On the quantum decomposition of the planet Mercury's orbit path

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Abstract

By quantizing the gravitational time dilation using various step sizes, one obtains a set of weighted paths. The precession associated with each weighted path combines to provide the same answer as the classical analytical solution.

1 On time dilation

The kinematic time dilation is:

$$\frac{d\tau}{dt} = \frac{\sqrt{c^2 - ||\vec{v}||^2}}{c} = \sqrt{1 - \frac{||\vec{v}||^2}{c^2}}. \tag{1}$$

The gravitational time dilation is:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{R_s}{r}}. \tag{2}$$

Quantized step size (e.g. epsilon) in general is $\epsilon = 2^{-m}$ where m is the number of mantissa bits in the floating point number.

2 Steps in spacetime

Where ℓ_s denotes the Sun's location at the origin, ℓ_o denotes the orbiter's location, and $\vec{d}$ denotes the direction vector that points from the orbiter toward the Sun:

$$\vec{d} = \ell_s - \ell_o, \tag{3}$$

$$\hat{d} = \frac{\vec{d}}{||\vec{d}||}, \tag{4}$$

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the Newtonian acceleration vector is:

$$\vec{g}_n = \frac{\hat{d}GM}{||\vec{d}||^2}. \tag{5}$$

One parameter is closely related to the kinematic time dilation:

$$\alpha = 2 - \sqrt{1 - \frac{||\vec{v}_o||^2}{c^2}}. \tag{6}$$

Another parameter is the gravitational time dilation:

$$\beta = \sqrt{1 - \frac{R_s}{||\vec{d}||}}. \tag{7}$$

Finally, the semi-implicit Euler integration for velocity and then location is:

$$\vec{v}_o(t + \delta_t) = \vec{v}_o(t) + \delta_t \alpha \vec{g}_n, \tag{8}$$

$$\ell_o(t + \delta_t) = \ell_o(t) + \delta_t \beta \vec{v}_o(t + \delta_t). \tag{9}$$

Note that Newtonian gravity is the result where $\alpha = \beta = 1$.

3 Precession

$$\delta_p = \frac{6\pi GM}{c^2(1 - e^2)a} \left(\frac{1}{\pi \times 180 \times 3600} \right) \left(\frac{365}{88} \times 100 \right) = 42.937 \tag{10}$$

References

[1] Halayka. On simulating the four Solar System tests of general relativity using two-parameter post-Newtonian gravitation with Euler integration. (2024)

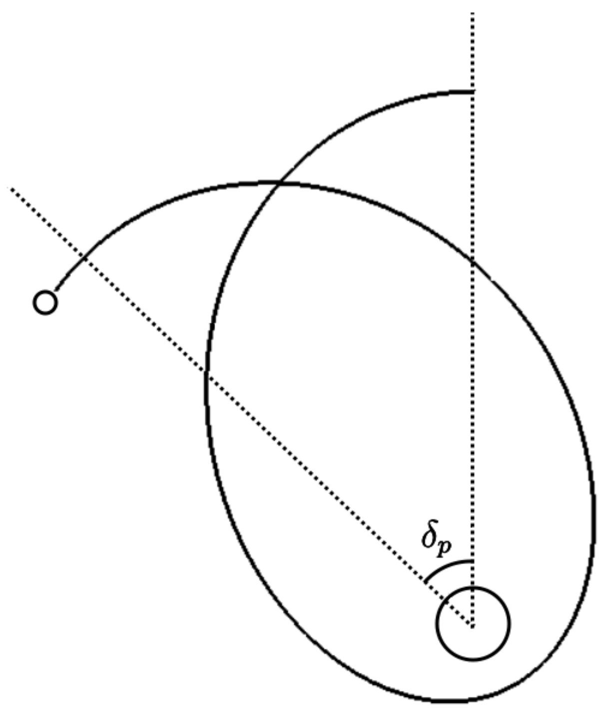

Figure 1: A diagram showing precession, where the orbit does not quite form a closed ellipse.