Learning and Earning Under Noise and Uncertainty

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Introduction

Sequential decision-making under uncertainty is central to a range of operations and marketing problems. In the face of an unknown environment, the decision-maker needs to strike a balance between learning the known environment ("learning") and selecting nearly optimal decisions ("earning"). For example, consider pricing a new product. If the retailer knew the *demand function* (i.e., the relationship between mean demand and the price), they would simply select the revenue-maximizing policy. However, the demand function may not be available in practice, so they need to experiment with different prices to learn the demand function, and they make pricing decisions based on the observed data.

The learning-earning, or exploration-exploitation trade-off can be captured by the *Multi-Armed Bandits* (MAB) framework, which has recently attracted significant attention from a range of communities. While most of the fundamental problems have been theoretically well understood, these algorithms have been rarely deployed in practice. This thesis serves as a preliminary step towards filling this gap by addressing three challenging aspects:
(i) **monotonicity constraint:** Markdown Pricing Under Unknown Demand (Section 1),
(ii) **combinatorial structures**: Optimal Decision Tree with Noise (Section 2), and
(iii) **large action space:** Short-lived High-volume Bandits (Section 3).
These challenges stem from fundamental areas at the operations-marketing interface, including pricing, survey design, and content recommendation. We offer theoretical insights with performance guarantees and empirical evidence demonstrating why our methodology outperform existing approaches.

1 Monotonicity Constraint

Dynamic pricing under unknown demand has been theoretically well understood, usually under the MAB framework. But in practice, these bandit-based policies are rarely deployed by real-world

retailers, largely because oscillating prices may cause customer dissatisfaction. For example, "1% price increase (in menu prices) leads to a 3% to 5% decrease in online ratings on average" [Luca and Reshef, 2021].

This motivates us to consider a dynamic **markdown** pricing problem, where the price sequence is required to be non-increasing. While both (a) markdown pricing under *known* demand and (a) *unconstrained* pricing (i.e., where the price is allowed to increase and decrease) under unknown demand have been extensively studied, little is known for markdown pricing under a unknown demand function. In particular, some basic questions remain open:

What is the optimal regret for markdown pricing under unknown demand?
And are they higher than the bounds for non-markdown pricing?

Thus motivated, we formulate a continuum-armed bandit problem where the arm sequence is required to be non-increasing. Each arm corresponds to a price, and the reward function $r(p)$ is the product of the price p and the demand function $d(p)$. The unknown demand function comes from a given class, which may be either nonparametric [Jia et al., 2021] or parametric Jia et al. [2023a].

	Due to	Family	Bdd. r''	Monotone	Regret
Nonparam.	Kleinberg [2005]	Lipschitz	no	no	$\tilde{\Theta}(n^{2/3})$
	Babaioff et al. [2015]	MHR	no	yes	$\tilde{O}(n^{3/4})$
	Jia et al. [2021], Chen [2021]	Lipschitz	no	yes	$\Theta(n)$
	Jia et al. [2021],Chen [2021]	unimodal	no	yes	$\tilde{\Theta}(n^{3/4})$
	Jia et al. [2021]	unimodal	yes	yes	$\tilde{\Theta}(n^{5/7})$
Param.	Broder and Rusmevichientong [2012]	k-crossing ($k \geq 1$)	yes	no	$\tilde{\Theta}(\sqrt{n})$
	Jia et al. [2023a]		yes	yes	$\tilde{\Theta}(n^{k/k+1})$
	Broder and Rusmevichientong [2012]	0-crossing	yes	no	$\Theta(\log n)$
	Jia et al. [2023a]		yes	yes	$\Theta(\log^2 n)$

Table 1: **Minimax Regret, With and Without Monotonicity.** Our results (colored **blue**) are **optimal** up to logarithmic (or constant) terms. Here, r'' is the second derivative of the revenue function r. Non-parametric results still hold if the initial inventory m is finite, with n replaced by $\max\{m, n\}$.

We provide a **complete** settlement by presenting novel learning policies with minimax optimal regret, summarized in Table 1. Notably, for parametric setting, we show that In the parametric setting, we introduced the term "k-crossing" which roughly means that any two curves in the family intersect k times at most. In particular, 0-crossing means any two curves are disjoint. We highlight that most of these bounds have higher asymptotic order than their unconstrained counterparts, except for the 1-crossing family (e.g., linear demand function).

2 Optimal Decision Tree Under Noisy Outcomes

Combinatorial structures in the problem instance, such as those enabling binary search, can sometimes be leveraged to speed up learning substantially. This is encapsulated by the *Optimal Decision Tree* (ODT) problem: Given a set of n *tests*, a set of m *hypotheses*, and an $m \times n$ table encoding the outcome for each pair of test and hypothesis, we aim to find a low-cost testing procedure (*decision tree*) that identifies the true hypothesis, with an acceptable misidentification probability.

Adaptivity	Assumption	Apxn. ratio	APX-hardness
non-adaptive	none	$\log m$	$\log m$
adaptive	**Few Unknowns:** $\leq r$ unknowns per row, $\leq c$ unknowns per column	$\max\{c, r\} + \log m$	$\log m$
	Few Knowns: $\lesssim \sqrt{n}$ knowns per row (hypotheses)	$\log m$	

Table 2: **Summary of Our Results.** Here, "row" and "column" refer to the outcome table. "Apxn. ratio" is the worst-case ratio (over all instances) between the expected cost of our algorithm and the optimal cost achievable by the same type of adaptivity (i.e., non-adaptive or adaptive).

The deterministic-outcome setting of this problem has been extensively studied. However, in many applications, the outcomes may be uncertain, which renders the ideas in the deterministic setting invalid. Despite the extensive literature on ODT, little is known about the noisy version from the perspective of approximation algorithms. There are two main reasons: First, the persistence of noise disables many statistical learning tools such as concentration bounds. Secondly, the structure of the optimal solution becomes significantly more complicated under noisy outcomes, posing substantial challenge for the analysis of approximation ratio.

Thus motivated, in Jia et al. [2019], we study a fundamental variant of the ODT problem where some test outcomes are uncertain (stochastically or adversarially), even in the persistent-noise setting. We design new approximation algorithms for both the non-adaptive setting, where the test sequence must be fixed *a-priori*, and the adaptive setting where the test sequence depends on the outcomes of prior tests. Our new approximation algorithms provide guarantees that are nearly best-possible and work for the general case of a large number of noisy outcomes per test or per hypothesis, with performance degrading smoothly with this number. Moreover, our numerical evaluations show that despite our theoretical logarithmic approximation guarantees, our methods give solutions with cost very close to the information theoretic minimum.

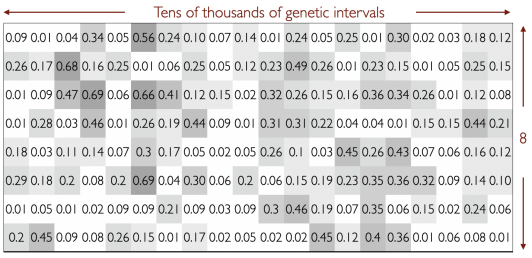

Figure 1: **Example Mutation Table:** *Each row/column correspond to a cancer type/genetic interval. The entries correspond to the mutation probability of a genetic interval (action) under a cancer type (hypothesis). The actual table entries are often much lower.*

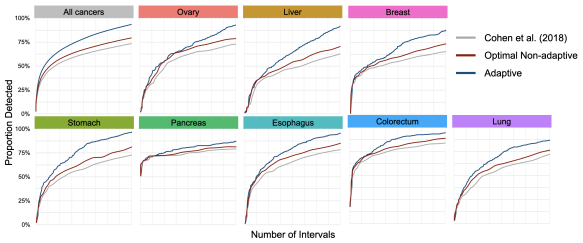

Figure 2: **Empirical Performance.** *Comparison of (non-adaptive) genomic panels from* **?** *with optimal non-adaptive panels and adaptive panels constructed using our partially adaptive greedy algorithm. The detection rate (on the COSMIC dataset) is plotted as a function of panel size.*

Inspired by recent advancements of liquid biopsies, in Gan et al. 2021 we study the *Active Sequential Hypothesis Testing* (ASHT) problem. Essentially, this a variant of the noisy ODT problem

with a prescribed misidentification probability: given an *error budget* $\delta > 0$, we aim to identify the true hypothesis with probability at least $1 - \delta$.

Partially Adaptive	**Brute Force**	**LP Heuristic**	**Our Algorithm**
Runtime	$\Omega(\|A\|!)$	LP of size $\Omega(\|A\|\|H\|^2)$ once	$O(\|A\|\|H\|)$ per iter.
Approximation Ratio	1	—	$O(\log(\|H\|))$
Fully Adaptive	**Brute Force**	**?**	**Our Algorithm**
Runtime	$\Omega(\|A\|^{2^{\|H\|}})$	LP of size $\Omega(\|A\|\|H\|)$ per iter.	$O(\|A\|\|H\|)$ per iter.
Approximation Ratio	1	—	$O(\log^2(\|H\|))$

Table 3: **Summary of Results:** *We compare the performance of our algorithms with benchmark algorithms, in terms of runtime and approximation guarantees. The bound of our algorithm is stated for fixed separation parameter and error tolerance.*

Motivated by applications where the number of hypotheses and actions is massive (e.g., genomics-based cancer detection), we propose efficient greedy algorithms and provide the first approximation guarantees for ASHT under two types of adaptivity; see Table 3. Notably, our guarantees are independent of the number of actions and logarithmic in the number of hypotheses. Moreover, on a real-world DNA mutation data (COSMIC), our algorithms substantially outperform previously proposed heuristic policies; see Figure 2. This work received the 2021 **Pierskalla Best Paper Award** in Healthcare Applications for its application in liquid biopsy, an emerging cancer detection method.

3 Short-Lived High-Volume Bandits

Recommendation tasks can be classified into four categories based on the *lifetime* and *volume* of contents generated, as shown in Figure 4. For persistent (long-lived) content, the problem is arguably straightforward: spend a small amount of time collecting sufficient data in the form of user feedback, and then apply suitable offline predictive model, which may range from basic collaborative filtering algorithms to deep neural networks (DNNs). Orthogonal to content lifetime, when there is a *low volume* of content relative to the number of users, the problem is also well-understood: Dedicated exploration methods (e.g. A/B testing) are sufficient for finding the right segments of users for which each content is most appealing.

The most challenging settings are where the content to be recommended is *short-lived* and *high-volume*. Such settings arise, for example, in content aggregation platforms (e.g., Apple News) and platforms with content that is entirely user-generated (e.g., TikTok). In these settings, both previous approaches are prone to failure: Offline predictive algorithms do not receive enough data on individual content to achieve meaningful accuracy due to the short lifetime, and dedicated exploration methods are ill-suited to high volume.

Our collaborator, *Glance* - a subsidiary the first unicorn, *Inmobi*, in India [Kadakia, 2023] - faces exactly this challenge. As a leading lockscreen content platform, their marketing team curates a large number of *content cards* per hour (see Figures 3 and 4), which will be sent to the users' phone. Most content cards have a short lifetime due to their transient nature, making the problem more challenging.

We tackle this challenge systematically in Jia et al. [2023b] by introducing a multiple-play Bayesian bandit problem that encapsulates the above key features. In each round, $O(n^\rho)$ new arms arrive.

Figure 3: **Sample Glance Cards:** *A Glance card typically consists of an image or graphic, a brief description, and a call-to-action (CTA). The contents vary from news, entertainment and advertisement.*

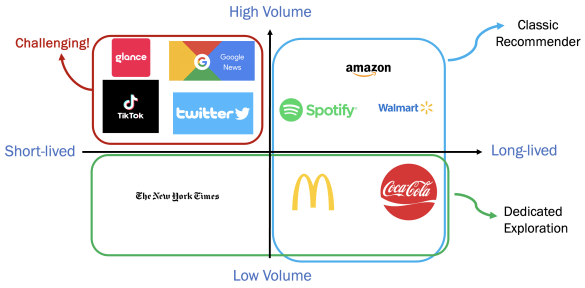

Figure 4: **Lifetime and Volume.** *Scenarios where items are long-lived (blue) or arrive in low volume (green) are relatively easy. This work focuses on the challenging scenario (red) where items are short-lived and arrive in high volume.*

Each arm is available for a short lifetime w (after which it expires), and has an unknown reward rate drawn from a distribution $\mathcal{D}$ that may vary over time. The learner selects a multi-set of n arms in each round and receives observable rewards. We present a policy whose *loss* (due to not knowing the reward rates) decreases in w, eventually converging to a lower bound as $n \to \infty$ for any fixed $\rho > 0$, assuming that $\mathcal{D}$ has a density function bounded above and below away from 0.

Finally, we validated the effectiveness of our policy through a large-scale field experiment on their real system, conducted on approximately 1% of their total traffic over 14 days in 2021. This involved approximately $510,000$ users and 18 million impressions. With certain practical adjustments, our policy outperformed their DNN-based recommender by 4.32% in total duration and 7.48% in total click-throughs.

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