数学公式编辑

一、球坐标

$$\left\{\begin{array}{lcl} x=r\sin\varphi\cos\theta, & &{0\leqslant\varphi\leqslant\pi}\\ y=r\sin\varphi\sin\theta, & &{0\leqslant\theta\leqslant 2\pi}\\ z=a\cos\varphi. & &{}\\ \end{array}\right.$$

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$\left\{
\begin{array}{lcl}
   x=r\sin\varphi\cos\theta, &  &{0\leqslant\varphi\leqslant\pi}\\
   y=r\sin\varphi\sin\theta, &  &{0\leqslant\theta\leqslant 2\pi}\\
   z=a\cos\varphi.          &    &{}\\
\end{array}
\right.$
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二、雅可比式

$$J=\cfrac{\partial (F,G)}{\partial (\mu ,\nu)}=\begin{vmatrix}\cfrac{\partial F}{\partial \mu}&\cfrac{\partial F}{\partial \nu}\\\cfrac{\partial G}{\partial \mu}&\cfrac{\partial G}{\partial \nu}\end{vmatrix}$$

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$J=\cfrac{\partial (F,G)}{\partial (\mu ,\nu)}=\begin{vmatrix}\cfrac{\partial
    F}{\partial \mu}&\cfrac{\partial F}{\partial \nu}\\\cfrac{\partial
    G}{\partial \mu}&\cfrac{\partial G}{\partial \nu}\end{vmatrix}$
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三、拉格朗日乘数法

$$\left\{\begin{array}{l} f_x(x,y)+\lambda\varphi_x(x,y)=0,\\ f_y(x,y)+\lambda\varphi_y(x,y)=0,\\ \varphi(x,y)=0. \end{array}\right.$$

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$\left\{
\begin{array}{l}
   f_{x}(x,y)+\lambda{\varphi}_{x}(x,y)=0,\\
   f_{y}(x,y)+\lambda{\varphi}_{y}(x,y)=0,\\
   \varphi (x,y)=0.
\end{array}\right.$
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四、二元函数的泰勒公式

$$\begin{aligned}
f(x_{0}+h,y_{0}+h)=&f(x_{0},y_{0})+(h\cfrac{\partial}{\partial x}+k\cfrac{\partial}{\partial y})f(x_{0},y_{0})+\\
&\cfrac{1}{2!}(h\cfrac{\partial}{\partial x}+k\cfrac{\partial }{\partial x})^{2}f(x_{0},y_{0})+\cdots +\cfrac{1}{n!}(h\cfrac{\partial}{\partial x}+k\cfrac{\partial }{\partial x})^{n}f(x_{0},y_{0})+\\
&\cfrac{1}{(n+1)!}(h\cfrac{\partial}{\partial x}+k\cfrac{\partial }{\partial x})^{n+1}f(x_{0}+\theta h,y_{0}+\theta k)\quad (x<\theta <1).
\end{aligned}$$

```
$\begin{aligned}
   f(x_{0}+h,y_{0}+h)=&f(x_{0},y_{0})+(h\cfrac{\partial}{\partial
      x}+k\cfrac{\partial}{\partial y})f(x_{0},y_{0})+\\
   &\cfrac{1}{2!}(h\cfrac{\partial}{\partial x}+k\cfrac{\partial }{\partial
      x})^{2}f(x_{0},y_{0})+\cdots +\cfrac{1}{n!}(h\cfrac{\partial}{\partial
      x}+k\cfrac{\partial }{\partial x})^{n}f(x_{0},y_{0})+\\
   &\cfrac{1}{(n+1)!}(h\cfrac{\partial}{\partial x}+k\cfrac{\partial
      }{\partial x})^{n+1}f(x_{0}+\theta h,y_{0}+\theta k)\quad (x<\theta <1).
\end{aligned}$
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五、极坐标中的二重积分公式

$$\iint\limits_{D} f(\rho\cos\theta,\rho\sin\theta)\rho d\rho d\theta = \int_{\alpha}^{\beta} d\theta \int_{\varphi_1(\theta)}^{\varphi_2(\theta)} f(\rho\cos\theta,\rho\sin\theta)\rho d\rho$$

六、球坐标中的三重积分公式

$$\iiint\limits_{D} f(x,y,z)dxdydz=\iiint\limits_{D} F(r,\varphi,\theta)r^{2}\sin\varphi drd\varphi d\theta$$

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$\displaystyle{\mathop{\iiint}\limits_{D}f(x,y,z)dxdydz=\mathop{\iiint}\limits_{D}
F(r,\varphi ,\theta)r^{2}\sin\varphi drd\varphi d\theta}$
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七、格林公式

$$\iint\limits_{D}(\cfrac{\partial Q}{\partial x}-\cfrac{\partial P}{\partial y})dxdy=\oint_{L}Pdx+Qdy$$

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$\displaystyle{\mathop{\iint}\limits_{D}(\cfrac{\partial Q}{\partial
    x}-\cfrac{\partial P}{\partial y})dxdy=\mathop{\oint}_{L}Pdx+Qdy}$
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八、对面积的曲面积分

$$\iint\limits_{\Sigma}f(x,y,z)dS=\iint\limits_{D_{xy}}f[x,y,z(x,y)]\sqrt{1+z_{x}^{2}(x,y)+z_{y}^{2}(x,y)}dxdy$$

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$\displaystyle{\mathop{\iint}\limits_{\Sigma}f(x,y,z)dS=\mathop{\iint}\limits
_{D_{xy}}f[x,y,z(x,y)]\sqrt{1+z_{x}^{2}(x,y)+z_{y}^{2}(x,y)}dxdy}$
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