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In [ ]: '''
Suppose that there are five drones parked at five marked positions. All drones take off at the same time, fly around and park at the marked positions randomly (i.e., all parking arrangements are equally likely). What is the expected number of drones parking at their original position? How about 10 drones with 10 positions? Conduct both theoretical analysis and simulation to find out the expectation.

Submit your Matlab or Python code and results.

Hint: You are asked to find out the expectation only, not the distribution. Let  $X_i$  be a Bernoulli random variable.  $X_i=1$  represents drone  $i$  parks at its original position and  $X_i=0$  otherwise. Drone  $i$  takes one of the five positions with equal probability.  $X_i$  and  $X_j$  are not independent for  $i \neq j$ . Let  $X$  be the number of drones parking at their original position. Then  $X=X_1+\dots+X_5$ . Note here  $X$  is NOT binomial since  $X_1, \dots, X_5$  are not independent.
'''
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Out[ ]: '\nSuppose that there are five drones parked at five marked positions. All drones take off at the same time, fly around and park at the marked positions randomly (i.e., all parking arrangements are equally likely).\nWhat is the expected number of drones parking at their original position? How about 10 drones with 10 positions?\n\nConduct both theoretical analysis and simulation to find out the expectation.\n\nSubmit your Matlab or Python code and results.\n\nHint: You are asked to find out the expectation only, not the distribution. Let  $X_i$  be a Bernoulli random variable.  $X_i=1$  represents drone  $i$  parks at its original position and  $X_i=0$  otherwise. Drone  $i$  takes one of the five positions with equal probability.  $X_i$  and  $X_j$  are not independent for  $i \neq j$ . Let  $X$  be the number of drones parking at their original position. Then  $X=X_1+\dots+X_5$ . Note here  $X$  is NOT binomial since  $X_1, \dots, X_5$  are not independent.\n'
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In [ ]: import numpy as np
import matplotlib.pyplot as plt
import random
def simulate_drones(num_drones, num_simulations):
    counts = []
    for _ in range(num_simulations):
        positions = list(range(num_drones))
        random.shuffle(positions)
        count = sum(i == pos for i, pos in enumerate(positions))
        counts.append(count)
    return counts

n_drones_5 = 5
n_drones_10 = 10
n_simulations = 1000000

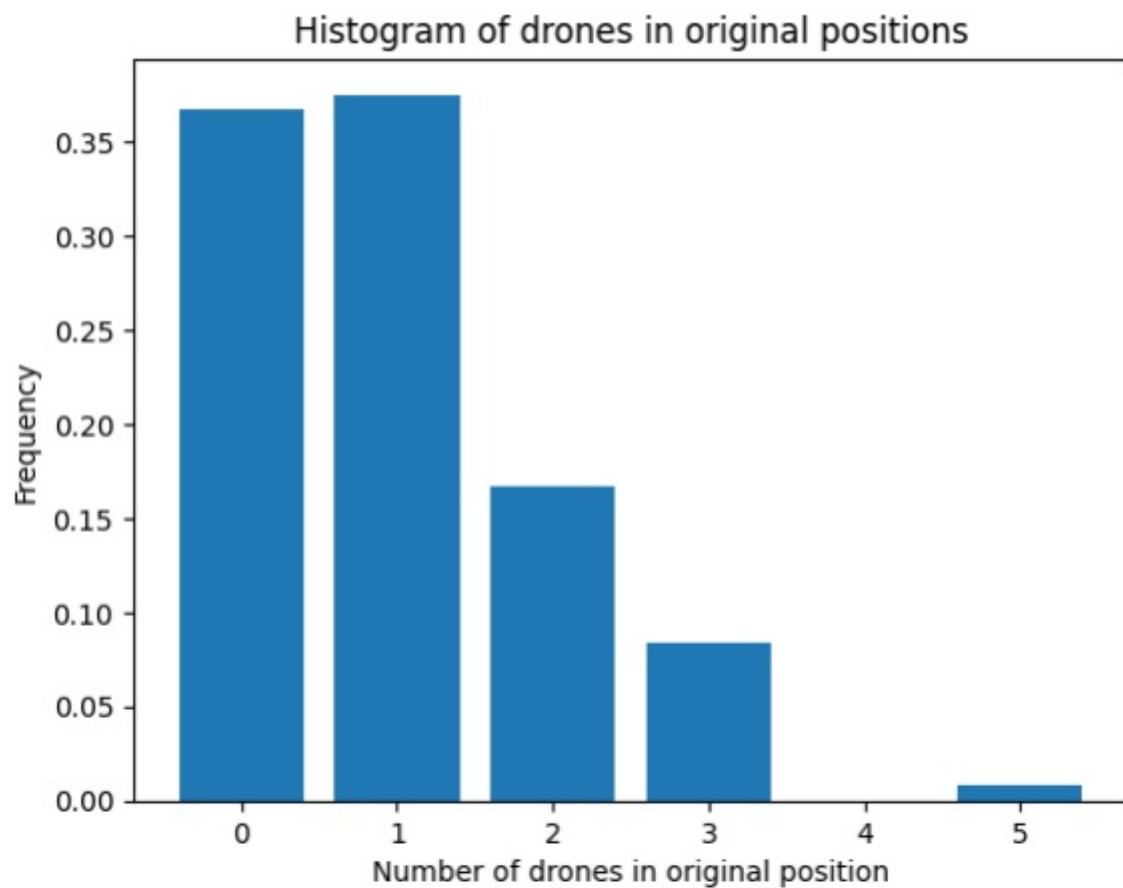
expected_5 = 1.0 * sum(simulate_drones(n_drones_5, n_simulations)) / n_simulations
expected_10 = 1.0 * sum(simulate_drones(n_drones_10, n_simulations)) / n_simulations

print(f"Expected number of drones in original position for 5 drones: {expected_5}")
print(f"Expected number of drones in original position for 10 drones: {expected_10}")
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Expected number of drones in original position for 5 drones: 1.000582
Expected number of drones in original position for 10 drones: 1.000455
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In [ ]: num_drones = 5
#num_simulations = 1000000
counts = simulate_drones(num_drones, num_simulations)

plt.hist(counts, bins=range(num_drones + 2), align='left', rwidth=0.8, density=True)
plt.xlabel('Number of drones in original position')
plt.ylabel('Frequency')
plt.title('Histogram of drones in original positions')
plt.show()
```



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In [ ]: num_drones = 10
#num_simulations = 1000000
counts = simulate_drones(num_drones, num_simulations)

plt.hist(counts, bins=range(num_drones + 2), align='left', rwidth=0.8, density=True)
plt.xlabel('Number of drones in original position')
plt.ylabel('Frequency')
plt.title('Histogram of drones in original positions')
plt.show()
```

Histogram of drones in original positions

