On the quantum decomposition of the planet Mercury's orbit path

S. Halayka*

Monday 3rd June, 2024 12:01

Abstract

By quantizing the gravitational time dilation using various step sizes, one obtains a set of weighted paths. The precession associated with each weighted path combines to provide the same answer as the classical analytical solution.

1 Time dilation

The kinematic time dilation is:

$$\frac{d\tau}{dt} = \frac{\sqrt{c^2 - ||\vec{v}||^2}}{c} = \sqrt{1 - \frac{||\vec{v}||^2}{c^2}}. \tag{1}$$

The gravitational time dilation is:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{R_s}{r}}. \tag{2}$$

In this paper we will be quantizing the kinematic and gravitational time dilation by casting them to a smaller floating point number. The quantized step size (e.g. epsilon) in general is $\epsilon = 2^{-m}$ where m is the number of mantissa bits in the floating point number.

2 Steps in spacetime

Where ℓ_s denotes the Sun's location at the origin, ℓ_o denotes the orbiter's location, and $\vec{d}$ denotes the direction vector that points from the orbiter toward the Sun:

$$\vec{d} = \ell_s - \ell_o, \tag{3}$$

$$\hat{d} = \frac{\vec{d}}{||\vec{d}||}, \tag{4}$$

*sjhalayka@gmail.com

the Newtonian acceleration vector is:

$$\vec{g}_n = \frac{\hat{d}GM}{||\vec{d}||^2}. \tag{5}$$

One parameter is closely related to the kinematic time dilation:

$$\alpha = 2 - \sqrt{1 - \frac{||\vec{v}_o||^2}{c^2}}. \tag{6}$$

Another parameter is the gravitational time dilation:

$$\beta = \sqrt{1 - \frac{R_s}{||\vec{d}||}}. \tag{7}$$

Finally, the semi-implicit Euler integration for velocity and then location is:

$$\vec{v}_o(t + \delta_t) = \vec{v}_o(t) + \delta_t \alpha \vec{g}_n, \tag{8}$$
$$\ell_o(t + \delta_t) = \ell_o(t) + \delta_t \beta \vec{v}_o(t + \delta_t). \tag{9}$$

Note that Newtonian gravity is the result where $\alpha = \beta = 1$.

3 Precession

$$\delta_p = \frac{6\pi GM}{c^2(1-e^2)a}\left(\frac{1}{\pi \times 180 \times 3600}\right)\left(\frac{365}{88} \times 100\right) = 42.937 \tag{10}$$

Here the bit count includes the number of mantissa bits, plus one sign bit.

Where initial speed is 25000 metres per second, and the analytical solution is 103.7:

Bits	Angle
20	17..04
21	17.04
22	17.04
23	47.8
24	38.95
25	34.8
26	34.96
27	34.9
28	35.07
29	35.08
30	35.06

If you add bit counts 22, 23, and 24 together (e.g. $17.04 + 47.8 + 38.95$) you get 103.79 arcseconds per Earth century. All of the other bit counts have a weight of zero.

Where initial speed is 20000 metres per second, and the analytical solution is 162.09:

Bits	Angle
21	27.86
22	76.62
23	52.9

If you add bit counts 21, 22, and 23 together, you get 157.38.

Where initial speed is 30000 metres per second, and the analytical solution is 72.04:

Bits	Angle
23	46.45
24	27.8

If you add bit counts 23 and 24 together, you get 74.25.

Where initial speed is 38858.47 metres per second, and the analytical solution is 42.9:

Bits	Angle
24	46.8

The numerical solution is 46.8.

Where initial speed is 42500 metres per second, and the analytical solution is 35.8:

Bits	Angle
26	51.7

The numerical solution is 51.7. Obviously, this solution's weight is less than 1 (e.g. $(35.8/51.7) < 1$).

References

[1] Halayka. On simulating the four Solar System tests of general relativity using two-parameter post-Newtonian gravitation with Euler integration. (2024)

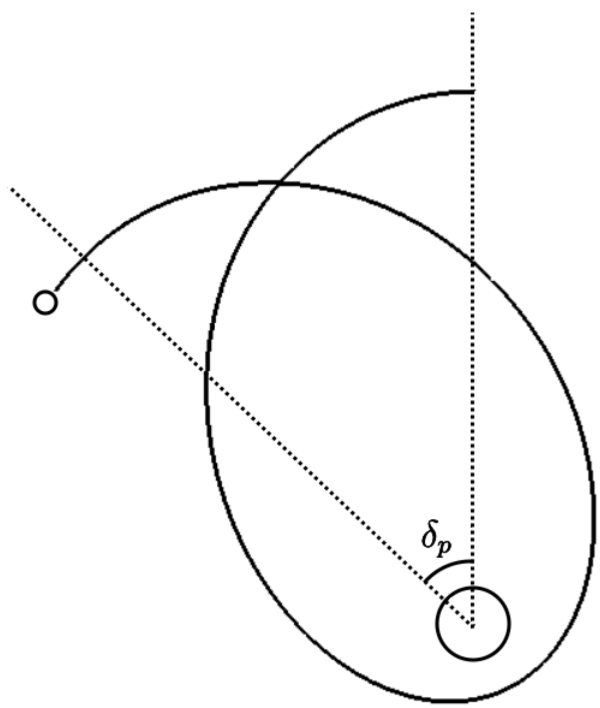

Figure 1: A diagram showing precession, where the orbit does not quite form a closed ellipse.