On the quantum decomposition of the planet Mercury's orbit path using post-Newtonian gravitation

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Abstract

By quantizing the kinematic and gravitational time dilation using various step sizes, one obtains a set of weighted orbit paths. The precession associated with each weighted orbit path combines to provide the same answer as the classical analytical solution.

1 Introduction

In a previous paper [1], we introduced a method of numerical simulation for the four Solar System tests of general relativity, in particular the precession of Mercury's orbit. There was a catch: one had to quantize the kinematic and gravitational time dilation by casting the relevant floating point variables from double-precision to single-precision. A first, this was taken to be a bug, but after careful consideration, it turns out to be a feature of reality. This paper will demonstrate how to decompose the orbit path of Mercury, to numerically obtain the relativistic orbit precession.

The C++ code for this paper is available. It uses the Boost multiprecision floating point library, so that we may specify an arbitrary number of mantissa bits.

2 Quantization of time dilation

In this paper, we will be quantizing the kinematic and gravitational time dilation by casting them to a lesser-precision floating point number.

The non-exponent bit count n includes the number of mantissa bits m, plus one sign bit. We generally used $n = 100$, except for the kinematic and gravitational time dilation, which uses a lesser, various precision (e.g. $n = 24$).

For subnormal numbers such as those used here, the smallest step size that can be represented is $\epsilon = 2 \times 2^{-b}$, where b is the largest exponent value (e.g. $2^7 - 1 = 127$ in single-precision floating point numbers, $2^{10} - 1 = 1023$ for doubles). For instance, we use 1023 where $n = 100$, and 127 otherwise (e.g. where $n = 24$).

As for m, it governs how many places there are after the decimal point.

See Figs. 1 and 2.

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3 Steps in spacetime

Where ℓ_s denotes the Sun's location at the origin, ℓ_o denotes the orbiter Mercury's location, and $\vec{d}$ denotes the direction vector that points from the orbiter toward the Sun:

$$\vec{d} = \ell_s - \ell_o, \tag{1}$$

$$\hat{d} = \frac{\vec{d}}{||\vec{d}||}, \tag{2}$$

the Newtonian acceleration vector is:

$$\vec{g}_n = \frac{\hat{d}GM}{||\vec{d}||^2}. \tag{3}$$

One parameter is closely related to the kinematic time dilation:

$$\alpha = 2 - \sqrt{1 - \frac{||\vec{v}_o||^2}{c^2}}. \tag{4}$$

Another parameter is the gravitational time dilation from the Schwarzschild solution:

$$\beta = \sqrt{1 - \frac{R_s}{||\vec{d}||}}. \tag{5}$$

Finally, the semi-implicit Euler integration for velocity and then location is:

$$\vec{v}_o(t + \delta_t) = \vec{v}_o(t) + \delta_t \alpha \vec{g}_n, \tag{6}$$
$$\ell_o(t + \delta_t) = \ell_o(t) + \delta_t \beta \vec{v}_o(t + \delta_t). \tag{7}$$

Note that Newtonian gravity is the result where $\alpha = \beta = 1$.
See Figs. 3 and 4.

4 Classical analytical calculation of Mercury's orbit precession

The classical orbit precession is:

$$\delta_p = \frac{6\pi GM}{c^2(1 - e^2)a} \left(\frac{1}{\pi \times 180 \times 3600} \right) \left(\frac{365}{88} \times 100 \right) = 42.937 \tag{8}$$

arcseconds per Earth century, where $e = 0.2056$ is the eccentricity, and $a = 5.7909 \times 10^{10}$ is the semi-major axis.
See Fig. 5.

5 Quantum path decomposition

Here we report the precession angle δ_p in terms of arcseconds per Earth century, after one full orbit. There is a highly visible pattern, which we shall point out.

We use an initial location that is 6.9817079×10^{10} metres from the Sun (e.g. the aphelion). We use various initial speeds.

Where initial speed is 25000 metres per second, and the analytical solution is 103.7:

Bits n	Angle δ_p
20	17..04
21	17.04
22	17.04
23	47.8
24	38.95
25	34.8
26	34.96
27	34.9
28	35.07
29	35.08
30	35.06

If we add bit counts 22, 23, and 24 together (e.g. 17.04 + 47.8 + 38.95) we get 103.79 arcseconds per Earth century. All of the other bit counts have a weight of zero.

Where initial speed is 20000 metres per second, and the analytical solution is 162.09:

Bits n	Angle δ_p
21	27.86
22	76.62
23	52.9

If we add bit counts 21, 22, and 23 together, we get 157.38.

Where initial speed is 30000 metres per second, and the analytical solution is 72.04:

Bits n	Angle δ_p
23	46.45
24	27.8

If we add bit counts 23 and 24 together, we get 74.25.

Where initial speed is 38858.47 metres per second (e.g. the actual speed at aphelion), and the analytical solution is 42.9:

Bits n	Angle δ_p
24	46.8

See Eq. 8. Note that a single-precision floating point number has 24 mantissa and sign bits.

Where initial speed is 42500 metres per second, and the analytical solution is 35.8:

Bits n	Angle δ_p
26	51.7

Obviously, this final solution's weight is less than 1 (e.g. $(35.8/51.7) < 1$).

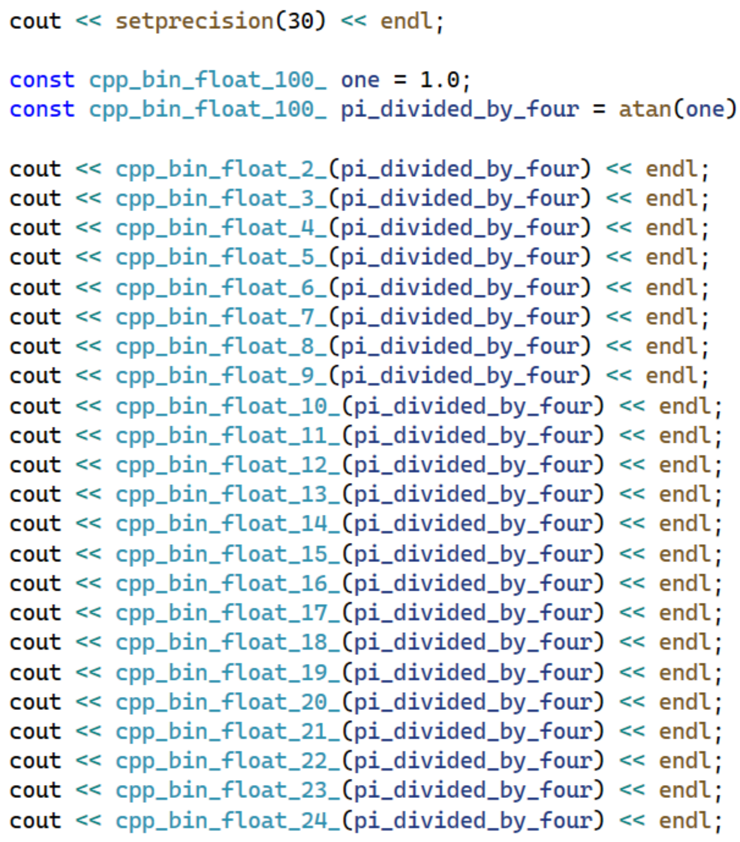
6 Conclusion

Here we have quantized the kinematic and gravitational time dilation. The result is a set of weighted orbit paths that add up to provide the same answer as the classical analytical solution.

References

[1] Halayka. On simulating the four Solar System tests of general relativity using two-parameter post-Newtonian gravitation with Euler integration. (2024)

[2] Misner et al. Gravitation. (1970)

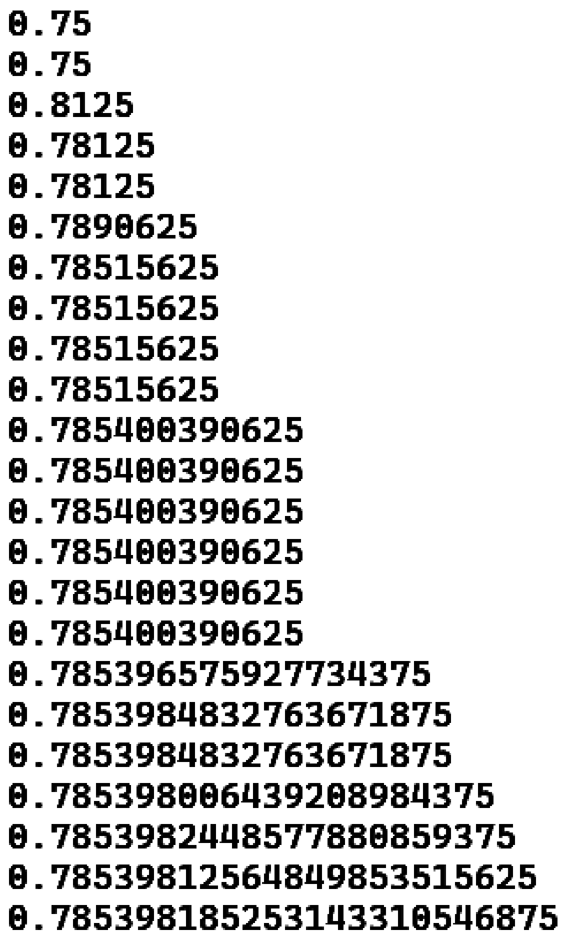


```
cout << setprecision(30) << endl;

const cpp_bin_float_100_ one = 1.0;
const cpp_bin_float_100_ pi_divided_by_four = atan(one);

cout << cpp_bin_float_2_(pi_divided_by_four) << endl;
cout << cpp_bin_float_3_(pi_divided_by_four) << endl;
cout << cpp_bin_float_4_(pi_divided_by_four) << endl;
cout << cpp_bin_float_5_(pi_divided_by_four) << endl;
cout << cpp_bin_float_6_(pi_divided_by_four) << endl;
cout << cpp_bin_float_7_(pi_divided_by_four) << endl;
cout << cpp_bin_float_8_(pi_divided_by_four) << endl;
cout << cpp_bin_float_9_(pi_divided_by_four) << endl;
cout << cpp_bin_float_10_(pi_divided_by_four) << endl;
cout << cpp_bin_float_11_(pi_divided_by_four) << endl;
cout << cpp_bin_float_12_(pi_divided_by_four) << endl;
cout << cpp_bin_float_13_(pi_divided_by_four) << endl;
cout << cpp_bin_float_14_(pi_divided_by_four) << endl;
cout << cpp_bin_float_15_(pi_divided_by_four) << endl;
cout << cpp_bin_float_16_(pi_divided_by_four) << endl;
cout << cpp_bin_float_17_(pi_divided_by_four) << endl;
cout << cpp_bin_float_18_(pi_divided_by_four) << endl;
cout << cpp_bin_float_19_(pi_divided_by_four) << endl;
cout << cpp_bin_float_20_(pi_divided_by_four) << endl;
cout << cpp_bin_float_21_(pi_divided_by_four) << endl;
cout << cpp_bin_float_22_(pi_divided_by_four) << endl;
cout << cpp_bin_float_23_(pi_divided_by_four) << endl;
cout << cpp_bin_float_24_(pi_divided_by_four) << endl;
```

Figure 1: Code, showing the quantization of a constant $\pi/4$.



```
0.75
0.75
0.8125
0.78125
0.78125
0.7890625
0.78515625
0.78515625
0.78515625
0.78515625
0.785400390625
0.785400390625
0.785400390625
0.785400390625
0.785400390625
0.785400390625
0.7853965759277734375
0.78539848327763671875
0.78539848327763671875
0.785398006439208984375
0.7853982448577880859375
0.78539812564849853515625
0.785398185253143310546875
```

Figure 2: Output from the code.

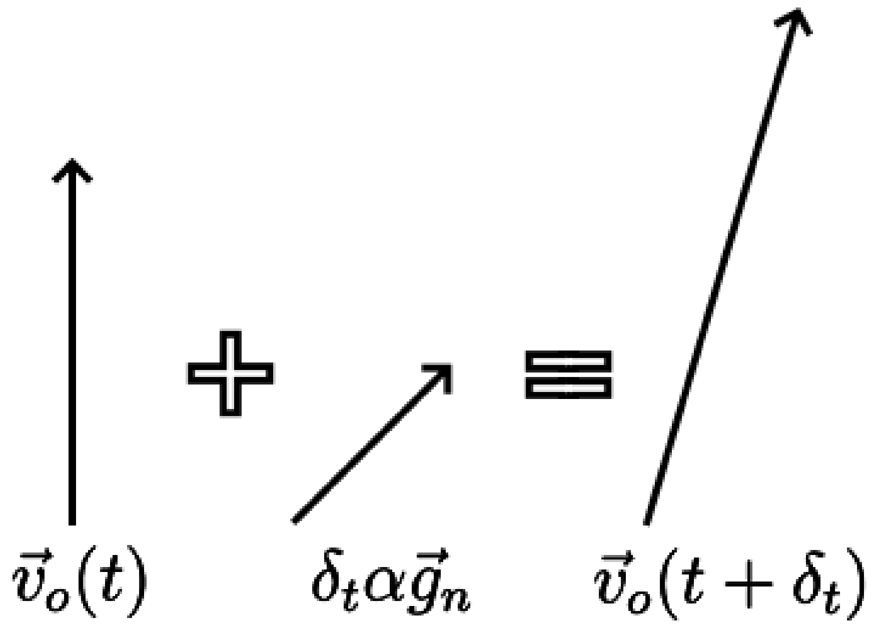


Figure 3: A diagram of the Euler integration of velocity.

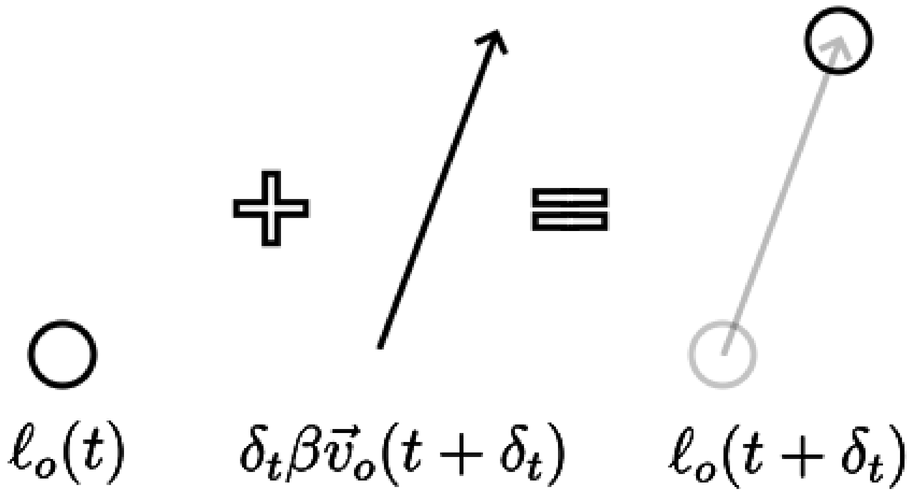


Figure 4: A diagram of the Euler integration of location.

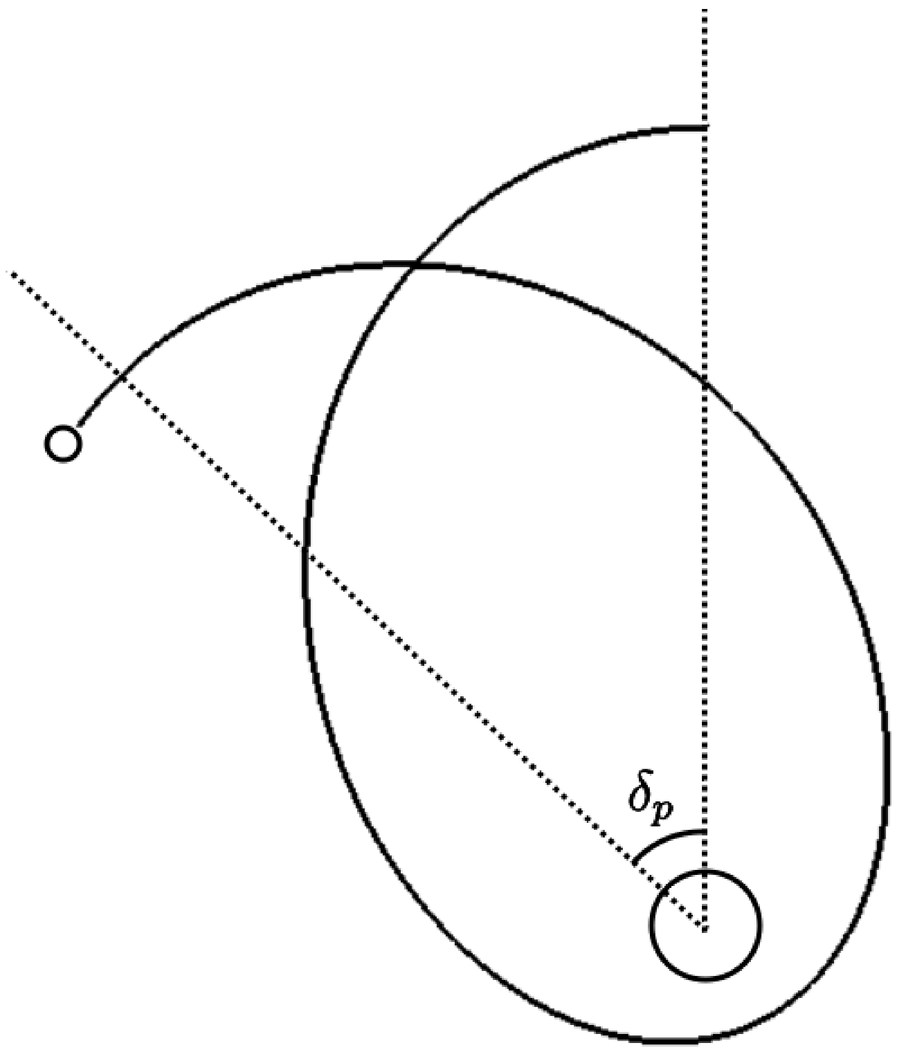


Figure 5: A diagram showing precession, where the orbit does not quite form a closed ellipse.