On the quantum decomposition of the planet Mercury's orbit path

S. Halayka*

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Abstract

By quantizing the gravitational time dilation using various step sizes, one obtains a set of weighted paths. The precession associated with each weighted path combines to provide the same answer as the classical analytical solution.

1 Time dilation

The kinematic time dilation is:

$$\frac{d\tau}{dt} = \frac{\sqrt{c^2 - ||\vec{v}||^2}}{c} = \sqrt{1 - \frac{||\vec{v}||^2}{c^2}}. \tag{1}$$

The gravitational time dilation is:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{R_s}{r}}. \tag{2}$$

In this paper, we will be quantizing the kinematic and gravitational time dilation by casting them to a lesser-precision floating point number.

The smallest step size that can be represented is $\epsilon = 2 \times 2^{-b}$, where b is the largest exponent value (e.g. $2^7 - 1 = 127$ in single-precision floating point numbers, $2^{10} - 1 = 1023$ for doubles).

The non-exponent bit count n includes the number of mantissa bits m, plus one sign bit. We generally used $n = 100$, except for the kinematic and gravitational time dilation, which uses a lesser, various precision (e.g. 24).

*sjhalayka@gmail.com

2 Steps in spacetime

Where ℓ_s denotes the Sun's location at the origin, ℓ_o denotes the orbiter's location, and $\vec{d}$ denotes the direction vector that points from the orbiter toward the Sun:

$$\vec{d} = \ell_s - \ell_o, \tag{3}$$

$$\hat{d} = \frac{\vec{d}}{||\vec{d}||}, \tag{4}$$

the Newtonian acceleration vector is:

$$\vec{g}_n = \frac{\hat{d}GM}{||\vec{d}||^2}. \tag{5}$$

One parameter is closely related to the kinematic time dilation:

$$\alpha = 2 - \sqrt{1 - \frac{||\vec{v}_o||^2}{c^2}}. \tag{6}$$

Another parameter is the gravitational time dilation:

$$\beta = \sqrt{1 - \frac{R_s}{||\vec{d}||}}. \tag{7}$$

Finally, the semi-implicit Euler integration for velocity and then location is:

$$\vec{v}_o(t + \delta_t) = \vec{v}_o(t) + \delta_t \alpha \vec{g}_n, \tag{8}$$

$$\ell_o(t + \delta_t) = \ell_o(t) + \delta_t \beta \vec{v}_o(t + \delta_t). \tag{9}$$

Note that Newtonian gravity is the result where $\alpha = \beta = 1$.

3 Classical analytical calculation of Mercury's orbit precession

$$\delta_p = \frac{6\pi GM}{c^2(1 - e^2)a} \left(\frac{1}{\pi \times 180 \times 3600} \right) \left(\frac{365}{88} \times 100 \right) = 42.937 \tag{10}$$

4 Quantum path decomposition

There is a pattern.

Where initial speed is 25000 metres per second, and the analytical solution is 103.7:

Bits n	Angle δ_p
20	17..04
21	17.04
22	17.04
23	47.8
24	38.95
25	34.8
26	34.96
27	34.9
28	35.07
29	35.08
30	35.06

If you add bit counts 22, 23, and 24 together (e.g. $17.04 + 47.8 + 38.95$) you get 103.79 arcseconds per Earth century. All of the other bit counts have a weight of zero.

Where initial speed is 20000 metres per second, and the analytical solution is 162.09:

Bits n	Angle δ_p
21	27.86
22	76.62
23	52.9

If you add bit counts 21, 22, and 23 together, you get 157.38.

Where initial speed is 30000 metres per second, and the analytical solution is 72.04:

Bits n	Angle δ_p
23	46.45
24	27.8

If you add bit counts 23 and 24 together, you get 74.25.

Where initial speed is 38858.47 metres per second, and the analytical solution is 42.9:

Bits n	Angle δ_p
24	46.8

The numerical solution is 46.8.

Where initial speed is 42500 metres per second, and the analytical solution is 35.8:

Bits n	Angle δ_p
26	51.7

The numerical solution is 51.7. Obviously, this solution's weight is less than 1 (e.g. $(35.8/51.7) < 1$).

References

[1] Halayka. On simulating the four Solar System tests of general relativity using two-parameter post-Newtonian gravitation with Euler integration. (2024)

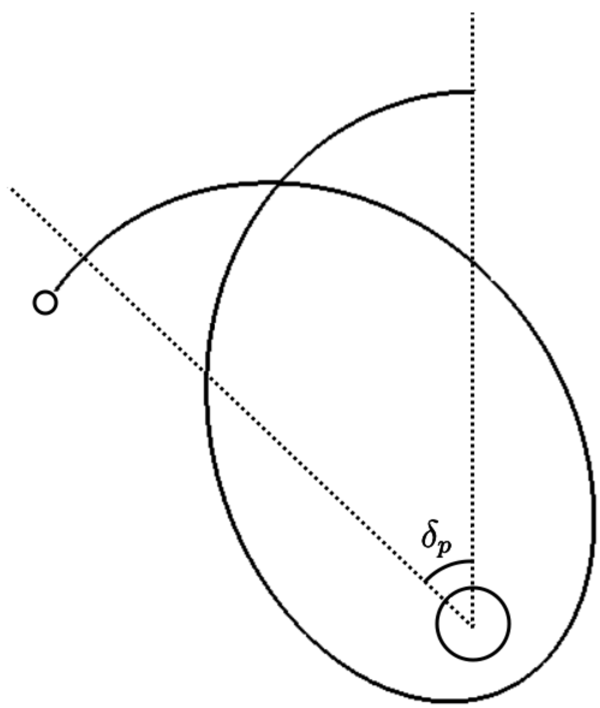

Figure 1: A diagram showing precession, where the orbit does not quite form a closed ellipse.