

Have you heard the "binary indexed tree"?

- <https://leetcode.com/problems/range-sum-query-mutable/>
- <http://www.csie.ntnu.edu.tw/~u91029/Sequence.html#8>

More hints in the next page ↓.

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 0 |   |   |   |   |   |   |   |
| 4 |   |   |   | 0 |   |   |   |
| 2 |   | 2 |   | 0 |   | 0 |   |
| 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |

Figure 1: The binary indexed tree

The upper level is:

- Addition of the lower 2 levels if they are both non-zero.
- 0, otherwise.

More hints in the next page ↓.

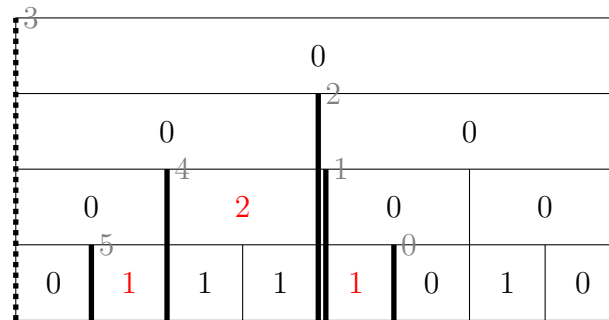


Figure 2: The binary indexed tree

| Check                      | Result  | Length | Loop invariant                   |
|----------------------------|---------|--------|----------------------------------|
| Try to add 1 to align to 2 | OK      | 1      | Align to 2                       |
| Try to add 2 to align to 4 | Already | 1      | Align to 4                       |
| Try to add 4 to align to 8 | Fail    | 1      | Align to 4, less than 4 more 1's |
| Try to add 2               | OK      | 3      | Less than 2 more 1's             |
| Try to add 1               | OK      | 4      | Less than 1 more 1's (done)      |

Table 1: The Algorithm

We traverse the tree bottom-up then top-down. This algorithm can handle at most  $(2^h - 1)$  1's where  $h$  is the height of the tree. So you have to choose  $h = 9$  to handle  $k = 500$ .

Note that we also illustrate the "OK", "Already" and initial state by bold black lines while "Fail" is dotted bold lines.

One more example in the next page ↓.

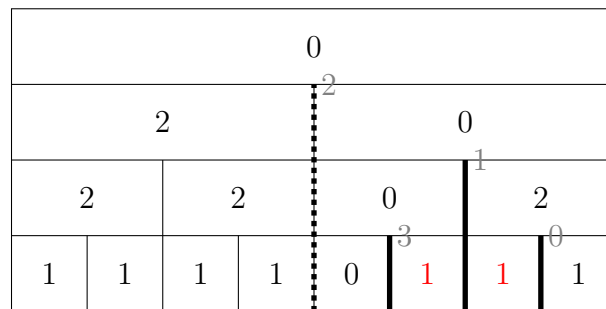


Figure 3: The binary indexed tree

| Check                      | Result | Length | Loop invariant                   |
|----------------------------|--------|--------|----------------------------------|
| Try to add 1 to align to 2 | OK     | 1      | Align to 2                       |
| Try to add 2 to align to 4 | Fail   | 1      | Align to 2, less than 2 more 1's |
| Try to add 1               | OK     | 2      | Less than 1 more 1's (done)      |

Table 2: The Algorithm

In this example we "Fail" at the lower level.

Sadly I have no more hint for you.