Seung Jun Baek

Topics: Discrete-time LTI systems

1. A discrete-time signal $x[n]$ is shown in Fig 1. Sketch and label carefully each of the following signals :

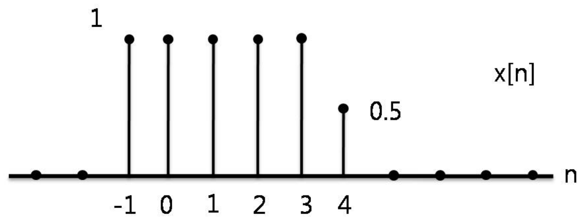

Figure 1:

   (a) $x[n-2]$
   (b) $x[4-n]$
   (c) $x[2n]$
   (d) $x[n]u[2-n]$
   (e) $x[n-1]\delta[n-3]$

   Sol: Sol) See Fig. 2

2. For each of the following systems, determine whether the system is (1) stable, (2) causal, (3) linear, and (4) time invariant.

   (a) $T(x[n]) = (\cos \pi n)x[n]$
   (b) $T(x[n]) = x[n^2]$
   (c) $T(x[n]) = x[n]\sum_{k=0}^{\infty} \delta[n-k]$
   (d) $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$

   Sol:

   (a) linear, causal, stable, not time-invariant – because $T(x[n-1]) = (\cos \pi n)x[n-1] \neq y[n-1] = (\cos \pi(n-1))x[n-1]$. In general, if $T(x[n])$ has the form $F(x[n], a[n])$ where F is some function, and $a[n]$ is a function of n that is independent of $x[n]$, in most cases T is not time-invariant. In particular if $a[n]$ is a function that depends on absolute time index n. For example, in this problem $F(x[n], a[n]) = x[n] \cdot a[n]$ where $a[n] = \cos \pi n$.
   (b) linear, stable, not causal, not time-invariant.
   (c) linear, causal, stable, not time-invariant. Note $\sum_{k=0}^{\infty} \delta[n-k] = u[n]$, so $T(x[n]) = x[n]u[n]$. Use similar arguments as part (a).

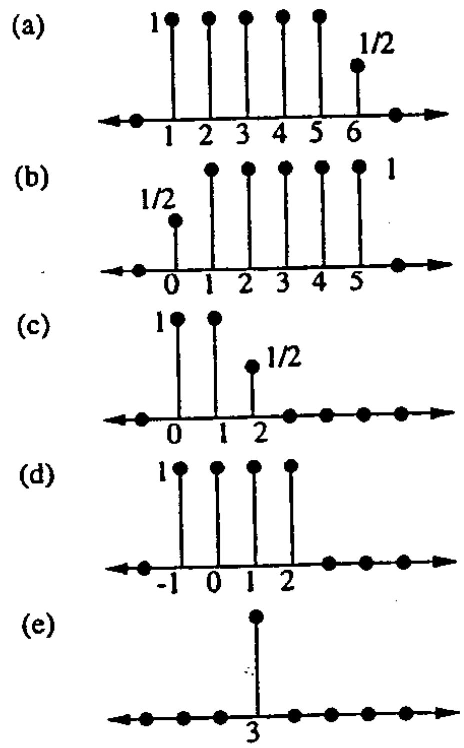

Figure 2:

(d) linear, time-invariant, not stable, not causal

3. The impulse response of a linear time-invariant system is shown in Fig 3. Determine and carefully sketch the response of this system to the input $x[n] = u[n-4]$.

Sol: Sol)

$$
\begin{aligned}
y[n] &= x[n] * h[n] \\
&= \sum_{k=0}^{\infty} x[k]h[n-k] \\
&= \sum_{k=0}^{\infty} u[k-4]h[n-k] \\
&= \sum_{k=4}^{\infty} h[n-k]
\end{aligned}
$$

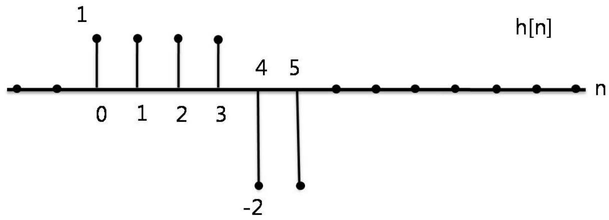

Figure 3: prob.1

Note that, $\sum_{k=0}^{5} h[k] = \sum_{k=0}^{\infty} h[k] = 0$

$$\begin{aligned} y[n] &= 0, \quad \text{if } n < 4, \ \text{ or } \ n > 8 \\ y[4] &= h[0] = 1 \\ y[5] &= h[1] + y[4] = 2 \\ y[6] &= h[2] + y[5] = 3 \\ y[7] &= h[3] + y[6] = 4 \\ y[8] &= h[4] + y[7] = 2 \end{aligned}$$

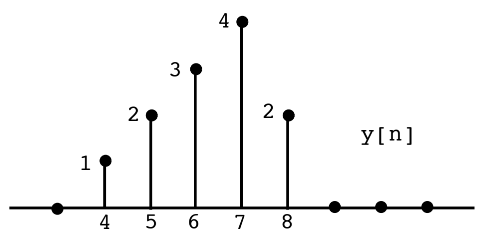

Figure 4: sol.1

4. Determine which of the following signals is periodic. If a signal is periodic, determine its period.

   (a) $x[n] = e^{j(2\pi n/5)}$

   (b) $x[n] = \sin(\pi n/19)$

   (c) $x[n] = ne^{j\pi n}$

   (d) $x[n] = e^{jn}$

   Sol: Sol)

   (a) Periodic. We need to check if $x[n] = x[n+N]$, and we see that $N = \frac{2\pi/1}{2\pi/5} = 5$.

   (b) Periodic. $N = \frac{2\pi/1}{\pi/19} = 38$.

   (c) Not Periodic.

   (d) Not Periodic.

5. The system L in Fig 5 is known to be linear. Shown are three output signals $y_1[n], y_2[n]$, and $y_3[n]$ in response to the input signals $x1[n], x_2[n]$, and $x_3[n]$, respectively.

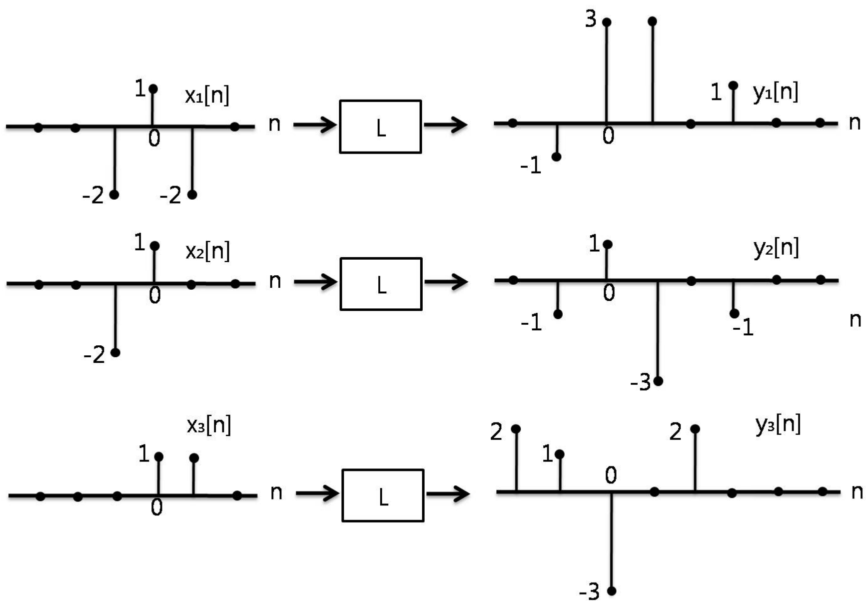

Figure 5: prob.3

(a) Determine whether the system L could be time invariant.

(b) If the input $x[n]$ to the system L is $\delta[n]$, what is the system response $y[n]$?

Sol: Sol)

$$\delta[n] = \frac{1}{2}x_1[n] - \frac{1}{2}x_2[n] + x_3[n]$$

(a)

$$\begin{aligned} L\{\delta[n]\} &= \frac{1}{2}y_1[n] - \frac{1}{2}y_2[n] + y_3[n] \\ \delta[n-1] &= -\frac{1}{2}x_1[n] + \frac{1}{2}x_2[n] \\ L\{\delta[n-1]\} &= \frac{1}{2}y_1[n] + \frac{1}{2}y_2[n] \end{aligned}$$

$$L\{\delta[n]\} \neq L\{\delta[n-1]\}$$

So, the system is not time invariant.

(b)

$$L\{\delta[n]\} = \frac{1}{2}y_1[n] - \frac{1}{2}y_2[n] + y_3[n]$$

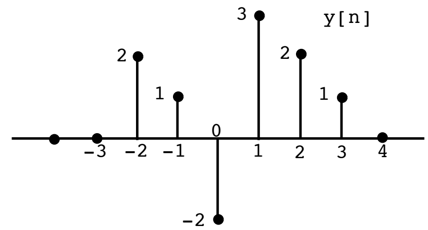

Figure 6: sol 6

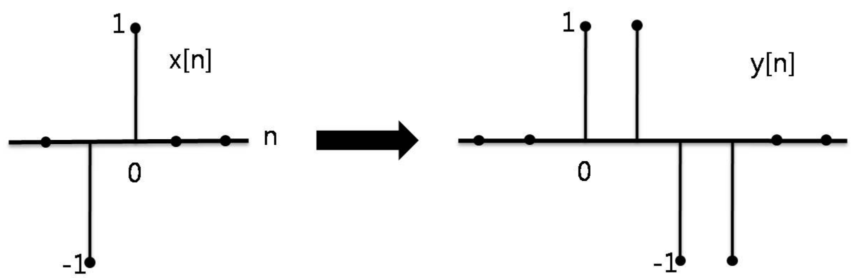

Figure 7:

6. The signals $x[n]$ and $y[n]$ shown in Fig 7. are the input and corresponding output for a causal LTI system.
   (a) Find the response of the system to the sequence $x_2[n]$ in Fig 8.

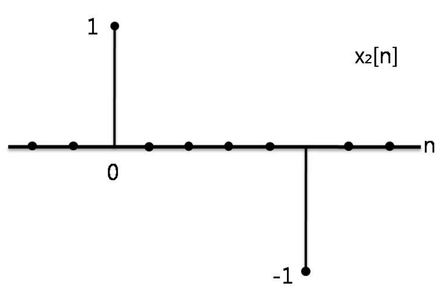

Figure 8:

   (b) Find the impulse response $h[n]$ for this LTI system.

Sol: Sol)

(a)

$$x_2[n] = -\sum_{k=1}^{5} x[n-k]$$
$$y_2[n] = -\sum_{k=1}^{5} y[n-k]$$

(b) After the convolution,

$$\begin{aligned} h[1] &= -1 \\ h[2] &= -2 \\ h[3] &= -1 \\ h[n] &= 0, \text{ if } n \neq 1, 2, 3 \end{aligned}$$