Numerical general relativity using Euler integration

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Abstract

Two numerical solutions for general relativity are provided. The topics under consideration are the deflection of light and neutrinos by the Sun, and the relativistic rotation of Mercury's orbit plane. To simplify the calculations, Euler integration is used.

1 Introduction

Euler is just fine for calculating deflection and precession.

In the case of deflection, the timeslice is simply:

$$\delta_{Timeslice} = 1. \tag{1}$$

Note that Euler integration automatically leads to a negative rotation in terms of Mercury's orbit plane, but given a small enough timeslice, this negative rotation becomes negligible:

$$\delta_{Timeslice} = \frac{c}{||\vec{v}_{Orbiter}||} \times 10^{-5}. \tag{2}$$

The analytical solution for the deflection angle is:

$$\delta_{Deflection} = \frac{4GM_{Sun}}{c^2} \left(\frac{1}{\pi \times 180 \times 3600} \right) = 1.75 \text{ arc seconds.} \tag{3}$$

The predicted amount is 1.75 arc seconds, which is practically identical to the amount given by the analytical solution.

The analytical solution for the precession of the perihelion of Mercury is:

$$\delta_{Precession} = \frac{6\pi GM_{Sun}}{c^2(1-e^2)a} \left(\frac{1}{\pi \times 180 \times 3600} \right) \left(\frac{365}{88} \times 100 \right) = 42.937 \text{ arc seconds per Earth century,} \tag{4}$$

where $e = 0.2056$ is the eccentricity and $a = 57.909 \times 10^9$ is the semi-major axis. Altogether the relativistic precession predicted is 46 arc seconds per Earth century, where the amount given by the analytical solution is 43 arc seconds per Earth century.

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Where ℓ denotes location, and $\vec{d}$ denotes the direction vector that points from Mercury toward the Sun:

$$\vec{d} = \ell_{Sun} - \ell_{Orbiter}, \tag{5}$$

$$\hat{d} = \frac{\vec{d}}{||\vec{d}||}. \tag{6}$$

The acceleration vector is:

$$\vec{g} = \frac{\hat{d} G M_{Sun}}{||\vec{d}||^2}. \tag{7}$$

Two important values are:

$$\alpha = 2 - \sqrt{1 - \frac{||\vec{v}_{Orbiter}||^2}{c^2}}, \tag{8}$$

where internal process is seen to be a resistance to gravitation, and

$$\beta = \sqrt{1 - \frac{R_{Schwarzschild}}{||\vec{d}||}}, \tag{9}$$

where

$$R_{Schwarzschild} = \frac{2GM_{Sun}}{c^2}. \tag{10}$$

The Euler integration is:

$$\vec{v}_{Orbiter} = \vec{v}_{Orbiter} + \alpha \vec{g} \delta_{Timeslice}, \tag{11}$$

$$\ell_{Orbiter} = \ell_{Orbiter} + \beta \vec{v}_{Orbiter} \delta_{Timeslice}. \tag{12}$$

References

[1] Misner et al. Gravitation. (1970)