COSE380 Digital Signal Processing Homework 2.5

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Topics: LCCDE

1. Consider the linear constant-coefficient difference equation

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n-1]$$

   Determine $y[n]$ for $n \geq 0$ when $x[n] = \delta[n]$ and $y[n] = 0, n < 0$.

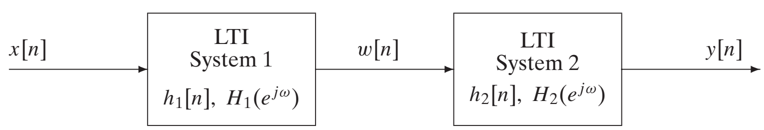
2. (a) Determine the frequency response $H(e^{j\omega})$ of the LTI system whose input and output satisfy the difference equation

$$y[n] - \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2]$$

   (b) (b) Write a difference equation that characterizes a system whose frequency response is

$$H(e^{j\omega}) = \frac{1 - \frac{1}{2}e^{-j\omega} + e^{-j3\omega}}{1 + \frac{1}{2}e^{-j\omega} + \frac{3}{4}e^{-j2\omega}}$$

3. Consider the cascade of LTI discrete-time systems shown in Figure. The first system is described by the frequency



   response

$$H_1(e^{j\omega}) = e^{-j\omega}\begin{cases} 0, & |\omega| \leq 0.25\pi \\ 1, & 0.25\pi < |\omega| \leq \pi \end{cases}$$

   and the second system is described by

$$h_2[n] = 2\frac{\sin(0.5\pi n)}{\pi n}$$

   (a) Determine an equation that defines the frequency response,$H(e^{j\omega})$, of the overall system over the range $-\pi \leq \omega \leq \pi$.

   (b) Sketch the magnitude, $|H(e^{j\omega})|$, and the phase, $\angle H(e^{j\omega})$, of the overall frequency response over the range $-\pi \leq \omega \leq \pi$.

   (c) Use any convenient means to determine the impulse response $h[n]$ of the overall cascade system.

4. A linear time-invariant system is described by the input-output relation

$$y[n] = x[n] + 2x[n-1] + x[n-2].$$

(a) Determine $h[n]$, the impulse response of the system.

(b) Is this a stable system?

(c) Determine $H(e^{jw})$, the frequency response of the system. Use trigonometric identities to obtain a simple expression for $H(e^{jw})$

(d) Plot the magnitude and phase of the frequency response.

(e) Now consider a new system whose frequency response is $H_1(e^{jw}) = H(e^{j(w+\pi)})$. Determine $h_1[n]$, the impulse response of the new system.