Hodgkin Huxley Model

MPhil in Computational Biology

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(Adapted from Dayan and Abbott textbook "Theoretical Neuroscience", chapter 5.)

As background reading, consult the Wikipedia page for the model: `http://en.wikipedia.org/wiki/Hodgkin-Huxley_model` and this great animation `http://tinyurl.com/matthews-channel`. The Hodgkin-Huxley model of action potential propagation is a system of differential equations that states how the membrane voltage V of a neuron changes over time:

$$c_{\mathrm{m}} \frac{\mathrm{d}V}{\mathrm{d}t} = -i_m + i_e \tag{1}$$

c_{m} is the membrane capacitance per unit area (i.e. per mm^2); i_e is the external input per unit area. i_m is the current flowing (per unit area) across the membrane due to three ionic channels:

- The slow potassium (K) channel.
- The fast sodium (Na) channel.
- A leak term (L) that encompasses other minor channels.

The current flow across the membrane due to three ionic channels is thus:

$$i_m = \bar{g}_{\mathrm{L}}(V - E_{\mathrm{L}}) + \bar{g}_{\mathrm{K}} n^4 (V - E_{\mathrm{K}}) + \bar{g}_{\mathrm{Na}} m^3 h (V - E_{\mathrm{Na}}) \tag{2}$$

The maximal conductances $\bar{g}_{\mathrm{L}}, \bar{g}_{\mathrm{K}}, \bar{g}_{\mathrm{Na}}$ and reversal potentials E_{L}, E_{K}, E_{Na} are constants and given in Table 1. m, n, h however are dynamic gating variables that control the flow of current due to each channel. They evolve over time according to:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \alpha_n(V)(1-n) - \beta_n(V)n \tag{3}$$

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \alpha_m(V)(1-m) - \beta_m(V)m \tag{4}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \alpha_h(V)(1-h) - \beta_h(V)h \tag{5}$$

Finally, the rate at which the sodium and potassium channels open and close are dependent on the membrane voltage, given by:

$$\alpha_n(V) = \frac{.01(V+55)}{1-\exp(-.1(V+55))} \tag{6}$$

$$\beta_n(V) = 0.125\exp(-0.0125(V+65)) \tag{7}$$

$$\alpha_m(V) = \frac{.1(V+40)}{1-\exp(-.1(V+40))} \tag{8}$$

$$\beta_m(V) = 4\exp(-.0556(V+65)) \tag{9}$$

$$\alpha_h(V) = .07\exp(-.05(V+65)) \tag{10}$$

$$\beta_h(V) = 1/(1+\exp(-.1(V+35))) \tag{11}$$

For initial values take: $V = -65$ mV, $m = 0.0529$, $h = 0.5961$, and $n = 0.3177$. Use an external current with $i_e = 200\ \mathrm{nA/mm^2}$ and plot V, m, h, and n as functions of time for a suitable interval.

Term	value
c_m	10 nF/mm^2
E_L	-54.387 mV
E_K	-77 mV
E_Na	+50 mV
$\bar{g}_\mathrm{L}$	0.003 mS/mm^2
$\bar{g}_\mathrm{K}$	0.360 mS/mm^2
$\bar{g}_\mathrm{Na}$	1.200 mS/mm^2

Table 1: Constants for the Hodgkin-Huxley model