Model Recognition as Planning

Diego Aineto and **Sergio Jiménez** and **Eva Onaindia** and **Miquel Ramírez**
Departamento de Sistemas Informáticos y Computación
Universitat Politècnica de València.
Camino de Vera s/n. 46022 Valencia, Spain
{dieaigar,serjice,onaindia}@dsic.upv.es

Abstract

Given a partially observed plan execution, and a set of possible planning models (models that share the same state variables but that update these variables according to different action models); *model recognition* is the task of identifying which model in the set produced/explains the given observation. The paper formalizes the *model recognition* task and introduces a novel method to estimate the probability of a STRIPS model to produce an observation of a plan execution. This method, that we called *model recognition as planning*, is built on top of off-the-shelf classical planning algorithms and is robust to missing intermediate states and actions in the observation of the plan execution. The effectiveness of *model recognition as planning* is shown in a set of STRIPS models encoding different kinds of *automata*. We show that *model recognition as planning* succeeds to identify the executed automata despite some state variables (e.g. the internal automata state) or the actual applied transitions are unobserved.

Introduction

Plan recognition is the task of predicting the future actions of an agent provided observations of its current behavior (Carberry 2001). *Goal recognition* is a closely related task that aims identifying the goals of the observed agent. Goal recognition is considered *automated planning* in reverse; while automated planning compute sequences of actions that accounts for a given goals, goal recognition compute goals that account for an observed sequence of actions (Geffner and Bonet 2013).

Diverse approaches has been proposed for plan/goal recognition such as *rule-based systems*, *parsing*, *graph-covering*, *Bayesian nets*, etc (Sukthankar et al. 2014). *Plan recognition as planning* is the model-based approach for plan/goal recognition that leverages the action model of the observed agent to compute its most likely goal (Ramírez 2012; Ramírez and Geffner 2009).

This paper formalizes the *model recognition* task where the object to recognize is not a goal but the *planning model* that is determining the behavior of the observed agent. Given a partially observed plan execution, and a set of possible planning models (models that share the same state variables but that update these variables with different action models); *model recognition* is the task of identifying the model in the set with the highest probability of producing/explaining the given observation.

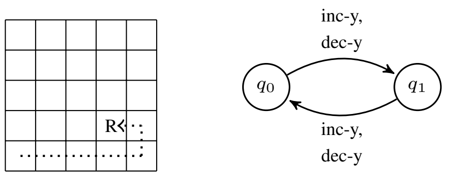

Figure 1: *(Left)* Robot navigating a 5×5 grid. *(Right)* Automata to control that the robot only increments its x-coordinate when q_0 holds (actions `inc-y` and `dec-y` update the robot y-coordinate and switch the automata state).

To better illustrate *model recognition*, imagine a robot in a $n \times n$ grid whose navigation is determined by the STRIPS model of Figure 2. According to this model the robot could increment its *x-coordinate* when $q0$ holds (i.e. at *even rows* if $q0$ holds initially) and decrement it when $q1$ holds (at *odd rows* if $q0$ holds initially). Apart from this particular navigation model, different action models could be defined within the same state variables (e.g. altering the way $q0$ and $q1$ are required and updated) and these models can determine different kinds of robot navigation. Given an observation of a plan execution (like the one illustrated at Figure 1) *model recognition* would aim here to identify which navigation model produced/explains that observation, despite key information is unobserved (e.g. the applied actions or the value of state variables $q0$ and $q1$).

Model recognition is of interest because once the planning model is recognized, then the model-based machinery for automated planning becomes applicable (Ghallab, Nau, and Traverso 2004). In addition, it enables identifying different kinds of automata by observing their execution. It is well-known that diverse automata representations, like *finite state controllers*, *push-down automata*, GOLOG *programs* or *reactive policies*, can be encoded as classical planning models (Baier, Fritz, and McIlraith 2007; Bonet, Palacios, and Geffner 2010; Ivankovic and Haslum 2015; Segovia-Aguas, Jiménez, and Jonsson 2017).

The paper also introduces *model recognition as planning*; a novel method to estimate the probability of a given STRIPS

```
(:action inc-x
  :parameters (?v1 ?v2)
  :precondition (and (xcoord ?v1) (next ?v1 ?v2) (q0))
  :effect (and (not (xcoord ?v1)) (xcoord ?v2)))

(:action dec-x
  :parameters (?v1 ?v2)
  :precondition (and (xcoord ?v1) (next ?v2 ?v1) (q1))
  :effect (and (not (xcoord ?v1)) (xcoord ?v2)))

(:action inc-y-even
  :parameters (?y1 ?y2)
  :precondition (and (ycoord ?y1) (next ?y1 ?y2) (q0))
  :effect (and (not (ycoord ?y1)) (ycoord ?y2)
               (not (q0)) (q1))

(:action inc-y-odd
  :parameters (?y1 ?y2)
  :precondition (and (ycoord ?y1) (next ?y1 ?y2) (q1)))
  :effect (and (not (ycoord ?y1)) (ycoord ?y2)
               (not (q1)) (q0)))

(:action dec-y-even
  :parameters (?y1 ?y2)
  :precondition (and (ycoord ?y1) (next ?y2 ?y1) (q0))
  :effect (and (not (ycoord ?y1)) (ycoord ?y2)
               (not (q0)) (q1)))

(:action dec-y-odd
  :parameters (?y1 ?y2)
  :precondition (and (ycoord ?y1) (next ?y2 ?y1) (q1))
  :effect (and (not (ycoord ?y1)) (ycoord ?y2)
               (not (q1)) (q0)))
```

Figure 2: Example of a STRIPS action model (codded in PDDL) for robot navigation in a $n \times n$ grid.

model to produce an observed plan execution. Our method is built on top of off-the-shelf classical planning algorithms and is robust to missing intermediate states and actions in the observed plan execution. We evaluate the effectiveness of *model recognition as planning* with sets of STRIPS models that represent different *automata*. All the *automata* in a set are defined within the same state variables but different *transition functions*. We show that *model recognition as planning* succeeds to identify the executed *automata* despite some state variables (e.g. the internal automata state) or the actual applied transitions are unobserved.

Background

This section formalizes the models for *classical planning* and for the *observation* of the execution of a classical plan.

Classical planning

We use F to denote the set of *fluents* (propositional variables) describing a state. A *literal* l is a valuation of a fluent $f \in F$, i.e. either $l = f$ or $l = \neg f$. A set of literals L represents a partial assignment of values to fluents (without loss of generality, we will assume that L does not assign conflicting values to any fluent). We use $\mathcal{L}(F)$ to denote the set of all literal sets on F, i.e. all partial assignments of values to fluents.

A *state* s is a full assignment of values to fluents and we explicitly include negative literals $\neg f$ in states; i.e. $|s| = |F|$, so the size of the state space is $2^{|F|}$. Like in PDDL (Fox and Long 2003), we assume that fluents F are instantiated from a set of *predicates* Ψ. Each predicate $p \in \Psi$ has an argument list of arity $ar(p)$. Given a set of *objects* Ω, the set of fluents F is induced by assigning objects in Ω to the arguments of predicates in Ψ; i.e. $F = \{p(\omega) : p \in \Psi, \omega \in \Omega^{ar(p)}\}$ such that Ω^k is the k-th Cartesian power of Ω.

A *classical planning frame* is a $\langle F, A\rangle$ pair, where F is a set of *fluents* and A is a set of *actions*. The semantics of actions $a \in A$ is specified with two functions: $\rho(s, a)$ that denotes whether the action a is *applicable* in a state s and $\theta(s, a)$ that denotes the *successor state* that results of applying the action a in a state s. In this work we compactly represent the ρ and θ functions following the STRIPS model. With this regard, an action $a \in A$ is defined by:

- $\mathsf{pre}(a) \in \mathcal{L}(F)$, the *preconditions* of a, is the set of literals that must hold for the action $a \in A$ to be applicable.
- $\mathsf{eff}^+(a) \in \mathcal{L}(F)$, the *positive effects* of a, is the set of literals that are true after the application of action $a \in A$.
- $\mathsf{eff}^-(a) \in \mathcal{L}(F)$, the *negative effects* of a, is the set of literals that are false after the application of the action.

We say that an action $a \in A$ is *applicable* in a state s iff $\mathsf{pre}(a) \subseteq s$. The result of applying a in s is the *successor state* denoted by $\theta(s, a) = \{s \setminus \mathsf{eff}^-(a)) \cup \mathsf{eff}^+(a)\}$.

A *classical planning problem* is a tuple $P = \langle F, A, I, G\rangle$, where I is an initial state and $G \in \mathcal{L}(F)$ is a goal condition. A *plan* π for P is an action sequence $\pi = \langle a_1, \ldots, a_n\rangle$ that induces the *trajectory* $\tau(\pi, P) = \langle a_1, s_1, \ldots, a_n, s_n\rangle$ such that $s_0 = I$ and, for each $1 \leq i \leq n$, a_i is applicable in s_{i-1} and generates the successor state $s_i = \theta(s_{i-1}, a_i)$. The *plan length* is denoted with $|\pi| = n$. A plan π *solves* P iff $G \subseteq s_n$, i.e., if the goal condition is satisfied at the last state reached after following the application of the plan π in the initial state I. A solution plan for P is *optimal* if it has minimum length.

Conditional effects

Conditional effects allow classical planning actions to have different semantics according to the value of the current state. This feature is useful for compactly defining our method for *model recognition as planning*.

An action $a \in A$ with conditional effects is defined as a set of *preconditions* $\mathsf{pre}(a) \in \mathcal{L}(F)$ and a set of *conditional effects* $\mathsf{cond}(a)$. Each conditional effect $C \rhd E \in \mathsf{cond}(a)$ is composed of two sets of literals $C \in \mathcal{L}(F)$, the *condition*, and $E \in \mathcal{L}(F)$, the *effect*.

An action $a \in A$ is *applicable* in a state s iff $\mathsf{pre}(a) \subseteq s$, and the *triggered effects* resulting from the action application are the effects whose conditions hold in s:

$$triggered(s, a) = \bigcup_{C \rhd E \in \mathsf{cond}(a), C \subseteq s} E,$$

The result of applying action a in state s is the *successor* state $\theta(s, a) = \{s \setminus \mathsf{eff}_c^-(s, a)) \cup \mathsf{eff}_c^+(s, a)\}$ where $\mathsf{eff}_c^-(s, a) \subseteq triggered(s, a)$ and $\mathsf{eff}_c^+(s, a) \subseteq triggered(s, a)$ are, respectively, the triggered *negative* and *positive* effects.

The observation model

Given a classical planning problem $P = \langle F, A, I, G\rangle$ and a plan π that solves P; the observation of the execution of π on P is $\mathcal{O}(\pi, P) = \langle a_1^o, s_1^o \ldots, a_l^o, s_m^o\rangle$, an interleaved combination of $1 \leq l \leq |\pi|$ observed actions and $1 \leq m \leq |\pi|$ observed states such that:

- Observed actions are *consistent* with π (Ramírez and Geffner 2009). This means that $\langle a_1^o, \ldots, a_l^o \rangle$ is a sub-sequence of the solution plan π.
- Observed states are a sequence of partial states $\langle s_1^o, \ldots, s_m^o \rangle$ that is *consistent* with the sequence of states $\langle s_0, s_1, \ldots, s_n \rangle$ traversed by the execution of π on P.

On the one hand, the initial state I is fully observed while the observed states in $\mathcal{O}(\pi, P)$ may be partial, i.e. the value of certain fluents in the intermediate states may be omitted ($|s_i^o| \leq |F|$ for every $1 \leq i \leq m$). On the other hand, the sequence of observed states $\langle s_1^o, \ldots, s_m^o \rangle$ in $\mathcal{O}(\pi, P)$ corresponds to the same sequence of states traversed by π but certain states may also be omitted ($0 \leq |s_i^o|$ for every $1 \leq i \leq m$). This means that the transitions between two consecutive observed states in $\mathcal{O}(\pi, P)$ may require the execution of more than a single action ($\theta(s_i^o, \langle a_1, \ldots, a_k \rangle) = s_{i+1}^o$, where $k \geq 1$ is unknown and unbound). Therefore we can conclude that having $\mathcal{O}(\pi, P)$ does not imply knowing the actual length of π.

Definition 1 (Φ-observation)**.** *Given a subset of fluents $\Phi \subseteq F$ we say that $\mathcal{O}(\pi, P)$ is a Φ-observation of the execution of π on P iff, for every $1 \leq i \leq m$, each observed state s_i^o only contains fluents in Φ.*

Figure 1 illustrates the six-state Φ-observation $\{$`<(xcoord 2)(ycoord 1)>, <(xcoord 3)(ycoord 1)>, <(xcoord 4)(ycoord 1)>, <(xcoord 5)(ycoord 1)>, <(xcoord 5)(ycoord 2)>, <(xcoord 4)(ycoord 2)>`$\}$. In this case Φ only contains fluents of the kind `(xcoord ?v)` and `(ycoord ?v)` so that the value of the remaining fluents, namely `(next ?v1 ?v2)`, `(q0)` and `(q1)`, is unknown.

Model Recognition

The *model recognition* task is a tuple $\langle P, M, \mathcal{O} \rangle$ where:

- $P = \langle F, A[\cdot], I, G \rangle$ is a *classical planning problem* s.t. the semantics of actions $a \in A[\cdot]$ is unknown (i.e. the corresponding ρ and/or θ functions are undefined).
- $M = \{\mathcal{M}_1, \ldots, \mathcal{M}_m\}$ is a finite non-empty *set of models* for the actions in $A[\cdot]$ s.t., each model in $\mathcal{M} \in M$, defines different function pairs $\langle \rho, \theta \rangle$ over the state variables F.
- $\mathcal{O}(\pi, P)$ is an *observation* of the execution of an unknown solution plan π for the planning problem P.

Model recognition can be understood as a *classification task* where each class is represented with a different planning model $\mathcal{M} \in M$ and the observed plan execution $\mathcal{O}(\pi, P)$, is the single example to classify. The planning model associated to each class acts as the corresponding *class prototype* and it summarizes any observation of a plan execution that could be synthesized with that model (i.e. all the examples that belong to that class).

We follow the *naive Bayes classifier* to assign a model $\mathcal{M} \in M$ to the given observation $\mathcal{O}(\pi, P)$ with respect to the expression:

$$argmax_{\mathcal{M} \in M} P(\mathcal{O}|\mathcal{M}) \times P(\mathcal{M}). \tag{1}$$

$$\begin{tikzcd} \mathcal{M} \arrow[r] & \mathcal{M}' \arrow[r] & \tau \arrow[r] & \mathcal{O} \end{tikzcd}$$

Figure 3: *Bayesian network* abstracting that model $\mathcal{M}$ can be *transformed* into a model $\mathcal{M}'$ that is able to produce a trajectory $\tau(\pi, P)$ that reaches the goals in P and is consistent with the observation $\mathcal{O}(\pi, P)$.

The *solution* to the *model recognition* task is the subset of models in M that maximizes expression (1).

Formulating the $P(\mathcal{O}|\mathcal{M})$ *likelihood*

The $P(\mathcal{M})$ probability expresses whether one model is known to be a priori more likely than the others. If this probability is not given as input it is reasonable to assume that, *a priori*, all models are equiprobable. The challenge in our formulation for *model recognition* is the definition of the $P(\mathcal{O}|\mathcal{M})$ *likelihood* that expresses the probability of observing $\mathcal{O}(\pi, P)$ when $\mathcal{M}$ is the planning model.

Our approach to formulate the $P(\mathcal{O}|\mathcal{M})$ *likelihood* is to assess the cost of *transforming* $\mathcal{M}$ into a model $\mathcal{M}'$ that can produce a trajectory $\tau(\pi, P)$ that that reaches the goals in P and is consistent with the observation $\mathcal{O}(\pi, P)$. Figure 3 shows the four-variable *Bayesian network* abstracting this process. Regarding this network we have the following formulation:

$$P(\mathcal{O}|\mathcal{M}) = \sum_{\mathcal{M}'} \sum_{\tau} P(\mathcal{M}'|\mathcal{M}) P(\tau|\mathcal{M}') P(\mathcal{O}|\tau), \tag{2}$$

where τ ranges over all the trajectories that can be synthesized with a model $\mathcal{M}'$ and $\mathcal{M}'$ ranges over all the models that can be generated *transforming* $\mathcal{M}$.

Edit distances are similarity metrics, traditionally computed over *strings* or *graphs*, and that has been proved successful for *pattern recognition* (Masek and Paterson 1980; Bunke 1997). We assess the cost of *transforming* a classical planning model $\mathcal{M}$ into a model $\mathcal{M}'$ formalizing and computing *edit distances* that, in this work, are referred to STRIPS planning models.

Recognition of STRIPS models

Here we analyze the particular instantiation of the *model recognition* task where the semantics of actions (i.e. ρ and θ functions) are specified with STRIPS action schema. We start formalizing the STRIPS schema and define the full space of possible STRIPS schema. Eventually, we introduce an *edit distance* for STRIPS schema to estimate the $P(\mathcal{O}|\mathcal{M})$ likelihoods for classical planning models.

Well-defined STRIPS action schema

STRIPS action schema provide a compact representation for specifying classical planning models. Figure 2 shows six STRIPS action schema, codded in PDDL, that determine a particular kind of robot navigation in $n \times n$ grids.

A STRIPS *action schema* ξ is defined by a list of *parameters* $pars(\xi)$, and three list of predicates (namely $pre(\xi)$, $del(\xi)$ and $add(\xi)$) that shape the kind of fluents that can appear in the *preconditions*, *negative effects* and *positive effects* of the actions induced from that schema.

We say that two STRIPS schemes ξ and ξ' are *comparable* iff $pars(\xi) = pars(\xi')$, both share the same list of parameters. For instance, we claim that the six action schema of Figure 2 are *comparable* while, for example, the `stack(?v1,?v2)` and `pickup(?v1)` schemes from the four operator *blocksworld* (Slaney and Thiébaux 2001) are not. Last but not least, we say that two STRIPS models $\mathcal{M}$ and $\mathcal{M}'$ are *comparable* iff there exists a bijective function $\mathcal{M} \mapsto \mathcal{M}^*$ that maps every action schema $\xi \in \mathcal{M}$ to a comparable schema $\xi' \in \mathcal{M}'$ and vice versa.

Given a STRIPS action schema ξ, let us define an additional set of objects ($\Omega \cap \Omega_\xi = \emptyset$), that we denote as *variable names*, and that contains one variable name for each parameter in $pars(\xi)$, that is $\Omega_\xi = \{v_i\}_{i=1}^{|pars(\xi)|}$. For any of the six schema defined in Figure 2, $|pars(\xi)| = 2$ so $\Omega_\xi = \{v_1, v_2\}$.

Given a STRIPS action schema ξ and a set of *predicates* Ψ that shape the propositional state variables. The set of FOL interpretations of Ψ over the corresponding Ω_ξ objects (*variable names* for schema ξ), confines the elements that can appear in the $pre(\xi)$, $del(\xi)$ and $add(\xi)$ lists. We denote this set of FOL interpretations as $\mathcal{I}_{\Psi,\xi}$. For any of the six schema defined in Figure 2 the $\mathcal{I}_{\Psi,\xi}$ set contains the same ten elements, $\mathcal{I}_{\Psi,\xi}$ ={`xcoord(`v_1`)`, `xcoord(`v_2`)`, `ycoord(`v_1`)`, `ycoord(`v_2`)`, `q0()`, `q1()`, `next(`v_1`,`v_1`)`, `next(`v_1`,`v_2`)`, `next(`v_2`,`v_1`)`, `next(`v_2`,`v_2`)`)}.

Despite any element from $\mathcal{I}_{\Psi,\xi}$ can *a priori* appear in the $pre(\xi)$, $del(\xi)$ and $add(\xi)$ of a schema ξ, the space of possible STRIPS schema is constrained by the set $\mathcal{C}$ that includes:

- *Syntactic constraints*. STRIPS constraints require negative effects appearing as preconditions, negative effects cannot be positive effects at the same time and also, positive effects cannot appear as preconditions. Formally, $del(\xi) \subseteq pre(\xi)$, $del(\xi) \cap add(\xi) = \emptyset$ and $pre(\xi) \cap add(\xi) = \emptyset$). Considering exclusively these syntactic constraints, the size of the space of possible STRIPS schema is given by $2^{2\times|\mathcal{I}_{\Psi,\xi}|}$. For every action schema in the navigation model of Figure 2 then $2^{2\times 10} = 1,048,576$.
- *Domain-specific constraints*. One can also introduce domain-specific knowledge to more precisely constrain the space of possible schema. For instance in a *robot navigation* model like the one in Figure 2, `q0()` and `q1()` are exclusive so they cannot hold at the same time in a $pre(\xi)$/$del(\xi)$/$add(\xi)$ list. Further, `next(`v_1`,`v_1`)` and `next(`v_2`,`v_2`)` will not appear at any of these lists because the `next` predicate is coding the *successor* function for *natural numbers*. These domain-specific constraints reduces the size of the space of possible action schema to $2^{2\times 7} = 16,384$ (for every schema).

Definition 2 (Well-defined STRIPS action schema)**.** *Given a set of* predicates Ψ*, a list of action* parameters $pars(\xi)$*, and set of FOL constraints* $\mathcal{C}$ *we say that* ξ *is a* **well-defined STRIPS action schema** *iff its three lists* $pre(\xi) \subseteq \mathcal{I}_{\Psi,\xi}$*,* $del(\xi) \subseteq \mathcal{I}_{\Psi,\xi}$ *and* $add(\xi) \subseteq \mathcal{I}_{\Psi,\xi}$ *only contain elements in* $\mathcal{I}_{\Psi,\xi}$ *and they satisfy all the constraints in* $\mathcal{C}$*.*

We say a classical planning action model is *well-defined* if all its corresponding STRIPS action schema are *well-defined*.

Edit distances for STRIPS action models

First, we define two edit *operations* on a schema $\xi \in \mathcal{M}$ that belongs to a STRIPS action model $\mathcal{M} \in M$:

- *Deletion*. An element is removed from any of these three lists $pre(\xi)$, $del(\xi)$ and $add(\xi)$ of the operator schema $\xi \in \mathcal{M}$ such that the resulting schema is a *well-defined* STRIPS action schema.
- *Insertion*. An element in $\mathcal{I}_{\Psi,\xi}$ is added to any of these three lists $pre(\xi)$, $del(\xi)$ and $add(\xi)$ of the operator schema $\xi \in \mathcal{M}$ s.t. the resulting schema is *well-defined*.

We can now formalize an *edit distance* that quantifies how similar two given STRIPS action models are. The distance is symmetric and meets the *metric axioms* provided that the two *edit operations*, deletion and insertion, have the same positive cost.

Definition 3 (Edit distance)**.** *Let* $\mathcal{M}$ *and* $\mathcal{M}'$ *be two* comparable *and* well-defined STRIPS *action models defined within the same set of predicates* Ψ*. The* **edit distance** $\delta(\mathcal{M},\mathcal{M}')$ *is the minimum number of* edit operations *that is required to transform* $\mathcal{M}$ *into* $\mathcal{M}'$*.*

Since $\mathcal{I}_{\Psi,\xi}$ are bound sets, the maximum number of edits that can be introduced to a given action model is bound as well. The **maximum edit distance** of a STRIPS model $\mathcal{M}$ built within predicates Ψ is $\delta(\mathcal{M},*) = \sum_{\xi\in\mathcal{M}} 3 \times |\mathcal{I}_{\Psi,\xi}|$ (note that, if we consider the syntactic STRIPS constraints, $\delta(\mathcal{M},*) = \sum_{\xi\in\mathcal{M}} 2 \times |\mathcal{I}_{\Psi,\xi}|$).

An observation $\mathcal{O}(\pi, P)$ of the execution of a plan generated with an action model $\mathcal{M}$, reflects *semantic knowledge* that constrains further the space of the possible schema $\xi \in \mathcal{M}$. In this case, we talk about a third kind of constraints that we call *observation constraints* and that can also be included into the $\mathcal{C}$ set. In addition, *observation constraints* allow us to define an edit distance to elicit the $P(\mathcal{O}|\mathcal{M})$ likelihoods. It can be argued that the shorter this distance the better the given model explains the given observation.

Definition 4 (Observation edit distance)**.** *Given* $\mathcal{O}(\pi, P)$*, an observation of the execution of a plan for solving* P *and a* STRIPS *action model* $\mathcal{M}$*, defined within the same predicates* Ψ*. The* **observation edit distance***,* $\delta(\mathcal{M},\mathcal{O})$*, is the minimal edit distance from* $\mathcal{M}$ *to any* comparable *and* well-defined *model* $\mathcal{M}'$ *s.t.* $\mathcal{M}'$ *produces a trajectory* $\tau(\pi, P)$ *that reaches the goals in* P *and is* consistent *with* $\mathcal{O}(\pi, P)$*;*

$$\delta(\mathcal{M},\mathcal{O}) = \min_{\forall \mathcal{M}' \rightarrow \mathcal{O}} \delta(\mathcal{M},\mathcal{M}')$$

Note that the *observation edit distance* could also be defined assessing the edition required by the observed plan execution to match the given model. This implies defining *edit operations* that modify the observation $\mathcal{O}(\pi, P)$ instead of $\mathcal{M}$ (Sohrabi, Riabov, and Udrea 2016). Our definition of the *observation edit distance* is more practical since normally $\mathcal{I}_{\Psi,\xi}$ is much smaller than F. In practice, the number of *variable objects* should be smaller than the number of objects in a planning problem.

Definition 5 (*Closest consistent models*). *Given a model $\mathcal{M}$. The set M^* of* **closest consistent models** *is the set of action models $\mathcal{M}'$, closest to $\mathcal{M}$ in terms of editions, and that can produce a trajectory $\tau(\pi, P)$ that reaches the goals in P and is* consistent *with $\mathcal{O}(\pi, P)$;*

$$\underset{\forall \mathcal{M}' \rightarrow \mathcal{O}}{\arg\min}\ \delta(\mathcal{M}, \mathcal{M}')$$

Approximating the $P(\mathcal{O}|\mathcal{M})$ *likelihood*

Following equation (2), the exact computation of $P(\mathcal{O}|\mathcal{M})$ is intractable. For most planning problems the set of trajectories consistent with an arbitrary observation can easily be huge (infinite in the case of planning problems without dead-ends). Even worse, the number of models $\mathcal{M}'$ that can be generated *editing* a given classical planning model $\mathcal{M}$ explodes combinatorially.

Instead, our approach is to formulate an effectively estimate of $P(\mathcal{O}|\mathcal{M})$.

Full observability of the executed plan. The *full observability of the executed plan* is a too strong assumption for *model recognition* but it allows us to understand how to build a reasonable estimate of the $P(\mathcal{O}|\mathcal{M})$ *likelihood* for the general case.

In this baseline scenario there is only a single possible trajectory $\tau^*(\pi, P)$ *consistent* with the given observation (because of full observability), so $P(\mathcal{O}|\tau^*) = 1$. Further, there is also a single model M^* that can exactly produce that trajectory (otherwise models are identical, at least, in the actions relevant to the observed trajectory so they can be considered the same model).

If there is a single possible trajectory and a single possible model $\mathcal{M}^*$ consistent with the input observation then, the probabilities of expression (2) are not added up and expression (1) simplifies to:

$$argmax_{\mathcal{M} \in M} P(\mathcal{M}^*|\mathcal{M}) \times P(\mathcal{M}). \tag{3}$$

Note that the term $P(\tau|\mathcal{M}^*)$ is taken out of the maximization because is independent of the model $\mathcal{M} \in M$.

Partial observability of the executed plan. In a similar way, an approximation of the $P(\mathcal{O}|\mathcal{M})$ *likelihood* can be built for the general case where the executed plan is partially observed. To deal with this general scenario we add the following two assumptions:

1. The *principle of rationality*: The expectation that intentional agents will tend to choose actions that achieve their goals most efficiently, given their knowledge of the actual world state.
2. *The best plans for P are unique or have different cost.*

Similar assumptions were taken in previous work for *goal recognition* (Ramírez 2012). They allow us to claim that the sum in equation (2) is dominated by its largest term; the term corresponding to the shortest trajectory τ^* (which is assumed to be unique) consistent with the observation, so other terms in the sum are not added up. Again, there is a single model M^* that can produce τ^*. With this regard, both $P(O|\tau^*)$ and $P(\tau^*|\mathcal{M}^*)$ are independent of $\mathcal{M} \in M$ so equation (1) simplifies once again to equation (3) under the previous assumptions.

Note that the more complete is the observation of the plan execution the more accurate is our estimate because it becomes more reasonable to assume that there is only one plan consistent with the given observation.

The $P(\mathcal{M}^*|\mathcal{M})$ probability distribution. $P(\mathcal{M}'|\mathcal{M})$ indicates the probability of *transforming* a classical planning model $\mathcal{M}$ into a model $\mathcal{M}'$ by exclusively using the two *edit operations* previously defined, *deletion* and *insertion*.

We know from *pattern recognition* that, if string symbols are uniformly random and independent, the distance of a given string to a fixed string follows a *Binomial distribution*. Moreover, the probability that a particular string will be within a distance D is the Cumulative Distribution Function (CDF) for the *Binomial distribution* (Wilcox 1981):

$$CDF(D) = \sum_{d=0}^{D} \binom{N}{d} \times p^d \times (1-p)^{N-d} \tag{4}$$

where N is the length of the string, and $p < 0.5$ since the cost of applying an *edit operation* is higher than do not applying it.

With this regard, and considering that STRIPS models $\mathcal{M} \in M$ can be encoded with a propositional representation of fixed length N, we formulate the $P(\mathcal{M}'|\mathcal{M})$ probability distribution mapping the distance $\delta(\mathcal{M}, \mathcal{M}')$ according to equation:

$$P(\mathcal{M}'|\mathcal{M}) = p^d \times (1-p)^{N-d} \tag{5}$$

where $d = \delta(\mathcal{M}, \mathcal{M}')$. Likewise we can formulate $P(\mathcal{M}^*|\mathcal{M})$ with this same equation (5) but instead, the parameter d is given by the *observation distance*, $d = \delta(\mathcal{M}, \mathcal{O})$.

In other words, we are modeling the edition of a STRIPS planning model as a *Bernoulli process* in which there is a sequence of N independent events representing N binary decisions (the N possible applications of the edition operations) such that for every event $P(X = \top) = p$ and $P(X = \perp) = 1 - p$.

Model Recognition as classical planning

This section shows that, for STRIPS planning models, $\delta(\mathcal{M}, \mathcal{O})$ can be computed (and hence an approximation of the $P(\mathcal{O}|\mathcal{M})$ likelihood) with a a classical planning compilation.

The compilation is an extension of the classical planning compilation for the learning of STRIPS planning models (Aineto, Jiménez, and Onaindia 2018). The intuition behind this compilation is that a solution to the resulting classical planning task is a sequence of actions that:

1. **Edits the action model $\mathcal{M}$ to build** $\mathcal{M}'$. A solution plan starts with a *prefix* that modifies the preconditions and effects of the action schemes in $\mathcal{M}$ using to the two *edit operations* defined above, *deletion* and *insertion*.
2. **Validates the edited model** $\mathcal{M}'$. The solution plan continues with a postfix that:

```
00 : (insert_pre_xcoord_v1_inc-x)        07 : (validate_0)
01 : (insert_pre_next_v1_v2_inc-x)       08 : (editable_inc-x 1 2)
02 : (insert_pre_q0_inc-x)               09 : (editable_inc-x 2 3)
03 : (delete_del_xcoord_v2_inc-x)        10 : (editable_inc-x 3 4)
04 : (delete_add_xcoord_v1_inc-x)        11 : (editable_inc-x 4 5)
05 : (insert_del_xcoord_v1_inc-x)        12 : (editable_inc-y-even 1 2)
06 : (insert_add_xcoord_v2_inc-x)        13 : (editable_dec-x 5 4)
                                         14 : (validate_1)
```

Figure 4: Plan for editing (steps [0-6]) and validating (steps [7-14]) the model of Figure2 when schema `inc-x` has no *preconditions* and positive/negative *effects* are swapped wrt Figure2.

(a) Induces a solution plan π for the original classical planning problem P.

(b) Validates that $\mathcal{O}(\pi, P)$ is an observation of the execution of π on the classical planning problem P.

Figure 4 shows the plan with a prefix (steps [0,6]) for editing the planning model of Figure2 when its schema `inc-x` is defined without preconditions and its positive/negative effects are swapped wrt Figure 2. The postfix of the plan (steps [7,14]) validates the edited action model at the observation of the plan execution illustrated at Figure 1. Note that our interest is not in $\mathcal{M}'$, the edited model resulting from the compilation, but in the number of required *edit operations* (insertions and deletions) required by $\mathcal{M}'$ to be validated. In the example of Figure 4, $\delta(\mathcal{M}, \mathcal{O}) = 7$.

A propositional encoding for STRIPS action schema

Given a STRIPS action schema ξ, a propositional encoding for the *preconditions*, *negative* and *positive* effects of that schema can be represented with fluents of the kind $[pre|del|add]\_e\_\xi$ such that $e \in \mathcal{I}_{\Psi,\xi}$ is a single element from the set of interpretations of predicates Ψ over the corresponding variable names Ω_ξ. Figure 5 shows the propositional encoding for the six action schema defined in Figure 2.

The interest of having a propositional encoding for STRIPS action schema is that, using *conditional effects*, it allows to compactly define *editable actions*. Actions whose semantics is given by the value of the $[pre|del|add]\_e\_\xi$ fluents at the current state. Given an operator schema $\xi \in \mathcal{M}$ its *editable* version is formalized as:

$$\begin{aligned}
\mathsf{pre}(\mathsf{editable}_\xi) =& \{pre\_e\_\xi \implies e\}_{\forall e \in \mathcal{I}_{\Psi,\xi}} \\
\mathsf{cond}(\mathsf{editable}_\xi) =& \{del\_e\_\xi\} \rhd \{\neg e\}_{\forall e \in \mathcal{I}_{\Psi,\xi}}, \\
& \{add\_e\_\xi\}\} \rhd \{e\}_{\forall e \in \mathcal{I}_{\Psi,\xi}}.
\end{aligned}$$

Figure 6 shows the PDDL encoding of the *editable* `inc-x(?v1,?v2)` schema for robot navigation in a $n \times n$ grid (Figure 2). Note that this editable schema, when the fluents of Figure 5 hold, behaves exactly as defined in Figure 2.

The compilation formalization

Conditional effects allow us to compactly define our compilation for computing $\delta(\mathcal{M}, \mathcal{O})$ and hence, estimate the $P(\mathcal{O}|\mathcal{M})$ likelihood. Given a STRIPS model $\mathcal{M} \in M$ and the observation $\mathcal{O}(\pi, P)$ of the execution of a plan for solving $P = \langle F, A, I, G\rangle$, our compilation outputs a classical planning task with conditional effects $P' = \langle F', A', I', G'\rangle$:

- F' extends the original fluents F with:

```
;;; Propositional encoding for inc-x(?v1 ?v2)
(pre_xcoord_v1_inc-x) (pre_next_v1_v2_inc-x)
(pre_q0__inc-x)
(del_xcoord_v1_inc-x) (add_xcoord_v2_inc-x)

;;; Propositional encoding for dec-x(?v1 ?v2)
(pre_xcoord_v1_dec-x) (pre_next_v2_v1_dec-x)
(pre_q1___dec-x)
(del_xcoord_v1_dec-x) (add_xcoord_v2_dec-x)

;;; Propositional encoding for inc-y-even(?v1 ?v2)
(pre_ycoord_v1_inc-y-even) (pre_next_v1_v2_inc-y-even)
(pre_q0__inc-y-even)
(del_ycoord_v1_inc-y-even) (del_q0__inc-y-even)
(add_ycoord_v2_inc-y-even) (add_q1__inc-y-even)

;;; Propositional encoding for inc-y-odd(?v1 ?v2)
(pre_ycoord_v1_inc-y-odd) (pre_next_v1_v2_inc-y-odd)
(pre_q0__inc-y-odd)
(del_ycoord_v1_inc-y-odd) (del_q1__inc-y-odd)
(add_ycoord_v2_inc-y-odd) (add_q0__inc-y-odd)

;;; Propositional encoding for dec-y-even(v1 ?v2)
(pre_ycoord_v1_dec-y-even) (pre_next_v2_v1_dec-y-even)
(pre_q0__dec-y-even)
(del_ycoord_v1_dec-y-even) (del_q0__dec-y-even)
(add_ycoord_v2_dec-y-even) (add_q1__dec-y-even)

;;; Propositional encoding for dec-y-odd(?v1 ?v2)
(pre_ycoord_v1_dec-y-odd) (pre_next_v2_v1_dec-y-odd)
(pre_q1__dec-y-odd)
(del_ycoord_v1_dec-y-odd) (del_q1__dec-y-odd)
(add_ycoord_v2_dec-y-odd) (add_q0__dec-y-odd)
```

Figure 5: Propositional encoding for the six schema from Figure 2.

  - Fluents $[pre|del|add]\_e\_\xi$ to model the possible STRIPS schema.
  - The fluents to code the *observation constraints*:
    * $F_\pi = \{plan(a_i, i)\}_{1 \leq i \leq l}$ to code the i^{th} action in $\mathcal{O}(\pi, P)$. The static facts $next_{i,i+1}$ and the fluents at_i, $1 \leq i < l$, are also added to iterate through the l actions in $\mathcal{O}(\pi, P)$.
    * The fluents $\{test_j\}_{1 \leq j \leq m}$, indicating the state observation $s_j \in \mathcal{O}(\pi, P)$ where the action model is validated.
  - The fluents $mode_{edit}$ and $mode_{val}$ to indicate whether the operator schema are edited or validated.
- I' extends the original initial state I with the fluent $mode_{edit}$ set to true as well as the fluents F_π plus fluents at_1 and $\{next_{i,i+1}\}$, $1 \leq i < l$, for tracking the plan step where the action model is validated. Our compilation assumes that initially $\mathcal{M}'$ is defined as $\mathcal{M}$. Therefore fluents $[pre|del|add]\_e\_\xi$ hold as given by $\mathcal{M}$.
- $G' = G \bigcup \{at_n, test_m\}$.
- A' comprises three kinds of actions with conditional effects:
  1. The *editable* version of the original actions given by $\mathcal{M}$. This actions have now an extra preconditions because they can only be applied in the *validation* mode (i.e. when $mode_{val}$ holds). When the observation $\mathcal{O}(\pi, P)$ includes observed actions, they also include the extra conditional effects $\{at_i, plan(a_i, i)\} \rhd \{\neg at_i, at_{i+1}\}_{\forall i \in [1,l]}$ to validate that actions are applied, exclusively, in the same order as they appear in $\mathcal{O}(\pi, P)$.

```
(:action editable_inc-x
  :parameters (?v1 ?v2)
  :precondition
    (and (or (not (pre_xcoord_v1_inc-x)) (xcoord ?v1))
         (or (not (pre_xcoord_v2_inc-x)) (xcoord ?v2))
         (or (not (pre_ycoord_v1_inc-x)) (xcoord ?v1))
         (or (not (pre_ycoord_v2_inc-x)) (xcoord ?v2))
         (or (not (pre_q0__inc-x)) (q0))
         (or (not (pre_q1__inc-x)) (q1)))
         (or (not (pre_next_v1_v1_inc-x)) (next ?v1 ?v1)))
         (or (not (pre_next_v1_v2_inc-x)) (next ?v1 ?v2)))
         (or (not (pre_next_v2_v1_inc-x)) (next ?v2 ?v1)))
         (or (not (pre_next_v2_v2_inc-x)) (next ?v2 ?v2))))
    :effect (and
       (when (del_xcoord_v1_inc-x) (not (xcoord ?v1)))
       (when (del_xcoord_v2_inc-x) (not (xcoord ?v2)))
       (when (del_ycoord_v1_inc-x) (not (xcoord ?v1)))
       (when (del_ycoord_v2_inc-x) (not (xcoord ?v2)))
       (when (del_q0__inc-x) (not (q0)))
       (when (del_q1__inc-x) (not (q1)))
       (when (del_next_v1_v1_inc-x) (not (next ?v1 ?v1)))
       (when (del_next_v1_v2_inc-x) (not (next ?v1 ?v2)))
       (when (del_next_v2_v1_inc-x) (not (next ?v2 ?v1)))
       (when (del_next_v2_v2_inc-x) (not (next ?v2 ?v2)))

       (when (add_xcoord_v1_inc-x) (xcoord ?v1))
       (when (add_xcoord_v2_inc-x) (xcoord ?v2))
       (when (add_ycoord_v1_inc-x) (xcoord ?v1))
       (when (add_ycoord_v2_inc-x) (xcoord ?v2))
       (when (add_q0__inc-x) (q0))
       (when (add_q1__inc-x) (q1))
       (when (add_next_v1_v1_inc-x) (next ?v1 ?v1))
       (when (add_next_v1_v2_inc-x) (next ?v1 ?v2))
       (when (add_next_v2_v1_inc-x) (next ?v2 ?v1))
       (when (add_next_v2_v2_inc-x) (next ?v2 ?v2)))
```

Figure 6: Editable version of the `inc-x(?v1,?v2)` schema for robot navigation in a $n \times n$ grid.

2. Actions for *editing* operator schema $\xi \in \mathcal{M}$. This includes the actions for *inserting* a new *precondition* into an action schema $\xi \in \mathcal{M}$ and for inserting a new *negative* or *positive* effect into the action schema $\xi \in \mathcal{M}$

$$\begin{aligned}\mathsf{pre}(\mathsf{insertPre}_{\mathsf{e},\xi}) =& \{\neg pre\_e\_\xi, \neg del\_e\_\xi, \\ & \neg add\_e\_\xi, mode_{edit}\}, \\ \mathsf{cond}(\mathsf{insertPre}_{\mathsf{e},\xi}) =& \{\emptyset\} \rhd \{pre\_e\_\xi\}.\end{aligned}$$

$$\begin{aligned}\mathsf{pre}(\mathsf{insertEff}_{\mathsf{e},\xi}) =& \{\neg del\_e\_\xi, \neg add\_e\_\xi, mode_{edit}\}, \\ \mathsf{cond}(\mathsf{insertEff}_{\mathsf{e},\xi}) =& \{pre\_e\_\xi\} \rhd \{del\_e\_\xi\}, \\ & \{\neg pre\_e\_\xi\} \rhd \{add\_e\_\xi\}.\end{aligned}$$

Besides these actions, A' also contains the actions for *deleting* a *precondition* and a negative/positive *effect*.

3. Actions for *validating* the edited models at the s_j observed states, $0 \leq j < m$.

$$\begin{aligned}\mathsf{pre}(\mathsf{validate_j}) =& s_j \cup \{test_{j-1}\}, \\ \mathsf{cond}(\mathsf{validate_j}) =& \{\emptyset\} \rhd \{\neg test_{j-1}, test_j, \\ & \{mode_{edit}\} \rhd \{\neg mode_{edit}, mode_{val}\}.\end{aligned}$$

Evaluation

To evaluate the empirical performance of *model recognition as planning* we collect a set M of possible STRIPS models, that share the same state variables but define different action models. Then, we randomly choose one of these models $\mathcal{M} \in M$ and use it to produce an observation $\mathcal{O}(\pi, P)$ of a plan execution. Finally, we follow our *model recognition as planning* method to identify the model $\mathcal{M} \in M$ that produced $\mathcal{O}(\pi, P)$. The experiment is repeated for models of different kind and different observability of the given plan execution.

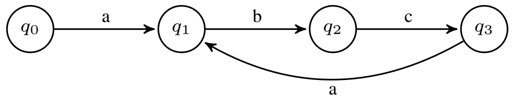
$$\begin{tikzcd} q_0 \arrow[r, "a"] & q_1 \arrow[r, "b"] & q_2 \arrow[r, "c"] & q_3 \arrow[lll, phantom, ""] \arrow[ll, bend left, "a"] \end{tikzcd}$$

Figure 7: Four-symbol four-state *regular automata* for recognizing the $\{(abc)^n : n \geq 1\}$ language ($\underline{q_3}$ is the acceptor state).

Reproducibility. MADAGASCAR is the classical planner we used to solve the instances that result from our compilations for its ability to deal with dead-ends (Rintanen 2014). Due to its SAT-based nature, MADAGASCAR can apply the actions for editing preconditions in a single planning step (in parallel) because there is no interaction between them. Actions for editing effects can also be applied in a single planning step, thus significantly reducing the planning horizon.

The compilation source code, evaluation scripts and benchmarks (including the used training and test sets) are fully available at this anonymous repository so any experimental data reported in the paper can be reproduced.

Recognition of *Regular Automata*. In this experiment the models in M represent different *regular automata*. Figure 7 illustrate a four-symbol four-state *regular automata* for recognizing the $\{(abc)^n : n \geq 1\}$ language. The *input alphabet* is $\Sigma = \{a, b, c, \Box\}$, and the machine states are $Q = \{q_0, q_1, q_2, \underline{q_3}\}$ (where $\underline{q_3}$ is the only acceptor state).

We generated a M set of different STRIPS models that encode different *regular automata*. Each $\mathcal{M} \in M$ encodes a different *regular automata*. Each automata transition is encoded with a planning action. For instance, the execution of the *regular automata* defined in Figure 7, with the sequence of input symbols $abcabc$, produces the following six-action plan $(a, q_0 \rightarrow q_1)$, $(b, q_1 \rightarrow q_2)$, $(c, q_2 \rightarrow q_3)$, $(a, q_3 \rightarrow q_1)$, $(b, q_1 \rightarrow q_2)$, $(c, q_2 \rightarrow q_3)$.

Here we assume that the actual applied transitions are unknown as well as the internal machine state. Assuming that the actual applied transitions is unknown means that the observation $\mathcal{O}(\pi, P)$ of the execution of a regular automata contains no actions, it is simply a sequence of states $\mathcal{O}(\pi, P) = \langle s_1, \ldots, s_m \rangle$. Assuming that the internal machine state is unknown means that $\mathcal{O}(\pi, P)$ is a Φ-observation and that the Φ subset does not contain (q) fluents, with $q \in Q$ and $q \neq q_0$.

Recognition of navigation models. In this experiment the given models in M represent different navigation models that are computed as the cross product of a regular automata with a four-operator STRIPS model for navigating a $n \times n$ grid.

In this case the regular automata constrain the applications of the navigation actions producing different navigation policies e.g. like the one in Figure 2. Given an observation of a plan execution, like the one illustrated at Figure 1, here the task is to identify which navigation model produced that observation, despite the the applied actions are unobserved. In addition, for each observed state, only the value of fluents encoding the x and y coordinates of the agent are known while the value of the regular automata conditioning the navigation policy is unknown.

Results

Conclusions

This paper formalized the *model recognition* task and proposed, *model recognition as planning*, a method built on top of off-the-shelf classical planning algorithms to estimate the probability of a STRIPS model to produce a partial observation of a plan execution. The paper shows the effectiveness of *model recognition as planning* in a set of STRIPS models encoding different kinds of *automata*. *Model recognition as planning* succeeds to identify the executed automata despite the internal machine state or actual applied transitions, are unobserved.

Previous work on the learning of STRIPS action models also defined semantic error metrics to quantify the errors of a model with respect to the observation of a plan execution (Yang, Wu, and Jiang 2007). Our approach for quantifying this error is based on the definition of a edit distance for the model which allow us to not accumulate the repetition of errors coming from the the same model flaw. A related approach is recently followed for *model reconciliation* (Chakraborti et al. 2017) where model edition is used to conform the PDDL models of two different agents. Also related to this paper is the work on *Goal Recognition Design* but in that work the action model is given in advance and fixed (Keren, Gal, and Karpas 2014).

References

Aineto, D.; Jiménez, S.; and Onaindia, E. 2018. Learning strips action models with classical planning.

Baier, J. A.; Fritz, C.; and McIlraith, S. A. 2007. Exploiting procedural domain control knowledge in state-of-the-art planners. In *ICAPS*, 26–33.

Bonet, B.; Palacios, H.; and Geffner, H. 2010. Automatic derivation of finite-state machines for behavior control. In *AAAI Conference on Artificial Intelligence*.

Bunke, H. 1997. On a relation between graph edit distance and maximum common subgraph. *Pattern Recognition Letters* 18(8):689–694.

Carberry, S. 2001. Techniques for plan recognition. *User Modeling and User-Adapted Interaction* 11(1-2):31–48.

Chakraborti, T.; Sreedharan, S.; Zhang, Y.; and Kambhampati, S. 2017. Plan explanations as model reconciliation: Moving beyond explanation as soliloquy. In *IJCAI*.

Fox, M., and Long, D. 2003. PDDL2.1: An extension to PDDL for expressing temporal planning domains. *Journal of Artificial Intelligence Research* 20:61–124.

Geffner, H., and Bonet, B. 2013. A concise introduction to models and methods for automated planning.

Ghallab, M.; Nau, D.; and Traverso, P. 2004. *Automated Planning: theory and practice*. Elsevier.

Ivankovic, F., and Haslum, P. 2015. Optimal planning with axioms. In *Twenty-Fourth International Joint Conference on Artificial Intelligence*.

Keren, S.; Gal, A.; and Karpas, E. 2014. Goal recognition design. In *ICAPS*.

Masek, W. J., and Paterson, M. S. 1980. A faster algorithm computing string edit distances. *J. Comput. Syst. Sci.* 20(1):18–31.

Ramírez, M., and Geffner, H. 2009. Plan recognition as planning. In *International Joint conference on Artifical Intelligence*, 1778–1783.

Ramírez, M. 2012. *Plan recognition as planning*. Ph.D. Dissertation, Universitat Pompeu Fabra.

Rintanen, J. 2014. Madagascar: Scalable planning with sat. *Proceedings of the 8th International Planning Competition (IPC-2014)*.

Segovia-Aguas, J.; Jiménez, S.; and Jonsson, A. 2017. Generating context-free grammars using classical planning. In *International Joint Conference on Artificial Intelligence, ICAPS-17*.

Slaney, J., and Thiébaux, S. 2001. Blocks world revisited. *Artificial Intelligence* 125(1-2):119–153.

Sohrabi, S.; Riabov, A. V.; and Udrea, O. 2016. Plan recognition as planning revisited. In *IJCAI*, 3258–3264.

Sukthankar, G.; Geib, C.; Bui, H. H.; Pynadath, D.; and Goldman, R. P. 2014. *Plan, activity, and intent recognition: Theory and practice*. Newnes.

Wilcox, R. R. 1981. A review of the beta-binomial model and its extensions. *Journal of Educational Statistics* 6(1):3–32.

Yang, Q.; Wu, K.; and Jiang, Y. 2007. Learning action models from plan examples using weighted max-sat. *Artificial Intelligence* 171(2-3):107–143.