Visualizing the escape paths of quaternion fractals

S. Halayka*

July 27, 2018

Abstract

The escape paths of the points in some quaternion fractal sets are visualized using OpenGL. C++ source code is provided.

1 Escape paths that do not reach infinity

As discussed in [1], a 3D scalar field of quaternion magnitudes (e.g. $|Z|$) results from calculating a quaternion fractal set when using a finite 3D lattice of regularly spaced points as input.

Here we will visualize the escape paths, using OpenGL, for those points that maintain a quaternion magnitude less than the infinity threshold value (e.g. 4.0) during the iteration process (e.g. 8 iterations).

The notion that the escape paths can make for nice visualizations was independently found by Paul Bourke [2]. Bourke's work was inspired by the Buddhabrot. The primary difference between Bourke's visualizations and the visualizations given here is that Bourke draws the paths for those points not in the set, where – oppositely – here we draw the paths for those points in the set. In essence, Bourke focuses on the escape paths that reach infinity, whereas here we are focused on escape paths that do not reach infinity.

Here we use Bezier curves and cylinders to draw the escape paths. The length of these 'shaggy' escape paths is shortened to only 20% of the total length, giving the impression of a isotropic buzz cut.

The C++ code for this paper can be found at [3].

*sjhalayka@gmail.com

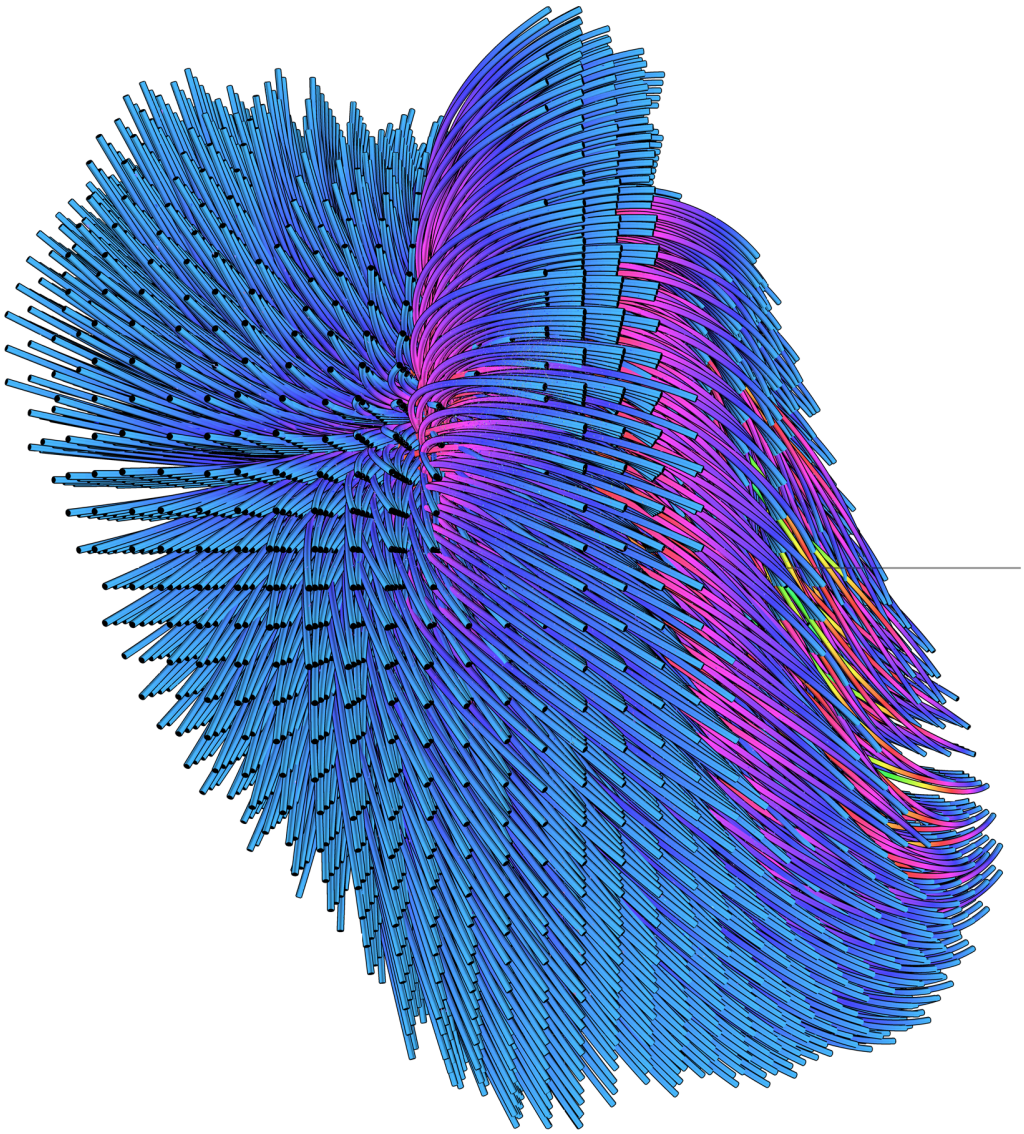

Figure 1: $Z' = Z^2 + C$, where $C_{xyzw} = 0.3, 0.5, 0.4, 0.2$.

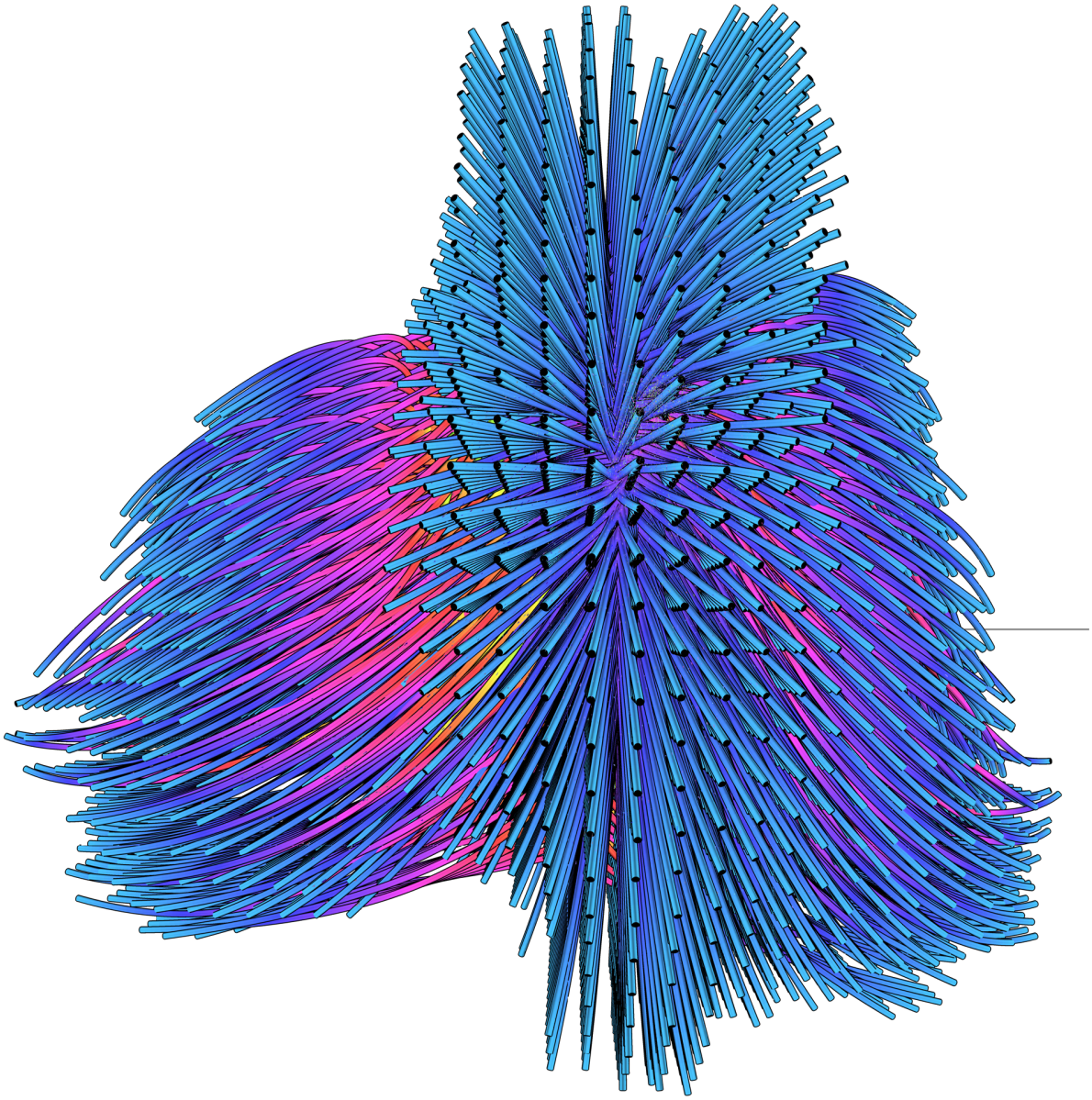

Figure 2: $Z' = Z^3 + C$, where $C_{xyzw} = 0.3, 0.5, 0.4, 0.2$.

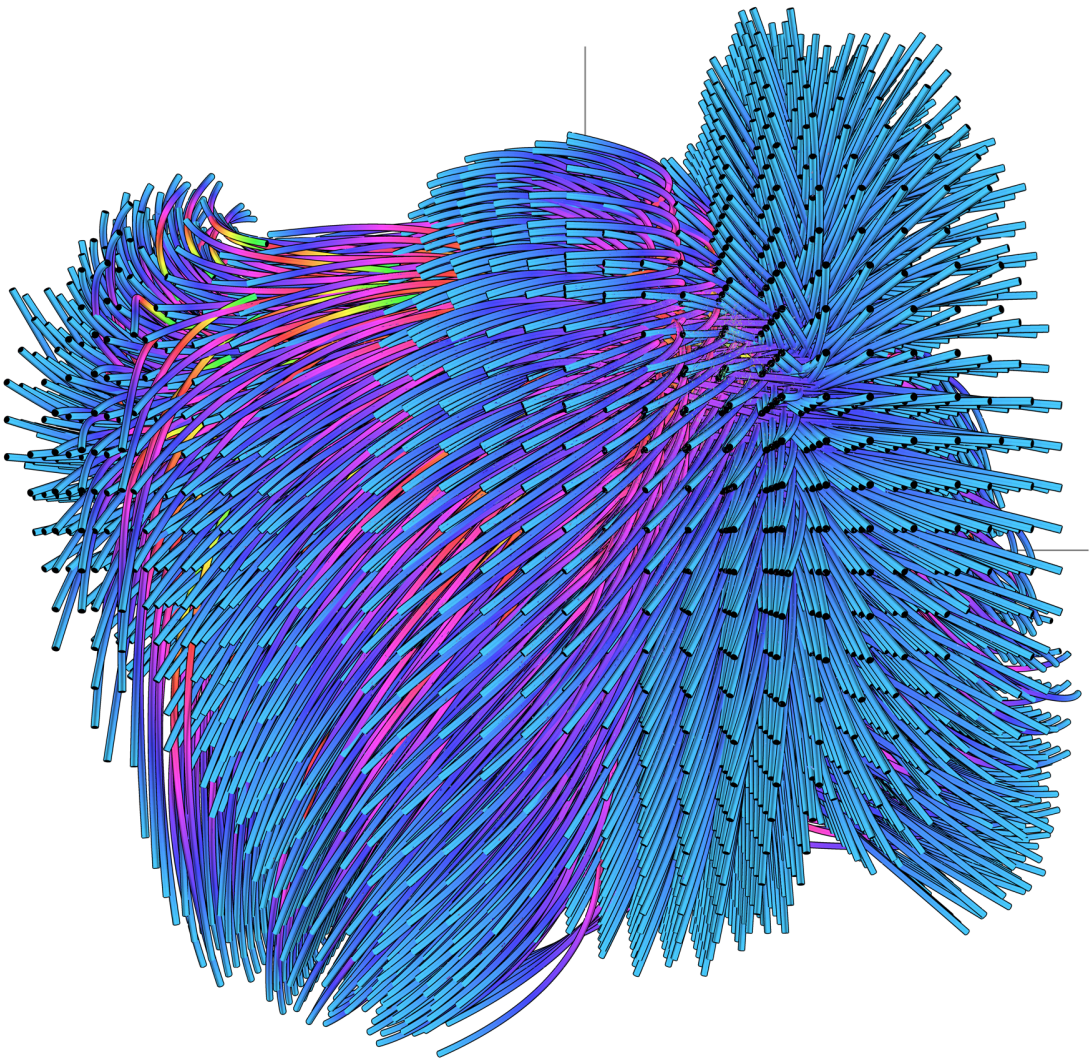

Figure 3: $Z' = Z^4 + C$, where $C_{xyzw} = 0.3, 0.5, 0.4, 0.2$.

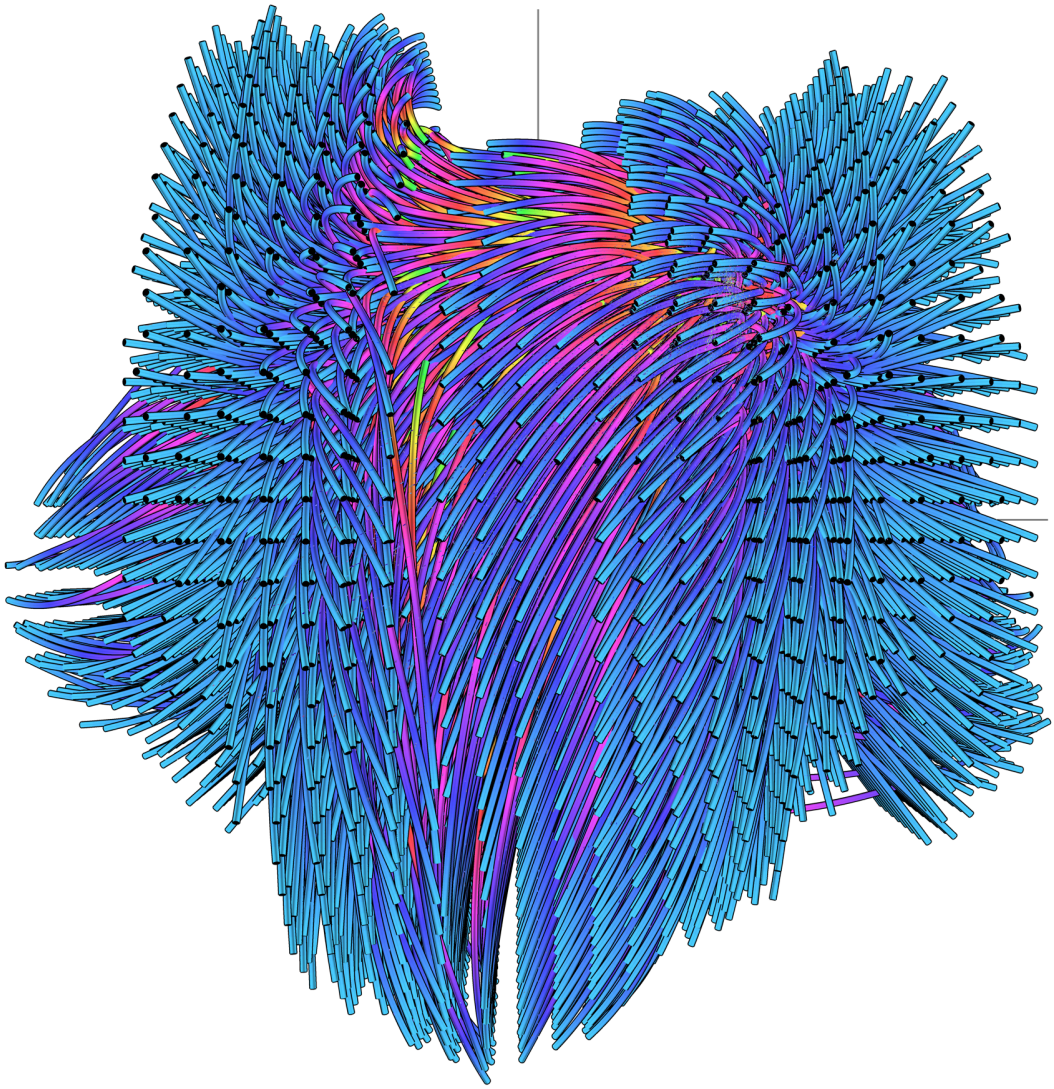

Figure 4: $Z' = Z^5 + C$, where $C_{xyzw} = 0.3, 0.5, 0.4, 0.2$.

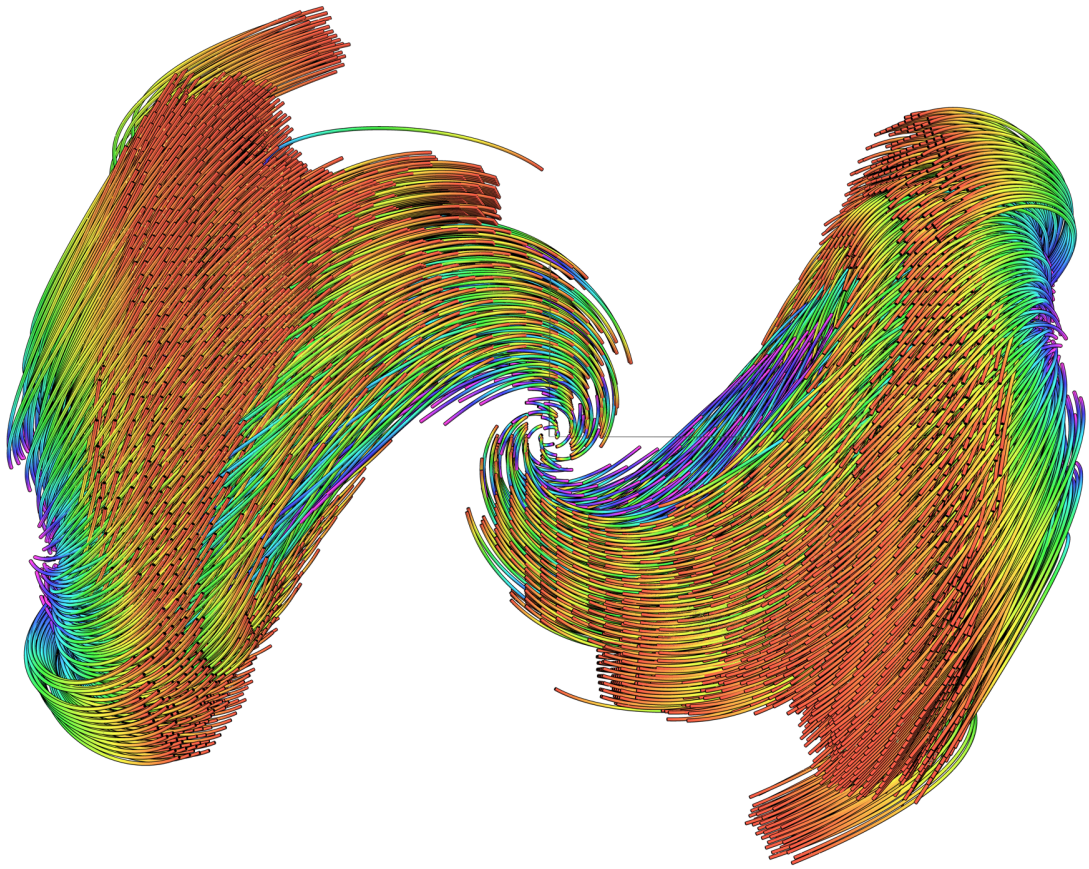

Figure 5: $Z' = \sin(Z) + C \cdot \sin(Z)$, where $C_{xyzw} = 0.3, 0.5, 0.4, 0.2$.

References

[1] Halayka S. *Some visually interesting non-standard quaternion fractal sets* Chaos, Solitons & Fractals Vol. 41, Issue 5

[2] `http://paulbourke.net/fractals/trajectories/`

[3] `https://github.com/sjhalayka/bezier_fractal`