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# MR Image Reconstruction from Spatial Frequency Domain(k-Space)

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# 1 Introduction

## 1.1 What is MRI?

Magnetic Resonance Imaging, commonly known as MRI is a technique used in the field of medicine to produce visuals of the internal structures within the body. By utilizing the Nuclear Magnetic Resonance Property of the abundantly available Hydrogen atom in our body, MRI generates cross-sectional images that provide valuable insights into our organs, tissues, and other structures. This non-invasive procedure aids, in diagnosing conditions and tracking the progression of diseases.
During an MRI scan patients lie on a table that moves into a machine resembling a tube. The main magnetic field aligns hydrogen atoms in tissues. When an orthogonal radio frequency pulse is applied these atoms are temporarily disrupted from their alignment. As they return to their state they emit radio signals that sensors within the MRI machine detect. These signals are then further processed to create images of structures such as soft tissues organs, muscles, bones, and blood vessels.
MRI is a tool, for identifying abnormalities in tissues as it offers excellent contrast between different types of soft tissues. Unlike X-rays or CT scans MRI does not use radiation making it a safer option, for repeated imaging needs. With customizable pulse sequences, MRI offers diverse contrast options to highlight different tissue properties, facilitating accurate diagnosis in a single scan. MRI also has the ability to provide images from various angles without repositioning the patient.
Overall, MRI plays a crucial role in modern medicine in making accurate diagnoses and treatment decisions based on detailed and high-quality images of the body's internal structures.

## 1.2 Image acquisition

A strong magnetic field is essential in MRI (Magnetic Resonance Imaging) because it aligns all the hydrogen nuclei within the body along the magnetic field. Later on, when a radiofrequency pulse is applied on this alignment, the nuclei emit signals that help to reconstruct detailed images.
The word "resonance" in MRI refers to the interaction between the main magnetic field, radiofrequency pulses, and hydrogen nuclei. When an external energy RF pulse is added to the main magnetic field, the magnetic vector is deflected. Magnetic resonance occurs when this external energy is injected into individual nuclear spin systems at the Larmor (resonance) frequency. Exposure of individual nuclei to the RF radiation at the Larmor frequency causes nuclei in the lower energy state to jump into the higher energy state and essentially "resonate" with the RF pulse.
An antenna or receiver coil is essential in MRI (Magnetic Resonance Imaging) as it detects and amplifies the weak radiofrequency signals emitted by hydrogen nuclei during the relaxation. The oscillating net magnetic flux from the excited spin system can be captured by the coil in which an induced electric current is generated. This current is then amplified, digitized, and filtered to extract frequency and phase information. Dif-

ferent coil designs optimize sensitivity for various tissues, aiding in creating detailed and accurate images for medical diagnosis. Specialized surface coils offer higher resolution for specific areas of interest.
The relationship between k-space data and image data is the Fourier transformation. The data acquisition matrix contains raw data before image processing. In 2-dimensional (2D) Fourier transform imaging, a line of data corresponds to the digitized MR signal at a particular phase encoding level. From a signal processing point of view, MRI is configured in such a way that the signal acquired from receiver coils is basically a 2D Fourier Transform of the MR images that we are seeking. In other words, if we do an inverse Fourier Transform of the signal acquired, we can reconstruct the MR Images.

## 1.3 Advantages and disadvantages

## Advantages of MRI:

- **Non-Ionizing Radiation:** Unlike X-rays and CT scans, MRI does not use ionizing radiation, making it safer for patients, especially for repeated or long-term imaging.
- **Soft Tissue Contrast:** MRI provides excellent soft tissue contrast, allowing for clear differentiation between various soft tissues, organs, and structures.
- **Multi-Planar Imaging:** MRI can produce images in multiple planes (sagittal, coronal, and axial), which aids in comprehensive visualization and assessment of anatomical structures.
- **No Bone Artifacts:** MRI is not affected by bone artifacts, unlike CT scans, which can have limitations in imaging structures near bones.
- **High Resolution:** MRI can achieve high spatial resolution, enabling the visualization of fine details and small structures.

## Disadvantages of MRI:

- **Time-Consuming:** MRI scans can take longer to complete compared to other modalities because of the magnetization and relaxation process takes time and need to be repeated several times in order to acquire sufficient signal.
- **Cost and Availability:** MRI machines are expensive to purchase and maintain, leading to higher costs for patients and healthcare facilities.
- **Metal Interference:** MRI is sensitive to metal objects, which can cause artifacts

and safety concerns. Patients with certain implants, devices, or metal fragments may not be eligible for MRI.

- **Limited in Bone Imaging:** While MRI excels in soft tissue imaging, it can be less effective in imaging dense bone structures.
- **MRI Artifacts:** The acquired MR signal is susceptible to a number of artifacts such as Chemical shift, T2 blurring etc.

Comparing MRI with other imaging modalities involves considering the specific medical condition, the information needed, patient factors, and potential risks. The choice of imaging modality is typically based on the clinical question, the type of tissue or structure being investigated, and the patient's medical history.

### 1.4 Overview of the Project

As discussed earlier, The MRI scanner acquires data in the spatial frequency domain, known as k-space. MR image reconstruction requires the inverse Fourier transform of the acquired k-space data. In this project, we are given this kSpace data in HDF5 format. The main objective of the project is to implement these necessary transformations using Java to reconstruct MR Images.
In doing so, the idea of complex numbers and its implementation comes handy because complex numbers could most suitably express the signal detected by the MR scanner. Finally, the project requires implementing some filtering and image compression features.

## 2 Methods

### 2.1 k-Space

k-space refers to a mathematical representation of the spatial frequency domain of the MR signal. In simpler terms, k-space is an array of numbers representing spatial frequencies in the MR image. It is critical to understand how MRI data is acquired and reconstructed to form MR images.
k-space is a grid-like space where each cell contains spatial frequency and phase information about every pixel in the final image. In other words, we can think of k-space as a sort of "data space" where the MRI machine collects raw signal data. These data points are then transformed using inverse Fourier transform to generate the final MR images that we see. In practice, k-Space data could be well expressed using complex numbers. The magnitude and phase of k-Space are defined by the formula 1 and 2 respectively, where a is the real part of the signal and b is the imaginary part. The magnitude represents the length of the vector in the k-Space plane and the phase ex-

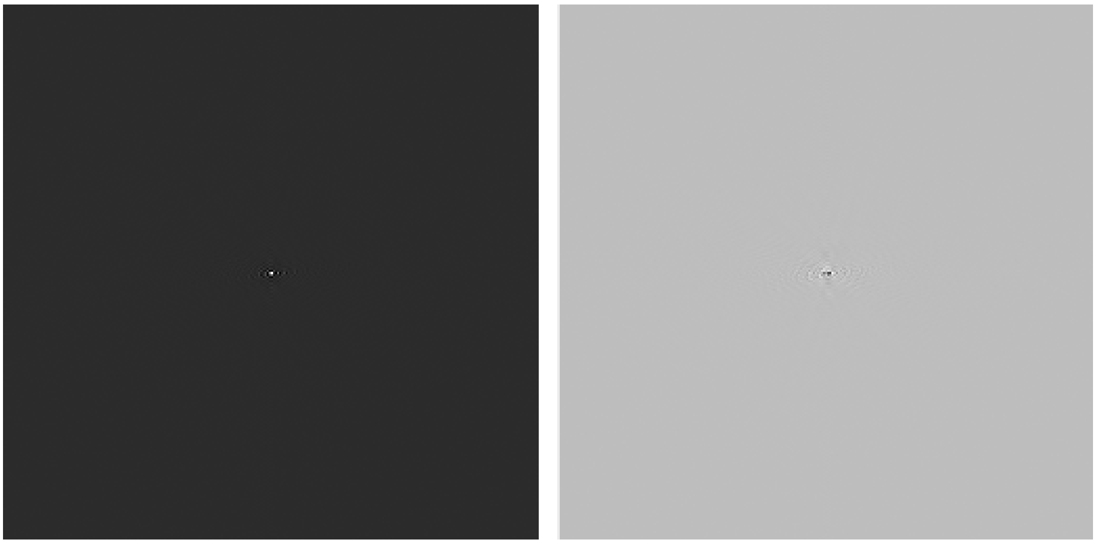

**Figure 1** *Real and imaginary part of the k-space*

presses the angle of the vector to the positive real axis.

$$r = |z| = \sqrt{a^2 + b^2} \tag{1}$$

$$\phi = \operatorname{atan2}(b/a) \tag{2}$$

The magnitude of k-space has a huge image intensity range between the center (low-frequency part, very large) and the periphery (high-frequency part, very small). Taking a logarithm of the magnitude of k-space (point-wise) can reduce the huge image intensity range for better visualization of the magnitude of k-space.

Figure 1 shows the real part (left) and imaginary part (right) of the k-Space data for the human knee. Figure 2 shows the magnitude (left), log-transformed magnitude(center) and the phase (right) of the k-Space signal.
Magnitude variations correspond to tissue properties and relaxation, while phase variations relate to spatial encoding and susceptibility effects. Due to their distinct roles and sensitivities to various factors, the intensity variations seen in magnitude images are not directly mirrored in phase images in MRI k-space.

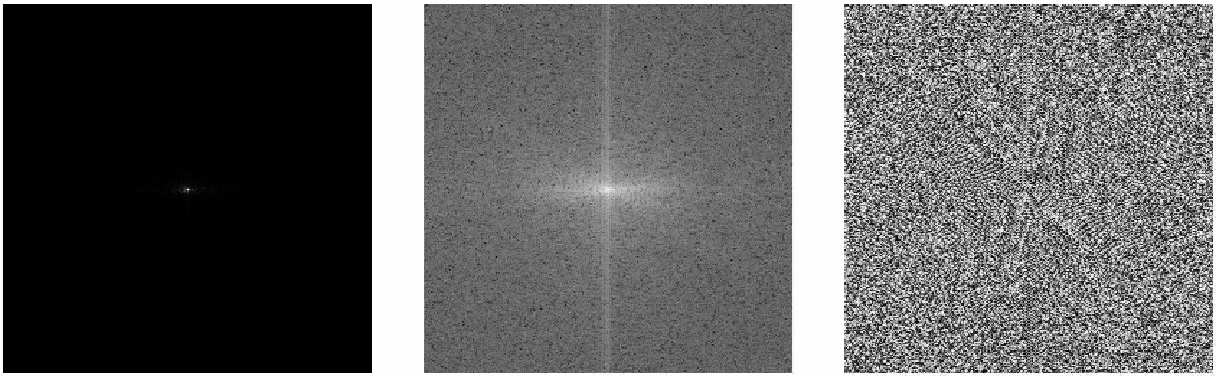

**Figure 2** *Magnitude, logarithmic magnitude, and phase of the k-space*

## 2.2 Reconstruction

Until now, we were dealing with k-Space data which is in the spatial domain. Reconstruction in MRI serves the purpose of converting this raw MR signal into meaningful images for medical diagnosis and treatment. This process takes the complex frequency and phase information encoded in k-space and transforms it into visual images.
In essence, MRI reconstruction translates raw MRI data into accessible and clinically relevant images. From a pure signal processing point of view, the sum of the total magnetization of the excited spins that are apparent in the measured volume can be manipulated by external magnet field gradients in such a way that the resulting signal is a Fourier transform of the image to be measured. So, the major step to reconstruct MR images is the inverse Fourier Transform of the measured k-Space signal. But we are missing a crucial step in between which is needed for this reconstruction, FFT Shift. Let's dive more into this, but first, let's talk a little about Fourier Transform.
The Fourier transform is a mathematical technique used to analyze signals and functions by expressing them in terms of their constituent frequencies. It essentially breaks down a signal or function into its individual frequency components. This transformation is particularly useful in various fields of science and engineering, including signal processing, image analysis, and physics. So as an outcome, we go from Time Domain to Frequency Domain for a given signal.
In the real world, we deal with discrete signals instead of continuous signals. That's why the Discrete Fourier Transform is an extremely important tool in all engineering contexts. DFT has an efficient implementation called the Fast Fourier Transform (FFT). The FFT has an interesting feature – it uses a representation of k-space where the DC component (which is the value when k=0) is placed at index 0 in the transformed array. However, in MRI, this DC component is at the center of the collected data. So, before applying the FFT algorithm, we have to rearrange the k-space. This "rearrangement" process is called FFT Shift.
In summary, in k-space measurement, the central portion of the matrix captures the low-frequency components. The concentration of signal intensities occurs around the DC component. Hence for implementation, a crucial step involves shifting the DC component to the array's initial index before initiating an inverse Fast Fourier Transform (iFFT).

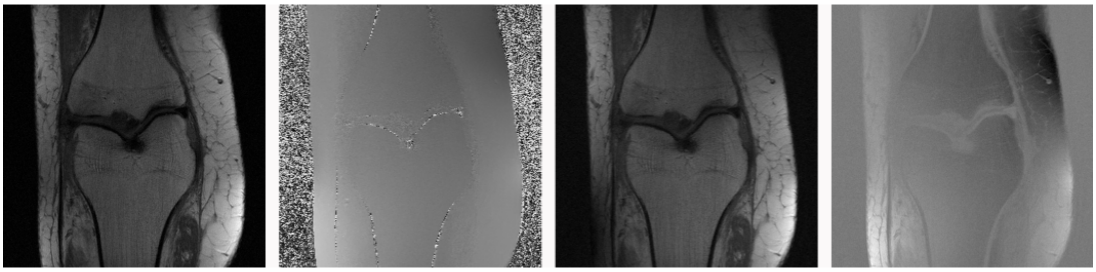

**Figure 3** *Reconstructed image of magnitude, phase, real, and imaginary part*

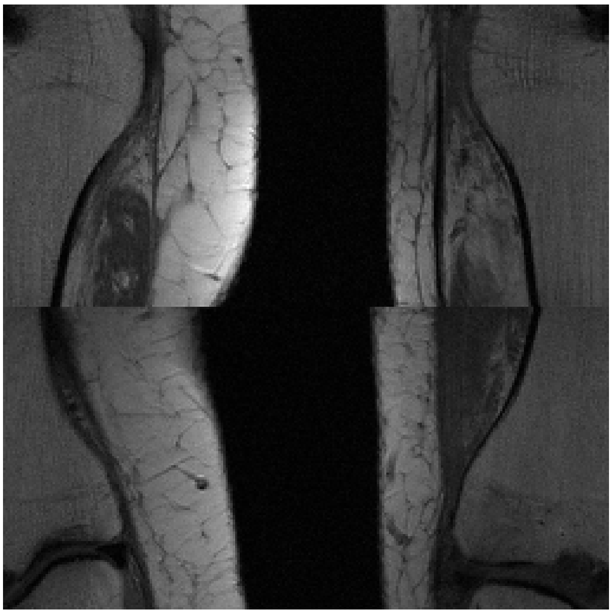

**Figure 4** *Reconstructed image without the second FFT Shift (post iFFT)*

### 2.2.1 MRI Image Reconstruction

Figure 3 shows the magnitude, phase, and real and imaginary parts of the reconstructed MR image. It's evident that the magnitude and real look more similar to each other compared to phase and imaginary. In radiology practice, mostly the magnitude reconstructed image is used for diagnosis.

It's worth mentioning the FFT Shift is done twice during the reconstruction process: once before the iFFT and once after the iFFT is done. If the second shift is not done, the image will not be visually comprehensive for any analysis further. This is because in such cases, the boundary part of the desired image will be in the center and vice-versa in both the x and y directions of the image plane. This will result in an output with four quadrants of the image mirrored across the center point. Figure 4 demonstrates this scenario.

It is possible to again go back to the spatial domain from the image domain. For this, all we have to do is Fast Fourier Transform of the image. Just like the reconstruction step, we need to take care of the rearrangement of the DC component and do two FFT shifts. Figure 5 shows the phase and the log-magnitude of the kSpace created from the already reconstructed MR image.

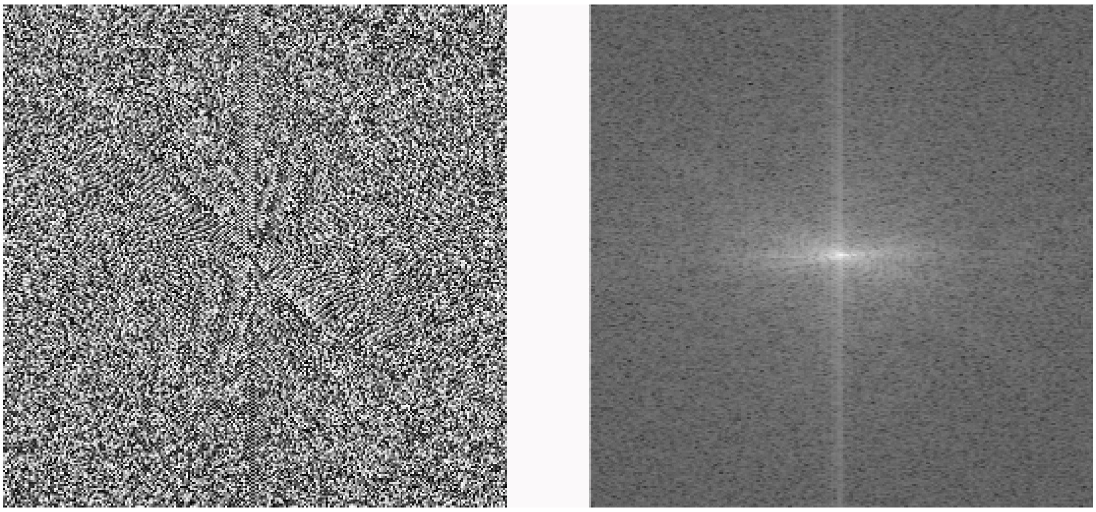

**Figure 5** *Reproduced images of the phase and the magnitude in log-scale*

## 2.3 Filters

Filters in image processing are operations applied to an image to enhance or modify certain aspects of it. These filters work by altering the pixel values of the image based on predefined rules or mathematical operations. Filters are commonly used to detect edges, compress resolution, remove noise, emphasize certain features, blur or sharpen images, and extract specific information.

### 2.3.1 Sinc-Filter

The sinc filter is a type of frequency-domain filter commonly used in signal processing and digital image processing. It's based on the mathematical function known as the sinc function, which is defined as the normalized sine of a radian angle divided by that angle. The normalized sinc function is given by the formula: sinc(x) = $\frac{\sin((\pi x)}{\pi x}$

In image processing, sinc filters can be used for tasks like interpolation, resizing, and smoothing. However, due to the computational complexities and potential artifacts associated with sinc-based filtering, other filter types like Gaussian filters or more advanced interpolation methods are often preferred in practice.

### 2.3.2 Box Multiplication

A black-and-white image is a 2D array of pixel values. Now, if we take another 2D array of the same dimension and put 1 in the cells associated with our area of interest in the image and 0 elsewhere, and then do point-wise multiplication, we get something called box multiplication. This technique could be used to manipulate the kSpace and adjust details within the reconstructed image. Figure 7 shows how box multiplication with 96

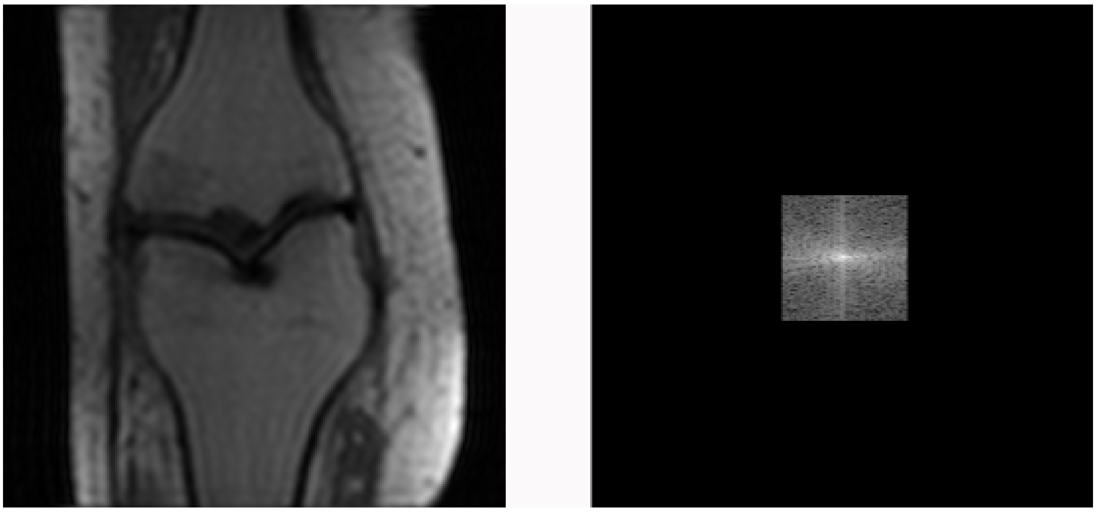

**Figure 6** *The Figure shows the magnitude image (left) and the magnitude of the k-space (right) after application of the box multiplication.*

lines(number of pixels from the periphery in box window set to 0) changes the kSpace image(magnitude) and the reconstructed image from it. As expected the outermost part is totally black due to this operation.

High-frequency components in k-space correspond to rapidly changing variations in the image. In the context of MRI or imaging, high-frequency components generally represent fine details, sharp edges, and rapid transitions between different intensities. These components capture local changes in the image, providing information about boundaries and textures. Whereas low-frequency components in k-space correspond to slowly varying variations in the image. They represent the overall intensity and large-scale structure of the image. These components capture global changes in the image, providing information about the overall shape and orientation of objects.
This is further demonstrated in Figure 7 where different configurations were used for box multiplication. We can see that the more the lines parameter, the more image loses its resolution and edge details without affecting the overall image shape and contrast. This is because more line number means we are retaining only the central area (low frequency) of the kSpace.

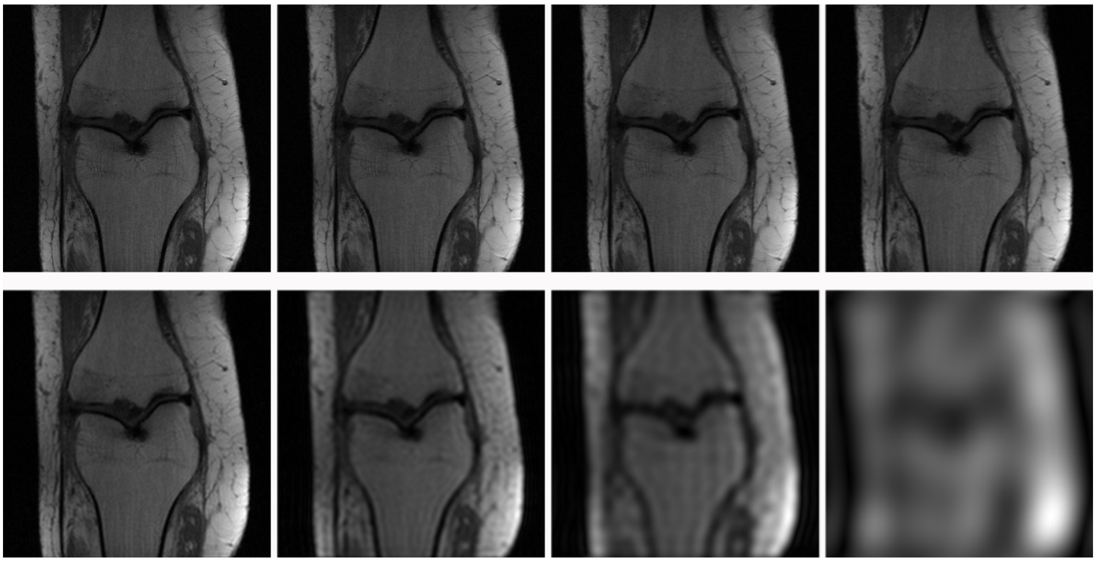

**Figure 7** *The Figure shows different levels of image degradation through zeroing k-space: From top left to bottom right lines used for box multiplication - 0,16,32,48,64,96,112,124*

## 2.4 Reducing the Image Size

### 2.4.1 Cropping k-space

k-Space could be cropped in order to reduce the overall image size. This crop is done by excluding or limiting certain portions of the k-space data. This is often done by excluding the outer regions of k-space, where higher-frequency information resides, and retaining the lower-frequency parts. These high-frequency components represent fine details and rapid changes in the image and low-frequency parts represent the overall shape, intensity, and contrast of the image.
This is why in Figure 8, we can see that even after cropping a $128 \times 128$ block from the center of the k-Space, we were still able to reconstruct a pretty good MR image.

### 2.4.2 Max Pooling

Max-pooling is a pooling technique that involves selecting the maximum value from a set of values or data points according to a defined criterion. This operation is often used to reduce the dimensionality of data or to extract salient information from a larger dataset. It uses parameterized window/kernel to iterate over desired data/ signal. Though it is mainly used in image processing, it is also applied to time series data, audio data, or even abstract data representations, etc. The iteration of the kernel also depends on another parameter called stride: how much the kernel shifts in each step in any direction during the iteration.
If there are incomplete blocks in the boundary of an image, the dimension of the output of the image will be adjusted to accommodate the max-pooling operation essentially reducing the the image size more.

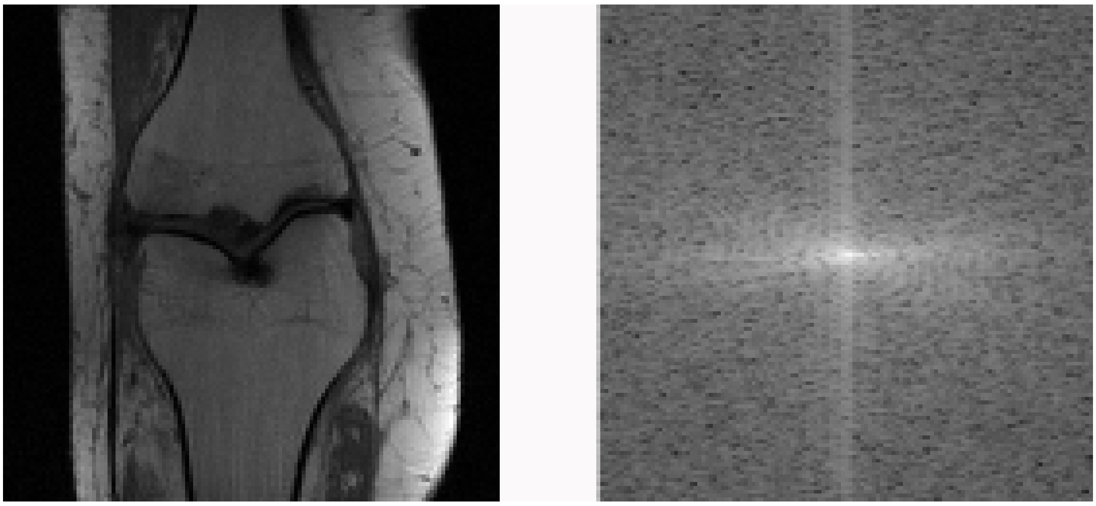

**Figure 8** *The Figure shows the magnitude image (left) and the log magnitude of the k-space (right) after cropping in k-space.*

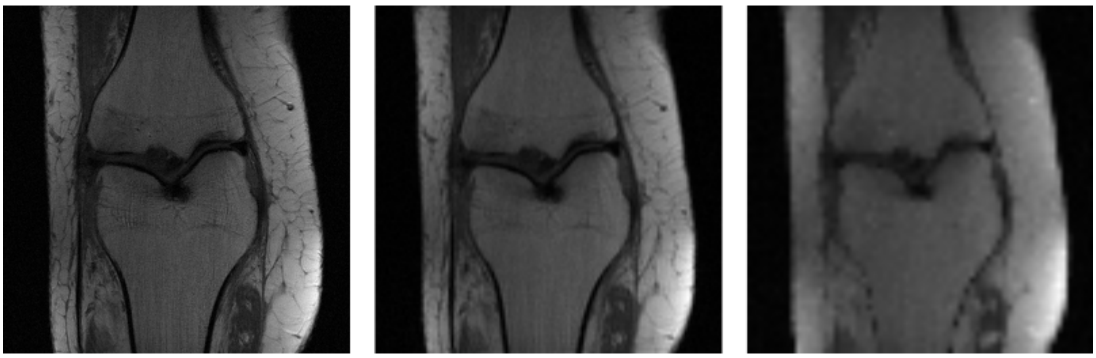

**Figure 9** *The Figure shows the original (left), the cropped k-space image (middle), and the scan after applying max pooling (right).*

Improve the resolution of the MR image could be achieved in several ways. Here are listed few ways to achieve the same,

- Increasing Field Strength of $B_0$
- Longer Scan Time
- Reducing Field of View
- Reducing the slice thickness

There is a trade-off between scan time, image resolution, and signal-to-noise ratio (SNR). Increasing resolution typically requires longer scan times, which can lead to increased motion artifacts and reduced SNR. Finding the right balance is crucial. Longer scan time also invokes more cost both computationally and monetarily.
Figure 9 shows both the image where k-Space cropping and maxpool have been applied. We used a $128 \times 128$ cropped k-Space and a $4 \times 4$ kernel for the maxpool. With this configuration, the cropped k-Space image seems better with image resolution and details. It is very important to remember that the cropping of the k-Space happens before the reconstruction whereas the maxpool is applied to the already reconstructed image.

# 3 Conclusion

With the recent advent of AI, techniques are being used to reduce imaging time and improve image quality. Especially deep learning-based algorithms are used both for pre-construction and post-construction processing.
This study [Souza et al., 2020] evaluates two-element U-nets, W-nets, for multi-channel magnetic resonance image reconstruction. Results show that networks in the image domain perform better independently, while dual-domain methods are better when processing all channels simultaneously.
Study[Ramzi, 2022] has been conducted to improve Magnetic Resonance Imaging (MRI) for undersampled data by introducing unrolled neural networks for acquisition scenarios. The research reviews neural networks, selects the best performer, extends it to fastMRI 2020 and 3D non-Cartesian data problems, addresses clinical applicability concerns, and introduces a new acceleration method, SHINE.
For the most part, AI-based reconstruction methods are vendor-dependent and highly proprietary. That's why it's difficult to determine if they are already operational in practice or not. Examples include Siemens' Deep Resolve.[Dee, ]
This project could be described as the basic template for how measured MR signals could be reconstructed into visual images. In summary, we are given measured MR signal in HDF5 format. Then we implemented classes using Java for different functionalities to reconstruct MR Images. Then we performed 2D FFT shift. Finally, the project requires implementing some filtering and image compression features. This project only implements the basic mathematical and signal processing framework needed to reconstruct MR images from k-Space data. It doe not attempt to optimize the recon-

struction process for better image quality. As discussed earlier, using various pre and post-processing techniques, this could be achieved. As a student of AI, I look forward to exploring those possibilities.

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