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CS 325
HW6

1. a. The shortest path is 16. Each vertex has a d placed in front of it to make the variable in the LP code. vertex1-vertex2 <= number means that the path betweent he vertexes is less than or equal to the number provided.

LP CODE

```
max dc
ST
      dg = 0
      dd - dg <= 2
      dh - dg <= 3
      de - dd <= 25
      df - dd <= 18
      db - dh <= 9
      da - dh <= 4
      df - da <= 10
      db - da <= 8
      de - db <= 10
      dc - db <= 4
      dc - df <= 3
      de - dg <= 2
      da - df <= 5
      db - df <= 7
      dg - de <= 7
      dd - de <= 9
      dd - dc <= 3
```

```
  Global optimal solution found.
  Objective value: 16.00000
  Infeasibilities: 0.000000
  Total solver iterations: 4
  Elapsed runtime seconds: 0.03

  Model Class: LP

  Total variables: 7
  Nonlinear variables: 0
  Integer variables: 0
  Total constraints: 18
  Nonlinear constraints: 0

  Total nonzeros: 31
  Nonlinear nonzeros: 0



Variable  Value        Reduced Cost
DC        16.00000     0.000000
DG        0.000000     0.000000
DD        2.000000     0.000000
DH        3.000000     0.000000
DE        0.000000     0.000000
DF        14.00000     0.000000
DB        12.00000     0.000000
DA        4.000000     0.000000

Row    Slack or Surplus    Dual Price
1      16.00000            1.000000
2      0.000000            1.000000
3      0.000000            0.000000
4      0.000000            1.000000
5      27.00000            0.000000
6      6.000000            0.000000
7      0.000000            1.000000
8      3.000000            0.000000
9      0.000000            0.000000
10     0.000000            0.000000
11     22.00000            0.000000
12     0.000000            1.000000
13     1.000000            0.000000
14     2.000000            0.000000
15     15.00000            0.000000
16     9.000000            0.000000
17     7.000000            0.000000
18     7.000000            0.000000
19     17.00000            0.000000
```

b. The shortest paths are as follows:
G -> A = 7
G -> B = 12
G -> D = 2
G -> E = 19
G -> F = 17
G -> G = 0
G -> H = 3

LP CODE

```
max da + db + dd + de + df + dg + de
ST
    dg = 0
    dh - dg <= 3
    dd - dg <= 2
    db - dh <= 9
    da - dh <= 4
    df - da <= 10
    db - da <= 8
    dc - db <= 4
    de - db <= 10
    dd - dc <= 3
    de - dd <= 25
    df - dd <= 18
    dc - df <= 3
    de - df <= 2
    da - df <= 5
    db - df <= 7
    dg - de <= 7
    dd - de <= 9
```

```
 Global optimal solution found.
 Objective value: 76.00000
  Infeasibilities: 0.000000
  Total solver iterations: 3
  Elapsed runtime seconds: 0.02

  Model Class: LP

  Total variables: 7
  Nonlinear variables: 0
  Integer variables: 0
  Total constraints: 18
  Nonlinear constraints: 0

  Total nonzeros: 36
  Nonlinear nonzeros: 0
Variable  Value        Reduced Cost
DA        7.000000     0.000000
DB        12.00000     0.000000
DD        2.000000     0.000000
DE        19.00000     0.000000
DF        17.00000     0.000000
DG        0.000000     0.000000
DH        3.000000     0.000000
DC        0.000000     0.000000

Row    Slack or Surplus    Dual Price
1      76.00000            1.000000
2      0.000000            7.000000
3      0.000000            5.000000
4      0.000000            1.000000
5      0.000000            1.000000
6      0.000000            4.000000
7      0.000000            3.000000
8      3.000000            0.000000
9      16.00000            0.000000
10     3.000000            0.000000
11     1.000000            0.000000
12     8.000000            0.000000
13     3.000000            0.000000
14     20.00000            0.000000
15     0.000000            2.000000
16     15.00000            0.000000
```

	17 12.00000 0.000000 18 26.00000 0.000000 19 26.00000 0.000000

2. The objective function would be 3.45s + 2.32p + 2.81b + 3.25c where s is silk, p is polyester, b is blend1 and c is blend2. This would give us the maximum profit. The constraints can be viewed in the LP Code section in the table below. There must be at least 6000 silk ties and at most 7000. Must be at least 10000 polyester ties and at most 14000 polyester ties. There must be at least 13000 blend1 ties and at most 16000 blend1 ties. There must be at least 6000 blend2 ties and at most 8500 blend2 ties. Each tie amount must be 0 or a positive number.

LP CODE

```
max 3.45s + 2.32p + 2.81b + 3.25c
ST
      .125s <= 1000
      .08p + .05b + .03c <= 2000
      .05b + .07c <= 1250
      s >= 6000
      s <= 7000
      p >= 10000
      p <= 14000
      b >= 13000
      b <= 16000
      c >= 6000
      c <= 8500
      s >= 0
      p >= 0
      b >= 0
      c >= 0
```

```
Global optimal solution found.
  Objective value: 120196.0
  Infeasibilities: 0.000000
  Total solver iterations: 2
  Elapsed runtime seconds: 0.02

  Model Class: LP

  Total variables: 4
  Nonlinear variables: 0
  Integer variables: 0

  Total constraints: 16
  Nonlinear constraints: 0

  Total nonzeros: 22
  Nonlinear nonzeros: 0

Variable   Value        Reduced Cost
S          7000.000     0.000000
P          13625.00     0.000000
B          13100.00     0.000000
C          8500.000     0.000000

Row    Slack or Surplus    Dual Price
1      120196.0            1.000000
2      125.0000            0.000000
3      0.000000            29.00000
4      0.000000            27.20000
5      1000.000            0.000000
6      0.000000            3.450000
7      3625.000            0.000000
8      375.0000            0.000000
9      100.0000            0.000000
10     2900.000            0.000000
11     2500.000            0.000000
12     0.000000            0.4760000
13     7000.000            0.000000
14     13625.00            0.000000
15     13100.00            0.000000
16     8500.000            0.000000
```

The maximum profit would be 120196
With 7000 silk ties
13625 polyester ties

13100 blend1 ties
and 8500 blend2 ties

3. With the following code each variable pxwx indicates a route between plant number x and warehouse number x. Each variable wxrx indicates a route between the warehouse number x and the retailer number x. There can be at most 150 refrigerators coming out of plant 1. At most 450 refrigerators coming out of plant 2, 250 refrigerators coming out of plant 3 and 150 coming out of plant 4. Retailer 1 can take at most 100 refrigerators, retailer 2 can take at most 150 refrigerators, retailer 3 100, retailer 4 200, retailer 5 200 retailer 6 150 and retailer 7 can take at most 100. All variables must be positive or 0.

LP CODE

```
min 10 p1w1 + 15 p1w2 + 11 p2w1
+ 8 p2w2 + 13 p3w1 + 8 p3w2 + 9
p3w3 + 14 p4w2 + 8 p4w3 + 5
w1r1 + 6 w1r2 + 7 w1r3 + 12
w2r3 + 10 w1r4 + 8 w2r4 + 14
w3r4 + 10 w2r5 + 12 w3r5 + 14
w2r6 + 12 w3r6 + 6 w3r7
ST
      p1w1 + p1w2 <= 150
      p2w1 + p2w2 <= 450
      p3w1 + p3w2 + p3w3 <= 250
      p4w2 + p4w3 <= 150
      w1r1 >= 100
      w1r2 >= 150
      w1r3 + w2r3 >= 100
      w1r4 + w2r4 + w3r4 >= 200
      w2r5 + w3r5 >= 200
      w2r6 + w3r6 >= 150
      w3r7 >= 100
      w1r1 + w1r2 + w1r3 + w1r4
- p1w1 - p2w1 - p3w1 = 0
      w2r3 + w2r4 + w2r5 + w2r6
- p1w2 - p2w2 - p3w2 - p4w2 = 0
      w3r4 + w3r5 + w3r6 + w3r7
- p3w3 - p4w3 = 0
      p1w1 >= 0
      p1w2 >= 0
      p2w1 >= 0
      p2w2 >= 0
      p3w1 >= 0
      p3w2 >= 0
      p3w3 >= 0
      p4w2 >= 0
      p4w3 >= 0
      w1r1 >= 0
      w1r2 >= 0
      w1r3 >= 0
      w2r3 >= 0
      w1r4 >= 0
      w2r4 >= 0
      w3r4 >= 0
      w2r5 >= 0
      w3r5 >= 0
      w2r6 >= 0
      w3r6 >= 0
```

```
  Global optimal solution found.
  Objective value: 17100.00
  Infeasibilities: 0.000000
  Total solver iterations: 12
  Elapsed runtime seconds: 0.02

  Model Class: LP

  Total variables: 21
  Nonlinear variables: 0
  Integer variables: 0

  Total constraints: 36
  Nonlinear constraints: 0

  Total nonzeros: 84
  Nonlinear nonzeros: 0



Variable    Value        Reduced Cost
P1W1        150.0000     0.000000
P1W2        0.000000     8.000000
P2W1        200.0000     0.000000
P2W2        250.0000     0.000000
P3W1        0.000000     2.000000
P3W2        150.0000     0.000000
P3W3        100.0000     0.000000
P4W2        0.000000     7.000000
P4W3        150.0000     0.000000
W1R1        100.0000     0.000000
W1R2        150.0000     0.000000
W1R3        100.0000     0.000000
W2R3        0.000000     2.000000
W1R4        0.000000     5.000000
W2R4        200.0000     0.000000
W3R4        0.000000     7.000000
W2R5        200.0000     0.000000
W3R5        0.000000     3.000000
W2R6        0.000000     1.000000
W3R6        150.0000     0.000000
W3R7        100.0000     0.000000

Row    Slack or Surplus   Dual Price
1      17100.00           -1.000000
2      0.000000            1.000000
3      0.000000            0.000000
```

w3r7 >= 0	4	0.000000	0.000000
	5	0.000000	1.000000
	6	0.000000	-16.00000
	7	0.000000	-17.00000
	8	0.000000	-18.00000
	9	0.000000	-16.00000
	10	0.000000	-18.00000
	11	0.000000	-21.00000
	12	0.000000	-15.00000
	13	0.000000	11.00000
	14	0.000000	8.000000
	15	0.000000	9.000000
	16	150.0000	0.000000
	17	0.000000	0.000000
	18	200.0000	0.000000
	19	250.0000	0.000000
	20	0.000000	0.000000
	21	150.0000	0.000000
	22	100.0000	0.000000
	23	0.000000	0.000000
	24	150.0000	0.000000
	25	100.0000	0.000000
	26	150.0000	0.000000
	27	100.0000	0.000000
	28	0.000000	0.000000
	29	0.000000	0.000000
	30	200.0000	0.000000
	31	0.000000	0.000000
	32	200.0000	0.000000
	33	0.000000	0.000000
	34	0.000000	0.000000
	35	150.0000	0.000000
	36	100.0000	0.000000

Where the optimal solution is 17100 (This is the minimum cost so $17,100)
And the optimal routes are:
P1->W1 = 150
P1->W2 = 0
P2->W2 = 200
P2->W2 = 250
P3->W1 = 0
P3->W2 = 150
P3->W3 = 100
P4->W2 = 0
P4->W3 = 150
W1->R1 = 100
W1->R2 = 150
W1->R3 = 100
W2->R3 = 0
W1->R4 = 0
W2->R4 = 200
W3->R4 = 0
W2->R5 = 200
W3->R5 = 0

W2->R6 = 0
W3->R6 = 150
W3->R7 = 100

4. With both part A and Part b t = tomato, l = lettuce, s = spinach, c = carrot, ss = sunflower seeds, st = smoked tofu, cp = chickpeas and o = oil.

a.

LP CODE

```
min 21t + 16l + 40s + 41c + 585ss +
120st + 164cp + 884o
ST
      .85t + 1.62l + 2.86s + .93c +
23.4ss + 16st + 9cp >= 15
      .33t + .2l + .39s + .24c + 48.7ss
+ 5st + 2.6cp + 100o >= 2
      .33t + .2l + .39s + .24c + 48.7ss
+ 5st + 2.6cp + 100o <= 8
      4.64t + 2.37l + 3.63s + 9.58c +
15ss + 3st + 27cp >= 4
      9t + 28l + 65s + 69c + 3.8ss +
120st + 78cp <= 200
      .6l + .6s - .4t - .4c - .4ss - .
4st - .4cp - .4o >= 0
      1t + .75l + .5s + .5c + .45ss +
2.15st + .95cp + 2o - cost = 0
```

```
  Global optimal solution found.
  Objective value: 114.7541
  Infeasibilities: 0.000000
  Total solver iterations: 3
  Elapsed runtime seconds: 0.02

  Model Class: LP

  Total variables: 9
  Nonlinear variables: 0
  Integer variables: 0

  Total constraints: 8
  Nonlinear constraints: 0

  Total nonzeros: 62
  Nonlinear nonzeros: 0



Variable  Value        Reduced Cost
T         0.000000     16.90164
L         0.5854801    0.000000
S         0.000000     14.51366
C         0.000000     36.28962
SS        0.000000     408.3880
ST        0.8782201    0.000000
CP        0.000000     97.55191
O         0.000000     886.4044
COST      2.327283     0.000000

Row    Slack or Surplus      Dual Price
1        114.7541           -1.000000
2        0.000000           -7.650273
3        2.508197            0.000000
4        3.491803            0.000000
5       0.2224824E-01        0.000000
6        78.22014            0.000000
7        0.000000           -6.010929
8        0.000000            0.000000
```

So you can see that a bit better here is the fomula:

```
min 21t + 16l + 40s + 41c + 585ss + 120st + 164cp + 884o //minimum energy or
calories
ST
      .85t + 1.62l + 2.86s + .93c + 23.4ss + 16st + 9cp >= 15 //at least 15 protien
      .33t + .2l + .39s + .24c + 48.7ss + 5st + 2.6cp + 100o >= 2 // at least 2 fat
      .33t + .2l + .39s + .24c + 48.7ss + 5st + 2.6cp + 100o <= 8 // at most 8 fat
      4.64t + 2.37l + 3.63s + 9.58c + 15ss + 3st + 27cp >= 4 //at least 4 carbs
      9t + 28l + 65s + 69c + 3.8ss + 120st + 78cp <= 200 // at most 200 sodium
```

```
      .6l + .6s - .4t - .4c - .4ss - .4st - .4cp - .4o >= 0 // mass of 40% leafy
greens
      1t + .75l + .5s + .5c + .45ss + 2.15st + .95cp + 2o - cost = 0 // figures
cost
```

This indicates that the salad with the lowest calories consists of Lettuce in the amount of .585 and Smoked Tofu in the amount of .878 (what a boring salad!) for a grand total of 114.75 calories (you'll be hungry in half an hour). This salad will cost $2.33

b.

LP CODE

```
min 1t + .75l + .5s + .5c + .45ss +
2.15st + .95cp + 2o
ST
      .85t + 1.62l + 2.86s + .93c +
23.4ss + 16st + 9cp >= 15
      .33t + .2l + .39s + .24c + 48.7ss
+ 5st + 2.6cp + 100o >= 2
      .33t + .2l + .39s + .24c + 48.7ss
+ 5st + 2.6cp + 100o <= 8
      4.64t + 2.37l + 3.63s + 9.58c +
15ss + 3st + 27cp >= 4
      9t + 28l + 65s + 69c + 3.8ss +
120st + 78cp <= 200
      .6l + .6s - .4t - .4c - .4ss - .
4st - .4cp - .4o >= 0
      21t + 16l + 40s + 41c + 585ss +
120st + 164cp + 884o - cals = 0
```

```
  Global optimal solution found.
  Objective value: 1.554133
  Infeasibilities: 0.000000
  Total solver iterations: 3
  Elapsed runtime seconds: 0.01

  Model Class: LP

  Total variables: 9
  Nonlinear variables: 0
  Integer variables: 0

  Total constraints: 8
  Nonlinear constraints: 0

  Total nonzeros: 62
  Nonlinear nonzeros: 0



Variable    Value         Reduced Cost
T           0.000000      1.002081
L           0.000000      0.4029122
S           0.8322983     0.000000
C           0.000000      0.4869142
SS          0.9608330E-01 0.000000
ST          0.000000      0.4056086
CP          1.152364      0.000000
O           0.000000      7.281258
CALS        278.4884      0.000000

Row    Slack or Surplus      Dual Price
1        1.554133           -1.000000
2        0.000000          -0.1312607
3        6.000000            0.000000
4        0.000000          0.5184714E-01
5        31.57633            0.000000
6        55.65109            0.000000
7        0.000000          -0.2413582
8        0.000000            0.000000
```

So you can see that a bit better here is the formula:

```
min 1t + .75l + .5s + .5c + .45ss + 2.15st + .95cp + 2o //lowest cost
ST
      .85t + 1.62l + 2.86s + .93c + 23.4ss + 16st + 9cp >= 15 // at least 15
protien
      .33t + .2l + .39s + .24c + 48.7ss + 5st + 2.6cp + 100o >= 2 //at least 2 fat
      .33t + .2l + .39s + .24c + 48.7ss + 5st + 2.6cp + 100o <= 8 //at most 8 fat
      4.64t + 2.37l + 3.63s + 9.58c + 15ss + 3st + 27cp >= 4 //at least 4 carbs
      9t + 28l + 65s + 69c + 3.8ss + 120st + 78cp <= 200 // at most 200 sodium
```

```
      .6l + .6s - .4t - .4c - .4ss - .4st - .4cp - .4o >= 0 //40% leafy greens
      21t + 16l + 40s + 41c + 585ss + 120st + 164cp + 884o - cals = 0
//calories/energy total
```

So the cheapest salad would consist of Spinach in the amount of .832, Sunflower Seeds in the amount of .096 and Chickpeas in the amount of 1.152(less boring, but not sure I would want to eat it...). This salad has quite a bit more calories at 278.49 calories and has a low low price of $1.55