

## COMPLEX NUMBER

### LEVEL-I

1. If  $z_1, z_2$  are two complex numbers such that  $\arg(z_1+z_2) = 0$  and  $\operatorname{Im}(z_1 z_2) = 0$ , then  
(A)  $z_1 = -z_2$  (B)  $z_1 = z_2$   
(C)  $z_1 = \bar{z}_2$  (D) none of these
2. Roots of the equation  $x^n - 1 = 0, n \in \mathbb{I}$ ,  
(A) form a regular polygon of unit circum-radius. (B) lie on a circle.  
(C) are non-collinear. (D) A & B
3. Which of the following is correct  
(A)  $6 + i > 8 - i$  (B)  $6 + i > 4 - i$   
(C)  $6 + i > 4 + 2i$  (D) None of these
4. If  $(1+i\sqrt{3})^{1999} = a+ib$ , then  
(A)  $a = 2^{1998}, b = 2^{1998}\sqrt{3}$  (B)  $a = 2^{1999}, b = 2^{1999}\sqrt{3}$   
(C)  $a = -2^{1998}, b = -2^{1998}\sqrt{3}$  (D) None of these
5. If  $z = 1 + i\sqrt{3}$ , then  $|\arg(z)| + |\arg(\bar{z})|$  equals  
(A)  $\pi/3$  (B)  $2\pi/3$   
(C) 0 (D)  $\pi/2$
6. The equation  $z(\overline{z+i+i\sqrt{3}}) + \bar{z}(z+1+i\sqrt{3}) = 0$  represents a circle with  
(A) centre  $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  and radius 1 (B) centre  $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$  and radius 1  
(C) centre  $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  and radius 2 (D) centre  $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$  and radius 2
7. Number of solutions to the equation  $(1-i)^x = 2^x$  is  
(A) 1 (B) 2  
(C) 3 (D) no solution
8. If  $\arg(z) < 0$ , then  $\arg(-z) - \arg(z) =$   
(A)  $\pi$  (B)  $-\frac{\pi}{4}$  (C)  $-\frac{\pi}{2}$  (D)  $-\frac{\pi}{2}$
9. The number of solutions of the equation  $z^2 + |z|^2 = 0$ , where  $z \in \mathbb{C}$  is  
(A) one (B) two (C) three (D) infinitely many
10. If  $\omega$  is an imaginary cube root of unity, then  $(1 + \omega - \omega^2)^7$  equals  
(A)  $128 \omega$  (B)  $-128 \omega$   
(C)  $128 \omega^2$  (D)  $-128 \omega^2$

11. If  $z_1$  and  $z_2$  be the  $n^{\text{th}}$  roots of unity which subtend right angle at the origin. Then  $n$  must be of the form  
 (A)  $4k + 1$  (B)  $4k + 2$   
 (C)  $4k + 3$  (D)  $4k$
12. For any two complex numbers  $z_1$  and  $z_2$   $|\sqrt{7} z_1 + 3z_2|^2 + |3z_1 - \sqrt{7} z_2|^2$  is always equal to  
 (A)  $16(|z_1|^2 + |z_2|^2)$  (B)  $4(|z_1|^2 + |z_2|^2)$   
 (C)  $8(|z_1|^2 + |z_2|^2)$  (D) none of these
13. If  $\alpha$  is an  $n^{\text{th}}$  root of unity other than unity itself, then the value of  $1 + \alpha + \alpha^2 + \dots + \alpha^{n-1}$  is .....
14. Locus of 'z' in the Argand plane is  $|z| = 2$ , then the locus of  $z + 1$  is -  
 (A) a straight line (B) a circle with centre (1, 0)  
 (C) a circle with centre (0, 0) (D) a straight line passing through (0, 0)
15. Value of  $\omega^{1999} + \omega^{299} + 1$  is  
 (A) 1 (B) 2  
 (C) 0 (D) -1
16. Square root(s) of '-1' is/ are -  
 (A)  $\frac{1}{\sqrt{2}}(1-i)$  (B)  $\frac{1}{\sqrt{3}}(i-1)$   
 (C)  $\pm \frac{1}{2}(1-i)$  (D)  $-\frac{1}{\sqrt{2}}(1-i)$
17. The real value of ' $\theta$ ' for which  $\frac{3+2i \sin \theta}{1-2i \sin \theta}$  is real is  
 (A)  $\theta = n\pi, n \in I$  (B)  $\theta = n\pi + \frac{\pi}{3}, n \in I$   
 (C)  $\theta = n\pi + \frac{\pi}{2}, n \in I$  (D)  $\theta = \frac{n\pi}{2}, n \in I$
18. Principal argument of  $z = -\sqrt{3} + i$  is  
 (A)  $\frac{5\pi}{6}$  (B)  $\frac{\pi}{6}$   
 (C)  $-\frac{5\pi}{6}$  (D) None
19. Which one is not a root of the fourth root of unity  
 (A)  $i$  (B) 1  
 (C)  $\frac{i}{\sqrt{2}}$  (D)  $-i$

20. If  $z^3 - 2z^2 + 4z - 8 = 0$  then

(A)  $|z| = 1$

(C)  $|z| = 3$

(B)  $|z| = 2$

(D) None

## LEVEL-II

1. If  $a, b, c$  are three complex numbers such that  $c = (1 - \lambda)a + \lambda b$ , for some non-zero real number  $\lambda$ , then points corresponding to  $a, b, c$  are  
 (A) vertices of a triangle (B) collinear  
 (C) lying on a circle (D) none of these
2. If  $z$  be any complex number such that  $|3z - 2| + |3z + 2| = 4$ , then locus of  $z$  is  
 (A) an ellipse (B) a circle  
 (C) a line-segment (D) None of these
3. If  $\arg(\bar{z}_1) = \arg(z_2)$ , then  
 (A)  $z_2 = k z_1^{-1}$  ( $k > 0$ ) (B)  $z_2 = k z_1$  ( $k > 0$ )  
 (C)  $|z_2| = |\bar{z}_1|$  (D) None of these.
4. The value of the expression  $2\left(1 + \frac{1}{\omega}\right)\left(1 + \frac{1}{\omega^2}\right) + 3\left(2 + \frac{1}{\omega}\right)\left(2 + \frac{1}{\omega^2}\right) + 4\left(3 + \frac{1}{\omega}\right)\left(3 + \frac{1}{\omega^2}\right) + \dots + (n+1)\left(n + \frac{1}{\omega}\right)\left(n + \frac{1}{\omega^2}\right)$ , where  $\omega$  is an imaginary cube root of unity, is  
 (A)  $\frac{n(n^2 + 2)}{3}$  (B)  $\frac{n(n^2 - 2)}{3}$  (C)  $\frac{n^2(n+1)^2 + 4n}{4}$  (D) none of these
5. For a complex number  $z$ ,  $|z - 1| + |z + 1| = 2$ . Then  $z$  lies on a  
 (A) parabola (B) line segment  
 (C) circle (D) none of these
6. If  $z_1$  and  $z_2$  are two complex numbers such that  $|z_1| = |z_2| + |z_1 - z_2|$ , then  
 (A)  $\operatorname{Im}\left(\frac{z_1}{z_2}\right) = 0$  (B)  $\operatorname{Re}\left(\frac{z_1}{z_2}\right) = 0$   
 (C)  $\operatorname{Re}\left(\frac{z_1}{z_2}\right) = \operatorname{Im}\left(\frac{z_1}{z_2}\right)$  (D) none of these.
7. If  $\left|\frac{z_1}{z_2}\right| = 1$  and  $\arg(z_1 z_2) = 0$ , then  
 (A)  $z_1 = z_2$  (B)  $|z_2|^2 = z_1 z_2$   
 (C)  $z_1 z_2 = 1$  (D) none of these.
8. Number of non-zero integral solutions to  $(3 + 4i)^n = 25^n$  is  
 (A) 1 (B) 2  
 (C) finitely many (D) none of these.
9. If  $|z| < 4$ , then  $|iz + 3 - 4i|$  is less than  
 (A) 4 (B) 5  
 (C) 6 (D) 9
10. If  $z$  is a complex number, then  $z^2 + \bar{z}^2 = 2$  represents

- (A) a circle  
(C) a hyperbola
- (B) a straight line  
(D) an ellipse
11. If  $\frac{1-i\alpha}{1+i\alpha} = A + iB$ , then  $A^2 + B^2$  equals to  
(A) 1  
(B) -1  
(B)  $\alpha^2$   
(D)  $-\alpha^2$
12. A, B and C are points represented by complex numbers  $z_1, z_2$  and  $z_3$ . If the circumcentre of the triangle ABC is at the origin and the altitude AD of the triangle meets the circumcircle again at P, then P represents the complex number  
(A)  $-\frac{z_1 z_2}{z_3}$   
(B)  $-\frac{z_2 z_3}{z_1}$   
(C)  $-\frac{z_3 z_1}{z_2}$   
(D)  $\frac{z_1 z_2}{z_3}$
13. If  $|z_1| = |z_2|$  and  $\arg(z_1) + \arg(z_2) = \pi/2$ , then  
(A)  $\arg(z_1^{-1}) + \arg(z_2^{-1}) = -\pi/2$   
(B)  $z_1 z_2$  is purely imaginary  
(C)  $(z_1 + z_2)^2$  is purely imaginary  
(D) All the above.
14. If  $z_1$  and  $z_2$  are two complex numbers satisfying the equation  $\left| \frac{z_1 + iz_2}{z_1 - iz_2} \right| = 1$ , then  $\frac{z_1}{z_2}$  is a  
(A) purely real  
(B) of unit modulus  
(C) purely imaginary  
(D) none of these
15. If the complex numbers  $z_1, z_2, z_3, z_4$ , taken in that order, represent the vertices of a rhombus, then  
(A)  $z_1 + z_3 = z_2 + z_4$   
(B)  $|z_1 - z_2| = |z_2 - z_3|$   
(C)  $\frac{z_1 - z_3}{z_2 - z_4}$  is purely imaginary  
(D) none of these
16. If  $\left| \frac{z_1 z - z_2}{z_1 z + z_2} \right| = k, (z_1, z_2 \neq 0)$  then  
(A) for  $k = 1$  locus of  $z$  is a straight line  
(B) for  $k \neq \{1, 0\}$   $z$  lies on a circle  
(C) for  $k = 0$   $z$  represents a point  
(D) for  $k = 1, z$  lies on the perpendicular bisector of the line segment joining  $\frac{z_2}{z_1}$  and  $-\frac{z_2}{z_1}$
17. If the equation  $|z - z_1|^2 + |z - z_2|^2 = k$  represents the equation of a circle, where  $z_1 \equiv 2 + 3i, z_2 \equiv 4 + 3i$  are the extremities of a diameter, then the value of  $k$  is  
(A)  $\frac{1}{4}$   
(B) 4  
(C) 2  
(D) None of these

18. If  $z$  be a complex number and  $a_i, b_i, (i = 1, 2, 3)$  are real numbers, then the value of the determinant  $\begin{vmatrix} a_1z + b_1\bar{z} & a_2z + b_2\bar{z} & a_3z + b_3\bar{z} \\ b_1z + a_1\bar{z} & b_2z + a_2\bar{z} & b_3z + a_3\bar{z} \\ b_1z + a_1 & b_2z + a_2 & b_3z + a_3 \end{vmatrix}$  is equal to
- (A)  $(a_1 a_2 a_3 + b_1 b_2 b_3) |z|^2$  (B)  $|z|^2$   
 (C) 0 (D) None of these
19. If  $z = x + iy$  satisfies the equation  $\arg(z-2) = \arg(2z+3i)$ , then  $3x-4y$  is equal to
- (A) 5 (B) -3  
 (C) 7 (D) 6
20. If a complex number  $x$  satisfies  $\log_{1/\sqrt{2}} \left( \frac{|z|^2 + 2|z| + 6}{2|z|^2 - 2|z| + 1} \right) < 0$ , then locus / region of the point represented by  $z$  is
- (A)  $|z| = 5$  (B)  $|z| < 5$   
 (C)  $|z| > 1$  (D)  $2 < |z| < 3$
21. If for a complex number  $z = x + iy$ ,  $\sec^{-1} \left( \frac{z-2}{i} \right)$  is an acute angle, then
- (A)  $x = 2, y = 1$  (B)  $x < 2, y < -1$   
 (C)  $xy < 0$  (D)  $x = 2, y > 1$
22. Number of solutions of  $\operatorname{Re}(z^2) = 0$  and  $|z| = a\sqrt{2}$ , where  $z$  is a complex number and  $a > 0$ , is
- (A) 1 (B) 2  
 (C) 4 (D) 8
23. If the area of the triangle formed by the points represented by,  $Z, Z + iZ$  and  $iZ$  is 200, then  $|Z|$  is \_\_\_\_\_
24. Let  $z$  is a variable complex number and  $a$  is a real constant. Then the solution set for  $z$ , satisfying the equation,  $|z-a| + |z+a| = |a|$  is \_\_\_\_\_
25. If  $Z_1, Z_2$  be two non zero complex numbers satisfying the equation  $\left| \frac{Z_1 + Z_2}{Z_1 - Z_2} \right| = 1$  then  $\frac{Z_1}{Z_2} + \overline{\left( \frac{Z_1}{Z_2} \right)}$  is \_\_\_\_\_.
26. If  $(x - iy)^{1/3} = a - ib$ , then  $\frac{x}{a} + \frac{y}{b}$  equals
- (A)  $-2(a^2 + b^2)$  (B)  $4(a + b)$   
 (C)  $4(a - b)$  (D)  $4ab$

27. If  $(\sqrt{3} + i)^n = 2^n$ , where  $n$  is an integer, then  
 (A)  $n$  is a multiple of 5 (B)  $n$  is a multiple of 6  
 (C)  $n$  is a multiple of 10 (D) none of these
28. If points corresponding to the complex numbers  $z_1, z_2$  and  $z_3$  in the Argand plane are A, B and C respectively and if  $\triangle ABC$  is isosceles, and right angled at B then a possible value of  $\frac{z_1 - z_2}{z_3 - z_2}$  is  
 (A) 1 (B) -1  
 (C)  $i$  (D) none of these
29. If  $z_1$  and  $z_2$  are two complex numbers satisfying the equation  $\left| \frac{z_1 + z_2}{z_1 - z_2} \right| = 1$ , then  $\frac{z_1}{z_2}$  is a number which is  
 (A) Real (B) Imaginary  
 (C) Zero (D) None of these
30. If  $|z| = 1$ , then  $|z-1|$  is  
 (A)  $< |\arg z|$  (B)  $> |\arg z|$   
 (C)  $= |\arg z|$  (D) None of these
31. If  $z_1, z_2$  and  $z_3, z_4$  are two pairs of conjugate complex numbers then  $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$  equals  
 (A)  $\frac{\pi}{2}$  (B)  $\pi$   
 (C)  $\frac{3\pi}{2}$  (D) 0
32. If  $||z + 2| - |z - 2|| = a^2$ ,  $z \in \mathbb{C}$  is representing a hyperbola for  $a \in S$ , then  $S$  contains  
 (A)  $[-1, 0]$  (B)  $(-\infty, 0]$   
 (C)  $(0, \infty)$  (D) none of these
33. If  $|z| = 1$  and  $z \neq \pm i$ , then  $\frac{z+i}{z-i}$  is  
 (A) purely real  
 (B) purely imaginary  
 (C) a complex number with equal real and imaginary parts  
 (D) none of these
34. The locus of  $z$  which satisfied the inequality  $\log_{0.5}|z-2| > \log_{0.5}|z-i|$  is given by  
 (A)  $x+2y > 1$  (B)  $x-y < 0$   
 (C)  $4x-2y > 3$  (D) none of these
35. Let  $Z_1$  and  $Z_2$  be the complex roots of  $ax^2 + bx + c = 0$ , where  $a \geq b \geq c > 0$ . Then

- (A)  $|Z_1 + Z_2| \leq 1$  (B)  $|Z_1 + Z_2| > 2$   
 (C)  $|Z_1| = |Z_2| = 1$  (D) none of these
36. If the roots of  $z^3 + az^2 + bz + c = 0$ ,  $a, b, c \in \mathbb{C}$  (set of complex numbers) acts as the vertices of an equilateral triangle in the argand plane, then  
 (A)  $a^2 + b = c$  (B)  $a^2 = b$   
 (C)  $a^2 + b = 0$  (D) none of these
37. If  $|z_1| = 4$ ,  $|z_2| = 4$ , then  $|z_1 + z_2 + 3 + 4i|$  is less than  
 (A) 2 (B) 5  
 (C) 10 (D) 13
38. If  $z = x + iy$  satisfies  $\operatorname{Re}\{z - |z - 1| + 2i\} = 0$ , then locus of  $z$  is  
 (A) parabola with focus  $\left(-\frac{1}{2}, \frac{1}{2}\right)$  and directrix  $x + y = \frac{1}{2}$   
 (B) parabola with focus  $\left(-\frac{1}{2}, \frac{1}{2}\right)$  and directrix  $x + y = -\frac{1}{2}$   
 (C) parabola with focus  $\left(0, \frac{1}{2}\right)$  and directrix  $y = -\frac{1}{2}$   
 (D) parabola with focus  $\left(\frac{1}{2}, 0\right)$  and directrix  $x = -\frac{1}{2}$
39. If  $|z + 1| = z + 1$ , where  $z$  is a complex number, then the locus of  $z$  is  
 (A) a straight line (B) a ray  
 (C) a circle (D) an arc of a circle
40. Length of the curved line traced by the point represented by  $z$ , when  $\arg \frac{z-1}{z+1} = \frac{\pi}{4}$ , is  
 (A)  $2\sqrt{2}\pi$  (B)  $\sqrt{2}\pi$   
 (C)  $\frac{\pi}{\sqrt{2}}$  (D) none of these
41. If  $8iz^3 + 12z^2 - 18z + 27i = 0$  then  
 (A)  $|z| = 3/2$  (B)  $|z| = 1$  (C)  $|z| = 2/3$  (D)  $|z| = 3/4$
42. If  $|z - i| \leq 2$  and  $z_1 = 5 + 3i$  then the maximum value of  $|iz + z_1|$  is  
 (A)  $2 + \sqrt{31}$  (B)  $\sqrt{31} - 2$  (C)  $\sqrt{31} + 2$  (D) 7
43.  $\sin^{-1}\left\{\frac{1}{i}(z - 1)\right\}$ , where  $z$  is not real, can be the angle of the triangle if  
 (A)  $\operatorname{Re}(z) = 1, I_m(z) = 2$  (B)  $\operatorname{Re}(z) = 1, -1 \leq I_m(z) \leq 1$   
 (C)  $\operatorname{Re}(z) + I_m(z) = 0$  (D) None of these



44. The value of  $\ln(-1)$   
 (A) does not exist (B)  $2\ln i$  (C)  $i\pi$  (D) 0
45. If  $n_1, n_2$  are positive integers then  $(1+i)^{n_1} + (1+i^3)^{n_2} + (1+i^5)^{n_1} + (1+i^7)^{n_2}$  is a real Number if and only if  
 (A)  $n_1 = n_2 + 1$  (B)  $n_1 + 1 = n_2$  (C)  $n_1 = n_2$  (D)  $n_1, n_2$  be +ve integers
46. Let  $z_1, z_2$  be two nonreal complex cube roots of unity and  $|z - z_1|^2 + |z - z_2|^2 = \lambda$  be the equation of a circle with  $z_1, z_2$  as ends of a diameter then the value of  $\lambda$  is  
 (A) 4 (B) 3 (C) 2 (D)  $\sqrt{2}$
47. The center of the arc  $\arg\left(\frac{3z-6-3i}{2z-8-6i}\right) = \frac{\pi}{4}$  is  
 (A) (4,1) (B) (1,4) (C) (2,5) (D) (3,1)
48. The value of  $\sum_{k=1}^6 \left( \sin \frac{2\pi k}{7} - i \cos \frac{2\pi k}{7} \right)$   
 (A)  $i$  (B)  $-i$  (C) 1 (D)  $-1$
49. The complex numbers  $z_1, z_2$  and  $z_3$  satisfying  $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1-i\sqrt{3}}{2}$  are the vertices of a triangle which is  
 (A) of area zero (B) right angled isosceles  
 (C) equilateral (D) obtuse angled isosceles
50. If  $|z| = 3$  then the number  $\frac{z-3}{z+3}$  is  
 (A) purely real (B) purely imaginary  
 (C) a mixed number (D) none of these
51. If  $iz^3 + z^2 - z + i = 0$ , then  $|z|$  is equal to .....
52. If  $\alpha$  and  $\beta$  are different complex numbers with  $|\beta| = 1$ , then  $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|$  is equal to
53. If the complex numbers  $z_1, z_2, z_3$  are in A.P., then they lie on a  
 (A) circle (B) parabola  
 (C) line (D) ellipse

54. If  $z_1$  and  $z_2$  are two  $n$ th roots of unity, then  $\arg\left(\frac{z_1}{z_2}\right)$  is a multiple of .....
55. The maximum value of  $|z|$  when  $z$  satisfies the condition  $\left|z + \frac{2}{z}\right| = 2$  is .....
56. All non-zero complex numbers  $z$  satisfying  $\bar{z} = iz^2$  are.....
57. Common roots of the equation  $z^3 + 2z^2 + 2z + 1 = 0$  and  $z^{1985} + z^{100} + 1 = 0$  is .....

### LEVEL-III

- If points corresponding to the complex numbers  $z_1, z_2, z_3$  and  $z_4$  are the vertices of a rhombus, taken in order, then for a non-zero real number  $k$

(A)  $z_1 - z_3 = i k (z_2 - z_4)$  (B)  $z_1 - z_2 = i k (z_3 - z_4)$   
 (C)  $z_1 + z_3 = k (z_2 + z_4)$  (D)  $z_1 + z_2 = k (z_3 + z_4)$
- If  $z_1$  and  $z_2$  are two complex numbers such that  $|z_1 - z_2| = ||z_1| - |z_2||$ , then  $\arg z_1 - \arg z_2$  is equal to

(A)  $-\pi/4$  (B)  $-\pi/2$   
 (C)  $\pi/2$  (D)  $0$
- If  $f(x)$  and  $g(x)$  are two polynomials such that the polynomial  $h(x) = x f(x^3) + x^2 g(x^6)$  is divisible by  $x^2 + x + 1$ , then

(A)  $f(1) = g(1)$  (B)  $f(1) \neq -g(1)$   
 (C)  $f(1) = g(1) \neq 0$  (D)  $f(1) = -g(1) \neq 0$
- Consider a square OABC in the argand plane, where 'O' is origin and  $A \equiv A(z_0)$ . Then the equation of the circle that can be inscribed in this square is; (vertices of square are given in anticlockwise order)

(A)  $|z - z_0(1+i)| = |z_0|$  (B)  $2 \left| z - \frac{z_0(1+i)}{2} \right| = |z_0|$   
 (C)  $\left| z - \frac{z_0(1+i)}{2} \right| = |z_0|$  (D) none of these .
- For a complex number  $z$ , the minimum value of  $|z| + |z - \cos \alpha - i \sin \alpha|$  is

(A)  $0$  (B)  $1$   
 (C)  $2$  (D) none of these
- The roots of equation  $z^n = (z+1)^n$

(A) are vertices of regular polygon (B) lie on a circle  
 (C) are collinear (D) none of these
- The vertices of a triangle in the argand plane are  $3 + 4i$ ,  $4 + 3i$  and  $2\sqrt{6} + i$ , then distance between orthocentre and circumcentre of the triangle is equal to,

(A)  $\sqrt{137 - 28\sqrt{6}}$  (B)  $\sqrt{137 + 28\sqrt{6}}$   
 (C)  $\frac{1}{2} \sqrt{137 + 28\sqrt{6}}$  (D)  $\frac{1}{3} \sqrt{137 + 28\sqrt{6}}$  .
- One vertex of the triangle of maximum area that can be inscribed in the curve  $|z - 2i| = 2$ , is  $2 + 2i$ , remaining vertices is / are

(A)  $-1 + i(2 + \sqrt{3})$  (B)  $-1 - i(2 + \sqrt{3})$   
 (C)  $1 + i(2 - \sqrt{3})$  (D)  $-1 - i(2 - \sqrt{3})$

9. If  $\left(\frac{3-z_1}{2-z_1}\right)\left(\frac{2-z_2}{3-z_2}\right) = k$ , then points  $A(z_1)$ ,  $B(z_2)$ ,  $C(3, 0)$  and  $D(2, 0)$  (taken in clockwise sense) will
- (A) lie on a circle only for  $k > 0$   
 (B) lie on a circle only for  $k < 0$   
 (C) lie on a circle  $\forall k \in \mathbb{R}$   
 (D) be vertices of a square  $\forall k \in (0, 1)$
10. Let 'z' be a complex number and 'a' be a real parameter such that  $z^2 + az + a^2 = 0$ , then
- (A) locus of z is a pair of straight lines  
 (B)  $\arg(z) = \pm \frac{2\pi}{3}$   
 (C)  $|z| = |a|$   
 (D) All
11. If  $z_1, z_2, z_3, \dots, z_{n-1}$  are the roots of the equation  $z^{n-1} + z^{n-2} + z^{n-3} + \dots + z + 1 = 0$ , where  $n \in \mathbb{N}$ ,  $n > 2$ , then
- (A)  $\omega^n, \omega^{2n}$  are also the roots of the same equation.  
 (B)  $\omega^{1/n}, \omega^{2/n}$  are also the roots of the same equation.  
 (C)  $z_1, z_2, \dots, z_{n-1}$  form a geometric series.  
 (D) none of these.
- Where  $\omega$  is the complex cube root of unity.
12. The value of  $i \log(x - i) + i^2 \pi + i^3 \log(x + i) + i^4 (2 \tan^{-1} x)$ ,  $x > 0$  (where  $i = \sqrt{-1}$ ) is
- (A) 0  
 (B) 1  
 (C) 2  
 (D) 3
13. If  $z = -2 + 2\sqrt{3}i$ , then  $z^{2n} + 2^{2n} z^n + 2^{4n}$  may be equal to
- (A)  $2^{2n}$   
 (B) 0  
 (C)  $3 \cdot 2^{4n}$   
 (D) none of these
14. The value of  $169e^{i\left(\pi + \sin^{-1} \frac{12}{13} + \cos^{-1} \frac{5}{13}\right)}$  is
- (A)  $119 - 120i$   
 (B)  $-i(120 + 119i)$   
 (C)  $119 + 120i$   
 (D) none of these
15. Let  $z_1$  and  $z_2$  be the complex roots of the equation  $3z^2 + 3z + b = 0$ . If the origin, together with the points represented by  $z_1$  and  $z_2$  form an equilateral triangle then the value of b is
- (A) 1  
 (B) 2  
 (C) 3  
 (D) None of these
16. If  $|z-2| = \min\{|z-1|, |z-3|\}$ , where z is a complex number, then
- (A)  $\operatorname{Re}(z) = \frac{3}{2}$   
 (B)  $\operatorname{Re}(z) = \frac{5}{2}$

$$(C) \operatorname{Re}(z) \in \left\{ \frac{3}{2}, \frac{5}{2} \right\}$$

(D) None of these

17. If  $x = 1 + i$ , then the value of the expression

$$x^4 - 4x^3 + 7x^2 - 6x + 3 \text{ is}$$

(A) -1

(B) 1

(C) 2

(D) None of these

18. If  $z$  lies on the circle centred at origin. If area of the triangle whose vertices are  $z$ ,  $\omega z$  and  $z + \omega z$ , where  $\omega$  is the cube root of unity, is  $4\sqrt{3}$  sq. unit. Then radius of the circle is

(A) 1 unit

(B) 2 units

(C) 3 units

(D) 4 units

19. If  $\theta_i \in [0, \pi/6]$ ,  $i = 1, 2, 3, 4, 5$  and  $\sin \theta_1 z^4 + \sin \theta_2 z^3 + \sin \theta_3 z^2 + \sin \theta_4 z + \sin \theta_5 = 2$ , then  $z$  satisfies.

(A)  $|z| > \frac{3}{4}$

(B)  $|z| < \frac{1}{2}$

(C)  $\frac{1}{2} < |z| < \frac{3}{4}$

(D) None of these

20. If  $\alpha$  is the angle which each side of a regular polygon of  $n$  sides subtends at its centre, then  $1 + \cos \alpha + \cos 2\alpha + \cos 3\alpha \dots + \cos (n-1)\alpha$  is equal to

(A)  $n$

(B) 0

(C) 1

(D) None of these

21. Triangle  $ABC$ ,  $A(z_1)$ ,  $B(z_2)$ ,  $C(z_3)$  is inscribed in the circle  $|z| = 2$ . If internal bisector of the angle  $A$  meets its circumcircle again at  $D(z_d)$  then

(A)  $z_d^2 = z_2 z_3$

(B)  $z_d^2 = z_1 z_3$

(C)  $z_d^2 = z_2 z_1$

(D) none of these

# ANSWERS

## LEVEL -I

- |       |       |       |       |
|-------|-------|-------|-------|
| 1. C  | 2. D  | 3. D  | 4. A  |
| 5. B  | 6. B  | 7. A  | 8. A  |
| 9. D  | 10. D | 11. D | 12. A |
| 13. 0 | 14. B | 15. C | 16. A |
| 17. A | 18. A | 19. C | 20. B |

## LEVEL -II

- |  |                        |                      |                    |
|--|------------------------|----------------------|--------------------|
| 1. B   | 2. C                   | 3. A                 | 4. C               |
| 5. B   | 6. A                   | 7. B                 | 8. D               |
| 9. D   | 10. C                  | 11. A                | 12. B              |
| 13. D  | 14. A                  | 15. A, B, C          | 16. A, B, C, D     |
| 17. B  | 18. C                  | 19. D                | 20. B              |
| 21. D  | 22. A                  | 23. 20               | 24. $\phi$         |
| 25. 0  | 26. A                  | 27. D                | 28. C              |
| 29. B  | 30. A                  | 31. D                | 32. A              |
| 33. B  | 34. C                  | 35. A                | 36. D              |
| 37. D  | 38. D                  | 39. B                | 40. D              |
| 41. A  | 42. D                  | 43. B                | 44. C              |
| 45. C  | 46. B                  | 47. A                | 48. A              |
| 49. C  | 50. B                  | 51. 1                |                    |
| 52. 1  | 53. C                  | 54. $\frac{2\pi}{n}$ | 55. $1 + \sqrt{3}$ |
| 56. $\left(\pm \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ | 57. $\omega, \omega^2$ |                      |                    |

## LEVEL -III

- |          |          |       |       |
|----------|----------|-------|-------|
| 1. A     | 2. D     | 3. A  | 4. B  |
| 5. B     | 6. C     | 7. B  | 8. A  |
| 9. C     | 10. D    | 11. C | 12. A |
| 13. B, C | 14. A, B | 15. A | 16. C |
| 17. B    | 18. D    | 19. A | 20. B |
| 21. A    |          |       |       |