

Γ : reflection coefficient at the load

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{130 + j90 - 50}{130 + j90 + 50} = \frac{80 + j90}{180 + j90} = \frac{120.416 \angle 41.63^\circ}{201.246 \angle 63.43^\circ} = 0.598 \angle 21.8^\circ$$

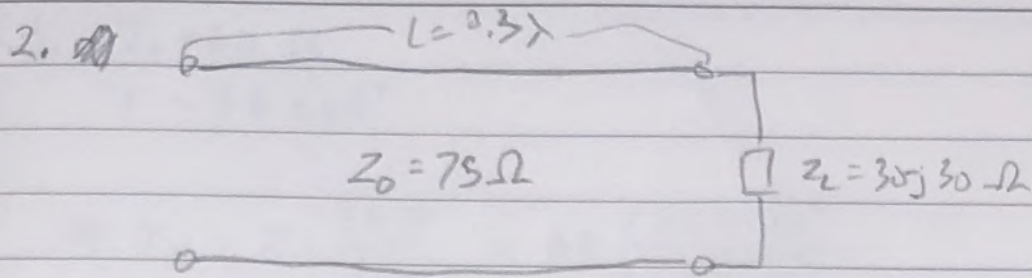
$\Gamma(z=-l)$: reflection coefficient at the input

$$\Gamma e^{-2j\beta l} = (0.598 \angle 21.8^\circ) e^{-j216} = 0.598 \angle 165.8^\circ$$

$$\beta l = \frac{2\pi}{\lambda} \cdot 0.3\lambda = 0.6\pi = 108^\circ$$

Input Impedance

$$\begin{aligned} Z_{in} &= Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = 50 \frac{130 + j90 + j50 \tan(108^\circ)}{50 + j(130 + j90) \tan(108^\circ)} \\ &= 50 \frac{130 + j(-63.884)}{50 + j(-400.1)} \\ &= 50 \frac{144.85 \angle -63.83^\circ}{516 \angle -9.26^\circ} \\ &= 14.8 \angle 24.57^\circ \\ &= 12.73 + j5.82 \Omega \end{aligned}$$



a) Γ : reflection coefficient at the load

$$\begin{aligned}\Gamma &= \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 - j30 - 75}{30 - j30 + 75} = \frac{-45 - j30}{105 - j30} \\ &= \frac{54.083 \angle 56.31^\circ}{109.202 \angle -74.05^\circ} \\ &= 0.495 \angle 130.36^\circ \\ &= -0.32 + j0.38\end{aligned}$$

b) $\Gamma (z = -\lambda)$: reflection coefficient at the input

$$\Gamma e^{-2j\beta L} = (0.495 \angle 130.36^\circ) e^{-j240}$$

$$\beta L = \frac{2\pi}{\lambda} \cdot 0.3\lambda = 0.6\pi = 108^\circ$$

$$\begin{aligned}&= 0.495 \angle 274.36^\circ \\ &= 0.04 - j0.49\end{aligned}$$

c) Input impedance

$$\begin{aligned}Z_{in} &= Z_0 \frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L} = 75 \cdot \frac{30 - j30 + j75 \tan(108^\circ)}{75 + j(30 - j30) \tan(108^\circ)} \\ &= 75 \frac{30 + j(-260.826)}{17.33 + j(-92.33)} \\ &= 75 \frac{262.546 \angle -6.561^\circ}{93.442 \angle 10.631^\circ} \\ &= 209.608 \angle 17.192^\circ \\ &= 200.24 + j61.95\end{aligned}$$

$$3. Z_0 = 60 \Omega$$

$$\Gamma = 0.4 \angle 60^\circ$$

$$a) Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma} = 60 \frac{1+0.4\angle 60^\circ}{1-0.4\angle 60^\circ}$$

$$= 60 \frac{1.25 \angle 16.26^\circ}{1.87 \angle -23.63^\circ}$$

$$= 86.207 \angle 39.89^\circ$$

$$= 66.14 + j55.29$$

$$b) \Gamma = |\Gamma| e^{j(60^\circ - 2\beta l)}$$

$$\beta l = \frac{2\pi}{\lambda} \cdot 0.3\lambda = 0.6\pi = 108^\circ$$

$$\Gamma = 0.4 e^{j(60^\circ - 216^\circ)}$$

$$= 0.4 e^{j(-156^\circ)}$$

$$c) Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = 60 \frac{66.14 + j55.29 + j60 \tan(108^\circ)}{60 + j(66.14 + j55.29) \tan(108^\circ)}$$

$$= 60 \frac{66.14 + j(-129.371)}{235.165 + j(-203.558)}$$

$$= 60 \frac{145.30 \angle -62.12^\circ}{307.27 \angle -41.49^\circ}$$

$$= 28.372 \angle 21.43^\circ$$

$$= 26.41 + j10.37$$

9.

$$Z_L = \frac{Z_1}{Z_0}$$

$$= \frac{40 + j20}{50}$$

$$= 0.8 + j0.4$$

- Plot Smith chart at (A)

104° from (B)

2.5λ & 2π rad/in $\rightarrow +\pi$ just other side of circle

$$2.5 \times 2\pi = 5\pi$$

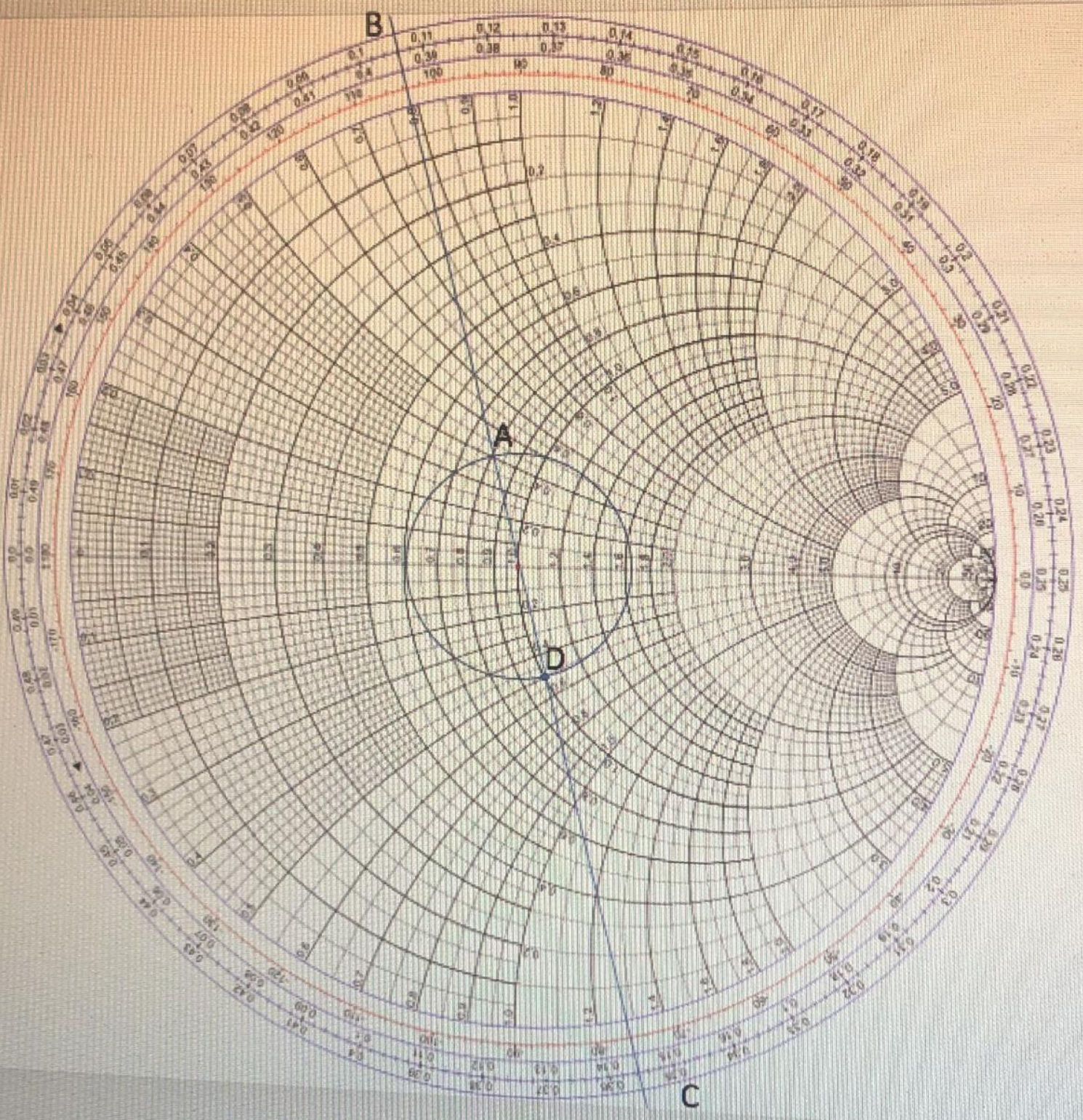
$$104^\circ + 5\pi = 1004^\circ = (360^\circ \times 3) - 76^\circ$$

-76° from (C)

$$Z_{in} = 1 - j0.5$$

from (D)

input impedance at P



5. $Z_L = \frac{Z_L}{Z_0}$

$$= \frac{100 - j100}{50}$$

$$= 2 - j2$$

Plot at (A)

a) $|r| = \frac{|0A|}{|0Z|} = \frac{5.1}{8.22} = 0.62$

$\theta_r = -30^\circ$

$$r = 0.62 \angle -30^\circ$$

$$= 0.54 + j0.31$$

b) -30° from (B)

$0.05 \text{ m} \quad \& \quad 2\pi/3 \text{ rad/m}$

$0.05 \times 2\pi/3 = 60^\circ$

$-30^\circ + 60^\circ$

-90° from (C)

not sure which way

30°

or

$Z_n = 0.45 - j0.9$ from (D)

$2 + j2$

