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Topics: Discrete-time LTI systems and DTFT

1. An LTI system has impulse response $h[n] = 5(1/2)^n u[n]$. Use the Fourier transform to find the output of this system when the input is $x[n] = (1/3)^n u[n]$.

2. Consider an LTI system with frequency response

$$H(e^{j\omega}) = \frac{1 - e^{-j2\omega}}{1 + \frac{1}{2}e^{-j4\omega}}$$

Determine the output $y[n]$ for all n if the input $x[n]$ for all n is

$$x[n] = \sin\left(\frac{\pi n}{4}\right)$$

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3. (a) Determine the Fourier transform of the sequence

$$r[n] = \begin{cases} 1, & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

(b) Consider the sequence

$$w[n] = \begin{cases} \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{M}\right)\right], & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

Sketch $w[n]$ and express its DTFT $W(e^{j\omega})$ in terms of $R(e^{j\omega})$ which is DTFT of $r[n]$.

(c) Sketch the magnitude of $R(e^{j\omega})$ and $W(e^{j\omega})$ for the case $M = 4$.

4. An LTI discrete-time system has frequency response given by

$$H(e^{j\omega}) = \frac{1}{1 - 0.8e^{-j\omega}} + \frac{e^{-j2\omega}}{1 - 0.8e^{-j\omega}}$$

(a) Use one of the above forms of the frequency response to obtain an equation for the impulse response $h[n]$ of the system.

(b) From the frequency response, determine the difference equation that is satisfied by the input $x[n]$ and the output $y[n]$ of the system.

(c) If the input to this system is

$$x[n] = 4 + 2\cos(\omega_0 n)$$

for what value of ω_0 will the output be of the form $y[n] = A =$ constant for $-\infty < n < \infty$? What is the constant A?

5. Which of the following disrete-time signals could be eigenfunctions of any stable LTI system?

(a) $5^n u[n]$

(b) e^{j2wn}

(c) $e^{jwn} + e^{j2wn}$

(d) 5^n

(e) $5^n e^{j2wn}$

6. Consider an LTI system with frequency response

$$H(e^{jw}) = e^{-j(w-\frac{\pi}{4})}\left(\frac{1 + e^{-j2w} + 4e^{-j4w}}{1 + \frac{1}{2}e^{-j2w}}\right)$$

Determine the output $y[n]$ for all n if the input for all n is

$$x[n] = \cos(\frac{\pi n}{2}).$$

7. For $X(e^{jw}) = 1/(1 - ae^{-jw})$, with $-1 < a < 0$, determine and sketch the following as a function of w :

(a) $Re\{X(e^{jw})\}$

(b) $Im\{X(e^{jw})\}$

(c) $|X(e^{jw})|$

(d) $Arg\{X(e^{jw})\}$

8. A sequence has the discrete-time Fourier transform

$$X(e^{jw}) = \frac{1 - a^2}{(1 - ae^{-jw})(1 - ae^{jw})}, \quad |a| < 1.$$

(a) Find the sequence $x[n]$.

(b) Calculate $\int_{-\pi}^{\pi} X(e^{jw}) \cos(w) dw / 2\pi$.

1. **(MATLAB):**

   Try to avodig using `for` or `while` loops in the code – use vectors and matrices if possible

   (a) **Plotting discrete signals:** Construct a sequence $x[n] = \left(\frac{1}{2}\right)^n u[n]$. Also consider impulse response of a moving average system

   $$h[n] = \begin{cases} \frac{1}{M+1} & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

   When you plot discrete sequences, always use `stem` function rather than the usual `plot` function. Let $N = 20$ and $M = 5$. Plot the sequences $x[n]$ and $h[n]$ for index vector $0 \le n \le N-1$ where $N = 20$. Also use `subplot` to plot two sequences in one page: $x[n]$ above and $h[n]$ below. (Hint: Some part of your code will look like
   '`subplot(2,1,1); stem(...); subplot(2,1,2); stem(...)`')

   (b) **Discrete convolution:** Let $y[n]$ be the output of the system with impulse response $h[n]$ and input sequence $x[n]$. We would like to find out $y[n]$, and you may want to use built-in `conv` function. Also you will find that the length of output $y[n]$ will be different from $x[n]$ or $h[n]$. Plot $y[n]$ against the index vector nn that starts from 0 and has the same length as $y[n]$. You might find `length` or `size` function useful.

   (c) **Computing DTFT:** Plot the magnitude and phase of the DTFT of $x[n]$, $h[n]$ and $y[n]$. Use subplot to plot magnitude and phase of each signal in one page respectively against ω ranging from -3π to 3π with the step size of 0.01π. You should use normal `plot` function for plotting in the frequency domain. (Warning: remember vectorized coding – you should use ONLY vectors and matrices to compute DTFT!) (Hint: When computing DTFT, try to express the complex exponentials as a *matrix*.)