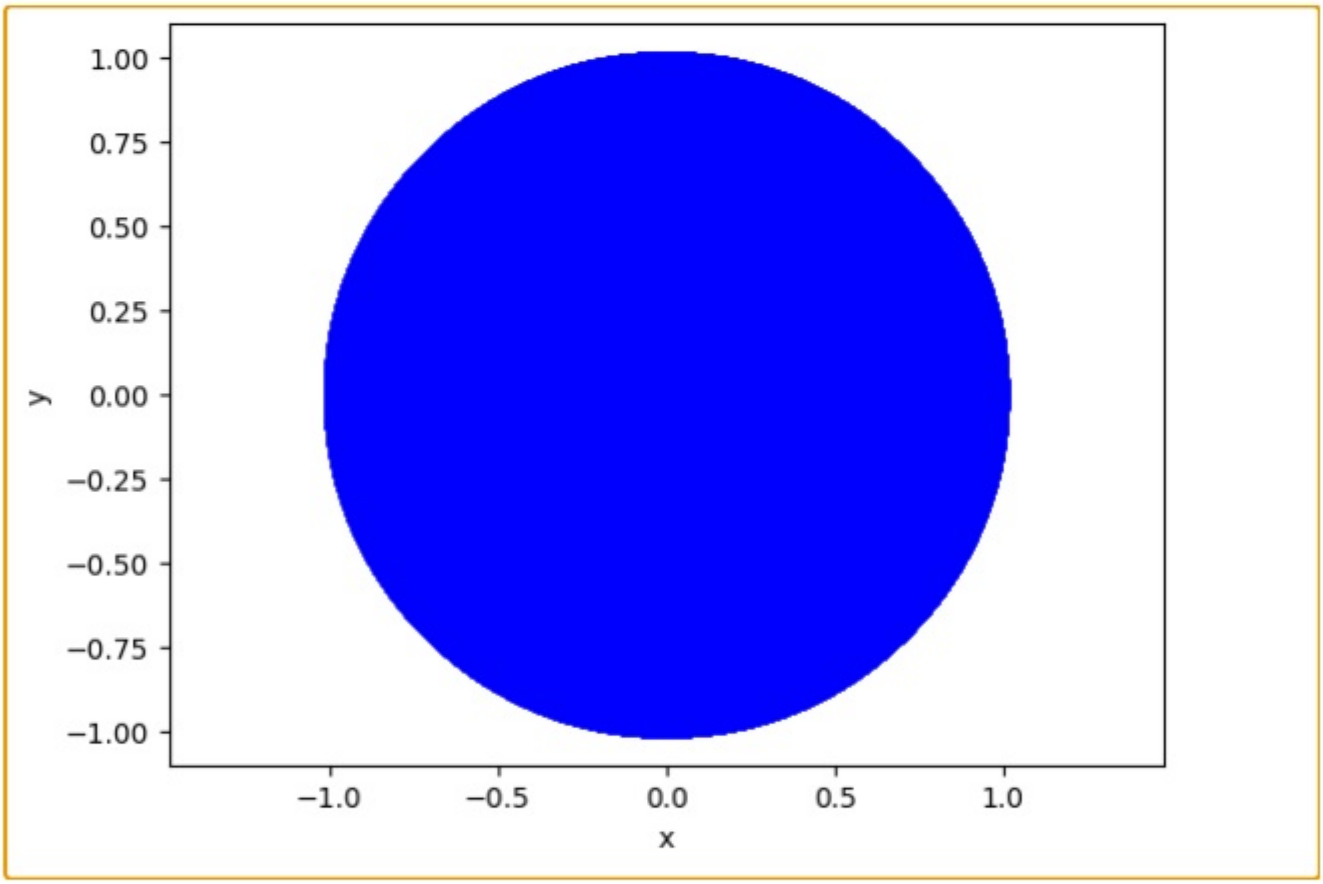
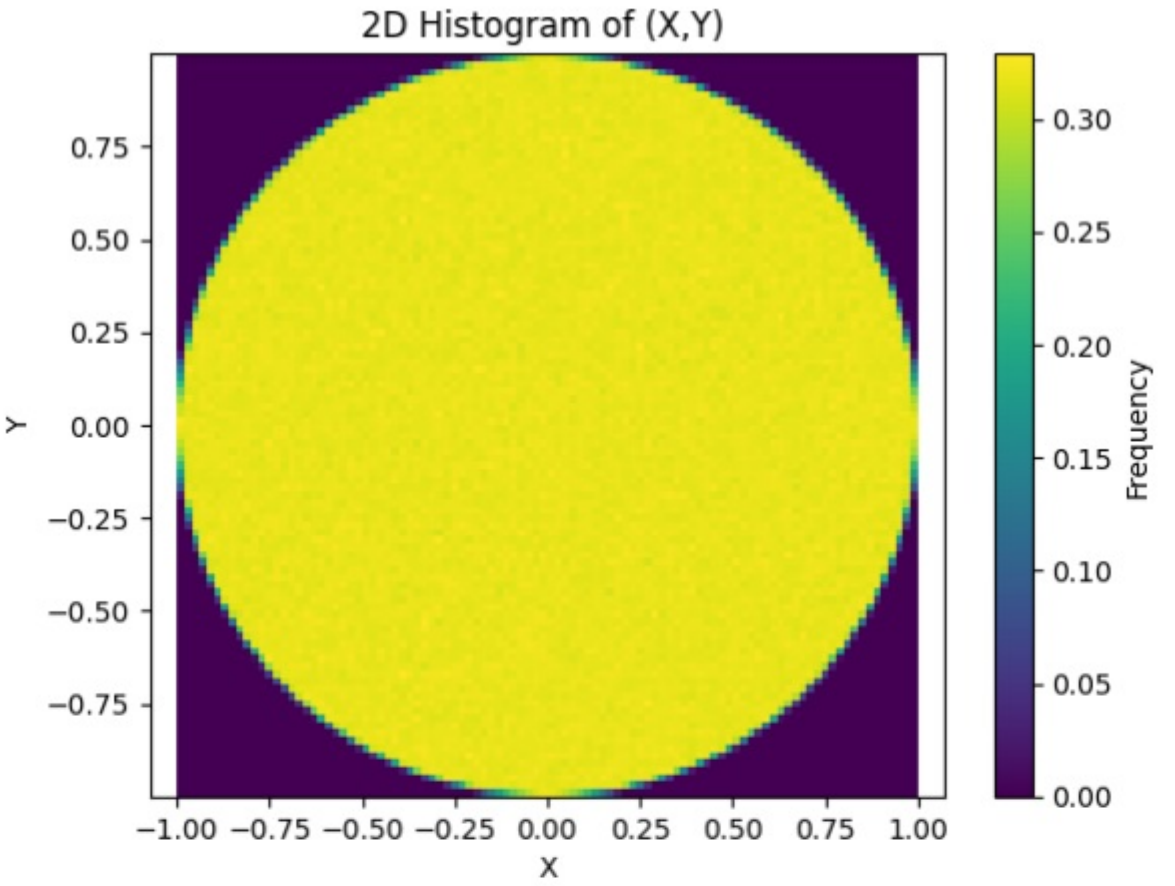
In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt

# Number of samples
num_samples = 100000000

# Generate random angles and radii
theta = 2 * np.pi * np.random.rand(num_samples)
r = np.sqrt(np.random.rand(num_samples))

# Calculate the x and y coordinates of the samples
x = r * np.cos(theta)
y = r * np.sin(theta)
# Plot the samples
plt.plot(x, y, 'b.')
plt.axis('equal')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

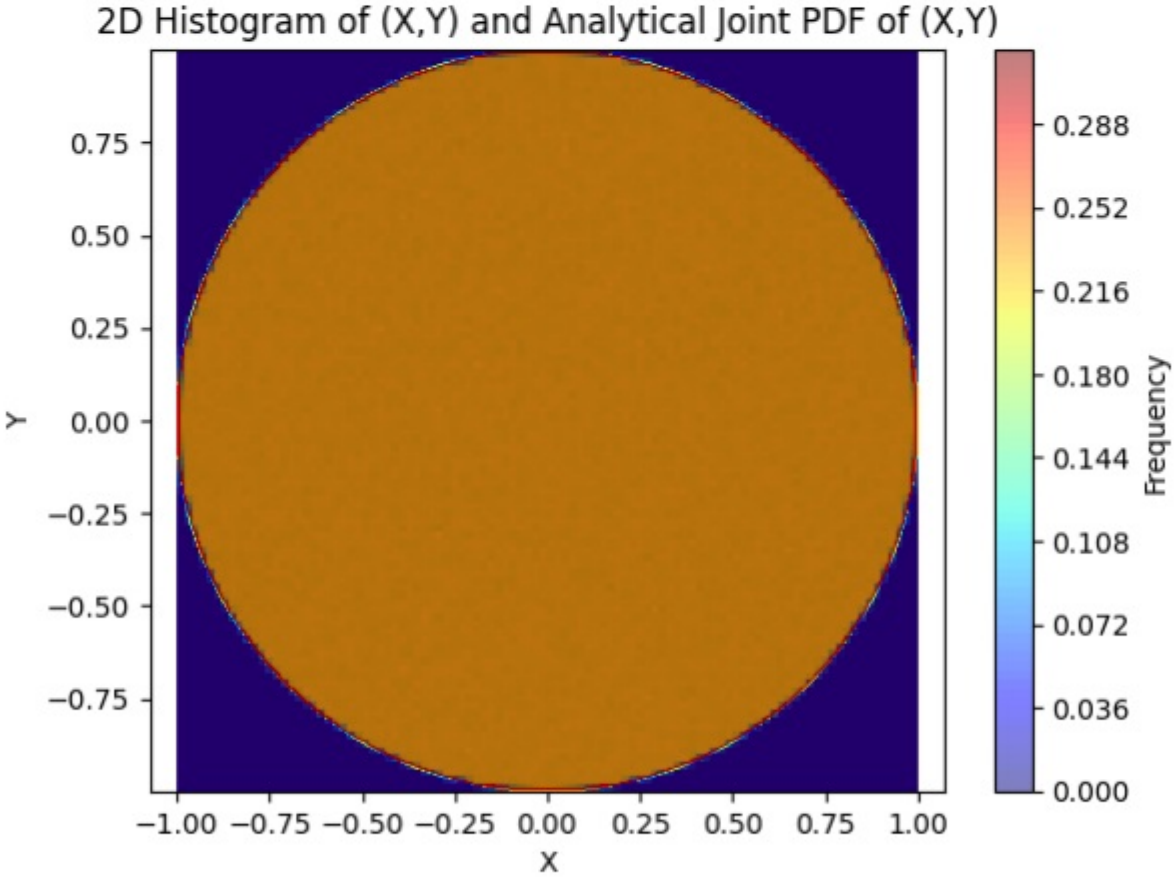


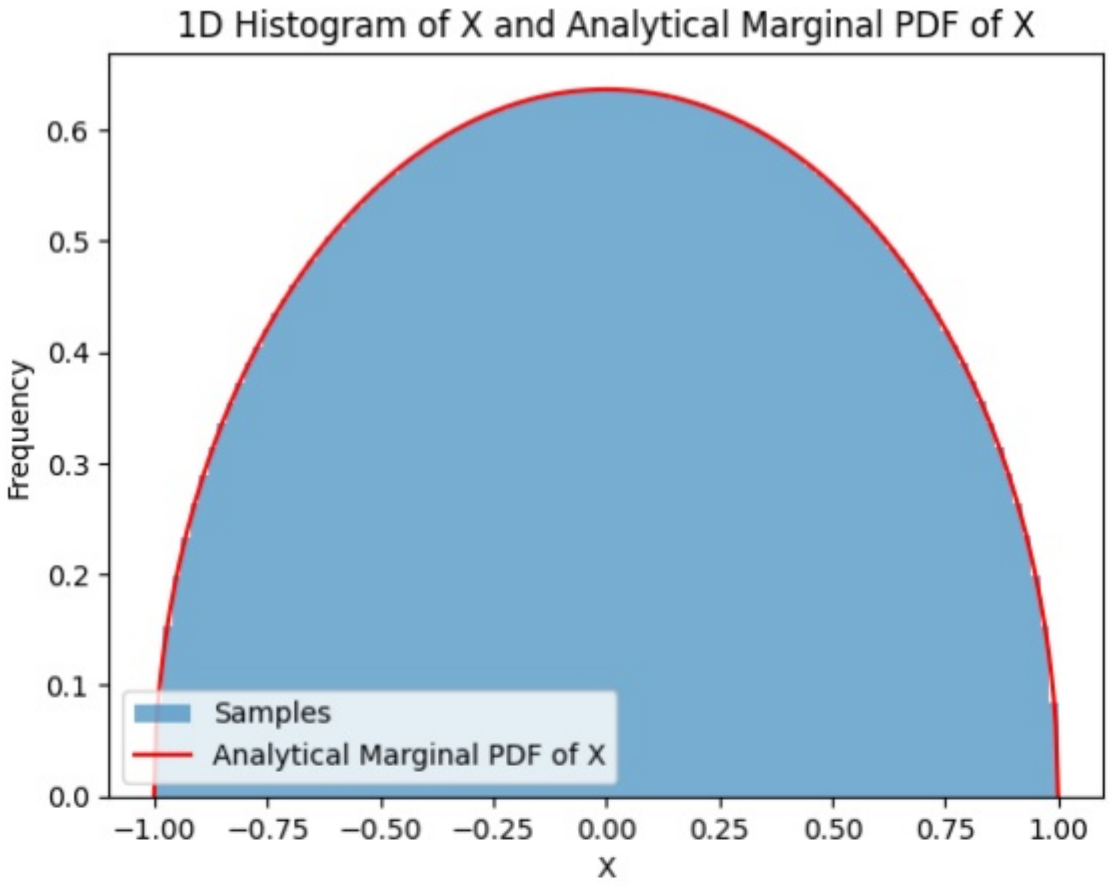
In [ ]:
```
# Evaluate the analytical joint pdf
x_values = np.linspace(-1, 1, 400)
y_values = np.linspace(-1, 1, 400)
x_mesh, y_mesh = np.meshgrid(x_values, y_values)

# For every point in the meshgrid, set the value to 1/pi if it's inside the unit c
z_mesh = np.where(x_mesh**2 + y_mesh**2 < 1, 1/np.pi, 0)
```

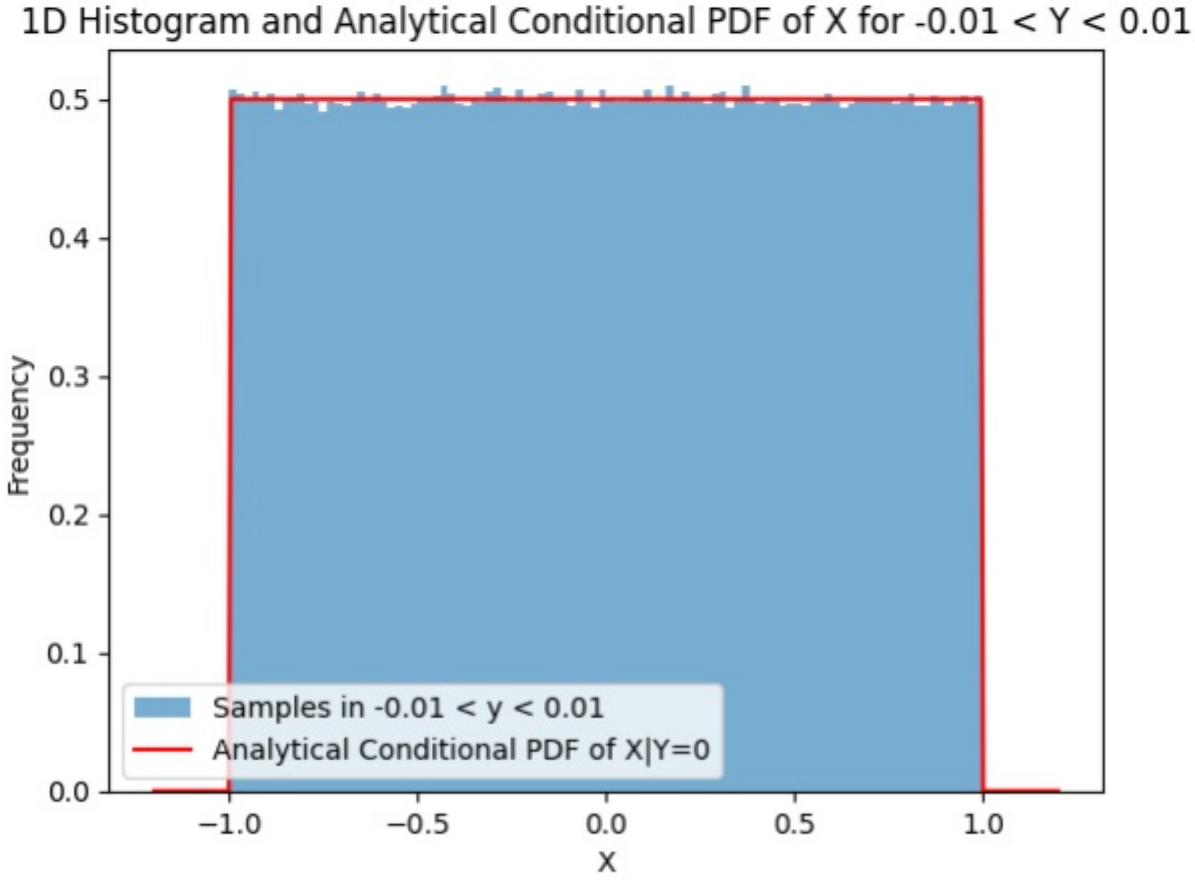
In [ ]:
```
# Figure 1: 2D-histogram of samples (x,y) and the analytical joint pdf
plt.figure()
plt.hist2d(x, y, bins=100, density=True)
#plt.contourf(x_mesh, y_mesh, z_mesh, levels=100, alpha=0.5, cmap='jet')

plt.colorbar(label='Frequency')
plt.title('2D Histogram of (X,Y) ')
plt.xlabel('X')
```

```
plt.ylabel('Y')
plt.axis('equal')
plt.show()


plt.figure()
plt.hist2d(x, y, bins=100, density=True)
plt.contourf(x_mesh, y_mesh, z_mesh, levels=100, alpha=0.5, cmap='jet')

plt.colorbar(label='Frequency')
plt.title('2D Histogram of (X,Y) and Analytical Joint PDF of (X,Y)')
plt.xlabel('X')
plt.ylabel('Y')
plt.axis('equal')
plt.show()

# Analytical marginal pdf of X
def marginal_pdf_x(x):
    return (2 * np.sqrt(1 - x**2)) / np.pi

x_values = np.linspace(-1, 1, 400)
pdf_values = marginal_pdf_x(x_values)


# Figure 2: 1D-histogram of samples x and the analytical marginal pdf of X
plt.figure()
plt.hist(x, bins=100, density=True, alpha=0.6, label='Samples')
plt.plot(x_values, pdf_values, 'r-', label='Analytical Marginal PDF of X')

plt.title('1D Histogram of X and Analytical Marginal PDF of X')
plt.xlabel('X')
plt.ylabel('Frequency')
plt.legend()
plt.show()

# Filter samples of x for which -0.01 < y < 0.01
x_cond = x[(y > -0.01) & (y < 0.01)]

# Analytical conditional pdf of X given Y=0
def conditional_pdf_x(x):
    return np.where((x > -1) & (x < 1), 0.5, 0)

x_values = np.linspace(-1.2, 1.2, 400)  # a bit extended to see where it touches z
pdf_values_conditional = conditional_pdf_x(x_values)
# Figure 3: 1D-histogram of samples x falling in the region -0.01 < y < 0.01
indices = np.where((y > -0.01) & (y < 0.01))
plt.figure()
plt.hist(x[indices], bins=100, density=True, alpha=0.6, label='Samples in -0.01 <
plt.plot(x_values, pdf_values_conditional, 'r-', label='Analytical Conditional PDF

plt.title('1D Histogram and Analytical Conditional PDF of X for -0.01 < Y < 0.01')
plt.xlabel('X')
plt.ylabel('Frequency')
plt.legend()
plt.show()
```


2D Histogram of (X,Y)
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
Y
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
X
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Frequency


2D Histogram of (X,Y) and Analytical Joint PDF of (X,Y)
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
Y
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
X
0.288
0.252
0.216
0.180
0.144
0.108
0.072
0.036
0.000
Frequency

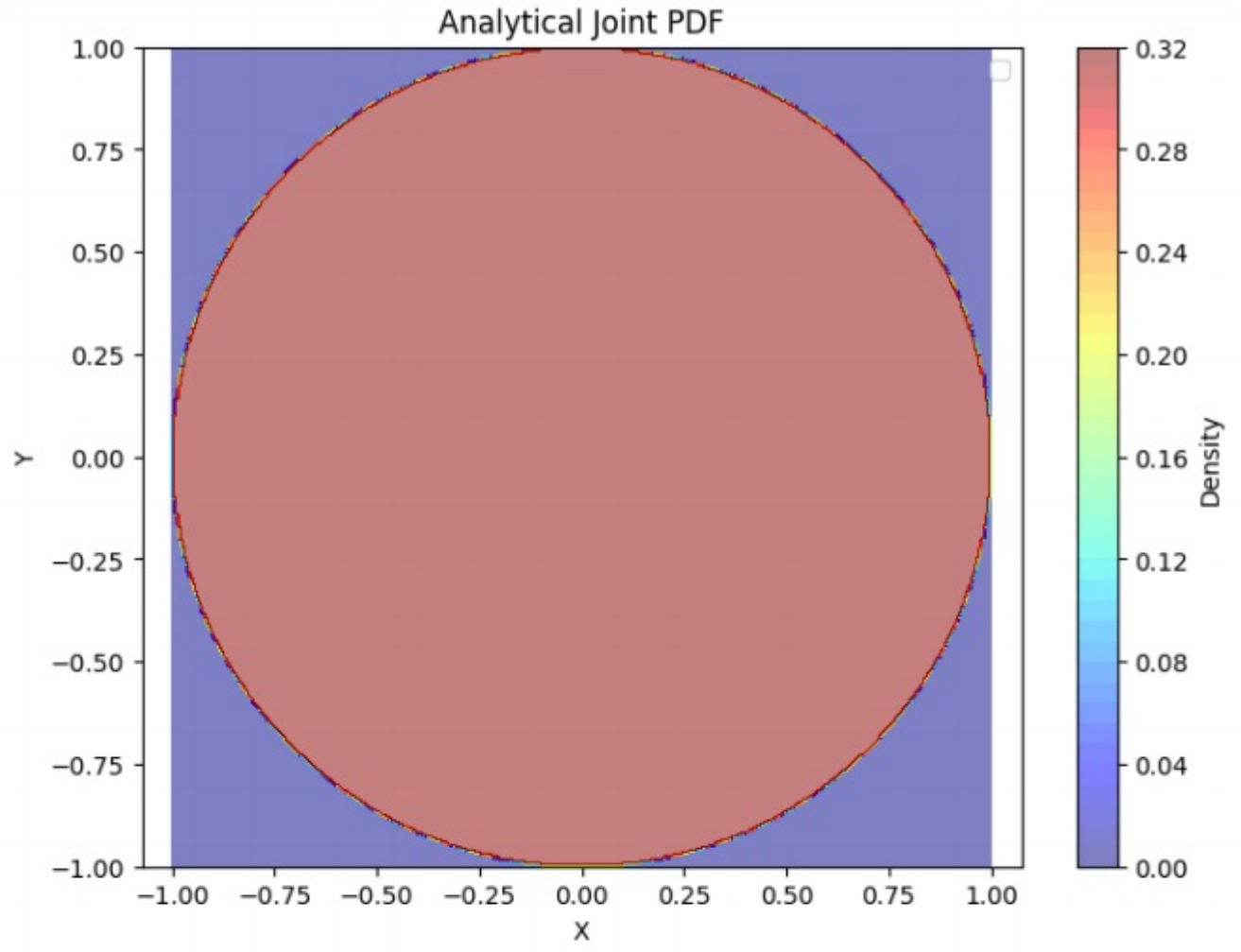




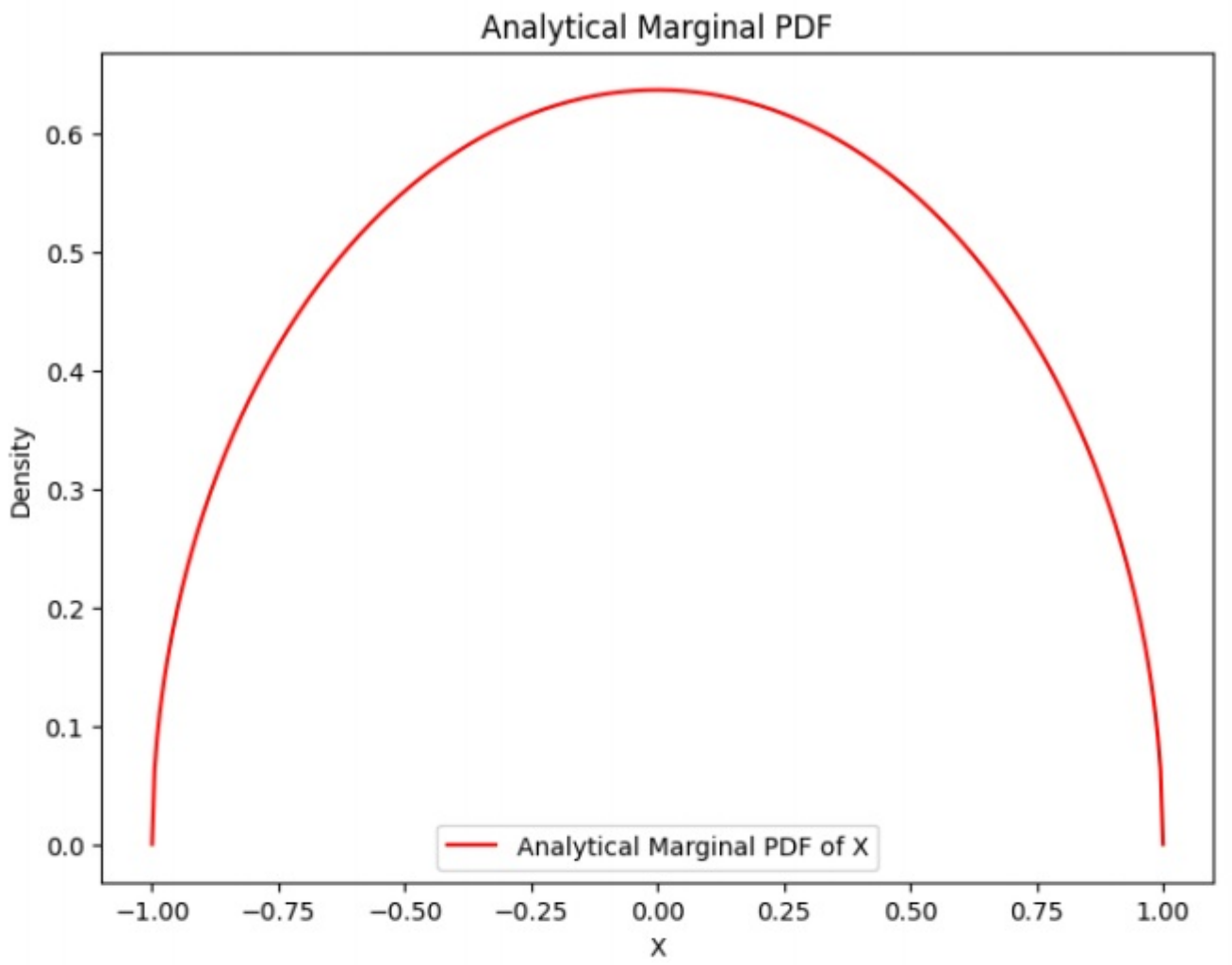
In [ ]:
```python
# Figure 1: 2D-histogram of samples (x,y) and the analytical joint pdf
plt.figure(figsize=(8,6))
#plt.hist2d(x, y, bins=100, density=True, alpha=0.5, label='Histogram')
plt.contourf(x_mesh, y_mesh, z_mesh, levels=50, alpha=0.5, cmap='jet')
plt.colorbar(label='Density')
plt.title('Analytical Joint PDF')
plt.xlabel('X')
plt.ylabel('Y')
plt.legend()
```

```
plt.axis('equal')
plt.show()
```

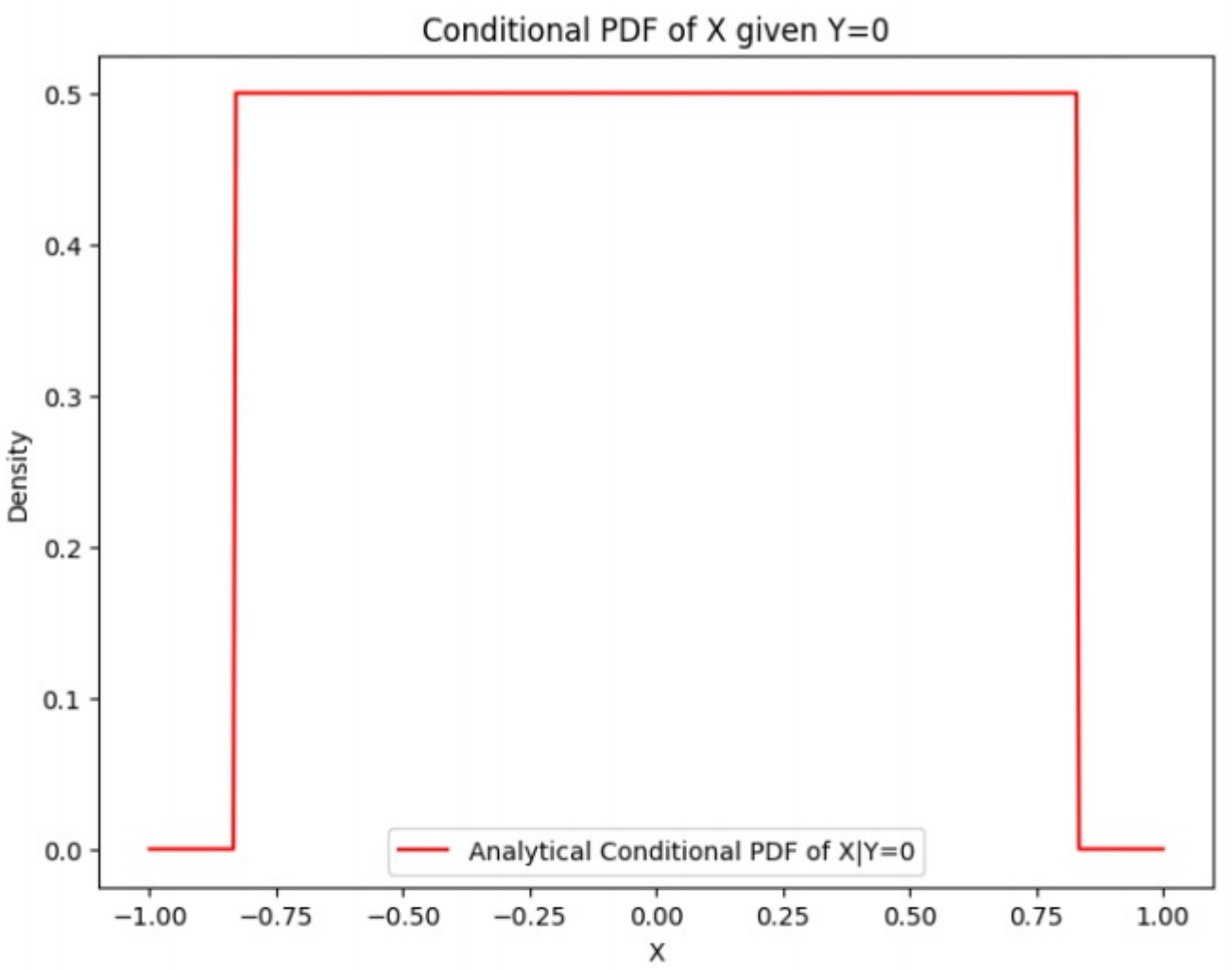
```
No artists with labels found to put in legend.  Note that artists whose label star
t with an underscore are ignored when legend() is called with no argument.
```



In [ ]:
```
# Figure 2: 1D-histogram of samples x and the analytical marginal pdf of X
plt.figure(figsize=(8,6))
#plt.hist(x, bins=50, density=True, alpha=0.6, label='Samples')
plt.plot(x_values, pdf_values, 'r-', label='Analytical Marginal PDF of X')
plt.title('Analytical Marginal PDF')
plt.xlabel('X')
plt.ylabel('Density')
plt.legend()
plt.show()
```



In [ ]:
```
# Figure 3: 1D-histogram of samples x falling in the region -0.01<y<0.01 and the a
plt.figure(figsize=(8,6))
#plt.hist(x_cond, bins=50, density=True, alpha=0.6, label='Samples with -0.01<y<0.
plt.plot(x_values, pdf_values_conditional, 'r-', label='Analytical Conditional PDF
plt.title('Conditional PDF of X given Y=0')
plt.xlabel('X')
plt.ylabel('Density')
plt.legend()
plt.show()
```


Conditional PDF of X given Y=0
Density
X
Analytical Conditional PDF of X|Y=0
0.0
0.1
0.2
0.3
0.4
0.5
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00