ENGR 5022 Engineering Analysis

Homework Chapter 3
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1. Eigenvalue Problem

Exercise 1.1. *Show that the eigenvectors of a symmetric matrix are orthogonal.*

Proof:

□

Exercise 1.2. *Consider the matrix in the controllable canonical form*

$$A = \begin{bmatrix} 0 & 1 & 0 & ... & 0 & 0 \\ 0 & 0 & 1 & ... & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 & 1 \\ -c_0 & -c_1 & -c_2 & ... & -c_{n-2} & -c_{n-1} \end{bmatrix}.$$

Show that the characteristic equation of this matrix is

$$\lambda^n + c_{n-1}\lambda^{n-1} + ... + c_1\lambda_1 = 0.$$

Proof:

□

Exercise 1.3. *Consider the matric in the observable canonical form*

$$A = \begin{bmatrix} 0 & 0 & 0 & ... & 0 & -c_0 \\ 1 & 0 & 0 & ... & 0 & -c_1 \\ \vdots & \vdots & \vdots & \ddots & 0 & -c_{n-2} \\ 0 & 0 & 0 & ... & 1 & -c_{n-1} \end{bmatrix}.$$

Show that the characteristic equation of this matrix is

$$\lambda^n + c_{n-1}\lambda^{n-1} + ... + c_1\lambda_1 = 0.$$

Proof:

□

Exercise 1.4. *Consider the matrix*

$$M = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \textbf{-}\boldsymbol{Q} & \textbf{-}\boldsymbol{A'} \end{bmatrix}$$

*where **A** and **Q** are square matrices. This matrix is known as the Hamiltonian matrix in control system theory. Show that the eigenvalues of **M** are symmetrically located with respect to the imaginary axis*

Proof:

$\square$

Exercise 1.5.

Proof:

$\square$

Exercise 1.6.

Proof:

$\square$

Exercise 1.7. *Given the matrix*

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix}.$$

Using Cayley-Hamilton theorem, find the matrix $\boldsymbol{B} = \sqrt{\boldsymbol{A}}$*. Also compute the eigenvalues of the matrices* $\boldsymbol{A}$ *and* $\boldsymbol{B}$ *and verify that* $\lambda_{\boldsymbol{B}} = \sqrt{\lambda_{\boldsymbol{A}}}$*.*

Proof:

$\square$

Exercise 1.8. *For the matrix given above, find the function* $\boldsymbol{B} = sin(\boldsymbol{A})$*. Also verify that* $\lambda_{\boldsymbol{B}} = sin(\lambda_{\boldsymbol{A}})$*.*

Proof:

$\square$

Exercise 1.9. *Compute the matrix exponentual* $e^{\boldsymbol{A}}$ *for the following matrices:*

$$\begin{bmatrix} 2 & -1 & 0 \\ 2 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Proof:

$\square$

Exercise 1.10. *Using the Cayley Hamilton theorem compute the inverse of the following matrices:*

$$\begin{bmatrix} 2 & -1 & 0 \\ 2 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Proof:

$\square$

Exercise 1.11. *Show that the eigenvalues of the matrices* $\boldsymbol{A}$ *and* $\boldsymbol{A}$*' are identical but the eigenvectors are different. Find a relation between the two sets of eigenvectors.*

Proof:

$\square$

Exercise 1.12. *Supposed $\boldsymbol{T}$ is a transformation matrix that diagonalizes the matrix $\boldsymbol{A}$, i.e. $\boldsymbol{D} = \boldsymbol{T}^{-1}\boldsymbol{A}\boldsymbol{T}$. Show that $\boldsymbol{A}^k = \boldsymbol{T}\boldsymbol{D}^k\boldsymbol{T}^{-1}$. Note that in this method the computation of matrix power becomes relatively simple since $\boldsymbol{D}^k$ is easily computed. Note: The same concept holds for any other matrix function as well.*

Proof:

□

Exercise 1.13. *Find the eigenvalues and eigenvectors of the matrix $\boldsymbol{A}^n$ where n is arbitrary*

$$\begin{bmatrix} 2 & -1 & 0 \\ 2 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

Proof:

□

Exercise 1.14. *Show that if λ is an eigenvalue of $\boldsymbol{A}$ then $f(\lambda)$ is an eigenvalue of the matrix $f(\boldsymbol{A})$, where f is analytic function of its argument. Hint: Use Cayley-Hamilton theorem and the basic definition of eigenvalue problem.*

Proof:

□

Exercise 1.15. *Show that for a matrix $\boldsymbol{A}$ with eigenvalues λ_i, $i = 1, 2, ..., n$, show that*

$$|\boldsymbol{A}| = \prod_{i=1}^{n} \lambda_i$$

$$Tr(\boldsymbol{A}) = \sum_{i=1}^{n} \lambda_i$$

Proof:

□

Exercise 1.16. *Suppose $\boldsymbol{A}$ is an $n \times n$ matrix whose eigenvalues are not necessarily distinct. Show that the set of eigenvectors and generalized eigenvectors form a basis for the space $\mathbb{R}^n$.*

Proof:

□

Exercise 1.17. *Find the eigenvalues an eigenvectors for the following matrices:*

$$\begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}, \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$$

Proof:

□

Exercise 1.18. *Suppose $\boldsymbol{A}$ be a block diagonal matrix given by*

$$A = \begin{bmatrix} \boldsymbol{M}_1 & 0 & 0 \\ 0 & \boldsymbol{M}_2 & 0 \\ 0 & 0 & \boldsymbol{M}_3 \end{bmatrix}$$

where the diagonal blocks are square matrices. Show that the eigenvalues of $\mathbf{A}$ *are those matrices* $\boldsymbol{M}_1$, $\boldsymbol{M}_2$, *and* $\boldsymbol{M}_3$

Proof:

□

Exercise 1.19.

Proof:

□

Exercise 1.20. *Show that the transfer function of a system is invariant under similarity transformation.*

Proof:

□

Exercise 1.21. *Let* λ *be an eigenvalue of the matrix* $\mathbf{A}$. *Then show that* λ^{ϕ} *is an eigenvalue of the matrix* $\mathbf{A}^{\phi}$, *where* ϕ *is not necessarily an integer.*

Proof:

□