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Parallel Computing for Engineers

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Homework #2: Numerical Simulation of 1D Couette Flow

Introduction:

This assignment solves a one-dimensional Couette flow numerically by using finite difference method written in C programming language. The governing physics required to solve this problem is in Equation 1.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

In order to solve this equation, finite difference is used to discretize the problem. The formula used is shown in Equation 2.

$$u_j^{n+1} = \Delta t\left[-\frac{1}{\rho}\frac{\partial P}{\partial x} + \mu\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{(\Delta y)^2}\right] + u_j^n \tag{2}$$

To have stability in the partial differential equation, stability criteria is found by making the delta t equal to Equation 3.

$$\Delta t \leq 0.5\left[\frac{(\Delta y)^2}{\nu}\right] \tag{3}$$

Since this is a time dependent problem, at some point, the problem will reach steady state. The time required to reach steady state is found in Equation 4.

$$\mathrm{T} = \frac{H^2}{\nu} \tag{4}$$

With this being only a 1D case of fluid flow, there is an exact solution for the steady state version of this problem as shown in problem 5. This solution is used to compare the numerical solution to verify the finite different method is setup correctly and obtains the same results.

$$u(y) = V_{plate}\frac{y}{H} + \frac{1}{2\mu}\left(\frac{dP}{dx}\right) + (y^2 - Hy) \tag{5}$$

Discussion/Results:

Figure 1 shows the results from the simulation of the exact values compared to the numerical simulation showing the same values on the plot. Reynolds number was varied between 1 and 10 to show various flows. Figure 2 shows a constant Reynolds number of 8 while varying the differential pressure with plots of -2, 0, and 2.

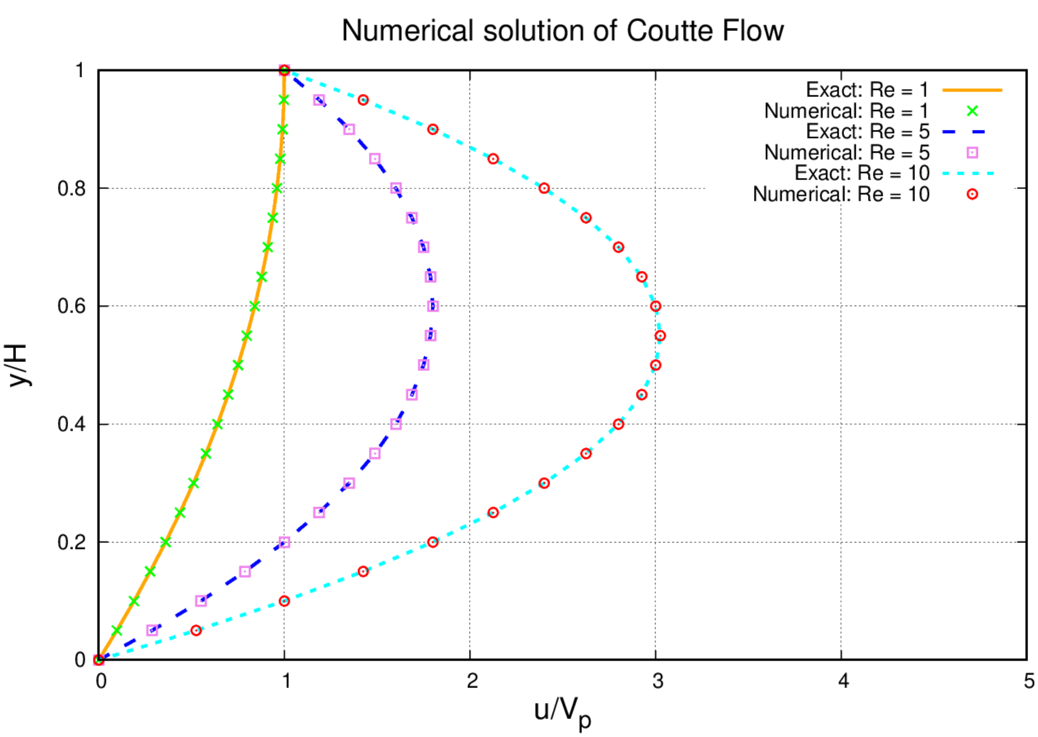

Figure 1: Comparison of a constant differential pressure of -2 with varying Reynolds Numbers

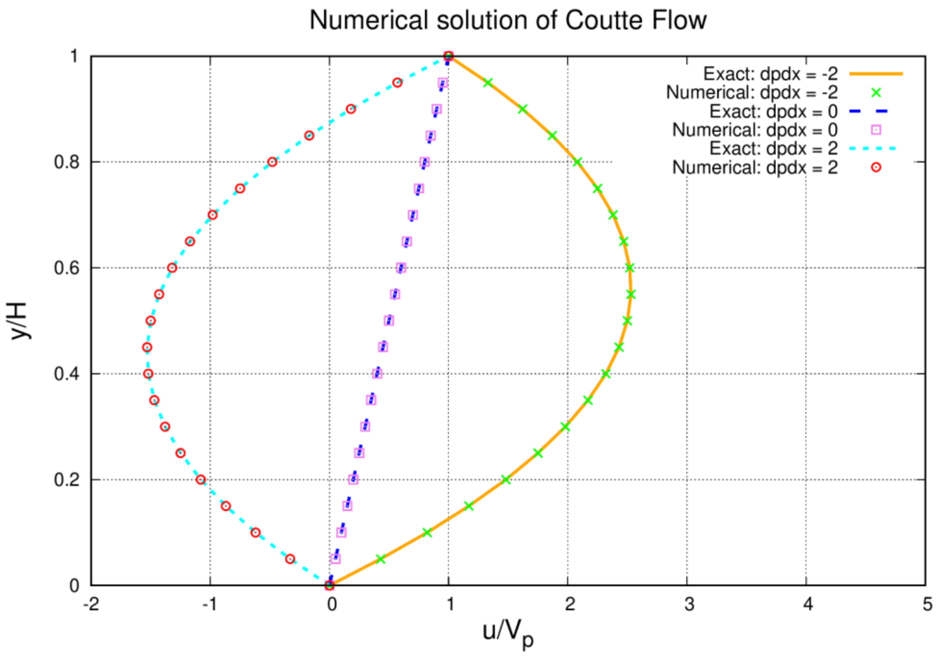

Figure 2: Constant Reynolds Number of 8 with varying differential pressure

Conclusion:

After reviewing the results of the numerical solutions with those of the exact solution and of textbooks, the results are as expected. The numerical solutions match very well with that of the exact solution proving the method and execution functioned as it should.