrical positions, and for non symmetrical position n = m – 1.

Case 3 : If $m = \frac{360}{\theta}$ is a fraction, no. of images formed will be equal to its integral part.

- Deviation caused by single reflection = $180^o - 2i$ (i = angle of incidence)

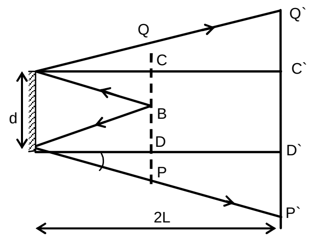
Exercise 1:

(i) ***A convergent beam of light is incident on a plane mirror. What will be the nature of image formed? Virtual or real ?***

(ii) ***What should be the minimum height of a mirror so that you can see your full image in it?***

(iii) ***Can a plane mirror ever form a real image of a real object?***

Illustration 1. *A point source of light B is placed at a distance L in front of the centre of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2L from it as shown. Find the greatest distance over which he can see the image of the light source in the mirror.*



Solution: The observer can see through the distance P′Q′, where reflected ray from B can meet the line.

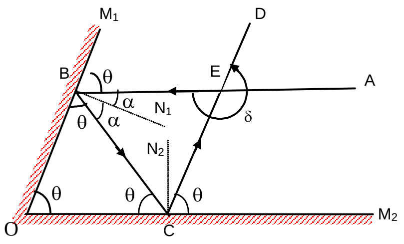
From geometry CD = d = C′D′

P′Q′ = C′D′ + P′D′ + Q′C′

= d + 2 PD + 2CQ = d + 2 (PD + QC) = d + 2d = 3d

Illustration 2. *Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror and parallel to the second is reflected from the second mirror parallel to the first mirror. Determine the angle between the two mirrors.*

Solution : Let θ be the angle between the two mirrors OM_1 and OM_2. The incident ray AB is parallel to mirror OM_2 and strikes the mirror OM_1 at an angle of incidence equal to α. It is reflected along BC; the angle of reflection being α. From figure we have



$\angle M_1BA = \angle OBC = \angle M_1OM_2 = \theta$

Similarly for reflection at mirror OM_2, we have

$\angle M_2CD = \angle BCO = \angle M_2OM_1 = \theta$

Now in triangle OBC, $3\theta = 180^o$, therefore, $\theta = 60^o$

Paraxial approximation

In all the lenses and mirrors, an approximation is made that the aperture of the mirror/lens is small compared to its radius of curvature. Thus, most of the incident rays are nearly parallel to the principal axis.

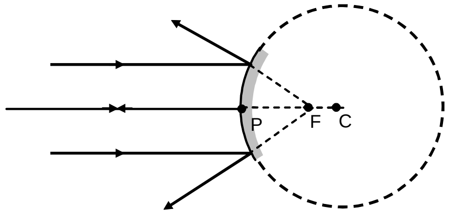
SPHERICAL MIRRORS

Mirrors which are part of a symmetrical curved surface like sphere, ellipsoid, cylindrical etc. are called curved mirrors and that which are from spheres are called spherical mirrors.

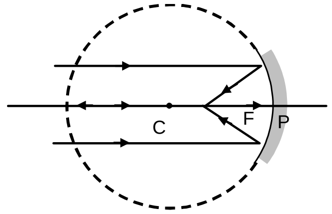
They are of two types: (i) convex (ii) concave

The mirrors in which the reflection takes place at the bulged surface are called **convex mirrors** and the mirrors in which reflection takes place at the depressed surface are called **concave mirrors**.

Convex mirror



Concave mirror



The following terms are frequently used in spherical mirrors.

Centre of curvature

It is the centre of the sphere of which the mirror/lens is a part.

Radius of curvature

It is the radius of the sphere of which the mirror is a part.

Pole

It is the geometrical centre of the circular cross section of the spherical reflecting surface.

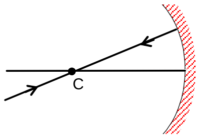
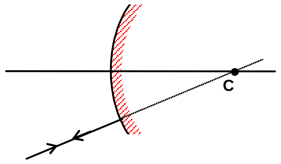
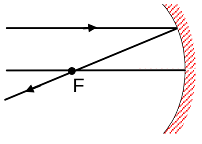
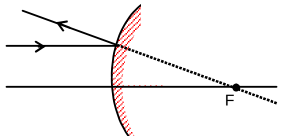
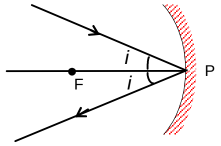
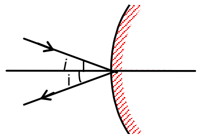
Principal axis

(For a spherical mirror): It is the straight line joining the centre of curvature to the pole.

Focus
When a narrow beam of rays of light, parallel to the principal axis and close to it, is incident on the surface of a mirror (lens), the reflected (refracted) beam is found to converge to or appear to diverge from a point on the principal axis. This point is as the focus.

Focal length
(For a mirror): It is the distance between pole and the focus.

RAY TRACING
In geometrical optics, to locate the image of an object, tracing of a ray as it reflects or refracts, is very important.

	Concave	Convex
1. Incident ray going through centre of curvature is reflected back along the same direction.		
2. Incident ray parallel to principal axis is reflected through the focus, and vice–versa. Also, mutually parallel rays after reflection intersect on the focal plane.		
3. Incident ray going to the pole and the reflected ray from it make equal angles with the principal axis.		

Sign convention
There are several sign–conventions. Students should try to follow any one of these. In this package, the new ***Cartesian Sign Convention*** has been used wherever required.

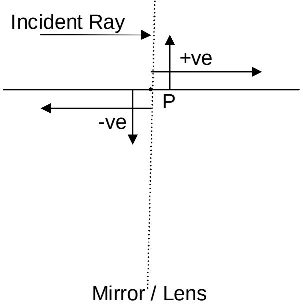
(i) All distances are measured from the pole P.

(ii) Distances measured along the direction of incident rays are taken as positive.

(iii) Distances measured along a direction opposite to the incident rays are taken as negative.

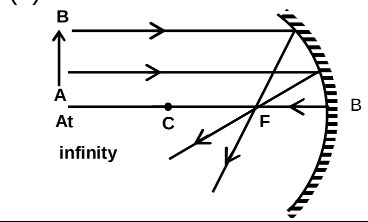
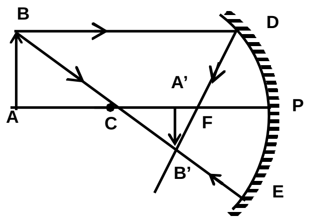
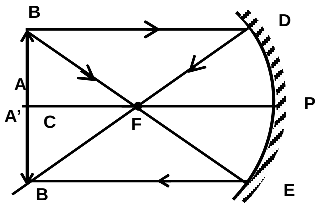
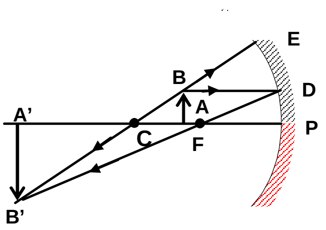
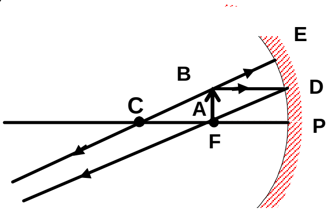
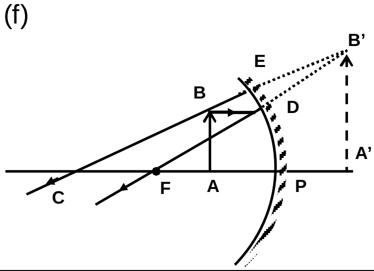
(iv) Distances above the principal axis are positive.

(v) Distances below the principal axis are negative.

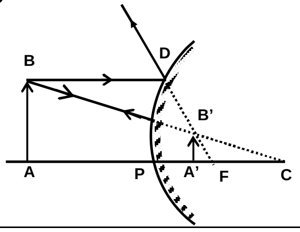
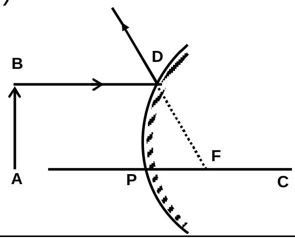


Angles when measured from the normal, in anti–clockwise direction are positive, while in clockwise direction are negative.

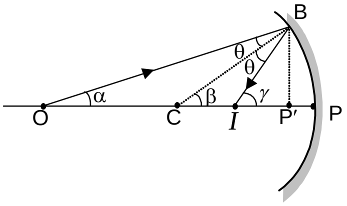
Images formed by a concave mirror

	Object position	Image position	Nature of Image
(a) 	At infinity	At the focus F	Real and point–sized
(b) 	Between infinity and the centre of curvature C	Between F and C	Real, smaller than the object, inverted
(c) 	At C	At C	Real, same size, inverted
(d) 	Between C and F	Between C and infinity	Real, enlarged, inverted
(e) 	At F	At infinity	Real or virtual, infinitely large, inverted
(f) 	Between the pole P and F	Behind the mirror	Virtual, enlarge, erect

Images formed by a convex mirror

	Object position	**Image position**	**Nature of Image**
(a) 	Between infinity and the pole	Between the focus and the pole	Virtual, smaller and erect
(b) 	At infinity	At the focus F	Virtual, point–sized

MIRROR FORMULA



Consider the shown figure where O is a point object and I is corresponding image.
CB is normal to the mirror at *B*.
By laws of reflection, $\angle OBC = \angle IBC = \theta$

$\alpha + \theta = \beta$, $\beta + \theta = \gamma$ $\Rightarrow \alpha + \gamma = 2\beta$

For small aperture of the mirror, $\alpha, \beta, \gamma \to 0$
(Paraxial approximation)

$\Rightarrow \quad \alpha \approx \tan\alpha$, $\beta \approx \tan\beta$, $\gamma \approx \tan\gamma$, $P' \to P$

$\Rightarrow \quad \tan\alpha + \tan\gamma = 2\tan\beta$

$$\Rightarrow \quad \frac{BP'}{OP'} + \frac{BP'}{IP'} = 2\frac{BP'}{CP'}$$

Take $OP' \approx OP$, $IP' \approx IP$, $CP' \approx CP$

Applying sign convention, $u = -OP$, $v = -IP$, $R = -CP$

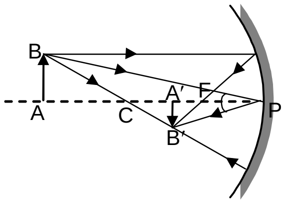
$$\Rightarrow \quad -\frac{1}{u} + \left(-\frac{1}{v}\right) = -\frac{2}{R}$$

If $u = \infty$, $\frac{1}{v} = \frac{2}{R}$, but by definition, if $u = \infty, v = f$.

Hence, $f = \frac{R}{2}$ and $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

For convex mirrors, an exactly similar formula emerges.

Lateral magnification



In new cartesian sign convention, we define magnification in such a way that a negative sign (of m) implies inverted image and vice–versa. A real image is always inverted one and a virtual one is always erect. Keeping these points in mind and that the real object and its real image would lie on the same sides in case of mirror and on opposite sides in case of lenses, we define m as in case of reflection by spherical mirror as :

$m = -\frac{v}{u}$

Illustration 3. *At what distance from a convex mirror of focal length 30 cm, should an object be placed, so that its image is formed 20 cm from the mirror on the other side.*

Solution: From mirror formula

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Here u = object distance, v (image distance) = 20cm, f (focal length of mirror) = 30 cm

$\frac{1}{u}+\frac{1}{20}=\frac{1}{30}$

u = – 60 cm

Hence object should be placed at a distance of 60 cm in front of mirror.

Illustration 4. *An object 3 cm high, is placed at a distance of 16 cm from a spherical mirror, which produces a virtual image of 4.5 cm height. Find*
(i) the focal length and nature of the mirror
(ii) the position of the image

Solution: (i) Calculation of position of image:

The height of image is 4.5 cm (h_I) and it is virtual hence magnification will be positive

$m=\frac{h_I}{h_0}=\frac{4.5}{3}=+1.5$

here h_0 = object height, Now $m=-\frac{v}{u}$

u(object distance) = – 16 cm

(Image distance) v = 1.5×16=+24 cm

Thus the position of image is 24 cm at the right side of mirror.

(ii) Calculation of focal length:

Here v = 24 cm, u = – 16 cm

$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$

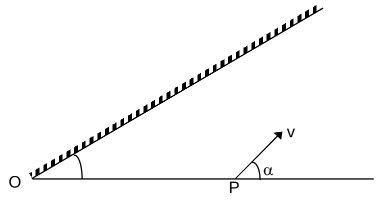
Here f= focal length of mirror

$\frac{1}{f}=-\frac{1}{16}+\frac{1}{24}$

f = – 48 cm

Hence focal length of mirror is 48 cm and minus sign shows that it is concave mirror.

Illustration 5. *A plane mirror is inclined at an angle θ = 60^0 with horizontal surface. A particle is projected from point P (see figure) at t = 0 with a velocity v at an angle α with horizontal. The image of the particle is observed from the frame of the particle projected. Assuming the particle does not*

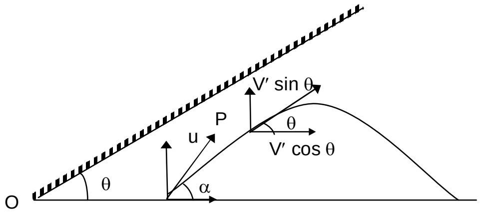


collide the mirror. The time when image will come momentarily at rest with respect to particle is

(A) $\frac{u\cos\alpha(\tan\alpha - \tan\theta)}{g}$ *(B)* $\frac{u\cos\alpha(\tan\alpha + \tan\theta)}{g}$

(C) $\frac{u\cos\alpha}{g}$ *(D)* $\frac{u\sin\alpha}{g}$

Solution: **(A)** Let at the time t the particle moves parallel to mirror.



$V'\sin\theta = u\sin\alpha - gt$ . . . (1)

And $v'\cos\theta = u\cos\alpha$

$v' = \frac{u\cos\alpha}{\cos\theta}$ . . . (ii)

from (i) and (ii)

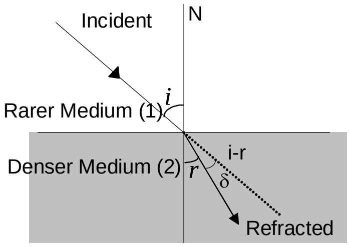
$\frac{u\cos\alpha}{\cos\theta}\cdot\sin\theta = u\sin\alpha - gt$

$t = \frac{u\cos\alpha(\tan\alpha - \tan\theta)}{g}$

Exercise 2:
(i) What is the focal length of a plane mirror?
(ii) What is the minimum distance between the real object and its real image in a concave mirror ?

REFRACTION

When a light ray passes from one medium to another such that it undergoes a change in velocity, refraction takes place. Hence, wavelength of light changes, but frequency remains the same. For non–normal rays, the refracted ray bends towards the normal on entering an optically denser medium from a rarer one.



The deviation is, $\delta = |i - r|$.

Laws of refraction

1. The incident ray, the refracted ray and the normal to the surface separating the two media – all lie in one plane.
2. **Snell's law :** For any two media the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a light beam of a particular frequency, i.e.

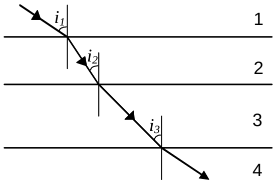
$$\Rightarrow \frac{\sin i}{\sin r} = {}_1\mu_2 = \frac{c_1}{c_2} = \frac{\text{Velocity of light in medium 1}}{\text{Velocity of light in medium 2}}$$

Refractive Index, ${}_1\mu_2$, by definition is,

$$_{\text{medium 1}}\mu_{\text{medium 2}} = {}_1\mu_2 = \frac{c_1}{c_2}$$

When merely μ of a medium is given, it implies with respect to vacuum / air.

$$\mu_{medium} = \frac{c_{vaccum}}{c_{medium}} \Rightarrow {}_1\mu_2 = \frac{c_{vaccum}/c_2}{c_{vaccum}/c_1} = \frac{\mu_2}{\mu_1}$$



As $\frac{\sin i}{\sin r} = {}_1\mu_2 = \frac{\mu_2}{\mu_1}$, when there are several medium we can say,

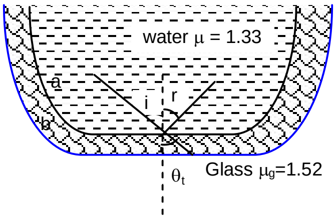
$$\frac{\sin i_1}{\sin i_2} = \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 \sin i_1 = \mu_2 \sin i_2$$

Hence, $\mu_1 \sin i_1 = \mu_2 \sin i_2 = \mu_3 \sin i_3$ = constant

or $\mu \sin i$ = constant.

This equation is found to be very useful for solving problems involving a variable refractive index.

Illustration 6. *Material a is water and b is glass with index of refraction 1.33 and 1.52, respectively. If the incidence ray makes an angle of 60° with the normal, find the directions of the reflected and refracted rays.*



Solution: From i = r

$\angle i = \angle r = 60°$

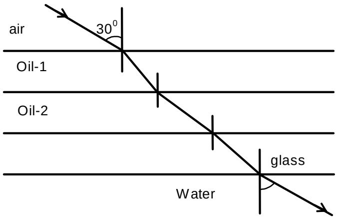
Again, from Snell's law

$$\frac{\sin i}{\sin \theta_t} = {}_1\mu_2 = \frac{\mu_g}{\mu_w}$$

$$\Rightarrow \sin \theta_t = \frac{\mu_w}{\mu_g} \times \sin i = \frac{1.33}{1.52} \times \sin 60^0 = 0.758$$

$\theta_t = 49.3^0$.

Illustration 7. *Light is incident from air on oil at an angle of 30°. After moving through oil–1, oil–2, and glass it enters water. If the refractive index of glass and water are 1.5 and 1.3, respectively, find the angle, which the ray makes with normal in water.*



Solution: As we know

$\mu \sin i$ = (constant)

$\Rightarrow \mu_{air} \sin i_{(air)} = \mu_{glass} \sin r_{(glass)}$

$$\sin r_{(glass)} = \frac{\mu_{air}}{\mu_{glass}} \sin i_{air} \quad \ldots(i)$$

Again $\mu_{glass} \sin i_{glass} = \mu_{water} \sin r_{water}$ …(ii)

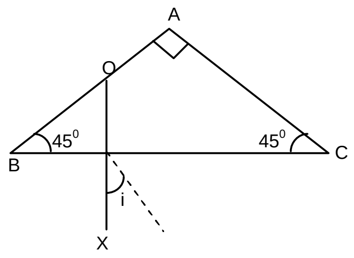
From (i) and (ii)

$= \sin 30 = 1.3 \sin r$

$$\sin r = \frac{1}{2 \times 1.3} = \frac{1}{2.6};\ r = \sin^{-1}\frac{1}{2.6}.$$

Illustration 8. *ABC is a right angled prism having refractive index $\sqrt{2}$. A ray is incident on face BC (hypotenuse) as shown in the figure. If emergent ray grazes the face AB then the angle of incidence is*

(A) zero *(B) 45^0*
(C) 90^0 *(D) none of these*



Solution: **(A)** Using Snell's law at face AB

$$\frac{\sin 90}{\sin r} = \sqrt{2}$$

$\Rightarrow$ $\sin r = \frac{1}{\sqrt{2}}$ $\Rightarrow r = 45°$

$\Rightarrow$ ray OX is $\perp$ to face BC

$\Rightarrow$ Angle of incidence is = 0

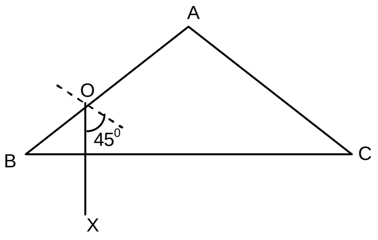
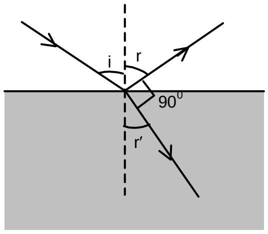


Illustration 9. *A ray of light is incident on a transparent glass slab of refractive index μ. If the reflected and refracted rays are mutually perpendicular, The angle of incidence is*

(A) $\tan^{-1}\mu$ *(B) $\sin^{-1}\mu$*
(C) $\cos^{-1}\mu$ *(D) none of these*



Solution : **(A)** Let the angle of incidence, angle of reflection and angle of refraction be i, r and r′ respectively.

Now as per the question 90 – r + 90 –r′ = 90

$\Rightarrow$ r′ = 90 – i (because i = r, in case of reflection

According to Snell's law ,

$1 \sin i = \mu \sin r'$

or $\sin i = \mu \sin r'$

or $\sin i = \mu \sin(90 - i)$

$\Rightarrow$ $\tan i = \mu$

or $i = \tan^{-1}\mu$.

Illustration 10. *Refractive index of glass with respect to water is 1.125. If the absolute refractive index of glass is 1.5, find the absolute refractive index of water.*

Solution : Here the refractive index of glass with respect to water

i.e. ${}_w\mu_g = 1.125$ and absolute refractive index of glass $\mu_g = 1.5$

We know that

$${}_a\mu_w = \frac{{}_a\mu_g}{{}_w\mu_g} = \frac{1.5}{1.125} = 1.33$$

Index of refraction and the wave aspects of light

The direction of a light ray changes when it passes from one material to another material with different index of refraction. What happens to the wave characteristics of the light?

First, the frequency f of the wave does not change when passing from one material to another.

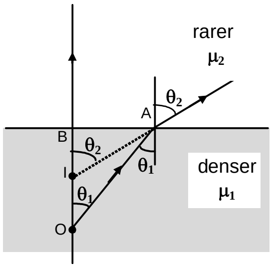
Second, in general the wavelength λ of the wave is different in different materials. This is because in any material, $v = f\lambda$; Since f is the same in any material as in vacuum and v is always less than the wave speed c in vacuum, λ also reduces correspondingly.

$\Rightarrow \lambda = \frac{\lambda_0}{\mu}$ (wavelength of light in a material)

Apparent shift of an object due to refraction

Due to bending of light at the interface of two different media, the image formation due to refraction creates an illusion of shifting of the object position.

Consider an object O in medium. After refraction, the ray at the interface bends. The bent ray when it falls on our eyes, is perceived as coming from **I**.



For **nearly normal** incident rays, θ_1 and θ_2 will be very small.

$$\tan\theta_1 = \sin\theta_1 = \frac{AB}{\text{Object distance from the refracting surface}}$$

Similarly, $\sin\theta_2 = \frac{AB}{\text{Image distance from the refracting surface}}$

$$\Rightarrow \quad \frac{\sin\theta_1}{\sin\theta_2} = {}_1\mu_2 = \frac{\mu_2}{\mu_1} \qquad \Rightarrow \frac{AB}{OB}\Big/\frac{AB}{BI} = \frac{\mu_2}{\mu_1}$$

$$\frac{BI}{OB} = \frac{\text{Apperant depth}}{\text{Real depth}} = \frac{\mu_2}{\mu_1}$$

So, Shift = Real depth – Apparent depth = Real depth $\left(1 - \frac{\mu_2}{\mu_1}\right)$

Case–I

If $\mu_1 < \mu_2$

Shift becomes negative. Image distance > object distance. Image is farther from the refracting surface

Case–II

If $\mu_1 > \mu_2$

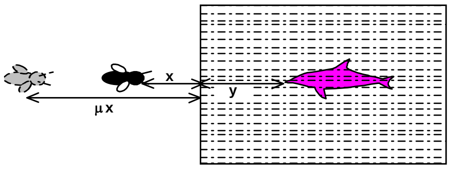
Shift becomes positive. Image distance < object distance. Image is closer to the refracting surface

Consider a special case

If $\mu_2 = 1$ or $\mu_1 = \mu$

Shift = Real depth $\left(1 - \frac{1}{\mu}\right)$

Illustration 11. *A fish in an aquarium, approaches the left wall at a rate of 3 m/s, and observes a fly approaching it at 8 m/s. If the refractive index of water is (4/3), find the actual velocity of the fly.*



Solution : For the fish the apparent distance of the fly from the wall of the aquarium is μx.

If x is actual distance, then apparent velocity will be $\frac{d(\mu x)}{dt}$

$(v_{app})_{fly} = \mu\, v_{fly}$

Now the fish observes the velocity of the fly to be 8 m/s

$\Rightarrow$ Apparent relative velocity = 8 m/s

$v_{fish} + (v_{app})_{fly} = 8$ m/s $\Rightarrow$ $3 + \mu v_{fly} = 8$

$v_{fly} = 5 \times \frac{3}{4}$ = 3.75 m/s

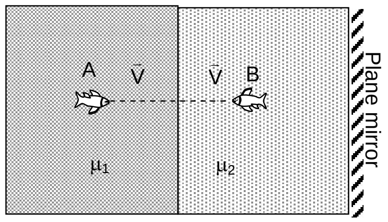
Illustration 12. *A layer of oil 3cm thick is floating on a layer of coloured water 5cm thick. Refractive index of coloured water is 5/3 and the apparent depth of the two liquids appears to be 36/7 cm. Find the refractive index of oil.*

Solution: Apparent depth (AI) = $\frac{t_1}{\mu_1} + \frac{t_2}{\mu_2}$

$\therefore \quad \frac{36}{7} = \frac{5}{5/3} + \frac{3}{\mu_2} \qquad \Rightarrow \qquad \frac{3}{\mu_2} = \frac{36}{7} - 3 = \frac{15}{7}$

or, $\mu_2 = \frac{7}{5} = 1.4$

Illustration 13. *An aquarium is bifurcated by a thin sheet of transparent material as shown in the figure. Each of the two portions contains different kinds of liquid (refractive indices'μ_1' and 'μ_2' respectively), two fish A and B swim along each other with their line of approach perpendicular to the interface. One of the side walls is a plane mirror. The velocity of separation of the two images of the fish B that are being observed by the fish 'A' is (Given that $\mu_1 < \mu_2$.)*



(A) $2\left(\frac{\mu_1}{\mu_2}\right)v$ *(B)* $\left(\frac{\mu_1}{\mu_2}\right)v$

(C) $3\left(\frac{\mu_1}{\mu_2}\right)v$ *(D) none of these*

Solution: **(A)** For A:–

The velocity of approach of the first image (by refraction)

$= v1 = v + \frac{\mu_1 v}{\mu_2} = v\left(1+\frac{\mu_1}{\mu_2}\right)$

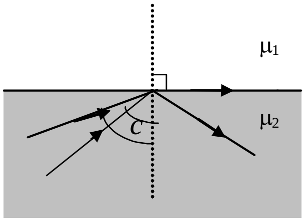
and the velocity of approach of the second image (by reflection)

$= v_2 = v - \frac{\mu_1 v}{\mu_2} = v\left(1-\frac{\mu_1}{\mu_2}\right)$

$\therefore$ Velocity of separation of the two images = $v_1 - v_2 = v\left(\frac{2\mu_1}{\mu_2}\right) = 2\left(\frac{\mu_1}{\mu_2}\right)v$

Critical angle and total internal reflection

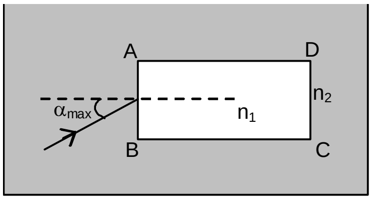
Consider a ray of light that travels from a denser medium to a rarer medium. As the angle of incidence increases in the denser medium the angle of refraction in the rarer medium increases. The angle of incidence for which the angle of refraction becomes 90° is called critical angle.



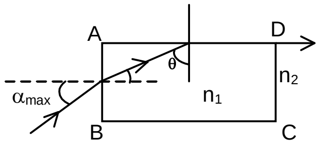
$$\frac{\sin c}{\sin 90^\circ} = {}_2\mu_1 = \frac{\mu_1}{\mu_2} \quad \Rightarrow \quad \sin c = \frac{1}{{}_1\mu_2}$$

When the angle of incidence of a ray travelling from denser to rarer medium is greater than the critical angle, no refraction occurs. The incident ray is *totally reflected* back into the same medium. Here the laws of reflection hold good. Some light is also reflected before the critical angle is achieved, but not totally.

Illustration 14. *A rectangular slab ABCD, of refractive index n_1, is immersed in water of refractive index n_2 ($n_1 > n_2$). A ray of light is incident at the surface AB of the slab as shown. Find the maximum value of incidence angle α_{max}, such that the ray comes out only from the other surface CD.*



Solution: For a maximum angle of incidence at surface AB there will be a minimum angle of incidence at the surface AD. For the ray to pass through the surface CD should not pass beyond AD i.e. it should not refract at AD, Hence, the angle θ should be the critical angle. By Snell's Law

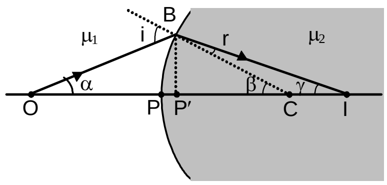


$\sin\theta = \frac{n_2}{n_1}$; $n_2 \sin\alpha_{max} = n_1 \sin(90^\circ - \theta)$

$\sin \alpha_{max} = \frac{n_1}{n_2}\cos\theta$; $\alpha_{max} = \sin^{-1}\left[\frac{n_1}{n_2}\cos\left[\sin^{-1}\left(\frac{n_2}{n_1}\right)\right]\right]$

Refraction at spherical surfaces

Consider the point object O placed in the medium with refractive index equal to μ_1.



if $\mu_2 > \mu_1$

As $\mu_1 \sin i = \mu_2 \sin r$, and for small aperture i, $r \to 0$

$\Rightarrow \mu_1 i = \mu_2 r$

$i = \alpha + \beta, \quad \beta = \gamma + r$

$\Rightarrow \mu_1(\alpha + \beta) = \mu_2(\beta - \gamma)$

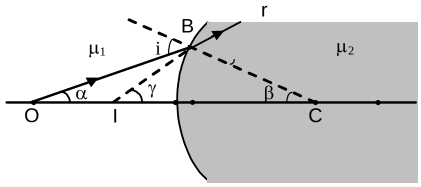
$\Rightarrow \mu_1\alpha + \mu_2\gamma = (\mu_2 - \mu_1)\beta$

As aperture is small,

$\alpha \approx \tan\alpha$, $\beta \approx \tan\beta$, $\gamma \approx \tan\gamma$

$\Rightarrow \mu_1 \tan\alpha + \mu_2 \tan\gamma = (\mu_2 - \mu_1)\tan\beta$

$$\Rightarrow \frac{\mu_1}{P'O} + \frac{\mu_2}{P'I} = \frac{\mu_2 - \mu_1}{P'C}$$



If $\mu_1 > \mu_2$

Take $OP \approx OP'$

$PC \approx P'C$

$PI \approx P'I$

Applying sign convention, i.e., $u = -P'O$, $v = P'I$, $R = P'C$

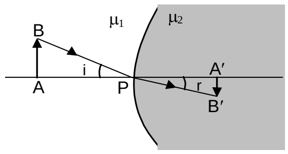
$$\therefore \text{For a spherical surface, } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

The symbols should be carefully remembered as: μ_2 – refractive index of the medium into which light rays are entering; μ_1 – refractive index of the medium from which light rays are coming. Care should also be taken while applying the sign convention to R.

Lateral magnification for refracting spherical surface

$$\text{Lateral magnification, } m = \frac{\text{Image height}}{\text{Object height}}$$

$$= \frac{-(A'B')}{AB}. \quad \text{Now, } \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$



For small angles of incidence and thus refraction $\sin i \approx \tan i$ and $\sin r \approx \tan r$

$$\Rightarrow \frac{\tan i}{\tan r} = \frac{\mu_2}{\mu_1}$$

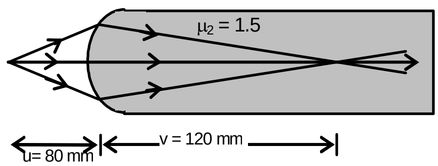
In triangles ABP and A′B′P we have

$$\frac{AB/PA}{A'B'/PA'} = \frac{\mu_2}{\mu_1} \qquad \Rightarrow -\frac{A'B'}{AB} = \frac{-PA'/\mu_2}{PA/\mu_1}$$

$$\text{Hence, } m = \frac{v/\mu_2}{u/\mu_1}.$$

Illustration 15. *One end of a cylindrical glass rod shown in the figure ends in a hemispherical surface of radius R = 20 mm.*

(a) Find the image distance of a point object on the axis of the rod, 80 mm to the left of the vertex. The rod is in air.



(b) Let the same rod be immersed in water of refractive index 1.33, the other quantities having the same values as before. Find the image distance.

Solution : (a) $\mu_1 = 1$, $\mu_2 = 1.5$, R = +20 mm, u = +80 mm

$$\frac{1.5}{v} - \frac{1}{-80} = \frac{1.5 - 1}{+20}$$

$v' = +120$ mm

The image is therefore formed at the right of the vertex (v is positive) and at distance of 120 mm from it.

(b) $$\frac{1.5}{v} - \frac{1.33}{-80} = \frac{1.5 - 1.33}{+20},$$

$v = -180$ mm.

The fact that v is negative which means that the rays, after refraction by the surface, are not converging but appear to diverge from a point 180 mm to the left of the vertex. In this illustration, then the surface forms a virtual image 180 mm to the left of the vertex.

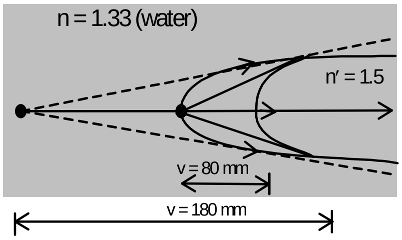
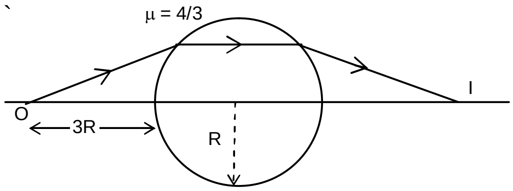


Illustration 16. *A thin walled glass sphere of radius R is filled with water ($\mu = 4/3$). An object is placed at distance 3R from the surface of the sphere. If the effect of the glass wall is neglected, find the distance of the final image from the centre of sphere.*

Solution: By relation,



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{4}{3v} - \frac{1}{-3R} = \frac{4/3 - 1}{R}$$

or $\frac{4}{3v} = 0$ $\Rightarrow$ $v = \infty$

Since image is formed at infinity, so refracted ray is parallel to principal axis.
Hence,
By symmetry the final image is also formed at 3R from the surface of sphere.
So distance of image from the centre = 3R + R = 4R

Illustration 17. *For an equilateral prism, it is observed that when a ray strikes graztingly at one face it emerges grazingly at the other. Calculate the critical angle and the refractive index.*

Solution: r = c, each surface

$A = r + r = 60°$

$r = 30°$

$\mu = \sin 90/\sin 30 = 2$

Illustration 18. *A lens is 5 cm thick and the radii of curvature of its convex surfaces are 10cm and 25 cm respectively. A point object is placed at a distance 12 cm from the surface whose radius of curvature is 10 cm. How far beyond the other surface image is formed?*

Solution: We know that $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$u = -12$ cm, $R = 10$ cm, $\mu_1 = 1$, $\mu_2 = 1.5$

$$\frac{1.5}{v} - \frac{1}{-12} = \frac{1.5 - 1}{10}$$

$\Rightarrow$ $v = -45$ cm

This image will serve as an object for the second surface.

For the second surface object distance

$u = 5 + 45 = 50$ cm

For the second surface again

$u = -50$ cm, $R = -25$ cm, $\mu_1 = 1.5$, $\mu_2 = 1$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

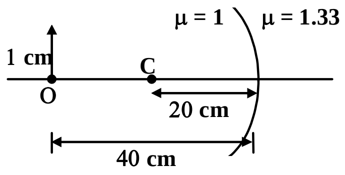
$$\Rightarrow \frac{1}{v} - \frac{1.5}{-50} = \frac{1 - 1.5}{-25}$$

or $v = -100$ cm

Final image will be at a distance –95 cm from the first surface on the same side as the objects.

Illustration 19. *Find the size of the image formed in the situation shown in figure.*

Solution : Here $u = -40$cm, $R = -20$ cm



$\mu_1 = 1$, $\mu_2 = 1.33$

We have,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.33}{v} = -\frac{1}{40} - \frac{0.33}{20}$$

$v = -32$ cm.

The magnification is $m = \frac{h_2}{h_1} = \frac{\mu_1 v}{\mu_2 u}$

$$\frac{h_2}{1} = -\frac{32}{1.33(-40)} \Rightarrow h_2 = 0.6 \text{ cm}$$

The image is erect.

Illustration 20. *A point source of light is placed at depth h below the surface of a large and deep lake. The percentage of light energy that escapes directly from the water surface is ($\mu_w = 4/3$)*

(A) zero *(B) 10 %*

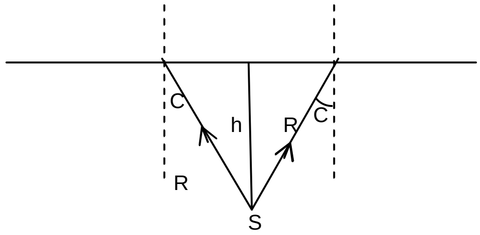
(C) 17 % *(D) 25 %*

Solution : **(C)** Fraction = $\dfrac{2\pi R^2(1-\cos\theta)}{4\pi R^2}$

$= \dfrac{1-\cos\theta}{2} = \dfrac{1}{2}\left\{1-\sqrt{1-\dfrac{1}{17^2}}\right\}$

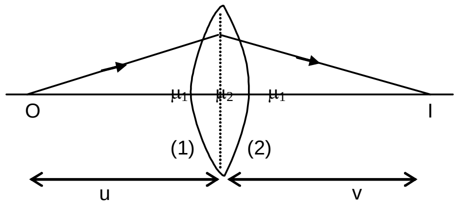
$= 0.17$

$\therefore$ Percentage = 17 %



THIN LENS

Consider a thin lens shown here with the two refracting surfaces having radii of curvature R_1 and R_2, respectively. The refractive indices of this surrounding medium and of the material of the lens are μ_1 and μ_2 respectively.



Now using the result that we obtained for refraction at single spherical surface we get,

For first surface, $\dfrac{\mu_2}{v_1} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R_1}$ . . . (1)

For second surface, $\dfrac{\mu_1}{v} - \dfrac{\mu_2}{v_1} = \dfrac{\mu_1 - \mu_2}{R_2}$ . . . **(**2)

Adding (1) and (2),

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $$\mu_1\left(\frac{1}{v} - \frac{1}{u}\right) = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\mu_1\left(\frac{1}{v} - \frac{1}{u}\right) = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad . . . (3)$$

Lensmaker's formula

In the above equation if the object is at infinity and image is formed at the focus. Then

$u = \infty$, $v = f$,

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad . . . (4)$$

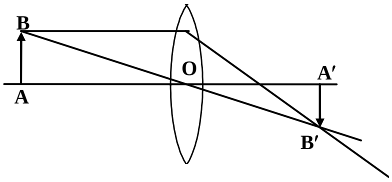
Thin lens formula

Now, comparing equations (3) and (4) we have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Magnification of a thin lens

$$m = \frac{v}{u} = \frac{-(OA')}{(AO)}$$

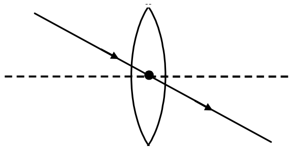
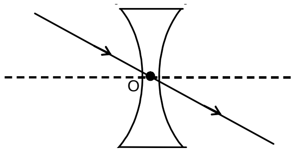
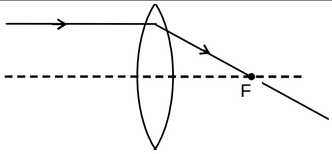
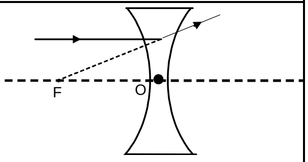


Power of a lens

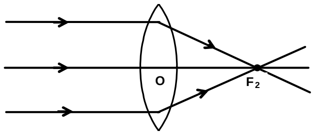
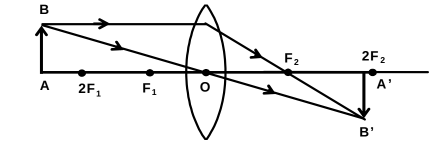
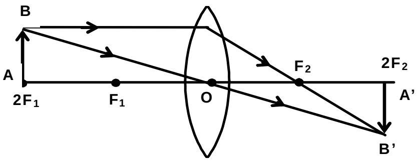
Power of a lens is expressed as the reciprocal of the focal length of the lens measured in meters. If the focal length f is in metres, then the power *P* of the lens is given by, $P = \frac{1}{f}$ dioptre.

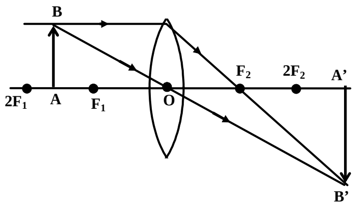
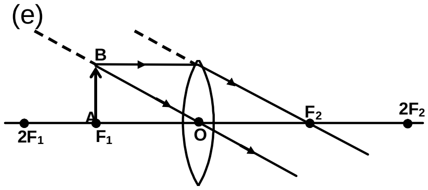
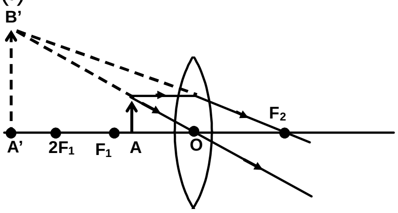
Note: A lens is converging if its focal length is positive and diverging if negative (in new–cartesian sign convention).

Ray tracing for lens

	Convex	**Concave**
1. A ray passing through optic centre goes undeviated.		
2. When a ray is incident parallel to principal axis, the refracted ray passes or appears to pass through the focus.		

Images formed by a convex lens

	Object position	**Image position**	**Nature of Image**
(a) 	At infinity	At F_2	Real and point–sized
(b) 	Between infinity and $2F_1$	Between F_2 and $2F_2$	Real, smaller inverted
(c) 	At $2F_1$	At $2F_2$	Real, same size, inverted

(d) 	Between $2F_1$ and F_1	Between $2F_2$ and infinity	Real, enlarged inverted
(e) 	At F_1	At infinity	Real or virtual, infinitely large, inverted
(f) 	Between F_1 and O	On the side of the object	Virtual, larger, erect

Images formed by a concave mirror

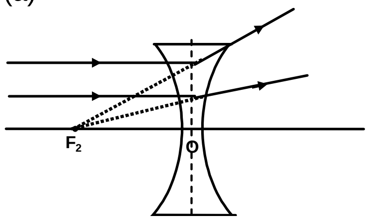
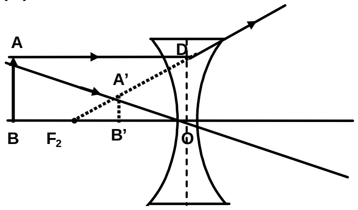
	Object position	Image position	Nature of Image
(a) 	At infinity	At F_2	Virtual, point–sized
(b) 	Between infinity and O	Between F_2 and O	Virtual, erect and smaller

Illustration 21. *A convex lens of refractive index 3/2 has got a focal length equal to 50 cm. The focal length of the lens, if it is immersed in ethyl alcohol of refractive index 1.36, is*

(A) 230 cm *(B) 242.75 cm*
(C) 260 cm *(D) 290 cm*

Solution: **(B)** According to lens makers formula

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots (1)$$

In ethyl alcohol

$$\frac{1}{f_{liquid}} = \left(\frac{1.5}{1.36} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots (2)$$

Dividing (1) and (2)

$$f_{liquid} = \frac{(1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{\left(\frac{1.5}{1.36} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} \times f = 242.75 \text{ cm}$$

Illustration 22. *A double convex lens forms a real image of an object on a screen which is fixed. Now the lens is given a constant velocity $v = 1ms^{-1}$ along its axis and away from the screen. For the purpose of forming the image always on the screen, the object is also required to be given an appropriate velocity. Find the velocity of the object at the instant its size is double the size of the image.*

Solution: Let us take the lens to be stationary and screen moving with velocity v away from the lens.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{v^2}\cdot\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\frac{du}{dt} = \frac{u^2}{v^2}\frac{dv}{dt}$$

$$\dot{u} = \frac{1}{m^2}\cdot\dot{v}$$

Thus, the object is moving with velocity $(1/m^2)V$ with respect to the lens and towards it (i.e., towards the screen). Velocity of the object with respect to the screen,

$v_{OS} = v - v/m^2$

As, $m = 1/2$, Hence $v_{OS} = \left[1 - \frac{1}{(1/2)^2}\right]v = -3v = 3 \text{ ms}^{-1}$ towards the screen.

Illustration 23. *An object is placed at a distance of 10 cm to the left on the axis of a convex lens L_1 of focal length 20 cm. A second convex lens L_2 of focal length 10 cm is placed co–axially to the right of the lens L_1 at a distance of 5 cm from it. Find the position of the final image and its magnification. Trace the path of the rays.*

Solution : Here for 1st lens,

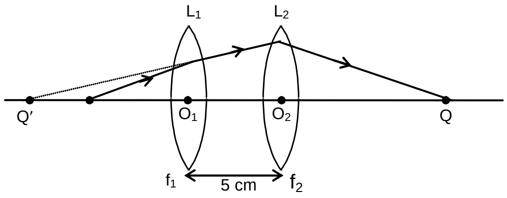
$u_1 = -10$ cm

$f_1 = 20$ cm.

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1} \quad \Rightarrow \quad \frac{1}{v_1} = \frac{1}{20} - \frac{1}{10}$$

$\Rightarrow v_1 = -20$ cm

i.e. the image is virtual and hence lies on the same side of the object. This will behave as an object for the second lens.



For 2nd lens, $\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$

Here, $u_2 = -(20 + 5)$, $f_2 = 10$ cm

$\frac{1}{v_2} + \frac{1}{25} = \frac{1}{10}$ $\Rightarrow v_2 = \frac{50}{3} = 16\frac{2}{3}$cm

i.e. the final image is at a distance of $16\frac{2}{3}$ cm on the right of the second lens.

The magnification of the image is given by,

$$m = \frac{v_1}{u_1}.\frac{v_2}{u_2} = \frac{20}{10}.\frac{50}{3\times 25} = \frac{4}{3} = 1.33.$$

Illustration 24. *A convergent lens of 6 dioptres is combined with a diverging lens of –2 dioptres. Find the power and focal length of the combination.*

Solution : Here $P_1 = 6$ diopters, $P_2 = -2$ dioptres

Using the formula $P = P_1 + P_2 = 6 - 2 = 4$ dioptres

$f = \frac{1}{P} = \frac{1}{4}$ m = 25 cm.

Illustration 25. *A lens has a power of +5 dioptres in air. What will be its power if completely immersed in water? [$_a\mu_w = 4/3$ and $_a\mu_g = 3/2$.]*

Solution : Let f_a and f_w be the focal lengths of the lens in air and water respectively, then

$P_a = \frac{1}{f_a}$ or $+5 = \frac{1}{f_a}$

$f_a = 0.2$ m = 20 cm

Now $\frac{1}{f_a} = ({}_a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ …(1)

and $\frac{1}{f_w} = ({}_w\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ …(2)

Dividing equation (1) by equation (2), we get,

$$\frac{f_w}{f_a} = \left[\frac{{}_a\mu_g - 1}{{}_w\mu_g - 1}\right]$$

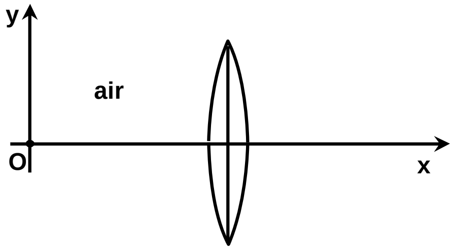
Again, ${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$

$\therefore \frac{f_w}{f_a} = \frac{(3/2) - 1}{(9/8) - 1} = \frac{(1/2)}{(1/8)} = 4$

$f_w = f_a \times 4 = 20 \times 4 = 80$ cm = 0.8 m

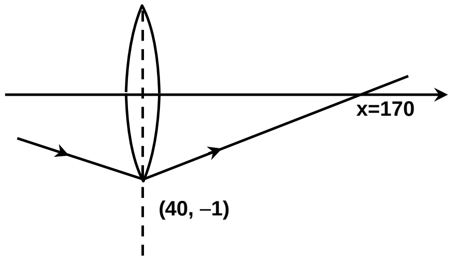
$$P_w = \frac{1}{f_w} = \frac{1}{0.8} = 1.25 \text{ dioptre.}$$

Illustration 26. *A thin equi–convex spherical glass lens (μ = 3/2) of focal length 30 cm is placed on the x–axis with its optical centre at x = 40 cm and its principal axis coinciding with the x–axis. A light ray, given by the equation: 39 y = – x + 1 (x, y in cm) is incident onto the lens, towards the positive x–axis.*



(a) Find the equation of the refracted ray.
(b) If the space on the right side of the lens (x > 40 cm) is filled with a liquid of refractive index 4/3, find the new equation of the refracted ray.

Solution: The object distance corresponding to this ray can be found from its intersection with the x–axis.



x = 1

$\therefore$ u = –(40 – 1) = –39

$\frac{1}{v} - \frac{1}{-39} = \frac{1}{30}$; v = 130 cm

The equation of the refracted ray is

$$y = \frac{1}{130}(x - 170)$$

If the space x > 40 cm, on the side of the lens is filled with a liquid of refractive index,

μ = 4/3, then we get,

$$\frac{1.5}{v_1} - \frac{1.0}{-39} = \frac{0.5}{30}$$

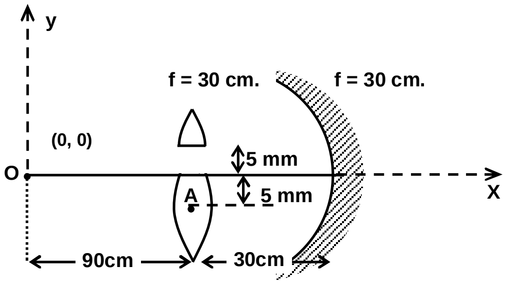
$$\frac{\frac{4}{3}}{v} - \frac{1.5}{v_1} = \frac{\frac{4}{3} - 1.5}{-30}$$

or, v = –390 cm

The equation of the refracted ray is

$$y = -\frac{1}{390}(x + 350)$$

Illustration 27. *A lens having focal length 30 cm. is cut along a plane parallel to the principal axis of the lens at a distance 5 mm above optical centre 'A' and upper part of lens is shifted by 5 mm from the x–axis as shown in the figure. A point object is placed on the principal axis of the mirror at origin O (0, 0). Find the number of images formed just after the reflection from the mirror and write their co–ordinates.*



Solution: Image formed by upper lens

$$\frac{1}{v_1} - \frac{1}{(-90)} = \frac{1}{30} \Rightarrow v_1 = 45\text{cm}$$

This will act as an object for mirror

$$\therefore \quad \frac{1}{v_2} + \frac{1}{15} = \frac{-1}{30}$$

$v_2 = -10$cm

Coordinates of image are (110, 0)

Image formed directly by mirror

$$\frac{1}{v} - \frac{1}{120} = \frac{-1}{30} \;\; ; \;\; \frac{1}{v} = \frac{-1}{30} + \frac{1}{120} = \frac{-3}{120} = -\frac{1}{40}$$

$v = -40$cm

$\therefore$ coordinates are (80, 0)

Image formed by lower lens.

$$\frac{1}{v_c} - \left(\frac{1}{-90}\right) = \frac{1}{30} \Rightarrow v_c = 45\text{cm}.$$

This acts as an objects for mirror

$$\therefore \quad \frac{1}{v_m} + \frac{1}{15} = \frac{-1}{30} \Rightarrow v_m = -10\text{cm}.$$

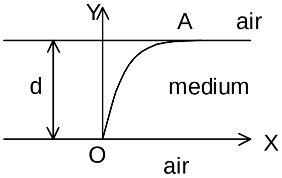
Magnification due to lens = $-\dfrac{45}{90} = -\dfrac{1}{2}$;

Coordinates of image formed by lower lens = (135, –0.75)

Magnification due to mirror = $\dfrac{+10}{15} = \dfrac{+2}{3}$

$\therefore$ Coordinates of final image = (110, –0.5)

Illustration 28. *A long rectangular slab of transparent medium of thickness d is placed on a table with its length parallel to the X-axis and width parallel to Y-axis. A ray of light travelling in air makes a near normal incidence on the slab as shown. Take the point of incidence as origin (0, 0, 0) and* $\mu = \dfrac{\mu_0}{1-(x/r)}$



where μ_0 and r(> d) are constants. $\mu_{air} = 1$

Determine the X–coordinate of the point A, where the ray intersects the upper surface of slab – air boundary.

Solution: $\because$ μ. sin θ_x = constant, θ_x = angle made with x-axis at any point.

$\therefore$ $\mu \sin \theta_x = \mu_0 \sin (\pi/2)$

$$\therefore \sin\theta_x = \frac{\mu_0}{\mu} = \frac{\mu_0}{\mu_0\{1-x/r\}^{-1}}$$

$$\therefore \frac{dy}{dx} = \tan\theta_x = \frac{1-x/r}{\sqrt{1-(1-x/r)^2}}$$

$$\therefore \int dy = \int \frac{(1-x/r)}{\sqrt{1-(1-x/r)^2}}\,dx$$

Putting $z = 1 - (1 - x/r)^2$

$\therefore \quad x^2 + y^2 - 2xr = 0$

Putting $y = d$ and $x = x_A$, $\quad x_A = r\{1 \pm \sqrt{1-(d/r)^2}\ \}$

Putting the condition, as $r \to \infty$, $x_A \to 0$, $x_A = r\{1 - \sqrt{1-(d/r)^2}\ \}$

Exercise 3:

(i). ***A substance has critical angle of 45° for yellow light. What is its refractive index ?***

(ii). ***What happens to the frequency when light passes from one medium to another ?***

(iii). ***The image of a small electric bulb fixed on the wall of a room is to be obtained on opposite wall 3m away by means of a large convex lens. What is the maximum possible focal length for this purpose ?***

(iv). ***For the same angle of incidence, the angle of refraction in medium P, Q and R are 35°, 25° and 15° respectively. In which medium velocity of light is minimum?***

(v). ***Why googgles (sun glasses) have zero power even though their surfaces are curved ?***

Equivalent focal length of two or more thin lenses in contact

If there are two lenses of focal lengths f_1 and f_2, the focal length F of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \cdots$$

Illustration 29. *Convex lens of 10 cm focal length is combined with a concave lens of 6 cm focal length. Find the focal length of the combination.*

Solution: Here $f_1 = 10$ cm, $f_2 = -6$ cm, $F = ?$

Use the formula $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{10} - \frac{1}{6} = -\frac{1}{15}$

$\therefore \quad F = -15$ cm

Silvering of lenses

If any surface of a lens is silvered, it will ultimately behave as a mirror and the power of mirror thus formed will be equal to the sum of powers of the optical lenses and the mirrors in between.

Lens with One Silvered Surface:

If the back surface of a lens is silvered and an object is placed infront of it, then the power of the silvered lens will be

$P = P_L + P_M + P_L$

Where $P_L = + \frac{1}{F_L} = \frac{1}{f_L} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

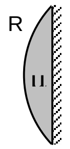
$P_M = -\frac{1}{f_M}$ with $f_M = \frac{R_2}{2}$

So the system will behave as a curved mirror of focal length 'F' given by

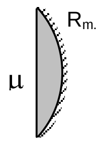
$$F = -\frac{1}{P}$$

Illustration 30. *Find the focal length of a plano–convex lens when :*
(i) The plane surface is silvered and the object is in-front of curved surface.
(ii) Curved surface is silvered and the object is in-front of plane surface.

Solution : (i) In this situation



(a)



(b)

$$\frac{1}{f_L} = (\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right) = \frac{\mu-1}{R}$$

and $F_M = \frac{\infty}{2} = \infty$

so $P_L = \frac{1}{f_L} = \frac{(\mu-1)}{R}$

and $P_M = -\frac{1}{f_M} = \frac{1}{\infty} = 0$

Hence power of the system

$P = P_L + P_M + P_L = 2P_L + P_M$

i.e. $P = \frac{2(\mu-1)}{R} + 0 = \frac{2(\mu-1)}{R}$

$\therefore \quad F = -\frac{1}{P} = -\frac{R}{2(\mu-1)}$

i.e. the lens will behave as a concave mirror of focal length.

$$\frac{R}{2(\mu-1)}$$

(ii) In this case

$$\frac{1}{f_L} = (\mu-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right) = \frac{(\mu-1)}{R}$$

and $F_M = \frac{(-R)}{2}$

So, $P_L = \frac{1}{f_L} = \frac{(\mu-1)}{R}$ and $P_M = -\frac{1}{f_M} = \frac{2}{R}$

Hence, power of system

$P = P_L + P_M + P_L = 2P_L + P_M$

$P = \frac{2(\mu-1)}{R} + \frac{2}{R} = \frac{2\mu}{R}$

$\therefore \quad F = -\frac{1}{P} = -\frac{R}{2\mu}$

i.e. the lens will be equivalent to a converging mirror of focal length $(R/2\mu)$

Illustration 31. *A pin is placed 10 cm in front of a convex lens of focal length 20cm made of a material having refractive index 1.5. The surface of the lens which is farther away from the pin is silvered and has a radius of curvature 22 cm. Determine the position of the final image. State the nature of the image.*

Solution: The curved silvered surface will behave as a concave mirror of focal length

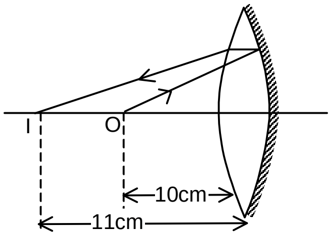
$f_m = \frac{R}{2} = -\frac{22}{2} = -11\text{cm} = -0.11\text{m}$

P_M = the power of the mirror $= -\frac{1}{f_M} = -\frac{1}{-0.11} = +\frac{1}{0.11}D$

Further as the focal length of lens is 20 cm i.e., 0.20m its power will be given by:

$P_L = \frac{1}{f_L} = \frac{1}{20}D$

Now as in image formation, light after passing through the lens will be reflected back by the curved mirror through the lens again.



$P = P_L + P_M + P_L = 2P_L + P_M$

$P = \frac{2}{0.20} + \frac{1}{0.11} = \frac{210}{11}$ D

So the focal length of equivalent mirror

$F = -\frac{1}{P} = \frac{11}{210}m = -\frac{110}{21}$ cm

i.e. the silvered lens behaves as a concave mirror of focal length (110/21) cm. So for object at a distance 10 cm in front of it

$\frac{1}{v} + \frac{1}{-10} = -\frac{21}{110}$ i.e., $v = -11$ cm

i.e. image will be 11cm infront of the silvered lens and will be real as shown in figure.

Illustration 32. *The convex surface of a plano convex lens is silvered whose radius of curvature is R. Find the focal length of the mirror thus formed.*

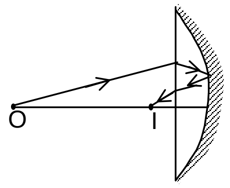
Solution : $P_{\text{final mirror}} = P_{\text{lens}} + P_{\text{concave mirror}} + P_{\text{lens}}$

As the ray of light first goes through lens then mirror and again lens.

$P_{\text{final mirror}} = \frac{1}{f_\ell} - \frac{1}{f_m} + \frac{1}{f_\ell} = \frac{2}{f_\ell} - \frac{1}{f_m}$ where f_ℓ is given as

$\frac{1}{f_\ell} = (n-1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = +\frac{(n-1)}{R}$

$\Rightarrow f_\ell = \frac{+R}{n-1} = 2R$ (for n = 1.5)

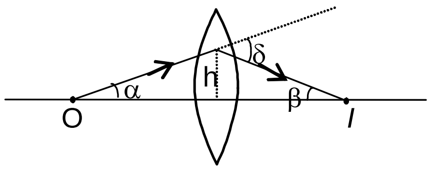


$f_m = -\frac{R}{2}$; $P_{\text{final mirror}} = \frac{2}{2R} + \frac{2}{R} = \frac{3}{R}$

$F_{\text{final mirror}} = -\frac{1}{P_{\text{finalmirror}}} = -\frac{R}{3}$ and hence the final mirror will be concave.

Angle of deviation of a ray when it passes through a lens

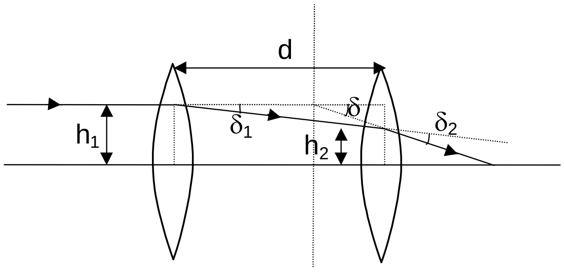
O represents the object **I** the image and δ is the angle of deviation.



$\delta = \alpha + \beta \approx \tan\alpha + \tan\beta$

$= \frac{h}{u} + \frac{h}{v} = h\left[\frac{1}{v} - \frac{1}{u}\right] \Rightarrow \delta = \frac{h}{f}$

Two thin lenses separated by a distance *d*



$\delta=\delta_1+\delta_2 \;\Rightarrow\; \tan\delta \approx \tan\delta_1+\tan\delta_2$

$\Rightarrow \dfrac{h}{F}=\dfrac{h_1}{f_1}+\dfrac{h_2}{f_2}.$

But, $h_1-h_2=d.\tan\delta_1=d.\delta_1=\dfrac{d.h_1}{f_1}$

$\Rightarrow h_2=h_1\left[1-\dfrac{d}{f_1}\right]$

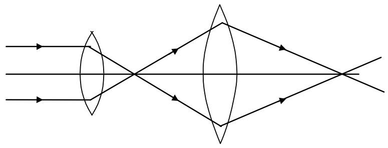
Substituting we get, $\dfrac{1}{F}=\dfrac{1}{f_1}+\dfrac{1}{f_2}-\dfrac{d}{f_1f_2}$

(This expression is valid for parallel rays. i.e. when object is placed at infinity)

A more powerful form of this formula is, $P=P_1+P_2-\dfrac{d}{\mu}P_1P_2$ (where P_1 and P_2 are optical powers of the two lenses, and μ is the refractive index of the medium in between them.)

Illustration 33. *Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. Find the image of a distant object formed by the combination by using thin lens formula separately for the two lenses.*

Solution : The first image is formed at the focus of the first lens. This is at 20 cm from the first lens and hence at u = –40 cm from the second. Using the lens formula for the second lens,



$$\frac{1}{v}=\frac{1}{u}+\frac{1}{f}=-\frac{1}{40}+\frac{1}{20} \qquad \text{or} \quad v = 40 \text{ cm}$$

The final image is formed 40 cm to the right of the second lens.

Focal length of a concave mirror and a convex lens using the u–v method.

In this method, one uses an optical bench and the convex lens (or the concave mirror) is placed on the holder.

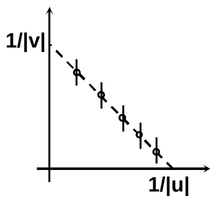
The position of the lens is noted by reading the scale at the bottom of the holder.

A bright object (a filament lamp or some similar object) is placed at a fixed distance (u) in front of the lens (mirror).

The position of the image (v) is determined by moving a white screen behind the lens until a sharp image is obtained (for real images).

For the concave mirror, the position of the image is determined by placing a sharp object (a pin) on the optical bench such that the parallax between the object pin and the image is nil.

A plot of |u| versus |v| gives a rectangular hyperbola. A plot of $\dfrac{1}{|v|}$ vs $\dfrac{1}{|u|}$ gives a straight line.



The intercepts are equal to $\dfrac{1}{|f|}$, where f is the focal length.

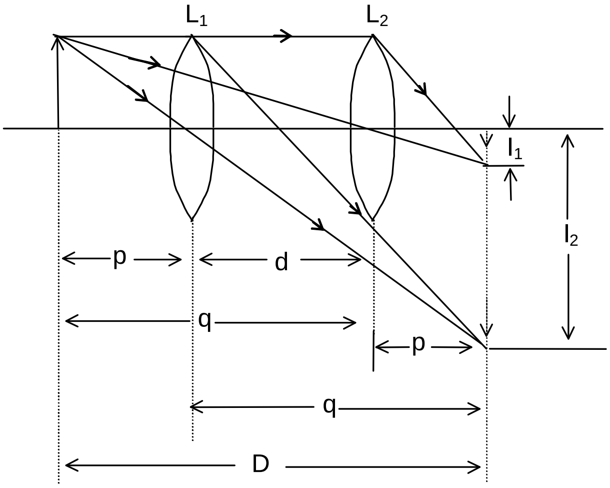
Error: The systematic error in this experiment is mostly due to improper position of the object on the holder. This error maybe eliminated by reversing the holder (rotating the holder by 180° about the vertical) and then taking the readings again. Then, the average are taken.

The equation for errors gives:

$$\left|\frac{\delta f}{f}\right| = \left|\frac{\delta u}{u}\right| + \left|\frac{\delta v}{v}\right| + \left|\frac{d(u+v)}{u+v}\right|$$

The errors δu, δv correspond to the error in the measurement of u and v.

Focal length of a convex lens by displacement method



O → Object size

I_1 & I_2 → image size of the two possible images for two positions of the lens at L_1 & L_2 for same object and image location.

In the first position L_1, the magnification of the lens is given by

$m_1 = \frac{v}{u} = \frac{I_1}{O}$

In the second position L_2, the magnification of the lenses given as

$m_2 = \frac{u}{v} = \frac{I_2}{O} \Rightarrow m_1 m_2 = \left(\frac{I_1}{O}\right)\left(\frac{I_2}{O}\right)$

$\Rightarrow \quad \left(\frac{v}{u}\right)\left(\frac{u}{v}\right) = \frac{I_1 \; I_2}{O^2}$

$\Rightarrow \quad O = \sqrt{I_1 \; I_2}$

Observing from the figure D–d = 2u

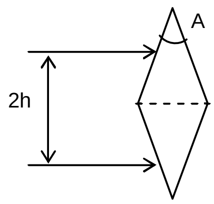
Since u + v = D $\Rightarrow$ v = D – u = $D - \frac{D-d}{2} = \frac{D+d}{2}$

Using Lens formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ & putting the values of u & v

We obtain, $\frac{1}{\left(\frac{D+d}{2}\right)} - \frac{1}{-\left(\frac{D-d}{2}\right)} = \frac{1}{f}$

$\Rightarrow \quad \frac{2(D+d+D-d)}{D^2-d^2} = \frac{1}{f} \qquad \Rightarrow \quad f = \frac{D^2-d^2}{4D}$.

Illustration 34. *Two identical thin isosceles prism of refracting angle A and refractive index μ are placed with their bases touching each other and this system can collectively acts as a crude converging lens. A parallel beam of light is incident on this system as shown. Find the focal length of this converging lens.*



Solution: $\tan\delta = h/f$

$f = \dfrac{h}{\tan\delta}$ for small δ, $\tan\delta = \delta$

further $\delta = (\mu - 1)A$

$f = \dfrac{h}{(\mu-1)A}$

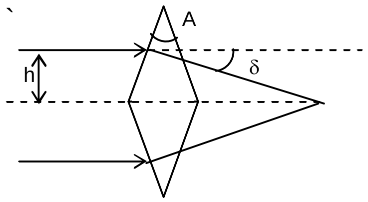
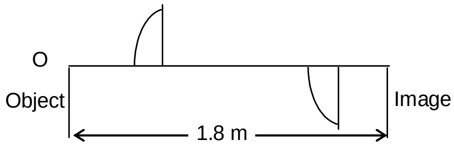


Illustration 35. *A thin plano convex lens of focal length f, is split into two halves. One of the halves is shifted along the principal axis as shown. The separation between an object placed on the axis and its image is 1.8 m. The magnification of the image formed by one of the half lenses is 2. Find the focal length of the combination and the separation between the two halves*



Solution: Since the magnification for L_1 is 2

$$\Rightarrow \quad \frac{v}{u} = -2 \;\Rightarrow\; \frac{\frac{D+d}{2}}{-\frac{D-d}{2}} = -2$$

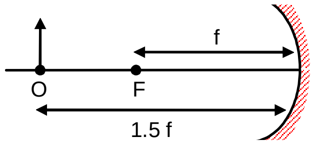
$$\Rightarrow \quad \frac{D+d}{D-d} = 2 \quad \Rightarrow \quad D = 1.8\text{ m},\ d = 0.6\text{ m}.$$

$$f = \frac{D^2-d^2}{4D} = \frac{(1.8+0.6)(1.8-0.6)}{4\times1.8} = 0.4\text{ m}.$$

Illustration 36. *An object of length 2.5 cm is placed at 1.5 f from a concave mirror where f is the focal length of the mirror. The length of the object is perpendicular to the principal axis. Find the length of the image. Is the image erect or inverted ?*

Solution : The focal length F = –f

and u = –1.5 f, we have,



$$\frac{1}{u}+\frac{1}{v}=\frac{1}{F} \qquad \text{or} \qquad \frac{1}{-1.5f}+\frac{1}{v}=-\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{1.5f}-\frac{1}{f}=-\frac{1}{3f}; \quad v = -3f$$

Now, $m = -v/u = -\dfrac{3f}{1.5f} = -2$

or $\dfrac{h_2}{h_1} = -2$ or $h_2 = -2h_1 = -5$ cm

The image is 5 cm long. The minus sign shows that it is inverted.

Illustration 37. *If a luminous point is moving at a speed v_0 towards spherical mirror of focal length f along its axis, find the speed of the image.*

Solution : As you know $\dfrac{1}{v}+\dfrac{1}{u} = \dfrac{1}{f}$

differentiating it with respect to time, We obtain,

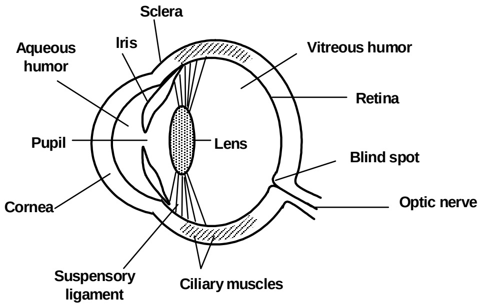
$$-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

or $-\frac{1}{v^2} v_{image} - \frac{1}{u^2} v_{Object} = 0 \Rightarrow \quad v_{Image} = -\left(\frac{v}{u}\right)^2 v_{Object}$

But, from mirror equation, $v = \frac{u\ f}{u - f}$

So, $v_{Image} = -\left(\frac{v}{u}\right)^2 v_{Object} = -\left[\frac{f}{u - f}\right]^2 v_O$

THE HUMAN EYE



The human eye is like a camera having a lens system forming an inverted, real image on a light sensitive screen inside the eye, called the retina. Light goes inside the eye through a thin membrane, called the cornea, which covers the transparent bulge on the surface of the eyeball. Eyeball is approximately sphere having diameter of about 2.3 cm. The cornea and the aqueous humor act as lens and both provide most of the refraction for the light, which enters in the eye.
The crystalline lens adjusts the focal length required to focus objects at different distances at the retina, behind cornea is the iris, which controls the size of the pupil. Iris can adjust its size, therefore, helps in regulating the light through pupil.

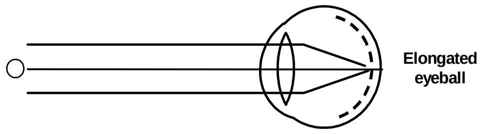
The light entering the eye is focussed by the eye lens and forms an inverted, real image of the object on the retina behind the lens. In the relaxed position the focal length of the eye lens is about 25 cm.
The eye lens can adjust its focal length and this property is called accommodation. However, the focal length of the eye lens cannot be decreased below a certain minimum limit.

Unlike a camera, the image formed on the retina of the eye is not permanent. Its impression remains on the retina for about $(1/16)^{th}$ of a second, even when the object has been removed. This is called the persistence of vision. A normal eye can accommodate for all distances between infinity (far point) and about 25 cm (near point).
The maximum variation in the power (1/f) of the eye-lens that can be achieved by the eye of a person is called its power of accommodation.

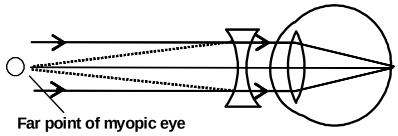
Defects of Vision

There are mainly four common defects of vision.

(i) ***Myopia- (Near-sightedness)***

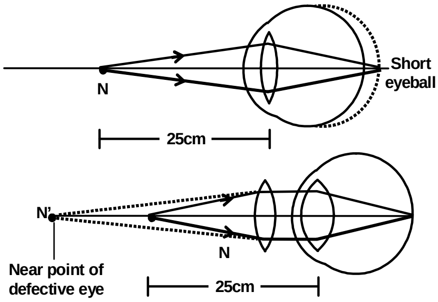


The eye defect in which a person is not able to see distant objects clearly is called near-sightedness or Myopia. This defect can be corrected by using a concave lens. In this type of defect, the image of a distant object is formed in front of the retina.



(ii). ***Hyper metropia-(Far sightedness)***

In this case the image of nearby objects is formed behind the retina. Thus nearby objects cannot be seen clearly, although distant objects can be seen. This defect is called far-sightedness, long sightedness or hypermetropia. This defect can be corrected by using a convex lens of appropriate focal length.



(iii). ***Presbyopia***

With growing age, the power of accommodation of the eye decreases. Due to this, most people cannot read comfortably and distinctly, without having eyeglasses. This defect is called Presbyopia.

(iv). ***Astigmatism***

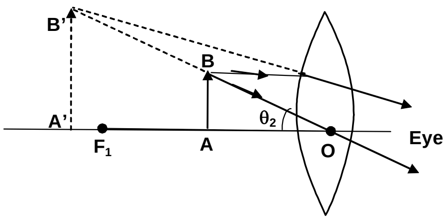
Sometimes the surfaces of the crystalline lens of the eye become uneven. Therefore, the light passing through this surface can't be focused sharply at single point. It is focussed at a number of closely placed points at varying distances. Therefore, the image looks blurred or distorted. Such type of defect is called astigmatism. To compensate this defect 'cylindrical glasses' are used.

Colour blindness

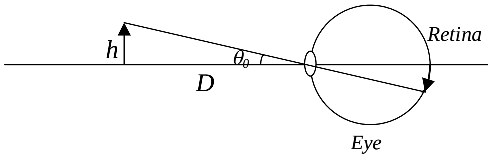
Colour blindness is the eye defect due to which the colour-blind person is unable to distinguish between certain colours. This is due to the absence of some cone cells that respond to certain colours. Colour blindness is a genetic disorder. It cannot be cured. The red-green colour blind person can not be able to distinguish between red and green colours. However, colour-blind person can see other colours perfectly well.

THE SIMPLE MICROSCOPE

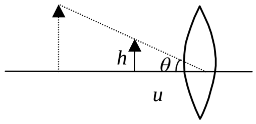
It is basically a convex lens of short focal length which is also called a magnifying glass. It works on the principal that when an object is placed within the focus of a convex lens, a magnified, virtual and erect image is produced. The ray diagram of a simple microscope is shown in the figure



Magnifying power of simple microscope is defined as, $m = \frac{\theta}{\theta_0}$, where θ_0 is the visual angle subtended by an object when it is seen at *D*, distance of distinct vision or near point, and θ is the visual angle using optical instrument.



$$\theta_0 = \frac{h}{D}$$



θ = h/u

m = $\frac{\theta}{\theta_0} = \frac{h}{u} \times \frac{D}{h}$

$\Rightarrow$ m = $\frac{D}{u}$

A simple microscope can be used in two standard adjustments, namely

(i) normal adjustment and

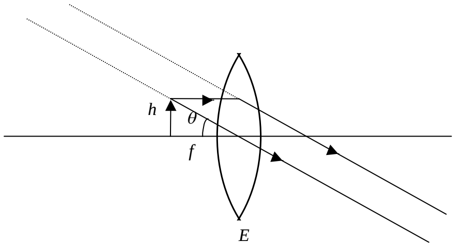
(ii) near–point adjustment.

In normal adjustment, the final image is seen at infinity and in near–point adjustment, the final image is seen at D, the distance of distinct vision. (D = 25 cm.)

Normal Adjustment

$\theta_0 = \frac{h}{D}$ and $\theta = \frac{h}{f}$

$m = \frac{\theta}{\theta_0} = \frac{h/f}{h/D} \Rightarrow m = \frac{D}{f}$



Near–Point Adjustment

Using, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}$

$\Rightarrow \frac{1}{u} = \left[\frac{1}{D} + \frac{1}{u}\right] \Rightarrow \frac{D}{u} = \left(1 + \frac{D}{f}\right)$

$m = \frac{\theta}{\theta_0} = \frac{h/u}{h/D} = \frac{D}{u} \Rightarrow m = 1 + \frac{D}{f}$

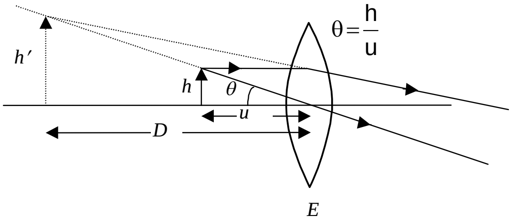


Illustration 38. *A simple microscope has a magnifying power of 3 when the image is formed at the near point (25 cm) of a normal eye.*

(a) What is its focal length?

(b) What will be its magnifying power if the image is formed at infinity ?

Solution: (a) For near point adjustment,

m = $1 + \frac{D}{f} \Rightarrow 3 = 1 + \frac{25}{f} \Rightarrow$ f = 12.5 cm.

(b) Image formed at infinity i.e. in normal adjustment

m = D/f

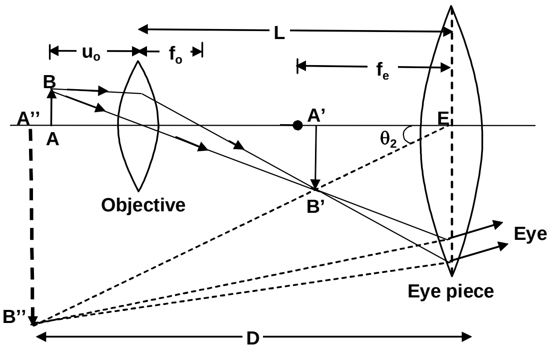
D = 25 cm, f = 12.5 cm ; m = 2.

COMPOUND MICROSCOPE

A simple microscope can be used when the required magnification is small. To achieve greater magnification, we use a compound microscope in which two convex lenses are used.

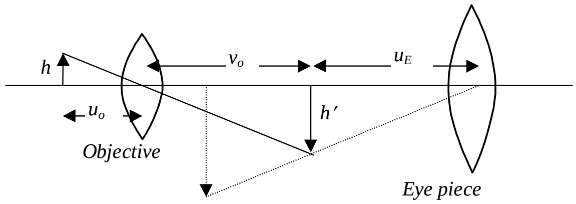
Figure shows a simplified version of compound microscope. It consists of two convex lenses mounted coaxially in separate tubes. In the given figure, an object AB is placed at a distance u_o from the objective, where u_o is slightly greater than the focal length (f_o) of the objective.

The objective forms a real, inverted and enlarged image A'B'. The separation between the lens is so adjusted such that the image A'B' lies within the focal length (f_e) of the eyepieces. A'B' acts as the object for the eyepiece. The eyepiece forms its virtual and enlarged image A"B". The final image is erect with respect to A'B', and hence, is inverted with respect to the object AB. By adjusting the separation between the objective and the eyepieces. A"B" can be formed at the least distance of distinct vision.



Compound Microscope

Magnifying power of a compound microscope



$$\theta_0 = \frac{h}{D} \quad , \quad \theta = \frac{h'}{u_E}$$

$$\Rightarrow \quad m = \frac{\theta}{\theta_0} = \frac{h'/u_E}{h/D} = \left(\frac{h'}{h}\right)\left[\frac{D}{u_E}\right]$$

By substituting from results derived for simple microscope,

from geometry $\frac{h'}{h} = \frac{v_0}{u_0} \quad \Rightarrow \quad m = \frac{v_0}{u_0}\left(\frac{D}{D_E}\right)$

for normal adjustment $u_E = f_E$

$$m = \frac{v_o}{u_o}\left[\frac{D}{f_E}\right] \text{ for normal adjustment, and,}$$

$$m = \frac{v_o}{u_o}\left[1 + \frac{D}{f_E}\right] \text{ for near-point adjustment}$$

Illustration 39. *A compound microscope consists of an objective of focal length 1.0 cm and an eyepiece of focal length 5 cm separated by 12.2 cm.*

(a) At what distance from the objective should an object be placed to focus it properly so that the final image is formed at the least distance of clear vision?

(b) Calculate the angular magnification.

Solution : For eyepiece, $v_E = -25$ cm. , $f_E = +5$ cm

$\Rightarrow \quad u_E = -4.17 \text{ cm} \approx -4.2$ cm

$L = v_o + |u_E| = 12.2$ cm , here $v_o = 8$ cm

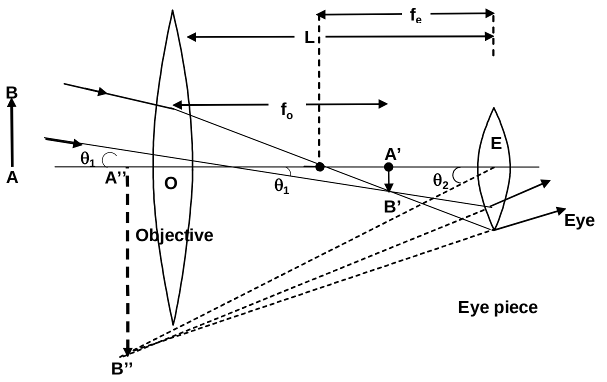
(a) $f_o = +1$ cm $\Rightarrow$ $u_o = -1.1$ cm.

(b) $m = \frac{v_o}{u_o}\left[1 + \frac{D}{f_E}\right] = \left(\frac{8}{-1.1}\right)\left[1 + \frac{25}{5}\right] \approx -44$

TELESCOPES

Astronomical Telescope

A telescope is used to see a magnified image of a distant object such as a star. A simplified diagram of an astronomical telescope is shown in the figure.

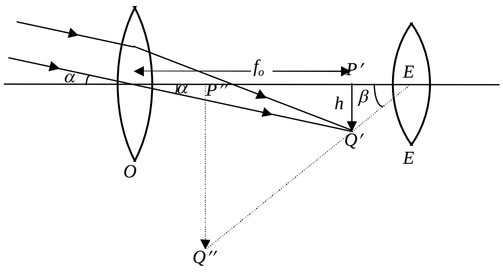


The lens facing the object is the objective and the one close to the eye is the eyepiece. In telescope, the focal length of the objective is larger than that of the eyepiece since the object AB is at infinity. The image A'B' is real, inverted, very small and almost at the focus of objective.

Also as the object AB is at infinity, the rays coming from point A and B are parallel to each other and make an angle θ_1 with the principle axis. Thus θ_1 is the angle subtended by AB on the lens. The distance between the lenses is adjusted such that the image A'B' falls within the focal length of the eyepiece. This image acts as the "object" for the eyepiece. The eyepiece then forms a large and virtual final image A"B". This image is erect with respect to A'B' hence it is inverted with respect to the object AB.

Magnifying Power Astronomical telescope

$$m=\frac{\beta}{\alpha}=\frac{h/EP'}{h/f_o}=\frac{f_o}{EP'}$$



For normal adjustment, $v_E = \infty$

$\Rightarrow$ $EP' = f_E$

$\therefore$ $m=\frac{f_o}{f_E}$ and $L=f_o+f_E$

For near point adjustment, $v_E = -D$, $u_E = -EP'$

$\Rightarrow$ $u_E=-\left[\frac{f_E D}{f_E+D}\right]$

$\therefore$ $m=\frac{f_o}{f_E}\left[1+\frac{f_E}{D}\right]$ and $L=f_o+\frac{f_E D}{f_E+D}$

Illustration 40. *An astronomical telescope has f_o = 200 cm and f_E = 4 cm. Telescope is focused to see an object 10 km from the objective. Final image is formed at infinity. Find length L of the tube and the angular magnification of telescope.*

Solution: $L = f_o + f_E = 200 + 4 = 204$ cm.

$$m=\frac{f_o}{f_e}=\frac{200}{4}=50$$

Galilean Telescope

$$m=\frac{\beta}{\alpha}=\frac{h/u_E}{h/f_o}=\frac{f_o}{u_E}$$

For normal adjustment, $u_E = f_E$

$$\therefore\ m=\frac{f_o}{f_E}\quad \text{and}\quad L=f_o-|f_E|$$

For near point adjustment, $v_E = -D$

$$\Rightarrow \frac{1}{u_E}=\frac{1}{f_E}-\frac{1}{D}$$

$$\therefore\ m=\frac{f_o}{f_E}\left[\frac{D-f_E}{D}\right]\quad \text{and}\quad L=f_o-u_E$$

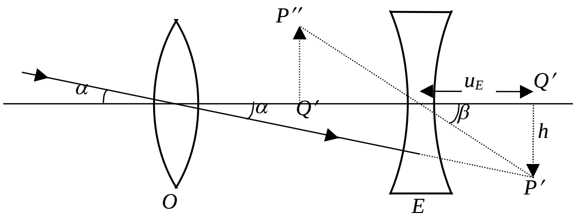


Illustration 41. *A Galilean telescope has f_o = 200 cm and f_E = 4 cm. The length the tube L and magnifying power if it is used to see object at a large distance in normal adjustment is*

(A) L = 45, m = 10 *(B) L = 35, m = 20*

(C) L = 15, m = 15 *(D) L = 25, m = 30*

Solution : **(A)** $L = f_o - |f_E|$ = 50 – 5 = 45 cm , $m=\frac{f_o}{f_E}$ = 10

Exercise 4:

(i) ***An astronomical telescope when in normal adjustment has magnifying power 5 and the two thin lenses lie 24 cm apart. Find the focal length of the lenses.***

(ii) ***A compound microscope has magnification of 30. The focal length of the eye–piece is 5 cm. Assuming the final image to be formed at the least distance of distinct vision (25 cm , calculate the magnification produced by the objective.***

(iii) ***A refracting telescope has an objective lens of focal length 1m and an eye–piece of focal length 0.2 m. A real image of sun 10 cm in diameter is formed on a screen 24 cm from the eyepiece. What angle does the sun subtend at the objective?***

PRISM

A prism is a portion of certain optical medium bounded by two inclined faces. The angle A between the inclined faces is known as angle of prism. Consider the prism where a ray PQ is incident on one face of the prism.

Angle of deviation = angle between incident ray and emergent ray (D).

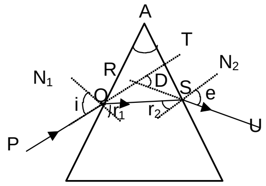
Let , *i* = incident angle, *e* = emergent angle

and r_1 and r_2 are angles of refraction.

From geometry it can be proved that,

$$A = r_1 + r_2$$

$$i+e = A+D$$



- ☞ Light obeys the principle of reversibility. If we send light along the emergent direction, it will emerge along the incident direction, maintaining the deviation.
- ☞ When deviation is minimum, $\angle i = \angle e$.

Hence, $\angle r_1 = \angle r_2 = \frac{\angle A}{2} \Rightarrow \quad \mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

Prism Formula

If the angle of prism A is very small, then $\sin\left(\frac{A+D_m}{2}\right) = \left(\frac{A+D_m}{2}\right)$ and $\sin\left(\frac{A}{2}\right) = \left(\frac{A}{2}\right)$

Then from the above prism formula we have, deviation for a small angled prism, $\delta = (\mu - 1)A$

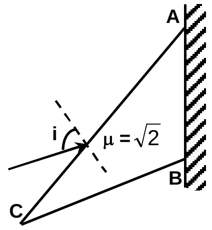
Illustration 42. *A ray of light falling at an angle of 50^o is refracted through a prism in minimum deviation position. The angle of the prism is 60^o. Calculate the refractive index of the prism. Given that $\sin 50^o = 0.766$.*

Solution : In minimum deviation position, angle of incidence, i = $\frac{A+\delta}{2}$

and angle of refraction r = A/2, where A is the angle of prism and δ is the angle of minimum deviation,

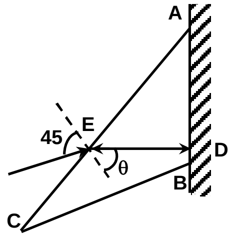
We know that, $\mu = \frac{\sin i}{\sin r} = \frac{\sin 50}{\sin 30} = \frac{0.766}{0.5} = 1.532$

Illustration 43. *One face of a prism ABC of $\mu = \sqrt{2}$ is silvered and the ray incident at angle 45° retraces its initial path. Find the angle $\angle$CAB.*



Solution: As incident ray retraces its path, the ray is incident normally on the silvered face.

From Snell's law at surface AC



$\frac{\sin 45}{\sin r} = \sqrt{2}$

$\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \sin r$

r = 30°

in ΔAED

$\angle$A + $\angle$(90–30) = 90°

$\angle$A = 30°

Illustration 44. *The principal section of a glass prism is an isosceles ΔPQR with PQ = PR. The face PR is silvered. A ray is incident perpendicularly on face PQ and after two reflections it emerges from base QR, normal to it. The angle of the prism is*

(A) 30^0 *(B) 45^0*
(C) 36^0 *(D) 60^0*

Solution: **(C)** $A + 2\theta = 180°$ … (i)

Also, $\theta = 2A$ … (ii)

From (i) and (ii)

$A = 36°$.

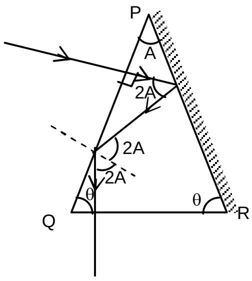
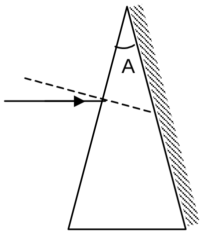


Illustration 45. *A thin prism is placed in the position of minimum deviation. If A be its refracting angle and μ the refractive index of the material of the prism, find the net deviation suffered by the ray on emergence from the prism, if the opposite refracting surface is silvered.*

(A) 90^0 *(B) 45^0*

(C) μ A *(D) $\pi - \mu A$*



Solution: **(D)** Path of the ray is shown in the figure refraction occurs at O and C while reflection takes place at B.

Deviation at O = $i - r = (\mu - 1) r = (\mu - 1)\dfrac{A}{2}$ (Clockwise)

Deviation of B = $\pi - 2r = \pi - A$ (clockwise)

Deviation at C = $(\mu - 1)\dfrac{3A}{2}$ (anticlockwise)

Net deviation $\delta = \delta_1 + \delta_2 - \delta_3 = \pi - \mu A$

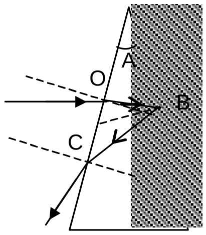


Illustration 46. *The cross–section of a glass prism has the form of an isosceles triangle. One of the refracting faces is silvered. A ray of light falling normally on the other refracting face, being reflected twice emerges through the base of the prism perpendicular to it. Find the angles of the prism.*

Solution : The incident ray BC at normal incidence is refracted along CD, suffers reflection at silvered face, along DE and at E it again suffers reflection along EF. Since the ray emerges normally from the base, therefore, the ray EF must fall normally on the base and emerge along FG.

We find i = A, also $\beta = \alpha$

Since EN_2 // CD, β = 2I (alternate angles)

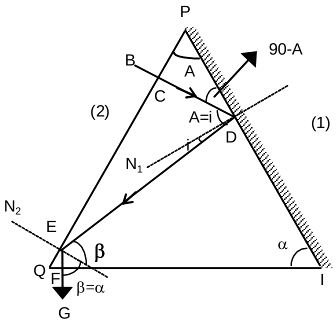
$\therefore \quad \alpha = 2A \quad (\because \beta = \alpha, i = A)$ …(1)

Also $2\alpha + A = 180°$

$(\because$ sum of angles of a triangle = 180°) …(2)

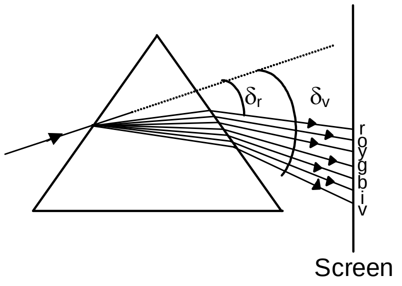
Solving (1) and (2) we get, A = 36° ,

$\alpha = 72°$



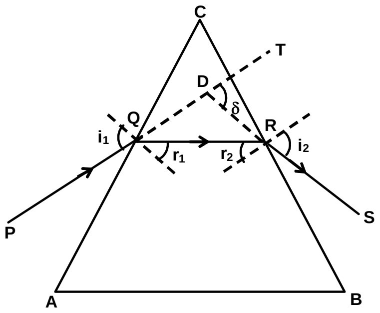
Dispersion

White light is actually a mixture of light of different colours. Therefore when white light gets refracted under certain conditions, its components bend by different amounts and separate out. This phenomenon of splitting of light into its component colours, due to different refractive indices for different colours, is called dispersion of light.

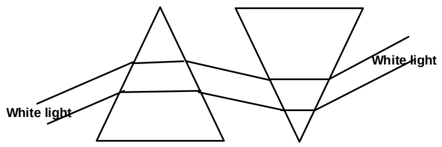


This phenomenon arises due to the fact that refractive index varies with wavelength. It has been observed for a prism that μ decreases with the increase of wavelength, i.e. $\mu_{blue} > \mu_{red}$.

If a narrow beam of light falls on one of the faces of glass prism, it emerges out from other face. We can observe that white light split into a band of different colours. In this band, one can distinguish seven prominent colours. These are Violet, Indigo, Blue, Green, Yellow, Orange and Red (acronym VIBGYOR). This band of colours is called spectrum. These colours can also be seen in a rainbow. We can observe that the violet light deviates most and the red light deviates the least. This indicates that the refractive index of glass is largest for violet and least for red.



We can also recombine these colours withthe help of another prism. Another prism can be kept inverted with respect to first. Then the final light obtained after the second prism is again a white light.



Angular dispersion, $\theta = \delta_v - \delta_r$

Dispersive power, Ratio of angular dispersion to mean deviation.

$$\omega = \frac{\delta_v - \delta_r}{\delta}$$ where δ is deviation of mean ray (yellow)

As $\delta_v = (\mu_v - 1)A$, $\delta_r = (\mu_r - 1)A$,

$\therefore$ $$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}; \text{ where } \mu_y = \frac{\mu_v + \mu_r}{2}$$

Illustration 47. *Find the dispersion produced by a thin prism of 18^o having refractive index for red light = 1.56 and for violet light = 1.68.*

Solution: We know that dispersion produced by a thin prism

$\theta = (\mu_v - \mu_R)A$

Here $\mu_v = 1.68$, $\mu_R = 1.56$ and $A = 18^o$.

$\theta = (1.68 - 1.56) \times 18^o = 2.16^o$.

Illustration 48. *Calculate the dispersive power for crown glass from the given data* *μ_v = 1.523, and μ_R = 1.5145.*

Solution : Here $\mu_v = 1.523$ and $\mu_R = 1.5145$

Mean refractive index, $\mu = \frac{1.523+1.5145}{2} = 1.51875$

Dispersive power ω is given by, $\omega = \frac{\mu_v - \mu_R}{(\mu-1)} = \frac{1.523-1.5145}{(1.51875-1)} = 0.1639$

Average deviation without dispersion

This means an achromatic combination of two prisms in which net or resultant dispersion is zero and deviation is produced. For the two prisms,

$$(\mu_v - \mu_r)A + (\mu_v' - \mu_r')A' = 0$$

$$\Rightarrow \quad A' = -\frac{(\mu_v - \mu_r)A}{(\mu_v' - \mu_r')} \quad \text{and} \quad \omega\delta + \omega'\delta' = 0$$

where ω and ω' are the dispersive powers of the two prisms and δ and δ' their mean deviations.

Total deviation, $\delta_{net} = (\mu_y - 1)A\left(1 - \frac{\omega}{\omega'}\right)$.

Illustration 49. *The dispersive power of crown and flint glasses are 0.03 and 0.05 respectively. If the difference in the refractive indices of blue and red colours is 0.014 for crown glass and 0.023 for flint glass, calculate the angle of the two prisms for a deviation of 10^o (without dispersion)*

Solution : The crown glass, $\omega = 0.03$ and $\mu_b - \mu_r = 0.014$

We know that $\omega = \frac{\mu_b - \mu_r}{\mu - 1}$

$\therefore \quad (\mu - 1) = \frac{0.014}{0.03} = 0.4667$ … (1)

For flint glass $\omega' = 0.05 \Rightarrow \mu_b' - \mu_r' = 0.023$

$\therefore \quad \mu' - 1 = \frac{0.023}{0.05} = 0.4600$ … (2)

Again $\frac{\delta}{\delta'} = \frac{\omega'}{\omega} = \frac{0.05}{0.03} = \frac{5}{3}$

$\delta = \frac{5}{3}\delta'$ … (3)

Given that, $\delta - \delta' = 10^o$

$\frac{5}{3}\delta' - \delta' = 10$ … (4)

From equations (3) and (4)

$\delta' = 15^o$

and $\delta = 25^o$

Now, using the formula, $\delta = (\mu - 1)A$

A (for crown glass) $= \frac{\delta}{(\mu-1)} = \frac{25}{0.4667} = 53.6^\circ$

A′ (for flint glass) = $\frac{\delta'}{\mu'-1} = \frac{15}{0.4600} = 32.6^o$

Dispersion without Average Deviation

A combination of two prisms in which deviation produced for the mean ray by the first prism is equal and opposite to that produced by the second prism is called a direct vision prism. This combination produces dispersion without deviation.

For deviation to be zero, $(\delta + \delta') = 0$

$\Rightarrow \quad (\mu - 1)A + (\mu' - 1)A' = 0$

$\Rightarrow \quad A' = -\frac{(\mu-1)A}{(\mu'-1)}$ (–ve sign ⇒ prism *A′* has to be kept inverted)

Total angular dispersion, $\theta = (\mu_y - 1)A\,(\omega - \omega')$.

Illustration 50. *A prism of crown glass refracting angle of 50^o and mean refractive index = 1.51 is combined with one flint glass prism of refractive index = 1.65 to produce no deviation. Find the angle of flint glass and net dispersion.*
Given : μ_v = 1.523, μ_R = 1.513 (for crown glass)
μ'_v = 1.665, μ_R' = 1.645 (for flint glass)

Solution : Let A′ be the angle of flint glass prism.

Here A = 5^o and μ = 1.51 for crown glass prism.

$\delta = (\mu - 1)A = (1.51 - 1)\times 5 = 2.55^o$

Deviation produced by flint glass

$\delta = (\mu' - 1)A' = (1.65 - 1)A' = 0.65A'$

For no deviation $\delta' = \delta$ or $0.65A' = 2.55$

$A' = \frac{2.55}{0.65} = 3.92^o$

Net dispersion, $(\mu_v - \mu_R)A - (\mu'_v - \mu'_R)A'$

= (1.523 – 1.513)5 – (1.655 – 1.645)3.92

= 0.05 – 0.0784 = –0.0284

Exercise 5:

(i). ***A small angled prism (μ = 1.62) gives a deviation of 4.8°. Find the angle of prism.***

(ii). ***If the refractive index of the material of a prism of refracting angle 8° is 1.532 for blue and 1.514 for red light. What is the angular dispersion by the prism.***

(iii). ***Calculate the angle of dispersion between red and violet colours produced by a flint glass prism of refracting angle of 60°. Given μ_v = 1.663 and μ_r = 1.662***

SCATTERING OF LIGHT

As sun light travels through the earth's atmosphere, it gets scattered by the large number of molecules present scattering represents basically change in the direction of light.

According to Rayleigh, Intensity of scattered light (I_s) varies inversely as the fourth power of the wavelength of incident light.

i.e. $I_s \propto \frac{1}{\lambda^4}$

These rays do not undergo any change in wavelength on scattering.

Examples:

(i) Blue colour of sky is due to scattering of sunlight. As the blue colour has a shorter wavelength than red, therefore blue colour is scattered much more strongly. Hence the sky looks blue.

(ii) At the time of sun rise and sunset, the sun is near the horizon. The rays from the sun have to travel a larger part of the atmosphere. As $\lambda_b < \lambda_r$ and intensity of scatted light $\propto \frac{1}{\lambda_4}$, therefore, most of the blue light is scattered away. Only red colour, which is least scatted enters our eyes and appears to come from the sun. Hence the sun looks red both at the time of sun rise and sun set.

MISCELLANEOUS EXERCISE

1. When a room has mirrors on two opposite walls, an infinite series of reflections can be obtained. Discuss this phenomenon in terms of images. Why do the distant images appear fainter?

2. If a spherical mirror is immersed in liquid, does its focal length change?

3. Refractive index of glass is 1.5. Calculate speed of light in the glass, if speed of light in vaccum is 3×10^8 m/s.

4. A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance will the pin appear to be raised, if it is viewed from the same point through a 15 cm thick glass sheet held parallel to horizontal (refractive index of glass is 1.5) ?

5. Calculate the distance at which an object should be placed in front of a convex lens of focal length 10 cm to obtain an image of double its size.

6. The wavelength of the red–light from He–Ne laser is 633 nm in air but 474 nm in the aqueous humour inside the eyeball. Calculate the index of refraction of the aqueous humour and the speed of the light in substance.

7. Radius of curvature of a lens is 10 cm for the both surfaces. The index of refraction is 1.52. What is the focal length of the lens ?

8. A converging lens of 6 dioptre is combined with a diverging lens of 2 dioptre. Find out the power and focal length of combination.

9. A convex lens of focal length 20 cm produces a real image of an object 30 cm away. Find the magnification and position of image.

10. A ray of light passes through an equilateral prism such that the angle of incidence is equal to the angle of the emergence. The angle of emergence is ¾ times the angle of prism. Calculate the refractive index of glass prism.

ANSWERS TO MISCELLANEOUS EXERCISE

3.	2×10^8 m/s.	4.	5 cm
5.	– 5 cm, –15 cm	6.	1.34, 2.25×10^8 m/s.
7.	9.6 cm	8.	4 D, +25 cm
9.	–2, 60 cm	10.	$\mu = \sqrt{2}$

SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob 1. *An object is placed 10 cm in front of a concave mirror of radius of curvature 15 cm. Calculate the position, nature and magnification of the image.*

Sol. f = $-\frac{15}{2}$ cm = – 7.5 cm

u = – 10 cm

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ or $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

$\frac{1}{v}=-\frac{1}{7.5}-\frac{1}{-10}$

$\Rightarrow$ v = – 30 cm

m = $\frac{-v}{u}=-\frac{-30}{-10}=-3$

Prob 2. *A small candle 2.5 cm in size is placed 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to receive a sharp image? Describe the nature and size of image. If the candle is moved closer to the mirror how would the screen have to be moved ?*

Sol. u = – 27 cm, f = R/2 = $-\frac{36}{2}\text{cm}$ = – 18 cm

$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=-\frac{1}{16}-\frac{1}{-27}$ $\Rightarrow$ v = – 54cm

Negative sign indicates that the image is formed in front of the mirror.

M = $\frac{I}{O}=-\frac{v}{u}$

$\frac{I}{2.5}=-\frac{-54}{-27}$ $\Rightarrow$ I = – 5 cm

If the candle is moved closer, the screen would have to be moved farther and farther. However, if the candle is closer than 18 cm from the mirror, the image would be virtual and therefore cannot be collected on screen.

Prob 3. *When an object is placed at a distance of 0.6 m from a convex spherical mirror, the magnification produced is ½. Where should the object be placed to get a magnification of 1/3?*

Sol. m = $\frac{f}{f-u}$

$\frac{1}{2}=\frac{f}{f+0.6}$ $\Rightarrow$ f = 0.6 cm

$\frac{1}{3}=\frac{0.6}{0.6-u}$ $\Rightarrow$ u = – 1.2 m.

Prob 4. *A concave mirror is held in water. What would be the change in focal length of the mirror.*

Sol. No change

Prob 5. *The bottom of a container is a 4 cm thick glass (μ = 1.5) slab. The container contains two immiscible liquids A & B of depth 6 cm and 8 cm respectively. What is the apparent position of a scratch on the outer surface of the bottom of the glass slab when viewed through the container ? Refractive indices of A and B are 1.4 and 1.3 respectively.*

Sol. Apparent position of scratch below upper liquid level

$$= \left(\frac{h_1}{\tau_1} + \frac{h_2}{\tau_2} + \frac{h_3}{\tau_3}\right)$$

$$= \frac{4}{1.5} + \frac{6}{1.4} + \frac{8}{1.3} = 13.11 \text{ cm}$$

IITJEE TYPE

Prob 6. *A ray of light is incident on a plane reflecting surface at an angle of 30°. Find the deviation in the incident ray. What will be the deviation if the ray suffers a reflection again at a surface inclined at 60° to the first surface ?*

Sol. $\angle B'CM_2 = 60°$, $\angle M_2CD = 60°$, $\angle B'CD = 120° = D_2$, $\angle DCN_2 = \angle N_2CB = 30°$

$\angle ABN_1 = \angle N_1BC = 30°$; $\angle CBA' = 120° = D_1$

The incident ray of light AB on the first mirror M_1 at an angle of incidence of 30° is reflected along BC at an angle of reflection 30°.

$\therefore$ Deviation in incident ray

= 60° + 60° = 120°

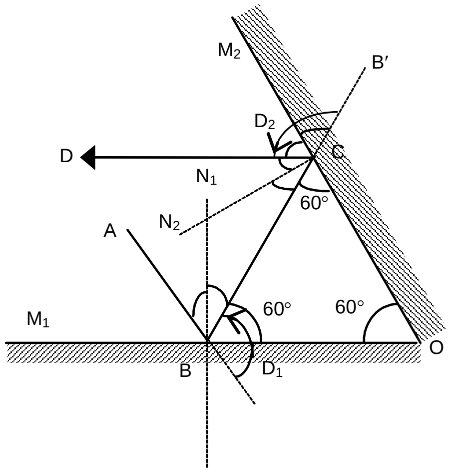
The reflected ray BC is incident on the second mirror M_2.

Since $\angle M_1OM_2 = 60°$, $\angle BCD = 60°$

$\therefore$ $\angle BCN_2 = \angle N_2CD = 30°$.

$\therefore$ Deviation D_2 of the ray

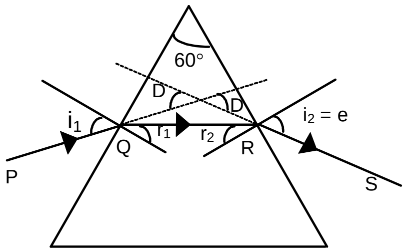
= 60° + 60° = 120°



$\therefore$ Total deviation due to the two mirrors = $D_1 + D_2$ = 240°

Prob 7. *A prism of angle 60° deviates a ray of light through 40° for two angles of incidence which differ by 11°. What is the refractive index of the glass of the prism ?*

Sol. The incident ray PQ is deviated through 40° and the principle of reversibility of ray, through optical system, shows that the ray SR would also suffer the same deviation (= 40°).



$\therefore$ The other angle of incidence causing same deviation = 40° is e = i_2

where $i_1 - i_2 = 11°$ (given)

Also $i_1 + i_2 = 100°$ $(\because i + e = A + D)$

$\therefore$ $i_1 = 55°30'$ and $i_2 = 44°30'$

Now $\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin i_2}{\sin r_2}$ where $r_2 = (60^\circ - r_1)$

$$\therefore \ \mu = \frac{\sin 55^\circ 30'}{\sin r_1} = \frac{\sin 44^\circ 30'}{\sin(60^\circ - r_1)}$$

We first solve this equation for r_1 and hence find μ.

$$\frac{0.8241}{\sin r_1} = \frac{0.7009}{\frac{\sqrt{3}}{2}\cos r_1 - \frac{1}{2}\sin r_1}$$

Dividing both sides in denominator by $\sin r_1$, we get

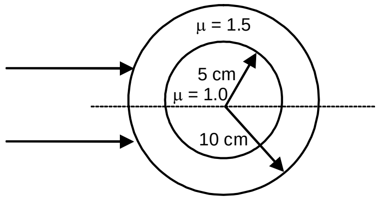
$$\frac{0.8241}{\frac{\sin r_1}{\sin r_1}} = \frac{0.7009}{\frac{\sqrt{3}}{2}\cot r_1 - \frac{1}{2}\frac{\sin r_1}{\sin r_1}}$$

$$\therefore \quad \frac{1}{2}\left(\sqrt{3}\cot r_1 - 1\right) = \frac{0.7009}{0.8241}$$

$$\therefore \quad \sqrt{3}\cot r_1 = 2.701 \ \therefore \quad \cot r_1 = 1.559 \Rightarrow r_1 = 32^\circ 40'$$

$$\therefore \quad \mu = \frac{\sin 55^\circ 30'}{\sin 32^\circ 40'} = \frac{0.8241}{0.5400} = 1.526$$

Prob 8. *A glass sphere with radius 10 cm has concentric spherical cavity of radius 5 cm. A narrow beam of parallel light is directed into the sphere, as shown in the figure. Find the position of the final image from the centre of the sphere.*



Sol. First surface,

$\mu_2 = 1.5$, $\mu_1 = 1.0$, $u = -\infty$, $R = +10$ cm.

$$\therefore \ \frac{1.5}{v} - \frac{1}{-\infty} = \frac{1.5 - 1.0}{10} \qquad \text{or} \qquad \frac{1.5}{v} = \frac{1}{20} \Rightarrow v = 30\text{cm}$$

Second surface ,

$\mu_2 = 1.0$, $\mu_1 = 1.5$, $u = 30 - 5 = 25$ cm, $R = +5$ cm.

$$\therefore \ \frac{1}{v} - \frac{1.5}{25} = \frac{1 - 1.5}{5}$$

$$\text{or} \ \frac{1}{v} - \frac{3}{50} = -\frac{1}{10} \qquad \Rightarrow \qquad v = -25\text{cm}$$

Third surface ,

$\mu_2 = 1.5$, $\mu_1 = 1.0$, $u = -25 - 10 = -35$cm, $R = -5$ cm.

$$\therefore \frac{1.5}{v} - \frac{1}{-35} = \frac{1.5 - 1.0}{-5} \qquad \text{or} \qquad \frac{1.5}{v} + \frac{1}{35} = -\frac{1}{10}$$

$\Rightarrow v = -\frac{35}{3}$

Fourth surface ,

$u = \frac{-35}{3} - 5 = -\frac{50}{3}$

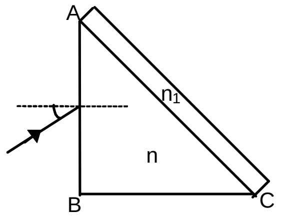
$\mu_2 = 1.0$, $\mu_1 = 1.5$, u = 9 – 4 = 5cm , R = –10 cm

$\therefore \frac{1}{v} + \frac{(1.5)3}{50} = \frac{1-1.5}{-10}$ or $\frac{1}{v} + \frac{9}{100} = \frac{1}{20}$

$\Rightarrow v = -25$ cm

The final position is 10 – 25 = –15 cm. (i.e. towards left)

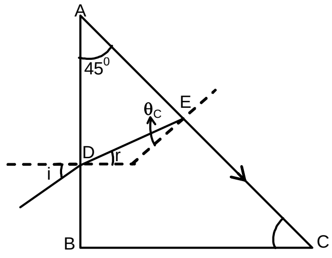
Prob 9. *A right angled prism ($45^o - 90^o - 45^o$) of refractive index n has a plate of refractive index n_1 ($n_1 < n$) cemented to its diagonal face. The assembly is in air. A ray is incident on AB.*



(a) Calculate the angle of incidence at AB for which the ray strikes the diagonal face at the critical angle.

(b) Assuming n = 1.352, calculate the angle of incidence at AB for which the refracted ray passes through the diagonal face undeviated.

Sol. (a) Let the critical angle at face AC be θ_C,
At face AB from Snell's law
1sini = n sinr
But from geometry, in the triangle ADE, we have,
$(90 - \theta_C) + (90 - r) + 45 = 180$,
i.e., $r = (45 - \theta_C)$
$\Rightarrow$ 1sini = n sin(45 – θ_C)



$= \frac{n}{\sqrt{2}} \left[\cos\theta_C - \sin\theta_C\right]$ . . . . (i)

But as here $\mu = \frac{n}{n_1}$

so $\sin\theta_C = \left[\frac{1}{\mu}\right] = \frac{n_1}{n}$ and $\cos\theta_C = \sqrt{1 - \sin^2\theta_C} = \frac{1}{n}\sqrt{n^2 - n_1^2}$

so, Eqn. (I) becomes

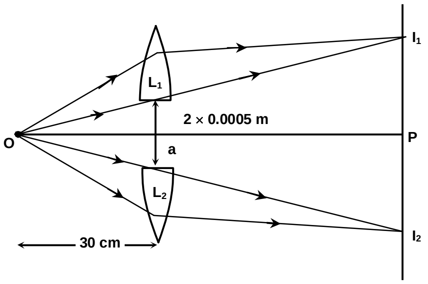
$\sin i = \frac{1}{\sqrt{2}}\left[\left(n^2 - n_1^2\right)^{\frac{1}{2}} - n_1\right]$

i.e. $i = \sin^{-1}\frac{1}{\sqrt{2}}\left[\left(n^2 - n_1^2\right)^{\frac{1}{2}} - n_1\right]$

(b) If the refracted ray passes undeviated through the diagonal face AC, the angle of refraction r at face AB by geometry of figure will be 45°. So by Snell's law at face AB

1sin i = 1.352 sin 45 i.e., $i = \sin^{-1}(0.9588) = 73.5°$

Prob 10. *A point object O is placed at a distance of 0.3 m from a convex lens (f = 0.2 m) cut into two halves each of which is displaced by 0.0005 m as shown in figure. Find the position of the image. If more than one image is formed, find their number and distance between them.*



Sol.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{-30}=\frac{1}{20}$$

$\Rightarrow$ v = 60 cm (Real image)

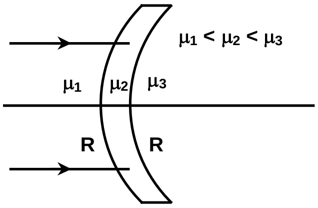
In similar Δ's OI_1I_2 and OL_1L_2

$$\frac{I_1I_2}{L_1L_2}=\frac{OP}{OQ}=\frac{v+u}{u}$$

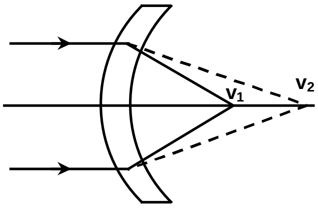
$\Rightarrow I_1\,I_2 = \frac{90}{30}(2\times 0.05)$

$\Rightarrow I_1\,I_2$ = 0.3 cm

Prob 11. *Find the focal length of the lens shown in the figure. The radii of curvature of both the surfaces are equal to R.*



Sol. For an object placed at infinity the image after first refraction will be formed at v_1



$$\frac{\mu_2}{v_1}-\frac{\mu_1}{-\infty}=\frac{\mu_2-\mu_1}{+R} \quad \ldots \text{(i)}$$

The image after second refraction will be found at v_2

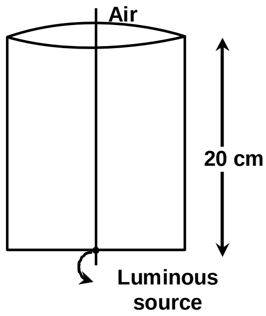
$$\frac{\mu_3}{v_2}-\frac{\mu_2}{v_1}=\frac{\mu_3-\mu_2}{+R} \quad \ldots \text{(ii)}$$

adding (i) and (ii)

$$\frac{\mu_3}{v_2}=\frac{\mu_3-\mu_1}{R}\Rightarrow\ v_2=\frac{\mu_3 R}{\mu_3-\mu_1}$$

Therefore focal length will be $\frac{\mu_3 R}{\mu_3-\mu_1}$.

Prob 12. *A thin equiconvex glass lens (μ_g = 1.5) is being placed on the top of a vessel of height h = 20 cm as shown in the figure. A luminous point source is being placed at the bottom of the vessel on the principal axis of the lens. When the air is on both the sides of the lens the image of luminous source is formed at a distance of 20 cm from the lens outside the vessel. When the air inside the vessel is being replaced by a liquid of refractive index μ_ℓ, the image of the same source is being formed at a distance 30 cm from the lens outside the vessel. Find the value of μ_ℓ.*



Sol. For the first case:

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{20}-\frac{1}{-20}=(1.5-1)\left(\frac{1}{R}-\frac{1}{-R}\right) \Rightarrow R = 10 \text{ cm}$$

For the second case. When the vessel is being filled with the liquid, then

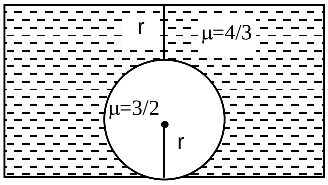
$$\frac{\mu_{air}}{v_1}-\frac{\mu_\ell}{u}=\frac{\mu_g-\mu_\ell}{R}+\frac{\mu_{air}-\mu_g}{-R}$$

$$\frac{1}{30}-\frac{\mu_\ell}{-20}=\frac{1.5-\mu_\ell}{10}+\frac{1-1.5}{-10}$$

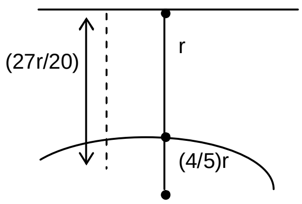
$$\frac{\mu_\ell}{20}+\frac{\mu_\ell}{10}=\frac{3}{20}+\frac{1}{20}-\frac{1}{30} \quad \Rightarrow \frac{3\mu_\ell}{20}=\frac{1}{6}$$

$$\mu_\ell = \frac{10}{9} = 1.11$$

Prob 13. *A glass sphere having refractive index (3/2) is having a small irregularity at its centre. It is placed in a liquid of refractive index $\frac{4}{3}$ such that surface of liquid is at a distance of above the sphere, where r is radius of sphere. If irregularity is viewed from above normally, calculate distance from centre where eye will observe the irregularity.*



Sol. Consider refraction at glass – water interface,



$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\Rightarrow \frac{4}{3v}-\frac{3}{-2r}=\frac{(4/3)-(3/2)}{-r}$$

$$\Rightarrow \frac{4}{3v}+\frac{3}{2r}=\frac{1}{6r}$$

$\therefore$ v = –r

Now refraction at water -air surface

u = – r – r = -2r

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{\infty} \quad \Rightarrow \frac{1}{v}+\frac{4/3}{-2r}=\frac{1-(4/3)}{\infty}$$

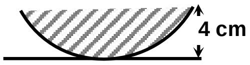
$$\frac{1}{v}=-\frac{4}{6r}=+\frac{2}{3r}$$

$v = -\frac{3r}{2}$

so height above centre $= 2r + \frac{3r}{2}$

$v = \frac{7r}{2}$.

Prob 14. *A plano–convex lens has a thickness of 4cm. When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom–most point of the lens is found to be 3cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face of the lens is found to be 25/8cm. Find the focal length of the lens.*



Sol. Apparent depth × refractive index = actual depth

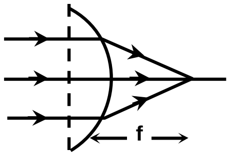
$\therefore\ \mu = \frac{4}{3}$

Again v = –25/8, u = –4 cm, $\mu_1 = 4/3$, $\mu_2 = 1$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

R = –25 cm

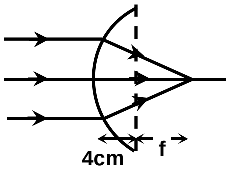
Case (i) When rays are falling on the plane surface first.



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$u \to \infty$ then $v \to f$

$$\frac{1}{f} - \frac{4}{3\times\infty} = \frac{1-4/3}{-25} \Rightarrow f = 75 \text{ cm}$$



Case (ii), when rays are falling on the curved surface first.

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$-\frac{4}{3v} - \frac{1}{\infty} = \frac{(4/3)-1}{25}$$

v = 100 cm

For refraction from plane surface

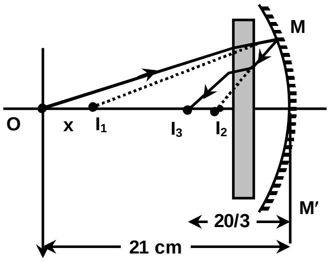
u = (100 – 4) = 96 cm

$$\frac{1}{v} - \frac{4}{3\times 96} = \frac{1-4/3}{\infty}$$

v = f = 72 cm

Prob 15. *An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror. Find the position of the final image formed. (You may take the distance of the near surface of the slab from the mirror to be 1 cm)*

Sol. (a) Shift in object position due to glass slab



$x = t\left(1-\frac{1}{\mu}\right)$ towards the mirror

$\therefore$ Shift = $3\left(1-\frac{1}{1.5}\right)$ = 1 cm

Object distance u = 21 – 1 = 20 cm

Applying mirror formula

$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$, we get

$\frac{1}{v}+\frac{1}{-20}=\frac{2}{-10}$

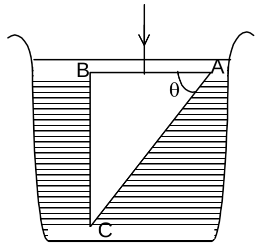
$\Rightarrow$ v = $-\frac{20}{3}$ cm

The outgoing ray will be shifted away from the mirror and shift in image position will be $3\left(1-\frac{1}{1.5}\right)$ = 1 cm

$\therefore$ Final position of image = $\frac{20}{3}+1=\frac{23}{3}$ = 7.67 cm

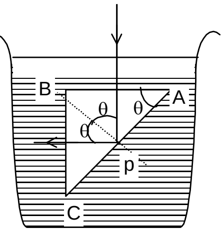
Objective:

Prob 1. *A glass prism of refractive index 1.5 is immersed in water (R.I. = 4/3). The beam of light incident normally on the face AB is totally reflected to reach the face BC, if*



(A) sin $\theta \ge 8/9$
(B) sin $\theta < 2/3$
(C) $2/3 <$ Sin $\theta < 8/9$
(D) None of these

Sol. The light ray is just reflected internally at the point P (say). Therefore, the critical angle $\cong \theta$. Applying Snell's Law of refraction at the critical angle of incidence (θ), we obtain



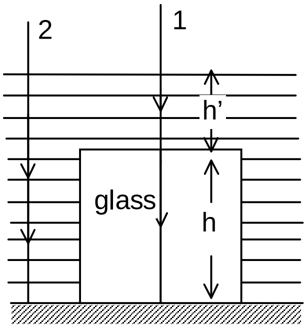
$$\frac{\sin\ \theta}{\sin\ 90} = \frac{n_w}{n_g} \qquad \Rightarrow \qquad \sin\theta = \frac{4/3}{3/2} = \frac{8}{9}$$

$$\Rightarrow \theta \ge \sin^{-1}\left(\frac{8}{9}\right) \qquad \Rightarrow \theta \ge \sin^{-1}(8/9)$$

or $\sin\theta \ge 8/9$.

$\therefore$ (A)

Prob 2. *Two beams of light are incident normally on water (R.I. = 4/3). If the beam 1 passes through a glass of height h as shown in the figure, the time difference for both the beams for reaching the bottom is*



(A) Zero *(B)* $\frac{h'}{6C}$

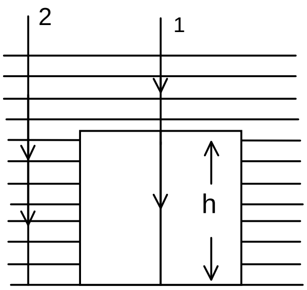
(C) $\frac{6h}{C}$ *(D)* $\frac{h}{6C}$

Sol. The refractive index of glass is greater than that of water. Therefore the speed of light in glass is lesser than that of water. It is given as

$v = \frac{c}{n}$ where $c = 3 \times 10^8$ m/sec.

v = speed of light in a medium of R.I. n

$\therefore$ The time difference for the rays



$$= t_1 - t_2 = \frac{h}{v_g} - \frac{h}{v_w}$$

$$\Rightarrow \quad \Delta t = \frac{h}{(C/n_g)} - \frac{h}{(C/n_w)} = \frac{h}{C}(n_g - n_w)$$

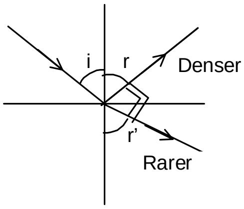
$$\Rightarrow \quad \Delta t = \frac{h}{C}\left(\frac{3}{2} - \frac{4}{3}\right) = \frac{h}{6C} \qquad \therefore \qquad \text{(D).}$$

Prob 3. *Focal length of a convex lens of refractive index 1.5 is 2 cm. Focal length of lens when immersed in a liquid of R.I. 1.25 will be*
(A) 5 cm *(B) 7.5 cm*
(C) 10 cm *(D) 2.5 cm*

Sol. $\frac{f_l}{f_a} = \frac{\mu_g - 1}{\frac{\mu_g}{\mu_l} - 1} \Rightarrow f_l = \frac{5}{2} \times f_a = \frac{5}{2} \times 2\text{ cm} = 5\text{ cm}$

$\therefore$ (A)

Prob 4. *A ray of light from a denser medium strikes a rarer medium at angle of incidence i. The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r & r′ respectively. The critical angle is*



(A) $\sin^{-1}$ (tan r) *(B) $\sin^{-1}$ (cot i)*
(C) $\tan^{-1}$ (Sin r) *(D) $\tan^{-1}$ (sin i)*

Sol. Applying Snell's law for refraction,

$\frac{\sin i}{\sin r'} = \frac{n_2}{n_1}$ …(1)

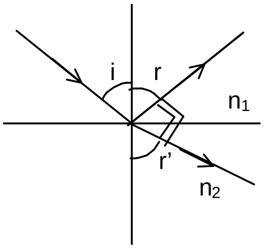
From the given condition, r + r′ = 90

$\Rightarrow$ Sin r′ = Cos r …(2)

From (1) and (2) yields, $\frac{\sin i}{\cos r} = \frac{n_2}{n_1}$ …(3)

According to the Law for reflection;

i = r …(4)



Using (3) and (4) we obtain

$\frac{\sin i}{\cos i} = \frac{n_2}{n_1}$

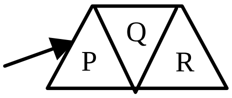
$\Rightarrow$ tan i = $\frac{n_2}{n_1}$ ….(5)

Since, at the time of total internal reflection

$\sin\theta_c = \frac{n_2}{n_1}$, Using (5) we obtain $\theta_c = \sin^{-1}$ (tan r)

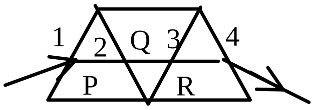
$\therefore$ (A)

Prob 5. *A given ray of light suffers minimum deviation in an equilateral prism P. Additional prisms Q and R of identical shape and of the same material as P are now added as shown in the figure. The ray will now suffer*



(A) greater deviation *(B) no deviation*
(C) same deviation as before *(D) total internal reflection*

Sol. No deviation occur on interfaces 2 and 3 as there is no change in medium. However, deviation at interface 4 is same as it was on interface 2 with only prism P.



$\therefore$ (C)

Prob 6. *A concave lens of glass, refractive index 1.5, has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a*
(A) convergent lens of focal length 3.5 R
(B) convergent lens of focal length 3.0 R
(C) divergent lens of focal length 3.5 R
(D) divergent lens of focal length 3.0 R

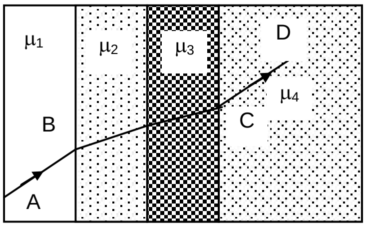
Sol. $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow \frac{1}{f} = \left(\frac{1.5}{1.75} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right)$

By solving, f = 3.5 R.

$\therefore$ (A)

Prob 7. *A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have*



(A) $\mu_1 = \mu_2$ *(B) $\mu_2 = \mu_3$*
(C) $\mu_3 = \mu_4$ *(D) $\mu_4 = \mu_1$*

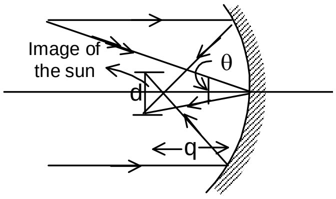
Sol. Considering Snell's law, $\mu \sin\theta$= constant, $\mu_1 = \mu_4$

Prob 8. *The sun (diameter D) subtends an angle of θ radians at the pole of a concave mirror of focal length f. The diameter of the image of the sun formed by the mirror is*

(A) $f\,\theta$ *(B)* $\frac{f\,2\theta}{D}$

(C) $2f\theta$ *(D)* $D\,\theta$

Sol. Since the sun is at very large distance, p = ∞



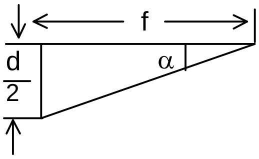
$\Rightarrow \frac{1}{\infty} + \frac{1}{q} = \frac{1}{f} \quad \Rightarrow \quad q = f$

If the diameter of the image be d,

$\frac{d/2}{q} = \alpha \Rightarrow d = (2\alpha)\,q$

Putting $2\alpha = \theta$ and q = f, we obtain, $d = f\,\theta$

$\therefore$ (A)



Prob 9. *A screen having a real image of magnification m_1 formed by a convex lens is moved a distance x. The object is then moved until a new image of magnification m_2 is formed on the screen. The focal length of the lens is*

(A) $\frac{x}{m_2 - m_1}$ *(B)* $\frac{x}{m_1 - m_2}$

(C) $\frac{x}{\sqrt{m_1 m_2}}$ *(D) None of these*

Sol. In first case, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ and $\frac{q}{p}=m_1$ $\Rightarrow$ $1+m_1=\frac{q}{f}$ . . . (1)

In the second case, $\frac{1}{q+x}+\frac{1}{p'}=\frac{1}{f}$ and $\frac{q+x}{p'}=m_2$

$\Rightarrow$ $m_2=\frac{q+x}{f}$ . . . (2)

From (1) and (2)

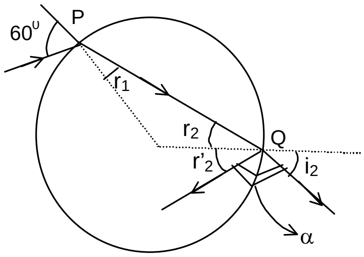
$\Rightarrow$ $m_2 - m_1 = x/f$ $\Rightarrow$ $f = \frac{x}{m_2-m_1}$

$\therefore$ (A)

Prob 10. *A ray enters a glass sphere of R..I. n = $\sqrt{3}$ at an angle of incidence 60° and is reflected and refracted at the farther surface of the sphere. The angle between the reflected and refracted rays at this surface is*

(A) 50° *(B) 60°*
(C) 90° *(D) 40°*

Sol. Refraction at P:



$\frac{\sin\ 60^0}{\sin\ r_1}=\sqrt{3}$ $\Rightarrow$ $\sin r_1 = \frac{1}{2}$

$\Rightarrow$ $r_1 = 30°$

Since $r_2 = r_1$ $\therefore$ $r_2 = 30°$

Refraction at Q $\frac{\sin\ r_2}{\sin\ i_2}=\frac{1}{\sqrt{3}}$

Putting $r_2 = 30°$ we obtain $i_2 = 60°$

Reflection at Q, $r_2' = r_2 = 30°$ $\therefore$ $\alpha = 180^0 - (r_2' + i_2)$

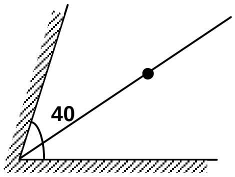
$= 180° - (30°+60°) = 90°$

$\therefore$ (C)

Fill in the Blanks

Prob 11. *An object is kept symmetrically between two plane mirrors making an angle of 40° between them. The number of images of the object in the mirrors is _______.*

Sol. **8**



Prob 12. *The minimum height of a mirror so that a man of height h can see his full image in it is _______.*

Sol. **h/2**

Prob 13. *The image formed by a convex mirror of a real object is ________ and ________.*

Sol. **virtual, erect**

Prob 14. *The image formed by a convex mirror of a real object is ________ than the object.*

Sol. **smaller**

Prob 15. *The surface above the water will look like a _______ to a fish.*

Sol. **cone**

True or False Type

Prob 16. *An equiconvex lens of focal length 15 cm is cut into two halves of thickness. The focal length of each half is 40 cm.*

Sol. **False**

Prob 17. *A convex lens forms a virtual image of an object. The position of the object is between optical centre and focus*

Sol. **True**

Prob 18. *A large–angle prism deviates a light ray more.*

Sol. **True**

***Prob 19.**.* *Dispersive power of a prism decreases with the increase in prism angle.*

Sol. **False**
As the deviation of the light ray increases with prism angle, so does dispersion.

Prob 20. *The luminous efficiency of a fluorescent lamp is less than that of common electric bulbs.*

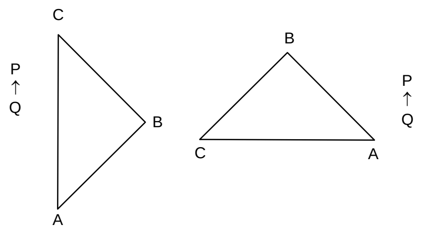
Sol. **False**
Most of the power in electric bulbs is dissipated as heat.

ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. An object is placed in front of a right angled prism ABC in two positions (a) and (b) as shown. The prism is made of crown glass with critical angle 41°. Trace the path of two rays from P to Q (i) in position (a) normal to the hypotenuse, and (ii) in position (b) parallel to the hypotenuse.



(a) (b)

2. Derive the relation between distance of object, distance of image and radius of curvature of a convex spherical surface, when refraction takes place from a rarer medium of refractive index μ_1 to a denser medium of refractive index μ_2 and the image produced is real, state assumptions and of signs used.

3. Draw a graph to show the variation of the angle of deviation 'D' with that of angle of incidence 'i' for a monochromatic ray of light passing through a glass prism of refractive angle 'A'. Hence deduce relation $\mu = \dfrac{\sin\dfrac{(D_m + A)}{2}}{\sin(A/2)}$.

4. Define critical angle with reference to the total internal reflection. Calculate the critical angle for glass-air surface if a ray of light which is incident in air on the glass surface is deviated through 15°, when angle of incidence is 45°.

5. Using the lens formula, show that a concave lens produces a virtual diminished image independent of location of object.

6. The magnifying power of an astronomical telescope in the normal adjacent positions is 100. The distance between the objective and eye piece is 101 cm. Calculate the focal length of the objective and eyepiece.

7. Draw a ray diagram of an astronomical telescope in the normal adjacent position. Write down the expression for its magnifying power.

8. Define magnifying power of an optical telescope. Write significance of diameter of the objective lens on the optical performance of a telescope.

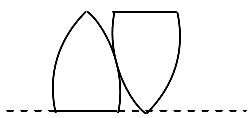
9. What is the difference between images formed by a large and a small mirror?

10. How will an air bubble inside water behave: As a converging lens or as a diverging lens?

11. Can a beam of white light passed through a hollow prism produce spectrum? Explain.

12. Explain why the rising (or setting) sun appears to be oval?

13. A lens of focal length f is cut into two equal halves and both the pieces are put in contact as shown. What will be the power of the lens system?



14. What change in focal length of (a) convex lens, and (b) convex mirror do you expect if blue light is replaced by red light?

15. If object and image distances are measured from the focus instead of the pole, what will be the relation between object distance and image distance?

16. If the two radii of a lens are not equal, will the focal length change with the change in surface facing light? Explain.

17. A concave mirror of radius of curvature r is dipped inside water. Find its focal length.

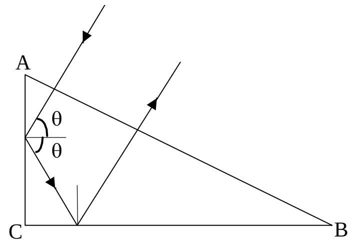
18. An object far away from a convex lens moves towards the lens. Does its image move faster, slower or at the same speed as compared to the object.

19. An object of 10 cm is placed at a distance of 20 cm from a lens of focal length 25 cm. The object is placed erect on the principal axis of the lens yet a crooked image was formed. Explain why.

20. A narrow beam of white light is incident on a glass slab. Will you be able to see the phenomenon of dispersion?

Level– I

1. A right prism is to be made by selecting a proper material and the angles A and B ($B \le A$), as shown in figure. It is desired that a ray of light incident normally on AB emerges parallel to the incident direction after two total internal reflections.
(a) What should be the minimum refractive index μ for this to be possible ?
(b) For μ = 5/3, is it possible to achieve this with the angle A equal to 60 degrees ?



2. A thin converging lens forms a magnified image (magnification k: p) of an object. The magnification factor becomes q when the lens is moved a distance ‘a’ towards the object. Find the focal length of the lens.

3. A convex lens of focal length 30 cm forms an image of height 2 cm of an object placed far away. A concave lens of focal length 20 cm is placed at a distance of 26 cm from the convex lens towards the image side. What is the final height of the image ?

4. An object is placed 21 cm in front of a concave mirror of radius of curvature 20cm. A glass slab of thickness 3 cm and refractive index 3/2 is placed close to the mirror in the space between the object and the mirror. Find the position of the final image formed. The distance of the nearer surface of the slab from the mirror is 10 cm.

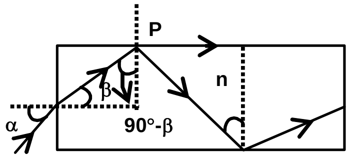
5. A parallel beam of light is incident normally onto a solid glass sphere of radius R and refractive index n. Find the distance of the image from the farther side of the glass sphere.

6. When an object is placed at a distance 25cm from a mirror, the magnification is m_1. The object is moved 15cm farther away with respect to the earlier position, and the magnification becomes m_2. If $\frac{m_1}{m_2}$ =4, then calculate the focal length of the mirror.

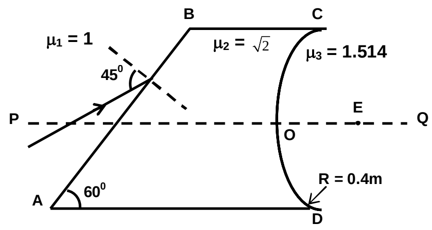
7. The refractive indices of the crown glass for blue and red lights are 1.51 and 1.49 respectively, and those of the flint glass are 1.77 and 1.73 respectively. An isosceles prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle of this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light. Determine the angle of the flint glass prism. Calculate the net dispersion of the combined system.

8. The convex surface of a thin concave–convex lens of glass of refractive index 1.5 has a radius of curvature 20cm. The concave surface has a radius of curvature 60cm. The convex side is silvered and placed at horizontal surface as shown in the figure. Where should a pin be placed on the optic axis such that its image is formed at the same place?

9. Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index n. Determine the least value of n so that the light entering the rod does not emerge from the curved surface of the rod irrespective of the value of α.



10. A light ray is incident on an irregular shaped slab of refractive index $\sqrt{2}$ at an angle of 45° with the normal on the incline face as shown in the figure. The ray finally emerges from the curved surface in the medium of the refractive index μ = 1.514 and passes through point E. If the radius of curved surface is equal to 0.4 m, find the distance OE correct upto two decimal places.



11. A compound microscope consists of an objective of focal length 1 cm and an eyepiece of focal length 5 cm separated by 12.2 cm. At what distance from the objective should an object be placed to focus it properly so that the final image is formed at least distance of clear vision ?

12. Yellow light is passed through a prism for which angle of minimum deviation is 40°. Calculate the refractive index of the material of the prism for yellow light if the angle of prism is 60°.

13. Two convex lenses of focal lengths 10 cm and 1 cm constitutes a telescope. The telescope is focused on a scale which is 1m away from the objective. Calculate the magnification produced and the length of the tube if the final image is formed at a distance of 25 cm from the eye.

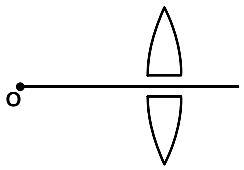
14. An air cavity in form of plano–convex lens is formed in the middle of water tube. The radius of curvature of its curved surface is 10 cm. while refractive index of water is 4/3. Find the focal length of this air lens.

15. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and μ = $\sqrt{2}$. Find angle of deviation of ray of right.

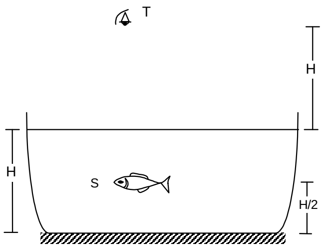
Level– II

1. A thin rod of length f/3 is placed along the optical axis of a concave mirror of focal length f such that its image which is real and elongated just touches the rod. Calculate the magnification.

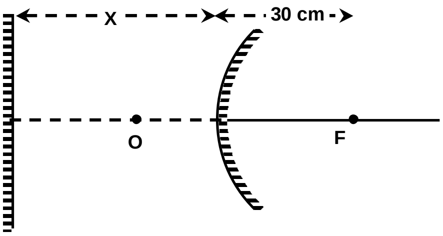
2. A point object is placed at a distance of 0.4 meters from a convex lens focal length 0.2 m. Cuts into two parts each of which is displaced vertically by 6 mm. away from the optic axis as shown in figure. Find,
   (a) the position of the image from the lens.
   (b) the number of images formed.
   (c) the distance between two images formed.



3. Consider the situation in figure. The bottom the pot is a reflecting plane mirror, S is a small fish and T is a human eye. Refractive index of water is μ.
   (a) At what distance(s) from itself will the fish see the image (s) of the eye?
   (b) At what distance(s) from itself will the eye see the image(s) of the fish?

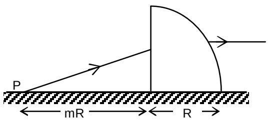


4. There is a convex mirror of focal length of 30 cm. A point object O is placed in front of the convex mirror at a distance of 15 cm. A plane mirror is kept at some distance from the convex mirror. If the distance between the two images (one formed by direct reflection at the plane mirror and the other formed by a reflection at convex mirror followed by a reflection at the plane mirror) of the object formed in the plane mirror is 25 cm, find the separation between the plane mirror and the convex mirror.

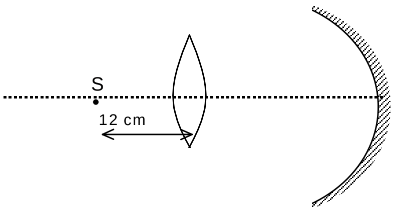


5. There are two spherical surfaces of radii R_1 = 30cm and R_2 = 60cm. In how many ways these surfaces may be arranged to get different lenses. If all the lenses are made of glass (μ = 1.5), find the focal length of each lens.

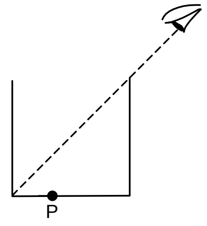
6. A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance of mR from it. Find the value of m for which a ray from P will emerge parallel to the table as shown in figure.



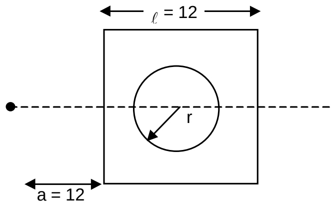
7. A converging lens of focal length 15 cm and a converging mirror of focal length 20cm are placed with their principal axis coinciding. A point source S is placed on the principal axis at a distance of 12 cm from the lens as shown in the figure. It is found that the final beam comes out parallel to the principal axis. Find the separation between the mirror and the lens.



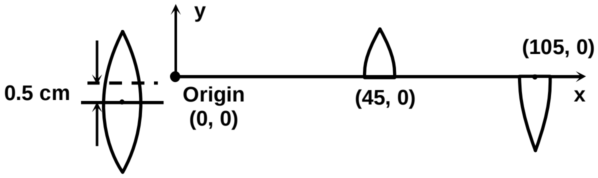
8. A cylindrical vessel, whose diameter and height both are equal to 30 cm, is placed on a horizontal surface and a small particle P is placed in it at a distance of 5.0 cm from the centre. An eye is placed at a position such that the edge of the bottom is just visible. The particle P is in the plane of drawing. Up to what minimum height should water be poured in the vessel to make the particle P visible?



9. A cubical block of glass (refractive index μ = 1.5) has a concentric spherical cavity of radius r = 3cm. Each edge of the cube is ℓ = 12cm long. A luminous point object is at a distance a = 12cm on left of left face of the cube as shown in figure. Calculate apparent position of the object when seen from right side of the cube.



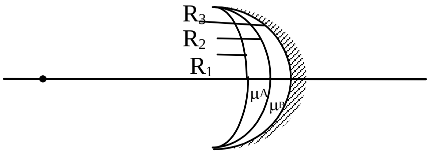
10. A thin convex lens whose focal length in air is known to be 30 cm is cut into two parts by a transverse plane parallel to the principal axis and 0.5 cm above the principal axis.



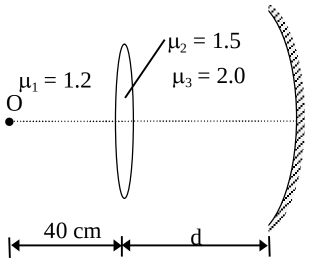
They are placed on the x–axis as shown in the figure. A point object is placed at the origin of a coordinate system. Find the co–ordinates of the final image formed due to the rays which get refracted by both the parts of lens, assuming paraxial ray approximations to be valid.

11. The x–y plane is the boundary between two transparent media. Medium –I with Z $\geq$ 0 has a refractive index $\sqrt{2}$ and medium –II with Z $\leq$ 0 has refractive index $\sqrt{3}$. A ray of light in medium –I given by the vector $\vec{A} = 6\sqrt{3}\vec{i} + 8\sqrt{3}\vec{j} - 10\vec{k}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in medium 2.

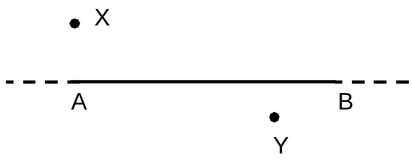
12. The figure shows a silvered lens. μ_A = 1.6 and μ_B = 1.2, R_1 = 80 cm, R_2 = 40 cm and R_3 = 20 cm. An object is placed at a distance of 12 cm from this lens. Find the image position.



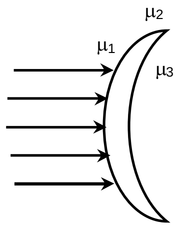
13. The figure shows an arrangement of a equi–convex lens and a concave mirror. A point object O is placed on the principal axis at a distance 40 cm from the lens such that the final image is also formed at the position of the object. If the radius of curvature of the concave mirror is 80 cm, find the distance d. Also draw the ray diagram. The focal length of the lens is 20 cm.



14. An image Y is formed of a point object X by a lens whose optical axis is AB as shown in the figure. Draw a ray diagram to locate the lens and its focus. If the image Y of the object X is formed by a concave mirror (having the same optic axis AB) instead of lens, draw another ray diagram to locate the mirror and its focus. Write down the steps of construction of the diagrams.



15. A meniscus lens is made of a material of refractive index μ_2. Both its surfaces have radii of curvature R. It has two different media of refractive indices μ_1 and μ_3 respectively, on its two sides (see figure). Calculate its focal length for $\mu_1 < \mu_2 < \mu_3$, when light is incident on it as shown.



Objective:

Level– I

1. A concave mirror is placed on a horizontal table, with its axis directed vertically upwards. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be
(A) real, and will remain at C
(B) real, and located at a point between C and ∞
(C) virtual, and located at a point between C and O.
(D) real, and located at a point between C and O.

2. A spherical convex surface separates object and image spaces of refractive indices 1 and 4/3 respectively. If radius of curvature of the surface is 0.1 m, its power is :
(A) 2.5 D (B) –2.5 D
(C) 3.3 D (D) –3.3 D

3. The minimum distance between a real object and its virtual image formed by a convex lens is:
(A) f (B) 4 f
(C) 0 (D) 2 f

4. A real image of a distant object is formed by a plano–convex lens on its principal axis. Spherical aberration
(A) is absent
(B) is smaller if the curved surface of the lens faces the object
(C) is smaller if the plane surface of the lens faces the object
(D) is the same whichever side of the lens faces the object

5. In the displacement method, a convex lens is placed in between an object and a screen. If the magnification in the two positions be m_1 and m_2 and the displacement of the lens between the two positions is X, then the focal length of the lens is :
(A) $X/m_1 \times m_2)$ (B) $X/(m_1 - m_2)$
(C) $X/(m_1 + m_2)$ (D) $X/(m_1 - m_2)^2$

6. A convex lens of focal length f is placed in between a real object and a screen. The distance between the object and the screen is X. If the numerical value of the magnification produced by the lens is m, then the focal length of the lens is :
(A) $mX/(m + 1)^2$ (B) $mX/(m - 1)^2$
(C) $(m + 1)^2/mX$ (D) $(m - 1)X/m$

7. A ray of light passes through an equilateral prism (refractive index = 1.5) such that the angle of incidence is equal to the angle of emergence and latter is equal to $3/4^{th}$ the angle of prism. The angle of deviation is :
(A) 45^o (B) 39^o
(C) 20^o (D) 30^o

8. A thin prism P_1 with angle 4^o and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is :
(A) 5.33^o (B) 4^o
(C) 3^o (D) 2.6^o

9. For which colour the angle of deviation is the least?
(A) Violet (B) Blue
(C) Red (D) Yellow

10. A liquid is placed in a hollow prism of angle 60^o. If angle of the minimum deviation is 30^o, what is the refractive index of the liquid?
(A) 1.41 (B) 1.50
(C) 1.65 (D) 1.95

11. A soldier directs a laser beam on an enemy by reflecting the beam from a mirror. If the mirror is rotated by an angle θ, by what angle will the reflected beam rotate?
(A) $\theta/2$ (B) θ
(C) 2θ (D) None of the these

12. Find the distance of an object from a concave mirror of focal length 20 cm so that the real image size is three times the size of the object.
(A) 40 cm (B) 60 cm
(C) 26.67 cm (D) 6.67 cm

13. When an object is at distances of u_1 and u_2 from the poles of a concave mirror, images of the same size are formed. The focal length of the mirror is
(A) $|u_1 + u_2|$ (B) $|u_1 - u_2|$
(C) $\left|\dfrac{u_1 + u_2}{2}\right|$ (D) $\left|\dfrac{u_1 - u_2}{2}\right|$

14. A short linear object of length ℓ lies along the axis of a concave mirror of focal length f at a distance u from the pole. What is the size of the image?
(A) $\dfrac{\ell f}{u}$ (B) $\dfrac{\ell f^2}{(u-f)^2}$
(C) $\dfrac{\ell f}{u+f}$ (D) $\dfrac{\ell f^2}{(u+f)^2}$

15. A bulb is placed at a depth of $2\sqrt{7}$ m in water and a floating opaque disc is placed over the bulb so that it is not visible from the surface. What is the minimum diameter of the disc ? (Refractive index of water is 4/3.)
(A) 6 m (B) 12 m
(C) 11 m (D) 24 m

Fill in the Blanks

16. A ray is reflected in turn by two plane mirrors mutually at right angles to each other. The angle between the incident and the reflected rays is ___________.

17. A monochromatic beam of light of wavelength 6000 Å in vacuum enters a medium of refractive index 1.5. In this medium, its wavelength is ___________ and its frequency is ___________.

18. When a ray of white light passes through a glass prism, red light is dispersed ___________ than violet light because refractive index of violet light is ___________ than that of red light.

19. A fish at a depth d below the water sees a bird flying at a height h above the ground at an apparent distance ___________.

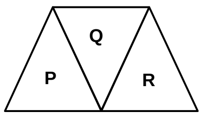
20. A glass (μ = 1.5) sphere when immersed in water will have its critical angle ___________.

True or False Type

21. Refraction occurs when light ray passes from one medium to another where the velocity of light is unequal.

22. A plane glass slab is placed over letters of various colours. Then yellow coloured letters appear to be raised the least.

23. A given ray of light suffers minimum deviation δ_m in an equilateral prism P. Additional prisms Q and R of identical shape and made of same material are now added as shown in the figure. The ray will now suffer a deviation of $2\delta_m$.

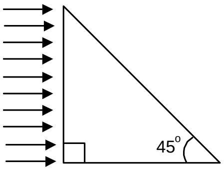


24. A concave mirror always forms a real image.

25. A convex mirror always forms a virtual image.

Level – II

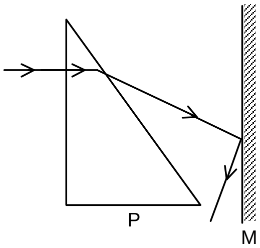
1. The focal lengths of a convex lens for blue and red colours of light are f_b and f_r, respectively, and those of a concave lens are F_b and F_r. Then;
(A) $f_b > f_r$ and $F_b < F_r$ (B) $f_b < f_r$ and $F_b > F_r$
(C) $f_b > f_r$ and $F_b > F_r$ (D) $f_b < f_r$ and $F_b < F_r$

2. A beam of light consisting of red, green and blue colours is incident on a right–angled prism. The refractive indices of the material of prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will:


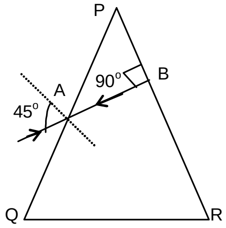
(A) separate part of the red colour from the green and blue colours
(B) separate part of the blue colour from the red and green colours
(C) separate all the three colours from one another
(D) not separate even partially any colour from the other two colours.

3. A prism can produce a minimum deviation δ in a light beam. If three such prisms are combined, the minimum deviation that can be produced in this beam is:
(A) 0 (B) δ
(C) 2δ (D) 3δ

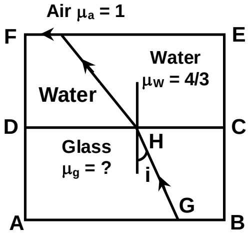
4. An equilateral triangular prism is made of glass (μ = 1.5). A ray of light is incident normally on one of the faces. The angle between the incident and emergent ray is :
(A) 60^o (B) 90^o
(C) 120^o (D) 180^o

5. P is a small angled prism of angle 3^o made of a material of refractive index 1.5. A ray of light is incident as shown in figure. M is a plane mirror. The angle of deviation for the ray reflected from the mirror M with respect to the incident ray is :


(A) 4.5^o (B) 175.3^o
(C) 177^o (D) 178.5^o

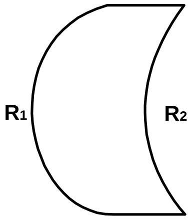
6. The face PR of a prism PQR of angle 30^o is silvered. A ray is incident on face PQ at an angle of 45^o as shown in figure. The refracted ray undergoes reflection on face PR and retraces its path. The refractive index of the prism is


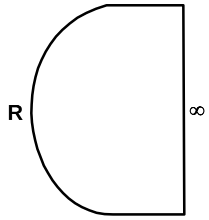
(A) $\sqrt{2}$ (B) $3/\sqrt{2}$
(C) 1.5 (D) 1.33

7. A ray of light (GH) is incident on the glass–water interface DC at an angle i. It emerges in air along the water-air interface EF (see figure). If the refractive index of water μ_w is 4/3, the refractive index of glass μ_s is


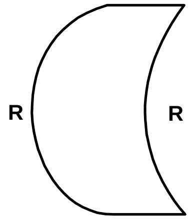
(A) (3/4sini) (B) 1/sini
(C) (4sini/3) (D) (4/3sini)

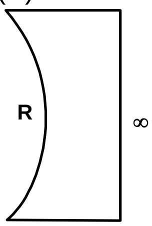
8. Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams.

(A)


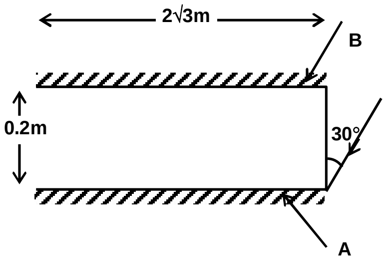
(B)


$R_1 \neq R_2$

(C)


(D)


9. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is



(A) 28 (B) 30
(C) 32 (D) 31

10. Two transparent media A and B are separated by a plane boundary. The speed of light in medium A is 2.0×10^8 m/s and in medium B is 2.5×10^8 m/s. The critical angle for which a ray of light going from A to B is totally internally reflected is

(A) $\sin^{-1}\left(\frac{1}{2}\right)$ (B) $\sin^{-1}\left(\frac{2}{5}\right)$
(C) $\sin^{-1}\left(\frac{4}{5}\right)$ (D) $\sin^{-1}\left(-\frac{1}{2}\right)$

11. A parallel beam of light travelling in water (μ = 4/3) is refracted by a spherical bubble of radius
R = 2 mm. Assuming the light rays to be paraxial, find the position of the final image from the centre.

(A) 2 mm to the left (B) 4 mm to the left
(C) 5 mm to the right (D) 8 mm to the left

12. A plano–concave lens is made of glass (R.I. = 1.5) and the radius of curvature of the curved face is 50 cm. The power of the lens is

(A) –1.0 D (B) –0.5 D
(C) +1.0 D (D) +0.5 D

13. Two plano–convex lenses of radius of curvature T and R.I. 1.5 will have a combined focal length equal to R when they are placed

(A) at distance R/4 apart (B) at distance R/2 apart
(C) at distance R apart (D) in contact with each other

14. A point object is placed at a distance of 25 cm from a convex lens of focal length 20 cm. If a glass slab of thickness t and R.I. 1.5 is inserted between the lens and the object, the image is formed at infinity. The thickness t of the glass slab is
(A) 15 cm (B) 5 cm
(C) 10 cm (D) 20 cm

15. An object and a screen are mounted on an optical bench and a converging lens is placed between them so that a sharp image is obtained on the screen. The linear magnification of the image is 25. The lens is now moved 30 cm towards the screen and a sharp image is again formed on the screen. Find the focal length of the lens.
(A) 14.0 cm (B) 14.3 cm
(C) 14.6 cm (D) 14.9 cm

16. A lens behaves as a converging lens in air and diverging lens in water $\left(\mu = \frac{4}{3}\right)$. The refractive index of the lens material may be
(A) 1 (B) 1.25
(C) 1.33 (D) 1.5

17. Spherical aberration in a thin lens can be reduced by
(A) using a circular annular mask over the lens (B) increasing the size of the lens
(C) using a monochromatic light (D) using a doublet combination

18. An achromatic combination of lenses produces
(A) highly enlarged images
(B) images unaffected by variation of R.I. with wavelength
(C) coloured images
(D) images in black and while

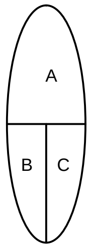
19. The focal length of a convex lens is 30 cm and the size of the image is half of the object. Then, the object distance is
(A) 60 cm (B) 90 cm
(C) 30 cm (D) 40 cm

20. An achromatic combination of lenses is formed by joining
(A) two concave lenses (B) two convex lenses
(C) convex and plane mirrors (D) one convex and one concave lens

21. A bird flies down vertically towards a water surface to a fish inside water, vertically below the bird, the bird will appear to
(A) be farther away than its actual distance (B) be closer than its actual distance.
(C) move faster than its actual speed (D) move slower than its actual speed

22. A solid, transparent sphere has a small, opaque dot at its centre when observed from outside. The apparent position of dot will be
(A) closer to the eye than its actual position.
(B) farther from the eye than its actual position
(C) the same as its actual position
(D) independent of the refractive index of the sphere.

23. A thin, symmetric double convex lens of power P is cut into three parts A, B and C as shown. The power of



(A) A is P
(B) A is 2P
(C) B is P/2
(D) B is P/4

24. If a converging beam of light is incident on a concave mirror, the reflected light
(A) may form a real image
(B) must form a real image
(C) may form a virtual image
(D) may be parallel beam

25. A point object P moves towards a convex mirror with a constant speed v, along its optic axis. The speed of the image
(A) may be < v
(B) may be v
(C) = v
(D) none of these

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – O

4. $i_c = 21.4°$

6. $f_0 = 100$ cm., $f_e = 1$ cm.

Level – I

1. (a) $\mu > \sqrt{2}$ (b) No total internal reflection does not takes place at the surface BC.
2. $\frac{apq}{q-p}$
3. 2.5 cm
4. Image will form at the object itself.
5. $\frac{R(2-n)}{2(n-1)}$
6. 20 cm
7. 4°, 0.04°
8. Object should be placed at 15 cm from the lens on the optic axis.
9. $n = \sqrt{2}$
10. OE = 6.056 m ≈ 6.06 m
11. – 1.1 cm
12. 1.532
13. M = –2.89, L = 12.1 cm
14. – 40 cm
15. 15°

Level – II

1. 3/2
2. (a) 0.4 meter. (b) 2 (c) 12 mm
3. (a) $H\left(\mu+\frac{1}{2}\right)$ above itself, $H\left(\mu+\frac{3}{2}\right)$ below itself
   (b) $H\left(1+\frac{1}{2\mu}\right)$ below itself, $H\left(1+\frac{3}{2\mu}\right)$ below itself
4. You may keep the plane mirror anywhere, the distance between the images formed in the plane mirror will always be 25 cm.
5. 4 ways, 40cm and 120cm (both converging & diverging)
6. 4/3
7. 40 cm.
8. 26.7 cm
9. 52/9 cm
10. [120 cm, –1 cm]
11. $\frac{\sqrt{2}}{10}\left[3\hat{i} + 4\hat{j} - 5\hat{k}\right]$
12. 24 cm to the left of the silvered lens.
13. 30 cm
15. $f = v = \frac{\mu_3 R}{\mu_3 - \mu_1}$

Objective:

Level – I

1.	D	2.	A
3.	C	4.	B
5.	B	6.	B
7.	D	8.	C
9.	C	10.	A
11.	C	12.	C
13.	C	14.	B
15.	B	16.	180°
17.	4000 Å, 5×10^{14}	18.	less, less
19.	d + μh	20.	increased
21.	True	22.	False
23.	False	24.	False
25.	True		

Level – II

1.	D	2.	A
3.	B	4.	C
5.	D	6.	A
7.	B	8.	C
9.	D	10.	C
11.	B	12.	A
13.	D	14.	A
15.	B	16.	B
17.	A	18.	B
19.	B	20.	D
21.	A, C	22.	C, D
23.	A, C	24.	A, C, D
25.	A, B, C		

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

ELECTROSTATICS

Electrostatics

Syllabus for IITJEE & Maharashtra Board:

Frictional electricity, charges and their conservation; Coulomb's law-Forces between two point electric charges. Forces between multiple electric charges; Superposition principle and continuous charge distribution.

Electric field and its physical significance, electric field due to a point charge, electric field lines; electric dipole, electric field due to a dipole and behaviour of a dipole in a uniform electric field.

Electric potential-physical meaning, potential difference, electric potential due to a point charge, a dipole and system of charges; Equipotential surfaces, Electrical potential energy of a system of two point charges and of electric dipoles in an electrostatic field.

Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Conductors and insulators, presence of free charges and bound charges inside a conductor; Dielectrics and electric polarization, general concept of a capacitor and capacitance, combination of capacitors in series and in parallel, energy stored in a capacitor, capacitance of a parallel plate capacitor with and without dielectric medium between the plates; Van de Graaff generator.

ELECTRIC CHARGE

You must be familiar with electric charge. To have a better understanding, do the following experiments.
(a) Rub a glass rod with silk and bring two such rods near each other. They repel each other.

(b) Now take a plastic rod instead of glass rod rub it with fur and repeat the above experiment. We find that they also repel each other.

(c) Now if we take a glass rod as (a) above and a plastic rod as (b) above and bring them near each other, we find that they attract each other.

We can conclude that a glass rod rubbed with silk produces one kind of charge and a plastic rod rubbed with fur creates another type of charge. Same type of charges repel each other and different type of charges attract each other. When the experiment is repeated we do not come across any other type of charge.

When bodies with opposite charges are brought into contact, charges disappear from both the bodies. This indicates that during rubbing one type of charge is transferred from one body to another. These get neutralised when the charged bodies are brought into contact.

Electric charge, like mass, is one of the fundamental attributes of the particle of which the matter is made. Charge is the physical property of certain fundamental particles (like electron, proton) by virtue of which they interact with the other similar fundamental particles. To distinguish the nature of interaction, charges are divided into two parts (i) positive (ii) negative. Like charges repel and unlike charges attract. SI unit of charge is coulomb and CGS unit is esu. $1\ C = 3 \times 10^9$ esu.
Magnitude of the smallest known charge is $e = 1.6 \times 10^{-19}$ C (charge of one electron or proton).
To understand the nature and behaviour of charge fully, atomic structure is to be kept in mind.

An atom consists of a nucleus (concentrated heavy portion in the centre) which comprises of positively charged particles protons and neutral particles neutrons. Negatively charged particles electrons revolve around nucleus. It is the transfer of these electrons which causes negative charge to pass from one body to another and in the process the first body gets positively charged.

Charging of a body

Charging can be done by two methods;

1. Conduction 2. Induction

Ordinarily, matter contains equal number of protons and electrons. A body can be charged by transfer of electrons or redistribution of electrons.

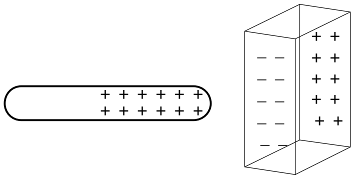
Charging by conduction

The process of charging from an already charged body can happen either by conduction or induction. Conduction from a charged body involves transfer of like charges. A positively charged body can create more bodies which are positively charged but the sum of the total charge on all positively charged bodies will be the same as the earlier sum.

Charging by induction

Induction is a process by which a charged body accomplishes creation of other charged bodies, without touching them or losing its own charge.

When a positively charged rod is brought near a plate, the free electrons are attracted by the +ve charge and move near to the rod.
Thus the portion nearer to rod becomes negatively charged and the portion farther from the rod becomes positively charged.



Further movement of electrons are repelled by the -ve charge already on the surface near the rod and finally this movement is stopped.

If the other end is grounded the +ve charge goes to the ground and, after removal of the rod, the plate has net –ve charge.

Conductors and insulators

We know that in some materials the charges flow easily. Such materials are called conductors. Atoms have equal number of protons and electrons which make them electrically neutral. When several atoms of a conductor unite, some of the outermost electrons do not remain attached to the atom and are free to move.

There are very few or no free electrons in an insulator. It causes restrictions in the movement of –ve charge and hence the movement of charge in insulators is restricted.

Semiconductors are in between conductors and insulators. They behave as insulators at low temperature but some of the electrons become free at higher temperatures and they start conducting to an extent.

PROPERTIES OF ELECTRIC CHARGE

Quantization of charge

Charge exists in discrete packets rather than in continuous amount. That is charge on any body is the integral multiple of the charge of an electron

$\Rightarrow$ $Q = \pm ne$, where n = 0, 1, 2, . . . where $e = 1.6 \times 10^{-19}$ C.

However, the step is so small that in most of the problems it can be taken as continuous variation.

Conservation of charge

Charge is conserved, i.e. total charge on an isolated system is constant. By isolated system, we mean here a system through the boundary of which no charge is allowed to escape or enter. This does not require that the amounts of positive and negative charges are separately conserved. Only their algebraic sum is conserved.

Within an isolated system, charges can be transferred from one part to another but the total charge is conserved.

Charges on a conductor

Static charges reside on the surface of the conductor.

Distribution of charges

The concentration of the charge is more on a surface with greater curvature.

Exercise 1:

(i) ***Two identical metallic spheres of exactly equal masses are taken, one is given a positive charge q and the other an equal negative charge. Their masses after charging are different. Comment on the statement.***

(ii) ***When a glass rod is rubbed with silk it gets___________(positively / Negatively) charged. Why does the rod get this charge ?***

(iii). ***Why is charge on an electron taken as the fundamental charge. Why can't we take charge on a proton to be as fundamental charge?***

COULOMB'S LAW:

Two point electric charges q_1 and q_2 at rest, separated by a distance r exert a force on each other whose magnitude is given by

$F = k\frac{q_1q_2}{r^2}$ where k is a proportionality constant.

If between the two charges there is free space then

$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2\ C^{-2}$, Where ε_0 is the absolute electric permittivity of the free space.

and $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 / N\text{-}m^2$

Exercise 2:

(i) ***A negatively charged particle is placed exactly midway between two fixed particles having equal positive charge. What will happen to the charge ?***

(a) if it is displaced at right angle to the line joining the positive charges?

(b) if it is displaced along the line joining the positive charges?

(ii) ***Is the Coulomb force between two given charges affected in anyway, if other charges are brought in the neighbourhood ?***

Illustration 1. *A polythene piece rubbed with wool is found to have negative charge of 3.2×10^{-7} C.*

(a) Estimate the number of electrons transferred from wool to polythene.

(b) Is there a transfer of mass from wool to polythene? If yes, how much?

Solution: (a) Let n be the number of e^- getting transferred.

$\Rightarrow n \times e = 3.2 \times 10^{-7}$

$\Rightarrow \quad 1.6 \times 10^{-19}\, n = 3.2 \times 10^{-7}$

$\Rightarrow n = 2 \times 10^{12} \quad \Rightarrow 2 \times 10^{12}$ electrons will get transferred.

(b) Mass transferred will be product of number of electrons and the mass of electron.

If m = mass getting transferred.

$\Rightarrow m = n \times (9.1 \times 10^{-31})$ kg

$m = 2 \times 10^{12} \times 9.1 \times 10^{-31}$

$m = 18.2 \times 10^{-31}$ kg

Illustration 2. Calculate force between two charges of 2 C each separated by 2 m in vacuum.

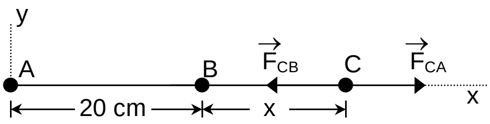
Solution: $F = \dfrac{kQ_1Q_2}{R^2} = \dfrac{9\times10^9 \times 2\times 2}{2^2}$

$\Rightarrow F = 9 \times 10^9$ N

Illustration 3. *Two particles A and B having charges 8×10^{-6} C and -2×10^{-6} C respectively are held fixed with a separation of 20 cm. Where should a third charged particle be placed so that it does not experience a net electric force?*

Solution: As the net electric force on C should be equal to zero, the force due to A and B must be opposite in direction. Hence, the particle should be placed on the line AB. As A and B have charges of opposite signs, C cannot be between A and B. Also A has larger magnitude of charge than B. Hence, C should be placed closer to B than A. The situation is shown in figure. Suppose BC=x and the charge on C is Q

$$\vec{F}_{CA} = \frac{1}{4\pi \in_0}\ \frac{(8.0\times10^{-6})\ Q}{(0.2+x)^2}\ \hat{i} \text{ and } \vec{F}_{CB} = \frac{-1}{4\pi \in_0}\ \frac{(2.0\times10^{-6})Q}{x^2}\ \hat{i}$$



$$\vec{F}_C = \vec{F}_{CA} + \vec{F}_{CB}$$

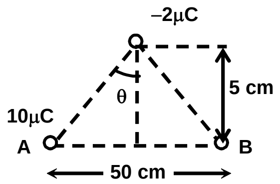
$$= \frac{1}{4\pi \in_0}\left[\frac{(8.0\times10^{-6})\ Q}{(0.2+x)^2} - \frac{(2.0\times10^{-6})\ Q}{x^2}\right] i$$

But $|\vec{F}_C| = 0$

Hence $\dfrac{1}{4\pi \in_0}\left[\dfrac{(8.0\times10^{-6})\ Q}{(0.2+x)^2} - \dfrac{(2.0\times10^{-6})\ Q}{x^2}\right] = 0$

Which gives x = 0.2 m

Illustration 4. *A charge of -2 μC is placed at the perpendicular bisector of the line joining two point charges of 10μC as shown in figure. What is the net force acting on the -2 μC charge ?*



Solution: Components of forces parallel to AB will cancel out.

$F = F_1 \cos\theta + F_2 \cos\theta$

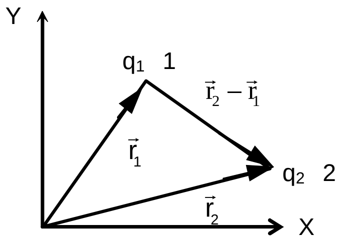
$$=2\cos\theta.\ \frac{1}{4\pi\varepsilon_0}\frac{10\times 2\times 10^{-12}}{(25^2+5^2)\times 10^{-4}}$$

$$= 9\times 10^9 \times \frac{5}{(25^2+5^2)^{1/2}}\frac{40\times 10^{-8}}{650}$$

= 1.086 N. (downward).

Exercise 3: ***Are positive and negative charges in a closed system separately conserved?***

Coulombs law in Vector Relations



Suppose two charges q_1 and q_2 are placed at points 1 and 2. The position vectors are $\vec{r}_1$ and $\vec{r}_2$, $\vec{r}_{21}=\vec{r}_2-\vec{r}_1$. As per Coulombs law the force on q_2 applied by q_1 will be

$$\vec{F}_{21}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$$

where $\hat{r}_{21}$ is the unit vector in the direction 1 to 2.
Similarly, force on q_1 applied by q_2 is

$$\vec{F}_{12}=-\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r}_{12}|^2}\hat{r}_{21}$$

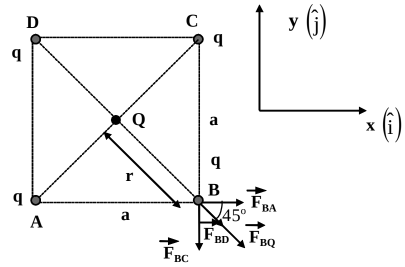
which is equal in magnitude and opposite in direction to the vector $\vec{F}_{21}$

PRINCIPLE OF SUPERPOSITION

This principle tells us that if charge Q is placed in the vicinity of several charges q_1, q_2 ….. q_n, then the force on Q can be found out by calculating separately the forces $\vec{F}_1,\vec{F}_2...\vec{F}_n$, exerted by q_1, q_2, ….. q_n respectively on Q and then adding these forces vectorially. Their resultant $\vec{F}$ is the total force on Q due to all of charges.

Illustration 5. *It is required to hold equal charges q* each *in equilibrium at the corners of a square of side a. What charge when placed at the centre of the square will do this?*

Solution : Let the charge be Q
As ABCD is a square of side a,



$$r=\frac{a\sqrt{2}}{2}=\frac{a}{\sqrt{2}}$$

$$\vec{F}_{BA}=\frac{kq^2}{a^2}\hat{i},\qquad \vec{F}_{BC}=\frac{-kq^2}{a^2}\hat{j}$$

$$\vec{F}_{BD}=\frac{kq^2}{(a\sqrt{2})^2}\left(\cos 45^\circ\hat{i}-\sin 45^\circ\hat{j}\right)$$

$$\vec{F}_{BQ}=\frac{kQq}{(a/\sqrt{2})^2}\left(\cos 45^\circ\hat{i}-\sin 45^\circ\hat{j}\right)$$

Here $\hat{i}$ and $\hat{j}$ have usual meaning. Net force on the charge at B is

$$\vec{F}_{Net} = \left(\frac{kq^2}{a^2} + \frac{kq^2}{\left(a\sqrt{2}\right)^2}\cos 45^\circ + \frac{kQq}{\left(a/\sqrt{2}\right)^2}\cos 45^\circ\right)\hat{i}$$

$$-\left(\frac{kq^2}{a^2} + \frac{kq^2}{\left(a\sqrt{2}\right)^2}\sin 45^\circ + \frac{kQq}{\left(a/\sqrt{2}\right)^2}\sin 45^\circ\right)\hat{j}$$

For charge q to be in equilibrium at B, the net force on it must be zero. Taking x-component

$\therefore \quad F_{Bx} = F_{BA} + F_{BD}\cos 45^o + F_{BQ}\cos 45^o = 0$

$\Rightarrow \quad k\left[\frac{q^2}{a^2} + \frac{q^2}{\left(a\sqrt{2}\right)^2}\cdot\frac{1}{\sqrt{2}} + \frac{Qq}{\left(a/\sqrt{2}\right)^2}\cdot\frac{1}{\sqrt{2}}\right] = 0$

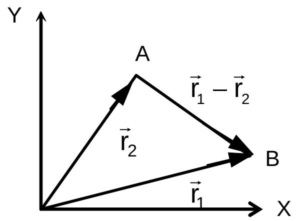
$\therefore \quad Q = -\frac{q}{4}(1+2\sqrt{2})$

Similarly, $F_{By} = 0, \text{if } Q = -\frac{q}{4}(1+2\sqrt{2})$.

ELECTRIC FIELD

Electric field due to a point charge is the space surrounding it, within which electric force can be experienced by another charge.

Electric field strength or electric field intensity $\left(\vec{E}\right)$ at a point is the electric force per unit charge experienced by a positive test charge at that point. Mathematically, $\vec{E} = \frac{\vec{F}}{q_0}$, where q_0 is positive test charge.



In vector form, electric field at B due to charge Q at A, $\vec{E} = k\frac{q}{\left|\vec{r}_1 - \vec{r}_2\right|^3}\left(\vec{r}_1 - \vec{r}_2\right)$

If electric field intensity is same at all points in the region, then the field is said to be uniform. Equispaced parallel lines represent uniform electric field. Arrow on the lines gives the direction of the electric field.

The electric field at a point due to several charges distributed in space is the vector sum of the fields due to individual charges at the point, $\vec{E} = \vec{E}_1 + \vec{E}_2 + ...\vec{E}_n$

The electric field due to continuous charge distribution at any point P, $\vec{E}_P = \frac{1}{4\pi\varepsilon_o}\int\frac{dq}{r^2}\,\hat{r}$

where dq is the charge on one element of the charge distribution and r is the distance from the element to the point under consideration and $\hat{r}$ is the unit vector directed from the position of elemental charge towards the point where electric field is to be found out.

Exercise 4 :

(i) ***Does an electric charge experience a force due to the field that the charge itself produces?***

(ii) ***Two point charges q and –q are placed at a distance d apart. What are the points at which resultant electric field is parallel to line joining the two charges?***

(iii) ***Three equal and similar charges q are placed on each corner of a square of side a. What is the electric field at the centre of the square?***

Illustration 6. Two point charges 5 μC and –5 μC are located at 50 cm apart in vacuum.

(a) What is the electric field at the mid point O of the line AB joining the two charges.

(b) If a negative test charge of magnitude 2×10^{-9} C is placed at the above point, what is the force experienced by the test charge?

Solution: (a) $\vec{E}_0 = 2\times\frac{kq}{R^2} = \frac{2\times9\times10^9\times5\times10^{-6}}{(1/4)^2}\hat{j}$

$$\vec{E}_0 = 16\times2\times9\times5\times10^3\,\frac{N}{C}\,\hat{j}$$

$$\vec{E}_0 = 144\times10^4\,\frac{N}{C}\hat{j}$$

(b) $\vec{F} = q\vec{E}_0 = 2\times10^{-9}\times144\times10^4(-\hat{i})N$

$\Rightarrow F = 288 \times 10^{-5}\,(-\hat{i})$ N

Physical significance of electric field:

The concept of electric field is not merely a mathematical construction but it is a real physical phenomenon. It is the force per unit charge that a positive test charge will feel when kept at a particular point in the field. The important points are

(i) It is assumed that no disturbance is produced on the charge distribution causing the field when the unit positive charge is brought to its position. This will happen when the test charge is very small.
(ii) The field at a point is independent of the test charge.
(iii) The field may vary from point to point.
(iv) If a charge q moves, the field produced by it at a point changes after a time lag equal to the distance divided by c (speed of light)

Illustration 7. *An electron falls through a distance of 4 m in a uniform field of strength 25×10^5 N/C. When the direction of electric field is reversed, a proton falls through the same distance. Calculate (a) acceleration of electron and proton (b) time of fall in each case.*
Given mass of electron (m_e) = 9.1×10^{-3} kg, mass of proton (m_p) = 1.67×10^{-27} kg.

Solution: (a) $|\vec{a}_e|$= acceleration of electron = $\frac{qE}{m}$

$$\Rightarrow \quad |\vec{a}_e| = \frac{1.6\times10^{-19}\times25\times10^5}{9.1\times10^{-31}}$$

$$\Rightarrow \quad |\vec{a}_e| = 4.4\times10^{17}\,m/s^2$$

$$|\vec{a}_p| = \frac{1.67\times10^{-19}\times25\times10^5}{1.67\times10^{-27}}$$

$$|\vec{a}_p| = 25\times10^{13}\,m/s^2$$

(b) $S = ut + \frac{1}{2}at^2$

$$\Rightarrow S = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2s}{a}}$$

$$t_e = \sqrt{\frac{2\times4}{4.4\times10^{17}}} = 0.426\times10^{-8}\text{ sec}$$

$$t_p = \sqrt{\frac{8}{25\times10^{13}}} = 0.1788\times10^{-6}\text{ sec}$$

Illustration 8. *3 point charges of Q coulomb each are kept on the vertices of equivalent tangent of side a. Find the electric field intensity at the centre of the triangle.*

Solution: Resultant field at P = $\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0$

As the 3 vectors $\vec{E}_1, \vec{E}_2$ and $\vec{E}_3$ are at 120° angle from each other and also their magnitudes are equal.

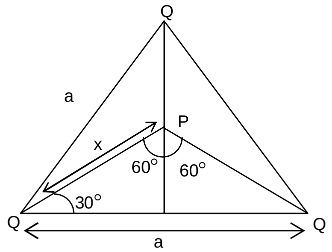
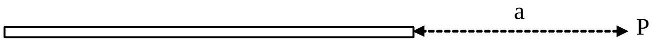
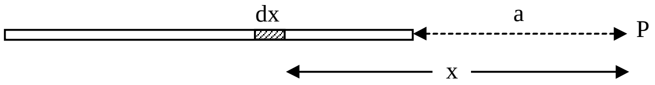


Illustration 9. *What is the electric field at any point on the axis of a uniformly charged rod of length `L' and linear charge density`λ'? The point is separated from the nearer end by a.*



Solution : Consider an element, dx at a distance, x from the point, P, where we seek to find the electric field. The elemental charge, dq =λdx



Then, $dE = k.\frac{\lambda dx}{x^2}$

or $E = k\lambda \int_a^{a+L} \frac{1}{x^2} dx = k\lambda \left[-\frac{1}{x}\right]_a^{a+L}$

$= k\lambda \left[\frac{-1}{a+L} + \frac{1}{a}\right]$

Thus, $E = \frac{\lambda}{4\pi \in_o}\left[\frac{1}{a} - \frac{1}{L+a}\right]$ $\left[k = \frac{1}{4\pi\varepsilon_0}\right]$

$\Rightarrow E = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{L}{a(L+a)}\right)$

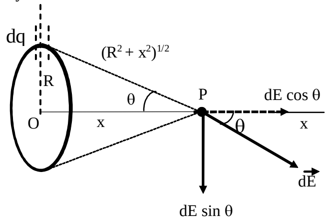
Illustration 10. *A ring shaped conductor with radius R carries a total charge q uniformly distributed around it. Find the electric field at a point P that lies on the axis of the ring at a distance x from its centre.*

Solution: Consider a differential element of the ring of length ds. Charge on this element is

$dq = \left(\frac{q}{2\pi R}\right) ds$

This element sets up a differential electric field $d\vec{E}$ at point P.

The resultant field $\vec{E}$ at P is found by integrating the effects of all the elements that make up the ring. From symmetry, this resultant field must lie along the right axis. Thus, only the component of $d\vec{E}$ parallel to this axis contributes to the final result.



$\vec{E} = \int d\vec{E}$ $\Rightarrow \vec{E} = \int d\vec{E} \cos\theta$

$$dE = \frac{1}{4\pi\in_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{qds}{2\pi R}\right)\frac{1}{\left(R^2+x^2\right)} \;;\quad \cos\theta = \frac{x}{(R^2+x^2)^{1/2}}$$

To find the total x-component E_x of the field at P, we integrate this expression over all segment of the ring.

$$E_x = \int d\vec{E}\cos\theta = \frac{1}{4\pi\in_0}\frac{qx}{2\pi R.\ \left(R^2+x^2\right)^{3/2}}\int ds$$

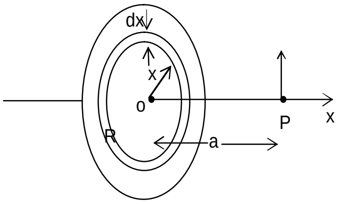
The integral is simply the circumference of the ring = $2\pi R$

$$E = \frac{1}{4\pi\in_0}\frac{qx}{\left(R^2+x^2\right)^{3/2}}$$

As q is positive charge, hence field is directed away from the centre of the ring along its axis.

Illustration 11. *A non conducting circular plate of radius R has a uniform surface charge density σ. Find the electric field intensity at a point P on the axis of the plate at a distance a from the centre of the plate.*

Solution:



$dA = \pi(x+dx)^2 - x^2 = 2\pi x\,dx$

$dQ_{ring} = \sigma dA = 2\pi\sigma x\,dx$

Let $d\vec{E}_p$ = field intensity at point P due to the small ring

$$\Rightarrow \int d\vec{E}_P = \int \frac{1}{4\pi\varepsilon_0}\frac{dQa}{(x^2+a^2)^{3/2}}$$

$$= \frac{\sigma a}{2\varepsilon_0}\int_0^R \frac{xdx}{(x^2+a^2)^{3/2}}$$

$$\vec{E}_p = \frac{\sigma a}{2\varepsilon_0}\left(\frac{1}{a} - \frac{1}{\sqrt{R^2+a^2}}\right)$$

$$\vec{E}_p = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{a}{\sqrt{R^2+a^2}}\right]$$

LINES OF FORCE

It has been found quite convenient to visualize the electric field pattern in terms of lines of force. The electric field pattern vector at a point is related to imaginary lines of force in two ways. The line of force in an electric field is a curve such that the tangent at any point on it gives the direction of the resultant electric field strength at that point.

(i) Tangent to the lines of force at a point gives the direction of $\vec{E}$.

(ii) These lines of force are so drawn that their number per unit cross-sectional area in a region is proportional to the intensity of electric field.

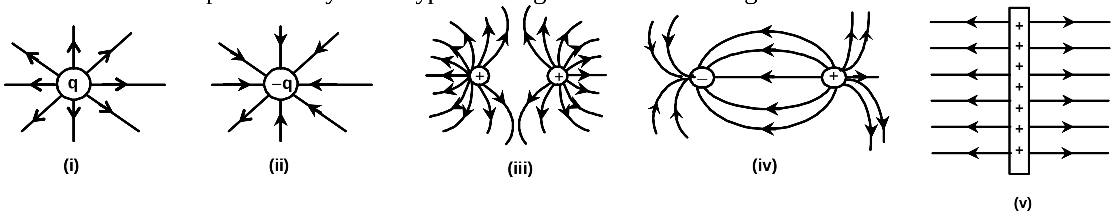
(iii) Electric lines of force can never be closed loops.

(iv) Lines of force are imaginary.

(v) They emerge from a positive charge and terminate on a negative charge.

(vi) Lines of force do not intersect.

Electric field lines produced by some typical charge distributions are given below.



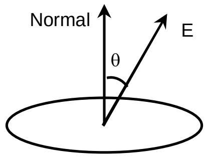
The concept of field lines was invented by Faraday. The field lines are constructed purely to visualise the electric field. They have no physical existence.

Note: When a conductor has a net charge that is at rest, the charge resides entirely on the conductor's surface and the electric field is zero everywhere within the material of the conductor.

Exercise 5: ***Electric lines of force never crosses each other . Why?***

GAUSS'S LAW

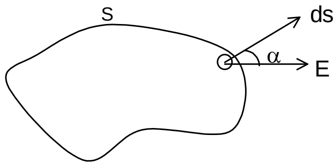
The net "flow" of electric field through a closed surface depends on the net amount of electric charge contained within the surface. This " flow" is described in terms of the electric flux through a surface, which is the product of the surface area and the component of electric field perpendicular to the surface.



Flux is a scalar quantity and is added on per scalar addition rules. For non-uniform field and / or surfaces which are not plane,

$$\phi = \oint \vec{E}.d\vec{s}$$

Direction of an area element is taken along its normal. Hence area can be treated as a vector quantity.



Gauss's law states that the total electric flux through a closed surface is proportional to the total electric charge enclosed within the surface. This law is useful in calculating fields caused by charge distributions that have various symmetry properties.

Mathematically $\oint \overrightarrow{E}.\overrightarrow{ds} = \dfrac{q}{\varepsilon_0}$.

$\oint$ $\rightarrow$ means integral done over a closed surface.

Gauss's law can be used to evaluate Electric field if the charge distribution is so symmetric that by proper choice of a Gaussian surface we can easily evaluate the above integral.

Some important aspects of application of Gauss's theorem

1. The surface which is chosen for application of Gausses law is called a Gaussian surface. Any Gaussian surface can be chosen but it should not pass through a discrete charge. However, it can pass through a continuous charge distribution.

2. Gauss's theorem is mostly used for symmetric charge distribution.

3. The term q on the right side of equation (q / ε_0) includes sum of all charges enclosed by the surface. Outside charges are not included.

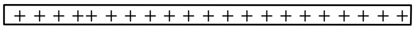
4. The electric field appearing on left (E) is the electric field due to all charges inside or outside the closed surface.

Gauss's theorem can be applied to continuous charge distribution. It may be linear charge density ($\lambda = \dfrac{\Delta Q}{\Delta \ell}$), surface charge density ($\sigma = \dfrac{\Delta Q}{\Delta A}$) and volume charge density ($\rho = \dfrac{\Delta Q}{\Delta v}$)

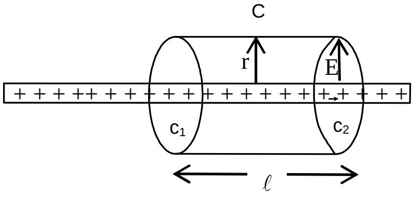
Integration can be performed all over the surface after putting the value of q accordingly.

Exercise 6 : ***Explain whether Gauss's law is useful in calculating the electric field due to three equal charges located at the corners of an equilateral triangle?***

Illustration12. *Figure shows a section of an infinite rod of charge having linear charge density λ which is constant for all points on the line. Find electric field E at a distance r from the line.*



Solution: From symmetry, $\vec{E}$ due to a uniform linear charge can only be radially directed. As a Gaussian surface, we can choose a circular cylinder of radius r and length l, closed at each end by plane caps normal to the axis.



$$\varepsilon_o \oint \vec{E}.\ d\vec{s} = q_{in}\ ; \qquad \varepsilon_0 \left[\int_c \vec{E}.d\vec{s} + \int_{c_1} \vec{E}.d\vec{s} + \int_{c_2} \vec{E}.d\vec{s}\right] = q_{in}$$

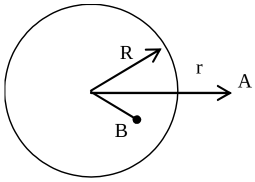
$$\varepsilon_o\, E\,(2\pi r l) + \varepsilon_0 E.ds.\cos 90^o + \varepsilon_0 E.ds.\cos 90^o = \lambda\ell$$

$$E = \frac{\lambda\ell}{\varepsilon_o 2\pi r\ell} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The direction of $\vec{E}$ is radially outward for a line of positive charge.

Illustration 13. *Figure shows a spherical symmetric distribution of charge of radius R. Find electric field $\vec{E}$ for points A and B which are lying outside and inside the charge distribution respectively.*

Solution: The spherically symmetric distribution of charge means that the charge density at any point depends only on the distance of the point from the centre and not on the direction. Secondly, the object can not be a conductor, or else the excess charge will reside on its surface.

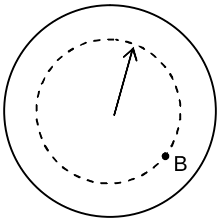


Now, apply Gauss's law to a spherical Gaussian surface of radius r

(r > R for point A), $\quad \varepsilon_o \oint \vec{E}.d\vec{s} = q_{en}$

$\Rightarrow \quad \varepsilon_o E(4\pi r^2) = q$

$E = \dfrac{1}{4\pi \in_0}\dfrac{q}{r^2}$ where q is the total charge



For point B (r<R), $\varepsilon_o \oint \overrightarrow{E}.d\vec{s} = \varepsilon_o E(4\pi r^2) = q'$

$$q' = \frac{q\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = q\left(\frac{r}{R}\right)^3 ;\ E = \frac{1}{4\pi\varepsilon_0}\frac{q\left(\frac{r}{R}\right)^3}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$$

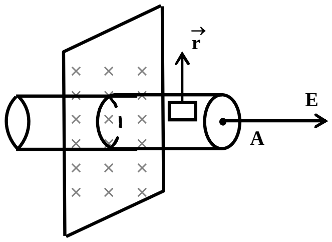


Illustration 14. *Determine the electric field due to a plane infinite sheet of charge.*

Solution: By symmetry, E will be perpendicular to the surface.
Gaussian surface is taken on a cylinder parallel to E.
$\oint E\,.\,ds = 2\,A.\,E$
Charged enclosed = σ . A, where σ is charge density.
As per Gauss's theorem
$2A.E = \dfrac{\sigma A}{\varepsilon_0}$ $\quad \therefore\ E = \dfrac{\sigma}{2\varepsilon_0}$
The direction of the field is perpendicular to eth surface.

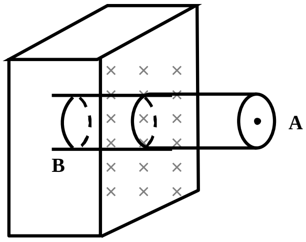


Illustration 15. *Determine the electric field near a large charged conducting surface ?*

Solution: Charged area being an equipotential surface, electric field will be perpendicular to the surface. Taking a cylindrical Gaussian surface as shown in the figure,
Flux through area A = $E\Delta S$
Flux through cylindrical surface is zero as the area is perpendicular to the field.
Flux through area B is zero as there is no field inside the conductor.

$$\therefore \qquad E\Delta S = \frac{q}{\varepsilon_0} = \frac{\sigma \Delta S}{\varepsilon_0}$$

$$\Rightarrow \qquad E = \frac{\sigma}{\varepsilon_0}$$

ELECTRIC POTENTIAL

Potential at a point in an electric field is the amount of work done by an external agent against electric forces in moving a unit positive charge with constant speed from infinity to that point. Electric potential is a scalar quantity.

Potential due to a point charge

Taking the position of the point charge as origin, suppose point P, where the potential is to be ascertained is at a distance r.
Work done by the external agent = – work done by electric force.

Hence the required potential $V = -\int \vec{E}.\ d\vec{\ell} = -\dfrac{q}{4\pi \in_0} \displaystyle\int_{\infty}^{r} \dfrac{dr}{r^2} = \dfrac{1}{4\pi \in_0}\ \dfrac{q}{r}$

The potential at a point due to a group of n point charges q_1 q_2, q_3 … q_n

$V = V_1 + V_2 + \ldots + V_n$ (Scalar Sum)

$= \displaystyle\sum_{i=1}^{n} \dfrac{1}{4\pi \in_0} \dfrac{q_i}{r_i}$

The electric potential due to a continuous charge distribution, $V = \displaystyle\int \dfrac{1}{4\pi \in_0} \dfrac{dq}{r}$

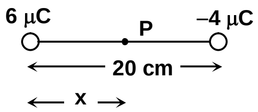


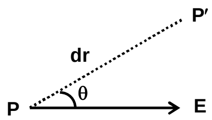
Illustration 16. *Two charges 6 μC and – 4 μC are placed on a line at a distance of 20 cm. Find out the position of point P where the potential is zero.*

Solution: Potential at point P = $\frac{6\times10^{-6}}{4\pi\varepsilon_0 x}+\frac{-4\times10^{-6}}{4\pi\varepsilon_0(20-x)}=0$

$\therefore \quad 6(20-x) = 4(x)$

$\Rightarrow \quad x = \frac{120}{10} = 12$ cm

Relation between field (E) and potential (V)



The negative rate of change of potential with distance along a given direction is equal to the component of the field along that direction.

i.e. $E_r = -\frac{dV}{dr}$

Illustration 17. *The electric field in a region is given by $\vec{E} = (A/x^3)\ \hat{i}$. What is the potential in the region?*

Solution: The electric potential in the region

$$V\ (x, y, z) = -\int_{\infty}^{(x,y,z)} \vec{E}.\ d\vec{r} = -\int_{\infty}^{(x,y,z)} \frac{A}{x^3}\ dx = \frac{A}{2x^2}$$

Illustration 18. *Kinetic energy of a charged particle decreases by 10 J as it moves from a point at potential 100 V to a point at potential 200 V. Find the charge on the particle.*

Solution: q (200 – 100) = 10

$\Rightarrow q = \frac{10}{100} = 0.1$ C

Illustration 19. *Potential in the x-y plane is given as $V = 5(x^2 + xy)$ volts. Find the electric field at the point (1, –2).*

Solution: $E_x = -\frac{\partial V}{\partial x} = -(10x + 5y) = -10 + 10 = 0$

$E_y = -\frac{\partial V}{\partial y} = -5x = -5$

$\therefore\ \vec{E} = -5\hat{j}\ V/m$.

Illustration 20. *Electric field in a region is given by $\vec{E} = (2\hat{i} + 3\hat{j} - 4\hat{k})$ V/m. Find the potential difference between points (0, 0, 0) and (1,2,3).*

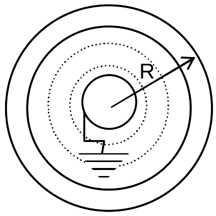
Solution: p.d. across the points = $-\vec{E}.\Delta\vec{r}$

$V_2 - V_1 = -(2\hat{i} + 3\hat{j} - 4\hat{k}).(\hat{i} + 2\hat{j} + 3\hat{k})$

= –2 – 6 + 12

= 4 volts.

Illustration 21. *N concentric thin conducting shells having radius R, R/2, $R/2^2$ …… is placed as shown in figure. N^{th} shell is earthed. If Q charge is given to the outer shell, find the charge on inner most shell.*

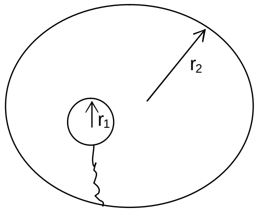


Solution:

$$\frac{kQ'}{R/2^{n-1}} + \frac{kQ}{R} = 0$$

$$Q' = -\frac{Q}{2^{n-1}}$$

Illustration 22. *A metal sphere having a radius r_1 charged to a potential ϕ_1 is enveloped by a thin walled conducting spherical shell of radius r_2 as shown in figure. Determine the potential ϕ_2 acquired by the sphere after it has been connected for a short time to the shell by a conductor.*



Solution: Initially, $q_1 = 4\pi\varepsilon_0 \phi_1 r_1$

After conducting, $\phi_2 = \dfrac{q_1}{4\pi\varepsilon_0 r_2} = \phi_1 \dfrac{r_1}{r_2}$

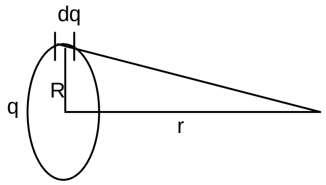
Exercise 7:

(i) ***If you know $\vec{E}$ at a given point, can you calculate V at that point ? If not, what further information do you need.***

(ii) ***If $\vec{E}$ equals zero at a given point, must V equal zero for that point?***

Potential due a uniformly charged ring

The element charge dq,



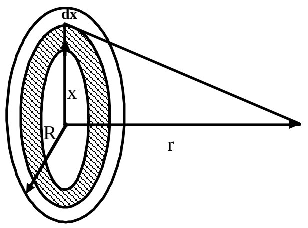
$$dv = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{r^2+R^2}}$$

$$v = \int dv = \frac{1}{4\pi\varepsilon_0\sqrt{r^2+R^2}}\int dq = \frac{q}{4\pi\varepsilon_0\sqrt{r^2+R^2}}$$

Potential due to a uniformly charged disc

If the charge on the disc = q, then $\sigma = \dfrac{q}{\pi R^2}$

The elemental ring's area = $2\pi x dx$

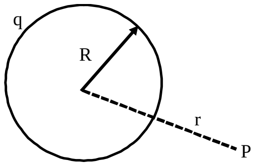


$$dV = \frac{1}{4\pi\varepsilon_o}\frac{\sigma(2\pi x dx)}{\sqrt{r^2+x^2}}$$

$$V = \int dV = \frac{\sigma}{2\in_o}\int_0^R \frac{x dx}{\sqrt{r^2+x^2}} = \frac{\sigma}{2\in_o}[\sqrt{R^2+r^2} - r]$$

Potential due to a uniformly charged spherical shell

If the charge on the shell = q



(i) For r > R, $E = kq/r^2$

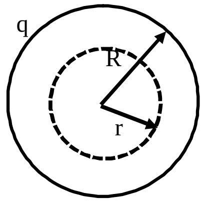
$$E = \frac{kq}{r^2}$$

Since $E = -\frac{dv}{dr}$; $v = -\int E dr$.

$\Rightarrow \quad V = -\int_{\infty}^{r} \frac{kq}{r^2} dr = \frac{kq}{r}$

(ii) For $r = R$, $V = \frac{kq}{R}$

(iii) For $r < R$, $E = 0$

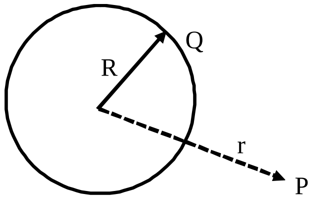


$\Rightarrow \quad V = -\left[\int_{\infty}^{R} E\, dr + \int_{R}^{r} E\, dr\right] = -\int_{\infty}^{R} \frac{kq}{r^2}\, dr - \int_{R}^{r} 0\, dr = \frac{kq}{R}$

Potential due to uniformly charged spherical volume

If the total charge = Q

(i) For $r > R$,



Volume charge density $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$

$E(r > R) = \frac{kQ}{r^2}$

$V = \int_{\infty}^{r} \frac{kq}{r^2}\, dr = \frac{kQ}{r}$

(ii) For $r = R$, $V = \frac{kQ}{R}$

$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$

(iii) $r < R$, $E(r < R) = \frac{\rho . r}{3\varepsilon_o} = \frac{Q.r}{\frac{4}{3}\pi R^3 . 3\varepsilon_o}$

$\Rightarrow \quad E(r < R) = \frac{Q.r}{4\pi\varepsilon_o R^3} = \frac{kQ.r}{R^3}$

$V = -\left[\int_{\infty}^{R} E\, dr + \int_{R}^{r} E dr\right] \equiv \int_{\infty}^{R} \frac{kQ}{r^2}\, dr - \int_{R}^{r} \frac{kQr}{R^3}\, dr = \frac{kQ}{2R}\left[3 - \frac{r^2}{R^2}\right]$.

For conducting solid sphere:

(i) $(r < R)$, $E = 0$, $V = \frac{kQ}{R}$

(ii) $(r = R)$, $E = \frac{kQ}{R^2}$, $V = \frac{kQ}{R}$

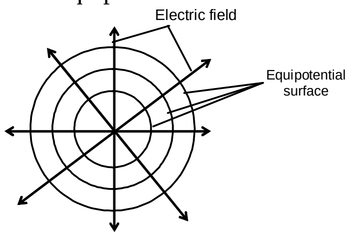
(iii) $(r > R)$, $E = \frac{kQ}{r^2}$, $V = \frac{kQ}{r}$

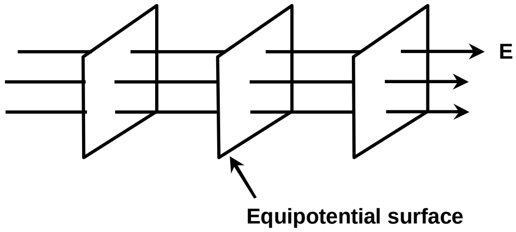
Exercise 8 :	***A closed metallic box is charged upto potential V_0, what will be the potential at the centre of the box.***

Equipotential Surfaces

The locus of points of equal potential is called an equipotential surface. The electric field is perpendicular to the equipotential at each point of the surface.

Examples of Equipotential surfaces





Exercise 9 : ***A solid metallic sphere is placed in a uniform electric field. Which of these A,B,C and D shows the correct path?***

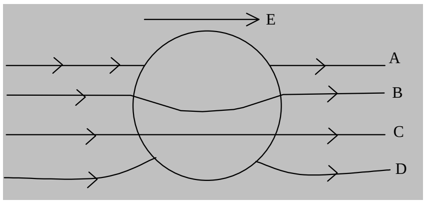


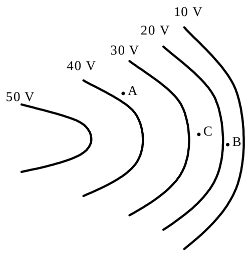
Illustration 23. *What is the electric potential on the axis of a uniformly charged ring of radius R containing charge q, at a point x away from the centre of the loop?*

Solution: As electric potential is a scalar quantity. Hence electric potential at P,

$$V = \int \frac{1}{4\pi \in_0} \frac{dq}{r} = \frac{1}{4\pi \in_0 r} \int dq$$

Hence $V = \dfrac{1}{4\pi \in_0} \dfrac{q}{(R^2 + x^2)^{1/2}}$

Illustration 24. *Given figure shows the lines of constant potential in a region in which an electric field is present. The value of potentials are written. At which of the points A, B and C is the magnitude of the electric field is the greatest?*



Solution: $\vec{E} = -\dfrac{dV}{dr}\hat{n}$

The potential difference between any two connective line $dV = V_1 - V_2 = 10$ V = constant and hence E will be maximum where the distance dr between the lines minimum. That is, at B where the lines are closest.

Illustration 25. *Some equipotential surfaces are shown in figure (1) and (2) What can you say about the magnitude and direction of the electric field.*

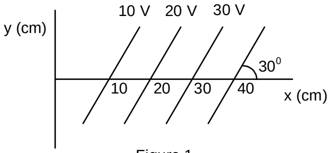


Figure 1

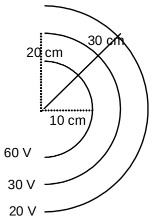


Figure 2

Solution: Electric field is perpendicular to the equipotential surface and in the direction of decreasing potential.

In Figure 1:

The electric field will be at an angle making an angle 120° with x-axis.

Magnitude of electric field along x-axis E cos 120° = $-\dfrac{(20-10)}{(20-10)\times10^{-2}}$

$$-E.\frac{1}{2} = -\frac{10}{0.10} \quad \Rightarrow \quad E = 200 \text{ V/m}$$

Figure 2:

Direction of electric field will be radially outward, similar to a point charge kept at the centre, i.e. $V = \dfrac{kq}{r}$ When $V = 60 \text{ V} = \dfrac{kq}{(0.1)}$

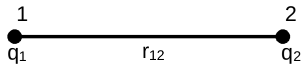
$\Rightarrow \quad kq = 6$

Hence potential at any distance from the center, $V(r) = \dfrac{6}{r}$

Hence $E = -\dfrac{dV}{dr} = \left(\dfrac{6}{r^2}\right)$ V/m

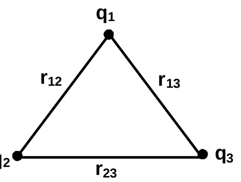
Electrostatic potential energy

The electric potential energy of a system of point charges is the amount of work done in bringing the charges from infinity in order to form the system. Two point charges q_1 and q_2 are separated at a distance r_{12}, Electric potential energy of the system q_1 and q_2



$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}$$

For three particle system q_1, q_2 and q_3



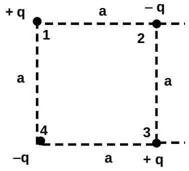
$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

We can define electric potential at any point P in an electric field as,

$V_P = U_P/q$; where U_p, is the change in electric potential energy in bringing the test charge q from infinity to point P.

Illustration 26. *Determine the interaction energy of the point charges of the following setup.*

Solution : As you know the interaction energy of an assembly of charges is

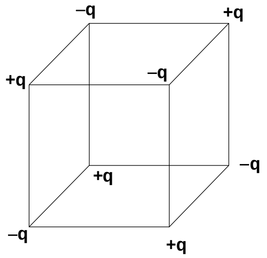


given by $\dfrac{1}{4\pi\varepsilon_0}\displaystyle\sum_{i\neq j}^{n}\frac{q_i q_j}{r_{ij}}$

$\therefore \quad U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

$$= -\frac{kq^2}{a}+\frac{kq^2}{\sqrt{2}\,a}-\frac{kq^2}{a}-\frac{kq^2}{a}+\frac{kq^2}{\sqrt{2}\,a}-\frac{kq^2}{a} = -\frac{\sqrt{2}kq^2}{a}[2\sqrt{2}-1] = \frac{q^2(4-\sqrt{2})}{4\pi\varepsilon_0 a}$$

Illustration 27. *Charges +q and –q are located at the corners of a cube of side a as shown in the figure. Find the work done to separate the charges to infinite distance.*



Solution: $W_{external} = \Delta PE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a}\left[-\dfrac{3}{1}+\dfrac{3}{\sqrt{2}}-\dfrac{1}{\sqrt{3}}\right]\times\dfrac{8}{2}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}.\frac{4}{\sqrt{6}}[3\sqrt{3}-3\sqrt{6}-\sqrt{2}].$$

Potential energy in an external field

Potential energy of a charge q placed in an external field E is $U = qV(r)$, where V(r) is the potential due to the field at point r.

Here potential of charge q at P is considered as work done in bringing a unit positive charge from infinity to the point P.

For two charges q_1 and q_2 at point r_1 and r_2 in field E, the potential energy will be U = Work done in bringing the charge q_1 from infinity to point r_1 in the field E + Work done in bringing charge q_2 from infinity to point r_2 in field E + work done in bringing the charge q_2 from infinitely to r_2 against the field of charge q_1

$$U = q_1(r_1) + q_2V(r_2) + \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}}.$$

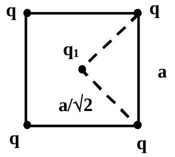


Illustration 28. *Four charges q are fixed at corners of a square of side a. Another charge q_1 is brought to the centre of the square. Find the potential energy of charge q_1*

Solution: Potential of four charges of kept at corners of the square at the centre of the square,

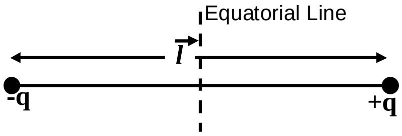
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a/\sqrt{2}}\times 4 = \frac{\sqrt{2}q}{a\pi\varepsilon_0}$$

Work done in bringing the charge q, at this point (centre of square) against the field is

$$= qV = \frac{\sqrt{2}q_1q}{\pi a\varepsilon_0}$$

ELECTRIC DIPOLE

A set of two equal and opposite charges separated by a finite distance form an electric dipole. It is characterised by dipole moment vector $\vec{p}$.



(1) Charges (+q) and (–q) are called the poles of the dipole.

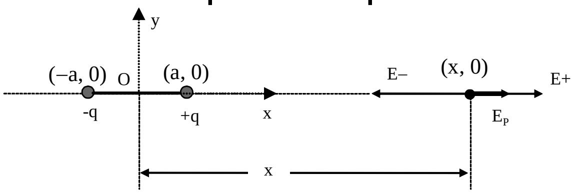
(2) $\vec{l}$ = the displacement vector from -ve charge to +ve charge.

(3) $\vec{p}$ = the dipole moment $= q\vec{l}$.

(4) The straight line joining the two poles (–q to q) is called axial line or the axis of the dipole.

(5) Perpendicular bisector of l is called equatorial line.

Electric field due to a dipole at axial point



Let the charges (–q) and +q are kept at point (–a, 0) & (a, 0) respectively in xy plane. The electric field at point P(x, 0) will be then;

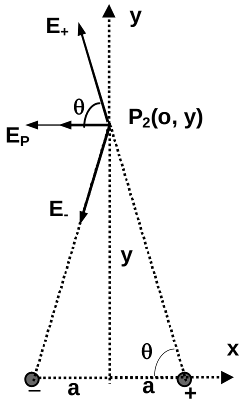
$$\vec{E}_{axial} = \vec{E}_{+q} + \vec{E}_{-q} = \frac{kq}{(x-a)^2}\hat{i} - \frac{kq}{(x+a)^2}\hat{i}$$, where $\hat{i}$ is the unit vector along axis.

$$= kq.\frac{\left((x+a)^2 - (x-a)^2\right)}{\left(x^2-a^2\right)^2}\hat{i} \quad = kq.\frac{(2x)(2a)}{\left(x^2-a^2\right)^2}\hat{i}$$

$$= \frac{2k\vec{p}x}{\left(x^2 - a^2\right)^2}\hat{i}, \text{as } x >> a, \vec{E} = \frac{2k\vec{p}}{x^3} \quad [p = 2aq]$$

$$\vec{E} = \frac{4aq}{4\pi\varepsilon_0 x^3}.\hat{i} = \frac{2\vec{p}}{4\pi\varepsilon_0 x^3}$$

Electric field on equatorial line



At P, $\vec{E}_+ = \frac{kq}{\left(y^2 + a^2\right)}(-\cos\theta\hat{i} + \sin\theta\hat{j})$

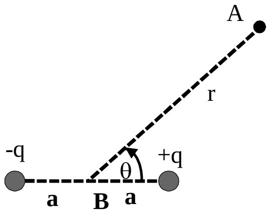
$\vec{E}_- = \frac{kq}{\left(y^2 + a^2\right)}(-\cos\theta\hat{i} - \sin\theta\hat{j})$; $\vec{E}_P = \vec{E}_+ + \vec{E}_- = \frac{-2kq}{\left(y^2 + a^2\right)}\cos\theta\hat{i}$

$\Rightarrow$ $\vec{E}_p = -2\left[\frac{kq}{y^2 + a^2}\right].\frac{a}{\left(a^2 + y^2\right)^{1/2}}\hat{i} = \frac{-k\left(2aq\right)}{\left(y^2 + a^2\right)^{3/2}}\hat{i} = \frac{-\vec{p}}{(y^2 + a^2)^{3/2}}$

For $a << y$, $E = \frac{kp}{y^3}$ or $\vec{E} = -\frac{k\vec{p}}{y^3}, = -\frac{\vec{p}}{4\pi\varepsilon_0 y^3}$

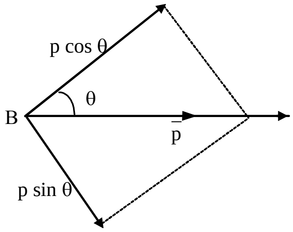
Resultant $\vec{E}$ is directed oppositely to $\vec{p}$.

Electric field due to a dipole at a point



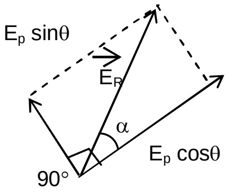
To get electric field at a general point due to a dipole, we use the earlier results.

We find electric field at A in terms of r and θ (r >> a).
The dipole moment can be vectorially resolved as:
P cos θ in the direction of A
P sin θ in the direction perpendicular to A.



Electric field at A due to ($\vec{p}$ cosθ) component $= \frac{2k\left(p\cos\theta\right)}{r^3}$

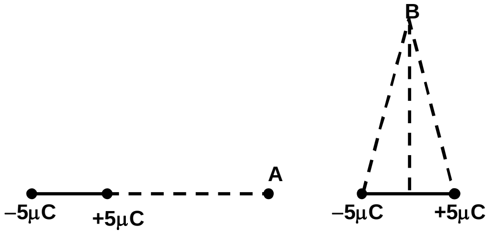
Electric field at A due to ($\vec{p}$ sinθ) component $= \frac{k\left(p\sin\theta\right)}{r^3}$



$\Rightarrow$ $E_R = \sqrt{E^2_{p\cos\theta} + E^2_{p\sin\theta}} = \frac{kp}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$

Resultant, $E_R = \frac{kp}{r^3}\sqrt{3\cos^2\theta + 1}$; $\tan\alpha = \frac{E_{p\sin\theta}}{E_{p\cos\theta}} = \frac{\tan\theta}{2}$

Illustration29. *A dipole is formed by two charges of 5μC and –5μC at a distance of 8 mm. Find electric field at*
(a) a point 25 cm away from dipole centre at its axis.
(b) a point 20 cm away on its a line perpendicular to the axis and passing through its centre



Solution: (a) $E = \frac{2p}{4\pi\varepsilon_0 r^3} = \frac{2\times5\times10^{-6}\times8\times10^{-3}\times9\times10^{9}}{(25)^3\times10^{-6}} = 4.6\times10^4$ N/C

(b) $E = -\frac{p}{4\pi\varepsilon_0 r^3} = -\frac{5\times10^{-6}\times8\times10^{-3}\times9\times10^{9}}{(20)^3\times10^{-6}} = -4.5\times10^4$ N/C

Dipole in an external electric field

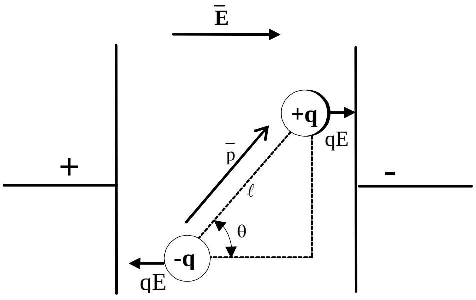
The net force on an electric dipole in a uniform external electric field is zero. However, the dipole in the presence of an external electric field experiences a torque and has tendency to align itself along the external electric field.

Torque on dipole = force x force arm

$= qE\,(l\sin\theta) = (ql)\,(E\sin\theta)$

$= pE\sin\theta$

or, $\vec{\tau} = \vec{p}\times\vec{E}$



The product of the charge q and separation ℓ is the magnitude of a quantity called the electric dipole moment denoted by p.

The direction of $\vec{p}$ is along the dipole axis from the negative charge to the positive charge as shown in the figure.

As $\vec{E}$ is a conservative field, work done by an external agent in changing the orientation of the dipole is stored as potential energy in the system of a dipole present in an external electric field.

$W = \int \tau\, d\theta$

$\Rightarrow \quad \int pE\sin\theta d\theta = -pE[\cos\theta]_{\theta_1}^{\theta_2}$

$\theta_2 = 0$, dipole is perpendicular to the field.

We assume, $\theta_1 = 90^o$ (as the datum for measuring potential energy can be chosen anywhere).

$\Rightarrow \quad U = -pE\cos\theta$ or $U = -\vec{p}.\vec{E}$

Illustration 30. *Two tiny spheres, each of mass M, and charges +q and -q respectively, are connected by a massless rod of length, L. They are placed in a uniform electric field at an angle θ with the $\vec{E}$ ($\theta \approx 0^o$). Calculate the minimum time in which the system aligns itself parallel to the $\vec{E}$.*

Solution : $\tau = pE\sin\theta$, (as $\theta\to0, \sin\theta\to\theta$)

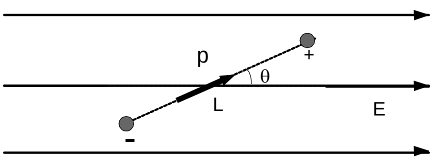
$\Rightarrow \tau = -(pE)\theta$ (If we assume angular displacement to be anti-clockwise, torque is clockwise)

$\Rightarrow \quad \alpha = -\left(\frac{pE}{I}\right)\theta = -\omega^2\theta$

As torque is proportional to 'θ' and oppositely directed, there will be an S.H.M.

Here, p = qL and moment of inertia,

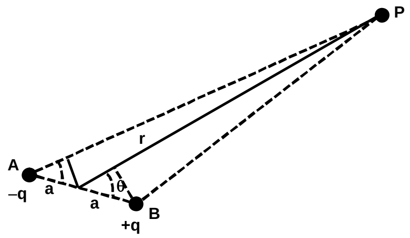
$I = M(L/2)^2 + M(L/2)^2 = ML^2/2$



$\therefore$ Time period, $T = 2\pi\sqrt{\dfrac{I}{pE}}$

The minimum time required to align itself along electric field is $\dfrac{T}{4}$

Potential due to an electric dipole



$$V_p = \frac{1}{4\pi \in_o}\left[\frac{q}{PB} - \frac{q}{PA}\right]$$

When r >> a, PB = r – a cosθ

PA = r + a cosθ

$$V_p = \frac{1}{4\pi\varepsilon_o} \times \frac{2qa\ \cos\theta}{\left(r^2 - a^2\cos^2\theta\right)}$$

As r>>a, $a^2\cos^2\theta$ can be neglected in comparison to r^2.

$$\therefore\ V_p = \frac{1}{4\pi\varepsilon_o}.\frac{2\ q\ a\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\varepsilon_o r^2}$$

Exercise 10: A electric dipole is placed in a non-uniform electric field. Is there a net force on it?

Physical significance of Dipole

To study electric properties of matter, dipoles play an important role. Matter is made of atoms and molecules which constitute positively charged protons, neutral neutrons and negatively charged electrons. If the centre of mass of positive charges does not coincide with the centre of mass of negative charges, the atom behaves like a dipole and has a dipole moment. In the absence of an electric field, the atoms or molecules are randomly oriented and hence the dipole moments have no effect. However, when an electric field is existing, the atoms realign themselves and orientation of dipoles becomes uniform and hence a net dipole moment is observed. The matter is called polarised in this case.

Even in case of non-polar molecules, the external field causes slight shift of centre of positive and negative charges and thus a dipole moment is produced in the molecules in the direction of the field. Hence, a net dipole moment is generated in the matter.
Effect of polarisation in matter is discussed in later chapters.

Electrostatics of conductor

We have studied earlier about conductors. They have fixed nuclei but floating electrons. The free outermost electrons can move freely which makes them good conductors of electric charges. There are some characteristic properties of conductors in electrostatics which are given below.

(a) Electrostatic field is zero inside a conductor.
When an electrostatic field exists, the free electrons in a conductor move in opposite direction and create an opposing field inside. As a result of redistribution of charges, there is no net electric field inside the conductor.

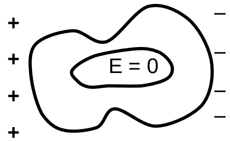
(b) Electrostatic potential is constant inside a conductor.
As there is no net field, the electrostatic potential inside a conductor is constant.

(c) Interior of a conductor has no excess charge. Excess charges reside on the surface of a conductor.
As there is no electrostatic field inside a conductor, it can be seen, with the help of Gaussian theorem that no net charge can exists inside a conductor.

(d) Field on the surface of a conductor is normal to the surface. By symmetry of charge distribution and by the fact that component of electric field parallel to the surface will cause free electrons to move, it is clear that field is normal to the surface.

(e) Electric field at the surface of a charged conductor is
$E = \sigma/\varepsilon_0$ normal to the surface. This has been explained earlier.

(f) **Electrostatic shielding.**
As the charges in a conductor reside on outer surface only and there is no field inside, any cavity inside a conductor is charge free and has no electric field even if the conductor is placed inside an external electric field. This fact is used in protecting sensitive instruments by providing electrostatic shielding by out side metal enclosure.



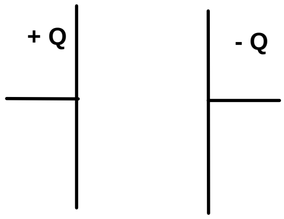
CAPACITANCE

If Q is the charge given to a conductor and V is the potential to which it is raised by this amount of charge, then it is found $Q \propto V$ or $Q = CV$, where C is a constant called capacitance of the conductor.

Capacitor

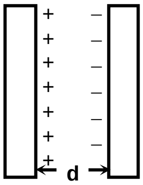
A pair of conductors separated by some insulating medium is called a capacitor. This medium is called dielectric of the capacitor. If Q units of the charge is given to one of the conductors, and thereby a potential difference V is set up between the conductors, the capacitance is then defined as

$$C = Q/V$$



Parallel Plate Capacitor

Parallel plate capacitor consists of two conductor plates kept at a small distance d. One plate is given a charge Q and the other one has a charge –Q. It is assumed that surface area A is much larger than separation d so that the effect of bending outward of electric field lines at the edges and the non-uniformity of σ (charge density) at the edges can be ignored.



Now taking field due to the surface charges, outside, on both sides of capacitor,

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

Inside, $E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$ $\quad \therefore \quad \frac{v}{d} = \frac{q}{A\varepsilon_0}$

or, $C = \frac{q}{v} = \frac{A\varepsilon_0}{d}$

This result is valid for vacuum between the capacitor plates. For other medium, then capacitance will be $C = \frac{kA\varepsilon_0}{d}$, where k is the dielectric constant of the medium,

$$\varepsilon_0 = \text{permittivity of free space} = 8.85 \times 10^{-12}\ C^2/N\text{-}m^2$$

Illustration 31. *A parallel plate capacitor is made of two square plats of 50 cm × 50 cm size and separation between the plates of 0.5 mm. Calculate its capacitance.*

Solution: $C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 0.25}{0.5 \times 10^{-3}} = 4.425 \times 10^{-9}$ F

$C = 4.425$ nF

That is the capacitance depends only on geometrical factors namely the plate area and plate separation.

Exercise 11: ***Why capacitance is generally measured in μ F and pF ?***

Illustration 32. *Find capacitance of a conducting sphere of radius R.*

Solution : Let charge Q is given to sphere. The field outside the sphere at distance r is $E = \frac{kQ}{r^2}$

$$\therefore -\frac{dV}{dr} = E$$

$$\therefore \int_0^V dV = -\int_\infty^R E dr$$

$$\Rightarrow V = kQ\left[-\frac{1}{r}\right]_\infty^R$$

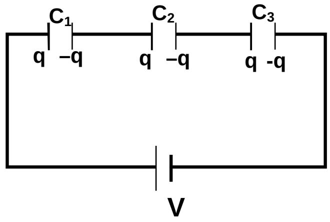
$$V = \frac{kQ}{R}$$

Since, $C = \frac{Q}{V} = \frac{R}{k} = \frac{R}{1/4\pi\varepsilon_0} = 4\pi \in_0 R$.

Combination of Capacitors

Series combination

In series combination, each capacitor has equal charge for any value of capacitance, provided that capacitors are initially uncharged.
Charge q and –q will appear on first plate of the first capacitor and the last plate of the last capacitor, provided by the battery.



Then right plate of first capacitor will have charge –q and hence left plate of 2nd capacitor will have charge q and so on.

As shown in the figure, charge q & –q will appear on all capacitors connected in series.
The total potential difference is the sum of potential differences on all capacitors

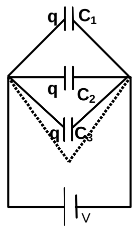
$$V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \quad \frac{1}{C_{eff}} = \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

This is true for any number of capacitors in series.

Parallel combination

In parallel combination the potential differences of the capacitors connected in parallel are equal for any value of capacitance. Charges q_1, q_2 and q_3 will appear on capacitors C_1, C_2 and C_3.



The total charge supplied by the battery,

$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V$, where V is the potential difference across the capacitor and is same for all capacitors.

$C_{eff} = Q/V = C_1 + C_2 + C_3$

This can be generalised to parallel combination of any number of capacitors.

Sometimes it may not be easy to find the equivalent capacitance of a combination using the equations for series parallel combinations. For any combination one can proceed as follows:

Step - 1 : Connect an imaginary battery between the points across which the equivalent capacitance is to be calculated. Send a positive charge +Q from the positive terminal of the battery and –Q from the negative terminal of the battery.

Step - 2 : Write the charges appearing on each plates using charge conservation principle say, Q_1, Q_2 ..

Step - 3 : Assume the potential of the negative terminal of the battery be zero and that of positive terminal to be V, and write the potential of each of the plates say V_1, V_2 . . .

Step - 4 : Write the capacitor equation Q = CV for each capacitor. Eliminate Q_1, Q_2 . . . V_1, V_2 . . . .etc to obtain the equivalent capacitance $C = \dfrac{Q}{V}$.

Illustration 33. *Four parallel plate capacitances of 4μF, 5μF, 6 μF and 2μF are connected (a) in series (b) in parallel*
Find equivalent capacitance.

Solution: (a) In series combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

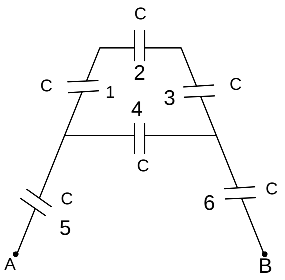
$$\frac{1}{C} = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{2} = \frac{67}{60}$$

$$C_{eff} = \frac{60}{67}\,\mu F$$

(b) In parallel combination,

$$C_{eff} = C_1 + C_2 + C_3 + C_4 = (4 + 5 + 6 + 2)\,\mu C = 17\,\mu C$$

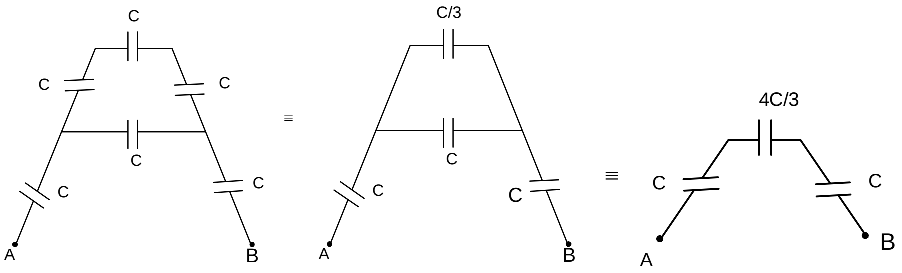
Illustration 34. *In the figure shown find the equivalent capacitance between A and B.*



Solution: Capacitors (i), (ii) and (iii) are in series

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

$$\Rightarrow C_4 = \frac{C}{3}$$



⇒

4 and 7 are in parallel

$$\Rightarrow C_4 = \frac{C}{3} + C = \frac{4C}{3}$$

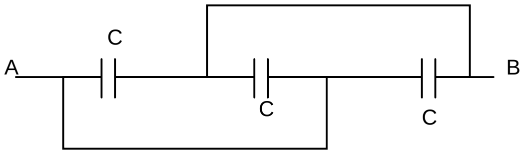
5, 6 and 8 are in series

$$\Rightarrow \frac{1}{C_4} = \frac{1}{C} + \frac{1}{C} + \frac{1}{\left(\frac{4C}{3}\right)} = \frac{2}{C} + \frac{3}{4C}$$

$$\Rightarrow \frac{1}{C_4} = \frac{11}{4C}$$

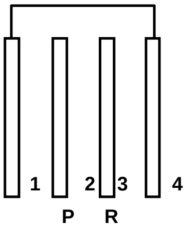
$$\Rightarrow C_4 = \frac{4C}{11}$$

Illustration 35. *3 capacitors are arranged as shown in figure. Find the equivalent capacitance between A and B.*

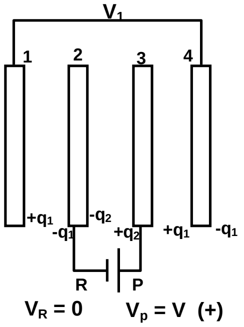


Solution: All the 3 capacitors are in parallel.
$\Rightarrow C_4 = C_1 + C_2 + C_3 = 3C$

Illustration 36. *Four identical metal plates are located in air parallel to each other and distances d from one another. The area of each plate is equal to A. The arrangement is shown in the figure. Find the capacitance of the system between points P and R.*



Solution: If we get the charge on each face of all the plates, then we can easily get the equivalent capacitance. To get charge on each face we will use Gauss's theorem and principle of conservation of charge. Let the point P and R have the potential V and 0 respectively and the plates 1 and 4 are at the potential V_1. Charge distribution is shown in the figure.



$\therefore$ $q_1 = C(V - V_1)$ from (4, 3) . . . (1)

$q_2 = CV$ from (3, 2) . . . (2)

$q_1 = CV_1$ from (2, 1) . . . (3)

From equation (1) and (2), $q_1 = \frac{CV}{2}$ . . . (4)

Charge supplied by the battery is

$Q = q_1 + q_2 = \frac{3}{2}V$

Using (2) and (4)

$\Rightarrow$ $CV = \frac{2}{3}Q$ . . . (5)

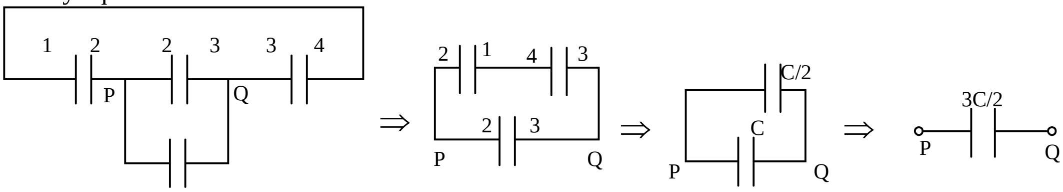
If equivalent capacitance be C′ then

$C'V = Q$ . . . (6)

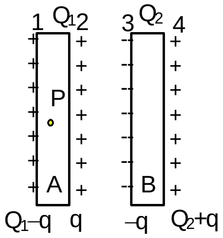
Dividing (5) by (6), $\frac{C}{C'} = \frac{2}{3}$

$$\Rightarrow \quad C' = \frac{3}{2}C = \frac{3}{2}\frac{\varepsilon_0 A}{d}$$

Aliter:
By equivalent circuit method



Plates of parallel plate capacitor given different charges



Two identical plates of parallel plate capacitor are given the charges Q_1 and Q_2. Let the charge appearing on the inner surface of Q_1 be q, then the charges appearing on other surfaces are as shown in the figure. If we take a point P inside the plate 1, then electric field at P should be zero. Suppose surface area of the each surface is A.

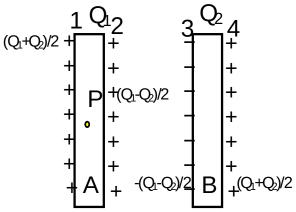
Using the equation $E = \dfrac{\sigma}{2\epsilon_0}$

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4,$$

$\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$ and $\vec{E}_4$ are the electric field due to surfaces 1, 2, 3 and 4.

$$\vec{E}_p = \frac{Q_1 - q}{2A\epsilon_0}\hat{i} - \frac{q}{2A\epsilon_0}\hat{i} + \frac{q}{2A\epsilon_0}\hat{i} - \frac{Q_2 + q}{2A\epsilon_0}\hat{i}$$

$$= \left(\frac{Q_1 - q}{2A\epsilon_0} - \frac{Q_2 + q}{2A\epsilon_0}\right)\hat{i}$$



But $|\vec{E}| = 0$, which gives, $q = \dfrac{Q_1 - Q_2}{2}$

Hence, charge distribution on each surface is shown in figure.

Note: The charge on inner surface of plate A is half of the difference of the charge on plate A and B (Q_1 and Q_2 respectively), i.e. $\dfrac{Q_1 - Q_2}{2}$, and on inner surface of B is opposite of the charge appearing on inner surface of A i.e. $-\left(\dfrac{Q_1 - Q_2}{2}\right)$; while the charge on outer surfaces of A and B is half of the sum of charge on A and B, i.e. $\dfrac{Q_1 + Q_2}{2}$

Exercise 12: ***If one of the plates of a capacitor is halved, will there be any change in the capacitance of the capacitor?***

DIELECTRICS

When a dielectric is introduced between conductors of a capacitor, its capacitance increases. A dielectric is characterised by a constant 'K' called dielectric constant.

Dielectric constant

When a dielectric is placed in an external electric field, polarization occurs and it develops an electric field in opposition to the external one. As a consequence total field inside it decreases. If E be the total field inside the dielectric when it is placed in an external field E_0, then its dielectric constant K is given as

$K = \frac{E_o}{E}$ (K > 1)

If a dielectric completely occupies the space between the conductors of a capacitor, its capacitance increases K times.

Hence in presence of a dielectric with dielectric constant K, the capacitance of a parallel plate capacitor $= \frac{K\varepsilon_o A}{d} = KC_0$ where C_0 is the capacitance without dielectric.

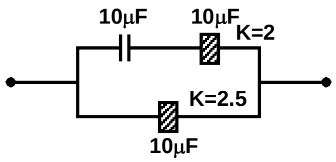


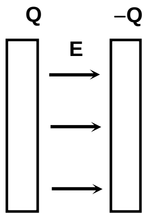
Illustration 37. *Three capacitors of 10μF each are connected as shown in the figure. Two of them are now filled with dielectric with K = 2, K = 2.5 as shown. Find the equivalent capacitance.*

Solution: After insertion of dielectrics,

$C_1 = 10\mu F$; $C_2 = KC_0 = 2 \times 10 = 20\mu F$; $C_3 = KC_0 = 2.5 \times 10 = 25\ \mu F$

$\therefore\ C_{eff} = \frac{10\times 20}{10+20} + 25 = 31\frac{2}{3}\ \mu F$

Force between the plates of a capacitor



Suppose charges Q and –Q are provided on plates of a capacitor.
Field due to charge Q on one plate is

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

Force on the plate with charge –Q will be

$$F = \frac{Q(-Q)}{2A\varepsilon_0} = -\frac{Q^2}{2A\varepsilon_0}$$

Considering magnitude, each plate applies a force of $\frac{Q^2}{2A\varepsilon_0}$ on the other plate.

Energy Stored in a Capacitor

Energy stored in a capacitor can be found out by calculating the work done while transferring total charge Q from one plate to another.

Suppose charge is being transferred from plate B to A. At a moment, charge on the plates are Q′ and –Q′. Then, to transfer a charge of dQ′ from B to A. the work done by an external force will be

$$dW = VdQ' = \frac{Q'}{C}dQ'$$

$$\text{Total work done} = \int_0^Q \frac{1}{C}Q'dQ' = \frac{Q^2}{2C}$$

$$\therefore\quad \text{Energy stored} = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

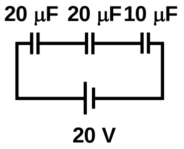


Illustration 38. *A battery of 20 V is connected to 3 capacitors in series as shown in the figure. Two capacitors are of 20μF each and one is of 10μF. Calculate the energy stored in the capacitors in the steady state.*

Solution: $\frac{1}{C_{eff}} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{4}{20} = \frac{1}{5}$

$C_{eff} = 5\mu F$

$\therefore$ Energy stored = $\frac{1}{2}CV^2 = \frac{1}{2}\times 5\times 10^{-6}\times 20^2 = 10^{-3}$ J

Also, as $C=\varepsilon_o A/d$ and V = E.d

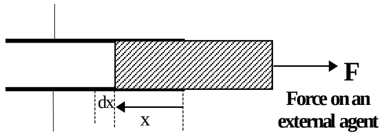
$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_o A}{d}\right)(Ed)^2 = \left(\frac{1}{2}\epsilon_o E^2\right)[Ad]$$

$$\Rightarrow U(\text{energy density}) = \text{Energy per unit volume} = \frac{1}{2}\epsilon_o E^2$$

If dielectric is introduced then, $U = \frac{1}{2}K\varepsilon_0 E^2$

This energy is stored in a capacitor in the electric field between its plates.

Force on a dielectric in a capacitor



Consider a differential displacement dx of the dielectric as shown in figure keeping the total force on it always zero.

When battery is disconnected,

$W_{Electrostatic} + W_F = 0$ (where W denotes the work done in displacement dx)

$W_F = -W_{Electrostatic}$

$W_F = \Delta U$

$$\Rightarrow \quad -F.dx = \frac{Q^2}{2}d.\left[\frac{1}{C}\right] \qquad [W = \frac{Q^2}{2C}]$$

$$\Rightarrow \quad -F.dx = \frac{-Q^2}{2C^2}.dC \Rightarrow F = \frac{Q^2}{2C^2}\left(\frac{dC}{dx}\right)$$

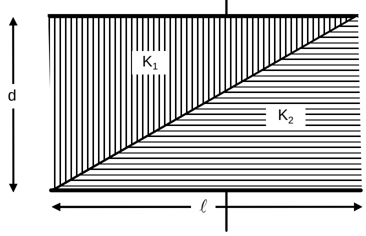
This is also true for the force between the plates of the capacitor.

If capacitor has battery connected to it, then as V= Q/C

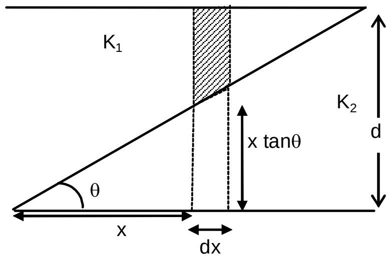
$$\Rightarrow F = \frac{1}{2}V^2\frac{dc}{dx}$$

Exercise 13: ***A dielectric slab is inserted in one end of a charged parallel plate capacitor. (The plates of the capacitor are horizontal and the charging battery having been disconnected). The electric slab is released. Describe what happens. Neglect friction.***

Illustration 39. *A capacitor is formed by two square metal plates of edge 'ℓ', separated by a distance d. Dielectrics of dielectric constant K_1 and K_2 are filled in the gap as shown in the figure. Find the capacitance.*



Solution : Take a differential element of length dx at a distance x from either end.



$$dC_1 = \frac{K_1\varepsilon_0\ell dx}{(d - x\tan\theta)} \quad \left[\because\ C = \frac{K\varepsilon_0 A}{d}\right]$$

$$dC_2 = \frac{K_2\varepsilon_0\ell dx}{(x\tan\theta)}$$

These two capacitors are connected in series

$$\therefore\quad \frac{1}{dC} = \frac{1}{\varepsilon_0\ell}\left[\frac{d - x\tan\theta}{K_1} + \frac{x\tan\theta}{K_2}\right]\frac{1}{dx}$$

$$\therefore\quad \tan\theta = d/\ell$$

$$\Rightarrow\quad \frac{1}{dC} = \frac{1}{\varepsilon_0\ell}\left[\frac{d - x.\frac{d}{\ell}}{K_1} + \frac{x.\frac{d}{\ell}}{K_2}\right]\frac{1}{dx} \quad \left[\because\ \tan\theta = \frac{d}{\ell}\right]$$

$$\Rightarrow\quad \int dC = \frac{K_1K_2\varepsilon_0 i^2}{d}\int_0^1 \frac{dx}{K_2\ell + (K_1 - K_2)x}$$

$$\Rightarrow C = \left[\frac{\varepsilon_0 AK_1K_2}{(K_2 - K_1)d}\right]\ln(K_2/K_1) \qquad [\,A = \ell^2]$$

Student should note that the "K" used here is different from $k = \dfrac{1}{4\pi\varepsilon_0}$ elsewhere.

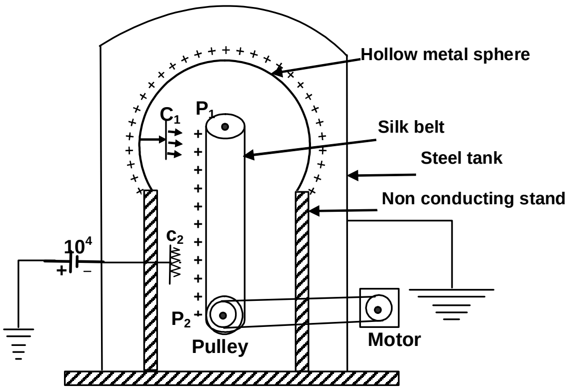
VAN-de-GRAAFF GENERATOR

It is a device which is used to generate very high voltage in the order of 10^7 volts which is used to accelerate charged particle like electrons and protons for atomic experiments.

Working:

When charge is given to the hollow conductor from inner surface, the total charge is transferred to the outer surface of the conductor howsoever large its potential is.

Positive charge is provided by an external source to comb C_2. Charge is passed from comb C_2 to belt by ionization of air.



Belt is rotated with the help of motor so charge reaches near comb C_1. The charge density at sharp points of C_1 becomes high due to which action at a distance starts and belt is neutralized. The positive charge is transferred to the outer surface of the shell. As its charge increases, the potential also increases $\left(V = \dfrac{q}{4\pi\varepsilon_o R}\right)$. In this way, very high potential is generated on the metal shell.

When potential at the surface of metal sphere becomes very high, dielectric breakdown of the surrounding air take place due to which potential can not be increased further. This limit is proportional to the radius $R(C = 4\pi\varepsilon_0 R)$ of the shell.

SUMMARY

Electric charge: It is the property by which a particle electrically interacts with other particles. These can be positive or negative. Like charges repel each other and charges with opposite sign attract each other.

Charges are quantized (minimum 1.6×10^{-19}C). Charge in an isolated system is conserved and the static charge resides on the surface of a conductor. Concentrations of charges is more on a smaller radius of curvature

Coulomb's law: Electrostatic force between two point charges q_1 and q_2 separated by a distance r is,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Here $\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ N - m^2/C^2$ and $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 / m^2 - N$.

It is the permittivity of free space.

Principle of Superposition

According to this principle, the net force acting on a charge q due to a number of charges $q_1, q_2, \ldots\ldots q_n$ is equal to the vector sum of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots \vec{F}_n$, where $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots \vec{F}_n$ are the forces on q due to $q_1, q_2 \ldots\ldots q_n$ respectively. Thus

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \ldots \vec{F}_n$$

Here, forces by charges are calculated separately without any effect of other charges.

Electric field

Electric field at $B(\vec{r}_2)$ due to a charge at $A(\vec{r}_1)$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1),$$

Electric field at a point is the force experienced by a unit positive charge kept at that point. Net field at a point is the vector sum of the fields due to individual charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + . \quad . \quad . + \vec{E}_n$$

$= \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^3}\vec{r}$ due to a continuous charge distribution.

Electric lines of force

The electric lines of force or field lines are imaginary and have no physical existence. Tangent at a point gives the direction of the net field at that point. Lines of force originate from positive charge and terminate on negative charge. They do not intersect each other. The number of lines of force per unit cross-sectional area placed perpendicular to the lines of force, is directly proportional to the intensity of electric field. Lines of force representing a uniform electric field are equi-spaced parallel straight lines.

Gauss' law

Gauss' law states that the total electric flux through a closed surface is proportional to the net charge inside the surface.

$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$

Force on a point charge in a Electric field

$$\vec{F} = q\vec{E}$$

Electric potential
Electric potential at a point in an electric field is the amount of work done by an external agent against electric force in moving a unit positive charge with constant speed from infinity to that point

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \text{ (Potential of a point r distance from charge Q)}$$

Relation between potential and Field

$$E = -\frac{dV}{dx}$$

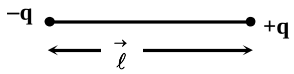
Electric field is along the line of maximum rate of decrease of potential.

Equipotential Surface
Equipotential Surface is the locus of points of equal potential in an electric field.

Electric field is always directed perpendicular to the equipotential surface.

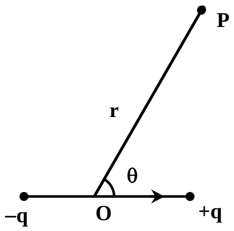
Electrostatic potential energy: It is the amount of work done in bringing the charges form infinity to form the system.

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}}\right) \text{ and so on.}$$

Electric dipole


It is a set of opposite charges separated by a small finite distance.

Dipole moment $\vec{p} = q\vec{\ell}$

Electric field due to a dipole


(a) On the axis of dipole (at distance x)

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{x^3}$, axis of dipole is the line joining the charges and the parallel to $\vec{P}$

(b) On an equatorial line

$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$ direction opposite to $\vec{P}$

(c) On any other point

$$E = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$$

Torque on a dipole in an external field
$\vec{\tau} = \vec{p} \times \vec{E}$

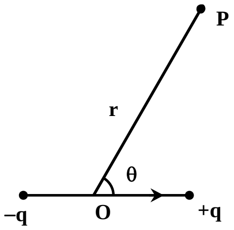
$\Rightarrow \tau = pE\sin\theta$

P.E. of a dipole in electrostatic field

$U = -\vec{p}\,.\,\vec{E}$

Potential due to a dipole

$$V_P = -\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$



Capacitance, capacitor

A capacitor consists of two metallic plates of area A at a distance d apart and opposite charges q and –q are given to the plates.

Capacitance, $C = \frac{Q}{V}$

For parallel plate capacitor $C = \frac{\varepsilon_0 A}{d}$

For spherical capacitor $C = 4\pi\varepsilon_0 R$

Capacitors connected in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots \frac{1}{C_n}$$

Capacitors connected in parallel

$$C_{eq} = C_1 + C_2 + C_3 + \ldots C_n$$

Potential energy stored in a capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

Energy stored per unit volume

$$U = \frac{1}{2}\varepsilon_0 E^2$$

$= \frac{1}{2}k\varepsilon E^2$ (with a dielectric of dielectric constant k introduced E is the net electric field in dielectric medium.

MISCELLANEOUS EXERCISE

1. Can two equipotential surfaces intersect?

2. If left free, will a charged particle necessarily move along the line of force?

3. If at a point V = 0, must E also be zero?

4. Mark true or false.
(a) A Gaussian surface must be symmetric.
(b) Electric field is calculated by Gauss law is the field due to charges inside the Gaussian surface only.
(c) For a certain distribution of charge, the flux through a spherical surface of radius 20 cm surrounding a certain charge is 60 V-m. Flux through a spherical surface of radius 40 cm will be less than 60 V-m.
(d) Increase in plate separation of an isolated charged capacitor will increase the stored energy.

5. Three capacitors of 4μF are available. What is the maximum capacitance which can be obtained using these 3 capacitors?

6. What are the lines of force? Mention their properties.

7. What is dipole moment?

8. Derive an expression for the electric field due to a dipole at a point on the axial line of the dipole at a distance r from the centre of the dipole.

9. Derive an expression for the electric field due to a dipole at an equatorial point r distance away from the centre of the dipole.

10. Derive an expression for torque on a dipole in a uniform electric field when angle between dipole and field is θ.

11. Derive an expression for potential energy of a dipole kept in a field at an angle θ from the field.

12. Derive an expression for capacitance of a parallel plates capacitor

13. State Coulomb's law and principle of superposition.

14. Derive an expression for electric field intensity at a point located on the axis of a disk of radius R having a surface charge density σ. The point is at a distance x from the centre of the disk

15. State Gauss's law. Is it applicable to any (uniform/ nonuniform) distribution of charges?

SOLVED PROBLEMS

Subjective:

Prob 1. *A charge of 15μC and x μC are placed 2 m apart. They attract each other with a force of 1.5 N. Find the value of x*

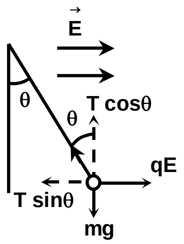
Sol. $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

$1.5 = 9 \times 10^9 \times \frac{15\times10^{-6}\times x\times10^{-6}}{2\times2}$

$x = \frac{6}{9\times15\times10^{-3}} = 44.44$

Prob 2. *A pendulum bob of mass 2 g has positive charge 5 μC on it. It is suspended in a horizontal field of 10 k-Vm^{-1}. What will be the angle θ made by the string with the vertical and tension in the string?*

Sol. $T\cos\theta = mg$



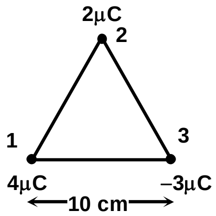
$T\sin\theta = qE$

$\tan\theta = \frac{qE}{mg} = \frac{5\times10^{-6}\times10^4}{2\times10^{-3}\times9.8} = 2.55$

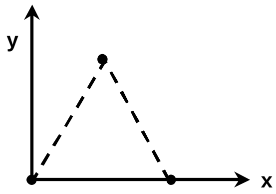
$\theta = 68.6°$

$T = \frac{mg}{\cos\theta} = \frac{2\times10^{-3}\times9.8}{0.365} = 0.05\text{ N}$

Prob 3. *Three charges 4μC, 2 μC and –3μC are kept on vertices of an equilateral triangle as shown in the figure. Find out the magnitude resultant force on the 2 μC charge.*



Sol. $F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{4\times2\times10^{-12}}{(0.1)^2} = 7.2\text{ N}$



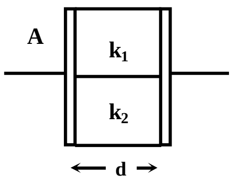
$F_{32} = \frac{1}{4\pi\varepsilon_0}\frac{(-3)\times2\times10^{-12}}{(0.1)^2} = -\frac{9\times10^9\times6\times10^{-12}}{0.01}\text{ N}$

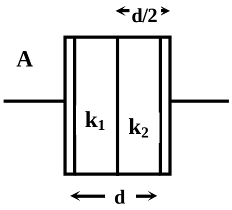
$= -5.4\text{ N}$

Resultant force $F_R = \sqrt{(7.2)^2+(5.4)^2-2\times7.2\times5.4\times\cos60}$

$F_R = 6.49\text{ N}$

Prob 4. *Two blocks of dielectric constants k_1 and k_2 are filled in a capacitor in the arrangements shown in the figure. Find out the capacitance in each case.*





Sol. (a) The capacitances are in parallel

$$C_1 = \frac{k_1\varepsilon_0(A/2)}{d} = \frac{k_1\varepsilon_0 A}{2d} \text{ and } C_2 = \frac{k_2\varepsilon_0 A}{2d}$$

$$\therefore \quad C_{eq} = C_1 + C_2 = \frac{\varepsilon_0 A}{2d}(k_1 + k_2)$$

(b) The two capacitors are in series

$$C_1 = \frac{k_1\varepsilon_0 A}{d/2} = \frac{2k_1\varepsilon_0 A}{d} \text{ and } C_2 = \frac{k_2\varepsilon_0 A}{d/2} = \frac{2k_2\varepsilon_0 A}{d}$$

$$\therefore \quad \frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2k_1\varepsilon_0 A} + \frac{d}{2k_2\varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\therefore \quad C_{eff} = \frac{2\varepsilon_0 A k_1 k_2}{d(k_1 + k_2)}.$$

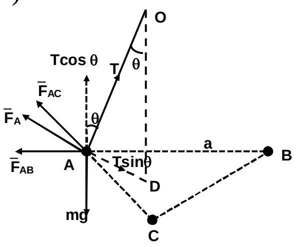
Prob 5. *Three particles, each of mass 1 gm and carrying a charge, q are suspended from a common point by insulated massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side 3 cm, calculate the charge, q on each particle (take g= 10 ms^{-2})*

Sol. $\vec{F}_A = \vec{F}_{AB} + \vec{F}_{AC} = \left[\frac{1}{4\pi \in_o}\frac{q^2}{a^2}\right] 2\cos\left[\frac{60^o}{2}\right]$ in the direction D to A

$= 2\left[\frac{1}{4\pi \in_o}\frac{q^2}{a^2}\right]\frac{\sqrt{3}}{2}$ in the direction D to A

For equilibrium, $T\cos\theta = mg$ .....(1) $T\sin\theta = F_A$ ... (2)

Dividing (2) by (1), and we get;



$\tan\theta = \frac{F_A}{mg}$

where , $\tan\theta \approx \sin\theta = \frac{AD}{AO} = \frac{\sqrt{3}\times 10^{-2}}{1}$

(AD = 2/3 of the median length, as D is the centroid)

or, $\quad 2\left[\frac{1}{4\pi \in_o}\right]\frac{q^2}{a^2}\frac{\sqrt{3}}{2} = 10^{-3} \times (\sqrt{3}\times 10^{-2})$

or, $\quad q = \sqrt{10}\times 10^{-9}\,C$

$\therefore \quad q = 3.16\times 10^{-9}\,C$

Prob 6. *Three charges q_1 , q_2 and q_3 are located at the vertices of an equilateral triangle of side a. Find the electric potential energy of the system.*

Sol. Taking datum at infinity. We can assume that the charges are brought one by one.

$$W = \Delta V q$$

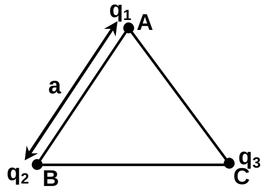
In absence of any charge, $V_A = 0$,

$\Rightarrow \quad W_1 = (V_A - V_\infty)q_1 = 0 \quad \ldots (1)$

$$W_2 = (V_B - V_\infty)q_2 = \left(\frac{Kq_1}{a} - 0\right)q_2 = \frac{Kq_1q_2}{a} \quad \ldots (2)$$

$\because \quad W_3 = (V_c - V_\infty)q_3$

$$= \frac{Kq_1q_3}{a} + \frac{Kq_1q_3}{a} \quad \ldots (3)$$



$W = W_1 + W_2 + W_3$

$$\Rightarrow \quad W = \frac{Kq_1q_2}{a} + \frac{Kq_2q_3}{a} + \frac{Kq_1q_3}{a} = \frac{1}{4\pi\varepsilon_0}[q_1q_2 + q_2q_3 + q_1q_3]$$

Prob 7. *A uniform electric field, E exists between the plates of a capacitor. The plate length is l and the separation of the plates is d.*

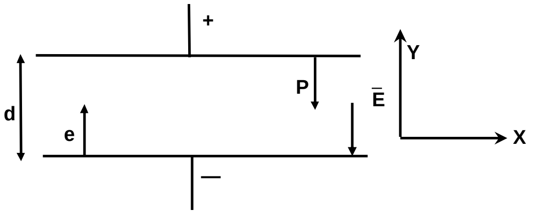
(a) An electron and a proton start from the negative plate and positive plate respectively and go to the opposite plates. Which of them wins this race?

(b) An electron and a proton start from the midpoint of the separation of plates at one end of the plates. Which of the two will have greater deviation when they start with the

(i) same initial velocity

(ii) same initial kinetic energy, and (iii) same initial momentum?

Sol. (a) In the chosen reference frame there is no force along x axis. The accelerations of the electron and the proton along y axis are as follows



$$a_e = \frac{-eE}{m_e}, \; a_p = \frac{eE}{m_p}$$

Using $S = ut + \frac{1}{2}at^2$

Here $S = d, u = 0$

$$\Rightarrow \quad t_e = \sqrt{\frac{2S}{a_e}} \text{ and } t_p = \sqrt{\frac{2S}{a_p}}$$

$$\text{or} \quad t_e = \sqrt{\frac{2dm_e}{eE}} \text{ and } t_p = \sqrt{\frac{2d.m_p}{eE}}$$

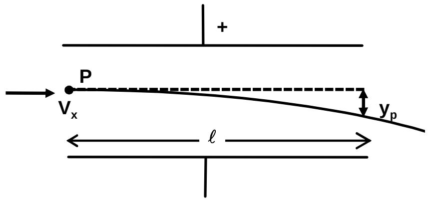
As $m_p > m_e$

$\Rightarrow t_p > t_e$

$\Rightarrow$ Electron takes less time to cross over than the proton.

(b) When proton moves parallel to the plates, it is deflected to the negative plate. i.e. electron wins this race.

Time taken by proton to cross over, $t = \frac{l}{V_x}$



During this time, deflection along vertical direction

$$y = \frac{1}{2}at^2$$

or, $y = \frac{1}{2}\frac{eE}{m}\left[\frac{l}{V_x}\right]^2$

(i) Thus $y_p = \frac{1}{2}\frac{eE}{m_p}\frac{l^2}{V_x^2}$ and $y_e = \frac{1}{2}\frac{eE}{m_e}.\frac{l^2}{V_x^2}$

As $m_p > m_e$, $y_p < y_e$. Of course, electron will be deflected more in the opposite direction.

(ii) Also $y = \frac{1}{2}\frac{eEl^2}{2\left[\frac{m}{2}v_x^2\right]} = \frac{1}{4}\frac{eEl^2}{K}$ where $K = \frac{1}{2}mv_x^2$ (initial kinetic energy)

$\Rightarrow y_p = y_e$

(iii) Also, as $K = \frac{p^2}{2m}$,where p = momentum,

$y = \frac{eEl^2}{4(\frac{p^2}{2m})}. = \frac{eEl^2.m}{2p^2} \Rightarrow y_p = \frac{eEl^2 m_p}{2p^2}, y_e = \frac{eEl^2 m_e}{2p^2}$

Thus for: $m_p > m_e$, $y_p > y_e$

Prob 8. *Two spherical cavities of radii, a and b, are hollowed out from the interior of a neutral conducting sphere of radius R. At the centre of each cavity, a point charge is placed. Call these charges q_a and q_b*

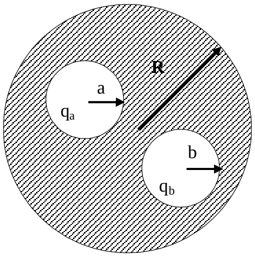
(a) Find the surface charge densities σ_a , σ_b and σ_R.

(b) What is the field outside the conductor?

(c) What is the field within each cavity?

(d) What is the force on q_a and q_b?

(e) Which of these answers would change if a third charge q_c were brought near the conductor?



Sol. (a) $\sigma_a = \frac{-q_a}{4\pi a^2}, \sigma_b = \frac{-q_b}{4\pi b^2}, \sigma_R = \frac{q_a + q_b}{4\pi R^2}$

(b) $E = \frac{k(q_a + q_b)}{r^2}, (r > R)$

(c) $E_a = \frac{kq_a}{r^2}, (r < a), E_b = \frac{kq_b}{r^2}, (r < b)$

(d) Force on q_a and q_b = 0 as both q_a and q_b are electrostatially shielded against each other.

(e) When a third charge is brought, only the field outside the conductor will change

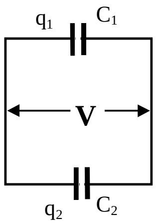
Prob 9. *A 4μf capacitor is charged to 150 V and another 6μf capacitor is charged to 200 V. Then they are connected across each other. Find the potential difference across them. Calculate the heat produced.*

Sol. 4 μf charged to 150 V would have $q_1 = C_1V_1 = 600$ μC

6 μf charged to 200 V would have $q_2 = C_2V_2 = 1200$ μC

After connecting them across each other, they will have a common potential difference V.

Charges will be redistributed as q_1' and q_2'



$$V = \frac{q_1'}{C_1} = \frac{q_2'}{C_2} \Rightarrow \frac{q_1' + q_2'}{C_1 + C_2} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{1800\mu C}{(4+6)\mu f} \quad \text{[conservation of charge]}$$

$V = 180$ volt.

Initial energy, $U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = \frac{1}{2}(4\mu f)(150\,V)^2 + \frac{1}{2}(6\mu f).(200\,V)^2 = 0.165\,J$

Final energy, $U_f = \frac{1}{2}[C_1 + C_2].V^2 = \frac{1}{2}(4\mu f + 6\mu f).(180)^2 = 0.161\,J$

Heat produced = $|U_f - U_i| = 0.003$ J

Prob10. *What charges will flow after the shorting of the switch S in the circuit given in the figure, through section A and B ?*

Sol. When S is open, C_1 and C_2 are in series and their equivalent capacitance is $\frac{C_1C_2}{C_1 + C_2}$

Charge on the plates 1 and 3 is

$Q = +\frac{C_1C_2}{C_1 + C_2}\varepsilon$

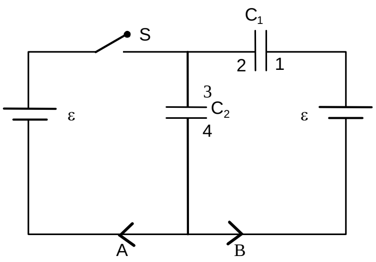
When S is closed P.D. across C_1 is zero.

$\Rightarrow$ Charge on the plate 1 is 0.

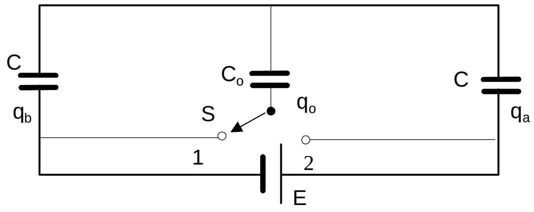
If charge flown through B is q, then Q + q = 0

$\Rightarrow q = -Q = -\frac{C_1C_2}{C_1 + C_2}\varepsilon$

Charge on the plate 3 is $C_2\varepsilon$, which is also equal to the charge flown through A.



Prob 11. *When switch is swapped from 1 to 2, find the heat produced in the circuit*



Sol. $q_a = \frac{C[C + C_o]}{2C + C_o}.E$

Now, $\frac{q_b}{C} = \frac{q_o}{C_o} = \frac{q_b + q_o}{C + C_o} = \frac{q_a}{C + C_o}$

$\Rightarrow \quad q_b = \frac{CE}{2C + C_o}$

When switch is shifted from 1 to 2, q_a and q_b get interchanged. This can be seen from symmetry

Potential energy of the system is same before and after.

A charge $\Delta q = q_a - q_b$ flows through the battery .

Work done by the battery = $(\Delta q).E = C.C_o.E^2/(2C + C_o)$.

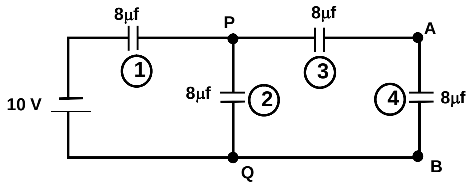
Applying the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

Here, $\Delta U = 0$,

$$\Rightarrow \quad \Delta Q = -\left[\frac{C.C_o}{2C + C_0}.E^2\right]$$

Negative sign means heat is liberated from the system.

Prob12. *In the above circuit, find the potential difference across AB.*



Sol. Let us mark the capacitors as 1, 2, 3 and 4 for identifications. As is clear, 3 and 4 are in series, and they are in parallel with 2. The 2, 3, 4 combine is in series with 1.

$C_{34} = \frac{C_3.C_4}{C_3 + C_4} = 4\mu f$, $\quad C_{2,34} = 8 + 4 = 12\ \mu f$

$C_{eq.} = \frac{8x12}{8+12} = 4.8\mu f$, $\quad q = C_{eq.}V = 4.8\times 10 = 48\ \mu C$

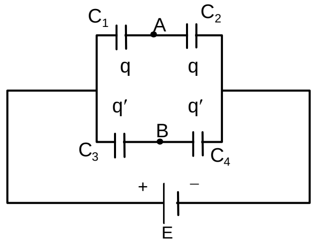
The `q' on 1 is 48 μC,

Thus $V_1 = \frac{q}{C} = \frac{48\mu C}{8\mu F} = 6$ V

$\Rightarrow V_{PQ} = 10 - 6 = 4$ V

By symmetry of 3 and 4, we can say, $V_{AB} = 2$ V.

Prob 13. *What is V_A–V_B in the arrangement shown? What is the condition such that $V_A - V_B = 0$?*



Sol. Let charge be as shown (Capacitors in series have the same charge)

Take loop containing C_1, C_2 and E

$$\frac{q}{C_1} + \frac{q}{C_2} - E = 0 \Rightarrow q = E\left[\frac{C_1C_2}{C_1 + C_2}\right]$$

From loop containing C_3, C_4 and E

Similarly,

$$\frac{q'}{C_3} + \frac{q'}{C_4} - E = 0$$

$$\Rightarrow \quad q' = E.\left[\frac{C_3C_4}{C_3 + C_4}\right]$$

Now, $$V_A - V_B = \frac{q}{C_2} - \frac{q'}{C_4} = E.\left[\frac{C_1}{C_1 + C_2} - \frac{C_3}{C_3 + C_4}\right]$$

$$V_A - V_B = E.\left[\frac{C_1.C_4 - C_3.C_2}{(C_1 + C_2).(C_3 + C_4)}\right]$$

Next for, $V_A - V_B = 0$

$\Rightarrow \quad C_1C_4 - C_2C_3 = 0$

or, $\quad C_1/C_2 = C_3/C_4$

Objective:

Prob 1. *A charge Q is kept on a spherical shell A of radius R and a charge q is kept on a spherical shell B of radius r inside the shell A. The potential difference between outer and inner shell is*

(A) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{r}\right)$ *(B)* $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{R}\right)$

(C) $\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{r}\right)$ *(D)* $\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}-\frac{1}{r}\right)$

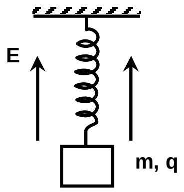
Sol. Potential at R $= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R}+\frac{q}{R}\right]$

Potential at r = $\frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R}+\frac{q}{r}\right]$

Potential difference = $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{r}\right]$

$\therefore$ **(A)**

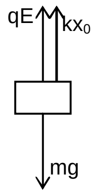
Prob 2. *A block of mass m having charge q, is hinged by a spring of spring constant k in a vertical electrostatic field E. The spring extension in equilibrium will be*



(A) $\frac{mg}{k}$ *(B)* $\frac{qE}{k}$

(C) $\frac{mg+qE}{k}$ *(D)* $\frac{mg-qE}{k}$

Sol. From the Free Body Diagram



$qE + kx_0 = mg$

$x_0 = \frac{mg-qE}{k}$

$\therefore$ **(D)**

Prob 3. *A charge +q is brought near an isolated metal cube having no charge initially*

(A) cube becomes positively charged.

(B) cube becomes negatively charged.

(C) external surface becomes negatively charged and interior becomes positively charged.

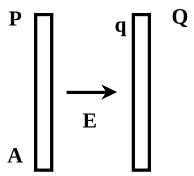
(D) Interior remains charge free and the surface gets non-uniform charge distribution.

Sol. Due to charge +q negative charge will be induced on the surface near to the external charge. Positive charge will shift on the other side of the cube.

Being metal conductor, the interior will remain charge free.

$\therefore$ **(D)**

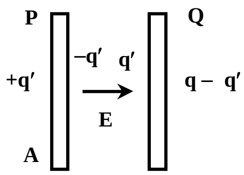
Prob 4. *Two conducting plates P and Q with large surface area A are placed as shown in figure. A charge q is given to plate P. The electric field between the plate at any point is*



(A) $\frac{q}{3A\varepsilon_0}$ *(B)* $\frac{q}{2A\varepsilon_0}$

(C) $\frac{q}{A\varepsilon_0}$ *(D)* $\frac{2q}{A\varepsilon_0}$

Sol. Charge distribution will be as shown

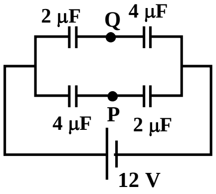


Field at point B

$= \frac{q-q'}{2A\varepsilon_0} - \frac{q'}{2A\varepsilon_0} + \frac{q'}{2A\varepsilon_0} - \frac{q'}{2A\varepsilon_0} = \frac{q}{2A\varepsilon_0}$

$\therefore$ (B)

Prob 5. *V_{PQ} in the figure.*



(A) 12 V *(B) 8 V*

(C) 4 V *(D) zero.*

Sol. C = $\frac{2\times4}{2+4} = \frac{4}{3}$ μF in one loop.

q = 12 × $\frac{4}{3}$ = 16 μC across one set of 2 μF & 4 μF capacitors.

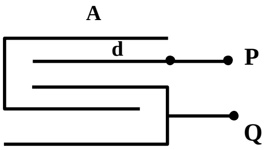
$\therefore$ $V_P = 12 - \frac{16}{4} = 8\,V$

$V_Q = 12 - \frac{16}{2} = 4$ V

$\therefore$ $V_P - V_Q = 8 - 4 = 4$ V

$\therefore$ **(C)**

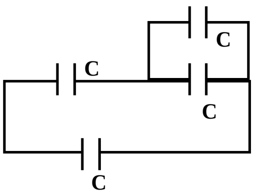
Prob 6. *The equivalent capacitance between P & Q in the figure is*



(A) $\frac{A\varepsilon_0}{2d}$ *(B)* $\frac{3A\varepsilon_0}{2d}$

(C) $\frac{2A\varepsilon_0}{d}$ *(D)* $\frac{5}{3}\frac{A\varepsilon_0}{d}$

Sol. Equivalent circuit is shown in figure

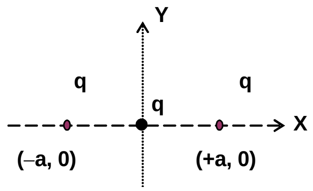


$C_{eq} = C + \frac{C\times 2C}{C+2C}$

$= C + \frac{2}{3}C = \frac{5}{3}C = \frac{5A\varepsilon_0}{3d}$

$\therefore$ **(D)**

Prob 7. *Two point charges each of charge +q are fixed at (+a, 0) and (–a, 0). Another positive point charge q placed at the origin is free to move along x- axis. The charge q at origin in equilibrium will have*



(A) maximum force and minimum potential energy

(B) minimum force & maximum potential energy

(C) maximum force & maximum potential energy
(D) minimum force & minimum potential energy.

Sol. The net force on q at origin is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = \frac{1}{4\pi\varepsilon_0}.\frac{q^2}{r^2}\hat{i} + \frac{1}{4\pi\varepsilon_0}.\frac{q^2}{r^2}(-\hat{i}) = 0$$

The P.E. of the charge q in between the extreme charges at a distance x from the origin along +ve x axis is

$$U = \frac{1}{4\pi\varepsilon_0}.\frac{q^2}{(a-x)} + \frac{1}{4\pi\varepsilon_0}.\frac{q^2}{(a+x)} = \frac{1}{4\pi\varepsilon_0}.q^2\left[\frac{1}{a-x} + \frac{1}{a+x}\right].$$

$$\frac{dU}{dx} = \frac{q^2}{4\pi\varepsilon_0}\left[-\frac{1}{(a-x)^2} + \frac{1}{(a+x)^2}\right]$$

For U to be minimum, $\frac{dU}{dx} = 0$, and $\frac{d^2U}{dx^2} > 0$,

$\Rightarrow (a - x)^2 = (a + x)^2$

$\Rightarrow a + x = \pm (a - x)$

$\Rightarrow x = 0$, because other solution is relevant.

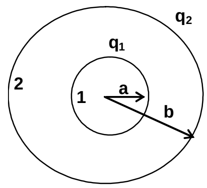
Thus, the charged particle at the origin will have minimum force and minimum P.E.

$\therefore$ **(D).**

Prob 8. *Two concentric conducting spherical shells having radii a and b are charged to q_1 & q_2 respectively. The potential difference between 1 & 2 will be*

(A) $\frac{q_1}{4\pi\varepsilon_0 a} - \frac{q_2}{4\pi\varepsilon_0 b}$ *(B)* $\frac{q_2}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$

(C) $\frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$ *(D) none of these.*



Sol. The potential on the surface of the sphere 1 is given by

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{b} \quad \ldots. (a)$$

The potential on the surface of the sphere 2 is given by,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{b} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{b}$$

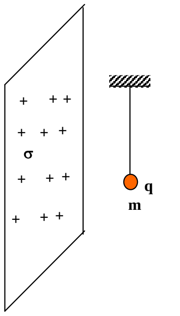
$V = V_1 - V_2$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{b}$$

$$\Rightarrow V = \frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$\therefore$ **(C)**

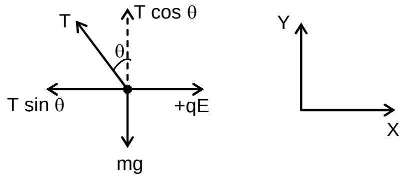
Prob 9. *A small charged particle of mass m and charge q is suspended by an insulated thread in front of a very large sheet of charge density σ. The angle made by the thread with the vertical in equilibrium is*



(A) $\tan^{-1}\left(\dfrac{\sigma q}{2\varepsilon_0 mg}\right)$ *(B)* $\tan^{-1}\left(\dfrac{\sigma q}{\varepsilon_0 mg}\right)$

(C) $\tan^{-1}\left(\dfrac{2\sigma q}{\varepsilon_0 mg}\right)$ *(D) zero*

Sol. In equilibrium, along x-axis,



T sin θ = qE

$\Rightarrow T\sin\theta = q\dfrac{\sigma}{2\varepsilon_0}$ . . . (1)

Where T is the tension in the string.

Along y-axis in equilibrium, T cos θ = mg . . . (2)

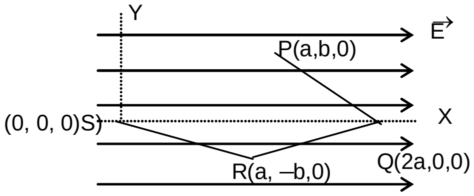
From (1) and (2) we obtain,

$\tan\theta = \dfrac{q\sigma}{2\varepsilon_0 mg}$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{\sigma q}{2\varepsilon_0 mg}\right)$

∴ **(A)**

Prob 10. *A point charge q moves from point P to point S along the path PQRS in a uniform electric field $\vec{E}$ pointing parallel to the positive direction of the x-axis. The coordinate of the points P,Q, R and S are (a, b, 0), (2a, 0, 0), (a, – b, 0) and (0, 0, 0) respectively. The work done by the field in the above process is given by the expression*



(A) qaE *(B) – qaE*

(C) $q(\sqrt{a^2+b^2})E$ *(D)* $3qE\sqrt{a^2+b^2}$

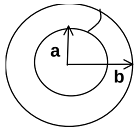
Sol. The work done is independent of the path followed and is equal to $(q\vec{E})\cdot\vec{r}$, where $\vec{r}$ =displacement vector $\overline{PS} = -a\hat{i} - b\hat{j}$, while $\overline{E} = E\hat{i}$

∴ Work = $q\overline{E}\cdot\overline{r} = -qaE$

∴ **(B)**

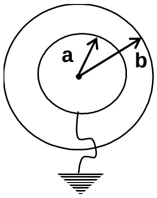
Fill up the blanks:

Prob 1. *The conducting spherical shells shown in the figure are connected by a conductor. The capacitance of the system is……………………………….*



Sol. As there will be no charge on the inner sphere, therefore the capacitance only will exist due to outer sphere. Hence the capacitance of the system is the capacitance due to outer sphere of radius b, therefore $C = 4\pi\varepsilon_0 b$.

Prob 2. *The figure shows a spherical capacitor with inner sphere earthed. The capacitance of the system is………………………*



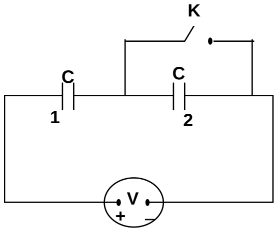
Sol. Let V be the potential on the outer sphere. Thus, we can consider two capacitors, between the outer sphere and inner sphere (a) and outer sphere and earth

Thus, $C_1 = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$; $C_2 = 4\pi\varepsilon_0 b$

$C = 4\pi\varepsilon_0 \dfrac{ab}{b-a} + 4\pi\varepsilon_0 b$

$\Rightarrow C = \dfrac{4\pi\varepsilon_0 b^2}{b-a}$

Prob 3. *The charge flowing across the circuit on closing the key K is equal to ……………………..*



Sol. When the key K is kept open the charge drawn from the source is $Q = C'V$

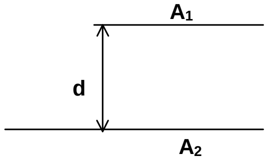
where C' is the equivalent capacitance given by $C' = C/2$

Therefore $Q = (C/2)V$

When the key K is closed, the capacitor 2 gets short circuited and thus the charge in that capacitor comes back to the source.

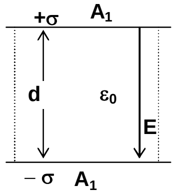
$\therefore$ Charge flowing is $Q = (C/2)V$

Prob 4. *The capacitance of the system of parallel plates shown in the figure is…………………….*

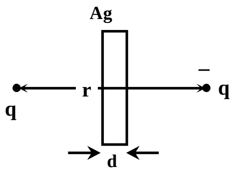


Sol. Since the electric field between the parallel charged plates is uniform and independent of the distance, neglecting the fringe effect, the effective area of the plate of area A_2 is A_1. Thus, the capacitance between the plates is

$C = \dfrac{\varepsilon_0 A_1}{d}$



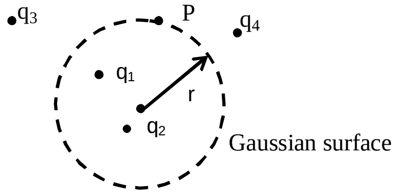
Prob 5. *Two point charges q are placed r distance apart. A silver block of thickness d is placed between them at the middle. The force between the charges will be…………………………….*



Sol. Field inside conductor is zero
Hence, no force will act between the charges.

True and False:

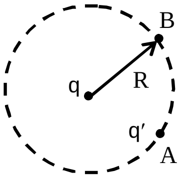
State whether the following statements are true or false.
Consider a spherical Gaussian surface that surrounds part of charge distribution shown in figure.



1. The flux through the Gaussian surface depends on q_4. [False]

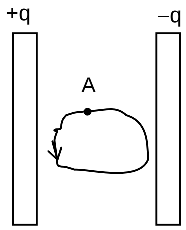
2. Field at point P depends on charge q_4 and q_3. [True]

3. In the figure shown, a circle of radius R is drawn with q as centre. The amount of work done in taking a charge q′ from A to B is independent of q, q′ and R



[True]

4. A uniform field exists between 2 charged plates as shown in figure. The amount of work does in taking a charge –q′ along as arbitrary closed path shown in figure is independent of charge on the plates.



[True]

5. Two equal charges are kept at x = a and x = – a. There can be only one point where the electric field will be zero. [True]

ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. Sketch the electric lines of force for two positive charges Q_1 and Q_2 ($Q_1 > Q_2$) separated by a distance d.

2. Name the physical quantity whose SI unit is (i) coulomb / volt (ii) newton / coulomb (iii) joule coulomb.

3. How does the force between two point charges change if the dielectric constant of the medium in which they are kept increasing?

4. What is the ratio of electric field intensities at any two points between the plates of a parallel plate capacitor ?

5. How much work is done in moving a 500 μC charge between two points on an equipotential surface ?

6. Two point charges of unknown magnitude and sign are placed a distance 'd' apart. The electric field intensity is zero at a point, not between the charges but on the line joining them. Write two essential conditions for this to happen.

7. Define intensity of electric field at a point. At what points is the electric dipole field intensity parallel to the line joining the charges?

8. Show that no electric field exists inside a hollow charged conductor.

9. State the Gauss's theorem in electrostatics. Apply this theorem to calculate the electric field due to an infinite plane sheet of charge.

10. Define 'electric potential'. Deduce an expression for the electric potential at a point distant 'r' from point charge Q (Q > 0).

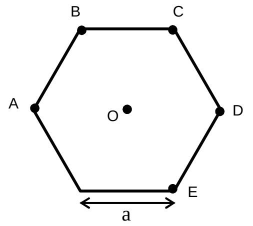
11. Prove that the energy stored in a parallel plate capacitor is $\frac{1}{2}CV^2$.

12. Define the term electric flux. State its unit.
    A sphere S_1, of radius r_1 encloses a charge Q. If there is another concentric sphere S_2 of radius r_2 ($r_2 > r_1$) and there be no additional charges between S_1 and S_2, find the ratio of electric flux through S_1 and S_2.

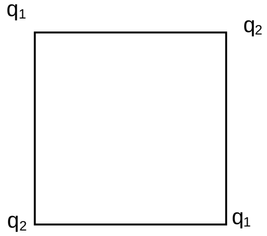
13. What is dielectric? A dielectric slab of thickness t, is kept between the plates of a parallel plate capacitor separated by distance d. Derive the expression for the capacity of the capacitor for t <<d.

Level - I

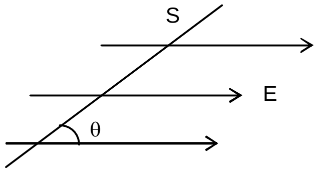
1. Five point charges, each of magnitude +q are kept at the corners A, B, C, D and E of a regular hexagon of side a. Find the electric field at the centre O of the hexagon.



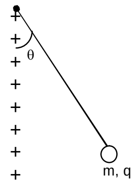
2. The charges, q_1 and q_2 are placed at the corners of a square as shown in the figure. Find q_2 such that the resultant force on q_1 is zero.



3. A plane surface of area S is inclined at an angle θ with a uniform field $\vec{E}$ as shown in the figure. Find the flux of $\vec{E}$ over S.



4. A small sphere of mass m carries a charge q. It hangs from a light inextensible thread of length ℓ making an angle θ with an infinite line of charge as shown in the figure. Find the linear charge density of the line charge.



5. A circular wire of radius R carries a total charge Q distributed uniformly over its circumference. A small length of wire subtending angle θ at the centre is cut off. Find the electric field at the centre due to the remaining portion.

6. Twenty-seven identical mercury drops are charged simultaneously to the same potential of 10 volt. What will be the potential if all the charged drops are made to combine to form one large drop ? Assume all drops to be spherical.

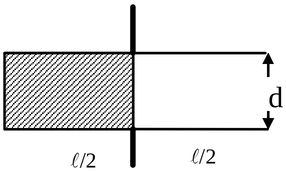
7. Two charged particles having charge +1 μC and –1μC and of mass 50 gm each are held at rest while their separation is 2 m. Find the speed of the particles when their separation is 1 m. Neglect the effect of gravity.

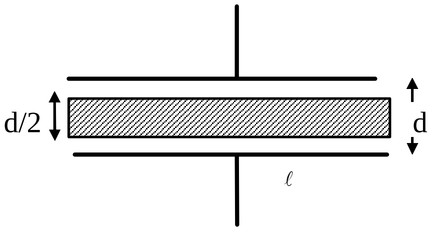
8. A certain charge \`Q' is to be divided into two parts, q and Q – q. What is the relationship of \`Q' to \`q' if the two parts, placed at a given distance \`r' apart, are to have maximum columbic repulsion? What is the work done in reducing the distance between them to half its value ?

9. A 1μF and a 2μF capacitor are connected in series across a 1200 V supply.
(a) Find the charge on each capacitor and voltage across each capacitor.

(b) The charged capacitor are disconnected from the line and from each other, and are now reconnected with terminals of like sign together. Find the final charge on each capacitor and the voltage across each capacitor.

10. The space between the plates of a parallel plate capacitor is filled with a dielectric as shown in figures (1) & (2). The area of each plate is A and permitivity of the dielectric is $\in_r$. Find the capacitance in each case.



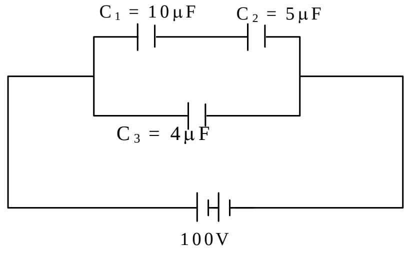


Level - II

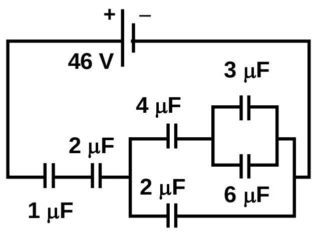
1. A 100 eV electron is projected directly toward a large metal plate that has a surface charge density of -2.0×10^{-6} C/m^2. From what distance must the electron be projected, if it is to just fail to strike that plate?

2. A 100 pF capacitor is charged to a potential difference of 24 V. It is connected to an uncharged capacitor of 20 pF. What will be the new potential difference across 100 pF ?

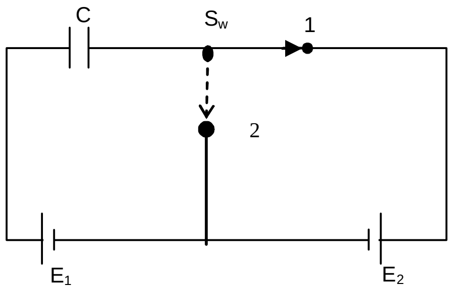
3. In the figure shown, find
(a) The charge
(b) The potential difference and
(c) The stored energy for each capacitor



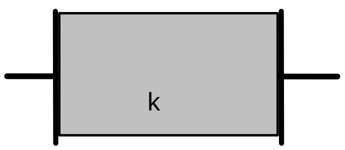
4. Find out the potential difference across the plates of 1 μF capacitor.



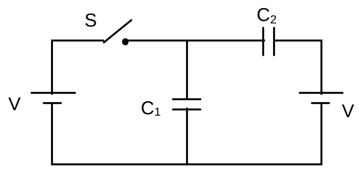
5. In the given circuit diagram, the switch S_W is shifted from position 1 to position 2. Find the amount of heat generated in the circuit.



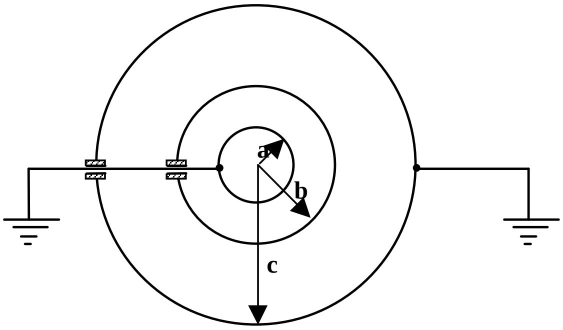
6. Figure shows a parallel plate capacitor having square plates of edge a and plate separation d. The gap between the plate is filled with a dielectric of dielectric constant k which varies from the left plate to the right plate as $k = k_o + \alpha x$. where k_o and α are positive constants and x is the distance from the left end. Calculate the capacitance.



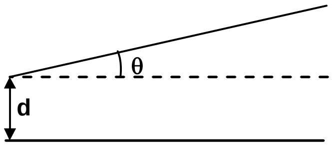
7. In the figure shown, find the charge on each capacitor (a) when the switch S is open (b) when the switch S is closed.



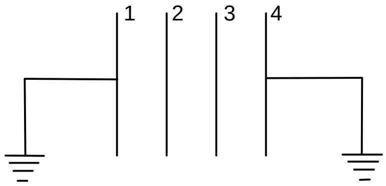
8. In the arrangement shown in figure, three concentric conducting shells are shown. The charge on the shell of radius b is q_0 . If the innermost and outermost shells are connected to the earth, find their charge densities and the potential on the shell of radius b in terms of a and q_0.
Given that a : b : c = 1 : 2 : 4



9. A capacitance has square plates, each of side a, making an angle θ as shown in the figure. Show that, for small θ, the capacitance is $C = \frac{\epsilon_0 a^2}{d}\left[1 - \frac{a\theta}{2d}\right]$



10. Four identical metallic plates, each having area of cross-section A, are separated by a distance d as shown in the figure. Plate 2 is given a charge Q. Find the potential difference between 2 & 3.



11. The electric field vector is given by $\vec{E} = a\sqrt{x}\ \hat{i}$. Find
(a) the flux of $\vec{E}$ through a cube bounded by surfaces
$x = \ell$, $x = 2\ell$, $y = 0$, $y = \ell$, $z = 0$ and $z = \ell$.
(b) The charge within the cube.

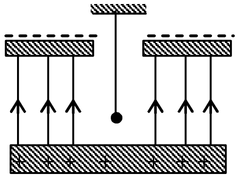
12. A spherical shell of radius r_1 with a uniform charge Q has a point charge q at its centre. Find the work done by an external agent in expanding the shell to a radius r_2.

13. An insulated conductor, initially free from charge, is charged by repeated contacts with a plate which, after each contact, is replenished to a charge Q from an electrophorus. If q is the charge on the conductor after the first operation, prove that the maximum charge which can be given to the conductor in this way is Qq/(Q – q).

Objective:

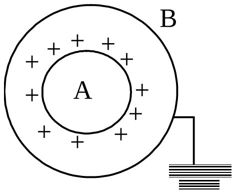
Level – I

1. A positively charged pendulum is oscillating in a uniform electric field as shown in figure. Its time period, as compared to that when it was uncharged:



(A) Will increase (B) Will decrease
(C) Will not change (D) Will first increase and then decrease

2. A and B are two concentric spheres If A is given a charge Q while B is earthed as shown in figure **:**

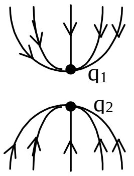


(A) The charge density of A and B are same
(B) The field inside and outside A is zero
(C) The field between A and B is not zero
(D) The field inside and outside B is zero

3. The maximum electric field intensity on the axis of a uniformly charged ring of charge q and radius R will be

(A) $\frac{1}{4\pi\varepsilon_0}\frac{q}{3\sqrt{3}R^2}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3R^2}$
(C) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3\sqrt{3}R^2}$ (D) $\frac{1}{4\pi\varepsilon_0}\frac{3q}{2\sqrt{2}R^2}$

4. The figure is a plot of the lines of force due to two charges $q_1$and q_2. The sign of the charges are



(A) both negative
(B) upper positive and lower negative
(C) both positive
(D) upper negative and lower positive

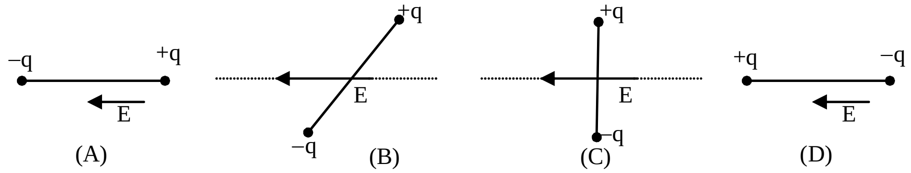
5. There are two concentric metal shells of radii r_1 and $r_2(>r_1)$. If the outer shell has a charge q and the inner shell is grounded, the charge on the inner shell is

(A) zero (B) $-(r_1/r_2)q$
(C) r_1r_2q (D) ∞

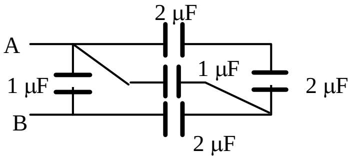
6. Electric charges q, q and $-2q$ are placed at the corners of an equilateral triangle ABC of side L. The magnitude of electric dipole moment of the system is

(A) qL (B) 2qL
(C) $(\sqrt{3})qL$ (D) 4qL

7. In which of the following states is the potential energy of an electric dipole maximum ?

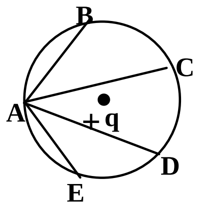


8. The effective capacitance between A and B will be



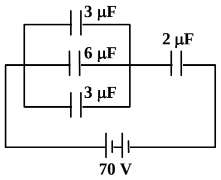
(A) 0.5 μF (B) 1.5 μF
(C) 2 μF (D) 2.5μF

9. In the electric field due to a point charge q, a test charge is carried from A to the points B, C, D and E lying on the same circle around q. The work done is



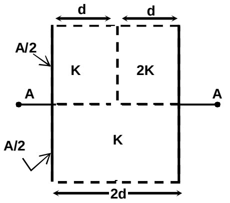
(A) the least along AB
(B) the least along AD
(C) zero along any one of the paths AB, AD, AC and AE
(D) the least along AE.

10. The potential difference across the capacitor of 2μF is



(A) 10 V (B) 60 V
(C) 28 V (D) 56 V

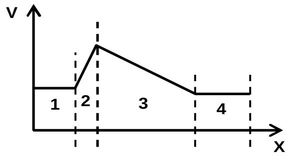
11. The capacitance of capacitor, of plate area A and separation 2d with dielectrics inserted as shown is



(A) $\frac{3k \in_0 A}{2d}$ (B) $\frac{5k \in_0 A}{12A}$
(C) $\frac{7k \in_0 A}{12d}$ (D) $\frac{k \in_0 A}{d}$

12. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved further apart by means of insulating handles. Which statement is ***incorrect*** ?.
(A) The charge on the capacitor increases.
(B) The voltage across the plates increases.
(C) The capacitance decreases.
(D) The electrostatic energy stored in the capacitor increases.

13. In which region magnitude of x component of electric field is maximum, if potential V_s, x graph is as shown



(A) 1 (B) 2
(C) 3 (D) 4

14. The minimum work required to bring charge +q from infinity to the centre of ring, of radius R and charge +Q uniformly distributed over it, is
(A) $\frac{Qq}{4\pi \in_0 R}$ (B) ∞
(C) $\frac{Qq}{8\pi \in_0 R}$ (D) None of these

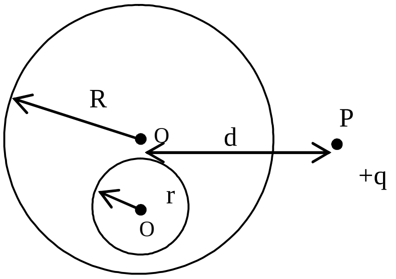
15. Two identical metal plates are given positive charges Q_1 and Q_2 ($<Q_1$) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C. The potential difference between the plates of the capacitor is

(A) $\frac{1}{2}\frac{Q_1 - Q_2}{C}$ (B) $\frac{1}{2}\frac{Q_1 + Q_2}{C}$

(C) $\frac{Q_1 - Q_2}{C}$ (D) $\frac{Q_1 + Q_2}{C}$

Level - II

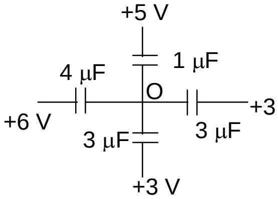
1. A solid conducting spheres of radius R having a spherical cavity of radius r as shown in the fig. Charge +q is distributed uniformly over the outer surface of the sphere. If a point P having charge +q and at distance d from the centre of the sphere, experiences a force F. What will be the force experienced by the point P, if a charge +q is placed at the centre of the cavity assuming charge remains distributed on sphere uniformly



(A) $F(R/r)^2$ (B) 2F

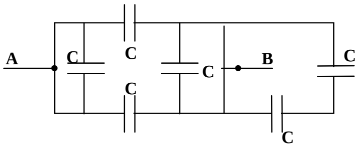
(C) F (D) $2F\left(\frac{d-R}{r}\right)^2$

2. What is the potential at point O,
(A) 4.27 V (B) 17 V
(C) zero (D) 34V

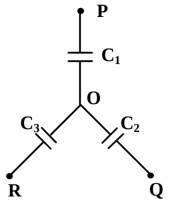


3. In any region the direction of electric field
(A) is always in the direction of increasing potential
(B) is always in the direction of decreasing potential
(C) depends on the nature of the field
(D) is in the direction of decreasing potential for +ve source charge and in the direction of increasing potential for –ve source charge.

4. The equivalent capacitance between point A and B of a combination shown in the figure is
(A) C (B) 2C
(C) C/2 (D) none of these



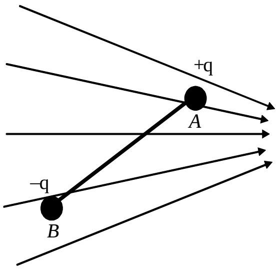
5. Three uncharged capactior of capacities C_1, C_2, C_3 are connected as shown in figure to one another and to points P, Q and R at potentials V_1, V_2 and V_3. The potential at O is



(A) $\frac{V_1+V_2+V_3}{3}$, (B) $\frac{C_1V_1+C_2V_2+C_3V_3}{C_1+C_2+C_3}$

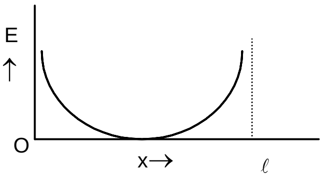
(C) $\frac{C_1V_1+C_2V_2+C_3V_3}{3}$ (D) 0.

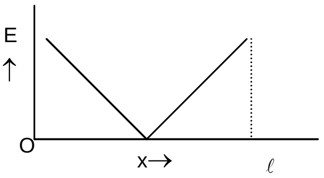
6. Two connected charges of +q and –q respectively are at fixed distance AB apart in a non uniform electric field whose lines of force are shown in the figure

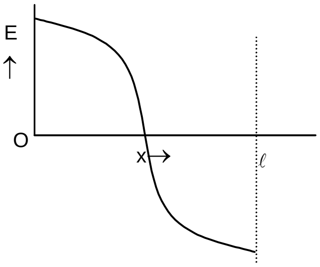


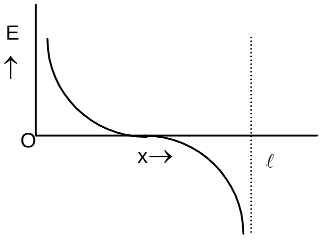
The resultant effect on the two charges is
(A) a torque vector in the plane of the paper and no resultant force.
(B) a resultant force in the plane of the paper and no torque.
(C) a torque vector normal to the plane of the paper and no resultant force.
(D) a torque vector normal to the plane of the paper and a resultant force in the plane of the paper.

7. Two identical point charges are placed at a separation of ℓ. P is a point on the line joining the charges, at a distance x from any one charge. The field at P is E. E is plotted against x for values of x from close to zero to slightly less than ℓ. Which of the following best represents the resulting curve?

(A)


(B)


(C)


(D)


8. The capacitance of two condensers in parallel is four times their capacitance when they are connected in series. The ratio of the individual capacitances will be

(A) 1:2 (B) 1:1
(C) 2:1 (D) 4:1

9. Find the capacitance of a system of three parallel plates each of area A separated by distances d_1 and d_2. The space between them is filled with dielectrics of relative permittivity ε_1 and ε_2. The dielectric constant of free space is ε_o .

(A) $\dfrac{\varepsilon_1\varepsilon_2\varepsilon_0 A}{\varepsilon_1 d_2+\varepsilon_2 d_1}$ (B) $\dfrac{\varepsilon_1\varepsilon_2\varepsilon_0 A}{\varepsilon_1 d_1+\varepsilon_2 d_2}$

(C) $\dfrac{\varepsilon_1\varepsilon_2 A}{\varepsilon_0(\varepsilon_1+\varepsilon_2)(d_1+d_2)}$ (D) $\dfrac{A}{\varepsilon_2\varepsilon_1\varepsilon_0(\varepsilon_1 d_1+\varepsilon_2 d_2)}$

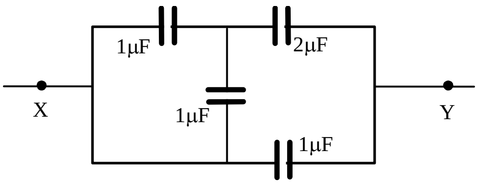
10. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the shell be V. If the shell is now given a charge of –3Q, the new potential difference between the same two surfaces is:

(A) V (B) 2 V
(C) 4 V (D) –2V

11. A parallel plate capacitor is connected across a source of constant potential difference. When a dielectric plate is introduced between the two plates then:

(A) some charge from the capacitor will flow back into the source
(B) some extra charge from the source will flow into the capacitor
(C) the electric field intensity between the two plates increases.
(D) energy of the capacitor does not change.

12. The figure shows a circuit of four capacitors. The effective capacitance between X and Y is

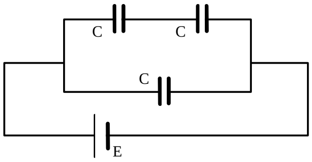


(A) 2 μF (B) 1 μF
(C) 3 μF (D) 1.5 μF

13. Two identical thin rings, each of radius R meters, are coaxially placed a distance R meters apart. If Q_1 C. and Q_2 C are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is

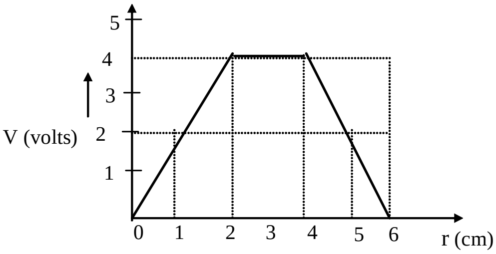
(A) zero
(B) $q(Q_1 - Q_2)(\sqrt{2} - 1) / (\sqrt{2}\, 4\pi\varepsilon_o R)$
(C) $q\sqrt{2}\,(Q_1 + Q_2) / (4\pi\varepsilon_o R)$
(D) $q(Q_1 + Q_2)(\sqrt{2} + 1) / (\sqrt{2}\, 4\pi\varepsilon_o R)$

14. Three capacitors are connected with the source of electromotive force E as shown in the figure. Then the energy drawn from the source is



(A) $\frac{1}{2}CE^2$ (B) $\frac{3}{2}CE^2$
(C) $\frac{1}{4}CE^2$ (D) $2CE^2$

15. The variation of potential with distance r from a fixed point is shown in figure. The electric field at r = 3 cm and r = 5 cm are, respectively,



(A) 0, 2 V/cm
(B) 2 V/cm, – 2 V/cm
(C) 0, –2 V/cm
(D) 2 V/cm, 0

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – I

1. $E = \frac{1}{4\pi\ \varepsilon_0}.\frac{q}{a^2}$ in the direction CO
2. $-\frac{q_1}{2\sqrt{2}}$
3. ES sin θ
4. $\frac{2\pi\varepsilon_0 mg\ell\sin\theta \quad \tan\theta}{q}$
5. $\frac{Q}{4\pi^2\varepsilon_0 R^2}\sin\left(\frac{\theta}{2}\right)$
6. 90 V
7. 0.37 m/s
8. $q = \frac{Q}{2}$ $\frac{Q^2}{16\pi\in_0 r}$.
9. (a) 800 μC, 800 V, 800 μC, 400 V. (b) 1600/3 V, 1600/3 μC, $\frac{3200}{3}$ μC
10. $\frac{A}{2d}\varepsilon_0(1+\varepsilon_r)$, $\frac{2\varepsilon_0\varepsilon_r A}{d(1+\varepsilon_r)}$

Level – II

1. 0.44 mm
2. 20 V
3. (a) $q_1 = q_2 = 3.3 \times 10^{-4}$ C, $q_3 = 4 \times 10^{-4}$ C, (b) $V_1 = 33.3$ V, $V_2 = 67.7$ V, $V_3 = 100$ V
   (c) $U_1 = 5.6 \times 10^{-3}$J, $U_2 = 11.9 \times 10^{-3}$ J, $U_3 = 2 \times 10^{-2}$J
4. 26 Volts
5. $\frac{1}{2}CE_2^2$
6. $\frac{\varepsilon_0 a^2\alpha}{\ln\left(1+\frac{\alpha d}{k_0}\right)}$
7. (a) $\left(\frac{C_1C_2}{C_1+C_2}\right)V$ (b) Zero on C_2, C_1V on C_1
8. $\sigma_{inner} = -\frac{q_0}{12\pi a^2}$, $\sigma_{outer} = -\frac{q_0}{96\pi a^2}$, $\frac{q_0}{24\pi\varepsilon_0 a}$
10. $\frac{Qd}{3\varepsilon_0 A}$
12. $\frac{Q(q+Q/2)}{4\pi\varepsilon_0}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$

Objective:

Level - I

1.	**A**	2.	**C**
3.	**C**	4.	**A**
5.	**B**	6.	**C**
7.	**A**	8.	**C**
9.	**C**	10.	**B**
11.	**C**	12.	**A**
13.	**B**	14.	**A**
15.	**A**		

Level - II

1.	B	2.	A
3.	B	4.	B
5.	**B**	6.	**D**
7.	**D**	8.	**B**
9.	**A**	10.	**A**
11.	**B**	12.	**A**
13.	**B**	14.	**B**
15.	**A**		

Learnaf
Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

ELECTROSTATICS

Electrostatics

sdadlmass

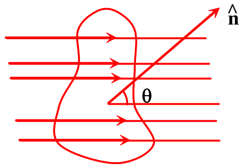
Syllabus for IITJEE and Maharashtra Board:

Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Conductors and insulators, presence of free charges and bound charges inside a conductor; Dielectrics and electric polarization, general concept of a capacitor and capacitance, combination of capacitors in series and in parallel, energy stored in a capacitor, capacitance of a parallel plate capacitor with and without dielectric medium between the plates; Van de Graaff generator.

ELECTRIC FLUX

The number of force lines passing perpendicularly through a closed surface placed in an electric field is called electric flux.



Mathematically,

$$\phi_e = \oint_s \vec{E}\cdot d\vec{s} = \oint_s \vec{E}\cdot \hat{n}\, dA = \oint_s E\cos\theta\, dA$$

Flux is also associated with all vector fields [e.g. magnetic field, gravitational field, etc.].
$\Rightarrow \Sigma Q_{in} = 0 \Rightarrow \phi_e = 0$, but electric field may or may not be zero.

If $\theta < 90^0$ then associated flux is positive and $\vec{E}$ lines emerge out of the area element.
If $\theta > 90^0$, then associated flux is negative and $\vec{E}$ lines enter the area element
If $\theta = 90^0$ then associated flux is zero and no $\vec{E}$ lines cross the area element normally.

As flu× measures $\vec{E}$ lines crossing the given surface normally therefore it is also called normal electric induction.

Gauss' Law

The electric flux ϕ_e coming out of any closed surface is equal to $1/\varepsilon_o$ times the total charge inside that surface, i.e.

$$\phi_e = \oint_s \vec{E}\cdot d\vec{s} = \frac{\sum Q_{in}}{\varepsilon_o}$$ ($\oint$ represents an integral taken over a closed surface)

Gauss's law is comparable to Coulomb's law, i.e. Coulomb's law can be derived from Gauss' law or vice-versa.

Gaussian Surface

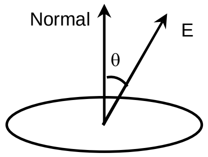
The hypothetical closed curve equipotential surface perpendicular to the electric field and having electric field $\vec{E}$ of constant magnitude through the surface is called Gaussian surface.

Symmetry	Nature of Gaussian surface
Spherical	Concentric sphere
Cylindrical	Co-axial cylinder
Plane	Pull-box which straddles the surface

Gauss's law can be used to evaluate electric field if the charge distribution is so symmetric that by proper choice of Gaussian surface we can easily evaluate the integral.

Gauss's Law

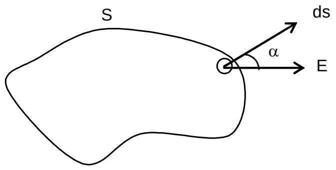
The net "flow" of electric field through a closed surface depends on the net amount of electric charge contained within the surface. This " flow" is described in terms of the electric flux through a surface, which is the product of the surface area and the component of electric field perpendicular to the surface.



Flux is a scalar quantity and is added on per scalar addition rules. For non-uniform field and / or surfaces which are not plane,

$$\phi = \oint \vec{E}.d\vec{s}$$

Direction of an area element is taken along its normal. Hence area can be treated as a vector quantity.



Gauss's law states that the total electric flux through a closed surface is proportional to the total electric charge enclosed within the surface. This law is useful in calculating fields caused by charge distributions that have various symmetry properties.

Mathematically $\oint \overrightarrow{E}.\overrightarrow{ds} = \frac{q}{\varepsilon_0}$.

$\oint \rightarrow$ means integral done over a closed surface.

Gauss's law can be used to evaluate Electric field if the charge distribution is so symmetric that by proper choice of a Gaussian surface we can easily evaluate the above integral.

In other words,

TNEI = $\frac{\Sigma q_{in}}{\varepsilon_0}$

TNEI means total normal electric induction.

Note: Some important aspects of application of Gauss's theorem

1. The surface which is chosen for application of Gausses law is called a Gausian surface. Any Gaussian surface can be chosen but it should not pass through a discrete charge. However, it can pass through a continuous charge distribution.

2. Gauss's theorem is mostly used for symmetric charge distribution.

3. The term q on the right side of equation (q / ε_0) includes sum of all charges enclosed by the surface. Outside charges are not included.

4. The electric field appearing on left (E) is the electric field due to all charges - inside or outside the closed surface.

   Gauss's theorem can be applied to continuous charge distribution. It may be linear ($\lambda = \frac{\Delta Q_0}{\Delta \ell}$), surface charge density ($\sigma = \frac{\Delta Q}{\Delta A}$) and volume charge density ($\rho = \frac{\Delta Q}{\Delta v}$)
   Integration can be performed all over the surface after putting the value of q accordingly.

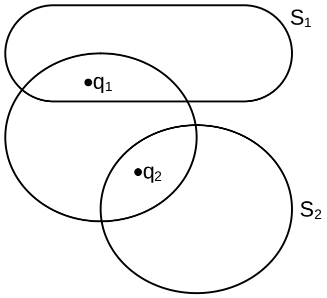
Illustration 1. *In a space a uniform electric field exists which is given by $\vec{E} = E_0\hat{j}$ and square plate of side a is oriented in three different wrap.*
(a) parallel to xy plane
(b) parallel to yz plane
(c) parallel to xz plane
Calculate electric flux linked with the plate in each case.

Solution: $\phi = \vec{E}.\Delta\vec{s}$
(i) $\phi = E_0\,\hat{i}.a^2\hat{k} = 0$
(ii) $\phi = E_0\,\hat{i}.a^2\hat{j} = E_0a^2$
(iii) $\phi = E_0\,\hat{i}.a^2\hat{j} = 0$

Illustration 2. *If Coulomb's law does not obey inverse square law does Gauss theorem hold good.*

Solution: Validity of Gauss theorem is based on the inverse square law therefore any departure would amount to failure.

Illustration 3. In the diagram below shows three Guassian surfaces and the position of point charges. The total electric flux (TNEI) associate with S_3 is



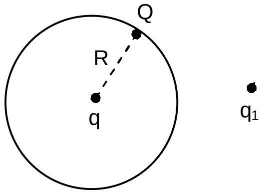
(A) q_1/ε_0 (B) zero
(C) q_2/ε_0 (D) $\frac{q_1 + q_2}{\varepsilon_0}$

Solution: Acceleration to Gauss theorem

$$\oint \vec{E}.d\vec{s} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

For flux linked with the S_3 will be equal to the $1/\varepsilon_0 \times$ total charge enclosed by the S_2
Thus the (D) option is correct.

Illustration 3. *A thin metallic shell contains charge Q on it. A point charge q is placed at the centre of the shell and another charge q_1 is placed outside it as shown in the adjacent figure. All charges are positive. The force on the charge at the centre is*



(A) towards left *(B) towards right*
(C) upward *(D) zero*

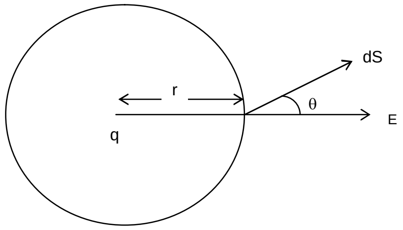
Solution: **D**

Exercise:
(i) Calculate TNEI linked with a cube if a point charge is placed
(a) at its centre
(b) any where with in the cube

(ii) Calculate TNEI with a face of a cube if a point charge is placed
(a) at its center
(b) at its vertex (any corner)

Proof of Gauss Theorem: (Flux due to internal charge)

Consider a point charge enclosed by a Gaussian surface of arbitrary shape.



Electric flux linked with the elemental area

$$d\phi = \vec{E}.d\vec{s}$$
$$= E\, ds \cos\theta$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} ds\cos\theta$$

From Colulomb's law

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

By the definition of solid angle,

$$\frac{ds\cos\theta}{r^2} = d\Omega$$

$$\therefore \quad d\phi = \frac{1}{4\pi\varepsilon_0} 9 d\Omega$$

Therefore total electric flux with entire Gaussian surface.

$$\oint \vec{E}.d\vec{s} = \frac{q}{4\pi\varepsilon_0}\oint d\Omega = \frac{q}{4\pi\varepsilon_0}4\Omega = \frac{q}{\varepsilon_0}$$

As total solid angle for any 3D surface is $4\Omega\ \delta r$

Flux due to external charge

$$d\phi = \vec{E}.d\vec{s}\cos\theta$$
$$= \frac{q}{4\pi\varepsilon_0}\frac{ds\cos\theta}{r^2}$$
$$d\phi = \frac{q}{4\pi\varepsilon_0}d\Omega$$
$$\oint d\phi = \oint \vec{E}.d\vec{s} = \frac{q}{4\pi\varepsilon_0}\oint d\Omega = 0$$

Flux due to system of charges, (few inside and remaining out sides)

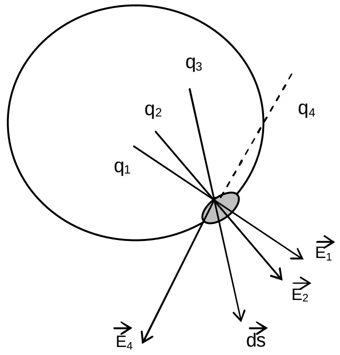
At any elemental area verticals resultant electric field is [From the superposition of electric fields]

$$d\phi = \vec{E}.d\vec{s}$$

$$\oint d\phi = \vec{E}.d\vec{s}$$

$$= \frac{q_1 + q_2}{\varepsilon_0}$$

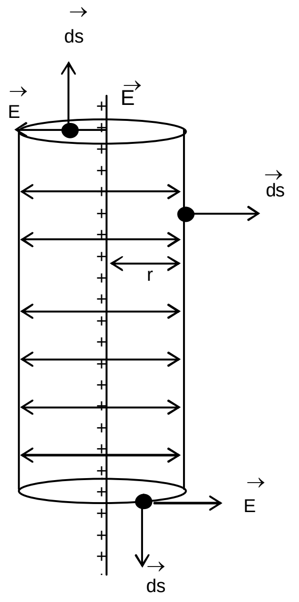
As outside charge do not contribute to net electric flux.



Application of Gauss theorem:

$\vec{E}$ due to a line charge (chare uniformly distributed and long)

Consider ℓ length of a Gussian surface of cylindrical geometry as $d\vec{s} + \vec{E}$ is cylindrically symmetries.



According to Gauss theorem

$$d\vec{s} = \oint \vec{E}.ds = \frac{\Sigma q_{in}}{\varepsilon_0}$$

$$\oint_{CS} \vec{E}.ds + \oint_{uS} \vec{E}.ds + \oint_{RS} \vec{E}.ds = \frac{\lambda \ell}{\varepsilon_0}$$

$$E \oint_{cs} ds = \frac{\lambda \ell}{\varepsilon_0}$$

$$E\ 2\pi r\ell = \frac{\lambda \ell}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$$

Chare density is $\lambda = \frac{dq}{d\ell}$

Its SI unit is m^{-1}

CS - curved surface

VS - upper surface

LS – lower surface

In vector form $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$

Note:

(i) If there is negative charge distributed on the line then $\vec{E}$ lines will be terminating normally on the line. And expression for $\vec{E}$ is vector form

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{-\lambda\ell}{r^2}\right)\vec{r}$$

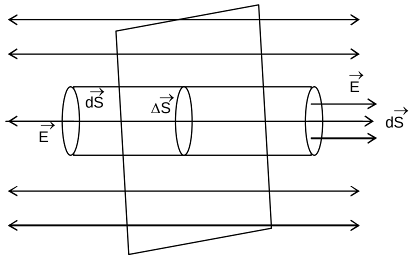
(ii) Electric field intensity at a point outside the uniformly charged cylinder is same as that for a line charge.

2. $\vec{E}$ due to infinitely large surface charged distribution.

Surface charge distribution is defined by surface charge density

$\sigma = \frac{dq}{ds}$

SI unit cm^{-2} electric field is having plane symmetry. Therefore choosing a Gaussian surface of cylindrical shape on both the side of plane sheet of charge.



According to Gauss theorem

$$\int \vec{E} \, . \, d\vec{s} \, = \frac{\Sigma q_{in}}{\varepsilon_0}$$

$$\int_{cs} \vec{E} \cdot ds + \int_{Ls} \vec{E} \cdot ds + \int_{Rs} \vec{E} \cdot ds = \frac{\Sigma qm}{\varepsilon_0}$$

At curved surface ($\vec{E} \perp d\vec{s}$), At left surface ($\vec{E} \,||\, d\vec{s}$), At right surface ($\vec{E} \,||\, d\vec{s}$)

$$0 + E \int ds + E \int ds = \frac{\sigma \Delta s}{\varepsilon_0}$$

$$2E\Delta s = \frac{\sigma DS}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Vector form,

$$\vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{n}$$

where $\hat{n}$ is unit vector normal to the plate.

If charge distribution is negative then

$$\vec{E} = -\frac{\sigma}{2\varepsilon_0} \hat{n}$$

Illustration 4. *A very long straight threed carrying uniform charge density (λ) = 4μC m^{-1}, along y –axis. Calculate intensity of electric field (only magnitude) at a point 8 mm along x-axis.*

Solution: $E = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{r}$

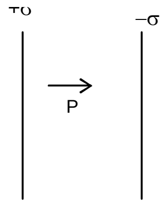
$= 9 \times 10^9 \times \frac{2 \times 4 \times 10^{-6}}{8 \times 10^{-3}}$

$= 9 \times 10^6$ Vm^{-1}

Illustration 5. *A dipole is lying between two large parallel plates carrying equal and opposite surface charge densities as shown in figure.*

(i) Find the force torque and potential energy in this case.

(ii) If dipole is rotated to such an orientation where angular acceleration is maximum (if released). Find the same.

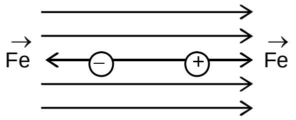


Solution : Between the plates electric field will be the superposition of electric fields to due to +ve and –ve lates both

$\vec{E} = E_+ + \vec{E}_-$

$= \frac{\sigma}{2\varepsilon_0}\hat{n} + \frac{\sigma}{2\varepsilon_0}(-\hat{n}) \;\; = \frac{\sigma}{\varepsilon_0}\hat{n}$

so electric field is uniform and is directed from +ve late to the –ve plate.



$\vec{P} \parallel \hat{n}$

(i) In uniform electric field net force on the dipole is zero. As the two electric force (Fe) are directed along the same line (i.e. the direction of $\vec{E}$) no couple is formed and therefore torque is zero.

For potential energy U = $\vec{P}.\vec{E}$

$= - \vec{P}.\frac{\sigma}{\varepsilon_0}\hat{n}$

$= - \frac{P\sigma}{\varepsilon_0}$

Electric potential energy is minimum and neither the force nor the torque acts this position of stable equilibrium.

(ii) Obviously angular acceleration will be maximum when torque on the dipole is maximum.

$\tau = P\,E \sin\theta$ i.e. $\tau_{max} = P.E.$

i.e. $\theta = 90^0$

Net force on the dipole is again zero. Electric potential energy u = $-PE\cos\theta = 0$

Illustration 6. *Between two plates carrying charges + σ and – σ there is a simple pendulum suspended from an insulating hook and thread bob of the pendulum is heavy metallic sphere electrically the motio of the pendulum will be*

(a) in left direction only.

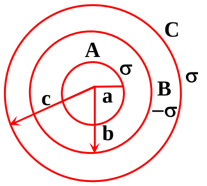
(b) in light direction only

(c) harmonic motion but not SHM

(d) SHM

Solution: (C) Harmonic motion but non SHM.

Illustration 6. *The adjacent figure shows charged spherical shells A, B and C having charge densities σ, $-\sigma$, σ and radii a, b, c, respectively. If $V_A = V_C$, then b equals to*



(A) a + c *(B)* $\sqrt{a^2 + c^2}$

(C) $\sqrt{ac}$ *(D) none of these*

Solution: **(D).** $V_A = \frac{\sigma}{\varepsilon_o}(a - b + c)$

$V_c = \frac{\sigma}{\varepsilon_o}\left(\frac{a^2 - b^2 + c^2}{c}\right)$

Since $V_A = V_C$, therefore

$b = c - a$

Illustration 6. *Three infinitely charged sheets are kept parallel to x–y plane having charge densities as shown in the adjacent figure. The value of electric field at 'P' is*

(A) $\frac{-4\sigma}{\varepsilon_0}\hat{k}$ *(B)* $\frac{4\sigma}{\varepsilon_0}\hat{k}$

(C) $\frac{-2\sigma}{\varepsilon_0}\hat{k}$ *(D)* $\frac{2\sigma}{\varepsilon_0}\hat{k}$

Solution: **(C).**

$$\vec{E}_P = \frac{\sigma}{2\varepsilon_0}\left(-\hat{k}\right) + \frac{(-2\sigma)}{2\varepsilon_0}\left(\hat{k}\right) + \frac{(-\sigma)}{2\varepsilon_0}\left(\hat{k}\right)$$

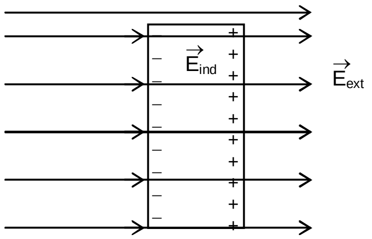
$$= \frac{-2\sigma}{\varepsilon_0}\hat{k}$$

Exercise:

(i) *Calculate force on an electric dipole P placed perpendicular to an uniformly charged straight wire λ at a distance much larger than the dimension of dipole.*
(ii) *Also calculate eth torque on the dipole.*
(iii) *Recognize the equi-potential surface for a line charge and plane sheet of charge.*

Electrostatic of conductor:

In a good conductor i.e. metal outermost electron (valance) are in very large number move randomly around fixed positive ions. Which an electric field is applied across a conductor body free electrons redistribute under the action of external field and then and internal electric field is set up.



As there is plenty of availability of free electron $\vec{E}_{ind}$ (induced electric field) soon becomes equal and opposite to external electric field and m equilibrium.

$$\vec{E}_{ext} + \vec{E}_{ind} = 0$$

then no further redistribution takes place and hence electrostatic condition are restored.

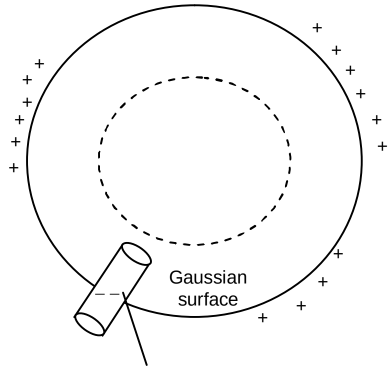
Thus within the conductor, electrostatic field is zero.

From the relation, $E = -\frac{dv}{dr} = 0$

$\Rightarrow$ V is constant

i.e. Electric potential is constant throughout the interior region of a conductor and is equal to the potential at its surface [equi-potential' and hence no work is done in transferring a point charge from its one point to other. No $\vec{E}$ lines penetrate the volume conductor. If a conductor is given charge if reside at its surface, electric field intensity vector is normal to the surface at every point as field in any other direction will produce a tangential.

Component which moves the charge along its surface and thus vibrates the c condition of electrostatics. Now, consider a conductor body which is given some excess charge it side at its surface this can be proved by Gauss theorem



$$\oint \vec{E}.d\vec{s} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

$$\int_{\text{within the cord}} \vec{E}\cdot ds + \int_{\text{outside the cord}} \vec{E}\cdot ds = \frac{\sigma \Delta S}{\varepsilon_0}$$

At curved surface ($\vec{E} \perp d\vec{s}$), At left surface ($\vec{E} \,||\, d\vec{s}$,

At right surface ($\vec{E} \,||\, d\vec{s}$)

E = $\frac{\sigma}{\varepsilon_0}$

In vector form, $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

When $\hat{n}$ is unit vector normal (outward)

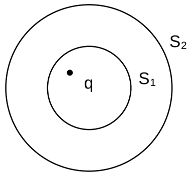
Concept of electrostatic shielding:

Note: Shielding does not work other way round i.e. if a charge is placed inside cavity outer space beyond the conductor is not shielded but if we put any amount of changed on the outer surface of conductor the volume inside the cavity is completely shielded.

This is the leagon a person while lightening should its inside a car instead of roaming in open.

Illustration 7. *A sphere S_1 of radius r_1 encloses a total charge q. If there is another concentric sphere S_2 of radius r_2 ($r_2 > r_1$) and there is no additional charges between S_1 and S_2. Find the ratio of electric flux through S_1 and S_2. How will the electric flux through S_1 charge if a medium of dielectric constant k is introduced in the space inside S_2 and out side S_1 and an additional charge 2q is placed between them.*

Solution: Acceleration to Gauss theorem



$\phi_1 = \frac{q}{\varepsilon_0}$ and $\phi_2 = \frac{q}{\varepsilon_0}$

$\therefore \quad \frac{\phi_1}{\phi_2} = \frac{1}{1}$

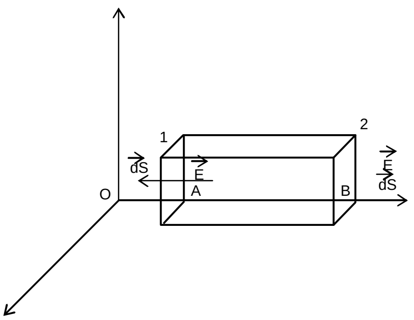
If we put 2q charge in addition to q total electric flux linked with S_2 will be

$\oint \vec{E}.d\vec{s} = \frac{\Sigma q_{in}}{\varepsilon_0}$

$\vec{E} = \frac{\vec{E}}{k}$ and $\Sigma\, q_m = q - 2q = 3q$

$\therefore$ new flux with $S_2 = \frac{3q}{\varepsilon_0 k}$

Illustration 8. *Calculate total charge enclosed by the cuboid as shown in figure if there exists an electric field whose component are*
$E_x = (100\ NC^{-1}\ m)\ \sqrt{x}$
$E_y = 0$
$E_z = 0$
(Given OA = a, AB = 2a, a = 0.1 m)



Solution: According to Gauss theorem,

$$\oint \vec{E} \cdot d\vec{s} = \frac{\Sigma q_{in}}{\varepsilon_0}$$

As electric field is along x-axis therefore only faces 1 and 2 will be having some flux and all other faces will be contributing zero flux as $\theta\ (\vec{E}.d\vec{s}) = 90^0$

flux linked with face 1 $= E_x\ a^2 \cos 180^0$

$= 100\ \sqrt{a}a^2(-1)$

$= -\ (100)\ (a^{5/2})$

Flux linked with face 2 $= 100\ \sqrt{3}aa^2 \cos 0$

$(100)\sqrt{3}\ \ a^{5/2}$

$\therefore$ total flux with the cuboid $= 100\ a^{5/2}\ (\sqrt{3}-1) = 0.23$ V.m.

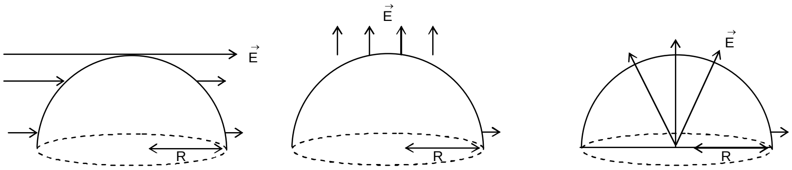
Now Σ qm = ε_0 total flux

$= 8.85 \times 10^{-12} \times 0.23 = 2.05 \times 10^{-12}$ C

Illustration 9. *A point charge causes TNEI of - 1.0 kV.m to pass through cylindrical Gaussian surface of length 50 cm and radius 10.0 cm.*
(a) If both the dimensions of cylindrical surface are doubled. How much flux would pass through the surface.
(b) What is the value of the point charge.

Solution: (a) As long as enclosed charge remains same the flux passing through a surface is constant whatever be its shape and size.
(b) Acceleration to Gauss theorem = (total flux)

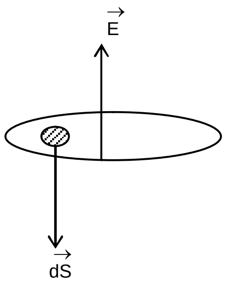
Illustration 10. *Find the electric flux linked with the curved surface in the three cases shown in figure.*



(a) (b) (c)

Solution: (a) Shown hemisphere has two surfaces.
(i) Curved surface and (ii) plane surface (PS)
As it does not contain any charge therefore flux is zero in accordance with the Gauss theorem.

$$\oint \vec{E}\cdot d\vec{s} = \frac{\Sigma q_{in}}{\varepsilon_0} = 0$$
$$\oint \vec{E}\cdot d\vec{s} = \int_{PS} \vec{E}.d\vec{s} + \int_{cs} \vec{E}.d\vec{s} = 0$$
$$0 = \int_{Ps} Eds\cos 180 + \int_{cs} \vec{E}.d\vec{s}$$
$$\int_{cs} \vec{E}.d\vec{s} = \int_{Ps} Eds = E\int_{Ps} ds = E.\pi R^2$$



(c) If electric field is radial then $\vec{E} \parallel d\vec{s}$ every where on the curved surface

$$\int_{cs} \vec{E}.d\vec{s} = \int_{Ps} Eds\cos\theta = E\int_{Ps} ds.i = E.2\pi R^2$$

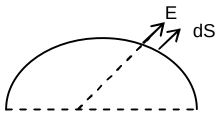
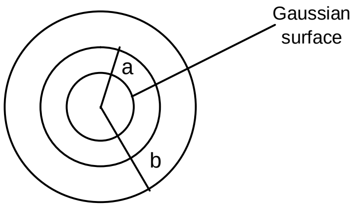


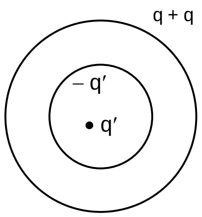
Illustration 11. *A spherical conducting shell of inner and outer radius a and b has a charge q. Find (i) Electric field inside the cavity is zero.*
(ii) a charge q is placed at the cente of the shell. What is the surface charged density on the inner and the outer surfaces of the shell ?
(iii) Is the electric field inside the cavity (which contains no charge) zero, even if the shell is not spherical, but has any irregular shape?

Solution: (i) Acceleration to Gauss theorem

$$\int \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$



As all given charges reside on the outer surface therefore $\Sigma q_{in} = 0$
(within the cavity)
$\therefore$ Flux zero $E = \vec{0}$ every whose inside the cavity.
(ii) When a charge q' is suspended inside the cavity by the process of induction charges –q charge on is induced at the inner surface of cavity and +q′ is induced at the surface of cavity.



(iii) If the shape of conductor is irregular change distribution (Density) on the outer surface but no charge inside the cavity and therefore $\vec{E}$ inside the cavity is still zero.

Illustration 12. *An uncharged metallic sphere is brought close to conductor body at the surface of which positive charnge is non uniformly distributed the electric force on the sphere will be*
(A) towards the plate *(B) away fro the plate*
(C) parallel to the plate *(D) zero*

Solution: *Due to induction equal and opposite charges are developed and the opposite aide of sphere*
As $|\vec{F}_A| > |\vec{F}_R|$
sphere is attracted towards the plate

Hence (A) is correct choice.

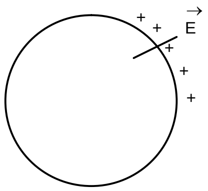


Illustration 13: *A cube of side a may enclose an uniformly an uniformly charged rod in any orientation (even partially) carrying change density λ, then the cubical box will be linked with*

(A) the maximum flux of $\dfrac{\sqrt{3}\,a\lambda}{\varepsilon_0}$

(B) any amount of flux upto $\dfrac{\sqrt{3}a\lambda}{\varepsilon_0}$

(C) any amount of flux upto $\dfrac{\sqrt{2}a\lambda}{\varepsilon_0}$

(D) any amount of flux more than zero up to a maximum value of $\dfrac{\sqrt{3}a\lambda}{\varepsilon_0}$

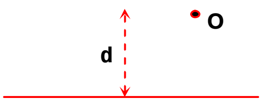
Solution: Maximum flux will be linked with the cube when changed red fits the diagonally opposite points is its $a\sqrt{3}$ length lies inside the cube and according to Gauss theorem

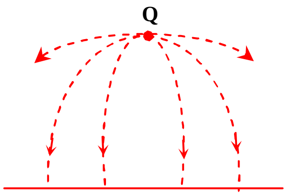
$\oint \vec{E}.d\vec{s} = \dfrac{\sigma q_{in}}{\varepsilon_0} = \dfrac{\lambda a\sqrt{3}}{\varepsilon_0}$, so option (A) is correct.

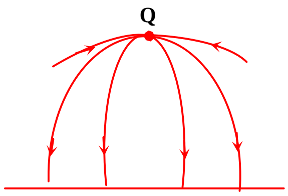
Now rod can be fitted in any orientation even partially but certainly some part inside the cube there fore flux will be necessarily more than zero upto if maximum value.

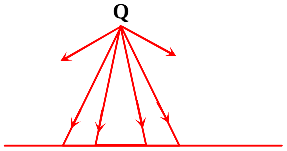
So option (D) is also correct.

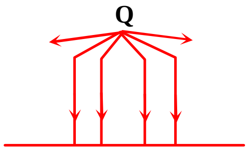
4. A charge Q is fixed at a distance d in front of an infinite metal plate. The lines of force are represented by



(A)


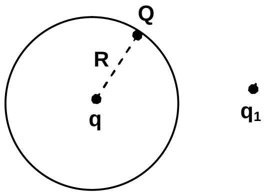
(B)


(C)


(D)


Sol. **(A).** Metal plate acts as an equipotential surface, therefore the field line should be normal to the surface of the metal plate.

5. A thin metallic shell contains charge Q on it. A point charge q is placed at the centre of the shell and another charge q_1 is placed outside it as shown in the adjacent figure. All charges are positive. The force on the charge at the centre is

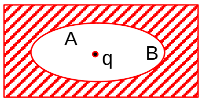


(A) towards left (B) towards right

(C) upward (D) zero

Sol. **(D).**

24. *An elliptical cavity is made within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the adjacent figure. Then,*



(A) electric field near A in the cavity = electric field near B in the cavity
(B) charge density at A = charge density at B
(C) potential at A = potential at B
(D) total electric field flux through the surface of the cavity is q/ε_0

Sol. ***(C) and (D)***

25. *A non-conducting solid sphere of radius R is uniformly charged. The magnitude of electric field due to the sphere at a distance r from its centre*
(A) increases as r increases, for r < R
(B) decreases as r increases, for $0 < r < \infty$
(C) decreases as r increases, for $R < r < \infty$
(D) is discontinuous at r = R

Sol. ***(A) and (C).***

$$|\vec{E}| = \begin{cases} \dfrac{kQ}{r^2} & \text{if } \infty > r > R \\ \dfrac{kQ}{R^3} & \text{if } 0 < r < R \end{cases}$$

Exercise: ***(i)*** *Draw $\vec{E}$ lines near and on the charged cubical body when it is placed inside an uniform external electric field pointing vertically downwards.*
(ii) *If 100 $\vec{E}$ lines are incident upon a dielectric cylindrical ros, then how many lines would pass through the cylinder if at all they do.*
(iii) *Calculate energy in the form of electric field set up by two lare plate carrying $+\sigma_1$ and $-\sigma_2$ changes A completely fill the gap*

Learnaf
Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

CURRENT ELECTRICITY

Current Electricity

Syllabus for IITJEE & Maharashtra Board:

Electric current, flow of electric charges in a metallic conductor, drift velocity and mobility and their relation with electric current; Ohm's law, electrical resistance, V–I characteristics, Exceptions of Ohm's law (Now–linear V–I characteristics), electrical resistivity and conductivity, classification of materials in terms of conductivity; Superconductivity (elementary idea); Carbon resistors, colour code for carbon resistors; combination of resistances–series and parallel. Temperature dependence of resistance. Internal resistance of a cell, Potential difference and emf of a cell, combination of resistances and cells in series and in parallel.

Kirchoff's laws – illustration by simple applications, Wheatstone bridge and its applications for temperature measurements, metre bridge – special case of Wheatstone bridge.

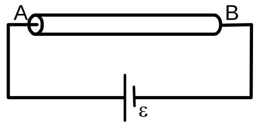
Potentiometer – principle and applications to measure potential difference and for comparing emf of two cells.

Electric power, thermal effects of current and Joule's law; Chemical effects of current–Faraday's laws of electrolysis; Electro–chemical cells–Primary and secondary cells, solid state cells.

Thermoelectricity–origin elementary ideas of Seebeck effect, Thermocouple, Thermo emf, neutral and inversion temperatures. Measurement of temperature using a thermo couple.

ELECTRIC CURRENT

Flow of electric charge constitutes electric current. For a given conductor AB, if 'δQ' charge flows through a cross–section of area A in time 'δt', then the electric current through the conductor AB, is given as



$$I = \frac{\delta Q}{\delta t}$$

The current so defined above, is the average current over the period δt. The instantaneous current is given as $I = \frac{dQ}{dt}$

Direction of electric current as defined above will be taken along the direction of flow of positive charge (although in majority of conductors the charge carrier is electron which is negatively charged and hence electric current would be in a direction opposite to that of flow of electrons).

Despite the direction that we associate with is electric current is not a vector quantity. Instead, we choose current density (j), that is current flowing through unit area of the cross–section, as a vector quantity.

Unit of Electric Current

The SI unit of electric current is ampere. It is denoted by A.

$$1 \text{ ampere (A)} = \frac{1\,\text{coulomb (C)}}{1\,\text{second (s)}} = 1 \text{ coulomb / second}$$

Current Density ($\vec{J}$)

To describe the flow of charge through a cross section of the conductor at a particular point, we use the term current density $\vec{J}$. The direction of flow of positive charge is same as that of the direction of the electric field E, and the direction of flow of –ve charge is opposite to the direction of the electric field. The direction of current density $\vec{J}$ is same as that of the electric field.

For each element of cross–section, the magnitude of current density J is equal to the current per unit area through that element.

The total current through the conductor is given as

$$i = \int \vec{J}.d\vec{A} \qquad \ldots (i)$$

where $d\vec{A}$ is the area vector of the element, perpendicular to it

If the current is uniform across the surface and parallel to $d\vec{A}$, then current density $\vec{J}$ is also uniform and parallel to area vector $d\vec{A}$. Hence, from equation (i)

$$i = \int \vec{J}d\vec{A} = JA$$

or $\quad J = i/A$

where A is the total cross–sectional area of the surface. S.I. unit for current density is the ampere per square meter (A/m^2).

Exercise 1:
(i) What is the relation between a coulomb and an ampere?
(ii) Why is current a scalar?

Illustration 1. *How many electrons pass through a wire in 1 minute if the current passing through the wire is 200 mA ?*

Solution: $I = \frac{q}{t} = \frac{ne}{t}$

or, $\quad n = \frac{It}{e}$

or, $\quad n = \frac{200\times10^{-3}\times60}{1.6\times10^{-19}} = 7.5 \times 10^{19}$.

Illustration 2. *One billion electrons pass through a conductor AB from end A to end B in 1 ms. What is the direction and magnitude of current?*

Solution : $i = \frac{Ne}{t} = \frac{(10^9)(1.6x10^{-19}C)}{10^{-3}} = 1.6x10^{-7}A$

$\Rightarrow$ i = 0.16 μ A. The current flows from B to A.

Illustration 3. *A particle having charge q coulomb describes a circular orbit. If radius of the orbit is R and frequency of the particle is f, then find the current in the orbit.*

Solution: Through any section of the orbit, the charge passes f times in one second. Therefore, through that section total charge passing in one second is fq. By definition i = fq.

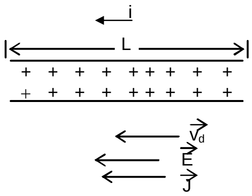
MECHANISM OF CURRENT FLOW IN METALLIC CONDUCTORS

Flow of current in metals is due to preferential flow of free electrons. In the absence of any externally applied emf (by means of a battery), the free electrons move randomly through the metal from one point to another giving zero net current.

When connected to a battery, the free electrons get accelerated due to the electric field (set up by the battery) and they gain velocity and energy. However, the passage is not smooth and the electrons collide with the lattice ions in which the ultimate gainer (of energy) is the ion. As we know the temperature of a body is related with the energy of vibrations of these ions, these collisions result in increase in temperature of the metal. The loss of energy of electrons in collision and their acceleration by the electric field, finally, results in drifting of electrons in a particular direction. (Although the actual motion of electrons is erratic, the overall effect is of drifting of electrons)

Drift Speed

When no current flows through a conductor, its conduction electrons move randomly, with no net motion in any direction. When the conductor does have a current through it, these electrons still move randomly but now they tend to drift with a drift speed v_d in the direction opposite to that of the applied electric field which causes the current. The drift speed is small compared to the speed in the random motion of the electrons.



Let us assume that the positive charge carriers move with the same drift speed v_d across the wire's cross–sectional area A as shown in the figure.

The number of charge carriers in a length L of the wire is nAL, where n is the number of charge carriers per unit volume. The total charge of the carriers, each with charge e, in the length L is

$$q = (nAL)e$$

Since all charge carriers move along the wire with speed v_d, therefore total charge moves through any cross section of the wire in the time interval,

$$t = \frac{L}{v_d}$$

$\therefore$ Current (i) = $\dfrac{q}{t}$

or, $i = nAev_d$

or , $v_d = \dfrac{i}{nAe}$

or, $\vec{v}_d = \dfrac{i}{nAe} = \dfrac{\vec{J}}{ne}$

Exercise 2: ***A bulb is connected by a pair of long straight conductors of effective resistance R, with a source of e.m.f. ε. What time after turning on the switch the bulb glows and after what time the electrons from the source reaches the bulb? Answer it qualitatively.***

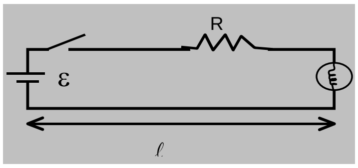


Illustration 4. *What is the drift velocity of electrons in a copper conductor having a cross–sectional area of 5×10^{-6} m^2 if the current is 10 A? Assume that there are 8×10^{28} electrons/m^3.*

Solution: Given that,

$A = 5 \times 10^{-6}$ m^2, I = 10 A, and n = 8×10^{28} electrons / m^3

Now, $v_d = \dfrac{I}{neA} = \dfrac{10}{8\times10^{28}\times16\times10^{-19}\times5\times10^{-6}}$ m/s

or, $v_d = 1.5625 \times 10^{-4}$ m/s.

Illustration 5. *A copper wire of cross–section 2 mm^2 carries a current of 30 A. Calculate the root-mean-square velocity (thermal velocity) of free electrons at 27°C. Also prove that v_d is very small compared to it.*

Data given: ρ_{Cu} = 8.9 gm/cc, Boltzmann constant k = 1.38×10^{-23} J/K

$m_e = 9.1\times10^{-31}$ kg, $N_A = 6.023 \times 10^{23}$, Atomic weight of Cu = 63.

Solution: $v_{rms} = \sqrt{\dfrac{3kT}{m}} = 1.17\times10^5 \text{ m/s}$**;** $v_d = \dfrac{I}{neA}$

63 gm of Cu = 6.023×10^{23} Cu atoms.

$8.9\,\text{gm of Cu} = \dfrac{6.023\times10^{23}\times8.9}{63}$ Cu atoms. = 1 cc of Cu atoms.

$\Rightarrow \quad 1\text{ m}^3 \text{ of Cu} = 8.5\times10^{28}$ Cu atoms.

Now, each Cu atom contributes one electron

$$\Rightarrow \quad n = 8.5\times10^{28}\frac{\text{electron}}{\text{m}^3}$$

$$\Rightarrow \quad v_d = \frac{I}{neA} = \frac{30}{n(1.6\text{x}10^{-19})(2\text{x}10^{-6})}$$

$$v_d = 1.1\text{x}10^{-3}\frac{\text{m}}{\text{s}}$$

It is clear that v_d is much smaller than $v_{thermal}$

Mobility

Mobility of a charge carrier is defined as the drift velocity of the charge carrier per unit electric field. It is generally denoted by μ. If v_d is drift velocity attained by free electrons on applying electric field E, then electron mobility is given by.

$$\mu = \frac{v_d}{E}$$

The SI unit of μ is $m^2\,V^{-1}s^{-1}$.

Substituting $v_d = \mu E$ in $I = neAv_d$. We get $I = neA\mu E$ this equation gives the relation between electron mobility and the current through conductor.

Relation between drift velocity and electric field

Due to random motion, the free electrons of metal collide with positive metal ions and undergo change in direction after every collision. So, the thermal velocities are randomly distributed in all possible directions. Therefore, the average velocity

$$\vec{u} = \frac{\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_n}{N} = \text{zero}$$

Here, $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n$ are the individual thermal velocities of the free electrons at any given time and N is the total number of free electrons in the conductor.

However, when some potential difference V is applied across the two ends of a conductor of length ℓ, an electric field is set up. which is given by, $E = V/\ell$

Since charge on an electron is –e, each free electron in the conductor experiences a force $\vec{F} = e\vec{E}$ in a direction opposite to the direction of electric field.

If m is the mass of the electron, then acceleration produced is $\vec{a} = -\dfrac{e\vec{E}}{m}$

At any given time, an electron has a velocity such that $\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$, where $\vec{u}_1$ is the thermal velocity and $\vec{a}\tau_1$ is the velocity acquired by the electron under the influence of the applied electric field. where τ_1 being the time that has elapsed since the last collision.

Similarly, the velocities of the other electrons are $\vec{v}_2, \vec{v}_3 ..... \vec{v}_N$, such that $\vec{v}_2 = \vec{u}_2 + a\vec{\tau}_2$, $\vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3$, . . ., $\vec{v}_n = \vec{u}_n + \vec{a}\tau_N$

The average velocity of all the free electrons in the conductor is equal to the drift velocity $\vec{v}_d$ of the free electrons. Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied electric field.

Now, $$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + ... + \vec{v}_N}{N}$$

or, $$\vec{v}_d = \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + ... + (\vec{u}_N + \vec{a}\tau_N)}{N}$$

or, $$\vec{v}_d = \frac{(\vec{u}_1 + \vec{u}_2 + ...\vec{u}_N)}{N} + \vec{a}\left(\frac{\tau_1 + \tau_2 + ... + \tau_N}{N}\right) \quad \text{But,} \quad \frac{\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_N}{N} = 0$$

$$\therefore \quad \vec{v}_d = \vec{a}\frac{\tau_1 + \tau_2 + ... + \tau_N}{N} \quad \text{or,} \quad \vec{v}_d = \vec{a}\tau .$$

Here, τ is the average time elapsed between two successive collisions. ^{o}

or, $$\vec{v}_d = -\frac{e\vec{E}}{m}\tau$$

Exercise 2:

(i) If steady current flows in a metallic conductor of non–uniform cross–section, then which of the following quantities is constant along the conductor ?
Current, current density, drift speed .

(ii) Are the paths followed by electrons between successive collisions (with the positive ions of the metal) straight lines in the
(a) Absence of electric field (b) Presence of electric field.

OHM'S LAW

It states that current flowing between two points in a conductor is directly proportional to the potential difference between the two points.

i.e. $I \propto V$, provided the temperature is constant

$$\Rightarrow \quad \frac{V}{I} = R$$

or, $V = IR$

where R is a constant.

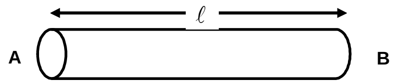
The constant 'R' is called resistance of the conductor. Its value depends upon the nature of conductor, its dimensions and the surrounding (e.g. temperature).

Ohm's law is not universal (i.e. all conductors do not obey Ohm's law). Conductors obeying Ohm's law are called Ohmic conductors. However, resistance is always defined as the ratio V/I.

For a conductor of cross–sectional area A, resistance between the sections A and B separated by length ℓ is given by, $R_{AB} = \rho\frac{\ell}{A}$

where ℓ = length of the conductor

A = Area of cross–section, and



ρ = resistivity or specific resistance of the conductor. (Its value depends upon the nature of the material of the conductor and its temperature.)

Unit of Resistance:
The SI unit of resistance is ohm. It is denoted by Ω. 1 ohm (Ω) = 1 volt amplere^{-1}

Conductance:
The reciprocal of resistance is called conductance. It is denoted by G.

$\therefore$ $G = \dfrac{1}{R}$; Its SI unit is ohm^{-1} or mho or siemen.

Unit of Resistivity:
We know that $R = \rho\ell/A$

$\therefore$ $\rho = RA/\ell$

In SI system, unit of resistivity = ohm $\times \dfrac{\text{metre}^2}{\text{metre}}$ = ohm–metre. or Ω – m

Conductivity:
It is defined as the reciprocal of resistivity and it is denoted by σ.

or, $\sigma = \dfrac{1}{\rho}$

The SI unit of conductivity is ohm^{-1} metre^{-1} or siemen m^{-1}.

Exercise 3: ***A rectangular metallic plate has its dimensions as shown in the figure. The resistance of the plate across the length, breadth and thickness are R_1, R_2 and R_3, respectively. Arrange these resistances in increasing order.***

Illustration 6. *A wire of resistivity 'ρ' is stretched to double its length. What will be its new resistivity?*

Solution: The resistivity of the wire does not depend on the length of the wire, so the resistivity will remain the same.

Illustration 7. *A wire of resistivity ρ is stretched to twice its length. If its initial resistance is R, then find its final resistance.*

Solution: $R = \dfrac{\rho\ell}{A}$

Volume of wire remains constant

$\Rightarrow$ $A\ell = A_1(2\ell) \Rightarrow$ $A_1 = \dfrac{A}{2}$

$R_1 = \dfrac{\rho\ell_1}{A_1} = \dfrac{\rho(2\ell)}{(A/2)} = \dfrac{4\rho\ell}{A} = 4R$

$\Rightarrow R_1 = 4R$.

Illustration 8. *Compare the resistances of two wires of same material. Their length are in the ratio 2 : 3 and their diameters are in the ratio 1 : 2.*

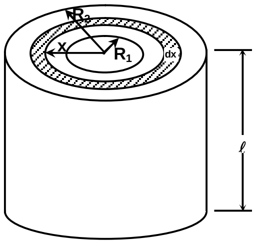
Solution:

$$R_1 = \frac{\rho\ell_1}{A_2} = \frac{\rho(2x)}{\pi(y/2)^2} = \frac{8\rho x}{\pi y^2}$$

$$R_2 = \frac{\rho\ell_2}{A_2} = \frac{\rho[3x]}{\pi\left(\frac{2y}{2}\right)^2} = \frac{3\rho x}{\pi y^2}$$

$$\Rightarrow R_1 : R_2 = 8 : 3$$

Illustration 9. *A cylindrical conductor of length ℓ and inner radius R_1 and outer radius R_2 has specific resistance ρ. A cell of emf ε is connected across the two lateral faces (inner and outer) of the conductor. Find the current drawn from the cell.*



Solution: To calculate current through the conductor, we have to calculate effective resistance between its two ends. So, we will consider the differential element of the cylindrical shell having radius x and thickness dx as shown in the figure.

$$dR = \rho \frac{dx}{2\pi x\ell}$$

$$\therefore R = \int_{R_1}^{R_2} \rho\frac{dx}{2\pi x\ell} \quad \Rightarrow R = \frac{\rho}{2\pi\ell}\ln\left(R_2/R_1\right) \quad (\because R = \rho\frac{\ell}{a})$$

$$I = \frac{\varepsilon}{R} \quad \Rightarrow I = \frac{2\pi\ell\varepsilon}{\rho\ln(R_2/R_1)}$$

Exceptions of Ohm's Law (Non–Linear V–I Characteristics)

The conductors which obey Ohm's law are called Ohmic conductors. The resistance of an Ohmic conductor does not depend upon potential difference or current.

The current is proportional to the applied potential difference by keeping the physical conditions constant. Thus the V–I graph for an Ohmic conductor is a straight line passing through the origin.

The conductors which do not obey ohm's law are called non–Ohmic conductors. For such conductors, the graph between potential difference and current is not a straight line passing through the origin, e.g. vacuum tubes, semiconductors, etc.

Microsopic form of Ohm's Law

We know that, $i = neAv_d$ and $v_d = \frac{eE}{m}\tau$

$$\therefore i = neA\left(\frac{eE}{m}\right)\tau$$

or, $$J = i/A = \frac{ne^2E}{m}\tau$$

or, $$J = \frac{E}{\frac{m}{ne^2\tau}}$$

or, $$J = \frac{E}{\rho}, \quad \text{where } \rho = \frac{m}{ne^2\tau}$$

or, $J = \sigma E \quad [\because \sigma = \frac{1}{\rho}]$

which is the microscopic form of Ohm's law.

Factors Affecting Electrical Resistivity

Let us consider a conductor of length ℓ and area of cross–section A. If n be the number of electrons per unit volume in the conductor and E is the applied electric field across the two ends of the conductor, then magnitude of drift velocity of electrons is

$$v_d = \frac{eE}{m}\tau \qquad \text{. . . (i)}$$

The current flowing through the conductor due to drift of electrons is

$$I = nAv_de \qquad \text{. . . (ii)}$$

From equations (i) and (ii)

$$I = \frac{nAe^2E\tau}{m} \qquad \text{. . . (iii)}$$

If V is the potential difference applied across the two ends of the conductor, then

$$E = \frac{V}{\ell} \qquad \text{. . . (iv)}$$

From equations (iii) and (iv)

$$I = \frac{nAe^2V\tau}{m\ell} \qquad \text{or, } \frac{V}{I} = \frac{m\ell}{ne^2\tau A}$$

or, $$R = \frac{m}{ne^2\tau}\frac{\ell}{A} \qquad [\because \quad R = \frac{V}{I}]$$

or, $$R = \rho\frac{\ell}{A}$$

It means the resistivity of the material of a conductor is

$$\rho = \frac{m}{ne^2\tau}$$

It shows that,

(i) $\rho \propto \frac{1}{n}$ [where n = number of free electrons per unit volume of the conductor]

(ii) $\rho \propto \frac{1}{\tau}$ [where τ = average relaxation time of free electrons in the conductor]

Variation of Resistivity with Temperature

ρ is independent of the shape and size of the conductor. It depends on temperature.
As temperature increases, ρ increases in case of Ohmic conductors.
At any temperature t, ρ is given by the following expression
$\rho_{(t)} = \rho_0 (1 + \alpha\Delta T)$,
where ρ_0 = the resistivity at 0°C, and α = temperature coefficient of resistivity.

Also, $$\alpha = \frac{(\rho - \rho_o)}{\rho_o.\Delta T} \qquad \Rightarrow \alpha = \frac{1}{\rho}\frac{d\rho}{dT}.$$

The resistivity of a semiconductor decreases rapidly with increasing temperature. We can explain these facts from the equation

$$\rho = \frac{m}{ne^2\tau} \qquad \text{. . . (i)}$$

(i) In case of conductors, the number of free electrons is fixed. Due to increase of temperature, the amplitude of vibration of atoms / ions increases. As a constant result of this, the collisions of electrons with the atoms become more effective and frequent. Therefore, τ decreases and hence ρ increases.

(ii) In case of insulators and semiconductors, the number of charge carriers at temperature T is given by

$$n(T) = n_0 e^{-E_g/k_BT} \quad \ldots \text{(ii)}$$

where E_g is the energy gap between valence and conduction bands in a solid.
Combining equations (i) and (ii),

$$\rho_T = \ell_0 e^{E_g/k_BT}$$

which shows that for semiconductors and insulators, resistivity increases with decreasing temperature.

Exercise 4: ***Why does the resistance of a conductor increase with increase in temperature?***

Illustration 10. *Calculate the resistance of a piece of a silver wire 0.50 m long and having diameter 2.74×10^{-4} m. Sp. resistance of silver $\rho = 1.66 \times 10^{-8}$ ohm metre.*

Solution: Given that, $\ell = 0.50$ m

$$r = \frac{2.74 \times 10^{-4}}{2} \text{ m} = 1.37 \times 10^{-4} \text{ m}$$

$$A = \pi r^2 = \pi(1.37 \times 10^{-4})^2 \text{ m}^2 ; \qquad \rho = 1.66 \times 10^{-8} \text{ ohm m.}$$

We know that, $R = \rho\ell/A$

or, $$R = \frac{1.66 \times 10^{-8} \times 0.50 \times 7}{22 \times (1.37 \times 10^{-4})^2}$$

$$R = 0.1407\ \Omega.$$

Illustration 11. *The resistance of a platinum wire of platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5.39 Ω. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.795 Ω. Calculate the temperature of the bath.*

Solution: $R_t = R_0(1 + \alpha t)$

or $R_t - R_0 = R_0 \alpha t \quad \ldots$ (i)

Also, $R_{100} = R_0(1 + 100\alpha)$

or, $R_{100} - R_0 = 100 R_0 \alpha. \quad \ldots$(ii)

Dividing equation (i) by equation (ii)

$$\frac{R_0 \alpha t}{100 R_0 \alpha} = \frac{R_t - R_0}{R_{100} - R_0} \times 100$$

or, $$t = \frac{5.795 - 5}{5.23 - 5} \times 100 = 345.65°\text{C}.$$

Exercise 5:
(i) ***A wire is cut into half. What is the effect on its specific resistance ?***
(ii) ***What is the order of resistivity of an insulator?***
(iii) ***How does the conductance of a semiconductor material change with rise in temperature?***

Illustration 12. *Resistance of a conductor is 1.72 Ω at a temperature of 20°C. Find the resistance at 0°C and 100°C. Given the coefficient of resistivity is α = 0.00393/°C.*

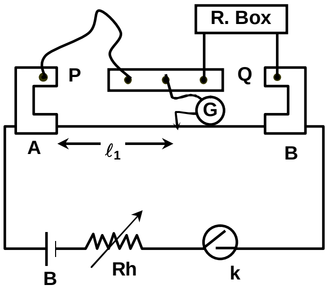
.

Solution: $R = R_0(1 + \alpha\,\Delta T)$

$R = 1.72\ \Omega\ [1 + 0.00393/°\text{C}\ (0°\text{C} - 20°\text{C})] = 1.58\ \Omega$, and at T = 100°C

$R = 1.72\ \Omega\ [1 + (0.00393/°\text{C}]\ (100°\text{C} - 20°\text{C})] = 2.26\ \Omega$

SPECIFIC RESISTANCE OF THE MATERIAL OF A WIRE USING A METER BRIDGE



A known length (L) of a wire is connected in one of the gaps (P) of a metre bridge, while a Resistance Box is inserted into the other gap (Q). The circuit is completed by using a battery (B), a rheostat (Rh), a key (K) and a galvanometer (G).

The balance length (ℓ) is found by closing key K and momentarily connecting the galvanometer until it gives zero deflection (null point).

Then, $\frac{P}{Q} = \frac{\ell}{100-\ell}$

(using the expression for the meter bridge at balance.)

Here, P represents the resistance of the wire while Q represents the resistance in the resistance box. The key K is kept open when the circuit is not in use.

The resistance of the wire, P = $\rho\frac{L}{\pi r^2} \Rightarrow \rho = \frac{\pi r^2}{L}P$

where r is the radius of wire and L is the length of the wire, r is measured using a screw gauge while L is measured with a scale.

Errors

The major systematic errors in this experiment are due to the (i) heating effect, (ii) end corrections introduced due to shift of the zero of the scale at A and B, (iii) stray resistances in P and Q, (iv) errors due to non–uniformity of the meter bridge wire.

Error analysis:

End corrections can be estimated by including known resistances P_1 and Q_1 in the two ends and finding the null point:

$$\frac{P_1}{Q_1} = \frac{\ell_1+\alpha}{100-\ell_1+\beta}$$ (where α and β are the end corrections.)

When the resistance Q_1 is placed in the left gap and P_1 in the right gap,

$$\frac{Q_1}{P_1} = \frac{\ell_2+\alpha}{100-\ell_2+\beta}$$ which gives two linear equations for finding α and β.

In order that α and β be measured accurately, P_1 and Q_1 should be as different from each other as possible.

For the actual balance point, $\frac{P}{Q} = \frac{\ell+\alpha}{100-\ell+\beta} = \frac{\ell'_1}{\ell'_2}$,

Errors due to non–uniformity of the meter bridge wire can be minimised by interchanging the resistances in the gaps P and Q.

$\therefore \quad \frac{\delta P}{P} = \left|\frac{\delta\ell'_1}{\ell'_1}\right| + \left|\frac{\delta\ell'_2}{\ell'_2}\right|$, where $\delta\ell'_1$ and $\delta\ell'_2$ are of the order of the least count of the scale.

The error is, therefore, minimum if $\ell'_1 = \ell'_2$ i.e. when the balance point is in the middle of the bridge.

The error in ρ is $\frac{\delta\rho}{\rho} = \frac{2\delta r}{r} + \frac{\delta L}{L} + \frac{\delta P}{P}$.

Illustration13. *With two resistances R_1 and $R_2(> R_1)$ in the two gaps of a metre bridge, the balanced point was found to be $\frac{1}{3}$m from the zero end. When a 6 Ω resistance is connected in series with the smaller of the two resistances, the point is shifted to $\frac{2}{3}$m from the same end. Calculate R_1 and R_2.*

Solution: $\frac{R_1}{R_2} = \frac{\ell}{1-\ell}$, where ℓ is in metre.

$$\frac{R_1}{R_2} = \frac{1/3}{1-1/3} = \frac{1}{2}$$

or, $R_2 = 2R_1$ . . . (i)

Again, $\frac{R_1+6}{R_2} = \frac{2/3}{1-2/3} = 2$

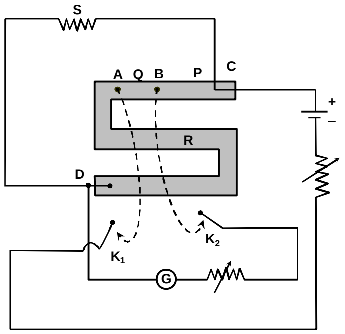
or, $R_1 + 6 = 2R_2$ . . . (ii)

From (i) and (ii)

$R_1 = 2\Omega$ and $R_2 = 4\ \Omega$.

Exercise 6: ***What is the principle of meter bridge ?***

MEASUREMENT OF UNKNOWN RESISTANCE USING A POST OFFICE BOX



A Post Office Box can also be used to measure an unknown resistance. It is a Wheatstone Bridge with three arms P, Q and R; while the fourth arm is the unknown resistance. P and Q are known as the ratio arms while R is known as the rheostat arm.

At balance, the unknown resistance

$$S = \left(\frac{P}{Q}\right)R \qquad . . . (1)$$

The ratio arms are first adjusted so that they carry 100 Ω each. The resistance in the rheostat arm is now adjusted so that the galvanometer deflection is in one direction, if $R = R_o$ (ohm) and in the opposite direction,

This implies that the unknown resistance, S lies between R_o and $R_o + 1$ (ohm).

Now, the resistances in P and Q are made 100 Ω and 1000 Ω, respectively, and the process is repeated.

Equation (1) is used to compute S. The ratio P/Q is progressively made 1 : 10, and then 1: 100. The resistance S can be accurately measured.

Errors

The major sources of error are the connecting wires, unclear resistance plugs, change in resistance due to Joule heating, and the insensitivity of the Wheatstone bridge.

These may be removed by using thick connecting wires, clean plugs, keeping the circuit on for very brief periods (to avoid Joule heating) and calculating the sensitivity.

In order that the sensitivity is maximum, the resistance in the arm P is close to the value of the resistance S.

Illustration14. *In an experiment with a post–office box, the ratio arms are 1000 : 10. If the value of the third resistance is 999 Ω,. find the unknown resistance.*

Solution: The ratio arms are 1000 : 10

$\therefore \quad \frac{P}{Q} = \frac{1000}{10} = 100$

Third resistance R = 999 Ω

Let x be the unknown resistance.

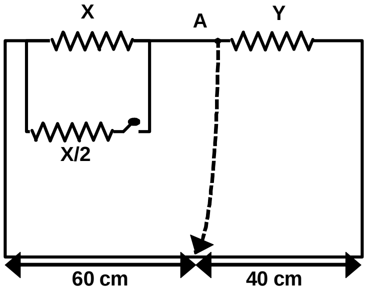
We know that,

$$\frac{P}{Q} = \frac{R}{X}$$

$\therefore \quad X = \frac{Q}{P} \times R = \frac{1}{100} \times 999 \ = 9.99\,\Omega$

Illustration15. *When two resistances X and Y are put in the left hand and right hand gaps in a meter bridge, the null point is at 60 cm. If X is shunted by a resistance equal to half of itself then find the shift in the null point.*

Solution : Arrangement is shown in the figure.



$\frac{X}{Y} = \frac{60}{40} = \frac{3}{2}$ . . . (1)

When X is shunted, then resistance in the left gap becomes

$X' = \frac{X.\frac{X}{2}}{X + \frac{X}{2}} = \frac{X}{3}$ . . . (2)

Now, $\frac{\left(\frac{X}{3}\right)}{Y} = \frac{l}{(100 - l)} \Rightarrow \frac{1}{3} \times \frac{3}{2} = \frac{l}{(100 - l)}$

$\Rightarrow \quad$ l = 33.3 cm

$\therefore \quad$ Shift = 60 – 33.3 = 26.7 cm

CLASSIFICATION OF MATERIALS IN TERMS OF CONDUCTIVITY:

On the basis of the conductivity, the materials can be classified into the following categories.

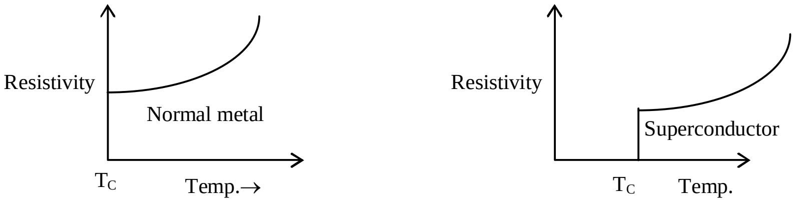
(i) In certain materials, the outermost electrons of atoms/molecules are loosely bound. These electrons are almost free to move throughout the body of the material.

These are called free electrons. These free electrons participate in electrical conduction, therefore these are also called conduction electrons. Due to their presence in large number, the materials have large value of conductivity. These materials are called conductors, e.g. copper, silver, etc.

(ii) Some materials have very low value of electrical conductivity. In these materials, the electrons are tightly bound to their respective atoms or molecules. These materials are called insulators or dielectrics, e.g. glass, rubber, etc.

(iii) In one class of materials, the conductivity is greater than insulators but less than that of conductors. These type of materials are called semiconductors. e.g. silicon, germanium etc.

Superconductivity
The resistivity of certain materials suddenly becomes zero below a certain temperature. This phenomenon is called superconductivity and the material showing such behaviour is called superconductor. Above the critical temperature T_C (at which such transition occurs), the resistivity of the metal follows the trend of a normal metal as shown in graph A.



Graph (A)

KIRCHHOFF'S LAW :
There are certain rules and techniques to solve the complicated circuits, containing resistances and batteries. These rules enable us to handle the complicated circuit systematically. The method given by Kirchoff's law is one of them. Kirchoff's law is incomplete without defining the basic terms.

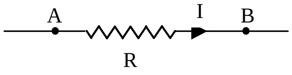
Branch Point
The branch point in the network is the point where three or more conducts are joined.

Loop
A loop is any closed conducting path.

Junction rule
It is based on the law of conservation of charge. At a junction in a circuit, the total incoming current is equal to the total outgoing current. In other words, the algebraic sum of the currents at a junction is zero. A junction in a circuit is neither acts as a sink nor as a source of charge.

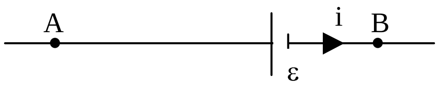
Loop rule
It is based on the law of conservation of energy. The algebraic sum of the potential drop around any closed path is zero.

- In case of a resistor of resistance 'R' potential will decrease in the direction of current. Hence, for the shown conductor, potential drop across a resistance is IR $V_B - V_A = -IR$



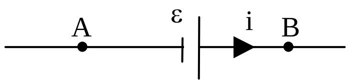
- For an emf source, the changes in potential will be obtained as illustrated below.

Emf = ε, internal resistance = r



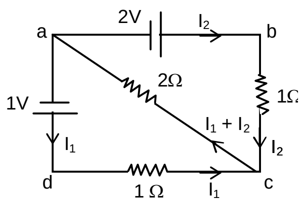
$v_B - v_A = -\varepsilon - ir$

Emf = ε, internal resistance = r



$v_B - v_A = \varepsilon - ir$

Illustration16. *Calculate current in each resistance in the electrical circuit shown in the figure. The internal resistances of the cells are negligible.*



Solution: Applying Kirchhoff's second law to the mesh adca, we get

$-I_1 \times 1 - 2(I_1 + I_2) + 1 = 0$

$I_1 + 2(I_1 + I_2) = 1$

$3I_1 + 2I_2 = 1$ . . . (i)

Applying Kirchhoff's second law to the mesh abca, we get

$2 - 1 \times I_2 - 2(I_1 + I_2) = 0$

or, $I_2 + 2(I_1 + I_2) = 2$

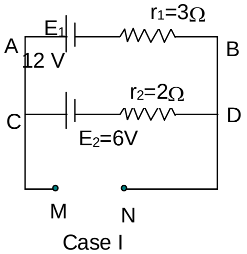
$2I_1 + 3I_2 = 1$ . . . (ii)

solving (i) and (ii), we get,

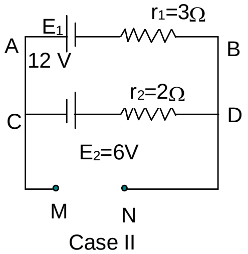
$I_1 = -0.2$ A and $I_2 = 0.8$ A

Current in 2Ω resistance $= (0.8 - 0.2)\text{A} = 0.6$ A.

Illustration 17. *What is the potential difference between the points M and N for the circuits shown in the figures, For case I and case II?*

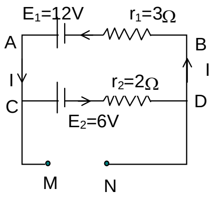


Case I



Case II

Solution : **Case I:**



Case I

$$I = \frac{E_1 - E_2}{r_2 + r_1} = \frac{12-6}{3+2} = 1.2 \text{ A}$$

So for cell E_1, $v_A - E_1 + Ir_1 = v_B$

i .e. $v_A - v_B = E_1 - Ir_1$

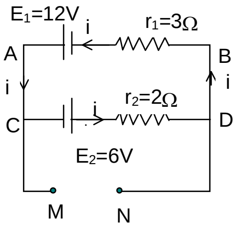
$= 12 - 1.2 \times 3 = 8.4$ V

For cell E_2, $v_C - E_2 - Ir_2 = v_D$

i.e. $v_C - v_D = 6 + 1.2 \text{ x } 2 = 8.4$ V

Hence, $v_C - v_D = v_A - v_B = v_M - v_N = 8.4$ V

Case II:



CaseI II

$$I = \frac{E_1 + E_2}{r_1 + r_2} = \frac{12+6}{3+2} = 3.6 \text{ A}$$

For cell E_1,

$v_A - E_1 + Ir_1 = v_B$ i.e. $v_A - v_B = E_1 - Ir_1$

$= 12 - 3.6 \times 3 = 1.2$ V

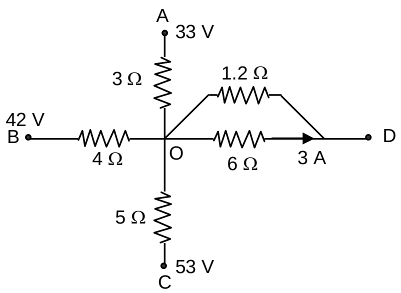
For cell E_2,

$v_c + E_2 - Ir_2 = v_D$

i.e. $v_C - v_D = -E_2 + Ir_2 = -6 + 3.6 \times 2 = 1.2$ V

Hence, $v_A - v_B = v_C - v_D = v_M - v_N = 1.2$ V

Illustration 18. *The given network is part of another larger circuit. Calculate the potential of point D.*



Solution : Let the potential of point O be x volts

Going from D to O, we get $V_O - V_D = 6 \times 3 = 18$ volt

$\Rightarrow\ V_D = x - 18$

Let us assume that the current goes away from point O to the points A, B, C and D through all branches

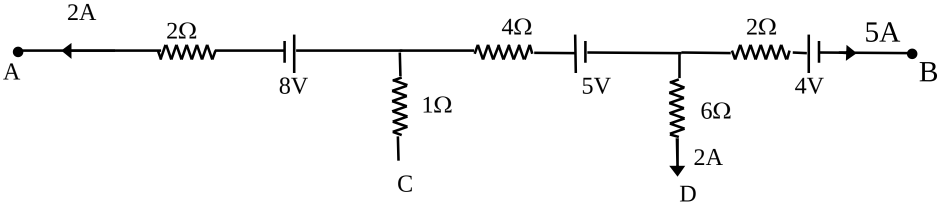
$\Rightarrow i_{OA} + i_{OB} + i_{OC} + i_{OD} = 0$

$$\Rightarrow \frac{V_O - V_A}{3} + \frac{V_O - VB}{4} + \frac{V_O - V_C}{5} + \frac{V_O - V_D}{6} + \frac{V_O - V_D}{1.2} = 0$$

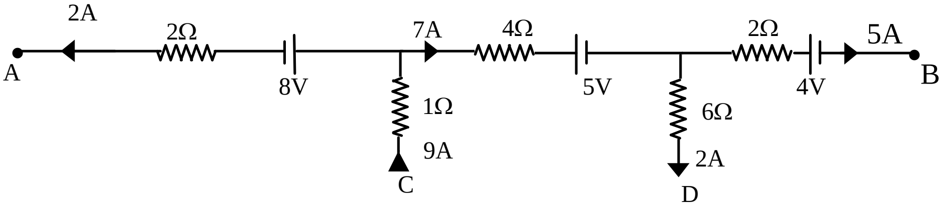
$$\Rightarrow \frac{x-33}{3} + \frac{x-42}{4} + \frac{x-53}{5} + 3 + \frac{18}{1.2} = 0 \qquad \Rightarrow x = 18\text{ V}$$

$\Rightarrow$ D is at a potential of 0 volts.

Illustration 19. The following figure shows part of certain circuit. Find
(i) power dissipated in 6 Ω resistance.
(ii) potential difference $V_C - V_B$
(iii) potential difference $V_A - V_D$

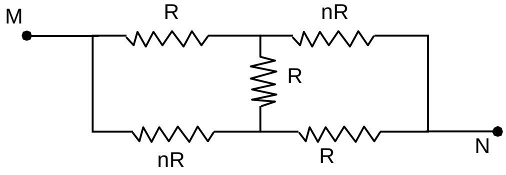


Solution:



Apply junction rule to calculate the current through each resistor
(i) $P = I^2R \quad \Rightarrow \quad P_{(6\Omega)} = 24$ watt
(ii) $V_C - V_B = V_{BC} = 4 + (5 \times 2) + 5 + (7 \times 4) + 9 \times 1 = 56$ V
(iii) $V_A - V_D = V_{DA} = -2 \times 2 - 8 + 7 \times 4 + 5 + 6 \times 2 = 33$ V

Illustration 20. *Find the equivalent resistance between M and N.*

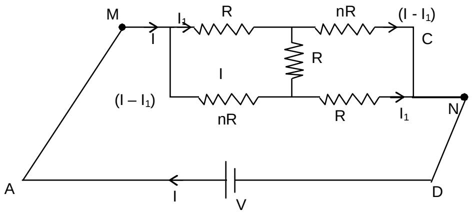


Solution : For loop 1:

$-I_1R + R(I - 2I_1) + nR(I - I_1) = 0$

$\Rightarrow I_1\,(R + 2R + nR) = I\,(R + nR)$

$$\Rightarrow I_1 = \frac{I(n+1)}{(n+3)}$$



For loop AMCNDA ; we have from Kirchoff's loop law

$-I_1R - (I - I_1)nR + V = 0$

$\Rightarrow I_1\,(R - nR) = V - nIR$

$$\Rightarrow \frac{I(n+1)}{(n+3)}\,R(1-n) + nIR = V$$

$$\Rightarrow IR\left[\frac{1-n^2}{n+3}+n\right] = V$$

$$\Rightarrow V = IR\left(\frac{1+3n}{n+3}\right)$$

$$\therefore\ R_{eq} = \left(\frac{3n+1}{n+3}\right)R$$

Carbon Resistors

To make a carbon resistor, carbon with a suitable binding agent is modulated into a cylinder. Wire leads are attached to this cylinder. The resistor is enclosed in a plastic or ceramic jacket. The resistor is connected to the circuit by means of two leads. Carbon resistors of different values are commercially available. They are widely used in electronic circuits of radio, amplifier, etc.

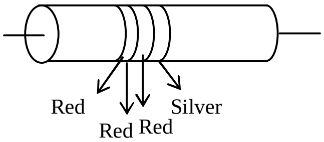
Colour Code for Carbon Resistors

The resistances of a carbon resistor are indicated by means of colour code printed on it. The resistor has a set of co–axial coloured rings on it with their significance as indicated in the table shown.

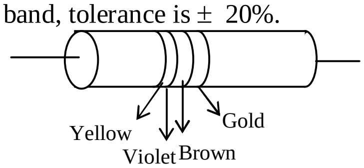
Colour	Number	Multiplier	Tolerance (%)
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			20

To read the value of a carbon resistance, the following sentence serves as an aid to memory.
BBROY Great Britain very good wife

From the left end, first two bands indicate the first two significant figures of the resistance in ohms. The third band indicates the decimal multiplier and the last band indicates the tolerance or possible variation in percent about the indicated value. In the absence of fourth band, tolerance is $\pm$ 20%.

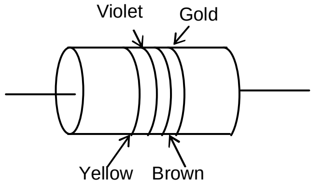


(a)



(b)

Illustration 21. The *figure shows a colour–coded resistor. What is the resistance of this resistor?*

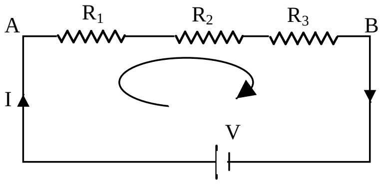


Solution: The number for yellow colour is 4. The number for violet colour is 7. Brown colour gives a multiplier of 10^1. Gold indicates a tolerance of 5%. So, the resistance of the given resistor is $47 \times 10\ \Omega \pm 5\%$.

Exercise 7: ***A carbon resistor is marked in coloured bands in the sequence blue, green, orange and gold. What is the resistance of the resistor ?***

GROUPING OF RESISTANCES

Series Combinations



Let the equivalent resistance between A and B equals R_{eq} , By definition,

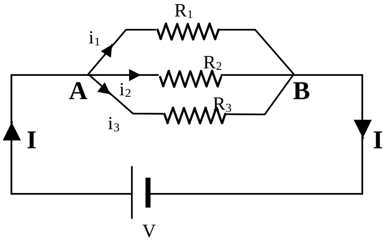
$$R_{eq} = \frac{V}{I} \qquad \text{. . . (1)}$$

Using Kirchoff's 2nd rule for the loop shown in figure,

$$V = IR_1 + IR_2 + IR_3 \qquad \text{. . . (2)}$$

From (1) and (2), $R_{eq} = R_1 + R_2 + R_3$

Parallel Combinations



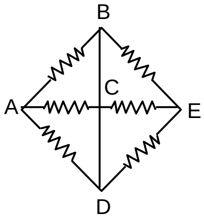
Here again, $R_{eq} = \frac{V}{I}$ . . .(1)

$$I = i_1 + i_2 + i_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \qquad \text{. . . (2)}$$

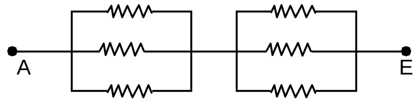
From (1) and (2)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Illustration 22. *Calculate the equivalent resistance between points A and E as shown in the figure. Each resistance is of 2 Ω.*



Solution: The points B, C and D are at the same potential. So, resistances AB, AC and AD are in parallel. Similarly, the resistances BE, CE and DE are in parallel. So, an equivalent of the given network is as under.

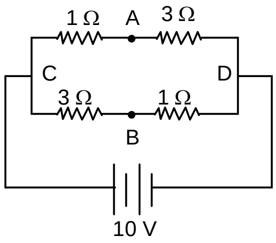


Parallel combination of 2 Ω, 2 Ω and 2 Ω gives $\frac{2}{3}\Omega$.

$$\therefore\ R_{AE} = 2 \times \frac{2}{3}\Omega = \frac{4}{3}\Omega$$

$$= 1.33\ \Omega.$$

Illustration 23. *A battery of emf 10 V is connected to resistances as shown in the figure. Determine the potential difference between A and B.*



Solution: Total resistance $= \dfrac{4\times 4}{4+4} = 2\Omega$

Current I = $\dfrac{10V}{2\Omega} = 5A$

Since the resistances of both the branches are equal, therefore the current of 5 A shall be equally distributed.

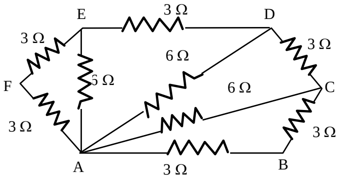
Current through each branch = $\dfrac{5}{2}A = 2.5A$

$V_c - V_A = 2.5 \times 1 = 2.5$ V

$V_c - V_B = 2.5 \times 3 = 7.5$ V.

$V_A - V_B = (V_c - V_B) - (V_c - V_A) = 7.5 - 2.5 = 5.0$ V

Illustration 24. *Find the effective resistance between the points A and B.*



Solution: Resistors AF and FE are in series with each other. Therefore, network AEF reduces to a parallel combination of two resistors of 6 Ω each.

$R_{eq} = \dfrac{6\ \times\ 6}{6+6} = 3\ \Omega.$

Similarly, the resistance between A and D is given, $\dfrac{6 \text{ x } 6}{6+6} = 3\ \Omega.$

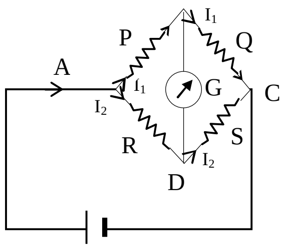
Now, resistor AC is in parallel with the series combination of AD and DC. Therefore, the resistance between A and C is $\dfrac{6 \text{ x } 6}{6+6} = 3\ \Omega.$

AC + CB = 3 + 3 = 6 Ω, since they are in series.

Resistance between A and B is given by, $\dfrac{1}{R} = \dfrac{1}{6} + \dfrac{1}{3} = \dfrac{3}{6}$ or $R_{AB} = 2\Omega.$

Wheatstone Bridge

For a certain adjustment of Q, $V_{BD} = 0$, then no current flows through the galvanometer.



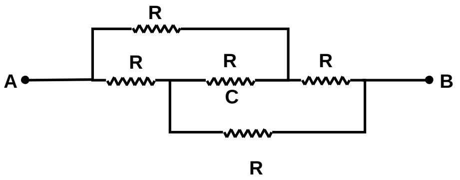
$\Rightarrow$ $V_B = V_D$ or $V_{AB} = V_{AD} \Rightarrow I_1.P = I_2.R$

Likewise, $V_{BC} = V_{DC} \Rightarrow I_1.Q = I_2.S$

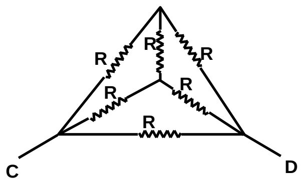
Dividing, we get, $\dfrac{P}{Q} = \dfrac{R}{S}$

Illustration 25. *What's the effective resistance of following circuits?*

(a)



(b)



Solution: (a) It is a Wheatstone bridge that is balanced. Hence, the central resistance labeled 'C' can be assumed as ineffective.

$\Rightarrow$ $R_{eq} = R$.

(b) The resistor R is in parallel with a balanced Wheatstone bridge.

$$\Rightarrow \quad R_{eq} = \frac{R.R}{R+R} = \frac{R}{2}$$

GROUPING OF IDENTICAL CELLS

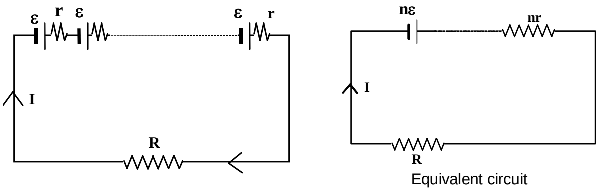
Series Grouping

E.m.f. of the cell is ε and internal resistance is r.

Applying Kirchhoff's Law

$\varepsilon - ir + \varepsilon - ir + ...(\text{to n times}) - iR = 0$

$$\Rightarrow \quad i = \frac{n\varepsilon}{R+nr} = \frac{\varepsilon}{(R/n)+r}$$



Equivalent circuit

Illustration 26. *A cell has an emf of 5.0 volt and an internal resistance of 1.0 Ω. Its terminals are joined through a 9 Ω resistor. Calculate the potential difference across the terminals.*

Solution: $I = \frac{E}{R+r} = \frac{5}{9+1}A = \frac{1}{2}A$

$V = E - Ir = 5 - \frac{1}{2}\times 1$

$V = 4.5$ volt.

Illustration 27. *Six lead–acid type of secondary cells, each of emf 2.0 V and internal resistance 0.015 Ω, are joined in series to provide a supply to a resistance of 8.5 Ω. Determine*

(i) current drawn from the supply and (ii) its terminal voltage.

Solution: emf of series combination of cells = (6×2)V = 12 V.

Total internal resistance of series combination = 6 × 0.015 Ω = 0.09 Ω
External resistance = 8.5 Ω

Total resistance of circuit = (8.5 + 0.09)Ω = 8.59 ohm.

Total current (I) = $\frac{\text{Total emf}}{\text{Total resistance}} = \frac{12}{8.59}$ A = 1.4 A

Terminal potential difference = (12 – 14 × 0.09) volt =11.9 volt.

Illustration 28. *In a series grouping of N cells, current in the external circuit is I. The polarity of how many cells should be reversed so that the current becomes (1/3)rd of the earlier value?*

Solution: Before reversing the polarity of the cells, the current is

$$I = \frac{NE}{R + Nr}$$

Let n cells be reversed in their polarity

∴ Net e.m.f. = (N – n)E – nE= (N–2n) E

Total resistance = Nr + R

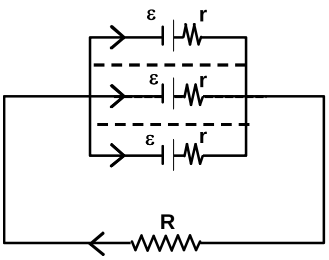
$$\Rightarrow i' = \frac{(N-2n)\ E}{Nr + R}$$

But, $\frac{I'}{I} = \frac{(N-2n)\ E/(R+Nr)}{(NE/Nr+R)} = \frac{N-2n}{N} \Rightarrow n = \frac{N}{3}$,

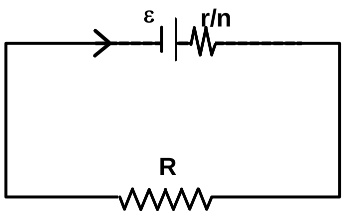
This is valid only when N is a multiple of 3, otherwise it cannot be done.

Parallel Grouping

Let us assume that there are n rows and each single row contains battery of emf E and internal resistance r, as shown in the figure.



⇒



Equivalent Circuit

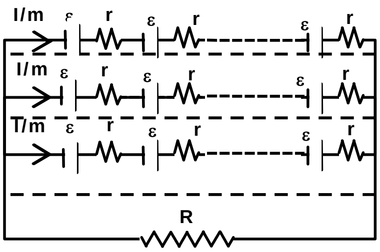
Applying Kirchhoff's law, to the equivalent circuit

$$\varepsilon - \frac{I}{n}r - IR = 0 \qquad \Rightarrow \qquad I = \frac{n\varepsilon}{r + nR}$$

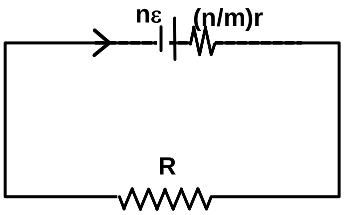
- To get maximum current, cells must be connected in series if effective internal resistance is less then external resistance, and in parallel if effective internal resistance is greater than external resistance.

Mixed grouping

Let us assume, that each branch of the circuit contains n batteries of emf E and internal resistance r and there are m such rows as shown in the figure.



$\Rightarrow$



Equivalent Circuit

Number of rows is m and number of cells in each row is n.
Applying Kirchhoff's law to the equivalent circuit.

$$n\varepsilon - n\frac{I}{m}r - IR = 0 \qquad \Rightarrow \quad I = \frac{mn\varepsilon}{mR + nr}$$

Effective grouping of cells
For effective grouping, current should be maximum

$\Rightarrow$ mR + nr should be minimum

Now, $mR + nr = \left(\sqrt{mR}\right)^2 + \left(\sqrt{nr}\right)^2 = \left(\sqrt{mR} - \sqrt{nr}\right)^2 + 2\sqrt{mnrR}$

The second term is non–zero, the term in the parenthesis $\left(\sqrt{mR} - \sqrt{nr}\right)^2 = 0$

$$\Rightarrow \quad mR = nr \Rightarrow \quad R = \frac{n}{m}r$$

Illustration 29. *There are 27 cells with an internal resistance 0.4 Ω and an external resistance 1.2 Ω. What is the most effective way of grouping them?*

Solution: Let there be n cells in series having m parallel branches

$mn = 27$ … (1)

$mR = nr$ … (2)

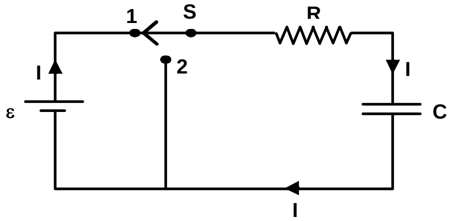
$1.2\,m = n\,(0.4)$

$\Rightarrow 3m = n$ … (3)

From equations (1) and (3) , we get m= 3 and n = 9

RC–CIRCUIT

Charging
Let us assume that the capacitor in the shown network is uncharged for t < 0. The switch is connected to position 1 at t = 0. Now, 'C' is getting charged.



If the charge on capacitor at time 't' is q, writing the loop rule,

$$\frac{q}{C} + IR - E = 0 \quad \Rightarrow \quad R\frac{dq}{dt} = E - \frac{q}{C}$$

$$\Rightarrow \quad RC\frac{dq}{dt} = EC - q$$

$$\Rightarrow \quad \frac{dq}{EC - q} = \frac{1}{RC}dt$$

Integrating, $\displaystyle\int_o^q \frac{dq}{EC - q} = \frac{1}{RC}\int_o^t dt \Rightarrow -\ln|EC - q|_o^q = \frac{1}{RC}.t$

$$\Rightarrow \quad \ln\left|\frac{EC - q}{EC}\right| = \frac{-t}{RC} \Rightarrow \quad q = EC[1 - e^{-t/RC}]$$

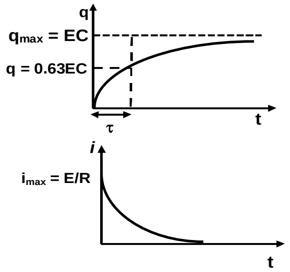
$\Rightarrow$ At t = 0, q = 0 and at t = ∞, q = EC (the maximum charge.)

Thus,

$$q = q_{max}\left[1 - e^{\frac{-t}{RC}}\right]$$

$$i = \frac{dq}{dt} = \frac{q_{max}}{RC}e^{-t/RC} = \frac{E}{R}e^{-t/RC}$$

$$i = i_{max}e^{-t/RC}, \qquad \text{where} \quad i_{max} = \frac{E}{R}$$



Time Constant (τ)

It is the time during which the charging would have been completed, had the growth rate been as it began initially. Numerically, it is equal to RC. The unit of time constant is sec., if resistance R is in ohm and capacitance C is in farad.

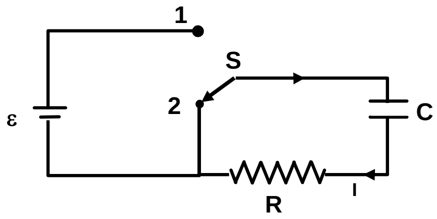
Discharging

Consider the same arrangement that we had in previous case with one difference that the capacitor has charge q_o for $t < 0$ and the switch is connected to position 2 at t = 0. If the charge on capacitor is q at any later moment t, then the loop equation is given as

$$\frac{q}{C} + IR = 0$$

$$\Rightarrow \quad R\frac{dq}{dt} = \frac{-q}{C}$$

$$\Rightarrow \quad \frac{dq}{q} = \frac{-1}{RC}dt$$



Integrating, at $t = 0$, $q = q_0$

$t = t$, $q = q$

$$\Rightarrow \quad \int_{q_0}^{q}\frac{dq}{q} = -\frac{1}{RC}\int_{o}^{t}dt$$

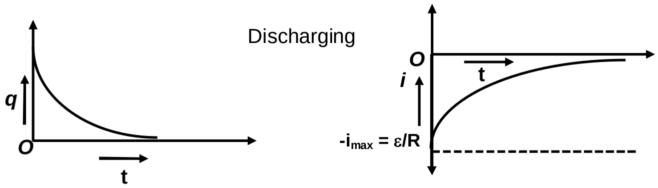
$$\Rightarrow \quad \ln\left|\frac{q}{q_o}\right| = -\frac{t}{RC}$$

or, $$q = q_0.e^{-t/RC}$$

$$i = -\frac{q_0}{RC}e^{-t/RC} = \frac{-EC}{RC}e^{-t/RC}$$

$$i = -i_0e^{-t/RC}$$

'–ve' sign indicates that the discharging current flows in a direction opposite to the charging current.



MEASURING INSTRUMENTS

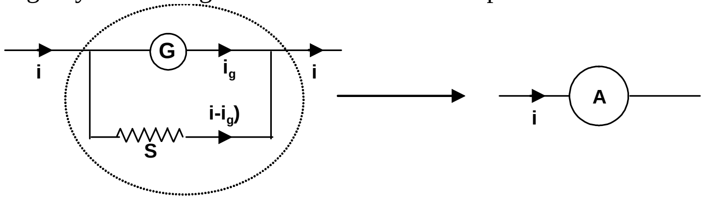
Galvanometer:

It is used to detect very small current. It has negligible resistance.

Ammeter
It is an instrument used to measure current. It is put in series with the branch in which current is to be measured. An ideal Ammeter has zero resistance. A galvanometer with resistance G and current rating i_g can be converted into an ammeter of rating I by connecting a suitable resistance S in parallel to it.

Thus, $S(i - i_g) = i_g G$

$\Rightarrow$ $S = \frac{i_g G}{i - i_g}$



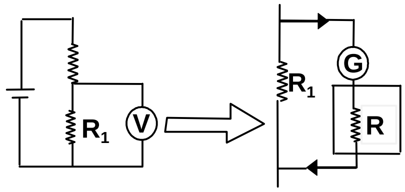
Exercise 8: ***An ammeter is always connected in series and voltmeter connected to parallel in any circuit element, why?***

Illustration 30. *A galvanometer having a coil resistance of 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into ammeter giving full scale deflection for a current of 10 A?*

Solution:
$$S = \frac{i_g.G}{i - i_g} = \frac{(10^{-3}A)(100\Omega)}{(10 - 10^{-3})A} = \frac{0.1}{9.99}$$

$$\Rightarrow \quad S = \frac{1}{99.99}\Omega \approx 10^{-2} \ \Omega$$

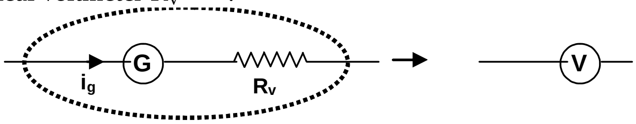
Voltmeter
It is an instrument to find the potential difference across two points in a circuit.



It is essential that the resistance R_v of a voltmeter be very large compared to the resistance of any circuit element with which the voltmeter is connected. Otherwise, the metre itself becomes an important circuit element and alters the potential difference that is measured.

$R_v \gg R$

For an ideal voltmeter $R_V = \infty$.



$i_g(G + R) = V \quad \Rightarrow R = \frac{V}{i_g} - G$

Illustration 31. *A galvanometer having a coil resistance of 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10 V?*

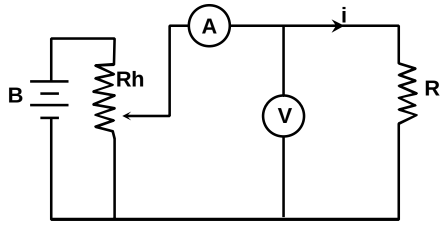
Solution: $V = I_g [G + R_v]$

$10 = (10^{-3})(100 + R_v)$

$$\Rightarrow \quad R_v = \left(\frac{10}{10^{-3}}\right) - 100 = 9{,}900\,\Omega = 9.9 \text{ k}\Omega$$

VERIFICATION OF OHM'S LAW USING VOLTMETER AND AMMETER

A voltmeter (V) and an ammeter (A) are connected in a circuit, alongwith a resistance R as shown in the figure, with a battery B and a rheostat Rh. Simultaneous readings of the current i and the potential drop V are taken by changing the resistance in the rheostat (Rh). A graph of V versus i is plotted and it is found to be linear (within errors).



The magnitude of R is determined by :

(a) taking the ratio $\frac{V}{i}$, (b) fitting to a straight line: V = iR, and is determining the slope R.

Errors

Systematic errors in this experiment arise from the current flowing through V (finite resistance of the voltmeter), the Joule heating effect in the circuit and the resistance of the connecting wires/ connections of the resistance. The effect of Joule heating may be minimsed by switching on the circuit for a short while only, while the effect of finite resistance of the voltmeter can be overcome by using a high resistance instrument or a potentiometer. The lengths of connecting wires should be minimised as much as possible.

Error analysis: The error in computing the ratio $R = \frac{V}{i}$ is given by $\left|\frac{\delta R}{R}\right| = \left|\frac{\delta V}{V}\right| + \left|\frac{\delta i}{i}\right|$

where δV and δi are of the order of the least counts of the instruments used.

Exercise 9: ***The following measurements were taken to measure a resistance R in an experiment involving Ohm's law.***

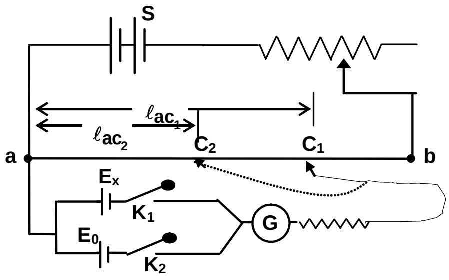
Voltmeter	*1.0 V*	*1.5 V*	*1.4 V*
Ammeter	*50 mA*	*75 mA*	*70 mA*

Calculate the value of R and estimate the error in R.

POTENTIOMETER

A potentiometer is an instrument that measures the terminal potential difference with high accuracy without drawing any current from the unknown source. It is based on the principle that if constant current is passed through a wire of uniform cross–section, then potential difference across any segment of the wire is proportional to its length.

The given diagram shows a typical arrangement to measure emf E_x of a battery.



The wire ab is of uniform cross–section and carries a constant current supplied by battery S. First the switch K_1, is closed and K_2 is kept open. The slider is moved on the wire ab till we get zero deflection in the galvanometer. If C_1 is the corresponding point in the wire, $E_x = V_{ac_1}$.

Now, the experiment is repeated with key K_1 open and K_2 closed. This time, if the null deflection is obtained with contact on wire at C_2,

$E_o = V_{ac_2}$ (E_o is known)

Now, $\frac{E_x}{E_o} = \frac{V_{ac_1}}{V_{ac_2}} = \frac{\ell_{ac_1}}{\ell_{ac_2}}$, where ℓ_{ac_1} and ℓ_{ac_2} are the lengths of segments ac_1 and ac_2 respectively.

Illustration32. *A 10 m long wire of uniform cross – section and 20 Ω resistance is fitted in a potentiometer. The wire is connected in series with battery of 5 volt, alongwith an external resistance of 480 Ω If an unknown emf E is balanced at 6.0 m length of this wire, calculate (i) the potential gradient of the potentiometer wire, (ii) the value of the unknown emf E.*

Solution: $I = \frac{5}{480+20} A = 0.01 A$

P.D. across the potentiometer wire = (0.01×20) V = 0.2 V

(i) Potential gradient = $\frac{0.2V}{10m}$ = 0.02 Vm^{-1}

(ii) E = (0.02×6.0) V = 0.12 volt.

Exercise 10: ***With a certain cell, the balance point is obtained at 65 cm from the end of a potentiometer wire. With another cell whose emf differs from that of first by 0.1 V, the balanced point is obtained at 60 cm. Find the emf of each cell.***

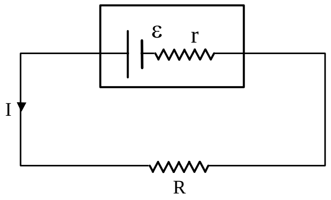
ENERGY, POWER AND HEATING EFFECT OF THE CURRENT

When a constant current I flows for time t from a source of emf E, then the amount of charge that flows in time t is Q = It. Electrical energy delivered W = Q.V = VIt

Thus, power given to the circuit = W/t = VI or V^2/R or I^2R

In the circuit, we can write, E. I = $I^2R + I^2 r$,

where EI is the rate at which chemical energy is converted to electrical energy, I^2R is power supplied to the external resistance R and I^2r is the power dissipated in the internal resistance of the battery.



An electrical current flowing through, conductor produces heat in it. This is known as Joule's effect. The heat developed in Joules is given by H = I^2.R.t, where I = current in Ampere, and R = resistance in ohms, t = time in seconds.

The equation H = I^2Rt is also known as Joule's law of heating.

Exercise 11:

(i) ***Which bulb has got more resistance (a) 100 W, 220 V (b) 0 W, 220 V.***

(ii) ***Under what condition is the heat produced in an electric circuit***
(a) directly proportional to the resistance of the circuit ?
(b) inversely proportional to the resistance of the circuit ?

(iii) ***What is the difference between Kilowatt and kilowatt hour ?***

(iv) ***Three resistors 2 Ω, 3 Ω and 4 Ω are connected to the same battery turn by turn in which resistor the power dissipation will be maximum ?***

(v) ***Which has a greater resistance 1 kW electric heater or a 100 W filament bulb (both marked for 220 V) ?***

Illustration 33. *A copper wire having a cross–sectional area of 0.5 mm^2 and a length of 0.1 m is initially at 25^{o}C and is thermally insulated from the surroundings. If a current of 10 A is set up in this wire,*

(a) find the time in which the wire starts melting. The change of resistance of the wire with temperature may be neglected.

(b) what will this time be, if the length of the wire is doubled?

Density of Cu = 9×10^3 kg m^{-3}, specific heat of Cu = 9×10^{-2} cal kg^{-1} ^{o}C^{-1}, M.P. (Cu) 1075 ^{o}C and specific resistance = $1.6 \times 10^{-8}\,\Omega$m.

Solution: (a) Mass of Cu = volume × density = $0.5 \times 10^{-6} \times 0.1 \times 9 \times 10^3 = 45 \times 10^{-5}$ Kg.

Rise in temperature = θ = 1075 – 25 = 1050 ^{o}C.

Specific heat = 9×10^{-2} kg^{-1} ^{o}C × 4.2 J

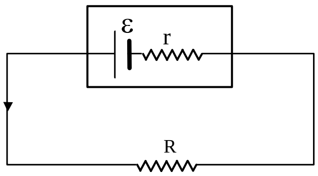
$$\Rightarrow \quad I^2Rt = mS\theta \qquad \Rightarrow \qquad t = \frac{mS\theta}{I^2.R}$$

but $$R = \frac{\rho L}{A} = \frac{1.6\times10^{-8} \times 0.1}{0.5 \text{x} 10^{-6}} = 3.2\times10^{-3}\ \Omega$$

$$\Rightarrow \qquad t = \frac{45\times4.2\times10^{-5}\times1050\times0.09}{10\times10\times3.2\times10^{-3}} = 0.558 \text{ s}$$

(b) When the length of wire is doubled, R is doubled and correspondingly mass is also doubled. Therefore, wire will start melting in the same time.

MAXIMUM POWER TRANSFER THEOREM



Consider the adjoining circuit. For the shown network power developed in resistance R equals

$$P = \frac{E^2.R}{(R+r)^2} \quad (\because I = \frac{E}{R+r} \text{ and } P = I^2R)$$

Now, for dP/dR = 0 (for P to be maximum $\frac{dP}{dR} = 0$)

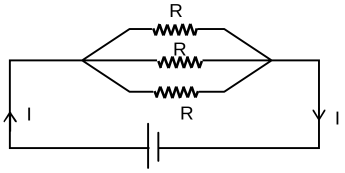
$$\Rightarrow \qquad E^2.\frac{(R+r)^2 - 2(R)(R+r)}{(R+r)^4} = 0 \qquad \Rightarrow (R+r) = 2R$$

or R = r

⇒ The power output is maximum when the external resistance equals the internal resistance.

Exercise 12: ***How can two resistances be connected to a constant voltage d.c. source to develop maximum power?***

Illustration 34. *Three equal resistances, each of R Ω, are connected as shown in figure. A battery of emf 2 V and internal resistance 0.1 Ω is connected across the circuit. Calculate the value of R for which the heat generated in the circuit is maximum?*



Solution: The given network is a parallel combination of three resistances.

Combined resistance R′ = R/3

Current (I) = $\dfrac{E}{R/3+r}$

Power (P) = $\left(\dfrac{E}{R/3+r}\right)^2 \dfrac{R}{3} = \dfrac{E^2R/3}{\left[\dfrac{R}{3}-r\right]^2+\dfrac{4Rr}{3}}$

For maximum power, $\dfrac{R}{3}-r=0$

or $R = 3r = 0.3\ \Omega$

Illustration 35. *An electric bulb rated 220 V and 60 W is connected in series with another electric bulb rated 220 V and 40 W. The combination is connected across a 220 volt source of e.m.f. Which bulb will glow brighter ?*

***Solution*:** $R = \dfrac{V^2}{P}$

$\therefore$ Resistance of first bulb is $R_1 = \dfrac{V^2}{P_1}$,

and resistance of the second bulb is $R_2 = \dfrac{V^2}{P_2}$

In series same current will pass through each bulb

$\therefore$ Power developed across first is $P_1^{'} = I^2\dfrac{V^2}{P_1}$

and that across second is $P_2^{'} = I^2\dfrac{V^2}{P_2}$

$\Rightarrow \dfrac{P_1^{'}}{P_2^{'}} = \dfrac{P_2}{P_1}$

As $P_2 < P_1 \Rightarrow \dfrac{P_2}{P_1} < 1$

$\Rightarrow \dfrac{P_1^{'}}{P_2^{'}} < 1 \Rightarrow P_1^{'} < P_2^{'}$

The bulb rated 220 V and 40 W will glow more.

FARADAY’S LAWS OF ELECTROLYSIS

(i) The mass of the ions deposited or librated in electrolysis is directly proportional to the quantity of electricity, i.e. charge passed through the electrolyte.

If m be the mass of the ions and Q the total charge passed, then

$m \propto Q$ or $m = ZQ$

where Z = electrochemical equivalent of the substance. Its value depends upon the nature of the substance.

If the electric current i flows for time t, then

$m = zit$

If $Q = 1C$, then
$Z = m$

Therefore, the electrochemical equivalent of a substance is defined as the mass of the substance librated when a charge of one coulomb passes through it.

(ii) When the same electric current is passed through several electrolytes for the same time, the masses of the various ions deposited or librated at each of the electrodes are proportional to their chemical equivalents i.e., equivalent weights.

$$\frac{\text{Mass of A deposited}}{\text{Mass of B deposited}} = \frac{\text{Chemical equivalent of A}}{\text{Chemical equivalent of B}}$$

The Unit “faraday” and Faraday Constant

The charge of 1 mole of electrons is called 1 faraday. So, faraday is a unit of charge and its relation with Coulomb is

$$1 \text{ faraday} = (1.6022 \times 10^{-19} C) \times (6.022 \times 10^{23}) = 96485 \text{ C}.$$

The quantity of charge per mole of electrons is called Faraday constant and is denoted by the symbol F.
Thus, $F = 96485$ C/mol. = 1 faraday/mole

Electro–Chemical Cells:

A system of electrodes and an electrolyte in which a chemical reaction either uses or produces a potential difference between the electrodes is called an electrochemical cell.

The electrode at higher potential is called the positive electrode and that at lower potential is called the negative electrode.

A cell is called primary if it is used only for discharge. A secondary cell on the other hand can be discharged as well as charged.

SOURCE OF EMF (CELLS)

An electrochemical cell or simply a cell may be defined as an arrangement which converts chemical energy into electrical energy at a steady rate.

Cells are of three types: Primary, secondary and fuel cells.
Primary cells cannot be recharged electrically: Once discharged, they have no further use.
Chemical reactions occurring in a secondary cell are reversible.

PRIMARY CELLS

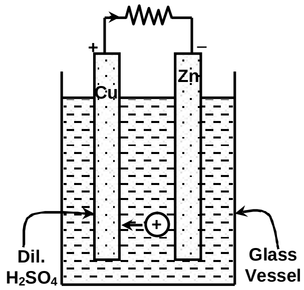
Simple Voltaic Cell

Construction: This cell consists of zinc and copper electrodes dipped in an electrolyte of dilute sulphuric acid.

$H_2SO_4 \rightarrow 2H^+ + SO_4^{2-}$

$Zn \rightarrow Zn^{2+} - 2e^-$

Copper is at a higher potential with respect to the electrolyte and zinc is at a lower potential with respect to the electrolyte.
The difference of potential between the two electrodes is nearly 1.08 volt.

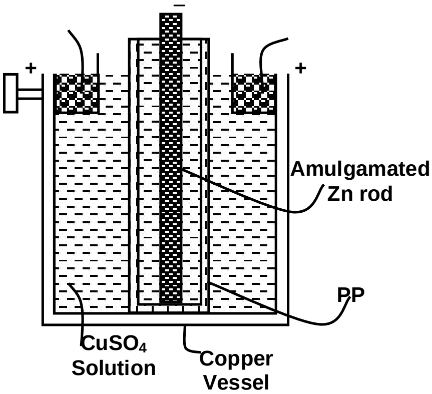


Daniel Cell

Construction: It consists of a cylindrical copper vessel containing concentrated copper sulphate ($CuSO_4$) solution. The copper vessel itself serves as the anode, i.e. positive electrode.
PP is porous earthen ware pot containing the electrolyte. It is 10% dilute sulphuric acid.

$Zn \rightarrow Zn^{2+} + 2e^-$
$H_2SO_4 \rightarrow 2H^+ + SO^{2-}$
$Zn^{2+} + SO_4^{2-} \rightarrow ZnSO_4$

$Zn + H_2SO_4 \rightarrow ZnSO_4 + 2H^+ + 2e^-$



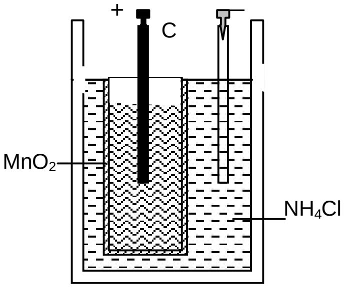
The hydrogen ions so formed diffuse through the porous pot and act on the copper sulphate forming sulphuric acid and liberating Cu^{2+} ions.

$$2H^+ + CuSO_4 \rightarrow H_2SO_4 + Cu^{++}$$

The $CuSO_4$ solution acts as the depolarizer. The emf of the Daniel cell is nearly 1.12 volt.

Leclanche Cell

It consists of a carbon and a zinc electrode in a solution of NH_4Cl. The carbon electrode is packed in a porous cup, containing MnO_2. The negative and positive terminals are made by the zinc and carbon electrodes. The Cl^- ions combine with zinc and the NH_4 +ions move towards the carbon electrode, when a current passes through the cell.



The NH_4 +ions produces ammonia and hydrogen

$$2\,NH_4^+ + 2e^- \longrightarrow 2NH_3 + H_2$$

The hydrogen reacts with solid MnO_2 to form manganese oxide (Mn_2O_3) and water.

Thus, MnO_2 prevents hydrogen from collecting on the anode which could otherwise stop the cell's function.

The absorbing hydrogen in some chemical reaction (depolarising action) is quite slow in Leclanche cell. Therefore, if the current is drawn from the cell continuously, hydrogen starts collecting at the carbon electrode and the cell stops functioning. Thus, the Leclanche cell is used when intermittent currents are needed. Its emf is about 1.5 V.

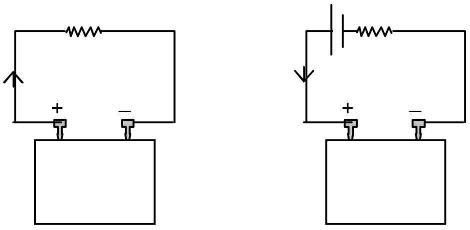
Dry Cell

This is a special kind of Leclanche cell in which both NH_4Cl and MnO_2 are prepared in the form of a paste. The paste is put in a zinc container which itself works as the negative electrode. The internal resistance of a dry cell is of the order of 0.1 Ω.

Secondary Cell

In a secondary cell, current can pass in both directions. In case of normal working, current leaves the cell at the positive terminal and enters at the negative terminal. This is called discharging of a cell. In this case chemical energy is converted into electrical energy.

In case of charging, the cell is connected to some other source of larger emf. Current may enter the cell at the positive terminal and leave it at the negative terminal. The electric energy is then converted into chemical energy and the cell gets charged. The most commonly used secondary cell is lead–accumulator.



Discharging Charging

Lead Accumulator

It consists of electrodes made of PbO_2 and of Pb immersed in dilute H_2SO_4 solution. The specific gravity of solution should lie between 1.20 to 1.28. Pb acts as the negative electrod and PbO_2 acts at the positive electrode.

Discharging: In this process SO_4^{--} ions move towards the Pb electrode, give up the negative charge and form $PbSO_4$. The H^+ ions will move to the PbO_2 electrode, give up the positive charge and reduce PbO_2 to PbO.

$$PbO_2 + 2H = PbO + H_2O$$

PbO reacts with H_2SO_4 to form $PbSO_4$ and water

$$PbO + H_2SO_4 = PbSO_4 + H_2O$$

Charging: It is the reverse of discharging. Current enters from the positive to the negative electrode inside the cell. The H^+ ions move towards the negative electrode and react with $PbSO_4$ (which was formed during discharging).

$$PbSO_4 + 2H = Pb + H_2SO_4$$

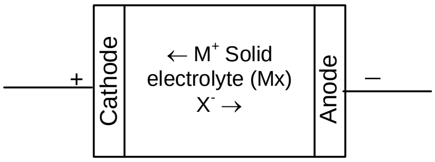
At the positive electrode,

$$PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4.$$

$PbSO_4$ deposited at the two electrodes is dissolved, Pb is deposited at the negative electrode and PbO_2 at the positive electrode. This restores the capacity of the cell to provide current.

Solid State Cells

In a solid state cell, the electrolyte is a solid in which ions can move. These electrolytes are commonly known as solid state electrolytes. Such materials are available in the form of gels polymers, composites etc.



The given figure shows the basic geometry of a solid state cell. It uses a solid electrolyte with mobile cations M^+ and anion X^-. Either one of these ions or both can move.

Exercise 13:

(i) ***Which has a greater resistance 1 kW electric heater or a 100 W filament bulb. Both marked for 220 V? Write the equations for chemical reactions taking place at the***
(a) anode, and
(b) cathode of a lead accumulator during its charging.

(ii) ***On what factors does the emf of a cell depend ?***

THERMOELECTRICITY

The phenomenon of production of electricity with the help of heat is called thermoelectricity and this effect is called thermoelectric effect. This effect comprises of three related effects:
(i) Seebeck effect
(ii) Peltier effect
(iii) Thomson effect

Seebek Effect

When the junctions of two different metals or semiconductors are maintained at different temperatures, an emf is generated in the circuit and very weak current flows in the circuit. This effect is called **Seebeck effect** or **thermoelectric effect**. The magnitude of the emf, known as **thermo–emf,** depends on the nature of the metals and the difference in temperature. Such a combination of two metals is called a thermocouple. The graph shows the nature of variation of the thermo–emf with the temperature of the hot junction. If the temperature of the cold junction is 0^oC and the hot junction be at θ^oC, then the thermo–emf is given by

$$E = a\theta + \frac{1}{2}b\theta^2 \qquad \ldots (i)$$

where a and b are the constants for a given pair of metals which constitute the thermocouple.

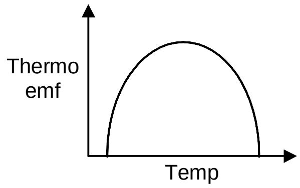


Illustration 36. *For copper–iron and a chrome–lumel thermocouple, the plots between thermoelectric emf and the temperature θ of the hot junction (when the cold junction is at 0^oC) are found to satisfy approximately the parabolic equation.*

Solution: $V = \alpha\theta + \frac{1}{2}\beta\theta^2$ with

$\alpha = 14\ \mu v\ c^o$, $\beta = -0.04\ \mu vc^{-2}$
$\alpha = 41\ \mu vc^{-1}$, $\beta = 0.002\ \mu vc^{-2}$

Wich of the two thermocouple would you use to measure temperature in the rang of about 500^oC to $600\ ^oC$.

The temperature θ_n (neutral temperature) corresponding to maximum emf is given by

$\frac{dV}{d\theta} = 0$, e. $\alpha + \beta\theta_n = 0$

or, $\theta_n = -\alpha / \beta$

For a copper–iron thermocouple, $\theta_n = -\frac{14}{-0.04}\ ^oC = 350^oC$.

Thermoelectric Series

The sequence of metals arranged in a series which is used to predict the direction of current in a thermocouple in the temperature range 0^oC to 100^oC is called the **thermoelectric series**. The series constituting the sequence of metals is as follows.

Antimony nchrome, iron, zinc, copper, gold, silver, lead, aluminium, mercury, platinum–rhodium, platinum, nickel, constantan, bismuth. The direction of current at the cold junction is from the metal coming earlier in the series. For example, in an antimony–bismuth thermocouple the current flows from antimony to bismuth at the cold junction. (Remember ABC as antimony to Bismuth at cold junction.) The series also gives an idea regarding the relative magnitude of thermo emf: farther apart the metal lie in the series, larger is the emf. Thus, $\varepsilon_{iron-copper} < \varepsilon_{iron-bismuth}$.

Thermoelectric Power

Differentiating E with respect to temperature θ,

$$\frac{dE}{d\theta} = a + b\theta$$

The quantity $\frac{dE}{d\theta}$ is called the thermoelectric power P at temperature θ. Thus, P = a + bθ represents the equation of a straight line, which is called a thermoelectric line.

Neutral Temperature: θ_n and Temperature of Inversion θ_i:

The temperature of the hot junction at which the thermo emf is maximum is called the neutral temperature (θ_n). Thus, at neutral temperature $\frac{dE}{d\theta} = 0$

$$\Rightarrow \quad \theta_n = -\frac{a}{b}$$

The temperature of the hot junction at which the thermo–emf changes its sign (direction of thermo–current is reversed) is called the temperature of inversion θ_i. If θ_c = temperature of the cold junction, then

$$\theta_n - \theta_c = \theta_i - \theta_n$$

Peltier Effect

When an electric current is passed through a thermocouple whose junctions were initially at the same temperature, heat is produced at one junction (temperature of the junction increases and is warmed up) and heat is absorbed at the other junction (temperature of the junction decreases and is cooled down). The effect is called Peltier effect and it is reverse of Seebeck effect. Peltier effect is a reversible effect which means that reversal of the direction of current produces reversal of the cooling and warming up of the junction. This means that the junction which was initially warmed up gets cooled down when current direction is reversed.

According to Peltier, $\Delta H \propto \Delta Q$,

or, $\Delta H = \pi \, \Delta Q$

where $\Delta Q = I \, \Delta t$ = quantity of charge passed through,

ΔH = heat produced or absorbed at the junction, and

π = Peltier emf (expressed in volt).

Comparison between Joule heat and Peltier heat:

1. In Joule heat, $H \propto I^2$, which always warms up, whereas Peltier heat $H = \pm \pi It$, which warms up or cools down the junction depending upon the direction of current.

2. Peltier heating or cooling occurs only at the junctions of the thermocouple whereas Joule heat is produced throughout the resistor.

3. $H_{joule} \propto I^2$ whereas $H_{Peltier} \propto I$
   (Irreversible) (Reversible)

Thomson Effect

When an electric current flows through a conductor, the ends of which are maintained at different temperatures (having a temperature gradient along the length), heat may either be evolved or absorbed in different sections of the conductor. If either the direction of current or the temperature gradient is reversed, heat is absorbed rather than being evolved. This heat is in addition to Joule heat (I^2 Rt) and is called Thomson heat. And the effect is called Thomson effect.

Thomson heat ΔH produced in a small section is given by

$\Delta H = \sigma.\ \Delta Q.\ \Delta\theta$

where ΔQ = quantity of charge passed through,

$\Delta\theta$ = temperature difference across the given section,

σ = Thomson coefficient for a given metal at a given temperature.

Hence, $\frac{\Delta H}{\Delta Q} = \sigma.\Delta\theta$ = Thomson emf

The Seebeek effect is in fact a combination of two effects: Peltier and Thomson effects.

Quantitatively,

Seebeek emf , $\varepsilon_{AB} = (\pi_{AB})_{\theta_2} - (\pi_{AB})_{\theta_1} + (\theta_2 - \theta_1)(\sigma_A - \sigma_B)$.

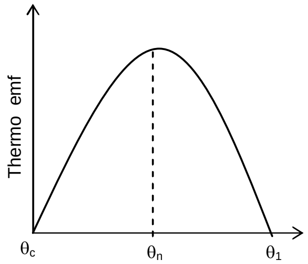
MEASUREMENT OF TEMPERATURE USING A THERMOCOUPLE

Thermoelectric Thermometer

It is a device used to measure both low and high temperatures. It is based upon Seebeck effect. A thermoelectric thermometer is basically a thermocouple.

By keeping cold junction at 0^oC, a graph between thermo–emf and the temperature of hot junction is plotted. This graph is called calibration curve of the thermocouple and shown in the figure below.

Now to measure the temperature of a hot body one junction is kept at 0^oC and the other junction is kept in contact with the hot body. The thermo–emf produced in the thermocouple is measured. Now, from the calibration curve read the temperature of the hot body against the measured thermo–emf.



(Temperature Hot Junction)

SUMMARY

Current: An electric current i in a conductor is given by $\mathbf{i} = \frac{dq}{dt}$

Current Density: Current is related to current density $\vec{J}$ by $i = \int \vec{J}.d\vec{A}$

Drift Speed: When an electric field $\vec{E}$ is established in a conductor, the charge carriers aquire a drift speed v_d in the direction of $\vec{E}$. The velocity $\vec{v}_d$ is related to the current density by

$\vec{J} = (ne)\vec{v}_d$, where ne = carrier charge density.

Mobility: $\mu = m^2v^{-1}s^{-1}$

Resistance of a conductor: $R = \frac{V}{i}$ (definition of R)

Resistivity and conductivity: $\rho = \frac{1}{\sigma} = \frac{E}{J}$

The resistance R of a conducting wire of length L and uniform cross–section is

$R = \frac{\rho L}{A}$, where A is the cross–sectional area.

Change of ρ with temperature: $\rho - \rho_0 = \rho_0\, \alpha\, (T - T_0)$

where T_0 and ρ_0 are reference temperature and resistivity, respectively.

Ohm's law: A given device obeys Ohm's law if its resistance R = V / i is independent of the applied potential difference V.

Superconductors: Superconductors lose all electrical resistance at low temperatures.

Kirchhoff's law:
(a) At any junction of circuit elements, the sum of currents entering the junction must equal the sum of currents leaving it.
(b) The algebraic sum of changes in potential around any closed loop must be zero.

Resistor:
(a) Total resistance of n resistors, each of resistance R, connected in series is given by

$$R = R_1 + R_2 + \ .\ .\ .\ . + R_n$$

(b) Total resistance of n resistors connected in parallel is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$$

Wheatstone Bridge: It is the arrangement of four resistances P, Q, R, S. The null point condition is given by

$$\frac{P}{Q} = \frac{R}{S}$$

Charging of capacitor:

$$q = q_{max}\,[1 - e^{t/RC}]$$
$$I = i_{max}\, e^{-t/RC}$$

Faraday’s laws of electrolysis:

(i) m = Zit

(ii) $\dfrac{\text{Mass of A deposited}}{\text{Mass of B deposited}} = \dfrac{\text{Chemical equivalent of A}}{\text{Chemical equivalent of B}}$

Seebeck Effect:

If two junctions of dissimilar conductors in a circuit are held at different temperatures, an emf develops causing a current to flow in the circuit. This effect is called **Seebeck effect**.

Peltier Effect:

When an electric current is passed through a junction of two different conductors, heat is either absorbed or released at the junction. This depends on the direction of the current. This effect is called Peltier effect.

Thomson effect:

Emf develops between two parts of a single metal when they are at different temperatures. This effect is called Thomson effect.

SOLVED PROBLEMS

Subjective:

Prob 1. *A copper wire of diameter 1.02 mm carries a current of 1.7 amp. Find the drift velocity (v_d) of electrons in the wire. Given n, number density of electrons in copper = 8.5×10^{28} /m^3.*

Sol. I = 1.7 A

J = current density

$= \frac{I}{\pi r^2} = \frac{1.7}{\pi \times (0.51 \times 10^{-3})^2} = nev_d$

$= 8.5 \times 10^{28} \times (1.6 \times 10^{-19}) \times v_d$

$\therefore v_d = \frac{1.7}{\pi \times (0.51 \times 10^{-3})^2 \times 8.5 \times 10^{28} \times 1.6 \times 10^{-19}}$

$= 1.5 \times 10^{-3}$ m/sec. = 1.5 mm/sec.

Prob 2. *A copper wire of resistance 4 Ω is stretched to thrice its original length. Find the resistance of stretched wire.*

Sol. Since volume of the wire does not change.

$\Rightarrow \ell_1 A_1 = \ell_2 A_2$, where ℓ_1 and A_1 are the initial length and cross–section of the wire, and ℓ_2 and A_2 are the final length and cross–section.

$$\Rightarrow \quad \frac{l_1}{l_2} = \frac{A_2}{A_1} \qquad \ldots (1)$$

$$\therefore \quad R_1 = \rho\frac{l_1}{A_1} \quad \text{and } R_2 = \rho\frac{l_2}{A_2}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \frac{l_1}{l_2} \times \frac{l_1}{l_2} \qquad \ldots (2)$$

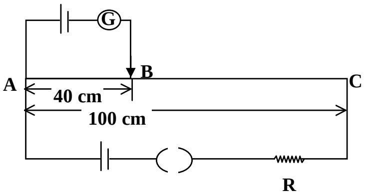
$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{l_1^2}{l_2^2} \text{ as } \ell_2 = 3\ell_1$$

$$\Rightarrow \quad R_2 = 9R_1 = (9 \times 4)\,\Omega = 36\,\Omega$$

Prob 3. *A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance and a cell of 2 volts emf and negligible internal resistance. A source of emf 10 mV is balanced against a length of 40cm of potentiometer wire. What is the value of external resistance?*

Sol. As shown in figure, if R is the unknown resistance, current in the circuit

$I = \frac{V}{(r+R)} = \frac{2}{(10+R)}$



Now, as 100 cm wire has a resistance of 10 Ω, the resistance of 40 cm wire will be 40 × (10/100) = 4 ohm.

Potential drop across 40 cm wire will be V = I ×4

But here V = 10 mV (given) and $I = \dfrac{2}{(10+R)}$

$\therefore\ \dfrac{10}{100} = \dfrac{2\times4}{10+R}$

i.e. R = 790 Ω.

Prob 4. *The junctions of a thermocouple are maintained at 0^oC and 100^oC. If the electric constants a and b for the couple be $16.3 \times 10^{-6}\ VC^{-1}$ and $-0.042 \times 10^{-6} V^oC^{-2}$ respectively, determine the seebeck emf produced.*

Sol. $\varepsilon = a\theta + \dfrac{1}{2}b\theta^2$

$= (16.3 \times 10^{-6} \times 100) + \dfrac{1}{2}(-0.042 \times 10^{-6} \times 10^4) = 1.42 \times 10^{-3}$ V.

Prob 5. *Find the neutral temperature and the temperature of inversion for a Cu – Fe thermocouple whose junctions are maintained at 0^oC and 40^oC.*

Given : $a_{Cu-Bi} = 2.76\ \mu V\,^oC^{-1}$ $\quad b_{cu-Bi} = 0.012\ \mu V\,^oC^{-1}$

$A_{Fe-Bi} = 16.6\ \mu V\,^oC^{-1}$ $\quad b_{Fe-bi} = 0.03\ \mu V^oC^{-1}$

Sol. $a_{cu-Fe} = a_{cu-Bi} - a_{Fe-Bi} = (2.76 - 16.6)\ \mu V\,^oC^{-1} = -13.84 \times 10^{-6}\ V^oC^{-1}$

$B_{cu-Fe} = b_{Cu-Bi} - b_{Fe-Bi}$

$= [0.012 - (-0.03)]\mu V^oC^{-2} = 0.042 \times 10^{-6}\ V\,^oC^{-2}$

Now neutral temperature $\theta_n = -\dfrac{a}{b}$

$= \dfrac{13.84\times10^{-6}}{0.042\times10^{-6}}\,^oC = 329.5^oC$

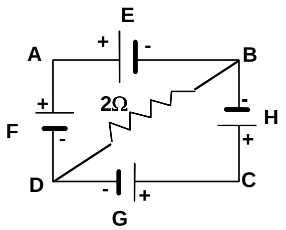
Again, temperature of inversion $\theta_I = -\dfrac{2a}{b} = 659\ ^oC$.

Prob 6. *In the series circuit shown, E, F, G ,H are cells of emf 2V, 1V, 3V and 1V respectively, and their internal resistances are 2, 1, 3 and 1Ω, respectively.*

Calculate

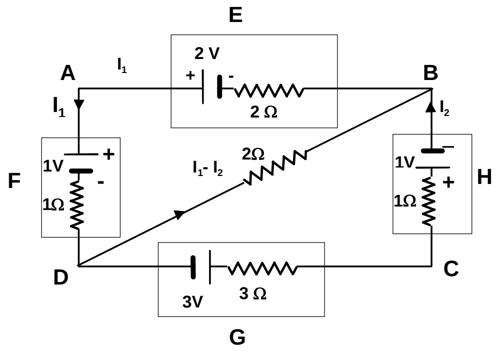
(i) the potential difference between B and D and

(ii) the potential difference across the terminals of each of the cells G and H.



Sol. Let us redraw the circuit.

At junction D, we have applied the junction rule, whereby we get current in DB as shown.



Loop BADB

$2I_1 - 2 + 1 + I_1 + 2(I_1 - I_2) = 0$

$\Rightarrow\ 5I_1 - 2I_2 = 1$

Loop DCBD

$-3 + 3I_2 + I_2 + 1 - 2(I_1 - I_2) = 0$

$\Rightarrow 6I_2 - 2I_1 = 2$

$\Rightarrow \quad I_1 = \frac{5}{13}\text{A}, \; I_2 = \frac{6}{13}\text{A}$

(i) $V_{BD} = 2(I_1) - 2 + 1 + I_1$

$= 3I_1 - 1 = 3\left(\frac{5}{13}\right) - 1 = \frac{2}{13}\ \text{V}$

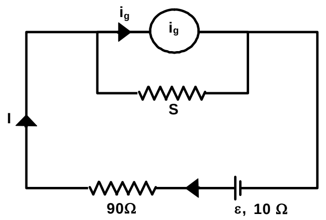
(ii) Terminal voltage of G = $|-3 + 3.I_2|$

$= \left|-3 + 3\left(\frac{6}{13}\right)\right| = \frac{21}{13}\ \text{V}$

Terminal voltage of H = $\left|\frac{6}{13}I + 1\right| = \frac{19}{13}\text{V}$.

Prob 7. *A galvanometer having 50 divisions provided with a variable shunt S is used to measure the current when connected in series with a resistance of 90Ω and a battery of internal resistance 10Ω. It is observed that when the shunt resistances are10Ω and 50Ω respectively the deflections are respectively 9 and 30 divisions. What is the resistance of the galvanometer? Further if the full scale deflection of the meter movement is 200 mA, find the emf of the cell.*

Sol.



$$I = \frac{\varepsilon}{\left(90 + 10 + \frac{SG}{S+G}\right)} = \frac{\varepsilon}{\left(100 + \frac{SG}{S+G}\right)} \qquad \ldots (1)$$

Applying Kirchhoff's law

We get, $i_g = \frac{IS}{S+G}$

$$\Rightarrow \qquad i_g = \frac{S}{S+G} \times \frac{\varepsilon}{\left(100 + \frac{SG}{S+G}\right)} \qquad \ldots (2)$$

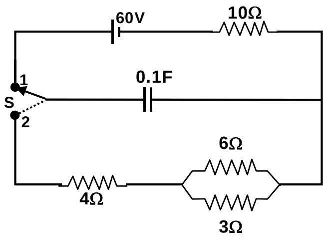
Let $i_g = I_1$ for S = 10Ω and $i_g = I_2$ for S = 50Ω

$$\frac{I_1}{I_2} = \frac{\left(\frac{10}{10+G}\right) \times \left(\frac{\varepsilon}{100 + \frac{10G}{10+G}}\right)}{\left(\frac{50}{50+G}\right) \times \left(\frac{\varepsilon}{100 + \frac{50G}{50+G}}\right)} \qquad \Rightarrow \qquad \frac{I_1}{I_2} = \frac{100+3G}{100+11G}$$

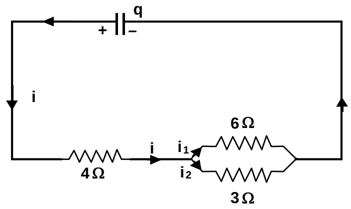
$\therefore$ Deflection is proportional to the current

$\Rightarrow \qquad \frac{9}{30} = \frac{100+3G}{100+11G}$, G = 233.3 Ω

Prob 8. *A two way switch S is used in the circuit shown in the figure. First the capacitor is charged by putting the switch in position 1. Calculate heat generated across each resistor when the switch is shifted to position 2.*



Sol. Initially, the switch was in position 1. Therefore, initially potential difference across capacitor was equal to the e.m.f. of the battery i.e., 60V.



$\Rightarrow$ Initially energy stored in the capacitor was

$U = \frac{1}{2}\ CV^2 = \frac{1}{2} \times 0.1 \times 60^2 J\ = 180\ J$

When the switch is shifted to position 2 capacitor begins to discharge and energy stored in the capacitor is dissipated in the form of heat in the resistor.

For a current I, applying Kirchoff's law, $i_1 + i_2 = i$

And $6i_1 - 3i_2 = 0$, or $i_2 = 2i_1$

$\Rightarrow\ i_1 = \frac{i}{3}$ and $i_2 = \frac{2}{3}i$

But heat generated per unit time in the resistance is i^2R

$\therefore$ The heat generated across 4Ω, 6Ω and 3Ω resistances are in ratio $4i^2\ :\ 6i_1^2\ :\ 3i_2^2\ = 6:1:2$

Total heat generated in the circuit per unit time = $P_1 + P_2 + P_3 = U$

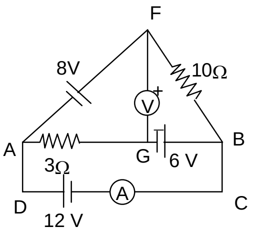
Heat generated across 4Ω resistor is $P_1 = 120$ J

Heat generated across 6Ω resistor is $P_2 = 20$ J

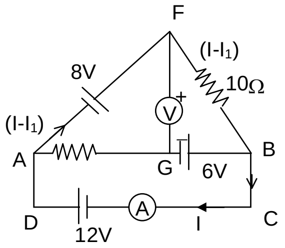
Heat generated across 3Ω resistor is $P_3 = 40$ J

Since during discharging, no current flows through 10Ω, therefore heat generated across it is equal to zero.

Prob 9. *Find the reading of ammeter A and voltmeter V shown in figure assuming the instruments to be ideal.*



Sol. Distributing the currents in the circuit according to Kirchhoff's current law as shown in the figure, and applying Kirchhoff's law in mesh ABCDA



$-3I_1 + 6 + 12\ = 0$ i.e. $I_1 = 6A$

Now, apply Kirchoff's law in AFBA

$8 - (I - I_1) \times 10 - 6\ + 3I_1\ = 0$

i.e $10\ I - 13I_1\ = 2$ or $I\ = \frac{2}{10} + \frac{13}{10} \times 6\ = 8A$

Hence reading of ammeter = 8A

Reading of the voltmeter $V = V_F - V_G$

Applying Kirchhoff's law in mesh AFGA

$$8 - V + 6 \times 3 = 0 \text{ i.e. } V = 26\ V$$

Hence, reading of voltmeter = 26 V.

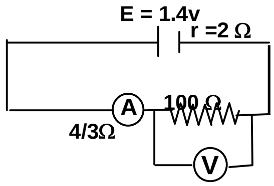
Prob 10. *A battery of emf 1.4 V and internal resistance 2Ω is connected to a 100 Ω resistor through an ammeter. The resistance of the ammeter is (4/3) Ω. A voltmeter is also connected to find the potential difference across the resistor.*

(i) Draw the circuit diagram.

(ii) The ammeter reads 0.02A. What is the resistance of the voltmeter?

(iii) The voltmeter reads 1.10 V. What is the error in reading?

Sol. (i)



(ii) Total resistance in the circuit = $\left[2+\frac{4}{3}+\frac{100R_v}{100+R_v}\right]$

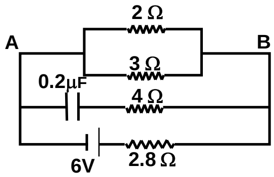
$$I = \frac{\text{Emf}}{\text{Total resistance}}$$

$$\Rightarrow \quad 0.02\,A = \frac{1.4\,V}{\left[2+\frac{4}{3}+\frac{100R_v}{100+R_v}\right]\Omega} \quad \Rightarrow \quad R_v = 200\ \Omega$$

(iii) Potential difference across the voltmeter = $0.02.\left[\frac{100\times200}{100+200}\right]$ = 1.33 V.

Voltmeter reading = 1.10 V $\Rightarrow$ Error = 1.33 – 1.10 = 0.23 V.

Prob 11. *Calculate the steady–state current in the 2Ω resistor shown. The internal resistance of the battery is negligible and the capacity of the condenser is 0.2 μF.*



Sol. The resistance of the parallel combination of 2Ω and 3Ω resistors is given by

$$\frac{1}{R} = \frac{1}{2}+\frac{1}{3} = \frac{5}{6} \quad \Rightarrow \quad R = 1.2\ \Omega$$

This resistance is in series with 2.8 Ω giving a total effective resistance

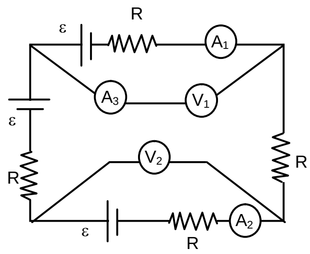
= (1.2 + 2.8) Ω = 4 Ω.

In the steady state, no current flows through capacitor C and hence no current passes through 4Ω resistor which is in series with the capacitor.

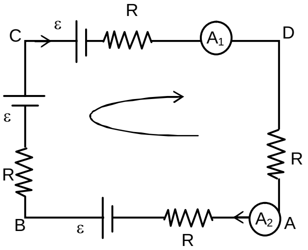
Thus the current through the circuit = 6/4 = 1.5 A,

V_{AB} = 1.5 × 1.2 = 1.8 V, I through 2Ω resistor = 1.8/2 = 0.9 A.

Prob 12. *If all the ammeter and voltmeters shown in the circuit are ideal, then find the readings in all of them.*



Sol. An ideal voltmeter offers very large resistance [infinite] and an ideal ammeter offers no resistance.
$\Rightarrow$ No current flows through V_1 and V_2, therefore the circuit becomes equivalent to



Applying KVL in ABCDA

$\Rightarrow -iR + \varepsilon - iR + \varepsilon - \varepsilon - iR - iR = 0$

$\Rightarrow i = \dfrac{\varepsilon}{4R}$

$\Rightarrow$ Reading of A_2 = reading of $A_1 = \dfrac{\varepsilon}{4R}$ amperes.

Reading of $A_3 = 0$

$$V_D + \left(\frac{\varepsilon}{4R}\right)R + \varepsilon = V_C$$

$$V_C - V_D = \frac{5\varepsilon}{4}$$

$\Rightarrow$ Reading of $V_1 = \dfrac{5\varepsilon}{4}$

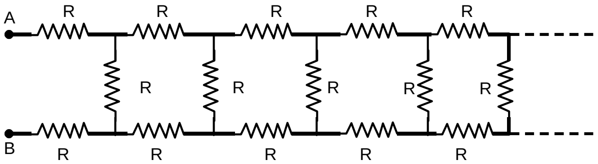
$$V_A - iR + \varepsilon = v_B$$

$\Rightarrow V_A - \dfrac{\varepsilon}{4} + \varepsilon = V_B$

$\Rightarrow V_B - V_A = \dfrac{3\varepsilon}{4}$

Reading of $V_2 = \dfrac{3\varepsilon}{4}$

Prob 13. *Find the equivalent resistance between points A and B.*



Sol. Let R_{eq} be the equivalent resistance between points A and B.

$\Rightarrow 2R + \dfrac{R_{eq} \times R}{R_{eq} + R} = R_{eq}$

$\Rightarrow 2R = R_{eq}\left[1-\dfrac{R}{R_{eq}+R}\right]$

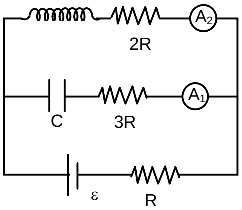
$\Rightarrow 2R = \dfrac{R_{eq}^2}{R_{eq}+R} \Rightarrow R_{eq}^2-(2R)R_{eq}-2R^2=0$

$\Rightarrow R_{eq} = \dfrac{2R\pm\sqrt{4R^2+8R^2}}{2} = R(1\pm\sqrt{3})$

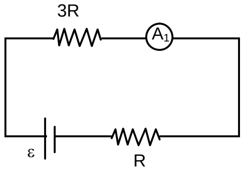
Leaving negative sign as resistance cannot be negative.

$\Rightarrow R_{eq} = (\sqrt{3}+1)R$

Prob 14. *Find the readings of A_1 and A_2 at*
(a) t = 0
(b) after a very long time.



Sol. (a) At t = 0, the capacitor will act like a short circuit and the inductor will behave like an open circuit.
Therefore, at t = 0
Reading of $A_2 = 0$ and the circuit reduces to

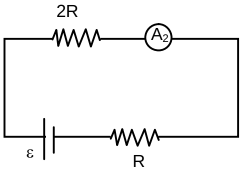


Current through A_1 will be $\dfrac{\varepsilon}{4R}$

$\Rightarrow$ At t = 0, reading of $A_2 = 0$

Reading of $A_1 = \dfrac{\varepsilon}{4R}$

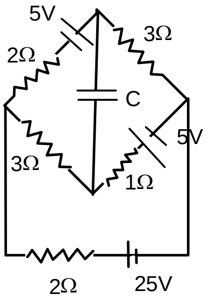
(b) After a very long time (i.e. $t\to\infty$), capacitor will act like short circuit.
$\therefore$ At $t\to\infty$, circuit reduces to



$\Rightarrow$ Reading of A_1 after a long time = 0

Reading of A_2 after a long time = $\dfrac{\varepsilon}{3R}$

Prob 15. *Consider the circuit in steady state as given. Internal resistances of all the batteries are zero.*
(a) Find current through the 1Ω resistance.
(b) Find charge on the capacitor C if C = (19/75) μF.



Sol. (a) Applying KVL to BCDEFB

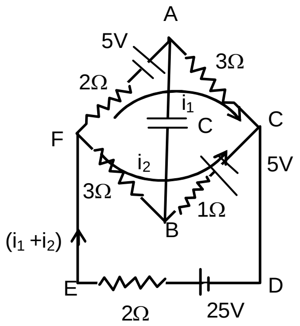
$25 - 2(i_1 + i_2) - 3i_2 - i_2 - 5 = 0 \quad \ldots \text{(i)}$

Applying KVL to ACDEFA

$25 - 2(i_1 + i_2) - 2i_1 + 5 - 3i_1 = 0 \quad \ldots \text{(ii)}$

Solving (i) and (ii) we get

$i_1 = \dfrac{70}{19}\text{A}, \quad i_2 = \dfrac{40}{19}\text{A}$



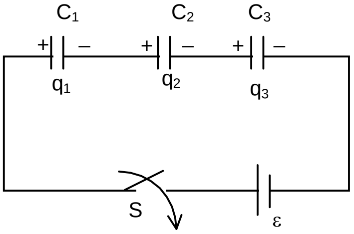
(b) $v_A - v_B = \dfrac{75}{19}$ volt

Hence, charge on capacitor $= \left(\dfrac{19}{75}\right)\left(\dfrac{75}{19}\right) = 1\ \mu\text{C}.$

Prob 16. *Charges q_1, q_2 and q_3 are placed on capacitors of capacitances C_1, C_2 and C_3 respectively, arranged in series as shown. Switch S is then closed. What are the final charges q'_1, q'_2 and q'_3 on the capacitors ? Given $q_1 = 30\mu C$, $q_2 = 20\ \mu C$, $q_3 = 10\mu C$;*

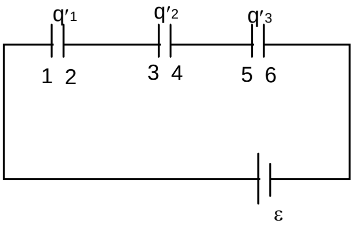
$C_1 = 10\mu F$, $C_2 = 20\mu F$, $C_3 = 30\mu F$

and $\varepsilon = 12$ volts



Sol. Applying Kirchoff's law on the loop,

$$\frac{1}{\left(\dfrac{M}{m} + \dfrac{r^2}{R^2}\right)} = \varepsilon \qquad \ldots(1)$$



Net charge on plates 2 and 3 will remain conserved.

$\therefore -q'_1 + q'_2 = -q_1 + q_2 \quad \ldots.(2)$

Also, net charge on plates 4 and 5 will remain conserved.

$\therefore -q_2' + q'_3 = -q_2 + q_3 \quad \ldots(3)$

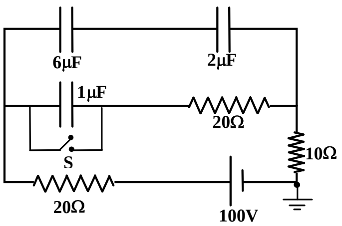
Using equations (1), (2) and (3) and putting values, we get

$q'_1 = \dfrac{790}{11}\mu\text{C}, \ q'_2 = \dfrac{680}{11}\mu\text{C}, \ q'_3 = \dfrac{570}{11}\mu\text{C}.$

Prob 17. *Calculate the charge on each capacitor and the potential difference across it in steady state in the circuit shown for the cases*

(i) switch S closed, and

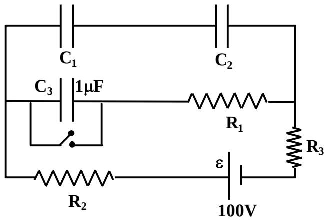
(ii) switch S open.



Sol. (i) With switch S closed, potential difference across $C_3 = 0$ and hence charge on $C_3 = 0$

If I be the current through the resistors ,

$$I = \frac{\varepsilon}{R_1 + R_2 + R_3} = \frac{100}{50} = 2\text{A}$$



$\Rightarrow$ P.d. across $R_1 = (20\ \Omega)(2A) = 40$ V

$\Rightarrow$ P.d. across branch containing C_1 and $C_2 = 40$ V

$\Rightarrow$ Charge on C_1 = charge on C_2 = Q, say

$= \left(\frac{C_1C_2}{C_1+C_2}\right)(40V) = \left(\frac{12}{8}\times 40\right)\mu\ C$

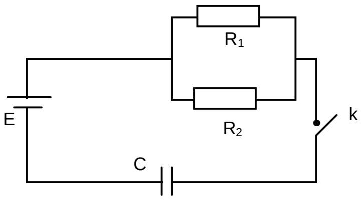
$= 60 \times 10^{-6}$ Coul.

Hence p.d. across $C_1 = \frac{60}{6}V = 10V$

and p.d. across $C_2 = \frac{60}{2}V = 30V$

(ii) With switch 'S' is open, as there is no current in any branch of the given circuit.
P.d. across $C_3 = 100$ V and charge on $C_3 = (100)(1)\ \mu C = 10^{-4}$ C.
Similarly charge on C_2 = charge on C_1 = [3/2 μF] [100 V] = 150 μC
P.d. across C_1 = (150 μC)/(6μF) = 25 v and p.d. across C_2 = (100 – 25) V = 75 V .

Prob 18. *In the circuit diagram shown in figure, the capacitor of capacitance C is uncharged when the key k is open. the key is closed over some time during which the capacitor becomes charged to a voltage V. Determine the amount of heat Q_2 liberated during this time in the resistor of resistance R_2 if the emf of the source is E, and its internal resistance can be neglected.*



Sol. Energy conservation theorem :

$E_q = \frac{q^2}{2C} + Q_T$

$Q_T = Q_1 + Q_2,\quad \frac{Q_1}{Q_2} = \frac{R_2}{R_1}$

$Q_2 = C\left(Ev - \frac{v^2}{2}\right)\frac{R_1}{R_1+R_2}$.

Objective:

Prob 1. *The negative Zn pole of a Daniell cell, sending a constant current through a circuit, decreases in mass by 0.13 g in 30 minutes. If the electrochemical equivalents of Zn and Cu are 32.5 and 31.5, respectively, the increase in the mass of the positive Cu pole in this time is*

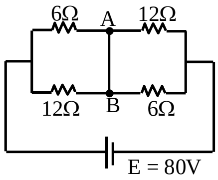
(A) 0.126 g *(B) 0.242 g*
(C) 0.180 g *(D) 0.141 g.*

Sol. $\frac{0.13}{m_{cu}} = \frac{32.5}{31.5}$

or $m_{cu} = \frac{0.13 \times 31.5}{32.5} g = 0.126g$

$\therefore$ **(A)**

Prob 2. *In the circuit shown, a wire is connected between points A and B. How much current will flow through the wire?*



(A) 5 A *(B) $\frac{10}{3}$A*
(C) $\frac{20}{3}$A *(D) $\frac{5}{3}$A*

Sol Current through battery

$I = \frac{80}{8} = 10$ amp. = 10 amp.

Current through $6\sigma_2$ resistance will be 20/3 amp. And current through 12 r^2 resistance will be 20/3 amp. Hence, current through wire AB will be 20/3 –10/3 = 10/3 amp.

$\therefore$ **(B)**

Prob 3. *In a gas discharge tube, if 3×10^{18} electrons are flowing per sec from left to right and 2×10^{18} protons are flowing per second from right to left through a given cross section, the magnitude and direction of current through the cross section is*

(A) 0.48 A, left to right *(B) 0.48 A, right to left*
(C) 0.80 A, left to right *(D) 0.80 A, right to left*

Sol. As current is the rate of flow of charge in the direction in which positive charge will move, the current due to electron will be

$i_e = \frac{n_e q_e}{t} = 3 \times 10^{18} \times 1.6 \times 10^{-19} = 0.48$ A (opposite to the motion of electrons, i.e. right to left)

Current due to protons, $i_p = \frac{n_p q_p}{t} = 2 \times 10^{18} \times 1.6 \times 10^{-19} = 0.32$ A (Right to left)

So total $I = i_e + i_p$

$\Rightarrow$ = 0.48 + 0.32 = 0.80 A (right to left)

$\therefore$ **(D)**

Prob 4. *The electron in a hydrogen atom moves in a circular orbit of radius 5×10^{-11} m with a speed of $0.6\,\pi \times 10^{6}$ m/s. Then*

(A) the frequency of the electron is 6×10^{15} rev/s
(B) the electron carries 1.6×10^{-19} C around the loop
(C) the current in the orbit is 0.96 mA
(D) the current flows in the opposite direction to the direction of motion of the electron

Sol. Electric current at a point on the circle

$i = fe$

$$f = \text{frequency} = \frac{\omega}{2\pi} = \frac{v}{2\pi r} = \frac{0.6\times10^{6}\times\pi}{2\pi\times5\times10^{-11}} = 6 \times 10^{15}\ \text{rev/s}$$

$i = 6 \times 10^{15} \times 1.6 \times 10^{-19} = 0.96$ mA

Hence correct options are **(A), (B), (C) and(D).**

Prob 5. *The current in a wire varies with time according to the equation I = 4 + 2t, where I is in ampere and t is in sec. The quantity of charge which has passed through a cross–section of the wire during the time t = 2 sec to t = 6 sec will be*

(A) 60 coulomb *(B) 24 coulomb*
(C) 48 coulomb *(D) 30 coulomb*

Sol. Let dq be the charge which has passed in a small interval of time dt,

then $dq = idt = (4 + 2t)dt$

Hence total charge passed between interval t = 2 sec and t = 6 sec

$$q = \int_2^6 (4+2t)dt = 48 \text{ coulomb},$$

$\therefore$ **(C)**

Prob 6. *The magnitude of momentum of electrons in a straight wire of copper of length 1 meter carrying a current of 16 ampere will be*

(A) 14.56×10^{-12} kg m sec^{-1} *(B) 29.12×10^{-12} kg m sec^{-1}*
(C) 18.2×10^{-11} kg m sec^{-1} *(D) 91×10^{-12} kg m sec^{-1}*

Sol. If n is the number of electrons per unit volume, then total number of free electrons = nLA

Hence, total mass of the electrons = nLA m_e

Total momentum of electrons = nLA $m_e \times v_d$

$$= nLA\, m_e \times \frac{I}{e}$$

$$= \frac{1\times9.1\times10^{-31}\times16}{1.6\times10^{-19}} = 91 \times 10^{-12}\ \text{kg sec}^{-1},$$

$\therefore$ **(D)**

Prob 7. *An electric current of 16 A exists in a metal wire of cross section 10^{-6} m^2 and length 1m. Assuming one free electron per atom, the drift speed of the free electrons in the wire will be (Density of metal $=5 \times 10^{3}$ kg/m^3, atomic weight = 60)*

(A) 5×10^{-3} m/s *(B) 2×10^{-3} m/s*
(C) 4×10^{-3} m/s *(D) 7.5×10^{-3} m/s*

Sol. According to Avogadro's hypothesis

$$\frac{N}{N_A} = \frac{m}{M}$$

So $n = \frac{N}{v} = N_A \frac{m}{VM} = \frac{N_A}{M}$

Hence, total number of atoms $n = \frac{6\times10^{23}\times5\times10^{3}}{60\times10^{-3}} = 5\times10^{28}$ /m^3

Hence, drift velocity $v_d = \frac{I}{n_e eA} = \frac{16}{5\times10^{28}\times1.6\times10^{-19}\times10^{-6}} = 2\times10^{-3}$ m/s.

$\therefore$ **(B)**

Prob 8. *If a copper wire is stretched to make it 0.1% longer, the percentage change in its resistance is*

(A) 0.2 % increase *(B) 0.2% decrease*

(C) 0.1 % increase *(D) 0.1 % decrease*

Sol. For a given wire, $R = \frac{\rho L}{s}$ with $L \times s$ = volume = V = constant

So $R = \rho \frac{L^2}{V}$

$\frac{\Delta R}{R} = 2\frac{\Delta L}{L} = 2\,(0.1\,\%) = 0.2\,\%$ (increase)

$\therefore$ **(A)**

Prob 9. *The mass of three wires of copper are in the ratio 1 : 3 : 5. And their lengths are in the ratio 5 : 3 : 1 . The ratio of their electrical resistances is*

(A) 1 : 3 : 5 *(B) 5 : 3 : 1*

(C) 1 : 15 : 125 *(D) 125 : 15 : 1*

Sol. $R = \frac{\rho\ell}{A} = \frac{\rho\ell^2}{A\ell} = \frac{\rho\ell^2}{V} = \frac{\rho\ell^2}{m/d}$

$R = \frac{\rho d\ell^2}{m}$ or $R \propto \frac{\ell^2}{m}$

$R_1 : R_2 : R_3 = \frac{\ell_1^2}{m_1} : \frac{\ell_2^2}{m_2} : \frac{\ell_3^2}{m_3}$

$= \frac{25}{1} : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$ $\therefore$ **(D)**

Prob10. *A battery of 10 volt is connected to a resistance of 20 ohm through a variable resistance R. The amount of charge which has passed in the circuit in 4 minutes, if the variable resistance R is increased at the rate of 5 ohm/min, is*

(A) 120 coulomb *(B) 120 $\log_e 2$ coulomb*

(C) $\frac{120}{\log_e 2}$ coulomb *(D) $\frac{60}{\log_e 2}$ coulomb*

Sol. $I = \frac{dq}{dt} = \frac{V}{R}$

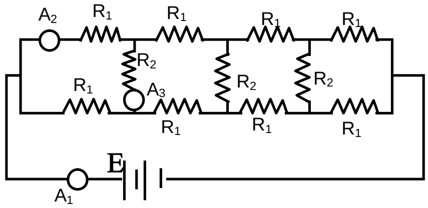
$\frac{dq}{dR}.\frac{dR}{dt} = \frac{V}{R}$; dq = 12 V

$q = 12\text{ V}\int_{20}^{40}\frac{dR}{R} = 12\text{ V}(\log_e 40 - \log_e 20)$

$= 12 \times 10 \times \log_e 2,$

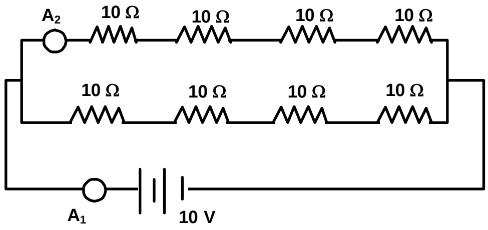
$\therefore$ **(B)**

Prob 11. *In the given circuit, given $R_1 = 10\ \Omega$, $R_2 = 6\ \Omega$ and $E = 10$ V. Then which of the following statements are correct ?*



(A) Effective resistance of the circuit is 20 Ω.
(B) Reading of A_1 is ½ amp.
(C) Reading of A_2 is ¼ amp.
(D) Reading of A_3 is 1/8 amp.

Sol. Potential difference across R_2 resistances is zero. Therefore, current in two branches is zero, therefore current in the two branches containing R_1 will be same, the simplified circuit will be

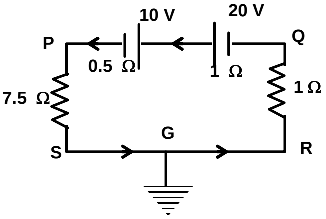


Effective resistance of the circuit, $R_{eff} = \frac{40\times 40}{40+40} = 20\ \Omega$

current through the circuit, $I = \frac{10}{20} = (1/2)$ amp

Hence, reading of A_1 = ½ amp.
Hence, reading of A_2 =1/4 amp.
Hence, correct answer is **(A), (B) and (C)**

Prob 12. *In the circuit shown:*



(A) the potential at P is -7.5 V
(B) the potential at Q is -1V
(C) the potential at R is zero
(D) the potential at S is zero

Sol. The current in the circuit $I = \frac{(20-10)}{(7.5+.5+1+1)} = \frac{10}{10} = 1$ amp.

The potential difference across PS $V_{PS} = 7.5 \times 1 = 7.5$ V
The potential difference across Q R, $V_{QR} = 1 \times 1 = 1$V

As point G is connected to earth, hence potentials of R and S are zero.
The direction of the current in the resistance is from P to S, hence point P is at higher potential.

$\therefore V_p = 7.5$ V

Similarly $V_Q = -1$V
Hence correct answer is **(B), (C) and (D).**

Prob13. *If same heat is produced when a resistor R_1 is connected to a battery for a given time as it is produced in resistor R_2 in the same time, then the internal resistance of the battery is*

(A) $\frac{R_1+R_2}{2}$ *(B)* $\sqrt{(R_1+R_2)R_1}$

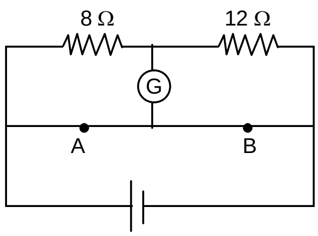
(C) $\frac{R_1-R_2}{2}$ *(D)* $\sqrt{R_1R_2}$

Sol. $$\frac{V_0^2}{(R_1+r)^2}R_1 = \frac{V_0^2 R_2}{(R_2+r)^2}$$

or, $r = \sqrt{R_1R_2}$

$\therefore$ **(D)**

Prob 14. *A potentiometer wire AB is 40 cm long. From point A, at what length null point will be obtained?*



(A) 8 cm *(B) 16 cm*

(C) 4 cm *(D) none of these*

Sol. $$\frac{8}{x} = \frac{12}{40-x} \text{ or, } \frac{320}{20} = 16 \text{ cm.}$$

$\therefore$ **(B)**

Prob15. *A fuse wire of radius 0.1 mm melts when a current of 10 A is passed through it. Find the current at which a fuse wire of 0.12 mm will melt.*

(A) 13.0 A *(B) 12.9 A*

(C) 13.2 A *(D) 12.0 A*

Sol. $$\frac{I_1}{I_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

or, $I_2 = I_1\left(\frac{r_2}{r_1}\right)^{3/2} = 10 \times (1.2)\sqrt{1.2} = 13.2$ A.

$\therefore$ **(C)**

ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. Write two special characteristics of wire of an electric heater.

2. Of metals and alloys, which has a greater coefficient of resistance. Will the resistance of a metallic conductor increase or decrease if the temperature of the conductor is increased ?

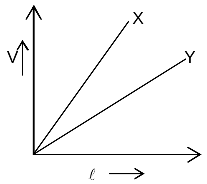
3. Give reasons why electrical conductance of electrolytes is less than that of metals.

4. Why is a voltmeter always connected in parallel with a circuit element across which voltage is to be measured?

5. Why is a potentiometer preferred over a voltmeter for measuring EMF ?

6. How does the drift velocity of electrons in a metallic conductor vary with increase in temperature ?

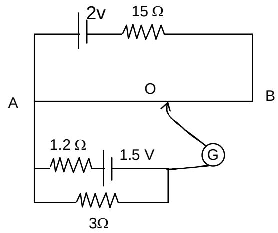
7. The variation of potential difference V with length ℓ in case of two potentiometers X and Y are as shown in the figure. Which one of these will you prefer for comparing EMF's of two cells and why ?



8. What is the difference between EMF of a cell and the terminal potential difference of a cell ?

9. Sequence of bands marked on a carbon resistor are: Red, Red, Red, Silver. Write the value of resistance with tolerance.

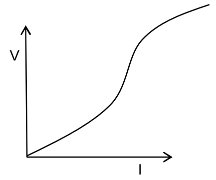
10. AB is a 1 m long uniform wire of 10 Ω resistance. The other data are as shows in the circuit diagram given below: Calculate
(i) potential gradient along AB.
(ii) length OA of the wire, when the galvanometer shows no deflection.



11. What is the largest voltage you can safely put across a resistor marked 98 Ω – 0.5 W ?

12. If potential difference V applied across a conductor is increased to 2V, how will the drift velocity of electrons change ?

13. A hypothetical voltage (V) vs current (I) graph is shown in the figure. What can be said about the resistance?

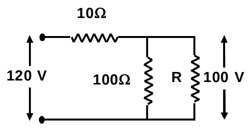


14. State Kirchoff's voltage law and prove that it is in accordance with law of conservation of energy.

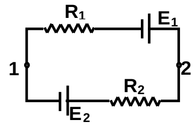
15. Show that the resistance of a conductor is given by $\frac{m\ell}{ne^2\tau}$, where the symbols have their usual meanings.

Level – I

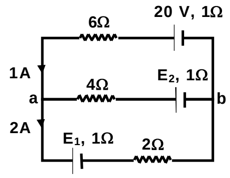
1. Find out the value of resistance R in the figure.



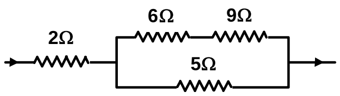
2. Find the potential difference between points 1 and 2 of the circuit shown in figure. $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $E_1 = 5.0$ V and $E_2 = 2.0$ V. The internal resistances of the cells are negligible.



3. Find the emf's E_1 and E_2 in the given network. Also find the potential difference between points 'a' and 'b'.



4. In the circuit shown; the 5 Ω resistor develops 45 J/s power due to current flowing through it. Calculate
(a) heat developed/sec through the 2Ω resistor.
(b) p.d. difference across the 6 Ω resistor.



5. An electric kettle has two heating–coils. When one of the coils is switched on, the water in the kettle boils in 6 minutes, and when the other is switched on, the water boils in 8 minutes. In what time will the water boil if both coils are switched on simultaneously.
(a) in series, and (b) in parallel ?

6. If a copper wire is stretched to make it 0.1 % longer, what is the percentage change in its resistance?

7. A power station supplies 100 kW to a load via cables of which resistance is 5 Ω. Find the power loss in the cable if the potential difference across the load is
(a) 10^4 V, (b) 2×10^5 V

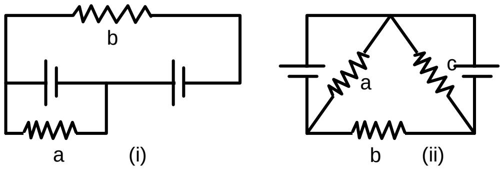
8. An ammeter with a scale ranging from 0 to 15 mA has a resistance of 5Ω. How should the instrument be connected together with a resistor (and with what resistance) to measure : (a) a current from 0 to 0.15 A, (b) a potential difference from 0 to 150 V.

9. An electric current of 5 A is passed through a circuit containing three wires arranged in parallel. If the length and radius of the wire are in the ratio 2 : 3 : 4 and 3 : 4 : 5, then find the ratio of current passing through the wires.

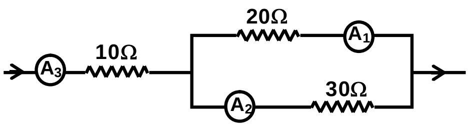
10. Two electric bulbs rated P_1 and P_2 watt at V volt, are connected in series across V volt mains. Calculate their total power consumption P.

Level – II

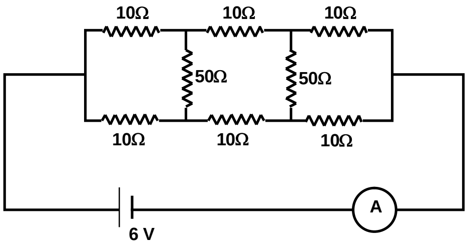
1. Each of the resistors shown in figure has a resistance of 10 Ω and each of the batteries has an emf of 10 V. Find the currents through the resistors a and b in the two circuits.



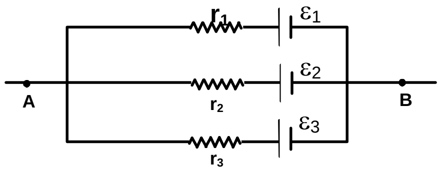
2. If the reading of ammeter A_1 in figure is 2.4 A, what will the ammeters A_2 and A_3 read ? Neglect the resistances of the ammeters.



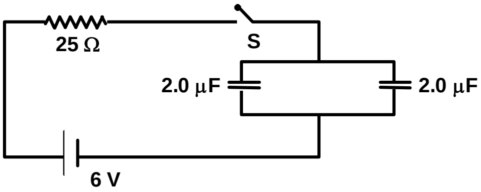
3. Find the current measured by the ammeter in the circuit shown in figure.



4. In the circuit shown in figure, $\varepsilon_1 = 3$ V, $\varepsilon_2 = 2$ V, $\varepsilon_3 = 1$V and $r_1 = r_2 = r_3 = 1\Omega$. Find the potential difference between the points A and B and the current through each branch.

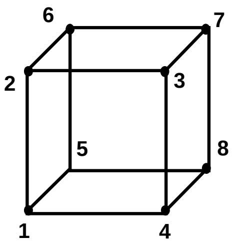


5. Find the charge on each of the capacitors 0.20 ms after the switch S is closed in figure.

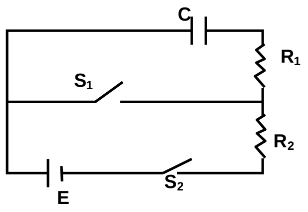


6. A capacitor of capacitance C is given a charge Q. At t = 0, it is connected to an ideal battery of emf ε through a resistance R. Find the charge on the capacitor at time t.

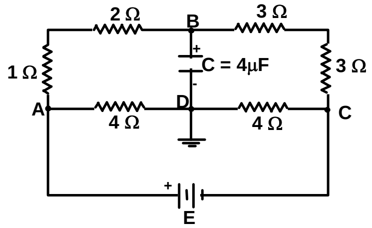
7. In the given figure, wires are arranged along the edges of a cube. Find the resistance of this wire frame, when measured between the points mentioned below.
(a) (1–7) body diagonal
(b) (1–2) edge
(c) (1–3) face diagonal
The resistance of each edge of the frame is R.



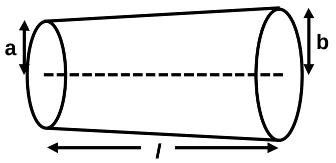
8. The capacitor shown in the figure has been charged to a potential difference of V volt so that it carries a charge CV with both the switches S_1 and S_2 remaining open. Switch S_1 is closed at t = 0. At $t = R_1C$, switch S_1 is opened and S_2 is closed. Find the charge on the capacitor at $t = 2R_1C + R_2C$.



9. Analyse the given circuit in the steady state condition. Charge on the capacitor in the steady state is $q_o = 16$ μC.
(a) Find the current in each branch.
(b) Find the e.m.f. of the battery.



10. Figure shows a conductor of length l having a circular cross–section. The radius of cross–section varies linearly from a to b. The resistivity of the material is ρ. Assuming that $b - a << l$, find the resistance of the conductor.

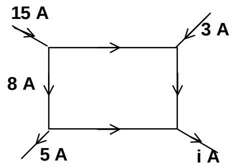


Objective:

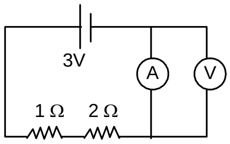
Level – I

1. A current passes through a wire of non–uniform cross section. Which of the following quantities depend upon cross section ?
(A) the charge crossing in a given time interval. (B) drift speed
(C) current (D) free electron density

2. The value of current i in the circuit shown in figure is
(A) 3 ampere (B) 13 ampere
(C) 23 ampere (D) –3 ampere

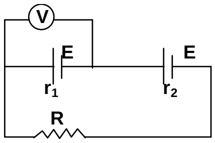


3. In the circuit shown in figure, the voltmeter reading would be (voltmeter and ammeter are ideal)
(A) 0 volt. (B) 0.5 volt.
(C) 1 volt. (D) 2 volt.



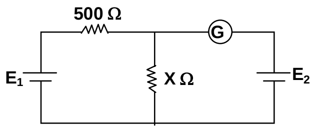
4. If two bulbs of 25 W and 100 W rated at 200 volts are connected in series across a 440 volts supply, then
(A) 100 watt bulb will fuse (B) 25 watt bulb will fuse
(C) none of the bulb will fuse (D) both the bulbs will fuse

5. In the adjoining figure, the reading of an ideal voltmeter is zero, then the relation between R, r_1 and r_2 is
(A) $R = r_2 - r_1$ (B) $R = r_1 - r_2$
(C) $R = r_1 + r_2$ (D) $R = \frac{r_1 r_2}{r_2 + r_1}$

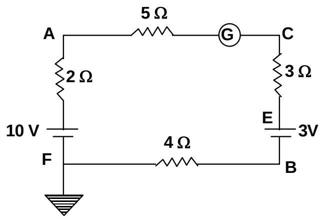


6. An external resistance R is connected to a cell of emf E and internal resistance r. The power developed in the external circuit is maximum when
(A) $R < r$ (B) $R > r$
(C) $R = r$ (D) $R = 0$

7. In the adjoining circuit, the battery E_1 has an E.M.F. of 12 volts and zero internal resistance, while the battery E_2 has an E.M.F. of 2 volts. If the galvanometer G reads zero, then the value of the resistance X, in ohms, is
(A) 10 (B) 100
(C) 14 (D) 200

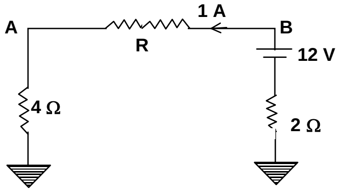


8. In the circuit shown in the figure, the point F is grounded. Which of the following is a wrong statement ?



(A) Potential at E is zero
(B) Potential at B is 2V
(C) The current in the circuit will be 0.5 A
(D) The current in the circuit is same whether or not F is grounded

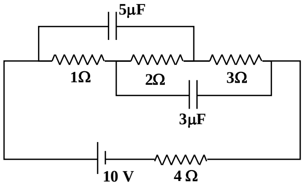
9. Which value of R is correct as shown in the figure ?



(A) R = 80 ohms
(B) R = 6 ohms
(C) R = 10 ohms
(D) R = 60 ohms

10. An electric heater has a resistance of 150 Ω and can bear a maximum current of 1A. if the heater is to be used on 220 V mains, the least resistance required in the circuit will be
(A) 70 Ω (B) 5Ω
(C) 2.5Ω (D) 1.4 Ω

11. In the shown network, charges on capacitors of capacitances 5 μF and 3 μF in steady state will be

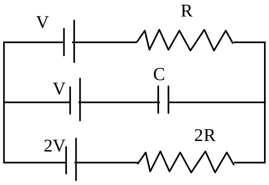


(A) 15 μC, 15 μC, (B) 15 μC, 5 μC,
(C) 5 μC, 15 μC, (D) 5 μC, 5 μC,

12. A wire has a resistance R. It is broken into two equal parts and these two parts are joined in parallel. The effective resistance is
(A) 2R (B) R
(C) R/2 (D) R/4

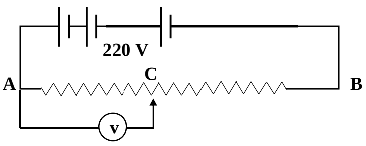
13. A wire of length ℓ and resistance R is bent in the form of a ring. The resistance between two points which are separated by angle θ is
(A) $\frac{R}{4\pi^2}(2\pi - \theta)\theta$ (B) $\frac{R\ell}{4\pi^2}(2\pi - \theta)\,\theta$
(C) $R(2\pi - \theta)$ (D) $R\,\theta$

14. In the given circuit, with steady current, the potential drop across the capacitor must be



(A) V (B) V/2
(C) V/3 (D) 2V/3

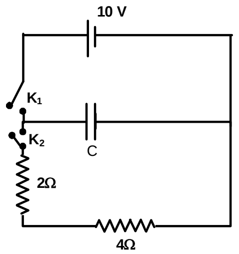
15. A potential difference of 220 volts is maintained across a 12000 Ω rheostat as shown. The voltmeter V has a resistance 6000 Ω and point C is at one–fourth of the distance from A to B. The reading of the voltmeter is



(A) 25 V (B) 40 V
(C) 50 V (D) 60 V

Level – II

1. A capacitor of capacitance C = 3 μF is first charged by connecting it across a 10 V battery by closing key K_1. It is then allowed to get discharged through 2Ω and 4Ω resistors by closing the key K_2. The total energy dissipated in the 2Ω resistor is equal to



(A) 0.5 mJ (B) 0.05 mJ
(C) 0.15 mJ (D) none of these.

2. The potential difference across the terminals of a battery is 8.5 V when there is a current of 3A in the battery from the negative to the positive terminal. When the current is 2A in the reverse direction, the potential difference becomes 11 V. The internal resistance of the battery is
(A) 2.5 Ω (B) 5.5 Ω
(C) 2.83 Ω (D) 0.5 Ω

3. A certain circuit with a resistance R is supplied with power simultaneously from N identical storage batteries. If the current in the circuit is to be same whether the batteries are connected in series or in parallel, the internal resistance of each of storage batteries should be equal to
(A) R/2 (B) R/4
(C) R (D) 2 R

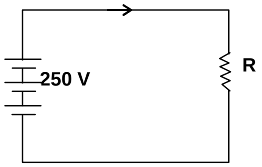
4. A cell of e.m.f. E and internal resistance r is connected in series with an external resistance nr. The ratio of the terminal potential difference to E.M.F. is
(A) $\frac{1}{n}$ (B) $\frac{1}{n+1}$
(C) $\frac{n}{n+1}$ (D) $\frac{n+1}{n}$

5. To measure the potential difference across a resistor of resistance RΩ, a voltmeter of resistance R_v is used. To measure the potential difference with a minimum accuracy of 95 %
(A) $R_v = 5\ R$ (B) $R_v = 15\ R$
(C) $R_v = 10\ R$ (D) $R_v \geq 19\ R$

6. A cell of E.M.F. E and internal resistance r supplies currents for the same time t through external resistances R_1 and R_2 respectively. If the heat produced in both cases is the same, then the internal resistance is
(A) $\frac{1}{r_1} = \frac{1}{R_1} + \frac{1}{R_2}$ (B) $r = \frac{R_1 + R_2}{2}$
(C) $r = \sqrt{R_1 R_2}$ (D) $R_1 + R_2$

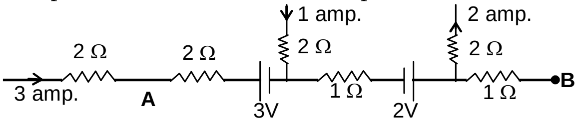
7. A uniform wire of resistance 36 Ω is shaped into a regular haxagon. The equivalent resistance between any two corners can have
(A) the minimum value of 5 ohm and maximum value 9 Ω
(B) the minimum value 6 ohm and maximum value of 9 Ω
(C) the maximum value 9 ohm and minimum value of 4 Ω
(D) the maximum value 6 ohm and minimum value of 4 Ω

8. In the circuit shown, the resistance R has a value that depends on the current. R is 20 ohms when I is zero and the amount of increase in resistance is numerically equal to one–half of the current. What is the value of current I in circuit ?



(A) 8.33 A (B) 10 A
(C) 12.5 A (D) 18.5 A

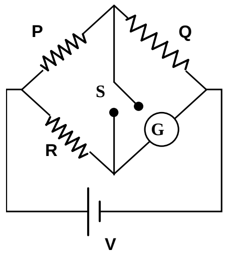
9. The potential difference between points A and B, in a section of a circuit shown, is



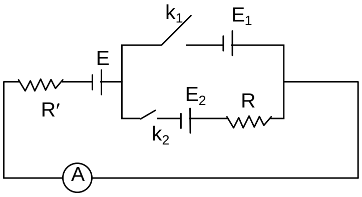
(A) 5 volts (B) 1 volts
(C) zero volts (D) 13 volts

10. Two conducting wires of same length are made from the same material. One wire is solid with diameter 1mm while the other is hollow with outer diameter 2mm and inner diameter 1mm. The ratio of their resistances will be
(A) 1 : 2 (B) 1: 3
(C) 3 : 1 (D) 2 : 1

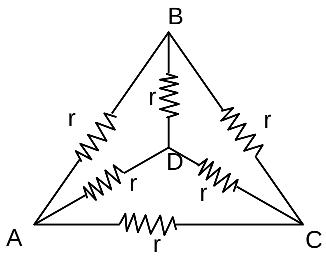
11. In the circuit shown, P ≠ R, and the reading of the galvanometer is same wheteher switch S is open or closed. Then
(A) $I_R = I_G$ (B) $I_P = I_G$
(C) $I_Q = I_G$ (D) $I_Q = I_R$



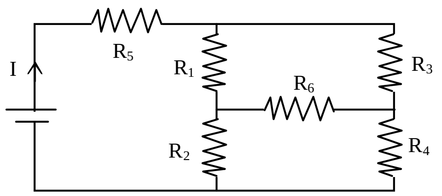
12. In the circuit given in the figure, the reading of ammeter is same in each case, either of k_1 or k_2 is closed. The reading of the ammeter is
(A) $\frac{E_1}{R}$ (B) $\frac{E_2 + E_1}{R}$
(C) $\frac{E_2 - E_1}{R}$ (D) data not sufficient.



13. In the given network, the equivalent resistance between A and D is
(A) $\frac{2}{5}r$ (B) $\frac{5r}{2}$
(C) r / 2 (D) none of these

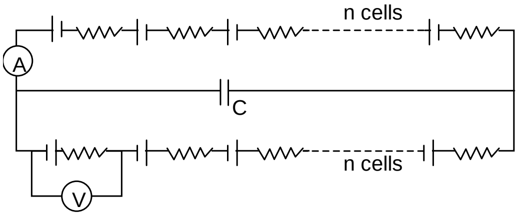


14. In the given circuit, it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy.



(A) $R_1R_2R_5 = R_3R_4R_6$
(B) $\frac{1}{R_5}+\frac{1}{R_6}=\frac{1}{R_1+R_2}+\frac{1}{R_3+R_4}$
(C) $R_1R_4 = R_2R_3$
(D) $R_1R_3 = R_2R_4 = R_5R_6$

15. 2n identical cells each having EMF ε and internal resistance r, and a capacitor of capacitance C are connected as shown in the figure. Assuming ammeter and voltmeter to be ideal. Choose the correct statement from the following.



(A) Charge in the capacitor is ncε.
(B) The reading of the ammeter is $2\frac{\varepsilon}{r}$.
(C) The reading of the voltmeter is ε.
(D) The reading of the voltmeter is zero.

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – I

1. 100 Ω
2. –4 V
3. $E_1 = 7$ V, $E_2 = 18$ V, $V_a - V_b = 13$ V
4. (a) 32J/s, (b) 6V
5. (a) 14 minutes, (b) 3 minutes 26 sec.
6. 0.2%
7. (a) 500 W, (b) 1.25 W
8. (a) R = 0.555 Ω in parallel with the instrument, (b) R = 9995 Ω in series with the instrument.
9. 54 : 64 : 75
10. $P_1P_2/(P_1+P_2)$

Level – II

1. (i) current in a = 1 A, current in b = 2 A, (ii) current in a = 1 A, current in b = 0
2. 1.6 A, 4.0 A
3. 0.4 A
4. 2 V, $i_1 = 1$A, $i_2 = 0$, $i_3 = -1$ A.
5. 10.4 μC
6. $[\,C\varepsilon(1-e^{-t/CR}) + Qe^{-t/CR}\,]$
7. (a) (5/6) R (b) (7/12)R (c) (3/4)R
8. $q_f = EC\left(1-\frac{1}{e}\right)+\frac{VC}{e^2}$
9. (a) $\frac{8}{3}$ A, 3 A (b) 24 V
10. $\frac{\rho\ell}{\pi ab}$

Objective:

Level – I

1.	**B**	2.	**B**
3.	**A**	4.	**B**
5.	**B**	6.	**C**
7.	**B**	8.	**A**
9.	**B**	10.	**A**
11.	**A**	12.	**D**
13.	**C**	14.	**C**
15.	**B**		

Level – II

1.	**B**	2.	**D**
3.	**C**	4.	**C**
5.	**D**	6.	**C**
7.	**A**	8.	**B**
9.	**D**	10.	**C**
11.	**A**	12.	**A**
13.	**C**	*14.*	**C**
15.	**D**		

Learnaf

Educational Revolution

PHYSICS

FITJEE

PINNACLE

For – JEE (Main/Advanced)

MAGNETICS

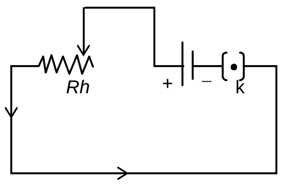
Magnetics

Syllabus of IIT-JEE and Maharastra Board:
Concept of magnetic field, Oersted's experiment, Biot−Savart law, magnetic field due to an infinitely long current carrying straight wire and a circular loop; Ampere's circuit law and its applications to straight and toroidal solenoids; force on a moving charge in uniform magnetic and electric fields; Cyclotorn, Force on current−carrying conductor in a uniform magnetic field. Forces between two parallel current−carrying conductors−definition of ampere; torque experienced by a current loop in a uniform magnetic field, moving coil galvanometer−its current sensitivity and conversion to ammeter and voltmeter.

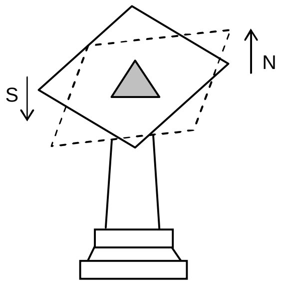
Current loop as a magnetic dipole and its magnetic dipole moment; Magnetic dipole moment of a revolving electron; Magnetic field intensity due to magnetic dipole (bar magnet) along the axis and perpendicular to the axis; Torque on a magnetic dipole (bar magnet) in a uniform magnetic field; Bar magnet as an equivalent solenoid, magnetic field lines; Earth's magnetic field and magnetic elements; Para−dia and ferro−magnetic substances with examples, Electro−magnets and permanent magnets.

MAGNETIC FIELD
A magnetic field is the space around a magnet or the space around a conductor carrying current in which magnetic influence can be experienced. In the latter case, the magnetic field disappears as soon as the current is switched off. It suggests that motion of electrons in the wire produces a magnetic field. In general, a moving charge is a source of magnetic field.

Oersted's Experiment
In this experiment, oersted showed that the electric current through a wire deflects the magnetic needle held below it. The direction of deflection of magnetic needle is reversed if the direction of the current in the wire is reversed. This shows that the magnetic field is associated with a current carrying wire.



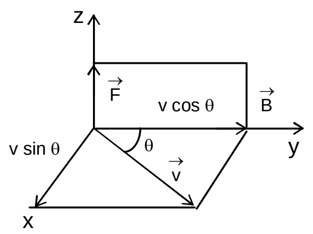
An electric current is due to the charges in motion. Such charges produce magnetic interaction. The magnetic field produced by a conductor carrying current thus interacts with the magnetic field needle and deflects it. The branch of physics which deals with the magnetism due to electric current is called electromagnetism.



A magnetic field in a region can be represented by magnetic line of force.

In order to define $\vec{B}$, we deduce an expression for the force on a moving charge in a magnetic field. $\vec{B}$ is often called magnetic induction or magnetic flux density.

Consider a positive charge q moving in a uniform magnetic field $\vec{B}$, with velocity $\vec{v}$. Let the angle between $\vec{v}$ and $\vec{B}$ be θ. Due to interaction between the magnetic field produced due to moving charge (i.e. current) and magnetic field applied, the charge q then experiences a force, which depends on the following factors.



(i) $F \propto q$
(ii) $F \propto v \sin\theta$
(iii) $F \propto B$
Combining the above factors, we get
$F \propto qvB\sin\theta$
$F = k\,qvB\sin\theta$
where k is a constant of proportionality. Its value is found to be one, i.e. k = 1

$\therefore \quad F = qvB\sin\theta$

$|\vec{F}| = q|\vec{v}\times\vec{B}|$

$\vec{F} = q(\vec{v}\times\vec{B})$

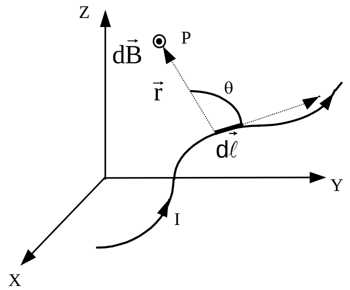
The direction of $\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. It is directed as given by Right Hand Rule.

If v = 1, q = 1 and sin θ = 1,
then $F = 1 \times 1 \times B \times 1 = B$

Thus, the magnetic field induction at any point in the field is equal to the force experienced by a unit charge moving with a unit velocity perpendicular to the direction of magnetic field at that point.

The Biot and Savart Law

Biot–Savart's Law is an experimental law predicted by Biot and Savart. This law deals with the magnetic field induction at a point due to a small current element.



Consider an infinitesimal element of length 'dl' of a wire carrying current I. The magnetic field $d\vec{B}$ at P because of $d\vec{\ell}$ is given by this law as:

$$d\vec{B} = \frac{\mu_0 I(d\vec{\ell}\times\vec{r})}{4\pi\ r^3}$$

Here, $d\vec{\ell}$ is a vector of length $d\ell$ which is along the direction of current. The product $Id\vec{\ell}$ is called current element, it is the smallest possible entity causing a magnetic field.

Note: μ_0 = permeability of free space.

Magnetic field induction at point P due to current through entire wire is

$$\vec{B} = \int \frac{\mu_0}{4\pi} I \frac{d\vec{\ell}\times\vec{r}}{r^3}$$

Biot–Savart's law in terms of current density J, states that

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{\vec{J}\times\vec{r}}{r^3}dv$$, where dv is volumetric element.

Biot–Savart's law in terms of charge (q) and its velocity v is

$$d\vec{B} = \frac{\mu_0}{4\pi} q \frac{(\vec{v}\times\vec{r})}{r^3}$$

Biot–Savart's law in terms of magnetising force or magnetic intensity (H) of the magnetic field

$$d\vec{H} = I\frac{d\vec{\ell}\times\vec{r}}{r^3}$$

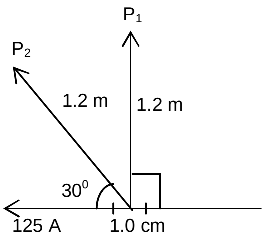
Some important features of Biot–Savart's law:

1. Biot Savart's law is valid for a symmetrical current distribution.
2. Biot Savart's law is applicable only to a very small length of conductor carrying current.
3. This law cannot be easily verified experimentally as the current carrying conductor of very small length cannot be obtained practically.
4. This law is analogus to Coulomb's law in electrostatics.
5. The direction of $d\vec{B}$ is perpendicular to both $d\vec{\ell}$ and $\vec{r}$.
6. If $\theta = 0°$, i.e. point P lies on the axis of the linear conductor carrying current, then

   $dB = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2} = 0$
7. If $\theta = 90°$, i.e. the point P lies at a perpendicular position w.r.t. to current, then

   $dB = \frac{\mu_0}{4\pi}\frac{Id\ell}{r^2}$, which is maximum.

Illustration 1. *A copper wire carries a steady current of 125 A to an electroplating tank. Find the magnetic field caused by a 1.0 cm segment of this wire at a point 1.2 m away from it (a) point P_1, straight out to the side of the segment; (b) point P_2, on a line at 30° to the segment.*

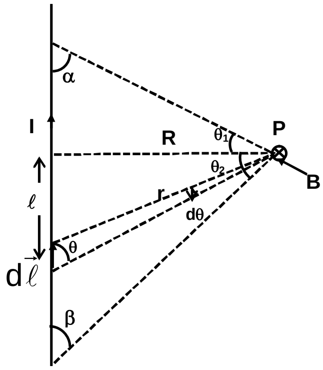


Solution: (a) $B = \frac{\mu_0}{4\pi}Id\ell\frac{\sin\phi}{r^2} = \frac{10^{-7}\times125\times1\times10^{-2}\times\sin90^0}{(1.2)^2} = 8.7 \times 10^{-8}$ T.

(b) $B = \frac{10^{-7}\times125\times1\times10^{-2}\times\sin30^0}{(1.2)^2} = 4.35 \times 10^{-8}$ T

Magnetic field due to current in a straight wire

The magnetic field due to a wire segment carrying current I at P, when the wire segment subtends angles α and β as shown, can be determined as follows:



$d\vec{B}$ at P, due to $d\vec{\ell}$ is: $dB = \frac{\mu_o}{4\pi}\frac{Id\ell\sin\theta}{r^2}$

Now, $\ell = R\cot\theta$, $d\ell = -R\,\text{cosec}^2\theta.d\theta$

$r = R\,\text{cosec}\,\theta$

$\Rightarrow \quad dB = \frac{\mu_o}{4\pi}\frac{I(\sin\theta)(-R\text{cosec}^2\theta d\theta)}{R^2.\text{cosec}^2\theta}$

$$B = \frac{-\mu_o I}{4\pi R}\int_{\beta}^{180-\alpha}\sin\theta d\theta = \frac{-\mu_o I}{4\pi R}\left[\cos\alpha + \cos\beta\right]$$

The magnitude of $B = \frac{\mu_o I}{4\pi R}[\cos\alpha + \cos\beta] = \frac{\mu_0 I}{4\pi R}\left[\sin\theta_1 + \sin\theta_2\right]$,

In the vector form, $\vec{B} = \frac{\mu_0 I}{4\pi R}\left[\sin\theta_1 + \sin\theta_2\right](-\hat{k})$

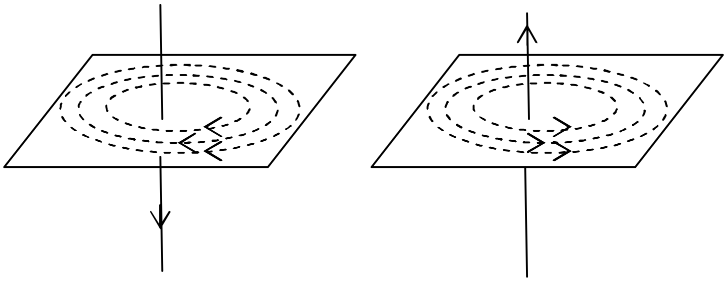
Special case:

(i) When the conductor is of infinite length and the point P lies near the centre of the current, then $\theta_1 = \theta_2 = 90°$.

So, $B = \frac{\mu_0 I}{4\pi R}(\sin 90° + \sin 90°) = \frac{\mu_0 (2I)}{4\pi R}$

(ii) When the conductor is of infinite length but the point P lies near the end, then $\theta_1 = 90°$ and $\theta_2 = 0°$

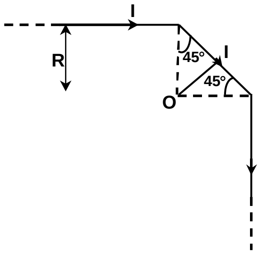
So, $B = \frac{\mu_0 I}{4\pi R}(\sin 90^0 + \sin 0^0) = \frac{\mu_0 I}{4\pi R}$



Direction of magnetic field:
The magnetic field lines due to a straight conductor carrying current are in the form of concentric circles with the conductor as centre, lying in a plane perpendicular to the straight conductor.

The direction of magnetic field lines can be given by right hand thumb rule or Maxwell's cork screw rule.

Illustration 2. *A long infinite current carrying wire is bent in the shape as shown is the figure. The magnetic induction at point O is*



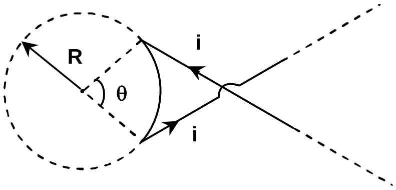
(A) $\frac{\mu_0 I}{\pi R} \otimes$ *(B)* $\frac{\mu_0 I}{2\pi R} \odot$

(C) $\frac{\mu_0 I}{2\pi R} \odot$ *(D)* $\frac{\mu_0 I}{2\pi R} \odot$

Solution: **(A)** $B_1 = \frac{\mu_0 I}{2\pi R}[\cos 0 - \cos 90] \times 2$

$B_2 = \frac{\mu_0 I}{4\pi R}\sqrt{2}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$

Illustration 3. *A current carrying wire has the configuration shown in the figure. Two semi–infinite straight sections, each tangent to the same circle, are connected by a circular arc, of angle θ along the circumference of the circle, with all sections lying in the same plane. What must be θ in order for B(magnetic field) to be zero at the centre of the circle?*



(A) $\theta = 1$ *rad* *(B)* $\theta = 2$ *rad*
(C) $\theta = 3$ *rad* *(D) none of the above*

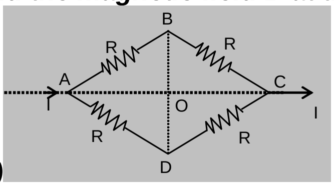
Solution: **(B)** For $\vec{B}$ to be zero at the centre, sum of field due to the ring and that due to the semi–infinite straight section should be equal and opposite

i.e. $\frac{\mu_0 i}{2\pi R} - \frac{\mu_0 i}{2\pi R}\left(\frac{\theta}{2}\right) = 0$

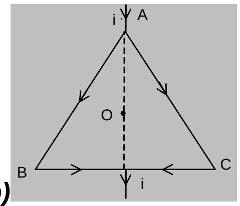
$\Rightarrow \frac{\mu_0 i}{2\pi R}\left(1 - \frac{\theta}{2}\right) = 0$

$\Rightarrow \theta = 2$ rad

Exercise 1 : ***(i) Find the magnetic field B at the point O.***



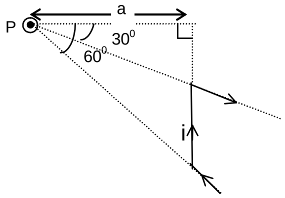
(a)



(b)

(ii) Twelve uniform wires of equal length ℓ and each of resistance r are connected to form a skeleton cube. A battery of e.m.f. E is connected between two diagonally opposite corners of the cube. Find the magnetic induction at the centre of the cube.

Illustration 4. *Find the magnitude and direction of magnetic field at point P due to the current carrying wire as shown.*



Solution : $\vec{B} = \frac{\mu_0 i}{4\pi R}\left[\sin\theta_1 + \sin\theta_2\right](\hat{k})$

Here, $\theta_1 = -30°$, $\theta_2 = 60°$,

putting these values , we get

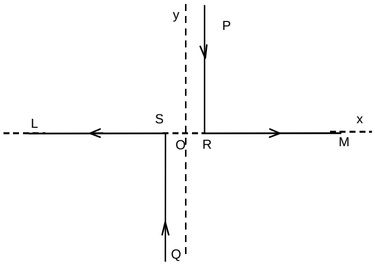
$\vec{B} = \frac{\mu_0 i}{4\pi a}\left[-1/2 + \sqrt{3}/2\right](\hat{k})$

Illustration 5. *A long straight conductor carries a current of 1.0 A. At what distance from the axis of the conductor is the magnetic field, caused by the current, equal to 0.5×10^{-4} T.*

Solution: $B = \frac{\mu_0 I}{2\pi r}$

$\Rightarrow r = \frac{\mu_0 I}{2\pi B} = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.5 \times 10^{-4}} = 4 \times 10^{-3}$ m = 4 mm.

Illustration 6. *A pair of stationary and infinitely long bent wires is placed in the x –y plane as shown in figure. Each wire carries current of 10 amp. The segments L and M are along the x–axis. The segments P and Q are parallel to the y–axis such that OS = OR = 0.02 m. Find the magnitude and direction of the magnetic induction at the origin O.*



Solution: As point O is along the length of segments L and M, so the field at O due to these segments will be zero. Also point O is near one end of a long wire.

The resultant field at O, $\vec{B}_R = \vec{B}_P + \vec{B}_Q$

$$\vec{B}_R = \frac{\mu_0 I}{4\pi RO}(\hat{k}) + \frac{\mu_0 I}{4\pi \;\; SO}(\hat{k})$$

But RO = SO = 0.02 m

Hence, $\vec{B}_R = 2 \times \frac{\mu_0}{4\pi} \times \frac{10}{0.02}(\hat{k}) = 2 \times 10^{-7}\,\frac{10}{0.02}(\hat{k}) = \; 10^{-4}\left(\frac{Wb}{m^2}\right)\;(\hat{k})$.

Illustration 7. *Two straight infinitely long and thin parallel wires are spaced 0.1 m apart and carry a current of 10 ampere each. Find the magnetic field at a point distant 0.1 m from both wires in the two cases when the currents are in the (a) same and (b) opposite directions.*

Solution: The point P is situated equidistant from the wires A and B. Hence for the given case the magnitude of the magnetic field at P due to both the wires will be same.

$B_A = B_B = B = \; \frac{\mu_0 I}{2\pi d} = 2 \times 10^{-7} \times \frac{10}{0.1} = 2 \times 10^{-5}\,T$

(a) If the wire carries current in the same direction, B_A and B_B will have the directions as shown in the figure.

The net magnetic field

$\vec{B}_R \; = \; 2 \;\; B \;\; \cos \;\; 30^0 \;\; (-\hat{i})$

$= 2\sqrt{3} \;\; x \;\; 10^{-5} \;\; T$ along the –x–axis.

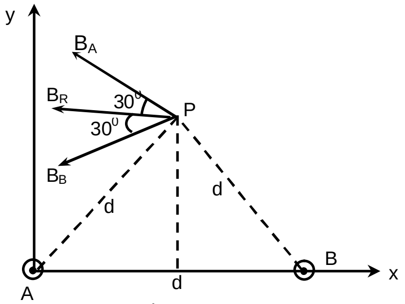


Figure - 1

(b) If the wires carry current in opposite directions, the magnetic field at P due to wires A and B will be as shown in figure.

The net magnetic field

$\vec{B}_R \; = \; 2 \;\; B \;\; \cos 60^0 \;\; (+\hat{j})$

$= 2 \times 10^{-5}$ T along y–axis.

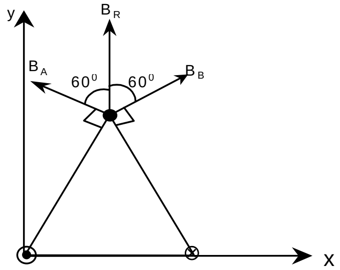
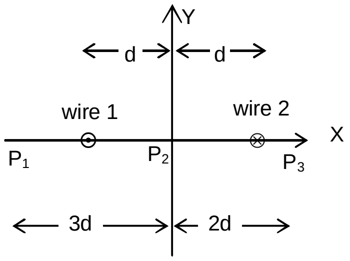
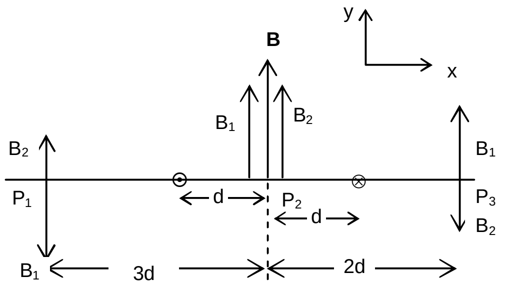


Illustration 8. *The figure gives an end view of two long, straight, parallel wires perpendicular to the X–Y plane, each carrying a current I but in opposite directions.*

(a) Find the magnitude and direction of $\vec{B}$ *at points P_1, P_2 and P_3.*



Solution: At point P_1



$$B_1 = \frac{\mu_0 I}{2\pi(2d)} = \frac{\mu_0 I}{4\pi d}(-\hat{j})$$

$$B_2 = \frac{\mu_0 I}{2\pi(4d)} = \frac{\mu_0 I}{8\pi d}(+\hat{j})$$

$$\vec{B}_{total} = \vec{B}_1 + \vec{B}_2$$

$$= \left(\frac{\mu_0 I}{4\pi d} - \frac{\mu_0 I}{8\pi d}\right)(-\hat{j})$$

$$= \frac{\mu_0 I}{8\pi d}(-\hat{j})$$

At point P_2

$$B_1 = B_2 = \frac{\mu_0 I}{2\pi d}$$

$$B_{total} = \frac{\mu_0 I}{2\pi d}\hat{j} + \frac{\mu_0 I}{2\pi d}\hat{j} = \frac{\mu_0 I}{\pi d}(\hat{j})$$

At point P_3

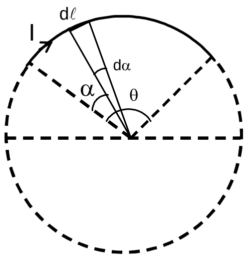
$$B_1 = \frac{\mu_0 I}{2\pi(3d)} = \frac{\mu_0 I}{6\pi d}$$

$$B_2 = \frac{\mu_0 I}{2\pi d}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{6\pi d}(\hat{j}) + \frac{\mu_0 I}{2\pi d}(-\hat{j}) = \frac{\mu_0 I}{3\pi d}(-\hat{j})$$

Magnetic field at the centre of a current carrying arc

The magnetic field due to the element $d\vec{\ell}$



$$d\vec{B} = \frac{\mu_o I}{4\pi}.\frac{d\vec{\ell} \times \vec{r}}{r^3}.$$

Here, $dB = \dfrac{\mu_o I}{4\pi}\dfrac{|d\vec{\ell}| \times r\sin 90^o}{r^3}$ (as $d\vec{\ell} \perp \vec{R}$)

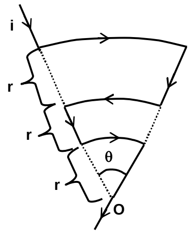
$\Rightarrow$ $dB = \dfrac{\mu_o I}{4\pi r}(d\alpha)$ (as $d\ell = r d\alpha$)

$$B = \int dB = \frac{\mu_o I}{4\pi r}\int_0^\theta d\alpha$$

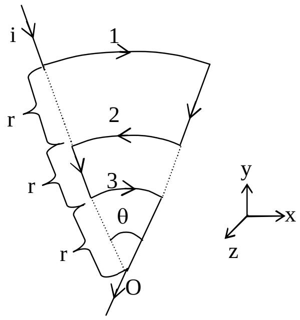
$B_{arc} = \dfrac{\mu_o I}{4\pi r}\theta$; where θ is the angle in the radian.

Therefore, B at the centre of a circular loop of radius R is $B = \dfrac{\mu_o I}{4\pi R}(2\pi) = \dfrac{\mu_o I}{2R}$.

Illustration 9. *Shown in the figure is a conductor carrying a current i. Find the magnetic field intensity at the point O.*



Solution: Since magnetic field at the centre of an arc is equal to, $B = \frac{\mu_0 I\theta}{4\pi r}$



Magnetic field due to arc 1, $\vec{B}_1 = \frac{\mu_0 \; I \; \theta}{4\pi \; . \; 3 \; r}\left(-\hat{k}\right)$

Magnetic field due to arc 2, $\vec{B}_2 = \frac{\mu_0 \; I \; \theta}{4\pi \; . \; 2 \; r}\left(\hat{k}\right)$

Magnetic field due to arc 3, $\vec{B}_3 = \frac{\mu_0 \; I \; \theta}{4\pi \; . \; r}\left(-\hat{k}\right)$

Net magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

Hence, net $\vec{B} = \frac{\mu_o I}{4\pi}\left[-\frac{1}{r}+\frac{1}{2r}-\frac{1}{3r}\right]\theta = -\frac{5\mu_o I\theta}{24\pi r}\;\hat{k}$.

Illustration 10. *A charge q moves in a circular path of radius r with a speed v. Calculate the induction of the magnetic field produced at the centre of the circle.*

Solution: The equivalent current in a circular path is

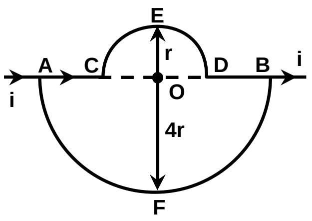
$$i = \frac{q}{t} = \frac{q}{2\pi(r/v)} = \frac{qv}{2\pi r}$$

Hence the induction of the magnetic field at the centre of the circle is

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi i}{r} = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi qv}{2\pi r^2}$$

$$= \left(\frac{\mu_0}{4\pi}\right)\frac{qv}{r^2}$$ Tesla, along the axis.

Illustration 11. *In the given circuit AC and BD are straight lines and CED and AFB are semicircular with radii r and 4r, respectively. The entire setup is lying in the same plane. If i is current entering at A what fraction of i will flow in ACEDB such that resultant magnetic field at O is zero.*



(A) i/5 *(B) 4i/5*
(C) 3i/5
(D) no current through bigger semicircular part.

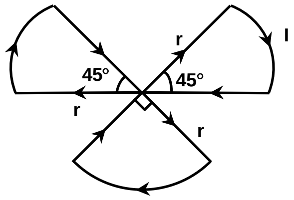
Solution: **(A)** *If i_1 is current in ACEDB, then*

$$\frac{\mu_0}{4}\cdot\frac{i_1}{r} = \frac{\mu_0}{4}\cdot\frac{i_2}{4r}$$

$$\frac{i_1}{i_2} = \frac{1}{4}$$

$$i_1 = \frac{1}{5}\times i = \frac{i}{5}$$

Illustration 12. *Calculate the magnetic field at point P due to three arcs as shown in the figure.*



(A) $\frac{\mu_0 I}{2r}$ (B) $\frac{\mu_0 I}{4r}$

(C) $\frac{\mu_0 I}{2\pi r}$ (D) zero

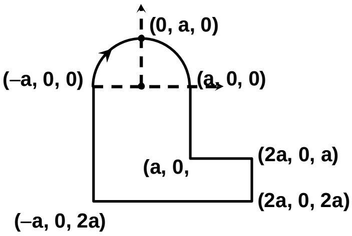
Solution: **(B)** By using the relation

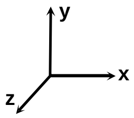
$B = \frac{\mu_0 i\theta}{4\pi r}$, where θ is in radian

Here, θ = 180°, i.e. π

So, $B_p = \frac{\mu_0 I}{4r}$.

Illustration 13. *The net magnetic field due to current I in loop at point (0, 0, 0) is $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$. Which of the following is incorrect?*





(A) $B_3 \neq 0$

(B) $B_2 \neq 0$

(C) $B_1 \neq 0$

(D) none of these

Solution: **(C)** The upper loop is x–y plane

Hence $B_3 \neq 0$

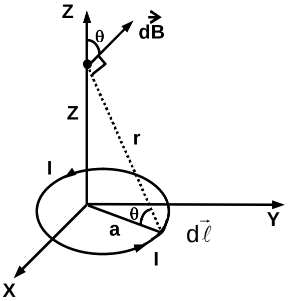
The lower loop is in z–x plane.

Hence, $B_2 \neq 0$

$B_1 = 0$

Magnetic field on the axis of a current carrying loop

Let the radius of the loop be a and axial distance be z, then noting that from symmetry, the resultant field at P must be along z, we have



$$dB_z = (dB)\cos\theta = \left(\frac{\mu_o I}{4\pi r^3} d\ell\, r \sin 90^o\right)\cos\theta$$

$$= \frac{\mu_o I}{4\pi r^3} d\ell\, r\, \frac{a}{r} = \frac{\mu_o I a}{4\pi r^3} d\ell = \frac{\mu_o I a d\ell}{4\pi(a^2+z^2)^{3/2}}$$

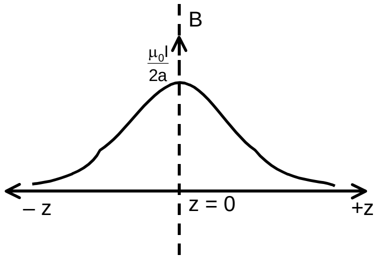
Adding all the elements around the loop, we get,

$\therefore$ $B_z = \int dB_z$

$$B_z = B = \frac{\mu_o I a^2}{2(a^2+z^2)^{3/2}} \text{ wb/m}^2$$

(a) B varies non linearly as shown in the plot between z and B and maximum when $z^2 = 0$, i.e. the point at the centre of the coil and then

$B = \dfrac{\mu_0 I}{2a}$, Which is same as magnetic field at the centre of a circular coil



(b) If z>>R, then

$$B = \frac{\mu_0 IR^2}{2z^3} = \frac{\mu_0}{2\pi}\frac{I\pi R^2}{z^3} = \frac{\mu_0}{4\pi}\frac{I.\ 2\pi R^2}{z^3}$$

but πR^2 = A = Area cross section of the coil

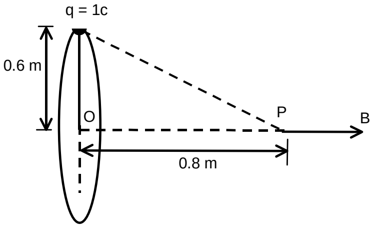
Then $B = \dfrac{\mu_0}{4\pi}\dfrac{2IA}{z^3} = \dfrac{\mu_0 2M}{4\pi z^3}$

or, $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{2\vec{M}}{z^3}$

Where $\vec{M} = I.(\vec{A}) =$ magnetic dipole moment of the loop. the direction of $\vec{M}$ is same as the direction of normal of the area of the loop.

Illustration 14. *A charge of 1C is placed at one end of a non–conducting rod of length 0.6m. The rod is rotated in a vertical plane about a horizontal axis passing through the other end of the rod with angular frequency $10^4\pi$ rad/s. Find the magnetic field at a point on the axis of rotation at a distance of 0.8 m from the centre of the path. Now, half of the charge is removed from one end and placed on the other end. The rod is rotated in a vertical plane about a horizontal axis passing through the mid point of the rod with the same angular frequency. Calculate the magnetic field at a point on the axis at a distance of 0.4m from the centre of the rod.*

Solution: A revolving charge is equivalent to a current



$$I = q.f = q.\frac{\omega}{2\pi} = 1x\frac{10^4\pi}{2\pi} = 5x10^3\ A$$

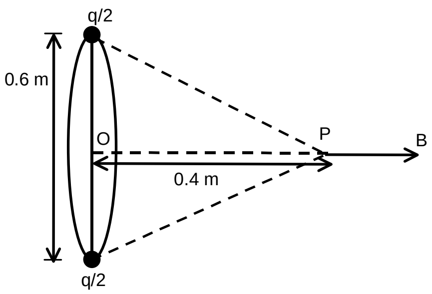
The field at a distance z from the centre of the axis of a current carrying coil is given by,

$$B = \frac{\mu_0 a^2 I}{2\left(a^2 + z^2\right)^{3/2}}$$

$$= \frac{4 \times 10^{-7} \times 5 \times 10^3 \left(0.6\right)^2}{2\left[\left(0.6\right)^2 + \left(0.8\right)^2\right]^{3/2}} = 1.13 \times 10^{-3}\ T$$

Equivalent current

$I' = \frac{q}{2}f + \frac{q}{2}f = qf = 5 \times 10^3 A$



Magnetic field at P in this case

$$B' = \frac{\mu_0 \ I \ a^2}{2\left(a^2 + z^2\right)^{3/2}}$$

Here, a = 0.3 m, z = 0.4 m

Which gives,

$B' = 2.26 \times 10^{-3}$ T

Illustration 15. *A coil consisting of 100 circular loops with radius 0.60 m carries a current of 5.0 A. (a) Find the magnetic field at a point along the axis of the coil, 0.8 m from the centre. (b) Along the axis, at what distance from the centre is the field magnitude 1/8 times its value at the centre ?*

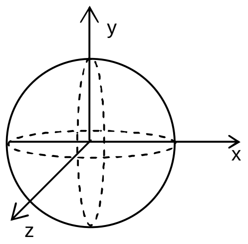
Solution: (a) $B_x = \frac{4\pi \times 10^{-7} \times 100 \times 5.0 \times (0.6)^2}{2[(0.8)^2 + (0.6)^2]^{3/2}} = 1.1 \times 10^{-4}$ T

(b) $B_x = \frac{\mu_0 N I \pi a^2}{2\pi(x^2 + a^2)^{3/2}}$

$$\frac{1}{(x^2 + a^2)^{3/2}} = \frac{1}{8(0^2 + a^2)^{3/2}}$$

$\Rightarrow x = \pm \sqrt{3}\, a = \pm 1.04$ m.

Illustration 16. *Three rings, each having equal radius R, are placed mutually perpendicular to each other and each having its centre at the origin of co–ordinate system. If current I is flowing through each ring, then the magnitude of the magnetic field at the common centre is*



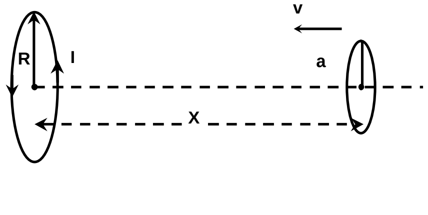
(A) $\sqrt{3}\,\frac{\mu_0 I}{2R}$ *(B) zero*

(C) $(\sqrt{2} - 1)\frac{\mu_0 I}{2R}$ *(D)* $\left(\sqrt{3} - \sqrt{2}\right)\frac{\mu_0 I}{2R}$

Solution: **(A)** $\vec{B} = \frac{\mu_0 I}{2R}(\pm\hat{i}) + \frac{\mu_0 I}{2R}(\pm\hat{j}) + \frac{\mu_0 I}{2R} \pm \hat{k}$

$\therefore |\vec{B}| = \frac{\mu_0 I}{2R}\sqrt{3}$

Illustration 17. *There, is a current carrying loop of radius R as shown in the figure. At a distance of X on the axis of the first loop with current in anticlockwise direction as seen from right, there, is another small loop of radius a whose plane is parallel to the plane of the first loop. Given that the smaller loop is given a*



velocity of V m/s towards the loop of radius R, the emf induced in the smaller loop, given that R<<X and r << R, is

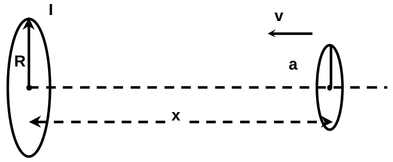
(A) $\frac{3}{2}\frac{\mu_0 IR^2a^2V\pi}{X^4}$, *clockwise.* (B) $\frac{3}{2}\frac{\mu_0 IR^2a^2V\pi}{X^4}$, *anticlockwise*

(C) $\frac{3}{2}\frac{\mu_0 IRa^2V\pi}{X^3}$, *clockwise.* (D) $\frac{3}{2}\frac{\mu_0 IR^2aV\pi}{X^3}$, *anticlockwise*

Solution: **(A)** $\phi = \frac{\mu_0 IR^2}{2X^3} \times \pi a^2 = \frac{\mu_0 IR^2a^2\pi}{2X^3}$

$$emf = -\frac{d\phi}{dt} = -\left[\frac{\mu_0 IR^2a^2\pi}{2}\frac{-3}{X^4}.\frac{dX}{dt}\right]$$

$$= \frac{3}{2}\frac{\mu_0 IR^2a^2\pi}{X^4}\left(\frac{dX}{dt}\right) = \frac{3}{2}\frac{\mu_0 IR^2a^2V\pi}{X^4},$$



As given by the Lenz's law, the direction is clockwise.

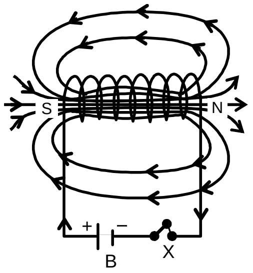
Magnetic field due to current in a solenoid

A solenoid consists of an insulated long wire closely wound in the form of a helix. Its length is very large as compared to its distance.

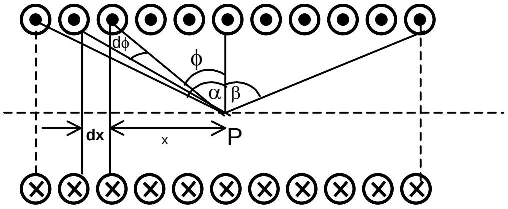
Consider a long straight solenoid having n turns per unit length and carrying current I. The magnetic field setup in the solenoid is as shown.

A linear solenoid carrying current is equivalent to a bar magnet. The magnetic field lines due to current carrying solenoid resemble exactly with those of a bar magnet.

The magnetic field induction at a point just outside the curved face of the solenoid carrying current is zero.



The field at a point on the axis of a solenoid can be obtained by superposition of fields due to a large number of identical coils all having their centre on the axis of solenoid.



Let us consider a coil of width dx at a distance x from the point P on the axis of solenoid as shown in the figure. The magnetic field due to this coil

$$dB = \frac{\mu_0 \; NIR^2}{2\left(R^2 + x^2\right)^{3/2}}$$

Here, N = ndx, x = R tanϕ and dx = R sec$^2\phi$ dϕ

Hence, $dB = \frac{\mu_0 \; ndx \; x \; IR^2}{2\left(R^2 + R^2\tan^2\phi\right)^{3/2}}$

$$B = \int dB = \frac{\mu_0}{2} nI \int_{-\alpha}^{\beta} \cos\phi \, d\phi \text{ i.e. } B = \frac{\mu_0 \, n \, I}{2}\left[\sin\alpha + \sin\beta\right]$$

Magnetic field due to current in a toroid

A toroid is an endless solenoid in the form of a ring. According to ampere's circuital law, the line integral of magnetic field induction $\vec{B}$ along the circular path of radius r is given by
$\oint \vec{B}.d\vec{\ell} = \mu_0 \times$ total current passing through circle of radius r.

Now, $\oint \vec{B}.d\vec{\ell} = B\,(2\pi r)$

Total current passing through circle of radius r = number of turns in the solenoid $\times$ I
$= 2\pi r\, nI$
$\therefore \quad B(2\pi r) = \mu_0\,(2\pi r nI)$
or $\quad B = \mu_0\, nI$

It is to be noted that the magnetic field B due to a toroid carrying current is independent of r but depends upon the current and number of turns per unit length of toroid. The magnetic field inside the toroid is constant and is always tangential to the circular closed path.

Exercise 2: ***A long straight wire of radius R carries a current distributed uniformly over its cross section. Find the locations where magnetic field is maximum and minimum.***

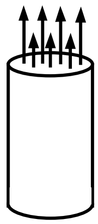
Ampere's Law

Similar to the Gauss's law of electrostatics, this law provides us shortcut methods of finding magnetic field in cases of symmetry. According to this law, the line integral of magnetic field over a closed path ($\oint \vec{B}\cdot d\vec{\ell}$) is equal to μ_0 times the net current crossing the area enclosed by that path. Mathematically, $\oint \vec{B}\cdot d\vec{\ell} = \mu_0\, i_{enclosed}$

Positive direction of current and the direction of the line integral are given by the right hand thumb and curling fingers, respectively.

In order to find magnetic field using Ampere's law, the closed path of line integral is generally chosen such that $\vec{B}$ is either parallel or perpendicular to the path line. Also, wherever $\vec{B}$ is parallel to the path, its value should be constant.

Illustration 18. *Suppose that the current density in a wire of radius a varies with r according to $J = Kr^2$, where K is a constant and r is the distance from the axis of the wire. Find the magnetic field at a point distant r from the axis when (i) r < a and (ii) r > a.*



Solution: Choose a circular path centred on the conductor's axis and apply Ampere's law.

(i) To find the current passing through the area enclosed by the path

$$dI = JdA = (Kr^2)\,(2\pi r dr)$$

i.e. $\quad I = k\int_o^r 2\pi r^3 dr = \frac{\pi k r^4}{2}$

Since $\int \vec{B}.d\vec{\ell} = \mu_o I$

$$\Rightarrow B\, 2\pi r = \mu_o . \frac{\pi k r^4}{2}$$

$$\Rightarrow \qquad B = \frac{\mu_o k r^3}{4}$$

(ii) If r > a, then net current through the Amperian loop is

$$I' = \int_0^a K r^2\, 2\pi r dr = \frac{\pi K a^4}{2}$$

Therefore, $B = \dfrac{\mu_o K a^4}{4r}$

Magnetic Force on the moving Charge

1. When a charged particle having charge 'q' is projected into a magnetic field, it experiences a magnetic force which is given by the expression

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

Here, $\vec{v}$ = velocity of the particle and $\vec{B}$ = magnetic field

On the basis of above expression, we can draw following conclusion.

(a) Stationary charge (i.e $\vec{v}$ = 0) experiences no magnetic force.

(b) If $\vec{v}$ is parallel or anti parallel to $\vec{B}$, then the charged particle experiences no–magnetic force.

(c) Magnetic force is always perpendicular to both $\vec{v}$ and $\vec{B}$.

(d) As the magnetic force is always perpendicular to $\vec{v}$, it does not deliver power to the charged particle.

(e) As magnetic force is always perpendicular to $\vec{v}$, this force will compel the charged particle to move in a circular path.

2. On the basis of expression $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$, the maximum value of magnetic force is equal to F = qvB, which occurs when the charge is projected perpendicular to the magnetic field. In this case path of charged particle is purely circular (in uniform $\vec{B}$) and magnetic force provides necessary centripetal force.

(a) If radius of the circular path is R then,

$\dfrac{mv^2}{R}$ = qvB, where m = mass of the particle

$$\Rightarrow \qquad R = \frac{mv}{qB}$$

Time taken to complete one revolution is $T = \dfrac{2\pi R}{v}$

$$\Rightarrow \qquad T = \frac{2\pi m}{qB}.$$

3. In general $\vec{v}$ can be resolved into two components, one along the $\vec{B}$ say $v_{||}$ (parallel component) and the other perpendicular to $\vec{B}$ say $v_\perp$. Due to $v_\perp$ it experiences a magnetic force and hence has a tendency to move on a circular path. Due to $v_{||}$ it experiences no force, and hence has a tendency to move on a straight path along the field. So in this case it moves along a helical path.

(a) Radius of the helix is R = $\dfrac{mv_\perp}{qB}$

(b) Time taken to complete one revolution is T = $\dfrac{2\pi m}{qB}$. (Note: T is independent of v)

(c) The distance moved by the charged particle along the magnetic field during one revolution is called pitch.

$\Rightarrow$ pitch = $v_{\parallel} \times T = \dfrac{2\pi m v_{\parallel}}{qB}$

Illustration 19. *A proton is moving with velocity v = 2.5 × 10⁵ m/s making an angle θ such that $v_x = 1.5 \times 10^5$ m/s, $v_y = 0$ and $v_z = 2 \times 10^5$ m/s at t = 0. A uniform magnetic field is directed along the x–axis with magnitude 0.005 T.*
(a) Find the force on the proton and acceleration.
(b) Find the radius of the helical path, the angular speed of the proton and the pitch of the helix.

Solution: (a) $\vec{F} = q(\vec{v} \times \vec{B})$

$= q(v_x\hat{i} + v_z\hat{k}) \times B\hat{i} = qv_zB\hat{j}$

$= 1.6 \times 10^{-14}$ N $\hat{j}$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{1.6\times10^{-14}\text{N}}{1.6\times10^{-27}\text{kg}} = 9.58\times10^{12}\ \text{m/s}^2\ \hat{j}$$

(b) at t = 0 the component of velocity perpendicular to $\vec{B}$ is v_z, so

$$R = \frac{mv_z}{qB} = \frac{1.67\times10^{-27}\times2\times10^5}{1.6\times10^{-19}\times0.5}$$

$= 4.18 \times 10^{-3} = 4.18$ mm

(c) $$\omega = \frac{qB}{m} = \frac{1.6\times10^{-19}\times0.5}{1.67\times10^{-27}} = 4.79\times10^7\ \text{rad/s}$$

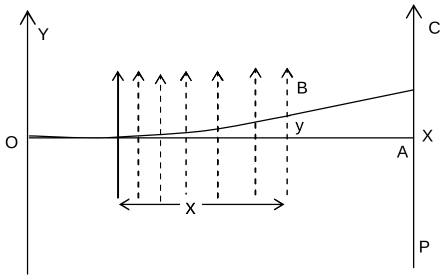
$$\Rightarrow T = \frac{2\pi}{4.79\times10^7} = 1.31\times10^{-7}\ \text{s}$$

pitch = v_xT

$= 1.5 \times 10^5 \times 1.31 \times 10^{-7} = 19.7$ mm.

Motion of a charged particle in a uniform electric field

Let the charged particle be subjected to a uniform electric field of strength E acting along OY. Due to electric field, the charged particle experiences a force,



$\vec{F} = qE(\hat{j})$ $\Rightarrow \vec{a} = \dfrac{qE}{m}\hat{j}$

since $t = \dfrac{x}{v}$ $\Rightarrow y = (0)t + \dfrac{1}{2}\dfrac{qE}{m}t^2$

$\Rightarrow y = \dfrac{1}{2}\dfrac{qE}{m}t^2$

or $x^2 = \dfrac{2mv^2}{qE}y$

or, $x^2 = ky$

This is an equation of a parabola. Hence, inside the electric field, the charged particle moves on a parabolic path OB and on leaving the field, it moves along a straight path BC, tangent to the curved path OB at B.

If a charged particle is describing a circular path of radius r under the effect of perpendicular electric field, then :

1. The force on the charge particle is inversely proportional to the square of radius of the circular path

   i.e. $F \propto \frac{1}{r^2}$

2. The velocity of the charged particle is inversely proportional to the square root of the radius of the circular path, i.e. $v \propto \frac{1}{\sqrt{r}}$

3. The square of a time period of revolution of the charged particle is directly proportional to the cube of the radius of circular path, i.e.
   $T^2 \propto r^3$

Lorentz force: The force experienced by a charged particle moving in space where both electric and magnetic fields exists is called Lorentz force.

Force due to electric field, $\vec{F}_e = q\vec{E}$

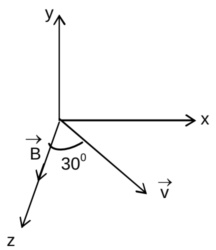
Force due to magnetic field, $\vec{F}_m = q(\vec{v} \times \vec{B})$

Due to both the electric and magnetic fields, the total force experienced by the charged particle will be given by

$$\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q(\vec{v} \times \vec{B})$$
$$= q\,(\vec{E} + \vec{v} \times \vec{B})$$

This is called Lorentz force.

Illustration 20. *A beam of protons moves at 3×10^5 m/s through a uniform magnetic field with magnitude 2.0 T that is directed along the positive z–axis. The velocity of each proton lies in the x–z plane at an angle of 30° to the +z axis. Find the force on a proton.*



Solution: Since the charge is positive, so the force is in the same direction as the vector product $\vec{v} \times \vec{B}$

$\vec{F} = 1.6 \times 10^{-19}(3 \times 10^5 \text{ m/s}) [(\sin 30\,\hat{i} + \cos 30\,\hat{k}) \times 2.0\text{T}\,\hat{k}]$

$= -4.8 \times 10^{-14}\text{ N}\,\hat{j}$

$= 4.8 \times 10^{-14}\text{ N}\,(-\hat{j})$

Illustration 21. *Two protons move parallel to x–axis in opposite directions at the same speed v. At the point where the distance between the two is minimum, find the electric and magnetic forces on the upper proton and determine the ratio of their magnitudes.*

Solution: $F_E = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$

$F_B = qv \times \frac{\mu_0 qv}{4\pi r^2} = \frac{\mu_0}{4\pi}\frac{q^2v^2}{r^2}$

Thus, $\frac{F_B}{F_E} = \frac{\mu_0 q^2v^2/4\pi r^2}{q^2/4\pi\varepsilon_0 r^2} \Rightarrow \frac{\mu_0 v^2}{1/\varepsilon_0} = \varepsilon_0\mu_0 v^2$

Using the relation $\varepsilon_0\,\mu_0 = 1/c^2$

$\Rightarrow \frac{F_B}{F_E} = \frac{v^2}{c^2}$

Illustration 22. *A charged particle moves in a uniform magnetic field perpendicular to it, with a radius of curvature 4 cm. On passing through a metallic sheet, it looses half of its kinetic energy. Then, the radius of curvature of the particle is*

(A) 2 cm *(B) 4 cm*
(C) 8 cm *(D) $2\sqrt{2}$ cm*

Solution: **(D)** $R = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2(KE)}{m}} = \frac{\sqrt{2m}}{qB}\sqrt{KE}$

$\Rightarrow \frac{R_1}{R_2} = \frac{\sqrt{KE}}{\sqrt{KE/2}}$

$\Rightarrow R_2 = \frac{R_1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\,\text{cm}$

Illustration 23. *An electron is projected with velocity v_0 in a uniform electric field E perpendicular to the field. Again it is projected with velocity v_0 perpendicular to a uniform magnetic field B. If r_1 is initial radius of curvature just after entering in the electric field and r_2 is initial radius of curvature just after entering in magnetic field, then the ratio $\frac{r_1}{r_2}$ is equal to*

(A) $\frac{Bv_0^2}{E}$ *(B) $\frac{B}{E}$*
(C) $\frac{Ev_0}{B}$ *(D) $\frac{Bv_0}{E}$*

Solution: **(D)** $r_1 = \frac{v_0^2}{eE}m_e$

$r_2 = \frac{m_e v_0}{eB}$

$\frac{r_1}{r_2} = \frac{v_0^2 m_e}{eE} \times \frac{eB}{m_e v_0} = \frac{Bv_0}{E}$

Illustration 24. *An electron of mass 9.1×10^{-31} kg enters a magnetic field of magnitude 1 tesla with a velocity of 2 m/s, at some angle θ to the magnetic field. If the distance moved by the particle in time 17.86 pico–seconds in the direction of magnetic field is 30.93 pico–meter, the angle θ at which the particle entered the magnetic field, is*

(A) 60° *(B) 45°*
(C) 90° *(D) 30°*

Solution: **(D)** Distance moved along the magnetic field in time t = v cos θt = X (say)

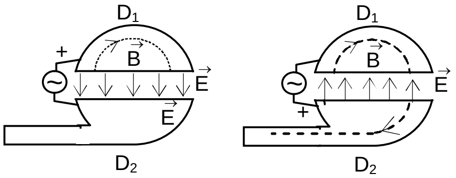
Now $\cos\theta = \frac{X}{vt}$

$= \frac{30.93}{17.86 \times 2} = 0.8659$

$\therefore \theta = \cos^{-1}(0.866) = 30^\circ$

THE CYCLOTRON

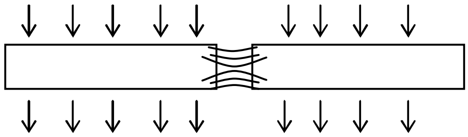
In electrostatic accelerators, the acceleration depends on the total potential difference ΔV. To produce high energy particles, ΔV must be very large. However, in a cyclic accelerator an electric charge may receive a series of accelerations by passing many times through a relatively small potential difference.



Essentially, a cyclotron consists of a cylindrical cavity divided into two halves each called dee and placed in a uniform magnetic field parallel to its axis. The two dees are electrically insulated from each other.

An ion source S is placed at the centre of the space between the dees. The system must be maintained within a high vacuum to prevent collisions between the accelerated particles and any gas molecule. An alternating potential difference of the order of 10^4 V is applied between the dees. When the ions are positive, they will be accelerated towards the negative dee. once the ions get inside a dee, they experience no electrical force, since the electric field is zero in the interior of a conductor. However, the magnetic field makes the ions describe a circular orbit with a radius given by $r = \frac{mv}{qB}$

and angular velocity $\omega = qB/m$



In this way, the potential different between dees is in resonance with the circular motion of the ions.

As the ion describes half a revolution, the polarity of dees is reversed. When the ions cross the gap between them. They receive another small acceleration. The next half-circle described then has a large radius but the same angular velocity. The process repeats itself several times, until the radius attains maximum value R, which is practically equal to the radius of the dees. The poles of the magnet are designed. Such that the magnetic field at the edge of dees decreases sharply and the ions move tangentially, escaping through a convenient opening. The maximum velocity v_{max} is related to radius R by

$R = \frac{mv_{max}}{qB}$

or $v_{max} = (q/m)BR$

$\Rightarrow E_k = \frac{1}{2}mv_{max}^2 = \frac{1}{2}q\left(\frac{q}{m}\right)B^2R^2$.

Illustration 25. *A magnetron in a microwave oven emits electromagnetic waves with frequency f = 2450 MHz. What magnetic field strength is required for electrons to move in circular paths with this frequency ?*

Solution: $B = \frac{m\omega}{q} = \frac{2\pi fm}{q}$

$= \frac{2\times\pi\times 2450\times 10^{6}\times 9.11\times 10^{-31}}{1.6\times 10^{-19}} = 0.0877\ T$

Exercise 3:

(i) ***If a charged particle is deflected either by an electric or a magnetic field, how can we ascertain the nature of the field ?***

(ii) ***If an electron is not deflected in passing through a certain region of space, can we be sure that there is no magnetic field in that space ? What possible conclusions could be drawn regarding the existence of electric and magnetic fields?***

(iii) ***A beam of protons is deflected sideways. Could this deflection be caused (a) by a constant electric field? (b) by a constant magnetic field? (c) If either is possible, how can you tell which one is present?***

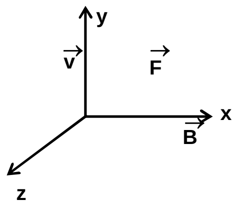
Illustration 26. *A uniform magnetic field of 30 mT exists in the + X direction. A particle of charge +e and mass 1.67 × 10^{-27} kg is projected through the field in the + Y direction with a speed of 4.8 × 10^{6} m/s.*

(a) Find the force on the charged particle in magnitude and direction.

(b) Find the force if the particle were negatively charged.

(c) Describe the nature of path followed by the particle in both the cases.

Solution: (a) Force acting on a charge particle moving in the magnetic field, $\vec{F} = q(\vec{v}\times\vec{B})$



Magnetic field $\vec{B}$ = 30(mT) $\hat{i}$

Velocity of the charge particle

$\vec{v} = 4.8\times 10^{6}$ (m/s) $\hat{j}$

$\vec{F} = 1.6\times 10^{-19}[(4.8\times 10^{6}\hat{j})\times(30\times 10^{-3})(\hat{i})$

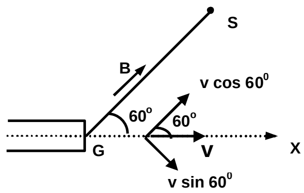
$\vec{F} = 230.4\times 10^{-16}(-\hat{k})$ *N.*

(b) If the particle were negatively charged, the magnitude of the force will be the same but the direction will be along (+ z) direction.

(c) As v $\perp$ B, the path described is a circle

$R = \frac{mv}{qB} = (1.67\times 10^{-27}) . (4.8\times 10^{6}) / (1.6\times 10^{-19}) . (30\times 10^{-3}) = 1.67$m.

Illustration 27. *An electron gun G emits electrons of energy 2 keV travelling in the positive x–direction. The electrons are required to hit the spot S where GS = 0.1 m, and the line GS makes an angle of 60^o with the x-axis, as shown. A uniform magnetic field B parallel to GS exists in the*



region outside the electron gun. Find the minimum value of B needed to make the electrons hit S.

Solution: Let the electron gun emits the electrons with initial velocity along x-axis.

Then, $\frac{1}{2}mv^2 = eV$ or, $v = \sqrt{\frac{2eV}{m}}$

Now, the component of velocity of the electron beam along the direction of the magnetic field B is $v_{\parallel}$ and that $\perp$ to B is $v_{\perp}$

$v_{\parallel} = v\cos\theta$, $v_{\perp} = v\sin\theta$ (where $\theta = 60°$)

GS = n × pitch where n = 1, 2,…….

$\Rightarrow$ $GS = n\frac{2\pi m}{qB}v\cos\theta$

$\Rightarrow$ $B = \frac{n2\pi m}{q.GS}.v\cos\theta$, where $v = \sqrt{\frac{2eV}{m}}$

For B to be minimum, n should be 1.

Substituting the values, $B = 4.7 \times 10^{-3}$ T

Force on a Current Carrying Wire

As $\vec{F} = q\left(\vec{v}\times\vec{B}\right)$,

We can say, $d\vec{F} = dq[\vec{v}\times\vec{B}]$

or $d\vec{F} = dq\left[\frac{d\vec{\ell}}{dt}\times\vec{B}\right] = \frac{dq}{dt}\left[d\vec{\ell}\times\vec{B}\right]$

$\Rightarrow$ $d\vec{F} = I(d\vec{\ell}\times\vec{B})$

Illustration 28. *A straight horizontal copper wire carries a current of 50 A from west to east in a region between the poles of a large electromagnet. In this region, there, is a horizontal magnetic field towards the north- east with magnitude 1.20 T as shown in the figure. Find the magnitude and direction of the force on a 1.00 m section of rod.*

Solution: $F = I\ell B\sin\phi$

$= 50 \times 1 \times 1.2 \times \sin 45°$

$= 42.4$ N

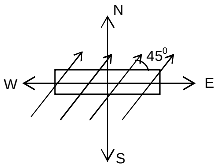


Illustration 29. *A wire of length 5.0 cm carries a current of 3.0 A; kept in an external uniform magnetic field of magnitude 10^{-3} Wbm^{-2}. Calculate the magnetic force exerted on the wire, if the wire is inclined at 30° with $\vec{B}$.*

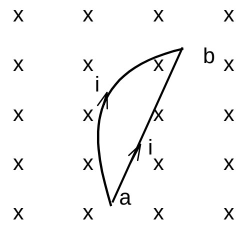
Solution: The force is given by the vector relation,

$\therefore$ F = i ℓ B sinθ where θ is the angle between $\vec{\ell}$ and $\vec{B}$

$F = (3.0A) \times (5 \times 10^{-2}\ m) \times (10^{-3}\ Wbm^{-2}) \times 0.5 = 7.5 \times 10^{-5}\ N$ $\vec{\ell}$ and $\vec{B}$

The direction of this force is perpendicular to the plane which contains both $\vec{\ell}$ and $\vec{B}$.

Illustration 30. *Show that the force on a straight wire between a and b of the figure given is the same as the force on a wire of arbitrary shape between the same two points when they carry the same current from a to b and are placed in the same magnetic field.*

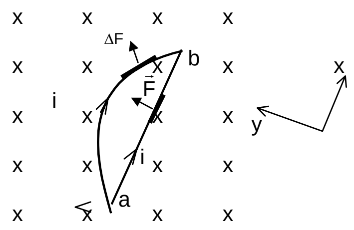


Solution: Force on the straight conductor

$\vec{F} = i\ell\, B\sin 90\ \hat{j} = i\ell\, B\,\hat{j}$

Where ab $= \ell$,

Now consider an element $\Delta\ell$ of the curved wire,



The force on the element $= \Delta F = i\,\Delta\ell B \sin 90 = i\,\Delta\ell B$

The direction of the force is at right angles to the element.

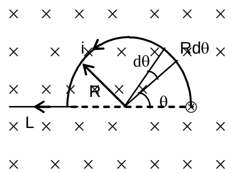
Then, $\Delta\vec{F} = i\Delta\ell\ B\cos\theta\,\hat{i}\ + i\Delta\ell\ B\sin\theta\,\hat{i}$

Hence net force $\vec{F} = \Delta\vec{F} =\ iB\left(\sum\Delta\ell\ \ \cos\theta\right)\ \hat{i}\ + iB\left(\sum\Delta\ell\ \ \sin\theta\right)\ \hat{j}$

$\sum\Delta\ell\ \ \cos\theta$ = sum of projection of $\Delta\ell$'s on ab = ℓ and $\sum\Delta\ell\ \ \sin\theta\ =\ 0$

Hence net force on $\vec{F}\ =\ i\ell\ B\,\hat{j}$, which is same the force on the straight–line conductor ab.

Illustration 31. *In the given below figure, the magnetic field $\vec{B}$ is uniform and perpendicular to the plane of the figure, pointing out. The conductor has a straight segment with length L perpendicular to the plane of the figure on the right, with the current opposite to $\vec{B}$; followed by a semi –circle with radius R; and finally another straight segment with length L parallel to x–axis as shown. The conductor carries a current I. Find the total magnetic force on the two segments of wire as shown.*



Solution: F_1 = ILB (upward)

In the semicircle, dF = I ($d\vec{\ell}\times\vec{B}$) radially outward from the centre

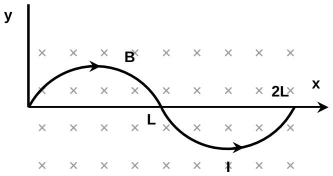
dF = I$d\ell$B = I (R$d\theta$)B

$dF_x = IRd\theta B\cos\theta \qquad \Rightarrow F_x = IRB\int_0^{\pi}\cos d\theta = 0$

$dF_y = IRd\theta\ B\sin\theta \qquad \Rightarrow F_y = IRB\int_0^{\pi}\sin\theta d\theta = 2IRB$ upward

$\Rightarrow\ \vec{F} = IB(L + 2R)\ \hat{j}$

Illustration 32. *A wire carrying current I is placed in a uniform magnetic field B in the form of a curve $y = a \sin\left(\frac{\pi x}{L}\right)$, $0 \le x \le 2L$ as shown below. The force upon the wire is*



(A) $2\pi ILB$ *(B) $\frac{\pi L^2}{4}BI$*

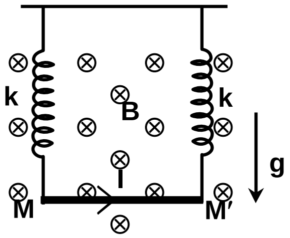
(C) 2IBL *(D) zero.*

Solution: **(*C*)** Since force on a wire is given by

$dF = I\,d\vec{\ell} \times \vec{B}$, so $F = \int dF = \int I\,d\vec{\ell} \times \vec{B}$

So, the total force on wire is 2IBL.

Illustration 33. *Consider the plane of the paper to be the vertical plane with direction of 'g' as shown in the figure. A rod (MM′) of mass 'M' and carrying a current I. Rod is, suspended vertically through two insulated springs of spring constant K at two ends. A uniform magnetic field is applied perpendicular to the plane of the paper such that the potential energy stored in the spring, in equilibrium position, is zero. If the rod is slightly displaced in the vertical direction from the position of equilibrium, then the time period of oscillation is*



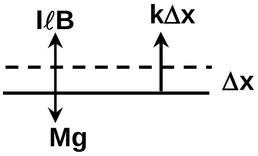
(A) $2\pi\sqrt{\frac{m}{K}}$ *(B) $2\pi\sqrt{\frac{m}{2K}}$*

(C) $2\pi\sqrt{\frac{K}{2m}}$ *(D) $2\pi\sqrt{\frac{m}{K}}.\frac{mg}{I\ell B}$*

Solution: **(B)** $B = \frac{mg}{I\ell}$

$\therefore -2K\Delta x = -m\omega^2\Delta x$

$\omega = \sqrt{\frac{2K}{m}}$



FLEMING'S LEFT-HAND RULE

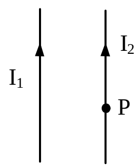
In the special case of straight wire of length ℓ in a uniform magnetic field B, $\vec{F} = I\left(\vec{\ell} \times \vec{B}\right)$

The direction of the force $\vec{\mathbf{F}} = \mathbf{I}\left(\vec{\ell} \times \vec{\mathbf{B}}\right)$ is given by the **Fleming's left hand rule**.

Close your left fist and then shoot your index finger in the direction of the magnetic field. Relax your middle finger in the direction of the current. The force on the conductor is shown by the direction of the erect thumb.

Force between two infinite parallel current carrying wires.

Let two infinite parallel wires carrying currents I_1 and I_2 be separated by a distance r.



If we take an arbitrary point. 'P' on the second wire, the angle α and β subtended by the other wire are:

$\alpha = 0$ and $\beta = 0$.

$$B_{21} = \frac{\mu_o I_1}{4\pi r}[\cos 0 + \cos 0] = \frac{\mu_o I_1}{2\pi r}$$

$$\bar{F}_{21} = I_2\left(\vec{\ell} \times \vec{B}_1\right) \Rightarrow F_{21} = \frac{I_2 \ell_2 \mu_o I_1}{2\pi r}$$

$$\frac{F_{21}}{\ell_2} = \frac{\mu_o I_1 I_2}{2\pi r}, \text{ and By symmetry } \frac{F_{12}}{\ell_1} = \frac{\mu_o I_1 I_2}{2\pi r}$$

$$\text{Force per unit length} = \frac{\mu_o I_1 I_2}{2\pi r}.$$

Note that wires carrying current in the same direction attract each other.

If $I_1 = I_2 = 1$ amp; $r = 1$ m

Then, $F = \frac{\mu_0}{4\pi} \times \frac{2I_1I_2}{r} = 10^{-7} \times 2 \times 1 \times 1 = 2 \times 10^{-7}$ N/m.

Thus, one ampere is that much current which when flowing through each of the two parallel uniform long linear conductors placed in free space at a distance of one meter from each other will attract or repel each other with a force of 2×10^{-7} N/m of their lengths.

Illustration 34. *Two straight parallel wires 4.5 mm apart carry equal currents of 15000 A in opposite directions. Calculate the force per unit length between the two.*

Solution:

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

$$= \frac{4\pi \times 10^{-7} \times (15000)^2}{2\pi(4.5 \times 10^{-3})} = 1 \times 10^4 \text{N/m}$$

Illustration 35. *A long straight wire carries a constant current of 5 Amp. At a distance of 10 cm from the straight wire, a small straight conductor of length 10 cm and parallel to the wire and carries a current of 3 amp having. If it is released from its position, the acceleration of the conductor just after being released is*

(A) $10^{-3}\ m/s^2$ *(B) $2 \times 10^{-3}\ m/s^2$*

(C) $4 \times 10^{-3}\ m/s^2$ *(D) $5 \times 10^{-3}\ m/s^2$*

Solution: **(A)** $F = \frac{\mu_0}{4\pi} \times \frac{2I_1I_2}{d}\ell$

$$\text{Acceleration, } a = \frac{10^{-7} \times 2 \times 5 \times 3 \times 0.1}{0.1} \times \frac{1}{3 \times 10^{-3}}$$

$$= 10^{-3}\,\text{m/s}^2$$

Torque on a current carrying planer loop in a uniform magnetic field

Case I:

When plane of the loop is perpendicular to magnetic field

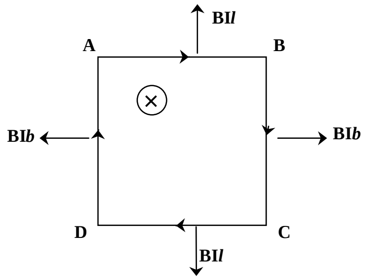
Length of AB = DC = ℓ and that of BC = AD = b

Forces experienced by all the sides are shown in the figure.

$\therefore$ Forces on AB and DC are equal and opposite to the each other same is the case with BC and AD.

$\Rightarrow \quad \Sigma F = 0$

Since the line of action of the forces on AB and DC is same and also the line of action of the forces BC and AD is same, therefore, torque is zero.



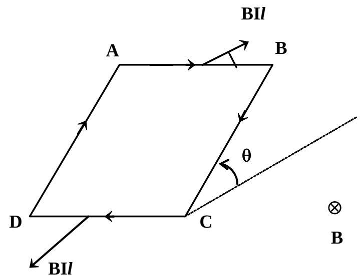
Case II:

When the plane of the loop is inclined to the magnetic field.

In this case again $\Sigma F = 0$

$\therefore$ Lines of action of the forces on AB and DC are different, therefore, this forms a couple and produces a torque. Side view of the loop is shown in the figure.

Torque = $B\ell I\,(b\sin\theta) = BI(lb)\sin\theta = BIA\sin\theta$.



If loop has N turns, then $\tau = BNIA \sin\theta$

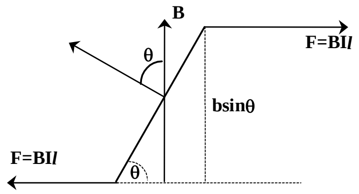
In vector form, $\vec{\tau} = \vec{\mu} \times \vec{B}$, where $\vec{\mu} = NI\vec{A}$

Energy needed to rotate the loop through an angle $d\theta$ is

$dU = \tau d\theta$

$$\Rightarrow \quad \Delta U = \int dU = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\theta_1}^{\theta_2} \mu B \sin\theta d\theta$$

$\Delta U = \mu B(\cos\theta_1 - \cos\theta_2)$,



if we choose θ_1 such that at $\theta = \theta_1$, $U_1 = 0$

This is the energy stored in the loop, $U = -\vec{\mu}_m.\vec{B}$

Illustration 36. *A circular coil 0.05 m in radius, with 30 turns of wire, lies in a horizontal plane. It carries a current of 5 A in a counter clockwise sense when viewed from above. The coil is in a uniform magnetic field directed towards the right, with magnitude 1.20 T. Find the magnitudes of the magnetic moment and the torque on the coil.*

Solution: The area of the coil is

$A = \pi r^2 = \pi(0.05)^2 = 7.85 \times 10^{-3}\ m^2$

$\mu_{total} = 30 \times 5 \times 7.85 \times 10^{-3} = 1.18\ A.\ m^2$

The angle ϕ between the direction of $\vec{B}$ and the direction of $\vec{\mu}$ is 90°.

$\therefore \ \tau = \mu B \sin\phi$

$= 1.18 \times 1.2 \text{ x } \sin 90°$

$= 1.41\ N - m$

Exercise 4 : ***A rectangular current loop is in an arbitrary orientation in an external magnetic field. Is any work required to rotate the loop about an axis perpendicular to its plane ?***

Illustration 37. *The coil of a galvanometer has 500 turns and each turn has an average area 3×10^{-4} m^2. Calculate the magnetic moment of the coil when a current of 0.5 A passes through it. If a torque of 1.5 Nm is required for this coil carrying same current to set it parallel to a magnetic field, calculate the strength of the magnetic field.*

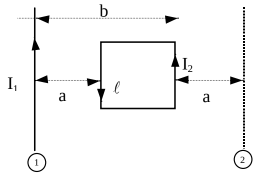
Solution: The magnetic moment of a current loop
μ = NI A = 500 x 0.5 x 3 x 10^{-4} = 0.075 Am^2.
Also $\vec{\tau} = \vec{\mu} \times \vec{B}$ or $|\vec{\tau}| = \mu B \sin\theta$
where θ = angle between B and A
Here, θ = 90°
$\therefore \quad \tau = \mu B \sin 90^0$

or, $B = \frac{\tau}{\mu} = \frac{1.5}{0.075} = 20$ T

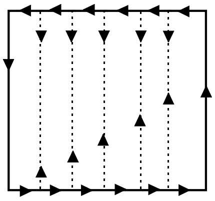
Illustration 38. *If a coil rotates from its initial position to a position where its magnetic moment is parallel to $\vec{B}$, what is the change in potential energy ?*

Solution: Initial PE = $-\mu B\cos\phi_1 = -(1.18 \times 1.2 \times \cos 90°) = 0$
Final PE = $-\mu B\cos\phi_2 = -(1.18 \times 1.2 \times \cos 0) = -1.41$
$\Delta V = -1.41$ J
Thus, P.E. decreases because the rotation is in the direction of the magnetic field.

Illustration 39. *What is the work done in transferring the wire from position (1) to position (2)?*



Solution: The loop can be thought of being made of elementary loops.
The net current in the dotted wires is 0 as currents in the neighbouring loops flowing through the same wire are opposite in direction. The magnetic moment of the elemental loop 'dμ' = $I_2\ell dr$.

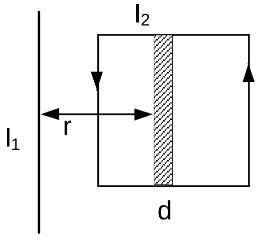


The magnetic field at that point due to straight wire = $\mu_0 I_1/2\pi r$.

$dU = -B.d\mu = -\frac{\mu_0 I_1}{2\pi r} I_2 \ell dr\ (\cos\pi)$

[As dμ is anti–parallel to B.]

$$U_1 = \int du = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\int_a^b \frac{1}{r}dr = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\ln\left(\frac{b}{a}\right)$$

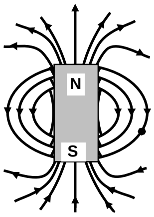


By symmetry, $U_2 = -U_1$

$$\Rightarrow \quad -\Delta U = \text{work done} = -(U_2 - U_1) = 2\frac{\mu_0 I_1 I_2 \ell}{2\pi}\ln\frac{b}{a}.$$

The work done in transferring the wire from position 1 to 2 = $\frac{\mu_0 I_1 I_2 \ell}{\pi}\ln\frac{b}{a}$.

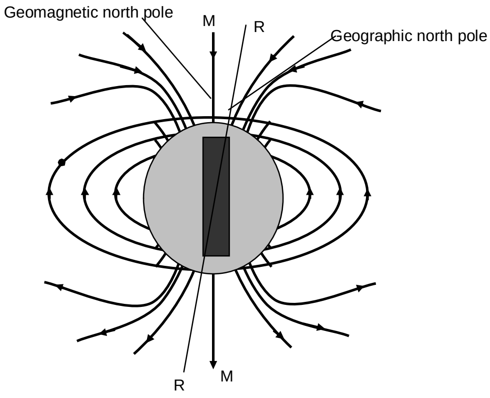
MAGNETS



A bar magnet is a magnetic dipole. The iron filling suggests the magnetic field lines. Iron filling sprinkled around such a magnet tends to align with the magnetic field of the magnet, and their pattern reveals the magnetic field lines. The clustering of the lines at the ends of the magnet suggests that one end is a source of the lines (the field diverges from it) and the other end is a sink of the lines (the field converges towards it). By convention, we call the source the North pole of the magnet and the opposite end the South pole, and we say the magnet, with its two poles, is an example of magnetic dipole.

THE MAGNETISM OF EARTH

Earth is a huge magnet; for points near earth's surface, its magnetic field can be approximated as the field of a huge bar magnet –a magnetic dipole that straddles the centre of the planet. The figure shown is an idealized symmetric depiction of the dipole field, without the distortion caused by passing charged particles from the sun.



Because earth's magnetic field is that of a magnetic dipole, a magnetic dipole moment $\vec{\mu}$ is associated with the field. For the idealized field, the magnitude of $\vec{\mu}$ is 8×10^{22} J/T and the direction of $\vec{\mu}$ makes an angle of 11.5° with the rotation axis (RR) of the earth. The dipole axis MM lies along $\vec{\mu}$ and intersects earth's surface at the geometric north pole in north west Greenland and the geometric south pole in Antarctica. The lines of the magnetic field $\vec{B}$ generally emerges in the southern hemisphere, and re–enter earth in the northen hemisphere,.

The direction of the magnetic field at any location on earth's surface is commonly specified in terms of two angles. The field declination is the angle (left or right) between geographic north (which is towards 90° latitude) and the horizontal component of the field. The field inclination is the angle (up or down) between a horizontal plane and the field's direction.

Magnetism and electrons

Magnetic materials are magnetic because of the electron within them. One way in which electrons can generate magnetic field is to send them through a wire as an electric current and their motion produces a magnetic field around the wire.

Spin magnetic dipole moment

An electron has an intrinsic angular momentum called its spin angular momentum or just spin $\vec{s}$; associated with the spin is an intrinsic spin magnetic dipole moment $\vec{\mu}_s$. $\vec{s}$ and $\vec{\mu}_s$ are related by

$$\vec{\mu}_s = -\frac{e}{m}\,\vec{s}$$

where e and m are charge and mass of electron, respectively. The minus sign means that $\vec{\mu}_s$ and $\vec{s}$ are oppositely directed.

Spin $\vec{s}$ is different from the angular momentum:

- $\vec{s}$ itself cannot be measured. However, its component along any axis can be measured.

- A measured component of $\vec{s}$ is quantized, which is a general term that means it is restricted to certain values. A measured component of $\vec{s}$ can have only two values which differ only in sign.

Let us assume that component of spin $\vec{s}$ is measured along the z–axis of a coordinate system.

Then, the measured component $S_z = m_s \frac{h}{2\pi}$ for $m_s = \pm \frac{1}{2}$

where m_s is called the spin magnetic quantum number.

When S_z is parallel to the z–axis, m_s is $+\frac{1}{2}$ and the electron is said to be spin up. When S_z is antiparallel to the z–axis, m_z is $-\frac{1}{2}$ and the electron is said to be spin down.

We can relate the component

$$\mu_{s,z} = -\frac{e}{m} s_z$$
$$= \pm \frac{eh}{4\pi m}$$

where the plus and minus signs corresponds to $\mu_{s,z}$ being parallel and antiparallel to the z–axis, respectively. This quantity is termed as Bohr magneton

$$\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} \text{ J/T}$$

Orbital magnetic dipole moment

When it is an atom, an electron has an additional angular momentum called its orbital angular momentum $\vec{L}_{orb.}$.

Associated with $\vec{L}_{orbt}$ is an orbital magnetic dipole moment $\vec{\mu}_{orbt}$,

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb}$$

The minus sign means that $\vec{\mu}_{orbt}$ and $\vec{L}_{orbt}$ have opposite directions.

Here, again $L_{orb,\,z} = m_L \frac{h}{2\pi}$ [for $m_L = 0, \pm 1, \pm 2 \ldots . \pm$ (limit)]

in which m_L is called the orbital magnetic quantum number and limit refers to some larger allowed integer value of m_L.

We can write the z– component $\mu_{orb,z}$ of the orbital magnetic dipole moment as

$$\mu_{orb,\,z} = -m_L \frac{e}{4\pi m}$$

and in terms of Bohr magneton as

$$\mu_{orb,\,z} = -m_L \mu_B$$

Magnetic Materials

Each electron in an atom has an orbital magnetic dipole moment and a spin magnetic dipole moment that combine vectorially. The resultant of these two vector quantities combines vectorially with similar resultants for all other electrons in the atom, and the resultant for each atom combines with those for all these magnetic dipole moments produces a magnetic field, the material is magnetic. There, are three general types of magnetism : diamagnetism; paramagnetism and ferromagnetism.

1. Paramagnetism is exhibited by materials containing transition elements, rare earth elements and actinide elements. Each atom of such a material has a permanent

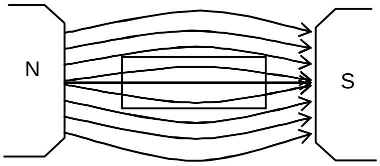
resultant magnetic dipole moment, but the moments are randomly oriented in the material and material as a whole lacks a net magnetic field. However, an external magnetic field $\vec{B}_{ext}$ can partially align the atomic magnetic dipole moments to give the material a net magnetic field. The alignment and thus its field disappear when $\vec{B}_{ext}$ is removed. The term paramagnetic material usually refers to materials that exhibit primarily paramagnetism.

2. In diamagnetism, weak magnetic dipole moments are produced in the atoms of the material when the material is placed in an external magnetic field $\vec{B}_{ext}$; the combination of all those induced dipole moments and thus their net field disappear when $\vec{B}_{ext}$ is removed. The term diamagnetic material usually refers to materials that exhibit only diamagnetism.

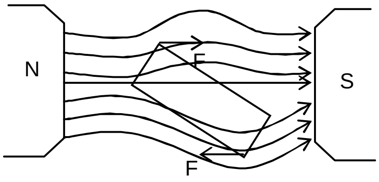
3. Ferromagnetism is a property of iron, nickel and certain other elements (and of compounds and alloys of these elements). Some of the electrons in these materials have their resultant magnetic dipole moment aligned, which produces regions with strong magnetic dipole moments. An external field $\vec{B}_{ext}$ can then align the magnetic moments of such regions, producing a strong magnetic field for a sample of the material; the field partially persists when $\vec{B}_{ext}$ is removed. The term ferromagnetic material and even the common term magnetic material is used to refer to materials that exhibit primarily ferromagnetism.

Paramagnetic substances:

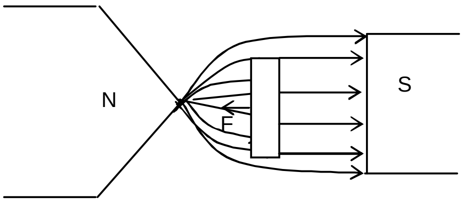
(i) The substance, when placed in magnetic field, acquires a very feeble magnetisation in the same sense as the applied field. Thus the magnetic induction inside the substance is slightly greater than outside.



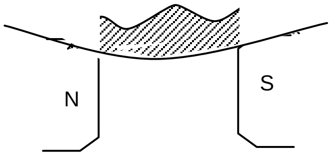
(ii) In a uniform magnetic field, these substances rotate until longest axis are parallel to the field.



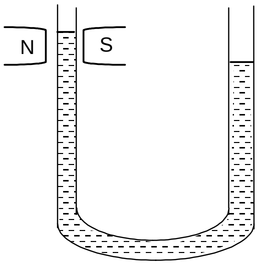
(iii) In non–uniform magnetic field, these substances are attracted towards stronger magnetic field.



If a paramagnetic liquid is placed in a watch glass resting on the poles of powerful electromagnets, the liquid is found to move so that the general depth at a points of greatest magnetic field.



(iv) If a paramagnetic material (liquid) is filled in a narrow U–tube and one limb is placed in between the pole pieces of an electromagnet such that the level of the liquid is in line with the field, then the liquid will rise as the field is switched on.

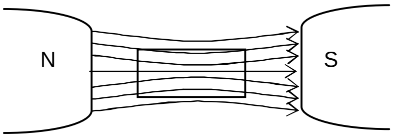


(v) The relative permeability k is slightly greater than one.

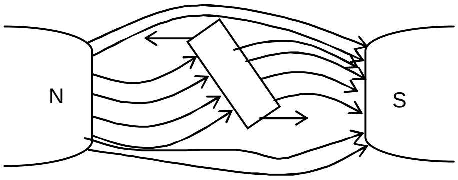
(vi) The susceptibility χ does not change with magnetising field at a particular temperature. But as temperature increases, χ decreases, i.e. it varies inversely as the absolute temperature.

Diamagnetic Substances:

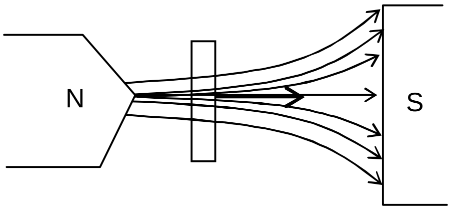
(i) These substances, when placed in a magnetic field, acquire feeble magnetisation in a direction opposite to applied field. Thus, the lines of induction inside the substance are smaller than outside it.



(ii) In a uniform magnetic field, these substances rotate until their longest axes are normal to the field.



(iii) In a non–uniform field, these substances move from stronger to weaker parts of the field.



If a diamagnetic liquid is placed in a watch glass resting on the pole of a powerful electromagnet, the liquid is found to accumulate on sides, where the field is weaker.

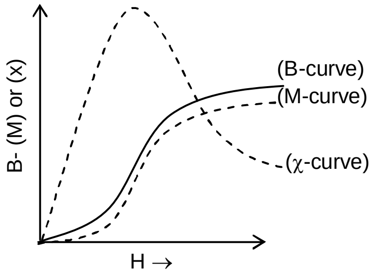
(iv) If a diamagnetic liquid is filled in a narrow U–tube and one–limb is placed between the pole pieces of a electromagnet, the level of the liquid depresses as and when the magnetic field is switched on.

(v) The relative permeability k is slightly less than 1.

(vi) The susceptibility χ of such substances is always negative. It is constant and does not vary with field or the temperature.

Ferromagnetic substances:
(i) These substances are strongly magnetised by even a weak magnetic field.
(ii) The relative permeability is very large and is of the order of thousands even.
(iii) The susceptibility is positive and very large.

(iv) The intensity of magnetization M is proportional to the magnetising field intensity H for its smaller values, increases rapidly for larger values and attains a constant value for large values of H.



(v) Permeability μ also varies as χ except at very high magnetic fields where μ decreases slowly in comparison to χ.

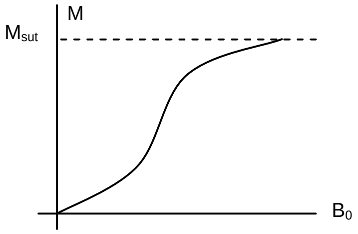
Hysteresis curves:

In the ferromagnetic materials, strong interactions between atomic magnetic moments cause them to line up parallel to each other in regions called magnetic domains, even when no external field is present.

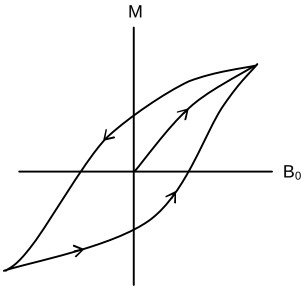
When there, is no externally applied field, the domain magnetization are randomly oriented. Application of $\vec{B}_0$, makes the domain to orient themselves parallel to the field.

As the external field is increased, a point is eventually reached out at which nearly all the magnetic moments in the materials aligns parallel to the external field. This condition is called the saturation magnetization.

For many ferromagnetic materials, the relation of magnetization to external magnetic field is different when the external field is increasing from when it is decreasing.



In the given example, when the material is magnetized to saturation and then the external field is reduced to zero some magnetization remains. This behaviour is characteristic of permanent magnets, which retain most of their saturation magnetization when the magnetic field is removed. To reduce magnetization to zero requires a magnetic field in reverse direction.



This behaviour is called hysteresis and the curves are called hysteresis loops. Magnetization and demagnetisation of a material involves dissipation of energy by heat loss.

Soft and hard magnetic materials:
The magnetic properties of a ferromagnetic substance can be obtained from the size and shape of the hysteresis loop. The study of curve for different materials gives the following information.

(i) **Susceptibility:** The susceptibility is the intensity of magnetisation per unit magnetisizing field (i.e. M/H) is greater for soft materials than for hard materials.

(ii) **Permeability:** The permeability, the magnetic induction per unit magnetizing field, (i.e. B/H) is greater for soft materials.

(iii) **Retentivity:** When a magnetic specimen is first magnetised and then the magnetising field is reduced to zero, the specimen retains intensity of the magnetisation (or the magnetic induction). This is known as retentivity. It is greater for soft material.

(iv) **Coercivity:** To demagnetise the magnetic specimen completely a –ve field is required. The value of reverse field H required to reduce the intensity of magnetisation to zero is known as the correcivity. It is less for soft materials.

Choice of magnetic materials:
The choice of a magnetic material for different uses can be decided from hysteresis curves of a specimen of the material.

(i)	Permanent magnets	Electromagnets	Choke coil, Armature coil and motors
Retentivity	High	–	–
Coactivity	High	–	–
Hysteresis loss	Immaterial	Low	Low
Permeability	–	High	High
Sp. resistance	–	–	High
Example	Steel, vicalloy	Iron	copper

Illustration 40. *A permanent magnet is made of a ferromagnetic material with a magnetization M of about 8×10^5 A/m. The magnet is in the shape of a cube of side 2 cm.*
(a) Find the magnetic dipole moment of the magnet.
(b) Estimate the magnetic field due to the magnet at a point 10 cm from the magnet along its axis.

Solution: The total magnetic moment is the magnetization multiplied by the volume.

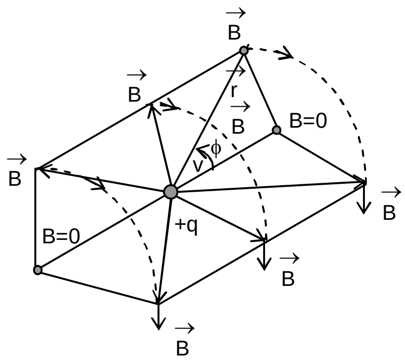
$M_M = (8 \times 10^5) \times (2 \times 10^{-2})^3 = 6\ \text{A.m}^2$

$$B = \frac{\mu_0 M_M}{2\pi(x^2 + a^2)^{3/2}} = \frac{4\pi \times 10^{-7} \times 6}{2\pi(0.1)^3} = 1 \times 10^{-3}\ \text{T} = 10\ \text{G}.$$

SUMMARY

The magnetic field $\vec{B}$ created by a charge q moving with velocity $\vec{v}$ depends on the distance r from the source point (the location of q) to the field point (where $\vec{B}$ is measured). the field $\vec{B}$ is perpendicular to $\vec{v}$ and to $\hat{r}$, the unit vector directed from the source point to the field point. The principle of superposition of magnetic fields states that the total field $\vec{B}$ produced by several moving charges is the vector sum of the fields produced by the individual charges.

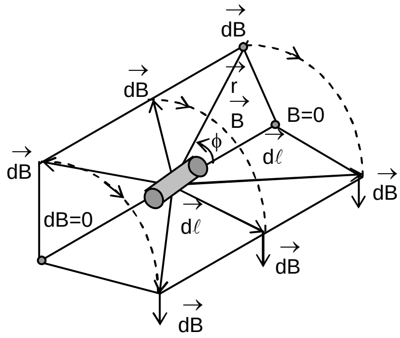
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$



The law of Biot and Savart gives the magnetic field $d\vec{B}$ created by an element $d\vec{\ell}$ of a conductor carrying current I.

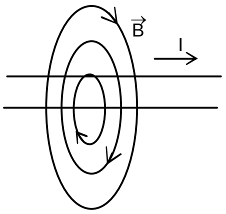
The field $d\vec{B}$ is perpendicular to both $d\vec{\ell}$ and $\hat{r}$, the unit vector from the element to the field point. The field $\vec{B}$ created by a finite current carrying conductor is the integral of $d\vec{B}$ over the length of the conductor.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{Iq\vec{\ell}\times\hat{r}}{r^2}$$



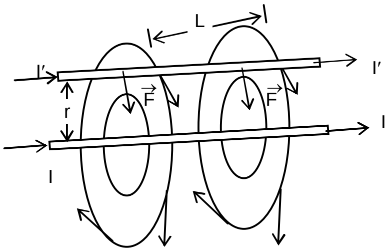
The magnetic field $\vec{B}$ at a distance r from a long, straight conductor carrying a current I has a magnitude that is inversely proportional to r. The magnetic field lines are circles coaxial with the wire, with directions given by the right hand rule.

$$B = \frac{\mu_0 I}{2\pi r}$$



Two long, parallel, current–carrying conductors attract if the currents are in the same direction and repel if the currents are in opposite directions. The magnetic force per unit length between the conductors depends on their currents I and I′ and their separation r. The definition of the ampere is based on this relation.

$$\frac{F}{L} = \frac{\mu_0 II'}{2\pi r}$$



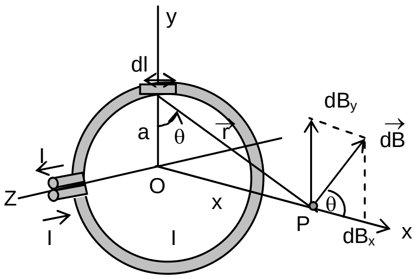
The law of Biot and Savart allows us to calculate the magnetic field produced along the axis of a circular conducting loop of radius a carrying current I. The field depends on the distance x along the axis from the centre of the loop to the field point. If there, are N loops, the field is multiplied by N. At the center of the loop, x = 0.

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

(circular loop)

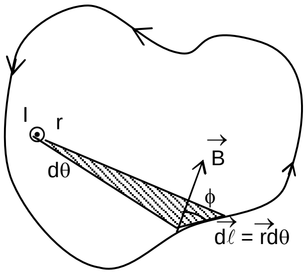
$$B_x = \frac{\mu_0 NI}{2a}$$

(centre of N circular loops)



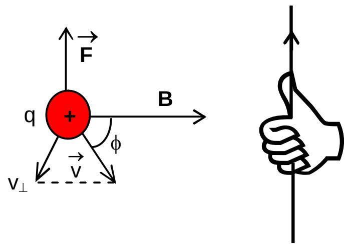
Ampere's law states that the line integral of $\vec{B}$ around any closed path equals μ_0 times the net current through the area enclosed by the path. The positive sense of current is determined by a right–hand rule.

$$\oint \vec{B}.d\vec{\ell} = \mu_0 I_{encl}$$

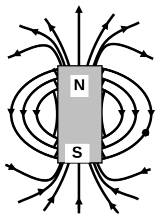


Magnetic interactions are fundamentally interactions between moving charged particles. These interactions are described by the vector magnetic field, denoted by $\vec{B}$. A particle with charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force $\vec{F}$ that is perpendicular to both $\vec{v}$ and $\vec{B}$. The SI unit of magnetic field is tesla (1T = 1 N/A. m)

$$\vec{F} = q\vec{v} \times \vec{B}$$

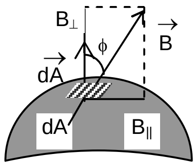


A magnetic field can be represented graphically by magnetic field lines. At each point a magnetic field line is tangent to the direction of $\vec{B}$ at that point. Where field lines are close together the field magnitude is large, and viceversa.



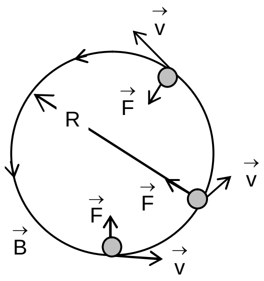
Magnetic flux ϕ_B through an area is defined in an analogous way to electric flux. The SI unit of magnetic flux is weber. The net magnetic flux through any closed surface is zero. As a result, magnetic field lines always close on themselves.

$$\phi_B = \oint B_\perp dA$$
$$= \int B\cos\phi dA$$
$$= \int \vec{B}.d\vec{A}$$
$$\int \vec{B}.d\vec{A} = 0 \text{ (closed surface)}$$

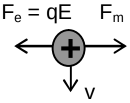


The magnetic force is always perpendicular to $\vec{v}$; a particle moving under the action of a magnetic field alone moves with constant speed. In a uniform field, a particle with initial velocity perpendicular to the field moves in a circle with radius R that depends on the magnetic field strength B, the particle mass m, speed v, and charge.

$$R = \frac{mv}{|q|B}$$



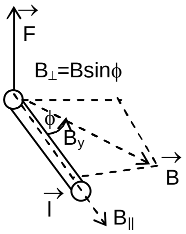
Crossed electric and magnetic fields can be used as velocity selector. The electric and magnetic forces exactly cancel when v = E/B.



A straight segment of a conductor carrying current I in a magnetic field B experiences a force F that is perpendicular to both B and the unit vector $\vec{\ell}$, which points in the direction of the current and has magnitude equal to the length of the segment. A similar relationship gives the force $d\vec{F}$ on an infinitesimal current–carrying segment $d\vec{\ell}$.

$$\vec{F} = I\vec{\ell} \times \vec{B}$$

$$d\vec{F} = Id\vec{\ell} \times \vec{B}$$



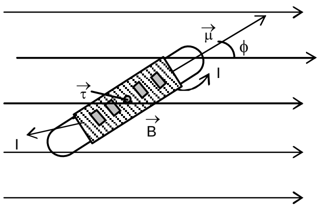
A current loop with area A and current I in a uniform magnetic field $\vec{B}$ experiences no net magnetic force, but does experience a magnetic torque of magnitude τ. The vector torque $\vec{\tau}$ can be expressed in terms of the magnetic moment $\vec{\mu} = I\vec{A}$ of the loop, as can be potential energy U of a magnetic moment in a magnetic field $\vec{B}$. The magnetic moment of a loop depends only on the current and the area; it is independent of the shape of the loop.

$$\tau = IBA \sin\phi$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu}.\vec{B}$$

$$= -\mu B \cos\phi$$



The following table lists magnetic fields caused by several current distributions. In each case the conductor is carrying current I.

Current Distribution	Point in Magnetic field	Magnetic Field Magnitude
Long, straight conductor	Distance r from conductor	$B = \frac{\mu_0 I}{2\pi r}$
Circular loop of radius a	On axis of loop At centre of loop	$B = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$ $B = \frac{\mu_0 I}{2a}$ (for N loops, multiply these expressions by N)

Long cylindrical conductor of radius R	Inside conductor, $r < R$ Outside conductor $r > R$	$B = \frac{\mu_0 Ir}{2\pi R^2}$ $B = \frac{\mu_0 I}{2\pi r}$
Long, closely wound solenoid with n turns per unit length, near its midpoint	Inside solenoid, near centre. Outside solenoid	$B = \mu_0 nI$ $B \approx 0$
Tightly wound toroidal solenoid (toroid) with N turns	Within the space enclosed by the windings, distance r from symmetry axis. Outside the space enclosed by the windings.	$B = \frac{\mu_0 NI}{2\pi r}$ $B = 0$

When magnetic material are present, the magnetization of the material causes an additional contribution to $\vec{B}$. For paramagnetic and diamagnetic materials, μ_0 is replaced in magnetic – field expressions by $\mu = K_m\mu_0$, where μ is the permeability of the material and K_m is its relative permeability. The magnetic susceptibility X_m is defined as $X_m = K_m - 1$. Magnetic susceptibilities for paramagnetic materials are small positive quantities; those for diamagnetic material are small negative quantities. For ferromagnetic materials, K_m is much larger than unity and is not constant. Some ferromagnetic materials are permanent magnets, retaining their magnetization even after the external magnetic field is removed.

MISCELLANEOUS EXERCISE

1. A charged particle moving in a perpendicular uniform magnetic field penetrates a layer of lead and thereby loses half of its kinetic energy. How will the radius of curvature of its path changes.

2. The energy of a charged particle moving in a uniform magnetic field does not change, why ?

3. State property of material of the wire used for the suspension of the coil in a moving cell galvanometer.

4. Can you accelerate a neutron by cyclotron ?

5. What is the radial magnetic field ?

6. Does the magnetic field exert a force on a static charge ?

7. What is the shape of magnetic field lines when current is straight or circular ?

8. Why does a solenoid tend to contract when a current passes through it ?

9. Which one has lowest resistance: ammeter, voltmeter and galvanometer ?

10. Name some magnetic and non–magnetic substances.

ANSWERS TO MISCELLANEOUS EXERCISE

1. $r \propto \sqrt{E_k}$, radius is reduced to $1/\sqrt{2}$ times

2. Force is perpendicular to the direction of motion.

3. It should have low torsional constant, high tensile strength, low temperature coefficient of resistance. It should be a non magnetic substance and a good conductor of electricity.

4. No

5. Where the plane of the coil always lies in the direction of magnetic field.

6. No

7. Circular, when current is straight;
   Straight, when current is circular.

8. Due to mutual attraction between the coil turns.

9. Ammeter

10. **Magnetic substance:** iron, cobalt, nickel, steel, etc.
    Non–magnetic substances: brass, paper, wood, etc.

SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob 1. *A charged particle is projected in a magnetic field $\vec{B}$ = (3i + 4j) $\times$ 10^{-2} T. The acceleration of the particle is found to be $\vec{a} = (-\frac{8}{3}\hat{i} + y\hat{j})$ m/s^2. Find the value of y.*

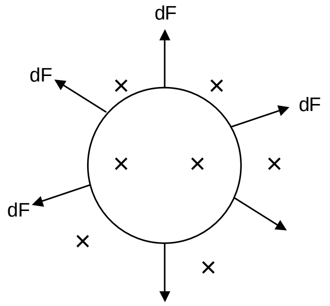
Sol. $F_m \perp B \rightarrow a \perp B$

$\vec{a}.\vec{B} = 0$

$(-8/3\hat{i} + y\hat{j}).(3\hat{i} + 4\hat{j}) = 0$

y = 2

Prob 2. *A circular loop of conducting wire of length 0.5 m lies in a magnetic field of 1.0 tesla perpendicular to the plane of the loop. Calculate the tension developed in the wire if the current flowing in the wire is $\pi/2$ ampere. Also find the direction of current.*



Sol. If T is the tension in the loop, for equilibrium of a small part of it

2T sin θ = dF = BidL

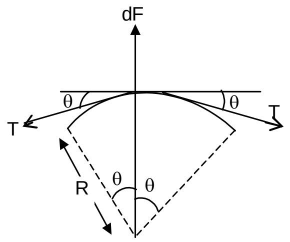
For an element, sin $\theta \approx \theta$ and d_2 = 2Rθ,

2Tθ = BI $\times$ 2Rθ

$\Rightarrow$ T = BIR [L = 2πR]

$T = \frac{BIL}{2\pi} = \frac{1 \times 1.57 \times 0.5}{2 \times 3.14} = 0.125N$

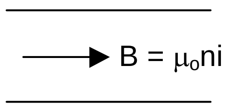
Current is clockwise.



Prob 3. *The magnetic field intensity inside a long solenoid is 1 T. If the current per turn of the winding is 1 A, Find the number of turns in one meter.*

Sol. $B = \mu_o ni$

$$n = \frac{B}{\mu_o i} = \frac{1}{4\pi \times 10^{-7} \times 1} = \frac{1}{4\pi} \cdot 10^7 \text{ turns / m}$$



At the ends, $B = \frac{\mu_0 ni}{2}$

Prob 4. *A particle of charge q and mass m is projected perpendicular to a magnetic field $\vec{B}$ and it is observed to rotate w.r.t. an axis whose direction is given by the* *vector* $\left(2\hat{i}+2\hat{j}-3\hat{k}\right)$*, with an angular speed of* $\sqrt{17}$ *rad/s. If the charge to mass ratio (q/m) of the particle is* $\sqrt{3}$ *C/kg, find the magnetic field* $\vec{B}$*.*

Sol. *If the particle is projected perpendicular to the* $\vec{B}$ *field. The angular velocity,* $\vec{\omega}$ *is*

$$\vec{\omega}=\frac{q\vec{B}}{m}$$

$$\sqrt{17}\times\left(\frac{2\hat{i}+2\hat{j}-3\hat{k}}{\sqrt{17}}\right)=\sqrt{3}\times\vec{B}$$

$$\therefore \vec{B}=\frac{2\hat{i}+2\hat{j}-3\hat{k}}{\sqrt{3}} \text{ tesla}$$

Prob 5. *A particle having mass m, charge q and velocity v passes through an electromagnetic field undeviated. If the electric field is* $E_0\hat{j}$*, what can we say about the magnetic field?*

Sol. Since the particle passes through the field undeviated, net force on it must be zero.

i.e. $q[E_0\ \hat{j}+v\hat{i}\times\vec{B}]=0$

At this point, one may be tempted to directly take the direction of $\vec{B}$ along z–axis to satisfy the equation. But that way we will not get a general solution to the Problem. For general solution one must proceed as follows:

$$q[E_0\hat{j}+v\hat{i}\times\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)]=0$$

$$\Rightarrow\quad E_0\hat{j}+vB_y\hat{k}-vB_z\hat{j}=0$$

$$\Rightarrow\quad \left(E_0-vB_z\right)\hat{j}+vB_y\hat{k}=0$$

$$\Rightarrow\quad B_z=\frac{E_0}{v},\ B_y=0$$

Here, with the given information we can only find y and z components of the magnetic field. The x component can have any value.

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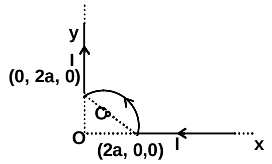
Prob 6. *Current I flows through the wire as shown in the figure. The semi–circular part of the wire is perpendicular to the x–y plane. Find the magnetic field at the centre C of the semi–circle.*



Sol . First, we determine the direction of the magnetic field due to the straight parts of the wire. Magnetic field due to the two straight parts is directed into the page, i.e. along negative z–axis. Magnetic field due to the semi–circular part is directed along CO. Now, magnitude of the magnetic field due to any one of the straight parts is

$$B_1=\frac{\mu_0\ I}{4\pi\ a}\left[\sin\ 90^o-\sin\ 45^o\right]=\frac{\mu_0\ I}{4\pi\ a}\left(1-1/\sqrt{2}\right)\qquad\text{(along –ve z–direction)}$$

Magnetic field due to the semi–circular part $B_Z=\dfrac{\mu_0\ I}{4\sqrt{2}\ a}$

(Radius of the semi circle is $\sqrt{2}\ a$)

Now, unit vector along CO = $-\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$

$\therefore$ The resultant magnetic field at C,

$$\vec{B} = 2\frac{\mu_0}{4\pi}\frac{I}{a}\left(\frac{1}{\sqrt{2}}-1\right)\hat{k} - \frac{\mu_0}{4}\frac{I}{a}\frac{(\hat{i}+\hat{j})}{2}$$

$$= -\frac{\mu_0}{4}\frac{I}{a}\left[\frac{\hat{i}+\hat{j}}{2} + \frac{(2-\sqrt{2})}{\pi}\hat{k}\right].$$

Prob 7. *A disc of radius R rotates at an angular velocity ω about the axis perpendicular to its surface and passing through its centre. If the disc has a uniform surface charge density σ, find the magnetic induction on the axis of rotation at a distance x from the centre.*

Sol. Consider a ring of radius r and width dr.

Charge on the ring, dq = $(2\pi r dr)\sigma$

Current due to ring is dI = $\frac{dq}{T} = \frac{\omega dq}{2\pi} = \sigma\omega\, r\, dr$

Magnetic field due to ring at point P is

$$dB = \frac{\mu_0 dI r^2}{2(r^2+x^2)^{3/2}}$$

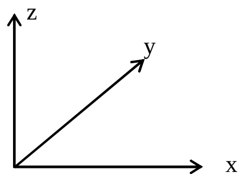
$$\text{or} \quad B = \int dB = \frac{\mu_0\sigma\omega}{2}\int_0^R \frac{r^3 dr}{(r^2+x^2)^{3/2}} \qquad \ldots \text{(i)}$$

Putting $r^2 + x^2 = t^2$ and 2r dr = 2tdt and integrating (i) we get

$$B = \frac{\mu_0\sigma\omega}{2}\left[\frac{R^2+2x^2}{\sqrt{R^2+x^2}} - 2x\right].$$

Prob 8. *A particle of mass 1×10^{-26} kg and charge $+1.6\times10^{-19}$ C travelling with a velocity 1.28×10^{6} ms^{-1} in the +x direction enters a region in which a uniform electric field E and a uniform magnetic field of induction B are present such that $E_x=E_y=0$, $E_z=-102.4$ kVm^{-1} and $B_x = B_z = 0$, $B_y = 8\times10^{-2}$ Wbm^{-2}. The particle enters this region at the origin at time t = 0. Determine the location (x, y and z coordinates) of the particle at $t = 5\times10^{-6}$ s. If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at $t = 7.45\times10^{-6}$ s ?*

Sol. Let $\hat{i}, \hat{j}$ and $\hat{k}$ be unit vectors along the positive directions of x, y and z axes.



Q = charge on the particle = 1.6×10^{-19} C,

$\vec{v}$ = velocity of the charged particle

= (1.28×10^{6}) ms^{-1} $\hat{i}$

$\vec{E}$ = electric field intensity;

= $(-102.4\times10^{3}\ Vm^{-1})\ \hat{k}$

$\vec{B}$ = magnetic induction of the magnetic field

= $(8\times10^{-2}\ Wbm^{-2})\ \hat{j}$

$\therefore$ $\vec{F}_e$ = electric force on the charge

$= q\vec{E} = \left[1.6\times10^{-19}(-102.4\times10^{3})N\right]\hat{k} = 163.84\times 10^{-16}\ N\ (-\hat{k})$

$\vec{F}_m$ = magnetic force on the charge $= q\vec{v}\times\vec{B}$

$= [1.6\times10^{-19}\ (1.28\times10^{6})(8\times10^{-2})N](\hat{i}\times\hat{j}) = (163.84\times 10^{-16}\ N)\ (\hat{k})$

The two forces $\vec{F}_e$ and $\vec{F}_m$ are along z–axis and equal, opposite and collinear.

The net force on the charge is zero, hence the particle does not get deflected and continues to travel along x–axis.

(a) At time $t = 5\times10^{-6}$ s

$x = (5\times10^{-6})(1.28\times10^{6}) = 6.4$ m

$\therefore$ Coordinates of the particle = (6.4 m, 0, 0)

(b) When the electric field is switched off, the particle is in the uniform magnetic field perpendicular to its velocity only and has a uniform circular motion in the x–z plane (i.e. the plane of velocity and magnetic force), anticlockwise as seen along +y axis.

Now, $\frac{mv^2}{r}$ =qvB, where r is the radius of the circle.

$\therefore\ r = \frac{mv}{qB} = \frac{(1\times10^{-26})(1.28\times10^{6})}{(1.6\times10^{-19})(8\times10^{-2})} = 1$

The length of the arc traced by the particle in [$(7.45-5)\times10^{-6}$ s]

$= (v)\ (t) = (1.28\times10^{6})(2.45\times10^{-6})$

$= 3.136\ m = \pi\ m = \frac{1}{2}$ circumference

$\therefore$ The particle has the coordinates (6.4m, 0, 2m) as (x, y, z).

Prob 9. *A proton of velocity 1.0×10^{7} m/s is projected at right angles to a uniform magnetic field of induction 100 wb/m^2.*

(a) How much is the particle path deflected from a straight line after it has traversed a distance of 1 cm in the direction of initial velocity.

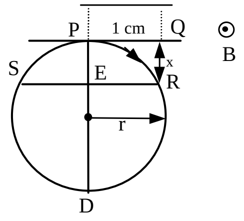
(b) How long does it take for the proton to traverse a 90^o arc?

($m_p = 1.65\times10^{-27}$ kg, $q = 1.6\times10^{-19}$ C).

Sol. (a) It is clear that the proton would describe a circular path under the given conditions.

The radius of the path is given by

$$r = \frac{mv}{qB} = \frac{(1.65\times10^{-27}kg)\ (1.0\times10^{7}\ m/s)}{(1.6\times10^{-19}C)\ (100\ T)} = \frac{1.65}{1.6}m$$



If QR = x be the deflection of the particle after traversing a distance 1 cm, then by the properties of the circle,

(SE). (RE) = (PE). ED

or $(0.01\ m)^2 = x(2r-x) = 2rx - x^2$.

Neglecting x^2 which is small compared to 2rx.

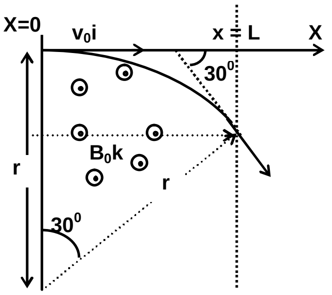
$$X = \frac{1\times10^{-4}}{2r} = \frac{1\times10^{-4}}{2\times1.6576} = 4.848\times10^{-5}\ m.$$

(b) Let 't' be the time taken by the particle to traverse a 90° arc. Then

$$t = \frac{\pi}{2}.\frac{m}{qB} = 1.619\times10^{-10}s.$$

Prob 10. *The region between x = 0 and x = L is filled with uniform, steady magnetic field $B_0\hat{k}$. A particle of mass m, positive charge q and velocity $v_0\,\hat{k}$ travels along x–axis and enters the region of magnetic field. Neglect gravity throughout the question.*
(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30° to the initial velocity.
(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the field now extents up to x = 2.1L.

Sol . (a) As the initial velocity of the particle is perpendicular to the field the particle will move along the arc of a circle as shown.



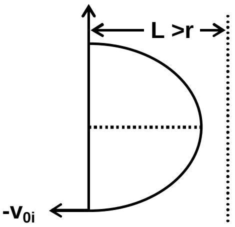
If r is the radius of the circle, then

$$\frac{m\ v_0^2}{r} = q\ v_0 B_0$$

Also from geometry, L = r sin 30°

$\Rightarrow$ r = 2L or $L = \dfrac{m\ v_0}{2q\ B_0}$

(b) In this case $L = \dfrac{2.1\ mv_0}{2q\ B_0} > r$

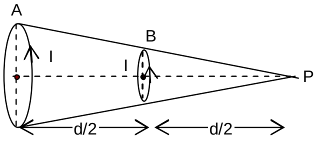


Hence the particle will complete semi–circular path and emerge from the field with velocity ($-v_0\,\hat{i}$) as shown.
Time spent by the particle in the magnetic field

$$T = \frac{\pi r}{v_0} = \frac{\pi m}{q\ B_0}$$

The speed of the particle does not change due to magnetic field.

Prob 11. *Two circular coils A and B subtend same solid angle at point P lying on the axis of the coils as shown in figure. Smaller coil B is midway between A and P. Both of the coils carry equal current in the same direction. Find the ratio of magnetic induction at point P due to coils A and B.*



Sol . Magnetic induction at point A,

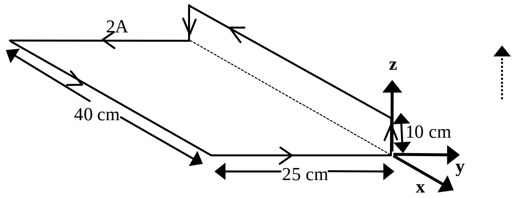
$$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi I(2r)^2}{[2r^2 + d^2]^{3/2}}$$

and due to coil B,

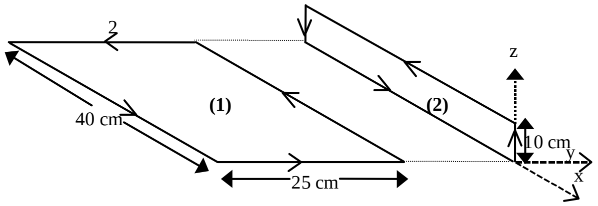
$$B_2 = \frac{\mu_0}{4\pi}.\frac{2\pi I r^2}{[r^2 + (d/2)^2]^{3/2}} \quad [\,r = \frac{d}{2}\tan\theta \text{ where } 2\theta = \text{solid angle}]$$

$$\therefore\ \frac{B_1}{B_2} = \frac{1}{2}.$$

Prob 12. *Find the magnitude and direction of magnetic dipole moment of the loop shown in figure and the torque acting on it, if there, exists a uniform magnetic field given by $\vec{B} = (0.2i + 0.5j + 3k)T$.*



Sol. The given loop can be considered as a combination of the two loops as shown in figure.



Total vector dipole moment

$$\vec{P} = \vec{P}_1 + \vec{P}_2 = iA_1\hat{k} - iA_2\hat{j}$$

Where $A_1 = 0.25 \times 0.4 = 0.1\ m^2$, and

$A_2 = 0.40 \times 0.10 = 0.04\ m^2$

$$\vec{\tau} = \vec{P} \times \vec{B} = -(-0.2\hat{k} + 0.08\hat{j}) \times (0.2\hat{i} + 0.5\hat{j} + 3\hat{k})$$

$$= -(0.34\hat{i} - 0.04\hat{j} - 0.016\hat{k})Nm$$

Prob 13. *A particle with charge q is projected successively along the x and y axes with same speed v. The force on the particle in these situations are $qvB(-3\hat{j} + 4\hat{k})$ and $qvB\ (3\hat{i})$ respectively. Find the unit vector in direction of $\vec{B}$.*

Sol . Let $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

Hence $q\left[v\hat{i} \times (B_1\hat{i} + B_2\hat{j} + B_3\hat{k})\right] = qvB(-3\hat{j} + 4\hat{k})$

$\Rightarrow\ B_2\hat{k} - B_3\hat{j} = -3B\hat{j} + 4B\hat{k}$

$\Rightarrow\ B_2 = 4B,\ \ B_3 = 3B$

Also $q\left[v\hat{j} \times (B_1\hat{i} + B_2\hat{j} + B_3\hat{k})\right] = qvB(3\hat{i})$

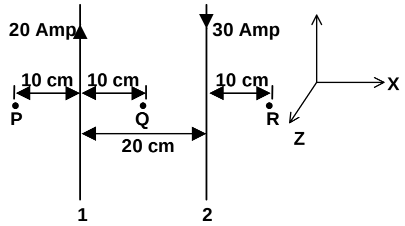
$\Rightarrow\ -B_2\hat{k} + B_3\hat{i} = 3B\hat{i}$

$\Rightarrow\ B_1 = 0$

Hence $\vec{B} = 4B\hat{j} + 3B\hat{k}$

$$\Rightarrow\ \hat{B} = \frac{\vec{B}}{|\vec{B}|} = \frac{4}{5}\hat{j} + \frac{3}{5}\hat{k}$$

Prob 14. *In the adjoining figure there, are two current carrying wires 1 and 2. Find the magnitudes and directions of the magnetic field B at the points P, Q and R.*



Sol. At P due to current in (1), magnetic field is in upward direction and due to current in (2), magnetic field is in downward direction.

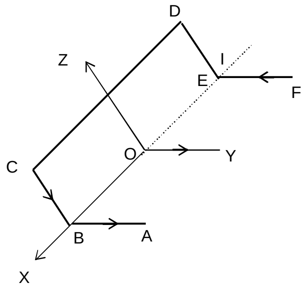
At Q due to current in (1) magnetic field is downward and due to current in (2) also, magnetic field is in downward direction.

Therefore, at P, $B_1 - B_2 = \frac{\mu_0}{2\pi}\frac{20}{0.1} - \frac{\mu_0}{2\pi}\frac{30}{0.3}$ = 2×10^{-5} N/Amp-meter, along positive z–axis

At Q, $B_1 + B_2 = \frac{\mu_0}{2\pi}\frac{20}{0.1} + \frac{\mu_0}{2\pi}\frac{30}{0.3}$ = 1×10^{-4} N/Amp-meter, along negative z–axis

At R, $B_2 - B_1 = \frac{\mu_0}{2\pi}\frac{30}{0.1} - \frac{\mu_0}{2\pi}\frac{20}{0.3}$ = 4.7×10^{-5} N/Amp-meter, along positive z–axis

Prob 15. *A wire ABCDEF (with each side of length L) bent as shown and carrying a current I is placed in a uniform magnetic induction $\vec{B}$ parallel to the +ve y–direction. Find the magnitude and direction of the force experienced by the wire.*



Sol. $\vec{F} = I(\vec{L} \times \vec{B})$

The angle between F and $\vec{B}$ is 180°

The angle between BA and $\vec{B}$ is 0°

For both these $\vec{L} \times \vec{B} = 0$

For ED, $\vec{F}_{ED} = I(\vec{L} \times \vec{B}) = ILB \sin 90^0\, \hat{i} = ILB\hat{i}$

Similarly, $\vec{F}_{CB} = I(\vec{L} \times \vec{B})$ = ILB sin 270° = – ILB $\hat{i}$

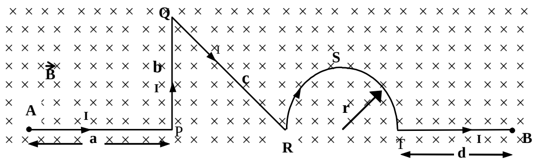
$\therefore\ \vec{F}_{ED} + \vec{F}_{CB} = 0$

$\vec{F}_{DC} = I(\vec{L} \times \vec{B})$ = $I[L\,\hat{i} \times B\hat{j}]$ = $ILB\hat{k}$

$\therefore\ \vec{F}_{AB} + \vec{F}_{BC} + \vec{F}_{CD} + \vec{F}_{DE} + \vec{F}_{EF} = ILB\hat{k}$

$\therefore$ The magnitude of the two required forces is ILB and it is directed along +ve z–axis.

Prob 16. *Calculate the force on a current carrying conductor in a uniform magnetic field as shown.*



Sol. The net force from A to B,

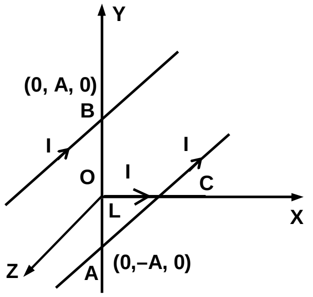
$$dF = I(dL \times B)$$
$$\int_A^B d\vec{F} = \int_A^P I\left[d\vec{L}_1 \times \vec{B}\right] + \int_P^Q I\left[d\vec{L}_2 \times \vec{B}\right] + \int_Q^R I\left[d\vec{L}_3 \times \vec{B}\right] + \int_R^T I\left[d\vec{L}_4 \times \vec{B}\right] + \int_T^B \left(d\vec{L}_5 \times \vec{B}\right)$$

The entire path can be broken down into elemental vectors joined to each other in sequence. We know, from polygon law of addition of vectors that vector joining the tail of the first vector to the head of the last vector is the resultant vector.

$\vec{F} = I\left(\vec{L} \times \vec{B}\right)$; where $\left|\vec{L}\right|$ = a + $\sqrt{c^2 - b^2}$ + 2r + d

$\Rightarrow$ F_{net} = I.B ($a + \sqrt{c^2 - b^2} + 2r + d$) and direction is upwards in the plane of paper.

Prob 17. *A straight segment OC (of length L meter) of a circuit carrying a current I amp is placed along the x – axis. Two infinitely long straight wires A and B, each extending from z = –∞ to +∞ are fixed at y = – a metre and y = + a metre, respectively, as shown in the figure. If the wires A and B each carry a current I amp into the plane of the paper, obtain the expression for the force acting on segment OC. What will be the force on OC if the current in the wire B is reversed?*

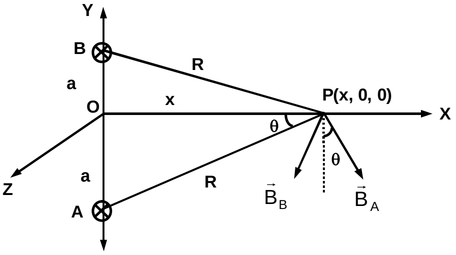


Sol. Magnetic field B_A produced at P(x, 0, 0) due to wire,

$B_A = \mu_0 I/2\pi R$,

$B_B = \mu_0 I/2\pi R$.

Components of B_A and B_B along x – axis cancel, while those along y – axis add up to give total field.



$$B = 2\left(\frac{\mu_0 I}{2\pi R}\right)\cos\theta = \frac{2\mu_0 I}{2\pi R}.\frac{x}{R} = \frac{\mu_0 I}{\pi}\frac{x}{(a^2 + x^2)}$$ (along – y direction)

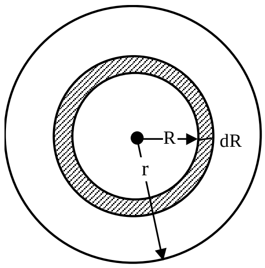
The force dF acting on the current element is $d\vec{F} = I\left(d\vec{\ell} \times \vec{B}\right)$

$\therefore$ dF = $\dfrac{\mu_0 I^2}{\pi}\dfrac{x dx}{a^2 + x^2}\left[\because \quad \sin 90^0 = 1\right]$

$\Rightarrow$ F = $\dfrac{\mu_0 I^2}{\pi}\displaystyle\int_0^L \dfrac{x dx}{a^2 + x^2}$ = $\dfrac{\mu_0 I^2}{2\pi}\ln\dfrac{a^2 + L^2}{a^2}$

If the current in B is reversed, the magnetic field due to the two wires would be only along x – direction and the force on the current along x – direction will be zero.

Prob 18. *A uniformly charged disc whose total charge has magnitude q and whose radius is r rotates with constant angular velocity of magnitude ω. What is the magnetic dipole moment?*



Sol. The surface charge density is $q/\pi r^2$. Hence, the charge within a ring of radius R and width dR is

$$dq = \frac{q}{\pi r^2}(2\pi R dR) = \frac{2q}{r^2}(R dR)$$

The current carried by this ring is its charge divided by the rotation period,

$$di = \frac{dq}{2\pi/\omega} = \frac{q\omega}{\pi r^2}[R.dR]$$

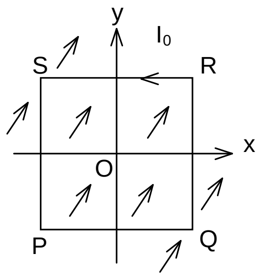
The magnetic moment contributed by this ring has the magnitude

dM = a |di|, where a is the area of the ring.

$$dM = \pi R^2\,|di| = \pi R^2 \times \frac{q\omega}{\pi r^2} R dR = \frac{q\omega}{r^2} R^3 dR$$

$$M = \int dM = \int_{r=0}^{r} q.\frac{\omega}{r^2}(R^3 dR) = q\omega\frac{r^2}{4}.$$

Prob 19. *A uniform, constant magnetic field $\vec{B}$ is directed at an angle of 45° to the x–axis in the xy–plane. PQRS is a rigid, square wire frame carrying a steady current I_0, with its centre at the origin O. At time t = 0, the frame is at rest in the position shown in the figure, with its sides parallel to the x and y axes. Each side of the frame is of mass M and length L.*



(a) What is the torque $\vec{\tau}$ about O acting on the frame due to the magnetic field?

(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt, and the axis about which this rotation occurs. (Δt is so short that any variation in the torque during this interval may be neglected). Given moment of inertia of the frame about an axis through its centre perpendicular to its plane is (4/3) ML^2.

Sol . (a) As magnetic field $\vec{B}$ is in x–y plane and subtends an angle of 45° with x–axis

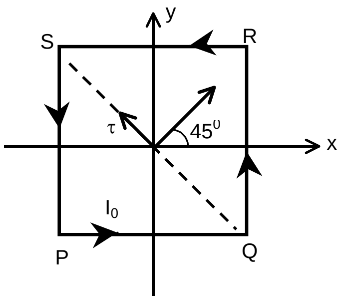
$B_x = B\cos 45° = B/\sqrt{2}$

And $B_{y=} = B\sin 45° = B/\sqrt{2}$

So in vector form

$\vec{B} = \hat{i}\left(B/\sqrt{2}\right) + \hat{j}\left(B/\sqrt{2}\right)$ and $\vec{M}$

$= I_0 S\hat{k} = I_0 L^2\hat{k}$,



where S = area of the loop

So, $\vec{\tau} = \vec{M} \times \vec{B} = I_0 L^2 \hat{k} \times \left(\frac{B}{\sqrt{2}}\hat{i} + \frac{B}{\sqrt{2}}\hat{j}\right)$

i.e. $\vec{\tau} = \frac{I_0 L^2 B}{\sqrt{2}} \times (-\hat{i} + \hat{j})$

i.e., torque has magnitude $I_0 L^2$ B and is directed along line QS from Q to S.

(b) As by theorem of perpendicular axes, moment of inertia of the frame about QS,

$I_{QS} = \frac{1}{2}\ I_z = \frac{1}{2}\left(\frac{4}{3}\ ML^2\right) = \frac{2}{3}\ ML^2$ and as $\tau = I\,\alpha$,

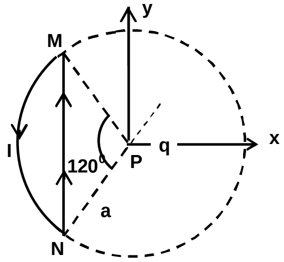
$\alpha = \frac{\tau}{I} = \frac{I_0L^2B\times 3}{2L^2M} = \frac{3}{2}\frac{I_oB}{M}$

As here, α is constant, equations of circular motion are valid and hence from

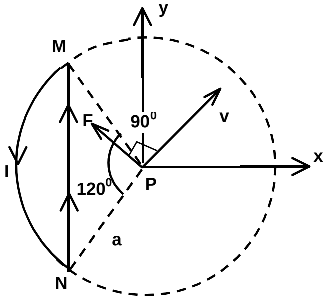
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ with $\omega_0 = 0$ we have

$\theta = \frac{1}{2}\alpha\ t^2 = \frac{1}{2}\left(\frac{3}{2}\frac{I_0B}{M}\right)(\Delta t)^2 = \frac{3}{4}\frac{I_0B}{M}\Delta t^2$.

Prob 20. *A wire loop carrying a current I is placed in the x–y plane as shown in the figure.*
(a) If a particle with charge q and mass m is placed at the centre P and given a velocity v along NP, find its instantaneous acceleration.
(b) If an external uniform magnetic induction $\vec{B} = B\hat{i}$ *is applied, find the force and torque acting on the loop.*



Sol . (a) As in case of current–carrying straight conductor and arc, the magnitude of B is given by



$B_\omega = B_1 = \frac{\mu_0}{4\pi}\frac{I}{d}\ (\sin\ \alpha\ +\ \sin\beta)$

and $B_{MN} = B_2 = \frac{\mu_0}{4}\frac{I\ \phi}{\pi\ r}$

So, in accordance with right hand screw rule,

$\left(\vec{B}_W\right) = \frac{\mu_0}{4\pi}\frac{I}{(a\ \ \cos\ \ 60)} x\, 2\sin 60(-\hat{k})$

The direction of magnetic field due to the arc MIN is along the positive z–direction.

and $\left(\vec{B}\right)_{MN} = \frac{\mu_0}{4\pi}\ \frac{I}{a}\ \ x\ \left(\frac{2}{3}\pi\right)\ (-\hat{k})$

and hence net $\vec{B}$ at P due to the given loop

$\vec{B} = \vec{B}_w + \vec{B}_A \qquad \Rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\sqrt{3} - \frac{\pi}{3}\right]\left(-\hat{k}\right) \qquad$ (i)

Now, as force on charged particle in a magnetic field is given by

$\vec{F} = q\left(\vec{v}x\vec{B}\right)$

So here, $\vec{F} = qvB\sin 90^0$ alongPF

i.e. $\vec{F} = \frac{\mu_0}{4\pi}\ \frac{2\ \ qvI}{a}\ \left[\sqrt{3}\ -\ \frac{\pi}{3}\right]$ along PF

and so $\vec{a} = \frac{\vec{F}}{m} = 10^{-7}\ \frac{2\ \ qvI}{a}\left[\sqrt{3}\ -\ \frac{\pi}{3}\right]$ along PF

(b) As $d\vec{F} = Id\vec{L}\times\vec{B}$, so $\vec{F} = \int Id\vec{L}\times\vec{B}$

As here, I and $\vec{B}$ are constant

$$\vec{F} = I\left[\oint d\vec{L}\right] \times \vec{B} = 0 \qquad \left[\text{as } \oint d\vec{L} = 0\right]$$

Further as area of coil,

$$\vec{S} = \left[\frac{1}{3}\pi a^2 - \frac{1}{2}.2a\sin 60^0 \, x a \cos 60^0\right]\hat{k} = a^2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{k}$$

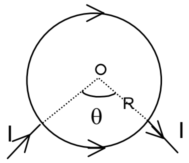
So, $I\vec{S} = Ia^2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{k}$

and hence $\vec{\tau} = \vec{M} \times \vec{B} = Ia^2B\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\left(\hat{k} \times \hat{i}\right)$

i.e. $\vec{M}\ \vec{\tau} = Ia^2B\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{j} N-m \qquad \text{as}\left(\hat{k} x \hat{i} = \hat{j}\right)$

Objective:

Prob 1. *Magnetic field intensity B at the centre of the circular loop is*



(A) zero *(B)* $\frac{\mu_0 I(2\pi-\theta)}{4\pi R}$

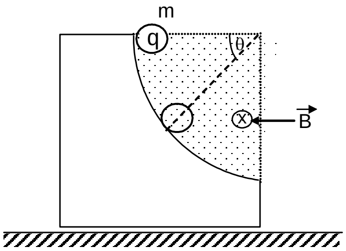
(C) $\frac{\mu_0 I\theta}{4\pi R}$ *(D)* $\frac{\mu_0 I^2(\pi-\theta)}{4\pi R}$

Sol. Since current in each segment is proportional to their resistance, B due to arcs are in opposite direction. Hence

$$B_{net} = \frac{\mu_o}{4\pi R}\left[\frac{i(2\pi-\theta)\ \theta}{2\pi} - \frac{i\theta(2\pi-\theta)}{2\pi}\right],$$

Hence **(A)** is correct.

Prob 2. *In the figure a charged sphere, of mass m and charge q starts sliding from rest on a vertical fixed circular track of radius R from the position shown. There, exists a uniform and constant horizontal magnetic field of induction B. The maximum force exerted by the track on the sphere, is*



(A) mg *(B) 3mg – qB*$\sqrt{2gR}$

(C) 3mg + qB$\sqrt{2gR}$ *(D) mg – qB*$\sqrt{2gR}$

Sol. F_m = qvB, and directed radially inward.

As N – mg sin θ + qvB = $\frac{mv^2}{R}$ $\Rightarrow N = \frac{mv^2}{R} + mg\sin\theta - qvB$

Hence, at θ = π/2

$\Rightarrow \ N_{max} = \frac{2mgR}{R} + mg - qB\sqrt{2gR}$

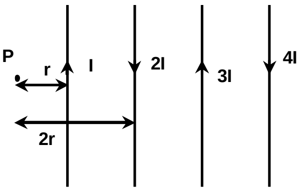
$= 3mg - qB\sqrt{2gR}$.

Hence **(B)** is correct.

Prob 3. *Twenty-seven infinitely long parallel wires are placed at one side of a point P at a perpendicular distances of r, 2r, 3r, ..., 27 r from P and carry current I, 2I, 3I, ... 27I, respectively. Calculate the magnetic field due to all these the wires at the point P if adjacent wires carry currents in opposite directions.*

(A) $\frac{27\mu_0 I}{2\pi r}$ *(B)* $\frac{\mu_0 I}{2\pi r}$

(C) $\frac{9\mu_0 I}{\pi r}$ *(D) zero.*



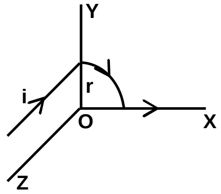
Sol. Magnetic field at P

$$B = \frac{\mu_0 I}{2\pi}\left[\frac{1}{r} - \frac{2}{2r} + \frac{3}{3r} - \frac{4}{4r}\ldots,\right]$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Hence **(B)** is correct.

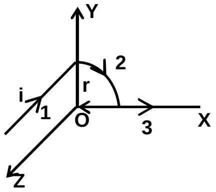
Prob 4. *Shown in the figure is a conductor carrying a current i. The magnitude of magnetic field at the origin is.*



(A) $\frac{\mu_o i}{4r}\left(\frac{1}{\pi}\hat{i} + \frac{1}{2}\hat{j}\right)$ *(B)* $\frac{\mu_o i}{4r}\left(\frac{1}{\pi}\hat{i} - \frac{1}{2}\hat{j}\right)$

(C) $\frac{\mu_o i}{4r}\left(\frac{1}{2}\hat{i} - \frac{1}{\pi}\hat{j}\right)$ *(D)* $\frac{\mu_o i}{4r}\left(\frac{2}{\pi}\hat{i} + \hat{j}\right)$

Sol . $\vec{B}_o = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$



Where $\vec{B}_1 = -\frac{\mu_0 i}{4\pi r}\hat{i}$ (Semi–infinite straight wire)

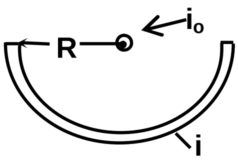
$B_2 = -\frac{1}{4}\left(\frac{\mu_o i}{2r}\right)\hat{j}$ (One-quarter of a circular loop)

and $B_3 = 0$ (since the line of the wire 3 passes through the origin)

$\Rightarrow B_o = -\frac{\mu_o i}{4r}\left(\frac{1}{\pi}\hat{i} + \frac{1}{2}\hat{j}\right)$.

Hence **(A)** is correct.

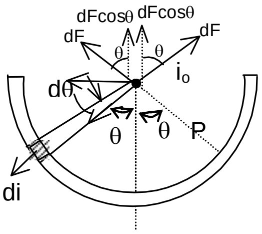
Prob 5. *Shown in the figure is a very long semicylindrical conducting shell of radius R and carrying a current i. An infinitely long straight current carrying conductor is lying along the axis of the semicylinder. If the current flowing through the straight wire be i_o, then the force on the semicylinder is .*



(A) $\frac{\mu_0 i i_o}{\pi R^2}$ *(B)* $\frac{\mu_0 i_o i}{\pi^2 R}$

(C) $\frac{\mu_0 i_o^2 i}{\pi^2 R}$ *(D) None of these*

Sol . The net magnetic force on the conducting wire



$$= F = \int 2dF\cos\theta \Rightarrow F = \int 2\left[\frac{\mu_o (di) i_o}{2\pi R}\right]\cos\theta$$

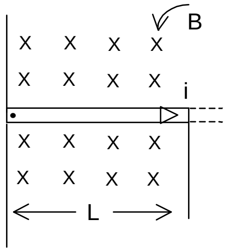
$$\Rightarrow F = \frac{\mu_o i_o}{\pi R}\int di\cos\theta$$

$$\text{when } di = \frac{i}{\pi R} \times R d\theta = \frac{i d\theta}{\pi}$$

$\Rightarrow\ F = \frac{\mu_o i_o}{\pi R}\int\frac{(id\theta)\cos\theta}{\pi} \Rightarrow\ F = \frac{\mu_o i_o i}{\pi^2 R}\int_0^{\pi/2}\cos\theta d\theta = \frac{\mu_o i_o i}{\pi^2 R}$.

Hence **(B)** is correct.

Prob 6. *A straight conductor of mass m and carrying a current i is hinged at one end and placed in a plane perpendicular to the magnetic field of intensity $\vec{B}$ as shown in the figure. At any moment if the conductor is let free, then the angular acceleration of the conductor will be (Assume gravity free region).*



(A) $\frac{2iB}{3m}$ *(B)* $\frac{3iB}{2m}$

(C) $\frac{iB}{2m}$ *(D)* $\frac{3i}{3mB}$

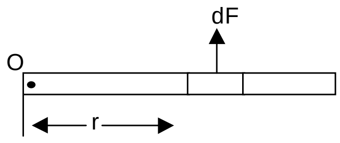
Sol. The force acting on the elementary portion of the current carrying conductor is given as,

$dF = i(d\ell)B \sin 90°$

$\Rightarrow dF = iBdr$

The torque applied by dF about O = $d\tau = rdF$

$\Rightarrow$ The total torque about O,



$\tau = \int d\tau = \int r(iBdr)$

$\Rightarrow\ \tau = iB\int_0^L rdr = \frac{iBL^2}{2}$

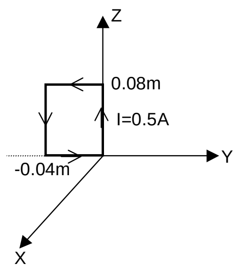
The angular acceleration $\alpha = \frac{\tau}{M.I.}$ (where M.I. = moment of inertia)

$\Rightarrow\ \alpha = \left(\frac{iBL^2}{2}\right)\Big/\left(\frac{mL^2}{3}\right)$

$\Rightarrow \alpha = \frac{3iB}{2m}$.

Hence **(B)** is correct.

Prob 7. *The rectangular coil having 100 turns is turned in a uniform magnetic field of* $\frac{0.05}{\sqrt{2}}\ \hat{j}$ *Tesla as shown in the figure. The torque acting on the loop is*



(A) 11.32×10^{-4} *(N–m)* $\hat{k}$ *(B)* 22.64×10^{-4}*(N–m)* $\hat{k}$

(C) 5.66×10^{-5} *(N–m)* $\hat{k}$ *(D) Zero*

Sol. The magnetic dipole moment of the current carrying coil is given by $\vec{M} = NIA\hat{n}$

$= 100 \times 0.5 \times (0.08) \times 0.04\ \hat{i} = 16 \times 10^{-2}\ Am^2\ (\hat{i})$

The torque acting on the coil is

$\vec{\tau} = \vec{m}\times\vec{B} = mB\ \ (\hat{i}\times\hat{j})$

$$= 16 \times 10^{-2} \times \frac{0.05}{\sqrt{2}}\ \hat{k}$$

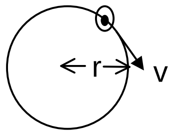
$= 5.66 \times 10^{-5}\,(N - m)\,\hat{k}$.

Hence **(C)** is correct.

Prob 8. *The magnetic moment of an electron orbiting in a circular orbit of radius r with a speed v is equal to*

(A) evr/2 *(B) evr*
(C) er/2v *(D) none of these*

Sol. Magnetic moment $\mu = niA$



where n = number of turns of the current loop,
i = current

Since the orbiting electron behaves as a current loop of current i, we can write

$$i = \frac{e}{T} = \frac{e}{\frac{2\pi r}{v}} = \frac{ev}{2\pi r}$$

A = area of the loop = πr^2

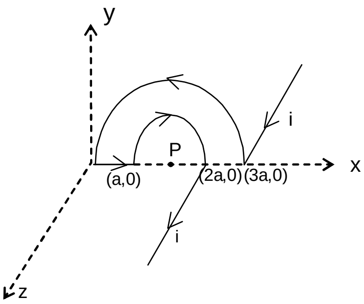
$$\Rightarrow \quad \mu = (1)\left(\frac{ev}{2\pi r}\right)\left(\pi r^2\right) = \frac{evr}{2}.$$

Hence **(A)** is correct.

Prob 9. *In the figure shown the magnetic field at the point P is*

(A) $\frac{2\mu_0 i}{3\pi a}\sqrt{4-\pi^2}$ *(B)* $\frac{\mu_0 i}{3\pi a}\sqrt{4+\pi^2}$

(C) $\frac{2\mu_0 i}{3\pi a}(4+\pi^2)$ *(D)* $\frac{2\mu_0 i}{3\pi a}(4-\pi^2)$



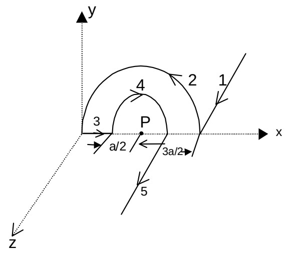
Sol.

$$\vec{B}_p = \left(\vec{B}_1\right)_p + \left(\vec{B}_2\right)_p + \left(\vec{B}_3\right)_p + \left(\vec{B}_4\right)_p + \left(\vec{B}_5\right)_p$$

where $\left(\vec{B}_1\right)_p = \frac{\mu_o i}{4\pi\left(\frac{3a}{2}\right)}\ (-\hat{j})$ (semi–infinite wire)

$$\left(\vec{B}_2\right)_p = \frac{\mu_o i}{4\left(\frac{3a}{2}\right)}\ (+\hat{k}), \qquad \left(\vec{B}_3\right)_p = 0$$

$$\left(\vec{B}_4\right)_p = \frac{\mu_o i}{4\left(\frac{a}{2}\right)}\ (-\hat{k}), \qquad \left(\vec{B}_5\right)_p = \frac{\mu_o i}{4\ \pi\left(\frac{a}{2}\right)}\ (-\hat{j})$$



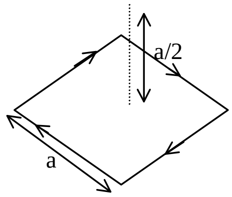
$$\Rightarrow \vec{B}_p = \frac{\mu_o i}{a}\left[-\left(\frac{1}{3\pi} + \frac{1}{\pi}\right)\hat{j} - \left(1 - \frac{1}{3}\right)\hat{k}\right]$$

$\Rightarrow \ \vec{B}_p = \frac{2\mu_o i}{3a}\left[\frac{2}{\pi}j + \hat{k}\right]$

$\Rightarrow \ \vec{B}_p = \frac{\mu_o i}{3\pi \ a}\ \sqrt{4 + \pi^2}$.

Hence **(B)** is correct.

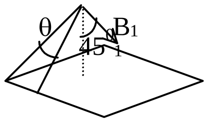
Prob 10. *The magnetic field due to a current carrying square loop of side 'a' at a point located symmetrically at a distance of a/2 from its centre (as shown) is*



(A) $\frac{\sqrt{2}\mu_0 i}{\sqrt{3}\pi a}$ *(B)* $\frac{2\mu_0 i}{\sqrt{3}\pi a}$

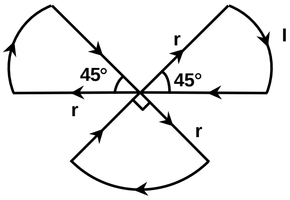
(C) $\frac{2\sqrt{2}\mu_0 i}{\sqrt{3}\pi a}$ *(D) Zero*

Sol. Magnetic field due to one side $B_1 = \frac{\mu_0 i}{4\pi(a/\sqrt{2})}$ [2 sin θ]



Here, sin θ = $1/\sqrt{3}$
Resultant B_1 = 4 B cos45°
Hence **(B)** is correct.

Prob 11. *Calculate the magnetic field at point P due to three arcs as shown in the figure.*



(A) $\frac{\mu_0 I}{2r}$ *(B)* $\frac{\mu_0 I}{4r}$

(C) $\frac{\mu_0 I}{2\pi r}$ *(D) zero*

Sol. By using the relation

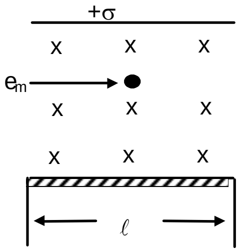
$B = \frac{\mu_0 i\theta}{4\pi r}$ where θ is in radian

Here, θ = 180°, i.e. π

So, $B_p = \frac{\mu_0 I}{4r}$

Hence **(B)** is correct.

Prob 12. *An electron moves straight inside a charged parallel plate capacitor of uniform surface charge density σ. The space between the plates is filled with constant magnetic field of induction $\vec{B}$. Neglecting gravity, the time of straight line motion of the electron in the capacitor is*



(A) $\frac{\sigma}{\varepsilon_0 \ell B}$ *(B)* $\frac{\varepsilon_0 \ell B}{\sigma}$

(C) $\frac{\sigma}{\varepsilon_0 B}$ *(D)* $\frac{\varepsilon_0 B}{\sigma}$

Sol . The net electric field

$\vec{E} = \vec{E}_1 + \vec{E}_2 \ \Rightarrow \ E = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o}$

The net force acting on the electron is zero because it moves with constant velocity

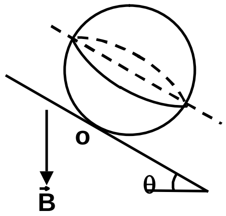
$\Rightarrow \vec{F}_{net} = \vec{F}_e + \vec{F}_m = 0$

$\Rightarrow \quad |\vec{F}_e| = |\vec{F}_m| \Rightarrow \quad eE = evB \quad \Rightarrow \quad v = \frac{E}{B} = \frac{\sigma}{\varepsilon_o B}$

$\therefore$ The time of motion inside the capacitor $= t = \frac{\ell}{v} = \frac{\varepsilon_o \ell B}{\sigma}$

Hence **(B)** is correct.

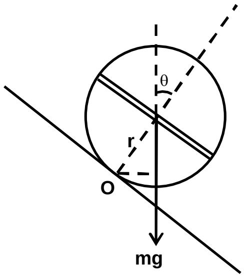
Prob 13. *In the figure shown a coil of single turn is wound on a sphere, of radius r and mass m. The plane of the coil is parallel to the inclined plane and lies in the equatorial plane of the sphere,. If sphere, is in rotational equilibrium the value of B is (current in the coil is i)*



(A) $\frac{mg}{\pi i r}$ *(B)* $\frac{mg \sin\theta}{\pi \quad i}$

(C) $\frac{mgr \sin\theta}{\pi i}$ *(D) none of these*

Sol . The gravitational torque must be counter balanced by the magnetic torque about O, for equilibrium of the sphere,.



The gravitational torque $= \tau_{gr} = mg \times r \sin\theta$

$\Rightarrow \tau_{gr} = mgr \sin\theta$

The magnetic torque $\tau_m = \vec{\mu} \times \vec{B}$

Where the magnetic moment of the coil

$= \mu = (i\,\pi r^2)$

$\Rightarrow \tau_m = \pi i r^2 B \sin\theta$

$\pi i r^2 B \sin\theta = mgr \sin\theta$

$\Rightarrow B = \frac{mg}{\pi i r}$

Hence **(A)** is correct.

Prob 14. *A particle with a specific charge s starts from rest in a region where the electric field has a constant direction, but whose magnitude increases linearly with time. The particle acquires a velocity v in time t. Then,*

(A) $v \propto s$ *(B)* $v \propto \sqrt{s}$

(C) $v \propto t$ *(D)* $v \propto t^2$

Sol. $E = \alpha t \quad (\alpha = \text{constant})$

$F = QE$

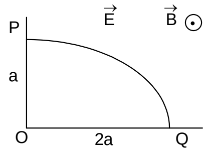
$a = F/m = QE/m = Es = \alpha s t.$

$\therefore \quad a = \frac{dv}{dt} = \alpha s t$

or $v = \frac{1}{2} \alpha s t^2 \qquad \therefore v \propto s$ and t^2

Hence **(A) & (D)** are correct.

Prob 15 . *A particle with charge +q and mass m, moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$, follows a trajectory from P to Q as shown. The velocities at P and Q are $v\hat{i}$ and $-2v\hat{j}$. Then,*



(A) $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

(B) The rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$

(C) The rate of work done by the electric field at P is 0.

(D) The rate of work done by both the fields at Q is 0.

Sol. In going from P to Q increase in kinetic energy

$= \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{1}{2}\ m(3v^2)$ = work done by electric field.

or $\frac{3}{2}mv^2 = Eq \times 2a$ or $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

The rate of work done by E at P = force due to E × velocity.

$= (qE)v = qv\left[\frac{3}{4}\left(\frac{mv^2}{qa}\right)\right] = \frac{3}{4}\left(\frac{mv^3}{a}\right)$

At Q, $\vec{v}$ is perpendicular to $\vec{E}$ and $\vec{B}$, and no work is done by either field.

Hence **(A), (B) & (D)** are correct.

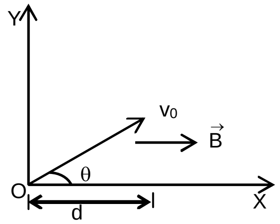
Prob 16. *A charged particle P leaves the origin with speed $v = v_0$, at some inclination with the x–axis. There, is a uniform magnetic field B along the x–axis. P strikes a fixed target T on the x–axis for a minimum value of $B = B_0$. P will also strike T if*

(A) $B = 2B_0$, $v = 2v_0$ *(B) $B = 2B_0$, $v = v_0$*

(C) $B = B_0$, $v = 2v_0$ *(D) $B = B_0/2$, $v = 2v_0$*

Sol . Let d = distance of the target T from the point of projection.

P will strike T if d an integral multiple of the pitch.



Pitch $= \left(2\pi\frac{m}{QB}\right)\ v\ \cos\theta$.

Here, m, Q and θ are constant

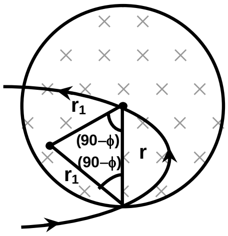
$\therefore$ pitch $= k\left(\frac{v}{B}\right)$ where k = constant

Initially, $d = k\left(\frac{v_0}{B_0}\right)$. Hence **(A) & (B)** are correct.

Prob 17. *A uniform magnetic field of intensity 1T is applied in a circular region of radius 0.1 m, directed into the plane of paper. A charged particle of mass 5×10^{-5} kg and charge q = 5×10^{-4} C enters the field with velocity $1/\sqrt{3}$ m/s making an angle of ϕ with a radial line of circular region in such a way that it passes through centre of applied field. The angle ϕ is*

(A) 60° *(B) 30°*

(C) 45° *(D) 90°*



Sol. $r_1 = \dfrac{mV}{qB} = \dfrac{5 \times 10^{-5} \times \left(1/\sqrt{3}\right)}{5 \times 10^{-5} \times 1} = \dfrac{1}{10\sqrt{3}}$

By sine rule

$$\frac{r_1}{\sin(90° - \phi)} = \frac{r}{\sin 2\phi}$$

$$\frac{1}{10\sqrt{3}\cos\phi} = \frac{0.1}{2\sin\phi\cos\phi}$$

$$\sin\phi = \frac{0.1 \times 10\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\phi = 60°$$

Hence **(A)** is correct.

ASSIGNMENT PROBLEMS

Subjective:

Level– O

1. Why do we not simply define the direction of the magnetic field $\vec{B}$ to be the direction of the magnetic force that acts on a moving charge ?

2. If an electron is not deflected in passing through a certain region of space, can we be sure that there, is no magnetic field in that region ?

3. A conductor, even though it is carrying a current, has zero net charge. Why then does a magnetic field exert a force on it ?

4. An alpha particle travels with a velocity $\vec{v}$ of magnitude 550 m/s through a uniform magnetic field $\vec{B}$ of magnitude 0.045 T. The angle between $\vec{v}$ and $\vec{B}$ is 30°. What are the magnitudes of car the force $\vec{F}_B$ acting on the particle due to the field and (b) the acceleration of the particle due to $\vec{F}_B$.

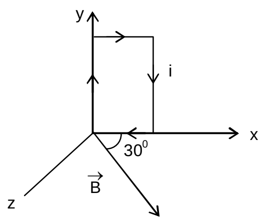
5. An electron is accelerated through a potential difference fo 1.0 kV and directed into a region between two parallel plates separated by 20 mm with a potential difference of 100 V between them. The electron is moving perpendicularly to the electric field of the plates when it enters the region between the plates. What uniform magnetic field, applied perpendicular to both the electron path and the electric field, will allow the electron to travel in a straight line ?

6. What uniform magnetic field, applied perpendicular to a beam of electron moving at 1.3×10^6 m/s, is required to make the electrons travel in a circular arc of radius 0.35 m.

7. In a certain cyclotron a proton moves in a circle of radius 0.5 m. The magnitude of the magnetic field is 1.2 T.
(a) What is the oscillatory frequency?
(b) What is the kinetic energy of proton?

8. A horizontal conducting wire carries a current of 5000 A from south to north. Earth's magnetic field (60.0 μT) is directed towards the north and is inclined downward at 70° to the horizontal. Find the magnitude and direction of the magnetic force on 100 m of the conductor due to Earth's field.

9. A rectangular loop of 20 – turn of dimensions 10 cm × 5cm is carrying a current of 0.10 amp and is hinged along one long side. it is mounted in x–y plane, at 30° to the direction of uniform magnetic field of magnitude 0.50 T. Find the magnitude and direction of the torque acting on the coil about the hinged line.

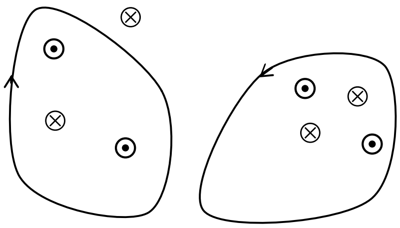


10. A circular coil of 160 turns has a radius of 1.90 cm.
(a) Calculate the current that results in a magnetic dipole moment of 2.30 A – m^2.
(b) Find the maximum torque, that the coil carrying this current, can experience in a uniform 35.0 mT magnetic field.

11. A wire carries 100 A of steady current, below which a magnetic compass is at placed 6.1 m.
(a) What is the magnetic field at the site of the compass due to wire.
(b) Will the Earth's magnetic field interfere with the compass readings ?

12. Two long parallel wires are 8.0 cm apart. What equal currents must be established in the wires for the magnetic field halfway between them to have a magnitude of 300 μT. Take (a) parallel current (b) antiparallel current.

13. Each of the eight conductors shown carries 2.0 Amp of current in or out of the page. Two paths are indicated for the line integral $\int \vec{B}.d\vec{\ell}$. What is the value of integral for the path. (a) At the left end, and (b) at the right ?



14. A wooden ring whose mean diameter is 14.0 cm is wound with a closely spaced toroidal windings of 600 turns. Compute the magnitude of the magnetic field at the centre of the cross–section of the windings when the current in the winding is 0.65 Amp.

15. A 200 - turns solenoid having a length of 25 cm and a diameter of 10 cm carries a current of 0.30 Amp. Calculate the magnetic dipole moment $\vec{\mu}$ of the solenoid.

Level– I

1. A potential difference of 600 V is applied across the plates of a parallel plate capacitor. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates, with a velocity of 2×10^6 m/s moves undeflected between the plates. Find the magnitude and direction of magnetic field in the region between the condenser plates.

2. A charge q is moving with a velocity $\vec{v}_1 = 1\hat{i}$ m/s at a point in a magnetic field and experiences a force $\vec{F}_1 = q[-1\hat{j} + 1\hat{k}]$ N. If the charge is moving with a velocity $\vec{v}_2 = 1\hat{j}$ m/s at the same point, it experiences a force $\vec{F}_2 = q\left[1\hat{i} - 1\hat{k}\right]$ N. Calculate the magnetic induction $\vec{B}$ at that point.

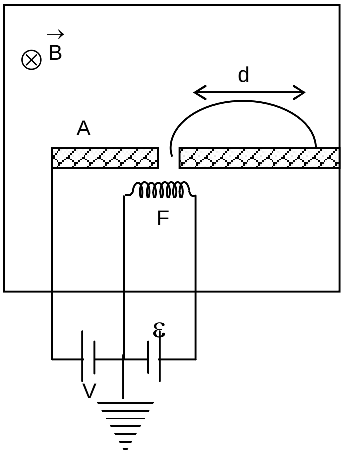
3. Two long, straight, horizontal parallel wires, one above the other, are separated by a distance a. If the wires carry equal currents I in (a) opposite directions, (b) in same direction, what is the field magnitude in the plane of the wires at a point:
(i) half way between them, and (ii) 'a' distance above the upper wire ?

4. Find the magnetic field B at the centre of a rectangular loop of length l and width b, carrying a current i.

5. A proton, a deuteron and an α–particle, accelerated through the same potential difference, enter a region of uniform magnetic field, moving at right angles to B. Compare the radii of their circular paths.

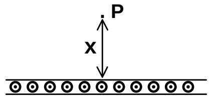
6. A circular coil of 100 turns and effective diameter 20 cm carries a current of 0.5 A. It is to be turned in a magnetic field B = 2T from a position r in which angle θ between magnetic field and axis of coil, equals zero to the position in which θ equals 180°. Calculate the work required in this process.

7. The value of $\frac{e}{m_e}$ can be obtained by using a specially designed vacuum tube. The figure contains a heated filament F and an anode A which is maintained at a positive potential relative to the filament by a battery of known voltage v. Electrons get emitted from the heated filament and are accelerated towards the anode, which has a small hole in the centre for the electrons to pass through into a region of constant magnetic field B, which points into the plain of the paper. The electrons then move in a semicircle of diameter d, hitting the detector (cathode) as shown. Prove that $\frac{e}{m_e} = \frac{8v}{B^2d^2}$.

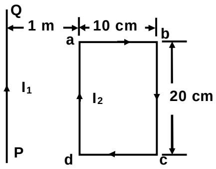


8. A circular loop of radius 4cm and carrying current 0.2A is placed in a uniform magnetic field of induction 0.4T and perpendicular to the plane of the loop. Calculate the force on the arc of the circle of length $\frac{4}{3}\pi$ cm.

9. Find the magnitude and direction of magnetic induction due to an infinite metallic sheet carrying a current of linear density j, at a point P at a distance x from the metal sheet.



10. The long straight wire PQ in the figure carries a current I_1 = 20 A. A rectangular loop 'abcd' whose longer sides are parallel to the wire, carries a current I_2 = 10 A. Find the magnitude and direction of the net force on the loop due to the currents.

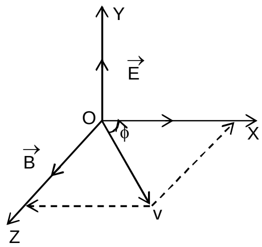


11. A straight wire of mass m can slide on two parallel plastic rails kept in a horizontal plane with a separation d. The coefficient of friction between the wire and the rails is μ. If the wire carries a current i, what minimum magnetic field should exist in the space in order to slide the wire on the rails.

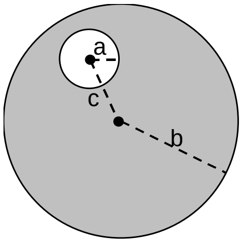
12. Two circular coils of radii 5.0 cm and 10 cm carry equal currents of 2.0 A. The coils have 50 and 100 turns, respectively, and are placed in such a way that their planes as well as the centres coincide. Find the magnitude of the magnetic field B at the common centre of the coils if the currents in the coils are (a) in the same sense, (b) in the opposite.

13. A plastic disk of radius R has a charge q uniformly distributed over its surface. If the disk is rotated at an angular frequency ω about its axis, show that the induction at the center of the disk is $B = \frac{\mu_o \omega q}{2\pi R}$.

14. A proton is moving with uniform velocity in x– y plane at an angle of 30° with x–axis, in the presence of electrostatic field $\vec{B} = 4kv/m\hat{j}$ and magnetic field $\vec{B}$ = 50 mT$\hat{k}$. Find the pitch of the helical trajectory followed by the proton when the electric field is switched off.

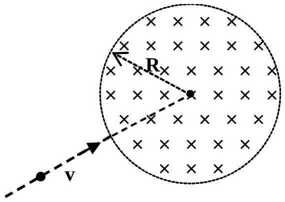


15. A long straight metal rod has a very long hole of radius 'a' drilled parallel to rod axis at a distance c from the axis of the rod as shown in the figure. If the rod carries a current I, find the magnetic field (a) on the axis of the rod, (b) on the axis of the hole.

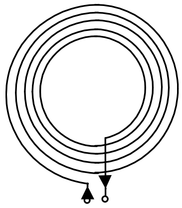


Level– II

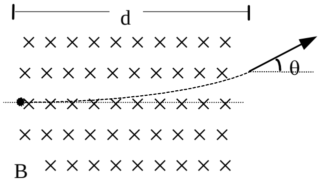
1. A particle of mass m having a charge q enters into a circular region of radius R with velocity v directed towards the centre. The strength of magnetic field is B. Find the deviation in the path of the particle.



2. A thin insulated wire forms a plane spiral of N = 100 tight turns carrying a current I = 8 mA. The radii of inside and outside turns (see fig.) are equal to a = 50 mm and b = 100 mm. Find:
(a) the magnetic induction at the centre of the spiral;
(b) the magnetic moment of the spiral with a given current.



3. A particle of mass m and charge q is projected into a region having a perpendicular magnetic field B. Find the angle of deviation (see fig.) of the particle as it comes out of the magnetic field if width d of the region is very slightly smaller than
(a) $\frac{mv}{qB}$ (b) $\frac{mv}{2qB}$ (c) $\frac{2mv}{qB}$.



4. A circular loop of radius R carries a current I_1. Another circular loop of radius r(<<R) carries a current I_2 and is placed at the centre of the larger loop. The planes of the two circles are at right angle to each other. Find the torque acting on the smaller loop.

5. The magnetic field existing in a region is given by
$$\vec{B} = B_o\left(1+\frac{x}{l}\right)\hat{k}.$$
A square loop of edge l and carrying a current i, is placed with its edges parallel to the X–Y axes. Find the magnitude of the net magnetic force experienced by the loop.

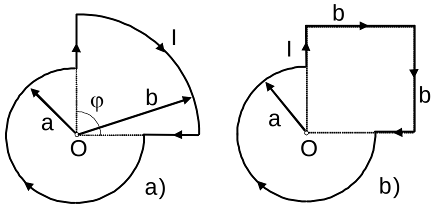
6. A particle having mass m and charge q is released from the origin in a region in which electric field and magnetic field are given by $\vec{B} = -B_o\,\vec{j}$ and $\vec{E}=E_o\vec{k}$. Find the speed of the particle as a function of its z–coordinate.

7. A non–conducting thin spherical shell of radius R has uniform surface charge density σ. The shell rotates about a diameter with constant angular velocity ω. Calculate
(a) magnetic induction B at the centre of the cell.
(b) magnetic moment of the sphere,.

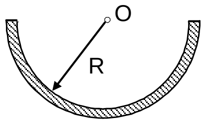
8. A regular polygon of n sides is formed by bending a wire of total length 2πr which carries a current i.
(a) Find the magnetic field B at the centre of the polygon.
(b) By letting n→ ∞, deduce the expression of the magnetic field at the centre of a circular current.

9. A hollow cylindrical conductor of radii r_1 and r_2 carries a current i uniformly spread over its cross–section. Find the magnetic field B in the region:
(a) $0 \le r \le r_1$ (b) $r_1 \le r \le r_2$ (c) $r \ge r_2$.

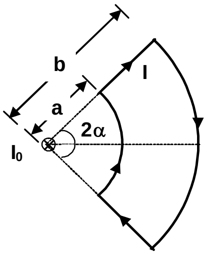
10. Find the magnetic induction of the field at the point O of a loop with current I, whose shape is illustrated
(i) in Figure a, the radii a and b, as well as the angle ϕ are known;
(ii) in Figure b, the radius a and the side b are known.



11. A current I flows in a long straight wire with cross–section having the form of a thin half–ring of radius R (see fig,). Find the induction of the magnetic field at the point O.

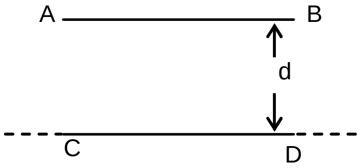


12. A loop, carrying a current I, lying in the plane of the paper, is in the field of a long straight wire with current I_0 (inwards) as shown in the figure. Find the torque acting on the loop.



13. A coil of radius R carries current I. Another concentric coil of radius r(r<<R) carries current i. Planes of two coils are mutually perpendicular and both the coils are free to rotate about common diameter. Find the maximum kinetic energy of the smaller coil when both the coils are released. Masses of coils are M and m, respectively.

14. A horizontal wire AB which is free to move in a vertical plane and carries a steady current, is in equilibrium at a height d over another parallel long wire CD, which is fixed in a horizontal plane and carries a steady current, as shown in figure. Show that when AB is slightly depressed and released, it executes simple harmonic motion. Find the period of oscillations.



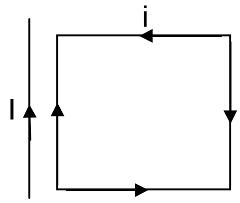
15. A particle of mass m and charge q is moving in a region where uniform constant electric and magnetic fields $\vec{E}$ and $\vec{B}$ are present. $\vec{E}$ and $\vec{B}$ are parallel to each other. At time t = 0, the velocity $\vec{v}_0$ of the particle is perpendicular to $\vec{E}$ (Assume that its speed is always << c, the speed of light in vacuum.) Find the velocity $\vec{v}$ of the particle at time t. You must express your answer in terms of t, q, m, the vectors $\vec{v}_0, \vec{E}$ and $\vec{B}$ and their magnitudes v_0, E and B.

Objective:

Level – I

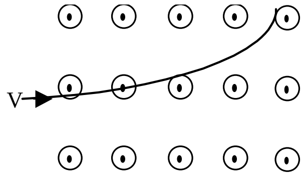
1. Two thin long parallel wires separated by a distance b are carrying a current i amp each. The magnitude of the force per unit length exerted by one wire on the other is:
(A) $\mu_0 i^2/b$ (B) $\mu_0 i^2/2\pi b$
(C) $2\mu_0 i^2/b$ (D) $\mu_0 i^2/\pi b$

2. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If a steady current I is established in the wire as shown in the figure, the loop will:



(A) rotate about an axis parallel to the wire (B) move away from the wire
(C) move towards the wire (D) remain stationary

3. A particle enters the region of a uniform magnetic field as shown in figure. The path of the particle inside the field is shown by dark line.
The particle is:



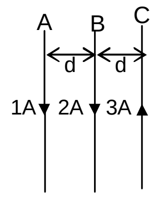
(A) electrically neutral
(B) positively charged
(C) negatively charged
(D) nothing definite can be said about the nature of the charge as the information given is inadequate

4. Considering magnetic field along the axis of a circular loop of radius R, at what distance from the centre of the loop is the field one-tenth of its value at the centre ?
(A) 4R (B) 1.9 R
(C) 3R (D) 6R

5. A proton (charge +e coulomb) enters in a magnetic field of strength B (Tesla) with speed v, parallel to the direction of magnetic lines of force. The force on the proton is
(A) evB (B) zero
(C) ∞ (D) evB/2

6. An electron (mass = 9×10^{-31} kg, charge = 1.6×10^{-19} C) moving with a velocity of 10^6 m/s enters a region where magnetic field exists. If it describes a circle of radius 0.10 m, the intensity of magnetic field must be:
(A) 1.8×10^{-4} T (B) 5.6×10^{-5} T
(C) 14.4×10^{-5} T (D) 1.4×10^{-6} T

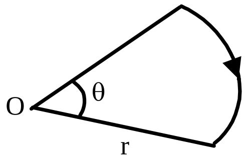
7. Three infinite straight wires A, B and C carry currents as shown in figure. The resultant force on wire B per metre length is



(A) $4\mu_0 / \pi d$ towards A
(B) $2\mu_0 / \pi d$ towards C
(C) $4\mu_0 / \pi d$ towards C
(D) zero

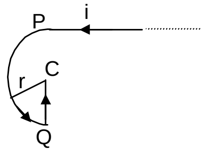
8. A strong magnetic field is applied on a stationary electron, then:
(A) the electron will move in the direction of the field
(B) the electron will move in an opposite direction
(C) the electron remains stationary
(D) the electron starts spinning

9. A wire bent in the form of a sector of radius r subtending an angle θ at centre, as shown in figure, is carrying a current i. The magnetic field at O is:



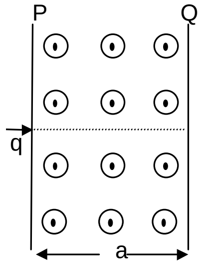
(A) $\frac{\mu_0 i}{2r}\theta$
(B) $\frac{\mu_0 i}{2r}(\theta/180)$
(C) $\frac{\mu_0 i}{2r}(\theta/360)$
(D) zero

10. An infinitely long straight conductor is bent into the shape as shown in the following figure. It carries a current i ampere and the radius of the circular loop is r metre. Then the magnetic induction B at the centre of the circular part is :



(A) zero
(B) infinite
(C) $\frac{\mu_0}{4\pi}\frac{2i}{r}\left(\frac{\pi}{2}+1\right)$
(D) $\frac{\mu_0}{4\pi}\frac{i}{r}(\pi+1)$

11. A charged particle having kinetic energy K and charge q enters into the region of a uniform magnetic field between two plates P and Q as shown in Figure. The charged particle just misses hitting the plate Q. The magnetic field in the region between the two plates is :



(A) mK/qa
(B) 2mk/qa
(C) $\sqrt{(mK)}$/qa
(D) $\sqrt{(2mK)}$/qa

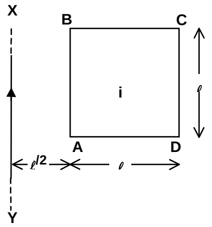
12. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then
(a) the magnetic field at all points inside the pipe is the same, but not zero.
(b) the magnetic field at any point inside the pipe is zero.
(c) the magnetic field is zero only on the axis of the pipe.
(d) the magnetic field is different at different point inside the pipe.

13. A particle of mass m and charge q moves with a constant velocity v along the positive x direction. It enters a region containing a uniform magnetic field B directed along the negative z direction, extending from x = a to x = b. The minimum value of v required so that the particle can just enter the region x>b is

(A) $\frac{qbB}{m}$ (B) $\frac{q(b-a)B}{m}$

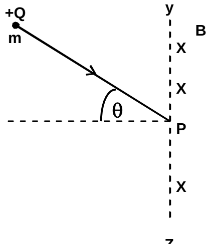
(C) $\frac{qaB}{m}$ (D) $\frac{q(b+a)B}{2m}$

14. A square loop ABCD, carrying a current i, is placed near and coplanar with a long straight conductor XY carrying a current **I**. Choose the



(A) There, is no net force on the loop.
(B) The loop will be attracted by the conductor only if the current in the loop flows clockwise.
(C) The loop will be attracted by the conductor only if the current in the loop flows anticlockwise.
(D) The loop will always be attracted by the conductor.

15. A particle with charge +Q and mass m enters a magnetic field of magnitude B, existing only to the right of the boundary YZ. The direction of the motion of the particle is perpendicular to the direction of B. Let $T = 2\pi \frac{m}{QP}$. Then, the time spent by the particle in the field will be

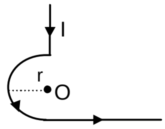


(A) $T\theta$ (B) $2T\theta$

(C) $T\left(\frac{\pi+2\theta}{2\pi}\right)$ (D) $T\left(\frac{\pi-2\theta}{2\pi}\right)$

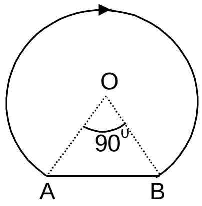
Level– II

1. In the given figure, what is the magnetic field induction at point O?



(A) $\frac{\mu_0 I}{4\pi r}$ (B) $\frac{\mu_0 I}{4r}+\frac{\mu_0 I}{2\pi r}$
(C) $\frac{\mu_0 I}{4r}+\frac{\mu_0 I}{4\pi r}$ (D) $\frac{\mu_0 I}{4r}-\frac{\mu_0 I}{4\pi r}$

2. A current i flows along a thin wire shaped as shown in figure. The radius of the curved part of the wire is r. The field at the centre O of the coil is

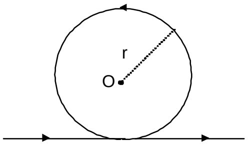


(A) $\frac{\mu_0}{4}\cdot\frac{i}{R}$ (B) $\frac{\mu_0}{2\pi R}$
(C) $\frac{\mu_0 i}{2\pi R}$ (D) none of these

3. Electrons at rest are accelerated by a potential of V volt. These electrons enter the region of space having a uniform, perpendicular magnetic induction field B. The radius of the path of the electrons inside the magnetic field is:
(A) $\sqrt{(meV)}/Be$ (B) $\sqrt{(2meV)}/Be$
(C) $\sqrt{(meV)}/2Be$ (D) none of these

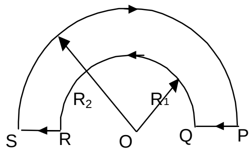
4. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2, respectively. The ratio of the mass of X to that of Y is:
(A) $\left(\frac{R_1}{R_2}\right)^{1/2}$ (B) $\frac{R_2}{R_1}$
(C) $\left(\frac{R_1}{R_2}\right)^{2}$ (D) $\frac{R_1}{R_2}$

5. An infinitely long straight conductor is bent into shape as shown in figure. It carries a current i amp and the radius of circular loop is r metre. Then, the magnetic induction at the centre of the circular loop is:



(A) zero (B) infinite
(C) $\frac{\mu_0 i}{2\pi r}(\pi+1)$ (D) $\frac{\mu_0 i}{2\pi r}(\pi-1)$

6. The wire loop shown in figure carries a current as shown. The magnetic field at the centre O is:

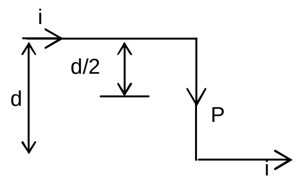


(A) zero (B) $\frac{\mu_0 i}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$
(C) $\frac{\mu_0 i}{4}\left(\frac{1}{R_2}+\frac{1}{R_1}\right)$ (D) $\frac{\mu_0 i}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

7. A straight cylindrical conductor of radius r carries current uniformly distributed over its cross section. The variation of the magnetic field B with distance x from the axis of the conductor is described by
(A) $B \propto 1/x$ for $0 \le x < \infty$
(B) $B \propto x$ for $0 \le x < \infty$
(C) B is uniform for $0 \le x \le r$ and $B \propto 1/x$ for $x \ge r$
(D) $B \propto x$ for $0 \le x \le r$ and $B \propto 1/x$ for $x \ge r$

8. A wire of length L is formed into a circular coil of n turns. If the coil carries a current i and placed in a given magnetic field B, the maximum torque developed is
(A) $\frac{nBiL^2}{4\pi}$ (B) $\frac{BiL^2}{4\pi n}$
(C) $\frac{BiL^2}{4\pi}$ (D) none of these

9. A wire in the shape shown in figure carries a current i. The magnetic field B at point P is



(A) $\frac{\mu_0 i}{\pi d}$ (B) $\frac{\mu_0 i}{2\pi d}$
(C) $\frac{2\mu_0 i}{\pi d}$ (D) zero

10. In a given length L of a wire, a current I may be established. The ratio of the magnetic fields at the centre, when the wire is formed into a circle and a square, respectively, is
(A) $\frac{\pi}{4}$ (B) $\frac{\pi^2}{8\sqrt{2}}$
(C) $\left(\frac{\pi}{4}\right)^2$ (D) $\frac{\pi^2}{8\sqrt{3}}$

State whether the given statements are True or False

11. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then, the magnetic field at all points inside the pipe is the same, but not zero.

12. No net force acts on a rectangular coil carrying a steady current when suspended freely in a uniform magnetic field.

13. There, is no change in the energy of a charged particle moving in a magnetic field although a magnetic force is acting on it.

14. A charged particle enters a region of uniform magnetic field at an angle of 85^o to the magnetic lines of force. The path of the particle is a circle.

15. A current is flowing in a circular loop of wire in clockwise direction. The magnetic field at the centre of the wire is directed downward.

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level – O

4. (a) 3.96×10^{-18} N (b) 6×10^{8} m/s

5. 0.27 mT

6. 21 μT

7. (a) 1.15×10^{8} rad/s (b) 1.72×10^{7} eV

8. 28.2 N, horizontally west

9. 4.3×10^{-3} N.m, along negative y–axis

10. (a) 1.26×10^{-3} (b) 8.05×10^{-2} Joule

11. (a) 3.3 μT (b) yes

12. (a) not possible (b) 0.30 A

13. (a) ($-$ 2.0 μ_0) Amp. (b) zero

14. 1.11×10^{-3} T

15. 0.47 A–m^2

Level – I

1. 0.1 Wb/m^2, perpendicular to the plane of paper (inwards).

2. (i + j + k) Wb/m^2

3. (a) $\frac{2\mu_o I}{\pi a}$, (b) $\frac{\mu_o I}{4\pi a}$, (i) zero, (ii) $\frac{3\mu_o I}{4\pi a}$

4. $\frac{2\mu_o i\sqrt{l^2+b^2}}{\pi lb}$

5. $r_p : r_d : r_\alpha :: 1 : \sqrt{2} : \sqrt{2}$

6. $2\pi J$

8. 3.2×10^{-3} N

9. $\frac{1}{2}\ \mu_0 \hat{j}$, Parallel to the sheet leftward.

10. 7.2×10^{-7} N

11. $\frac{\mu mg}{id\sqrt{1+\mu^2)}}$

12. (a) $8\pi \times 10^{-4}$ T (b) zero

14. 6 cm.

15. (a) $\frac{\mu_0}{2\pi}\ \frac{Ia^2}{c\left(b^2-a^2\right)}$ (b) $\frac{\mu_0}{2\pi}\ \frac{Ic}{\left(b^2-a^2\right)}$

Level – II

1. $\pi - 2\tan^{-1}\left(\dfrac{mv}{qBR}\right)$
2. (a) $7\,\mu T$; (b) $15\,mA.m^2$
3. (a) $\pi/2$ (b) $\pi/6$ (c) π
4. $\dfrac{\mu_o \pi I_1 I_2 r^2}{2R}$
5. $iB_0\,\ell$
6. $\sqrt{\dfrac{2qE_o z}{m}}$
7. (a) $\dfrac{2}{3}\mu_0\sigma\omega R$ (b) $\dfrac{4}{3}\pi\sigma\omega R^4$
8. (a) $\dfrac{\mu_o n^2 i}{2\pi^2 r}\sin\left(\dfrac{\pi}{n}\right)\tan(\pi/n)$ (b) $\dfrac{\mu_0 I}{2r}$
9. (a) zero, (b) $\dfrac{\mu_0 i}{2\pi(r_2^2 - r_1^2)}.\dfrac{r^2 - r_1^2}{r}$ (c) $\dfrac{\mu_0 i}{2\pi r}$
10. (i) $B = \dfrac{\mu_o I}{4\pi}\left(\dfrac{2\pi-\varphi}{a}+\dfrac{\varphi}{b}\right)$; (ii) $B = \dfrac{\mu_o I}{4\pi}\left(\dfrac{3\pi}{2a}+\dfrac{\sqrt{2}}{b}\right)$
11. $B = \mu_o I/\pi^2 R$
12. $\dfrac{\mu_0 I I_0 \sin\alpha}{\pi}(b-a)$
13. $\dfrac{\mu_0\pi \;\; Ii \;\; MRr^2}{2\left(MR^2 + mr^2\right)}$
14. $2\pi\sqrt{d/g}$
15. $\vec{v} = \cos\left(\dfrac{qB}{m}t\right)(\vec{v}_0) + \left(\dfrac{q}{m}t\right)\vec{E} + \left[\sin\left(\dfrac{qBt}{m}\right)\right]\left(\dfrac{\overrightarrow{V_o}x\vec{B}}{B}\right)$

Objective:

Level – I

1.	**B**	2.	**C**
3.	**C**	4.	**B**
5.	**B**	6.	**B**
7.	**A**	8.	**C**
9.	**C**	10.	**D**
11.	**D**	12.	**B**
13.	**B**	14.	**D**
15.	**C**		

Level – II

1.	**C**	2.	**D**
3.	**B**	4.	**C**
5.	**C**	6.	**B**
7.	**D**	8.	**B**
9.	**D**	10.	**B**
11.	**False**	12.	**True**
13.	**True**	14.	**False**
15.	**True**		

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Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

Electromagnetic Induction & Alternating Current

Syllabus for IITJEE and Karnataka Board:

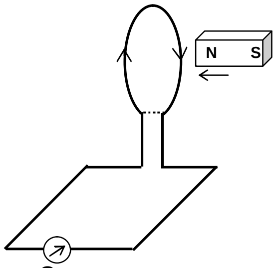
Electromagnetic induction, Faraday's laws, Induced emf and current, Lenz's law. Eddy currents, self and mutual inductance.

Alternating current, peak and rms value of alternating current/voltage, reactance and impedance; LC oscillations, LCR series circuit (Phasor diagram)–Resonant circuits and Q–factor; power in AC circuits, wattless current
AC generator and Transformer.

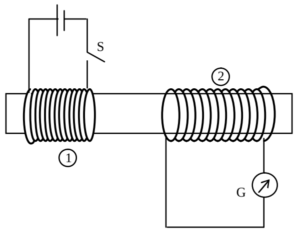
Electromagnetic Induction

We have studied that moving charges or current produces magnetic field. Then normally we get a doubt whether the converse of this is also true? Does nature permit such a symmetrical relation? Two physicists Micheal Faraday and Josheph Henry demonstrated the fact that changing magnetic fields and electric currents with the help of the following simple experiments.

If we bring a magnet near a coil connected with a galvanometer, the galvanometer shows deflection indicating flow of current in the coil. This current flow as long as the magnet is moving, i.e. the magnetic flux through the coil is changing. Once the magnet becomes stationary, the current stops.

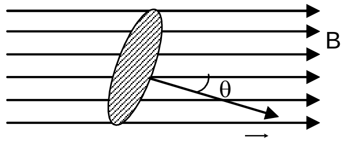


If we take two coils wound on an iron cylinder, one fitted with a battery and a switch, the other fitted with a galvanometer, when switch is closed, current increases from zero to its maximum value. During this brief time, the galvanometer deflects showing flow of current. The changing currents in coil (1) gives rise to a changing magnetic field which induces an emf and a current in the coil (2). This phenomenon of inducing electricity by changing magnetic field is known as electromagnetic induction.



Magnetic Flux

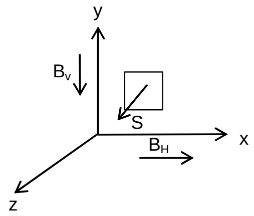
The magnetic flux ϕ_B through an area $d\vec{S}$ in a magnetic field $\vec{B}$ is defined as, $\phi_B = \int \vec{B}.d\vec{S}$



Exercise 1. ***A thin conducting spherical shell is placed in a uniform magnetic field. Find the flux associated with the shell.***

Illustration 1. *At a given place, horizontal and vertical components of earth's magnetic field B_H and B_V are along x and y axes respectively as shown in the figure. What is the total flux of earth's magnetic field associated with an area S, if the area S is in the* *(a) x–y plane (b) y–z plane?*

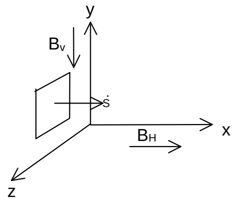
Solution: The earth's magnetic field



$\vec{B} = \left(B_H\hat{i} - B_v\hat{j}\right)$

(a) Given the area $\vec{A} = S\hat{k}$

$\phi_{xy} = \vec{B}.\vec{A} = \left(B_H\hat{i} - B_v\hat{j}\right).S\hat{k} = 0$



(b) For the case $\vec{A} = S\hat{i}$

$\phi_{yz} = \left(B_H\hat{i} - B_v\hat{j}\right).S\ \hat{i} = B_H S$

FARADAY'S LAWS

(i) When the flux of magnetic induction through a loop is changing, an electromotive force (emf) is induced in the loop. It lasts as long as the magnetic flux changes.

(ii) This induced e.m.f. is equal to the negative rate of change of flux, i.e.,

$\mathcal{E} = \frac{-d\Phi}{dt}$. where $\Phi = n\int \vec{B}.d\vec{S}$, n = number of turns

$\vec{B}$ = magnetic induction , $d\vec{S}$ = area element

If the resistance of the loop is R, the current in the loop will be $i = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\phi}{dt}$.

Illustration 2. *A conducting circular loop having a radius of 5.0 cm, is placed perpendicular to a magnetic field of 0.50 T. It is removed from the field in 0.50 s. Find the average emf produced in the loop during this time.*

Solution: Radius = 5 cm

$\therefore$ Area (S) = $25\,\pi \times 10^{-4} m^2$

$\phi = \vec{B}.\vec{S} = BS\cos 0^0$

$= 3.927 \times 10^{-3}$

$\therefore$ Average induced emf = $\frac{\Delta\phi}{\Delta t} = 7.85 \times 10^{-3}$ volt

Illustration 3. *A coil of area 500 cm^2 having 1000 turns is placed such that the plane of the coil is perpendicular to a magnetic field of magnitude 4×10^{-5} weber/m^2. If it is rotated by 180° about an axis passing through one of its diameter in 0.1 sec, find the average induced emf.*

(A) zero *(B) 30 mV*
(C) 40 mV *(D) 50 mV*

Solution: **(C)**

Total flux through the loop is

$\phi = B.NA = 4 \times 10^{-5} \times 1000 \times 500 \times 10^{-4}$

Since loop is rotated by 180°

Total change in flux = 2ϕ

i.e. emf $= 2\phi/0.1$ = change in flux / time = $\frac{2\times4\times5\times10^{-4}}{0.1}$ = 40 mV

Illustration 4. *A ring of radius 10 cm is placed in a circular magnetic field, which is varying at the rate of 10 tesla/sec. The electric field intensity at any point on the circumference of the ring is*

(A) 1 N/C *(B) 1.5 N/C*
(C) 0.5 N/C *(D) zero*

Solution: **(C)**

Magnetic flux $\phi = \vec{B}\cdot\vec{A}$

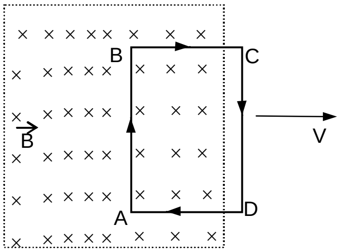
$e = \frac{d\phi}{dt} = \pi r^2 \frac{dB}{dt}$ and

$e = \oint E.d\ell = 2\pi rE$

$\Rightarrow \quad E = \frac{1}{2} r \frac{dB}{dt} = 0.5\ \text{N/C}$

Lenz's law

It states that the polarity of the induced e.m.f. and the direction of induced current is such that it opposes the very cause which produces it. Consider the figure shown. A rectangular loop ABCD is being pulled out of the magnetic field directed into the plane of the paper and perpendicular to the plane of the paper. As the loop is dragged out of the field the flux associated with the loop which is directed into the plane of the paper decreases.



The induced current will flow in the loop in the sense to oppose the decreasing of this flux. For this to happen magnetic field due to induced current in the loop must be directed into the plane of the paper. Hence the current in the loop must flow in the clockwise sense.

Exercises 2. ***(i) A metal ring is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring. Will the acceleration of the falling magnet be equal to, greater than, or less than due to gravity?***

(ii) What is the direction of induced current in the loop as shown in figure, if the current in the straight wire from A to B is

(a) Constant (b) Increasing

(c) Decreasing

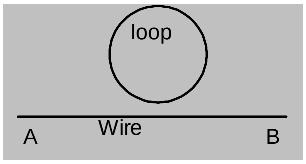


Illustration 5. *A circular loop of a radius a having n turns is kept in a horizontal plane. A uniform magnetic field B exists in a vertical direction as shown in the figure. Find the e.m.f. induced in loop if the loop is rotated with a uniform angular velocity ω about*

(a) an axis passing through the centre and perpendicular to the plane of the loop.

(b) a diameter.

Solution: (a) The e.m.f. induces when there is change of flux. As in this case there is no change of flux, hence no e.m.f. will be induced in the coil.

(b) If the loop is rotated about a diameter there will be a change of flux with time. In this case e.m.f. will be induced in the coil. The area of the loop is $A = \pi a^2$. If the normal of the loop makes an angle $\theta = 0$ with the magnetic field at t = 0, this angle will become $\theta = \omega t$ at time t. The flux of the magnetic field at this time is $\phi = n.B.\pi a^2 \cos\theta$

$= n.B.\pi a^2 \cos\omega t$

The induced e.m.f. is

$$\varepsilon = -\frac{d\phi}{dt} = \pi n a^2 B\omega \sin\omega t$$

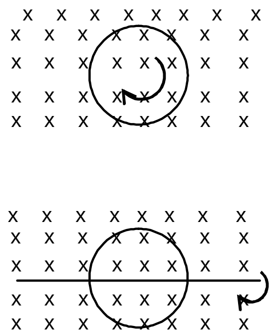


Illustration 6. *A square–shaped copper coil has edges of length 50 cm and contains 50 turns . It is placed perpendicular to a 1.0 T magnetic field. It is removed from the magnetic field in 0.25 s and restored in its original place in the next 0.25 s. Find the magnitude of the average emf induced in the loop during*

(a) Its removal,

(b) its restoration

(c) its entire motion in the field

Solution: No. of turns = 50, Area of square = 0.25 m^2

Magnitude of magnetic field = 1 T

(a) Average rate of complete removal = $\Delta s / \Delta t$ = 1m/s^2

$\therefore$ average induced emf = $NB\frac{\Delta S}{\Delta t}$ = 50 volt

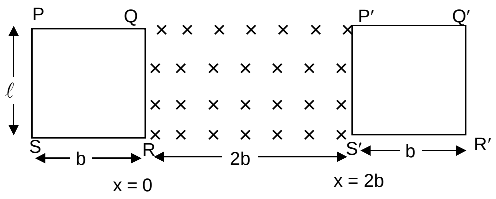
(b) Average rate of complete restoration = $\Delta s / \Delta t$ = 1m/s^2

$\therefore$ average induced emf = $NB\frac{\Delta S}{\Delta t}$ = 50 volt

(c) For entire motion in the field $\Delta S = 0$

$\therefore$ average induced emf = $NB\frac{\Delta S}{\Delta t}$ = 0 volt

Illustration 7. *A rectangular flat loop of wire with dimensions ℓ and b has N turns and a total resistance R. The loop moves with constant velocity v from position PQRS to P′Q′R′S′ through a region of constant magnetic field B as shown in figure*

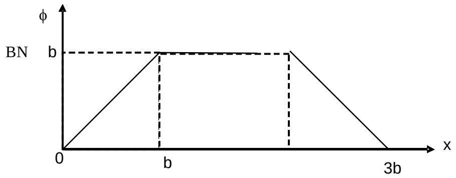


(a) Plot the graph of the flux linked with loop vs x. (Where x is the distance moved by the loop)

(b) Plot the graph of the emf induced in the loop vs x

Solution : (a) $\phi = \int \vec{B}.d\vec{S} = B.S$

(b) $\varepsilon = -\dfrac{d\phi}{dt} = \dfrac{B.dS}{dt}$



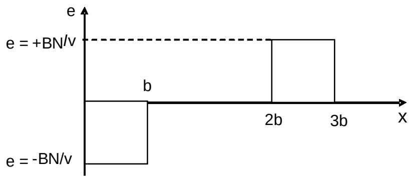
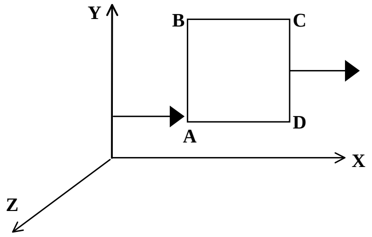


Illustration 8. *In the co–ordinate system shown in the figure magnetic field is directed along negative z–axis and its magnitude varies as $B = B_0/x$, where B_0 is a positive constant. A square loop ABCD of side 'a' and resistance per unit length 'λ' is moved with constant speed v with its plane parallel to x–y plane. Initially side AB was on the y–axis. Find the current induced in the loop as a function of time.*



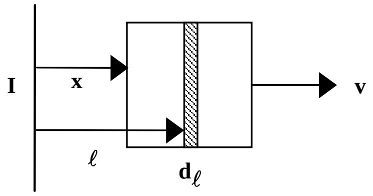
Solution:

$$\phi = \int_x^{a+x} \frac{B_0}{\ell} a d\ell = B_0 a \ \ell n\left(\frac{a+x}{x}\right)$$

$$= B_0 a \ \ell n\left(1 + \frac{a}{vt}\right)$$

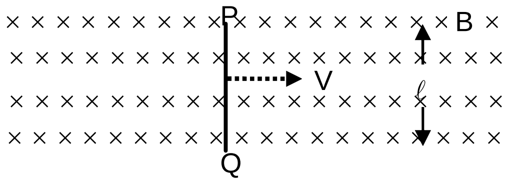
$$\varepsilon = -\frac{\phi_B}{dt} = B_0 a \frac{vt}{(a+vt)} \frac{a}{vt^2}$$

$$\therefore I = \frac{B_0 a}{4(a+vt)t\lambda}.$$



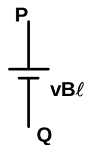
Motional e.m.f.

Consider a straight conductor PQ moving in a magnetic field. The electrons inside it experience a force $\vec{F} = -e\vec{v} \times \vec{B}$ and accumulate at the end of the conductor near Q. Thus, an electric field is established across its ends. Then $-e\vec{v} \times \vec{B}$ is balanced by $-e\vec{E}$ in the opposite direction, at equilibrium.



$\Rightarrow -e\vec{v} \times \vec{B} - e\vec{E} = 0$,

$\varepsilon = -\int \vec{E}.d\vec{\ell} = \int (\vec{v} \times \vec{B}).d\vec{\ell} = \int \vec{B}.(d\vec{\ell} \times \vec{v})$



As $d\vec{\ell}\times\vec{v}$ is the area swept per unit time by length $d\ell$ and hence $\vec{B}.(d\vec{\ell}\times\vec{v})$ is the flux of induction through this area. Therefore, the motional emf is equal to the flux of induction cut by the conductor per unit time. If the ℓ, B and v are mutually perpendicular to each other then $\varepsilon = B\ell v$

We can replace the moving rod by a battery of emf $vB\ell$ with positive terminal at P and the negative terminal at Q.

Exercise 3. ***A rod of length ℓ is dropped along the axis of a circular current carrying conductor of radius r with current I. What is the e.m.f. developed across the length of the rod?***

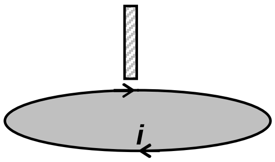
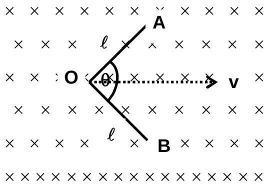


Illustration 9. *An angle ∠AOB made of a conducting wire moves along its bisector through a magnetic field B as suggested by figure. Find the emf induced between the two free ends if the magnetic field is perpendicular to the plane at the angle.*



Solution: The rod OA is equivalent to a cell of emf $vB\ell\sin\theta/2$. The positive charges shift towards 'A' due to the force $q\vec{v}\times\vec{B}$. The positive terminal of the equivalent cell appears towards A. Similarly, the rod OB is equivalent to a cell of emf $vB\ell\sin\theta/2$ with the positive terminal towards O. The equivalent circuit is shown in figure. Clearly, the emf between the points A and B is $2B\ell v\sin\theta/2$.

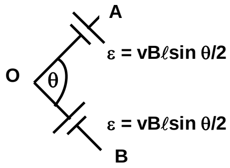
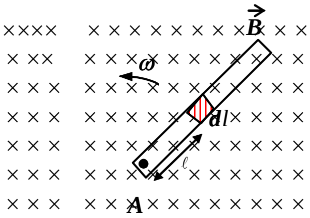


Illustration 10. *A copper rod of length 'ℓ' rotates at an angular velocity 'ω' in a uniform magnetic field B as shown. What is the induced emf across its ends?*

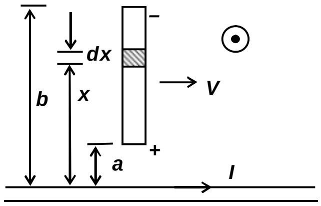


Solution: $\varepsilon = -\int\vec{E}.d\vec{\ell} = \int(\vec{v}\times\vec{B}).d\vec{\ell} = \int Bvd\ell$

As $v = \omega\ell$, $\varepsilon = \omega B\int_0^{\ell}\ell d\ell = \frac{1}{2}\omega B\ell^2$

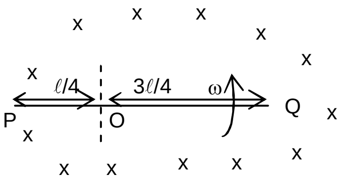
The rod AB may be replaced by a battery of emf = $\frac{1}{2}B\omega\ell^2$ with positive terminal towards A.

Illustration 11. *A copper rod moves with a constant velocity 'v' parallel to a long straight wire carrying a current 'i'. Calculate the induced emf in the rod, if the ends of the rod from the wire are at distances 'a' and 'b'.*



Solution: $\varepsilon = \int \vec{B}.(d\vec{l} \times \vec{v}) = \int_a^b \frac{\mu_o i}{2\pi x} v\, dx = \frac{\mu_o i v}{2\pi}.\ln\left(\frac{b}{a}\right)$

Illustration 12. *A conducting rod of length ℓ is rotating with constant angular velocity ω about point O in a uniform magnetic field B as shown in the figure. The emf induced between ends P and Q will be*



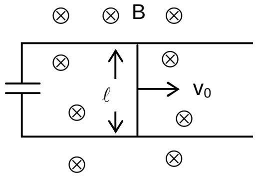
(A) $\frac{1}{4}B\omega\ell^2$ *(B)* $\frac{5}{10}B\omega\ell^2$

(C) zero *(D)* $\frac{1}{2}B\omega\ell^2$

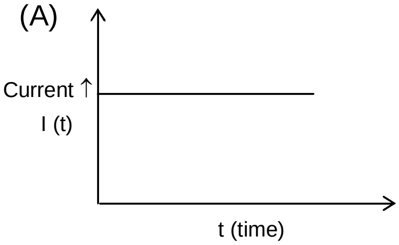
Solution: **(A)**

$$\varepsilon = \int_{-\ell/4}^{3\ell/4} B\omega x dx = \frac{1}{4}B\omega\ell^2$$

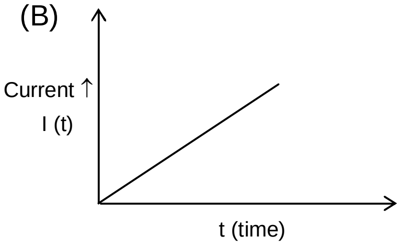
Illustration 13. *Two infinitely long conducting parallel rails are connected through a capacitor of capacitance C as shown in the figure. A conductor of length ℓ is moved with constant speed v_0. Which of the following graph truly depicts the variation of current through the conductor with time ?*



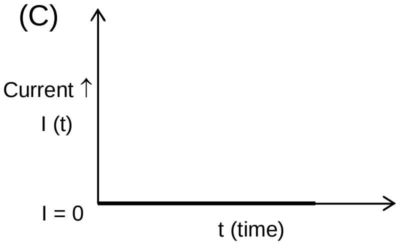
(A)



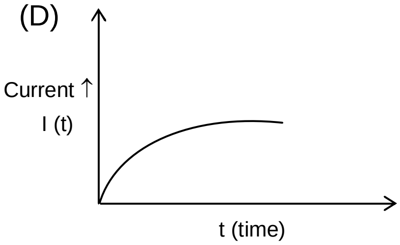
(B)



(C)



(D)



Solution: **(C)**

$q = C\,(B\ell v_0) = \text{const.}$

$\Rightarrow$ q is constant

$$I = \frac{dq}{dt} = 0$$

$\therefore$ Current is zero.

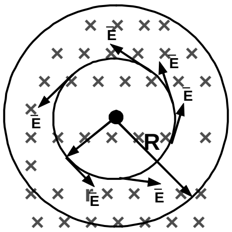
Fleming's right hand rule for direction of induced emf

Stretch your right hand thumb, the index finger and middle finger such that all the three are mutually perpendicular to each other. If the thumb represents the direction of the motion of conductor, the index finger the direction of magnetic field, then the middle finger represents the direction of the current.

Time varying magnetic field

If a conducting loop is placed in a time varying magnetic field, this changing magnetic field acts as a source of electric field and hence induces an emf; infact the electric field is induced even when no conductor is present. Faraday discovered that

$$\oint \vec{E}.\, d\vec{\ell} = -\frac{d\phi}{dt}$$



This field $\vec{E}$ differs from an electrostatic field, it is non–conservative. We can not define the potential corresponding to this field in the usual sense i.e..$dV = \vec{E}.d\vec{r}$ does not hold here.

$\vec{E}$ has to have a direction shown when $\vec{B}$ is increasing, because $\oint \vec{E}.d\vec{\ell}$ has to be negative when $\frac{d\phi}{dt}$ is positive.

Exercise 4. ***(i) Can electric lines of force ever be closed curves?***

(ii) Consider a cylindrical magnetic field which is increasing with time. How are the electric field at points P and Q related?

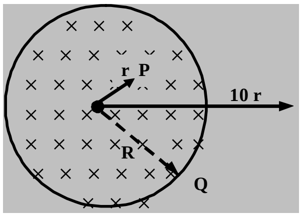
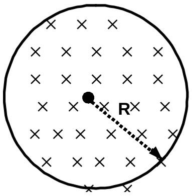
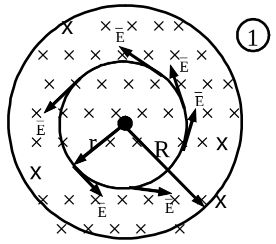


Illustration 14. *Consider a cylindrical magnetic field which increases with time. Find out the electric field at a distance r from its centre (i) r < R, and (ii) r > R.*



Solution : **(i)** Consider (r < R) or loop 1.



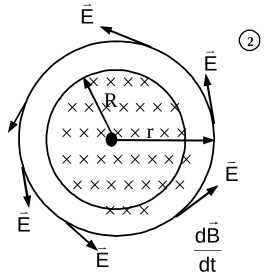
E at all points in the loop has the same value due to symmetry.

$$\text{Using} \quad \oint \vec{E}.d\vec{l} = \frac{-d\Phi}{dt}$$

$$E(2\pi r) = -(\pi r^2)\frac{dB}{dt}$$

$$\text{or} \qquad E = -\frac{r}{2}\frac{dB}{dt} \qquad \text{(for cylindrical region only)}$$

(ii) Consider (r > R) or loop 2.



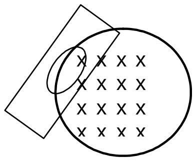
E at all points on the loop has the same value from symmetry.

$$\text{Using,} \ \oint \vec{E}.d\vec{l} = \frac{-d\Phi}{dt}$$

$$\text{or } E.(2\pi r) = -\pi R^2.\frac{dB}{dt} \Rightarrow E = -\frac{R^2}{2}\left(\frac{dB}{dt}\right).\frac{1}{r}$$

(for cylindrical region only)

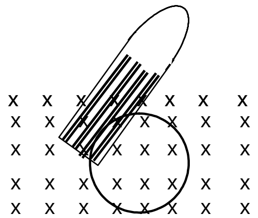
Eddy currents: If a metal plate, e.g. copper is passed through a magnetic field (see figure). During entry into the field and exit from the field, the magnetic flux through a loop (Consider an arbitrary loop as shown in the figure) changes. This change in flux cause current to be set up in the loop. There may be many such loops and currents will flow through them. These are called eddy currents.



Eddy currents flow in many loops in a plate and cause heating. This thermal energy is produced by conversion of kinetic energy and thus the plate slows down. This is called electromagnetic damping.

To avoid eddy currents, slots are cut in the plate due to which the flow of eddy current is broken.

To reduce the losses due to eddy currents, conducting parts are made in large number of thin layers , separated by lacqueran insulator. These are called laminations. These break paths of eddy currents.



Uses of eddy currents:

1. Used for braking systems in trains.
2. For electromagnetic shielding.
3. Used in speedometers
4. Used in induction furnaces

Self Induction

A changing current in a circuit causes a change in the magnetic flux associated with itself, which induces an opposing e.m.f. in it. The net magnetic flux linked with itself is proportional to the current in the loop.

Thus $\phi = Li$

Where L is a constant called coefficient of self–induction or self inductance. Also e.m.f induced in loop is $\varepsilon = -L\frac{di}{dt}$

The induced emf in case of self inductance opposes the change in the current. Physically it is analogous to inertia in mechanics.

Self inductance of an ideal solenoid

For a solenoid of length ℓ and cross–sectional area A having number of turns N, the flux linked is given by $N\phi$, where ϕ is the flux linked by each turn.

$\Rightarrow \quad N\phi = n\ell BA \quad (n = N/\ell)$

The magnetic induction of a solenoid is given by $B = \mu_0 nI \Rightarrow N\phi = \mu_0 n^2\ell AI$

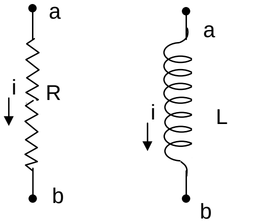
But, $\quad N\phi = LI \Rightarrow L = \mu_0 n^2 \ell A$.

If the inside of a solenoid is filled with a material of relative magnetic permeability μ_r (e.g. soft iron)

then, $L(\mu_r) = \mu_r \mu_0 n^2 A\ell$

Self inductance of a coil depends on its geometry and on μ_r of the medium inside it.

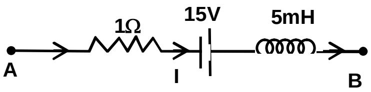
Note: When a current flows from a to b through a resistor, V_{ab} is always positive; the potential is higher at a than b. When a current flows from a to b through an inductor of negligible resistance, V_{ab} is positive for an increasing current, negative for a decreasing current, and zero for a constant current.



$V_{ab} = iR$ $\quad V_{ab} = L(di/dt)$

Exercise 5. ***Does the coefficient of self induction depend upon the rate at which the current is changing through it?***

Illustration 15. *The network shown in the figure is part of a complete circuit. What is the potential difference $V_B - V_A$, when the current I is 5A and is decreasing at a rate of 10^3A/s?*



Solution: In accordance with law of potential distribution, for the given network,

$$V_A - IR + E - L\frac{dI}{dt} = V_B$$

And as here I is decreasing (dI/dt) is negative.

$$V_B - V_A = -5 \times 1 + 15 - 5 \times 10^{-3}(-10^3)$$

$$V_B - V_A = -5 + 15 + 5 = 15 \text{ V}$$

Illustration 16. *An average induced emf of 0.20 V appears in a coil when the current in it is changed from 5.0A in one direction to 5.0A in the opposite direction in 0.20s. Find the self-inductance of the coil.*

Solution: As $\varepsilon = -L\frac{di}{dt}$

$$\frac{di}{dt} = \frac{(-5.0)-(5.0)}{0.20} = -50\,A/s$$

Self inductance $L = -\frac{\varepsilon}{(di/dt)} = -\frac{0.20}{(-50)} = 4$ mH

Illustration 17. *An average emf of 20 V is induced in an inductor when the current in it is changed from 2.5 amp in one direction to the same value in the opposite direction in 0.1 s. The self–inductance of the inductor is*

(A) zero *(B) 200 mH*

(C) 400 mH *(D) 600 mH*

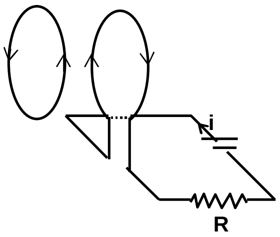
Solution: As $\varepsilon = -L\frac{di}{dt}$

$$\frac{di}{dt} = \frac{(-2.5)-(2.5)}{0.10} = -50\,A/s$$

Self inductance $L = -\frac{\varepsilon}{(di/dt)} = \frac{20}{50} = 400$ mH

Mutual Inductance

A changing current in one circuit causes a changing magnetic flux and an induced emf in a neighboring circuit. The net flux linked with the second circuit is proportional to current in first circuit .



i.e. $N_2\phi_2 = Mi_1$

The proportionality factor is called mutual inductance.

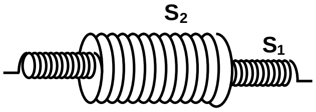
Also $M = \frac{N_2\phi_2}{i_1} = \frac{N_1\phi_1}{i_2}$

Or, $\varepsilon_2 = -M\frac{di_1}{dt}$ & $\varepsilon_1 = -M\frac{di_2}{dt}$

Note : Proceeding in the same way as in self inductance the mutual inductance M of solenoid of length ℓ and area of cross–section A and with number of turns N_1 and N_2 in primary and secondary coils is found to be $M = \mu N_1N_2A/\ell$.

Exercise 6. ***Is it possible to have mutual inductance without self inductance? What about self–inductance without mutual inductance?***

Illustration 18. *A solenoid S_1 is placed inside another solenoid S_2 as shown in the figure. The radii of the inner and the outer solenoids are r_1 and r_2 respectively and the number of turns per unit length are n_1 and n_2 respectively. Consider a length ℓ of each solenoid, calculate the mutual inductance between them.*

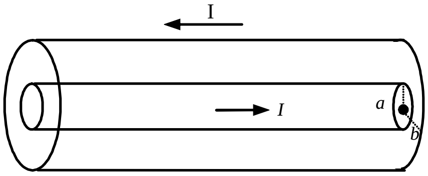


Solution: Suppose a current i is passed through the inner solenoid S_1. A magnetic field $B = \mu_0 n_1 i$ is produced inside S_1 whereas the field outside it is zero. The flux through each turn of S_2 is, $B\,\pi r_1^2 = (\mu_0 n_1 i)\,\pi r_1^2$

The total flux through all the turns in a length ℓ of S_2 is,

$$\phi_2 = (\mu_0 n_1 i \pi r_1^2)\ n_2\ell = (\mu_0 n_1 n_2 \pi r_1^2 \ell)\ i\ ;\ \text{Thus, } M = \mu_0 n_1 n_2 \pi r_1^2 \ell$$

Illustration 19. *What is the self inductance of a system of co–axial cables carrying current in opposite directions as shown? Their radii are 'a' and 'b' respectively.*



Solution: The 'B' between the space of the cables is, $B = \mu_0 I/2\pi r$.
Ampere's law tells that 'B' outside the cables is zero, as the net current through the amperian loop would be zero.
Taking an element of length ℓ and thickness 'dr', the magnetic flux through it is

$$d\Phi = \frac{\mu_o I}{2\pi r}.\ell dr \Rightarrow \quad \Phi = \frac{\mu_o I\ell}{2\pi}\int_a^b \frac{1}{r}dr = \frac{\mu_o \ell I}{2\pi}.\ln\frac{b}{a}$$

$$L = \frac{\Phi}{I} = \frac{\mu_o \ell}{2\pi}\ln\left(\frac{b}{a}\right)$$

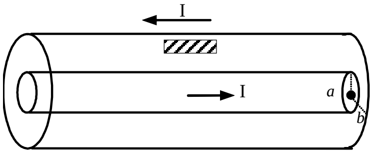
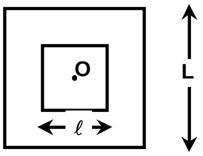


Illustration 20. *The coefficient of mutual induction between the primary and secondary of a transformer is 5 H. Calculate the induced emf in the secondary when 3 ampere current in the primary is cut off in 2.5×10^{-4} second.*

Solution: Induced emf in the secondary $\varepsilon_S = -M\frac{di_P}{dt} = -5\frac{3}{1/4000} = -6\times 10^4$ V

The negative sign merely indicates that the emf opposes the change.

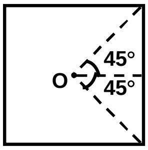
Illustration 21. *A very small square loop of side 'ℓ' is placed inside a large square loop of wire of side L as shown loops are coplanar. What is the mutual inductance of system?*



(A) zero. *(B)* $\frac{\mu_0 \ell^2}{\pi L}$

(C) $\frac{\mu_0 4\sqrt{2}\ell^2}{\pi L}$ *(D)* $\frac{\mu_0 2\sqrt{2}\ell^2}{\pi L}$

Solution: **(D)**
Using relation



$$B_0 = \frac{4\mu_0 I}{4\pi r}\left[\sin\alpha + \sin\beta\right]$$

$$= \frac{\mu_0 I}{\pi r}[\sin 45° + \cos 45°]$$

Also r = L/2

$$B_0 = \frac{\mu_0 I}{\pi L}\left(\frac{2}{\sqrt{2}}\right) = \frac{\mu_0 I 2\sqrt{2}}{\pi L}$$

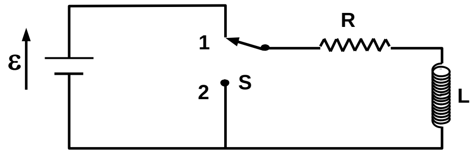
$$\phi = B_0\ell^2 = \frac{\mu_0 I 2\sqrt{2}}{\pi L}\ell^2 = LI = \text{flux through smaller loop}$$

$$\Rightarrow L = \frac{\mu_0 2\sqrt{2}\ell^2}{\pi L}$$

R – L Circuit

Growth of current in an inductor

When current is allowed through an inductor the growing current induces an e.m.f. which opposes the growth of current in the inductor. When the switch is connected to the terminal 1, the current grows in the inductor.



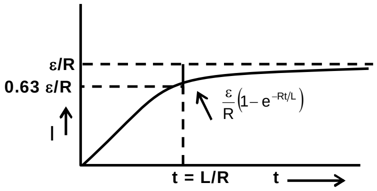
At position 1 of the switch, applying Kirchhoff's law to the closed circuit.

$$-IR - \frac{LdI}{dt} + \varepsilon = 0$$

Solving the differential equation, we get,

$$I = \frac{\varepsilon}{R}\left[1 - e^{\frac{-Rt}{L}}\right], \quad \frac{L}{R} = \tau$$

where τ is called the time constant of the circuit.



Time constant

It measures the rapidity with which the final state or the steady state is approached and may be defined as the time in which the steady state would have been reached if the current were allowed to increase at the initial rate.

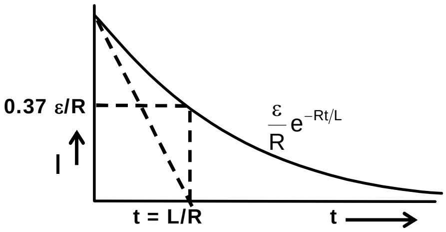
Decay of current in an inductor

As the switch is turned off, i.e. the emf is disconnected the current decays in the inductor through the resistor. It decays as the switch is turned to 2.

Applying Kirchhoff's law to the closed circuit thus formed we have, $\frac{LdI}{dt} + IR = 0$

Solving we get, $I = \left[I_o e^{\frac{-Rt}{L}}\right]$,

Where I_0 = current in the circuit at t = 0



Note: The time constant may also be defined in the same way as we have done in growth

Exercise 7. A real inductor has some resistance. Can it ever be possible that the induced e.m.f. in an inductance be greater than e.m.f. applied across it?

Energy stored in the magnetic field of an inductor

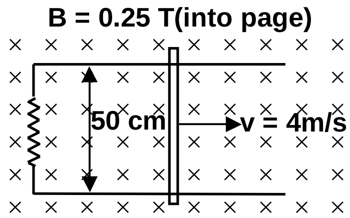
As, $\varepsilon = IR + \frac{LdI}{dt},\ \varepsilon I = I^2R + LI\frac{di}{dt}$

εi is the power supplied by the battery, I^2R is the electrical power dissipated in the resistance and $LI\ \frac{dI}{dt}$ is the rate of energy stored in the inductor. $\Rightarrow$ $\varepsilon Idt = I^2Rdt + LIdI$

$\Rightarrow$ Energy stored in the inductor is $U_B = \int_0^I LIdI = \frac{1}{2}LI^2$

Note: It is important not to confuse with the behavior of resistor and inductors where energy is concerned. Energy flows into a resistor whenever a current passes through it, whether the current is steady or varying and this energy is dissipated in the form of heat. By contrast, energy flows into an ideal, zero resistance inductor only when the current in the inductor increases. This energy is not dissipated; it is stored in the inductor and released when the current decreases. When a steady current flows through an inductor, there is no energy flow, in or out. This energy is associated with magnetic field of the inductor. If the field is in vacuum, the magnetic energy density u (energy per unit volume) is given by, $U = \frac{B^2}{2\mu_0}$.

Illustration 22. *As shown in the figure, a metal rod makes contact with a partial circuit and completes the circuit. The circuit area is perpendicular to a magnetic field with B = 0.25T. If the resistance of the total circuit is 3Ω, what force is needed to move the rod with a constant speed of 4 m/s as indicated in the figure ?*



Solution: The induced emf in the rod causes a current to flow counter clockwise in the circuit. Because of this current in the rod, it experiences a force to the left due to the magnetic field. In order to pull the rod to the right with constant speed, the force must be balanced by the puller.

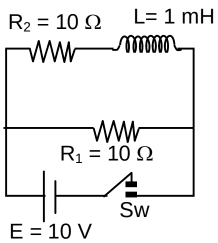
The induced emf in the rod is

$|\varepsilon| = BLv = (0.25)\ (0.5)(4) = 0.5\ V$

$I = \varepsilon/R = 0.5/3\ A$

$F = iLB\sin 90^o = (0.05)(0.5/3)(0.25) = (0.0625/3)N = 0.021\ N$

Illustration 23. *Find the current provided by the source immediately after the switch is closed at t = 0 and also at t = infinity.*



Solution: At t = 0 current through inductor will be zero.

Therefore current provided by the source $I = E/R_1 = 10/10 = 1$ amp.

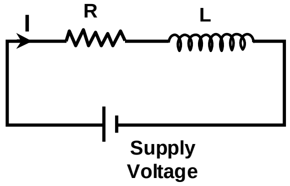
At $t = \infty$, inductor will be shorted.

Therefore current provided by the source $I' = \frac{E}{\frac{R_1R_2}{R_1+R_2}} = 2$ amp.

Illustration 24. *When current (I) in R–L series circuit is constant where L is a pure inductor. The following statements are given*
(i) voltage across R is RI.
(ii) voltage across L is equal to voltage across R.
(iii) voltage across L is equal to supply voltage.
(iv) magnetic energy stored is $\frac{1}{2}Li^2$

(A) (i), (ii) and (iv) are true. *(B) (i), (ii) and (iv) are true.*
(C) (i) and (iv) are true. *(D) (i), (ii), (iii) and (iv) are true.*

Solution: **(C)**



$V_R = RI$

Energy stored in inductor = $\frac{1}{2}LI^2$

Voltage across inductor [v_L] = 0 as here $\frac{dI}{dt} = 0$

Illustration 25. *A solenoid of inductance 100 mH and resistance 20 Ω is connected to a cell of emf 10 V at t = 0. The energy stored in the inductor when the time t = 5 ln2 milliseconds is*

(A) $\frac{1}{640}$ J *(B)* $\frac{1}{320}$ J
(C) $\frac{1}{180}$ J *(D)* $\frac{1}{80}$ J

Solution: **(B)**

The current at any time can be given by the expression, $i = i_0\,(1 - e^{-t/\tau})$

Where $\tau = \frac{L}{R} = \frac{100\text{mH}}{20\Omega} = 5 \times 10^{-3}$ s

$$i = 0.5\text{A}\left(1 - e^{-\frac{5\ln 2}{5}}\right)$$

$$= i_o\left(1 - \frac{1}{e^{\ln 2}}\right) = i_o\left(1 - \frac{1}{2}\right) = \frac{i_o}{2}$$

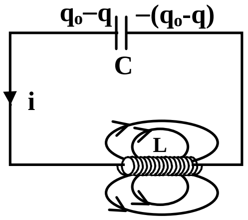
Where $i_0 = \frac{10\text{V}}{20\Omega} = 0.5\text{A}$

$\therefore$ The energy stored = $\frac{1}{2}Li^2$

$$= \frac{1}{2} \times \left(100 \times 10^{-3}\right)\left(\frac{1}{4}\right)^2 = \frac{1}{320}\text{ J.}$$

L.C. Oscillations

A capacitor is charged to a p.d. of V_0 by connecting it across a battery and then it is allowed to discharge through a pure inductor of inductance L.

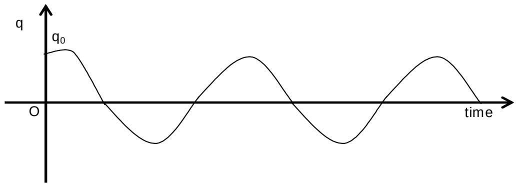


At any instant, let the charge on the capacitor be q and current in the circuit be i.

$$\frac{q^2}{2C}+\frac{1}{2}Li^2 = \text{const} \qquad (1)$$

Differentiating w. r. t. time we get

$$-\frac{1}{2C}.2q\ \frac{dq}{dt}+\frac{1}{2}L\ \ 2i\frac{di}{dt^2}=0$$



i.e. $\frac{d^2q}{dt^2}=-\frac{1}{LC}q = -\ \omega^2\ q$

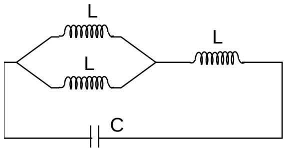
(where, $\omega = 2\pi f$)

which gives f = $\frac{1}{2\pi\ \sqrt{LC}}$

$\therefore$ Current i in the circuit and the charge q on the plates of the capacitor vary sinusoidally as

$i = i_o \sin \omega t$; $q = q_o \cos \omega t$.

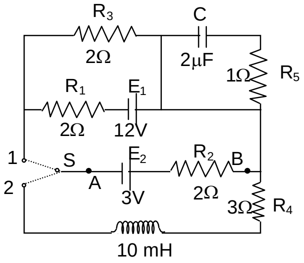
Illustration 26. *(a) Find the frequency of the LC circuit shown in figure*



(b) A circuit containing a two position switch S is shown in figure.

(i) The switch S is in position 1. Find the potential difference $V_A - V_B$ and the rate of production of joule heat in R_1.

(ii) If now the switch S is put in position 2 at t = 0, find the time when the current in R_4 is half the steady value. Also calculate the energy stored in the inductor L at that time.



Solution: (a) Equivalent Inductance = $\frac{L.L}{L+L} + L = \frac{3}{2}L$

$$\Rightarrow \quad f = \frac{1}{2\pi(\sqrt{3/2})LC}\ m$$

(b) (i) Applying kirchhoff's second law to meshes (1) and (2)

$2I_2 + 2I_1 = 12 - 3 = 9$

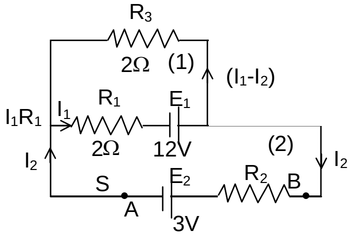
and $2I_1 + 2(I_1 - I_2) = 12$

Solving $I_1 = 3.5$ A, $I_2 = 1$A

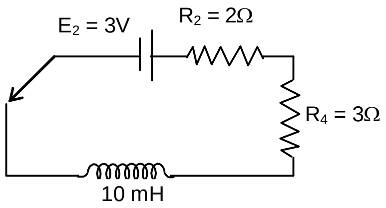
P.D.between AB $= 2I_2 + 3 = 5$ volt

Rate of production of heat

$$\left(\frac{dQ}{dt}\right) = i_1^2R_1 = 24.5\ \text{J}$$



(ii) $i = \frac{E}{R}\left[1 - e^{-Rt/L}\right] = i_0\left[1 - e^{-Rt/L}\right]$



$i = i_0/2 \quad \Rightarrow \frac{Rt}{L} = \log_e 2$

$\Rightarrow t = 0.0014$ sec.

Energy stored $= \frac{1}{2} Li^2 = 0.00045$

J.

ALTERNATING CURRENT

Voltage and Currents in AC Circuits

Up to now we have considered voltage source and current in one direction only. In many cases we come across situations where the direction of current changes with time and the source provides voltage varying with time. One such case is when voltage and current vary like a sine function with time. These are called alternating voltage (a.c. voltage) and alternating current (a.c. current). Electricity supply provided at our homes and offices fall in this category. Main advantage of using this a.c. voltage and a.c. current is that a.c. voltage can easily be converted to lower or higher value by use of transformers and these can be economically transmitted over long distances.

Mean Value of Voltage and Current

The mean value of sinusoidal current or voltage in one complete cycle is zero. For half cycle, the mean value can be found as given below.

$$I = I_0 \sin \omega t;\ \ I_{mean} = \left[\frac{\int_0^{T/2} I\, dt}{\int_0^{T/2} dt}\right] = \frac{1}{T/2}\left[-\frac{I_0}{\omega}\cos\omega t\right]_0^{T/2}$$

$$= \frac{2}{T}\frac{I_0 T}{2\pi}\left[1-(-1)\right] = \frac{2I_0}{\pi}\text{, Similarly } v_{mean} = \frac{2V_o}{\pi}$$

Root Mean square value of voltage and current (V_{rms} and I_{rms})

$$I = I_0 \sin \omega t;\qquad I_{rms} = \left[\frac{\int_0^{T} I^2 dt}{\int_0^{T} dt}\right]^{1/2}$$

$$I_{rms}^2 = \frac{1}{T}\int_0^T I_0^2 \sin^2 \omega t dt = \frac{I_0^2}{2T}\int_0^T (1-\cos 2\omega t)dt$$

$$= \frac{I_0^2}{2T}\left[T - \frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{I_0^2}{2}$$

$$\therefore\ I_{rms} = \frac{I_0}{\sqrt{2}}\text{, similarly } V_{rms} = \frac{V_0}{\sqrt{2}}.$$

Exercise 8. ***What is the value of current measured by a hot wire ammeter attached to a.c. supply (peak value, rms value or mean value)?***

Impedance

In any circuit the ratio of the effective voltage to the effective current is defined as the impedance Z of the circuit. Its unit is ohm.

Phasors and phasor diagrams

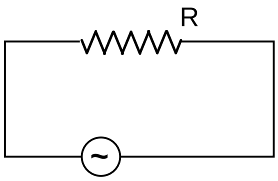
In the study of A.C. circuits, we shall come across alternating voltage and currents which have the same frequency but differ in phase with each other. It is found that the study of A.C. circuits becomes simple, if alternating currents and voltages are treated as rotating vectors or more correctly as 'phasors'. The phase angle between the two quantities is also represented in the vector diagram.

A diagram representing alternating voltage and current as vectors with the phase angle between them is known as phasor diagram.

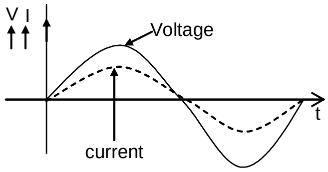
AC circuit with a resistor

Instantaneous current

$$i = \frac{E}{R} = \frac{e_0}{R}\sin\omega t$$
$$= i_0 \sin\omega t$$



$e = e_0 \sin\omega t$



where $i_0 = \frac{e_o}{R}$ = current amplitude. Thus the voltage and the current in an A.C. circuit containing pure resistance are in phase.

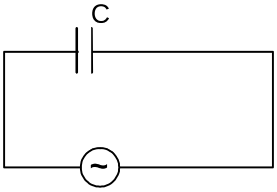
AC circuit with a capacitor

Instantaneous charge on the Capacitor

$Q = CV = Ce_0 \sin\omega t$, $I = \frac{dQ}{dt} = \omega\, Ce_0 \cos\omega t = i_0 \cos\omega t$

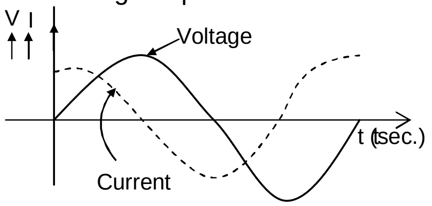
$= i_0 \sin(\omega t + \pi/2)$ where $i_0 = \omega\, Ce_0 = \frac{e_0}{1/\omega C} = \frac{e_0}{X_c}$

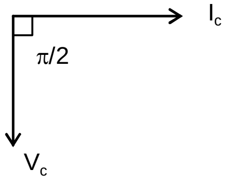
where $X_c = \frac{1}{\omega C}$ is known as capacitive reactance.



$e = e_0 \sin\omega t$

The following diagrams show graphical representation and phasor treatment of current and voltage illustrating the phase difference between them.





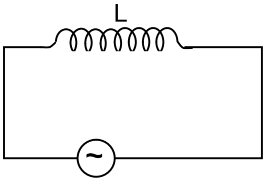
In capacitor, voltage lags the current or the current leads the voltage by $\pi/2$

AC circuit with an inductor

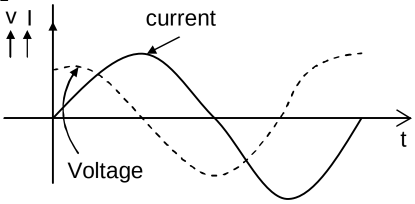
L = inductance in an ac circuit, $e = e_0 \sin(\omega t) = L\frac{di}{dt}$

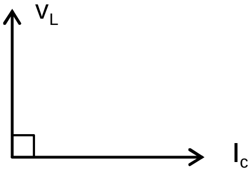
$i = -\frac{e_0}{L\omega}\cos(\omega t) + c = -i_0 \cos(\omega t) + c$

$= i_0 \sin(\omega t - \pi/2)$ where $i_0 = \frac{e_0}{\omega L} = \frac{e_0}{X_L}$



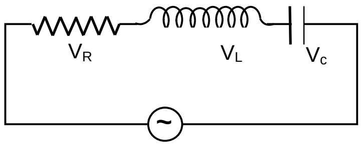
[$\because$ Voltage is sinusoidal so current should be also sinusoidal so c = 0)
where X_L is known as inductive reactance

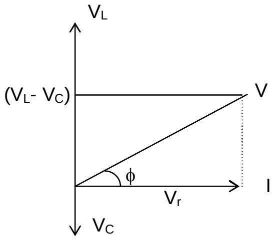




In an inductor voltage leads the current by $\pi/2$

L–C–R Circuit





$\vec{V} = \vec{V}_R + \vec{V}_L + \vec{V}_C$

$V = \sqrt{(V_L - V_C)^2 + V_R^2} = I\sqrt{(X_L - X_C)^2 + R^2}$,

where $X_L=\omega L$ & $X_C= \dfrac{1}{\omega C}$

$\Rightarrow V = IZ$, where $Z = \sqrt{(X_L - X_C)^2 + R^2}$ is known as the impedance of the circuit.

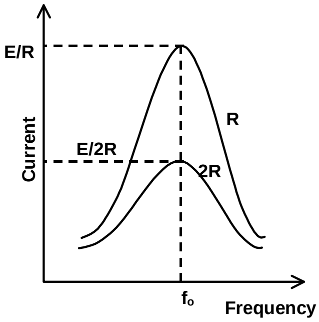
Phase angle ϕ, $\tan\phi = \dfrac{V_L - V_C}{V_R} = \dfrac{X_L - X_C}{R}$

Let us study the phase relationship between current and e.m.f. in L.C.R. series circuit in the following case

1. Where $\omega L > \dfrac{1}{\omega C}$, it follows that $\tan\phi$ is positive, i.e. ϕ is positive. Hence, in such a case, voltage leads the current.
2. When $\omega L < \dfrac{1}{\omega C}$, it follows that $\tan\phi$ is negative, i.e. ϕ is negative. Hence in such a case, voltage lags behind the current.
3. When $\omega L = \dfrac{1}{\omega C}$, it follows that $\tan\phi$ is zero, i.e. ϕ is zero. Hence in such a case, current and voltage are in phase with each other.
4. In fact, when $\omega L = \dfrac{1}{\omega C}$, the impedance of the circuit would be just equal to R (minimum). In other words, the LCR–series circuit will behave as a purely resistive circuit. Due to the minimum value of impedance, the current in LCR–series circuit will be maximum. This condition is known as resonance.

   $\Rightarrow \omega_o = \dfrac{1}{\sqrt{LC}}$ where $\omega = \omega_o$ resonant frequency; $f_o = \dfrac{1}{2\pi\sqrt{LC}}$

If R, L and C are constant, and the frequency f of the applied emf is raised continuously from zero, the peak current varies as shown in figure. At first the current is very small, increases to maximum when the frequency increase to the resonance value f_o, and then falls again.



It is interesting to note that before resonance the current leads the applied emf, at resonance it is in phase, and after resonance it lags behind the emf. Series resonant circuit is also called acceptor circuit.

Illustration 27. *An LCR circuit has L = 20 mH, C = 50 μF and R = 10 Ω connected to a power source of 100 sin 400 t volt. Find out the rms value of current in the circuit.*

Solution: $L = 20 \times 10^{-3}$ H, $C = 50 \times 10^{-6}$ F and R = 10 Ω, ω = 400 rad/sec.

Impedance

$$z = \sqrt{(400\times 20\times 10^{-3} - \frac{1}{400\times 50\times 10^{-6}})^2 + 10^2} = 43.17\ \Omega$$

$$i_{rms} = \frac{100}{\sqrt{2}}.\frac{1}{43.17} = 43.17\text{ A} = 1.64\text{ A}$$

POWER

In an a.c. circuit the instantaneous power is the product of the instantaneous value of the current and the voltage. $e = e_0 \sin \omega t$; ; $I = i_0 \sin(\omega t - \phi)$ $-90^0 \le \phi \le 90^0$

$P = e_0\, i_0 \sin \omega t \sin(\omega t - \phi)$

$$P_{av} = \frac{\int_0^T P dt}{\int_0^T dt} = \frac{1}{2}E_0 I_0 \cos\phi = E_{rms}\ I_{rms}\ \cos\phi = \text{apparent power x } \cos\phi$$

where cos ϕ is known as power factor.

Power factor in L–C–R circuit is $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

If $\cos\phi = 0$, or $\phi = \pi/2$ $\Rightarrow P_{av} = 0$

i.e. when the current and the voltage differ in phase by 90°,

Under this condition, the current is known as wattless current, because the average power consumed in the circuit is zero.

Exercise 9. Through what a.c. components a wattless current can be attained?

Illustration 28. *A series LCR circuit containing a resistance of 120 Ω has angular resonance frequency 4 $\times$ 10^5 rad s^{-1}. At resonance the voltages across resistance and inductance are 60 V and 40 V respectively. Find the values of L and C. At what frequency the current in the circuit lags the voltage by 45^0 ?*

Solution : At resonance as X = 0, $I = \frac{V}{R} = \frac{60}{120} = \frac{1}{2}$A and $V_L = IX_L = I\omega L$,

$$L = \frac{V_L}{I\omega} = \frac{40}{(1/2)\times 4\times 10^5} = 2 \times 10^{-4}\text{ Henry}$$

at resonance, $\omega L = \frac{1}{\omega C}$ so $C = \frac{1}{\omega^2 L}$ i.e.,

$$C = \frac{1}{0.2\times 10^{-3}\times(4\times 10^5)^2} = \frac{1}{32}\mu F$$

Now in case of series LCR circuit, $\tan\phi = \frac{X_L - X_C}{R}$

For the current to lag the applied voltage by 45^0

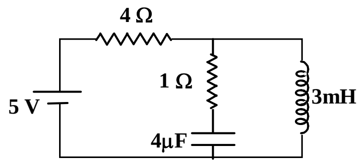
$$\tan 45 = \frac{\omega L - \frac{1}{\omega C}}{R}$$

i.e., $1 \times 120 = \omega \times 2 \times 10^{-4} - \frac{1}{\omega(1/32)\times 10^{-6}}$

i.e., $\omega^2 - 6 \times 10^5\, \omega - 16 \times 10^{10} = 0$

i.e., $\omega = \frac{6\times 10^5 + 10\times 10^5}{2} = 8 \times 10^5 \frac{rad}{s}$.

Illustration 29. *In the figure shown the steady state current through the inductor will be*



(A) zero *(B) 1A*
(C) 1.25 A
(D) can not be determined

Solution: **(C)**

At steady state, current through the capacitor = 0
Therefore the current through the inductor = 5/4 = 1.25 A.

Illustration 30. *A coil of inductance L = 300 mH and is connected to a constant voltage source. Current in the resistance R = 140 mΩ coil will reach to 50% of its steady state value after t is equal to*

(A) 15 s *(B) 0.75 s*
(C) 0.15 s *(D) 1.5 s*

Solution: **(D)**

At steady state $i = \varepsilon/R$

At any time t, $I = \varepsilon/R\,(1 - e^{-Rt/L}) = \varepsilon/2R$

$\therefore\ t = \frac{L}{R}\ln 2 = 1.5\,\text{sec}.$

CHOKE COIL

A coil with low resistance and high inductance can reduce the current in an a.c. circuit without an appreciable heat loss. Such coils are called choke coils. Choke coil is preferred as it does not dissipate power. Physically a choke coil is a coil of insulated copper wire which offers a high reactance ($L\omega$) to A.C. (but a low D.C. resistance), thus reducing the A.C. appreciably without the loss of energy.

In L–R circuit, current amplitude $I_o = \frac{e_o}{Z} = \frac{e_o}{\sqrt{R^2 + \omega^2 L^2}}$

The average power consumed in A.C. circuit is $P = \frac{E_0 i_0}{2}\cos\phi$

And the power factor is given by $\cos\phi = \frac{R}{\sqrt{(L^2\omega^2 + R^2)}}$

The inductance 'L' of the choke coils is quite large on account of the large number of turns and the high permeability of iron core, while the resistance R is very small. Thus $\cos\phi \approx 0$, therefore the power absorbed by the coil is extremely small. Thus the choke coil reduces current without any appreciable loss of energy. The wastage of energy is only due to the hysteresis loss in soft iron core. Eddy current loss is reduced by laminating the soft iron core.

A.C. GENERATOR

An AC generator is a machine which converts mechanical energy to electrical energy based on electromagnetic induction.

Principle:

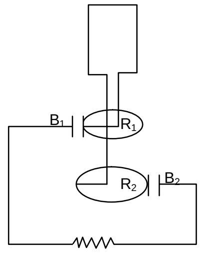
When the magnetic flux through a closed loop changes, it induces an emf in the coil. It lasts as long as the change in flux continues. A coil forming a closed loop is rotated in a magnetic field and change in flux due to rotation of coil produces induced emf.

Construction:

1. Armature: Armature is a rectangular coil. It consists of a large no. of turns of insulated copper wire wound over a soft iron core. The soft iron core is used to increase the magnetic flux.

2. Field Magnet: A strong electromagnet is provided with a magnetic field of the order of 1–2 tesla. It has concave north and south poles. The armature rotates between these poles.

3. Slip Rings: These are two hollow metallic rings. Ends of the armature coil are connected to these two rings R_1 and R_2 separately. These provide moving contact hence these are called slip rings.



4. Brushes: Graphite brushes B_1 and B_2 keep contact with slip rings $R_1 \propto R_2$ and pass on current from the armature coil to the external resistance load.

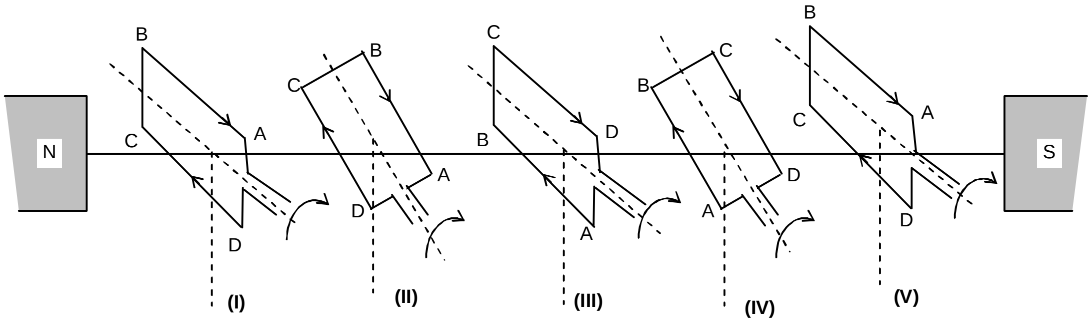
Working:

As the coil rotates between the N– S poles, magnetic flux linked with the coil changes. When normal to the coil area is parallel to the magnetic field lines, flux through it is maximum but rate of change of flux is minimum hence induced emf is zero.

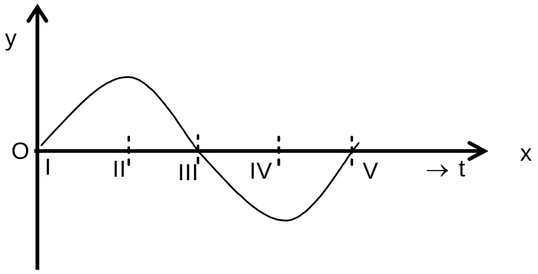
In the position where normal to the coil area is perpendicular to the magnetic field lines, rate of change of flux is maximum as coil wire move perpendicular the field lines. Hence at this position, induced emf is maximum.

The direction of induced emf can be determined by Fleming's right hand rule.

After half the rotation, the directions of current changes as the motion of arms are just opposite to the motion in first half. Applying Flemings right hand rule, the current direction is reversed, position III to V as compared to position I to III.



(Figure1: Variation of induced emf with time (I, II, III and IV are positions of coil as shown in the figure I)



(figure –2)

If the magnetic field is B, the angular of the coil is ω (i.e. $\frac{d\theta}{dt} = \omega$), there are n no. of coils and area of coils is A, magnetic flux.

ϕ = nBA cos θ = nBA cosωt $\quad\therefore\quad \frac{d\phi}{dt} = -nBA.\omega \sin\omega t$

Induced emf, $e = -\frac{d\phi}{dt}$

$e = nBA\omega \sin \omega t = e_0 \sin \omega t$

Here $e_0 = nBA\omega$

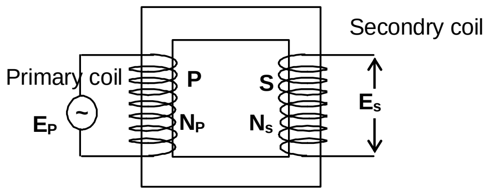
$e_{max} = nBA\,\omega$

Current $i = \frac{e}{R} = \frac{e_0}{R}\sin\omega t = i_0 \sin\omega t$ (here $i_0 = e_0/R$)

TRANSFORMER

A transformer is an electrical device used to convert AC current from low voltage & high current to high voltage and low current and vice–versa.

A transformer which increases the ac voltage is called step up transformer and which decreases the AC voltage is called step down transformer.



N_P. = No. of turns, of primary coil

N_s = No. of turns of secondary coil

Principle:

A transformer works on the principle of mutual induction. When two coils are indirectely coupled, change in current or magnetic flux in one coil induces an emf in the other coil.

Construction:

Two coils Primary P and secondary S are wound around a soft iron core. A.C. input is applied to the primary coil and A.C. output is taken at the secondary coil. Laminated soft iron sheets are used in core to minimise eddy current and increase magnetic flux.

Step up transformers: Number of turns in secondary coil are more than those in the primary coil
($N_s > N_p$). It converts a low voltage high current to a high voltage low current.

Step down transformer: Number of turns in secondary coil are less than those in the primary coil. It converts a high voltage, low current to a low voltage, high current.

Working

When primary coil is connected to AC source, the magnetic flux linked with the primary changes. The magnetic flux of primary is passed through the secondary through the iron core. Therefore magnetic flux through secondary also changes. Due to the change in magnetic flux the induced emf is produced across the ends of secondary coil.

N_p=no of turns in primary

N_s=no of turns in secondary

E_p=voltage across primary

E_s=voltage across secondary

Since magnetic flux in directly proportional to no. of turns

$\phi \propto N$

$$\frac{\phi_s}{\phi_p} = \frac{N_s}{N_p}$$

$$\phi_s = \frac{N_s}{N_p} \times \phi_p$$

$$E_s = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}\left(\frac{N_s}{N_p} \times \phi_p\right) = -\frac{N_s}{N_p}\left(\frac{d\phi_p}{dt}\right)$$

$$E_s = \frac{N_s}{N_p} \times E_p$$

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

If $N_s < N_p$ then $E_s < E_p$, such type of transformer is called step down transformer.

Energy Losses in a Transformer:

Various types of losses are

(a) **Flux losses:** – Flux of primary does not get 100 % linked up with secondary coil.

(b) **Copper losses:** – Energy lost as heat due to resistance in copper coils.

(c) **Iron Losses:** – Loss as heat in iron core due to eddy current losses.

(d) **Hysteresis losses:** Energy lost due to repeated magetization and demagnetization of iron core.

(e) **Humming losses:** Due to alternating current, the iron core vibrates producing humming noise. Some energy is lost in this.

Use of transformers:

Use of transformer are

1. Transmission of AC over long distances: Transmission losses are less when transmitted at very high voltage.
2. In induction furnaces – to heat metallic parts.
3. For welding where low voltage high current is required.
4. Voltage regulators in electrical devices.

SUMMARY

Electromagnetic induction: The production of electromotive force in a conductor when there is change in the magnetic flux linkage with a coil or when there is a relative motion of the conductor across a magnetic field.

Magnetic flux: Magnetic flux through an area $d\vec{S}$ in a magnetic field $\vec{B}$ is

$$\phi_B = \int \vec{B}.d\vec{S}$$

Faradays Law: When the flux of a magnetic field through a loop changes, an emf is induced in the loop which is given by

$$\varepsilon = -\frac{d\phi}{dt}$$

where $\phi = \int \vec{B}.d\vec{S}$ is the flux through the loop.

This emf lasts as long as the magnetic field changes.

Lenz's law: Lenz's law states that the polarity of the induced emf and the direction of the induced current is such that it opposes the change that has induced it.

Motional emf: When a conducting rod of length ℓ moves with a constant velocity v in a magnetic field B such that B, v and ℓ are mutually perpendicular then the induced emf is

$$\varepsilon = B\ell v$$

This is called motional emf. If the circuit is completed the direction of the current can be worked out by Lenz's law.

Induced Electric field: A time varying magnetic field induces an electric field. If a conductor is placed in this field, an induced emf is produced.

$\varepsilon = \oint \vec{E}.d\vec{\ell}$

also, $\varepsilon = -\dfrac{d\phi}{dt}$

or, $\oint \vec{E}.d\vec{\ell} = -\dfrac{d\phi}{dt}$

Eddy current: Changing magnetic field induces currents in closed loops of irregular shapes in a conductor. These are called eddy currents. These dissipate thermal energy at the cost of kinetic energy, thus causing electromagnetic damping.

Self induction: A current carrying loop produces a magnetic flux through the area

$\phi = Li$ where L is self inductance of the loop.

If current changes in the loop the induced emf ε is given by

$$\varepsilon = -\frac{d\phi}{dt} = -L\frac{di}{dt}$$

For a solenoid of n turns,

$$\varepsilon = -n\frac{d}{dt}\int \vec{B}.d\vec{s}$$

self inductance of a long solenoid is

$$L = \mu_r\mu_0 n^2 \ell A$$

where μ_r is relative magnetic permeability of the core, n is no. of turns per unit length, ℓ is the length of the solenoid and A is the cross–sectional area of the solenoid.

Mutual inductance: A changing current in one circuit causes a changing flux and hence an induced emf in a neighboring circuit.

$N_2\phi_2 = Mi_1$

where M is mutual inductance of ciols 1 $\propto$ 2.

$$M = \frac{N_2\phi_2}{i_1} = \frac{N_1\phi_1}{i_2}$$

or, $\varepsilon_2 = -M\frac{di_1}{dt}$, $\varepsilon_1 = -M\frac{di_2}{dt}$

Alternating voltage & Alternating current: Voltage and current varying sinusoidally with time are called alternating voltage (Alternating voltage) and alternating current (Alternating current)

$V = V_0 \sin\omega t$

$I = I_0 \sin \omega t$

Mean values of voltage and current:

(a) In one complete cycle,

$$\bar{V} = 0,\ \bar{I} = 0$$

(b) In half cycle,

$$\bar{V} = \frac{2V_0}{\pi},\ \bar{I} = \frac{2I_0}{\pi}$$

(c) Root mean square values

$V_{rms} = \frac{V_0}{\sqrt{2}}, I_{rms} = \frac{I_0}{\sqrt{2}}$ where v_0 and I_0 are the peak voltage and current.

AC circuit with a resistor: $E = E_0 \sin \omega t = \frac{E_0}{R}\sin\omega t$

Resistance: R

AC circuit with a capacitor: $V = E_0 \sin\omega t$

$I = \frac{E_0}{1/\omega C}\sin(\omega + \pi/2)$ (voltage lagging)

$x_c = \frac{1}{\omega C}$ is capacitive reactance.

AC circuit with an inductor: $V = E_0 \sin \omega t$

$I = I_0 \sin (\omega t - \pi/2)$ (voltage leading)

$X_c = \omega L$ inductive reactance

LCR circuit: Impedance, $z = \sqrt{(\omega L - \frac{1}{\omega C})^2 + R^2}$

for phase angle ϕ, $\tan\phi = \frac{X_L - X_C}{R}$

Resonance:

$\omega L = \frac{1}{\omega c}$

the impedance of the circuit is equal to R only.

$\omega_0 = \frac{1}{\sqrt{LC}}$

$f_0 = \frac{1}{2\pi\sqrt{LC}}$ (resonant frequency)

Power:

Average power, $P_{av} = E_{rms}\, I_{rms} \cos\phi = \frac{1}{2} E_0 I_0 \cos\phi$

where $\cos \phi$ is power factor

Power factor in LCR circuit,

$$\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{(\omega L - \frac{1}{\omega C})^2 + R^2}}$$

Choke coil:

A coil with low resistance and high inductance used to reduce current in AC circuit without much heat loss is called a choke coil.

Power factor, $\cos\phi = \frac{R}{\sqrt{\omega^2 L^2 + R^2}}$

R is low and L is high thus reducing current.

MISCELLANEOUS EXERCISE

1. Define magnetic flux. What is the magnetic flux through an area A kept at an angle θ from the magnetic field B ?

2. Describe Faraday's Law of Induction.

3. What is Lenz's law ? Explain with an example.

4. Explain how motional emf is generated starting with the force acting on a moving charge. How do you decide the polarity of the induced emf depending on the direction of motion of conductor and the direction of magnetic field.

5. Can we define a potential associated with an induced electric field due to a changing magnetic field? Is the induced electric field a conservative field ?

6. Describe an expression for self induction of a solenoid of length ℓ, cross–sectional area A and having n turns per unit length.

7. Derive an expression for the energy stored in the magnetic field of an inductor of inductance L.

8. Define resonance in an LCR circuit. Why is current maximum at resonance ?

9. Derive an expression for power factor in an LCR circuit. What is wattless current?

10. Explain how a choke reduces current without appreciable loss of energy.

SOLVED PROBLEMS

Subjective:

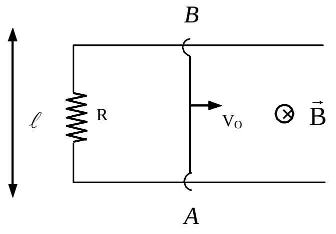
BOARD TYPE

Prob 1 . *A connecting rod AB of mass m slides without friction over two long conducting rails separated by a distance ℓ.*

Initially, the rod is moving with a velocity v_0 to the right. Find :

(a) The distance covered by the rod until it comes to rest.

(b) The amount of heat generated in the resistance R during the process.



Sol. (a) $\varepsilon_t = B\ell v_t$; $I_t = \frac{\varepsilon_t}{r} = \frac{B\ell v_t}{R}$; F(retarding) $= I_t \ell B = \frac{B^2\ell^2 v_t}{R}$

$$\Rightarrow \frac{-mdv_t}{dt} = \frac{B^2\ell^2 v_t}{R} \qquad \Rightarrow \frac{mdv_t}{ds}\frac{ds}{dt} = -\frac{B^2\ell^2}{R}v_t$$

$$\Rightarrow ds = \frac{-Rm}{B^2\ell^2}.dv \qquad \Rightarrow \int ds = -R\frac{m}{B^2\ell^2}\int_{v_0}^{0} dv$$

$$\Rightarrow s = \frac{Rmv_0}{B^2\ell^2}$$

(b) Heat generated = loss in K.E. = $\frac{1}{2}mv_0^2$.

Prob 2 . *A closed circular coil having a diameter of 50 cm made of 200 turns of wire with a total resistance of 10Ω is placed with its plane at right angles to a magnetic field of strength 10^{-2} tesla. Calculate the quantity of electric charge passing through it when the coil is turned through 180° about an axis in its plane.*

Sol. Area of the coil = A = $\pi r^2 = \pi(0.25)^2 m^2$

Number of turns of coil = n = 200; Field strength = B = 10^{-2} Wbm^{-2}

Since the plane of the coil is perpendicular to the magnetic field, the magnetic flux through the coil is ϕ_1 = nAB when the coil is turned through 180° about an axis in its plane, the magnetic flux through the coil is ϕ_2 = –nAB.

$\therefore$ Change in the magnetic flux = $d\phi = |\phi_2 - \phi_1|$

$\therefore$ $d\phi = 2nAB = 2 \times 200 \times \pi \times (0.25)^2 \times 10^{-2} = \frac{\pi}{4}$ Wb

$\therefore$ Quantity of electric charge = $\frac{\text{change of magnetic flux}}{\text{resistance}} = \frac{\frac{\pi}{4}}{R} = \frac{\pi}{4R}$

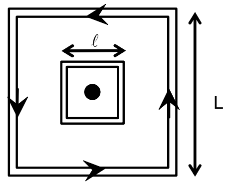
It is independent of the time taken to turn the coil.

Resistance of the coil = 10Ω

$\therefore$ Quantity of electric charge passing through the coil = $\frac{\pi}{4\times 10}$ =0.078 C

Prob 3. *A small square loop of wire of side ℓ is placed inside a large square loop of wire of side L (>>ℓ). The loops are coplanar and their centres coincide. What is the mutual inductance of the system?*

Sol. Considering the larger loop to be made up of four rods each of length L , the field at the centre, i.e., at a distance L/2 from each rod, will be



$$B = 4x\frac{\mu_0 I}{4\pi d}\left[\sin\alpha + \sin\beta\right]$$

i.e. $B = 4x\frac{\mu_0 I}{4\pi(L/2)}2\sin 45^0$

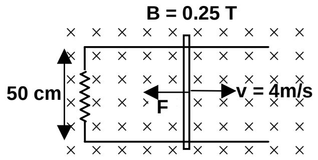
i.e. $B_1 = \frac{\mu_0 8\sqrt{2}}{4\pi L}I$

So the flux linked with smaller loop

$$\phi_2 = B_1 S_2 = \frac{\mu_0}{4\pi}\frac{8\sqrt{2}}{L}\ell^2 I$$ and

hence, $M = \frac{\phi_2}{I} = 2\sqrt{2}\frac{\mu_0 \ell^2}{\pi L}$

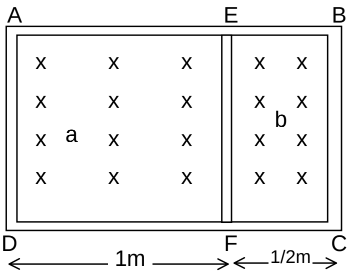
Prob 4 . *A conducting rod makes contact with a partial circuit and completes the circuit as shown. The circuit area is perpendicular to a magnetic field with B = 0.25 T. If the resistance of the total circuit is 5 Ω, how large force is needed to move the rod as indicated with a constant speed of 4 m/s apart from the force F = 1/80N already acting on it in the direction shown ?*

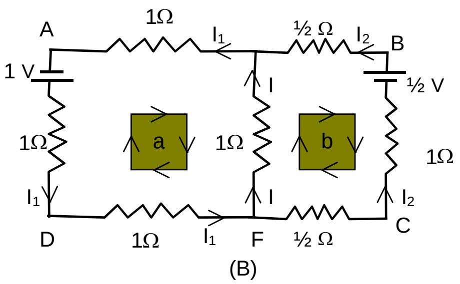


Sol. The induced emf in the rod is
$|\varepsilon|$ = BLv = (0.25) (0.5)(4) = 0. 5 V
I = ε/R = 0.1 A
F = ILBsin90^o = 1/80 N
Thus the total force acting on the rod on left is 1/40 N.
Hence 1/40 N of force is to be applied on the rod in the right side to move it with a constant velocity.

Prob 5. *A rectangular frame ABCD made of a uniform metal wire has a straight connection between E and F made of the same wire as shown in figure (A) AEFD is a square of side 1 m and EB = FC = 0.5m. The entire circuit is placed in a steadily increasing uniform magnetic field directed into the plane of the paper and normal to it. The rate of change of the magnetic field is 1 Ts^{-1}. The resistance per unit length of the wire is $1\Omega m^{-1}$. Find the magnitude and direction of the currents in the segments AE, BE and EF.*

Sol. As in case of changing field, induced emf





$$E = \frac{d\phi}{dt} = \frac{d}{dt}(BS) = S\frac{dB}{dt}$$

So for loops a and b we have, e_1 = (1x1) x 1 = V and

$$e_2 = \left(\frac{1}{2} \times 1\right) \times 1 = \frac{1}{2}\ V$$

The direction of induced emfs e_1 & e_2 and currents I_1 & I_2 in the two loops in accordance with Lenz's law of the junction are shown in Fig. (B)

Now by Kirchhoff's law at junction E, $I + I_2 - I_1 = 0$ i.e., $I = I_1 - I_2$

And by Kirchhoff's II law in mesh a,

$$I_1 + 1(I_1 - I_2) \times 1 + I_1 \times 1 + I_1 \times 1 - 1 = 0 \text{ i.e., } 4I_1 - I_2 = 1 \ldots (I)$$

While in mesh b,

$$I_2 \times \frac{1}{2} + I_2 x1 + I_2 x\frac{1}{2} - (I_1 - I_2)x1 - \frac{1}{2} = 0 \qquad \text{i.e., } -I_1 + 3I_2 = \frac{1}{2} \ldots (II)$$

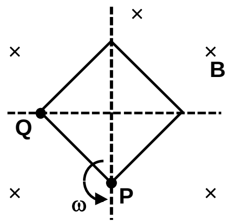
Solving Eqn. (i) and (ii), $I_1 = \frac{7}{22}$ A and $I_2 = \frac{6}{22}$ A

So current in segment AE, $I_1 = \frac{7}{22}$ A from E to A while in BE,

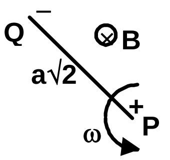
$I_2 = \frac{6}{22}$ A from B to E and in EF, $I = I_1 - I_2 = \frac{7}{22} - \frac{6}{22} = \frac{1}{22}$ from F to E.

IIT-JEE TYPE

Prob 6. *A conducting square loop of side $a\sqrt{2}$ is rotated in a uniform magnetic field B about P in the plane of the paper as shown in the figure. Find the induced emf between P and Q and indicate the relative polarity of the points P and Q.*



Sol. The rotation of the ring about point P generates an emf. The ring within P & Q is equivalent to a rod of length PQ.



Now PQ = $a\sqrt{2}$ as given

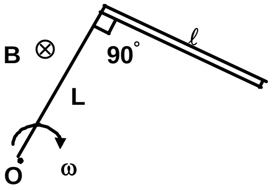
As we know the emf across a rod of length ℓ

rotating with angular velocity ω is = $\frac{1}{2}$ B $\omega\ell^2$

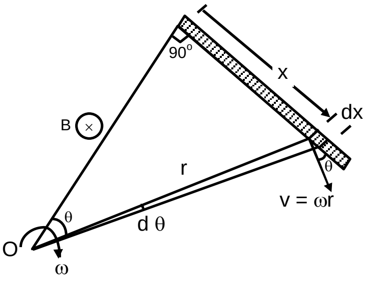
Then emf between P and Q is given by

$$\varepsilon = \frac{1}{2}B\omega\left(a\sqrt{2}\right)^2 = B\omega a^2$$

Prob 7. *A conducting rod of length ℓ attached to a rod of insulating material of length L is rotated with constant angular speed in a plane normal to the uniform magnetic field B, as shown in the figure.*
Find the emf produced across the ends of the conducting rod.



Sol. Consider a small elemental length dx of the rod at a distance x from the end of the rod as shown in the diagram.



The emf across the elemental rod will be

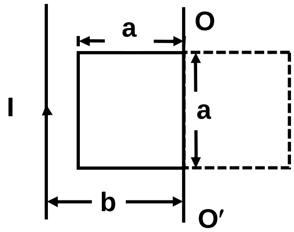
$E = \int(d\bar{x}\times\bar{v}).\bar{B}$; $|d\bar{x}\times\bar{v}| = \omega r dx \sin\theta$

$(d\bar{x}\times\bar{v}).\bar{B} = B\omega(r\sin\theta)dx = B\omega x\,dx$

$\therefore E = \omega B\int_o^{\ell} x\,dx = \frac{1}{2}\omega B\ell^2$

The result is same as if the rod is rotated about one of its ends.

Prob 8. *A square wire frame with side a and a straight conductor carrying a constant current I are located in the same plane as shown in the figure. The inductance and the resistance of the frame are equal to L and R respectively. The frame was turned through 180° about the axis OO' which is located at a distance b from the current–carrying conductor. Find the electric charge which passes through the frame.*



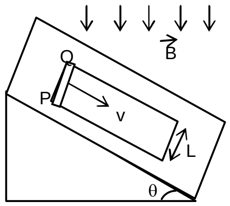
Sol. Electric charge through a loop = $\frac{\text{change in flux}}{\text{resistance}}$

$\Rightarrow\ q = \frac{1}{R}(\phi_f - \phi_i);$ since $d\phi = \frac{\mu_o I}{2\pi r}adr \quad \Rightarrow \phi_i = \frac{\mu_o Ia}{2\pi}\int_{b-a}^{b}\frac{dr}{r}$

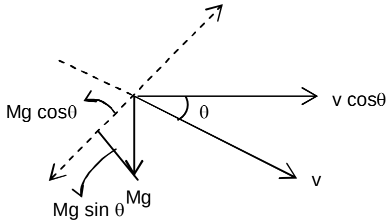
$\phi_i = \frac{\mu_o Ia}{2\pi}\ln\left(\frac{b}{b-a}\right)$; $\phi_f = -\frac{\mu_o Ia}{2\pi}\ln\left(\frac{b+a}{b}\right)$

$q = \frac{1}{R}\left[\frac{\mu_o Ia}{2\pi}\ln\left(\frac{b+a}{b-a}\right)\right]$

Prob 9. *A metallic cylindrical rod PQ of resistance R slides without friction on a rectangular circuit composed of perfectly conducting wires fixed on inclined plane as shown in the figure. A vertical magnetic field $\vec{B}$ exists in the region of the above mentioned setup. Find the velocity of the rod PQ when it starts moving without any acceleration ?*



Sol. Induced emf in the rod $\varepsilon = BL\,(v\cos\theta)$



$i = \varepsilon/R = \dfrac{BLv}{R}\cos\theta$

$\therefore\ F = BiL = BL\left(\dfrac{BLv}{R}\cos\theta\right)$

for uniform velocity force on rod up the plane = force on rod down the plane

$\Rightarrow\ \dfrac{B^2L^2v}{R}\cos^2\theta = mg\sin\theta$

$v = \dfrac{Rmg}{B^2L^2}\left(\dfrac{\sin\theta}{\cos^2\theta}\right)$

$v = \dfrac{Rmg}{B^2L^2}\tan\theta\sec\theta$

Prob 10. A very small circular loop of radius a is initially coplanar and concentric with a much larger circular loop of radius b (a < < b). A constant current i is passed in large loop, which is kept fixed in space, and the small loop is rotated with angular velocity ω about a diameter. The resistance of the small loop is 'R'. If its self inductance is negligible. Find

(a) current in small loop as a function of time.

(b) induced emf in large loop as a function of time.

Sol. (a) Magnetic induction due to large loop at its centre is

$B = \dfrac{\mu_0 i_b}{2b}$

and $\phi_B = \overline{B}.\overline{A} = \left(\dfrac{\mu_0 i_b}{2b}\right)\pi a^2\cos\theta$

where 'θ' is angle between loops and $\theta = \omega t$

$\therefore$ induced emf $\varepsilon = -\dfrac{d\phi}{dt} = \dfrac{\pi a^2\mu_0 i_b\omega}{2b}\sin\omega t$

$\therefore$ current $i_a = \dfrac{\varepsilon}{R} = \left[\dfrac{\pi a^2\mu_0 i_b\omega}{2bR}\right]\sin\omega t$ … (i)

(b) Mutual inductance $M = \dfrac{d\phi}{dt} = \dfrac{\pi a^2\mu_0}{2b}\cos\omega t$

$\therefore\ \varepsilon = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(Mi_a)$

from (i)

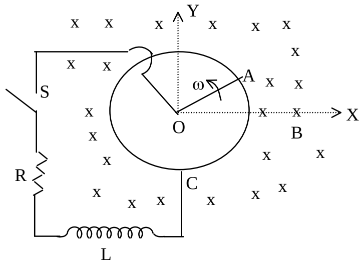
$\varepsilon = -\dfrac{d}{dt}\left[\dfrac{\pi a^2\mu_0}{2b}\cos\omega t.\dfrac{\mu a^2\mu_0 i_b\omega\sin\omega t}{2bR}\right]$

$= -\dfrac{i_b}{R}\left(\dfrac{\pi a^2\mu_0}{2b}\right)^2\dfrac{d}{dt}\left[\dfrac{\sin 2\omega t}{2}\right]$

$= -\dfrac{\omega i_b}{R}\left(\dfrac{\pi a^2\mu_0}{2b}\right)^2 2\omega\dfrac{\cos 2\omega t}{2}$

$$\varepsilon = -\frac{i_b}{R}\left(\frac{\pi\omega a^2\mu_0}{2b}\right)^2 \cos 2\omega t$$

Prob 11. *A conducting rod 'OA' of mass 'm' and length 'l' is kept rotating in a vertical plane about a fixed horizontal axis passing through 'O'. The free end 'A' is arranged to slide on a fixed conducting ring without any friction. A uniform and constant magnetic field 'B' perpendicular to the plane of rotation is applied. The point 'O' and the point 'C' (on the ring) are connected by a series combination of' a resistor 'R' and an inductor 'L' through a switch 'S'. The angular frequency of the rod is ω. Initially the switch is opened. Neglect any other resistance.*



(a) Find the e.m.f. induced across the length of the rod.
(b) The switch is closed at time t = 0 .
(i) Obtain an expression for the current in the resistor as a function of time
(ii) In the steady state find the torque needed to maintain the constant angular speed of the rod. The rod was initially along the positive X–axis.

Sol. A conducting rod 'OA' of mass 'm' and length 'l' is kept rotating in a vertical plane . . . . . any other resistance.

(a) $\frac{1}{2}Bl^2\omega = \varepsilon$

(b) $E = iR + L\frac{di}{dt} \quad \Rightarrow \int \frac{dt}{L} = \int \frac{di}{E - iR}$

$\Rightarrow -\frac{Rt}{L} = \log(E - iR) + c$

$\Rightarrow E - iR = Ee^{-Rt/L}$

$\Rightarrow i = \frac{E}{R}\left[1 - e^{-Rt/L}\right]$

$i = \frac{1}{R}\left[\frac{1}{2}Bl^2\omega\right]\left[1 - e^{-Rt/L}\right]$

$i = \frac{Bl^2\omega}{2R}$ At $t \to \infty$ steady state

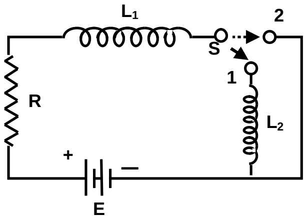
Power = Torque (ω)

$i^2 R = J\omega$

$J = \frac{i^2R}{\omega} = \frac{B^2l^4\omega^2R}{4R^2\omega}$ + torque due to weight of the rod

$J = \frac{B^2l^4\omega}{4R}$ + torque due to weight of the rod $= \frac{B^2l^4\omega}{4R}$ + Mg (ℓ/2)cos ωt

Prob 12. *In the circuit shown, if the switch S is suddenly shifted to position 2 from 1 at t = 0. Find the current in the circuit as a function of time. Assume that, initially the circuit is in a steady state condition and the connection is switched from 1 to 2 in a continuous manner.*



Sol. Let I_0 be the initial current in the steady state condition, $I_0 = \frac{E}{R}$

Since inductors have the tendency to maintain the flux constant, therefore,

$$\phi = I_0(L_1 + L_2) = I_0' L_1$$

Where I_0' is the current in the circuit at t = 0 when switch position is changed.

If I is the instantaneous current in the circuit, then applying Kirchoff's voltage law,

$$L_1 \frac{dI}{dt} + IR = E;$$

or $$\int_{I_0'}^{I} \frac{dI}{E - IR} = \frac{1}{L_1}\int_0^t dt$$

or $$\ln\left[\frac{E - IR}{E - I_0' R}\right] = \frac{R}{L} t ;$$

or, $$E - IR = \left(E - I_0' R\right)e^{-\frac{R}{L_1}t}$$

or, $$I = \frac{1}{R}\left[E - \left(E - I_0' R\right)e^{-\frac{R}{L_1}t}\right]$$

Since $$I_0' = I_0\left(\frac{L_1 + L_2}{L_1}\right) = \frac{E}{R}\left(\frac{L_1 + L_2}{L_1}\right)$$

$$\therefore \quad I = \frac{E}{R}\left[1 + \frac{L_2}{L_1}e^{\frac{-Rt}{L_1}}\right]$$

Prob 13. *A solenoid of resistance 50Ω and inductance 80 henry are connected to a 200V battery. How long will it take for the current to reach 50% of its final equilibrium value? Calculate the maximum energy stored.*

Sol. The current i in R L circuit is given by

$$i = \frac{E}{R}\left[1 - e^{-Rt/L}\right], \quad \text{When} \quad t \to \infty, \; i = \frac{E}{R}$$

Suppose that the current reaches half of this value in a time t_0.

$$\frac{1}{2}\frac{E}{R} = \frac{E}{R}\left[1 - e^{-Rt_0/L}\right]$$

$$\frac{1}{2} = \frac{E}{R}\left[1 - e^{-Rt_0/L}\right]; \quad \left[1 - e^{-Rt_0/L}\right] = 1 - \frac{1}{2} = \frac{1}{2}$$

$$t_0 = \frac{L}{R}\log_e(2) = \frac{80}{50} \times 0.693 = 1.11\text{s}$$

$$E_{max} = \frac{1}{2}Li^2 = \frac{1}{2} \times 80 \times \left(\frac{200}{50}\right)^2 = 640\text{J}$$

Prob 14. *A 10μF capacitor is connected with 1–henry inductance in series with a 50Hz source of alternating current. Calculate the impedance of the combination.*

Sol. The impedance of the L–C circuit is

$$Z = \left(L\omega \sim \frac{1}{C\omega}\right) = \left[1\times 2\pi\times 50 \sim \frac{1}{10\times 10^{-6}\times 2\pi\times 50}\right]$$

$$Z = \left[100\pi \sim \frac{10^3}{\pi}\right] = \left[\frac{100\pi^2 \sim 10^3}{\pi}\right]$$

or $Z = 4.15\ \Omega$

Prob 15. *200V A.C. is applied at the ends of an LCR circuit. The circuit consists of an inductive reactance $X_L = 50\Omega$, capacitive reactance $X_C = 50\Omega$ and ohmic resistance $R = 10\Omega$. Calculate the impedance of the circuit and also potential differences across L and R. What will be the potential difference across L–C?*

Sol. The impedance of L–C–R circuit is

$$Z = \sqrt{(X_L \sim X_C)^2 + R^2} = \sqrt{(50-50)^2 + (10)^2} = 10\Omega$$

Now the r.m.s. current flowing in the circuit is

i.e. $\frac{E}{Z} = \frac{200}{10} = 20$ amp

$\therefore$ The potential difference across the inductance is

$V_L = iX_L = 20\times 50 = 1000V$, $V_R = iR = 20\times 10 = 200V$

The potential difference across L–C is $(50 - 50)\times 20 = 0$

Prob 16. *To a circuit of 1Ω resistance and 0.01 henry inductance is connected a 200 volt line of frequency 50 cycles/ second. Calculate the reactance, the impedance and the current in the circuit and also the lag in phase between alternating voltage and current.*

Sol. The reactance of the circuit = $X_L = L\omega = 2\pi fL$

$\therefore$ $X_L = 0.01\times 2\pi\times 50 = \pi = 3.14\Omega$

The impedance of the circuit = $\sqrt{X_L^2 + R^2}$

$\therefore$ $Z = \sqrt{(3.14)^2 + 1^2} = \sqrt{10.86} = 3.3\Omega$

The phase difference (lag) is, $\tan\phi = L\omega/R = \pi/1 = \pi$

Prob 17. *A 20 volts 5 watt lamp is used on AC mains of 200 volts 50 cps. Calculate the value of*

(i) capacitance, (ii) inductance to be put in series to run the lamp.

(iii) how much pure resistance should be included in place of the above device so that lamp can run on its voltage

Sol. For lamp, $i = \frac{5}{20} = 0.25$ A, $R = \frac{20}{0.25} = 80\ \Omega$

Current through the lamp should be 0.25 A

(i) when condenser C is placed in series

$$i = \frac{200}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = 0.25$$

Putting the value of $\omega = 2\pi \times 50$ $\quad\therefore C = 4.0\ \mu F$

(ii) When inductor is used, $I = \frac{200}{\sqrt{R^2 + (\omega L)^2}} = 0.25 \Rightarrow L = 2.53\ H$

(iii) When resistance is used, $I = \frac{200}{R + r} = 0.25 \Rightarrow r = 720\ \Omega$

Prob 18. *When a 15 V dc source was applied across a choke coil then a current of 5 Amp flows in it. If the same coil is connected to a 15 V, 50 rad/s AC source, a current of 3 A flows in the circuit. Determine the inductance of the coil. Also, find the power developed in the circuit and its resonance frequency if a 2500 μf capacitor is connected in series with the coil.*

Sol. For a coil, $Z = \sqrt{R^2 + \omega^2 L^2}$, $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$

For dc source, $\omega = 0$ $\quad I = \frac{V}{R}$ i.e., $R = \frac{15}{5} = 3\Omega$ . . . (i)

When ac is applied

$I = \frac{V}{Z}$ i.e. $Z = \frac{15}{3.0} = 5\Omega$

$\therefore R^2 + x_L^2 = 25$

$x_L^2 = 25 - 9 = 16\Omega \Rightarrow x_L = 4\Omega$

$L = \frac{4}{40} = 0.08$ Henry.

Now when the capacitor is connected is series.

$\therefore x_C = \frac{1}{\omega C} = \frac{1}{50 \times 2500 \times 10^{-6}} = 8\Omega$

$Z = \sqrt{R^2 + (x_L - x_C)^2} = \sqrt{3 + (4-8)^2} = 5\ \Omega$

$\therefore I = \frac{15}{5} = 3A \quad \therefore P_{av} = v_{rms} I_{rms} \cos\phi = I_{rms}^2 R = (3)^2 \times 3 = 27$ W.

Resonance frequency:

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.08) \times 2500 \times 10^{-6}}} = \frac{1000}{2\pi \times 5 \times 2\sqrt{2}}$$

$$= \frac{25\sqrt{2}}{\pi} = 11.25\ \text{Hz}$$

***Prob* 19.** *A current of 4A flows in a coil when connected to a 12 V DC source. If the same coils is connected to a 12 V, 50 rad/s, AC. source, a current of 2.4 A flows in the circuit.*

Determine the inductance of coil. Also find the power developed in the circuit if a 2500 μF condenser is connected in series with the coil.

Sol. When connected to d.c. source

$R = \frac{V}{I} = \frac{12}{4} = 3\ \Omega$ ...(1)

When connected to a.c. source

$Z = \frac{V_{rms}}{I_{rms}} = \frac{12}{2.4} = 5\ \Omega$ ...(2)

$Z = \sqrt{R^2 + (\omega L)^2}$ ...(3)

From (1), (2) and (3)

L = 0.08 H

When the condenser is also connected.

$$Z' = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{3^2 + \left(50 \times 0.08 - \frac{1}{50 \times 2500 \times 10^{-6}}\right)^2}$$

$= 5\ \Omega$

$\cos\phi = \frac{R}{Z'} = 0.6$

$\therefore P = V_{rms}.I_{rms} \cos\phi$

$= 12 \times 2.4 \times 0.6 = 17.28$ volt

Prob 20. *(a) A 100 V potential difference is suddenly applied to a coil of inductance 100 mH and resistance 50 Ω. Find the rate at which the current increases after one second.*

(b) The current in the circuit is given by $I = I_0 (t/\tau)$. Calculate the rms current for the period t = 0 to t = τ.

Sol. (a) $\tau_L = \frac{L}{R} = \frac{100 \times 10^{-3}}{50} = 2 \times 10^{-3}$ sec.

$\therefore\ I = I_0 (1 - e^{t/\tau}) = \ = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$

$\frac{dI}{dt} = \frac{v}{\tau.R} e^{-t/\tau} \qquad = \frac{100}{2 \times 10^{-3} \times 50}.e^{-\frac{0.001}{2 \times 10^{-3}}}$

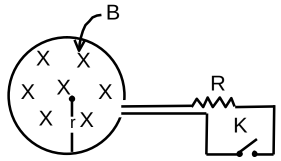
$= 10^3 (0.606) = 606$ Amp/sec.

(b) Heat produced, $H = \int_0^\tau I_0^2 (t/\tau)^2 R dt = \frac{1}{3} I_0^2 R\tau$

$\therefore\ I_{rms} = \sqrt{\frac{(1/3) I_0^2 R\tau}{R\tau}} = \frac{I_0}{\sqrt{3}}$.

Objective:

Prob 1. *Shown in the figure is a circular loop of radius r and resistance R. A variable magnetic field of induction $B = e^{-t}$ is established inside the coil. If the key (K) is closed at t = 0, the electrical power developed is equal to*



(A) $\frac{\pi r^2}{R}$ (B) $\frac{10r^3}{R}$

(C) $\frac{\pi^2 r^4 R}{5}$ (D) $\frac{10r^4}{R}$

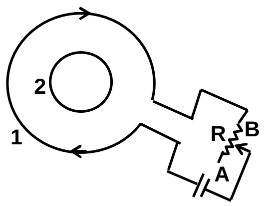
Sol. The induced emf, $\varepsilon = -\frac{d\phi}{dt}$

$= -A\,\frac{dB}{dt} = -(\pi r^2)\,\frac{d}{dt}\,(e^{-t}) = \pi r^2 e^{-t}$

$\Rightarrow \varepsilon_0 = \pi\, r^2\, e^{-t}\big|_{t=0}$ **=** $\pi\, r^2$ $[\,\pi^2 \approx 10]$

$\therefore$ The electrical power developed in the resistor just at the instant of closing the key $= P = \frac{\varepsilon_0^2}{R} = \frac{\pi^2\ r^4}{R} \cong \frac{10\ r^4}{R}$**.** Hence **(D)** is correct.

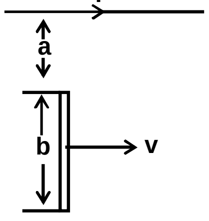
Prob 2. *Shown in the figure is a small circular loop that is co–axial with the bigger circular loop. If the slider moves from A to B, then*



(A) current flow in both the loops will be opposite.
(B) clockwise current in loop 1 and anti–clockwise current in loop 2 flow.
(C) no current flows in loop 2.
(D) clockwise current flows in loop 2.

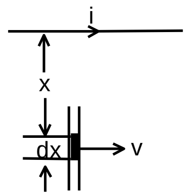
Sol. When the slider moves towards B, the resistance of the circuit (bigger loop) decreases. Therefore, the current in the bigger loop increases. The increasing current results in increasing flux ($\phi\ \alpha$ i) linked by the smaller coil. Consequently, induced emf will be generated in the smaller loop causing an induced current so as to oppose the increase in flux. Therefore, the current flows anticlockwise in the inner loop. Hence **(B)** is correct.

Prob 3. *A rod of length b moves with a constant velocity v in the magnetic field of a straight long conductor that carries a current i as shown in the figure. The emf induced in the rod is*



(A) $\frac{\mu_o iv}{2\pi}\ \tan^{-1}\frac{a}{b}$ (B) $\frac{\mu_0 iv}{2\pi}\ln(1+\frac{b}{a})$

(C) $\frac{\mu_o iv\sqrt{ab}}{4\pi(a+b)}$ (D) $\frac{\mu_o iv(a+b)}{4\pi ab}$



Sol. The induced emf between two ends of a segment

$dx = dE = Bvdx$

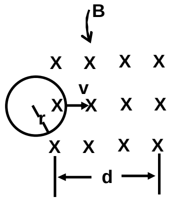
$\therefore$ where B = magnetic field due to straight current carrying wire at the segment dx = $\frac{\mu_0 i}{2\pi x}$ $\Rightarrow$ dE = $\frac{\mu_0 i v dx}{2\pi\ x}$

$\therefore$ The induced emf between the ends of the rod

$$= E = \int dE = \frac{\mu_0 iv}{2\pi}\int_a^{a+b}\frac{dx}{x}$$

$\Rightarrow$ $E = \frac{\mu_0 iv}{2\pi}\ \ln\left(\frac{b}{a}+1\right)$ Hence **(B)** is correct.

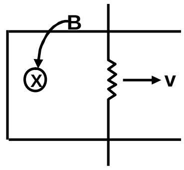
Prob 4. *A conducting loop is pulled with a constant velocity towards a region of constant (steady) magnetic field of induction B as shown in the figure. Then the current involved in the loop is (d > r)*



(A) Clockwise *(B) Anti–clockwise*
(C) Zero *(D) All of these*

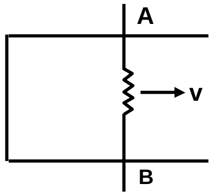
Sol. When the loop is drawn into the magnetic field, the area of the portion of the loop in the magnetic field will increase. That means, the flux linkage increases. Therefore, an (anticlockwise) current is induced in the loop so as to oppose the change. When the loop is fully inside. When the loop emerges out of the magnetic field the flux decrease, following the previous argument, the direction of induced in it will be eversed (clockwise).
Hence **(D)** is correct.

Prob 5. *A conducting bar pulled with a constant speed v on a smooth conducting rail. The region has a steady magnetic field of induction B as shown in the figure. If the speed of the bar is doubled then the rate of heat dissipation will be*



(A) constant. *(B) quarter of the initial value.*
(C) four fold. *(D) doubled.*

Sol. The induced emf between A and B = ε = $B\ell v$



$\Rightarrow$ The induced current = i = $\frac{\varepsilon}{R}$ $\Rightarrow$ i = $\frac{B\ \ell\ v}{R}$

The electrical power = P = i^2R = $\frac{B^2\ \ell^2\ v^2}{R}$

Since, v is doubled, the electrical power, becomes four times. Since heat dissipation per second is proportional to electrical power, it becomes four fold.
Hence **(C)** is correct.

Prob 6. *If all the linear dimensions of a cylindrical coil are doubled, the inductance of the coil will be (assuming complete winding over the core)*
(A) doubled *(B) four fold*
(C) eight times *(D) remains unchanged*

Sol. The inductance of a coil is given as L = $\frac{\mu_0 N^2 A}{\ell}$ where N = total number of turns of the coil; A = area of cross section of the coil = π r^2; r = radius of the core of the coil, ℓ = length of the coil. If ℓ is doubled the total number of turns will be doubled

$$\Rightarrow \frac{L_2}{L_1} = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{A_2}{A_1}\right)\left(\frac{\ell_1}{\ell_2}\right) = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{\pi r_2^2}{\pi r_1^2}\right)\left(\frac{\ell_1}{\ell_2}\right)$$

$$\Rightarrow \frac{L_2}{L_1} = (2)^2(2)^2\left(\frac{1}{2}\right) = 8 \qquad \left(\because \frac{N_2}{N_1} = \frac{r_2}{r_1} = \frac{\ell_2}{\ell_1} = 2\right).$$

Hence **(C)** is correct.

Prob 7. *A sinusoidal voltage $V_o \sin\omega t$ is applied across a series combination of resistance R and inductor L. The amplitude of the current in the circuit is*

(A) $\frac{V_o}{\sqrt{R^2+\omega^2\ell^2}}$ *(B)* $\frac{V_o}{\sqrt{R^2-\omega^2\ell^2}}$

(C) $\frac{V_o}{\sqrt{R^2+\omega^2\ell^2}}\sin\omega t$ *(D)* V_o/R

Sol. Impedance of the circuit = $\sqrt{R^2+\omega^2 L^2}$

Amplitude of voltage = V_o

$\therefore$ Amplitude of current = $\frac{V_o}{\sqrt{R^2+\omega^2 L^2}}$.

Hence **(A)** is correct.

Prob 8. *An ideal choke takes a current of 8 A when connected to an a.c. source of 100 volt and 50Hz. A pure resistor under the same conditions takes a current of 10A. If two are connected in series to an a.c. supply of 100V and 40Hz, then the current in the series combination of above resistor and inductor is*

(A) 10A *(B) 8A*

(C) $5\sqrt{2}$ *A* *(D)* $10\sqrt{2}$ *A*

Sol. $X_L = \frac{100}{8}$, $R = \frac{100}{10} = 10\ \Omega$; $L \times 100\pi = \frac{100}{8}$ or $L = \frac{1}{8\pi}$ H

$$Z = \sqrt{\left(\frac{1}{8\pi}\times 2\pi\times 40\right)^2 + 10^2} = 10\sqrt{2}$$

$$I = \frac{E}{Z} = \frac{100}{10\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}A$$

Hence **(C)** is correct.

Prob 9. *A resistor R, an inductor L and a capacitor C are connected in series to a source of frequency n. If the resonant frequency is n_r then the current lags behind voltage, when*

(A) n = 0 *(B) $n < n_r$*

(C) $n = n_r$ *(D) $n > n_r$*

Sol. Below resonant frequency the current leads the applied e.m.f., at resonance it is in phase with applied e.m.f. and above resonance frequency it lags the applied e.m.f., Hence **(D)** is correct.

Prob 10. *An ac source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. The ratio of reactance to resistance at the original frequency ω will be*

(A) $\sqrt{\frac{3}{5}}$ (B) $\sqrt{\frac{5}{3}}$

(C) $\frac{3}{5}$ (D) $\frac{5}{3}$

Sol. According to given Problem,

$$I = \frac{V}{Z} = V / [R^2 + (1/C\omega^2)]^{1/2} \qquad \ldots (1)$$

and $$\frac{I}{2} = \frac{V}{[R^2 + (3/C\omega)^2]^2} \qquad \ldots (2)$$

Substituting the value of I from equation (1) in (2),

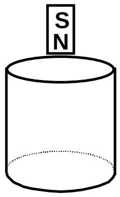
$$4\left(R^2 + \frac{1}{C^2\omega^2}\right) = R^2 + \frac{9}{C^2\omega^2} \qquad \text{i.e., } \frac{1}{C^2\omega^2} = \frac{3}{5}R^2$$

So that $$\frac{X}{R} = \frac{(1/C\omega)}{R} = \frac{[(3/5)R^2]^{1/2}}{R} = \sqrt{\frac{3}{5}}$$

Hence **(A)** is correct.

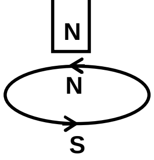
MULTIPLE CHOICE QUESTIONS (MORE THAN ONE CHOICE ARE CORRECT)

Prob 11. *When a magnet is released from rest along the axis of a hollow conducting cylinder situated vertically as shown in the figure,*



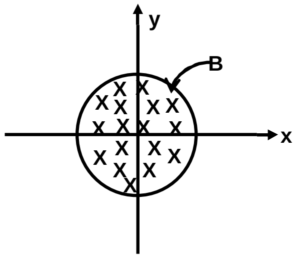
(A) The direction of induced current in the cylinder is anti−clockwise as seen from the above
(B) the magnet moves with an acceleration less than g = 9.8 m/s^2
(C) the cylinder gets heated.
(D) the magnet attains a terminal speed inside the cylinder if the cylinder is very long.

Sol. Initially the current induced in the hollow conductor is zero because the magnet was at rest. When the magnet falls, due to the variation of magnetic flux, induced current develop in the hollow conductor so as to resist the cause of increment of flux; that means the velocity of the magnet is resisted.



In the other word we can say that the magnet is retarded before entering into the hollow conducting cylinder. Consequently its acceleration will be less than g because magnetic force opposes the gravitational force, equally after sometime when the magnet attains acting on the magnet. If the cylinder is very long, the magnetic force opposes the gravitational force, terminal velocity. Hence **(A), (B), (C) & (D)** all are correct.

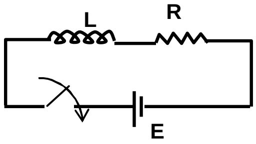
Prob 12. *A loop is kept so that its center lies at the origin of the coordinate system. A magnetic field has the induction B pointing along Z axis as shown in the figure*



(A) No emf and current will be induced in the loop if it rotates about Z axis
(B) Emf is induced but no current flows if the loop is a fiber. When the loop is rotated about y–axis.
(C) Emf is induced and induced current flows in the loop if the loop is made of copper
(D) If the loop moves along Z axis with constant velocity, no current flows in it.

Sol. If the loop rotates about Z axis, there is no variation of flux linkage. Therefore, no emf is induced. Consequently, no current flows in the loop
When rotated about y axis, its flux linkage changes. Therefore an emf is induced. Since the loop is a fiber it is non conducting. Therefore induced current is zero.
If the loop is made of copper, it is conductive therefore induced current is set up.
If the loop moves along the Z axis variation of flux linkage is zero. Therefore the induced emf and current will be equal to zero.
Hence **(A), (B), (C) and (D)** all are correct.

Prob 13. *Shown in the figure is an R–L circuit. Just after the Key (K) is closed*



(A) the current in the circuit is zero
(B) no potential drop across the resistor
(C) potential drop across the inductor is E
(D) no heat is dissipated in the circuit

Sol. The current in the circuit at the instant of closing the Key (K) is equal to i_o. Current at any time in transient state is given as

$i = i_0\left(1-e^{-\frac{t}{\tau}}\right)$ where $\tau = \dfrac{L}{R}$ Putting t = 0, i = 0

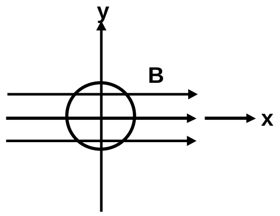
$\Rightarrow$ The voltage drop across the inductor is E′ that oppose the applied emf E
$\Rightarrow$ E′ = E (numerically).
Since the circuit current is zero, the heat loss (proportional to i_0^2 R) will be zero
Hence **(A), (B), (C), and (D)** are correct.

Prob 14. *A constant current flows in R–L circuit. Then*
(A) $V_R = 0$ and $V_L \neq 0$ (V = potential difference)
(B) Energy lost per second in the resistor is equal to energy gained by the inductor per second
(C) $V_{AB} = i\,R$ (numerically)
(D) $V_R \neq 0$ and $V_L = 0$

Sol. When i = constant, $\frac{di}{dt} = 0$ $\Rightarrow$ Induced emf across the inductor = $E_L = -L\frac{di}{dt} = 0$

$\Rightarrow$ Potential drop across the inductor is zero $\Rightarrow$ $V_L = 0$
Therefore potential drop across the resistor = $V_R = i\,R$ $\Rightarrow V_{AB} = V_R + V_L = i\,R$
Electrical power = P = i^2 R, $\Rightarrow$ Energy loss per second in the resistor α i^2 R
The energy stored in the inductor = $U_L = \frac{1}{2}Li^2$ = constant $\Rightarrow \frac{d\,U_L}{dt} = 0$
$\Rightarrow$ The energy gained by the inductor per second = 0
Hence **(C) & (D)** are correct.

Prob 15. *A conducting loop of resistance R and radius r has its center at the origin of the co–ordinate system in a magnetic field of induction B along x–axis axis when it is rotated about Y–axis through 90°, the induced charge in the coil is directly proportional to*



(A) B *(B) R*
(C) r^2 *(D) r*

Sol. Induced emf loop when the variation of flux $d\phi$ during time dt is given as

$$E = \frac{d\phi}{dt} \Rightarrow \int_{\phi_1}^{\phi_2} d\phi = \Delta\phi = \int E dt \qquad \ldots(1)$$

$\Rightarrow$ The total charge induced in the loop = $q = \int i\ dt$

$$q = \int \frac{E}{R}\ dt \qquad \ldots(2)$$

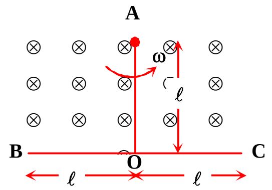
Using (1) and (2), $q = \frac{\Delta\phi}{R}$

Where $\Delta\phi$ = change in flux given as $\Delta\phi = \phi_2 - \phi_1 = B.(\pi r^2)$ because initially the flux is linked with the coil and it has maximum flux linkage $\phi_2 = B\,\pi r^2$ when turned through 90°.

$\Rightarrow$ $q = \frac{\pi\ B\ r^2}{R}$ $\Rightarrow$ i.e. $q \propto B$
$q \propto r^2$
$q \propto (1/R)$.
Hence **(A) & (C)** are correct.

Fill in the blanks

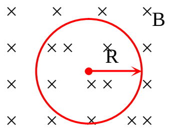
Prob 16. *A T–shaped rod is rotating about AO with angular velocity ω in a magnetic field as shown. The emf induced across AC is ___________________________.*



Sol. $\frac{1}{2}\omega B\ell^2$

Induced emf across AO is zero
$\therefore E_{BO} = E_{OC}$ = zero

Prob 17. *A conducting loop of radius R is present in a uniform magnetic field B perpendicular to the plane of the ring. If radius R varies as a function of time t as $R = R_0 + t$ the emf induced in the loop is __________________*

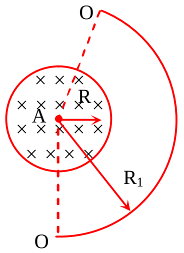


Sol. $2\pi(R_0 + t)$ anticlockwise

$A = \pi r^2$

$\phi = \pi BR^2$, $e = \frac{d\phi}{dt} = 2\pi\left(R_0 + t\right)B$

Prob 18. *There is a uniform magnetic field B in a circular region of radius R as shown in the figure whose magnitude changes at a rate $\frac{dB}{dt}$. The emf induced across the ends of a circular concentric conducting arc of radius R_1 having an angle θ as shown _________________________($\angle OAO' = \theta$)*



Sol. $QR^2\left(\frac{dB}{dt}\right)$

Required emf = $\pi R^2 \frac{dB}{dt}\frac{\theta}{2\pi} = \frac{R\theta}{2}\left(\frac{dB}{dt}\right)$

$\therefore \varepsilon = \frac{R^2}{2\pi R}\left(\frac{dB}{dt}\right)$

ASSIGNMENT PROBLEMS

Subjective:

Level – O

1. Derive the value of v_{rms} for a power source, $v = v_0 \sin\omega t$.

2. Explain phasor and phasor diagram in relation to alternating voltage and associated alternating current in a circuit.

3. Fill in the blanks:
In an LCR circuit having inductance L, capacitance C and resistance R connected in series to a source $v_0 \sin\omega t$,
(a) If $\omega L = \frac{1}{\omega C}$, voltage ………………current.
(b) If $\omega L > \frac{1}{\omega C}$, voltage……………..current.
(c) If $\omega L < \frac{1}{\omega C}$, voltage ………………current.

4. A rod of length ℓ rotates with a constant angular velocity ω in a uniform magnetic field B, plane of motion of the rod is perpendicular to the magnetic field. Find the induced emf generated between the ends of the rod.

5. A square coil of side 10 cm having 200 turns is placed in a magnetic field of 1 W/m^2 such that its plane is perpendicular to the field. The coil is rotated by 180^0 in 0.5 seconds. What is the induced emf.

6. A railway track running north– south has two parallel rails 1.0 m apart. Calculate the value of induced emf between the rails when a train passes at a speed of 90 km/h. Horizontal component of earth's field at the place is 0.3×10^{-4} Wb/m^2 and angle of dip 60^0.

7. A current i = 0.5 sin 300 t is passed through a coil with self inductance 50 mH. What will be the maximum magnitude of induced emf in the coil.

8. The self inductance of an inductance coil having 100 turns is 20 mH. Find the magnetic flux through the cross–section of the coil corresponding to a current of 4 milli–ampere. Also, find the total flux.

9. Circular coil A with radius 10 mm and 50 turns and coil B with radius 0.5 m and 600 turns are placed coaxially. Find out mutual inductance of the coil.

10. An air–cored solenoid is of length 0.3 m, area of cross–section 1.2×10^{-3} m^2 and has 2500 turns. Around its central section, a coil of 350 turns is wound. The solenoid and the coil are electrically insulated from each other. Calculate the emf induced in the coil, if initial current of 3A in the solenoid is reversed in 0.25 sec.

11. A wire is bent to form a semicircle of 40 cm radius. It is moved perpendicular to a magnetic field B = 0.25 T with a velocity 25 m/s. Find out the induced emf across its ends.

12. In figure 1 and 2, find the direction of induced current
(a) Figure 1 wire loop of irregular shape turning into a regular shape.
(b) A circular loop turning into a narrow straight wire.

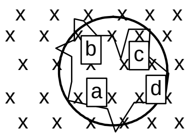


fig. 1

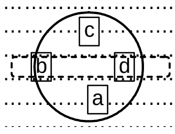
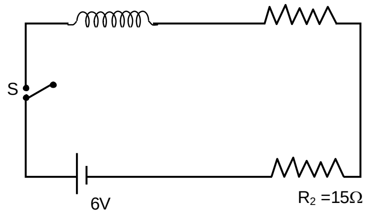


fig. 2

13. In a circuit, capacitance 60 μF, inductance 15 mH and resistance 20 Ω are connected to a power source having frequency 100 Hz. What are
(a) Capacitative reactance (b) inductive reactance
(c) total impedance of the circuit.

14. A resistance of 15 ohm, capacitance and an ammeter of negligible resistance are connected in series to a source of 110 V – 60 Hz. If the reading of ammeter shows 5 A, what is the reactance of the capacitance?

15. In the circuit shown, the switch is operated to complete the circuit at time t = 0. Calculate the time required for current in R_1 to become half of the steady state current. What is the energy stored in the inductor when the current reaches steady state.



16. A lamp having a hot resistance of 25 Ω is not allowed to pass current more than 5A. Find the value of inductance which must be used in series with the lamp, which is supplied by an AC of maximum rms 325 V at 50 Hz.

17. A resistance of 60 Ω, capacitance of 20 μF and an inductance of 0.25 H are connected in series to a source of 30 V, 50 Hz. Find out the time by which the temperature of the resistance will rise by 15^0C. Thermal capacity of resistance is 1.8 J/C.

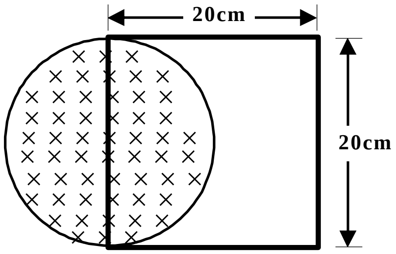
18. An ac source of 220 volts and 60 Hz is connected in series with an inductance of 1.0 H and a resistance of 377 ohm. Find the wattless component of current in the circuit.

19. A circuit containing 8 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. If the circuit has a resistance of 15 ohm, obtain the average power transferred to each element of the circuit and the total power absorbed.

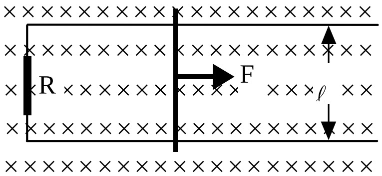
20. An AC generator consists of a coil of 2000 turns each of area 80 cm^2 and rotating at an angular speed of 200 rpm in a uniform magnetic field of 4.8×10^{-2} T. Calculate the peak and rms value of emf in the coil.

Level – I

1. A uniform magnetic field B exists in a cylindrical region of radius 10 cm as shown in figure. A uniform wire of length 80 cm and resistance 4.0 Ω is bent into a square frame and is placed with one side along a diameter of the cylindrical region. If the magnetic field increases at a constant rate of 0.010 T/s, find the current induced in the frame.



2. The figure shows a wire sliding on two parallel, conducting rails placed at a separation 'ℓ'. A magnetic field B exists in a direction perpendicular to the plane of the rails. What force is necessary to keep the wire moving at a constant velocity *v*?



3. An inductor–coil of inductance 20 mH having resistance 10 Ω is joined to an ideal battery of emf 5.0 V. Find the rate of change of the induced emf at
   (a) t = 0
   (b) t = 10 ms and
   (c) t = 1.0 s.

4. A metal disc of radius 2 m is rotated at a constant angular speed of 60 rad s^{-1} in a plane at right angles to an external uniform field of magnetic induction 0.05 Wbm^{-2}. Find the emf induced between the centre and a point on the rim.

5. An LR circuit having a time constant of 50 ms is connected with an ideal battery of emf ε. Find the time elapsed before
   (a) the current reaches half its maximum value,
   (b) the power dissipated in heat reaches half its maximum value and
   (c) the magnetic field energy stored in the circuit reaches half its maximum value.

6. Find the inductance of a coil in which a current of 0.1 A increasing at the rate of 0.5 A/s represents a power flow of $\frac{1}{2}$W.

7. If the voltage in an ac circuit is represented by the equation, V = 220$\sqrt{2}$ sin(314t–ϕ) V. Calculate (a) peak and rms value of the voltage (b) average voltage
   (c) frequency of AC.

8. A coil of resistance 300Ω and inductance 1.0 henry is connected across an alternately voltage of frequency 300/2π Hz. Calculate the phase difference between the voltage and current in the circuit.

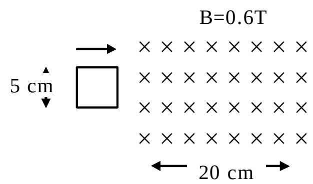
9. A 0.21H inductor and a 12Ω resistance connected in series to a 220V, 50Hz AC source. Calculate the current in the circuit and the phase angle between the current and the source voltage.

10. A 100mH inductor, a 25μF capacitor and a 15Ω resistor are connected in series to a 120V, 50Hz AC source. Calculate
(a) impedance of the circuit at resonance.
(b) current at resonance.
(c) Resonant frequency.

11. Find the value of an inductance which should be connected in series with a capacitor of 5μF, a resistance of 10Ω and an ac source of 50Hz so that the power factor of the circuit is unity.

12. A voltage of 10V and frequency 1000Hz is applied to a 0.1μF capacitor in series with a resistor of 500Ω. Find the power factor of the circuit and the average power dissipated.

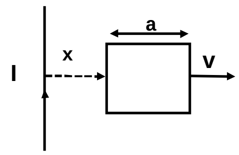
13. Fig. Shows a square loop of side 5 cm being moved towards right at a constant speed of 1 cm/s. The front edge enters the 20 cm wide magnetic field at t=0. Find the emf induced in the loop at
(a) t = 2s, (b) t = 10 s, (c) t = 22 s and (d) t = 30 s.



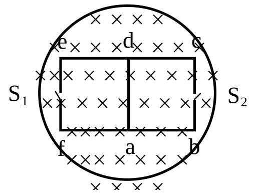
Find the total heat produced during the interval 0 to 30 s if the resistance of the loop is 4.5 mΩ.

14. A long solenoid of radius 2 cm has 100 turns/cm and carries a current of 5A. A coil of radius 1 cm having 100 turns and a total resistance of 20 Ω is placed inside the solenoid coaxially. The coil is connected to a galvanometer. If the current in the solenoid is reversed in direction, find the charge that flows through the galvanometer.

15. A square frame with side a and a long straight wire carrying a current I are located in the same plane as shown in (Fig) . The frame translates to the right with a constant velocity v. Find the emf induced in the frame as a function of distance x.



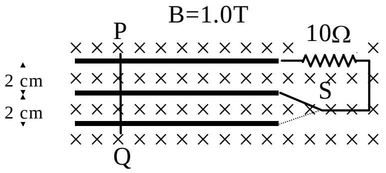
16. The magnetic field in the cylindrical region shown in the figure. increases at a constant rate of 20.0 mT/s. Each side of the square loop abcd and defa has a length of 1.00 cm and a resistance of 4.00 Ω. Find the current (magnitude and sense) in the wire and if
(a) the switch S_1 is closed but S_2 is open,
(b) S_1 is open but S_2 is closed,
(c) both S_1 and S_2 are open and
(d) both S_1 and S_2 are closed.



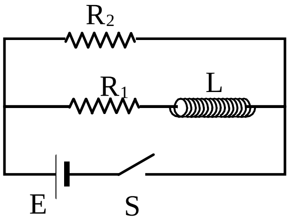
17. An inductor having inductance L and resistance R carries a current I. Show that the time constant is equal to twice the ratio of energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

18. The magnetic field in a region is given by $\vec{B} = \vec{k}\left(\frac{B_o}{L}y\right)$ where L is a fixed length. A conducting rod of length L lies along the Y–axis between the origin and the point (0, L, 0). If the rod moves with a velocity $v = v_o\,\vec{i}$, find the emf induced between the ends of the rod.

19. Consider the situation shown in the figure. The wire PQ has a negligible resistance and is made to slide on the three rails with a constant speed of 5 cm/s. Find the current in the 10 Ω resistor when the switch S is thrown to
(a) the middle rail (b) the bottom rail.



20. Consider the circuit shown in fig.
(a) Find the current through the battery a long time after the switch S is closed.
(b) Suppose the switch is again opened. What is the time constant of the discharging circuit?
(c) Find the current through the inductor, after one time constant, during discharging.



Level – II

1. A flat coil with radius 8 mm has fifty loops of wire on it. It is placed in a magnetic field = 0.3 T so that the maximum flux goes through it. Later it is rotated in 0.02 s to a position such that no flux goes through it. Find the average emf induced between the terminals of the coil.

2. It is desired to set up an undriven L – C circuit in which the capacitor is originally charged to a difference of potential of 100.0 V. The maximum current is to be 1.0 A, and the oscillation frequency is to be 1000 Hz. What are the required values of L and C?

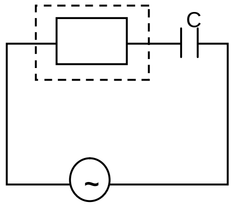
3. A 50W, 100V lamp is to be connected to an ac mains of 200V, 50Hz. What capacitance is essential to be put in series with the lamp.

4. A coil of negligible resistance is connected in series with 90Ω resistor across a 120V – 60Hz line. A voltmeter reads 36V across the resistance. Find the voltage across the coil and inductance of the coil.

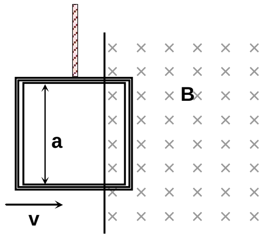
5. An alternating current of 1.5mA and angular frequency ω = 300 rad/s flows through 10KΩ resistor and a 0.50μF capacitor in series. Find the r.m.s. voltage across the capacitor and impedance of the circuit?

6. The inductance of a choke–coil is 0.2 henry and its resistance is 0.50Ω. If a current of 2.0 ampere (rms value) and frequency 50Hz be passed through it, what will be the potential difference across its ends?

7. In the circuit shown there is a box and capacitance C connected to alternating power source of angular frequency of 2 rad/s. Box has power factor $1/\sqrt{2}$ and circuit has overall power factor 1. Find the impedance of the box.



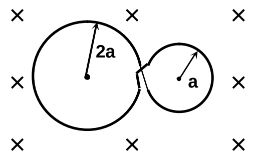
8. A coil of wire with n loops swings with speed v into a uniform magnetic field B as shown in the figure. If the ends of the coil are connected together and the resistance of the coil is R, find the force exerted on the coil by the field when the coil is in the position shown.



9. A long solenoid that has 800 loops per meter carries a current i = 3 sin(400t) A. Find the electric field inside the solenoid at a distance 2 mm from the solenoid axis. Consider only the field tangential to a circle having its center on the axis of the solenoid.

10. A circular ring of diameter 20 cm has a resistance of 0.01Ω. How much charge will flow through the ring if it is turned in a uniform magnetic field of 2.0 T from an initial position perpendicular to the field to a position parallel to the field ?

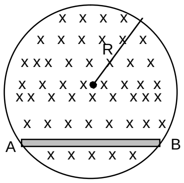
11. A thin wire of cross–sectional area A is bent into the shape of a numerical figure 8, as shown in the figure. A time varying magnetic field perpendicular to the plane of the figure is applied. $B = B_0 \sin\omega t$. If ρ is the resistivity of the wire, find the maximum current through the loop.



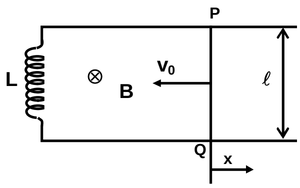
12. A closed coil having 100 turns is rotated in a uniform magnetic field $B = 4.0 \times 10^{-4}$ T about a diameter which is perpendicular to the field. The angular velocity of rotation is 300 revolutions per minute. The area of the coil is 25 cm^2 and its resistance is 4.0 Ω. Find
(a) the average emf developed in half a turn from a position where the coil is perpendicular to the magnetic field,
(b) the average emf in a full turn and
(c) the net charge displaced in part (a).

13. Two parallel wires of radius r, whose centres are a distance d apart, carry equal currents in opposite directions. Neglecting the flux within the wires, find the inductance of length ℓ of such a pair of wires.

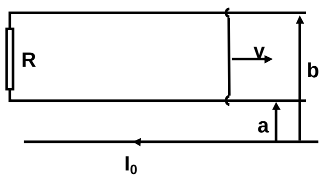
14. A metal rod AB of length L is placed in a magnetic field $\bar{B}$ as shown in the figure. If the rate of change of B with respect to time is $\frac{dB}{dt}$, find the emf produced across AB.



15. A conducting rod PQ of mass m is free to slide on frictionless rails in the horizontal plane as shown in the figure. At t = 0 the rod is given an initial velocity v_0. Find the variation of x with respect to time. Neglect the resistance of the rod, rails and the inductor L. Assume that the rod remains within the magnetic field $\bar{B}$.



16. A long straight wire carries a current I_0. At distances a and b from it there are two other wires, parallel to the former one, which are interconnected by a resistance R (Fig.). A connector slides without friction along the wires with a constant velocity v. Assuming the resistances of the wires, the connector, the sliding contacts, and the self–inductance of the frame to be negligible, find:
(a) the magnitude and the direction of the current induced in the connector;
(b) the force required to maintain the connector's velocity constant.



17. A LCR circuit has L =10 mH, R = 3 ohms and C = 1 μF connected in series to a source of 15 cos ωt volts. Calculate the current amplitude and the average power dissipated per cycle at a frequency that is 10 % lower than the resonance frequency.

18. A current of 4 A flows in a coil when connected to a 12 V dc source. If the same coil is connected to a 12 V 50 rad/s ac source a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF capacitor is connected in series with the coil.

19. A box P and a coil Q are connected in series with an ac source of variable frequency. The emf of the source is constant at 10 V. Box P contains a capacitance of 1 μF in series with a resistance of 32 Ω. Coil Q has a self–inductance 4.9 mH and a resistance of 68 Ω in series. The frequency is adjusted so that the maximum current flows in P and Q. Find the impedance of P and Q at this frequency. Also find the voltage across P and Q respectively.

20. An LCR series circuit with 100Ω resistance is connected to an ac source of 200 V and angular frequency 300 rad/s. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current and the power dissipated in the LCR circuit.

Objective:

Level – I

1. A capacitor of capacitance C farad is charged by a battery of e.m.f. V_0 volt. The battery is then disconnected and a pure inductor of L henry is connected across it so that LC oscillations are set up. Then the frequency of oscillations is :

(A) $2\pi\sqrt{(LC)}$ (B) $\frac{1}{2\pi\sqrt{(LC)}}$

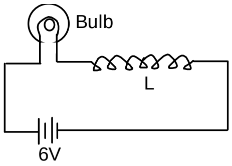
(C) $\frac{1}{2\pi}\sqrt{\left(\frac{L}{C}\right)}$ (D) $\frac{1}{2\pi}\sqrt{\left(\frac{C}{L}\right)}$

2. A rectangular loop of wire is placed in a uniform magnetic field B acting normally to the plane of the loop. If we attempt to pull it out of the field with a velocity v, the power needed is :

(A) $Bi\ell\cdot v$ (B) $\frac{B^2\ell^2v^2}{R}$

(C) $\frac{B\ell^2v^2}{R}$ (D) $\frac{Bv\ell}{R}$

3. A bulb is connected in series with an inductance and a 6 V DC source as shown in figure. A soft iron core is inserted in the coil quickly. During this process, the intensity of bulb:



(A) remains unaltered (B) increases
(C) decreases
(D) may increase or decrease depending on the size of core

4. A coil of resistance R and inductance L is connected to a battery of emf E volts. Then the final current in the coil is :

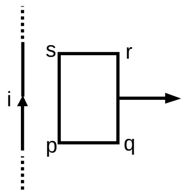
(A) E/R (B) E/L

(C) $\sqrt{\left[E/\left(R^2+L^2\right)\right]}$ (D) $\sqrt{\left[EL/\left(R^2+L^2\right)\right]}$

5. If L and R represent inductance and resistance respectively, then the dimensions of L/R will be:

(A) $M^0L^0T^{-1}$ (B) M^0LT
(C) M^0L^0T (D) cannot be represented in terms of M, L & T

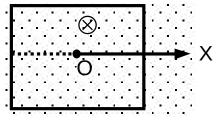
6. A rectangular coil pqrs is moved away from an infinite, straight wire carrying a current as shown in figure. Which of the following statements is correct?



(A) There is no induced current in coil pqrs
(B) The induced current in coil pqrs is in the clockwise sense
(C) The induced current in the coil pqrs is in anticlockwise direction

(D) None of the above

7. A rectangular coil is placed in a region having a uniform magnetic field B, perpendicular to the plane of the coil. An e.m.f. will not be induced in the coil if the:

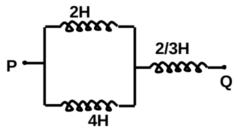


(A) magnetic field increases uniformly.
(B) coil is rotated about an axis perpendicular to the plane of the coil and passing through its centre O, the coil remaining in the same plane.
(C) coil is rotated about the axis OX.
(D) magnetic field is suddenly switched off.

8. A metal disc of radius R rotates with an angular velocity ω about an axis perpendicular to its plane passing through its centre in a magnetic field of induction B acting perpendicular to the plane of the disc. The induced e.m.f. between the rim and axis of the disc is
(A) $B\pi R^2\omega$ (B) $BR^2\omega$
(C) $B\pi R^2\omega/2$ (D) $BR^2\omega/2$

9. In LC circuit the capacitance is changed from C to 4C. For the same resonant frequency, the inductance should be changed from L to:
(A) 2L (B) L/2
(C) L/4 (D) 4L

10. The equivalent inductance between points P and Q in figure is :



(A) 2 H (B) 6 H
(C) 8/3 H (D)4/9 H

11. The frequency for which a 5.0μF capacitor has a reactance of 1000Ω is given by
(A) $\frac{1000}{\pi}$ cycles/sec (B) $\frac{100}{\pi}$ cycles/sec
(C) 200 cycle /sec (D) 5000 cycles /sec

12. In an AC circuit V and I are given by V = 50 sin50t volt and I = 100 sin(50t + π/3) mA. The power dissipated in the circuit
(A) 2.5 kW (B) 1.25 kW
(C) 5.0 kW (D) 500 watt

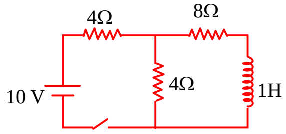
13. The root–mean–square value of an alternating current of 50Hz frequency is 10 ampere. The time taken by the alternating current in reaching from zero to maximum value and the peak value of current will be
(A) 2×10^{-2} sec and 14.14 amp. (B) 1×10^{-2} sec and 7.07 amp.
(C) 5×10^{-3} sec and 7.07 amp. (D) 5×10^{-3} sec and 14.14 amp.

14. A coil of resistance 2000Ω and self–inductance 1.0 henry has been connected to an a.c. source of frequency 2000/2π Hz. The phase difference between voltage and current is
(A) 30° (B) 60°
(C) 45° (D) 75°

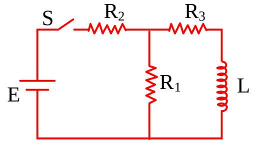
15. In a series resonant circuit, the AC voltage across resistance R, inductance L and capacitance C are 5V, 10V and 10V, respectively. The AC voltage applied to the circuit will be

(A) 20V (B) 10V
(C) 5V (D) 25V

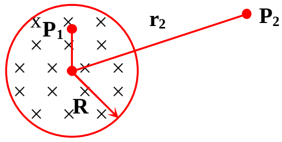
Fill in the blanks



16. The switch in the figure is close date time t = 0. The current in the inductor and current through the switch as function of time these after are ______________and __________________

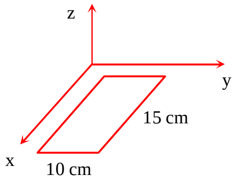


17. For the circuit shown, E = 50 V, R_1 = 10 Ω, R_2 = 20Ω, R_3 = 30Ω, L = 2.0 mH. The current through R_1 and R_2 are _________________________ and ____________________ immediate after the switch is closed.



18. For the situation described in the figure. The magnetic field changes with according to B = $(2t^3 - 4t^2 + 0.8)$T and r_2 = 2R = 5 cm. Then the force on an electron located at P_2 at t = 2.0 sec is ____________________.

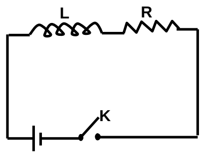
19. In the above question the magnitude and direction of the electric field at P_1 when t = 3.00 and r_1 = 0.02 m are ___________________________ and _____________________.



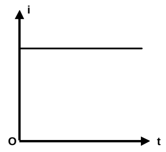
20. A rectangular loop (10 × 15 cm) lies in x–y plane as shown in the figure. A time dependent magnetic field $\vec{B} = B_0\left(x\sin\omega t\hat{j} + y\cos\omega t\,\hat{k}\right)$, with B_0 = 120 gauss/m and ω = 10000 rad/sec exist within the loop. Induced emf in the loop due to the changing magnetic field is ___________________

Level – II

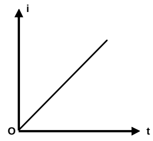
1. Which one of the following graphs represent correctly the variations of current (i), time (t) when key k is pressed in the circuit shown in the figure:



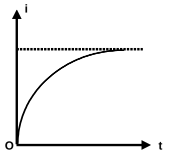
(A)



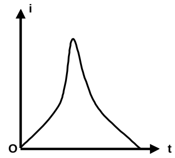
(B)



(C)



(D)



2. A metal rod of resistance 20 Ω is fixed along a diameter of a conducting ring of radius 10 cm and lies on the x–y plane. There is a magnetic field $\vec{B} = 50(\text{T})\hat{k}$. The ring spins with an angular velocity 20 rad/s about its axis. An external resistance of 10 Ω is connected across the centre of the ring and rim. The current through external resistance is

(A) $\frac{1}{3}$ A (B) $\frac{5}{3}$ A

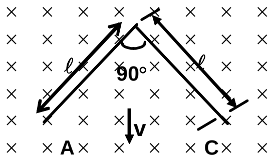
(C) $\frac{1}{4}$ A (D) $\frac{1}{2}$ A

3. A metallic wire is folded to form a square loop of side a. It carries a current i and is kept perpendicular to the region of uniform magnetic field B. If the shape of the loop is changed from square to an equilateral triangle without changing the length of the wire and current. The amount of work done in doing so is

(A) $Bia^2\left(1-\frac{4\sqrt{3}}{9}\right)$ (B) $Bia^2\left(1-\frac{\sqrt{3}}{9}\right)$

(C) $\frac{2}{3}Bia^2$ (D) Zero

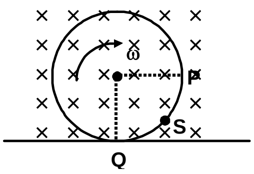
4. A rigid conducting wire bent as ∧ shaped, is released to fall freely in a horizontal magnetic field which is perpendicular to the plane of the conductor. If magnetic field strength is B then the emf induced across the point A and C when it has fallen from rest through a distance h will be



(A) $B\ell\sqrt{2gh}$ (B) $B\ell\sqrt{gh}$

(C) $2B\ell\sqrt{gh}$ (D) $2B\ell$

5. A conducting ring of radius r is rolling without slipping with a constant angular velocity ω . If the magnetic field strength is B and is directed into the page then emf induced across PSQ is

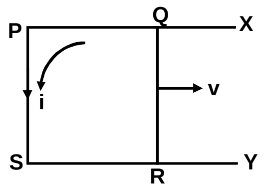


(A) $B\omega r^2$ (B) $B\omega r^2/2$
(C) $4 B\omega r^2$ (D) $B\omega r^2/4$

6. A coil of inductance 1H and negligible resistance is connected to a source of supply whose voltage is given by V = 4t volt. If the voltage is applied when t = 0 and switched off at time 4 sec., the energy stored in the coil will be
(A) 512 J (B) 256 J
(C) 1024 J (D) 144 J

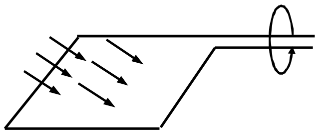
7. Two coils A and B have 200 and 400 turns respectively. A current of 1 A in coil A causes a flux per turn of 10^{-3} Wb to link with A and a flux per turn of 0.8×10^{-3} Wb through B. The ratio of self–inductance of A and the mutual inductance of A and B is
(A) 5/4 (B) 1/1.6
(C) 1.6 (D) 1

8. QR is a wire moving with a speed v as shown in figure. The conducting rails PX and SY are smooth. The induced current is in anticlockwise direction. The magnetic field in the region is :



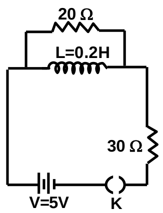
(A) directed from left to right in the plane of page.
(B) directed from right to left in the plane of page.
(C) perpendicular to the plane of page and directed away from reader.
(D) perpendicular to the plane of page and directed towards reader.

9. A plane rectangular coil of one turn and area A is spun at a frequency n about one of its sides, which is placed at 90^o to a uniform magnetic field having flux density B. The peak e.m.f. induced in the coil is :



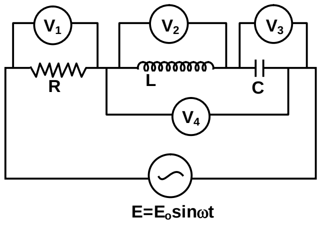
(A) BAn (B) πBAn
(C) 2BAn (D) 2πBAn

10. An ideal inductance of 0.2 H is connected as shown in figure. After the key k is pressed, the current in 20 Ω resistance reaches a steady value of



(A) 0.1 A (B) 0.25 A
(C) 0.017 A (D) zero

11. In the given figure, which voltmeter will read zero voltage at resonant frequency ω rad/sec?



(A) V_1 (B) V_2
(C) V_3 (D) V_4

12. In a R, L, C circuit, three elements is connected in series by an AC source. If frequency is less than resonating frequency then net impedance of the circuit will be
(A) capacitive (B) inductive
(C) capacitive or inductive. (D) pure resistive.

13. Using an AC voltmeter, the potential difference in the electrical line in a house is read to be 234 volts. If the line frequency is known to be 50 cycles per second, the equation for the line voltage is
(A) $V = 165 \sin(100\pi t)$ (B) $V = 331 \sin(100\pi t)$
(C) $V = 234 \sin(100\pi t)$ (D) $V = 440 \sin(100\pi t)$

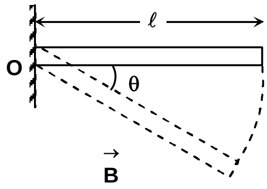
14. In LCR circuit the inductance is changed from L to 4L. For the same resonant frequency, the capacitance should be changed from C to
(A) 2C (B) C/2
(C) C/4 (D) 4C

15. In an AC circuit, the current is $I = 5 \sin(100-\pi/2)$ A and the AC potential is $V = 200 \sin(100t)$ volt. Then the power consumption is
(A) 20 watts (B) 40 watts
(C) 1000 watts (D) 0 watts

16. Two long solenoids having their radii R_1 and R_2 and number of turns N_1 and N_2 carry current i_1 and i_2 respectively. If the ratio of $R_1/R_2 = \frac{1}{4}$, $N_1/N_2 = \frac{4}{1}$ and $i_1/i_2 = \frac{1}{2}$ the ratio of their self inductances L_1/L_2 will be (ignore mutual inductance),
(A) 1:2 (B) 2:1
(C) 1:4 (D) 1:1

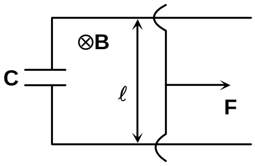
17. A given LCR series circuit satisfies the condition for resonance with a given AC source. If the angular frequency of the AC source is increased by 100%, then in order to establish resonance, and without changing the value of inductance the capacitance must be
(A) Increased by 100 % (B) Reduced by 50 %
(C) Increased by 75 % (D) Reduced by 75 %.

18. A conducting rod of length ℓ is hinged at point O. It is free to rotate in vertical plane. There exists a uniform magnetic field $\vec{B}$ in horizontal direction. The rod is released from position shown in the figure. Potential difference between two ends of the rod is proportional to



(A) $\ell^{3/2}$ (B) ℓ^2
(C) $\sin\theta$ (D) $(\sin\theta)^{1/2}$

19. A conducting wire of length ℓ and mass m can slide without friction on two parallel rails and is connected to capacitance C. Whole system lies in a magnetic field B and a constant force F is applied to the rod. Then



(A) the rod moves with constant velocity
(B) the rod moves with an acceleration of $\frac{F}{m+B^2\ell^2c}$
(C) there is constant charge on the capacitor.
(D) charge on the capacitor increases with time

20. A circular loop of radius r, having N turns of a wire, is placed in a uniform and constant magnetic field B. The normal of the loop makes an angle θ with the magnetic field. Its normal rotates with an angular velocity ω such that the angle θ is constant. Choose the correct statement from the following.
(A) emf in the loop is $\frac{NB\omega r^2}{2}\cos\theta$.
(B) emf induced in the loop is zero.
(C) emf must be induced as the loop crosses magnetic lines.
(D) emf must not be induced as flux does not change with time.

ANSWERS TO ASSIGNMENT PROBS

Subjective:

Level – O

3. (a) in phase with (b) leads (c) lags
4. $\frac{1}{2}B\omega\ell^2$
5. 8 volts
6. 1.3×10^{-3} V
7. 7.5 V
8. 8×10^{-5} Wb, 8×10^{-3} wb
9. 1.18×10^{-5} H
10. 0.1056 V
11. 5 V
12. (a) along abcda (b) along abcda
13. (a) 26.5 Ω, (b) 9.42 Ω, (c) 26.3 Ω
14. 16.09 Ω
15. 1.39 ms, 1.44 mJ
16. 0.19 H.
17. 5.06 s.
18. 0.29 A
19. $P_{av}(R) = 789.6$ W, $P_{av}(L) = 0$, $P_{av}(c) = 0$, $P_{av}(\text{total}) = 789.6$ W.
20. 16.085 V, 11.375 V

Level– I

1. 3.9×10^{-5} A
2. $\frac{B^2l^2v}{R}$
3. (a) 2.5×10^3 V/s (b) 17 V/s (c) 0 V/s
4. 6 V
5. (a) 35 ms, (b) 61ms, (c) 61 ms
6. 10 H
7. (a) 311V, 220V (b) 0 (c) f = 50Hz
8. $\pi/4$
9. I = 3.28A, $\phi = 79.7°$
10. (a) 15Ω (b) 8A (c) 100.7Hz
11. $\frac{20}{\pi^2} \cong 2$H
12. (a) PF = 0.3, $P_{AV} = 0.018$ W
13. (a) 3×10^{-4} V, (b) zero, (c) 3×10^{-4} V and (d) zero. 2×10^{-4} J
14. 1.967×10^{-4} C
15. $E = \frac{\mu_o}{4\pi}\frac{2Ia^2v}{x(x+a)}$.
16. (a) 1.25×10^{-7} A, a to d, (b) 1.25×10^{-7} A, d to a, (c) zero (d) zero
18. $\frac{B_o v_o L}{2}$
19. (a) 0.1 mA (b) 0.2 mA
20. (a) $\frac{E(R_1+R_2)}{R_1R_2}$, (b) $\frac{L}{R_1+R_2}$ (c) $\frac{E}{R_1e}$

Level – II

1. 0.15 V
2. 15.9 mH, 1.59 μF
3. $C = 9.2\mu F$
4. V_l = 114V, L = 0.76H
5. 10V, $1.2\times10^4\Omega$
6. 125.6V
7. $\frac{1}{C\sqrt{2}}$
8. $n^2a^2v\ B^2/R$
9. $960\mu_o \cos(400t)$V/m.
10. 6.28 C
11. $\frac{\omega aAB_0}{2\rho}$
12. (a) 2.0×10^{-3} V (b) zero (c) 5.0×10^{-5} C
13. $\frac{\mu_0\ell}{\pi}\ln\left(\frac{d-r}{r}\right)$
14. $\frac{dB}{dt}\frac{L}{2}\sqrt{R^2-\left(\frac{L}{2}\right)^2}$
15. $x = -\frac{v_0}{\omega}\sin\omega t$ where $\omega=\sqrt{\frac{B^2\ell^2}{ML}}$
16. (a)$I=\frac{\mu_o v I_o}{2\pi R}\ln\frac{b}{a}$ (b)$F=\frac{v}{R}\left(\frac{\mu_o I_o}{2\pi}\ln\frac{b}{a}\right)^2$
17. 0.704 A, 5.2×10^{-3} J
18. 0.08H, 17.28 W
19. $Z_P = 77\ \Omega$, $Z_Q = 97.6\ \Omega$, $V_P = 7.7$ V and $V_Q = 9.76$ V
20. 2A, 400W

Objective:

Level – I

1.	**B**	2.	**B**
3.	**C**	4.	**A**
5.	**C**	6.	**B**
7.	**B**	8.	**D**
9.	**C**	10.	**A**
11.	**B**	12.	**B**
13.	**D**	14.	**C**
15.	**C**		

Fill in the blanks

16. $0.5(1 - e^{-10t})$A, $1.50 - 0.25e^{-10t}$ A
17. 1.67 A, 1.67 A
18. 8×10^{-21} N (downward and to the right perpendicular to r_2)
19. 0.3 V/m, upward and to the left perpendicular to r_1
20. $0.009 \sin \omega t$ V

Level– II

1.	**C**	2.	**C**
3.	**A**	4.	**C**
5.	**A**	6.	**A**
7.	**B**	8.	**C**
9.	**D**	10.	**D**
11.	**D**	12.	**A**
13.	**C**	14.	**C**
15.	**D**	16.	**D**
17.	**D**	18.	**A, D**
19.	**B, D**	20.	**B, D**

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

ELECTROMAGNETIC WAVES

Electromagnetic Waves

Syllabus for IIT-JEE and Karnataka Board:
Electromagnetic waves and their characteristics (qualitative ideas only); Transverse nature of electromagnetic waves. Electromagnetic spectrum (Radio−microwaves, infra−red, optical, ultraviolet, X−rays, gamma rays) including elementary facts about their uses; Propagation of electromagnetic waves in atmosphere.

SOURCE OF ELECTROMAGNETIC WAVE

How are electromagnetic waves produced?
Neither stationary charges nor charges in uniform motion (steady currents) can be sources of electromagnetic waves. The former produces only electrostatic fields, while the latter also produces magnetic fields that however, do not vary with time. It is an important result of Maxwell's theory that accelerated charges radiate electromagnetic waves.

Consider a charges oscillating with some frequency. This produces an oscillating electric field in a space which produces an oscillating magnetic field which in turn is a source of oscillating electric field and so on. The oscillating electric and magnetic fields thus regenerate each other, so to speak, as the wave propagates through the space. The frequency of electromagnetic wave naturally equals the frequency of oscillation of the charge. The energy associated with the propagating wave comes at the expense of energy of the source – the accelerated charge.

How electromagnetic wave in propagate in a medium.

The propagation of electromagnetic wave is also due to inertial and elastic properties of the medium. Every medium (including vacuum) has inductive property described by what we call the magnetic permeability μ of the medium. This property provides the magnetic inertia of the medium. The elasticity of the medium is provided by the capacitive property called the electrical permittivity ε of the medium. Permeability μ states the magnetic energy and permittivity ε stores the electric field energy. This electromagnetic energy propagates in the medium in the form of electromagnetic waves.

MAXWELL'S EQUATION

Four basic laws of physics: Gauss law in electrostatics, Gauss law in magnetism, Faraday's law of electromagnetic induction and Ampere's circuital law were stated by Maxwell in the form of four integral and differential equations called Maxwell's equations.

Gauss' law: It states that total electric flux through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

$$\oint \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0}$$

Gauss's law in magnetism: It states that net magnetic flux crossing any closed surface is zero.
$$\oint \vec{B}.d\vec{s} = 0$$

Faraday's law of electromagnetic induction
It states that changing magnetic flux induces an electric field.

Maxwell's displacement current:

It was maxwell who recognised that a changing electric field is equivalent to an elastic current. He introduced the concept of what is called displacement current density which is defined as

$$I_D = \frac{\partial \Delta}{\partial t} \quad \ldots \text{(i)}$$

Where D is the displacement vector.

The displacement current I_D over any surface S is defined in a ---------- analogous to the definition of the conventional current i.

$$I_D = \int_D I_D.ds \quad \ldots \text{(ii)}$$

From (i) and (ii)

$$I_D = \int \frac{\partial \Delta}{\partial t}.ds = \frac{\partial}{\partial t}\int \varepsilon_0 E.ds$$

$$I_D = \varepsilon_0 \frac{\partial}{\partial t}\int_f E.ds$$

$$I_d = \varepsilon_0 \frac{d\phi}{dt}.$$

Ampere's law:

$$\oint B.d\vec{\ell} = \mu_0 I$$

where I is the electric current crossing a surface bounded by a closed curve and line integral of $\vec{B}$ is calculated along that closed curve. This equation is valid only when the electric field at the surface does not change with time.

As an example, consider a parallel plate capacitor with circular plates being charged by a battery. If we place a compass needle in the space between the plates, the needle gets deflected. This shows that there is a magnetic field in the region.

As there is no current between plates, hence in this case

$$\oint \vec{B}.d\vec{\ell} \neq \mu_0 I$$

Source of this magnetic field is the changing electric field. As the capacitor gets charged, the electric field between the plates changes and this changing electric field produce magnetic field. So, Maxwell modified Ampere's law.

Maxwell Ampere's circuital law

It states that the line integral of magnetic field along a closed path is equal to μ_0 times the total current.

$$\oint \vec{B}.d\vec{l} = \mu_0 I = \mu_0 \left(I_c + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

I_c = conduction current (due to flow of charge), and

$I_D = \varepsilon_0 \frac{d\phi_E}{dt} =$ displacement current (due to charging electric field)

The displacement current arises due to varying electric field.

So, in 1865, Maxwell pointed out "There is a great symmetry in nature, i.e. change in either field (electric or magnetic) with time produces the other field". This idea led Maxwell to conclude that the variation in electric and magnetic field vectors perpendicular to each other constitute an electromagnetic wave, which propagates in space in a direction perpendicular to direction of both fields.

Exercise 1: What is Maxwell displacement current ?

Illustration 1: *A parallel plate capacitor has circular plates, each of radius 5.0 cm. It is being charged so that electric field in the gap between its plates rises steadily at the rate of 10^{12} V/m. What is the magnitude of displacement current ?*

Solution: $I_D = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \pi r^2 \frac{dE}{dt} = 8.85 \times 10^{-12} \times \pi \times (5 \times 10^{-2})^2 \times 10^{12} = 0.07$ A.

Illustration 2: *How would you establish an instantaneous displacement current of 1 A in the space between the two parallel plates of a 2 μF capacitor?*

Solution: Here, $I_D = 1$ Amp, and $C = 2\ \mu$F

So $\frac{dV}{dt} = \frac{I_D}{C} = \frac{1}{2\times10^{-6}} = 5\times10^5$ volt / sec.

Thus, a displacement current of 1 Amp. can be set up by changing the potential difference across the parallel plates of the capacitor at the rate of 5×10^5 volt /sec.

TRANSVERSE NATURE OF ELECTROMAGNETIC WAVE

An electromagnetic wave consists of sinusoidal time varying electric and magnetic fields acting at right angle to each other as well as at right angle to the direction of propagation of the wave.

Two mutually perpendicular time varying electric and magnetic field may be represented by

$E = E_m \sin(kx - \omega t)$

$B = B_m \sin(kx - \omega t)$

where E and B are the sinusoidally varying electric and magnetic fields at the position x at time t, ω is angular frequency, k is called wave number.

$k = \frac{2\pi}{\lambda}$

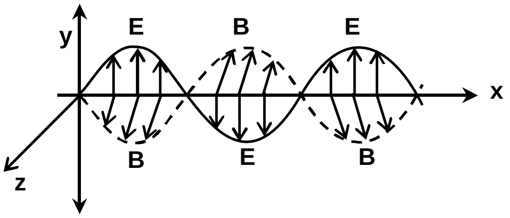
If the wave propagates with speed c,

$c = \frac{\omega}{k}$

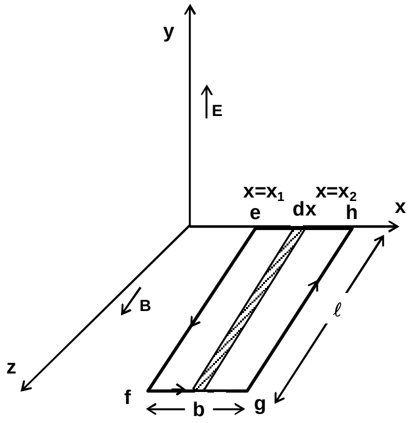
E_m and B_m are called amplitudes of E and B_y respectively.

So the two fields combine to constitute an electromagnetic wave, which propagates in space in a direction perpendicular to the direction of both the fields.

The figure below shows the propagation of a plane electromagnetic wave with velocity c along x–axis. The electric fields vector is along y–axis, while magnetic field vector is along z–axis. Since the electric field, magnetic field and propagation of wave are perpendicular to each other, therefore electromagnetic waves are transverse in nature.



Velocity of electromagnetic waves:



Consider a plane electromagnetic wave propagating along positive direction of x–axis in space with speed c.

Let us consider the rectangular path efgh in the x–z plane as shown in figure.

$$\oint \vec{B}.d\vec{\ell} = \int_e^f \vec{B}.d\vec{\ell} + \int_f^g \vec{B}.d\vec{\ell} + \int_g^h \vec{B}.d\vec{\ell} + \int_h^e \vec{B}.d\vec{\ell}$$

$$= B_0\ell\,[\sin \omega(t - x_1/c) - \sin \omega(t - x_2/c)] \qquad \text{. . . (i)}$$

$$\phi_E = \oint \vec{E}.d\vec{s} = \int_{x_1}^{x_2} E(x)dx\,\ell$$

$$= E_0\ell \int_{x_1}^{x_2} \sin\omega(t - \frac{x}{c})dx$$

$$= -\frac{c}{\omega}E_0\ell\left[-\cos\omega(t - \frac{x_2}{c}) + \cos\omega(t - \frac{x_1}{c})\right]$$

$$\frac{d\phi_E}{dt} = -cE_0\ell[\sin\omega(t - \frac{x_2}{c}) - \sin\omega(t - \frac{x_1}{c}) \qquad \text{. . . (ii)}$$

The Ampere's law for vacuum is

$$\oint \vec{B}.d\vec{\ell} = \mu_0\varepsilon_0\frac{d\phi_E}{dt} \qquad \text{. . . (iii)}$$

From (i), (ii) and (iii)

$$B_0 = \mu_0\,\varepsilon_0\,cE_0$$

$$\mu_0\varepsilon_0 = \frac{B_0}{cE_0}$$

But $\frac{E_0}{B_0} = c$

$\therefore \qquad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Putting the values of μ_0 and ε_0, we get

$c = 2.99793 \times 10^8$ m/s, which is same as the speed of light.

Important facts about the electromagnetic waves:

(i) The electromagnetic waves are produced by accelerating or oscillating charge.

(ii) Electromagnetic waves do not require any material medium for their propagation.

(iii) These waves travel in free space with a speed 3×10^8 m/s given by the

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

(iv) These waves are transverse in nature like light waves.

(v) The variation in the amplitudes of electric and magnetic fields in the electromagnetic waves always take place at the same time and at same point in the space.

Thus, the ratio of the amplitudes of electric and magnetic fields is always constant and it is equal to velocity of electromagnetic waves.

$$\frac{E_0}{B_0} = c$$

(vi) These waves obey superposition principle.

(vii) The energy in electromagnetic waves is divided equally between electric and magnetic field vectors.

Average electric energy density = $\frac{1}{4}\varepsilon_0 E^2$

Average magnetic energy density = $\frac{1}{4\mu_0}B^2$

Average energy density of an electromagnetic wave is $\frac{1}{2}\varepsilon_0 E^2$ or $\frac{B^2}{2\mu_0}$

Intensity: Energy crossing per unit area per unit time perpendicular to the direction of propagation is called intensity of the wave.

$I = \frac{U}{A\Delta t} = U_{av}c = \frac{1}{2}\varepsilon_0 E_0^2 c$

Momentum: If a portion of an electromagnetic wave of energy U is propagating with speed c, then linear momentum of the electromagnetic wave is given by

$$p = \frac{U}{c}$$

If the incident electromagnetic wave is completely absorbed by a surface it delivers energy U and momentum $\frac{U}{c}$ to the surface. If the electromagnetic wave is completely reflected, then the momentum delivered to the surface is $\frac{2U}{c}$. As the momentum of electromagnetic wave changes from p to – p, this shows that the electromagnetic wave exerts a force on the surface on which it is incident.

Illustration 3. *What is the wavelength of a television station which can transmit vision on 500 MHz ? Given c = 3 × 10⁸ m/s.*

Solution: $\nu = 500\text{ MHz} = 500 \times 10^6$ Hz.

$c = 3 \times 10^8$ m/s

$\lambda = \frac{c}{\nu} = 0.6\text{m}$

Illustration 4. *A light beam travelling in the x–direction is described by the electric field: $E_y = 270 \sin \omega(t - x/c)$. An electron is constrained to move along the y–direction with speed of 2×10^7 m/s. Find maximum electric force and maximum magnetic force on the electron.*

Solution: $E = E_0 \sin \omega(t - x/c)$

$E_0 = 270$ V/m

$B_0 = \frac{E_0}{c} = 9\times10^{-7}$ T

$F_e = qE_0 = 1.6 \times 10^{-19} \times 270 = 4.32 \times 10^{-17}$ N

$F_{mag} = qvB_0 = 2.88 \times 10^{-18}$ N.

Illustration 5. *A magnetic field in a plane electromagnetic wave is given by $B_y = 3 \times 10^{-7} \sin (0.314 \times 10^3 x + 3.14 \times 10^{11} t)$ Tesla. Write down an expression for the electric field.*

Solution: $E_0 = cB_0 = (3 \times 10^8) \times (3 \times 10^{-7}) = 90$ V /m.

$\therefore\ E_z = E_0 \sin\frac{2\pi}{\lambda}(x + vt)$

$\therefore\ E_z = 90 \sin(0.314 \times 10^3 x + 3.14 \times 10^{11} t)$ V /m

Illustration 6. *The photon energy, in eV, for electromagnetic waves of wavelength 40 m is*

(A) 3.1×10^{-8} *(B) 4.8×10^{-27}*
(C) 6.2×10^{-8} *(D) 9.6×10^{-27}*

Solution: $E = hc/\lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{40\times e} = 3.1 \times 10^{-8}$ *eV*

Hence (A) is correct option.

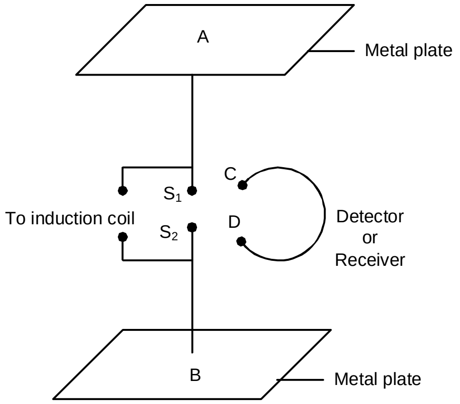
Exercise 2: ***Give the ratio of velocities of light rays of wavelengths 4500 A° and 7500 A°***

Exercise 3: ***Light can travel in vacuum whereas sound cannot do so, why.***

HERTZ EXPERIMENT

Hertz experiment was based on the fact that an oscillating electric charge radiates electromagnetic waves and these waves carry energy which is being supplied at the cost of kinetic energy of the oscillating charge. The detailed study revealed that the electromagnetic radiation is significant only if the distance to which the charge oscillates is comparable to the wavelength of radiation.

Hertz Apparatus



The experimental setup used by Hertz for the production and detection of electromagnetic wave: A and B are two large square metal plates of copper and zinc placed about 60 cm apart. They are connected to two highly polished metallic spheres S_1 and S_2 through thick copper wires. A high potential difference of several thousand volts is applied across the square using induction coil. Due to high potential difference across S_1 and S_2, the air between the spheres gets ionized and provides a path for discharge of plates. Due to it a spark is produced between S_1 and S_2 and electromagnetic waves of high frequency are radiated. Here, the two plates act as a capacitor having small capacitance C and connecting wires provide a low inductance L. The high frequency of oscillations of charges between the plate is given by

$$\nu = \frac{I}{2\pi\sqrt{LC}}$$

The frequency of oscillations is of the order of 5×10^7 Hz. With this arrangement Hertz could obtaine radiation of wavelength about 6 m.

ELECTROMAGNETIC SPECTRUM

After the experimental discovery of electromagnetic waves by Hertz, many other electromagnetic waves were discovered by different ways of excitation.

The orderly distribution of electromagnetic waves (according to wavelength or frequency) in the form of distinct groups having widely different properties is called electromagnetic spectrum.

Main parts of electromagnetic spectrum

The electromagnetic spectrum may be divided into following main parts.

Radiowaves

Radiowaves are produced by oscillating electric circuits having an inductor and capacitor. The range of frequency of radio–waves is from 5×10^5 Hz to 10^9 hz.

Uses: Radiowaves are used for wireless communication purposes. They are used to transmit radio and TV signals.

Microwaves

The frequency of microwaves varies from 1 GHz to 300 GHz. They are produced by oscillating electronic circuits.

Uses: Microwaves are used in RADAR system, long distance telephone communication and for cooking purpose.

Infrared Waves

The frequency of these waves varies from 3×10^{14} Hz to 4×10^{11} Hz. Infrared waves are sometimes called as heat waves. Infrared waves are produced by hot bodies and molecules.

Uses: (i) Infrared rays are used in solar water heaters and cookers.
(ii) Infrared ray photographs are used for weather forecasting.
(iii) Infrared rays are used for taking photographs during the condition of fog.

Visible Light

The frequency of these waves varies from 4×10^{14} Hz to 8×10^{14} Hz. It is produced due to atomic excitation.

Ultraviolet Rays

The frequency of these waves varies from 8×10^{14} Hz to 5×10^{17} Hz. The ultraviolet rays are produced by sun, special lamps and hot bodies. The ultraviolet rays in large quantity produce harmful effects on human being.

Uses:

(i) Ultraviolet rays are used for checking the mineral samples by making use of its property of causing fluorescence.
(ii) It is used in the study of molecular structure.
(iii) It can cause photoelectric effect.
(iv) Ultra–violet rays destroy bacteria and hence they are used for sterilizing surgical instruments.

X–RAYS

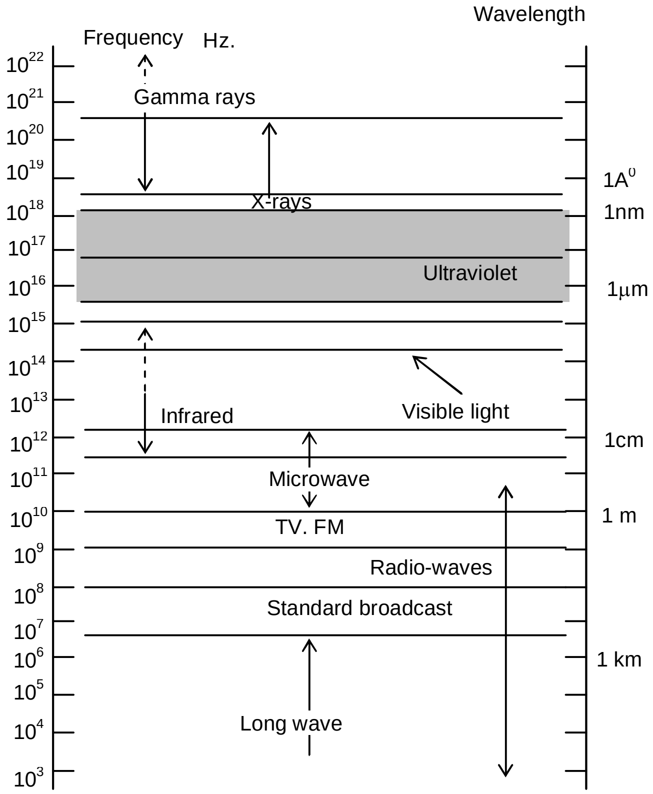
The frequency: of these waves varies from 10^{16} Hz to 3×10^{21} Hz. X–ray can be produced when high energy electrons are stopped suddenly on a metal of high atomic number. X–rays have high penetrating power.

Uses:
(i) **In surgery:** For the detection of fractures, foreign bodies like bullets and stones in the human body.
(ii) **In engineering:** for detecting faults, cracks and holes in final metal product.
(iii) **In scientific research:** for the investigation of structure of crystals arrangement of atoms and molecules in the complex substances.

γ –rays

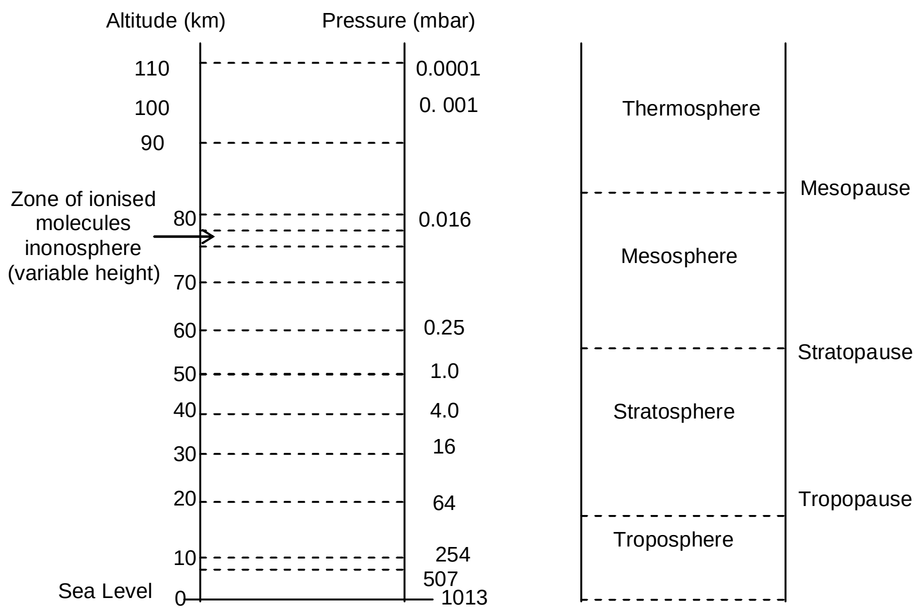
The frequency of these waves varies from 3×10^{18} Hz to 5×10^{22} Hz. These can be produced by nucleus of radioactive substances.

Uses: (i) It can be used to treat cancer and tumours.
(ii) It can be used to produce nuclear reactions.
(iii) It can be used to preserve the food stuffs for a long time.



PROPAGATION OF ELECTROMAGNETIC WAVES IN ATMOSPHERE

The atmosphere is a gaseous envelope surrounding our earth. It is retained to the earth due to gravitational attraction. As we go up, the air thins out gradually and air pressure decreases. The atmosphere can be divided into various layers as shown in figure below. The layers are known by different names and with tops denoted by pauses.



(The earth's atmosphere)

The troposphere includes the layer close to the earth and extends upto about 12 km. This layer is responsible for all the important weather phenomena affecting our environment. The next layer, called the stratosphere, extends from about 10 – 16 km to about 50 km. The mesosphere extends from about 50 km to about 80 km. The thermosphere extends from 80 km to the edge of the atmosphere. It receives energy directly from the solar radiation. The ozone layer is in the lower stratosphere and extends from 15 km to about 30 km. This ozone results from the dissociation of molecular oxygen by solar ultraviolet radiation in the upper atmosphere, called ionosphere, which is composed partly of electrons and positive ions. The rest of the atmosphere is composed mostly of neutral molecules.

The sun is main source of electromagnetic waves of different wavelengths towards the earth. As the electromagnetic waves propagate through earth's atmosphere, most of infra–red radiation is absorbed by atmosphere and it gets heated. A small part of visible light is also absorbed in atmosphere.

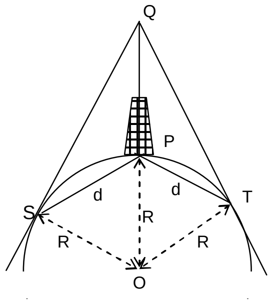
Ultraviolet radiation is harmful to planets and living cells, so ozone layer absorbs most of ultraviolet radiation. Also ozone layer converts the ultraviolet radiation to infra – red which further heats up the atmosphere and the earth's surface.

The propagation of radio waves depends on the wavelength of the waves. Medium frequency (MF) waves (300 kHz–3 MHz) are largely absorbed and the high frequency (HF) waves (3–30 MHz) are reflected back by the ionosphere. In the range 30 MHz to 3 GHz, waves are transmitted from one place to another either by direct line of sight using tall towers, or by beaming to artificial satellites and rebroadcasting from there.

Exercise 4: ***Why are microwaves used in RADAR.***

Exercise 5: ***What is the role of ozone layer in atmosphere.***

Height of transmitting antenna:
Suppose PQ is a TV transmitting antenna of height h located at P on the surface of earth. Due to finite curvature of the earth, the signal transmitted from Q can not be received beyond the tangent point T and S.



The effective reception range of broadcast is essentially the region from S to T, which is covered by the line of sight during TV. transmission. Let PS = PT = d. The distance d is limited by the curvature of earth therefore; the TV signals will be received in a circle of radius d.
PQ = h
In triangle QPS,
$(SQ)^2 = PS^2 + PQ^2 = d^2 + h^2$
In triangle OSQ,
$OQ^2 = OS^2 + SQ^2$
$(R + h)^2 = h^2 + d^2 + h^2$
$d = \sqrt{2Rh}$.
For TV signal area covered = $\pi d^2 = 2\pi Rh$
Population covered = population density $\times$ area covered

Illustration 7. *What should be the height of transmitted antenna if the TV telecast cover a radius of 120 km? If the average population density around the tower is 500/km^2, how much population is covered ? $R_e = 6 \times 10^6$ m*

Solution: Height of transmitted antenna

$$h = \frac{d^2}{2R_e}$$

$$= \frac{(120\times10^3)^2}{2\times6\times10^6} = 1200 \text{ m.}$$

Total population covered $= \pi d^2 \times$ population density
$= 2.26 \times 10^7$

Illustration 8. *A TV tower has a height 50 m if the average population density around the tower is 500 per km^2 then the population covered by TV broadcast is*
(A) 40 lac *(B) 20 lac*
(C) 60 lac *(D) 10 lac*

Solution: $d = \sqrt{2hR}$
Area covered $= \pi d^2 = 2\pi hR = 637\pi$ km^2
Population covered $= 637\pi \times 500$ = 10 lacks
Hence (D) is correct option.

Wein's displacement law

This law gives the relation between the wavelength of radiation emitted from a body to maximum intensity and the temperature of the body

$\boldsymbol{\lambda_m T}$ **= constant = b**

where λ_m = wavelength (in meter) of radiation emitted from a body corresponding to maximum intensity, T = temperature of the body in Kelvin, and b = Wein's constant $= 2.889 \times 10^{-3}$ mK

Illustration 9. *Obtain the temperature ranges for ultraviolet part of radiation of EM waves.Take frequency of ultraviolet part of radiation as 8×10^{14} Hz to 5×10^{17} Hz.*

Solution: The corresponding wavelength to the frequency 8×10^{14} Hz is $\lambda_1 = c / \nu_1$
$= 3.75 \times 10^{-7}$ m.
The corresponding wavelength to the frequency 5×10^{17} Hz is $\lambda_2 = c / \nu_2$
$= 6 \times 10^{-10}$ m
As $\lambda_m \times T = 2.9 \times 10^{-3}$ so $T_1 = 7.73 \times 10^3$ K and $T_2 = 4.83 \times 10^6$ K
Temperature ranges 7.73×10^3 K to 4.83×10^6 K

Illustration 10. *The temperature (approx) of the body emitting wavelength 0.3 μm corresponding to maximum intensity is*

(A) 2×10^4 K *(B) 10^4 K*
(C) 2×10^3 K *(D) 10^3 K*

Solution: $\lambda_m T = b$ (Wien's displacement law)

So, $T = \dfrac{2.9 \times 10^{-3}}{3 \times 10^{-7}} \approx 10^4$ K

Hence, (B) is correct option.

SUMMARY

Displacement current:

$\mathbf{I_D = \varepsilon_0}\ \frac{d\phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt}$

$= \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}$

or $I_D = C\frac{dV}{dt}$ $\left(\because E = \frac{V}{d} \text{ and } C = \frac{\varepsilon_0 A}{d}\right)$

Maxwell's displacement current:

$$\oint \vec{B}.d\vec{\ell} = \mu_0 (I + I_D)$$

Energy density of electromagnetic waves

- Energy of a photon, $E = h\nu = hc/\lambda$

- $E_0 = B_0 c$,

Where E_0 = amplitude (or maximum value) of electric field and
B_0 = amplitude (or maximum value) of amplitude

- Average energy density of electric field, $u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{E_0}{\sqrt{2}}\right) = \frac{1}{4}\varepsilon_0 E_0^2$

and average energy density of magnetic field $u_B = \frac{1}{2\mu_0}B^2 = \frac{B_0^2}{4\mu_0}$

Total average energy density $= u_E + u_B = \frac{B_0^2}{2\mu_0}$

Height of antenna

$d = \sqrt{2hR}$

where d is the distance in meter, h is height of tower in meter and R is the radius of earth in meter.

Area covered = $\pi d^2 \times$ population density

Wein's displacement law

$\lambda_m T$ = constant = b

where λ_m = wavelength (in meter) of radiation emitted from a body corresponding to maximum intensity, T = temperature of the body in Kelvin, and b = Wein's constant = 2.889×10^{-3} mK

MISCELLANEOUS EXERCISE

1. Which part of electromagnetic spectrum is used in operating a RADAR ?

2. Which of the following has shortest wavelength, microwave, ultraviolet wave and X–rays?

3. Name the electromagnetic radiations used for viewing the objects through haze and fog.

4. Give the ratio of velocities of light rays of wavelengths 4000 A° and 6000 A° in vacuum.

5. What is the limit of frequency upto which signals using sky waves can be transmitted.

6. From which layer of atmosphere radio and microwaves are reflected back ?

7. Which are relevant waves in telecommunication ?

8. What oscillates in electromagnetic waves ?

9. What is the name given to the part of electromagnetic spectrum which is used for taking photographs of earth under foggy conditions from great heights ?

10. State two applications of ultraviolet radiation.

ANSWERS TO MISCELLANEOUS EXERCISE

1. Microwaves
2. X–rays (1A°)
3. Infrared radiations.
4. One
5. 1.5 MHz. to 40 MHz.
6. Ionosphere
7. Microwaves
8. The electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$.
9. Infrared rays
10. (i) To preserve the food stuff,
    (ii) For sterilizing the surgical instruments

SOLVED PROBLEMS

Subjective:

Problem 1. *What is the wavelength of electromagnetic wave having frequency 1000 MHZ.*

Solution: v = 1000×10^6 Hz. c = 3×10^8 m/s

$\lambda = \frac{c}{v} = 0.3\text{m}$

Problem 2. *The voltage between the plates of a parallel plate capacitor of capacitance 1.0 μF is changing at rate of 1.0 V/s. Find displacement current.*

Solution: Given C = 1.0 μF = 10^{-6} F, $\frac{dV}{dt} = 5\frac{v}{5}$

$I_D = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{VA}{d}\right)$

$= \frac{\varepsilon_0 A}{d}\frac{dV}{dt} = C\frac{dV}{dt} = 1 \times 10^{-6} = 10\ \mu A$

Problem 3. *In a plane electromagnetic wave, the electric field oscillates sinusoidaly at a frequency of 2×10^{10} Hz and amplitude 48 V/m. Find wavelength of the wave.*

Solution: $\lambda = \frac{c}{v} = \frac{3\times10^8}{2\times10^{10}} = 1.5 \times 10^{-2}$ m.

Problem 4. *In the above question what is the amplitude of the oscillating magnetic field.*

Solution: $B_0 = \frac{E_0}{C} = \frac{48}{3\times10^8} = 1.6 \times 10^{-7}$ T.

Problem 5. *A plane electromagnetic wave in the visible region is moving along the z–direction. The frequency of the wave is 10^{15} Hz and the electric field at any point is varying sinusoidally with him with an amplitude 0.5 V/m. Find average values of the densities of the electric and magnetic fields.*

Solution: $\langle u_E \rangle = \frac{1}{4}\varepsilon_0 E^2 = \frac{1}{4}\times 8.85\times10^{-12}\times\frac{1}{4} = 0.55\times10^{-12}\,J/m^3$

$\langle u_B \rangle = \frac{B_0^2}{4\mu_0} = \frac{1}{4}\frac{E_0^2}{4\mu_0 c^2} = 0.55 \times 10^{-12}$ J/m^3

Problem 6. *Electromagnetic waves travel in a medium at a speed of 2×10^8 m/s. The relative permeability of medium is 1. Find relative permittivity.*

Solution: $c' = \frac{1}{\sqrt{\mu c}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$

$\varepsilon_r = \frac{c^2}{c'^2\mu_r} = \left(\frac{3}{2}\right)^2 = 2.25$.

Problem 7. *The electric field in an electromagnetic wave is given by*

$$E = 50 \sin \frac{2\pi}{\lambda}(ct - x) N/C$$

Find the energy contained in a cylinder of cross–section 10 cm^2 and length 50 cm along the x–axis.

Solution: Average energy density of the electromagnetic wave is

$$< U_{av} > \ = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2} \times 8.55 \times 10^{-12} \times (50)^2 = 1.1 \times 10^{-8}\, Jm^{-3}$$

Volume = 10 (cm)^2 × 50 cm = 5 × 10^{-4} m^3

U = volume × energy density

= 5 × 10^{-4} × 1.1 × 10^{-8} = 5.5 × 10^{-12} J.

Problem 8. *The permittivity and permeability of free space are $\varepsilon_0 = 8.85 \times 10^{-12}\ c^2N^{-1}m^{-2}$ and $\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$ respectively. Find velocity of electromagnetic wave.*

Solution: $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ m/s

Problem 9. *Suppose that the electric field amplitude of an electromagnetic wave is E_0 = 120 N/c and its frequency is v = 50 MHZ. Find B_0, ω, k and λ (b) Find expression for $\vec{E}$ and $\vec{B}$.*

Solution: $B_0 = \dfrac{E_0}{c} = \dfrac{120}{3 \times 10^8} = 4 \times 10^{-7}\, T$

$\omega = 2\pi v = 3.14 \times 10^8$ rad/s

$\lambda = \dfrac{c}{v} = \dfrac{3 \times 10^8}{5 \times 10^7} = 6m$

$k = \dfrac{2\pi}{\lambda} = 1.05$ rad/m

$E = E_0 \sin (kx - \omega t)$

$= 120 \sin (1.05\,x - 3.14 \times 10^8 t)$

$B = B_0 \sin (kx - \omega t)$

$= 4 \times 10^{-7} \sin (1.05x - 3.14\, t)$

Problem 10. *How would you established an instantaneous displacement current of 4A in the space between the two parallel plates of 2μF capacitor.*

Solution: I_D = 4A, C = 2μF = 2 × 10^{-6}

$I_D = \varepsilon_0 A \dfrac{dE}{dt} = \varepsilon_0 A \dfrac{d}{dt}\left(\dfrac{v}{d}\right)$

$= \dfrac{\varepsilon_0 A}{d}\dfrac{dv}{dt}$

$\dfrac{dv}{dt} = \dfrac{I_D}{C} = \dfrac{4}{2 \times 10^{-6}} = 2 \times 10^6\, v/s$

So, by varying the voltage between plate at rate of 2 × 10^6 v/s, displacement current of 4A can be established.

Objective:

Problem 1. *The ratio of velocities of light waves of wavelengths 2000 A° and 3000 A° in vacuum is*

(A) 1 : 1 *(B) 2 : 3*
(C) 3/2 *(D) none of these*

Solution: Velocity of all electromagnetic wave in vaccum is same (= 3×10^8 m/s)

$\therefore$ (A)

Problem 2. *The charging current for a capacitor is 0.2 A. Find displacement current.*

(A) 0.2 A *(B) 0.4 A*
(C) 0.1 A *(D) zero*

Solution: Displacement current = charging current

$\therefore$ (A)

Problem 3. *Find the wavelength of electromagnetic waves of frequency 3×10^{12} Hz in free space*

(A) 10^{-4} m *(B) 10^4 m*
(C) 10^{-2} m *(D) 10^2 m*

Solution: $\lambda = c/\nu = \dfrac{3\times10^8}{3\times10^{12}} = 10^{-4}$ m

$\therefore$ (A)

Problem 4. *A plane electromagnetic wave is incident on a material surface. The wave delivers momentum P and energy E. Then*

(A) $P \neq 0, E \neq 0$ *(B) $P = 0, E = 0$*
(C) $P \neq 0, E = 0$ *(D) $P = 0, E \neq 0$*

Solution: (A)

Problem 5. *Given wave function for a wave to be $\psi(x,t) = 10^3 \sin (3 \times 10^6 x - 9 \times 10^{14} t)$*
The speed of the wave is

(A) 3×10^8 m/s *(B) 2×10^8 m/s*
(C) 10^8 m *(D) 6×10^8 m/s*

Solution: $c = \omega/k = \dfrac{9\times10^{14}}{3\times10^6} = 3 \times 10^8$

$\therefore$ (A)

Problem 6. *In an apparatus, the electric field was found to oscillate with an amplitude of 18 V/m. The magnitude of the oscillating magnetic field will be*

(A) 4×10^{-6} T *(B) 6×10^{-8} T*
(C) 9×10^{-9} T *(D) 11×10^{-11} T*

Solution: $B_0 = \dfrac{E_0}{c}$

$\therefore$ (B)

Problem 7. *The velocity of light is maximum in*
(A) diamond *(B) water*
(C) glass *(D) vacuum*

Solution: (D)

Problem 8. *The frequency of x- rays, γ - rays and ultraviolet rays are respectively a, b and c. Then*
(A) $a < b, \; b > c$ *(B) $a > b, \; b > c$*
(C) $a > b, \; b < c$ *(D) $a < b, \; b < c$*

Solution: (A)

Problem 9. *The velocity of electromagnetic wave is parallel to*
(A) $\vec{B} \times \vec{E}$ *(B)* $\vec{E} \times \vec{B}$
(C) $\vec{E}$ *(D)* $\vec{B}$

Solution: Electromagnetic wave is perpendicular to both $\vec{E}$ and $\vec{B}$.
$\therefore$ (B)

Problem 10: *Which of the following are not electromagnetic waves ?*
(A) β – rays *(B) gamma rays*
(C) C – rays *(D) infrared – rays*

Solution: (A)

ASSIGNMENT PROBLEMS

Subjective:

Level– O

1. Define electromagnetic waves.
2. What is meant by electromagnetic spectrum ? Give its four uses.
3. What is X–ray ?
4. What is γ–ray ?
5. Give two uses of (i) Infrared rays, and (ii) Ultraviolet rays.
6. State two properties of electromagnetic waves.
7. What is Maxwell's equation ?
8. What is the main difference between x–rays and γ–rays ?
9. Write down the expression for speed of an electromagnetic wave in free space.
10. What is common between different types of electromagnetic radiation.
11. What oscillates in an electromagnetic wave?
12. Which are relevant waves in telecommunication?
13. Give the ratio of velocities of light rays of frequencies 4×10^{13} and 12×10^{12} Hz in vacuum.
14. Explain that microwaves are better carrier of signal than radio–waves.
15. Derive the expression for velocity of an electromagnetic wave in vaccum.

Level– I

1. Find the wavelength of electromagnetic waves of frequency 6×10^{12} Hz in free space.

2. In a plane electromagnetic wave, magnetic field oscillates with amplitude 1.6×10^{-11} T. Find the maximum value of electric field.

3. Find energy stored in a 30 cm length of a laser beam operating at 6 mW.

4. Electromagnetic waves travel in a medium at speed of 2×10^{8} m/s. If relative permittivity is 2.25, find relative permeability of the medium.

5. If wavelength of an electromagnetic waves in free space is 0.075m, find its frequency.

6. The charging current for a capacitor is 0.05 A. What is the displacement current ?

7. The amplitude of the magnetic field part of harmonic electromagnetic wave in vacuum is B_0 = 510 nT. What is the amplitude of the electric field part of the wave ?

8. A charge particle oscillates about its mean equilibrium position with a frequency of 10^{9} Hz. What is the frequency of the electromagnetic waves produced by the oscilltator.

9. Give the ratio of velocities of light rays of frequencies 5×10^{12} Hz and 2.5×10^{12} Hz.

10. Find the ratio of frequencies of light waves of wavelengths 4000 $\mathring{A}$ and 8000 A°.

Level – II

1. A parallel plate capacitor has circular plates each of radius 5.0 cm. It is being charged so that electric field in the gap between its plates rises steadily at the rate of $2 \times 10^{12}\ Vm^{-1}s^{-1}$.

2. The voltage between the plates of a parallel-plate capacitor of capacitance 2 μF is changing at the rate of 10 V/s. Find displacement current.

3. A parallel capacitor is being charged by an external source show that conduction current and displacement current have same value in the circuit.

4. A radio can tune to any station in the 7.5 MHz to 12 MHZ band. What is corresponding wavelength band.

5. In an electromagnetic wave show that the average energy density of E field equals the average energy density of the B– field.

6. The electric field in an e.m. wave is given by
$E = 100 \sin \frac{2\pi}{\lambda}(ct - x)\ NC^{-1}$
Find the energy contained in a cylinder of a cross section 100 cm^2 and length 50 cm along the x–axis.

7. What should be the height of transmitting antenna if the TV telecast is to cover a radius of 128 km ?

8. A laser beam has intensity 2.5×10^{14} w/m^2. Find the amplitudes of electric and magnetic fields in the beam.

9. The magnetic field in a plane electromagnetic wave is given by
$B = 200\ (\mu T) \sin 4 \times 10^{-5}\ s^{-1}\ (t - x/c)$. Find the maximum magnetic and electric fields.

10. In the Q. No. 9, find average energy density corresponding to electric and magnetic fields.

Objective:

Level – I

1. The ozone layer absorbs
(A) infrared radiations
(B) ultraviolet radiations
(C) X-rays
(D) γ-rays

2. According to Maxwell's hypothesis, a changing electric field gives rise to
(A) an emf
(B) electric current
(C) magnetic field
(D) pressure radiation

3. The amplitudes of electric and magnetic fields are related to each other as
(A) $E_0 = B_0$
(B) $E_0 = cB_0$
(C) $E_0 = \frac{B_0}{c}$
(D) $E_0 = \frac{c}{B_0}$

4. An accelerated charge produces
(A) α – rays
(B) β – rays
(C) em rays
(D) all of the above

5. Ozone layer blocks the radiation of wavelength
(A) less than 3×10^{-7} m
(B) equal to 3×10^{-7} m
(C) more than 3×10^{-7} m
(D) none of these

6. The dimensional expression of $\frac{1}{2}\varepsilon_0 E^2$ is
(A) MLT^{-1}
(B) ML^2T^{-2}
(C) $ML^{-1}T^{-2}$
(D) ML^2T^{-1}

7. Speed of electromagnetic waves in vacuum is equal to
(A) $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$
(B) $\sqrt{\varepsilon_0\mu_0}$
(C) $\varepsilon_0\mu_0$
(D) $\frac{1}{\mu_0\varepsilon_0}$

8. If the magnetic monopole existed then which of the following maxwell's equation would be modified
(A) $\oint \vec{E}.d\vec{s} = \frac{q}{\varepsilon_0}$
(B) $\oint \vec{B}.d\vec{s} = 0$
(C) $\oint \vec{E}.d\vec{\ell} = -\frac{d\phi}{dt}$
(D) $\oint \vec{B}.d\vec{\ell} = \mu_0\varepsilon_r \frac{d\phi}{dt} + \mu_0 I$

9. An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx - \omega t)$ which of the following are independent of wavelength.
(A) k
(B) $\frac{k}{\omega}$
(C) $k\omega$
(D) ω

10. Unit of $\varepsilon_0 \frac{d\phi_E}{dt}$ is of
(A) potential
(B) charge
(C) capacitance
(D) current

Level – II

1. Energy of e.m. wave is due to their
(A) wavelength (B) frequency
(C) electric and magnetic field (D) none of the above

2. The average value of electric energy density in an electromagnetic wave is
(A) $\frac{1}{2}\varepsilon_0 E^2$ (B) $\frac{E^2}{2\varepsilon_0}$
(C) $\varepsilon_0 E^2$ (D) $\frac{1}{4}\varepsilon_0 E^2$

3. Electromagnetic waves are
(A) transverse (B) longitudinal
(C) longitudinal or transverse (D) none of these

4. In a plane electromagnetic wave the electric field oscillates sinusoidaly at frequency 3×10^{10} Hz and amplitude 30 V/m. The wavelength of the wave is
(A) 1.5 m (B) 1.5×10^{-1} m
(C) 10^{-2} m (D) 1.5×10^{-3} m

5. In the above question amplitude of oscillating magnetic field
(A) 1.6×10^{-5} T (B) 1.6×10^{-6}T
(C) 10^{-7}T (D) 1.6×10^{-8} T

6. In the above question the total energy density of the electromagnetic field of the wave is
(A) 1.0×10^{-7} J/m^3 (B) 1.0×10^{-8} J/m^3
(C) 4×10^{-9} J/m^3 (D) 1.0×10^{-10} J/m^3

7. Wave used for telecommunication is
(A) visible light (B) ultraviolet
(C) α – rays (D) microwave

8. If v_g, v_x and v_m are the speeds of gamma rays, X–rays and microwaves respectively in vacuum then
(A) $v_g < v_x < v_m$ (B) $v_g > v_x > v_m$
(C) $v_g > v_x > v_m$ (D) $v_g = v_x = v_m$

9. An electromagnetic wave going through vacuum is described by
$E = E_0 \sin(kx - \omega t)$, $B = B_0 \sin(kx - \omega t)$
(A) $E_0B_0 = \omega k$ (B) $E_0\omega = B_0k$
(C) $E_0k = B_0\omega$ (D) none of these

10. There are three wavelengths 10^7 m, 10^{-10} m, 10^{-7} m. Find their respective name.
(A) Radiowaves, X –rays, visible rays
(B) X– ray, visible ray, radiowave,
(C) Visible ray, α – ray, β – ray
(D) X–ray, γ – ray, radiowave.

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level– I

1. 5×10^{-5} m
2. 4.8×10^{-3} V/m
3. 6×10^{-12} J
4. 1
5. 4×10^{9} Hz.
6. 0.05 A
7. 153 NC^{-1}
8. 10^{9} Hz.
9. 1 : 1
10. 2 : 1

Level – II

1. 0.14 A
2. 20 μA
4. 40 m – 25 m
6. 22×10^{-11} J
7. 1280 m
8. 4.3×10^{8} V/m, 1.45 T
9. (200 μT, 6×10^{4} V/m)
10. 8×10^{-3} J/m^3, 8×10^{-3} J/m^3

Objective:

Level – I

1.	**B**	2.	**C**
3.	**B**	4.	**C**
5.	**A**	6.	**C**
7.	**A**	8.	**B**
9.	**B**	10.	**D**

Level – II

1.	**C**	2.	**D**
3.	**A**	4.	**C**
5.	**C**	6.	**C**
7.	**D**	8.	**D**
9.	**C**	10.	**A**

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

MODERN PHYSICS

Modern Physics

Syllabus:

Alpha-particle scattering experiment, size of the nucleus, composition of nucleus-protons and neutrons. Nuclear instability- Radiactivity-Alpha, Beta and Gamma particles/rays and their properties, radioactive decay law, simple explanation of α-decay, β-decay and γ-decay. Mass-energy relation, mass defect, Binding Energy per nucleon, its variation with mass number, Nature of nuclear forces, nuclear reaction-Nuclear fission and Nuclear fusion.

Photo-electric effect, Einstein's Photo-electric equation-particle nature of light, photo-cell. Matter waves-wave nature of particles, De-Broglie relation, Davison and Germer experiment. Characterstic and continuous X-rays, Moseley's law, Bohr's theory of Hydrogen like atoms.

Wave Particle Duality - Matter Waves and Particle Nature of Light

Despite the wave nature, electromagnetic radiations, have properties similar to those of particles. Thus, electromagnetic radiation emerges as an emission with a dual nature having both wave and particles aspects. In particular, the energy conveyed by an electromagnetic wave is always carried in units whose magnitude is proportional to frequency of the wave. These units of energy are called photons.

Energy of a photon is E = h.f, where h is Planck constant, and f is frequency of wave.

According to de-Broglie, nature loves symmetry; as wave behaves like matter, matter also behaves like wave. According to him, the wavelength of the matter wave is given by $\lambda = \frac{h}{p} = \frac{h}{mv}$, where m is the mass and v is velocity of the particle.

If an electron is accelerated through a potential difference of V volt, then

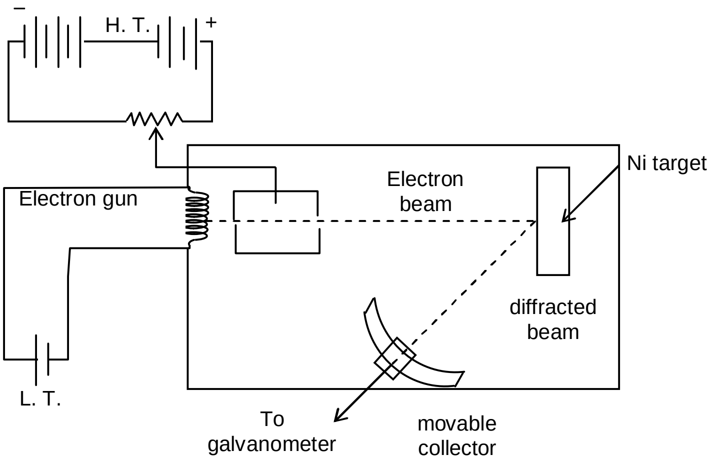
$$\frac{1}{2}m_e v^2 = eV \qquad \text{or} \qquad v = \sqrt{\frac{2eV}{m_e}} \qquad \therefore\ \lambda = \frac{h}{m_e v} = \frac{h}{\sqrt{2eVm_e}}$$

(It is assumed that the voltage V is not more than several tens of kV)

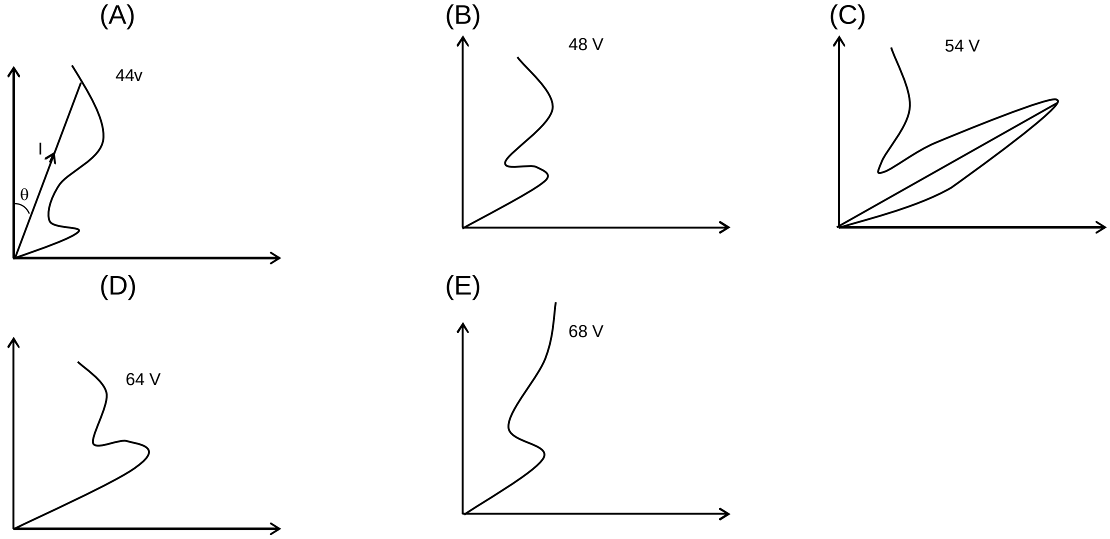
Davisson and Germer Experiment:

This experiment was conducted by Davisson and Germer in 1927. It shows the wave nature of electrons.

In this experiment, an electron beam is accelerated through a high tension field and then bombarded on a Nickel crystal. The electrons are scattered due to diffraction (see figure). The scattered electrons are collected by a movable collector which is free to move along a graduated circular scale. Angle θ is measured by the scale.
The intensity of diffracted electron beam is measured with the help of a galvanometer at different angles (θ).



The intensity is plotted against θ for various values of accelerating voltage and the pattern observed is as shown below.



It is observed that highest intensity is obtained at 54 V and $\theta = 50^0$. This diffraction maxima occurs due to constructive interference of electrons scattered by different layers of atoms in the metal plate. From electron diffraction measurements, the wavelength of the matter waves was found to be 0.165 nm. The de-Broglie's wavelength λ associated with electron is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mVe}} = \frac{1.227}{\sqrt{V}}\text{nm} = \frac{1.227}{\sqrt{54}} = 0.166 \text{ nm.}$$

which is in excellent agreement with the results of the experiment.

Hence, the Davisson and Germer experiment confirms the wave nature of electrons and the de-Broglie relation.

Illustration 1. *What is the energy and wavelength of a thermal neutron at a temperature of 20^0 C.*

Solution: By definition, a thermal neutron is a free neutron in a neutron gas at about 20^oC (293 K).

$$KE = \frac{3}{2}kT = \frac{3}{2}\left(1.38\times10^{-23}\right)\left(293\right) = 6.07\times10^{-21}J$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_o\left(KE\right)}} = \frac{6.63\times10^{-34}}{\sqrt{2\left(1.67\times10^{-27}\right)\left(6.07\times10^{-21}\right)}} = 0.147 \text{ nm}$$

Illustration 2. *(a) How many photons of a radiation of wavelength $\lambda = 5 \times 10^{-7}$ m must fall per second on a blackened plate in order to produce a force of 6.62×10^{-5} N.*

(b) At what rate will temperature of the plate rise if its mass is 19.8 kg and specific heat is equal to 2500 J/kg K ?

Solution: (a) As seen, the momentum of photon = h/λ

If n is the number of photons falling per second on the plate, then total momentum per second of the incident photons is

$$P = n \times \frac{h}{\lambda}$$

Since the plate is blackened, all photons are absorbed by it.

$$\therefore \quad \frac{\Delta P}{\Delta t} = n\frac{h}{\lambda}$$

Since, $F = \frac{\Delta P}{\Delta t} = n\frac{h}{\lambda}$ $\qquad \therefore n = \frac{F.\lambda}{h}$

or $n = \frac{6.62 \times 10^{-5} \times 5 \times 10^{-7}}{6.62 \times 10^{-34}} = 5 \times 10^{22}$

(b) Energy of each photon = hc/λ

Since n photons fall on the plate per second, total energy absorbed by the plates in one second is $E = n \times \frac{hc}{\lambda} = 19890$ J/s

i.e. $\frac{dQ}{dt} = 19890$ J/s ; $\qquad$ m.c. $\frac{dT}{dt} = 19890$ J/s

$\therefore \quad \frac{dT}{dt} = 19890/(19.8 \times 2500) = 0.4°C/s$

Illustration 3. *If the stationary proton and α-particle are accelerated through same potential difference, the ratio of de-broglie's wavelengths will be*

(A) 2 *(B) 1*

(C) $2\sqrt{2}$ *(D) None of these*

Solution: The gain in K.E. of a charged particle after moving through a potential difference of V is given as eV, that is also equal to $\frac{1}{2}$ mv^2, where v is the velocity of the charge particle. Disregarding the relativistic effect,

$$\frac{1}{2}mv^2 = qV \qquad \Rightarrow \qquad v = \sqrt{\frac{2qV}{m}}$$

$$\therefore \quad mv = \sqrt{2mqV}$$

$\Rightarrow$ De-Brogle's wavelength = $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2\ mqV}}$

$\therefore \quad \dfrac{\lambda_p}{\lambda_\alpha} = \sqrt{\dfrac{m_\alpha\ \ q_\alpha V_\alpha}{m_p q_p V_p}}$

Putting $V_\alpha = V_p$, $\dfrac{\lambda_p}{\lambda_\alpha} = \sqrt{\dfrac{(4)\ \ (2)}{(1)\ \ (1)}} = 2\sqrt{2}$

$\therefore$ (C).

Illustration 4. *What will be the energy of a photon whose*
(a) wavelength is 0.8 A^0,
(b) frequency is 500 kHz.

Solution: (a) $\lambda = 0.8\ A^0 = 0.8 \times 10^{-10}$ m

$E = \dfrac{hc}{\lambda} = \dfrac{6.63\times10^{-34}\times3\times10^{8}}{0.8\times10^{-10}}$ J

$= 1554$ eV = 1.554 keV.

(b) $E = hf = 6.63 \times 10^{-34} \times 5 \times 10^5$ J

$= \dfrac{33.15\times10^{-29}}{1.6\times10^{-19}} = 2.07\times10^{-9}$ eV

Illustration 5. *An electron in a hydrogen like atom is in an excited state. It has a total energy of $-$3.4 eV. Calculate*
(i) the kinetic energy, and
(ii) the de-Broglie wavelength of the electron.

Solution: (i) U + K = E, where K = – E and U = 2E

$\therefore$ K = – (– 3.4 eV) = 3.4 eV

(ii) $\lambda = \dfrac{h}{P} = \dfrac{h}{\sqrt{2mK}} = \dfrac{6.63\times10^{-34}\,J-s}{\sqrt{2\times(9.1\times10^{-31}kg)\times(3.4\times1.6\times10^{-19})}} = 6.63A^0$

Photon Theory of Light

Light has wave character as well as particle character. Depending on the situation, one of the two characters dominates. The particles of light are called photons. Some of the important properties of a photon are given below.

(a) A photon always travels at a speed $c \simeq 3.0\times10^8$ m/s in vacuum and it is independent of any form of reference frame used to observe the photon.

(b) Rest mass of a photon is zero

(c) Each photon has a definite energy and a definite linear momentum.
$E = hf = hc/\lambda$ & $p = h/\lambda = E/c$
where h is Planck's constant & has a value 6.626×10^{-34} Js.

(d) A photon may collide with a material particle. The total energy and the total momentum remain conserved in such a collision. The photon may get absorbed and/or a new photon may be emitted. Thus number of photons may not be conserved.

(e) If the intensity of light of given wavelength is increased there is an increase in the number of photons crossing a given area in a given time. The energy of each photon remains the same.

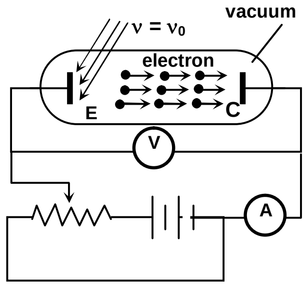
Photoelectric Effect

When light of sufficiently small wavelength is incident on a metal surface, electrons are ejected from the metal. This phenomenon is called the photoelectric effect. The electrons ejected from the metal are called photoelectrons.
There are large number of free electrons in a metal which wander throughout the body of the metal. However these electrons are not free to leave the surface of the metal. As they try to come out of the metal, the metal attracts them back. A minimum energy, equal to the work function (W_0), must be given to an electron so as to bring it out of the metal. The electron after receiving the energy, may lose energy to the metal in course of collisions with the atoms of the metal. If electron is given an energy E which is greater than W_0 and it makes the most economical use of it, it will have a maximum kinetic energy.

$KE_{max} = E - W_0$.

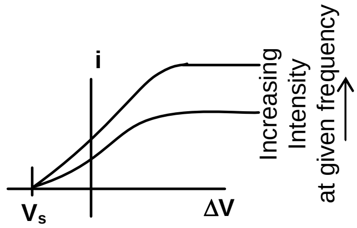
Thus electrons with K.E. ranging from 0 to KE_{max} will be produced.

Laws of photoelectric effect
(i) The emission of photoelectrons is instantaneous.
(ii) The number of photoelectrons emitted per second is proportional to the intensity of the given incident light.
(iii) The maximum velocity with which electrons emerge is dependent solely on the frequency and not on the intensity of the incident light.
(iv) There is always a lower limit of frequency called threshold frequency below which no emission takes place, however high the intensity of the incident radiation may be.



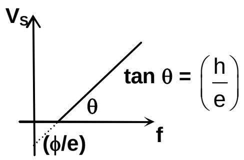
The figure shown is an experimental arrangement for studying photoelectric effect. If we set up a suitable potential difference ΔV between E (Emitter) and surface C (collector), we can sweep the ejected photoelectrons and measure them as a photoelectric current I in the external circuit.
If the collector is at a higher potential than the emitter and if ΔV is large enough, the photoelectric current reaches constant saturation value at which all electrons emerging from emitter are collected. If we reduce ΔV to zero, the photoelectric current does not drop to zero because the electrons are emitted with definite range of speeds. However, if we reverse the sign of the potential difference and make ΔV large enough, we eventually reach a point at which even the most energetic emitted electrons are turned back before they strike the collector and photoelectric current i does indeed drop to zero. The magnitude of this stopping potential difference is called the stopping potential V_S.
Conservation of energy gives,

$$eV_S = KE_{max}$$



Einstein's photoelectric equation

If the photon incident on a metal surface makes most efficient use of its energy and is just sufficient to liberate the electron. (with the Kinetic Energy of the electron being zero).
i.e. $hf_0 = W_0$
where f_0 is the threshold frequency and W_0 is the work function. If the frequency of incident light is less than f_0, no photoelectric emission takes place.
Now, suppose the frequency of incident photon is f.



Maximum Kinetic Energy of the photoelectron is
$\Delta KE_{max} = hf - hf_0 = h(f - f_0)$
$eV_s = hf - \phi$
$= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = \left[12400\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)eV\right]$. where λ is in Angstroms (A^0)

Illustration 6. *In a beam of light with wavelength 700 nm and intensity 100 W/m^2*
(a) what is the momentum of each photon ?
(b) How many photons cross 2 cm^2 area perpendicular to the beam in one second ?

Solution: (a) Energy of each photon,

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{700\times10^{-9}\times1.6\times10^{-19}} = 1.77eV$$

$$p = \frac{E}{c} = \frac{1.77eV}{3\times10^{8}m/s}$$

$p = 5.9 \times 10^{-9}$ eV – s/m.

(b) Energy crossing 2 cm^2 in 1 sec.
$E' = 100\ W/m^2 \times 2\times10^{-4}\ m^2$
$= 2\times10^{-2}$ J
No. of photons crossing 2 cm^2 in 1 sec.

$$= \frac{2\times10^{-2}J}{1.77\times1.6\times10^{-19}J/photon}$$

$= 7.06 \times 10^{16}$.

Illustration 7. *Photoelectric threshold of silver is λ = 3800 A^0. Ultra-violet light of λ = 2600 A^0 is incident on silver surface. Calculate*
(a) the value of work function in joule and in eV.
(b) maximum kinetic energy of the emitted photoelectrons.
(c) the maximum velocity of the photo electrons.
(Mass of the electron = 9.11 × 10^{-31} kg)

Solution: (a) $\lambda_0 = 3800\ \mathring{A}$

$$W = hf_0 = h\frac{c}{\lambda_o} = \frac{6.63\times10^{-34}\times3\times10^{8}}{3800\times10^{-10}}J = 5.23\times10^{-19}J$$

$= 3.27$ eV

(b) Incident wavelength, $\lambda = 2600\ \mathring{A}$

$\therefore$ f = incident frequency = $\frac{3\times10^{8}}{2600\times10^{-10}}$Hz

Then $KE_{max} = hf - W_o$

$$hf = \frac{6.63\times10^{-34}\times3\times10^{8}}{2600\times10^{-10}} = 7.65\times10^{-19}J = 4.78\ eV$$

$KE_{max} = hf - W_0 = 4.78\ eV - 3.27\ eV = 1.51$ eV.

(c) $KE_{max} = \frac{1}{2}mv_{max}^2$

$$\therefore \quad v_{max} = \sqrt{\frac{2\ \ KE_{max}}{m}} = \sqrt{\frac{2\times 2.42\times 10^{-19}}{9.11\times 10^{-31}}} = 0.7289\times 10^{6}\,ms^{-1}$$

Illustration 8. *In a photoelectric setup, the radiations from the Balmer series of hydrogen atom are incident on a metal surface of work function 2 eV. The wavelength of incident radiation lies between 450 nm and 700 nm. Find the maximum kinetic energy of photoelectron emitted. (Given hc = 1242 eV-nm).*

Solution: $\Delta E = 13.6\left[\frac{1}{4}-\frac{1}{n^2}\right] = \frac{hc}{e\lambda} = \frac{1242}{\lambda} \Rightarrow \lambda = \frac{1242\times 4n^2}{13.6(n^2-4)}$

λ_{min} which lies between 450 nm and 700 nm is for transition from n = 4 to n = 2 and is equal to 487.05 nm.

For maximum K.E. of photoelectron

$\frac{hc}{\lambda_{min}} - \phi$ = K.E.$_{max} \Rightarrow$ K.E$_{max}$ = $\frac{13.6\times 12}{4\times 16} - 2 = 0.55$ eV.

Illustration 9. *Photoelectrons are emitted when 400 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing α-particles. A maximum energy electron combines with an α-particle to form a He^+ ion, emitting a single photon in this process. He^+ ions thus formed are in their fourth excited state. Find the energies in eV of the photons, lying in the 2 to 4eV range, that are likely to be emitted during and after the combination.*

Solution: W = work function =1.9 eV

λ = wavelength of photon = 400 nm

E = energy of photons = $\frac{hc}{\lambda} = 3.1$eV

Now E = W + K, where K = maximum kinetic energy of photoelectrons

$\therefore$ K = E −W = (3.1 − 1.9)eV = 1.2 eV

Now, initial energy of α particle = 0

$\therefore$ Initial energy of the system (electrons + α particles) = K = 1.2 eV

The He^+ ions formed are in 4^{th} excited state i.e. n = 5

$\therefore$ Final energy of He^+ ions = − 13.6 $Z^2\left(\frac{1}{n^2}\right)$ = − 2.176 eV ($\because$ z = 2, n = 5)

Hence, the energy of photons emitted during combination

= 1.2 – (–2.176) = 3.376 eV

After combination energy of emitted photons that lie in the given range are for electron transitions

n = 5 $\rightarrow$ n = 3 and ΔE = 3.86 eV

n = 4 $\rightarrow$n = 3 and ΔE = 2.64 eV

After combination energies are 3.868 eV and 2.644 eV.

Illustration 10. *In an experiment on photoelectric emission following observations are made*
(1)wavelength of incident light = 1.98 $\times 10^{-7}$ m
(2) stopping potential = 2.5 volt find (a) thresold frequency (b) work function and (c) energy of photoelectron with maximum speed

Solution: Let V_0 = stopping potential

$\frac{1}{2}mV_{max}^2 = eV_0 = 4\times 10^{-19}$ J ($\because$ e = 1.6 $\times$ 10^{-19} C, V_0 = 2.5 Volt)

$E = \frac{hc}{\lambda} = 10^{-18}$ Joule (∵ λ 1.98×10^{-7} m)

W_0 = Work function = $\frac{hc}{\lambda} - \frac{1}{2}mV_{max}^2$ = 6×10^{-19} J

thresold frequency = $\frac{W_0}{h}$

= 9.1×10^{14} Hz.

Illustration 11. *A metallic surface is illuminated alternately with light of wavelength 3000Å. It is observed that maximum speeds of the photoelectrons under these illumination are in the ratio 3:1 Calculate work function of the metal and the maximum speed of the photoelectron*

Solution: According to Einstein photoelectron equation

$$\frac{hc}{\lambda} = \frac{1}{2}mV^2 + W_0$$

for λ_1 $\frac{hc}{3000 \times 10^{-10}} = \frac{1}{2}m(3V^2) + W_0$ ....(i)

for λ_2 $\frac{hc}{6000 \times 10^{-19}} = \frac{1}{2}mV^2 + W_0$ .....(ii)

from (i) and (ii) $W_0 = 2.896 \times 10^{-19}$

$3V = 9 \times 10^5$ m/s = maximum speed.

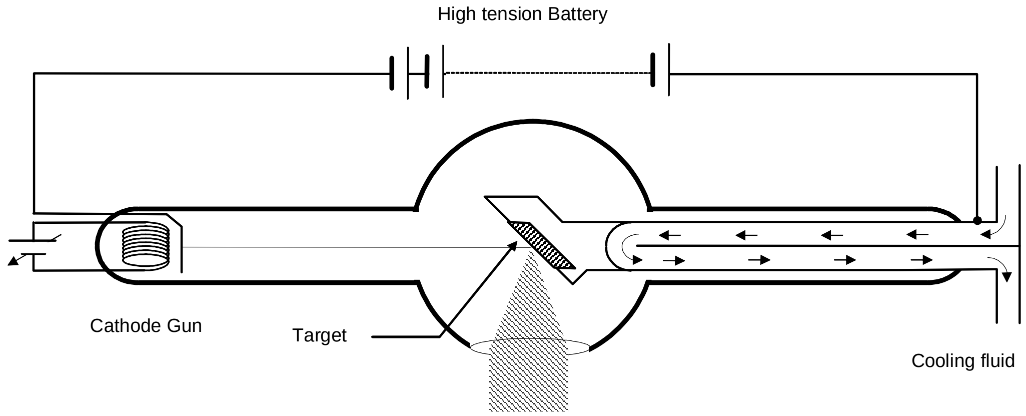
X-Rays

Characteristic and production of x-rays

(i) It is also known as inverse photoelectric effect, as energetic electrons produce electro-magnetic radiations.

(ii) Their wavelength is of the order of 1 A^o.

(iii) They are produced with an apparatus called Coolidge tube.



In a Coolidge tube, electrons are emitted by thermionic emission, accelerated across a very high potential difference V and made to hit a target. X-rays are produced and emerge out of a window. Water is circulated in the target to keep it cool.

Continuous and characteristic x-rays

(i) When an accelerated electron hits the target, the electron loses its energy in two processes. One process gives rise to continuous X-rays and the other process gives rise to characteristic X-rays.

(ii) When the electron loses its kinetic energy in several collisions with the atoms, a fraction of the lost kinetic energy is converted into electromagnetic radiations. This fraction can range from zero to one.

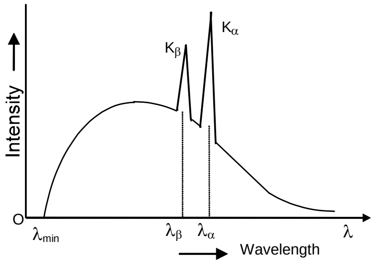
(iii) Corresponding to the maximum kinetic energy lost by an electron, we have the largest frequency of the X-ray or the shortest wavelength.

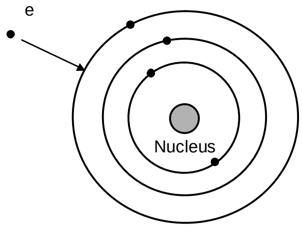
$$\frac{hc}{\lambda_{min}} = K_{max.} = eV \qquad \Rightarrow \qquad \lambda_{min} = \frac{hc}{eV}$$

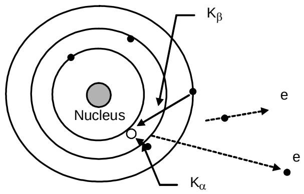
The emitted wavelengths range from a minimum value all the way to infinity. Thus the name continuous spectrum.

(iv) The other possibility is that the accelerated electron might knock out the inner electron of the target atom, whereby a vacancy is created in the inner orbit. Electrons from the higher orbit jump in to fill this vacancy, while releasing the energy difference as electro-magnetic radiations.

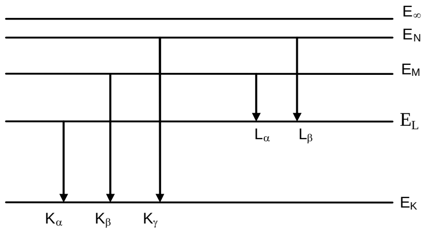
These wavelengths are characteristic of the material from which they are emitted. Hence the name Characteristic Spectra.







(v) To discuss energy transitions in X-rays, we take the ground state atom with all electrons intact as the zero-level. The atom with a vacancy in the K-shell is the one with the highest energy, as a lot of energy is required to dismiss a K-shell electron. Therefore we have the following energy diagram.



$$\lambda = \frac{hc}{E_K - E_L} \text{ for } K_\alpha \text{ wavelength.}$$

$$\lambda = \frac{hc}{E_L - E_M} \text{ for } L_\alpha \text{ wavelength and so on.}$$

Exercise 1: ***(i) Why do we have observe de Broglie wave in daily life ?***
(ii) How penetrating power of x-rays can be increased?
(iii) Why k_α line for constituent elements in an alloy or compound do not change when it is made target material in Coolidge tube.
(iv) If the intensity of incident radiation on a metal surface is doubled what happens to the kinetic energy of electrons emitted?
(v) if the intensity of the incident light on a photocell is increased, how does the stopping potential vary.

Moseley's law

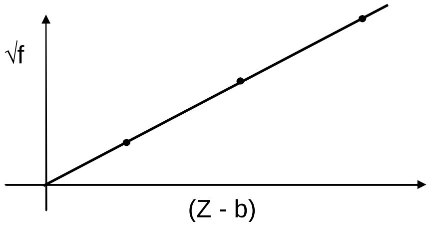
Moseley gave an empirical law.

$\sqrt{f} = a(Z - b)$

Where, f = frequency of X-ray,

Z = atomic number of target atom,

a, b are constants.



It was proposed before the Bohr's theory, and we see that it agrees with the Bohr's model.

For K_α -line, the L-electron jumps to a vacancy in the K-level. So for the L electron, there are Z protons in the nucleus and an electron in the K-shell which screens of the positive charge. So the net charge the L electron experiences can be taken as (Z - 1)e.

$$\text{Now, } E = Rhc(Z-1)^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right] \quad ; \qquad hf = Rhc\left(Z-1\right)^2.\left[\frac{3}{4}\right]$$

$$\sqrt{f} = \sqrt{\frac{3}{4}.R.c}\ (Z-1)$$

If we compare the above with Moseley's formula, we see that for K_α line, $a = \sqrt{\frac{3}{4}Rc}$ and b = 1.

Properties of x-rays

(i) X-rays do not get deflected by electric or magnetic fields.

(ii) When passed through gases, they produce ionisation.

(iii) X-rays diffraction is used to study crystal structures.

(iv) They have high penetrating power and are used in radiographs.

Illustration 12. *Find the maximum frequency of the X-rays emitted by an X-ray tube operating at 30 kV.*

Solution: For maximum frequency, the total kinetic energy [eV] should be converted into an X-ray photon.

Thus, hf = eV

$$f = \frac{eV}{h} = \frac{1.6\times10^{-19}\times30\times10^{3}}{6.63\times10^{-34}} = 7.2 \times 10^{18}\text{ Hz}$$

Illustration 13. *The wavelength of K_α X-rays produced by an X-ray tube is 0.76 A^o. The atomic number of the anticathode material is*

(A) 82 *(B) 41*

(C) 20 *(D) 10*

Solution: For K_α X-ray line,

$$\frac{1}{\lambda_\alpha} = R(Z-1)^2 \left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R(Z-1)^2 \left[1 - \frac{1}{4}\right]$$

$$\Rightarrow \frac{1}{\lambda_\alpha} = \frac{3}{4} R(Z-1)^2 \qquad \ldots(1)$$

With reference to given data,

$\lambda_\alpha = 0.76\,\overset{o}{A} = 0.76 \times 10^{-10}\,m$

$R = 1.097 \times 10^7$ m

Putting these values in equation (1)

$$(Z-1)^2 = \frac{4}{3}\ \frac{1}{0.76 \times 10^{-10} \times 1.097 \times 10^7}$$

$\cong$ 1600

$\Rightarrow$ Z-1 = 40 $\Rightarrow$ Z = 41

$\therefore$ (B).

Illustration 14. *An X-ray tube, operated at a potential difference of 40 kV, produces heat at the rate of 720 W. Assuming 0.5% of the energy of the incident electrons is converted into X-rays. Calculate*

(a) the number of electrons per second striking the target.

(b) the velocity of the incident electrons.

Solution: (a) Heat produced per second at the target is

P = 0.995 VI ($\because$ 0.5% of energy is converted into X - rays)

$$\therefore\ I = \frac{P}{0.995\ V} = \frac{720}{(0.995)(40 \times 10^3)} = 0.018\ A$$

The number of electrons per second incident on the target

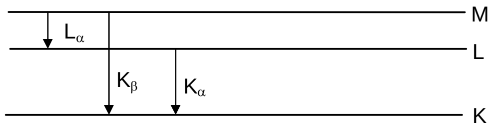
$$n = \frac{I}{e} = \frac{0.018}{1.6 \times 10^{-19}} = 1.1 \times 10^{17}\ \text{electrons}$$

(b) Energy of incident electrons $\frac{1}{2}mv^2 = eV$

$$\text{or}\quad v = \sqrt{\frac{2eV}{m}} = 1.2 \times 10^8\,m/s$$

Illustration 15. *Show that the frequency of K_β X-ray of a material equals to the sum of frequencies of K_α and L_α X-rays of the same material.*

Solution: The energy level diagram of an atom with one electron knocked out is shown above.



Energy of K_α X-ray is

$E_{K\alpha} = E_L - E_K$ of K_β X-ray is

$E_{K_\beta} = E_M - E_K$ and,

L_α X-ray is $E_{L\alpha} = E_M - E_L$

thus, $E_{K_\beta} = E_{K\alpha} + E_{L\alpha}$

or $f_{K_\beta} = f_{K_\alpha} + f_{L_\alpha}$

Illustration 16. *An X-ray tube is operated at 10 kV and the current through the tube is 1 mA. Find out*
(a) no. of electrons falling on the anode every second.
(b) energy falling on anode per second.
(c) cut - off wavelength of the X-rays produced.

Solution: (a) Current = $\frac{q}{t} = \frac{ne}{t} = \frac{\text{coulomb}}{\text{sec}}$

$\therefore$ Electrons falling on anode,

n = $\frac{i}{e} = \frac{1\times10^{-3}\,c/s.}{1.6\times10^{-19}\,c}$ = 6.25×10^{15} per sec.

(b) Energy falling = nev = iv = i $\times$ v
= i $\times 10^{-3} \times 10 \times 10^{3}$ = 10 J/s.

(c) $\frac{hc}{\lambda_{min}}$ = eV

$\lambda_{min} = \frac{hc}{eV} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times10\times10^{3}}$

$\lambda_{min} = 1.24 \times 10^{-10}$ m = 0.124 nm.

Alpha Particle Scattering Experiment:

1. Ernest Rutherford, in a series of experiments between 1906 and 1911 obtained valuable information on the structure of atom.
In the alpha particle scattering experiment carried out by Rutherford, a beam of alpha particles was bombarded on a thin foil of gold. The observations were:
1. Most of the α - particles passed through undeviated or with minor deviation.
2. Some of the α - particles were deflected through large angles. Some of them (1 in 8000) were deviated by more than 90^0. They were turned back.

Based on the above observations, Rutherford concluded that
- atom contains positively charged tiny particles at the centre called nucleus of the atom.
- negatively charged particles electrons move around nucleus.
- space between nucleus and electrons is empty and this determines the size of an atom.

Bohr's theory of hydrogen like atoms

Consider an electron of charge -e and mass m, orbiting, with a speed v, a central hydrogen nucleus of charge +e. In classical electromagnetism, charges undergoing acceleration emit radiation and, therefore, they would lose energy. Therefore the electron should spiral towards the nucleus and the atom should collapse. In order to overcome this difficulty, Bohr suggested that in those orbits where the angular momentum is a multiple of $\frac{h}{2\pi}$, the energy is constant.

Twelve years later, de Broglie proposed that a particle such as an electron may be considered to behave as wave of wavelength $\lambda = \frac{h}{p}$.

If electron jumps from a higher energy shell to another shell, it emits energy in the form of radiation, which is equal to energy difference of the two shells.

If the electron can behave as a wave, it must be possible to fit a whole number of wavelengths in an orbit. In this case a standing wave pattern is set up and the energy in the wave is confined to the atom. A progressive wave would imply that the electron is moving from the atom and is not in a stationary orbit.

If there are n waves in the circular orbit and λ is the wavelength.

$$n\lambda = 2\pi r$$

$$\therefore \quad \lambda = \frac{2\pi r}{n} = \frac{h}{mv} \quad \Rightarrow \quad mvr = \frac{h}{2\pi}(n)$$

The centripetal force is provided by the electrostatic attraction.

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_o \; r^2} \quad \therefore \quad r = \frac{\varepsilon_o h^2 n^2}{\pi m e^2}$$

Velocity of the electron in the nth orbit

$$v = \frac{e^2}{2\varepsilon_o hn}$$

Kinetic energy of the electron in the nth orbit

$$K = \frac{1}{2}mv^2 = \frac{me^4}{8\varepsilon_o^2 h^2 n^2}$$

Potential energy of the electron

$$V = -\frac{me^4}{4\varepsilon_o^2 h^2 n^2}$$

$\therefore$ Total energy of the electron

$$E = K + V = -\frac{me^4}{8\varepsilon_o^2 h^2 n^2}$$

For hydrogen like-atoms with nuclear charge +Ze, the following expression holds true :

(i) Radius of the nth orbit, $r_n = \dfrac{\varepsilon_o h^2 n^2}{\pi m e^2 Z} = r_o \dfrac{n^2}{Z}$

r_o = radius of the hydrogen atom = 0.53 Å (n = 1)

(ii) Velocity of the electron in the nth orbit, $v_n = \dfrac{Ze^2}{2\varepsilon_o hn} = v_o \dfrac{Z}{n}$

(iii) Energy of the electron in the nth shell, $E_n = -\dfrac{mZ^2e^4}{8\varepsilon_o^2 h^2 n^2} = E_o \dfrac{Z^2}{n^2}$

E_o = Energy of the electron (n = 1) in hydrogen atom, E_o = -13.6 eV

(iv) Kinetic energy of an electron = -Total energy = $-\dfrac{1}{2} \times$ potential energy

Illustration 17. *Radiations of wavelength between 100 nm and 200 nm pass through a sample of hydrogen gas. All atoms in the hydrogen gas are assumed to be in the ground state. Which wavelengths will be of low intensity in the transmitted radiations?*

Solution: If the energy of photon is equal to the difference between energy levels of electrons in ground state and an excited state, it will have high probability of being absorbed and hence will have low intensity in the transmitted beam.

Energy photons range from

$E_1 = \frac{1242\,\text{nm} - \text{eV}}{100\text{nm}}$ = 12.42 eV

$E_2 = \frac{1242\,\text{nm} - \text{eV}}{200\text{nm}}$ = 6.21 eV

Range of energy of photons is 6.21 to 12.42 eV

[Here hc = $\frac{6.63\times10^{-34}\times3\times10^{8}\times10^{9}}{1.6\times10^{-19}} = 1242$ nm – eV

Difference between energy levels of first excited state and ground state of electron in hydrogen gas.

$E_2 - E_1 = 13.6\ (1 - \frac{1}{4})$

= 10.2 eV.

Similarly, for 2nd excited state,

$E_3 - E_1 = 13.6\ (1 - \frac{1}{9})$

= 12.1 eV

and $E_4 - E_1 = 13.6\ (1 - \frac{1}{16})$ = 12.75 eV.

Energy difference 10.2 eV and 12.1 eV fall in the range of the radiations falling, corresponding to them

$\lambda_1 = \frac{1242\,\text{nm} - \text{eV}}{10.2}$ = 122 nm

$\lambda_2 = \frac{1242\,\text{nm} - \text{eV}}{12.1}$ = 103 nm

Hence low intensity wave length will be 122 nm and 103 nm.

Illustration 18. *The de-Broglie's wavelength of an electron in first orbit of Bohr's hydrogen is equal to*

(A) Radius of the orbit *(B) Perimeter of the orbit*
(C) Diameter of the orbit *(D) semi perimeter of the orbit*

Solution: The de-Brogle's wavelength $\lambda = \frac{h}{mv}$ …(1)

where the angular momentum of an orbiting electron is given as on

$mvr = \frac{nh}{2\pi}$

for first orbit n = 1

$\Rightarrow\ mv = \frac{h}{2\pi r}$ …(2)

Using (1) and (2) we obtain

$\lambda = \frac{h}{\left(\frac{h}{2\pi r}\right)} = 2\pi r$ (where $2\pi r$ = perimeter of the orbit)

$\therefore$ (B)

Illustration 19. *A single electron orbits a stationary nucleus of charge +Ze, where Z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second orbit to third Bohr orbit. Find*

(a) the value of Z
(b) the energy required to excite the electron from the third to the fourth Bohr orbit.
(c) the wavelength of electromagnetic radiation required to remove the electron from first Bohr orbit to infinity.
(d) Find the K.E. , P.E. and angular momentum of electron in the 1st Bohr orbit.
(e) the radius of the first Bohr orbit.
[The ionisation energy of hydrogen atom = 13.6 eV, Bohr radius = 5.3×10^{-11} m, velocity of light = 3×10^{8} m/s, Planck's constant = 6.6×10^{-34} J.s.]

Solution: The energy required to excite the electron from n_1 to n_2 orbit revolving round the nucleus with charge +Ze is given by

$$E_{n_2} - E_{n_1} = \frac{Z^2me^4}{8\varepsilon_0^2h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $E_{n_2} - E_{n_1} = Z^2 \times 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ electron volt.

(a) Since 47.2 eV energy is required to excite the electron from n_1 =2 to n_2=3 orbit.

Therefore, $47.2 = Z^2 \times 13.6\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

$Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 25$ or $Z = 5$.

(b) The energy required to excite the electron from $n_1 = 3$ to $n_2 = 4$ orbit is given by

$E_4 - E_3 = 25 \times 13.6 \times \left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \frac{25 \times 13.6 \times 7}{144} = 16.53\text{eV}$

(c) The energy required to remove the electron from the first Bohr orbit to infinity (∞) is given by

$E_\infty - E_1 = 13.6 \times Z^2 \left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = 13.6 \times 25$ eV

In order to calculate the wavelength of radiation, we use frequency relation

$E = \frac{hc}{\lambda} = 13.6 \times 25 \times (1.6 \times 10^{-19})$ J

or $\lambda = \frac{(6.6 \times 10^{-34}) \times (3 \times 10^{8})\text{m}}{13.6 \times 25 \times (1.6 \times 10^{-19})}$ = 36.5 Å = 340 eV

(d) K.E. = $\frac{1}{2}mv_1^2 = \frac{1}{2} \times \frac{Ze^2}{4\pi\varepsilon_o r_1} = 340 \times 1.6 \times 10^{-19}$ J

P.E. = $-2 \times$ K.E. = -1086.8×10^{-19} J

Angular momentum = $mv_1r_1 = h/2\pi = 1.06 \times 10^{-34}$ Js

(e) The radius r_1 of the first Bohr's orbit is given by

$r_1 = \frac{\varepsilon_o h^2}{\pi me^2}\frac{1}{Z} = \frac{0.53 \times 10^{-10}}{5}$ $\left(\because \frac{\varepsilon_o h^2}{\pi me^2} = 0.53 \times 10^{-10}\right)$

$= 0.106 \times 10^{-10}$ m = 0.106 Å

Exercise 2: ***(i) What will happen if we do not consider nucleus in Bohr atom infinitely heavy.***
(ii) Can a hydrogen atom absorb a photon having energy more than 13.6 eV.
(iii) How is impact parameter related to angle of scattering

Illustration 20. *Hydrogen atom in its ground state is excited by means of mono chromatic radiation of wavelength 975 Å. How many different lines are possible in the resulting spectrum. Calculate the longest wavelength among them. You can assume the ionization energy for hydrogen atom as 13.6eV.*

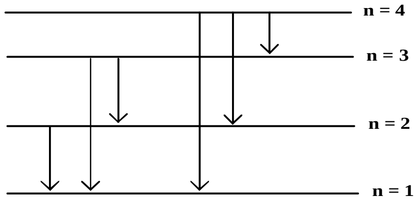
Solution: $\therefore$Ionization energy of hydrogen atom = 13.6eV

$$\therefore 13.6\left(\frac{1}{1^2}-\frac{1}{n^2}\right)=12.75 \qquad (\because\ E = \frac{12400}{975}\text{eV} = 12.75\text{eV})$$

$$\Rightarrow \frac{1}{n^2}=1-\frac{12.75}{13.6}=\frac{0.85}{13.6}=\frac{1}{16}$$

$$\therefore \frac{1}{n}=\frac{1}{4}\Rightarrow n=4$$

$\therefore$ No. of lines possible in the resulting spectrum = ${}^4C_2$ = 6



The longest wavelength will be emitted for transition from 4th orbit to 3rd orbit with an energy

$$\Delta E = (-13.6\text{eV})\left(\frac{1}{4^2}-\frac{1}{3^2}\right)=13.6\times\frac{7}{144}\text{eV}$$

$$\therefore \text{ longest wavelength } \lambda=\frac{hc}{\Delta E}=18800 \text{ Å}$$

Illustration 21. *A gas of identical hydrogen like atoms has some atoms in the lowest energy level A. and some atoms in a particular upper energy level B, and there are no atoms in any other energy level. The atoms of the gas make transition to a higher energy level by aborting monochromatic light of photon energy level 2.7eV. Subsequently the atoms metrication of only six different photon energies. Some of the emitted photons have less energy 2.7eV, some have energy more and some have less than 2.7eV.*
(i) the principal quantum number of the initially excited level B
(ii) the ionization energy of the gas atom
(iii) the maximum and the minimum energy of the emitted photons.

Solution: Energy of the nth orbit $E_n = (-13.6\text{eV})\ \frac{z^2}{n^2}$

This comes from the fact that the ionization potential of hydrogen is 13.6eV

Final excitation level can be only 4 as for this no only six de-excitation are possible as $^4C_2 = 6$

Thus, available orbit nos. are 1, 2, 3, 4. As some de-excitations involve energies less than 2.7eV others just 2.7eV and same other more than 2.7eV, so excitation from 3 to 4 and 2 to 3 is not possible.

Thus, excitation is from n = 1, n = 3

Now, $(E_n)_{n-2} - (E_n)_{n-1} = 13.6 \times Z^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right)\text{eV}$

$= 13.6 \times Z^2 \times \frac{8}{9}\text{eV} = 2.7\text{eV}\,(\text{given})$

$\therefore Z = \sqrt{\frac{9}{40}}$

But Z has to be a whole no. and more than one so, this transition is not possible.

Similarly transition n = 1, n = 2 is not possible.

Now we try the excitation n = 2, n = 4

$\left(E_n\right)_{n-1} - (E_n)_{n-2} = 13.6\,Z^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right)\text{eV}$

$= 13.6\,Z^2 \times \frac{3}{16}\text{eV} = 2.7\,\text{eV}$

$\Rightarrow Z = \sqrt{\frac{16}{15}} \approx 1$

$\therefore$ Z = 1 and it is hydrogen gas.

The above calculation shows that the principal quantum numbers of level B = 2

And ionization potential of the atoms = 13.6eV

Maximum energy of emitted photons = $13.6\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 12.8\,\text{eV}$

And , minimum energy of emitted photons

$= 13.6\left(\frac{1}{9} - \frac{1}{16}\right) = 0.63\,\text{eV}$

Nuclear Physics

The nucleus is positively charged and it is located at the centre of the atom. Almost the entire mass of the atom is concentrated in its nucleus.

Composition of Nucleus

A nucleus consists mainly of two types of particles, protons and neutrons. Collectively these two particles are sometimes referred to as nucleons.

(i) **Atomic Number:** It is the number of protons present in the nucleus. It is equal to the number of electrons in the atom.

(ii) **Mass Number :** Total number of protons and neutrons is the mass number.

If the atomic number of an atom is Z, and number of neutrons present in the nucleus of the atom is N, then its mass number can be given by the expression,
A = Z + N, where A = mass number.

Mass defect:
The actual mass of a nucleus is experimentally observed to be smaller than the sum of the masses of free nucleons constituting it. The difference between the experimental mass, $m({}_{Z}X^{A})$, of a nucleus and the sum of the masses of free nucleons (Z protons, and A-Z neutrons) is known as the mass defect :

$$\Delta m = Zm_p + (A-Z)\, m_n - m\ ({}_{Z}X^{A})$$

Illustration 22. *The element Curium ${}^{248}_{96}Cm$ has a mean life of 10^{13} seconds. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. The masses involved in decay are as follows:*
${}^{248}_{96}Cm$ = 248.072220 u, ${}^{244}_{94}Pu$ = 244.064100 u and ${}^{4}_{2}He$ = 4.002603 u. Calculate the power output from a sample of 10^{20} Cm atoms. (1u = 931MeV/c^2)

Solution: The reaction involved in α decay is
$${}^{248}_{96}Cm \to {}^{244}_{94}Pu + {}^{4}_{2}He$$
Mass defect = Δm
Δm = 248.072220 – 244.064100 – 4.002603
= 0.005517 u.
Therefore, energy released in α - decay will be
$E_\alpha = (0.005517 \times 931)\text{MeV} = 5.136\text{ MeV}$
Similarly $E_{fission} = E_f = 200$ MeV (Given)
Disintegration constant = $\lambda = \dfrac{1}{t_{mean}} = 10^{-13}\,s^{-1}$
Rate of decay at the moment when number of nuclei are $10^{20} = \lambda N = 10^{-13} \times 10^{20}$
= 10^7 disintegrations per second.
Out of these disintegrations, 8 % are in fission and 92 % are in α - decay.

Therefore, energy released per second
= $(0.08 \times 10^7 \times 200 + 0.92 \times 10^7 \times 5.136)$ MeV
= 2.072×10^8 MeV
$\therefore$ Power out put = 2.072×10^8 MeV
= 3.32×10^{-5} watt.

Size of nucleus:
Experiments have shown that average radius R of a nucleus can be given by
$$R = R_0\, A^{1/3}$$
where $R_0 = 1.1 \times 10^{-15}$ m = 1.1 fm.
and A is the mass no. of the atom.
Volume of the nucleus can be calculated as below.
$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$$
It may be noted that density of the nucleus, $d = \dfrac{M}{A}$ is constant as $M \propto A$.

Illustration 23. *Calculate radius of nucleus $_{26}Fe^{56}$*

Solution: $R = R_0\, A^{1/x3} = 1.1 \times (56)^{1/3}$
$R = 1.1 \times 3.826$
$R = 4.208$ fm

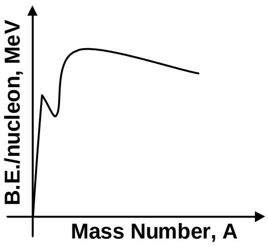
Nuclear Forces: Nuclear forces operate between all the nucleons i.e. between proton - proton, neutron-proton and neutron – neutron.
- these forces are short ranged. They operate at distances of the order of femtometer or less.
- they are much stronger than electromagnetic forces (50 - 60 times)
- they are independent of charge.

Nuclear force depends not only on centre distance but also on spin. It is stronger if spins of nucleons are parallel.

Binding energy

The amount of energy needed to disintegrate a nucleus into its constituent free nucleons is called the binding energy. It is the energy equivalent to the mass defect.
B.E. = Δmc^2, where c = velocity of light.
If mass is measured in amu then B.E. = (Δm $\times$ 931.5)MeV. As 1 amu = 931.5 MeV/C^2

Binding Energy per nucleon



The binding energy per nucleon for a given nucleus is found by dividing its total binding energy by the number of nucleons it contains. The given figure show the binding energy per nucleon plotted against the number of nucleon for various atomic nuclei. The greater the binding energy per nucleon, the more stable the nucleus is. The graph has its maximum of 8.8 MeV/nucleon when number of nucleons is 56.
Two remarkable conclusions can be drawn from the curve in the figure shown. The first that if we can somehow split a heavy nucleus into two medium sized ones, each of the new nuclei will have more binding energy per nucleon than the original nucleus.

The other notable conclusion is that joining two light nuclei together to give a single nucleus of medium size also means more binding energy per nucleon in the new nucleus. Splitting a heavy nucleus is called Nuclear fission, while combining two nuclei is called Nuclear fusion.

Illustration24. *What is the binding energy of $_6C^{12}$?*

Solution: One atom of $_6^{12}C$ consists of 6 protons, 6 electrons and 6 neutrons. The mass of the uncombined protons and electrons is the same as that of six $_1^1H$ atoms (if we ignore the very small binding energy of each proton-electron pair).

Mass of six $_1^1H$ atoms = 6 $\times$ 1.0078		=	6.0468 u
Mass of six neutrons = 6 $\times$ 1.0087		=	6.0522 u
Total mass of component particles		=	12.0990 u
Mass of $_6^{12}C$ atom		=	12.0000 u
Mass defect	= 12.0990 – 12.0000	=	0.0990 u
Binding energy = (931)(0.099)		=	92 MeV

Illustration 25. *Find out the binding energy per nucleon of an α particle in MeV. It is given that the masses of α particle, proton and neutron are respectively 4.00150 amu, 1.00728 amu and 1.00867 amu.*

Solution: An α particle consists of 2 protons and 2 neutrons.

Mass of constituents

$= 2 \times 1.00728 + 2 \times 1.00867 = 4.0319$ amu.

Mass defect = $\Delta m = 4.0319 - 4.00150$

$= 0.0304$ amu

Binding energy $= 0.0304 \times 931$ MeV

28.30 MeV

Nuclear reaction - Fission & Fusion:

A large amount of energy is liberated by either breaking a heavy nucleus into two nuclei of middle weight or by combining two light nuclei to form a middle weight nucleus. These processes are called nuclear fission and nuclear fusion respectively.

Nuclear Fission: Middle weight nuclei are more tightly bound and have more binding energy. When a heavy nucleus breaks into two middle weight nuclei, binding energy increase. Hence the rest mass decreases. This extra energy is liberated as K.E. or some other form.

Example: $^{236}_{92}U \rightarrow ^{137}_{53}I + ^{97}_{39}y + 2n$

and $^{236}_{92}U \rightarrow ^{140}_{56}Ba + ^{94}_{39}kr + 2n$

Nuclear Fusion

Light weight nuclei are less tightly bound than middle weight nuclei. When two light weight nuclei combine and form a middle weight nucleus, binding energy is increased and rest mass is decreased. This energy is liberated as K.E. or some other form.

Conservation law's in Nuclear Reactions.

1. Conservation of Mass Energy: Mass energy of the reactants is equal to mass energy of the products.
2. Conservation of Linear Momentum: The sum of momentum of the reactants and products remains conserved.
3. Conservation of mass number: Total mass number does not change in nuclear reactions.
4. Conservation of electric charge: Total electric charge remains conserved.

Energy of a nuclear reaction

Initial **Final**

Initial energy $E_i = m_1c^2 + m_2c^2 + K_1 + K_2 + hf_o$

Final energy $E_f = m_3c^2 + m_4c^2 + K_3 + K_4 + hf$

Since $E_i = E_f$

$\therefore [(m_1 + m_2) - (m_3 + m_4)]c^2 = (K_3 + K_4) - (K_1 + K_2) + hf - hf_o$

The energy that is released or absorbed in a nuclear reaction is called the Q - value or disintegration energy of the reaction.

$$Q = [(m_1 + m_2) - (m_3 + m_4)]c^2$$

If Q is positive, rest mass energy is converted to kinetic mass energy, radiation mass - energy or both, and the reaction is exogenic.

If Q is negative, the reaction is endoergic. The minimum amount of energy that a bombarding particle must have in order to initiate an endoergic reaction, is called threshold Energy E_{th}

$$E_{th} = -Q\left(\frac{m_1}{m_2}+1\right)$$

Where m_1 = mass of the projectile and m_2 = mass of the target

Illustration 26. *Find the Kinetic energy of the α. particle emitted in the alpha decay of $_{88}Ra^{226}$.*

$$\left[\text{Given}: m\left({}^{226}\text{Ra}\right) = 226.0254024\,u, m\left({}^{222}\text{Rn}\right) = 222.017571u, m\left({}^{4}\text{He}\right) = 4.00260\,u\right]$$

Solution: From the question we have

$${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + \alpha$$

$$Q = \left[m\left({}^{226}\text{Ra}\right) - m\left({}^{222}\text{Rn}\right) - m\left({}^{4}\text{He}\right)\right]c^2$$

$$= [\,226.0254024 - 222.017571 - 4.00260\,] \times (931.5\ \text{MeV})/u$$

$$= 4.871\ \text{MeV}$$

Illustration 27. *Neon - 23 β decays in the following way :*

$$_{10}Ne^{23} \rightarrow {}_{11}Na^{23} + {}_{-1}e^{0} + \bar{\nu}$$

Find the minimum and maximum KE that the β particle can have. The atomic masses of ^{23}Ne and ^{23}Na are 22.9945 u and 22.9898 u, respectively.

Solution: Since the mass of an atom includes the masses of the atomic electrons, the appropriate number of electron masses must be subtracted from the given values.

Mass of Reactant	Mass of Products
${}^{23}_{10}$Ne 22.9945 - 10m_e	${}^{23}_{11}$Na 22.9898 - 11 m_e, ${}^{0}_{-1}$e -m_e
Total mass = 22.9945 - 10m_e	Total mass = 22.9898 - 10m_e

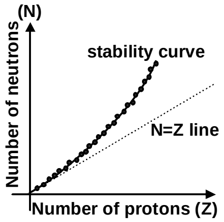
Mass defect = 22.9945 - 22.9898 = 0.0047 u

$$Q = (0.0047)(931) = 4.4\ \text{MeV}$$

The β - particle and anti - neutrino share this energy. Hence the energy of the β-particle can range from 0 to 4.4 MeV.

Stable Nuclei:

A stable nucleus maintains its constitution all the time. The figure shows a plot of neutron number (N) versus proton number (Z) for the nuclides observed. For light stable nuclides, the neutron number is equal to the proton number so that ratio N/Z is equal to 1. The ratio N/Z increases for the heavier nuclides and becomes about 1.6 for heaviest stable nuclides. Protons are positively charged and repel one another electrically. This repulsion becomes so great in nuclei with more than 10 protons or so that



an excess of neutrons, which produce only attraction forces, is required for stability. Thus stability curve departs more and more from N = Z line as Z increases.

Radioactivity

Radioactivity was discovered in 1896 by Antoine Becquerel. Three extraordinary features of Radioactivity are:

1. When a nucleus undergoes α or β decay its atomic number Z changes and it becomes the nucleus of different element. Thus the elements are not immutable.
2. The energy liberated during radioactive decay comes from within individual nuclei without external excitation, unlike the case of atomic radiation.
3. Radioactive decay is a statistical process that obeys the laws of chance. No. cause-effect relationship is involved in the decay of a particular nucleus, only a certain probability per unit time.

Alpha Decay

An α - particle is a helium nucleus, i.e. a helium atom which has lost two electrons. It has a mass about four times that of a hydrogen atom and carries a charge +2e. They have very little penetrating power but have a very high ionising power. An α decay reduces the Z and the N of the original nucleus by two each.

Beta Decay

β - particles are electrons moving at high speeds. These have greater (compared to α-particles) penetrating power but less ionising power. Their velocities are close to the velocity of light. Unlike α particles, they have a wide spectrum of energies, i.e., beta particles possess energy from a certain minimum to a certain maximum value. In negative beta decay a neutron is transformed into a proton and an electron is emitted, this happens when nucleus has either too large a neutron/proton ratio for stability. In positive beta decay, a proton becomes a neutron and a positron is emitted.

Gamma rays (γ)

γ - rays are electromagnetic waves of wavelength of the order of ~ 10^{-11} m. They have the maximum penetrating power and the least ionising power. γ - rays are emitted due to the transition of an excited nucleus from a higher energy state to a lower energy state.

Soddy's displacement law

The emission of an particle reduces the atomic number by 2 and the mass number by 4 and, it displaces the element by two columns to the left in the periodic table.

$${}_{a}^{b}X = {}_{a-2}^{b-4}Y + \alpha$$

α – decay: ${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He$

β – decay: ${}_{0}^{1}n \rightarrow {}_{1}^{1}P + {}_{-1}^{0}e^{-}$

β^{+} decay: ${}_{1}^{1}H \rightarrow {}_{0}^{1}n + {}_{1}^{0}e^{+}$

Electron capture: ${}_{1}^{1}H + {}_{-1}^{0}e^{-} \rightarrow {}_{0}^{1}n$

Radioactive decay law

In a radioactive decay the number of nuclei disintegrating per second $\left(\frac{dN}{dt}\right)$ is directly proportional to the number of undecayed nucleus atoms (N) present at that instant.

$\frac{dN}{dt} = -\lambda N$ where λ is the radioactivity decay constant.

If N_o is the number of radioactive nucleus present at a time t = 0, and N is the number at the end of time t, then $N = N_o e^{-\lambda t}$

The term $\left(-\frac{dN}{dt}\right)$ is called the activity of a radioactive substance and is denoted by 'A'.

Units of activity: 1 becquerel (Bq) = 1 disintegration per second (dps)
1 curie (Ci) = 3.7×10^{10} dps
1 rutherford = 10^6 dps

Half life

The time interval during which a radioactive substance decays to half its original value. It is denoted by $T_{1/2}$.

$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$

Mean life

It is the average of the lives of all the nuclei. $T_{av} = \frac{\int_0^\infty N_o e^{-\lambda t} dt}{N_o} = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693}$

Exercise 3:

(i) ***When Boron nucleus ($_5B^{10}$) is bombarded by neutrons, α particle is emitted. What is the mass number and name of the resulting element?***

(ii) ***A reaction between Proton and $_8O^{18}$ that produces $_9F^{18}$, will librate a particle . What is that particle?***

(iii) ***In the nuclear process $_6C^{11} \rightarrow {_5B^{11}} + \beta^+ + X$. what is X?***

(iv) ***What is the ratio of radii of two nuclei of mass numbers A_1 and A_2 ?***

(v). ***Why is the energy distribution of β-rays continuous?***

(vi). ***why electron capture is more in heavy atoms?***

Illustration 28. *Masses of two isobars $_{29}Cu^{64}$ and $_{30}Zn^{64}$ are 63.9298 u and 63.9292 u respectively. It can be concluded from these data that*

(A) Both the isobars are stable

(B) Zn^{64} is radioactive decaying to Cu^{64} through β - decay.

(C) Cu^{64} is radioactive decaying to Zn^{64} through γ-decay.

(D) Cu^{64} is radioactive decaying to Zn^{64} through β-decay.

Solution: Cu^{64} is radioactive after β-decay its atomic number increases by one and its mass number doesn't change.

$\therefore$ it decays to Zn^{64}

$\therefore$ (D)

Illustration 29. *The half-life of Cobalt - 60 is 5.25 years. After what time would its activity have decreased to about one-eigth of its original value ?*

Solution: The activity is proportional to the number of undecayed atoms.
In each half-life, half the remaining sample decays.

Since $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{8}$, therefore, three half-lives or 15.75 years are required for the sample to decay to $\frac{1}{8}$th of its original strength.

Illustration 30. *If a stationary electron gets annihilated due to the association with a stationary positron (hypothetically), the wavelength of the resulting radiation will be*

(A) $\frac{h}{m_0 c}$ *(B)* $\frac{2h}{m_0 c}$

(C) $\frac{h}{2m_0 c}$ *(D) None of these*

where m_0 = rest mass of an electron (or positron), c = speed of light

Solution: Conserving the momenta of the system just before and after the event (annihilation) we see that two identical photons will be resulted and travel in opposite directions with equal magnitude of momenta and energy $\frac{hc}{\lambda}$ where λ = wavelength of radiation.

Conservation of energy yields, $\frac{hc}{\lambda}+\frac{hc}{\lambda}=m_0c^2+m_0c^2$

$\Rightarrow \frac{hc}{\lambda}=m_0c^2 \quad \Rightarrow \quad \lambda=\frac{h}{m_0c}$

$\therefore$ (A)

Illustration 31. *The half-life of radium is 1620 years. How many radium atoms decay in 1s in a 1gm sample of radium. The atomic weight of radium is 226 kg/k mol.*

Solution: Number of atoms in 1gm sample is

$N=\left(\frac{0.001}{226}\right)\left(6.02\times10^{26}\right)=2.66\times10^{21}$ atoms.

The decay constant is

$\lambda=\frac{0.693}{T_{1/2}}=\frac{0.693}{(1620)\left(3.16\times10^{7}\right)}=1.35\times10^{-11}s^{-1}$

(taking one year = 3.16×10^{7} s)

Now, $\frac{\Delta N}{\Delta t}=\lambda N=\left(1.35\times10^{-11}\right)\left(2.66\times10^{21}\right)=3.6\times10^{10}s^{-1}$

Thus, 3.6×10^{10} nuclei decay in one second.

Illustration 32. *A rock is 1.5×10^{9} year old the rock contains ^{238}U which disintegrates to from ^{206}Pb. Assume that there was no ^{206}Pb in the rock initially and it is only stable product formed by the decay calculate the ratio of numbers of nuclei of ^{238}U to that of ^{206}Pb in the rock. Half life of ^{238}U is 4.5×10^{9} years [$2^{1/3}=1.259$]*

Solution: Let N_0 be the initial number of nuclei of ^{235}U.After time t

$$N_U = N_0\left(\frac{1}{2}\right)^n$$

Here n = number of half lives = $\frac{t}{t_{1/2}} = \frac{1.5\times10^9}{4.5\times10^9} = \frac{1}{3}$

$$\therefore N_U = N_0\left(\frac{1}{2}\right)^{\frac{1}{3}}$$

and $N_{pb} = N_0 - N_U = N_0\left[1-\left(\frac{1}{2}\right)^{1/3}\right]$

$$\frac{N_U}{N_{pb}} = \frac{(1/2)^{1/3}}{1-(1/2)^{1/3}} = 3.861$$

Illustration 33. *A radioactive nucleus X decays to a nucleus Y with a decay constant λ_x = 0.1 sec^{-1}. Y further decays to a stable nucleus Z with a decay constant λ_Y = 1/30 sec^{-1}. Initially, there are only X nuclei and their number is $N_0 = 10^{20}$. Set up the rate equations for the populations of X, Y and Z. The population of the Y nucleus as a function of time is given by $N_Y(t) = \{N_0\lambda_X/(\lambda_X - \lambda_Y)\}\{\exp(-\lambda_Y t) - \exp(-\lambda_X t)\}$. Find the time at which N_Y is maximum and determine the population X and Z at that instant.*

Solution: $dN_X/dt = -\lambda_x N_x$

$$\therefore N_x(t) = N_0 e^{-\lambda_x t}$$

$dN_y/dt = \lambda_x N_x - \lambda_y N_y$ and $dN_z/dt = \lambda_y N_y$

where $N_y(t) = N_0\lambda_x\{e^{-\lambda_y t} - e^{-\lambda_x t}\}/(\lambda_x - \lambda_y)$

For maximum N_y, $dN_y/dt = 0$

$$\Rightarrow \lambda_x e^{-\lambda_x t} = \lambda_y e^{-\lambda_y t}$$

$\Rightarrow$ t = 16.48 sec.

$\Rightarrow N_x = 1.924 \times 10^{19}$ and $N_z = \int\lambda_y N_y\,dt$

$$= N_0\left[1 - \frac{\lambda_x}{\lambda_x - \lambda_y}e^{-\lambda_y t} + \frac{\lambda_y}{\lambda_x - \lambda_y}e^{-\lambda_x t}\right] = 2.302\times10^{19}$$

Illustration 34. *10^{-3} kg of a radioactive isotope (atomic mass 226 emits 3.72×10^{10} $\propto$ particles in a second. Calculate the average life of the isotope. If 4.2×10^2 Joules is released in one hour, in this process. What is the average of the $\propto$ particles ?*

Solution: $N = N_0\, e^{-\lambda t}$

$N = N_0\, e^{-\lambda/t}$ where t = 1/λ = average life

$$\Delta N = N_0\, e^{-t/t}\ \frac{\Delta t}{T}$$

$$\therefore \frac{\Delta N}{N} = \frac{\Delta t}{T}$$

N = Number of radioactive atoms in 1 gm of isotope

$$= \frac{10^{-3}}{226}\times 6.023\times10^{26} = 2.7\times10^{21}$$

Given that $\frac{\Delta N}{\Delta t} = 3.72 \times 10^{10}$ /sec

$\therefore T = \frac{N}{(\Delta N / \Delta t)} = \frac{2.7 \times 10^{21}}{3.72 \times 10^{10}} = 7.16 \times 10^{10}$ sec

Half life t1/2 = 0.693 T
= $0.693 \times 7.16 \times 10^{10}$
= 1573 years

similarly Δ N = $N \frac{\Delta t}{T} = y$

$2.7 \times 10^{21} \times \frac{3600}{7.16 \times 10^{10}}$

1.34×10^{14} /hour

Energy per decay $= \frac{4.2 \times 10^{2} \text{ Joule/hour}}{1.34 \times 10^{14} \text{ /hr}}$

= 3.12×10^{-12} Joule
= 19.5 MeV

SUMMARY

Wave–Particle Duality

$\therefore \quad \lambda_e = \dfrac{h}{m_e v} = \dfrac{h}{\sqrt{2eVm_e}}$

Photoelectric Effect

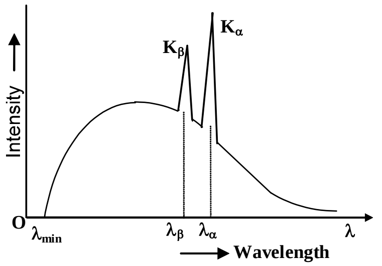
$$eV_s = K_{max} = \frac{hc}{\lambda} - \phi$$

where V_s = stopping potential, λ = wavelength of incident radiation, and ϕ = work function.

X-Rays

When highly energetic electrons are made to strike on a metallic target, electromagnetic radiation called X-rays are emitted.

Continuous and Characteristic X-Rays



Continuous X-Rays

When the electron loses its kinetic energy during several collisions with the other atoms, a fraction of the lost kinetic energy is converted into electromagnetic radiations. This fraction can range from 0 to 1.

Corresponding to the maximum kinetic energy lost by an electron, we have the largest frequency of the X-ray or the shortest wavelength.

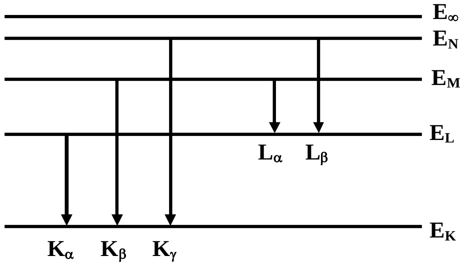
$$\frac{hc}{\lambda_{min}} = K_{max} = eV$$

$$\Rightarrow \quad \lambda_{min} = \frac{hc}{eV}$$

The emitted wavelengths ranges from a minimum value all the way to infinity. Thus, the name continuous spectrum.

Characteristic X-Rays

The accelerated electron might knock out the inner electron of the target atom, whereby a vacancy is created in the inner orbit. Electrons from the higher orbit jump in to fill this vacancy, while releasing the energy difference as electromagnetic radiations, these wavelengths are characteristic of the material from which they are emitted. Hence, the name Characteristic Spectra.

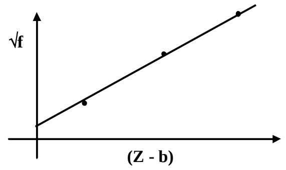


$$\lambda = \frac{hc}{E_K - E_L} \text{ for } K_\alpha \text{ wavelength.}$$

$$\Rightarrow \quad \lambda = \frac{hc}{E_L - E_M} \text{ for } L_\alpha \text{ wavelength and so on.}$$

Moseley's Law

f = frequency of X-ray,
Z = atomic number of target atom,
a, b are constants.
$\sqrt{f} = a(Z - b)$



Bohr's Theory

This theory is applicable for hydrogen and similar atoms.

Bohr's theory for hydrogen-like atoms

$$mvr = \frac{nh}{2\pi} \qquad \text{and} \qquad \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze.e}{r^2}$$

$$\Rightarrow \qquad r_n = \frac{\varepsilon h^2 n^2}{\pi m Z e^2} \qquad \text{and} \qquad E_n = \frac{-Z^2 m e^4}{8h^2\varepsilon_0^2 n^2} = -\frac{Z^2 Rhc}{n^2}$$

If an electron makes a transition from $n = n_2$ to $n = n_1$

$$\frac{hc}{\lambda} = Z^2 Rhc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\Rightarrow \qquad v = \frac{Z}{n}v_0$$

$r_0 = 0.53\,\overset{o}{A}$; $\quad r = \dfrac{r_o n^2}{Z}$,

$E_0 = -13.6$ eV; $E = \dfrac{E_o Z^2}{n^2}$

Also, K.E. = $-$P.E./2

E = P.E. + K.E. = P.E./2 = $-$K.E.

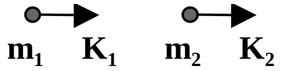
Nuclear Physics

(a) α decay $= {}_A^Z X \rightarrow {}_{A-2}^{Z-4}Y + {}_2^4 He + \text{energy}$

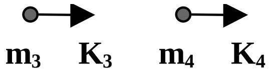
(b) β^- decay $= {}_A^Z X \rightarrow {}_{A+1}^{Z}Y + \beta^- + \gamma + \text{energy}$

(c) β^+ decay $= {}_A^Z X \rightarrow {}_{A-1}^{Z}Y + \beta^+ + \gamma + \text{energy}$; where A = atomic number, Z = mass number

Energy Produced in a Nuclear Reaction



Initial



Final

Initial energy: $E_i = m_1c^2 + m_2c^2 + K_1 + K_2 + hf_o$

Final energy: $E_f = m_3c^2 + m_4c^2 + K_3 + K_4 + hf$

Since $E_i = E_f$, therefore $Q = [(m_1 + m_2) - (m_3 + m_4)]c^2$

If Q is positive, the rest mass energy is converted to kinetic mass energy or radiation mass energy or both, and the reaction is exoergic.

If Q is negative, the reaction is endoergic. The minimum amount of energy that a bombarding particle must have in order to initiate an endoergic reaction, is called Threshold Energy E_{th}.

$E_{th} = -Q\left(\dfrac{m_1}{m_2} + 1\right)$, where m_1 = mass of the projectile and m_2 = mass of the target.

Binding Energy

It is the energy required to break up the nucleus into its constituent nucleus and place them infinitely apart at rest.

Radioactive Decay Law

If nuclei A is decaying to nuclei B

$$-\frac{dN}{dt} = \lambda N$$

$\Rightarrow$ $N = N_0 e^{-\lambda t}$

$$t = \frac{2.303}{\lambda}\log\frac{N_0}{N}$$

Activity: $A = \left|\frac{dN}{dt}\right| = \lambda N$

Half Life

The time interval during which a radioactive substance decays to half its original value. It is denoted by $T_{1/2}$.

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Mean Life

$$T = \frac{\int_0^\infty N_0 e^{-\lambda t}dt}{N_0} = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693}$$

Units of activity: 1 becquerel (Bq) = 1 disintegration per second (dps)
1 curie (Ci) = 3.7×10^{10} dps
1 rutherford (rd) = 10^6 Bq = 10^6 dps

Chain Reaction

Theoretically; it can be represented as

$$A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C$$

$$-\frac{dN_A}{dt} = \lambda_A N_A \quad \text{. . . (1)}$$

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \quad \text{. . . (2)}$$

$$\frac{dN_C}{dt} = \lambda_B N_B$$

Solving Eqs. (1), (2) and (3), we can get the values N_A, N_B, N_C at any time t.

MISCELLANEOUS EXERCISE

1. The total energy of an electron in the first excited state of the hydrogen atom is –3.4 eV. What is the potential energy of the electron in this state?

2. The wavelength of first member of Lyman series is 1216 Å. Calculate the wavelength of 2^{nd} member of Balmer series.

3. Calculate the longest wavelength in the Balmer series of hydrogen atom ($R = 1.097 \times 10^7\ m^{-1}$)

4. Calculate the maximum frequency of continuous X-rays from an X-ray tube. Whose operating voltage is 50,000 volt. [Given $e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ Js]

5. The wavelength of $k\alpha$ line is1216 A^0. Calculate the ionization potential of k shell electron in the atom.

6. After a certain lapse of time, the fraction of radioactive polonium undecayed is found to be 12.5 % of the initial quantity. What is the duration of this time lapse if the half life of polonium is 18 days?

7. A radioactive substance decays to $\left(\frac{1}{32}\right)^{th}$ of its initial activity in 25 days. Calculate its half life.

8. Determine the amount of ${}_{84}Po^{210}$ necessary to provide a source of alpha particles of 5 mci strength. The half life of polonium is 138 days.

9. Light incident on a metal surface has wavelength in the range 300 – 400 nm. The stopping potential varies from 0.82 to 1.85 volts. Find out the value of Planck's Consnat (in units eV - s) from this data.

10. Find the number of photons emitted per second by a 25 W source of monochromatic light of wavelength 6000 A^0.

ANSWERS TO MISCELLANEOUS EXERCISE

1.	–6.8 eV	2.	4864Å
3.	6563 Å	4.	1.21×10^{19} Hz
5.	13.62 eV	6.	54 days
7.	5 days	8.	1.11×10^{-6} gm
9.	4.12×10^{-15} eV -s	10.	7.55×10^{19}.

SOLVED PROBLEMS

Subjective:

BOARD TYPE

Prob 1. *The work function of Cesium is 2.14 eV. Find (a) the threshold frequency for Cesium and (b) the wavelength of the incident light if the photo current is brought to zero by a stopping potential of 0.60 V. Given h = 6.63 $\times$ 10^{-34} Js.*

Sol. (a) Threshold frequency, $f_0 = \frac{\phi}{h} = \frac{2.14\times1.6\times10^{-19}\,J}{6.63\times10^{-34}\,J-s}$

$= 5.16 \times 10^{15}$ Hz.

(b) Energy of photon, $E - \phi = eV_0$ or $E = ev_0 + \phi$

Wavelength, $\lambda = \frac{hc}{E} = \frac{hc}{ev_0+\phi}$

$= \frac{6.63\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times0.6+2.14\times1.6\times10^{-19}}$

$= 4.53 \times 10^{-7}$ m

= 453 nm.

Prob 2. *Light of wavelength 5000 A^0 falls on a sensitive plate with photoelectric work function = 1.90 eV. Find (i) energy of photon in eV, (ii) kinetic energy of the photoelectron emitted and (iii) stopping potential. Given h = 6.62 $\times$ 10^{-34} Js, 1 eV = 1.6 $\times$ 10^{-19} J.*

Sol. (i) Energy $E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^8}{5\times10^{-7}}J$

$= 3.97 \times 10^{-19} \times \frac{1}{1.6\times10^{-19}}eV = 2.48$ eV

(ii) K.E. of photons

K. E. $= E - \phi = (2.48 - 1.9)$ eV = 0.58 eV

(iii) Stopping potential = v_0

$eV_0 = 0.58$ eV

$v_0 = 0.58$ eV

= 0.58 Volts.

Prob 3. *Obtain the binding energy of the nuclei $_{26}Fe^{56}$ and $_{83}Bi^{209}$ in units of MeV from the following data; mass of hydrogen atom = 1.0007825 amu, mass of neutron = 1.008665 amu, mass of $_{26}Fe^{56}$ atom = 55.934 939 amu and mass of $_{83}Bi^{209}$ atom = 208.980388 amu.*

Sol. (a) B.E. of $_{26}Fe^{56}$ = $(26 \times 1.007825 + 30 \times 1.008665 - 55.934939) \times 931.5$ MeV.

= 492.3 MeV

$\therefore$ B. E. per nucleon = $\frac{492.3}{56} = 8.79\text{MeV}$

(b) B. E. of $_{83}Bi^{209}$

= (83 × 1.007825 + 126 × 1.008665 – 208.980388) × 931.5 MeV

= 1640.3 MeV

$\therefore$ Binding energy per nucleon = $\frac{1640.3}{209}$ = 7.85 MeV.

Prob 4. *Calculate the half life period of a radioactive substance if its activity drops to $\frac{1}{16}$ th of the initial value in 30 years.*

Sol. $A = A_0 e^{-\lambda t}$

$\frac{1}{16} = e^{-\lambda x 30}$ when t is in years.

$\therefore$ $30\lambda = \ln 16$; $\lambda = \frac{\ln 16}{30}$

half life period,

$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{30\ln 2}{\ln 16}$.

$= \frac{30\times\ln 2}{4\ln 2}$ = 7.5 years.

Prob 5. *X- rays of wavelength 0.82 A^0 fall on a metal plate. Find the wavelength associated with photoelectron emitted. Neglect work function of the metal. Given h = 6.6 × 10^{-34} J-s, C = 3 × 10^8 m/s.*

Sol. Here, $\lambda = 0.82\ A^0$

$= 0.82 \times 10^{-10}$ m, $W_0 = 0$

KE of electron, $\frac{1}{2} mv^2 = h\nu - W_0$

or $mv^2 = 2h\nu = \frac{2hc}{\lambda}$ $(\because\ W_0 = 0)$

or $mv = \sqrt{\frac{2hcm}{\lambda}}$

$\therefore$ $\lambda' = \frac{h}{mv} = \frac{h}{\sqrt{2hcm/\lambda}} = \sqrt{\frac{h\lambda}{2cm}}$

$= \sqrt{\frac{6.6\times10^{-34}\times0.82\times10^{-10}}{2\times3\times10^{8}\times9.1\times10^{-31}}}$

$= 0.099 \times 10^{-10}$ m $= 0.099\ A^0$

IIT-JEE TYPE

Prob 6. *In an experiment, the activity of 1.2 mg radioactive potassium chloride solution was found to be 170 s^{-1}. Taking molar mass of kcl to be 0.075 kg/mol, find the no. of k-40 atoms in the sample and hence find the half life of k - 40. Given Avagadro's number = 6.023 × 10^{23} mol^{-1} .*

Sol. no. of moles of kcl = $\frac{1.2\times10^{-6}\,kg}{0.075\,kg/mol}$

no. of k – 40 atoms in sample = $\frac{1.2\times10^{-6}}{0.075}\times6.023\times10^{23}$

= 9.637×10^{18}.

A = $\frac{dN}{dt} = 170 s^{-1}$ = λN

$\therefore$ $\lambda = \frac{170}{N} = \frac{170}{9.637\times10^{18}}$

= $1.764 \times 10^{-17}\ s^{-1}$.

$\therefore$ half life of k – 40

$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1.764\times10^{-17}}$

$t_{1/2} = 3.93 \times 10^{16}$ s.

Prob 7 . *In an ore containing uranium, the ratio of U^{238} to Pb^{206} nuclei is 3. Calculate the age of the ore assuming that all the lead present in the ore is the final stable product of U^{238}. Take the half life of U^{238} to be 4.5×10^9 years.*

Sol. In this Problem it is assumed that the half life of the intermediate products is very small in comparison to that of U^{238}.

Let the required age is 't'. If N_0. be the initial number of U^{238} nuclei then

$\frac{N_0.e^{-\lambda t}}{N_0.(1-e^{-\lambda t})} = 3 \quad \Rightarrow \quad 4\,e^{-\lambda t} = 3$

$\Rightarrow \quad \lambda t = \ln(4/3)$

$t = \frac{\ln(4/3)}{\lambda} = \frac{\ln(4/3)}{\ln 2}T = 1.87 \times 10^9$ years**.**

Prob 8. *A small quantity of a solution containing Na radio nuclei (half life 15 hrs) of activity 1.0 micro-curie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hrs shows an activity of 296 disintegration per minute. Determine the total volume of blood in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person.*

Sol. If the radioactive nuclei spread uniformly in the blood, and $N \propto v$

or $A \propto v$ (where A = activity and v = volume of blood)

Now, total activity at time t = $\left[\frac{A(t)}{v}\right]V$ Also, $A(t) = A_o e^{-\lambda t}$

we get, $V = \frac{A_o e^{-\lambda t}}{[A(t)/v]}$

$= \frac{(3.7\times10^4)e^{-\frac{(5\times3600)\ln 2}{(15\times3600)}}}{296\times(1/60)}\ cm^3 = \frac{3.7\times60\times10^4}{296\times2^{1/3}}\ cm^3 = 5.95 \times 10^3\ cm^3$.

Prob 9. *The binding energy per nucleon for deuteron ($_1H^2$) and helium ($_2He^4$) are 1.1 Mev and 7.0 MeV respectively. What is the energy released when two deutrons fuse to form a helium nucleus.*

Sol. $Q = [(\sum m)_{initial} - (\sum m)_{final}]c^2$
the given values of B.E. help us to get $(\sum m)_{initial}$ & $(\sum m)_{final}$.
$M_{deutron}c^2 = (M_n + m_p)c^2 = (1.1\text{ MeV})2$ . . . (i)
similarly $M_{helium}\, c^2 = 2\,(m_n + m_p)c^2 = (7\text{ Mev})4$ . . . (ii)
using (I) and (ii) we get
$(2M_{deutron} - M_{helium})c^2 = (28 - 4.4)\text{Mev}$
$= 23.6$ Mev.

Prob 10. *A monochromatic beam of light (λ = 4900 Å) incident normally upon a surface produces a pressure of 5×10^{-7} N/m^2 on it. Assuming that 25% of the light incident is reflected and the rest absorbed, find the number of photons falling per second on a unit area of this surface.*

Sol. Momentum of light falling on the surface per second
$P = [2(0.25) + 0.75]\dfrac{I}{c} = 1.25\dfrac{I}{c}$
$\therefore$ Intensity of light, $I = \dfrac{cP}{1.25} = \dfrac{(3\times10^8)(5\times10^{-7})}{1.25} = 120\text{ Wm}^{-2}$
Energy of photon, $E = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34})(3\times10^8)}{0.49\times10^{-6}} = 4\times10^{-19}$ J
$\therefore$ Number of photons incident per unit area per second
$n = \dfrac{I}{E} = \dfrac{120}{4\times10^{-19}} = 3\times10^{20}\text{m}^{-2}\text{s}^{-1}$

Prob 11. *When the voltage applied to an X-ray tube is increased from V_1 = 10 kV to V_2 = 20 kV, the wavelength interval between the K_α line and the short-wave cut off of the continuous X-ray spectrum increases by a factor η = 3.0. Find the atomic number of the element of which the tube's anticathode is made.*

Sol. $\lambda_\alpha = \dfrac{4}{3R(Z-1)^2}$
Cut off wavelengths : $\lambda_1 = \dfrac{hc}{eV_1}$; $\lambda_2 = \dfrac{hc}{eV_2}$
Using given condition : $(\lambda_2 - \lambda_\alpha) = 3(\lambda_1 - \lambda_\alpha)$
or $3\lambda_1 - \lambda_2 = 2\lambda_\alpha$ or $\dfrac{hc}{e}\left[\dfrac{3}{V_1} - \dfrac{1}{V_2}\right] = \dfrac{8}{3R(Z-1)^2}$
Taking $V_1 = 10\times10^3$ V ; $V_2 = 20\times10^3$ V
$Z = 29$.

Prob 12. *A radioactive nucleus X decays to a nucleus Y with a decay constant λ_x = 0.1 sec^{-1}. Y further decays to a stable nucleus Z with a decay constant*

λ_Y = 1/30 sec^{-1}. Initially, there are only X nuclei and their number is $N_0 = 10^{20}$. Set up the rate equations for the populations of X, Y and Z. The population of the Y nucleus as a function of time is given by $N_Y(t) = \{N_0\lambda_X /(\lambda_X - \lambda_Y)\}$ $\{\exp(-\lambda_Y t) - \exp(-\lambda_X t)\}$. Find the time at which N_Y is maximum and determine the population X and Z at that instant.

Sol. $dN_X/dt = -\lambda_x N_x$

$\therefore N_x(t) = N_0 e^{-\lambda_x t}$

$dN_y/dt = \lambda_x N_x - \lambda_y N_y$ and $dN_z/dt = \lambda_y N_y$

where $N_y(t) = N_0\lambda_x\{e^{-\lambda_y t} - e^{-\lambda_x t}\}/(\lambda_x - \lambda_y)$

For maximum N_y, $dN_y/dt = 0$

$\Rightarrow \lambda_x e^{-\lambda_x t} = \lambda_y e^{-\lambda_y t}$

$\Rightarrow$ t = 16.48 sec.

$\Rightarrow N_x = 1.924 \times 10^{19}$ and $N_z = \int \lambda_y N_y \, dt$

$$= N_0\left[1 - \frac{\lambda_x}{\lambda_x - \lambda_y}e^{-\lambda_y t} + \frac{\lambda_y}{\lambda_x - \lambda_y}e^{-\lambda_x t}\right] = 2.302 \times 10^{19}$$

Prob 13. *An electron in Bohr's hydrogen atom has an energy of –3.4 eV. What is the angular momentum of the electron?*

Sol. The energy of an electron in an orbit of principal quantum number n is given as

$E = \dfrac{-13.6}{n^2}$ eV $\quad \Rightarrow -3.5 \text{ eV} = \dfrac{-13.6}{n^2}$ eV

$\Rightarrow n^2 = 4 \quad \Rightarrow n = 2$

The angular momentum of an electron in nth orbit is given as $L = \dfrac{nh}{2\pi}$

Putting n = 2, We obtain $L = \dfrac{2h}{2\pi} = \dfrac{h}{\pi}$

Prob 14. *A doubly ionised Lithium atom is hydrogen like with atomic number 3.*

(a) Find the wavelength of radiation required to excite the electron in Li^{++} from the first to the third Bohr Orbit. (Ionization energy of the hydrogen atom equals 13.6 eV).

(b) How many spectral lines are observed in the emission spectrum of the above excited system?

Sol. (a) $E_n = \dfrac{-13.6\,Z^2}{n^2}$

Excitation energy = $\Delta E = E_3 - E_1 = -13.6 \times (3)^2\left[\dfrac{1}{3^2} - \dfrac{1}{1^2}\right]$

$= +13.6 \times (9)\,[1 - 1/9] = 13.6 \times (9)\,(8/9) = 108.8$ eV.

Wavelength $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(6.63\times10^{-34})\,(3\times10^{8})\text{m}}{108.8\,(1.6\times10^{-19})}$ = 114.3 A°

(b) From the excited state (E_3), coming back to ground state, there can be ${}^3C_2$ = 3 possible radiations.

Prob 15. *How much energy must a bombarding proton possess to cause the reaction?*

$${}^7Li_3 + {}^1H_1 \rightarrow {}_4Be^7 + {}_0n^1$$

Sol. Since the mass of an atom includes the masses of the atomic electrons, the appropriate number of electron masses must be subtracted from the given values.

Reactants		**Products**	
${}^7_3Li$	7.01600 - 3 m_e	${}^7_4Be$	1.01693 - 4m_e
${}^1_1H$	1.0783 - 1 m_e	${}^1_0n$	1.0866
Total	8.02383 - 4m_e	Total	8.02559 - 4m_e

Mass defect = 8.02383 – 8.02559 = – 0.00176

The Q-value of the reaction

$$Q = -0.00176 \text{ u} = -1.65 \text{ MeV}$$

The energy is supplied as KE of the bombarding proton. The incident proton must have more than this energy because the system must possess some KE even after the reaction, so that momentum is conserved. With momentum conservation taken into account, the minimum KE that the incident particle must have can be found with the formula.

$$E_{th} = -\left(1+\frac{m}{M}\right)Q = -\left(1+\frac{1}{7}\right)(-1.65) = 1.89\text{MeV}$$

Objective:

Prob 1. *The speed of an electron in the orbit of a hydrogen atom in the ground state is*
(A) c/10 *(B) c/137*
(C) c/2 *(D) c*

Sol. In H-atom with n =1,
Speed of electron, $v = \frac{e^2}{2h\varepsilon_0} = c\frac{e^2}{2ch\varepsilon_0}$
$\frac{e^2}{2ch\varepsilon_0} = \frac{(1.6\times10^{-19})^2}{2\times3\times10^8\times6.63\times10^{-34}\times8.85\times10^{-12}} = \frac{1}{137}$
$v = \frac{c}{137}$
$\therefore$ (B)

Prob 2. *Consider the spectral line resulting from the transition n = 2 to n = 1 in the atoms and ions given below, the shortest wavelength is produced by*
(A) hydrogen atom *(B) deuterium atom*
(C) single ionised helium *(D) doubly ionised lithium*

Sol. $\frac{1}{\lambda} = z^2R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$
for λ to be min, z should be maximum.
Hence doubly ionised lithium will produce shortest wavelength.
$\therefore$ (D)

Prob 3. *The longest wavelength that a singly ionised helium atom in its ground state will absorb is*
(A) 3.03 A^0 *(B) 30.3 A^0*
(C) 303 A^0 *(D) 3030 A^0*

Sol. $\frac{1}{\lambda} = z^2R(1)\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$
For longest wavelength, value of R.H.S. of the above equation should be minimum. Hence n_1 and n_2 will be 1 & 2 respectively.
$\therefore\ \frac{1}{\lambda} = 4\times1.1\times10^7\left(\frac{1}{1}-\frac{1}{4}\right)$
$= \frac{4\times1.1\times10^7\times3}{4} = 3.3\times10^7$
$\lambda = \frac{1}{3.3}\times10^{-7} = 0.303\times10^{-7}$ m = 303 A^0.
$\therefore$ (C)

Prob 4. *The half life of radium is 1600 years. The fraction of a sample of radium that would decay is 6400 years is*

(A) 1/8 *(B) ¼*
(C) 7/8 *(D) 15/16*

Sol. Amount left after 6400 years is after 4 half lives

$$= \left(\frac{1}{2}\right)^4 N = \frac{1}{16}N$$

$\therefore$ Fraction of amount that would decay = $1 - \frac{1}{16} = \frac{15}{16}$

$\therefore$ (D)

Prob 5. *A particular material can decay in two ways emitting α and β particles. Half lives for α and β decay separately are T_1 and T_2 respectively. If the material decays for α and β simultaneously, then half life of the material is*

(A) $T_1 + T_2$ *(B) $T_1 - T_2$*
(C) $\frac{T_1 + T_2}{T_1 - T_2}$ *(D) $\frac{T_1T_2}{T_1 + T_2}$*

Sol.

$$\left(\frac{dN}{dt}\right)_1 = -\lambda_1 N$$

$$\left(\frac{dN}{dt}\right)_2 = -\lambda_2 N$$

$$\frac{dN}{dt} = -\lambda_1 N + -\lambda_2 N \quad \therefore \ \lambda_e = \lambda_1 + \lambda_2$$

where λ_e is effective λ for the material for simultaneous decay.

hence, $\frac{\ln 2}{T_e} = \frac{\ln 2}{T_1} + \frac{\ln 2}{T_1} \quad \therefore \ T_e = \frac{T_1T_2}{T_1 + T_2}$

$\therefore$ (D)

Prob 6. *The ratio of ionization energy of Bohr's hydrogen atom and Bohr's hydrogen like Lithium atom is*

(A) 1:1 *(B) 1:3*
(C) 1:9 *(D) None of these*

Sol. Energy of an electron in ground state of an atom (Bohr's hydrogen atom like) is given as

$E = -13.6\ Z^2$ eV where Z = atomic number of the atom

$\Rightarrow$ The ionization energy of that atom = $E_{ion} = 13.6\ Z^2$

$\Rightarrow (E_{ion})_H / (E_{ion})_{Li} = \frac{(Z_H)^2}{(Z_{Li})^2}$

$$\Rightarrow \frac{(E_{ion})_H}{(E_{ion})_{Li}} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$\therefore$ (C)

Prob 7. *The ratio of minimum to maximum wavelengths in the Lyman Series of radiation that an electron causes in a Bohr's hydrogen atom is*

(A) 1/2 *(B) Zero*

(C) 3/4 *(D) 27/32*

Sol. Energy of radiation that corresponds to the energy difference between two energy levels n_1 and n_2 is given as

$$E = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\text{eV}$$

E is minimum when $n_1 = 1$ & $n_2 = 2$

$$\Rightarrow\ E_{min} = 13.6\left(\frac{1}{1}-\frac{1}{4}\right)\ \text{eV} = 13.6\times\frac{3}{4}\ \text{eV}$$

E is maximum when $n_1 = 1$ & $n_2 = \infty$ (the atom is ionized, that is known as ionization energy)

$$\Rightarrow\ E_{max} = 13.6\left(1-\frac{1}{\infty}\right) = 13.6\ \text{eV}.$$

$$\therefore\ \frac{E_{min}}{E_{max}} = \frac{3}{4} \Rightarrow \frac{hc/\lambda_{max}}{hc/\lambda_{min}} = \frac{3}{4}$$

$$\Rightarrow\ \frac{\lambda_{min}}{\lambda_{max}} = \frac{3}{4}$$

$\therefore$ (C)

Prob 8. *An electron in Bohr's hydrogen atom has an energy of –3.4 eV. The angular momentum of the electron is*

(A) h/π *(B) $h/2\pi$*

(C) $nh/2\pi$ (n is an integer) *(D) $2h/\pi$*

Sol. The energy of an electron in an orbit of principal quantum number n is given as

$E = \dfrac{-13.6}{n^2}$ eV $\Rightarrow$ -3.4 eV = $\dfrac{-13.6}{n^2}$ eV

$\Rightarrow\ n^2 = 4$ $\Rightarrow\ n = 2$

The angular momentum of an electron in nth orbit is given as

$L = \dfrac{nh}{2\pi}$, Putting n = 2

We obtain $L = \dfrac{2h}{2\pi} = \dfrac{h}{\pi}$

$\therefore$ (A)

Prob 9. *A monochromatic radiation of wavelength λ_1 is incident on a stationary atom as a result of which the wavelength of the photon after the collision becomes λ_2 and the recoiled atom has De Broglie's wavelength λ_3. Then,*

(A) $\lambda_3 = \sqrt{\lambda_1\lambda_2}$ *(B) $\lambda_1 = \dfrac{\lambda_2\lambda_3}{\lambda_2+\lambda_3}$*

(C) $\lambda_1 = \sqrt{\lambda_1^2+\lambda_2^2}$ *(D) $\lambda_3 = \sqrt{\lambda_1^2+\lambda_2^2}$*

Sol. Conservation of momentum yields

$$\frac{h}{\lambda_1}+0=\frac{h}{\lambda_2}+mv$$

$\Rightarrow \frac{h}{\lambda_1}-\frac{h}{\lambda_2}=mv$ $\Rightarrow \frac{1}{\lambda_1}-\frac{1}{\lambda_2}=\frac{mv}{h}$

Since, $\frac{h}{mv}=\lambda_3$ $\Rightarrow \frac{1}{\lambda_1}-\frac{1}{\lambda_2}=\frac{1}{\lambda_3}$

$\Rightarrow \frac{1}{\lambda_1}=\frac{1}{\lambda_2}+\frac{1}{\lambda_3}$ $\Rightarrow \lambda_1=\frac{\lambda_2\lambda_3}{\lambda_2+\lambda_3}$

$\therefore$ (B)

Prob 10. *A ray of light of wavelength 6630 A^o is incident on a totaly reflecting surface. The momentum delivered by the ray is equal to*

(A) 6.63 x 10^{-27} kgm/s *(B) 2 x 10^{-27} kgm/s*

(C) 10^{-27} kgm/s *(D) None of these*

Sol. The momentum of the incident radiation is given as P = h/λ. When the light is totally reflected normal to the surface the direction of the ray is reversed. That means it reverses the direction of its momentum without changing its magnitude.

$\Rightarrow$ Change in momentum has a magnitude $\Delta P = 2P = \frac{2h}{\lambda}$

$\Rightarrow \Delta P = \frac{2(6.63\times10^{-34}\,J-sec)}{(6630\times10^{-10}m)}$

$\Rightarrow \Delta P = 2 \times 10^{-27}$ kgm/s

$\therefore$ (B)

SHORT ANSWER TYPE QUESTION

Prob 11. *Which spectral series was observed first and why?*

Sol. Balmer series. It is in visible, region.

Prob 12. *Is bohr's theory applicable to any atom? If not what is the necessary criterion*

Sol. No. Only to single electron species

Prob 13. *Deduce the bohr's quantization postulate using de broglie wavelength.*

Sol. $\lambda = \frac{h}{mv}$ . . . (i)

But $2\pi r = n\lambda$ . . . (ii)

$\therefore 2\pi r = n\frac{h}{mv}$

$\Rightarrow mvr = \frac{nh}{2\pi}$

Prob 14. *List some of the shortcomings of bhor's model.*

Sol. It failed to explain
(i) Zeeman effect
(ii) Heisenberg's uncertainty principle

True / false

Prob 15. *As the value of n increase the energy gap between consecutive orbits increase*

Sol. [False.]
$$\frac{1}{\tau} = Z^2R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Prob 16. *Bohr's quantization theory is applicable only to the single electron species*

Sol. [True.]

Prob 17. *Electron velocity for same orbit n is greater in hydrogen compared to Li^{+2}.*

Sol. [False.]
$$\simeq \left(\frac{C}{137}\right)\left(\frac{Z}{\tau}\right)$$

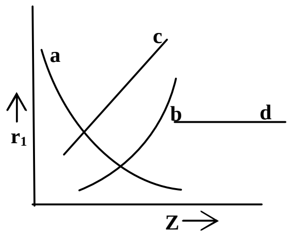
Fill in the blanks

Prob 18. *The ionization energy of L_i^{2t} in ground state is -----------------------*

Sol. $$13.6\text{ev}\frac{(3^2)}{(1)^2} = -13.6\times 9 = -122.4\text{ev}$$

Prob 19. *Which of the following curve gives the relationship between radius of first orbit and atomic no. z. for ground state of all hydrogen like atoms.*

Sol. (A)
For n = 1
R_1 z = constant



Prob 20. *The no. of different transitions possible from n = 4 to down the orbits is ---------*

Sol. $${}^4C_2 = \frac{L^4}{L^2L^2} = 6$$

Prob 21. *If the total energy of a system is 10 J then it's K.E. is ------------------ and its P.E. is --------------*

Sol. 10 J, –20 J

ASSIGNMENT PROBLEMS

Subjective:

Level- O

1. Electron is a particle. Does it behave as wave in some situations? If yes, write an expression of wavelength of electron in terms of its momentum.

2. Describe the phenomenon of photoelectric effect. Which theory of light is based on ?

3. Describe briefly Bohr's postulates on structure of atom.

4. Find out the wavelength of electrons emitted when x-rays of wavelength 0.75 A^0 fall on a metal. Assume zero work function of the metal.

5. Describe production of x-rays in a Coolidge tube. Why does the metal anode become very hot in the process ?

6. What are continuous and characterstic x-rays ? Show them in a graph of intensity vs wavelength.

7. Describe what are isotopes isotones and isobars.

8. Calculate the binding energy per nucleon of ${}_{20}C^{40}$ nucleus. Given mass of ${}_{20}C^{40}$ nucleus = 39.962589 amu, mass of proton = 1.007825 amu, mass of neutron = 1.008665 amu and 1 amu = 931 MeV.

9. Draw the graph showing the variation of binding energy per nucleon with mass no. Give the reason for binding energy per nucleon for nuclei with high mass number?

10. With the help of diagram explain how the neutron to proton ratio changes during α-decay of a nucleus.

Level - I

1. How many alpha and beta particles are emitted when uranium $_{92}U^{238}$ decays to lead $_{82}Pb^{206}$?

2. The half life of radon is 3.8 days. After how many days will only one twentieth of radon sample be left over ?

3. An electron in a hydrogen like atom is in excited state. It has a total energy of -3.4eV. Calculate (i) the kinetic energy and (ii) the de Broglie wavelength of the electron.

4. Monochromatic light of wave length 5×10^{-5} cm illuminates a metal surface (work function $\varphi = 2.0$ eV). Calculate
   (a) threshold wavelength
   (b) maximum energy of photoelectrons
   (c) minimum retarding potential to prevent photo emission.

5. A hydrogen like ion has the wavelength difference between the first lines of Balmer and Lymen series equal to 59.3 nm. Identify the ion.

6. An X-rays tube operates at 20 kV. Find the maximum speed of the electrons striking the anticathode, given the charge of electron = 1.6×10^{-19} coulomb and mass of electron is equal to 9×10^{-31}kg.

7. The binding energy of $_{17}Cl^{35}$ nucleus is 298MeV. Find its atomic mass. The mass of hydrogen atom ($_1H^1$) is 1.008143 u. and that of a neutron is 1.008986 a.m.u. Given 1 u. = 931MeV.

8. Find the amount of energy produced in joules due to fission of 1 gram of uranium assuming that 0.1 percent of mass is transformed into energy.
   Take 1 u = 931 MeV, Mass of Uranium = 235 u, Avogadro Number $N_A = 6.02 \times 10^{23}$.

9. An electron is shot down in an X-ray tube across a potential difference 30 kV. If it collides with a heavy massive atom of the target and loses all its energy., what is the wave length of the X-ray produced ?

10. If the wavelength of the light falling on a surface is increased from 3000 Å to 3040 Å, then what will be the corresponding change in the stopping potential? (Given that hc = 12.4×10^3 eV Å)

11. A hydrogen atom in its ground state is excited by incident light having a continuous spectrum with all wavelengths upto 975 A^0. What is the highest energy level reached by the electron? How many lines are observed in the resulting spectrum?

12. Which state of a triply ionised beryllium has the same orbital radius as that of ground state of hydrogen? Also compare the energies of two states.

13. Light is incident on the cathode of a photocell and the stopping voltages are measured for lights of two different wavelengths as shown in data below. Determine the work function of the metal of cathode in eV. Also determine the value of the planck constant (h) from given data.

λ :	4000Å	4500 Å
V :	1.3 V	0.9 V

14. A beam of light has three wavelengths 4144 Å, 4972 Å and 6216 Å with a total intensity of 3.6×10^{-3} W/m^2, equally distributed amongst the three wavelengths. The beam falls on an area 1 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photo electrons liberated in 2 seconds.

15. A P^{32} radionuclide with half life 14.3 days is produced in a reactor at a constant rate q = 10^9 per second. How soon after the beginning of production of radionuclide will its activity be equal to A = 1×10^8 disintegration/sec.

Level - II

1. A hydrogen-like atom of atomic number Z is in an excited state of quantum number 2n. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) for this atom . Also, calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is -13.6 eV.

2. Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82eV. Find
(a) the energy of the photons causing the photoelectric emission,
(b) the quantum numbers of the two levels involved in the emission of these photons.
(c) the change in the angular momentum of the electron in the hydrogen atom in the above transition, and
(d) the recoil speed of the photon emitting hydrogen atom assuming to be at rest before the transition. (Ionization potential of hydrogen is 13.6 V).

3. A stationary He^+ ion emitted a photon corresponding to the first line of the Lyman series. That photon liberated a photoelectron from a stationary hydrogen atom in the ground state. Find the velocity of the photoelectron.

4. There is a stream of neutrons with a kinetic energy of 0.0327eV. If the half life of neutrons is 700 seconds, what fraction of neutrons will decay before they travel a distance of 10 km. (mass of the neutron = 1.675×10^{-27}kg)

5. Find the binding energy of the nucleus of lithium isotope $_3Li^7$ and hence find the binding energy per nucleon in it. Given $_3Li^7$ atom = 7.016005 amu;
$_1H^1$ atom = 1.007825 amu; $_0n^1$=1.008665 amu.

6. A nucleus at rest undergoes a decay emitting an α particle of de-Broglie wavelength $\lambda = 5.76 \times 10^{-15}$ m. If the mass of the daughter nucleus is 223.610 a.m.u. and that of the α particle is 4.002 a.m.u., determine the total kinetic energy in the final state. Hence, obtain the mass of the parent nucleus in a.m.u. (1 a.m.u. = 931.470 MeV/c^2)

7. A star has 10^{40} deutron. It produces energy via, the process
$_1H^2 + {_1H^2} \longrightarrow {_1H^3} + P$
$_1H^2 + {_1H^3} \longrightarrow {_2He^4} + n$
If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in how much time ? The masses of the nuclei are as follows :
$M(_1H^2)$ = 2.014 amu ; M(p) = 1.007 amu ; M(n) = 1.008 amu; $M(He^4)$ = 4.001 amu

8. A sample of uranium is a mixture of three isotopes $_{92}U^{234}$, $_{92}U^{235}$ and $_{92}U^{238}$ present in the ratio of 0.006%, 0.71% and 99.284% respectively. The half lives of three isotopes are $2.5x10^5$ years, $7.1x10^8$ years and $4.5x10^9$ years respectively. Calculate the contribution to total activity (in%) coming from each isotope.

9. The ionization energy of a hydrogen like Bohr atom is 4 Rydberg.
(i) What is the wavelength of radiation emitted when the electron jumps from the first excited state to the ground state ?

(ii) What is the radius of the first orbit for this atom ?
Given that Bohr radius of hydrogen atom $= 5 \times 10^{-11}$ m and
1 Rydberg = 2.2×10^{-18} J

10. In a photoelectric experiment set up, photons of energy 5 eV falls on the cathode having work function 3 eV. (a) If the saturation current is $i_A = 4\mu A$ for intensity 10^{-5} W/m^2, then plot a graph between anode potential and current. (b) Also draw a graph for intensity of incident radiation 2×10^{-5} W/m^2.

11. When a beam of 10.6 eV photons of intensity 2.0 W/m^2 falls on a platinum surface of area 1.0×10^{-4} m^2 and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum amd maximum energies (in eV). Take 1 eV = 1.6×10^{-19} J .

12. Ultraviolet light of wavelength 800Å and 700Å when allowed to fall on hydrogen atom in their ground state is found to liberate electrons with kinetic energy 1.8eV, and 4.0eV respectively. Find the value of Planck's constant.

13. A doubly ionized Lithium atom is hydrogen like with atomic number 3 ;
(a) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit. (Ionisation energy of hydrogen atom equals 13.6 eV).
(b) How many spectral lines are observed in the emission spectrum of the above excited system.

14. At what minimum kinetic energy must a hydrogen atom move for its inelastic head-on collision with another, stationary, hydrogen atom to make one of them capable of emitting a photon? both atoms are supposed to be in the ground state prior to the collision.

15. Radiation falls on a target kept within a solenoid with 20 turns per cm, carrying a current 2.5 A. Electrons emitted move in a circle with a maximum radius of 1 cm. Find the wavelength of radiation, given that the work function of the target is 0.5 Electron volts, $e = 1.6 \times 10^{-19}$ coulomb, $h = 6.625 \times 10^{-34}$ J–s, $m = 9.1 \times 10^{-31}$ Kg.

Objective:

Level - I

1. If h is Planck's constant, the momentum of a photon of wavelength 0.01Å is :
(A) 10^{-2} h
(B) h
(C) 10^{2} h
(D) 10^{12} h

2. The ionisation potential of hydrogen atom is 13.6 volt. The energy required to remove an electron from the second orbit of hydrogen is:
(A) 3.4 eV
(B) 6.8 eV
(C) 13.6 eV
(D) 27.2 eV

3. The radius of Bohr's first orbit is a_0. The electron in n^{th} orbit has a radius:
(A) na_0
(B) a_0/n
(C) n^2a_0
(D) a_0/n^2

4. The ionisation energy of 10 times ionised sodium atom is :
(A) 13.6 eV
(B) 13.6×11 eV
(C) (13.6/11) eV
(D) $13.6 \times (11^2)$ eV

5. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation:
(A) the intensity increases
(B) the minimum wave length increases
(C) the intensity remains decreases
(D) the minimum wave length decreases

6. In producing X-rays, a beam of electrons accelerated by a potential difference V is made to strike a metal target. For what value of V of the following X-rays have the lowest wavelength:
(A) 10 KV
(B) 20 KV
(C) 30 KV
(D) 40 KV

7. The X-ray beam coming from an X-ray tube will be:
(A) monochromatic
(B) having all wavelengths smaller than a certain maximum wave length
(C) having all wavelengths larger than a certain minimum wave length
(D) having all wavelengths lying between a minimum and a maximum wave length.

8. When high speed electrons strike a target:
(A) only heat is produced
(B) only continuous X-rays are emitted
(C) only continuous and characteristic X-rays are emitted
(D) heat is produced and simultaneously continuous and characteristic X-rays are emitted.

9. The relation between half-life T of a radioactive sample and its mean life τ is:
(A) $T = 0.693\ \tau$ (B) $\tau = 0.693\ T$
(C) $\tau = T$ (D) $\tau = 2.718\ T$

10. If the stationary proton and $\propto$ particle are accelerated through same potential difference the ratio of their wavelength will be
(A) 2 (B) 1
(C) $2\sqrt{2}$ (D) none of these

11. The transition from the state n = 3 hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the translation.
(A) $2 \to 1$ (B) $3 \to 2$
(C) $4 \to 2$ (D) $5 \to 4$

12. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly iodized Li atom (Z = 3) is
(A) 1.51 (B) 13.6
(C) 40.8 (D) 122.4

13. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential is
(A) 2 (B) 4
(C) 6 (D) 10

14. A particle of mass M at rest decays into two particles of mass m_1 and m_2, having nonzero velocities. The ratio of the de Broglie wavelengths of the particles, λ_1 / λ_2, is
(A) m_1 / m_2 (B) m_2 / m_1
(C) 1.0 (D) $\sqrt{m_2} / \sqrt{m_1}$

15. Find de Broglie wavelength of a 3 kg object moving with a speed of 2 m/s
(A) 1.1×10^{-34} (B) 2.2×10^{-34}
(C) 10^{-34} (D) none

Fill in the blanks

16. The splitting of hydrogen spectra in pressure of magnetic field and electric field is called -----------effect

17. A photon is incident on a plate and is elastically reflected back. If the wavelength of the photon is λ. Then the change in momentum due to this collision is.----------------

18. The momentum of a photon having energy E is -------------------

19. The rest mass of photon is -------------------------

20. In photoelectric effect , the photoelectric current does not depend on -------------------- of photon but depend on ----------------------of incident light

True / False

21. Stopping potential depends on the intensity of incident light.

22. Matter waves are electromagnetic waves.

23. Momentum of photon of wavelength λ is h/λ.

24. The number of photoelectrons emitted is proportional to intensity of light.

25. Stopping potential decreases with decrease in intensity of light.

Level - II

1. Radius of the second Bohr orbit of singly ionised helium atom is
(A) 0.53 A^0 (B) 1.06 A^0
(C) 0.265 A^0 (D) 0.132 A^0

2. If E_1, E_2 and E_3 represent respectively the kinetic energies of an electron, an alpha particle and a proton each having same de Broglie wavelength then:
(A) $E_1 > E_3 > E_2$ (B) $E_2 > E_3 > E_1$
(C) $E_1 > E_2 > E_3$ (D) $E_1 = E_2 = E_3$

3. If the electron in the hydrogen atoms is excited to n = 5 state, the number of different frequencies of radiation which may be emitted is :
(A) 4 (B) 5
(C) 8 (D) 10

4. The ratio of magnetic dipole moment of an electron of charge e and mass m in Bohr's orbit in hydrogen to its angular momentum is:
(A) e/m (B) e/2m
(C) m/e (D) 2m/e

5. The wave length λ of K_α-ray line of an anticathode element of atomic number Z is nearly proportional to:
(A) Z^2 (B) $(Z-1)^2$
(C) $\frac{1}{(Z-1)}$ (D) $\frac{1}{(Z-1)^2}$

6. X-rays will not show the phenomenon of :
(A) diffraction (B) polarisation
(C) deflection by electric field (D) interference

7. In the nuclear reaction given by $_2He^4 + {}_7N^{14} \rightarrow {}_1H^1 + {}_gX^P$ then the nucleus X is :
(A) Nitrogen of mass 16 (B) Nitrogen of mass 17
(C) Oxygen of mass 16 (D) Oxygen of mass 17

8. If 10% of a radioactive material decays in 5 days, then the amount of the original material left after 20 days is approximately:
(A) 60% (B) 65%
(C) 70% (D) 75%

9. 14×10^{15} electrons reach the anode per second. If the power consumed is 448 milli-watts, the anode voltage is :
(A) 150 V (B) 200 V
(C) 14×448 V (D) 448/14V

10. Consider the spectral line resulting from the trastion n = 2 $\rightarrow$ n = 1 in the atoms and ions given below, the shortest wavelength is produced by
(A) Hydrogen atom (B) deuterium atom
(C) Singly ionized helium (D) doubly ionized lithium

True / False

11. Stopping potential depends on frequency of incident light

12. Energy and momentum of photon are related as P = E/c.

13. The velocity of photoelectrons is inuasly proportional to the wavelength of the incident light

14. Photoelectric effect is based on conservation of mass.

15. Photoelectric current increases with intensity of light.

Fill in the blanks

16. The de broglie's wavelength of particle of kinetic energy k is λ, the wavelength of the particle when its kinetic energy was k/4 is ……………….

17. A proton will move……………..than electron, when they have same de broglie wavelength.

18. Work function of a metal is …………… eV. If its threshold wavelength is 6800 A^0

19. The wavelength of photon corresponding to energy (10^7 eV) is …………………

20. ………………… photons are required to emit one photoelectron.

ANSWERS TO ASSIGNMENT PROBS

Subjective:

Level- O

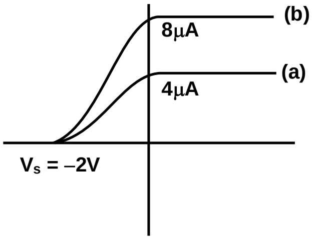
1. Yes, $\lambda = h/p$
2. Particle theory of light.
4. 0.095 A^0.
8. 8.547 MeV

Level - I

1. 8 - α particles , 6 - β particles
2. 16. 43 days
3. 3.4 eV, 6.659 Å
4. (a) 6.2×10^{-7} m (b) 7.78×10^{-20} J (c) 0.49 V
5. Li^{2+} ion
6. 8.4×10^7 m/s
7. 34.98 u
8. 8.97×10^{10} J
9. 0.414 Å
10. -0.055 V
11. n = 4, number of line = 6
12. n = 2; 4 : 1
13. 2.3 eV; 7.68×10^{-34} J-s
14. 1.1×10^{12} electrons
15. 2.17 days

Level - II

1. z = 4, n = 2, E_1 = - 217.6 eV 10.58 eV
2. 2.55 eV, n_i = 4 and n_f = 2; 2.1×10^{-34} J-s ; 0.814 m/s
3. 3.1×10^6 m/s
4. 0.004
5. 39.2 MeV; 5.6 MeV
6. 6.36 MeV, 227.6188 amu
7. 1.3×10^{12} sec.
8. 51.06 %, 2.13 %, 46.8 %
9. (i) 3×10^{-8} m (ii) 2.5×10^{-11} m
10.



11. 6.25×10^{11} per sec., zero, 5 eV.

12. 6.57×10^{-34} J - s

13. (a) 114 Å, (b) Three

14. 20.4 eV

15. 35.67 A^0

Objective:

Level - I

1.	**D**	2.	**A**
3.	**C**	4.	**D**
5.	**D**	6.	**D**
7.	**D**	8.	**D**
9.	**A**	10.	**C**
11.	**D**	12.	**D**
13.	**B**	14.	**C**
15.	**A**		
16.	Zeeman & stark electron		
17.	$\frac{2h}{\lambda}$		
18.	$\left[\frac{E}{C}\right]$	19.	Zero
20.	frequency, intensity	21.	False.
22.	False	23.	True
24.	True.	25.	False

Level - II

1.	**B**	2.	**A**
3.	**D**	4.	**B**
5.	**D**	6.	**C**
7.	**D**	8.	**B**
9.	**B**	10.	**D**
11.	True.	12.	True
13.	False.	14.	False
15.	True	16.	2λ
17.	slower	18.	1.825
19.	0.00124 A^0	20.	only one

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PHYSICS

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PINNACLE

For – JEE (Main/Advanced)

SOLIDS & SEMICONDUCTOR DEVICES

Solids and Semiconductor Devices

Syllabus:

Energy bands in solids (qualitative ideas only), difference between metals, insulators and semiconductors using band theory; Intrinsic and extrinsic semi-conductors, P-n junction, semiconductor diode - characteristics in forward and reverse bias, diode as a rectifier, solar cell, photo-diode, LED, zener diode as a voltage regulator; Junction transistor, transistor action, characteristics of a transistor; Transistor as an amplifier (common emitter configuration) and oscillator; Logic gates (OR, AND, NOT, NAND and NOR); Elementary ideas about I.C.

Introduction

There is a large number of free electrons in a conductor which wander randomly in the whole of the body. Whereas in an insulator all the electrons are tightly bound to same nucleus or the other. Free electrons experience force due to an electric field $\vec{E}$ established inside a conductor and acquire a drift speed. This result in an electric current. The ratio of resulting current density $\vec{J}$ and the electric field $\vec{E}$ is the conductivity σ. Larger the conductivity σ, better is the material as a conductor. The relation between these quantities is $\vec{J} = \sigma\vec{E}$.

$$\rho = \frac{1}{\sigma}$$

Further on the basis of the relative values of electrical conductivity (σ) or resistivity ($\rho = 1/\sigma$) the solids are broadly classified as:

(a) Metals: They posses high conductivity or very low resistivity

$\sigma \sim 10^2 - 10^8 \text{ sm}^{-1}$

$\rho \sim 10^{-2} – 10^{-8}\ \Omega\text{m}$

(b) Insulators: They have low conductivity and high resistivity.

$\sigma \sim 10^{-8} \text{ sm}^{-1}$

$\rho \sim 10^8\ \Omega$ m.

(c) Semicoudnctors: They have conductivity or resistivity intermediate to metals and insulators.

$\sigma \sim 10^{-5} - 10^0 \text{ sm}^{-1}$

$p \sim 10^5 – 10^0\ \Omega\text{m}$

Here please note that relative values of the resistivity are not the only criteria for distinguishing metals, insulators and semiconductors from each other. Our interest in this chapter is in the study of semiconductors which could be

(a) Elemental semiconductor: Si and Ge

(b) Compound seiconductors: examples are

Inorganic: CdS, GaAs, CdSe InP etc.

Organic: anthracene, doped pthalocyanines etc.

Organic Polymers: Polypyrrole, polyaniline, polythiophene etc.

In this chapter, we will restrict ourselves to the study of inorganic semiconductors, particularly elemental semiconductors Si and Ge.

There is another classification scheme on the basis of the source and nature of the charge carriers. Such a scheme divides semiconductors as intrinsic and extrinsic semiconductors.

(a) Intrinsic Semi-Conductors: These are pure semi conductor materials (impurity less than 1 part in 10^{10}). The electrical conduction is by means of mobile electrons and holes (hole as positive charge carriers)

(b) Extrinsic Semi-Conductors: These are obtained by adding or doping the pure semiconductor material with small amounts of certain specific impurities with valency different from that of the atoms of the parent material. Consequently, the number of mobile electrons / holes gets drastically changed. So, the electrical conductivity in such materials is essentially due to the foreign atoms or in other words extrinsic in nature.

Electrical Conduction in Semiconductors

The energy of an electron in an isolated atom is decided by the orbit in which it is revolving but because of the presence of many atoms placed close to each other these electron energies would be different.

In a solid, the atoms are held together closely in a well defined 3-dimensional array or lattice by strong forces as the separation between them is quite small. At such separations, the outer electronic orbits of the neighbouring atoms overlap considerably and hence get significantly distorted. In fact, some valence electrons may be shared by several atoms and it becomes difficult to say which electron goes with which atom. In conducting metals like Al, Cu etc. the outer orbit electrons are shared by all the atoms and the metallic crystal can be visualised as positive inner cores embedded in a regular fashion in a sea of shared electrons as shown in fig. 1

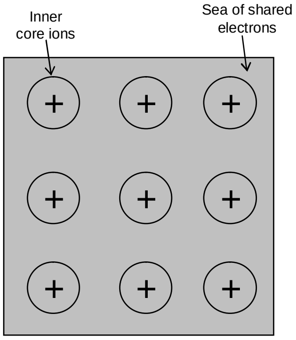


Fig.1
(inner core ions are embedded at fixed positions in the sea of shared electrons)

Such electron are obviously free to travel (free electrons) throughout the material randomly. The number of mobile electrons will be of the order of ~ 10^{29} /m^3 which is very large. A small electric field results in the flow of these free electrons in the direction of the +ve potential and gives low resistivity (or high conductivity to metals)

For insulators, the outermost electrons remain bound to their parent atoms at all temperatures and very high energies are required for these electrons to breakaway. Mobile charges are not available, the conductivity is low (or resistivity is high).

In semiconductors at low temperatures the conductivities of most of the semiconductors is low and comparable to those of insulators but at higher temperature give moderate values of conductivity.

Valance - Bond Description: Semiconductors Ge and Si whose lattice structure is shown in figure -2. Each atom is surrounded by four nearest neighbours as shown in the dotted curve. Si and Ge have four valence electrons and share one of its four valence electrons with each of its four nearest neighbour atoms. These shared electron pairs are referred to as forming a covalent bond or simply a valence bond.

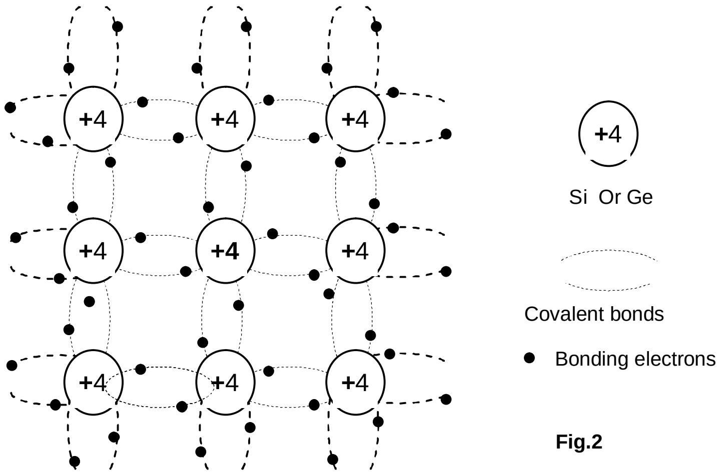


Fig.2

As the temperature increases, some of these electrons may break –away (becoming free electrons and leaves a vacancy with the effective positive electronic charge is called a hole.)

The number of free electrons (n_e) produced as a result of such an ionisation has been found theoretically to be given by

$$n_e = c \exp\left(\frac{-E_g}{2kT}\right)$$

Where C is a constant, k is the Boltzmann constant and T is the absolute temperature. For a given E_g, n_e increases as the temperature increases.

In intrinsic semiconductor, the number of free electrons (n_e) is equal to the number of holes (n_h). In semiconductors apart from electrons, the holes also move. The motion of hole may be looked upon as a transfer of ionisation from one atom to another carried out by the motion of the bound electrons between their covalent bonds . The total current is the sum of the electron current I_e due to the thermally generated conduction electrons and the hole current I_h.

$$I = I_e + I_h$$

Exercise 1: ***In a pu re semiconductor, the number of conduction electron is 6×10^{19} per cubic metre. How many holes are there in a sample of size 4 cm $\times$ 1 cm $\times$ 2 mm.***

Valance bond description of Extrinsic semiconductors

In extrinsic semiconductors a small amount say, ~ 1 ppm of suitable impurity is added to the pure semiconductor. The deliberated addition of a desirable impurity is called doping and the impurity atoms are called dopants. A size of the dopant and the semiconductor atmost should be nearly same so that dopant does not distort the original pure semiconductor lattice and preferably substitute some original semiconductor atom.

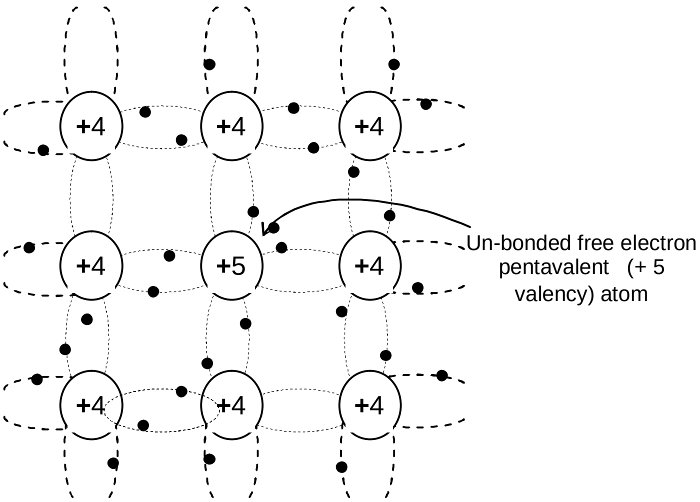
There are two types of dopants used in doping semiconductors like Si or Ge.

(i) Trivalent (Valency 3) like. Indium (In), Boron(B), Aluminium ($A\ell$) etc.

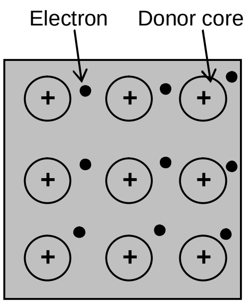
(ii) Pentavalent: (Valency 5): like Arsenic (As), Antimony (Sb), Phosphorus (P) etc.

The pentavalent and trivalent dopants in Si or Ge give two entirely different types of semiconductors.

(a) n–type semiconductor: If we dope Si or Ge (valancy 4) with a pentavalent (valency 5) element as shown in figure -3.



N-type semiconductor with one additional effective positive charge and its associated extra electron



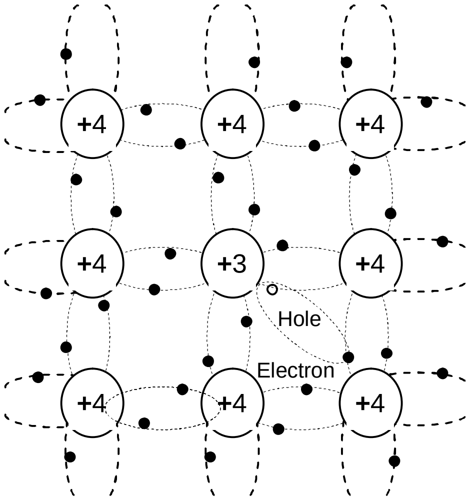
Schematic representation of n-type semiconductor

Fig.3

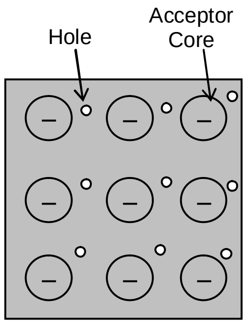
Four of its electrons make bond with the four silicon neighbours while the fifth electron is free to move and only a very small ionisation energy is required to set free this extra electron. Obviously the numbers of conduction electrons are now more than the number of holes. Hence the majority carriers are negatively charged electrons. These materials are known as n-type semiconductors.

(b) p- type semiconductor:

When Si or Ge (tetravalent) is doped with group - III trivalent impurities like Al, B, In etc. as shown in figure - 4. The dopant has one outer electron less than Si or Ge. This atom fails to form bond on one side. Therefore some of the outer bound electrons in the neighbourhood have a tendency to slide into this vacant bond leaving a vacancy or hole at its own site. Therefore these holes are in addition to the thermally generated holes. Thus, for such a material the holes are the majority carriers.



P-type semiconductor with one effective additional negative charge and its associated holes.



Schematic representation of p-type semiconductor

Fig.4

Note that the crystal maintains an overall charge neutrality.

Note:

(i) In a doped semiconductor, the number density of electrons and holes is not equal. But it can be established that

$$n_e n_h = n_i^2$$

where n_e and n_h are the number density of electrons and holes respectively and n_i is number density of intrinsic carriers (i.e. electrons or holes) in a pure semiconductor.

(ii) In n-type semiconductor, the number density of electrons is nearly equal to the number density of donar atoms N_d and is very large as compared to number density of holes. Hence

$$n_e \approx N_d >> n_h$$

(iii) In p-type semiconductor, the number density of holes is nearly equal to the number density of acceptor atoms N_a and is very large as compared to number density of electrons. Hence

$$n_h \approx N_a >> n_e$$

Exercise 2: (i) C, Si and Ge have same lattice structure. Why is C insulator while Si and Ge instrinsic semiconductor ?
(ii)An n-type semiconductor has a large number of electrons but still it is electrically neutral. Explain.

Illustration 1. *Pure Si at 300 K has equal electron (n_e) and hole (n_h) concentration of 1.5×10^{16} m^{-3}. Doping by indium increases n_h to $4.5 \times 10^{22} m^{-3}$. Calculate n_e in the doped silicon.*

Solution: In a doped semiconductor, the number density of electrons and holes is not equal. But it can b established that

$n_e n_h = n_i^2$

n_e = number density of electrons

n_h = number density of holes

n_i = number density of intrinsic carries (i.e. electron or holes in a pure semiconductor)

$$n_e = \frac{n_i^2}{n_h} = \frac{(1.5\times10^{16})^2}{(4.5\times10^{22})} = 5 \times 10^9 \text{ m}^{-3}$$

Energy band description of solids - Metals, Insulators and Semiconductors:

Due to strong overlapping of the orbitals. It is difficult to ascribe any one electron (or hole) belonging to any particular atom. Electron wave motion is in fact, quantum mechanical in nature is more closely related with the energy and momentum concepts rather than the space - localisation and velocity concepts so far used in the bond picture. This alternative approach is termed as energy - band description of solids.

Electron energies in solids in view of strong overlap of different atomic orbitals, will be more like an energy band instead of discrete energy levels of single isolated atoms.

The overlap (or interaction) will be more felt by the electrons in the outermost orbit while the inner orbit or core electron energies may remain unaffected.

For Si, the outermost orbit is the third orbit (n = 3), while for Ge it is the fourth orbit (n = 4). The number of electrons in the outermost orbit is 4 (2s and 2p electrons). Hence, the total number of outer electrons in the crystal is 4N. The maximum possible number of outer electrons in the orbit is 8 (2s + 6p electrons). So out of the 4N electrons, 2N electrons are in the 2N S – states (orbital quantum number ℓ = 0) and 2N electrons are in the available 6 N p - states obviously some p - electron states are empty as shown in the extreme right of figure. This is the case of well separated or isolates atoms

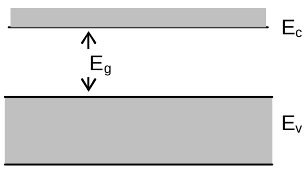


Fig.5

The energies of these electrons in the outermost orbit may change (both increase and decrease) due to the interaction between the electrons of different atoms. The 6N states for ℓ = 1, which originally had identical energies in the isolated atoms, spread out and form an energy band (region B in figure 5). Similarly, the 2N states for ℓ = 0 having identical energies in the isolated atoms split into a second band, separated from the first one by an energy gap.

At still smaller spacing, however, there comes a region in which the bands merge with each other. No energy gap exists where the upper and lower energy states get mixed.

Finally, if the distance between the atoms further decreases, the energy bands again split apart and are separated by an energy gap E_g as shown in figures. The total number of available energy states 8 N has been re-apportioned between the two bands (4N states each in the lower and upper energy bands).

The lower band is called the valence band which is completely filled. The upper band is called the conduction band which is completely empty.

At equilibrium spacing between the lowest conduction band energy is E_c and highest valence band energy is E_v. Above E_c or below E_v there are a large number of closely spaced energy levels as show in figure 6. Pauli's exclusion principle says that maximum number of two electrons can be in each energy level.

This gap between the top of the valence band and bottom of the conduction band is called the energy band gap. It may be large small or zero depending upon the material.

In case of conductors, E_g > 3eV, insulators, because electrons cannot be easily excited from the valence band to the conduction band by any external stimuli.

In case of E_g = 0, where the conduction and valence bands, are overlapping. This situation makes a large number of electrons available for electrical conduction and the resistance of such materials is low or the conductivity is high.

E_g < 3 eV is the case of finite but smaller band gap exists. Because of the small band gap, some electrons can be thermally excited to the conduction band and can move in the conduction band. This is the case of semiconductors. All the three cases are shown in figure -6.

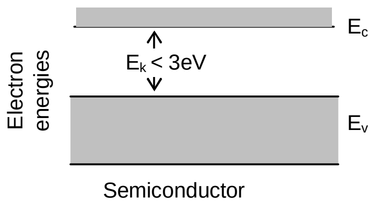


Fig.6

An intrinsic semiconductor will behave like an insulator at T = 0 K. At higher temperature thermally excited electrons, partially occupy some states in the conduction band. Therefore, the energy band diagram of an intrinsic semiconductor will be as shown in figure - 7.

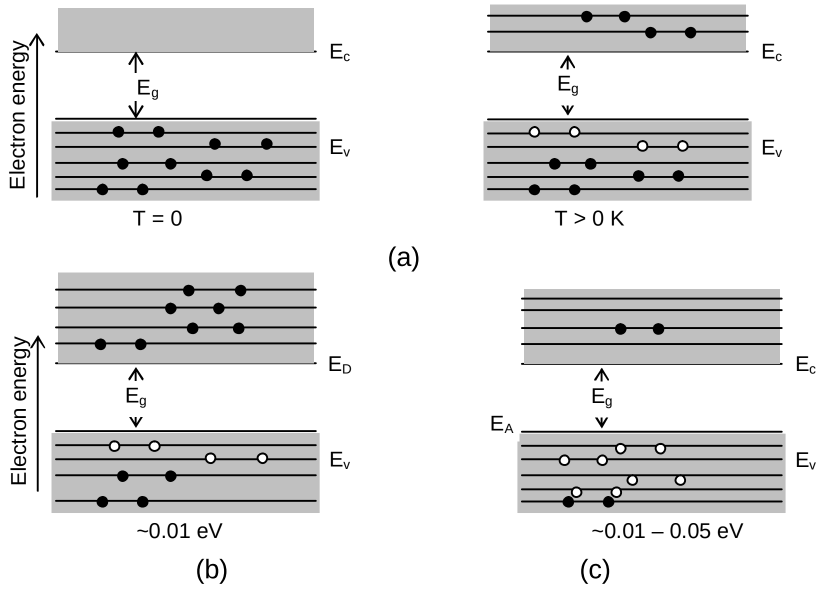


Fig.7

In the case of extrinsic semiconductor, additional energy states, apart from E_C and E_v due to donor impurities (E_D) and acceptor impurities (E_A) also exist. We have already discussed that in the n – type semiconductor very small energy (~ 0.1 eV) is required for the electrons to be released from the donor impurity.

Similarly, for p - type semiconductor an acceptor energy level E_A is obtained as shown in figure. The position of E_A is very near to the top of the valence band because an electron added to the acceptor impurity to complete its bonding within the semiconductor structure, comes easily from the valence band electrons of some other semiconductor atoms in the lattice. The above description is grossly approximate and hypothetical but it helps in understanding the difference between metals, insulators and semiconductors.

PN Junction

A p-n junction is basic building block of almost all semiconductor devices, a single p-n junction acts as a rectifying diode, two such junctions (viz. p – n followed by n – p) makes p-n-p transitor. Therefore detailed understanding of the p-n junction is required before going into the details of various other devices.

When a semi-conducting material such as Germanium or silicon is doped with impurity in such a way, that one side has a large number of acceptor impurities and the other side has a large number of donar impurities. We obtained a p-n junction. By diffusing a donar impurity to a pure semiconductor so that the entire sample becomes n-type. The acceptor impurity may then be diffused in higher concentration from one side to make that side p-type. Typical p-n junction structure is shown below in figure.

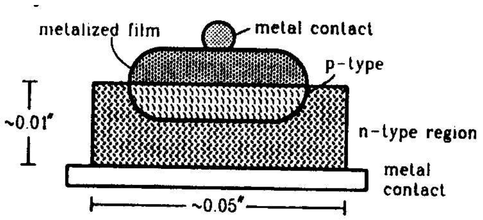


Fig.8

Diffusion current: Because of the concentration difference, holes try to diffuse from p-side to the n-side. However, the electric field at the junction exerts a force on the holes towards the left as they come to the depletion layer. Only these holes with a high kinetic energy are able to cross the junction. Similarly those electrons which start towards the left with high kinetic energy are able to cross the junction. This diffusion results in an electric current from the p-side to the n-side known as diffusion current.

Drift Current: Occasionally a covalent bond is broken (because of thermal collisions) and the electron jumps to the conduction band. An electron-hole pair is thus created. Also occasionally a conduction electron fills up a vacant bond so that an electron – hole pair is destroyed. These processes continue in every part of the material. However, if an electron hole pair is created in the deplection region, there is almost no chance of recombination of a hole with an electron in the deplection region because the electron is quickly pushed by the electric field towards the n-side and the hole towards the p-side. As electron – hole pairs are continuously created in the depletion region, there is a regular flow of electrons towards the n-side and of holes towards the p-side. This makes a current from the n-side to the p-side. This current is called the drift current.

Description of p- n junction without external applied voltage of Bias:

Suppose a p- n junction has just been formed. There are more electrons on the n –side while the number of holes on the p-side is larger. Because of concentration gradient holes from the p-side will go towards the n-side and electrons from n – side will diffuse towards the p-side. Diffusion of holes towards the right and diffusion of electrons towards the left make the right half positively charged and the left half negatively charged. This creates an electric field near the junction from the right to the left and will opposes any further diffusion of the majority carriers from either sides.fig. 8(b) is the appearance of a junction potential. This potential acts as a barriers and opposes any further diffusion of the majority carriers from either sides. Hence is known as Barrier Potential (V_B)

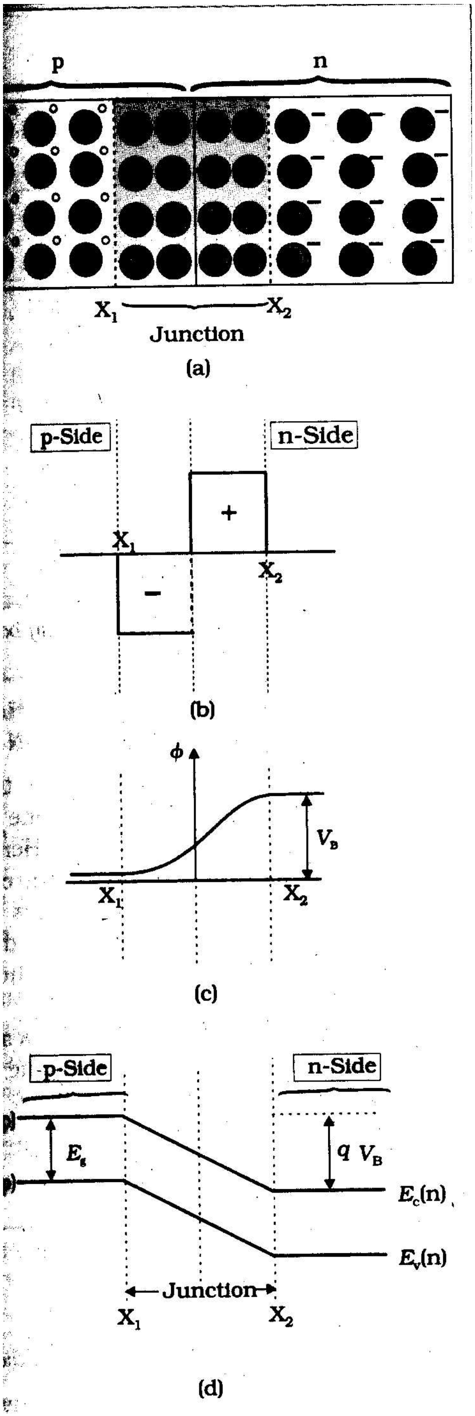


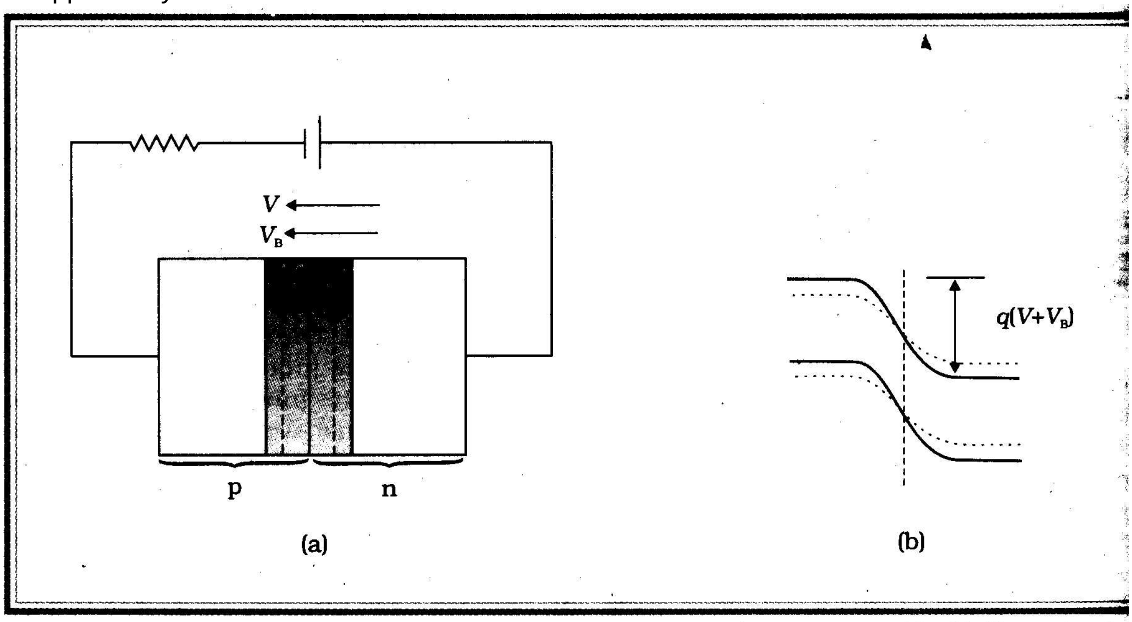
Fig. 9

Behaviour of p – n junction with an external applied voltage or Bias:

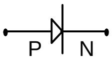
If the positive terminals of the battery is connected to the p-side and the negative terminal to the n-side, junction is forward biased. Due to the forward bias connection, the potential of the p-side is raised and hence the height of the potential barrier decreases. The width of the depletion region is also reduced in forward bias.

Diffusion current increases by connecting a battery in forward bias. The drift current remains almost unchanged because the rate of formation of new electron hole pairs is fairly independent of the electric field unless the field is too large.

Reverse bias: The applied voltage V on the n-side is positive and is negative on the p-side, the junction is said to be reverse biased. In this case, the potential barrier becomes higher as the battery further raised the potential of the n-side. The width of the depletion region is increased Diffusion becomes more difficult and hence the diffusion current decreases. The drift current is not appreciably affected and hence it exceeds the diffusion current.

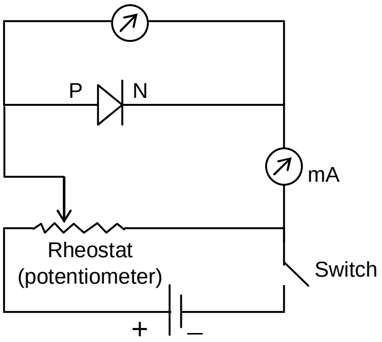


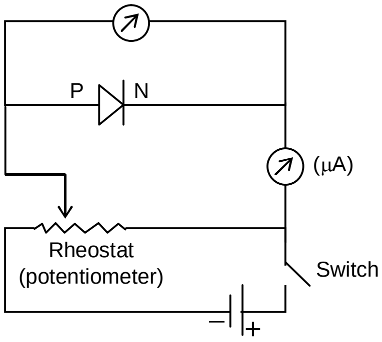
Voltage- current (V – I) characteristics of a p-n junction diode

In the p-n junction discussed above, there are two electrode connections one on the p-side and another on the n-side. Hence it is generally called diode (di + ode; di – means two and ode come from electrode). The diode is symbolised as 

. The arrow points from the p-side to the n-side. This junction offers a little resistance if we try to pass an electric current from the p-side to the n-side and offers a large resistacne if the current is passed from the n-side to the p-side.

The circuit arrangements for studying the V-I characteristic of a diode is shown in figure. The voltage applied to the diode can be changed through a potentiometer.





and for different values of voltages, the value of the current is noted.

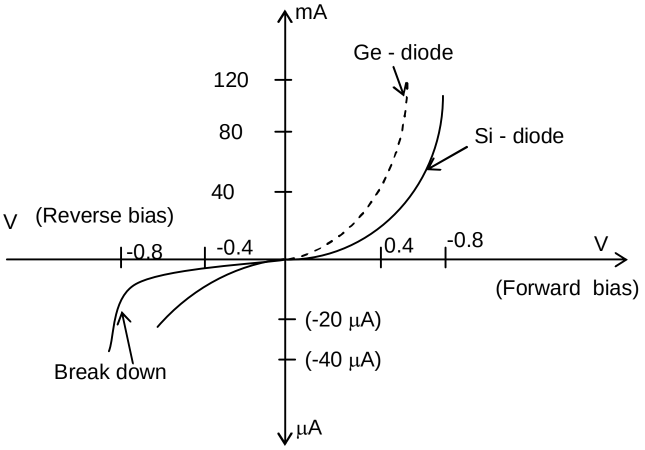


Figure shows a qualitative plot of current versus potential difference for a p - n junction. This is known as an I -V characteristic of the p- n junction. Note that the scales are different for the positive and negative current.

In forward biasing, the current first increases very slowly almost negligible till the voltage across the diode crosses a certain value. After this diode current increases significantly. This voltage is called the threshold voltage or cut in voltage (~ 0.2 V for germanium diode and ~ 0.7 V for silicon diode)

For the diode in reverse bias the current is very small (~ μA) and almost remains constant with bias. It is called reverse saturation current. However for special cases, at very high reverse bias (break down voltage) the current suddenly increases. The general purpose diodes are not used beyond the reverse saturation current region.
Primarily p – n diode restricts the flow of current only in one direction (forward bias)

Note:
The relation for the current I in the junction diode is given by

$$I = I_0\left[\exp\left(\frac{eV}{kT}\right)^{-1}\right]$$

where k, Boltzman constant, $k = 1.38 \times 10^{-23}$ J mole^{-1} k^{-1}

To reverse saturation current. In forward bias, v is positive and low then forward current.

$$I_f = I_0\left(e^{\frac{eV}{kT}} - 1\right)$$

In reverse bias, v is negative, so
$e^{ev/kT} << 1$, then reverse current $I_r = I_0$

Illustration 2. *In a p-n junction diode the reverse saturation current is 10^{-5} A at 27^0C. Find the forward current for a voltage of 0.2V.*
Given exp (7.62) = 2038.6, $k = 1.4 \times 10^{-23}$ jk^{-1}.

Solution: The current in the junction diode in given by $I = I_0\,(e^{ev/kt} - 1)$
where I_0 = reverse saturation current = 10^{-5} A

$$I = 10^{-5}\exp\left[\frac{1.6\times10^{-19}\times0.2}{1.4\times10^{-23}\times300}\right]$$

$= 10^{-5}\,(e^{7.62} - 1)$

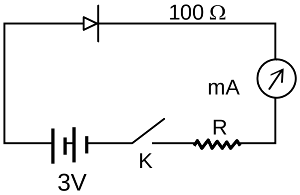
$= 10^{-5}\,(2038.6 - 1)$

Dynamic Resistance or A.C. Resistance of the Junction Diode:
It is defined as the ratio of a small change in voltage Δv applied across the junction to a small change in junction current ΔI .i.e.

$$R_d = \frac{\Delta v}{\Delta I}$$

Illustration 3. *Draw the circuit to (a) forward biased diode. (The supply voltage is 3V, 100 mA battery) (b) If the diode is made of silicon and the knee voltage is 0.7 V, and a current of 20 mA, passes through the diode, find the wattage of the resistor and the diode.*

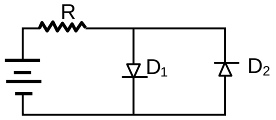
Solution: (a) Circuit for forward biased diode is shown in figure.



(b) Here, emf of battery = 3V
knee voltage, V_k = 0.7 V
current in circuit,
I = 20 mA = 20×10^{-3} A
$\therefore$ Voltage drop across
R = 3 – 0.7 = 2.3 V
Wattage of resistance R = voltage drop $\times$ current
= $2.3 \times (20 \times 10^{-3}) = 0.046$ W
Wattage of diode = voltage drop $\times$ current
= $0.7 \times 20 \times 10^{-3} = 0.014$ W.

Exercise 3:

(i) ***Identify forward biased or reverse biased diodes in circuit 1.***



(ii) ***Identify which of the diodes are forward biased or reverse biased during positive and negative half cycle.***

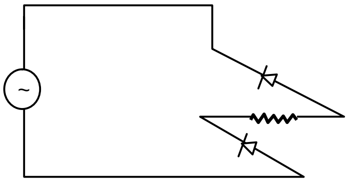
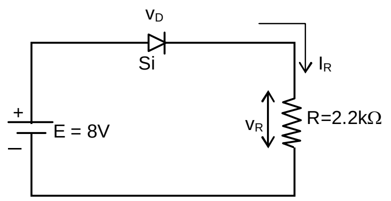


Illustration 4. *In the circuit shown determine v_D, v_R and I_D when (a) diode is forward biased (b) diode is reverse biased. (Given forward voltage drop of si diode is 0.7V)*



Solution: $v_R = E - v_D = 8 - 0.7\text{ V} = 7.3\text{ V}$

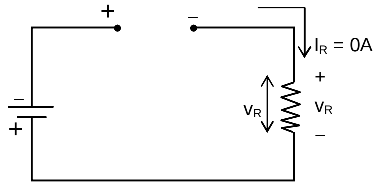
$I_D = I_k = \frac{v_R}{R} = \frac{7.3V}{2.2k\Omega} = 3.23\text{ mA}$

Reversed biased: In this case, diode circuit is open circuit

$-E - v_D - v_R = 0$

$v_R = I_R \times R = 0\text{ V}$

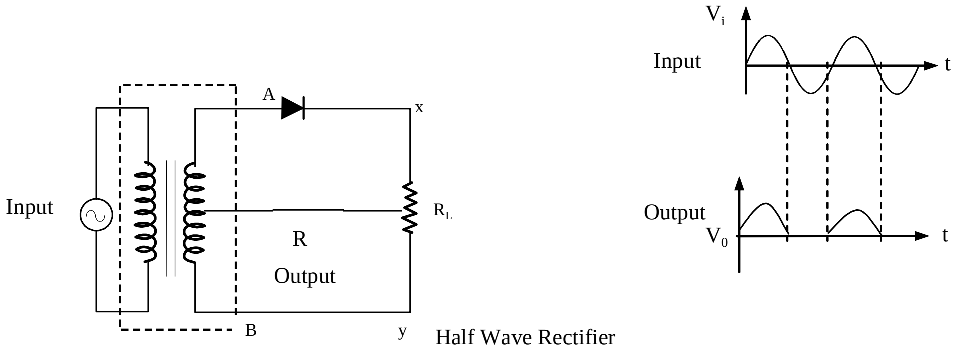
$v_D = -8\text{ V}$



p- n diode as a rectifier:

The forward bias resistance is low as compared to the reverse bias resistance this diode property has been used to restricts the voltage variation of ac to one direction only, a phenomenon known as rectification. A simple rectifier circuit called half wave rectifier, using only one diode is shown in figure.

The secondary of the transformer supplies the desired ac voltage across A and B. The diode is forward biased when the voltage at A is positive and it conducts. Diode is reversed biased, when A is negative and it does not conduct. Therefore, in the positive half cycle of ac there is a current through the load resistor R_L and we get an output voltage as shown in figure.

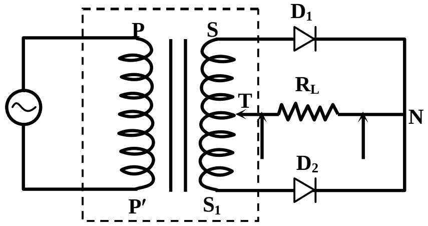


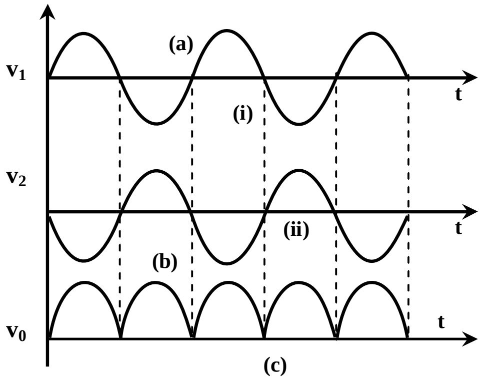
Half Wave Rectifier

The output voltage, though still varying, is restricted to only one direction and is said to be rectified. We get a voltage in the output for only one half cycle. Therefore such a circuit is known as Half Wave Rectifier.

Using two diodes as shown in the circuit arrangement gives rectified voltage corresponding to the positive as well as negative half of the ac cycle. Therefore it is known as Full Wave Rectifier. The circuit uses centre tap transformer and two diodes. The secondary of the transformer is would in to two equal parts as shown in the figure.

Note that voltage at any instant at ‘A’ (input of diode D_1) and B (input of diode D_2) with respect to the centre tap are out of phase with each other. This means when the diode D_1 gets forward biased and conducts (while D_2 is not conducting). Hence during positive half cycle we get an output current (and a consequent output voltage across the load resistor R_L) as shown in figure. At another instant, when the voltage at A becomes negative then the voltage at B would be +ve.

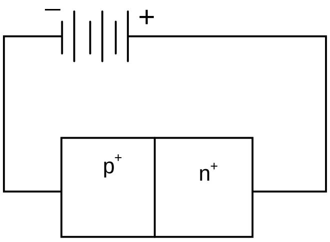




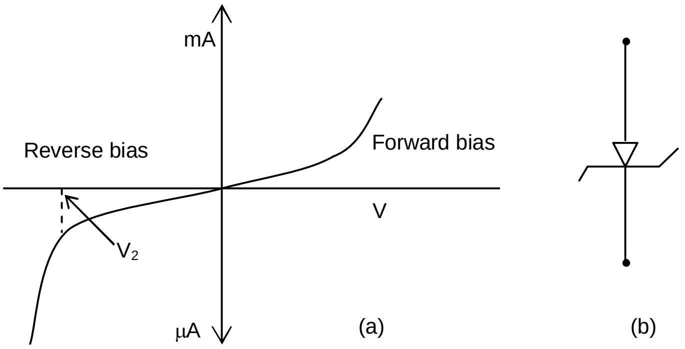
Now diode D_2 conducts and the diode D_1 does not conduct giving an output current and output voltage (across R_L) during the negative half cycle of the input ac. Thus, we get output voltage during the +ve as well as the –ve half of the cycle (during the full wave). This circuit is known as FULL WAVE RECTIFIER. Obviously this is more efficient circuit for getting rectified voltage or current.

Exercise 4: ***In half wave rectification if the input frequency is 50 Hz, what is the output frequency. What is the output frequency of a full wave rectifier for the same frequency?***

Zener Diode: If the reverse –bias voltage across a p-n junction diode is increased. The holes in the n-side and the conduction electrons in the p-side are accelerated due to the reverse-bias voltage. If these minority carrriers acquire sufficient kinetic energy from the electric field and collide with a valence electron the bond will be broken and the valence electron will be taken to the conduction band. Thus a hole electron pair will be created. Breakdown occurring in this manner is called avalanche breakdown.



Breakdown may also be produced by direct breaking of a valance bonds due to high electric field. When breakdown occurs in this manner it is called zener breakdown. At this voltage the rate creation of hole – electron pairs is increased leading to the increased current.



(a) V – I characteristics of a Zener Diode.
(b) The symbolic representation of a Zener diode

Once the breakdown occurs, the potential difference occurs the diode does not increases even if the applied battery potential is increased. This characteristic of diode is used to obtain constant voltage output. In other words, for widely different Zener currents, the voltage across the Zener diode remains constant.

Figure shows a typical circuit which gives constant voltage V_0 across the load resistance R_L. Even if there is a small change in the input voltage V_i, the current through R_L remains almost the same. The voltage across zener diode remais essentially the same for the change in current through the diode.

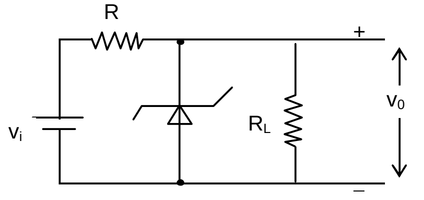
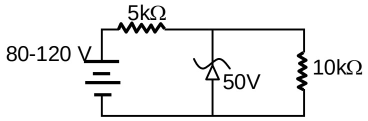


Illustration 5. *For the circuit shown in figure, Find the maximum and minimum values of Zener diode current.*



Solution: $v_z = 50$ V

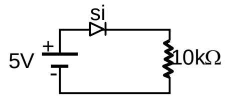
Load current $I_L = \frac{V_{out}}{R_L} = \frac{50}{10k\Omega}$ = 5 mA

The Zener current will be maximum when input voltage is maximum and corresponding current through series resistance (5 kΩ)

$I_{smax} = \frac{V_{smax} - V_{out}}{R_s} = \frac{120-50}{5} = 14mA$

corresponding Zener current I_{zmax} = 14 – 5 = 9 mA

Illustration 6. *What is the current in the following circuit given that barrier potential for silicon diode is 0.7 V.*



Solution: This barrier potential acts in opposite direction to the applied voltage

$\therefore$ current through the circuit $I = \frac{V - V_0}{R} = \frac{5-0.7}{10} = 0.43$

Photonic p-n junction Devices

We can have semiconductor electronic devices, in which the light photons have also a role to play in the overall performance of the device. Such devices are called photonic or opto – electronic devices. Further they can be classified as:

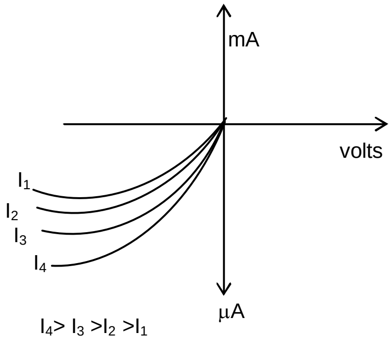
(i) Photo -detectors for detecting optical signals (e.g. photodiodes and photo-conducting cell)

(ii) Photo voltaic devices for converting optical radiation into electricity (e.g. solar cells)

(iii) Devices for converting electrical energy into light (e.g., light emitting diodes and diode lasers)

(a) Photodiodes: The general principle of all semiconductor based photo-detectors is the electron excitation from the valence band to the conduction band by photons. Suppose an optical photon of frequency ν. Corresponding energy $h\nu$ is incident on a semiconductor and its energy is greater than the band gap of the semiconductor (i.e $h\nu > E_g$). This photon will excite an electron from the valence band to the conduction band. Thus an electron hole pair is generated. These are additional charge carriers termed as photogenerated charge carriers which obviously increase the conductivity of the semiconductor. Increase in conductivity of the semiconductors is proportional to incident intensity of light.. Therefore, by measuring the change in the conductance of the semiconductor, one can measure the intensity of the optical signal. Such photoelectrons are known as photoconductive cells. However, more commonly used photodetecting devices are photodiodes.

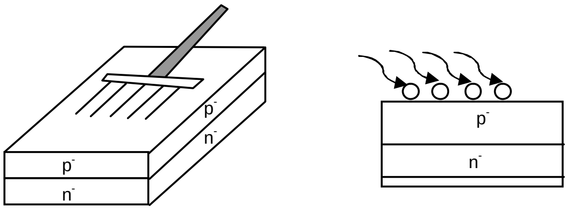
The diodes are generally reverse biased when used as photodiode. Suppose a p-n diode is illuminated with light photons having energy $h\nu > E_g$ and intensities I_1, I_2, I_3, etc. There would be a change in the reverse saturation current as shown in figure. Hence, a measurement of the change in the reverse saturation current on illumination can give the values of the light intensity.



$I_4 > I_3 > I_2 > I_1$

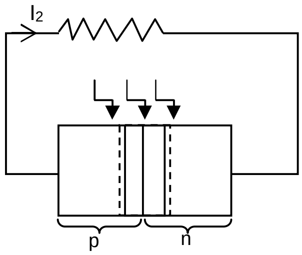
(b) Solar cell: Solar cell is based on similar principle as junction photodiode (generation of voltage due to the bombardment of optical photons).

Let us take a simple p-n junction solar cell shown in figure. A n-type semiconductor substrate (backed with a current collecting metal electrode) is taken, over which a thin p-layer is grown (e.g. by diffusion of a suitable acceptor impurity or by vapour deposition). On top of the p-layer, metal finger electrodes are prepared so that there is enough space between the fingers for the light to reach p-layer (and the underlying p-n junction) to be able to produce photo-generated holes and electrons.



When light (with $h\nu > E_g$) falls at the junction, electron-hole pairs are generated which move in opposite directions due to the junction field. They would be collected at the two sides of the junction giving rise to a photo-voltage.

When external load is connected as shown in figure, a photo-current I_2 flows. The materials most commonly used for solar cells are silicon (Si) and gallium arsenide (Ga As).



(c) Light emitting Diode (LED)

When a conduction electron makes a transition to the valence band to fill up a hole in a p-n junction, the extra energy may be emitted as a photon. If the wavelengths of this photon is in the visible range(380 nm – 780 nm), one can see the emitted light. Such a p-n junction is known as light emitting diode abbreviated as LED.

The semiconductor used in LED is chosen according to the required wavelength of emitted radiation. Visible LED's are available for red, green and orange.
LED's have the following advantages over conventional incandescen lamps.
(i) Low operational voltage and less power
(ii) Fast action and no warm up time required.
(iii) Long life and ruggedness

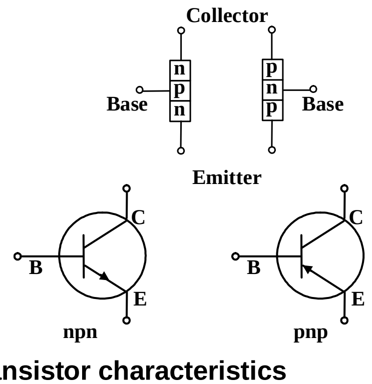
Transistor
It is a three layer semiconductor device consisting of either two N- and one P-type semiconductors or two P-type and one N- type. In former case it is called N-P-N transistor (P-type semiconductor is sand wiched between two N-type semiconductors). In the latter case it is called P-N-P transistor (N-type semiconductor is sandwiched between two P-type semiconductors). Three distinct regions thus formed are called emitter, base and collector.

Transfer of a current from a low resistance to a high resistance circuit produces the basic amplifying action of transistor. A combination of the words " transfer and resistance" gave the term "transistor".

Transfer + resistor = Transistor

In actual design, the middle layer is very thin ($\approx$ 1 μm) as compared to the widths of the two layers at the sides. The middle layer is called the base and is very lightly doped with impurity. One of the outer layers is heavily doped and is called emitter. This supplies a large number of majority carriers for current flow through the transistor. The other outer layer is moderately doped and is called collector. Which collects a major portion of the majority carriers supplied by the emitter. Emitter is of moderate size and collector is larger in size as compared to the emitter.

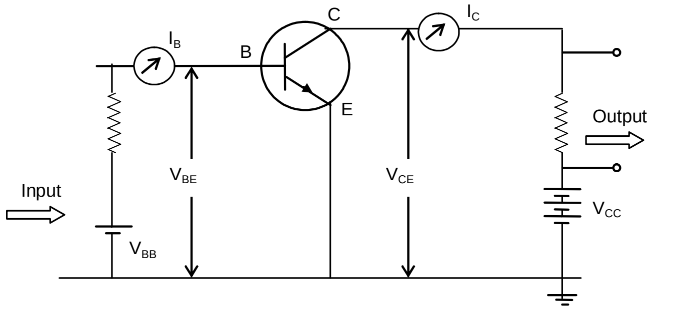
Terminals come out from the emitter, the base and the collector for external connections. Thus, a transistor is a three- terminal device. Figure shows the symbols used for a junction transistor.



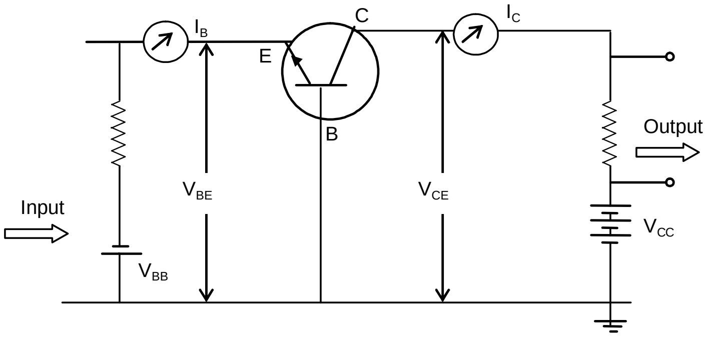
Basic transistor circuit configurations and transistor characteristics
In any electronic circuit or device there has to be two terminals for input and two terminals for out put, In a transistor, only three terminals are available viz. Emitter (E), Base (B) and collector (C). Therefore in a circuit the input/output connections have to be such that one of these (E, B or C) is common to both the input and the output. Therefore, the transistor can be connected in either of the three following configurations.
(a) Common Emitter (CE) (b) Common base (CB) (c) Common collector (CC)

In normal operation of a transistor, the emitter – base junction is always forward – biased, whereas the collector - base junction is reverse –biased. The arrow on the emitter line shows the direction of the current through the emitter –base junction. In an n-p-n transistor, there are a large number of conduction electrons in the emitter and a large number of holes in the base. If the junction is forward –biased, the electrons will diffuse from the emitter to the base and holes will diffuse from the base to the emitter. The direction of electric current at this junction is therefore, from the base to the emitter. This is indicated by the outward arrow on the emitter line in figure. Similarly, for a p-n-p transistor the current is from the emitter to the base when this junction is forward - biased which is indicated by the inward arrow in figure.

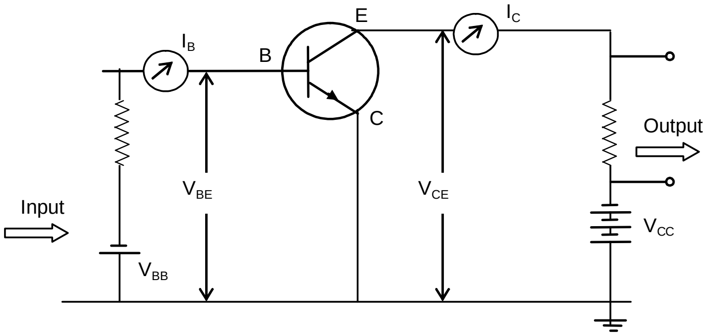
Biasing: To operate the transistor suitable potential differences should be applied across the two junctions is called biasing of the transistor.
Common Emitter, Common Base and Common Collector configuration are shown in figure, (a), (b) and (c) respectively.



(a)



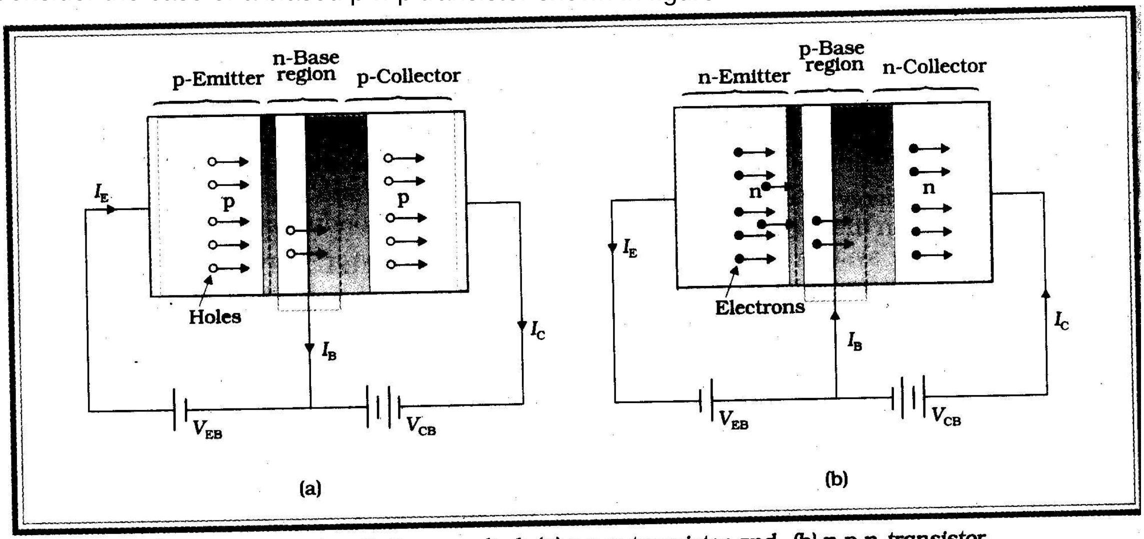
(b)



(c)

In common – emitter mode, the emitter is kept at zero potential and the other two terminals are given appropriate potentials. Similarly in common- base mode, the base is kept at zero potential, whereas in common – collector mode, the collector is kept at zero potential but we shall restrict ourselves only to CE configuration.

Consider the case of a biased p-n-p transistor shown in figure



Bias Voltage applied: (a) p-n-p *transistor and (b)* n-p-n *transistor.*

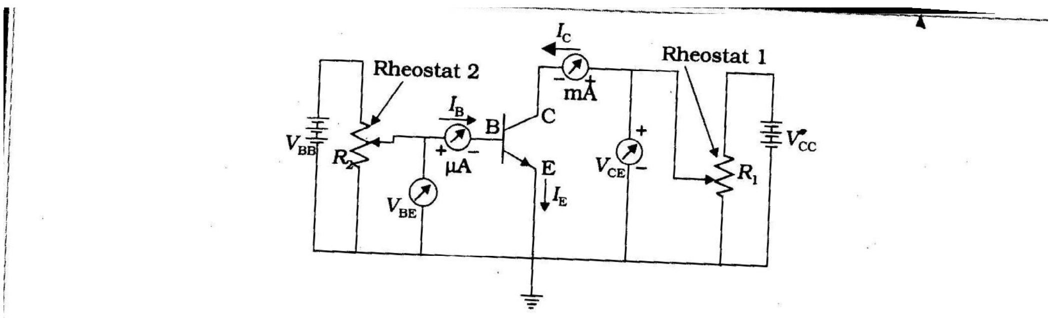
As the emitter base junction is forward biased, a large number of holes (majority carriers) from p-type emitter block flow towards the base. These holes have a tendency to combine with the electrons in the n- region of the base. Only a few holes (less than 5 %) are able to combine with the electrons in the base-region (giving only a very small base current , I_B) because the base is lightly doped and very thin. In the collector region, these holes see the favourable negative potential at the collector and hence they easily reach the collector terminal to constitute the collector current I_C. From Kirchhoff's current law.

$I_E = I_C + I_B \qquad (I_C >> I_b)$

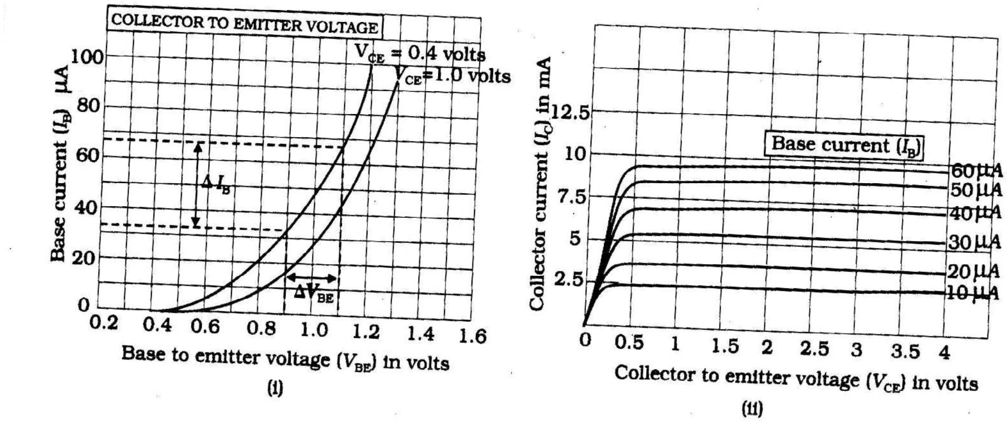
Similar description can be made for a biased n-p-n transistor as shown in figure. Here, the electrons are the majority carriers supplied by the n-type emitter region.

Any variation in the voltages on the input and output sides results in a change in the input and output currents. The variation of current on the input side with input voltage (I_E versus V_{BE}) is known as input characteristics while the variation in the output current with output voltage (I_C versus V_{CE}) is known as output characteristics.

A simple circuit for drawing the input and output characteristics of an n-p-n transistor is shown in figure.



(a)



(b)

The voltage applied to the base – emitter junction i.e. in the input section is V_{BE}. When the input current I_B is plotted against the voltage V_{BE} between the base and the emitter. The input characteristics shown in figure are like those of a forward – biased p-n junction. If the biasing voltage is small as compared to the height of the potential barrier at the junction, the current I_B is very small. Once the voltage is more than the barrier height, the current rapidly increases. However, since most of the electrons diffused across the junction go to the collector, the net base current is very small even at large values of V_{BE}.

To draw the output characteristics, we change the value of V_{CE} and note the values of I_C. For small values of the collector voltage, the collector – base junction is reverse biased because the base is at a more positive potential. The current I_C is then small. As the electrons one forced from the emitter side, the current I_C is still quite large as compared to a single reverse biased p-n junction. As the voltage V_C is increased, the current rapidly increases and becomes roughly constant once the junction is forward – biased. For higher base currents, the collector current is also high and increases more rapidly even in forward bias.

Input output characteristics are used to calculate the important transistor parameters as follows.

(i) Input Resistance (r_i) This is defined as the ratio for change in base-emitter voltage (ΔV_{BE}) to the resulting change in base current (ΔI_B) at constant collector – emitter voltage (V_{CE}).

$$r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$$

The value of r_i is of the order of a few hundred ohms.

(ii) Output resistance (r_0): This is defined as the ratio of change in collector –emitter voltage (ΔV_{CE}) to the change in collector current (ΔI_C) at constant base current I_B.

$$r_0 = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B = \text{constant}}$$

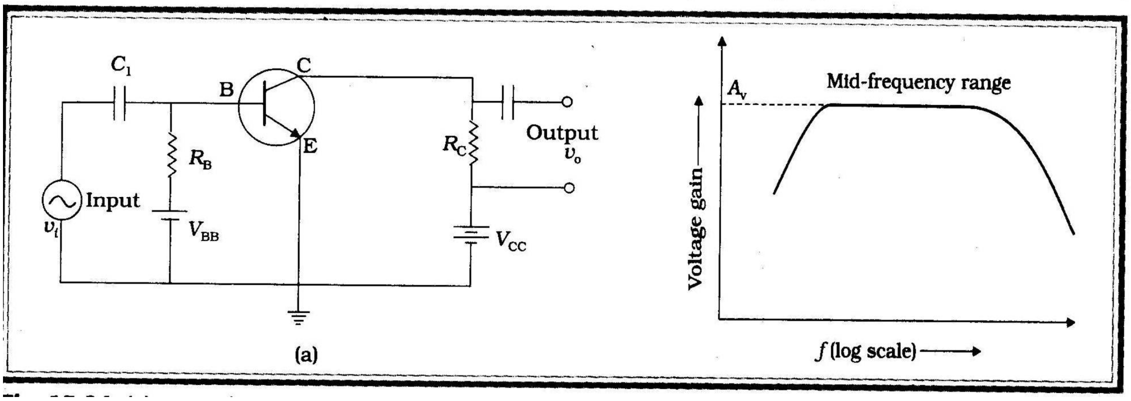
The values of r_0 are very high (of the order of 50 to 100 kΩ)

(iii) Current amplification factor (β): This is defined as the ratio of the change in collector current (output current) to the change in base current.

$$\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CB}}$$

This is also known as current gain. We normally work in the region in which I_C is almost independent of V_{CE} (or varies very slowly with V_{CE}). This is called the active region.

Transistor as an Amplifier (CE – configuration) : Figure shows an amplifier circuit using an n-p-n transistor in common – emitter mode. The battery v_{BB} provides the biasing voltage V_{BE} for the base – emitter junction. A potential difference V_{CC} is maintained between the collector and the emitter by the battery V_{CC}. The input signal voltage V_i is connected to the input side through a capacitance so that the biasing dc voltage V_{BB} is blocked from going towards the source of signal. Similarly the dc voltage V_{CC} in the output is blocked with the help of capacitor C_2.



Without signal, a dc current I_B flows through R_B white I_C is the dc collector current. If V_i is applied to the input base – emitter side, it will change I_B to $I_B + i_B$ where i_B is due to the signal voltage V_i. The collector current would also change to $I_C + i_C$ where i_C is the collector current due to the input signal. The effective input signal (due to v_i or i_B) to the transistor is the voltage across R_B (input resistance)

$$V_i = i_B R_B$$

The output signal voltage (v_0) across R_c would be $v_0 = i_C R_C$.
Therefore, the voltage gain (A_v) of the amplifier (output signal divided by input signal voltage) is given by $A_v = \left(\frac{v_0}{v_i}\right) = \left(\frac{i_c R_c}{i_B R_B}\right)$

i_c and i_B are respective collector and base current due to the signal input voltage. Further we know $\beta = \frac{i_c}{i_B}$

or, $\quad A_v = \beta(R_C/R_B)$

Thus, the voltage gain is related to the current amplification factor of the transistor, externally connected collector and base resistances. The above expression is without considering the effect of transistor parameters like base – emitter resistance, base collector resistance, junction capacitances etc.

Power gain = voltage gain $\times$ current gain
$= \beta^2 \dfrac{R_C}{R_B}$

Illustration 7. *An n-p-n transistor in a common emitter mode is used as a simple voltage amplifier with a collector current of 4 mA. The terminal of an 8 V battery is connected to the collector through a resistance R_B. The collector emitter voltage V_{ce} = 4V, base emitter voltage V_{be} = 0.6 V and base current amplification factor β_{dc} = 100. Calculate the values of R_L and R_B.*

Solution: Potential difference across R_L
$= 8V - V_{ce} = 8 - 4 = 4V$
Now $I_c R_L = 4V$
or, $R_L = \dfrac{4}{4\times10^{-3}} = 10^3\,\Omega = 1k\Omega$
Here, $I_b = I_c/\beta = \dfrac{4\times10^{-3}}{100} = 4 \times 10^{-5}$ A
Potential difference across R_B is
$= I_b R_B = 8 - V_{be} = 8 - 0.6 = 7.4$ V
or, $R_B = \dfrac{7.4}{I_b} = \dfrac{7.4}{4\times10^{-5}}$
$= 1.85 \times 10^5\ \Omega = 185\ k\Omega$.

Exercise 5:
(i) ***A common emitter transistor amplifier has a current gains of 50. If the load resistance is 4 kΩ and the input resistance is 500 Ω. Find the voltage gain of the amplifier.***

(ii) ***In an npn transistor circuit, the collector current is 10 mA. If only 90 % of emitter electron reach the collector, what is the emitter current ?***

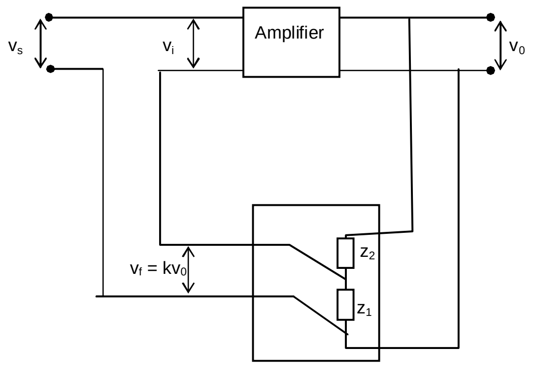
Transfer conductance: Transfer conductance g_m is defined as

$$g_m = \frac{\Delta I_C}{\Delta V_{BE}}$$

Which indicates how output current will change with change in V_{BE}. Further to have a large amplification, a small change in V_{BE} should result in a large change in the collector current I_C.

Transistor as an oscillator: In an amplifier, we have seen that input signal (ac) appears as an amplified output signal. and an external input was necessary for an amplifier. In an oscillator, we get ac output without any external input signal for same a portion of the output voltage of an amplifier is feedback to its input terminal in phase, this process is termed as positive feedback as shown in figure. The voltage across the z_1 is feedback in the input of oscillator.

We consider the circuit in which the feedback is accomplished by inductive coupling from one coil winding (T_1) to another coil winding (T_2). Coils T_2 and T_1 are wound on the same core therefore inductively coupled through their mutual inductance. For simplicity reason detailed biasing circuits have been emitted.

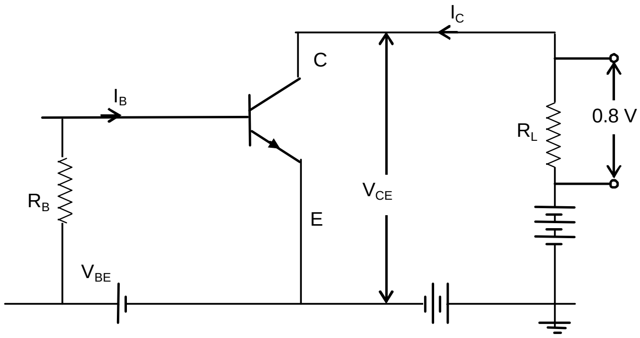


The feedback can be achieved by inductive coupling (through mutual inductance or LC or RC networks).

To understand how oscillations are built. Suppose switch S_1 is put on to apply proper bias for the first time. Current flows through the coil T_2 and increases from X to Y as shown in figure. Current flows in the emitter circuit because of inductive coupling between coil T_2 and coil T_1 (positive feed back) This current in T_1 (emitter current) also increases from X′ to Y′. The current in T_2 (collector circuit acquires the value y. When the transistor becomes saturated and there is no further change in collector current. Now there will be no further feedback (field becomes static) from T_2 to T_1. The emitter current begins to fall and decreases from Y downwards z. A decrease of collector current causes the magnetic field to decay around the coil T_2. Thus T_1 is now seeing a decaying field in T_2 (opposite from what it saw at the initial start operation). Thus causes a further decrease in the emitter current fill it reaches z when the transistor is cut off. Now both I_E and I_C cease to flow and the transistor has reverted back to its original state. The whole process now repeats itself. That is the transistor is driven to saturation to cut – off and back is determined by the constants of the tank circuit or tuned circuit i.e. inductance L of coil T_2 and C connected in parallel to it. The resonance frequency (f) of this tuned circuit determines the frequency at which the oscillator will oscillate

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Illustration 8. *An N-P-N transistor is connected in common – emitter configuration in which collector supply is 8 V and the voltage drop across the load resistance of 800 Ω connected in the collector circuit is 0.8 V. If current amplification factor is (25/36), determine collector – emitter voltage and base current. If the internal resistance of the transistor is 200 Ω, calculate the voltage gain and power gain.*



Solution: Collector current $I_C = \dfrac{\text{voltage drop across } R_L}{R_L} = \dfrac{0.8}{800} = 10^{-3}\,\text{amp}$

Now $v_{CE} = 8 - 0.8 = 7.2$volt

current gain $\beta = \dfrac{I_c}{I_B}$

$\dfrac{25}{26} = \dfrac{10^{-3}}{I_B}$ $\therefore$ $I_B = \dfrac{26}{25} \times 10^{-3} = 1.04 \times 10^{-3}$ amp.

Now $v_{CE} = 8 - 0.8 = 7.2$ volt

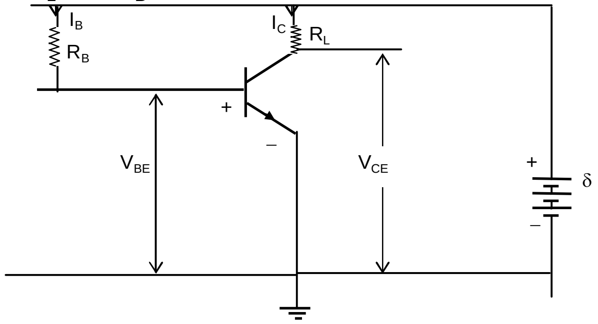
current gain $\beta = \dfrac{I_c}{I_B}$

$\dfrac{25}{26} = \dfrac{10^{-3}}{I_B}$ $\therefore$ $I_B = \dfrac{26}{25} \times 10^{-3} = 1.04 \times 10^{-3}$ amp.

voltage gain $= \beta.\ \dfrac{R_L}{R_c} = \dfrac{25}{26} \times \dfrac{800}{26} = 3.846$

Power gain $= \beta^2 \dfrac{R_L}{R_c} = \left(\dfrac{25}{26}\right)^2 \times \dfrac{800}{200} = \left(\dfrac{25}{26}\right)^2 \times 4 = 3.698$.

Illustration 9. *A n-p-n transistor in a common emitter mode is used as a simple voltage amplifier with a collector current of 4 mA. The terminal of an 8 V battery is connected to the collector through a load resistance R_L and to the base through a resistance R_B. The collector emitter voltage v_{CE} = 4V, base emitter voltage v_{be} = 0.6 v and base current amplification factor pdc is 100, calculate the values of R_L and R_B.*



Solution: Potential difference across R_L = 8 – 4 = 4V

$I_c R_L = 4V$

$R_L = \dfrac{4}{4 \times 10^{-3}} = 10^3\,\Omega = 1k\Omega$

$\beta = \dfrac{I_c}{I_B} \Rightarrow I_B = \dfrac{4 \times 10^{-3}}{100} = 4 \times 10^{-5}$ A

Potential across R_B is = 8 – 0.6 = 7.4 V

$R_B = \dfrac{7.4}{I_B} = \dfrac{7.4}{4 \times 10^{-5}} = 1.85 \times 10^5\,\Omega = 185\,k\Omega$

Illustration 10. *The current gain for common emitter amplifier is 59. If the emitter current is 6.0 mA, find the base current and collector current.*

Solution: Here, $\beta = 59$; $I_c = 6.0$ mA;

I_b = ? and I_c = ?

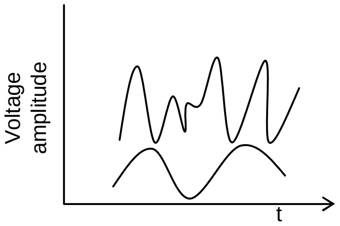
$\beta = \frac{I_c}{I_b} = \frac{I_e - I_b}{I_b}$

or, $I_b = \frac{I_e}{1+\beta} = \frac{6.0}{1+59} = 0.1\text{mA}$

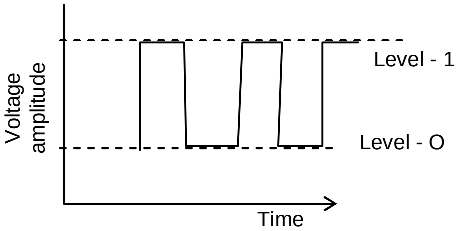
$I_c = I_e - I_b = 6.0 - 0.1 = 5.9$ mA.

Digital Electronics and logic Gates:

The wave form shown in figure, a continuous range of values of voltages are possible.



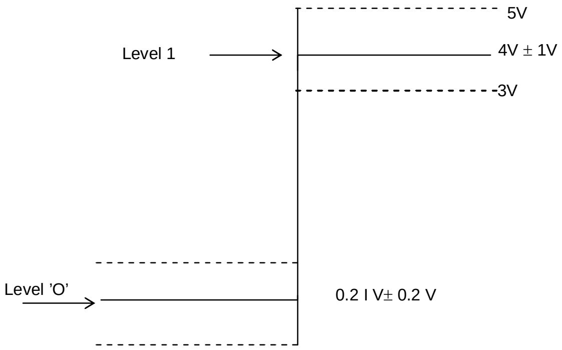
These are analog signals but in a pulse waveform in which only discrete values of voltages are possible. The high level is termed as '1' while the low level is called '0'. Further we know there are a number of questions which have only two answers either yes or No. There are a number of objects which can remains in either of two states only. An electric bulb can either be ON or OFF.



Above is closely related to the binary system of digits which we are already familiar.

Using the two levels of a signal like that in figure to represent binary digits 0 and 1 (called bits).

Please remember that 0 and 1 levels are not the same as a 0 and 1 V. In practice the bit 0 or 1 is recognised by the presence or absence of the pulse(i.e. either at high or at low levels). Further, the digital high and low levels are specified in certain voltage levels as shown in figure.



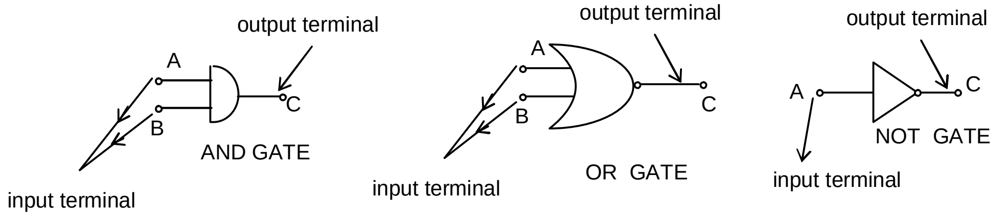
Logic Gates:

A logic gate evaluates a particular logical function using an electronic circuit. The circuit has one or more input terminals and an output terminal. A fixed positive potential V (say + 5V) denotes the logical value 1 and a potential zero (in general equal to the earth's potential) denotes the logical value 0. If zero potential is applied to an input terminal, the corresponding independent variable takes the value 0′.

If the positive potential V is applied to the terminal, the corresponding variable takes the value 1. The output terminal denotes the value of the function. Value of the function is O if the potential is zero. It is 1 if the potential is V.

A gate may have more than one output terminals then each output terminal represents a separate function and the same circuit may be used to evaluate more than one functions.

Figure shows the symbols for different logic gates with the terminals shown on the left are the input terminals and the terminals on the right is the output terminals.

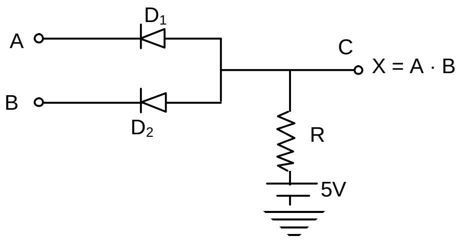


AND Gate:
An AND gate has two or more inputs and one output. The output C of AND gate is 1 if all the inputs is simultaneously have the state 1. We say that inputs A and B are ANDed to get C. It is also denoted by the symbol of dot. Thus C = A. B. C is a function of A and B, If we give the value of C for all possible combinations of A and B, the function is completely specified.

Truth table for AND function is as given below.

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

Realisation of AND gate with diodes:
Figure shows the construction of AND gate with two p-n junction diodes.



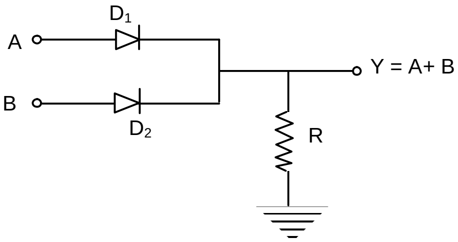
Suppose the potential at A and B are both zero so that both the diodes are forward biased. The potential at C is equal to the potential at A or B because forward biased diode offer no resistance. Thus C = 0. Now suppose, A = 0 and B = 1, Now diode D_1 is forward biased. the potential at C is equal to the potential at A which is zero. Thus, if A = 0 and B = 1 then C = 0. Similarly, when A = 1 and B = 0 then C = 0.

Now suppose A = B = 1. Now both diodes are reversed biased. As the diodes are not conducting, there will be no current through R and the potential at X will be equal to 5 V, i.e. C = 1.

OR Gate:
An OR gate has two or more inputs with one output. The output C will be 1 when the input A or B or both are 1. It is also represented by the symbol of plus. Thus C = A + B. This statement can also be given in the form of a table known as truth table as given below.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Realisation of OR gate with diodes: Figure shows the construction of an OR gate using two p-n junction diodes.

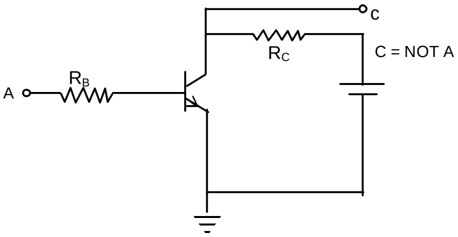


If input A = B = 0, there is no potential difference any where in the circuit so Y = 0. If A = 1 and B = 0. The diode D_1 is forward –biased and offers no resistance. Therefore potential at C is equal to the potential at A, i.e., 5V. Same is the case if A = 0 and B = 1, then Y = 1.
It both A and B are 1, both the diodes are forward - biased and the potential at Y is the same as the common potential at A and B which is 5 V. This also gives Y = 1.
NOT Gate: It has a single input (A) and a *single output* (C). The output is not the same as the input. It performs a negation operation on the input (output is the inverse of the input). The truth table is given below.

Truth table of NOT gate

A	C
0	1
1	0

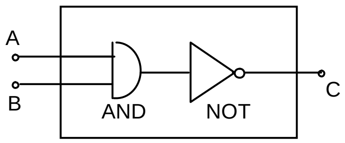
Realisation of NOT gate:
A NOT gate can not be constructed with diodes. For realising this, we use a simple transistor inverter circuit given in figure.

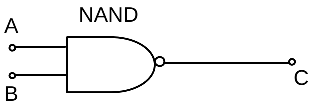


If A = 0, the emitter – base junction is unbiased and there is no current through it.
Therefore there is no current through the resistance R_C and the potential at C is 5 V. Thus, if A = 0, C = 1. Please note that the collectors base junction is also reverse – biased.

If the potential at A is 5V, the base – emitter junction is forward biased and there is a large current in the circuit. There is a potential drop across R and its value at C becomes zero. Thus if A = 1, C = 0.

NAND gate:
It is a combination of NOT and AND gates in which the negation operation (NOT) is applied after AND gate as shown in figure. This simply means that for input C = $\overline{A \cdot B}$

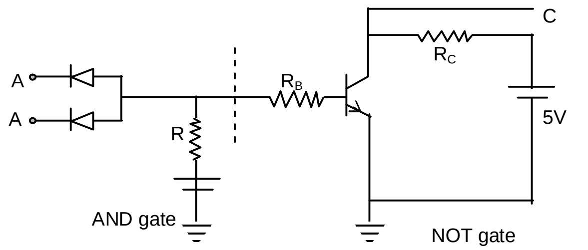




Conditions giving 1 output in AND gate will give O output in NAND gate and vice-versa. Hence truth table for NAND gate will be as under.

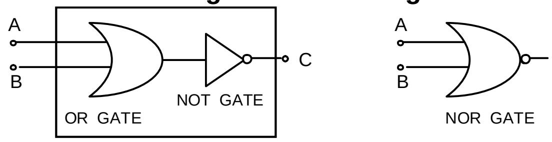
Truth table for NAND gate

A	B	C
0	0	1
0	1	1
1	0	1
1	1	0



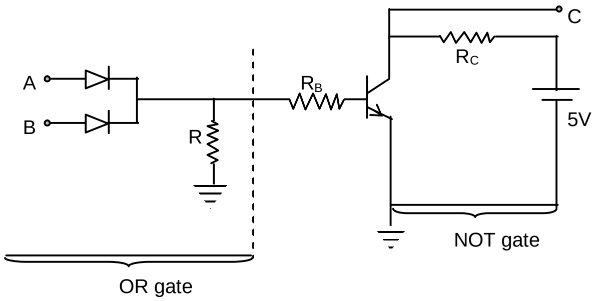
Circuit diagram for NAND gate

NOR Gate: A negation (NOT-operation) applied after OR gate gives a NOT-OR gate (or simply NOR gate). The symbolic representation for NOR gate is given in figure



Truth table for NOR gate

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

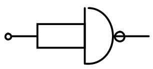


Circuit diagram for NOR gate

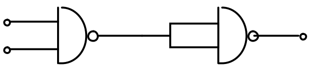
NAND and NOR as the basic building blocks:

NAND and NOR gates are considered as universal gates because you can obtain all the gates like AND, OR, NOT by using NOR and NAND gate.

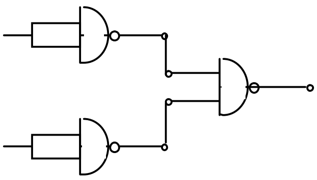
Figure shows the construction of NOT, AND and OR gates using NAND gates.



(a) NOT



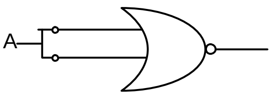
(b) AND



or gate

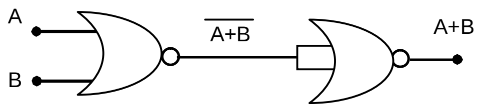
Illustration 11. *NOR gate is a universal gate. Prove that NOR gate can be used to realize OR, AND and NOT gates.*

Solution: As NOT Gate



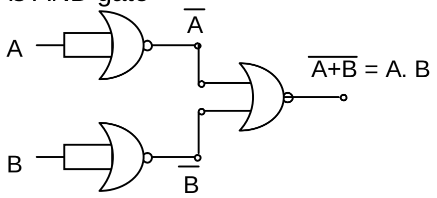
outputs obtained are just inversions of inputs. Which is Boolean expression for NOT gate.

As OR Gate



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

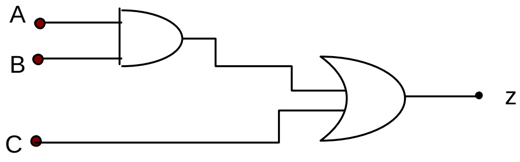
out put Y is same as of OR gate
As AND gate



A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Output is same as of AND gate.

Illustration 12. *Find the boolean equations for the output of the logic circuit shown in figure.*

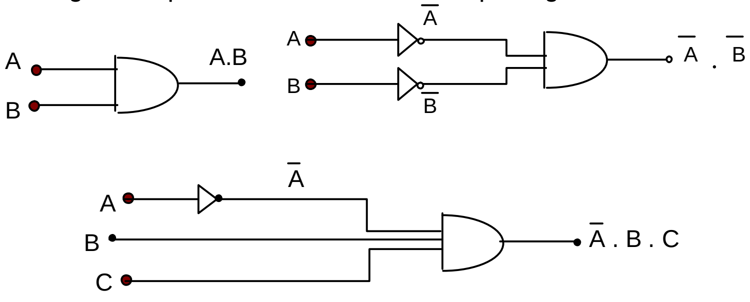


Solution: A and B are ANDed so their output is B. It then becomes one of the inputs for the 2-input OR gate. When AB ORed with C.
z = AB + C

Illustration 13. *Draw the logic circuit represented by the expression*

$$z = AB + \overline{A}\overline{B} + \overline{A}BC$$

Solution: In the given expression there are three input logical variables and z is the output.



- The first terms AB is obtained by ANDing A with B.
- The second term $\overline{A}\ \overline{B}$ is obtained by 2 inverters and one AND gate.
- The last term is obtained by using one INVERTER and one AND gate and connecting them.

Now the complete logic expression is realized by ORing the three outputs.

Integrated circuits

The most significant technological development of twentieth century is INTEGRATED CIRCUIT. Integrated circuits reduced product dimensions and cost, while ensuring greater reliability.

To overcome the drawbacks of space and reliability of separately manufactured components such as resistors, inductors, capacitors, diodes, transistors etc. joined by wires or plated conductors on printed boards engineers started a drive for miniaturised circuits. This led to the development of integrated circuits in 1958.

It is a complete electronic circuit consisting of both the active and passive components (including their inter connections) fabricated on an extremely tiny single chip of silicon.

In IC technology all the circuit elements including their interconnections are fabricated at a time.

Major limitations of the integrated circuit is it is not possible to fabricated inductors and trans-formes on the semi-conductor chip surface and it is neither convenient nor economical to fabricate capacitances exceeding 30 pF.

Classification of IC'S

(i) On the basis of fabrication techniques used, the ICs can be divided into following three classes.

(i) Monolithic Ic's (ii) Thin and Thick films Ic's (iii) Hybrid or Multi-Chip Ic's

Hybrid or Multi chip IC's:

As the name implies, the circuit is fabricated by interconnecting a number of individual chips. The active components are diffused transistors or diodes. The passive components may be group of diffused resistors or capacitors on a single chip, or they may be thin film components. Wiring or a metallised pattern provides connections between chips.

Thick film IC'S: The most widely used technology is the monolithic integrated circuit. Depending upon the level of integration (i.e. the number of circuit components of logic gates) the Ics are termed as small scale integration, SSI (logic gates $\leq$ 10); medium scale integration, MSI (logic gates $\leq$ 100); Large scale integration (logic gates $\leq$ 1000) and very large scale integration, VLSI (logic gates > 1000)

The basic structures of an IC consist of the four layers of different materials. The bottom most layer, typically 200 μm thick is a P-type silicon and serves as the body or substrate upon which the complete IC is built.

The second layer, typically 25 μm thick is of N-type silicon material which is grown as a single crystal extension of the substrate. All active and passive elements like diodes transistors, resistors and capacitors are fabricated or built in this layer employing series of p-type and n-type impurity diffusions.

The third layer is of silicon dioxide (SiO_2) material and acts as a barrier in the selective diffusion in the second layer. This SiO_2 layer is etched away from the region where diffusion is desired to be permitted, the rest of the water remaining protected against diffusion. The etching is done by subjecting the SiO_2 layer to a graphic process. This SiO_2 layer serves the additional function of protecting the silicon wafer against contamination.

The fourth layer is a metallic layer of aluminium. This is added at the top in selected region regions to permit interconnections between the different fabricated components.

The basic production process for the monolithic Ics are given below.

Epitaxial growth : On the high resistivity P-type subtract a low resistivity 25 μm thick layer of N-type is epitaxially grown. It is this epitaxial layer that all active and passive components of an IC are formed.

Insulation layer: In order to prevent the contamination of the epitaxial layer, a thin layer of SiO_2 is formed over the entire surface. The SiO_2 layer is grown by exposing the epitaxial layer to an oxygen atmosphere to about 1000^0C. This surface layer of SiO_2 will prevent any impurities from entering the N-type epitaxial layer.

MISCELLANEOUS EXERCISE

1. In the working of a transistor, emitter base junction is forward biased while collector – base junction is reverse biased. Why ?

2. Two amplifiers are connected one after the other in series (cascaded). The first amplifier has a voltage gain of 10 and the second has a voltage gain of 20. If the input signal is 0.01 V, calculate the output a.c.signal.

3. A common emitter amplifier is designed with n-p-n transistor (α = 0.99). The input impedance is 1 kΩ and load is 10 kΩ. Find the voltage gain and power gain.

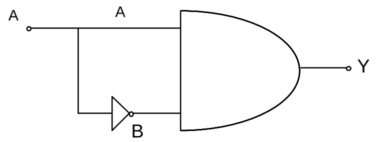
4. Why is the semiconductor damaged by a strong current ?

5. Why a transistor cannot be used as rectifier ?

6. In the working of a transistor, emitter base junction is forward while collector- base junction is reverse biased. Why?

7. Write down the truth table of NAND gate.

8. Find output y in the following circuit



9. In p-n-p transistor circuit, the collector current is 10 mA. If 90 % of the holes reach the collector. Find emitter and base currents.

10. What is the phase difference in a common base amplifier, between the input signal voltage and the output voltage.

ANSWERS TO MISCELLANEOUS EXERCISE

2. 2V

3. 99 C, 98010

8. O

9. 11 mA, 1 mA

10. O

SOLVED PROBLEMS

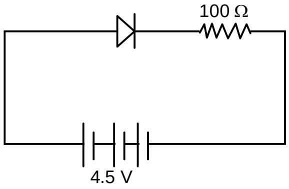
Subjective:

Problem 1. *The number densities of electron and holes in pure silicon at 27^0C are equal and its value is 1.5×10^{16} m^{-3}. On doping with indium, the hole density increases to 4.5×10^{22} m^{-3}. What is the type of semiconductor ? What is the electron density in doped silicon ?*

Solution: Since in a doped semiconductor $n_h >> n_e$, So the semiconductor is of p-type semiconductor. The electron density in doped semiconductor,

$$n_e = \frac{n_i^2}{n_h} = \frac{(1.5\times10^{16})^2}{4.5\times10^{22}} = 5 \times 10^9 \text{ m}^{-3}.$$

Problem 2. *Figure shows a diode connected to an external resistance and an emf. Assuming that the barrier potential developed in diode is 0.5 V, obtain the value of current in the circuit in milliampere.*



Solution: Given, E = 4.5 V, R = 100 Ω

Voltage drop across p-n junction = 0.5 V.

Effective voltage in the circuit

$$V = 4.5 – 0.5 = 4.0 \text{ V}$$

Current in the circuit

$$I = \frac{V}{R} = \frac{4.0}{100} = 0.04 \text{ A}$$

$$= 0.04 \times 1000 \text{ mA} = 40 \text{ mA}.$$

Problem 3. *A p-n-p transistor is used in common - emitter mode in an amplifier circuit. A change of 40 μA in the base current brings a change of 2 mA in collector current and 0.04 V in base – emitter voltage. Find the*

(i) input resistance R_{inp} and

(ii) The base current amplification factor β

If a load of 6 kΩ is used, then also find the voltage gain of the amplifier.

Solution: Given

$\Delta I_B = 40 \times 10^{-6}$ A, $\Delta v_{BE} = 0.04$ volt

$\Delta I_c = 2 \times 10^{-3}$ A, $R_L = 6 \times 10^3\ \Omega$

(i) input resistance

$$R_{inp} = \frac{\Delta v_{BE}}{\Delta I_B} = \frac{0.04}{40\times10^{-6}} = 1 \text{ k}\Omega$$

(ii) Current amplification factor:

$$\beta = \frac{\Delta I_c}{\Delta I_B} = \frac{2\times10^{-3}}{40\times10^{-6}} = 50$$

(iii) voltage gain

$$A_v = \beta\frac{R_L}{R_{inp}} = 50\frac{6\times10^3}{1\times10^3} = 300$$

Problem 4. *In an n-p-n transistor 10^{10} electrons enter the emitter in 10^{-6} s. 2% of the electrons are lost in the base. Calculate the current amplification factor.*

Solution: $I_E = \frac{N_e}{t} = \frac{10^{10} \times (1.6 \times 10^{-19})}{10^{-6}} = 1.6\text{mA}$

The base current I_B is given by

$I_B = \frac{2}{100} \times 1.6 = 0.032\text{mA}$

In a transistor

$I_E = I_B + I_c$

$I_c = I_E - I_B = 1.6 - 0.032 = 1.568$ mA

Current amplification factor $= \frac{I_c}{I_B} = 49$.

Problem 5. *The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. Find the band gap of the semiconductor. Given h = 6.63 $\times$ 10^{-34} Js, 3.0 $\times$ 10^{8} m/s and 1 eV = 1.6 $\times$ 10^{-19} J*

Solution: The band gap correspondents to the minimum energy required to push an electron from the valence band to the conduction band.

$\Delta E = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{2480 \times 10^{-9}} = 8.02 \times 10^{-20}$ J

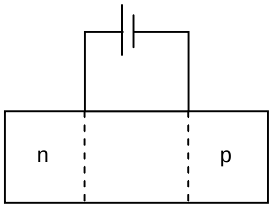
Problem 6. *A potential barrier of 0.3 V exists across P-N junction*

(a) If the depletion region is 1 μm wide, what is the intensity of the electric field in this region.

(b) An electron with speed 5 $\times$ 10^{5} m/s approaches this p-N junction from n-side what will be its speed on entering the p-side.

Solution: (a) Electric field strength

$$E = \frac{v}{d} = \frac{0.3}{1 \times 10^{-6}} = 3 \times 10^{5}\, v/m$$



(b) Electron is retarded on entering in the direction of electric field.

Retardation, $a = \frac{eE}{m}$

Final speed v is given by $v^2 = u^2 - 2\frac{eE}{m}d$

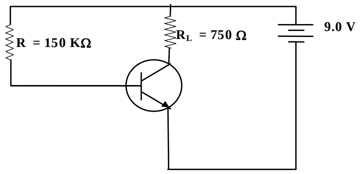
$= (5 \times 10^5)^2 - \frac{2 \times 1.6 \times 10^{-19} \times 3 \times 10^{5}}{9 \times 10^{-31}} \times 1 \times 10^{-6}$

$= 25 \times 10^{10} - 10.7 \times 10^{10}$

$= 14.3 \times 10^{10}$

$v = 3.8 \times 10^{5}$ m/s.

Problem 7. *In the transistor circuit shown in figure direct current gain of the transistor is 80*
Assuming $V_{BE} \approx 0$, calculate
(a) Base current I_B
(b) Potential difference between collectors and emitter terminals.



Solution:

$I_CR_L + V_{CE} = 9$ V

$RI_B + V_{BE} = 9$

$$\Rightarrow I_B = \frac{9}{R} = \frac{9}{150 \times 10^3} = 6 \times 10^{-5}\text{A} \text{ (since } V_{BE} \approx 0)$$

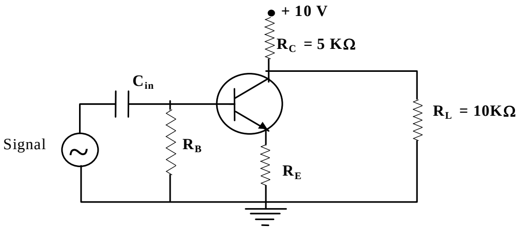
Since $\beta = \frac{I_C}{I_B}$

$\therefore I_C = \beta I_B = 80 x 6 x 10^{-5} = 4.8 x 10^{-3}$A

$\therefore V_{CE} = 9 - I_CR_L$

$= 9 - 4.8 x 10^{-5} x 750 = 5.4$ V

Problem 8. *In a transistor amplifier, when the input signal changes by 0.02V, the base current changes by 10 μA and collector current by 1mA. If collector load R_C = 5 kΩ and R_L= 10kΩ. Find*



(a) current gain *(b) input impedence* *(c) voltage gain*
(d) power gain *(e) output voltage if V_{in} = 1 mV*

Solution:

(a) Current gain $\beta = \frac{\Delta I_c}{\Delta I_B} = \frac{1 \text{ mA}}{10 \times 10^{-3} \text{ mA}} = 100$

(b) Input impedance $R_{in} = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{0.02\text{V}}{10 \ \mu\text{A}} = 2 \ \text{K}\Omega$

(c) Voltage gain $A_v = \beta \ \frac{R_{AC}}{R_{in}}$

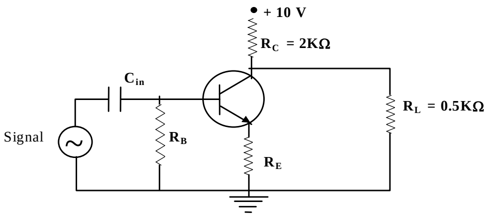
$R_{AC} = \frac{R_cR_L}{R_c + R_L} = 3.3$ KΩ

$\therefore A_v = 100 \times \frac{3.3 \ \text{K}\Omega}{2 \ \text{K} \ \Omega} = 165$

(d) Power gain $A_p = A_V.\beta = 165 \times 100 = 16500$

(e) $V_{out} = V_{in} \times A_V = 1 \text{ mV} \times 165 = 165$ mV

Problem 9. *In the circuit shown in the figure, find voltage gain of amplifier given that β = 60 and input resistance is R_{in} =1kΩ.*

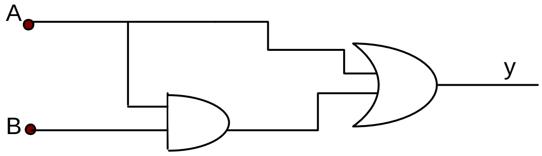


Solution: So for as voltage gain of the circuit is concerned, we need only R_{AC}, β and R_{in}.

$\therefore$ Effective load $R_{AC} = \dfrac{R_C R_L}{R_c + R_L}$ = 0.4 KΩ

$\therefore$ Voltage gain = $\beta \dfrac{R_{AC}}{R_{in}} = \dfrac{60 \times 0.4\ K\Omega}{1\ K\Omega}$ =24

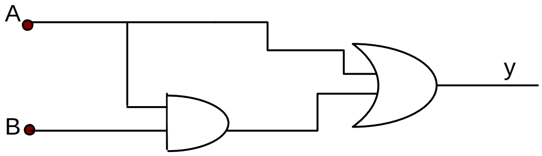
Problem 10. *Express by a truth table the output y for all possible inputs A and B in the following circuit.*



Solution: The circuit and truth table are shown in figure.

Truth table

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1



y = A. B + A

Objective:

Problem 1. *A Ge specimen is doped with Al. The concentration of acceptor atoms is $\approx 10^{21}$ atoms/m^3. Given that the intrinsic concentration of electrons in the specimen is 10^{19} / m^3. The new electron concentration is*

(A) 10^{17} /m^3 *(B) 10^{15} /m^3*
(C) 10^4 /m^3 *(D) 10^2 /m^3*

Solution: When Ge specimen is doped with Aℓ, then concentration of acceptor atoms is also called concentration of holes.

Using formula

$$n_i^2 = n_e n_h$$

n_i = concentration of electron hole pair = 10^{19} / m^3

n_e = concentration of electrons

n_h = concentration of holes = 10^{21} atoms/m^3

$\therefore$ $(10^{19})^2 = 10^{21} \times n_e$

$n_e = 10^{17}$ /m^3

$\therefore$ **(A)**

Problem 2. *For a transistor, the current amplification factor is 0.8. The transistor is connected in common emitter configuration. The change in the collector current when the base current changes by 6 mA is*

(A) 6 mA *(B) 4.8 mA*
(C) 24 mA *(D) 8 mA*

Solution: Given that,

$\beta = 0.8$

$\Delta I_b = 6$ mA $\quad \because \Delta I_c = \beta \Delta I_b$

$\therefore \Delta I_c = 0.8 \times 6$ mA $= 4.8$ mA

$\therefore$ **(B).**

Problem 3. *The transfer ratio β of a transistor is 50. The input resistance of the transistor when used in common – emitter configuration is 1kΩ. The peak value of the collector AC current for an AC input voltage of 0.01 V peak is*

(A) 100 μA *(B) 250 μA*
(C) 500 μA *(D) 800 μA*

Solution: Given that,

$V_I = 0.01$ volt

$R_I = 1\ k\Omega = 10^3\ \Omega$

$$\therefore\ I_b = \frac{V_I}{R_I} = \frac{0.01}{1 \times 10^3} = 0.01 \times 10^3\,\Omega = 0.01 \text{ mA}$$

Further, $I_C = \beta I_b = 50 \times 0.01$ mA

$= 0.5$ mA $= 500\ \mu$A

$\therefore$ **(C).**

Problem 4. *What is the voltage gain in a common emitter amplifier, where input resistance is 3 Ω and load resistance 24 Ω? Take β = 0.6.*

(A) 8.4 *(B) 4.8*
(C) 2.4 *(D) 1.2*

Solution: Given that,

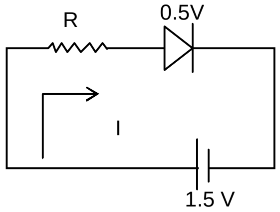
$R_L = 24\ \Omega$ and $R_I = 3\Omega$

$\therefore$ Resistance gain = $\frac{R_L}{R_I} = \frac{24}{3} = 8$

$\therefore$ Voltage gain = current gain $\times$ resistance gain
= $0.6 \times 8 = 4.8$

$\therefore$ **(B).**

Problem 5. *The diode used in the circuit shown in the figure has a constant voltage drop of 0.5 V at all currents and a maximum power rating of 100 milliwatts. What should be the value of the resistor R, connected in series with the diode, for obtaining maximum current?*



(A) 1.5 Ω *(B) 5 Ω*
(C)6.67 Ω *(D) 200 Ω*

Solution: Current in the circuit

I = P/V

$\therefore$ I = $\frac{100\times10^{-3}}{0.5}$ = 0.2 amp.

Now, R = $\frac{\text{voltage drop}}{0.5} = \frac{1}{0.2} = 5\Omega$

$\therefore$ **(B)**

Problem 6. *The following truth table belongs to which one of the following four gate ?*

A	*B*	*Y*
1	*1*	*0*
1	*0*	*0*
0	*1*	*0*
0	*0*	*1*

(A) OR *(B) NAND*
(C) XOR *(D) NOR*

Solution: Above given truth table belongs to NOR gate

i.e. y = $\overline{A+B}$

$\therefore$ **(D)**

Problem 7. *The valence band and conduction band of a solid overlap at low temperature, the solid may be*

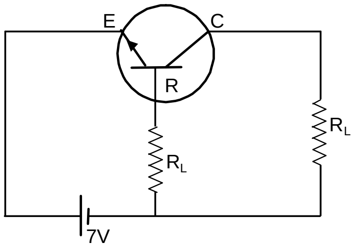
(A) a metal *(B) a semiconductor*

(C) an insulator (D)none of these

Solution: According to band theory of solids, in the valence band and the conduction band overlap in conductors.

$\therefore$ **(A)**

Problem 8. *In the given transistor circuit, the base current is 35 μA. the value of R_b is*



(A) 100 kΩ *(B) 200 kΩ*
(C) 300 kΩ *(D) 400 kΩ*

Solution: Given that,

$I_L = 35\ \mu A = 85 \times 10^{-6}$ A

and voltage in the circuit = 7 volt

We know that resistance offered by the junction in forward bias is zero.

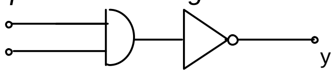
Since, the base emitter junction is forward biased therefore junction resistance is almost zero. We also know that the base resistance

$$R_b = \frac{V}{I_b} = \frac{7}{35 \times 10^{-6}}$$

R_b = 200 kΩ

$\therefore$ **(B)**

Problem 9. *Following diagram performs the logic function of*



(a) OR gate *(B) XOR gate*
(C) AND gate *(D) NAND gate*

Solution: We know that combination of NAND gate and NOT gate is known as AND gate since the given symbol contains NAND gate and NOT gate, therefore it is a NAND gate.

$\therefore$ **(D)**

Problem 10. *When p-n junction diode is forward biased then*
(A) the depletion region is reduced and barrier height is increased.
(B) the depletion region is widened and barrier height is reduced.
(C) both the depletion region and barrier height are reduced.
(D) both the depletion region and barrier height are increased.

Solution: In the forward biasing, the applied potential difference opposes the potential barrier present across p-n junction diode.

$\therefore$ **(C)**

ASSIGNMENT PROBLEMS

Subjective:

Level - O

1. Is the ratio of number of holes and number of conduction electrons in a n type extrinsic semiconductor more than, less than or equal to 1 ?

2. Doping in silicon with indium leads to which type of semiconductor.

3. Draw an energy –band diagram for an intrinsic semiconductor.

4. How does the forbidden energy gap of an intrinsic semiconductor vary with the increase in temperature.

5. Distinguish between n-type and p-type semiconductors on the basis of energy – band diagram.

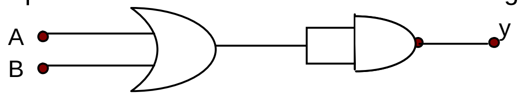
6. The number of silicon atoms per m^3 is 5×10^{28}. This is doped simultaneously with 5×10^{22} atoms per m^3 of Arsenic and 5×10^{20} per m^3 atoms of indium. Calculate the number of electrons and holes. Given that $n_i = 1.5 \times 10^{16}\ m^{-3}$. Is the material n-type or p-type ?

7. The resistance of p-n junction is low when forward biased and is high when reversed biased. Explain.

8. If the emitter and base of a n-p-n transistor have same doping concentration, explain how will the collector and base currents be affected.

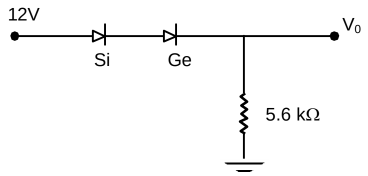
9. When a transistor is used as an oscillator, why is it necessary to feedback energy to L-C circuit?

10. Identify the gate represented by the block diagram of figure shown. Write its Boolean expression the truth table and name of the gate it works.

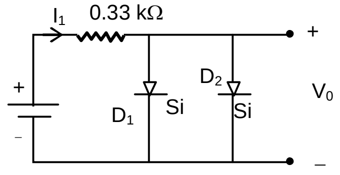


Level- I

1. In the network shown in figure determine V_0 and I_D

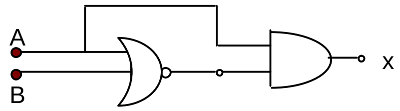


2. In the circuit shown in figure, determine V_0, I_{D1}, I_{D2}, I_1

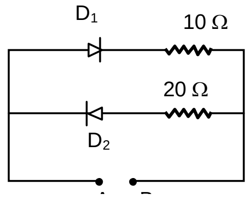


3. A p-type semiconductor has acceptor levels 57 meV above the valence band. Find the maximum wavelength of light which can create a hole.

4. Write the truth table for the function X of A and B represented by figure.

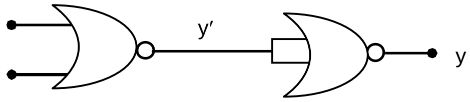


5. A 2V battery may be connected across the points A and B as shown in figure. Assume that the resistance of each diode is zero in forward bias and infinity in reverse bias. Find the current supplied by the battery if the positive terminal of the battery is connected to (a) the point A (b) the point B.

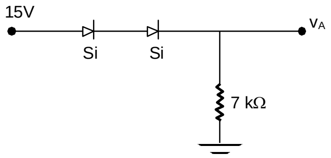


6. Suppose the energy liberated in the recombination of a hole-electron pair is converted into electromagnetic radiation. If the maximum wavelength emitted is 820 nm. What is the band gap.

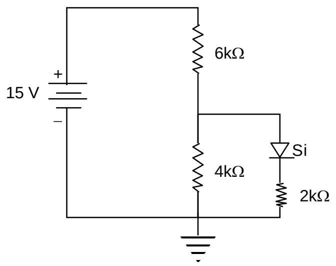
7. The output of an OR gate is connected to both the inputs of a NOR gate. Draw the logic of this combination and write the truth table



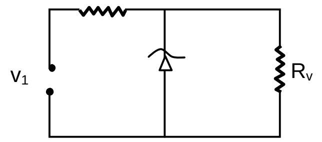
8. Find the voltage v_A of the circuit given in figure given barrier potential for silicon diode is 0.7V.



9. Determine the current through 2kΩ resistors in the circuit show in figure.

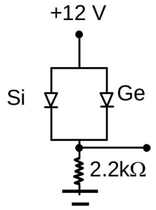


10. A 10 V zener diode along with a series resistance in connected across a 40 V supply. Calculate the minimum value of the resistance required. If the maximum Zener current is 50 mA.

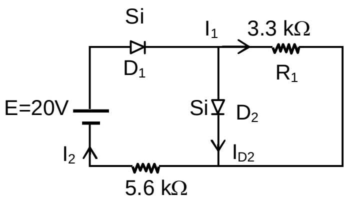


Level – II

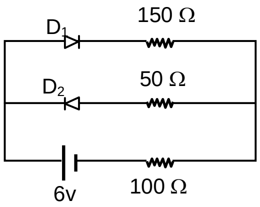
1. In the circuit shown determine the voltage V_0 for the circuit network (forward biased voltage of Si is 0.7 V and Ge is 0.3).



2. In the circuit shown determine the currents I_1, I_2 and I_{D2}



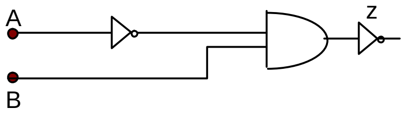
3. The circuit shown in figure contains two diodes each with forward resistances of 50 ohm and with infinite reverse resistance. If the battery voltage is 6 V, find the current through the 100 ohm resistance.



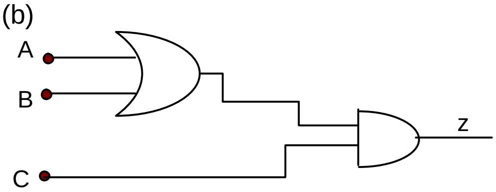
4. A p-n junction diode can withstand currents upto 10 mA. Under forward bias the diode has a potential drop of 0.5 V across it which is assumed to be independent of current. What is the maximum voltage of the battery used to forward bias the diode when a resistance of 200 Ω is connected in series with it.

5. Find the Boolean expression for the output of the logic circuits shown in figure.

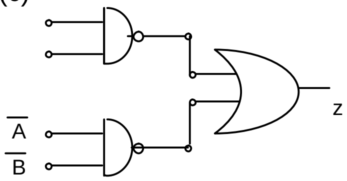
(a)



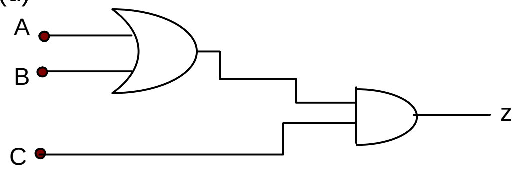
(b)



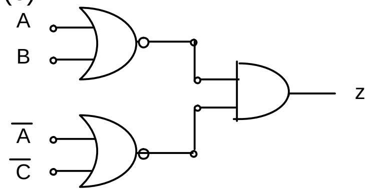
(c)



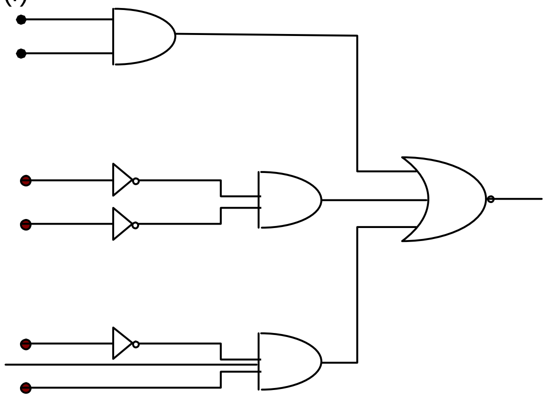
(d)



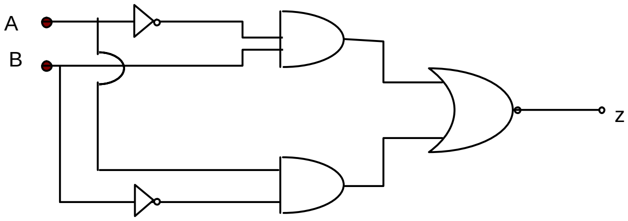
(e)



(f)



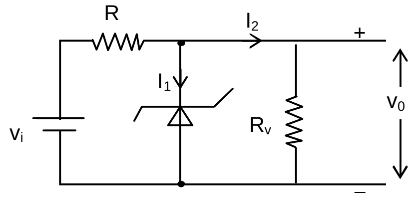
6. Write the expression for the output of the circuit given below.



7. Draw the logic circuits represented by each of the following Boolean equations.
(a) z = AB + C (b) AB + CD (c) $\overline{A}.B + A.\overline{B}$
(d) (A+B) $\overline{A}.B$

8. In a p-n-p transistor circuit, the collector current is 10 mA. If 90 % of the holes reach the collector, find emitter and base currents.

9. A 10 V Zener diode along with a series resistacne is connected across a 40 V supply calculate the minimum value of the resistance required, if the maximum zenod current is 50 mA.



10. A n-p-n transistor is connected in common emitter configuration in which collector supply is 8 V and the voltage drop across the load resistance of 800 Ω connected to the collector circuit is 0.8 V. If the current amplification factor α is $\frac{25}{26}$, determine collector emitter voltage and base current.

Objective:

Level - I

1. An oscillator is nothing but an amplifier with
(A) positive feedback (B) large gain
(C) no feedback (D) negative feedback

2. An n-type semiconductor is formed by adding impurity materials
(A) aluminium, boron or selenium (B) aluminium, boron or indium
(C) phosphorus, antimony or arsenic (D) cobalt, aluminium or selenium

3. In a good conductor, the energy gap between the conduction band and the valence band is
(A) infinity (B) wide
(C) narrow (D) zero

4. Bonding in germanium crystal is
(A) metallic (B) ionic
(C) vander Waal's type (D) covalent

5. When a semiconductor is heated, its resistance
(A) decreases (B) increases
(C) remains the same
(D) may increase or decrease depending upon the semiconductor

6. When the temperature of a semiconductor is increased, its electrical conductivity
(A) increases (B) decreases
(C) remains the same (D) increase at first and then decreases.

7. Holes can exist in
(A) metals (B) insulators
(C) semiconductors (D) transistors

8. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of
(A) each of them increases
(B) each of them decreases
(C) copper increases and germanium decreases
(D) copper decreases and germanium increases

9. In a semiconductor, the forbidden energy gap between the valence band and the conduction band is of the order of
(A) 2 eV (B) 5 eV
(C) 1 keV (D) 1 MeV

10. In an insulator the forbidden energy gap between the valence band the conduction band is of the order of
(A) 1 MeV (B) 0.1 MeV
(C) 1 eV (D) 5 eV

11. A small impurity is added to germanium to get a p-type semiconductor. This impurity is a
(A) trivalent substance (B) pentavalent substance
(C) bivalent substance (D) monovalent substance

12. The forbidden gap in the energy bands of germanium at room temperature is about
(A) 1.1 eV (B) 0.1 eV
(C) 0.67 eV (D) 6.7 eV

13. In a p-type semiconductor, the majority charge carriers are
(A) electrons (B) holes
(C) neutrons (D) protons

14. In p-type semiconductors, the majority and minority charge carries are, respectively,
(A) protons and neutrons (B) electrons and protons
(C) electrons and holes (D) holes and electrons

15. In a semiconductor crystal, if current flows due to breakage of crystal bonds, then the semiconductor is called
(A) acceptor (B) donor
(C) intrinsic semiconductor (D) extrinsic semiconductor

16. To obtain a p-type germanium semiconductor, it must be doped with
(A) arsenic (B) antimony
(C) indium (D) phosphorus

17. Depletion layer in a p–n junction consists of
(A) electron (B) holes
(C) positive and negative ions fixed in their position
(D) both electrons and holes

18. Zener diode is used for
(A) rectification (B) amplification
(C) stabilisation (D) modulation

19. Zener diode functions in
(A) forward biased condition (B) reverse biased condition
(C) both forward and reverse biased conductions (D) none of the above

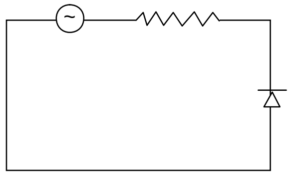
20. When a p–n junction diode is reverse biased, the flow of current across the junction is mainly due to
(A) diffusion of charges (B) depends on a nature of material
(C) drift of charges (D) both drift and diffusion of charges

Level - II

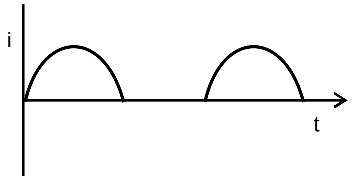
1. A photodiode is essentially a
(A) p-type semiconductor (B) n-type semiconductor
(C) p-n junction (D) none

2. A p-n junction obeys Ohm's law.
(A) True (B) False
(C) May be true (D) In some cases

3. A diode is used in the adjacent circuit of a half-wave rectifier. The waveform is represented by



(A)



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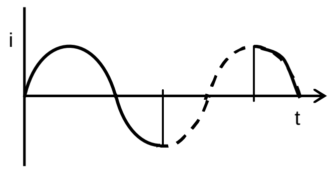
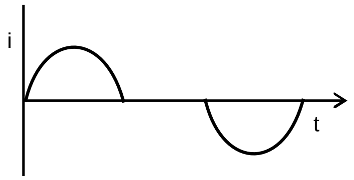
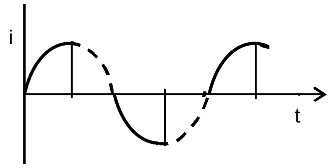


(C)



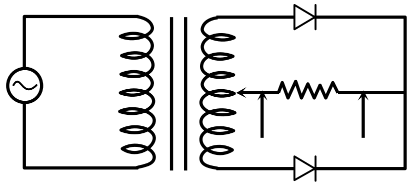
(D)

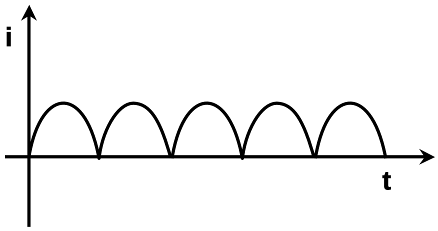


4. A transistor can be used as
(A) an amplifier (B) a voltage control device
(C) a logic gate (D) all the above

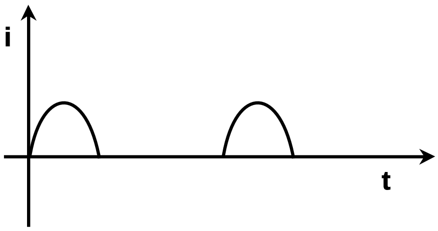
5. A transistor is generally operated at …………………part of its characteristics.
(A) straight part (B) curved part
(C) both (D) none

6. A rectifier circuit is shown in the adjacent figure. If the input current is sinusoidal AC current, then the output voltage pattern is

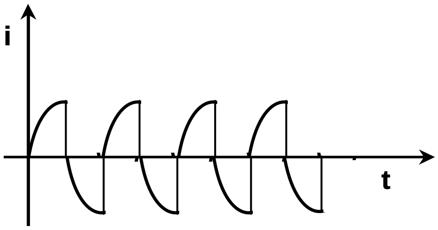




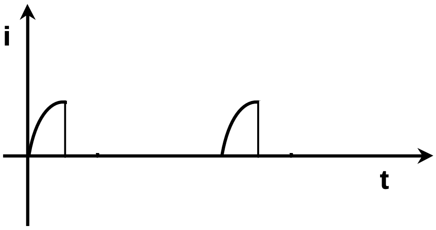
(A)



(B)



(C)



(D)

7. A transistor is a
(A) single terminal device (B) double terminal device
(C) three terminal device (D) none

8. The middle layer of transistor is called
(A) emitter (B) base
(C) collector (D) none

9. In the depletion layer, the electric field crate is from
(A) n side to p side
(B) p side to n side
(C) it charges randomly
(D) it depends on the biasing.

10. The cuase of the potential barrier in a p-n junction diode is
(A) depletion of positive charge near the junction
(B) concentration of positive charge near the junction
(C) depletion of negative charge near the junction
(D) concentration of positive and negative charges near the junction

11. A semiconductor device is connected in a series circuit with a battery and a resistance. A current is found to pass through the circuit. If the polarity of the battery is reversed, the current drops to almost zero. The device may be
(A) a p-n junction (B) an intrinsic semiconductor
(C) a p-type semiconductor (D) an n-type semiconductor

12. In a common-base transistor amplifier
(A) both junctions are forward biased
(B) both junctions are reverse biased
(C) biasing is immaterial
(D) emitter N base junction is forward biased and the collector N base junction is reverse biased.

13. The part of a transistor which is heavily doped to produce a large number of majority carriers is
(A) base (B) emitter
(C) collector (D) none of these

14. A common-emitter transistor amplifier has a current gain of 50. If the load resistance is 4 kΩ and input resistance is 50 Ω, Then the voltage gain of the amplifier is
(A) 160 (B) 200
(C) 300 (D) 400

15. In an intrinsic semiconductor at room temperature, number of electrons and holes are
(A) unequal (B) equal
(C) infinite (D) zero

16. The valence band and conduction band of a solid overlap at low temperature. The solid may be
(A) a metal (B) a semiconductor
(C) an insulator (D) none of these

17. The dominant contribution to current comes from holes in case of
(A) metals (B) intrinsic semiconductors
(C) p-type extrinsic semiconductors (D) n-type extrinsic semiconductors

18. Current gain is
(A) $\beta = \frac{\Delta I_B}{\Delta I_E}$ (B) $\beta = \frac{\Delta I_E}{\Delta I_B}$
(C) $\beta = \frac{\Delta I_B}{\Delta I_C}$ (D) $\beta = \frac{\Delta I_C}{\Delta I_B}$

19. Avalanche breakdown is due to
(A) high speed electrons colliding with the valence electrons
(B) direct breaking of bands due to the high electric field
(C) temperature rise
(D) noise effect

20. Conductivity is
(A) directly proportional to mobility and is independent of charge density.
(B) directly proportional to both
(C) inversely proportional to both
(D) independent of both

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level- O

1. less than one.
2. p-type of semiconductor

6. 4.95×10^{22} m^{-3}, 4.54×10^{9} m^{-3}, n- type

10. $y = \overline{A+B}$, NOR GATE

Truth table

A	B	A+B	$\overline{A+B}$
0	0	0	1
1	0	1	0
0	1	1	0
1	1	1	0

Level- I

1. $V_0 = 11$ V, $I_D = 1.96$ mA

2. $I_1 = 28.18$ mA
$I_{D_1} - I_{D_2} = 14.09$ mA
$v_D = 0.7$ V

4.

	A	B	x
	0	0	0
	1	0	0
	1	0	1
	1	1	1

5. (a) 0.2 A (b) 0.1 A

6. 1.5 eV

7.

A	**B**	**Y**
0	0	1
0	1	0
1	0	0
1	1	0

9. 1.205 mA

10. 60 Ω

Level – II

1. 11.7 v

2. $I_1 = 0.212$ mA
   $I_2 = 3.32$ mA
   $I_3 = 3.108$ mA

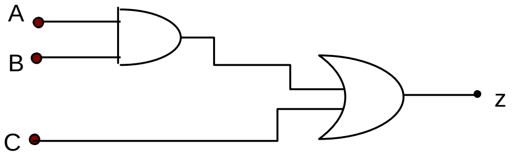
3. 0.02A

4. 2.5 volt

5. (a) $z = \overline{A \cdot B}$

   (b) z = (A + B).C
   (c) $z = A \,.\, B + \bar{A}\,\bar{B}$ (d) z = (A + B). C
   (e) z = (A + B) . (A + C)
   (f) $z = AB + \bar{A}\bar{B} + \bar{A}BC$

6. $Z = \bar{A}.B + A.\bar{B}$

7. (a)


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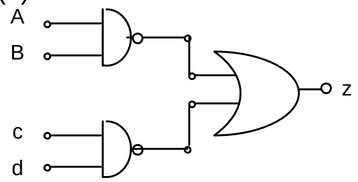
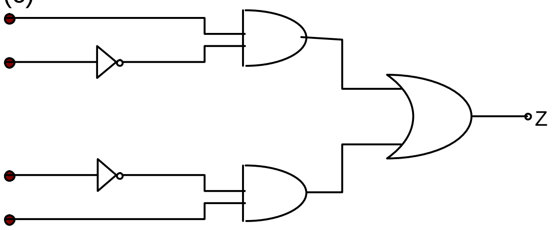


(c)


8. $I_e = 11$ mA
   $I_b = 1$ mA

9 600 Ω

10. $V_c = 7.2$ V ; $I_E = \frac{26}{25}$mA

Objective:

Level - I

1.	**A**	2.	**C**
3.	**D**	4.	**D**
5.	**A**	6.	**D**
7.	**C**	8.	**C**
9.	**A**	10.	**D**
11.	**A**	12.	**B**
13.	**B**	14.	**D**
15.	**C**	16.	**C**
17.	**C**	18.	**C**
19.	**B**	20.	**D**

Level - II

1.	**C**	2.	**B**
3.	**A**	4.	**D**
5.	**A**	6.	**A**
7.	**C**	8.	**B**
9.	**A**	10.	**D**
11.	**A**	12.	**D**
13.	**B**	14.	**C**
15.	**B**	16.	**A**
17.	**A**	18.	**D**
19.	**A**	20.	**B**

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

PRINCIPLES OF COMMUNICATION

Principles of Communication

Syllabus:

Elementary idea of analog and digital communication; Need for modulation; Modulation-amplitude, frequency and pulse modulation; Elementary idea about demodulation, Data transmission and retrieval-Fax and Modem.

Space communication: Propagation of E.M. waves in atmosphere. Sky and space wave propagation. Satellite communication. Applications in Remote Sensing.

Line Communication: 2-wire lines, cables, telephone links; optical communication (optical fibre, Lasers), elementary principle of light modulation.

Introduction

With the diversification of society there arose the need of individuals to communicate with their relatives, friends or associates at distant points around the globe. The science of communication involving long distances is called telecommunication. Technically speaking, the term communication signifies transmission, reception and processing of information by means of electric signals.

Basic constituents of communication system

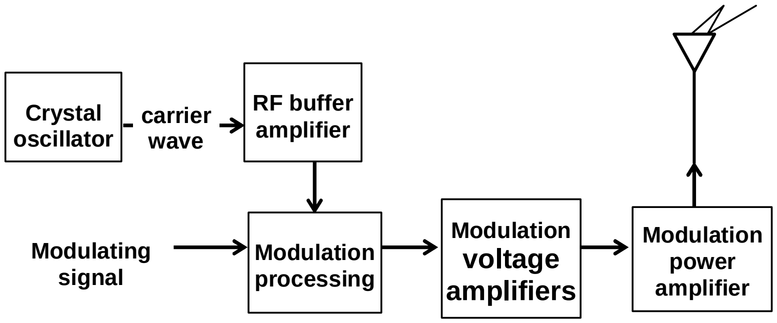
(a) Information

Any communication system serves to communicate a message of information source. The set of massage consist of various messages distinguishable from one another. Out of the total massage usually only a part is converged. The part of massage which is converged is called information. The amount of information contained in any massage is expressed in 'bets' or 'bits' and depends upon the no of choices that must be made.

(b) Transmitter

The information in the incoming signal is first converted into electrical variations, e.g., a sound signal has to be converted to electrical signal, then its audio frequencies are restricted into audio frequency range. A transmitter processes and encodes the incoming information so as to make it suitable for transmission and subsequent reception. In a transmitter information is impressed on a high frequency carrier waves through, a process called modulation, then amplified before feeding to transmitting antenna. According to requirements, there are different modulation techniques as amplitude modulation, frequency modulation, pulse modulation

Block Diagram of a typical transmitter

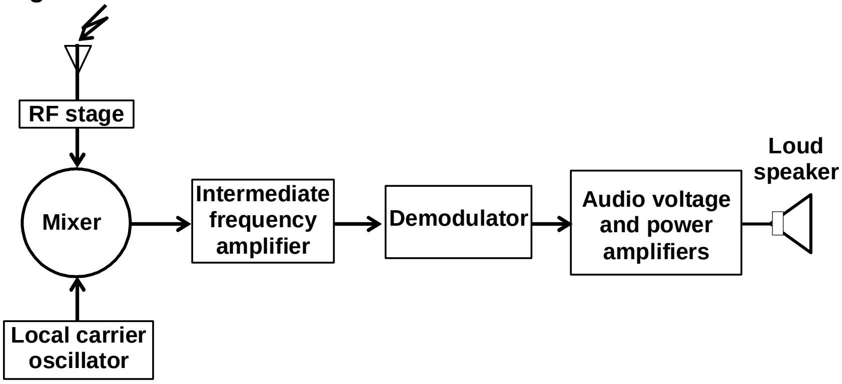


(c) Channel and noise source

The term channel refers to the frequency range allocated to a particular service or transmission. During transmission some deterioration may occur in the information due to distortion in the system. Electrical disturbance sometimes interfere with signals, producing noise. Noise is unwanted induction of energy, usually of random character, tending to interfere with the proper reception, reproduction of the information in a communication system. Noise may produce distortion in the picture in television receivers; it may produce unwanted pulses or cancel out wanted pulses. The sensitivity of receivers and bandwidth of a system may be reduced by noise and has its greatest effect when the signal is weak.

(d) Receiver
The signals received by receiver are quite weak; therefore the receiver first amplifies the signal. The signal may be accompanied by noise or a lot of other neighboring frequencies; thus the actual signal has to be selected. The most important function of receiver is demodulation and sometimes decoding as well which is the reverse of modulation in a transmitter. The output of a receiver may be fed to a loudspeaker, video display unit, teletypewriter, T.V. picture tube, pen recorder or computer. The transmitter and receiver must be in synchronization with the modulation and coding methods use. Figure shows block diagram of an AM receiver.

Block Diagram of a receiver



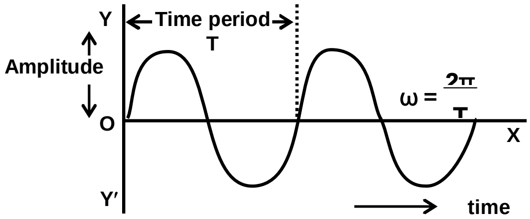
Message signals
Message signal are electrical signals generated from the original information to be transmitted, using an appropriate transducer. A message signal is a single valued function of time that conveys the information. This function has unique value at every instant of time. These signals are of two types:
1. Analog signals. 2. Digital signals.

(a) Analog communication
(i) Analog communication system involves analog electronic circuit, where the out put voltage changes continuously according to input voltage variations.
(ii) In this communication, the output voltage can have an infinite number of values.
(iii) A continuously varying signal (voltage or current) is called an analog signal.
(iv) Due to many–valued output, the analog operation is less reliable.

In the simplest form of an analog signal, amplitude of the signal varies sinusoidally with time. It is represented by the equation

$$E = E_0 \sin(\omega t + \phi)$$

where E_0 is max. value of voltage, called the amplitude, T is time period and $\omega = \frac{2\pi}{T}$ is angular frequency of the signal, In figure, ϕ represents the phase angle, such signals can have all sorts of values at different instants, but these values shall remain within the range of a maximum value ($+E_0$) and a minimum value ($-E_0$).

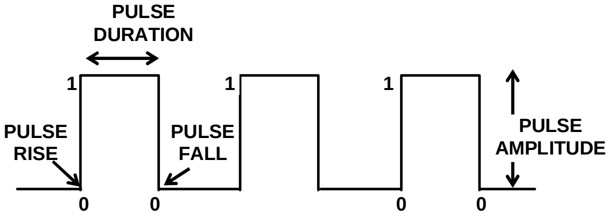


Examples of analog signals are speech, music, sound produced by a vibration tuning form, variations in light intensity etc. These are converted into current/voltage variations using suitable transducers. The information bearing signals are called base band signals. The frequency of analog signals associated with speech or music varies over a range of 20Hz to 20 kHz. The range over which the frequency of information bearing signals varies is called the band width of the signal.

(b)Digital communication
(i) Modern communication systems involve digital electronic circuits and digital signals.
(ii) A signal that can have only two discrete values is called a digital signal.
(iii) A square wave is a digital signal, because this signal has only two values, viz., +5 V and 0 V.
(iv) Digital operation has only two states (i.e., ON or OFF).
(v) Digital operation is more reliable than many – valued analog operation.
(vi) A digital circuit expresses the values in digits 1's or 0's. Hence, the name digital is given.

Digital signals: A digital signal is a discontinuous function of time, in contrast to an analog signal, wherein current or voltage value varies continuously with time.

Such a signal is usually in the form of pulses. Each pulse has two levels of current or voltage, represented by 0 and 1. Zero (0) of a digital signal refers to open circuit and (1) of a digital signal refers to closed circuit. Zero (0) is also referred to as 'No' or space and (1) is refereed to as 'Yes' or mark. Both 0 and 1 are called bits. A typical digital signal is shown in figure.



The significant characteristics of a digital signal are: Pulse amplitude; Pulse Duration or Pulse Width and Pulse Position, representing the time of rise and time of fall of the pulse amplitude, as shown in figure
Examples of digital messages are
(i) letters printed in this book
(ii) listing of any data,
(iii) output of a digital computer,
(iv) Electronic transmission of a document at a distant place via telephone line i.e. FAX etc.

An analog signal can be converted suitably into a digital signal and vice – versa.

Need for Modulation
(a) Frequency of signal
The audio frequency signals (20 Hz to 20 kHz) cannot be transmitted without distortion over long distance due to less energy carrier by low frequency audio waves. The energy of a wave is directly proportional to square of its frequency. Even if the audio signal is converted into electrical signal, the later cannot be sent very far without employing large amount of power.

(b) Height of antenna
For efficient radiation and reception, the height of transmitting and receiving antennas should be comparable to a quarter wavelength of the frequency used.

$$\text{Wavelength} = \frac{\text{Velociy}}{\text{Frequency}} = \frac{3\times10^8}{\text{frequency}(\text{Hz})}\text{ metre}$$

For 1 MHz it is 75 metre; for 15 kHz frequency, the height of antennas has to be about 5000 metre, which size is unthinkable.

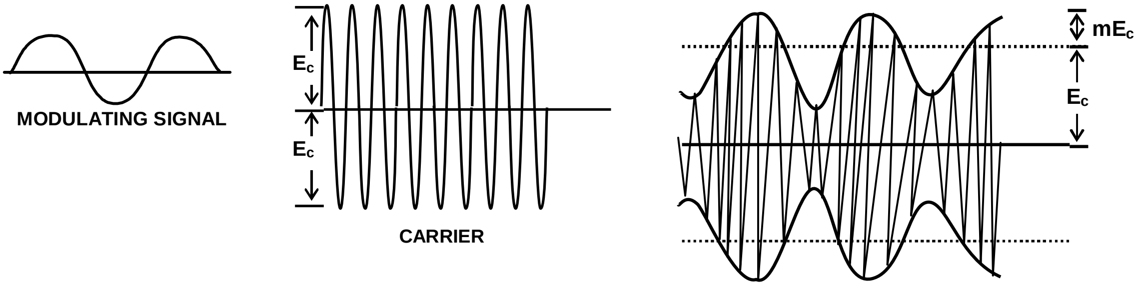
(c) Number of channels
Audio frequencies are concentrated in the range 20 Hz to 20 kHz. This range is so narrow that there will be overlapping of signals. In order to separate the various signals it is necessary to convert all of them to different portions of the electromagnetic spectrum.

Modulation

The radiation of electromagnetic energy is practicable only at high frequencies, e.g., above 20 kHz. The high frequency signals can be sent over large distances even with comparatively small power. This is achieved by superimposing electrical audio signal on high frequency carrier. The resultant waves are known as modulated waves or radio waves and the process is called modulation.

(a) Amplitude Modulation

When the amplitude of high frequency carrier wave is changed in accordance with the intensity of the modulation signal, it is called amplitude modulation. The frequency of the modulated wave is equal to carrier frequency. Figure shows the principle of amplitude modulation. The amplitudes of both positive and negative half-cycle of carrier wave are changed in accordance with the instantaneous values of the modulating signal. When the signal is increasing in the positive sense, the amplitude of carrier wave also increases; similarly during negative half-cycle of the signal, the amplitude of carrier wave decreases. Amplitude modulation is done by an electronic circuit called modulator.



(b) In case of amplitude modulation

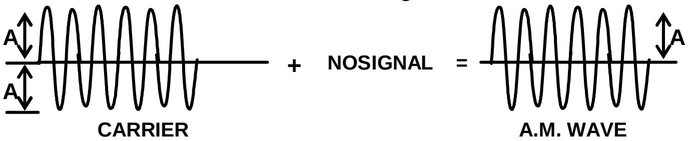
(1) The amplitude of the carrier wave changes according to the intensity of the signal.
(2) The amplitude variation of the carrier wave is at the signal frequency f_s.
(3) The frequency of the amplitude modulated wave is equal to carrier frequency f_s.

(i) Modulation factor

The ratio of change of amplitude of carrier wave to the amplitude of normal carrier wave is called the modulation factor or index of modulation (m).

$$\text{Modulation factor, (m)} = \frac{\text{Amplitude change of carrier wave}}{\text{Amplitude of normal (unmodulated) carrier wave}}$$

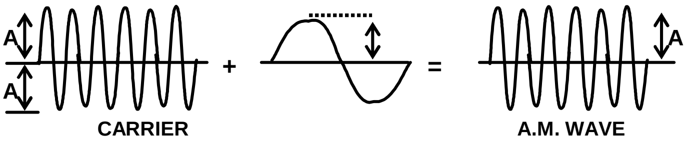
(a) (i) When signal amplitude is zero, the carrier wave is not modulated as shown in Figure. The amplitude of carrier wave remains unchanged.

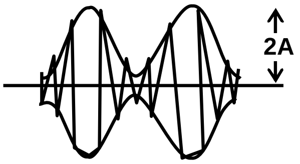


Amplitude change of carrier wave = 0
Amplitude of normal carrier wave = A
$\therefore$ Modulation factor, m = 0/A = 0 or 0%

(b) (ii) When signal amplitude is equal to the carrier wave amplitude as shown in figure the amplitude of carrier wave varies between 2A and zero.
Amplitude change of carrier wave = 2A – A = A

$$\therefore \text{ Modulation factor, (m)} = \frac{\text{Amplitude change of carrier wave}}{\text{Amplitude of normal carrier wave}} = \frac{A}{A} = 1 \text{ or } 100\%$$

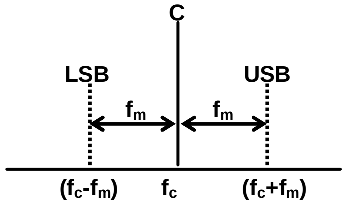




A.M.WAVE

(ii) Frequency spectrum AM waves
A detailed study of amplitude modulation reveals that the amplitude modulated wave consists of three discrete frequencies, as shown in figure of these, the central frequency is the carrier frequency (f_c), which has the highest amplitude. The other two frequencies are placed symmetrically about it. Both these frequencies have equal amplitudes-which never exceeds half the carrier amplitude. These frequencies are called side band frequencies.

$$F_{SB} = f_c \pm f_m$$



$\therefore$ Frequency of lower side band is

$$f_{LSB} = f_c - f_m \qquad \ldots(8)$$

and frequency of upper side band is

$$f_{USB} = f_c + f_m \qquad \ldots(9)$$

Bandwidth of amplitude modulated wave is

$$f_{USB} - f_{LSB}$$
$$= (f_c + f_m) - (f_c - f_m) = 2f_m \qquad \ldots(10)$$

Bandwidth = twice the frequency of the modulating signal.

(iii) Power and Current Relation in AM wave
Average power/cycle in the unmodulated carrier wave is

$$P_c = \frac{E_c^2}{2R} \qquad \ldots(11)$$

Where R is resistance (of antenna) in which power is dissipated. It can be shown that total power/cycle in the modulated wave is

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right) \qquad \ldots(12)$$

$$\therefore \qquad \frac{P_t}{P_c} = 1 + \frac{m_a^2}{2}$$

But $\quad P_t = I_t^2 R$ and $P_c = I_c^2 R$

$$\therefore \qquad \frac{I_t^2}{I_c^2} = \left(1 + \frac{m_a^2}{2}\right)$$

or, $$\frac{I_t}{I_c} = \sqrt{1 + \frac{m_a^2}{2}} \qquad \ldots(13)$$

Here I_t is rms value of total modulated current and I_c is the rms value of unmodulated carrier current.

Illustration 1. *An audio signal of amplitude one half the carrier amplitude is used in amplitude modulation. Calculate the modulation index?*

Solution: Here, $E_m = 0.5\, E_c$

$$\therefore E_{max} = E_c + E_m = E_c + 0.5E_c = 1.5E_c$$
$$E_{min} = E_c - E_m = E_c - 0.5E_c = 0.5E_c$$

$$m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$
$$= \frac{1.5E_c - 0.5E_c}{1.5 + 0.5E_c} = \frac{E_c}{2.0E_c} = 0.5$$

Illustration 2. *Show that the minimum length of antenna required to transmit a radio signal of frequency 10 MHz is 30 m.*

Solution: Here, f = 10 MHz = 10^7Hz
$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10^7} = 30\text{ m}$$

Exercise 1: *(i) A transmitter transmits a power of 20 kW, when modulation is 50%. Calculate power of the carrier wave.*

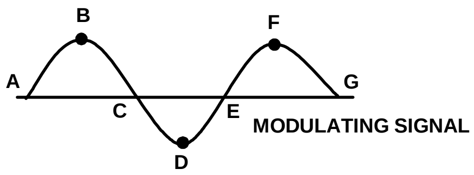
(ii) When a broadcast AM transmitter is 50% modulated, its antenna current is 12 A. What would be the carrier current?

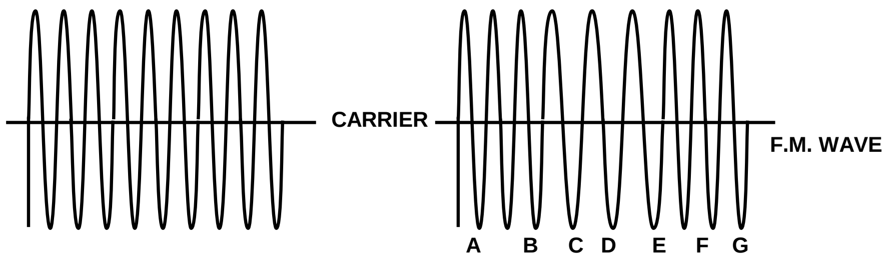
(iii) When the modulation percentage is 75% an AM transmitter produces 10 kW. How much of this is carrier power?

(b) Frequency Modulation

When the frequency of carrier wave is changed in accordance with the instantaneous value of the (modulating) signal it is called frequency modulation. In frequency modulation the amplitude of the modulated wave remains the same, as of carrier wave. The frequency variations of carrier wave depend upon the instantaneous amplitude of the signal as shown in figure. When the signal voltage is zero as at A, C, E, the carrier frequency is unchanged.

When the signal approaches its positive peaks as at B, the carrier frequency is increased to maximum as shown by the closely spaced cycles. When the modulating signal attains its negative peak as at D, the carrier frequency is reduced to minimum as shown by the widely spaced cycles. This type of modulation gives noiseless reception. Since noise is a form of amplitude variations, a FM receiver will reject such variations.





(b) (i) Analysis of FM Carrier Wave

Let the carrier and modulating voltage waves be represented as

$v_c = V_c \cos(\omega_c t + \theta)$ …(1)

and $v_m = V_m \cos\omega_m t$ …(2)

where v_c, V_c, ω_c are the instantaneous value, peak value, angular frequency of the carrier and v_m, V_m and ω_m are the instantaneous value, peak value and the angular frequency of the modulation signal. In frequency modulation, the frequency of the carrier wave varies with time in accordance with the instantaneous value of the modulating voltage. If f instantaneous frequency of carrier wave at instant t, then fractional deviation in frequency

$$\delta = \frac{f - f_c}{f_c} \qquad \ldots(3)$$

As deviation in frequency is proportional to modulating voltage, so

$$\delta = kV_m \cos\omega_m t \qquad \ldots(4)$$

where k, constant of proportionality

From eqns. (3) and (4)

$$\frac{f - f_c}{f_c} = kV_m \cos\omega_m t$$

or $$f = f_c\,(1 + kV_m \cos\omega_m t) \quad \ldots(5)$$

For maximum deviation, $\cos\omega_m t = \pm 1$

$$\therefore \qquad \delta_{max} = \frac{f - f_c}{f_c} = \pm kv_m$$

The modulation index for FM wave is defined as,

$$m_f = \frac{\text{maximum frequency deviation}}{\text{modulating frequency}} = \frac{f - f_c}{f_m} = \pm\, k\frac{v_m f_c}{f_m}.$$

This implies that modulation index increases with decreases in modulating signal frequency.

(ii) Frequency spectrum of FM wave

(i) The output of an FM wave consists of carrier frequency (f_c) and almost an infinite number of side bands, whose frequencies are ($f_c \pm f_m$), ($f_c \pm 2f_m$), ($f_c \pm 3f_m$), ….and so on. The sidebands are thus separated from the carrier by f_m, $2f_m$, $3f_m$ etc. i.e. they have a recurrence frequency of f_m.

(ii) The number of sidebands depends on the modulation index (m_f). The number of sidebands increases, when frequency deviation (δ) increases, keeping (f_m) constant. Similarly, number of sidebands decreases, when frequency of modulating signal (f_m) increases keeping frequency deviation constant.

(iii) The sidebands are disposed symmetrically about the carrier. Further, sidebands at equal distances from the carrier frequency. When there distance from carrier frequency increases, their amplitude decreases. Therefore, number of significant sideband pairs is limited.

(iv) As the distance of sidebands from carrier frequency increases, their amplitude decreases. Therefore, number of significant sideband pairs is limited.

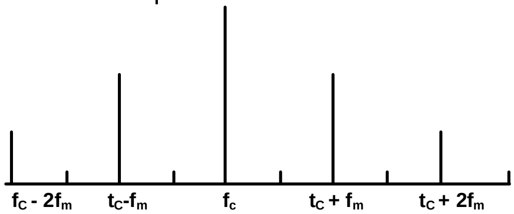
(v) In frequency modulated wave, the information (audio signal) is contained in the sidebands only. Since the sidebands are separated from each other by the frequency of the modulating signal (f_m), therefore,

$$\boxed{\text{Band width} = 2n \times X(f_m)} \quad \ldots.(24)$$

Where n is the number of the particular sideband pair.

(iii) FM Sidebands

From equation, it is obvious that the FM wave consists of the carrier frequency (f_c) plus a series of sidebands of decreasing amplitude, spaced about the carrier by the modulating frequency, i.e., f_c, $\left(f_c \pm 2f_m\right),\left(f_c \pm 3f_m\right)$ and so no.



Theoretically the number of sidebands are infinite, their strength becomes negligible after a few sidebands. Sidebands at equal distances on either side from f_c have equal amplitudes as illustrated in figure. Therefore side band distribution is symmetrical about f_c.

Illustration 3. *An audio signal of 2.8 kHz modulates a carrier of frequency 84 MHz and produces a frequency deviation of 56 kHz. Calculate*
(i) frequency modulation index
(ii) frequency range of FM wave.

Solution: Here, f_m = 2.8 kHz, f_c = 84 MHz
δ = 56 kHz

(i) Frequency modulation index $= m_f = \dfrac{\delta}{f_m}$

$= \dfrac{56}{2.8} = 20$

(ii) Frequency range of FM wave
$= f_c \pm f_m$
$= \left(84 \pm 2.8 \times 10^{-3}\right)$MHz
= 84.0028 MHz and 83.9972 MHz.

Exercise 2: ***A 500 Hz modulating voltage fed into FM generator produces a frequency deviation of 2.35 kHz. What is the modulation index?***

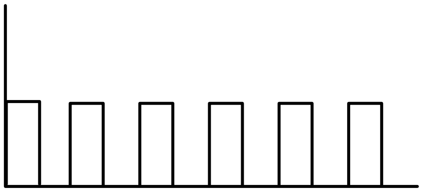
(c) Pulse Modulation
In pulse modulation, the carrier wave consists of a pulse train instead of a continuous sinusoidal wave (as in the case of analog modulation.) Some parameter of the pulse train carrier is varied in accordance with the instantaneous value of the modulating signal.

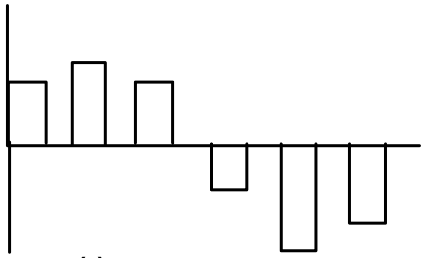
Types of pulse modulation
(c) (i) PAM (Pulse Amplitude Modulation)
Here, the amplitude of the pulses of the carrier pulse train is varied in accordance with the instantaneous value of the modulating signal, as

(ii) Pulse Time Modulation
Here, the timing of the pulses of the carrier pulse train is varied in accordance with the instantaneous value of the modulating signal. The pulse time modulation is further of two types:

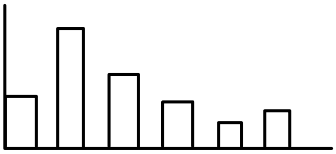

(a) MODULATING SIGNAL

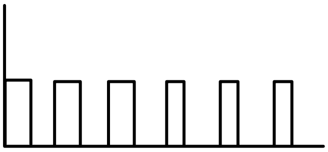
(a) Pulse Width Modulation (PWM). Here, the width or duration of the pulses of the carrier pulse train is varied in accordance with the instantaneous value of the modulating signal. It is also called Pulse Duration Modulation (PDM). This is shown in figure. As is clear from pulse width or pulse duration of the original unmodulated carrier is constant. In the PWM or PDM as shown in figure the pulse width/duration is large, when the amplitude of the modulating sinusoidal signal is large and vice-versa. Thus in PWM or PDM, information about the baseband signal lies in the trailing edge of the pulse.

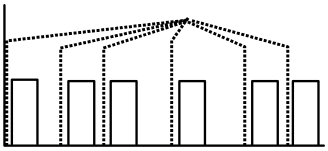

(b)UNMODULATED CARRIER PULSE TRAIN


(c)PAM (DOUBLE POLARITY)

(b) Pulse Position Modulation (PPM). Here, the position of the pulses of the carrier pulse train is varied in accordance with the instantaneous value of the modulating signal. Note that pulse position represents the time of rise or fall of the pulse. Figure represents pulse position modulated signal. The dashed lines in this figure shown the original positions of the pulses. Obviously, in PPM, these positions have shifted in time. The shift is more if modulating signal amplitude is high and vice-versa. The information about the baseboard signal lies in the leading edge of the pulse.

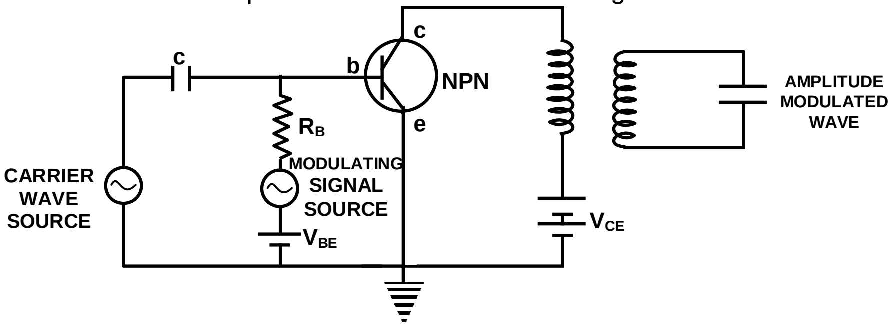

(d)PAM (single polarity)


(e)PWM or PDM


(f)PPM

Demodulation
Demodulation is the reverse process of modulation, which is performed in a receiver to recover the original modulating signals.

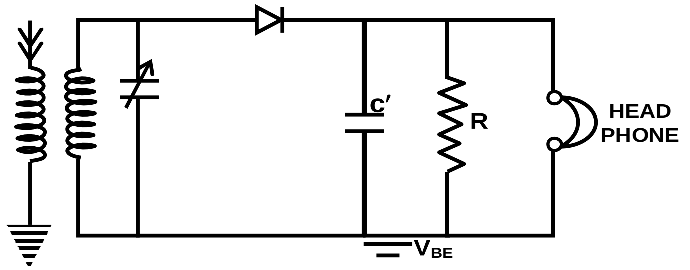
(a) Amplitude Modulator
The basic circuit of an amplitude modulator is shown in figure.

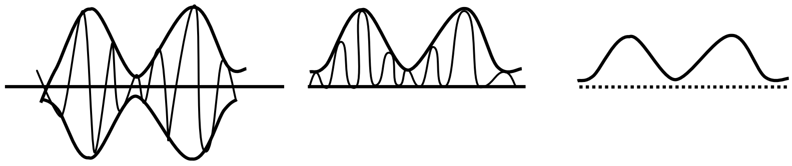


It is essentially a common emitter amplifier for the carrier wave signal. The base biasing voltage, in this case, is a sum of d.c. and the modulating signal. As the base biasing voltage changes, amplification changes. Therefore, out put voltage will be a carrier wave varying in amplitude in accordance with the biasing modulating voltage. This is the amplitude modulated wave.
Demodulator performs the following functions
(i) Selecting the desired signal and rejecting the unwanted signal
(ii) Amplifying the desired signal
(iii) Displaying the demodulated signal in a desired manner.

Basic circuit of a demodulator
Figure shows the basic circuit of a tunned demodulator, wherein a function diode acts as the detector. The input circuit consist of a turned circuit. This circuits selects the desired amplitude modulated wave signal from the different signals picked up by receiving antenna. When this signal is passed through junction diode we obtain rectified modulated wave containing only
half cycles. This occurs due to rectifying action of the junction diode.

The rectified out put is then fed to the parallel combination of a capacitor C" and a resistor R. The value of C' is so chosen that its reactance $\left(X_{C'} = \frac{1}{\omega C'}\right)$ to the high frequency carrier wave is low. This reactance will obviously be high for the low frequency modulating signal. Therefore, the capacitor C' acts as a bypass for the carrier waves and audio frequency modulating signal voltage appears across R. This sends current through headphone, and the original speech or music is reproduced.





AMPLITUDE MODULATED WAVE | RECTIFIED MODULATED WAVE | AUDIO OUTPUT (ACROSS R)

Note: The essential condition for demodulation is that time period of high frequency carrier wave must be much smaller than the time constant of RC circuit, i.e.

$$T_C << \tau \text{ or } \frac{I}{f_c} << RC$$

where f_c is frequency of carrier waves.

Illustration 4. *In a diode AM detector, the output circuit consists of R = 1kΩ and C = 10 pF. A carrier signal of 100 kHz is to be detected. Is it good? If yes, then explain why? If not, what value of C would you suggest?*

Solution: Here, $R = 1k\Omega = 10^3\Omega$,
$C = 10 \text{ pF} = 10 \times 10^{-12} \text{ F} = 10^{-11} \text{ F}$
$\therefore RC = 10^3 \times 10^{-11} \text{ s} = 10^{-8} \text{ s}$
and $\frac{1}{f_c} = \frac{1}{100 \times 10^3} \text{s} = 10^{-5} \text{s}$

We find that $\frac{1}{f_c}$ is not less than RC, as is required for demodulation. Therefore, the arrangement is NO GOOD.

For a satisfactory arrangement, let us try
$C = 1\mu \text{ F} = 10^{-6}\text{F}.$
$\therefore RC = 10^3 \times 10^{-6} \text{ s} = 10^{-3}\text{s}$
Now, $\frac{1}{f_c}\left(=10^{-5}\text{s}\right) << RC\left(=10^{-3}\text{s}\right)$
$\therefore$ The condition is satisfied. This is good enough for demodulation.

Exercise 3: ***In a diode detector, output circuit consist of R = 1 MΩ smf C= 1 F. Calculate the carrier frequency it can detect.***

(b) Data Transmission and Retrieval

In digital communication, the modulating signals are discrete. They are coded representation of the message signal or information to be transmitted. To transmit the digital data through space, following three modulation techniques are used:
Amplitude Shift Keying (ASK),
Frequency Shift Keying (FSK),
Phase Shift Keying (PSK).

(c) Modem

The term modem is a contraction of the term MO – dulater and DEM-odulater. In the transmitting mode the modem accepts digital data and converts it to analog signals for use in modulating a carrier signal. The modem at the receiving end of the system demodulates the carrier. Modems are placed at both ends of the communication circuits. Modems function at different speeds e.g., kbps modems are used for sending and receiving e-mail. Higher and lower speed modems are used for other application. A modem which provides transmission in only one direction is referred to as operating in the simplex mode. It uses only one unidirectional channel. In some modems data transfer can take place in both the directions, but the data flow takes turns, with flow in one direction at one time and in the opposite direction next time. This type of modem operation is called half – duplex. It requires only one bi-directional transmission channel. The speed of transmission is reduced because of necessity of sharing the same circuit. Full-duplex operation permits transmission in both directions simultaneously.

Generally the data circuits utilize telephone channels. For this type of service, the hard-wired modem and the acoustically coupled data set are used. Modems can perform this work at a variety of speeds, depending on the technology and availability of access lines. Some of the important features of modems are:

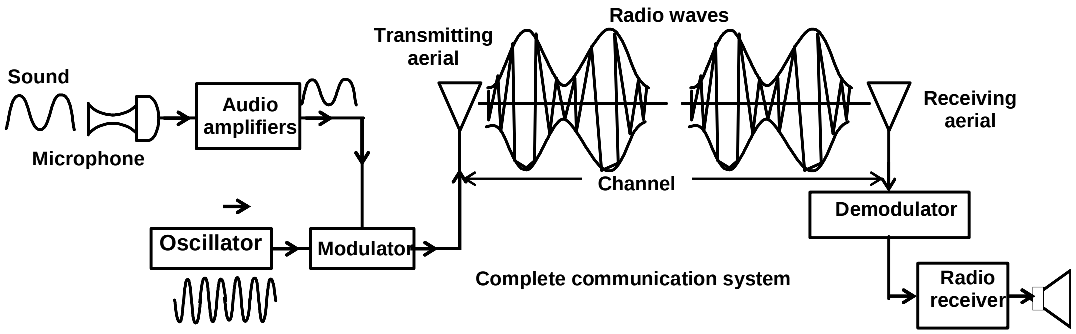
Bits per second (bps). This is the rate at which a modem can transmit and receive data. For sending and receiving electronic mail or faxes, kbps modems are often used. The fastest modems run at kbps. Still higher data transfer rates are achieved by data compression.

Voice/data mode. In voice mode, the modem acts like a regular telephone and in data mode, the mode acts like a regular modem.

Auto answer. An auto answer modem enables our computer to receive calls in our absence. These are suitably recorded and can be reproduced.

(f) Fax

This term is used for electronic transmission of a document (text, graphics, picture etc) to a distant place over telephone lines and its reproduction at the far end. Its basic components are: optical scanner at the sending end to read the document; Encoder for the black and white spots read by the scanner; Decoder at the receiving end to interpret the encoded signals and conversion to print dark and white spots on blank paper.



Complete communication system

Space Communication

The term space communication refers to the sending, receiving and processing of information through space. The information in the form of electrical signal can be sent from one place to another by superimposing it on undamped electromagnetic waves of high frequency called carrier waves through various methods of modulation and then releasing it into space by transmitting antenna as electromagnetic waves.

(a) Earth Atmosphere
The gaseous envelope surrounding the earth is called earth's atmosphere. The earth atmosphere has no sharp boundary. It has been divided into various regions as given below.

Troposphere. It extends upto a height of 12 km. The atmospheric air in this region has maximum density which varies from 1kg/m^3 at the surface of earth to 0.1 kg/m^3 at the top of this layer. The electrical conductivity of this region is least as compared to other regions of earth's atmosphere.

Stratosphere. It extends from 12 km to 50 km from the surface of earth. The density of air of this region varies from 0.1 kg/m^3 to 10^{-3} kg/m^3. There is an ozone layer in this region in between 30 km to 50 km from the surface of earth, which absorbs a large portion of ultraviolet radiations radiated by sun or coming from outer space.

Mesosphere. It extends from 50 km to 80 km from the surface of earth. The density of air in this layer varies from 10^{-3} kg/m^3 to 10^{-5} kg/m^3. The temperature of this region falls from 280 K to 180 K with height.

Ionosphere. It extends from 80 km to 400 km from the surface of the earth. The density of this region varies from 10^{-5} kg/m^3 to 10^{-10}kg/m^3. In this region temperature increases with height from 180 K to 700 K, that is why, it is called thermosphere. Ionosphere is the outermost part of the earth's atmosphere. It is composed of ionized matter (i.e. electrons and positive ions) which plays an important role in space communication.

(b) Behaviour of atmosphere toward Electromagnetic wave

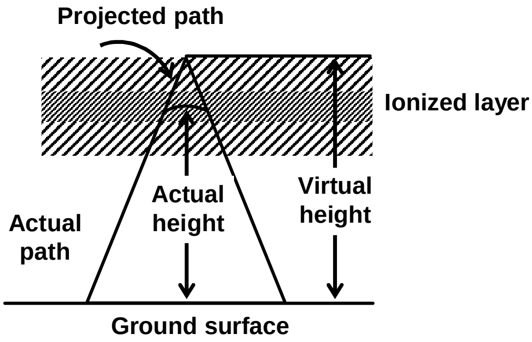
The behaviour of atmosphere is different for electromagnetic waves of different frequencies. The atmosphere is transparent to electromagnetic waves of visible region of wavelength range $4000\overset{0}{\text{A}}$ to $8000\overset{0}{\text{A}}$, as we can see the sun and the stars through it clearly. The electromagnetic waves belonging to infrared region of wavelength range 8×10^{-7} m to 3×10^{-5} m are not allowed to pass through atmosphere rather they get reflected by atmosphere. The ozone layer of earth's atmosphere blocks the electromagnetic waves of ultraviolet region of wavelength range $6\overset{0}{\text{A}}$ to $4000\overset{0}{\text{A}}$. The behaviour of earth's atmosphere towards electromagnetic waves of wavelength 10^{-10} m and higher is of special interest in space communication. The lower part of atmosphere is more or less transparent to electromagnetic waves of wavelength 20 m and higher used in radio communication but the top most layer, the ionosphere does not allow these waves to penetrate but reflects them back towards earth. Beyond a certain frequency, above 40 MHz, the ionosphere bends any incident electromagnetic wave but does not reflect it back towards earth.

(c) Space Communication
The term space communication refers to sending, receiving and processing of information through space. We have the following modes of space communication:
Ground or surface wave propagation.
Space wave or troposphere wave propagation.
Sky wave propagation.
Satellite communication.

Ground Wave or Surface Wave Propagation
It is a mode of propagation in which the ground waves or surface waves progress along the surface of earth. The ground wave transmission becomes weaker as its frequency increases, because more absorption of ground waves takes place at higher frequency during propagation. Due to which the strength of ground wave signal is reduced by absorption to such an extent that it becomes useless beyond an area of a few kilometers around the transmitting antenna.

(d) Sky wave propagation
The key waves are the radio wave of frequency range between 2 MHz to 30 MHz. The ionosphere reflects these radio waves propagation is also known as ionosphere propagation. The sky waves are used for very long distance radio communication at medium and thigh frequencies (i.e., at medium waves and short waves). The radio waves can cover a distance approximately 4000 km in a single reflection from the ionosphere. The sky wave propagation can cover a very long distance and so round – the – globe communication is possible.

Important terms for sky wave propagation:
(i) The virtual height
As shown in the figure as the wave is refracted, it is bent down gradually rather than sharply. Below the ionized layer the incident and refracted rays follow the same path as in the case of reflection from a surface at a greater height, called virtual height. If the virtual height is known then we can calculate the angle of incidence required for the wave to return to ground at a selected point.



(ii) Critical frequency
It is that highest frequency of radio wave, which when sent straight (i.e. normally) towards the layer of ionosphere gets reflected from ionosphere and returns to the earth. If the frequency of the radiowave propogated vertically is more than critical frequency, it will not be reflected by ionosphere. The critical frequency of a sky wave for reflection from a layer of atmosphere is given by $v_c = 9\,(N_{max})^{1/2}$, where N_{max} is the number density of electron/m^2. The number density per cubic metre of electrons for D, E, F_1 and F_2 layers are 10^9, 10^{11}, 5×10^{11} respectively.

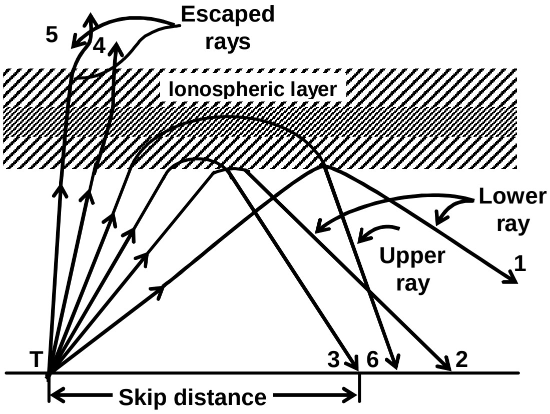
(iii) Maximum usable frequency
It is the limiting frequency of radio waves which when sent at some angle of incidence θ towards the ionosphere, get reflected and returns to the earth.

$$\text{MUF} = \frac{\text{critical frequency}}{\cos\theta}$$

(iv) Skip distance
It is the smallest distance between the transmitting antenna and the point R_v where the sky wave of a fixed frequency, more than critical frequency is first received after refection from ionosphere.

$$D_{skip} = 2h\sqrt{\left(\frac{\nu_{max}}{\nu_c}\right)^2 - 1}$$

where h is the height of reflecting layer of atmosphere, ν_{max} is the maximum frequency of electromagnetic waves and v_c is the critical frequency for that layer of ionosphere.



(v) Fading
It is the fluctuation in the strength of a signal at a receiver due to interference between two waves which left the same source but arrived at the destination by different paths. Fading is more at higher frequencies. Fading causes an error in data transmission and retrieval.

(e) Space wave propagation
The space waves are the radio waves of very high frequency (i.e., between 30 MHz to 300 MHz or more). The space waves can travel through atmosphere from transmitter antenna to receiver antenna either directly or after reflection from ground in the earth's troposphere region. That is why the space wave propagation is also called as Tropospherical propagation.

The space wave propagation is utilized in very high frequency (V.H.F.) bands (between 30 MHz to 3000 MHz), ultra high frequency (U.H.F) bands and microwaves because at such high frequencies, both the sky wave and ground wave and ground wave propagation fail. The space wave propagation is also called as line of sight propagation. This propagation is limited (i) to the line of sight distance (ii) by the curvature of the earth. The line of sight distance is the distance between transmitting antenna and receiving antenna at which they can see each other, which is also called range of communication. Thus the range of communication can be increased by increasing the heights of transmitting and receiving antennas.

The space wave propagation is utilized in television transmission, radar communication, etc.

Height of transmitting antenna
If PQ is a T.V. transmitting antenna of height h, the signals transmitted from it can be received within a circle of radius QS (or QT0 on the surface of earth. This distance is limited due to the curvature of the surface of earth.

Let R = radius of the earth

QT = QS = d, PQ = h

0Q = OP = PQ = R + h

R>> d, so that TQ is almost tangent to the surface of earth at T. From the right-angled triangle OTQ, we have

$(OQ)^2 = OT^2 + QT^2$

or $(R + h)^2 = R^2 + d^2$

or $d^2 = h^2 + 2Rh$

Since h << R, therefore $d^2 = 2hR$

or $d = \sqrt{2hR}$

Area covered for T.V. transmission

$= \pi d^2 = \pi\, 2hR$

Population covered = population density X area covered

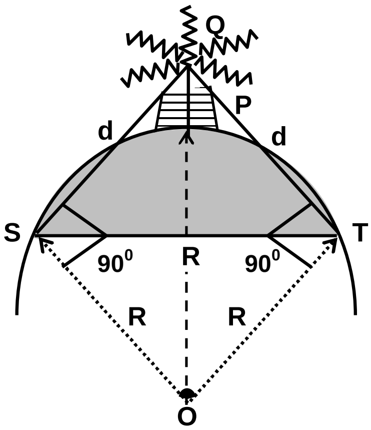


Illustration 5. *What should be the height of transmitting antenna if the T.V telecast is to cover a radius of 128 km? $R_e = 6.4 \times 10^6$ m. If the average population density around the tower is 1000/km^2, how much population is covered?*

Solution: Height of transmitting antenna

$$h = \frac{d^2}{2R} = \frac{\left(128 \times 10^3\right)^2}{2 \times 6.4 \times 10^6} = 1280 \text{ m.}$$

Total population covered

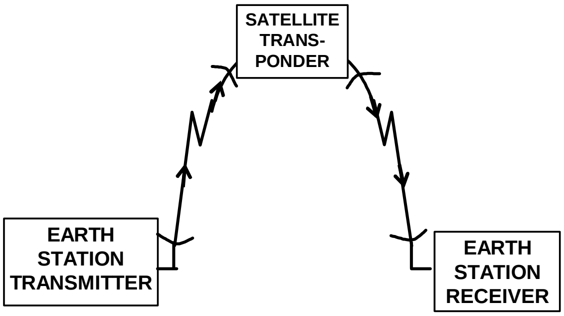
$= \pi d^2 \times$ population density

$= 3.14 \times (128)^2 \times 1000 = 5.14 \times 10^7$.

Exercise 4: ***A TV tower has a height of 100 m. By how much the height of tower be increased to triple it coverage range.***

Satellite Communication

The satellite communication is a mode of communication of signal between transmitter and receiver through satellite. The satellite communication is like the line of sight microwave communication. The basic principal of satellite communication is schematically shown in figure. A communication satellite is a space craft we can classify satellites into two types:



Passive satellites simply act as reflector which can be a metallic sphere. Moon, the natural satellite of earth, is a passive satellite.

The active satellite has antenna system, transmitter, receiver and power supply. It works as an active microwave repeater in the sky called transponder. The transponder fitted on the satellite, amplifies the microwave signals emitted by the transmitter from the surface of earth and sends them to the receiving stations on earth. Sputnik launched by Russia in 1957 was the first active satellite. The line of sight microwave communication through satellite is possible if the communication satellite is always at a fixed location with respect to the earth (geostationary satellite). A geostationary satellite has the same time period of revolution around the earth as the time period of rotation of earth about its polar axis, i.e. 24 hours.

The orbit in which the geostationary satellite revolves around the earth is known as geosynchronous orbit. The height of geosynchronous orbit from the surface of the earth is about 36000 km from the equatorial plane. This height gives a satellite the same angular velocity as the earth, as a result of which it appears to be stationary over a fixed point on the equator. A communication link between two stations on earth at a large distance apart can be achieved through a geostationary satellite. A single satellite cannot cover the entire surface of the earth as the large part of the earth is out of sight due to curvature of the earth. At least three geostationary satellites which are 120^0 apart from each other are required to have the communication link over the entire globe of earth.

Remote Sensing

'Remote sensing' is a technique of obtaining information about an object/area from a distance, without being in physical contact with it.

There are two classes of electromagnetic sensor systems: passive systems and active systems. Passive system measure radiant energy reflected or emitted by an object. Reflected energy falls mostly in the visible light and near – infrared regions, whereas emitted energy lies in the longer thermal infrared region. The most familiar instrument of the passive type is the camera using film sensitive to reflected energy at wavelengths in the visible range. Active systems use a beam of wave energy as a source, sending the beam is recorded by a detector. A simple analogy would be the use of a spot light on a dark night to illuminate a target, which reflects light back the eye. Remote sensing satellite systems provide data for weather forecasting, agriculture production forecasting resource explanation and environmental monitoring.

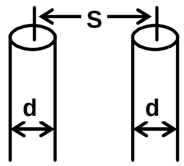
Line communication

In many communication systems, it is often necessary to interconnect points that are some distance apart from each other. For example, interconnection between (i) a transmitter and receiver, (ii) a transmitter and antenna, (iii) an antenna and a receiver. The wire systems which have properties that cannot be ignored are called transmission lines. The sizes of the wires and their general layout become significant, when transmission is at high frequencies of low wavelengths. The most commonly used two wire lines are

(i) parallel wire lines (ii) Twisted pair wire lines (iii) Co – axial wire lines.

(a) Parallel wire line

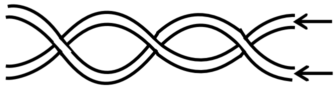
The parallel wire line is used where balanced properties are required, e.g., in connecting a folded – dipole antenna with the T.V. receiver set. These are in the form of a block ribbon where the spacing between the conductors and insulation is chosen according to the power to be handled.



(a) Parallel - wire

(b) Twisted pair line

It consists of two insulated copper wires twisted around each other, often used to connect telephone systems. Twisting help in minimizing electrical interference. These wire line can transmit analog and digital signals, although they cannot transmit signals over very large distances.



(c) Co-axial wire lines:

These wires are used when unbalanced properties are needed, often used to interconnect a transmitter and an earthed antenna. A co-axial wire line consists of inner and outer conductors separated by low dielectric insulators, e.g., polyethylene and Teflon. Co-axial line wires can also be used

Primary constants of transmission line

(a) The four line parameters R, L, G and C are referred to as primary constants of the transmission line. We may define them as follows:

1. Resistance (R) is defined as loop resistance per unit length of line. It is equal to sum of resistance of both the wires for unit line length.

2. Inductance (L) is defined as loop inductance per unit length of the line. It is equal to sum of inductance of both wires for unit line length.

3. Capacitance (C) is defined as shunt capacitance between two conductors per unit length of line.

4. Conductance (G) is defined as shunt conductance between the two conductors per unit length of line.

5. The series impedance of transmission line per unit length is

$Z = R + j\,\omega L$

6. The shunt admittance of the transmission line per unit length is

$Y = G + j\,\omega\, C$

(c) Velocity factor of a transmission line

All electromagnetic waves travel through air/vacuum with a speed = 3×10^8 m/s. In any other medium, this velocity reduces and is given by

$$v = \frac{c}{\sqrt{K}} = \frac{3\times10^8\ m/s}{\sqrt{K}}$$

where c = velocity of light in vacuum

v = velocity of light in any other medium

K = dielectric constant of the medium (=1 for vacuum and very nearly 1 for air)

The velocity factor (V.F.) of a transmission line is defined as the velocity reduction ratio, i.e.,

$$V.F. = \frac{v}{c} = \frac{1}{\sqrt{K}}$$

The dielectric constants of materials commonly used in transmission lines range from 1.2 to 2.8. Therefore, the velocity factor of a transmission line ranges from 0.9 to 0.6.

Illustration 6. *The dielectric constant of insulation used in a transmission line is 2.6. Calculate the velocity factor of the line.*

Solution: Here, k = 2.6, V.F. =?

As $V.F. = \frac{1}{\sqrt{k}} = \frac{1}{\sqrt{2.6}} = \frac{1}{1.61}$

$\therefore$ V.F. = 0.62

Exercise 5: ***A parallel wire line has air as dielectric. What can be the minimum characteristic impedance of such a line.***

Telephone link

A telephone link can be established ground waves, sky waves, microwaves, Co–axial cables and optical fiber cables.

Simultaneous transmission of a number of messages over a single channel without their interfering with one another is called multiplexing. Two types of multiplexing techniques are in use:

Frequency division multiplexing uses analog modulation of message signals.
Time division multiplexing makes use of pulse modulation of message signals.

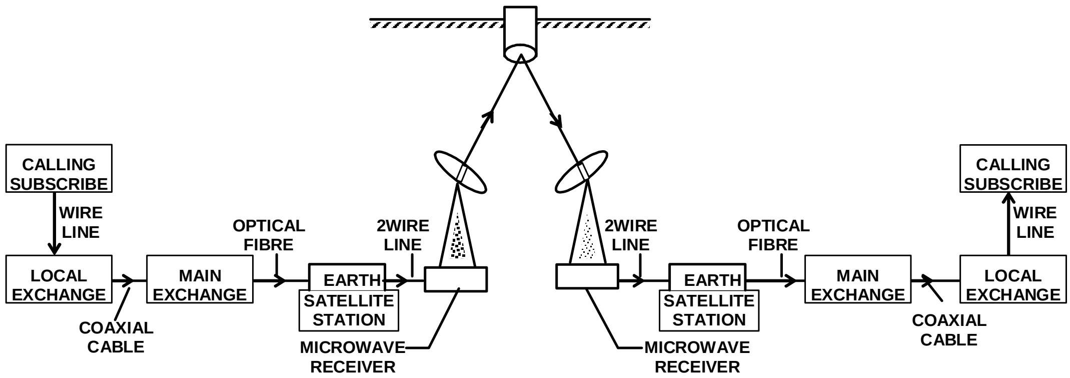
Twisted pair wire lines provide a band width of 2 MHz, while co – axial cable provides a band width of 20 MHz. For further increases in band width, we use
Microwave link.
Communication satellite link.

Microwave Link:

In a microwave transmission system, the transmitter and receiver of the system, which are mounted on high towers must be in the line of sight. This is because high frequency microwaves (used for carrying message signals) cannot bend across the obstacles like mountains and top of buildings etc. in their path. Further, curvature in the surface of earth also poses a problem. That is why range of microwave transmission is limited to 50 km only.

Communication Satellite Link:

In microwave communication link, the line of sight is intercepted on account of curvature in the surface of earth and presence of mountains and other tall structures. Therefore, several repeaters are normally required for long distance transmission. The cost of transmission increases exorbitantly. This problem has been overcome using communication satellites. Basically, a communication satellite is a microwave relay station, placed in geostationary orbit around the earth at a height of about 36000 km. Such a satellite stays over a particular place on the earth, and is accessible from any place.

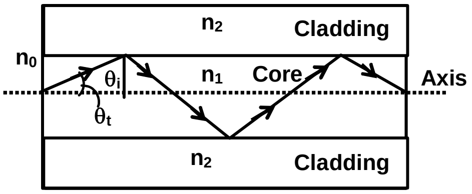


Optical Communication

It is a mode of communication by which we can transfer the information from one place to another through optical carrier waves. The information carrying capacity of a communication system is directly proportional to its band width. The wider the band width, the greater is its information carrying capacity. , the band width of a communication system is the measure of percentage of its carrier frequency; which is about 10% of the carrier frequency. For a VHF radio system operating at 100 MHz, band width is equal to 10 MHz (i.e. 10% of the carrier frequency). A microwave radio system operating at 6 GHz, has a band width equal to 0.6 GHz (= 600 MHz). It means, higher the frequency of carrier waves, the wider is the band width possible and consequently, the greater is the information capacity.

(a) Optical fibre

An optical fibre consists typically of a transparent core fibre of glass of index of refraction n_1 surrounded by a transparent glass sheath or cladding of slightly lower index n_2, with both enclosed in an opaque protective jacket. Figure shows a cross-section through the axis of an optical fibre. A ray entering the core from an external medium of index n_0 at an angle θ_e will make an angle θ_t with respect to axis inside the core.



From Snell's law

$$\sin\theta_t = \frac{n_0}{n_1}\sin\theta_e \qquad \text{.........(1)}$$

The ray continuing in the core will be incident on the core-cladding boundary at an angle θ_i. If $\theta_I > \theta_{ic'}$ where θ_{ic} is the critical angle, the ray will be totally internally reflected and continue to propagate inside the core.

$$\sin\theta_{ic} = \frac{n_2}{n_1} \qquad \text{..........(2)}$$

we have $$\sin\theta_e = \frac{n_1}{n_0}\sin\theta_t = \frac{n_1}{n_0}\sin\left(90^0 - \theta_{ic}\right) = \frac{n_1}{n_0}\cos\theta_{ic} \qquad \text{..........(3)}$$

or $$\sin\theta_e = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \qquad \text{..........(4)}$$

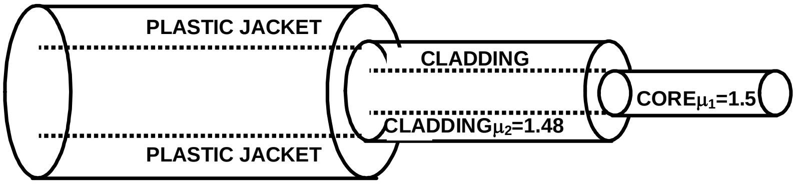
where θ_e = entrance angle on the axis of the core,
n_1 = index of refraction of core,
n_2 = index of refraction of cladding,
n_0 = index of refraction of external medium.
For air as the external medium ($n_0 = 1$), Equation (4) reduces to
$\sin\theta_e = \sqrt{n_1^2 - n_2^2}$

(b)Types of optical fibre

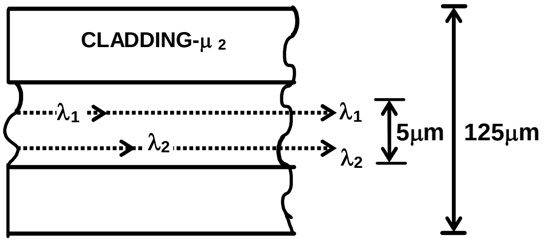
There are three types of optical fibre configurations.
(a) Single – mode step-index fibre
(b) Multi-mode step-index fibre
(c) Multi-mode graded-index fibre.

(i) Single-mode step-index fibre:

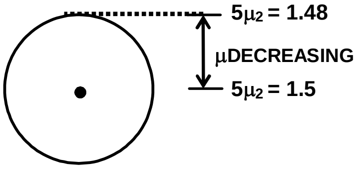
This fibre has a very narrow central core of glass of diameter about 5 μm to 10 μm. Its refractive index is $\mu_1 = 1.5$. The outside cladding is relatively big, about 125 μ m in diameter and of refractive index $\mu_2 = 1.48$

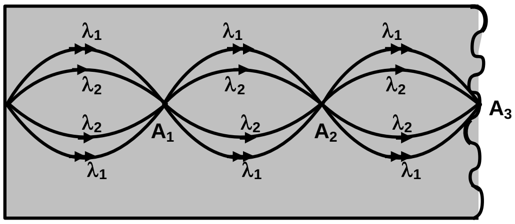


(ii) Multi-mode step-index fibre: Its construction is similar to that of single-mode step-index fibre with the only difference its central core is much bigger and is of diameter about 50μm. The refractive index μ_1 of the central core is uniform and constant throughout the core. The cladding thickness is less than that of single – mode step – index fibre, figure.



(iii) Multi-mode graded-index fibre: In this fibre, there is no clearcut separation between core and cladding. The refractive index of core and cladding is non-uniform which is maximum at the central axis of core and decreases smoothly towards the outer surface of the fibre (i.e. cladding) Figure.





(c) Numerical aperture (NA) of optical fibre

It is the figure of merit, which is used to describe the light collecting ability of an optical fibre. The larger is the magnitude of numerical aperture of a fibre, the greater wil be the amout of light collected by the fibre from the external source of light.

For step index fibre

$$NA = \sqrt{\mu_1^2 - \mu_2^2} = \mu\left(2\Delta\right)^{\frac{1}{2}}$$

Δ = relative core cladding difference

Illustration 7. *A silicon optical fibre with a core diameter large enough has a core refractive index of 1.50 and a cladding refractive index 1.47. Determine*
(i) the critical angle at the core cladding interface,
(ii) the numerical aperture for the fibre
(iii) the acceptance angle in air for the fibre.

Solution: Here, μ_1 = 1.50; μ_2 = 1.47;
μ_0 = 1

(i) Critical angle θ_c at the core – cladding interface is given by

$$\theta_c = \sin^{-1}\frac{1.47}{1.50} = 78.5^0.$$

(ii) Numerical aperture, $NA = \left(\mu_1^2 - \mu_2^2\right)^{\frac{1}{2}}$

$$= \left[\left(1.50\right)^2 - \left(1.47\right)^2\right]^{\frac{1}{2}} = (2.25 - 2.16)^{1/2} = 0.30.$$

(iii) Acceptance angle $\theta_a = \sin^{-1}(NA)$
$= \sin^{-1}(0.30) = 17.4^0$.

Exercise 6: ***Find relation between acceptance angle and numerical aperture of optical fibre.***

(d) Laser

Laser stands for light amplification by stimulated Emission of Radiation. The laser shoots the photons in forward direction in a very high concentration with pure and uniform wavelength, whereas normal light source shoots out stray random photons in all directions. Laser beam have high intensity and strength, which are used for variety of purposes. The working of a laser is based on stimulated emission of radiation and population inversion.

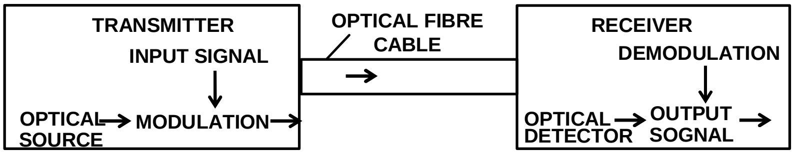
Main components of a laser

To produce a laser beam the laser system has three main components.
Active medium (A)
Pump (P)
Resonator guide.

Light modulation

It is process of varying the intensity of light directly in accordance with the information signal (like speech, music, digital code etc.), available in the form of electrical signal. The light modulation is achieved by varying the current/voltage through the optical source (LED or diode laser) by biasing it, corresponding to the information signal voltage produces a modulated light signal, emerging out of the optical source whose intensity varies in accordance with the information signal.

For a proper light modulation, the LED or diode laser is biased near its threshold voltage so that the small variation in the information signal voltage produces the desired light intensity modulation.



MISCELLANEOUS EXERCISE

1. What is the range of values of modulation index of an AM wave?

2. What does a modem present?

3. What is the expression for band width in FM transmission ?

4. What is the significances of modulation index ?

5. What is meant by 'sampling' of an analog signal ?

6. A 2000 kHz carrier is modulated with 2 Hz audio sine waves. What are the side band frequencies? What are the side band frequencies of the first pair?

7. Calculate the length of half wave dipole antenna at 50 MHZ.

8. What is a transducer ? Give two examples.

9. Why is an FM signal less susceptible to noise than an AM signal ?

10. What is varied in an FM wave ?

ANSWERS TO MISCELLANEOUS EXERCISE

2. It is a device that modulates and demodulates.

3. In frequency modulation,
Band width = 2n $\times$ frequency of audio modulating signal
Where n = number of significant side bands

6. 2002 kHz, 1998 kHz.

7. 3m

SOLVED PROBLEMS

Subjective:

Problem 1. *An audio signal of 15000 Hz modulates a carrier generated by a tank circuit containing 1 nF capacitor and 10 microhenry inductor. Calculate the frequencies of first pair of side bands.*

Solution: Here, f_m = 15000 Hz = 15 kHz
C = 1 nF = 10^{-9}F, f_{SB} = ?
L = 10μ H = 10 $\times$ 10^{-6} H = 10^{-5} H

$$f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times\frac{22}{7}\sqrt{10^{-5}\times10^{-9}}}$$

$$= \frac{10^7\times7}{44}\text{Hz} = 1592\,\text{kHz}$$

$f_{SB} = f_C \pm f_m$
= 1592 $\pm$ 15 = 1607 kHz or 1577 kHz.

Problem 2. *Calculate modulation index of an FM signal in which the modulating frequency is 2 kHz and maximum deviation is 10 kHz.*

Solution: Here, f_m = 2kHz, δ_{max} = 10kHz

As $\quad m_f = \dfrac{\delta_{max}}{f_m}$

$\therefore \quad m_f = \dfrac{10}{2} = 5.$

Problem 3. *A T.V. tower has a height of 100 m. How much population is covered by the T.V. broadcast if the average population density around the tower is 1000 per sq. km. Radius of the earth 6.37 $\times$ 10^6 m. By how much height of the tower be increased to double its coverage range?*

Solution : Here, h= 100 m;
R = 6.37 $\times$ 10^6 m;
Population density, ρ == 1000 km^{-2}
= 1000 $\times$ 10^{-6} m^{-6} m^{-2} = 10^{-3} m^{-2}
Population covered = $\rho\,\pi\,d^2 = \rho\,\pi\,(2hR)$

$$= 10^{-3}\times\frac{22}{7}\times2\times100\times6.37\times10^6$$

= 40 $\times$ 10^5 = 40 lakhs
Now, d' = $\sqrt{2h'R}$
$= 2d = 2\sqrt{2hR}$
or h' = 4h = 4 $\times$ 100 = 400 m
Increase in height of tower = h' – h
= 400 – 100 = 300 m.

Problem 4. *A step index fibre has a relative refractive index difference of 0.9%. Estimate the critical angle at the core-cladding interface, when the core index is 1.46 find the numerical aperture, if the source to fiber medium is air.*

Solution : Here, $\mu_1 = 1.46$;

$$\frac{\mu_1 - \mu_2}{\mu_1} = \frac{0.9}{100} = 0.009$$

or $$1 - \frac{\mu_2}{\mu_1} = 0.009$$

or $$\frac{\mu_2}{\mu_1} = 1 - 0.009 = 0.991$$

Critical angle,

$$\theta_c = \sin^{-1}\frac{\mu_2}{\mu_1} = \sin^{-1} 0.991 = 82.3^0$$

$$\mu_2 = 0.991 \times \mu_1 = 0.991 \times 1.46 = 1.45$$

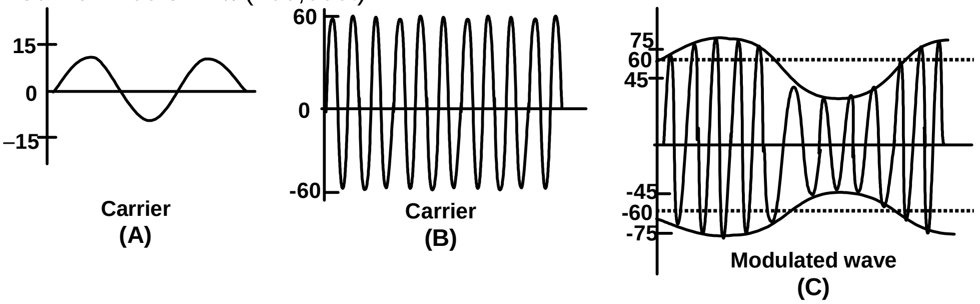
$$NA = \sqrt{\mu_1^2 - \mu_2^2}$$

$$= \sqrt{(1.46)^2 - (1.45)^2} = 0.17$$

Problem 5. *An audio signal 15 sin 2π (1500t) amplitude modulates a carrier 60 sin 2π (100,000t):*

(a) Sketch the audio signal.
(b) Sketch the carrier.
(c) Construct the modulated wave.
(d) Determine the modulation factor and per cent modulation.
(e) What are the frequencies of the audio signal and the carrier?
(f) What frequencies would show up in a spectrum analysis of the modulated wave?

Solution: Given: Audio signal = 15 sin 2π (1500t)
Carrier = 60 sin 2π (100,000t)



Carrier (A) **Carrier (B)** **Modulated wave (C)**

(a) Locate the amplitude of the modulated wave:
(b) Using the amplitude of the carrier as an axis, lay in the audio signal.
(c) Note that the envelope has been determined, a signal having an amplitude defined by the envelope found above and having frequency of the carrier is laid in within the envelope.
(d) Using the following equation for modulation factor,

$$m = \frac{\text{audio amplitude}}{\text{carrier amplitude}} = \frac{B}{A}$$

$$=\frac{15}{60}=\frac{1}{4}=0.25$$

Converting modulation factor to per cent modulation,

M = m $\times$ 100

= 0.25 $\times$ 100 = 25%

(e) Since Audio signal = B sin $2\pi f_a t$

= 60 sin 2π (100,000 t)

f_c = 100,000 Hz

(f) The frequency spectrum of an amplitude – modulated wave consists of

f_c, $f_c + f_a$ and $f_c - f_a$

f_c = 100,000 Hz

$f_c + f_a$ = 100,000 – 1500 = 98,500 Hz

The frequencies of the modulated wave are 100,000 Hz, 101, 500 Hz, 98.500 Hz

Problem 6. *What should be the transmission bandwidth of an FM signal with 75 kHz deviation and highest frequency of modulation 15 kHz?*

Solution : Frequency deviation, Δf = 75 kHz

Highest frequency of modulation, f_m = 15 kHz

The bandwidth occupied by the spectrum, BW = $2nf_m$ where n is the highest order of the significant sideband.

However, approximate expression for spectrum band width is

BW = $2(1 + m_f) f_m$

$$=2\left(1+\frac{\Delta f}{f_m}\right)f_m$$

$$=2\left(\Delta f+f_m\right)=2\left(75+15\right)=180\text{kHz}$$

Problem 7. *How many 600 kHz waves can be on a 5 mile transmission line simultaneously?*

Solution : Given: f = 600 kHz; d = 5 mile

First determine the wavelength of the 600 kHz signal using, $f\lambda = c$

Use 186000 mile per second as the speed of light since the line length is given in mile and no figure is given for the velocity factor.

Substitution numerical values,

$600\times10^3\lambda = 186000$

$$\lambda=\frac{186000}{600\times10^3}=0.31\text{ mile}$$

Knowing the wavelength of the signal and the length of the line, the number of cycle on the line can be found from

$$n=\frac{d}{\lambda}=\frac{5}{0.31}=16.13$$

Problem 8. *Determine the characteristic impedance of a transmission line which has a capacitance of 35 pF/ft and an inductance of 0.25 μH/ft.*

Solution: Given: C = 35 pF/ft; L = 0.25 μH/ft

Using the equation relation Z_0, L and C,

$Z_0 = \sqrt{L/C}$

Substituting numerical values, $Z_0=\sqrt{\frac{0.25\times10^{-6}}{35\times10^{-12}}}=84.5\Omega$

Problem 9. *An optical communication system, having an operating wavelength, λ (in metres), can use only x% of its source frequency as its channel band width. The system is to be used for transmitting TV signal requiring a band width of F hertz. How many channels can this system transmit simultaneously?*

Solution : Optical source frequency, $v = \frac{c}{\lambda}$ Hz.

Band width of channel = x% of v

$$= \frac{x}{100}v = \frac{x}{100}\frac{c}{\lambda}$$

Number of channels

$$= \frac{\text{total band width of channels}}{\text{band width for one channel}}$$

$$= \frac{xc}{100\lambda F}$$

Problem 10. *Consider an optical communication system operating at λ = 600 nm. Suppose, only 1% of the optical source frequency is the available channel band width for optical communication. How many channels can be accommodated for transmitting:*
(a) audio – signals requiring a band width of 9 kHz and
(b) video T.V. signals requiring an approximate band with of 5 MHz.

Solution : Here, $\lambda = 600 \text{ nm} = 600 \times 10^{-9}$ m

Optical source frequency, $v = \frac{c}{\lambda}$

$$= \frac{3\times10^{8}}{600\times10^{-9}} = \frac{1}{2}\times10^{15}\text{Hz}$$

Total band width of channel = 1% of source frequency

$$= \frac{1}{100}\times\frac{1}{2}\times10^{15} = \frac{1}{2}\times10^{13}\text{Hz}$$

Number of channels

$$= \frac{\text{total band width of channel}}{\text{band width needed per channel}}$$

(a) Number of channels for audio signal

$$= \frac{1}{2}\times10^{13}/\left(9\times10^{3}\right) \approx 5.6\times10^{8}$$

(b) Number of channels for video TV signal

$$= \frac{1}{2}\times10^{13}/5\times10^{6} \approx 6.7\times10^{5}.$$

Problem 11. *A photodiode is made from a semiconductor $In_{0.53}\,Gu_{0.47}$ As, with E_g = 0.73 eV. What is the maximum wavelength which it can detect? $h = 6.63\times10^{-34}$ J_S.*

Solution: The maximum wavelength (λ), which a photodiode can detect corresponds to the energy E_g. So $\frac{h_c}{\lambda} = E_g$

or $\lambda = \frac{h_c}{E_g} = \frac{6.63\times10^{-34}\times\left(3\times10^{8}\right)}{0.73\times1.6\times10^{-19}}$

$= 1703 \times 10^{-9}$ m = 1703 nm

Problem 12. *What is the modulation index of an FM signal having a carrier swing of 100kHz when the modulating signal has a frequency of 8 kHz?*

Solution: Given: Carrier Swing = 100 kHz

$f_a = 8$ kHz

From the defining equation, $m_f = \frac{\Delta f}{f_a}$

First determining Δf, $\Delta f = \frac{\text{Carrier Swing}}{= 50\,\text{kHz}} = \frac{100\times10^{3}}{2}$

Now substituting into the equation for m_f,

$$m_f = \frac{50\times10^{3}}{8\times10^{3}} = 6.25$$

Problem 13. *How many AM broadcast stations can be accommodated in a 100 kHz bandwidth if the highest frequency modulating a carrier is 5 kHz?*

Solution : Given: Total BW = 100 kHz

$F_{a\,max.} = 5$ kHz

Any station being modulated by a 5 kHz signal will produce an upper – side frequency 5 kHz above its carrier and a lower – side frequency 5 kHz below its carrier, thereby requiring a bandwidth of 10 kHz. Thus,

Number of stations accommodated $= \frac{\text{Total BW}}{\text{BW per station}} = \frac{100\times10^{3}}{10\times10^{3}}$

Number of stations accommodated = 10 stations

Problem 14. *A lossless coaxial cable has a capacitance of 7×10^{-11} F and an inductance of 0.39 μH. Calculate characteristic impedance of the cable.*

Solution: Here, $C = 7 \times 10^{-11}$ F,

$L = 0.39 \times 10^{-6}$ H

Z_0 = ? As the cable is lossless,

$\therefore Z_0\sqrt{\frac{L}{C}} = \sqrt{\frac{0.39\times10^{-6}}{7\times10^{-11}}} = 75\,\text{ohm}$

Problem 15. *The velocity factor of a transmission line is 0.62. Calculate dielectric constant of the insulation used.*

Solution : Here, V.F. = 0.62, k =?

As $k = \frac{1}{(\text{V.F.})^2}\ \therefore k = \frac{1}{(0.62)^2} = 2\cdot6.$

Objective:

Problem 1. *The velocity of electromagnetic waves in a dielectric medium (ε_r = 4) is:*
(A) 3×10^8 metre/second *(B) 1.5×10^8 metre/second*
(C) 6×10^8 metre/second *(D) 7.5×10^7 metre/second*

Solution : $\mu = \frac{c}{v} \Rightarrow v = 7.5 \times 10^7 \, m/sec$
$\therefore$ **(D)**

Problem 2. *For television broadcasting, the frequency employed is normally.*
(A) 30-300 MHz *(B) 30-300 GHz*
(C) 30-300 kHz *(D) 30-300 Hz*

Solution : $\therefore$ **(B)**

Problem 3. *An 'antenna' is:*
(A) inductive
(B) capacitive
(C) resistive above its resonance frequency
D) none of the above

Solution : $\therefore$ **(A)**

Problem 4. *The characteristic impedance of a co-axial cable is of the order of:*
(A) 50Ω *(B) 200Ω*
(C) 270Ω *(D) none of these*

Solution: At audio frequency

$$Z_0 = \sqrt{\frac{R + j\omega L}{C4 + j\omega C}}$$

At radio frequency $z = \sqrt{\frac{L}{C}}$

$\therefore$ **(C)**

Problem 5. *Long distance short-wave radio broadcasting uses:*
(A) ground wave *(B) ionospheric waves*
(C) direct wave *(D) sky wave*

Solution: $\therefore$ **(B)**

Problem 6. *Propagation constant of a transmission line is:*
(A) $\frac{R + j\omega L}{G + j\omega C}$ *(B)* $\sqrt{(R + j\omega L)(G + j\omega C)}$
(C) $\sqrt{1/LC}$ *(D)* $\sqrt{(R - j\omega L)(G - j\omega C)}$

Solution: $\therefore$ **(B)**

Problem 7. *The T.V. transmission tower in Delhi has a height of 240 m. The distance up to which the broadcast can be received (taking the radius of earth to be 6.4 $\times 10^6$m) is:*

(A) 100 km *(B) 60 km*
(C) 55 km *(D) 50 km*

Solution: $h = \dfrac{d^2}{2R}$

$d^2 = 240 \times 2 \times 6.4 \times 10^6$ m

$d \equiv 55\,\text{km}$

$\therefore$ **(C)**

Problem 8. *Broadcasting antennas are generally:*

(A) omnidirectional type *(B) vertical type*
(C) horizontal type *(D) none of these*

Solution: $\therefore$ **(C)**

Problem 9. The characteristic impedance of a transmission line is:

(A) $\sqrt{\dfrac{L}{C}}$ (B) $\sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$
(C) $\sqrt{(R + j\omega L)(G + j\omega C)}$ (D) $\sqrt{C/L}$

Solution: $\therefore$ **(B)**

Problem 10. *The AM wave contains three frequencies, viz.:*

(A) $\dfrac{f_c}{2}, \dfrac{f_c + f_s}{2}, \dfrac{f_c - f_s}{2}$ *(B)* $2f_c, 2(f_c + f_s), 2(f_c - f_s)$
(C) $f_{c'}(f_c + f_s), (f_c - f_s)$ *(D)* $f_{c'}\, f_{c'}\, f_c$

Solution: $\therefore$ **(C)**

Problem 11. *In AM waves, the amplitude of each side band frequency is:*

(A) E_c *(B)* mE_c
(C) $\dfrac{mE_c}{2}$ *(D)* $2mE_c$

Solution: $\therefore$ **(C)**

Problem 12. *In AM wave, carrier power is given by:*

(A) $P_c = \dfrac{2E_c^2}{R}$ *(B)* $P_c = \dfrac{E_c^2}{R}$
(C) $P_c = \dfrac{E_c^2}{2R}$ *(D)* $P_c = \dfrac{E_c^2}{\sqrt{2}R}$

Solution: $\therefore$ **(C)**

Problem 13. *In AM wave, total power of side bands is given by:*

(A) $P_s = \frac{E_c^2}{4R}$ *(B)* $P_s = \frac{4E_c^2}{R}$

(C) $P_s = \frac{m^2E_c^2}{4R}$ *(D)* $P_s = \frac{E_c^2}{4m^2R}$

Solution: $\therefore$ **(C)**

Problem 14. *Fraction of total power carried by side bands is given by:*

(A) $\frac{P_s}{P_T} = m^2$ *(B)* $\frac{P_s}{P_T} = \frac{1}{m^2}$

(C) $\frac{P_s}{P_T} = \frac{2+m^2}{m^2}$ *(D)* $\frac{P_s}{P_T} = \frac{m^2}{2+m^2}$

Solution: $\therefore$ **(D)**

Problem 15. *When m = 1, power carried by side bands is:*

(A) 11.1 % of the total power of AM wave
(B) 22.2 % of the total power of AM wave
(C) 33.3 % of the total power of AM wave
(D) 44.4 % of the total power of AM wave

Solution: $\therefore$ **(C)**

ASSIGNMENT PROBLEMS

Subjective:

Level- O

1. What is meant by channel and channel noise?

2. What are the band widths of an A.M. radio station and an FM radio station?

3. What is full form of AGC? Where is it used?

4. What is the basic difference between an analog communication system and a digital communication system?

5. What is the significance of modulation index?

6. Explain why TV transmission towers are usually made high.

7. Why the sky waves are not used in T.V transmission

8. Why we have better high frequency reception at night?

9. In a satellite communication, what is the minimum angular spacing of two satellites using the same frequency band.

10. What do you understand by (a) medium wave band (b) short wave band?

11. Explain the "green house effect" of earth's atmosphere.

12. If the maximum values of signal and carrier waves are 4 volt and 5 volt respectively, then find (i) the maximum and the minimum value of the modulated amplitude in volt (ii) the percentage of modulation.

13. Explain briefly the principle of transmitting signals using a satellite. State two main advantages of using a satellite for transmitting signals.

14. What is meant by 'remote sensing'? Briefly explain, how it is carried out. Mention any two applications of remote sensing.

15. Deduce an expression for the distance from which the T.V. signals can directly be received from a T.V. tower of height h.

16. Explain why an optical fiber transmits light.

17. What is meant by 'switching circuit' in a telephone system?

18. Long distance radio broadcasts use short wave bands. Why?

19. If the earth did not have atmosphere, would its average surface temperature be higher or lower than it is now?

Level - I

1. T.V. tower has a height of 70 m. How much population is covered by the T.V. broadcast if the average population density around the tower is 1000 km^{-2}? Radius of the earth is 6.4×10^6 m.

2. Calculate capacitance of a co-axial cable when its inductance is 0.4 μH and the characteristic impedance is 160 ohm.

3. A parallel wire line has air as dielectric. What can be the minimum characteristic impedance of such a line?

4. T.V. transmission tower at a particular station has a height of 160 m:
   (a) What is its coverage range?
   (b) How much population is covered by transmission, if the average population density around the tower is 1200 km^{-2}?
   (c) By how much the height of tower be increased to double its coverage range? Given radius of earth = 6400 km.

5. T.V. tower has a height 100m. What is the population density around the T.V. tower if the total population covered is 50 lac? Radius of earth = 6.4×10^6 m.

6. The dielectric constant of an insulator used in a transmission line is 2.25. Calculate velocity factor of the transmission line.

7. A step index fibre has a relative refractive index difference of 0.86%. Estimate the critical angle at the core-cladding interface.

8. The velocity of light in the core of a step index fibre is 2×10^8 m/s and the critical angle at the core-cladding interface is 80^0. Find the numerical aperture and the acceptance angle for the fibre in air. The velocity of light in vacuum is 3×10^8 m/s.

9. A ground receiver station is receiving a signal at (a) 5 MHz and (b) 100 MHz, transmitted from a ground transmitter at a height of 400 m located at a distance of 125 km. Identify whether it is coming via space wave or sky wave propagation or satellite transponder. Radius of earth = 6.4×10^6m; maximum number density of electrons in ionosphere $= 10^{12}\ m^{-3}$.

10. On a particular day, the maximum frequency reflected from the ionosphere is 9 MHz. On another day, it was found to increase by 1 MHz. Calculate, the ratio of the maximum electron densities of the ionosphere on the two days.

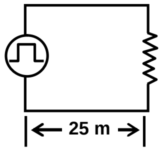
Level - II

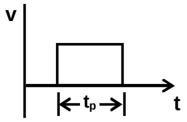
1. A 75 MHz carrier having an amplitude of 50 V is modulated by a 3 kHz audio signal having an amplitude of 20 V:
   (a) Sketch the audio signal.
   (b) Sketch the carrier.
   (c) Construct the modulated wave.
   (d) Determine the modulation factor and per cent modulation.
   (e) What frequencies would show up in a spectrum analysis of the modulated wave?
   (f) Write trigonometric equations for the carrier and the modulating waves.

2. On a particular day, the maximum frequency reflected from the ionosphere is 10 MHz. On another day, it was found to increase to 11MHz. Calculate the ratio of the maximum electron densities of the ionosphere on the two days. Point out a plausible explanation for this.

3. When the modulation percentage is 75, an AM transmitter produces 10 kW. How much of this is carrier power?

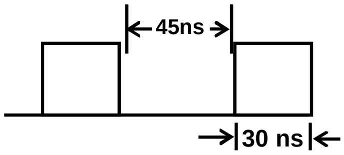
4. Determine the required pulse duration of a pulse so that when the pulse travels on a 25 metre line, the trailing edge occurs at the generator end of the line just as the leading edge reaches the load. Assume that the speed of the pulse on the line is the same as its free – space velocity (3×10^8 metre/s).





5. A loss-less co-axial cable has a capacitance of 7×10^{-11} F and an inductance of 0.39 μH. Calculate the characteristic impedance of the cable.

6. A pulse train is transmitted along a transmission line which is 200 metre long. The pulse train consists of pulses with a duration of 30 ns each and separated by 45 ns. How many pulses can be on the line at any given time? Assume the speed of e.m. waves to be the same as in free space.



7. An optical fibre has a numerical aperture of 0.20 and a cladding refractive index of 1.59. Determine
   (a) the acceptance angle for the fibre in water which has a refractive index of 1.33.
   (b) the critical angle at the core-cladding interface.

8. Calculate the bit rate for a signal, which has a sampling rate of 8 kHz and quantisation levels used are 32.

9. A T.V. tower has a height of 75 m. What is the maximum distance and area upto which this T.V. transmission can be received? Take radius of the earth as 6.4×10^6 m.

10. A T.V. tower has a height of 150 m. How much population is covered by the T.V. broadcast if the average population density around the tower is 1000 km^{-2}? Radius of the earth is 6.4×10^6m.

Objective:

Level - I

1. The characteristic impedance of a loss – less transmission line is given by:
(A) $Z_0 = \sqrt{LC}$
(B) $Z_0 = \sqrt{L/C}$
(C) $Z_0 = \sqrt{C/L}$
(D) $Z_0 = LC$

2. In an amplitude modulated wave for audio-frequency of 500 cycle/second, appropriate carrier frequency will be:
(A) 50 cycle/sec
(B) 100 cycle/sec
(C) 500 cycle/sec
(D) 50,000 cycle/sec

3. If there were no atmosphere, the average temperature on the surface of earth would be:
(A) lower
(B) higher
(C) same as now
(D) 0^0C

4. The Q of a resonant transmission line is:
(A) $Q = \frac{\omega}{LR}$
(B) $Q = \frac{\omega R}{L}$
(C) $Q = \frac{L}{R}$
(D) $Q = \frac{\omega L}{R}$

5. Radio waves of constant amplitude can be generated with:
(A) Filter
(B) Rectifier
(C) FET
(D) Oscillator

6. Ozone layer above earth's atmosphere will:
(A) prevent infrared radiations from sun reaching earth
(B) prevent infrared rays reflected from earth from escaping earth's atmosphere
(C) prevent ultraviolet rays from sun
(D) reflect back radio waves

7. A digital signal possesses:
(A) continuously varying values
(B) only two discrete values
(C) only four discrete values
(D) none of the above

8. A digital signal:
(A) is less reliable than analog signal
(B) is more reliable than analog signal
(C) is equally reliable as the analog signal
(D) none of the above

9. Modern communication system uses:
(A) analog circuits
(B) digital circuits
(C) combination of analog and digital circuits
(D) none of the above

10. The audio signal:
(A) can be sent directly over the air for large distance
(B) cannot be sent directly over the air for large distance
(C) posses very high frequency
(D) none of above

11. If μ_1 and μ_2 are the refractive indices of the materials of core and cladding of an optical fibre, the loss of light due to its leakage can be minimized by having
(A) $\mu_1 > \mu_2$ (B) $\mu_1 < \mu_2$
(C) $\mu_1 = \mu_2$ (D) none of the above

12. The frequencies of electromagnetic waves employed in space communication vary over a range of
(A) 10^4 Hz to 10^7 Hz (B) 10^4 Hz to 10^{11} Hz
(C) 1 Hz to 10^4 Hz (D) 1 Hz to 10^{11} Hz

13. Broadcasting antennas are generally
(A) omni-directional type (B) vertical type
(C) horizontal type (D) none of the above

14. Radio waves of constant amplitude can be generated with
(A) Filter (B) Rectifier
(C) FET (D) Oscillator

15. Which of the following is not transducer ?
(A) Loudspeaker (B) Amplifier
(C) Microphone (D) All of these

Level - II

1. The number of telephone conversations by a fibre of band width 40 MHz, with much less intensity loss, are:
(A) 1800 (B) 2000
(C) 2200 (D) 2400

2. Distance of propagation from antenna of height 20 m is
(A) 80 km (B) 8 km
(C) 16 km (D) 6.8 km

3. If critical frequency for ionosphere is 30 MHz. Calculate the maximum frequency which can come back on earth if transmitted at an angle 60^0 from vertical
(A) 30 MHz (B) 60 MHz
(C) 40 MHz (D) 15 MHz

4. What should be the length of dipole antenna for a carrier wave having frequency 3×10^8 Hz?
(A) 5 m (B) 0.5 m
(C) 1.5 m (D) 1 m

5. Which of the following is not a transducer?
(A) Loud speaker (B) Amplifier
(C) Microphone (D) All of these

6. Intel set satellite work as
(A) transmitter (B) receiver
(C) absorber (D) repeater

7. If critical frequency is 20 MHz for ionosphere, calculate the angle of transmission if 30 MHz frequency is transmitted.
(A) $\cos^{-1}(2/3)$ (B) $\sin^{-1}(2/3)$
(C) $\cos^{-1}(1/3)$ (D) $\sin^{-1}(1/3)$

8. A carrier wave of amplitude A is used to modulate (amplitude) the signal. Modulated signal amplitude swings from 0.1 A to 1.9A, calculate modulation index
(A) 1.8 (B) 0.9
(C) 0.5 (D) 0.75

9. Long distance short wave radio broadcasting uses
(A) ground wave (B) ionospheric wave
(C) direct wave (D) sky wave

10. The length of half wave dipole at 30 MHz is
(A) 10 m (B) 50 m
(C) 5 m (D) 100 m

11. A step index fibre has a relative refractive index of 0.88 %. What is the critical angle at the core-cladding interface ?
(A) 60^0 (B) 75^0
(C) 45^0 (D) none of the above

12. The maximum range of ground or surface wave propagation depends on
(A) the frequency of the radiowaves only
(B) power of the transmitter only
(C) both of them
(D) none of them

13. The length of a half wave dipole at 30 MHz is
(A) 10 m (B) 50 m
(C) 5 m (D) 100 m

14. A laser is a coherent source because it contains
(A) many wavelengths
(B) uncoordinated wave of a particular wavelength
(C) coordinated wave of many wavelengths
(D) coordinated waves of a particular wavelength

15. Intelset satellite works as a
(A) transmitter (B) receiver
(C) absorber (D) repeater

ANSWERS TO ASSIGNMENT PROBLEMS

Subjective:

Level - I

1. 2813440
2. 1.56 pF
3. 83.1 Ω
4. (a) 45255 m (b) 77.24 (c) 480 m
5. 1242.9 k/m^2
6. 0.67
7. 82.4^0
8. 0.264 ; 15.3^0
9. (a) 5 kg (b) space
10. 0.81

Level – II

1. (a), (b), (c) (d) 0.4, 40 % , (e) 74.997 MHz., 75 MHz, 75.003 MHz
(f) $E_c = 50 \sin(150 \times 10^6 \pi t)$, $E_m = 20 \sin(6\pi \times 10^3 \pi t + \phi)$
2. $\frac{N_1}{N_2} = 0.83$
3. 7.81 kW
4. 8.33 nano sec.
5. 74.64 Ω
6. 9 pulses
7. (a) 8.6^0 (b) 81.9^0
8. 40000 bits/sec
9. 30984 m, 3014.4 km^2
10. 6028800

Objective:

Level - I

1.	**B**	2.	**D**
3.	**A**	4.	**C**
5.	**D**	6.	**A, B, C**
7.	**B**	8.	**B**
9.	**C**	10.	**B**
11.	**A**	12.	**B**
13.	**B**	14.	**D**
15.	**B**		

Level - II

1.	**B**	2.	**C**
3.	**B**	4.	**B**
5.	**B**	6.	**D**
7.	**A**	8.	**B**
9.	**B**	10.	**C**
11.	**D**	12.	**C**
13.	**C**	14.	**D**
15.	**D**		

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

GRAND ASSIGNMENT PHASE - II

Grand Assignment (Phase – II)

Subjective

1. How will you define the term work. Calculate the work done by a constant force. Is work a scalar or a vector quantity?

2. Show that
$w = \int \vec{F}.\vec{ds}$ and $P = \vec{F}.\vec{v}$, where w, F, P, v and s have usual meanings.

3. Explain how can we find the work done by a variable force.

4. Define kinetic energy. Derive an expression for the kinetic energy of a body moving with a uniform velocity?

5. Show that the total mechanical energy of a body falling freely under gravity is conserved.

6. Derive expression for the potential energy stored in a system of block attached to a massless spring, when the block is pulled from its equilibrium position,

7. A ball bounces to 80% of its original height. What fraction of its mechanical energy is lost in each bounce? Where does this energy go?

8. A body of mass 1 kg falls from a height of 240m with an initial velocity of 14m/s and penetrates into sand to a depth of 0.2m. Determine the mean resistive force of the sand by applying the law of conservation of energy.

9. An automobile is moving at 100 km/hr and is exerting attractive force of 400 kgf. What horse power must the engine develop, if 20% of the power developed is wasted?

10. A spring requires 20J of work to stretch itself through 0.1m. Find the force constant of the spring. Also calculate the work done to stretch it further by 0.1m.

11. Determine the centre of mass of a homogeneous, semicircular plate of radius R.

12. Derive the law of conservation of momentum from Newton's second law.

13. Derive law of conservation of momentum from Newton's third law of motion.

14. What do you understand by impulse of a force? State and prove impulse-momentum theorem?

15. Describe the graphical method for the measurement of impulse in the following cases:
(i) When a constant force acts on a body.
(ii) When a variable force acts on a body.

16. What do you understand by the term collision? Is it necessary for the colliding particles to touch each other?

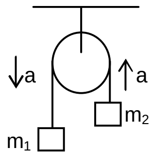
17. Distinguish between a head on and oblique collision.

18. What are the types of collision based on energy conservation?

19. Write down the characteristics of elastic and inelastic collision.

20. Elaborate the concept of equilibrium of concurrent forces acting on a body.

21. Two bodies of mass m_1 and $m_2(<m_1)$ are connected to the ends of a massless cord and allowed to move as shown. The pulley is both massless and frictionless. Determine the acceleration of centre of mass.



22. A bomb at rest explodes into three fragments of masses m each. Two fragments fly off at right angle to each other with velocities of 9 m/s and 12 m/s. Calculate the speed of the third fragment.

23. Ten 50 paise coins are put on the top of each other on a table. Each coin has a mass m gm. Find the normal reaction force between 7^{th} and 8^{th} coin from top.

24. Why are shockers used in the vehicles?

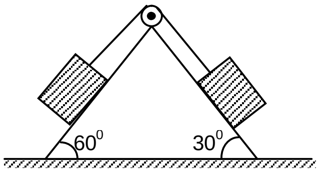
25. Why is it advisable to hold a pistol tight in one's hand when it is being fired?

26. Throwing mud on the wall is an example of perfectly inelastic collision. Explain.

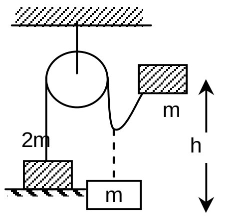
27. A bomb is projected at an angle θ from the horizontal with initial velocity u. In the mid-air, the bomb explodes. Can we conserve the linear momentum even in the presence of gravitational force?

28. A block is sliding on a frictional surface and collides with another body of same mass and moving in the same direction with half the velocity of other one. Can we conserve the linear momentum, even in the presence of frictional surface, if yes, then How?

29. Two blocks of equal masses are tied with a light string which passes over a massless pulley as shown in the figure. Calculate the magnitude of acceleration of centre of mass of both the blocks?



30. A mass 2m rests on a horizontal table. It is attached to a light inextensible string which passes over a smooth pulley and carries a mass m at the other end. If the mass m is raised vertically through a distance h and is then dropped then calculate the speed with which the mass 2m begins to rise?



31. Compare linear and rotational motion. Make a table indicating equivalent terms in linear and rotational motion.

32. Define radius of gyration. Find radius of gyration about central axis for following.
(a) A ring of radius R (b) A cylinder of radius R
(c) A thin hollow sphere of radius R (d) A solid sphere with radius R

33. Describe, with illustration (two each) the following
Parallel axis theorem
Perpendicular axis theorem

34. Derive an expression for moment of inertia for the following
(a) a thin hollow cylinder
(b) a thin hollow sphere
(c) a solid sphere.

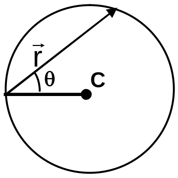
35. Describe physical meaning of angular momentum. Derive an expression for angular momentum in terms of mass and Areal velocity.

36. Define rolling motion, what is the required condition for pure rolling?

37. Answer in one/two lines only
(a) How is it possible that a particle is moving with a uniform speed and yet it is accelerated?
(b) What provides centripetal force to an electron revolving around a nucleus?
(c) How centripetal force is provided to a car taking a turn?
(d) Why do planets go around sun in orbits in fixed planes?
(e) Why is it that a stationary bicycle falls down whereas a moving one doesn't?
(f) How is a ballet dancer able to change her rotational angular velocity during the dance?
(g) Why are there two propellers in a helicopter?
(h) Is it possible to have pure rolling of a cylinder on a smooth surface? If no, why? If yes, how?
(i) What is the purpose of providing banking to a road at turning zone.
(j) If a bucket full of water is rotated in a vertical plane at sufficiently high angular speed, explain why the water doesn't fall.

38. A vehicle is traveling with a speed of 40 m/s on a circular road of radius 400 m. Its speed is increasing at the rate of 3 m/s^2 what is the magnitude of its acceleration?

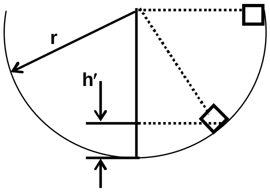
39. A particle moves along a circle of radius R = 60cm in such a way that its position vector $\vec{r}$ with respect to a fixed point 'P' on the circumference rotates with a constant angular speed $\omega_0 = 2$ rad/s. Find the magnitude and direction of its total acceleration



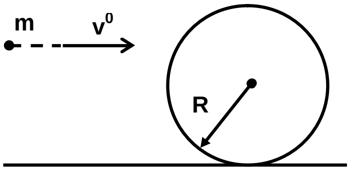
40. An object of mass m is weighed with the help of a spring balance in a moving train. The train is moving on a circular track of radius R with a uniform linear speed of V. What will be the weight indicated by the spring balance?

41. A hollow thin cylinder of radius R is rotating about its central axis. A block of mass m is kept touching the internal surface. Find out the minimum angular speed of the cylinder to prevent the block from falling. Coefficient of static friction between the surface of cylinder and the block is μ

42. A hemispherical bowl of radius r is fixed with its central axis vertical. A block of mass m is thrown horizontally along the wall of the bowl and it just reaches the top. The initial height of the block from the bottom is h'. Find the initial speed of the block.



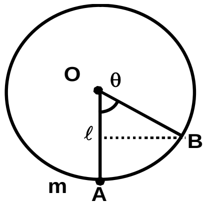
43. A particle of mass m moving with a uniform horizontal speed of v_0 collides with a sphere of mass M lying on a rough surface and sticks to it. The particle strikes the sphere at a height h from the ground. Neglect change in centre of mass of the sphere/ combined system. Find out



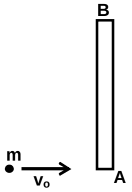
(a) Linear speed of the system just after collision
(b) Angular speed of the system just after the collision
(c) Height h if the sphere starts pure rolling just after the collision.

44. A sphere of mass m and radius R rolls down an inclined plane with angle θ from a height h.
(a) What will be its speed when it reaches the bottom?
(b) What will be the magnitude and direction of frictional force acting on the sphere during its motion on the incline?

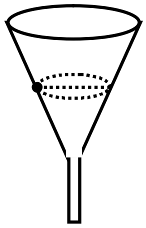
45. A particle of mass m is tied to a massless string of length ℓ. The other end of the string is fixed at 'O'. It is projected horizontally with speed $\sqrt{g\ell}$. At an instant during the motion, the tension in the string equals the weight of the particle.
Find at that instant
(a) angle of the string from vertical
(b) speed of the particle



46. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m traveling along the surface hits the end A with a velocity v_o in a direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest.
(a) Find the ratio m/M
(b) a point P on the rod is at rest immediately after the collision. Find the distance AP.
(c) Find the linear speed of the point P at a time $\pi L/(3v_o)$ after the collision.



47. A particle describes a horizontal circle on the smooth inner surface of a conical funnel as shown in the figure. If the height of the plane of the circle above the vertex is 9.8 cm, find the speed of the particle?



48. Give the dimensional formula of the following

(a) gravitational constant $\left[F = G\frac{m_1 m_2}{r^2}\right]$

(b) universal gas constant (PV=nRT)

(c) surface tension (T = F/ℓ)

(d) velocity gradient $\left(\frac{dv}{dZ}\right)$

(e) young's modulus $\left(\frac{FL}{A\Delta\ell}\right)$

49. Solve with due consideration to significant figure

(i). $\frac{0.8798 \times 2.73}{1.591}$

(ii). $\frac{6.67 \times 0.565}{123.13}$

50. The resistance values measured in an experiment are on follows

R_1=7$\pm$0.5 ohm

R_2=11$\pm$0.4 ohm

Find the values of total resistance in

(i) Series and

(ii) parallel

with limits of possible percentage errors.

Objective

1. If the stone is thrown up vertically and return to ground, its potential energy is maximum:
(A) during the upward journey. (B) at the maximum height
(C) during the return journey (D) at the bottom

2. Two springs have their force constant as K_1 and K_2 ($K_1>K_2$). When they are stretched by the same force:
(A) No work is done is case of both the springs.
(B) equal work is done in case of both the spring
(C) More work done in case of second spring
(D) More work is done in case of first spring

3. A body of mass 0.1 kg moving with a velocity of 10 m/s hits a spring (fixed at the other end) of force constant 1000 N/m and comes to rest after compressing the spring. The compression of the spring is
(A) 0.01 m (B) 0.1 m
(C) 0.2 m (D) 0.5 m

4. When the K.E. of a body is increased by 300% the momentum of the body is increased by
(A) 20% (B) 50%
(C) 100% (D) 200%

5. Mechanical advantage in a machine where a resistance W is overcome by applying an effort P is
(A) $W\times P$ (B) W/P
(C) P/W (D) W – P

6. A 50 kg girl is swinging on swing form rest. Then the power delivered when moving with a velocity of 2 m/s upwards in a direction making an angle 60^0 with the vertical is
(A) 980 W (B) 490 W
(C) $490\sqrt{3}$ W (D) 245 W

7. Two masses of 1 gm and 4 gm are moving with equal kinetic energies. The ratio of the magnitude of their linear momentum is
(A) 4 : 1 (B) $\sqrt{2}$: 1
(C) 1 : 2 (D) 1 : 16

8. A ball is allowed to fall from a height of 10m. If there is 40% loss of energy due to impact, then after one impact the ball will go up to
(A) 10 m (B) 8 m
(C) 4 m (D) 6 m

9. A ball of mass 10 kg is moving with a velocity of 10 m/s. It strikes another ball of mass 5 Kg, which is moving in the same direction with a velocity of 4 m/s. If the collision is elastic, their velocity after collision, will be respectively,
(A) 6 m/s, 12 m/s (B) 12 m/s, 6m/s
(C) 12 m/s, 10 m/s (D) none of the above

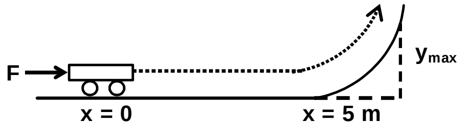
10. Which of the following statement is true?
(A) momentum is conserved in elastic collisions but not in inelastic collisions.
(B) total energy is conserved in elastic collisions but momentum is not conserved in elastic collisions.
(C) total kinetic energy is not conserved but momentum is conserved in inelastic collisions.
(D) Kinetic energy and momentum are conserved in all typed of collisions.

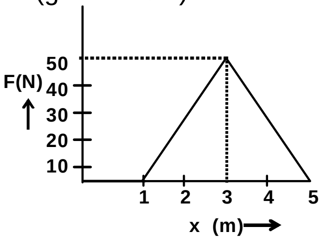
11. The same retarding force is applied to stop a train. It the speed is doubled, then the distance will be
(A) the same (B) doubled
(C) half (D) four times.

12. One fourth chain is hanging down from a table work done to bring the hanging part of the chain on the table is (mass of chain = M and length = L)
(A) $\frac{MgL}{32}$ (B) $\frac{MgL}{16}$
(C) $\frac{MgL}{8}$ (D) $\frac{MgL}{4}$

13. A body of mass m accelerates uniformly from rest to v_1 in time t_1. As a function of time t_1, the instantaneous power delivered to the body is
(A) mv_1t / t_1 (B) mv_1^2t / t_1
(C) $mv_1 t^2 / t_1$ (D) $mv_1^2 t / t_1^2$

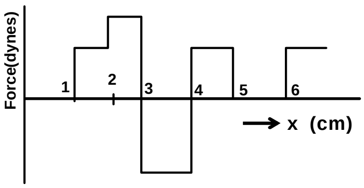
14. A force shown in the F-x graph is applied to 5 kg cart, which then coasts up a ramp as shown. The maximum height, y_{max} reached by the cart is (g = 10 m/s^2)





(A) 1 m (B) 2 m
(C) 3 m (D) 4 m

15. The relationship between force and position is as shown in figure. The work done by the force is displacing the body form x = 1 cm to x = 5 cm is



(A) 20 ergs
(B) 60 ergs
(C) 70 ergs (D) 700 ergs

16. If a body of mass 3 kg is dropped from the top of a tower of height 25m. Then its kinetic energy after 3 sec will be
(A) 1296 J (B) 1048 J
(C) 746 J (D) 557 J

17. A bag (Mass M) hangs by a long thread and bullet (mass m) comes horizontally with velocity v and gets caught in the bag. Then for the combined (bag + bullet) system:

(A) Momentum is mvM / (M + m)
(B) Kinetic energy is $\frac{mv^2}{2}$
(C) Momentum is $\frac{mv(M+m)}{M}$
(D) Kinetic energy is $\frac{m^2v^2}{2(M+m)}$

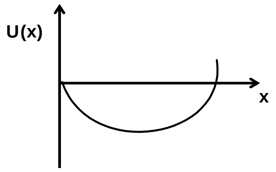
18. A body of mass 100 gm is rotating in a circular path of radius r with constant velocity. The work done in one complete revolution is

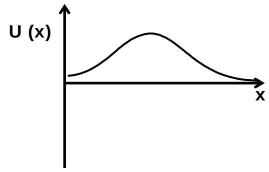
(A) 100 r J
(B) $\frac{r}{100}$ J
(C) $\frac{100}{r}$ J
(D) zero

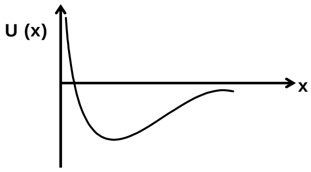
19. A bullet weighing 10 gm and moving with a velocity of 300 m/s strikes a 5 kg. block of ice and drops dead. The ice block is resting on a frictionless level. The speed of the block after the collision is

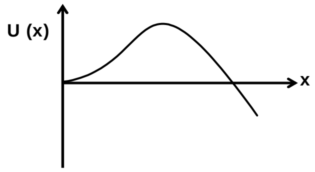
(A) 6 cm/s
(B) 60 cm/sec
(C) 6 m/s
(D) 0.6 cm/sec

20. A particle, which is constrained to move along the x-axis, is suspended to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here K and a are positive constant. For $x \geq 0$, the functional form of the potential energy U (x) of the particle is

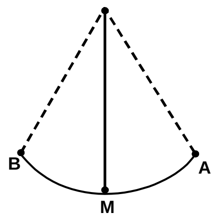
(A)


(B)


(C)


(D)


21. What is the velocity of the bob of a simple pendulum at its mean position, if it is able to rise to vertical height of 10cm ($g = 9.8$ m/s^2)



(A) 0.6 m/s
(B) 1.4 m/s
(C) 1.9 m/s
(D) 2.2 m/s

22. A car moving with a velocity of 10 m/s can be stopped by the application of a constant focal F in a distance of 20m. If the velocity of the car is 30 m/s, it can be stopped by this force in
(A) 20/3 m (B) 20 m
(C) 60 m (D) 180 m

23. A mass m moving horizontally with velocity v_0 strikes pendulum of mass m. If the two masses stick together after the collision, then the maximum height reached by the pendulum is
(A) $v_0^2/8g$ (B) $v_0^2/2g$
(C) $\sqrt{2v_0g}$ (D) $\sqrt{v_0g}$

24. A ball of mass m_1 makes a head on elastic collision with a ball of mass m_2 which is initially at rest. The transfer of kinetic energy to the second ball is maximum when
(A) $m_1 >> m_2$ (B) $m_1 = m_2$
(C) $m_1 << m_2$ (D) $m_1 \gtreqless m_2$

25. Two identical blocks A and B, each of mass m resting on smooth floor are connected by a light spring of natural length L and spring constant k, with the spring in its natural length. A third identical block of mass m moving with a speed v along the line joining A and B callides with A. The maximum compression in the spring is
(A) $v\sqrt{\frac{m}{2k}}$ (B) $m\sqrt{\frac{v}{2K}}$
(C) $\sqrt{\frac{mk}{v}}$ (D) $\sqrt{\frac{mv}{2k}}$

26. A body of mass 5 kg initially at rest is subjected to a force of 20N. The kinetic energy acquired by the body at the end of 10 sec is
(A) 4000 J (B) 400 J
(C) 40 J (D) 4 J

27. A mass M is lowered with the help of a string by a distance x at a constant acceleration g/2. The work done by the string will be
(A) Mg x (B) $\frac{Mg\,x}{2}$
(C) $\frac{1}{2}Mg\,x^2$ (D) $Mg\,x^2$

28. A body is dropped from a height h. If it acquires a momentum p, then the mass of the body is
(A) $\frac{p}{\sqrt{2gh}}$ (B) $\frac{p^2}{2gh}$
(C) $\frac{2gh}{p}$ (D) $\sqrt{\frac{2gh}{p}}$

29. If a spring extends by x on loading, then the energy stored by the spring is (T is the tension in the spring and K is force constant)

(A) $\frac{2x}{T^2}$ (B) $\frac{T^2}{2K}$

(C) $\frac{2K}{T^2}$ (D) $\frac{T^2}{2x}$

30. One man takes 1 minute to raise a box to a height of 1 meter and another man takes ½ minute to do so. The energy of the two is

(A) different (B) same

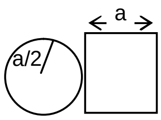
(C) energy of the first is more (D) energy of the second is more

31. All the particles of a body are situated at a distance R from the origin. The distance of the centre of mass of the body from the origin is

(A) = R (B) ≤ R

(C) > R (D) ≥ R

32. A circular plate of diameter a is kept in contact with a square plate of edge a as shown. The density of the material and the thickness are same everywhere. the centre of mass of the composite system will be



(A) inside the circular plate (B) inside the square plate

(C) at the point of contact (D) outside the system

33. Consider a system of two identical particles. One of the particle is at rest and other has an acceleration $\vec{a}$. The centre of mass has an acceleration

(A) zero (B) $\frac{1}{2}\vec{a}$

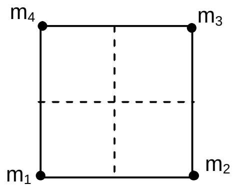
(C) $\vec{a}$ (D) $2\vec{a}$

34. A body has its centre of mass at the origin. The x-coordinates of the particles

(A) may be all positive (B) may be all negative

(C) may be all non-negative (D) none of the above.

35. Four particles of mass $m_1 = 2m$, $m_2 = 4m$ and $m_3 = m$ are placed at three corners of the square. What should be the value of m_4 of so that the centre of mass of all the four particles are exactly at the centre of the square?



(A) 2m (B) 8m

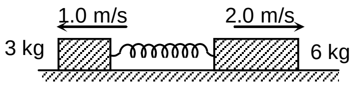
(C) 6m (D) none of these

36. The centre of mass of a non-uniform rod of length L whose mass per unit length $\lambda = \frac{kx^2}{L}$, where k is a constant and x is the distance from the one end is

(A) $\frac{3L}{4}$ (B) $\frac{L}{8}$

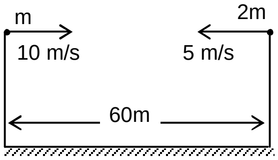
(C) $\frac{k}{L}$ (D) $\frac{3k}{L}$

37. Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant k = 200 N/m. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be



(A) 30 cm (B) 25 cm
(C) 20 cm (D) 15 cm

38. Two particles one of mass m and the other of mass 2m are projected horizontally towards each other from the same level above the ground with velocities 10 m/s and 5 m/s respectively. They collide in air and sticks to each other. The distance from A where the combined mass finally is

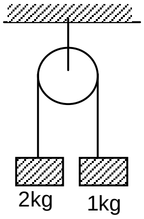


(A) 40 m (B) 20 m
(C) 30 m (D) zero

39. Internal force can change
(A) The linear momentum but not the kinetic energy.
(B) The kinetic energy but not the linear momentum.
(C) Linear momentum as well as kinetic energy.
(D) Neither linear momentum not kinetic energy.

40. In an elastic collision, which option is incorrect?
(A) the kinetic energy remains constant.
(B) the linear momentum remains constant.
(C) the final kinetic energy is equal to the initial kinetic energy.
(D) the final linear momentum is equal to the initial linear momentum.

41. Two blocks of masses 2 kg and 1 kg respectively are tied to the ends of a string which passes over a light frictionless pulley. The masses are held at rest at the same horizontal level and then released. The distance traversed by centre of mass in 2 seconds is ($g = 10\ m/s^2$)



(A) 1.42 m (B) 2.22 m
(C) 3.12 m (D) 3.33 m

42. A boy of mass 60 kg is standing over a platform of mass 40 kg placed over a smooth horizontal surface. He throws a stone of mass 1 kg with velocity u = 10 m/s at an angle of 45^0 with respect to ground. The displacement of platform (with boy) on the horizontal surface when the stone lands on the ground is ($g = 10\ m/s^2$)
(A) 25 cm (B) 5 cm
(C) 10 cm (D) 50 cm

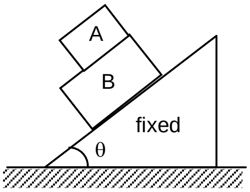
43. A particle of mass m collides with a stationary particle and continues to move at an angle of 45^0 with respect to the original direction. The second particle also recoils at an angle of 45^0 to this direction. The mass of the second particle is (collision is elastic)
(A) m (B) $\sqrt{2}$ m
(C) $\frac{m}{\sqrt{2}}$ (D) $\frac{m}{2}$

44. A gun fires a shell and recoils horizontally, if the shell travels along the barrel with speed v, the ratio of recoil speed of gun if (i) the barrel is horizontal to (ii) inclined at angle 30^0 with the horizontal is

(A) 1 (B) $\frac{2}{\sqrt{3}}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

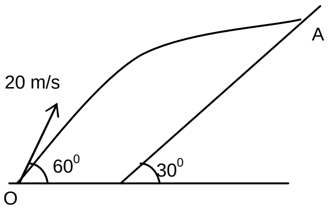
45. A block A slides over another block B which is placed over a smooth inclined plane. The coefficient of friction between the two blocks A and B is μ. Mass of block B is two times the mass of block A. The acceleration of the centre of mass of two blocks is



(A) g sin θ (B) $\frac{g\sin\theta - \mu g\cos\theta}{3}$

(C) $\frac{g\sin\theta}{3}$ (D) $\frac{2g\sin\theta - \mu g\cos\theta}{3}$

46. A ball is projected from the point O with velocity 20 m/s at an angle of 60^0 with horizontal. At highest point of its trajectory it strikes a smooth plane of inclination 30^0 at point A. The collision is perfectly inelastic. The maximum height from the ground attained by the ball is ($g = 10\ m/s^2$)

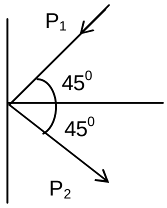


(A) 15 m (B) 22.5 m
(C) 18.75 m (D) 20.25 m.

47. A bullet of mass 0.04 kg moving with a speed of 600 m/s penetrates a heavy wooden block. It stops after a penetration of 500 cm. The resistive force exerted by the block on the bullet is
(A) 1440 N (B) 1480 N
(C) 270 N (D) 310 N.

48. A stream of water flowing horizontally with a speed of 15 m/s gushes out of a tube of cross-sectional area $10^{-2}\ m^2$ and hits at a vertical wall. The force exerted on the wall by the impact of water is
(A) 225 N (B) 2250 N
(C) 520 N (D) none of these

49. A ball moving with a momentum of 5 kg m/s strikes against a wall at an angle of 45^0 and is reflected at the same angle. The change in momentum will be (kg-m/s)



(A) 7.07 (B) 14.14
(C) 8.00 (D) 7.5

50. A rubber ball of mass 200g falls from a height of 2m and rebounds to a height of 1.5m. The impulse between the ball and the ground if the time during which they are in contact was 0.2 seconds is
(A) 2.5 N – s (B) 2.0 N – s
(C) 2.33 N – s (D) 3.2 N- s

51. A car speeding at 30km/h is stopped in 8m by applying brakes. If the same car were travelling at 90 km/hr, it can be brought to rest with the same breaking power in
(A) 32 m (B) 18 m
(C) 72 m (D) 12 m.

52. A football player kicks a = 0.25 kg ball and imparts it a velocity of 10 m/s. The contact between foot and the ball is only $\frac{1}{50}$th of a second . the kicking force is
(A) 250 N (B) 125 N
(C) 0 N (D) 3.78 N

53. Two skaters A & B of mass M and 2M are standing together on a frictionless ice surface. They push each other apart. The skater B moves away from A with a speed of 2 m/s relative to ice. The separation between the two skaters after 5 seconds will be
(A) 30 m (B) 32 m
(C) 40 m (D) 28 m

54. An object of mass 40 kg having velocity $4\hat{i}$ m/s collides with another object of mass 60 kg having velocity $2\hat{i}$ m/s. If the collision is perfectly inelastic, then the loss of mechanical energy is
(A) 24 J (B) 96 J
(C) 48 J (D) 6 J

55. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 m/s. How long does the body take to stop?
(A) 0.6 sec. (B) 6 sec.
(C) 3 sec. (D) 9 sec.

56. A gun weighing 10kg fires a bullet of 30 g with a velocity 330 m/s. With what velocity does the gun recoil ?
(A) 1.99 m/s (B) 1 m/s
(C) 99 m/s (D) 0.99 m/s

57. A machine gun has a mass of 200 kg. It fires 35 g bullets at the rate of 40 bullets per sec with a speed of 400 m/s. The force needed to the applied on the gun to keep in position?
(A) 560 N (B) 56 N
(C) 5.6 N (D) 0.56 N

58. A bomb of mass m is thrown with velocity v. It explodes and breaks into two fragments. One fragment of mass m′, immediately after the explosion, comes to rest. The speed of the other fragment will be

(A) $\frac{mv}{m-m'}$ (B) $\frac{mv}{m'}$

(C) v (D) $\frac{mv}{m+m'}$

59. A man weighing 80 kg is standing on a trolley weighing 320 kg. The trolley is resting on frictionless horizontal rails. If the man starts walking on the trolley along the rails at speed 1 m/s, then after 4 sec, his displacement relative to the ground will be

(A) 5 m (B) 4.8 m

(C) 3.2 m (D) 3.0 m

60. A cracker rocket is ejecting 0.05 kg of gases per sec at a velocity of 400 m/s. The accelerating force on the rocket is

(A) 20 dynes (B) 20 N

(C) 20 kg – wt. (D) none of the above

61. If a force of 200 N acts on a body. The momentum acquired is 100 kg m/s. What is the period for which force acts on the body ?

(A) 0.2 sec. (B) 0.5 sec.

(C) 0.4 sec. (D) 0.25 sec.

62. A player caught a cricket ball of mass 150 gm moving at the speed of 20 m/s. If the catching process be completed in 0.1 sec, the force of the blow exerted by the ball on the hands of the player is

(A) 0.3 N (B) 30 N

(C) 300 N (D) 3000 N

63. A hammer of mass M strikes a nail of mass m with velocity of u m/s and drives it a meter into fixed block of wood. The average resistance of wood during penetration of nail is

(A) $\left[\frac{M}{m+M}\right]\frac{u^2}{2a}$ (B) $\left[\frac{M^2}{(m+M)^2}\right]\frac{u^2}{2q}$

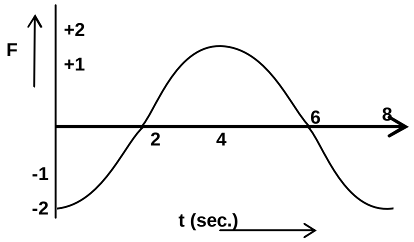
(C) $\left[\frac{M^2}{M+m}\right]\frac{u^2}{2a}$ (D) $\left[\frac{M+m}{m}\right]\frac{u^2}{2a}$

64. A ball of mass m is connected to a ball of mass M by means of a massless spring. The balls are pressed so that the spring is compressed. When released, ball of mass m move with an acceleration a. the magnitude of the acceleration of mass M will be

(A) $\frac{ma}{M}$ (B) $\frac{Ma}{m}$

(C) $\frac{ma}{M+m}$ (D) $\frac{Ma}{M+m}$

65. A force-time (F-t) graph for a linear motion is shown. The segment showed as sinusoidal. The linear momentum gained between 0 to 8 sec is
(A) - 2π N – s.
(B) zero N- s.
(C) + 4π N – s.
(D) + 6π N-s.



66. Which of the following has least moment of inertia?
(A) a hollow sphere
(B) a solid sphere
(C) a disc about its axis perpendicular to its plane
(D) a ring about its axis perpendicular to its plane

67. The centre of mass of a system does not depend on
(A) position of particles
(B) relative distances between the particles
(C) masses of the particles
(D) forces on the particles

68. If angular momentum of a system is constant, which of the following will be zero
(A) force
(B) torque
(C) linear impulse
(D) linear momentum

69. Moment of inertia of a body comes into play
(A) in a linear motion
(B) in motion along a curved path
(C) in rotational motion
(D) none of the above

70. Moment of momentum is called
(A) torque
(B) impulse
(C) couple
(D) angular momentum

71. Moment of inertia of a thin hollow sphere of mass M and radius R about an axis passing through its centre is
(A) MR^2
(B) $MR^2/2$
(C) 2/3 MR^2
(D) 2/5 MR^2

72. A solid sphere, a disc and a hollow sphere, all of same mass and same radius roll down from an inclined plane, then
(A) solid sphere will reach the bottom first
(B) disc will reach the bottom first
(C) solid cylinder will reach the bottom first
(D) all of them will reach the bottom at the same time

73. Three identical solid sphere (1, 2 and 3) are allowed to roll down from three inclined plane of angles θ_1, θ_2 and θ_3 respectively starting at time t = 0 where $\theta_1 > \theta_2 > \theta_3$, then
(A) sphere 1 will reach the bottom first
(B) sphere 2 will reach the bottom first
(C) sphere 3 will reach the bottom first
(D) all the sphere will reach the bottom at the same time

74. In which situation a body can not roll
(A) a smooth horizontal surface
(B) a moving smooth horizontal surface
(C) a smooth inclined surface
(D) a rough inclined surface

75. The parallel axis theorem can be applied to
(A) any two parallel axes
(B) any two parallel axes of which one must lie within the body
(C) any two parallel axes of which one must pass through the centre of mass of the body.
(D) any two parallel axes lying in the plane of the body

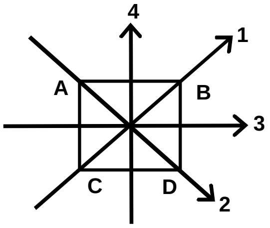
76. Considering a body of mass m, radius R rotating with angular speed ω about the centre of the mass and with a velocity v of centre of mass, the most appropriate definition of rolling motion will be
(A) $v = R\omega$
(B) the point of contact is stationary
(C) the point of contact is stationary with respect to the surface
(D) the body purely rotates about the point of contact.

77. A wheel starts from rest and rotates with a constant angular acceleration. It rotates by angel θ_1 in first second and by θ_2 in another second. Then the ratio θ_2/θ_1 is
(A) 1
(B) 4
(C) 2
(D) 3

78. The radius of gyration of a disc about its axis passing through its centre and perpendicular to its plane is
(A) $R/\sqrt{2}$
(B) $R/2$
(C) $\sqrt{2}\,R$
(D) $2R$

79. A particle attached to a string is rotating with a constant angular velocity and its angular momentum is L. If the string is halved and angular velocity is kept constant, the angular momentum will be
(A) L/4
(B) L/2
(C) L
(D) 2L

80. The moment of inertia of a thin square plate ABCD (see figure) of uniform thickness about an axis passing through the centre O and perpendicular to the plane of plate is not equal to



(A) I_1+I_2
(B) I_3+I_4
(C) I_1+I_3
(D) $I_1+I_2+I_3+I_4$

where I_1, I_2, I_3 and I_4 are moments of inertia about axes 1, 2, 3 and 4 respectively which are in the plane of paper.

81. Relationship between angular momentum L and kinetic energy E of a rigid body pure rotation is
(A) $E = L^2/2I$ (B) $E^2 = 2I/L$
(C) $L = E^2/2I$ (D) $E = 2IL$

82. A body rotating about a fixed axis has a kinetic energy of 360 J when its angular speed is 30 rad/sec. The moment of inertia of the wheel about its axis of rotation is
(A) 0.6 kgm^2 (B) 0.15 kgm^2
(C) 0.8 kgm^2 (D) 0.75 kgm^2

83. A solid cylinder of mass m and radius r rolls down an inclined plane with height h without slipping. The speed of its centre of mass when it reaches at bottom is
(A) $\sqrt{2gh}$ (B) $\sqrt{\frac{4}{3}gh}$
(C) $\sqrt{\frac{3}{4}gh}$ (D) $\sqrt{\frac{4g}{h}}$

84. A motorcyclist crosses with speed v, an over bridge of a canal which is in the form of a concave circular arc of radius r. The thrust on the road at the lowest position will be
(A) $\left(\frac{mv^2}{r}\right)mg$ (B) $\frac{\left(\frac{mv^2}{r}\right)}{mg}$
(C) $mg-\left(\frac{mv^2}{r}\right)$ (D) $mg+\left(\frac{mv^2}{r}\right)$

85. Centripetal acceleration of a particle of mass m moving in a circular of radius r is $4/r^2$. What is the momentum of the particle?
(A) 4m/r (B) $4m/\sqrt{r}$
(C) $2m/\sqrt{r}$ (D) 2m/r

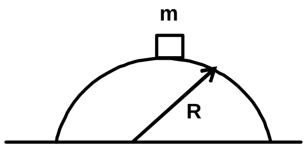
86. A particle of mass 2kg is tied to a massless string 1.5 m long. It is rotated in a vertical plane with a constant angular speed of 3 m/s. The tension in the string when it is horizontal is
(A) 47 N (B) 27 N
(C) 7 N (D) 20 N

87. A disc is rolling on a surface without slipping. What is the ratio of its transnational to rotational kinetic energies?
(A) 5:2 (B) 1:1
(C) 3:2 (D) 3:2

88. Two rings having same moment of inertia have their radii in the ratio 1:4.Their masses will be in the ratio
(A) 4:! (B) 16:1
(C) 1:4 (D) 2:1

89. Moment of inertia of a thin rod of mass m and length ℓ about an axis passing through a point L/4 from one end and perpendicular to the rod is
(A) $\frac{m\ell^2}{12}$ (B) $\frac{m\ell^2}{3}$
(C) $\frac{7m\ell^2}{48}$ (D) $\frac{m\ell^2}{9}$

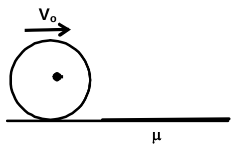
90. A particle of mass m starts sliding from top of a smooth hemisphere of radius R. The particle will loose contact of the hemisphere at the height



(A) $\frac{v^2}{2g}$ (B) $\frac{2R}{3}$
(C) $\frac{3R}{2}$ (D) $\frac{R}{\sqrt{2}}$

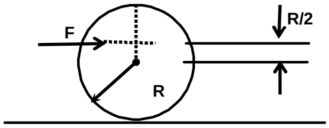
91. When a mass is rotating in a plane about a fixed point, its angular momentum is directed along
(A) the radius (B) tangent to the orbit
(C) line perpendicular to the plane of motion (D) none of these

92. A ring of mass m sliding on a smooth surface with velocity v_o enters rough surface with coefficient of kinetic friction μ_k, then



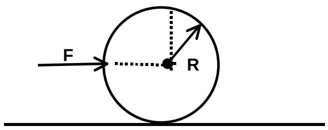
(A) the linear distance moved by centre of mass before the ring starts pure rolling is $\frac{3v_0^2}{8\mu_k g}$
(B) the gain in kinetic energy is $\frac{mv_0^2}{8}$ (C) the loss in kinetic energy is $\frac{mv_0^2}{4}$
(D) the work done by friction force is $-\frac{3mv_0^2}{8}$

93. A horizontal force F acts at a height of R/2 from centre of a sphere lying on a rough surface as shown in the figure. The direction of the friction force acting on the sphere will be



(A) in the direction of F (B) zero
(C) in opposite direction of F (D) can not be ascertained

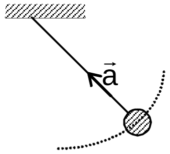
94. A force F is applied horizontally on a cylinder in the line of centre as shown in the figure. The cylinder is on a rough surface of coefficient of friction μ. The direction of the friction force acting on the cylinder will be

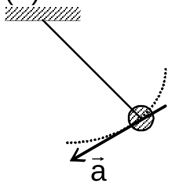


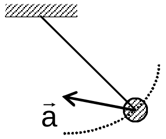
(A) in the direction of F (B) zero
(C) in opposite direction of F (D) can not be ascertained

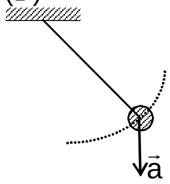
95. A particle of mass m tied to a string of length r is rotated in a vertical plane with a uniform linear speed of v. The tension in the string at the lowest point of the circle will be
(A) mg (B) $\frac{mv^2}{r}$
(C) $\left(\frac{mv^2}{r}\right)-mg$ (D) $\left(\frac{mv^2}{r}\right)+mg$

96. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown is

(A)


(B)


(C)


(D)


97. A vehicle is moving with velocity v on a curved road of width b and radius of curvature R. For counteracting the centrifugal force on the vehicle, the difference in elevation of outer edge of the road and the inner edge of the road is

(A) $\frac{v^2 b}{Rg}$ (B) $\frac{vb}{Rg}$

(C) $\frac{vb^2}{Rg}$ (D) $\frac{v^2 b}{R^2 g}$

98. A sphere roll down on an inclined plane of inclination θ. What is the acceleration as the sphere reaches bottom?

(A) $\frac{5}{7} g\sin\theta$ (B) $\frac{3}{5} g\sin\theta$

(C) $\frac{2}{7} g\sin\theta$ (D) $\frac{2}{5} g\sin\theta$

99. A thin uniform rod of mass m and length ℓ is hinged at the lower and of a level floor and stands vertically. It is now allowed to fall, then its upper end will strike the floor with a velocity given by

(A) $\sqrt{mg\ell}$ (B) $\sqrt{3g\ell}$

(C) $\sqrt{5\ mg\ell}$ (D) $\sqrt{2mg\ell}$

100. A string of length ℓ is fastened to a fixed point O. A mass m_1 is fastened at a distance $\ell/3$ from 'O' and another mass m_2 is fastened at the other end of the string. The system rotates about 'O' such that two portions of the string always remain in the same straight line. The ratio of tensions of two parts of the string will be

(A) $\frac{m_1 + m_2}{m_2}$ (B) $\frac{m_1 + 3m_2}{3m_2}$

(C) $\frac{3m_2}{m_1 + m_2}$ (D) $\frac{m_2}{m_1 + m_2}$

ANSWERS TO GRAND ASSIGNMENT

Subjective:

11. $\frac{4R}{3\pi}$

21. $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$

22. 15 m/s

23. 7 mg

29. $\left(\frac{\sqrt{3}-1}{4\sqrt{2}}\right)g$

30. $\frac{\sqrt{2gh}}{3}$

32. (a) R (b) $\frac{R}{\sqrt{2}}$ (c) $R\sqrt{\frac{2}{3}}$ (d) $R\sqrt{\frac{2}{5}}$

34. (a) $\frac{1}{2}mR^2$ (b) $\frac{2}{3}mR^2$ (c) $\frac{2}{5}mR^2$

38. 5 m/s^2

39. 9.6 m/s^2, towards centre of the circle

40. $m\sqrt{\left(\frac{v^4}{R^2}\right)+g^2}$

41. $\sqrt{\frac{g}{\mu R}}$

42. $r\sqrt{\frac{2g}{r-h'}}$

43. (a) $\frac{mv_0}{M+m}$ (b) $\frac{mv_0(h-R)}{\left[\frac{2}{5}M+m\right]R^2}$ (c) $\left[\frac{\frac{7}{5}M+2m}{M+m}\right]R$

44. (a) $\sqrt{\frac{10gh}{7}}$ (b) $\frac{2}{7}mg\sin\theta$ upwards along the incline.

45. (a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\sqrt{\left(\frac{g\ell}{3}\right)}$

46. (a) $\frac{1}{4}$ (b) $\frac{2}{3}L$ (c) $\frac{V_0}{2\sqrt{2}}$

47. 0.98 m/s

48. (a) $M^{-1}L^3T^{-2}$ (b) $ML^2T^{-2}K^{-1}$ (c) ML^0T^{-2} (d) $M^0L^0T^{-1}$ (e) $ML^{-1}T^{-2}$

49. (i). 1.51 (ii) 0.0306

50. (i) 18 ohm $\pm$ 5% (ii) 4.28 ohm $\pm$ 15.8%

Objective:

1.	**B**	2.	**C**	3.	**B**	4.	**C**	5.	**B**
6.	**C**	7.	**C**	8.	**D**	9.	**A**	10.	**C**
11.	**B**	12.	**A**	13.	**D**	14.	**B**	15.	**A**
16.	**A**	17.	**D**	18.	**D**	19.	**B**	20.	**D**
21.	**B**	22.	**D**	23.	**A**	24.	**B**	25.	**A**
26.	**A**	27.	**B**	28.	**A**	29.	**B**	30.	**C**
31.	**B**	32.	**B**	33.	**B**	34.	**B**	35.	**B**
31.	**B**	32.	**B**	33.	**B**	34.	**B**	35.	**B**
31.	**B**	32.	**B**	33.	**B**	34.	**D**	35.	**D**
36.	**A**	37.	**A**	38.	**A**	39.	**B**	40.	**A**
41.	**B**	42.	**A**	43.	**A**	44.	**C**	45.	**A**
46.	**C**	47.	**A**	48.	**B**	49.	**A**	50.	**C**
51.	**C**	52.	**B**	53.	**A**	54.	**C**	55.	**B**
56.	**D**	57.	**A**	58.	**A**	59.	**C**	60.	**B**
61.	**B**	62.	**B**	63.	**C**	64.	**A**	65.	**B**
66.	**A**	67.	**D**	68.	**B**	69.	**C**	70.	**D**
71.	**C**	72.	**A**	73.	**D**	74.	**C**	75.	**C**
76.	**C**	77.	**D**	78.	**A**	79.	**A**	80.	**D**
81.	**A**	82.	**C**	83.	**B**	84.	**D**	85.	**C**
86.	**B**	87.	**D**	88.	**B**	89.	**C**	90.	**B**
91.	**C**	92.	**A**	93.	**A**	94.	**C**	95.	**D**
96.	**C**	97.	**A**	98.	**A**	99.	**B**	100.	**B**

Learnaf

Educational Revolution

PHYSICS

FIITJEE

PINNACLE

For – JEE (Main/Advanced)

GRAND ASSIGNMENT PHASE - IV

Grand Assignment (Phase - IV)

Subjective

1. Length, breadth and thickness of a rectangular block are 3.176 m, 2.513 m and 2.12 m respectively. Find the area and volume to correct significant figures.

2. The centripetal force is given by $F = \frac{mv^2}{R}$. The mass (m), velocity (v) and radius (R) of the circular path of an object are 1 kg, 5 m/s and 0.4 m respectively. If m, v and R are measured to accuracies of 0.02 kg, 0.05 m/s and 0.01 m respectively, find percentage error in the centripetal force.

3. Three particles each of mass m are situated at the vertices of an equilateral triangle of side length a. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining separation a. Find the initial velocity that should be given to each particle and also the time period of the circular motion.

4. A satellite of mass 100 kg is placed initially in a temporary orbit 800 km above the surface of the earth. The satellite is to be placed now in a permanent orbit at 2000 km above the surface of the earth. Find the amount of work done to move satellite from temporary to permanent orbit. the radius of the earth is 6400 km.

5. A cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to the x-y plane) with its axis fixed along the y-axis and free to rotate about its axis. The platform is given a motion in x-direction given by $x = A \cos \omega$. There is no slipping between the cylinder and platform. What is maximum torque acting on the cylinder during its motion.

6. A sphere of mass m rolls without slipping on an inclined plane of inclination θ. Find the linear acceleration of the sphere and the force of friction acting on it. What should be the minimum coefficient of static friction to support pure rolling ?

7. A particle of mass 10^{-2} kg is moving along the +ve x-axis under the influence of a force $F(x) = -k/2x^2$, where $k = 10^{-2}$ Nm^2. At time t = 0, it is at x = 1.0 m and its velocity is v = 0
(a) Find its velocity when it reaches x = 0.50 m.
(b) Find tehtime at which it reaches x = 0.25 m.

8. A body falling freely from a height H hits an inclined plane in its path at a height h. As a result of this impact the direction of the velocity of the body becomes horizontal. For what value of (h / H) the body will take maximum time to reach the ground ?

9. A heavy particle is suspended by a string of length ℓ. The particle is given a horizontal velocity v_0. The string becomes slack at some angle and the particle proceeds on a parabola. Find the value of v_0 if the particle passes through the point of suspension.

10. A small ball A slides down the quadrant of a circle as shown in figure and hits the ball B of equal mass which is initially at rest. Find the velocities of both balls after collision, neglect the effect of friction and assume the collision to be elastic.

11. A pendulum bob of mass 10^{-2} kg is raised to a height 5×10^{-2} m then released. At the bottom of its swing, it picks up mass 10^{-3} kg. To what height will the combined mass rise?

12. One gram mole of oxygen at 27^0C and one atmospheric pressure is enclosed in a vessel. Assuming the molecules to be moving with v_{rms}. Find the number of collisions per second which the molecules make with one square metre area of the vessel wall.

13. Two rods of equal length and equal cross-sectional area are arranged end to end between fixed rigid supports at a certain temperature. If the temperature is increased by T, Then determine the stress at the interface, Take Y_1, Y_2 and α_1, α_2 the elastic constants and the coefficient of linear expansion respectively.

14. A clock which keeps correct time at 25^0 C has a pendulum made of brass whose coefficient of linear expansion is 0.000019. How many seconds a day will it gain if the temperature fall to 0^0C ?

15. Find the minimum attainable pressure of ideal gas in the process $T = T_0 + \alpha V^2$, where T_0 and α are positive constants and V is the volume of one mole of gas.

16. A chamber, volume v = 87 litre is evacuated by a pump whose evacuation rate equals C = 10 litre / s. How soon will the pressure in the chamber decreases by η = 1000 times.

17. The density of carbon dioxide gas at 0^0C and at a pressure of 1.0×10^5 N/m^2 is 1.98kg/m^3. Find the root mean square velocity of its molecules at 0^0C and 30^0C. Pressure is constant.

18. Calculate the ratio of the speed of sound in neon to that in water vapour at any temperature (molar weight of neon = 2.02×10^{-2} kg/mole and molar weight of water vapour 1.8×10^{-2} kg-mole)

19. An astronaut takes a cylinder of volume 10 litres containing of nitrogen gas at a temperature of 27^0C and pressure 50 atmosphere. He makes a hole of area 1 square cm in this cylinder and places it in open space. Estimate the time it would take for the cylinder to become empty.
(Boltzmann constant = 1.38×10^{-23} J/k)

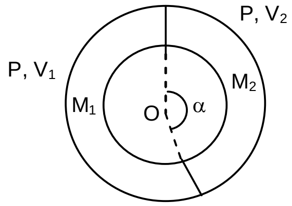
20. 2m^3 value of a gas at a pressure of 4×10^5 N/m^2 is compressed adiabatically so that the volume becomes 0.5 m^3. Find the new pressure. Compare this with the pressure that would result if the compression was isothermal. Calculate the work done in each process. (γ = 1.4)

21. Three moles of an ideal gas at temperature 300 K are isothermally expanded to five times its volume and heated at this constant volume so that the pressure is raised to its initial value before expansion. In the whole process 83.14 kJ heat is required. Calculate the ratio c_p/c_v of gas ($\log_e 5 = 1.61$)

22. Two moles of an ideal monoatomic gas are confined within a cylinder by a massless and frictionless by a massless and frictionless spring loaded piston of cross-sectional area 4×10^{-3} m^2. The spring is initially in its relaxed status. Now the gas is heated by an electric heater, placed inside the cylinder for some time. During this time, the gas expands and does 50 J of work in moving the piston through a distance 0.10 m. The temperature of the gas increases by 50 k. Calculate the spring constant and the heat supplied by the heater.

23. A body initially at 80^0C cools to 64^0C in 5 minutes and 52^0C in 10 minutes. What will be the temperature after 15 minutes and what is the temperature of the surroundings ?

24. A ring shaped tube contains two ideal gases with equal masses M_1 =32 and M_2 = 28. The gases are separated by one fixed partition P and another movable stopper S, which can move freely without friction inside the ring as shown in figure. Find the value of the angle α between the fixed partition and the movable stoppers.



25. Two thermometers are constructed in the same way, except that one has a spherical bulb and the other a cylindrical bulb. Which will respond quickly to temperature changes?

26. The average translational kinetic energy of O_2 molecules (relative molar mass 32) at a particular temperature is 0.48 eV. What will be the translational kinetic energy of N_2 molecules (relative molar mass 28) in eV at the same temperature?

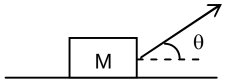
27. Why is it impossible for a ship to use the internal energy of sea water to operate the engine ?

28. Milk is poured into a cup of tea and is mixed with a spoon. Is this an example of reversible process? Give reason for answer.

29. An ideal gas a specific heat at constant pressure $C_p = \frac{5R}{2}$. The gas is kept in a closed vessel of volume 0.0083 m^3 at a temperature of 300 K and a pressure of 1.6×10^6 N/m^2. An amount of heat 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.

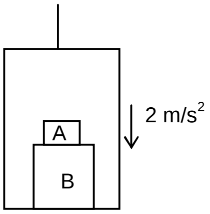
30. Why does the temperature of a gas decrease, when it is allowed to expand adiabatically?

31. A block of mass M is pulled on a smooth horizontal table by a string making an angle θ with the horizontal as shown. If the acceleration of the block is a, find the force applied by the string and by the table on the block.

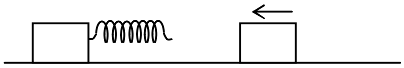


32. A bullet moving at 250 m/s penetrates 5cm into a tree limb before coming to rest. Assuming that the force exerted by the tree limb is uniform, find its magnitude. Mass of the bullet is 10 gm.

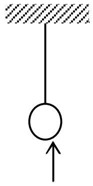
33. An elevator descending with an acceleration of 2 m/s^2. The mass of the block is 0.5 kg. What force is exerted by block A on the block B ?



34. Each of the block shown in the figure has mass 1 kg. The rear block moves with a speed of 2 m/s towards the front block kept at rest. The spring attached to the front block is light and has a sprig constant 50 N/m. Find the maximum compression of the string.

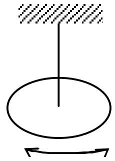


35. A bullet of mass 50 g is fired from below into the bob of mass 450 g of a long simple pendulum. The bullet remains inside the bob and the bob rises through a height of 1.8m. Find the speed of the bullet.

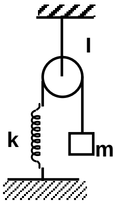


36. A ball falls on an inclined plane of inclination θ from a height h above the point of impact and makes a perfect elastic collision. Where will it hit the plane again.

37. A uniform disc of radius 50 cm and mass 200 gm is fixed at its centre to a metal wire, the other end of which is fixed with a clamped. The hanging disc is rotated about the wire through an angle and is released. If the disc makes torsional oscillations with time period 0.2 s, find the torsional constant of the wire.



38. The string and the spring shown in the figure are light. Moment of inertia of the pulley is I. Mass of the block is m. Find the time period of the mass m.



39. In a typical ultrasound scan, the wave travels with a speed of 1500 m/s. For a good detailed image, the wavelength should be no more than 1.0 mm. What frequency is required.

40. A certain transverse wave is described by

$$y(x, t) = (6.50\text{ mm}) \cos 2\pi \left(\frac{x}{28.0\text{cm}} - \frac{t}{0.0360\text{ s}}\right)$$

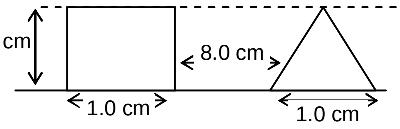
Determine the wave (a) amplitude (b) wavelengths (c) frequency (d) speed of propagation and (e) direction of propagation.

41. One end of a 14.0 m long rubber tube, with total mass 0.800 kg, is fastened to a fixed support and object with a mass of 7.50 kg. The tube is struck a transverse blow at one end. Find the time required for the pulse to reach at other end.

42. A point source is emitting sound of intensity 0.026 W/m^2 at a distance of 4.3 m from the source.
(a) What is the intensity at a distance of 3.1 m from the sound (b) how much energy does the source emit in one hour if its power output remains constant.

43. A tornado warning siren on top of a tall pole radiates sound waves uniformly in all directions. At a distance of 15.0 m the intensity of sound is 0.250 w/m^2. What is the total output of the siren ?

44. Two pulses on a stretched string travel towards each other, each with a speed of 2.0 cm/s. If the leading edges of the pulse are 8.0 cm apart at t = 0, sketch the shake of the spring at t = 1.0, 2.0, 3.0, 4.0 sec.

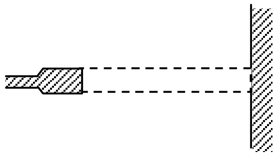


45. A wire with mass 40.0 g is stretched so that its ends are tied down at points 80.0 cm apart. The wire vibrates in its fundamental mode with frequency 60.0 Hz and with an amplitude at the antinodes of 0.30 cm (a) what is the speed of propagation of transverse waves. (b) compute the tension in the wire.

46. What intensities correspond to 28 dB and 92 dB.

47. Find the intensity of sound wave with $p_{max} = 3.0 \times 10^{-2}$ Pa. Assume the temperature is 20^0C so that the density of air is $\rho = 1.20$ kg/m^3 and the speed of sound is v = 344 m/s.

48. A unidirectional sound source aims a sound wave of wavelength λ at a wall. At what distances from the wall could you stand and hear no sound at all ?



49. On a day the speed of sound is 345 m/s, the fundamental frequency of a stopped organ pipe is 220Hz. (a) How long is this stopped pipe (b) the second overtone of this pipe has the same wavelength as the third harmonic of an open pipe. How long is the open pipe.

50. A stopped organ pipe is sounded near a guitar, causing one of the spring to vibrate with large amplitude. the string is 80 % as long as the stopped pipe. If both the spring and string vibrate at their fundamental frequency, calculate the ratio of wave speed on the string to the speed of sound in air.

51. Two small loudspeakers, A and B are driven by the same amplifier and emit pure sinusoidal wave in phase. If the speed of sound is 350 m/s. For what frequencies does constructive interfere occur at any point.

52. In a liquid with density 1300 kg/m^3, longitudinal waves with frequency 400 Hz are found to have wavelength 8.00 m. Calculate the bulk modulus of the liquid.

53. A metal bar with a length of 1.50 m has density 6400 kg/m^3. Longitudinal sound waves take 3.90×10^{-4} s to travel from one end of the bar to other end. What is Young's modulus for this metal.

54. Two sinusoidal sound waves with frequency 108 Hz and 112 Hz arrive at detector simultaneously. Each wave has an amplitude of 1.5×10^{-8} m (a) Find the beat frequency (b) the amplitude of maximum and minimum intensities.

55. If the siren is moving away from the listener with a speed of 45 m/s relative to air and listener is moving towards the siren with a speed of 15 m/s relative to the air. What frequency does the listener hear.

56. A railroad train is travelling at 25.0 m/s in still air. the frequency of the note emitted by the locomotive whistle is 400 Hz. What is the wavelength of sound (a) in front of the locomotive (b) behind the locomotive. What is frequency of the sound heard by a stationary listener (c) in front (d) back of the locomotive.

57. A source producing 1 kHz waves moves towards a stationary listener at ½ the speed of the sound. What is the frequency will the listener hear ?

58. A railroad train is travelling at 30.0 m/s in still air. The frequency of the note emitted by the train whistle is 262 Hz. what frequency is heard by a passenger on a train moving in the opposite direction to the first at 18.0 m/s (a) approach the first ? (b) r

Objective

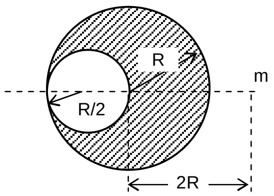
1. While measuring acceleration due to gravity by simple pendulum, a student makes a positive error of 1 % in the length of the pendulum and a negative error 3 % in the value of the time period. His actual percentage error in the measuremet of the value of g will be
(A) 2 % (B) 4 %
(C) 7 % (D) 10 %

2. In the equation, $(P + \frac{a}{v^2})(v - b)$ the unit of a is
(A) dyne cm^5 (B) dyne cm^4
(C) dyne cm^3 (D) dyne cm^{-2}

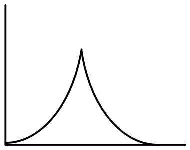
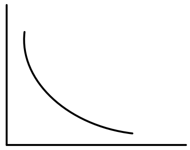
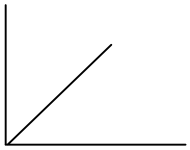
3. A satellite A of mass m is at a distance r form the centre of the earth. Another satellite B of men 2m is at a distance 2r from the earth's centre. Their time periods are in the ratio
(A) 1: 2 (B) 1 : 16
(C) 1: 32 (D) $1 : 2\sqrt{2}$

4. Radius of earth is 6400 km. The radius of the orbit of a geostationary satellite is about
(A) 36000 km (B) 29600 km
(C) 42400 km (D) 6400 km

5. From a sphere of radius R, a solid sphere of R/2 is cut out as shown in the figure. If initial mass of the sphere was M, the gravitational force on a point mass m at a distance 2R from the centre of initial solid sphere on the opposite side of the cut out (see figure) will be



(A) $\frac{GMm}{8R^2}$ (B) $\frac{GMm}{R^2}$
(C) $\frac{23}{100}\frac{GMm}{R^2}$ (D) $\frac{17}{100}\frac{GMm}{R^2}$

6. A graph of acceleration due to gravity v-s distance from earth's centre will look like
(A)

(B)

(C)

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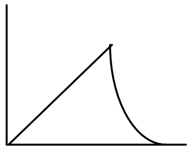
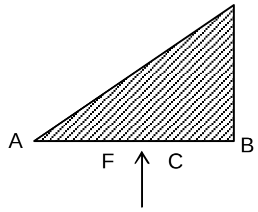


7. **State true or false for following statements.**

(i). A body is subjected to a number of forces which are parallel. The body will be in equilibrium if the algebraic sum of all the forces is zero or the algebraic sum of the moments of all the forces about any point is zero.

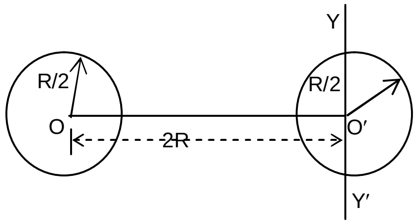
(ii). There may be no mass at the centre of mass of system.

(iii). A triangular plate of uniform thickness and density is made to rotate about an axis perpendicular to the plane of paper (a) passing through A, (b) passing through B, by application of the same force F at C (mid point of AB) as shown in figure. The angular acceleration in both the cases will be the same.



(iv). A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of same dimensions but of mass M/4 is placed gently on the first disc coaxially. The angular velocity of the system now is $2\omega/\sqrt{2}$.

8. Two spheres each of mass M and radius R/2 are connected with a massless rod of length 2R as shown in the figure, what will be the moment of inertia of the system about an axis passing through the centre of one of the spheres and perpendicular to the rod ?



(A) $\frac{21}{5}MR^2$ (B) $\frac{2}{5}MR^2$
(C) $\frac{5}{2}MR^2$ (D) $\frac{5}{21}MR^2$

9. A spherical ball rolls on a rough surface without slipping. The fraction of its total energy associated with translatory motion is
(A) 2/7 (B) 2/5
(C) 3/5 (D) 5/7

10. A rope wound around a solid cylinder of mass M and radius R is pulled with a force F. The angular acceleration of the cylinder will be
(A) MRF (B) $\frac{MR}{F}$
(C) $\frac{F}{MR}$ (D) $\frac{2F}{MR}$

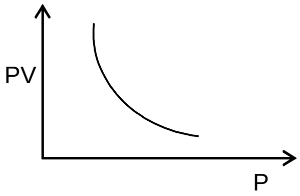
11. The internal energy of 1 mole of an ideal gas depends on
(A) only volume (B) only temperature
(C) only pressure (D) temperature

12. The correct relation between v_{rms}, v_{av} and v_{mp} is
(A) $v_{rms} > v_{mp} > v_{av}$ (B) $v_{rms} < v_{av} < v_{mp}$
(C) $v_{rms} > v_{av} > v_{mp}$ (D) $v_{rms} < v_{av} > v_{mp}$

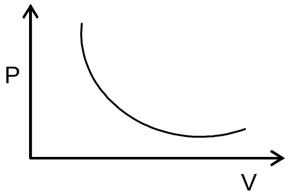
13. The mass of O_2 molecule is 16 times that of H_2 molecules. The rms velocity of O_2 molecules at room temperature is c_{rms}. The rms velocity of H_2 molecule at the same temperature will be
(A) 16 C_{rms} (B) 4 C_{rms}
(C) $\frac{C_{rms}}{4}$ (D) $\frac{C_{rms}}{16}$

14. Which of the following curves is not correct at constant temperature ?

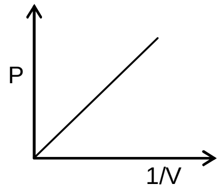
(A)



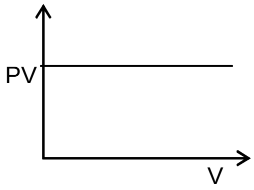
(B)



(C)



(D)



15. The correct expansion for pressure extracted by a gas on wall of a container is

(A) $P = \frac{mn}{3\ell^3}\sqrt{C^2}$ (B) $P = \frac{mnc^2}{3\ell^3}$
(C) $\frac{3\ell^2 C^2}{mn}$ (D) $P = \frac{mc^2}{3\ell^2}$

16. When a molecule moving with velocity u collides normally with the wall of container, then the change in its velocity and momentum will be
(A) – u and mu (B) 2u and 2mu
(C) 2u and mu (D) u and –mu

17. The direction of flow of heat between two bodies is determined by
(A) internal energy (B) kinetic energy
(C) total energy (D) none of these

18. The velocity of three molecules are 3V, 4 V and 5 V respectively. Their rms speed will be

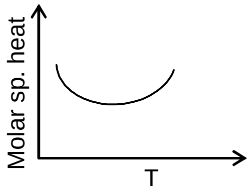
(A) $\frac{3}{15}V$ (B) $\sqrt{\frac{3}{15}}V$
(C) $\frac{50}{3}V$ (D) $\sqrt{\frac{50}{3}}V$

19. When a gas filled in a closed vessel is heated through 1^0C, its pressure increase by 0.04 %. The initial temperature of the gas was
(A) 25^0C (B) 250^0C
(C) 250 k (D) 2500 k

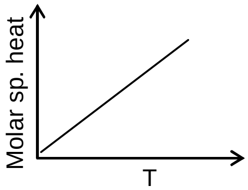
20. The temperature of 5 moles of a gas which was held at constant volume was changed from 100^0C to 120^0C. The change in the internal energy of the gas was found to be 80 Joule, the total heat capacity of the gas at constant volume will be equal to
(A) 0.4 Jk^{-1} (B) 4Jk^{-1}
(C) 0.8 Jk^{-1} (D) 8 Jk^{-1}

21. Variation of molar specific heat of a metal with temperature is best depicted by

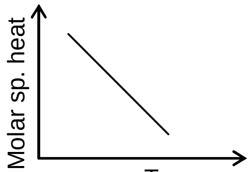
(A)



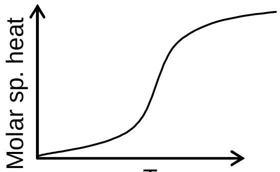
(B)



(C)



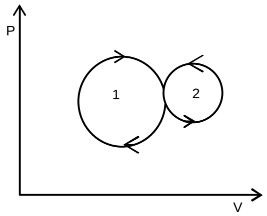
(D)



22. A mono atomic gas is supplied heat Q very slowly keeping the pressure constant. The work done by the gas is

(A) $\frac{2}{5}Q$
(B) $\frac{3}{5}Q$
(C) $\frac{Q}{5}$
(D) $\frac{2}{3}Q$

23. The net amount of work done in the following indicator diagram is



(A) zero
(B) positive
(C) negative
(D) infinite

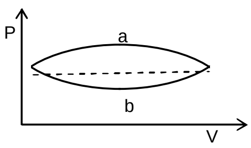
24. The maximum efficiency of an engine operating between the temperature 400^0C and 60^0C is
(A) 55 %
(B) 75 %
(C) 95 %
(D) none of these

25. When a liquid is heated, retaining its liquid sate, then its molecules gain
(A) Kinetic energy
(B) potential energy
(C) heat energy
(D) both kinetic energy and potential energy

26. What is the value of dp/p for adiabatic expansion of the gas ?
(A) $\gamma dV/V$
(B) $-dV/V$
(C) dV/V
(D) $-\gamma dV/V$

27. In free expansion of a gas the internal energy of the system
(A) increases
(B) decreases
(C) is unchanged
(D) changes

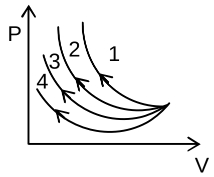
28. Figure represents two process a and b for a given sample of gas. Let ΔQ_1 and ΔQ be the heat absorbed by the system in the two cases respectively. Which of the following relation is correct?



(A) $\Delta Q_1 = \Delta Q_2$
(B) $\Delta Q_1 > \Delta Q_2$
(C) $\Delta Q_1 \le \Delta Q_2$
(D) $\Delta Q_4 < \Delta Q_2$

29. An ideal gas heat engine operates in Carnot cycle between 272^0C and 127^0C. It absorbs 6.0×10^4 cal at the higher temperature. The amount of heat converted into work is equal to
(A) 1.2×10^4 cal (B) 1.6×10^4 cal
(C) 3.5×10^4 cal (D) $4.8 \times 10_4$ cal.

30. Which of the following shows adiabatic compression?
(A) 4 (B) 2
(C) 3 (D) 1



31. For the Boyle's law to hold good, the necessary condition is
(A) isothermal (B) adiabatic
(C) isobaric (D) isochoric

32. During isothermal expansion at 800 K, the working substance of a Carnot's engine releases 480 calories of heat. If the sink is at 300 K then the work done by the working substance during isothermal expansion will be
(A) 480 cal (B) 300 cal
(C) 270 cal (D) 190 cal

33. For which process the relation dQ = dU is true ?
(A) isobaric (B) isochoric
(C) isothermal (D) adiabatic

34. The temperature of a black body increases form T to 2T. The factor by which the rate of emission will increase is
(A) 2 (B) 4
(C) 8 (D) 16

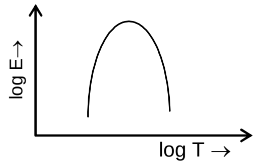
35. If the pressure of a gas is doubled then its thermal conductivity will
(A) remain constant (B) decreases
(C) decrease exponentially (D) increase

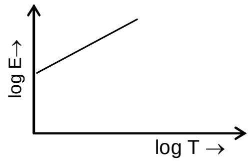
36. The diameter of ball P is three time that of another ball Q of the same material P and Q are heated to same temperature and allowed to cool up to same temperature. The relation between their rates of cooling will be
(A) $R_p = R_Q/3$ (B) $R_p = 3R_Q$
(C) $R_p = 9R_Q$ (D) $R_p = R_Q$

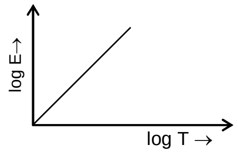
37. How the velocity of thermal radiations (V) related to the velocity of light (C)
(A) v < C (B) v > C
(C) v = C
(D) the relation depends upon the wave of the radiations.

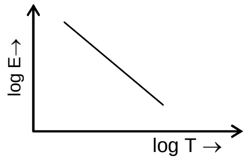
38. In the steady state, the two ends of a metre rod are at 30^0 C and 20^0C. The temperature at the 60th cm is
(A) 22^0C (B) 23^0C
(C) 24^0C (D) 25^0C

39. If the rate of emission of radiation by a body at temperature T(K) is E then the graph between log E and log T will be

(A)


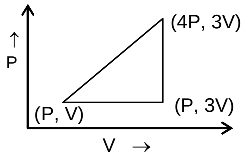
(B)


(C)


(D)


40. A body of surface area 5 cm^2 and temperature 727^0C emits 300 joules of energy per minute. Its emissivity will be
(A) 0.18 (B) 0.28
(C) 0.81 (D) 1

41. A sample of ideal mono-atomic gas is taken round the cycle ABCA as shown in the figure. The work done during the cycle is
(A) zero (B) 3PV
(C) 9PV (D) 6 PV



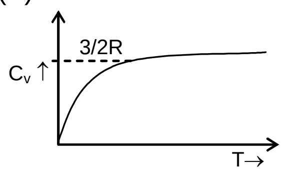
42. Consider a gas with density ρ and c as the root mean square velocity of its molecule contained in a volume, If the system moves as a whole with velocity v, then the pressure exerted by the gas is

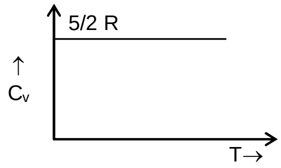
(A) $\frac{1}{3}\rho(\bar{c})^2$ (B) $\frac{1}{3}\rho(\bar{c}+v)^2$

(C) $\frac{1}{3}\rho(\bar{c}^2-v)$ (D) $\frac{1}{3}\rho(\bar{c}^2-v)^2$

43. When an ideal monoatomic gas is heated at constant pressure, fraction of heat energy supplied which increases the internal energy of gas is
(A) 2/5 (B) 3/5
(C) 3/7 (D) 3/4

44. Graphs of specific heat at constant volume for a mono-atomic gas is

(A)


(B)


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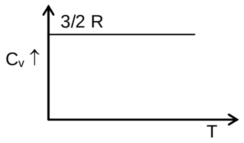
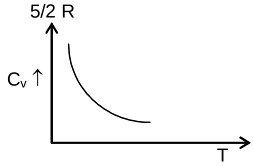


(D)


45. A sample of gas expands from volume V_1 to V_2. The amount of work done by the gas is greatest when the expansion is
(A) isothermal (B) isobaric
(C) adiabatic (D) equation in al case

46. A balloon contains 500 m^3 of helium at 27^0C and 1 atmosphere pressure. the volume of the helium at – 3^0C temperature and 0.5 atmosphere pressure will be
A) 500 m^3 (B) 700 m^3
(C) 900 m^3 (D) 1000 m^3

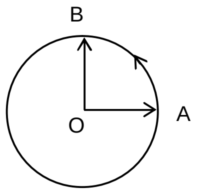
47. An ideal gas at 27^0C is compressed adiabatically to 8/27 of its original value. The rise in temperature is
(take γ = 5/3)
(A) 475^0C (B) 150^0C
(C) 275^0C (D) 375 ^0C

48. Which of the following is not true about the processes ?
(A) for isothermal process dT = 0 (B) for isobaric process dP = 0
(C) for isobaric process dE = 0 (D) for adiabatic process dQ = 0

49. A liquid tabes 5 minutes to cool from 70^0C to 60^0C. How long will it take to cool from 60^0C to 50^0C ?
(A) 5 minutes (B) more than 5 minutes
(C) less than 5 minutes
(D) less or more than 5 mintues depending on the nature of liquid.

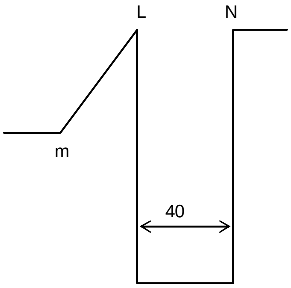
50. Two rods of length d_1 and d_2 coefficients of thermal conductivities and k_2 are kept touching each other. Both have the same area of cross-section. The equivalent of thermal conductivity is
(A) $k_1 + k_2$ (B) $k_1d_1 + k_2d_2$
(C) $\frac{d_1k_2 + d_2k_1}{d_1 + d_2}$ (D) $\frac{d_1 + d_2}{(d_1/k_1) + (d_2/k_2)}$

51. A particle starts from centre O towards OA then moves along AB and stops at B. If R=100 m the displacement of the particle is



(A) 100 m (B) $100\sqrt{2}$ m
(C) $\frac{100}{\sqrt{2}}$ m (D) none of these

52. A body is projected up a smooth inclined plane with velocity v from the point m as shown in the figure. The angle of inclination is 45^0C and the top is connected to a well of diameter 40 m. If the body just manage to cross the well . What is the value of v ?
(Length of the inclined plane is $20\sqrt{2}$ m)



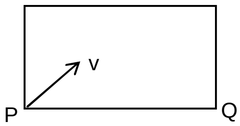
(A) 20 m/s (B) $20\sqrt{2}$ m/s
(C) 40 m/s (D) $40\sqrt{2}$ m/s

53. A particle moves in the x-y plane according to the law x = kt and y = kt(1 – dt) where k and d are positive constnts and t is time. What is the trajectory of the particle ?
(A) $y = -\frac{x^2d}{k}$ (B) y = xd
(C) $y = x - \frac{x^2d}{k}$ (D) y = kx

54. A particle is placed on a sphere of radius r. What minimum horizontal velocity be imparted to it so that it leaves sphere without touching the sphere ?

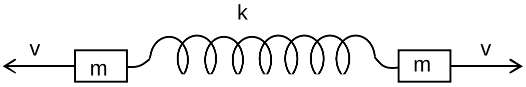
(A) $\sqrt{\frac{gr}{2}}$ (B) $\sqrt{gr}$
(C) $\sqrt{2gr}$ (D) none

55. A huge rectangular component falls vertically down with acceleration a. A gun fixed at corner P requires to hit Q. Bullet fired will hit Q if



(A) a < g (B) a > g
(C) a = g (D) any of (a) , (b) or (c)

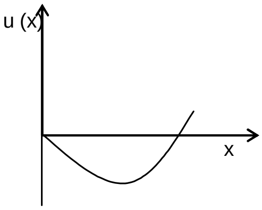
56. Two blocks of mass m each are connected to a spring of spring constant as shown in figure. The maximum displacement in the block

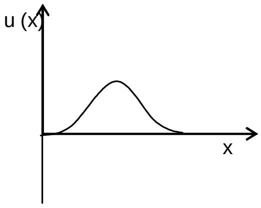


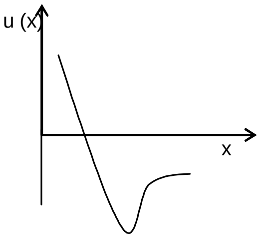
(A) $2\sqrt{\frac{mv^2}{k}}$ (B) $\sqrt{\frac{mv^2}{k}}$
(C) $\sqrt{\frac{2mv^2}{k}}$ (D) none of these

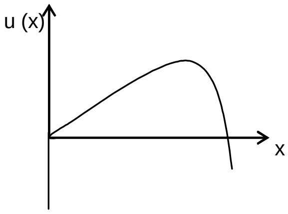
57. A body moving at 2 m/s can be stopped over a distance x. If the kinetic energy of the body is doubled, how long will it go before coming to the rest, if the retrading forces remains unchanged ?
(A) 8 x (B) 4x
(C) 2x (D) x

58. A particle constrained to move along the x-axis is subjected to a force in the same direction which varies with distance x of the particles from the origin. $F(x) = -kx + ax^3$ where k and a are positive constants. For $x \geq 0$, the functional form of the PE u(x) of the particle is

(A)


(B)


(C)


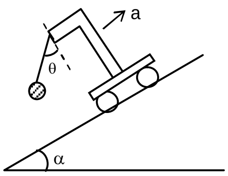
(D)


59. Three identical masses are placed at the vertices of an equilateral triangle of side ℓ. Find the workdone to displacement the masses to the vertices of an equilateral triangle of side 2ℓ

(A) $\frac{3Gm^2}{\ell}$ (B) $\frac{3Gm^2}{2\ell}$
(C) $\frac{3Gm^2}{4\ell}$ (D) $\frac{26Gm^2}{3\ell}$

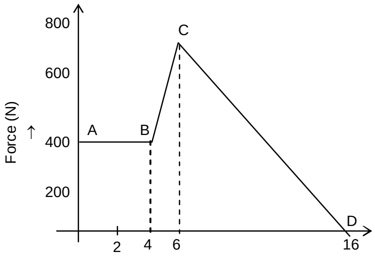
60. If the kinetic energy of a particle increases by 44 %. Find the increases in momentum
(A) 44 % (B) 20 %
(C) 12 % (d) 32 %.

61. A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string when the trolley roles up a plane of inclination α with acceleration a_0 is



(A) $\theta = \tan^{-1}\alpha$ (B) $\theta = \tan^{-1}(a_0 / g)$
(C) $\theta = \tan^{-1}(g/a_0)$ (D) $\theta = \tan^{-1}\left(\frac{a_0 + g\sin\alpha}{g\cos\alpha}\right)$

62. The magnitude of force (in N) acting on a body varies with time t (in μ s) as shown in figure. AB, BC and CD are straight line segments. The magnitude of the total impulse of the force on the body from t = 4μs to t = 16 μs is



(A) 6.8×10^{-3} Ns (B) 6.8×10^{-4} Ns
(C) 6.8×10^{-2} Ns (D) 6.8×10^{-1} Ns

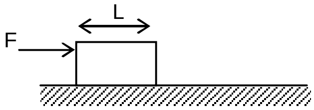
63. A block of metal weighing 2 kg is resting on a frictionless plane. It is struck by a jet of water at a rate of 1 kg/s at a speed of 5 m/s. The initial acceleration of the block is

(A) $\frac{2}{5} m/s^2$ (B) $\frac{5}{2} m/s^2$
(C) 5 m/s^2 (D) $\frac{1}{5} m/s^2$

64. Two particles of mass m_1 and m_2 and velocities u_1 and αu_1 makes an elastic head on collision. If the initial kinetic energies of the two particles are equal and m_1 comes to rest after collision, then

(A) $\frac{u_1}{u_2} = \sqrt{2} + 1$ (B) $\frac{u_1}{u_2} = \sqrt{2} - 1$
(C) $\frac{m_2}{m_1} = 3 + \sqrt{2}$ (D) $\frac{m_2}{m_1} = 3 - 2\sqrt{3}$

65. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is slightly high so that the block does not slide before toppling, the minimum force required to topple the block is



(A) Infinitesimal (B) mg(1 - μ)
(C) $\frac{mg}{2}$ (D) $\frac{mg}{4}$

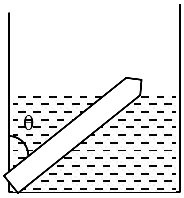
66. Two perfectly elastic balls of same mass m are moving with velocities u_1 and u_2. They collide elastically n times. The kinetic energy of the system finally is

(A) $\frac{1}{2}\frac{m}{n}u_1^2$
(B) $\frac{1}{2}\frac{m}{n}\left(u_1^2 - u_2^2\right)$
(C) $\frac{1}{2}m\left(u_1^2 + u_2^2\right)$
(D) $\frac{1}{2}mn\left(u_1^2 + u_2^2\right)$

67. A rubber cord of unstretched length 10 cm with area of cross section 1 mm^2 is used in a catapult. If is stretched to 12 cm and then released to project a stone of mass 5 g to hit a bird taking $Y_{ruber} = 5 \times 10^8$ N/m^2, the velocity of projection is

(A) 2 cm/s
(B) 20 cm /s
(C) 2 m/s
(D) 20 m/s

68. A wooden plank of length 1m and uniform cross- section is hinged at one end to the bottom of tank. The tank is filled with water upto a height of 0.5m. The specific gravity of the plank is 0.5. The angle made by the plank in equilibrium position is



(A) 30^0
(B) 45^0
(C) 60^0
(D) 90^0

69. A uniform rod of mass M and length L is pivoted about one end and oscillates in a vertical plane. The period of oscillation, if the amplitude of motion is small, will be

(A) $T = 2\pi\sqrt{\frac{L}{3g}}$
(B) $T = 2\pi\sqrt{\frac{3L}{2g}}$
(C) $T = \pi\sqrt{\frac{2L}{3g}}$
(D) $T = \pi\sqrt{\frac{3L}{2g}}$

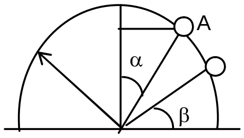
70. A circular hoop of radius R is hung over a nail. The period of oscillation is equal to that of a simple pendulum of length

(A) R
(B) 2R
(C) 3R
(D) 3R/2

71. A spring has an unstretched length ℓ and has force constant k. It is cut into two pieces of force constant k_1 and k_2 such that the length of first piece ℓ_1 is η times the length of second piece ℓ_2 (n > 1)

(A) $k_1 = nk_2$
(B) $k_2 = nk_1$
(C) $k_1 = k\left(\frac{n+1}{n}\right)$, $k_2 = (n+1)k$
(D) $k_1l_2 = k_2l_1$

72. A particle moves from rest at A on the surface of a smooth circular cylinder of radius r as shown. At B it leaves the cylinder. The equation relating α and β is



(A) $3 \sin \alpha = 2 \cos \beta$
(B) $2 \sin \alpha = 3 \cos \beta$
(C) $3 \sin \beta = 2 \cos \alpha$
(D) $2 \sin \beta - 3 \cos \alpha$

73. A particle of 1gm moving with a velocity $\vec{u}_1 = (3\hat{i} - 2\hat{j})$ m/s experiences a perfectly inelastic collision with another particle of mass 2g and velocity $\vec{u}_2 = (4\hat{j} - 6\hat{k})$ m/s. The velocity of the combined particle is

(A) $\hat{i}+2\hat{j}-4\hat{k}$ (B) $\hat{i}-2\hat{j}+4\hat{k}$
(C) $\hat{i}-2\hat{j}-4\hat{k}$ (D) $\hat{i}+3.3\hat{j}+4\hat{k}$

74. A ball of mass m is thrown upward with a velocity v. If air exerts an average resisting force F, the velocity with which the ball returns to the thrower is
(A) $v\sqrt{\frac{mg}{mg+F}}$ (B) $v\sqrt{\frac{F}{mg+F}}$
(C) $v\sqrt{\frac{mg-F}{mg+F}}$ (D) $v\sqrt{\frac{mg+F}{mg-F}}$

75. A particle of mass m is joined to a very heavy mass M by a light string passing over a light and frictionless pulley. Both the bodies are then set free. The total thrust on the pulley is
(A) mg (B) 2mg
(C) 4 mg (D) much greater than mg

76. A sine wave is travelling in the air. The minimum distance between the two particles, always having same speed, is
(A) $\lambda/4$ (B) $\lambda/3$
(C) $\lambda/2$ (D) λ

77. Which of the following equations represents a wave travelling along y-axis ?
(A) $x = A \sin (ky - \omega t)$ (B) $y = A \sin (kx - \omega t)$
(C) $y = A \sin ky \cos \omega t$ (D) $y = A \cos ky \sin \omega t$

78. Two waves of equal amplitude A and equal frequency travel in the same direction in a medium. The amplitude of resultant wave is
(A) 0 (B) A
(C) 2A (D) between zero and 2A

79. A cork floating in a calm pond executes simple harmonic motion of frequency ν when a wave generated by a boat passes by it. The frequency of the wave is
(A) ν (B) $\nu/2$
(C) 2ν (D) $\sqrt{2}\,\nu$

80. Two sine waves travel in the same direction in a medium. The amplitude of each wave is A and the phase difference between the two waves is 120^0. the resultant amplitude will be
(A) A (B) A
(C) 2A (D) $\sqrt{2}$ A

81. The fundamental frequency of a string is proportional to
(A) inverse of its length (B) the diameter
(C) the tension (D) the density

82. A tuning fork of frequency 500 Hz is used to vibrate a sonometer wire having natural frequency 250 Hz. The wire will vibrate with a frequency of
(A) 250 Hz (B) 500 Hz
(C) 750 Hz (D) will not vibrate

83. A transverse wave travels along the z-axis. The particles of the medium must move
(A) along the z-axis (B) along x-axis
(C) along the y-axis (D) in the x-y plane

84. A standing waves is produced on a string clamped at one end and free at other. The length of the string
(A) must be an integral multiple of $\lambda/4$
(B) must be an integral multiple of $\lambda/2$
(C) must be an integral multiple of λ
(D) must be an integral multiple of $\lambda/2$

85. In a standing wave,
(A) all the particles of the medium vibrate in phase
(B) all the antinodes vibrate in phase
(C) the nodes vibrate in phase
(D) all the particles between consecutive nodes vibrate in phase.

86. When sound wave is refracted from air to water, which of the following will remain unchanged?
(A) wave number
(B) wavelength
(C) wave velocity
(D) frequency

87. A tuning fork sends sound waves in air. IF the temperature of the air increases, which of the following parameter will change ?
(A) displacement amplitude
(B) frequency
(C) wavelength
(D) time period

88. When two waves with same frequency and constant phase difference interfere,
(A) there is a gain of energy
(B) there is a loss of energy
(C) the energy is redistributed and the distribution changes with time
(D) the energy is redistributed and the distribution remains constant in time.

89. An open pipe, open at both ends, contains
(A) longitudinal stationary waves
(B) longitudinal travelling waves
(C) transverse stationary waves
(D) transverse travelling waves.

90. The engine of a train sounds a whistle at frequency ν. The frequency heard by a passenger depends upon
(A) the speed of source
(B) the speed of the observer
(C) the frequency of the source
(D) separation between the source and the observer.

91. A listener is at rest with respect to the source of sound. A wind starts blowing along the line joining the source and the observer. Which of the following quantities do not change?
(A) frequency
(B) velocity of sound
(C) wavelength
(D) time period

92. Two harmonic waves travelling in the same medium have frequency in the ratio 1 : 2 and intensity in the ratio 1 : 36. Their amplitude ratio is
(A) : 6
(B) 1 : 8
(C) 1 : 72
(D) 1 : 3

93. A wave travelling in a stretched string is described by the equation y = A sin (ωt - kx). The maximum particle velocity is
(A) $A\omega$
(B) ω/k
(C) $\frac{d\omega}{dk}$
(D) $\frac{x}{t}$

94. Any progressive wave equation in differential form is

(A) $\frac{1}{\omega^2}\frac{\partial y^2}{\partial t^2} = \frac{1}{k^2}\frac{\partial^2 y}{\partial x^2}$
(B) $\frac{1}{\omega}\frac{\partial y}{\partial t} = -\frac{1}{k^2}\frac{\partial y}{\partial x}$
(C) $\frac{1}{\omega^2}\frac{\partial^2 y}{\partial t^2} = -\frac{1}{k^2}\frac{\partial^2 y}{\partial x^2}$
(D) $\frac{1}{\omega}\frac{\partial y}{\partial t} = \frac{1}{k}\frac{\partial y}{\partial x}$

95. A wave equation is given by y = A cos (ωt – kx), where symbols have their usual meanings. If v_P is the maximum particle velocity and v is the wave velocity of the wave then

(A) v_p can never be equal to v
(B) v_p = v for $\lambda = 2\pi A$
(C) v_p = v for $\lambda = A/2\pi$
(D) v_p = v for $\lambda = A/\pi$

96. A stretched rope having linear mass density 5×10^{-2} kg/m is under a tension of 80 N. The power that has to be supplied to the rope to generate harmonic waves at a frequency of 60 Hz and an amplitude of 6 cm is

(A) 215 W
(B) 251 W
(C) 512 W
(D) 521 W

97. A metal string is fixed between two rigid supports. Initially a negligible tension exists in the string. If the string has density d, Young's modulus Y and coefficient of thermal expansion α and allowed to cool through a temperature t then the transverse waves will move along it with a speed

(A) $t\sqrt{\frac{y\alpha}{d}}$
(B) $\sqrt{\frac{y\alpha t}{d}}$
(C) $\alpha t\sqrt{\frac{y}{d}}$
(D) $y\sqrt{\frac{\alpha t}{d}}$

98. A wave y = A cos (ωt – kx) passes through a medium. If v is the particle velocity and a is the particle acceleration than

(A) y, v and a all are in the same phase
(B) y lags behind v by a phae angle of $\pi/2$
(C) θ leads by a phae angle of π
(D) v leads a by a phae angle of $\pi/2$

99. A plane progressive wave of frequency 25 Hz and amplitudes 2.5×10^{-5}m and initial phase zero propagates along negative x-direction with a velocity of 300 m/s. At any instant, the phase difference between the oscillations at two points 6m apart along the line is ϕ and corresponding amplitude difference is A

(A) A = 0
(B) $\phi = 0$
(C) A = 2.5×10^{-5} m
(D) $\phi = \pi/2$

100. The ratio of the velocities of hydrogen and carbon dioxide at STP is

(A) $\sqrt{\frac{49}{45}\times 22}$
(B) $\sqrt{\frac{22}{10}\times 21}$
(C) $\sqrt{\frac{20\times 22}{11}}$
(D) $\sqrt{\frac{21}{25}\times 22}$

101. Two gases with different densities are mixed in proportions V_1 and V_2 by volume. if v be the velocity of sound in the mixture and v_1 and v_2 be velocities of sound in pure gases respectively then

(A) $\frac{V_1v_1 + V_2v_2}{v_1 + v_2}$ (B) $v = \frac{V_1v_2 + V_2v_1}{v_1 + v_2}$

(C) $v = v_1v_2\sqrt{\frac{V_1 + V_2}{V_1v_2^2 + V_2v_1^2}}$ (D) cannot calculate

102. The driver of a car approaching a vertical wall notices that the frequency of the horn of his car changes from 400 Hz to 450 Hz after being reflected from wall. Assuming speed of sound to be 340 m/s the speed of approach of car towards the wall is
(A) 10 m/s (B) 20 m/s
(C) 30 m/s (D) 40 m/s

103. A racing car moving towards a huge wall blows the horn. The driver listens that the reflected sound wave has a frequency one octave of the actual frequency. If v is the velocity of sound then velocity of car is
(A) v / 4 (B) v / 2
(C) v / 3 (D) 2v/3

104. The wavelength of light coming from a distance galaxy is found to be 0.5 % more than that coming from a source on earth. the velocity of galaxy is
(A) 1.5×10^6 m/s towards the earth (B) 1.5×10^6 m/s away from the earth
(C) 3×10^6 m/s towards the earth (D) 3×10^6 m/s away form earth

105. A train moves towards stationary observer with 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the trains speed is reduced to 17 m/s the frequency registered is f_2. if the speed of sound is 340 m/s, then the ratio f_1 / f_2 is
(A) 18 / 19 (B) 1 / 2
(C) 2 (D) 19 / 18

106. Two waves y_1 = a sin (ωt - kx) and y_2 = a cos (ωt – kx) superimpose at a point,
(A) the resultant amplitude is $\sqrt{2}$ a
(B) If I si the intensity of each source then I_{max} = 2J
(C) the resultant wave leads y_1 by a phase angle of $\pi/2$
(D) all are correct.

107. A metal bar clamped at its centre resonates in its fundamental mode to produce longitudinal wave of frequency 4 kHz. Now the clamp is moved to one end. If f_1 and f_2 be the frequencies of first overtone and second overtone respectively then,
(A) $3f_2 = 5f_1$ (B) $3f_1 = 5f_2$
(C) $f_2 = 2f_1$ (D) $2f_2 = f_1$

108. A tunning fork and a sonometer gives 5 bps both when the length of wire is 1m and 1.05 m. The frequency of fork is
(A) 210 Hz (B) 205 Hz
(C) 410 Hz (D) 420 Hz.

109. A cylindrical tube open at both ends has a frequency ν in air. The tube is dipped vertically in water so that half of it is in waters. The fundamental frequency of the air column is now
(A) $\nu/2$ (B) ν
(C) $3\nu/4$ (D) 2ν

110. The speed of longitudinal wave is 100 times the speed of transverse wave in a taut brass wire. If the Young's modulus of wire is 10^{11} N/m^2, then the stress in the wire is
(A) 10^5 N/m^2 (B) 10^6 M/m^2
(C) 10^7 N/m^2 (D) 10^8 N/m^2

111. A uniform string (length L, linear mass density μ and tension F) is vibrating with amplitude ane and frequency in its nth mode. The total energy of oscillation E is given by
(A) $E = 2\pi^2 f_n^2 A_n^2 \mu L$
(B) $E = 4\pi^2 f_n^2 A_n^2 \mu L$
(C) $E = \pi^2 f_n^2 A_n^2 \mu L$
(D) $E = \frac{1}{2}\pi^2 f_n^2 A_n^2 \mu L$

112. Two waves travelling in opposite directions produce a standing wave. The individual wave functions are given by $y_1 = 4 \sin (3x - 2t)$ cm and $y_2 = 4 \sin (3x + 2t)$cm, where x and y are in cm.
(A) the maximum displacement of the motion at x = 2.3 cm is 4.63 cm
(B) the maximum displacement of the motion at t = 2.3s is 4.63cm
(C) nodes are format at 0, $\frac{\pi}{4}, \frac{2\pi}{4}, \frac{3\pi}{4}, \pi$ . . ..
(D) Antinode are formed at values givenby $\pi / 8, 3\pi / 8, 5\pi / 8$ . . . . .

113. Two waves of same frequency but different amplitudes superpose at a point
(A) the resultant intensity varies periodically as a function of time.
(B) there will be no interference.
(C) there will be interface in which the minimum intensity will not be zero.
(D) there will be interference in which the minimum intensity is zero.

114. Waves from two sources of intensity I_0 and $4I_0$ interfere at a point. The resultant intensity at a point where the phase difference is $\pi/2$ is
(A) $9I_0$
(B) $5I_0$
(C) $3I_0$
(D) I_0

115. The displacement of a particle in a medium due to a wave travelling in the x-direction through the medium is given by $y = A \sin (\alpha t - \beta x)$ where t is time in seconds, α and β are constants.
(A) The frequency of the wave is α.
(B) the time period of the waves is $2\pi/\alpha$.
(C) the wavelength of the wave is $\pi/2\beta$.
(D) the velocity of the wave is α/β.

* * *

ANSWERS TO GRAND ASSIGNMENT

Subjective:

1. $A = 7.981\ m^2$, $v = 16.9\ m^3$
2. 5.5 %.
3. $\sqrt{\frac{Gm}{a}}$, $\frac{2\pi a^{3/2}}{\sqrt{3Gm}}$
4. 4.06×10^8 J
5. $\frac{1}{2}$ MRAω^2
6. $a = \frac{5}{7}g\sin\theta$, $f = \frac{2}{7}mg\sin\theta$, $\mu > \frac{2}{7}\tan\theta$
7. (a) – i m/s (b) $\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right)$ sec.
8. h/H = ½
9. $v_0 = \left[g\ell(2+\sqrt{3})\right]^{1/2}$
10. $v_A = 0$, $v_B = 140$ cm/sec.
11. 0.041 m (approx)
12. 1.97×10^{27}
13. $\frac{Y_1Y_2}{Y_1+Y_2}(\alpha_1+\alpha_2)\,\Delta\theta$
14. 20.52 sec.
15. $P_{in} = 2R\sqrt{\alpha T_0}$
17. $(v_{rms})_{30}$ = 410 m/sec.
18. 1.06.
19. 0.39 sec.
20. 14.75×10^5 J, 1.1×10^6 J
21. $\gamma = 1.42$
22. 1295 J
23. 43^0C, 16^0C
29. 6.15 K, 3.6×10^6 N/m^2.
31. N = Mg – Ma tanθ, T = Ma secθ
32. 6250 N
33. 4 N
34. 0.2 m
35. 60 m/s
36. 8 h sin θ along the plane
37. 0.25 kg – m^2/s^2
38. $2\pi\sqrt{\frac{(m+I/r^2)}{k}}$
39. 1.5 MHz.
40. (a) 6.50 mm (b) 28.0 cm. (c) 27.77 Hz (d) 777.7 cm/s (e) – x direction
41. 0.390 sec.
42. (a) 0.050 W W/m^2 (b) 2.2×10^4 J
43. 707 W
45. (a) 96.0 m/s (b) 461 N
46. 6.3×10^{-10} W/m^2, 1.6×10^{-3} W/m^2
47. 1.1×10^{-6} W/m^2
48. $\lambda/4$, $3\lambda/4$, $5\lambda/4$ . . . . . .
49. (a) 0.392 m (b) 0.470 m
50. $\frac{v_s}{v_a} = 0.40$
51. Integer multiple of 1000 Hz.
52. 1.33×10^{10} Pa
53. 9.47×10^{10} Pa
54. (a) 4 beats/s (b) 3.0×10^{-8} m
55. 277 Hzs.
56. (a) 0.798 m (b) 0.922 m (c) 431 Hz. (d) 373 Hz
57. 2000 Hz
58. (a) 302 Hz (b) 228 Hz

Objective:

1.	**C**	2.	**B**	3.	**D**	4.	**C**	5.	**C**
6.	A	7.	(i). False, (ii) True, (iii) False (iv) False					8.	**A**
9.	**D**	10.	**D**	11.	**B**	12.	**C**	13.	**B**
14.	**A**	15.	**B**	16.	**B**	17.	**D**	18.	**D**
19.	**C**	20.	**B**	21.	**D**	22.	**D**	23.	**A**
24.	**D**	25.	**A**	26.	**D**	27.	**C**	28.	**B**
29.	**B**	30.	**B**	31.	**A**	32.	**B**	33.	**B**
34.	**D**	35.	**A**	36.	**C**	37.	**C**	38.	**C**
39.	**B**	40.	**A**	41.	**B**	42.	**A**	43.	**B**
44.	**C**	45.	**B**	46.	**C**	47.	**D**	48.	**D**
49.	**B**	50.	**D**	51.	**A**	52.	**B**	53.	**C**
54.	**A**	55.	**C**	56.	**C**	57.	**C**	58.	**D**
59.	**B**	60.	**B**	61.	**D**	62.	**A**	63.	**B**
64.	**A**	65.	**C**	66.	**C**	67.	**D**	68.	**B**
69.	**A**	70.	**B**	71.	**B**	72.	**C**	73.	**A**
74.	**C**	75.	**C**	76.	**C**	77.	**A**	78.	**D**
79.	**A**	80.	**A**	81.	**A**	82.	**B**	83.	**D**
84.	**A**	85.	**D**	86.	**D**	87.	**C**	88.	**C**
89.	**A**	90.	**C**	91.	**A**	92.	**D**	93.	**A**
94.	**A**	95.	**B**	96.	**C**	97.	**B**	98.	**B**
99.	**A**	100.	**A**	101.	**C**	102.	**B**	103.	**C**
104.	**A**	105.	**D**	106.	**D**	107.	**A**	108.	**B**
109.	**B**	110.	**C**	111.	**C**	112.	**A**	113.	**C**
114.	**B**	115.	**B**						