

# Newtonian gravitation for C++ programmers

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## Abstract

...

## 1 Typedefs

```
typedef long double real_type;
```

or

```
#include <boost/multiprecision/cpp_bin_float.hpp>
using namespace boost::multiprecision;
```

```
typedef number<
    backends::cpp_bin_float<
        237,
        backends::digit_base_2,
        void,
        std::int32_t,
        -262142,
        262143>,
    et_off> cpp_bin_float_oct;

typedef cpp_bin_float_oct real_type;
```

## 2 Constants

```
const real_type dt = 10000; // 2.77777 hours
```

```
const real_type pi = 4.0 * atan(1.0);
```

```
const real_type G = 6.67430e-11;
```

```
const real_type c = 299792458;
```

```
const real_type c2 = c * c;
```

```
const real_type c3 = c * c * c;
```

---

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```

const real_type c4 = c * c * c * c;
const real_type h = 6.62607015e-34;
const real_type hbar = h / (2.0 * pi);

```

### 3 Brute force: integer field line count

Where  $r$  is the receiver radius,  $R$  is the distance from the centre of the emitter,  $\beta$  is the get intersecting line count function, and  $n$  is the field line count, the gradient is:

$$\alpha = \frac{\beta(R + \epsilon) - \beta(R)}{\epsilon}. \quad (1)$$

The gradient strength is:

$$g = \frac{-\alpha}{r^2}. \quad (2)$$

```

long long unsigned int get_intersecting_line_count(
    const vector<vector_3>& unit_vectors ,
    const vector_3& sphere_location ,
    const real_type sphere_radius)
{
    long long unsigned int count = 0;

    vector_3 cross_section_edge_dir(sphere_location.x, sphere_radius, 0);
    cross_section_edge_dir.normalize();

    vector_3 receiver_dir(sphere_location.x, 0, 0);
    receiver_dir.normalize();

    const real_type min_dot = cross_section_edge_dir.dot(receiver_dir);

    for (size_t i = 0; i < unit_vectors.size(); i++)
        if (unit_vectors[i].dot(receiver_dir) >= min_dot)
            count++;

    return count;
}

```

```

int main(int argc, char** argv)
{
    // Field line count
    const size_t n = 1000000000;

    cout << "Allocating memory for field lines" << endl;
    vector<vector_3> unit_vectors(n);

    for (size_t i = 0; i < n; i++)
    {
        unit_vectors[i] = RandomUnitVector();

        static const size_t output_mod = 10000;
    }
}

```

```

        if (i % output_mod == 0)
            cout << "Getting pseudorandom locations: "
                << static_cast<float>(i) / n << endl;
    }

    string filename = "newton.txt";
    ofstream out_file(filename.c_str());
    out_file << setprecision(30);

    const real_type start_distance = 10.0;
    const real_type end_distance = 100.0;
    const size_t distance_res = 1000;

    const real_type distance_step_size =
        (end_distance - start_distance)
        / (distance_res - 1);

    for (size_t step_index = 0; step_index < distance_res; step_index++)
    {
        const real_type r =
            start_distance +
            step_index * distance_step_size;

        const vector_3 receiver_pos(r, 0, 0);
        const real_type receiver_radius = 1.0;

        const real_type epsilon = 1.0;

        vector_3 receiver_pos_plus = receiver_pos;
        receiver_pos_plus.x += epsilon;

        const long long signed int collision_count_plus =
            get_intersecting_line_count(
                unit_vectors,
                receiver_pos_plus,
                receiver_radius);

        const long long signed int collision_count =
            get_intersecting_line_count(
                unit_vectors,
                receiver_pos,
                receiver_radius);

        const real_type gradient =
            static_cast<real_type>
            (collision_count_plus - collision_count)
            / epsilon;

        const real_type gradient_strength =
            -gradient
            / (receiver_radius * receiver_radius);

        cout << "r: " << r << " gradient strength: "
            << gradient_strength << endl;
    }

```

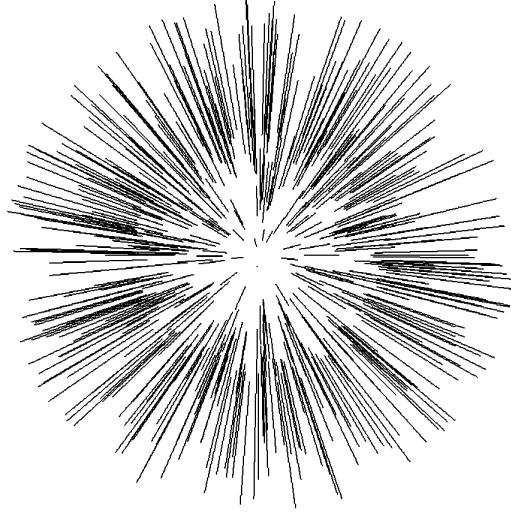


Figure 1: Example of an isotropic emitter. The emitter is spherical. The field line starting locations are placed pseudorandomly on a 2-sphere, and the normals (e.g. field line directions) are calculated using the same sphere.

```

        out_file << r << " " << gradient_strength << endl;
    }

    out_file.close();

    return 0;
}

```

While this method works, it is both memory and processor intensive. Thus, this method is meant to be a stepping stone for the next section.

## 4 Heuristic: real field line count

Where  $r$  is the receiver radius,  $R$  is the distance from the centre of the emitter,  $\beta$  is the get intersecting line count function, and  $n$  is the field line count, the gradient is:

$$\alpha = \frac{\beta(R + \epsilon) - \beta(R)}{\epsilon}. \quad (3)$$

Here we assume that the maximum number of field lines is given by the holographic principle:

$$n = \frac{Ac^3}{4G\hbar \log 2}. \quad (4)$$

The gradient strengths are:

$$g = \frac{-\alpha}{r^2} \approx \frac{n}{2R^3}, \quad (5)$$

$$g_N = \frac{gR\hbar \log 2}{2\pi M} = \frac{nc\hbar \log 2}{4\pi MR^2} = \frac{Ac^4}{16\pi GMR^2} = \frac{GM}{R^2}. \quad (6)$$

```

real_type get_intersecting_line_count(
    const real_type n,
    const vector_3& sphere_location,
    const real_type sphere_radius)
{
    const real_type big_area =
        4 * pi * sphere_location.x * sphere_location.x;

    const real_type small_area =
        pi * sphere_radius * sphere_radius;

    const real_type ratio =
        small_area / big_area;

    return n * ratio;
}

int main(int argc, char** argv)
{
    const real_type emitter_radius = 1.0;

    const real_type emitter_area =
        4.0 * pi * emitter_radius * emitter_radius;

    // Field line count
    // re: holographic principle:
    const real_type n =
        (c3 * emitter_area)
        / (log(2.0) * 4.0 * G * hbar);

    const real_type emitter_mass = c2 * emitter_radius / (2.0 * G);

    // 1.73502e+70 is the 't Hooft-Susskind constant:
    // the number of field lines for a black hole of
    // unit Schwarzschild radius
    //
    //const real_type G_ =
    //    (c3 * pi)
    //    / (log(2.0) * hbar * 1.73502e+70);

    const string filename = "newton.txt";
    ofstream out_file(filename.c_str());
    out_file << setprecision(30);

    const real_type start_distance = 10.0;
    const real_type end_distance = 100.0;
    const size_t distance_res = 1000;

    const real_type distance_step_size =
        (end_distance - start_distance)
        / (distance_res - 1);

```

```

for (size_t step_index = 0; step_index < distance_res; step_index++)
{
    const real_type r =
        start_distance + step_index * distance_step_size;

    const vector_3 receiver_pos(r, 0, 0);
    const real_type receiver_radius = 1.0;

    const real_type epsilon = 1.0;

    vector_3 receiver_pos_plus = receiver_pos;
    receiver_pos_plus.x += epsilon;

    // https://en.wikipedia.org/wiki/Directional\_derivative
    const real_type collision_count_plus =
        get_intersecting_line_count(
            n,
            receiver_pos_plus,
            receiver_radius);

    const real_type collision_count =
        get_intersecting_line_count(
            n,
            receiver_pos,
            receiver_radius);

    const real_type gradient =
        (collision_count_plus - collision_count)
        / epsilon;

    real_type gradient_strength =
        -gradient
        / (receiver_radius * receiver_radius);

    const real_type gradient_strength_ =
        n / (2.0 * pow(receiver_pos.x, 3.0));

    const real_type newton_strength =
        n * c * hbar * log(2.0)
        /
        (pow(receiver_pos.x, 2.0)
         * emitter_mass * 4.0 * pi);

    const real_type newton_strength_ =
        c4 * emitter_area
        / (16.0 * pi * G
         * pow(receiver_pos.x, 2.0) * emitter_mass);

    const real_type newton_strength__ =
        G * emitter_mass / pow(receiver_pos.x, 2.0);

    const real_type newton_strength___ =
        gradient_strength_ * receiver_pos.x

```

```

        * c * hbar * log(2)
        / (2 * pi * emitter_mass);

        //cout << newton_strength-- / newton_strength << endl;

        cout << "r: " << r << " gradient strength: "
             << gradient_strength << endl;

        out_file << r << " " << gradient_strength << endl;
    }

    out_file.close();

    return 0;
}

```

This method is faster and less memory intensive when compared to the integer field count method. This method is meant to be a stepping stone for the next section.

## 5 Application: modeling Mercury's orbit using numerical integration

The initial conditions are:

```

vector_3 Mercury_pos(0, 69817079000.0, 0);
vector_3 Mercury_vel(-38860, 0, 0);

```

The orbit code is:

```

vector_3 Newtonian_acceleration(
    const real_type emitter_mass,
    const vector_3& pos, // Receiver pos
    const real_type G)
{
    vector_3 grav_dir = vector_3(0, 0, 0) - pos;
    const real_type distance = grav_dir.length();
    grav_dir.normalize();

    vector_3 accel = grav_dir * G * emitter_mass / pow(distance, 2.0);

    return accel;
}

```

Here we show the Euler integration, which is extremely simple:

```

void proceed_Euler(
    vector_3& pos,
    vector_3& vel,
    const real_type G,
    const real_type dt)
{
    vector_3 accel =
        Newtonian_acceleration(
            emitter_mass,

```

```

        pos,
        G);

    vel += accel * dt;
    pos += vel * dt;
}

```

And so the passage of time is computed as:

```

void idle_func(void)
{
    proceed_Euler(Mercury_pos, Mercury_vel, G, dt);
}

```

On the other hand, rather than using Euler integration, the order-4 symplectic integration does a better job at conserving energy, but at a speed cost:

```

void proceed_symplectic_order_4(
    vector_3& pos,
    vector_3& vel,
    real_type G,
    real_type dt)
{
    static const real_type cr2 =
        pow(2.0, 1.0 / 3.0);

    static const real_type c[4] =
    {
        1.0 / (2.0 * (2.0 - cr2)),
        (1.0 - cr2) / (2.0 * (2.0 - cr2)),
        (1.0 - cr2) / (2.0 * (2.0 - cr2)),
        1.0 / (2.0 * (2.0 - cr2))
    };

    static const real_type d[4] =
    {
        1.0 / (2.0 - cr2),
        -cr2 / (2.0 - cr2),
        1.0 / (2.0 - cr2),
        0.0
    };

    pos += vel * c[0] * dt;
    vel += Newtonian_acceleration(
        emitter_mass,
        pos,
        G) * d[0] * dt;

    pos += vel * c[1] * dt;
    vel += Newtonian_acceleration(
        emitter_mass,
        pos,
        G) * d[1] * dt;

    pos += vel * c[2] * dt;
    vel += Newtonian_acceleration(

```



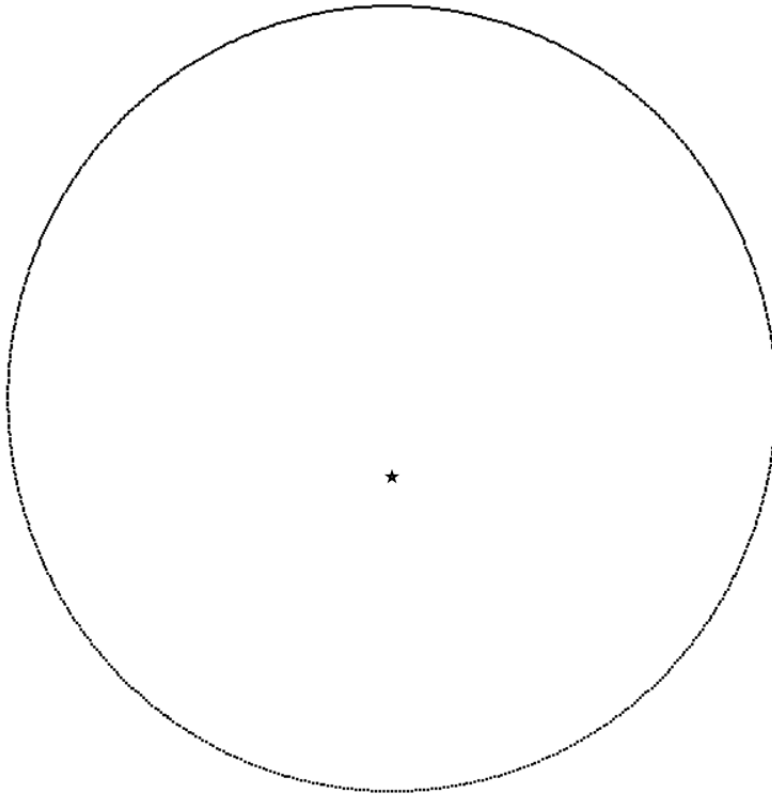


Figure 2: Mercury in orbit around the Sun.

```
        emitter_mass ,  
        pos ,  
        G) * d[2] * dt ;  
  
    pos += vel * c[3] * dt ;  
    // last element d[3] is always 0  
}
```

## 6 Final code

A final code, which models the orbit of Mercury, is at:

[https://github.com/sjhalayka/mercury\\_orbit\\_glut](https://github.com/sjhalayka/mercury_orbit_glut)

## References

- [1] Fiedler. Fix Your Timestep! (2004)
- [2] Fiedler. Integration Basics. (2004)