Simple Update Accuracy

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February 14, 2024

two ways to perform simple update:
(1) first get converged $D = 2$ simple update state, then use this $D = 2$ as initial state for $D = 4$ and further perform SU and get converged $D = 4$ SU state. Continue this process, we can gradually get larger D SU states. Denote this process as increasing D: $D = 2 \to 4 \to 6 \to \cdots \to 20$.
(2) first get converged $D = 2$ simple update state, then use this $D = 2$ as initial state for $D = 8$ and further perform SU and get converged $D = 8$ SU state. Similarly we can use this $D = 8$ SU states as initial state for $D = 20$ SU. After get converged $D = 20$ SU state, similarly we can get converged $D = 18$ SU state, then $D = 16$ SU state... Denote it as decreasing D: $D = 20 \to 18 \to 16 \to \cdots \to 2$. (This is also the way of femionic SU for the Hubbard model.)

The following two figures Figs. 1 and 2 show the results for 4×4 and 10×10 Heisenberg model with open boundary conditions. The cutoff $\chi = 2D$ is used to guarantee the χ convergence. 50000 Monte Carlo samples are used, with an energy (persite) variance smaller than 0.0001. Indeed it shows the simple update accuracy suffer a plateau, with an limitation 0.4% for 4×4 and 0.3% for 10×10.

For the 4×4 Heisenberg model, typically a fully optimized $D = 8$ PEPS is enough to converge the ground state, with zero local magnetic momentum $\langle S_i^z \rangle$ (recover the SU(2) symmetry). But here $D = 20$ SU state still has an obvious nonzero $\langle S_i^z \rangle$, around 0.15. (Note $\langle S_i^z \rangle$ here is shown just for understanding. If system size is large enough, $\langle S_i^z \rangle$ could not be zero because spontaneous symmetry broken, even if the ground state energy is very accurate.)

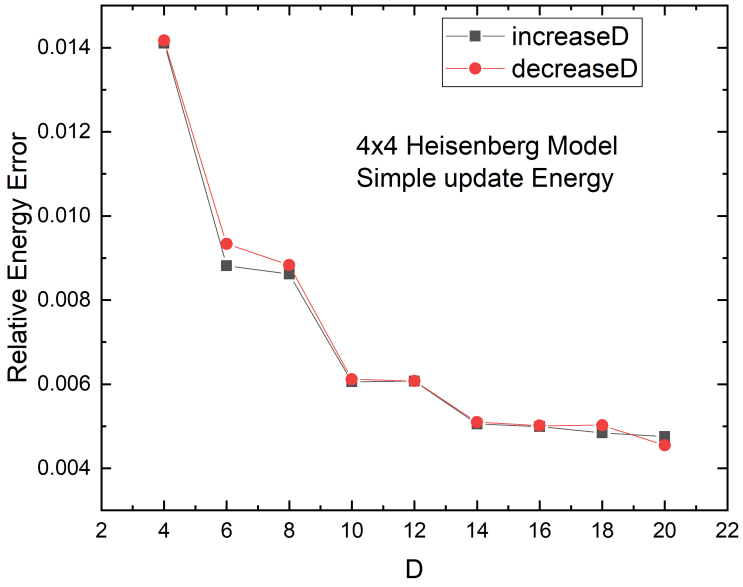

Figure 1: 4×4 Heisenberg model energies from simple update with $D = 4 - 20$.

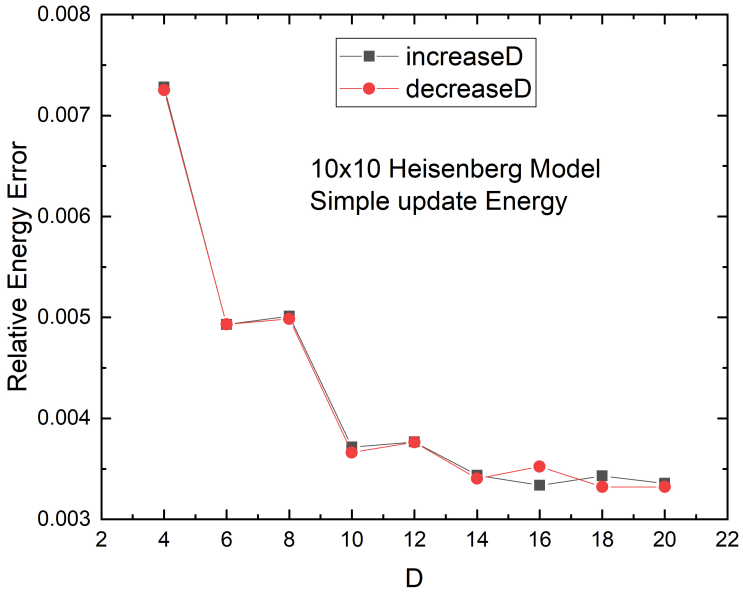

Figure 2: 10×10 Heisenberg model energies from simple update with $D = 4-20$.

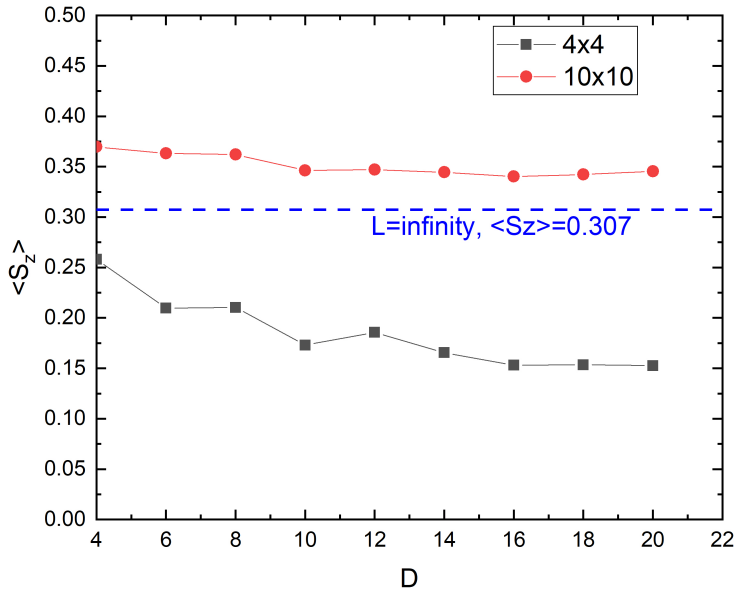

Figure 3: Local $\langle S_i^z \rangle$ at the center of 4×4 and 10×10 from simple update with $D = 4 - 20$.