# Communicating Intensity of Preference: A Theory of Political Action, Expression, and Identities

Seth J. Hill\*

November 16, 2018

**Abstract:** Political citizens take many actions of unclear personal benefit such as turning out to vote in large elections, accommodating corruption, expressing biased opinions favoring in-group candidates, and making statements inconsistent with democratic norms. Existing theories often attribute such actions to group identities. I present a theory that instead connects political behaviors of personal cost to the difficulty of communicating intensity of preference. Intensity of preference is often unobserved and thus citizens may exaggerate how much they care about the issue when communicating with politicians. The theory here interprets costly political actions as tools used by individuals to faithfully communicate how much they care. I show in a strategic model that costly political actions can improve societal welfare without assuming psychological, sociological, or affective benefits. The theory rationalizes apparently perverse pre-election political choices, explains policy implemented that is opposed by majorities of citizens, proposes causes of political polarization, suggests that survey respondents may not sincerely answer questions posed, explains why even citizens of successful representative democracies might act as if driven by political identities, and describes conditions where citizens demand malfeasant politicians. Costly political action communicates what citizens actually want.

**Keywords**: political representation; collective action; political identities; issue intensity; political economy; asymmetric information.

<sup>\*</sup>Department of Political Science, University of California, San Diego, 9500 Gilman Drive #0521, La Jolla, CA 92093-0521; sjhill@ucsd.edu, http://www.sethjhill.com.

Social scientists have long puzzled over why citizens undertake costly political actions unlikely to directly change political outcomes. Individuals turn out to vote in large elections despite near zero chance that one vote changes the result. Many volunteer for campaigns, attend rallies, make individual political donations, or participate in public meetings despite being little more than token contributions. Many express unusual attitudes when responding to survey questions or in interpersonal discussions, exhibiting favoritism towards in-groups such as political parties and expressing biased reports about the state of the world even on matters of fact. Why do citizens incur these costly behaviors and report these odd expressions?

In this essay, I present a theory to explain these phenomena by considering the difficulties communicating intensity of political preference between citizens and politicians. On issues contested in the political arena, citizens vary both in what they want and in how much they want it. First, citizens have different ideal policies, what they would implement if given authority for the decision. Second, they vary in how much they prefer their ideal policy over alternatives. Some may be almost as happy with a policy different from their ideal while others may find any departure highly distasteful.

If the goal of those who implement policy is to provide good public policy – either due to benevolence or due to career incentives – they may want to weight policy towards those who care more about the outcome relative to those who care less. In order to achieve these goals, officials need to know both citizens' ideal policies and the intensity with which they prefer ideal over alternatives. Unfortunately, when intensity of preference is hard to observe by others, individuals may not voluntarily reveal intensity. Knowing that officials cannot observe intensity, those with less intense preferences might claim their preferences are intense so that officials would implement policy at their (weakly-preferred) ideal policy. Of course political officials, knowing the indeterminacy of citizen statements on intensity, cannot learn much from what citizens say about intensity.

I present here a theory where a central feature of political contestation is citizens with

heterogeneous, hard to observe intensities of preference communicating with political officials. I connect this interaction and the challenge of communication to longstanding puzzles in politics regarding why citizens undertake costly individual actions unlikely to directly change political outcomes. Because citizens with intense preferences want to distinguish themselves from citizens with less intense preferences, they seek instruments to communicate how much they care about policy. The theory interprets costly political behaviors, expressions, and identities as actions chosen by citizens in pursuit of political goals.

Existing theories suggest that costly actions and biased expressions follow from individual attachments to political groups, often called "political identities." These theories are supported by a variety of empirical evidence. Identifiers seem willing to support candidates with unseemly backgrounds and overlook corrupt behavior if that candidate is part of their political group. Identifiers appear to learn about the political world – even on matters of fact – with perceptual bias, learning more from information that is favorable to their political group than from information unfavorable. Some claim that identifiers blindly adopt the issue positions of their favored political group when they learn that position, even when prior to that knowledge they had taken a position opposite. Identities may be manipulated to the political benefit of elites rather than citizens, leading to outcomes such as limited political accountability, inter- and intra-national violence, and the maintenance of authoritarian regimes. Many wonder why, if political identities lead to political behaviors with negative political consequences, nearly all societies are populated with citizens that take costly behavior and seem to exhibit political identities?

I depart from previous theories of costly political action by arguing such behaviors serve as tools of communication rather than manifestation of group identities. As an instantiation of this theory, I present a game-theoretic model where a set of costly political actions are chosen by each individual and observed by political candidates. By costly, I mean behaviors where the personal costs undertaken are greater than the immediate personal benefits, for example turning out in large elections. In the model, I show that such costly behaviors are

supported even without benefits from any cognitive, affective, or social group influences. Importantly, I show that these costly actions can increase welfare of society relative to a setting where citizens vary in intensity of preference but do not possess such instruments of communication.

The benefit of focus on this citizen-to-official communication challenge is that the theory of political action, expression, and identities helps explain a variety of puzzling political patterns of politics. Without resort to apolitical motives, the theory provides a coherent account of heterogeneity in political activism and participation, apparent bias towards candidates of an identifier's group, responding favorably to candidates making offensive statements, limited voter responsiveness to evidence of corruption, biased learning about political facts, negative out-group affect, blind adoption of the policy positions of elites, giving biased or embellished answers to survey questions, and making pecuniary contributions to candidates and interest groups. The theory also explains outputs of the political system such as policy implemented against majority preferences (issue incongruence) and variable election polls with predictable election results, and provides novel explanations for political polarization.

The theory provides an explanation for the correlation between group identities and costly political behaviors through three possibilities. Group identities might cause variation in intensity of preferences. Alternatively, what is classified as "identity" could be part of the portfolio of costly actions chosen by individuals. Identity might also be incidentally correlated with intensity of preference (see discussion below). The theory here provides a more general account of the etiology of and variation in costly political action, even for those who don't have political identities.

The game-theoretic model considers the case of voters with differing intensity of preference and differing ideal policies at an election with two competing parties. To briefly highlight results, I show first that an equilibrium exists where high-intensity voters on both

<sup>&</sup>lt;sup>1</sup> I note the validity of some of these claims is disputed in the literature.

the majority and minority side of a policy choose costly political actions and expressions in pursuit of policy goals, and that the parties contesting the election respond to these choices when choosing policy. Even with clear majorities on an issue, members of the majority still choose costly political behavior to maintain implementation of majority policy if preferences for the policy are sufficiently strong. Additionally, costly political behavior sometimes allows the minority to gain policy representation when they care more deeply about that policy than the majority *even* when the parties know the majority's preference. Second, I show that when intensity of preference is hard to communicate, a system with costly political action and expression can improve electorate welfare relative to a system without such opportunities to communicate. This finding is somewhat striking as only the minority benefits from the ability to communicate yet I find aggregate welfare can be improved because the benefits to the minority outweigh costs incurred by the majority.

Third, in settings where opportunities for costly behaviors are limited, voters may demand new opportunities for costly communication, which could take the form of seeking out corrupt or morally deficient candidates or other forms of deleterious political actions. Fourth, in settings with limited opportunities for costly behaviors, exogenous changes to communication technology can alter magnitude of costly identities chosen and policy output of the system without changes to the preferences of voters or politicians. This result suggests that polarization in political behaviors, with some individuals taking more costly action than previously, can follow from exogenous changes to communication technology but without any changes to preferences or electoral structure. I discuss implications below.

Following model results, I provide a set of stylized facts from conventional wisdom in political science on political action, expression, and identities. I interpret each stylized fact in the context of the theory, which suggests new ways to think about these patterns and future theoretical and empirical work. The analysis implies that biased or costly political behaviors should not always be taken as evidence that individuals benefit directly from that behavior. When voters turn out in large elections or volunteer for campaigns, we need not

believe voters think the influence of their efforts on the result outweights costs. If voters maintain support for unseemly candidates, parrot party propaganda, or falsely claim the economy faltered under the other party's incumbent, we should not necessarily conclude that these actions or expressions reflect true preferences or beliefs.

Rather, the theory suggests we consider citizens part of a process of strategic communication that at times may provide them incentive to take apparently perverse action. Much of empirical political science has focused on measuring these perverse actions and, having done so, comes to negative conclusions on the citizens' aptitude for creating political accountability. This essay suggests that these perverse behaviors may be to the benefit of representation and that we consider each behavior taken as part of a portfolio of costly actions chosen by individuals.

The theory suggests revision to how we interpret expressions of political attitudes in opinion surveys. Responses to survey questions might be considered part of the costly communication environment rather than sincere expressions of attitudes and beliefs. In the lens of the theory, the meaning of survey responses depends centrally on how costly and how public those citizens who give them believe their responses to be. If citizens believe survey responses costly and observed, responses may not sincerely measure the target of survey questions (see also Meirowitz, 2005). The perspective here suggests we consider that survey responses may be communicating something other than unmediated sincere reply to the question posed.

Finally, political science remains uncertain on what exactly motivates individuals to volunteer for campaigns, make donations to candidates or interest groups, participate in nomination contests, or even to vote in elections. The theory here suggests we consider these actions chosen from a set of costly behaviors available to individuals as they choose how to communicate intensity of preference. The stronger the intensity and the smaller the minority on an issue, the more costly behaviors an individual is willing to assume. We might interpret choices to participate as choices to undertake costly action in communication.

The essay proceeds as follows. I first offer intuition and anecdotal argument about the challenge faced by citizens and officials when intensity varies across individuals but is not observed. I then formalize the simple example and argument into a game theoretic representation and present equilibrium results. Following, I show how the core insights of the theory help explain a set of stylized facts about political action, expression, and identities, and finally offer concluding remarks toward future work.

# 1 Intuition for the theory

Imagine that due to population growth a community must build a new school. The new school will include an adjacent athletic field and the community must decide whether or not to build large elevated lights to allow athletic events to be played after dark. Some voters prefer the athletic field with the lights anticipating its many benefits to students and the community, while others do not want the disturbances that come from athletic events after dark.

Assume that the school board surveys the community and believes it evenly divided over the lights with no compromise available. Members of the school board care about winning reelection. Knowing that voters cast ballots based both on the lights issue and on other issues, they want to side with the half of the community who feels more intensely about the lights. Voters with intense preferences are more consequential in the event the election is otherwise close because their votes are more likely swayed by the lights.

Unfortunately, it is very difficult to determine which group of voters cares more intensely. Both halves know that members of the school board want to choose the policy preferred by the half who cares more intensely. They also know that there are no obvious indicators for strength of preference. If the school board surveys voters about how strongly they feel about the issue and the response to the survey is of negligible cost, even those who care only a little about the issue might claim it is important to them, that they will definitely vote against the incumbents and be highly dissatisfied if the decision goes

against their preference. Knowing this dynamic, the school board knows it cannot trust the responses given by the community about how intensely they feel about the lights issue.<sup>2</sup> Board members are in a bind.

At some point, either a member of the community with strong preferences or a savvy member of the school board realizes how to determine who cares more about the lights. Members of the board up for election seek volunteers for their campaigns. Although volunteering has some social benefits, at a level of commitment where the benefits are outweighed by time and transportation expenses, only those members of the community with intense preferences on the lights issue are willing to undertake the costs of volunteering. Undertaking high costs reveals they care intensely about the lights, and the more the individual cares about the lights issue, the higher the volunteering costs they are willing to incur to communicate their intensity.

The school board members observe who spends how many hours volunteering to infer which group in the community cares more intensely about the lights. If many more volunteering hours are given by opponents of lights, members can infer opponents feel more intensely. If many more volunteering hours are given by supporters, members can infer supporters feel more intensely.

A political scientist might investigate this situation by surveying respondents to learn what they think about the lights issue. They would note volunteering hours correspond to patterns of support for lights and candidates. If they asked respondents "Do you generally identify with Candidate A, Candidate B, or neither?," researchers might find correspondence between the strength of identity reported and reports of costly behaviors like volunteering.

The challenges of communicating issue intensity related in this anecdote are those of a wider class of problems in aggregating preferences. When candidates for office, bureaucrats, dictators, or other leaders cannot distinguish those who care only a little about an

<sup>&</sup>lt;sup>2</sup> Because the voters know the school board members know the responses are not useful, any pattern of responses about intensity might arise from the survey, not necessarily everyone claiming intense preferences. This is an important observation for interpretation of survey responses in general, to be discussed below.

issue from those who care a great deal, those who care only a little may pretend to care a great deal to motivate implementation of their (weakly) preferred policy. Knowing this indeterminacy of communication, officials realize they cannot learn about issue intensity from costless communication.

One solution to this setting of asymmetric information is communication that is costly. Those who care more deeply about the issue are willing to incur more costs in order to communicate their more intense preference. Empirical research suggests that many costly behaviors are common in electoral politics: supporting an unseemly candidate of moral turpitude, participating in time-intensive rallies, meetings, or nomination politics, volunteering for campaigns, parroting the policy positions and propaganda of a party or candidate, discriminating against members of out-groups, contacting representatives, making pecuniary contributions to parties, interest groups, or candidates, giving slanted answers to public opinion surveys consistent with party propaganda, consuming news media that covers events with bias, making socially-abhorrent public statements, and even engaging in violence.

#### 1.1 Related work

The political science literature on strategic models of communication in domestic politics generally considers the problem of asymmetric information when candidates or incumbents possess knowledge that the voter does not (e.g. Canes-Wrone, Herron, and Shotts, 2001; Fox and Van Weelden, 2015; Patty, 2016). Models with private information held by citizens include Lohmann (1993), who models costly political action as communication for citizens who possess private knowledge about the state of the world, Shotts (2006), who considers voter choice at a first election as communication to influence the beliefs of candidates at a second, and Meirowitz (2005), who models opinion survey responses as the communication technology. Gause (N.d.) argues that low-resource groups benefit from protests as costly communication more than high-resource groups due to higher relative costs. In legislative politics, Wawro and Schickler (2006, ch. 2) argue pre-1917 dilatory tactics in the U.S. Senate

were costly behaviors used to communicate how intensely senators felt about the bill being obstructed.

The literature explaining costly political action arising from political identities such as party identification, ethnic identity, and nationalism is extensive. Seminal game-theoretic models from political economy model 'party affinity' as a bias towards one of the parties in voters' electoral choices and exogenous to voter or candidate actions (e.g., Lindbeck and Weibull, 1987; Dixit and Londregan, 1996; Persson and Tabellini, 2000). Psychological approaches interpret partisan identities as stable psychological orientations that bring order to voters' efforts to make sense of the political environment (Campbell et al., 1960), or as objects of self-identification serving individual motivations to attach to social group stereotypes for self-esteem (Green, Palmquist, and Schickler, 2002). Political identities with non-party groups have also been widely studied, such as ethnic, national, class, and religious (e.g., Laitin, 1998).

### 2 Strategic model of political action and intensity of preference

To illuminate the theory, I present analysis of a game theoretic interaction between two parties competing to win election before an electorate with heterogeneous ideal policies and heterogeneous strength of preference over policy choices, similar to the athletic field anecdote above. This is a simplified form of the theory, but provides a careful representation of its logic. After the model, I connect results to stylized empirical facts from existing work.

#### 2.1 Primitives and payoffs

The game has two parties, A and B, and an electorate of three voters.<sup>3</sup> Voters have preferences over a binary policy s and are of type  $\tau$  = 0 (type-0) or  $\tau$  = 1 (type-1) preferring s = 0 or s = 1. Voter policy type is common knowledge for all voters and parties A and B.

<sup>&</sup>lt;sup>3</sup> Readers may imagine, without loss to intuition, a much larger electorate categorized into three groups of similar size with heterogeneous intensity. The choice of three here is for simplicity. The collective action problem for large electorates is considered below and should not affect results.

Assume that  $\tau_i$  = 1 for i = {1,2} and  $\tau_3$  = 0 so s = 0 is the minority position.

In addition to ideal policy type, voters vary in the *intensity* with which they care about the issue, represented by  $\beta_i \in \{1, \bar{\beta}\}$ ,  $\bar{\beta} > 1$ , representing low and high intensity. If policy is set at the voter's preference (e.g., s = 0 for a type-0 voter), their payoff is either  $\beta_i = \bar{\beta}$  if high intensity or  $\beta_i = 1$  if low-intensity such that they gain higher utility from policy if high intensity. If policy is set opposite their preference, their payoff is zero. Intensity  $\beta_i$  is private knowledge for each voter i. However, the rate q that voters are high intensity is common knowledge,  $\Pr(\beta_i = \bar{\beta}) = q$  and  $\Pr(\beta_i = 1) = 1 - q$ ,  $q \in [0, 1]$ .

Voter utility is also influenced by an independent election-shock  $\delta$  representing the net benefit of election factors separate from policy s from Party B. The election shock may be election-specific economic or foreign policy conditions or features of the parties not easily modified within a single election such as sticky policy reputations. The election shock is revealed at the time of the election and drawn according to the Uniform(c, d) distribution with c and d common knowledge.

Parties care about rents from holding office, normalized to 1 for the elected winner and 0 otherwise. In one extension below, Party B is also allowed to have preferences that the policy takes the value of 0, s = 0 (policy-motivated parties, e.g., Calvert, 1985). B receives benefit  $\omega$  when s = 0, zero otherwise, with  $\omega$  common knowledge to all players.<sup>4</sup> Other than this extension, I assume  $\omega$  = 0.

#### 2.2 Actions and payoffs

Parties each take one action, simultaneously proposing binding policy platforms  $s_A$  and  $s_B \in \{0,1\}$ . Party A's payoffs are 1 if A wins, 0 if B wins. Party B's payoffs are  $1 + \omega(1 - s_B)$  if B wins and  $\omega(1 - s_A)$  if A wins reflecting B's payoff of  $\omega$  when s = 0.

Voters take two actions. At the election, voters select their preferred party deterministically through a common knowledge ballot function with arguments  $\tau_i$ ,  $\beta_i$ ,  $\delta$ ,  $s_A$ , and  $s_B$ . The ballot function compares the benefit to the voter of the two parties' policy platforms and

<sup>&</sup>lt;sup>4</sup> Policy benefits are assumed only for Party B for simplicity.

the election shock. Type- $\tau$  intensity- $\beta$  voter payoffs to policy are  $u_i(s) = \tau_i \beta_i s + (1 - \tau_i) \beta_i (1 - s)$ . The voters prefer Party A to Party B when

$$\beta_{\mathbf{i}} \mathsf{s}_{\mathsf{A}} > \beta_{\mathbf{i}} \mathsf{s}_{\mathsf{B}} + \delta \quad \rightarrow \quad \beta_{\mathbf{i}} (\mathsf{s}_{\mathsf{A}} - \mathsf{s}_{\mathsf{B}}) > \delta \text{ (Voters 1 and 2)},$$

$$\beta_{\mathbf{i}} (\mathsf{1} - \mathsf{s}_{\mathsf{A}}) > \beta_{\mathbf{i}} (\mathsf{1} - \mathsf{s}_{\mathsf{B}}) + \delta \quad \rightarrow \quad \beta_{\mathbf{i}} (\mathsf{s}_{\mathsf{B}} - \mathsf{s}_{\mathsf{A}}) > \delta \text{ (Voter 3)}. \tag{1}$$

Because  $\delta$  is continuous and the electorate finite, each voter strictly prefers one party and there are no ties.

The voters' second action is choice over magnitude of political actions of (net) cost to the individual  $\lambda_i \in \mathbb{R}_{\geq 0}$ . Through a diversity of available costly actions, voters display continuous  $\lambda_i$ .<sup>5</sup>  $\lambda_i$  is observed by the two parties prior to platform choices  $s_A$  and  $s_B$ . Table 1 summarizes players, actions, and payoffs.

Table 1: Payoffs and Actions to the Game

Players	Voter 1	Voter 2	Voter 3	Party A	Party B
Actions	$\lambda_1$	$\lambda_2$	$\lambda_3$	s <sub>A</sub>	s <sub>B</sub>
Payoffs, A wins:	$\beta_1$ s <sub>A</sub> - $\lambda_1$	$\beta_2$ s <sub>A</sub> – $\lambda_2$	$\beta_3$ (1 – s <sub>A</sub> ) – $\lambda_3$	1	ω(1 – s <sub>A</sub> )
Payoffs, B wins:	$\beta_1$ s <sub>B</sub> - $\lambda_1$ + $\delta$	$\beta_2$ s <sub>B</sub> – $\lambda_2$ + $\delta$	$\beta_3$ (1 – s <sub>B</sub> ) – $\lambda_3$ + $\delta$	0	1+ ω(1 - s <sub>B</sub> )

#### 2.3 Timing

The timing of the game is

- 1. Nature independently draws  $\beta_i$ ,  $i \in 1, 2, 3$  from  $\{1, \bar{\beta}\}$  given q.
- 2. Voters privately observe  $\beta_i$  and then simultaneously choose pre-election actions  $\lambda_i$ .
- 3. Parties propose binding policy platforms  $s_A$  and  $s_B$  having observed actions  $\{\lambda_1,\lambda_2,\lambda_3\}$ .
- 4. Nature reveals  $\delta$  drawn from the uniform interval [c, d] and an election is held through the voters' ballot function. The party with the majority wins office.
- 5. Winning party implements policy and all payoffs are realized.

<sup>&</sup>lt;sup>5</sup> Voters have a variety of costly political behaviors of near-continuous intensity, for example monetary donations, that justify assuming  $\lambda$  continuous. In equilibrium, any value of  $\lambda$  on an interval can support separation, and so even a non-continuous  $\lambda$  with support in the equilibrium interval can support separation. I discuss potentially important implications for political polarization of non-continuous  $\lambda$  below.

#### 2.4 Strategies and beliefs

A voter's strategy is limited to choice of  $\lambda_i$  because vote choice is deterministic through the ballot function. The strategy is a function  $\sigma_V(\beta_i,\omega):\{1,\bar{\beta}\}\times\mathbb{R}_{\geq 0}\to\mathbb{R}_{\geq 0}$  mapping issue intensity and Party B's policy preference into a political action  $\lambda_i$ . For both parties, a strategy is a function  $\sigma_p(\lambda_1,\lambda_2,\lambda_3,\omega):\mathbb{R}^3_{\geq 0}\times\mathbb{R}_{\geq 0}\to\{0,1\},\ p\in\{A,B\}$ , mapping observed political actions and policy preferences of Party B into a policy platform  $s_p$ .

The relevant beliefs for the game are those of the parties.<sup>6</sup> Given knowledge of  $\vec{\lambda}$  and  $\omega$ , the parties use Bayes' Rule to update beliefs about  $\vec{\beta}$ . Because the parties' information and learning technologies are equivalent, so too are their beliefs.

#### 2.5 Solution concept

I use Perfect Bayesian Equilibrium (PBE) as solution concept. For a PBE, each party's policy strategy must be a best response given the other party's policy strategy and the parties' beliefs about  $\vec{\beta}$ . The voter strategy must be a best response for the voter given the party strategies and beliefs and that other voters are also playing best responses. I focus on equilibria in pure strategies.

# 3 Minority representation through political actions

I begin the analysis by showing that an equilibrium exists where a minority in the electorate with high-intensity policy preferences communicates those preferences through costly political actions and where the parties represent this minority policy position when the majority voters do not choose costly actions. Majority voters, too, choose costly political actions when they have intense preferences, though at a more narrow range of values than does minority Voter 3.

Assume that both parties are strictly office-motivated ( $\omega$  = 0) and that the parties' estimate of the intensity of Voter i after observing  $\lambda_i$  is  $\hat{\beta}_i$ . On first glance it might seem that the

<sup>&</sup>lt;sup>6</sup> Because voters display  $\lambda_i$  simultaneously and because choice between the two parties solely depends upon the policy platforms and the election shock, voter learning about the issue intensity of others is not relevant to voter actions by assumption.

two parties should always select  $s_p = 1$  given the majority support for that policy. However, because there are values of the unknown election shock  $\delta$  of sufficient magnitude that the vote choice of low-intensity voters is determined by the shock regardless of policy while the choice of high-intensity voters is not, Lemma 1 in the Appendix shows that the best response for both parties is to set  $s_p = 0$  when  $\hat{\beta_1} + \hat{\beta_2} \leq \hat{\beta_3}$  else  $s_p = 1.7$ 

Because  $s_A^* = s_B^* = 0$  when  $\hat{\beta_1} + \hat{\beta_2} \leq \hat{\beta_3}$ , both majority and minority voters have an incentive for parties to believe they are high-intensity, as in the athletic field anecdote above. In equilibrium, Voter 3's costly political behaviors convince the parties he or she is high-intensity, which induces the parties to propose equilibrium policy platforms  $s_A^* = s_B^* =$ 0 when they believe both majority voters are low-intensity. This is stated in Proposition 1 and proved in Appendix Section A:

**Proposition 1** (Minority policy representation through political actions). An equilibrium exists when q is small enough and  $\bar{\beta} > 2$  where minority Voter 3 chooses costly political actions  $\lambda_3$  =  $\lambda^* > 0$  when high-intensity ( $\beta_3$  =  $\bar{\beta}$ ) and  $\lambda_3$  = 0 when low-intensity ( $\beta_3$  = 1) at all values of q. At higher values of q, a separating equilibrium exists where Majority Voters 1 and 2 join Voter 3 in separating behavior. The two parties propose the policy preferred by minority Voter 3 when they believe Voter 3 is high-intensity and Voters 1 and 2 are low-intensity.

Minority Voter 3 always chooses costly political actions when high-intensity knowing that the only chance for the policy they (strongly) prefer is to take actions to reveal to the parties they are high-intensity.<sup>8</sup> When low-intensity they do not benefit enough to merit undertaking costs  $\lambda^* > 0$ .

Majority Voters 1 and 2 have different considerations than Voter 3. They each know that the other voter in the majority may be high intensity, lessening their incentive to choose costly political actions when high intensity and making their action depend upon q. Appendix Section A shows that the best response for both Voters 1 and 2 is to choose polit-

<sup>&</sup>lt;sup>7</sup> The parties always converge to the same policy when they care only about winning office (Calvert, 1985). 

<sup>8</sup> The minority voter plays a separating best response when  $q^2 - 2q + 1 \le \lambda^* \le \bar{\beta}(q^2 - 2q + 1)$ , which always holds because  $\bar{\beta} > 1$  and  $q^2 - 2q + 1 \ge 0 \ \forall \ q \in [0, 1]$ .

ical action  $\lambda^*>0$  when  $-q^2+q\leq \lambda^*\leq -\bar{\beta}q^2+\bar{\beta}q$ . Combining the equilibrium values for Voters 1, 2, and 3, the separating equilibrium holds when  $q^2-2q+1\leq \lambda^*\leq \bar{\beta}(q^2-2q+1)$  and  $-q^2+q\leq \lambda^*\leq -\bar{\beta}q^2+\bar{\beta}q$ . A little algebra shows that the supported values of  $\lambda^*$  follow the lower bound of the minority voter and the upper bound of the majority voter when q<0.5 and the reverse when q>0.5, defining the range of  $\lambda^*$  consistent with the separating equilibrium.

In the Appendix, I consider what values of  $\lambda^*$  in the supported range would be chosen by voters when they have agency over its magnitude. Lemma 2 shows that although a wider range of magnitudes can support separation, voters minimize  $\lambda^*$  by following the lower bounds described in the previous paragraph.

As an example, consider the following parameters:  $\bar{\beta}=4$ , q=0.25,  $\lambda^*=0.5625$ , c=-1, d=1. Note that the cost  $\lambda^*$  is less than the benefit a high-intensity voter attains with desired policy,  $\bar{\beta}=4$ , making costly actions of potential net benefit. Would any voter benefit from deviating from the separating equilibrium in Proposition 1 at these parameter values? Start with Voter 3 and set aside the expected value of the election shock  $(Pr(Bwins)*E(\delta|Bwins))$ , which is constant across voter strategies given Lemma 1. We are interested in the equilibrium that generates costly action when Voter 3 is high intensity. A high-intensity Voter 3's expected benefit when choosing costly action is  $(1-q)^2[\bar{\beta}]-\lambda^*$ , where the first term is the probability that Voters 1 and 2 are both low-intensity times the benefit to Voter 3 when s=0, and the second term is costly political action paid regardless of the policy eventually implemented.

At the parameter values of the last paragraph, Voter 3's expected benefit is about 1.7 ([1 - q][1 - q] = 0.5625). If Voter 3 were to deviate from this equilibrium and not pay the costs of action even when high-intensity, policy would be implemented at s = 1 with certainty with expected benefit to Voter 3 of zero. As 1.7 > 0, Voter 3 is better off not deviating. Continuing with Voters 1 and 2, whenever either plays  $\lambda^*$  policy is implemented at s = 1 because  $\hat{\beta}_1 + \hat{\beta}_2 > \hat{\beta}_3$  when  $\hat{\beta}_1$  or  $\hat{\beta}_2$  is  $\bar{\beta}$ . The expected benefit to a high-intensity

Voter 1 or 2 playing  $\lambda^*$  in the separating equilibrium is  $\bar{\beta} - \lambda^* = 3.4375$ . If the two majority voters deviated, their expected benefit would depend on whether Voter 3 was high- or low-intensity (probabilities q and 1 – q),  $q*0+(1-q)*\bar{\beta}=3$ . Voters 1 and 2 do not benefit from deviating, maintaining the separating equilibrium.

This example shows the logic that generates the equilibrium. Each player chooses to incur costly political actions when high-intensity to increase the probability – guarantee in the case of the majority – that policy is implemented at their preference. They make these choices when the benefit to policy is sufficiently high to merit the risk that they will pay the costs of political actions but not gain policy benefits. Aggregate welfare is also higher with political actions in this example when defined as aggregate electorate expected benefit. With the separating strategies, expected benefits are 1.7 + 2 \* 3.4375 = 8.575 versus without 0 + 2 \* 4 = 8. I prove welfare benefits below.

Proposition 1 shows that  $\lambda^*$  in equilibrium is strictly less than  $\bar{\beta}$  and its minimum and maximum are decreasing in q. High-intensity voters choose costly political actions in pursuit of policy benefits but do not choose behaviors of greater cost than the benefit from policy  $\bar{\beta}$ . Note that we do not observe political contributors bankrupting themselves making political donations nor campaign volunteers working to prostration. The range of  $\lambda^*$  in equilibrium also depends on the base rate of intensity in the electorate. The more unlikely intense preferences are to exist, the more value to communicating intensity and thus more willingness to engage in costly behaviors.

These results also show that in a large electorate voters do not choose to shirk. Imagine that Voters 1, 2, and 3 in the model represent groups of individual voters in a much larger electorate and that the parties' beliefs about the aggregate intensity of each group follows from the aggregate magnitude of costly political actions chosen by members of each group. Each individual member of the group might shirk and not choose costly action, but this

<sup>&</sup>lt;sup>9</sup> There is likely an asymmetric Nash equilibrium where one of Voter 1 or 2 plays a strategy of never- $\lambda^*$  and the other plays  $\lambda^*$  when high-intensity. Applying the reasoning of this paragraph to the asymmetric equilibrium, the "communicating" majority voter would not benefit from joining his fellow member of the majority in staying silent compared to remaining in the costly action equilibrium.

choice decreases the parties' estimate of aggregate intensity of the group. Although the decrement from shirking by one member is small, because the equilibrium magnitude of  $\lambda^*$  is strictly less than  $\bar{\beta}$ , there are parameter values where each individual has an incentive to make their individual contribution in pursuit of expected policy benefits. This may be an explanation for the non-zero yet non-universal levels of turnout observed across modern democracies.

# 4 Costly political actions increase welfare

In this section, I show that costly political actions as instrument of communication can increase expected welfare for the electorate. It might seem at first glance that because the majority is made worse off by the ability of the minority to choose costly action (see next paragraph) the majority's loss would make electorate welfare decline. I show below, however, that when policy benefits to high-intensity types are sufficiently greater than to low-intensity types, and as rate high-intensity q decreases, net gains to the minority can exceed net losses to the majority. This result suggests one reason we observe costly political actions, expressions, and identities in many different societies and times. Not because costly actions follow from group identities but because such costly instruments of communication make societies better off.

Define expected aggregate welfare as total expected utility for Voters 1, 2, and 3. To evaluate welfare, I compare expected welfare in a setting with opportunity to choose costly political actions to a setting where political actions are unavailable and the parties' only knowledge about  $\vec{\beta}$  is ex ante expected rate high-intensity q. Start with expected welfare without opportunity to communicate. The policy equilibrium  $s_A^* = s_B^* = 0$  occurs when  $\hat{\beta}_1 + \hat{\beta}_2 \leq \hat{\beta}_3$  (Lemma 1) with probability (1 - q)(1 - q)(q), which is strictly less than its complement. Without costly actions,  $s_A^* = s_B^* = 1$  in pure strategies. Because Voters 1 and 2 are ex ante just as likely to be high intensity as Voter 3, the parties are better off siding with the majority.

Political actions benefit the electorate when expected aggregate welfare is greater with opportunities for costly action than without. I prove in Appendix Section C that when the policy benefit for high intensity voters is sufficiently large or when the rate of high intensity preferences q is sufficiently small, aggregate welfare is improved when voters have the opportunity to engage in costly political behavior:

**Proposition 2** (Electorate welfare can be improved with costly political actions). The separating equilibrium from Proposition 1 leads to higher expected electorate welfare than in a setting without costly political actions when  $\bar{\beta} \geq 2$  and  $\lambda^* \leq \frac{1}{3}(\bar{\beta} - 2)(q - 1)^2$ . The welfare advantage to costly political actions is increasing in the size of the benefit for high-intensity voters  $\bar{\beta}$  and decreasing in the probability a voter is high-intensity q.

The benefits to the minority voter of costly communication are clear: without communication they never attain their preferred policy. Expected welfare for the majority, however, is strictly lower with communication (Appendix Section C). For electorate welfare to be improved, then, the net gains to the minority must exceed the net losses to the majority. The majority loses with communication through two channels. First, majority voters must take costly political action when high-intensity in pursuit of their preferred policy, costs they need not incur in the no-communication regime. Second, with communication policy is sometimes set to the preference of the minority.

The magnitude of loss to the majority through the first channel depends upon the value  $\lambda^*$  and its size relative to  $\bar{\beta}$ . Improvement in total welfare is increasing in  $\bar{\beta}$  not only because this benefits the minority, but also because it helps offset losses to the majority in having to incur  $\lambda^*$ . Likewise, improvement in welfare is decreasing in q partially because  $\lambda^*$  is decreasing in q (Proposition 1).

The magnitude of loss to the majority through the second channel depends on how often minority policy is implemented, which occurs when both majority voters are low-intensity and the minority voter is high-intensity. This occurs with probability (1-q)(1-q)(q), which is a decreasing function of q for two thirds of its range (q > 1/3).

In sum, political actions are more likely to benefit aggregate welfare with gains to minority welfare offsetting losses to majority welfare (a) with larger differences in preference for the policy outcome between high- and low-intensity citizens, and (b) when high-intensity preferences are less common. As a brief observation, if the electorate were able to *choose* whether or not political actions were allowed (this is outside of the model) the majority would be opposed. It may be worthwhile to explore what institutional arrangements might be implemented by majorities to prevent opportunities for minorities to communicate. Perhaps this would be one way to interpret efforts such as France's controversial 2010 "Act prohibiting concealment of the face in public space." 10

# 5 Demand for unseemly or corrupt candidates, offensive rhetoric, opportunities to participate, and new costly political behaviors

Consider a situation where opportunities to exhibit costly political behaviors are limited, relaxing the assumption of the model that any  $\lambda \in \mathbb{R}_{\geq 0}$  can be chosen by any citizen. This could be due to limits on available methods of communication between citizens and representatives, constraints on citizen time or resources, or cognitive or transportation challenges. If a citizen may only select a  $\lambda \in [0, \bar{\lambda}]$ , Proposition 2 implies that aggregate welfare can be improved by new opportunities for costly political actions when  $\bar{\lambda}$  is too small.

Subtly, if  $\lambda$  is categorical rather than continuous due to limited opportunities for costly behavior (i.e.,  $\lambda \in \{\lambda_1, \lambda_2, \dots, \lambda_k\}$ , k finite), available categorical values may not fall within the range of  $\lambda^*$  supporting separation. In such a setting, citizens might demand new avenues of costly behavior to indicate their preferences are *less intense* than current costly behaviors can communicate.

In a setting with limited costly behaviors, high-intensity citizens would benefit from and may therefore actively work to increase the number of costly behaviors available to them

<sup>&</sup>lt;sup>10</sup> See https://www.legifrance.gouv.fr/affichTexte.do?cidTexte=JORFTEXT000022911670. Of course, this may have an opposite effect of increasing the cost and thus increasing the communication value of concealing the face.

so that they may choose a  $\lambda^* > \bar{\lambda}$  not currently available. They might write petitions or organize protests, lobby for electoral institutions where candidate nominations are opened to the electorate, lobby for more costly requirements for registration or voting, or lobby to remove limits on campaign donations.

Likewise, costly political behaviors newly available due to exogenous changes could open up new ranges of  $\lambda^*$ . Citizens might nominate an unseemly or corrupt candidate when available over a more decorous candidate at a primary election. If corruption or moral turpitude were revealed about a candidate over the course of a campaign, high-intensity citizens may be *more* likely to publicly display support for that candidate – attend rallies, answer survey questions, display yard signs – exactly *because* that candidate is more costly to support. This incentive may induce candidates to purposefully take unpopular stances or reveal unseemly character traits (for a related observation in legislatures, see Patty, 2016).

Further, if there is change in (beliefs about) the costliness of behaviors, this could induce an arms race to develop new costly behaviors. If, for example, a national politician begins to say things that were previously thought to inspire violence and discrimination and thus said only at great social cost, the citizenry may change their beliefs about the costliness of making such statements. This could lead some citizens who previously did not make such statements – they were more costly than would benefit their policy interests – to begin making such (now lower-cost) statements to communicate intensity. Likewise, citizens who were previously making statements at great social cost might seek out new costly statements to maintain separation in communicating intensity. This dynamic could cause political polarization in discourse or political vandalism and violence.

In sum, if political polarization is changes in the number and magnitude of costly political behaviors chosen, then Propositions 1 and 2 suggest a new interpretation. Polarization in political behavior may occur not because policy preferences have changed and not because incentives of parties and candidates have changed, but instead due to changes in the set of costly political behaviors available. New behaviors might become available following

changes in communication technology, reformed electoral institutions, or level of revealed decency of candidates contesting elections. These behaviors broaden the space of communication and may lead some who before chose no costly behaviors to begin taking such actions.

# 6 Parties offer divergent policy proposals

To this point, parties have cared only about winning election and, given similar beliefs and payoffs, converge to the same policy platform (see Calvert, 1985). While office-seeking behavior is a reasonable simplifying assumption, for a variety of reasons parties may care about what policies they implement when in office, or even about what policies their opponent implements if the opponent is elected. In this section, I explore the consequences for choice of political actions when Party B cares both about winning office and about having policy implemented at their (arbitrary) preferred s=0.11

Start with the parameter values from Proposition 1. Adding a policy-motivated party yields the following statement of the divergent policy equilibrium, proved in Appendix Section D:

**Proposition 3** (Divergent policy platform equilibrium). A separating equilibrium exists where the parties propose different platforms  $s_A^* = 1$  and  $s_B^* = 0$  when  $\hat{\beta_1} + \hat{\beta_2} \geq \hat{\beta_3} \geq \hat{\beta_1} + \hat{\beta_2} - [3\omega d]/[\omega + 1]$ . Party B sacrifices some probability of victory to increase the chance that s = 0 is implemented. The range of the equilibrium depends upon the parameters  $\omega$ , d, and  $\bar{\beta}$ .

Appendix Section D presents the parameter values where Voters 1 and 2 and Voter 3 choose costly political actions when high-intensity and  $\lambda$  = 0 when low-intensity. Party B being policy-motivated changes the equilibrium behavior of all three voters because they know Party B is more likely to propose  $s_B$  = 0, all else equal.

Party A's behavior does not change in response to Party B's motive for policy (there

<sup>&</sup>lt;sup>11</sup> We might think of Party B's policy motives arising from intra-party competition, needs for campaign donors or volunteers, opportunities for quid pro quo corruption, or the types of individuals who are motivated to run for office (Besley and Coate, 1997).

are no opportunities for exchange between the two). Party B, however, is more likely to have a best response of proposing policy platform  $s_B=0$ . To simplify presentation of results, assume that Party B derives benefit from policy s=0 of  $\omega=0.2$ , that is, the party benefits from policy 20 percent as much as they benefit from holding office. Also assume that c=-1 and d=1. At these parameter values, B's best response to  $s_A=1$  is  $s_B=0$  when  $\beta_3 \geq \beta_1 + \beta_2 = 0.5$ . If, for example,  $\bar{\beta}=1.6$  and the parties believe that only Voter 3 is high-intensity after observing political actions  $\vec{\lambda}$ , Party A proposes  $s_A=1$  while Party B proposes  $s_B=0$ .

A corollary to the divergent policy equilibrium is that when a political party moves from being motivated solely by office to being partially motivated by office and partially by policy, voters may increase costly political actions even if their preference or strength of preference over policy have not changed. Changes to the incentives of the parties can cause increases in the costly political behaviors chosen by voters.

This section provides two key results when one of the parties is policy-motivated. First, policy motivation can lead to parties proposing divergent policy platforms, one with the majority and one with the minority, rather than converging as in Downs (1957). Second, parties with policy motivation can lead to more costly political actions chosen by the electorate even when ideal policies and intensity of preference have not changed.

# 7 Empirical implications for electoral behavior

The previous sections explored a game-theoretic representation of the theory of political action, expression, and identities. I now connect six empirical implications of this exploration to a set of stylized facts from political science research on electoral behavior and representation. Common interpretations of these stylized facts are that voters are limited, ineffective, or deterred by group identities from pursuing their interests. The alternative

<sup>12</sup> I show in Appendix Section E that when Party B is motivated by policy at  $\omega = 0.2$  and  $\bar{\beta} \geq 5$ , the lower bound on the equilibrium range of  $\lambda^*$  is greater than the lower bound in the equilibrium of Proposition 1.

<sup>&</sup>lt;sup>13</sup> The list of findings is not exhaustive and surely limited by my own expertise.

tive interpretations presented here suggest different conclusions about voter behavior.

- 1. Incidence of political behaviors with personal costs that appear counterproductive or even undemocratic are increasing in intensity of preference. This implication suggests an alternative interpretation to a set of commonlyh-observed costly behaviors:
  - Turnout by large numbers of voters in elections where the likelihood of being pivotal and directly changing the election result is near zero.
  - Other forms of political participation that are questionable in the context of the collective action problem, for example volunteering, making individual contributions to candidates and interest groups, or attending meetings or rallies.
  - Favoritism towards a political group in processing of factual political information,
     even with incentives to learn accurately (e.g., Hill, 2017).
  - Making offensive public statements, consuming slanted news media, discriminating against out groups, accepting behavior inconsistent with democratic norms, and maintaining support for unseemly candidates (if we believe these actios and expressions costly).
- 2. When a minority feels sufficiently more intense about an issue than a majority, policy proposed and implemented by political parties can be contrary to majority preference, even when the majority preference is common knowledge. This implication suggests an alternative interpretation of "issue incongruence" in representative democracies: The majority preference measured in opinion surveys is sometimes inconsistent with the policy delivered by representative bodies.
  - Issue incongruence is often attributed to ill informed voters, representatives
    or parties captured by donors or activists, institutional barriers, differential resources or participation, voter suppression, or voters acting on identities rather
    than policy interests.

- The theory here suggests that current measures of issue preferences may not
  measure what citizens want. Citizens might even pro-actively support candidates who advocate policies against their ideal point on one issue as a costly
  action to communicate how much they care about another.
- 3. Costly behavior is correlated with policy interests. Voters in the minority are more likely to choose costly behavior (Proposition 1). If policy interests are correlated with group memberships and group membership corresponds to expression of group identities, then costly political behavior is correlated with group identities. A stylized fact is that partisan and other group identities are widespread and structure political and electoral behavior by voters in developed democracies. Partisan and group identities are correlated with costly political behaviors such as turnout, donations, activism, participation in collective political activities, and, perhaps, social and economic discrimination against out-group members.
  - One common interpretation is that group identities "cause vote choice [through
    a] social-psychological inclination to support a given party ... rooted in group attachments formed early in adulthood that predate a voter's awareness and evaluation of the specific candidates and issues that emerge in subsequent election
    contests (Green and Baltes, 2017)."
  - With the theory of political action, expression, and identities, actions such as biased attitudes and beliefs expressed in surveys, hyperbole in social discourse, and biased evaluations of candidates could be interpreted as costly behaviors.
     Because parties in equilibrium respond to such behaviors with policy platforms, at the election there is correspondence between costly political behaviors, expression of group identities, and vote choice.

#### 4. Election campaigns involve strategic communication between voters and parties. A

<sup>&</sup>lt;sup>14</sup> Consequential party attachment may not be limited to developed democracies, see Carlson (2016).

stylized fact in American politics (due to Gelman and King, 1993) is that early election polls are not particularly informative to the outcome, but as the election campaign progresses polls increase in accuracy. At the same time, forecasting models do a better job of predicting the eventual outcome even when the polls do not. The Gelman and King (1993) interpretation is that voters increasingly pay attention and learn fundamentals of the election as the campaign progresses, including being reminded of underlying partisan or political dispositions. This process leads voters to their "enlightened preferences," estimated by forecasts.

The model here suggests that, early in the campaign, voter forecasts of vote intentions may be inaccurate either due to incomplete strategic communication or due to strategic misreporting of vote intention (if reports are costly and observable, see item [5] below). As the campaign comes to a close, however, communication options diminish, platforms solidify, and vote intentions become more accurate.

5. Responses to opinion survey questions may be strategic communication where voters give responses that are not necessarily sincere. This implication depends on features of the opinion survey that, in my view, are not widely agreed upon. Whether opinion survey responses are part of the strategic communication environment depends on whether survey takers believe their responses are (a) costly, and (b) observable to political officials.

On (a), to date there is no benchmark to measure unambiguously how costly citizens perceive survey responses, with recent evidence questioning the costliness respondents perceive (e.g., Bullock et al., 2015; Prior and Lupia, 2008; Prior, Sood, and Khanna, 2015). On (b), the results of opinion surveys are often reported to the public and paid attention by candidates for office and the media. It may be reasonable for respondents to believe their responses publicly observed.

In my view, the possibility cannot be dismissed that the conditions for strategic communication exist in survey responses. If so, a variety of stylized facts derived from

citizen responses to opinion surveys could be interpreted as costly political actions, expressions, or identities rather than sincere reflections of beliefs or preferences:

- Citizens respond to cues from (follow the lead of) political elites and partisan and group identities in giving responses to opinion survey questions about preferred policies (e.g., Zaller, 1992), even when they had previously given a different response or when the issue seems clearly inconsistent with their interests. Citizens give group-congenial survey responses in ignorance or against own interests e.g., "cheerleading" (Bullock et al., 2015) and respond to group cues in survey experiments.
- On survey instruments, citizens appear to process arguments and information
  with perceptual bias or motivated reasoning (Kunda, 1990), where they argue
  against information discordant with their preferred belief and accept less critically information consistent with their preferred belief.
- 6. Candidates with negative valence attributes such as corruption, criminal backgrounds, or moral failings may capture the votes of those with intense preferences because support separates intense from weak preferences. Voters often appear to accommodate corrupt behavior by politicians instead of voting it out (for a review, see De Vries and Solaz, 2017). This has been attributed to challenges such as information asymmetries, group loyalties, side payments, or attribution ambiguities (see De Vries and Solaz, 2017, p. 392-3). In the theory of political action, expression, and identities, supporting a corrupt official or candidate with clear deadweight loss to society is a costly action that may communicate intensity for other policy.

In sum, the empirical implications of the theory help explain a set of stylized facts from political science on electoral behavior and representation. The alternative understanding of these actions and expressions provided by the model opens new avenues of empirical and theoretical research.

# 8 Implications for party polarization

The model shows that increasing magnitudes of issue intensity  $\bar{\beta}$  can increase the magnitude of costly political behaviors  $\lambda^*$  chosen by high intensity types. Changes in magnitude of costly behaviors might be interpreted as polarization in behavior by groups. But the theory offers other implications for causes of party polarization involving changes to communication technology and party policy motivation.

The model highlights that new instruments of costly communication can change the magnitude of costly political behaviors chosen and the equilibrium policy delivered by the parties. If a society moves from limited opportunities for costly behavior to more continuous opportunities, new ranges of intensity are uncovered that can support the parties learning intensity of citizen preferences, and thus can lead to policy representation of minorities with intense preferences (Proposition 1). Party polarization might occur in response not to changing preferences or intensities of the electorate, but rather due to changes in communication technology. Suggestively, party polarization in the American Congress (McCarty, Poole, and Rosenthal, 2006) fell after the implementation of the secret ballot (new barrier to communication with parties) and then rose again during the post-World War II period of increasing access to communication technology.

A second result relevant to party polarization is that policy-motivated parties can cause increases in costly voter behaviors without change to voter preferences. If, for example, one party were partially captured by an interest group with strong policy goals that led the party to begin sacrificing electoral prospects in pursuit of that policy, it could increase the incentives for both supporters and opponents of the policy to increase costly behaviors. This could manifest as increases in polarization in participation or expression without corresponding change in citizen policy preferences.

These observations suggest one path to polarization-reducing reforms. If the costly behavior observed in polarized partisan environments such as the early 21st century United States are efforts to communicate policy intensity, creating opportunities for costly actions

with fewer negative externalities available could be of the benefit of society. One such idea is to dramatically increase the contribution limits available to Political Action Committees (PACs). This idea traces to Cain (2015), who argues that increasing the contribution limits of PACs would allow larger PACs with more heterogeneous memberships. From the perspective of the theory here, a variety of large issue-oriented PACs would allow voters in the population to make costly pecuniary contributions to communicate intensity across different issues. Notably, these contributions allow nearly-continuous  $\lambda$ , the absence of which can lead to less effective communication.

#### 9 Discussion

This paper has focused on taking heterogeneity in intensity of preference as a central part of the interaction between citizens and political officials and connecting this interaction to the prevalence of costly political actions, expression, and identities. The theory of this essay offers a different interpretation of political action. Instead of arising out of psychological or social identities, costly political actions help individuals who care deeply about policy distinguish themselves from those who care modestly. If individuals vary in intensity of preference, so too does incidence of costly political behaviors. The theory of political action, expression, and identities provides an explanation for actions of questionable instrumental value.

Proposition 1 from the strategic model offers two key results. First, in equilibrium high-intensity voters on both the majority and minority side of a policy choice choose costly political actions in pursuit of their policy goals. This suggests that even on an issue with a clear majority, members of the majority might still choose costly political actions to maintain implementation of majority policy. Second, in equilibrium the parties propose and implement the policy favored by the minority when the preference intensity of the minority is sufficiently larger than the intensity of the majority. This result predicts that costly political actions allow the minority to gain policy representation when they care more deeply about

policy. Thus, observations of failure of issue congruence – policy implemented with the majority – should not necessarily lead to conclusion of failure of political representation. The focus on heterogeneity in intensity also highlights an advantage of representative democracy over direct democracy by simple majority: more utilitarian policy when minorities feel more strongly on the issue than the majority.

The anecdote of the athletic field along with Proposition 2 and its discussion present three key findings. First, when intensity of preference is hard to communicate, a system with costly political actions can improve electorate welfare relative to a system without such actions. This suggests the prevalence of political action, expression, and identities may be because citizens are better off, not due to non-political causes of actions or elite manipulation. Further, the result suggests that citizens may maintain rather than suppress opportunities for such communication. Second, in settings where opportunities for costly behaviors are limited, voters may demand new costly opportunities, which could take the form of seeking out corrupt or morally deficient candidates. Perhaps this explains why politicians are so often enmeshed in scandal. Third, in settings with limited opportunities for costly behaviors, exogenous changes to communication technology can alter magnitude of costly actions chosen and policy output of the system without changes to the preferences of voters or politicians. This third result has implications for political polarization across time.

Because these results depend upon a simple theory and an abstract and stylized model, it is worth revisiting the central assumptions. First and foremost is the assumption that citizens vary in their intensity for policy and yet are not able to communicate that intensity without costly political actions. Variation in preferences is almost certain to hold in large societies with heterogeneous economic, social, and cultural conditions. The second component is perhaps more in question. The theory rests on an assumption that those with weak preferences have an incentive to indicate their preferences are strong when doing so is not too costly. This also seems likely to hold in many settings but merits future

exploration.

If there is important variation in intensity that is hard to observe, it is useful to speculate if variation is harder to observe on some issues than others. It may be, for example, intensity is easier to observe in the realm of economic policy because there are many opportunities for officials to observe and estimate voters' elasticities to income. Social policy on moral issues seems a more likely setting where opportunities to observe intensity are limited. This may be why social policy seems to generate larger polarization in rhetoric and behavior.

The model abstracts to an electorate of three individual voters, which might lead some to be concerned about collective action problems in a larger electorate. However, in equilibrium voters choose political behaviors of cost strictly less than the policy benefit they would personally receive. Because of this, each individual with intense preferences in a large electorate has an incentive to take on costs somewhere between zero and their expected personal policy benefit depending upon its communication value. Therefore, assuming costly behaviors are observed by candidates in expectation, the incentive to shirk is not present and voters with intense preferences would choose costly action despite not being individually pivotal.

Finally, I model a single election, with communication happening during one campaign period. A direction for future work is to consider the game over multiple elections. Repeated stages may explain why citizen political action, expressions, and identities and party issue positions often appear relatively stable across consecutive elections, e.g. habituation. This is not in the scope of the current model, but perhaps multiple periods with costly observation for parties would lead to separating equilibria with stable actions chosen by voters and parties.

The theory suggests need for more careful measurement of intensity. Most survey work on policy preferences considers only the individual's ideal policy or some reduced form representation of the full utility curve across policy alternatives. The discipline would benefit from better measures (see Fowler and Hall, 2016; Tausanovitch, N.d., for recent efforts).

However, if one takes seriously the argument presented above and one believes respondents view opinion surveys as costly and observable, respondents may have incentive to exaggerate or misreport their intensity on surveys. This suggests new measures may require careful observational work on costly political behaviors and party policy responses. When citizens and political officials engage in communication in the presence of asymmetric information, care must be taken in empirical observation and in interpretation of the role of political action, expression, and identities in that system.

#### References

- Besley, Timothy, and Stephen Coate. 1997. "An Economic Model of Representative Democracy." *Quarterly Journal of Economics* 112(1): 85–114.
- Bullock, John G., Alan S. Gerber, Seth J. Hill, and Gregory A. Huber. 2015. "Partisan Bias in Factual Beliefs About Politics." *Quarterly Journal of Political Science* 10(4): 519–578.
- Cain, Bruce E. 2015. Democracy More or Less. Cambridge: Cambridge University Press.
- Calvert, Randall L. 1985. "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence." *American Journal of Political Science* 29(1): 69–95.
- Campbell, Angus, Philip E. Converse, Warren E. Miller, and Donald E. Stokes. 1960. *The American Voter*. New York: Wiley.
- Canes-Wrone, Brandice, Michael C. Herron, and Kenneth W. Shotts. 2001. "Leadership and Pandering: A Theory of Executive Policymaking." *American Journal of Political Science* 45(3): 532–550.
- Carlson, Elizabeth. 2016. "Finding Partisanship Where We Least Expect it: Evidence of Partisan Bias in a New African Democracy." *Political Behavior* 38(1): 129–154.
- De Vries, Catherine E., and Hector Solaz. 2017. "The Electoral Consequences of Corruption." *Annual Review of Political Science* 20: 391–408.
- Dixit, Avinash, and John Londregan. 1996. "The Determinants of Success of Special Interests in Redistributive Politics." *Journal of Politics* 58(4): 1132–1155.
- Downs, Anthony. 1957. An Economic Theory of Democracy. New York: Harper Collins.
- Fowler, Anthony, and Andrew B. Hall. 2016. "The Elusive Quest for Convergence." *Quarterly Journal of Political Science* 11: 131–149.
- Fox, Justin, and Richard Van Weelden. 2015. "Hoping for the best, unprepared for the worst." *Journal of Public Economics* 130: 59–65.

- Gause, LaGina. N.d. The Advantage of Disadvantage: Legislative Responsiveness to Collective Action by the Politically Marginalized. University of California, San Diego: Book manuscript.
- Gelman, Andrew, and Gary King. 1993. "Why Are American Presidential Election Campaign Polls So Variable When Votes Are So Predictable?" *British Journal of Political Science* 23(October): 409–451.
- Green, Donald P., and Susanne Baltes. 2017. "Party Identification: Meaning and Measurement." In SAGE Handbook of Electoral Behavior, ed. Kai Arzheimer. pp. 287–312.
- Green, Donald P., Bradley Palmquist, and Eric Schickler. 2002. *Partisan Hearts and Minds: Political Parties and the Social Identities of Voters*. New Haven: Yale University Press.
- Hill, Seth J. 2017. "Learning Together Slowly: Bayesian Learning About Political Facts." *Journal of Politics* 79(4): 1403–1418.
- Kunda, Ziva. 1990. "The Case for Motivated Reasoning." *Psychological Bulletin* 108(3): 480–498.
- Laitin, David D. 1998. *Identity in Formation*. Ithaca, NY: Cornell University Press.
- Lindbeck, Assar, and Jorgen W. Weibull. 1987. "Balanced-Budget Redistribution as the Outcome of Political Competition." *Public Choice* 52(3): 273–297.
- Lohmann, Susanne. 1993. "A Signalling Model of Informative and Manipulative Political Action." *American Political Science Review* 87(2): 319–333.
- McCarty, Nolan, Keith T. Poole, and Howard Rosenthal. 2006. *Polarized America: The Dance of Ideology and Unequal Riches*. Cambridge, MA: The MIT Press.
- Meirowitz, Adam. 2005. "Polling games and information revelation in the Downsian framework." *Games and Economic Behavior* 41: 464–489.
- Patty, John W. 2016. "Signaling through Obstruction." *American Journal of Political Science* 60(1): 175–189.
- Persson, Torsten, and Guido Enrico Tabellini. 2000. "Electoral Competition." In *Political Economics: Explaining Economic Policy*. Cambridge, MA: MIT Press.
- Prior, Markus, and Arthur Lupia. 2008. "Money, Time, and Political Knowledge: Distinguishing Quick Recall and Political Learning Skills." *American Journal of Political Science* 52(1): 169–183.
- Prior, Markus, Gaurav Sood, and Kabir Khanna. 2015. "You Cannot be Serious: The Impact of Accuracy Incentives on Partisan Bias in Reports of Economic Perceptions." *Quarterly Journal of Political Science* 10(4): 489–518.
- Shotts, Kenneth W. 2006. "A Signaling Model of Repeated Elections." *Social Choice Welfare* 27: 251–261.

Tausanovitch, Chris. N.d. "Measuring Preference Intensity." Working paper, UCLA.

Wawro, Gregory J., and Eric Schickler. 2006. *Filibuster: Obstruction and Lawmaking in the U.S. Senate.* Princeton: Princeton University Press.

Zaller, John. 1992. *The Nature and Origins of Mass Opinion*. New York: Cambridge University Press.

#### **Appendix**

The following lemma characterizes best responses for Parties A and B when office-motivated, given their beliefs about the issue intensities of the voters.

**Lemma 1** (Party best responses). The best response for both parties is to propose the policy preferred by minority Voter 3,  $s^* = 0$ , when  $\hat{\beta}_1 + \hat{\beta}_2 \leq \hat{\beta}_3$ , otherwise to propose the policy preferred by the majority Voters 1 and 2,  $s^* = 1$ .

*Proof.* Begin by specifying the expected vote share for the two parties. Let the parties' estimates of the policy intensity of Voters 1, 2, and 3 after observing  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  be  $\hat{\beta}_1$ ,  $\hat{\beta}_2$ , and  $\hat{\beta}_3$ . Given the ballot function from Eq. (1) and the uniform distribution on  $\delta$ , the expected vote share for Party A is

$$p^{A} = \left(\frac{\hat{\beta}_{1}(s_{A} - s_{B}) - c}{d - c} + \frac{\hat{\beta}_{2}(s_{A} - s_{B}) - c}{d - c} + \frac{\hat{\beta}_{3}(s_{B} - s_{A}) - c}{d - c}\right)/3,$$

$$= \left([\hat{\beta}_{1} + \hat{\beta}_{2}][s_{A} - s_{B}] + \hat{\beta}_{3}(s_{B} - s_{A})\right)/3(d - c) - c/(d - c),$$

with  $p^B = 1 - p^A$ .

To identify best responses for the parties, note that when  $\omega$  = 0 the expected benefit for each party is the expected probability they are elected times the rents to office (normalized to one). Given that optimization problem, and suppressing the hats indicating posterior beliefs about  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  for convenience, Party A's best response to  $s_B$  = 0 is  $s_A$  = 0 when

$$\begin{array}{lll} \mathsf{U}^{\mathsf{A}}(0|\mathsf{s}_{\mathsf{B}}=0) & \geq & \mathsf{U}^{\mathsf{A}}(1|\mathsf{s}_{\mathsf{B}}=0), \\ -\mathsf{c}/(\mathsf{d}-\mathsf{c}) & > & (\beta_{\mathsf{1}}+\beta_{\mathsf{2}}-\beta_{\mathsf{3}})/3(\mathsf{d}-\mathsf{c})-\mathsf{c}/(\mathsf{d}-\mathsf{c}) \Rightarrow \beta_{\mathsf{3}} > \beta_{\mathsf{1}}+\beta_{\mathsf{2}}. \end{array}$$

Party A's best response to  $s_B = 1$  is  $s_A = 0$  when

$$U^{A}(0|s_{B} = 1) \geq U^{A}(1|s_{B} = 1),$$

$$(-\beta_{1} - \beta_{2} + \beta_{3})/3(d-c) - c/(d-c) \geq -c/(d-c) \Rightarrow \beta_{3} \geq \beta_{1} + \beta_{2}.$$

Likewise, Party B's best response to  $s_A = 0$  is  $s_B = 0$  when

$$\begin{array}{lll} \mathsf{U}^{\mathsf{B}}(0|\mathsf{s}_{\mathsf{A}}=0) & \geq & \mathsf{U}^{\mathsf{B}}(1|\mathsf{s}_{\mathsf{A}}=0), \\ 1+\mathsf{c}/(\mathsf{d}-\mathsf{c}) & \geq & 1-(-\beta_1-\beta_2+\beta_3)/3(\mathsf{d}-\mathsf{c})+\mathsf{c}/(\mathsf{d}-\mathsf{c}), \\ 0 & \geq & (\beta_1+\beta_2-\beta_3)/3(\mathsf{d}-\mathsf{c}) \Rightarrow \beta_3 \geq \beta_1+\beta_2. \end{array}$$

Party B's best response to  $s_A = 1$  is  $s_B = 0$  when

$$\begin{array}{rcl} & U^B(0|s_A=1) & \geq & U^B(1|s_A=1), \\ 1-(\beta_1+\beta_2-\beta_3)/3(d-c)+c/(d-c) & \geq & 1+c/(d-c), \\ & (-\beta_1-\beta_2+\beta_3)/3(d-c) & \geq & 0 \Rightarrow \beta_3 \geq \beta_1+\beta_2. \end{array}$$

Therefore, the best response for both parties is to propose the policy preferred by mi-

nority Voter 3 when  $\hat{\beta_1} + \hat{\beta_2} \leq \hat{\beta_3}$ .

# A Proof to Proposition 1

*Proof.* When  $\bar{\beta} < 2$ , the parties always choose s\* = 1 because always  $\beta_1 + \beta_2 > \beta_3$  (Lemma 1). For a separating equilibrium to exist,  $\bar{\beta}$  must be greater than two, and so I make that assumption for the remainder of this proof.

Suppose a separating equilibrium exists where  $\bar{\beta}>2$  and when  $\beta_i=\bar{\beta}, \lambda_i=\lambda^*>0$ , and when  $\beta_i=1, \lambda_i=0.15$  Then, it must also be that

$$\begin{array}{lll} \mathsf{U}^\mathsf{A}(\mathsf{s}_\mathsf{A} = \tau_\mathsf{i} | \lambda_\mathsf{i} = \lambda^*) & \geq & \mathsf{U}^\mathsf{A}(\mathsf{s}_\mathsf{A} = 1 - \tau_\mathsf{i} | \lambda_\mathsf{i} = \lambda^*), \text{ and} \\ \mathsf{U}^\mathsf{B}(\mathsf{s}_\mathsf{B} = \tau_\mathsf{i} | \lambda_\mathsf{i} = \lambda^*) & \geq & \mathsf{U}^\mathsf{B}(\mathsf{s}_\mathsf{B} = 1 - \tau_\mathsf{i} | \lambda_\mathsf{i} = \lambda^*), \text{ and} \\ \mathsf{U}^\mathsf{i}(\lambda^* | \beta_\mathsf{i} = \bar{\beta}) & \geq & \mathsf{U}^\mathsf{i}(\mathsf{0} | \beta_\mathsf{i} = \bar{\beta}), \text{ and} \\ \mathsf{U}^\mathsf{i}(\mathsf{0} | \beta_\mathsf{i} = 1) & \geq & \mathsf{U}^\mathsf{i}(\lambda^* | \beta_\mathsf{i} = 1). \end{array} \tag{A1}$$

The first two inequalities of (A1) hold. If either Voter 1 or Voter 2, or both, choose costly political action  $\lambda^*$ , it follows that  $\beta_1+\beta_2>\beta_3$  and from Lemma 1 both parties propose policy s = 1 as a best response. If Voter 3 chooses costly political action and neither of the voters of the majority choose costly action, then  $\beta_1+\beta_2<\beta_3$  and from Lemma 1 both parties propose policy s = 0 as a best response. In either case, the parties are better off setting policy to the preference of the voter choosing costly political action  $\lambda^*$ , and each of the two inequalities holds in equilibrium.

Continuing with the third and fourth inequalities of (A1) for Voter i, consider first Voter 3 with  $\tau_3=0$ . Voter 3 takes expectations over the strategies of Voters 1 and 2, who in the separating equilibrium play  $\lambda_1=\lambda_2=\lambda^*$  with probability  $q^2$ , play  $\lambda_1=\lambda_2=0$  with probability  $(1-q)^2$ , and play  $\lambda_1+\lambda_2=\lambda^*+0$  with probability  $1-q^2-(1-q)^2=2(q-q^2)$  where which of Voter 1 or 2 plays  $\lambda^*$  is irrelevant to Voter 3. Voter 3's expected benefit from  $\lambda_3=\lambda^*$  and  $\lambda_1=0$  is

$$\begin{array}{lll} \mathsf{U}^{3}(\lambda^{*}|\beta_{3}) & = & \mathsf{q}^{2}\mathsf{U}^{3}(\lambda^{*}|\lambda_{1}=\lambda_{2}=\lambda^{*},\beta_{3}) + (1-\mathsf{q})^{2}\mathsf{U}^{3}(\lambda^{*}|\lambda_{1}=\lambda_{2}=0,\beta_{3}), \\ & + & 2(\mathsf{q}-\mathsf{q}^{2})\mathsf{U}^{3}(\lambda^{*}|\lambda_{1}+\lambda_{2}=\lambda^{*},\beta_{3}), \\ \mathsf{U}^{3}(0|\beta_{3}) & = & \mathsf{q}^{2}\mathsf{U}^{3}(0|\lambda_{1}=\lambda_{2}=\lambda^{*},\beta_{3}) + (1-\mathsf{q})^{2}\mathsf{U}^{3}(0|\lambda_{1}=\lambda_{2}=0,\beta_{3}), \\ & + & 2(\mathsf{q}-\mathsf{q}^{2})\mathsf{U}^{3}(0|\lambda_{1}+\lambda_{2}=\lambda^{*},\beta_{3}). \end{array}$$

Given the party best response functions to  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$ , the components to the two

 $<sup>^{15}</sup>$  Assume high-intensity types choose the lowest cost  $\lambda$  that successfully separates. For off path beliefs, then, assume that when costly action is below the threshold to separate, the parties believe the voter is low intensity, and when costly action is higher than expected, the parties believe the voter high intensity.

expressions above are

$$\begin{split} & \mathsf{U}^3(\lambda^*|\lambda_1=\lambda_2=\lambda^*,\beta_3) = (-\mathsf{c}/(\mathsf{d}-\mathsf{c}))(\mathsf{0}) + (1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}) - \lambda^*, \\ & \mathsf{U}^3(\lambda^*|\lambda_1=\lambda_2=\mathsf{0},\beta_3) = (-\mathsf{c}/(\mathsf{d}-\mathsf{c}))(\beta_3) + (1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\beta_3+\mathsf{d}^2/2\mathsf{d}) - \lambda^*, \\ & \mathsf{U}^3(\lambda^*|\lambda_1+\lambda_2=\lambda^*,\beta_3) = (-\mathsf{c}/(\mathsf{d}-\mathsf{c}))(\mathsf{0}) + (1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}) - \lambda^*, \\ & \mathsf{U}^3(\mathsf{0}|\lambda_1=\lambda_2=\lambda^*,\beta_3) = (-\mathsf{c}/(\mathsf{d}-\mathsf{c}))(\mathsf{0}) + (1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}), \\ & \mathsf{U}^3(\mathsf{0}|\lambda_1=\lambda_2=\mathsf{0},\beta_3) = (-\mathsf{c}/(\mathsf{d}-\mathsf{c}))(\mathsf{0}) + (1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}), \\ & \mathsf{U}^3(\mathsf{0}|\lambda_1+\lambda_2=\lambda^*,\beta_3) = (-\mathsf{c}/(\mathsf{d}-\mathsf{c}))(\mathsf{0}) + (1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}), \end{split}$$

where d<sup>2</sup>/2d is the expected value of  $\delta$  conditional on Party B winning, i.e.  $\delta > 0.16$  The third inequality of (A1) holds for Voter 3 when

$$\begin{array}{l} \mathsf{U}^{3}(\lambda^{*}|\beta_{3}=\bar{\beta})\geq \mathsf{U}^{3}(0|\beta_{3}=\bar{\beta}),\\ \mathsf{q}^{2}\mathsf{U}^{3}(\lambda^{*}|\lambda_{1}=\lambda_{2}=\lambda^{*},\bar{\beta})+(1-\mathsf{q})^{2}\mathsf{U}^{3}(\lambda^{*}|\lambda_{1}=\lambda_{2}=0,\bar{\beta})+2(\mathsf{q}-\mathsf{q}^{2})\mathsf{U}^{3}(\lambda^{*}|\lambda_{1}+\lambda_{2}=\lambda^{*},\bar{\beta}),\\ \geq \mathsf{q}^{2}\mathsf{U}^{3}(0|\lambda_{1}=\lambda_{2}=\lambda^{*},\bar{\beta})+(1-\mathsf{q})^{2}\mathsf{U}^{3}(0|\lambda_{1}=\lambda_{2}=0,\bar{\beta})+2(\mathsf{q}-\mathsf{q}^{2})\mathsf{U}^{3}(0|\lambda_{1}+\lambda_{2}=\lambda^{*},\bar{\beta}),\\ \mathsf{q}^{2}[(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^{2}/2\mathsf{d})-\lambda^{*}]+(1-\mathsf{q})^{2}[(-\mathsf{c}/(\mathsf{d}-\mathsf{c}))(\bar{\beta})+(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\bar{\beta}+\mathsf{d}^{2}/2\mathsf{d})-\lambda^{*}]\\ +2(\mathsf{q}-\mathsf{q}^{2})[(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^{2}/2\mathsf{d})]+(1-\mathsf{q})^{2}[(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^{2}/2\mathsf{d})]\\ +2(\mathsf{q}-\mathsf{q}^{2})[(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^{2}/2\mathsf{d})],\\ (1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^{2}/2\mathsf{d})+(1-\mathsf{q})^{2}[\bar{\beta}]-\lambda^{*}\geq (1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^{2}/2\mathsf{d}),\\ \Rightarrow \lambda^{*}\leq \bar{\beta}\mathsf{q}^{2}-2\bar{\beta}\mathsf{q}+\bar{\beta}. \end{array} \tag{A2}$$

The fourth inequality of (A1) holds for Voter 3 when

$$\begin{split} & \mathsf{U}^3(0|\beta_3=1) \geq \mathsf{U}^3(\lambda^*|\beta_3=1), \\ & \mathsf{q}^2 \mathsf{U}^3(0|\lambda_1=\lambda_2=\lambda^*,1) + (1-\mathsf{q})^2 \mathsf{U}^3(0|\lambda_1=\lambda_2=0,1) + 2(\mathsf{q}-\mathsf{q}^2) \mathsf{U}^3(0|\lambda_1+\lambda_2=\lambda^*,1) \\ & \geq \mathsf{q}^2 \mathsf{U}^3(\lambda^*|\lambda_1=\lambda_2=\lambda^*,1) + (1-\mathsf{q})^2 \mathsf{U}^3(\lambda^*|\lambda_1=\lambda_2=0,1) + \\ & 2(\mathsf{q}-\mathsf{q}^2) \mathsf{U}^3(\lambda^*|\lambda_1+\lambda_2=\lambda^*,1), \\ & \mathsf{q}^2[(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d})] + (1-\mathsf{q})^2[(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}), \\ & + 2(\mathsf{q}-\mathsf{q}^2)[(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d})] \\ & \geq \mathsf{q}^2[(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}) - \lambda^*] + (1-\mathsf{q})^2[(-\mathsf{c}/(\mathsf{d}-\mathsf{c}))(1) \\ & + (1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}) - \lambda^*] + 2(\mathsf{q}-\mathsf{q}^2)[(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}) - \lambda^*], \\ & (1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}) \geq (1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}) + (1-\mathsf{q})^2[1] - \lambda^*, \\ & \Rightarrow \mathsf{q}^2 - 2\mathsf{q} + 1 \leq \lambda^*. \end{split}$$

Thus, the separating equilibrium holds for Voter 3 at any value of  $\lambda^*$  such that  $q^2 - 2q + 1 \le \lambda^* \le \bar{\beta}q^2 - 2\bar{\beta}q + \bar{\beta}$ , which always holds because  $\bar{\beta} > 1$  and  $q^2 - 2q + 1 \ge 0 \ \forall \ q$ .

$$\frac{16}{16}$$
 E( $\delta$ |c, d, B wins) =  $\int_0^d \frac{x}{d-0} dx = \frac{1}{d} \frac{x^2}{2} \Big|_{x=0}^{x=d} = \frac{d^2}{2d}$ .

Continuing the proof, consider the third and fourth inequalities of (A1) for Voters 1 and 2. Voters 1 and 2 have the same payoffs, beliefs, and actions, so the proof for Voter 1 is the same as for  $2.^{17}$  Voter 1 takes expectations over the strategies of Voters 2 and 3, who in the separating equilibrium play  $\lambda_2 = \lambda_3 = \lambda^*$  with probability  $q^2$ ,  $\lambda_2 = \lambda_3 = 0$  with probability  $(1-q)^2$ ,  $\lambda_2 = 0$ ,  $\lambda_3 = \lambda^*$  with probability  $(q-q^2)$ , and  $\lambda_2 = \lambda^*$ ,  $\lambda_3 = 0$  with probability  $(q-q^2)$ . Note, however, that whenever Voter 1 chooses costly political action at  $\lambda^*$ , the parties propose  $s^* = 1$  in equilibrium regardless of the values of  $\lambda_2$  or  $\lambda_3$ . In fact,  $s^*$  can only be zero at the combination of  $\lambda_{\{1,2,3\}} = \{0,0,\lambda^*\}$ , which occurs with probability  $q-q^2$  when Voter 1 chooses action  $\lambda_1 = 0$ . Voter 1's expected benefit from  $\lambda_3 = \lambda^*$  and  $\lambda_1 = 0$  is

$$\begin{split} U^{1}(\lambda^{*}|\beta_{1}) &= U^{1}(\lambda^{*}|\beta_{1},s^{*}=1) \\ &= (-c)/(d-c)(\beta_{1}) + (1+(c)/(d-c))(\beta_{1}+d^{2}/2d) - \lambda^{*}, \\ U^{1}(0|\beta_{1}) &= (q-q^{2})U^{1}(0|\beta_{1},s^{*}=0) + (1-q+q^{2})U^{1}(0|\beta_{1},s^{*}=1) \\ &= (q-q^{2})(1+(c)/(d-c))(d^{2}/2d) + (1-q+q^{2})[(-c)/(d-c)(\beta_{1}) \\ &+ (1+(c)/(d-c))(\beta_{1}+d^{2}/2d)], \end{split}$$

where  $d^2/2d$  is the expected value of  $\delta$  conditional on Party B winning. The third inequality of (A1) holds for Voter 1 when

$$\begin{array}{l} U^{1}(\lambda^{*}|\beta_{1}=\bar{\beta})\geq U^{1}(0|\beta_{1}=\bar{\beta}),\\ (-c)/(d-c)(\bar{\beta})+(1+(c)/(d-c))(\bar{\beta}+d^{2}/2d)-\lambda^{*}\geq \\ (q-q^{2})(1+(c)/(d-c))(d^{2}/2d)+(1-q+q^{2})[(-c)/(d-c)(\bar{\beta})+(1+(c)/(d-c))(\bar{\beta}+d^{2}/2d)],\\ (d-c)^{-1}[-c\bar{\beta}+c\bar{\beta}+cd^{2}/2d]+\bar{\beta}+d^{2}/2d-\lambda^{*}\geq (q-q^{2})[d^{2}/2d+cd^{2}/2d]+cd^{2}/2d(d-c)]+(1-q+q^{2})[(d-c)^{-1}(-c\bar{\beta}+c\bar{\beta}+cd^{2}/2d)+\bar{\beta}+d^{2}/2d],\\ cd^{2}/2d(d-c)+d^{2}/2d+\bar{\beta}-\lambda^{*}\geq \\ cd^{2}/2d(d-c)+d^{2}/2d+(1-q+q^{2})[\bar{\beta}],\\ \bar{\beta}-\lambda^{*}\geq (1-q+q^{2})[\bar{\beta}],\\ \Rightarrow \lambda^{*}\leq -\bar{\beta}q^{2}+\bar{\beta}q. \end{array}$$

 $<sup>^{17}</sup>$  There is likely an asymmetric equilibrium where one of Voter 1 or 2 plays a never-communicate strategy. The other majority voter's best response to such a strategy would be to communicate when high intensity, likely at a larger range of  $\lambda^*$  than in the symmetric equilibrium. As strategies of Voters 1 and 2 are not of central interest, I set the asymmetric case aside.

The fourth inequality of (A1) holds for Voter 1 when

$$\begin{split} & \mathsf{U}^1(0|\beta_1=1) \geq \mathsf{U}^1(\lambda^*|\beta_1=1), \\ & (\mathsf{q}-\mathsf{q}^2)(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(\mathsf{d}^2/2\mathsf{d}) + (1-\mathsf{q}+\mathsf{q}^2)[(-\mathsf{c})/(\mathsf{d}-\mathsf{c})(1)+(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(1+\mathsf{d}^2/2\mathsf{d})] \\ & \geq (-\mathsf{c})/(\mathsf{d}-\mathsf{c})(1)+(1+(\mathsf{c})/(\mathsf{d}-\mathsf{c}))(1+\mathsf{d}^2/2\mathsf{d})-\lambda^*, \\ & (\mathsf{q}-\mathsf{q}^2)[\mathsf{d}^2/2\mathsf{d}+\mathsf{c}\mathsf{d}^2/2\mathsf{d}(\mathsf{d}-\mathsf{c})]+(1-\mathsf{q}+\mathsf{q}^2)[(\mathsf{d}-\mathsf{c})^{-1}(-\mathsf{c}+\mathsf{c}+\mathsf{c}\mathsf{d}^2/2\mathsf{d})+1+\mathsf{d}^2/2\mathsf{d}] \\ & \geq (\mathsf{d}-\mathsf{c})^{-1}[-\mathsf{c}+\mathsf{c}+\mathsf{c}\mathsf{d}^2/2\mathsf{d}]+1+\mathsf{d}^2/2\mathsf{d}-\lambda^*, \\ & \mathsf{d}^2/2\mathsf{d}+\mathsf{c}\mathsf{d}^2/2\mathsf{d}(\mathsf{d}-\mathsf{c})+(1-\mathsf{q}+\mathsf{q}^2)\geq \mathsf{c}\mathsf{d}^2/2\mathsf{d}(\mathsf{d}-\mathsf{c})+1+\mathsf{d}^2/2\mathsf{d}-\lambda^*, \\ & -\mathsf{q}+\mathsf{q}^2\geq -\lambda^*, \\ & \Rightarrow -\mathsf{q}^2+\mathsf{q}<\lambda^*. \end{split}$$

Therefore, a separating equilibrium is supported when  $q^2$  –  $2q+1 \le \lambda^* \le \bar{\beta}q^2$  –  $2\bar{\beta}q+\bar{\beta}$  (Voter 3) and – $q^2+q \le \lambda^* \le -\bar{\beta}q^2+\bar{\beta}q$  (Voters 1 and 2), and an equilibrium exists at intermediate ranges of q and  $\bar{\beta}$  where Voter 3 exhibits separating behavior but Voters 1 and 2 do not.

### B Lemma 2

The following lemma shows that the Voters select the minimum value of  $\lambda^*$  on the supported range of  $\lambda^*$  in the separating equilibrium.

**Lemma 2** (Choice of  $\lambda^*$ ). When the voters have continuous choice over  $\lambda^*$ , they select the minimum value that supports separation consistent with Proposition 1. This value follows the discontinuous function  $\lambda^* = q^2 - 2q + 1$  when q < 0.5 and  $-q^2 + q$  when q < 0.5. The magnitude of costly action chosen in equilibrium is decreasing in q because it takes less to convince the parties a Voter is high-intensity the higher the ex ante rate of intensity (q) in the population.

*Proof.* Consider first the choice of  $\lambda^*$  for Voter 1 (and 2). When high-intensity, Voter 1's expected benefit to  $\lambda^*$  is summarized in (A3), where it is clear that Voter 1 and 2 prefer to minimize  $\lambda^*$ . Similarly, Voter 3 also prefers to minimize  $\lambda^*$  consistent with their expected benefit when high-intensity stated on the left hand side of the inequality (A2). Both minority and majority voters prefer the lowest value of  $\lambda^*$  that provides separation between high and low intensity in equilibrium.

The lowest value of  $\lambda^*$  supported in equilibrium depends on the parameters to Proposition 1. The lower bound for  $\lambda^*$  is  $q^2 - 2q + 1$  for Voter 3 and  $-q^2 + q$  for Voters 1 and 2. A bit of algebra shows that the Voter 3's lower bound is larger when q < 0.5, and Voter 1 and 2's when q > 0.5. Therefore, when the voters have continuous choice over  $\lambda^*$ ,  $\lambda^* = q^2 - 2q + 1$  when q < 0.5 and  $-q^2 + q$  when q < 0.5.

# C Proof to Proposition 2

*Proof.* Define expected aggregate welfare as the sum of the expected benefit to Voters 1, 2, and 3,

$$W = EU^{1} + EU^{2} + EU^{3}$$
.

To evaluate the welfare consequences of costly political action, consider W in two regimes,  $R_{\lambda}$  where costly political actions are available to voters and observed by parties, and  $R_0$ , where one or both conditions do not hold.

Start first with  $W_0$ , expected welfare under  $R_0$ . As in the models above, policy preference is common knowledge and issue intensity is private knowledge, but here costly political action is not observed. Lemma 1 shows that in equilibrium  $s^*=0$  when  $\hat{\beta_1}+\hat{\beta_2}\leq\hat{\beta_3}$ , else  $s^*=1$ . In  $R_0$ , party beliefs about  $\vec{\beta}$  follow from q.  $\beta_1+\beta_2\leq\beta_3$  only obtains when  $\beta_{\{1,2,3\}}=\{1,1,\bar{\beta}\}$ , which occurs with probability (1-q)(1-q)(q). Thus, with probability 1-(1-q)(1-q)(q) the parties' best response is  $s^*=1$ . As 1-(1-q)(1-q)(q) is strictly greater than (1-q)(1-q)(q) for all  $q\in[0,1]$ , the pure strategy equilibrium for the no-action regime is  $s^*_A=s^*_B=1$ .

Expected welfare for the three voters when  $s_A^* = s_B^* = 1$  is

$$\begin{array}{lll} U^{1}(s=1) & = & (q)(\bar{\beta}) + (1-q)(1) + (1+c/(d-c))(d^{2}/2d) \\ U^{2}(s=1) & = & (q)(\bar{\beta}) + (1-q)(1) + (1+c/(d-c))(d^{2}/2d) \\ U^{3}(s=1) & = & 0 + (1+c/(d-c))(d^{2}/2d), \text{ and} \\ W_{0} & = & 2q\bar{\beta} + 2(1-q) + 3(1+c/(d-c))(d^{2}/2d), \end{array} \tag{A4}$$

where the first term of  $(1+c/(d-c))(d^2/2d)$  is  $p^B$  and the second is the expected value of  $\delta$ . Welfare in  $R_{\lambda}$  is more complicated. Begin by noting the equilibrium described in Appendix Section A shows that

$$U^3(\lambda^*|\beta_3 = \bar{\beta}) = (1 + (c)/(d - c))(d^2/2d) + (1 - q)^2[\bar{\beta}] - \lambda^*$$
, and  $U^3(0|\beta_3 = 1) = (1 + (c)/(d - c))(d^2/2d)$ .

These two outcomes obtain with probabilities q and 1 – q, so Voter 3's expected welfare in  $R_{\lambda}$  is

$$U_{\lambda}^{3} = (1 + (c)/(d - c))(d^{2}/2d) + q(1 - q)^{2}[\bar{\beta}] - q\lambda^{*}.$$

For Voters 1 and 2 in  $R_{\lambda}$ , Section A shows

$$U^{1}(\lambda^{*}|\beta_{1} = \bar{\beta}) = (1 + (c)/(d - c))(d^{2}/2d) + \bar{\beta} - \lambda^{*}, \text{ and}$$

$$U^{1}(0|\beta_{1} = 1) = (1 + (c)/(d - c))(d^{2}/2d) + 1 - q + q^{2}.$$

Note that Voter 1 and 2's expected welfare is higher than Voter 3 because they obtain their desired policy of s = 1 if either or both are high-intensity.  $U^1(\lambda^*|\beta_1=\bar{\beta})$  occurs with probability q and  $U^1(0|\beta_1=1)$  1 – q, so Voter 1 and 2's expected welfare in  $R_{\lambda}$  is

$$U_{\lambda}^{1,2} = 2(1 + (c)/(d - c))(d^2/2d) + 2q[\bar{\beta} - \lambda^*] + 2(1 - q)[1 - q + q^2].$$

The welfare of Voters 1 and 2 is always lower in  $R_{\lambda}$  than in  $R_0$  if

$$\begin{split} &2(1+(c)/(d-c))(d^2/2d)+2q[\bar{\beta}-\lambda^*]+2(1-q)[1-q+q^2]\leq\\ &2q\bar{\beta}+2(1-q)+2(1+c/(d-c))(d^2/2d),\\ &-2q\lambda^*+2(1-q)[-q+q^2]\leq0,\\ &4q^2-2q^3-2q(1+\lambda^*)\leq0, \end{split}$$

which is true across the range of q because  $\lambda^* > 0$ .

Combining  $U_{\lambda}^{1,2}$  and  $U_{\lambda}^{3}$  yields  $W_{\lambda}$ 

= 
$$2(1 + (c)/(d - c))(d^2/2d) + 2q[\bar{\beta} - \lambda^*] + 2(1 - q)[1 - q + q^2] + (1 + (c)/(d - c))(d^2/2d) + q(1 - q)^2[\bar{\beta}] - q\lambda^*$$
  
=  $3(1 + (c)/(d - c))(d^2/2d) + [3q - 2q^2 + q^3][\bar{\beta}] - 3q\lambda^* + 2(1 - q)[1 - q + q^2].$  (A5)

Combining (A4) and (A5), society welfare is higher with costly political action when

$$\begin{split} &3(1+(c)/(d-c))(d^2/2d)+[3q-2q^2+q^3][\bar{\beta}]-3q\lambda^*+2(1-q)[1-q+q^2]\\ &\geq 3(1+c/(d-c))(d^2/2d)+2q\bar{\beta}+2(1-q),\\ &[3q-2q^2+q^3][\bar{\beta}]-3q\lambda^*+2-2q-2q+2q^2+2q^2-4q^3\geq 2q\bar{\beta}+2-2q,\\ &[1-2q+q^2][\bar{\beta}]-2+4q-4q^2\geq 3\lambda^*,\\ &\Rightarrow \frac{1}{3}(\bar{\beta}-2)(q-1)^2\geq \lambda^*. \end{split} \tag{A6}$$

Welfare is higher with political action per (A6) when  $\lambda^* \leq \frac{1}{3}(\bar{\beta}-2)(q-1)^2$ . As  $\lambda^*$  must be greater than zero, social welfare is only higher with action when high-intensity voters care sufficiently more than low-intensity voters ( $\bar{\beta} > 2$ ). The welfare benefits to political action are increasing in  $\bar{\beta}$  (derivative w.r.t.  $\bar{\beta} = (q-1)^2/3$ ) and q (derivative w.r.t.  $q = (\bar{\beta}-2)(2q-2)/3$  and  $\bar{\beta} > 2$ ).

# D Proof to Proposition 3

*Proof.* The proof to Proposition 3 (policy-motivated parties) is similar to that for Proposition 1. Voters do not differ in their ballot function with policy-motivated parties, so the expected vote share from Proposition 1 holds:

$$p^{A} = \left(\frac{\hat{\beta}_{1}(s_{A} - s_{B}) - c}{d - c} + \frac{\hat{\beta}_{2}(s_{A} - s_{B}) - c}{d - c} + \frac{\hat{\beta}_{3}(s_{B} - s_{A}) - c}{d - c}\right)/3$$
$$= \left([\hat{\beta}_{1} + \hat{\beta}_{2}][s_{A} - s_{B}] + \hat{\beta}_{3}(s_{B} - s_{A})\right)/3(d - c) - c/(d - c).$$

Party A continues to care only about collecting rents to office and so has the same best responses and follows the same strategy as in Proposition 1, proposing policy platform  $s_A = 0$  when  $\beta_3 \ge \beta_1 + \beta_2$ , else  $s_A = 1$ .

Party B has mixed incentives, being interested both in the rents to office and in the policy implemented by whichever party wins office. Given the payoffs for B in Table 1,

Party B's best response to  $s_A = 0$  is  $s_B = 0$  when

$$\begin{split} & \mathsf{U}^B(0|s_A=0) \geq \mathsf{U}^B(1|s_A=0), \\ & \mathsf{p}^A(0,0)\omega + [1-\mathsf{p}^A(0,0)][1+\omega] \geq \mathsf{p}^A(0,1)\omega + [1-\mathsf{p}^A(0,1)][1], \\ & (-\omega c)/(\mathsf{d}-c) + [1+(c)/(\mathsf{d}-c)][1+\omega] \geq \\ & \omega \big( -\hat{\beta_1} - \hat{\beta_2} + \hat{\beta_3} - 3c \big)/3(\mathsf{d}-c) + [1-\big( -\hat{\beta_1} - \hat{\beta_2} + \hat{\beta_3} - 3c \big)/3(\mathsf{d}-c)], \\ & \omega + (\omega c)/(\mathsf{d}-c) \geq [\omega-1][\hat{\beta_3} - \hat{\beta_1} - \hat{\beta_2}]/3(\mathsf{d}-c), \\ & \omega d \geq [\omega-1][\hat{\beta_3} - \hat{\beta_1} - \hat{\beta_2}]/3, \\ & \Rightarrow [\omega-1][\hat{\beta_3}] < 3\omega d + [\omega-1][\hat{\beta_1} + \hat{\beta_2}], \end{split}$$

where  $p^A(\cdot, \cdot)$  is the expected vote share of Party A given arguments  $s_A$  and  $s_B$ . Comparative statics depend on whether  $\omega$  is greater than or less than 1. Party B's best response to  $s_A = 1$  is  $s_B = 0$  when

$$\begin{split} & U^B(0|s_A=1) \geq U^B(1|s_A=1), \\ & [1-p^A(1,0)][1+\omega] \geq [1-p^A(1,1)][1], \\ & [1-\left(\hat{\beta}_1+\hat{\beta}_2-\hat{\beta}_3-3c\right)/3(d-c)][1+\omega] \geq 1+c/(d-c), \\ & 1+\omega+[1+\omega][3c/3(d-c)]-[1+\omega][\left(\hat{\beta}_1+\hat{\beta}_2-\hat{\beta}_3\right)/3(d-c)] \geq 1+c/(d-c), \\ & \omega d \geq [\omega+1][(\hat{\beta}_1+\hat{\beta}_2-\hat{\beta}_3)/3], \\ & \Rightarrow \hat{\beta}_3 \geq \hat{\beta}_1+\hat{\beta}_2-[3\omega d]/[\omega+1]. \end{split}$$

Party B is increasingly likely to propose  $s_B = 0$  as  $\omega$  increases.

The separating equilibrium of interest is when the two parties diverge in proposed policy, where Party A proposes  $s_A = 1$ , Party B proposes  $s_B = 0$ , and Voters 1, 2, and 3 choosing political action  $\lambda^*$  when high-intensity, zero when low-intensity. Suppose such an equilibrium exists. Then, it must be that

$$\begin{array}{lll} \mathsf{U}^{\mathsf{A}}(\mathsf{s}_{\mathsf{A}}=1|\vec{\lambda}) & \geq & \mathsf{U}^{\mathsf{A}}(\mathsf{s}_{\mathsf{A}}=0|\vec{\lambda}), \text{ and} \\ \\ \mathsf{U}^{\mathsf{B}}(\mathsf{s}_{\mathsf{B}}=0|\vec{\lambda},\omega) & \geq & \mathsf{U}^{\mathsf{B}}(\mathsf{s}_{\mathsf{B}}=1|\vec{\lambda},\omega), \text{ and} \\ \\ \mathsf{U}^{\mathsf{i}}(\lambda^{*}|\beta_{\mathsf{i}}=\bar{\beta}) & \geq & \mathsf{U}^{\mathsf{i}}(0|\beta_{\mathsf{i}}=\bar{\beta}), \text{ and} \\ \\ \mathsf{U}^{\mathsf{i}}(0|\beta_{\mathsf{i}}=1) & \geq & \mathsf{U}^{\mathsf{i}}(\lambda^{*}|\beta_{\mathsf{i}}=1). \end{array} \tag{A7}$$

The first inequality of (A7) holds when  $\hat{\beta}_3 \leq \hat{\beta}_1 + \hat{\beta}_2$  as in Proposition 1 (see Section A above). Neither Party A's incentives nor beliefs vary with  $\omega$ .

The second inequality of (A7) depends upon the values of  $\omega$  and  $\bar{\beta}$ . To establish that an equilibrium exists, assume that  $\omega$  = 0.2, c = -1, and d = 1. At these values, Party B's best response to  $s_A$  = 1 is  $s_B$  = 0 if  $\hat{\beta_3} \geq \hat{\beta_1} + \hat{\beta_2} - 0.5$  else  $s_B$  = 1; at  $\omega$  = 0.2,  $\bar{\beta}$  must be greater than 1.5 to support a divergent policy equilibrium.

The strategies of Parties A and B support the divergent policy equilibrium when  $\omega = 0.2$ 

<sup>&</sup>lt;sup>18</sup> Unlike Proposition 1, a separating equilibrium may exist with  $\bar{\beta}$  less than 2.

and  $\bar{\beta} \geq 1.5$ ,  $\hat{\beta_3} \leq \hat{\beta_1} + \hat{\beta_2}$ , and  $\hat{\beta_3} \geq \hat{\beta_1} + \hat{\beta_2} - 0.5$ . It now must be established that the strategies of Voters 1, 2, and 3 can also support this equilibrium. The equilibrium is supported when

$$\begin{array}{lll} \mathsf{U}^{\{1,2\}}(0|\beta_{\{1,2\}}=1) & \geq & \mathsf{U}^{\{1,2\}}(\lambda^*|\beta_{\{1,2\}}=1) \text{, and} \\ \\ \mathsf{U}^{\{1,2\}}(\lambda^*|\beta_{\{1,2\}}=\bar{\beta}) & \geq & \mathsf{U}^{\{1,2\}}(0|\beta_{\{1,2\}}=\bar{\beta}) \text{, and} \\ \\ \mathsf{U}^{3}(\lambda^*|\beta_3=\bar{\beta}) & \geq & \mathsf{U}^{3}(0|\beta_3=\bar{\beta}) \text{, and} \\ \\ \mathsf{U}^{3}(0|\beta_3=1) & \geq & \mathsf{U}^{3}(\lambda^*|\beta_3=1). \end{array} \tag{A8}$$

Starting with the first inequality of (A8), Voters 1 and 2 have the same payoffs, beliefs, and actions, so the proof for Voter 1 is the same as for 2. Voter 1 takes expectations over the strategies of Voters 2 and 3, who in the separating equilibrium play  $\lambda_2 = \lambda_3 = \lambda^*$  with probability  $q^2$ ,  $\lambda_2 = \lambda_3 = 0$  with probability  $(1-q)^2$ ,  $\lambda_2 = 0$ ,  $\lambda_3 = \lambda^*$  with probability  $(q-q^2)$ , and  $\lambda_2 = \lambda^*$ ,  $\lambda_3 = 0$  with probability  $(q-q^2)$ . Voter 1's expected benefit from  $\lambda_1 = 0$  with  $\beta_1 = 1$ ,  $\omega = 0.2$ , c = -1, and d = 1 (E[ $\delta$ |c = -1, d = 1] = 0) is

$$\begin{split} U^1(0|\beta_1=1) &= q^2[p_A(1,1)+1-p_A(1,1)](1)+(1-q)^2[p_A(1,1)+1-p_A(1,1)](1)+\\ & (q-q^2)[p_A(1,0)(1)+[1-p_A(1,0)](0)]+(q-q^2)[p_A(1,1)+1-p_A(1,1)](1),\\ &= q^2+(1-q)^2+(q-q^2)([\hat{\beta}_1+\hat{\beta}_2-\hat{\beta}_3]/3(d-c)-c/[d-c])+(q-q^2),\\ &= q^2+1-2q+q^2-(q-q^2)\bar{\beta}/6+5(q-q^2)/6+q-q^2,\\ &= 1+(1+\bar{\beta})q^2/6-(1+\bar{\beta})q/6. \end{split}$$

Voter 1's expected benefit from  $\lambda_1$  =  $\lambda^*$  with  $\beta_1$  = 1 is

$$\begin{split} U^1(\lambda^*|\beta_1=1) &= q^2[p_A(1,1)+1-p_A(1,1)](1)+(1-q)^2[p_A(1,1)+1-p_A(1,1)](1)+\\ & (q-q^2)[p_A(1,1)+1-p_A(1,1)](1)+(q-q^2)[p_A(1,1)+1-p_A(1,1)](1)-\lambda^*,\\ &= 1-\lambda^*. \end{split}$$

Voters 1 and 2 support the separating equilibrium when low-intensity when  $U^1(0|\beta_1 = 1) \ge U^1(\lambda^*|\beta_1 = 1)$ , or

$$1 + (1 + \bar{\beta})q^{2}/6 - (1 + \bar{\beta})q/6 \ge 1 - \lambda^{*},$$
  
 
$$\Rightarrow (1 + \bar{\beta})q/6 - (1 + \bar{\beta})q^{2}/6 \le \lambda^{*}.$$

Continuing with the second inequality in (A8), Voter 1 takes expectations over the strategies of Voters 2 and 3, who play  $\lambda_2 = \lambda_3 = \lambda^*$  with probability  $q^2$ ,  $\lambda_2 = \lambda_3 = 0$  with probability  $(1-q)^2$ ,  $\lambda_2 = 0$ ,  $\lambda_3 = \lambda^*$  with probability  $(q-q^2)$ , and  $\lambda_2 = \lambda^*$ ,  $\lambda_3 = 0$  with probability  $(q-q^2)$ . Voter 1's expected benefit from  $\lambda_1 = 0$  with  $\beta_1 = \bar{\beta}$ ,  $\omega = 0.2$ , c = -1,

and d = 1 is

$$\begin{array}{lll} \mathsf{U}^1(0|\beta_1=\bar{\beta}) &=& \mathsf{q}^2[\mathsf{p}_A(1,1)+1-\mathsf{p}_A(1,1)](\bar{\beta})+(1-\mathsf{q})^2[\mathsf{p}_A(1,1)+1-\mathsf{p}_A(1,1)](\bar{\beta})+\\ && (\mathsf{q}-\mathsf{q}^2)[\mathsf{p}_A(1,0)(\bar{\beta})+[1-\mathsf{p}_A(1,0)](0)]+(\mathsf{q}-\mathsf{q}^2)[\mathsf{p}_A(1,1)+1-\mathsf{p}_A(1,1)](\bar{\beta}),\\ &=& \bar{\beta}\mathsf{q}^2+\bar{\beta}(1-\mathsf{q})^2+\bar{\beta}(\mathsf{q}-\mathsf{q}^2)([\hat{\beta}_1+\hat{\beta}_2-\hat{\beta}_3]/3(\mathsf{d}-\mathsf{c})-\mathsf{c}/[\mathsf{d}-\mathsf{c}])+\bar{\beta}(\mathsf{q}-\mathsf{q}^2),\\ && \mathsf{Note} : \mathsf{Even} \ \mathsf{though} \ \lambda_1=0, \ \mathsf{at} \ \mathsf{time} \ \mathsf{of} \ \mathsf{ballot}, \ \mathsf{true} \ \beta_1=\bar{\beta} \ \mathsf{generates} \ \mathsf{p}^A.\\ &=& \bar{\beta}\mathsf{q}^2+\bar{\beta}(1-\mathsf{q})^2+\bar{\beta}(\mathsf{q}-\mathsf{q}^2)([\bar{\beta}+1-\bar{\beta}+3]/6)+\bar{\beta}(\mathsf{q}-\mathsf{q}^2),\\ &=& \bar{\beta}-\bar{\beta}\mathsf{q}/3+\bar{\beta}\mathsf{q}^2/3. \end{array}$$

Voter 1's expected benefit from  $\lambda_1 = \lambda^*$  with  $\beta_1 = \bar{\beta}$  is

$$\begin{array}{lll} U^{1}(\lambda^{*}|\beta_{1}=\bar{\beta}) & = & q^{2}[p_{A}(1,1)+1-p_{A}(1,1)](\bar{\beta})+(1-q)^{2}[p_{A}(1,1)+1-p_{A}(1,1)](\bar{\beta})+\\ & & (q-q^{2})[p_{A}(1,1)+1-p_{A}(1,1)](\bar{\beta})+(q-q^{2})[p_{A}(1,1)+1-p_{A}(1,1)](\bar{\beta})-\lambda^{*},\\ & = & \bar{\beta}-\lambda^{*}. \end{array}$$

Voters 1 and 2 support the separating equilibrium when high-intensity and when  $U^1(\lambda^*|\beta_1 = \bar{\beta}) \ge U^1(0|\beta_1 = \bar{\beta})$ , or

$$\bar{\beta} - \lambda^* \ge \bar{\beta} - \bar{\beta}q/3 + \bar{\beta}q^2/3,$$
 $\bar{\beta}q/3 - \bar{\beta}q^2/3 \ge \lambda^*,$ 

$$\Rightarrow \lambda^* \le \bar{\beta}q/3 - \bar{\beta}q^2/3.$$

Combining the two requirements for Voters 1 and 2, the separating equilibrium can be supported when

$$(1 + \bar{\beta})q/6 - (1 + \bar{\beta})q^2/6 \le \lambda^* \le \bar{\beta}q/3 - \bar{\beta}q^2/3.$$

This inequality holds for all  $q \in [0, 1]$ .

Finally, the strategies of Voter 3 must support the equilibrium. For the third and fourth inequalities of (A8), Voter 3 takes expectations over the strategies of Voters 1 and 2, who in the separating equilibrium play  $\lambda_1=\lambda_2=\lambda^*$  with probability  $q^2$ , play  $\lambda_1=\lambda_2=0$  with probability  $(1-q)^2$ , and play  $\lambda_1+\lambda_2=\lambda^*+0$  with probability  $(1-q)^2=(1-q)^2=2(q-q^2)$  where which of Voter 1 or 2 plays  $\lambda^*$  is irrelevant to Voter 3. Voter 3's expected benefit from  $\lambda_3=\lambda^*$  with  $\beta_3=\bar{\beta}$ ,  $\omega=0.2$ , c=-1, and d=1 (E( $\delta$ ) = 0 when c=-1 and d=1) is

$$\begin{split} \mathsf{U}^3(\lambda^*|\beta_3 = \bar{\beta}) &= \mathsf{q}^2(0) + (1-\mathsf{q})^2[\mathsf{p}_\mathsf{A}(1,0)](0) + [1-\mathsf{p}_\mathsf{A}(1,0)]\bar{\beta} + \\ &\quad 2(\mathsf{q} - \mathsf{q}^2)[\mathsf{p}_\mathsf{A}(1,1) + 1-\mathsf{p}_\mathsf{A}(1,1)](0) - \lambda^*, \\ &= (1-\mathsf{q})^2[1-([\hat{\beta}_1+\hat{\beta}_2-\hat{\beta}_3+3]/6)]\bar{\beta} - \lambda^*, \\ &= (1-\mathsf{q})^2[(\bar{\beta}+\bar{\beta}^2)/6] - \lambda^*. \end{split}$$

When Voter 3 chooses political action  $\lambda_3=0$ , the parties always propose  $s^*=1$  when  $\omega=0.2$ , and so  $U^3(0|\beta_3=\bar{\beta})=0$ . Voter 3 supports the separating equilibrium when

$$\begin{split} \mathsf{U}^{3}(\lambda^{*}|\beta_{3} = \bar{\beta}) \geq \mathsf{U}^{3}(0|\beta_{3} = \bar{\beta}), \\ (1 - \mathsf{q})^{2}[(\bar{\beta} + \bar{\beta}^{2})/6] - \lambda^{*} \geq 0, \\ (1 - 2\mathsf{q} + \mathsf{q}^{2})[(\bar{\beta} + \bar{\beta}^{2})/6] \geq \lambda^{*}, \\ \Rightarrow \lambda^{*} < (\bar{\beta} + \bar{\beta}^{2})/6 - \mathsf{q}(\bar{\beta} + \bar{\beta}^{2})/3 + \mathsf{q}^{2}(\bar{\beta} + \bar{\beta}^{2})/6. \end{split}$$

Voter 3's expected benefit from  $\lambda_3$  = 0 with  $\beta_3$  = 1 is zero because, as before, the parties both propose s\* = 1 when  $\omega$  = 0.2. Voter 3's expected benefit from  $\lambda_3$  =  $\lambda^*$  with  $\beta_3$  = 1 is

Voters 3 support the separating equilibrium when low-intensity when  $U^3(0|\beta_3=1) \ge U^3(\lambda^*|\beta_3=1)$ , or

$$0 \ge (1 - q)^2/3 - \lambda^*,$$
  
 $\Rightarrow \lambda^* > (1 - q)^2/3.$ 

Combining the two requirements for Voter 3, the separating equilibrium can be supported when

$$(1-q)^2/3 \le \lambda^* \le (\bar{\beta} + \bar{\beta}^2)/6 - q(\bar{\beta} + \bar{\beta}^2)/3 + q^2(\bar{\beta} + \bar{\beta}^2)/6.$$

This inequality holds for all  $q \in [0, 1]$ .

Thus, the divergent policy separating equilibrium holds at  $\omega$  = 0.2 when  $\bar{\beta} \geq$  1.5 and

$$(1-q)^2/3 \leq \lambda^* \leq (\bar{\beta}+\bar{\beta}^2)/6 - q(\bar{\beta}+\bar{\beta}^2)/3 + q^2(\bar{\beta}+\bar{\beta}^2)/6, \text{ and } (1+\bar{\beta})q/6 - (1+\bar{\beta})q^2/6 \leq \lambda^* \leq \bar{\beta}q/3 - \bar{\beta}q^2/3.$$

# E Policy-motivated parties can increase costly action

This section provides a corollary to Proposition 3 that shows policy-motivated parties can increase the magnitude of political action chosen by high-intensity voters relative to office-motivated parties. I show that the lower bound on the range of supported political action  $\lambda^*$  in the equilibrium to Proposition 1 can be lower than the lower bound on  $\lambda^*$  in the equilibrium to Proposition 3. This means that magnitudes of action that can support separation with parties motivated only by election are too small to support separation when Party B has policy motivation. Thus, voters with high-intensity preferences choose more costly action in equilibrium when Party B has policy motivation.

*Proof.* Begin by noting the lower bound of  $\lambda^*$  supported in the divergent policy separating equilibrium requires combining the two lower bounds at the end of Section D. The lower

bound for Voters 1 and 2 is greater than the lower bound for Voter 3, and thus a binding lower bound for the full separating equilibrium, when

$$(1 + \bar{\beta})(q - q^2)/6 \ge (1 - q)^2/3$$
,

else the lower bound for Voter 3 is greater than the lower bound for Voters 1 and 2.

Next, the lower bound from Proposition 3 is greater than the lower bound from Proposition 1 when

$$(1+\bar{\beta})(q-q^2)/6 \ge q^2-2q+1, \text{ and } (1+\bar{\beta})(q-q^2)/6 \ge (1-q)^2/3, \text{ and } q<0.5$$
(A9) or

$$(1-q)^2/3 \ge q^2 - 2q + 1$$
, and  $(1+\bar{\beta})(q-q^2)/6 \le (1-q)^2/3$ , and  $q < 0.5$ , (A10)

$$(1 + \bar{\beta})(q - q^2)/6 \ge q - q^2$$
, and  $(1 + \bar{\beta})(q - q^2)/6 \ge (1 - q)^2/3$ , and  $q > 0.5$ , (A11)

$$(1-q)^2/3 \ge q-q^2$$
, and  $(1+\bar{\beta})(q-q^2)/6 \le (1-q)^2/3$ , and  $q>0.5$ . (A12)

If any of the four conditions A9 to A12 hold at some value of  $\bar{\beta}$  and q, the lower bound of  $\lambda^*$  from Proposition 1 is lower than the lower bound from Proposition 3 and separation requires more costly political action in the latter. Consider condition A11. The first inequality, dividing both sides by  $(q-q^2)$ , holds when  $(1+\bar{\beta})/6 \geq 1$  or  $\bar{\beta} \geq 5$  for any value of q, consistent with the third inequality.<sup>19</sup> The second condition holds at  $\bar{\beta}=5$  when  $(q-q^2) \geq (1-q)^2/3$ , which a bit of algebra shows holds when  $q \geq 1/4$ , which is consistent with the third inequality.

Therefore, the inequalities of condition A11 hold and it is proved that the introduction of policy motives for Party B can increase the magnitude of costly political action chosen by voters even with the preferences of voters have not changed.

### F Costly political actions with an even split on policy

To show that an equilibrium of costly political actions does not depend on minority and majority, I present a version of the model with an even split in the electorate. Consider, contrary to the assumptions on  $\vec{\tau}$  in the main model, a split in electorate support for s=0 and s=1 by setting  $\beta_2$  to zero, common knowledge. The electorate is divided into one third preferring s=1, one third voting solely on the election shock ( $\beta_2=0$ ), and one third preferring s=0. Intensities  $\beta_1$  and  $\beta_3$  remain private knowledge up to q. Assume that both parties are strictly office-motivated ( $\omega=0$ ) and that the bounds on the election shock are c=-1 and d=1. These assumptions are relaxed subsequently, but this version of the game presents the basic intuition of the model.

The parties' estimates of the intensities of Voters 1 and 3 after observing  $\lambda_1$  and  $\lambda_3$  are  $\hat{\beta_1}$  and  $\hat{\beta_3}$ . Given the probability that each voter prefers Party A over B from Eq. (??) and the distribution of  $\delta$ , the (parties') expected vote share for Party A is

<sup>&</sup>lt;sup>19</sup> Recall that the proof to Proposition 3 assumed  $\omega$  = 0.2, so  $\bar{\beta}$  = 5 should not be ascribed any special meaning.

$$p^{A} = \left(\frac{\hat{\beta}_{1}(s_{A} - s_{B}) + 1}{2} + \frac{1}{2} + \frac{\hat{\beta}_{3}(s_{B} - s_{A}) + 1}{2}\right)/3$$

$$= \left(\hat{\beta}_{1}(s_{A} - s_{B}) + \hat{\beta}_{3}(s_{B} - s_{A})\right)/6 + 1/2,$$
(A13)

with B's expected vote share  $p^B = 1 - p^A$ . With Eq. A13, the best response for both parties is to set  $s_p = 0$  when  $\hat{\beta_1} \leq \hat{\beta_3}$ , else  $s_p = 1$  (proof in Appendix Section ??). Without policy motivation for Party B and with equal shares of type-0 and type-1 voters, the two parties converge to offer the policy they expect to yield the high-intensity voter(s). When they believe Voter 1 and Voter 3 have the same intensity for policy, high or low, either policy platform pair  $s_A = s_B = 0$  or  $s_A = s_B = 1$  can be supported in equilibrium. However, when they believe Voter 1 (Voter 3) is the only high-intensity voter, they propose policy s = 1 (s = 0). This provides an incentive for voters to communicate to parties that they are high-intensity types and leads to the following formal description of the equilibrium.

**Proposition 4** (Choice of costly action with a split electorate). With an even split in the electorate between those on two sides of a policy, a separating equilibrium exists where high-intensity voters ( $\beta_i = \bar{\beta}$ ) choose political action  $\lambda_i = \lambda^* > 0$  and low-intensity voters ( $\beta_i = 1$ ) choose  $\lambda_i = 0$ , revealing that voter(s) with  $\lambda_i = \lambda^*$  are high intensity. The two parties converge to offer the policy preferred by the voter(s) with political action  $\lambda^*$ .

*Proof.* Begin by specifying the expected vote share for the two parties. Let the parties' estimates of the policy intensity of Voters 1 and 3 after observing  $\lambda_1$  and  $\lambda_3$  be  $\hat{\beta}_1$  and  $\hat{\beta}_3$ . Given the probability that each voter prefers Party A over B from Eq. (??) and the uniform distribution on  $\delta$ , the expected vote share for Party A is

$$p^{A} = \left(\frac{\hat{\beta}_{1}(s_{A} - s_{B}) + 1}{2} + \frac{1}{2} + \frac{\hat{\beta}_{3}(s_{B} - s_{A}) + 1}{2}\right)/3$$
$$= \left(\hat{\beta}_{1}(s_{A} - s_{B}) + \hat{\beta}_{3}(s_{B} - s_{A})\right)/6 + 1/2$$

with  $p^{B} = 1 - p^{A}$ .

To identify best responses for the parties, note that when  $\omega=0$  the expected benefit for each party is the expected probability they are elected times the rents to office (normalized to one). Given that optimization problem, and suppressing the hats indicating posterior beliefs about  $\beta_1$  and  $\beta_3$  for convenience, Party A's best response to  $s_B=0$  is  $s_A=0$  when

$$U^{A}(0|s_{B} = 0) \geq U^{A}(1|s_{B} = 0),$$
  
 $1/2 \geq (1/6)(\beta_{1} - \beta_{3}) + 1/2 \Rightarrow \beta_{3} \geq \beta_{1}.$ 

Party A's best response to  $s_B = 1$  is  $s_A = 0$  when

$$U^{A}(0|s_{B} = 1) \geq U^{A}(1|s_{B} = 1),$$
  
 $(1/6)(-\beta_{1} + \beta_{3}) + 1/2 \geq 1/2 \Rightarrow \beta_{3} \geq \beta_{1}.$ 

Likewise, Party B's best response to  $s_A = 0$  is  $s_B = 0$  when

$$\begin{array}{rcl} \mathsf{U}^{\mathsf{B}}(0|\mathsf{s}_{\mathsf{A}}=0) & \geq & \mathsf{U}^{\mathsf{B}}(1|\mathsf{s}_{\mathsf{A}}=0), \\ 1-1/2 & \geq & 1-(1/6)(-\beta_1+\beta_3)-1/2, \\ 0 & \geq & (1/6)(\beta_1-\beta_3) \Rightarrow \beta_3 \geq \beta_1. \end{array}$$

Party B's best response to  $s_A = 1$  is  $s_B = 0$  when

$$\begin{array}{rcl} \mathsf{U}^\mathsf{B}(0|\mathsf{s}_\mathsf{A}=1) & \geq & \mathsf{U}^\mathsf{B}(1|\mathsf{s}_\mathsf{A}=1), \\ 1-(1/6)(\beta_1-\beta_3)-1/2 & \geq & 1-1/2, \\ (1/6)(-\beta_1+\beta_3) & \geq & 0 \Rightarrow \beta_3 \geq \beta_1. \end{array}$$

Suppose a separating equilibrium exists where when  $\beta_i = \bar{\beta}$ ,  $\lambda_i = \lambda^* > 0$ , and when  $\beta_i = 1$ ,  $\lambda_i = 0$ . Then, it must also be that

$$\begin{array}{lll} \mathsf{U}^\mathsf{A}(\mathsf{s}_\mathsf{A} = \tau_\mathsf{i} | \lambda_\mathsf{i} = \lambda^*) & \geq & \mathsf{U}^\mathsf{A}(\mathsf{s}_\mathsf{A} = 1 - \tau_\mathsf{i} | \lambda_\mathsf{i} = \lambda^*), \text{ and} \\ \mathsf{U}^\mathsf{B}(\mathsf{s}_\mathsf{B} = \tau_\mathsf{i} | \lambda_\mathsf{i} = \lambda^*) & \geq & \mathsf{U}^\mathsf{B}(\mathsf{s}_\mathsf{B} = 1 - \tau_\mathsf{i} | \lambda_\mathsf{i} = \lambda^*), \text{ and} \\ \mathsf{U}^\mathsf{i}(\lambda^* | \beta_\mathsf{i} = \bar{\beta}) & \geq & \mathsf{U}^\mathsf{i}(\mathsf{0} | \beta_\mathsf{i} = \bar{\beta}), \text{ and} \\ \mathsf{U}^\mathsf{i}(\mathsf{0} | \beta_\mathsf{i} = 1) & \geq & \mathsf{U}^\mathsf{i}(\lambda^* | \beta_\mathsf{i} = 1). \end{array}$$

The first two inequalities hold. When one voter is revealed high-intensity through choice of political action, the parties' best responses are to set policy at that voter's preference. When both voters reveal high- or low-intensity through choice of action, the parties are indifferent over choice of policy.

Continuing with the third and fourth inequalities for Voter i, consider Voter 3 with  $\tau_3$  = 0 but with symmetry to Voter 1. Voter 3 expects Voter 1 in the separating equilibrium to be playing  $\lambda_1$  =  $\lambda^*$  with probability q and  $\lambda_1$  = 0 with probability 1 – q. Voter 3's expected benefit from  $\lambda_3$  =  $\lambda^*$  and  $\lambda_1$  = 0 is

$$U^{3}(\lambda^{*}|\beta_{3}) = qU^{3}(\lambda^{*}|\beta_{1} = \bar{\beta}, \beta_{3}) + (1 - q)U^{3}(\lambda^{*}|\beta_{1} = 1, \beta_{3}),$$

$$U^{3}(0|\beta_{3}) = qU^{3}(0|\beta_{1} = \bar{\beta}, \beta_{3}) + (1 - q)U^{3}(0|\beta_{1} = 1, \beta_{3}).$$

Assume that the parties randomize s when they are indifferent, i.e. when  $\hat{\beta}_1 = \hat{\beta}_3$ .<sup>20</sup>

<sup>&</sup>lt;sup>20</sup> In Proposition 1 and 3, this assumption is unncessary.

Then, given the party best response functions to  $\lambda_1$  and  $\lambda_3$ ,

$$\begin{array}{llll} \mathsf{U}^{3}(\lambda^{*}|\beta_{1}=\bar{\beta},\beta_{3}=\bar{\beta}) & = & (1/2)(\bar{\beta}/2) + (1/2)(\bar{\beta}/2 + \delta) - \lambda^{*}, \\ \mathsf{U}^{3}(\lambda^{*}|\beta_{1}=1,\beta_{3}=\bar{\beta}) & = & (1/2)\bar{\beta} + (1/2)(\bar{\beta}+\delta) - \lambda^{*}, \\ \mathsf{U}^{3}(0|\beta_{1}=\bar{\beta},\beta_{3}=\bar{\beta}) & = & (1/2)\delta - 0, \\ \mathsf{U}^{3}(0|\beta_{1}=1,\beta_{3}=\bar{\beta}) & = & (1/2)(\bar{\beta}/2) + (1/2)(\bar{\beta}/2 + \delta) - 0, \\ \\ \mathsf{U}^{3}(\lambda^{*}|\beta_{1}=\bar{\beta},\beta_{3}=1) & = & (1/2)(1/2) + (1/2)(1/2 + \delta) - \lambda^{*}, \\ \mathsf{U}^{3}(\lambda^{*}|\beta_{1}=1,\beta_{3}=1) & = & (1/2)(1) + (1/2)(1 + \delta) - \lambda^{*}, \\ \mathsf{U}^{3}(0|\beta_{1}=\bar{\beta},\beta_{3}=1) & = & (1/2)(0) + (1/2)\delta - 0, \\ \mathsf{U}^{3}(0|\beta_{1}=1,\beta_{3}=1) & = & (1/2)(1/2) + (1/2)(1/2 + \delta) - 0. \end{array}$$

The first inequality  $U^3(\lambda^*|\beta_3 = \bar{\beta}) \ge U^3(0|\beta_3 = \bar{\beta})$  holds when

$$\begin{split} & \mathsf{q}[(1/2)(\bar{\beta}/2) + (1/2)(\bar{\beta}/2 + \delta) - \lambda^*] + (1 - \mathsf{q})[(1/2)\bar{\beta} + (1/2)(\bar{\beta} + \delta) - \lambda^*] \geq \\ & \mathsf{q}[(1/2)\delta] + (1 - \mathsf{q})[(1/2)(\bar{\beta}/2) + (1/2)(\bar{\beta}/2 + \delta)], \\ & \mathsf{q}[\bar{\beta}/2] + (1 - \mathsf{q})[\bar{\beta}] - \lambda^* \geq (1 - \mathsf{q})[\bar{\beta}/2], \\ & \lambda^* \leq \bar{\beta}/2. \end{split}$$

The second inequality  $U^3(0|\beta_3 = 1) \ge U^3(\lambda^*|\beta_3 = 1)$  holds when

$$q[(1/2)\delta] + (1 - q)[(1/2)(1/2) + (1/2)(1/2 + \delta)] \ge q[(1/2)(1/2) + (1/2)(1/2 + \delta) - \lambda^*] + (1 - q)[(1/2)(1) + (1/2)(1 + \delta) - \lambda^*],$$

$$\lambda^* \ge q[(1/2)] + 1 - q - (1/2) + q/2,$$

$$1/2 \le \lambda^*.$$

Therefore, a separating equilibrium exists when  $1/2 \le \lambda^* \le \bar{\beta}/2$ .