

Commutative Algebra and Algebraic Geometry.

Math 9014. Fall 2013

- **Professor:** Vasily Dolgushev
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- **Office:** Wachman Hall, Room 530.
- **Office hours:** Tuesday and Thursday 2:30-3:30pm and by appointment.
- **Lectures:** Monday and Wednesday, 10:30-11:50am in Wachman Hall Rm. 527.
- **Web-site:** Go to the *Blackboard*

<https://blackboard.temple.edu/webapps/login/>

Please, **visit the blackboard often** for homework assignments and for announcements.

- **Prerequisites:** Math 8011, 8012 or permission of instructor. It would be helpful to know rudiments of point-set topology.
- **Course Description:** This is a one-semester course on fundamental concepts of commutative algebra. Topics of this course include: ideals of commutative rings, modules, Noetherian and Artinian rings, Noether normalization, Hilbert's Nullstellensatz, rings of fractions, primary decomposition, discrete valuation rings and rudiments of the dimension theory.
- **Homeworks:** Assignments of homeworks are posted on the blackboard. They will be collected on Wednesdays in class. You may work together on homework assignments, but I expect everybody to write up their own solutions; obvious copies will not be accepted. The writing component is an essential part of this course. The lowest grade for the homework assignment will be dropped.
- **Final project:** In mid-November, I will list 5-7 original research papers related to commutative algebra. Each student will have to choose one of the papers, read it and write a one-page review. This review should describe the results of the paper and what the student learnt from this paper. Several students may choose the same paper. In fact all students may choose the same paper. But the reviews should be written independently. The reviews are due on December 13 (Friday).

- **Attendance Policy:** Attendance will not be monitored, but you are strongly encouraged to attend class regularly and to take complete class notes.
- **Grading:** The maximal total score is 300 pts. It is split as follows:
 1. Homeworks: $20 \times 12 = 240$ pts.
 2. Final project: 60 pts.

A homework assignment with the lowest score will be dropped.

The grading guide is similar to the one you had in your college

A/A-	90-100%
B+/B/B-	80-89%
C+/C/C-	70-79 %
D	60-69 %
F	below 60 %

- **Written assignments.** You have to **justify all your steps** in your written assignments. Also, please, **write legibly!** Not justifying steps or not writing legibly may reduce your score.
- **Crucial dates:**
 - The first day of classes is Monday, August 26
 - The last day to drop/add (tuition refund available) is Monday, September 9
 - Thanksgiving is on Thursday, November 28
 - The last day to withdraw (no refund) is Tuesday, October 22
 - The last day of classes is Wednesday, December 4

Content of the course:

1. Basics: rings, ideals, prime and maximal ideals, zero-divisors, nilpotents, radical of an ideal, local rings.
2. Modules over commutative rings: the Cayley-Hamilton theorem, the determinant trick, Nakayama's lemma, exact sequences.
3. Noetherian rings. Noetherian modules. The Hilbert basis theorem.
4. Finite extensions and Noether normalization: finite and integral A -algebras, tower laws, integral closure, non-singularity and normal rings, Noether normalization and the weak Nullstellensatz.

5. Hilbert's Nullstellensatz and $\text{Spec}(A)$. Definition of a variety. Irreducible varieties.
6. Rings of fractions $S^{-1}A$. Localization and its properties.
7. Primary decomposition.
8. Discrete valuation rings and normal integral domains.
9. Dimension theory for commutative rings: Hilbert function. Dimension theory for Noetherian local rings. Regular local rings. Transcendental dimension.

Optional complementary topics:

- Homological methods in commutative algebra.
- Depth. Codimension and Cohen-Macaulay rings.

Textbooks and complementary reading:

- M.F. Atiyah and I.G. Macdonald, *Introduction to commutative algebra*. Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont. 1969.
- D. Eisenbud, *Commutative algebra: With a view toward algebraic geometry*, Graduate Texts in Mathematics, **150** Springer-Verlag, New York, 1995.
- Ernst Kunz, *Introduction to Commutative algebra and algebraic geometry*, Birkhauser 1985,
- H. Matsumura, *Commutative algebra*, W. A. Benjamin, Inc., New York 1970.
- J.S. Milne, A Primer of Commutative Algebra, available openly at <http://www.jmilne.org/math/xnotes/ca.html>

G O O D L U C K !