

In-class assignment

1. Suppose you were presented with the following twenty test question responses from a test-taker where 1 indicates correct and 0 indicates incorrect:

$$y = (1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1).$$

You believe these test questions are well-summarized by the item-response theory (IRT) probability model

$$\Pr(y_{ij} = 1) = \Phi(\beta_j \psi_i - \alpha_j); \Pr(y_{ij} = 0) = 1 - \Phi(\beta_j \psi_i - \alpha_j)$$

with item parameters

$$\alpha = (0.05, 0.1, 0.07, 0.08, -0.14, -0.2, 0.15, -0.05, 0.11, -0.04, \\ 0.19, -0.19, -0.07, -0.17, -0.01, 0.01, -0.14, 0.14, 0.04, 0.14)$$

and

$$\beta = (-0.2, 0.3, -0.3, 0.4, 0.5, -0.6, -0.2, -0.4, 0.2, 0.4, \\ 0.5, 0.3, 0.1, 0.4, 0.1, -0.1, -0.3, 0.8, 0.5, 0.5).$$

Given this data and model, answer the following:

- (a) What is the likelihood of this set of test responses if the test-taker's latent intelligence is $\psi = 1$?
 - (b) What is the likelihood of this set of test responses if the test-taker's latent intelligence is $\psi = 0$?
 - (c) Given these two likelihoods, is it more likely the test-taker's intelligence is zero or 1?
 - (d) Write an algorithm (loop or optimizer) to identify the value of ψ that maximizes the likelihood of these responses. Show your work.
2. Estimate an IRT model of ideology on members of the 89th (1965-1966) and 117th (2021-2022) U.S. Houses of Representatives. Data is available at www.voteview.com. You may take a random sample of 100 votes to speed up estimation. With the results,
 - (a) Plot histograms of the estimated ideology of members for each party in each congress.
 - (b) Calculate means of the estimated ideology of members for each party in each congress.
 - (c) Comment on how the distribution and central tendency of ideology appears to differ by party in the two congresses.