COSE221
Digital Logic Design

College of Informatics
Korea University

PEARSON
Education Canada

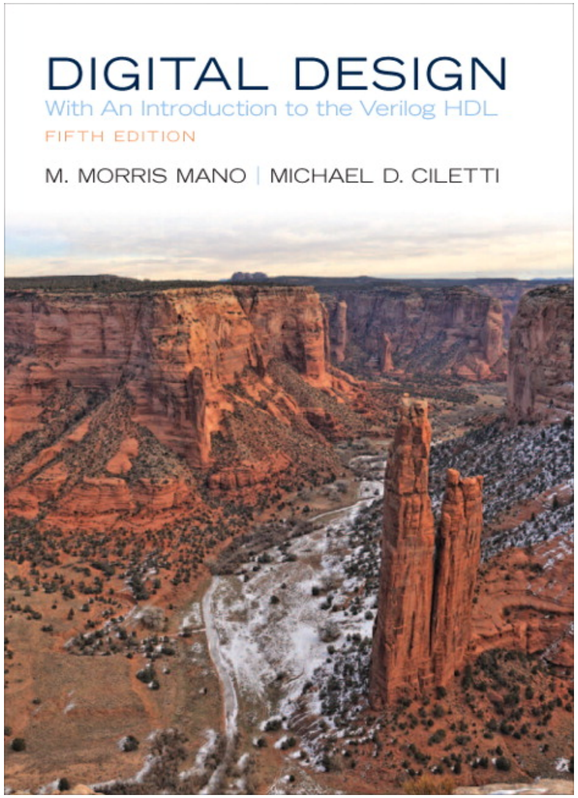
Syllabus

- Official course name and code
  - 논리회로설계(영강) – Digital Logic Design, COSE221
- Meeting time
  - Tue 6, Thur 6
- Instructor
  - Seung Jun Baek (sjbaek@korea.ac.kr)
  - Office: Science Library 404C
  - tel. 3290-4847
- Teaching Assistant
  - Chankyu Pyoung (pyoung1101@korea.ac.kr)
  - Seung Hyun Kwawk (kwaksh2319@naver.com)

Textbooks

- Main textbook
  - "Digital Design with introduction to Verilog HDL"

    by Mano & Cletti
- Reference textbooks
  - Fundamentals of logic design by Roth
  - Digital Design Principles by Wakerly



Grading

- Homework
  - Every 1-2 weeks hand-written homeworks will be assi gned
  - Collaboration is OK
    - Final submission must be your own work
  - Work must be done in <u>English</u>
  - HW submission is due at the <u>beginning</u> of class
    - <u>No late submission allowed</u>

- **<u>Work hard on homeworks, because exam proble ms will look similar to those assignments!</u>**

Grading

- Exams
  - 1 Midterm, 1 Final
  - All exams are closed book, closed notes
  - All exams will be administered in class. Absence at the exams will be pardoned only under excruciating circumstances, and the student must get consent from the instructor BEFORE the exam. Otherwise the exam will be graded as zero.
  - When the absence is approved, your score will be the average of your rest of the exam, or the class average, whichever is smaller (0 for non-approved absence)
  - If you get 0 in your exam, it is simply counted as zero towards total score, but it does not necessarily mean F

Grading

- Grading
  - HW 5%, Exams 95%
  - After your grade is assigned in the end of semester, it <u>cannot</u> be <u>changed</u> unless there is some serious error
  - This, of course, means that you cannot ask for grade r aise.
  - Also means that you cannot ask for lowering the grad e!
    - EX) I cannot take C from the class, so I want rather F => NE VER allowed
  - These requests may be reported to the univer sity

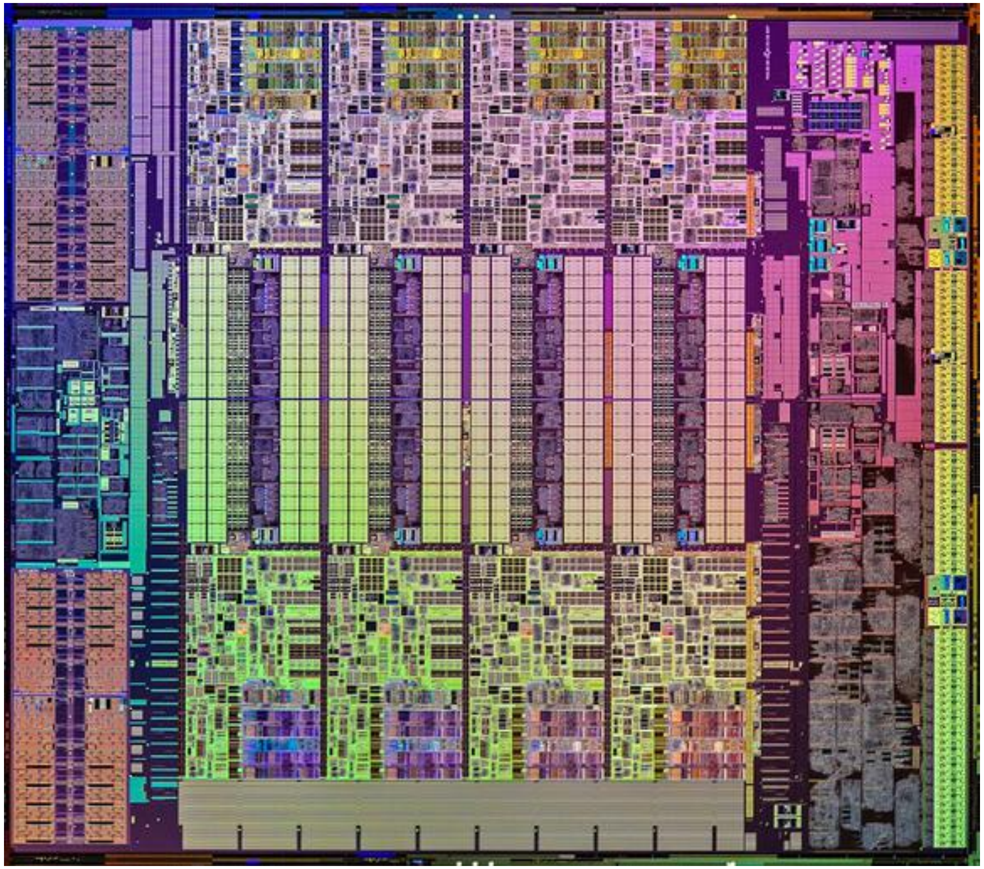
Grading

- Update your email address at BlackBoard (BB):
  - Throughout this course, e-mail will be the primary sou rce of communications, for example: important annou ncements.
  - Update your primary email on Black- Board so that yo u check the mail on a daily basis (some e-mail provid ers are sometimes unreachable, please check it).

- Course slides: Slides will be uploaded at BlackBoard wit hout notice, so please check them occasionally

Why learn digital logic design?

- Computer technology lies in the heart of modern industry revolution
- Digital logic is the very basic component of computer hard ware theory



Die photograph of 8-core Intel Haswell famliy server processor

Why learn digital logic design?

- What is implemented in a massive scale on this silicon die – millions of switches are being “On” or “Off”

- “On-Off” represents the state of voltage levels
  - Circuit level

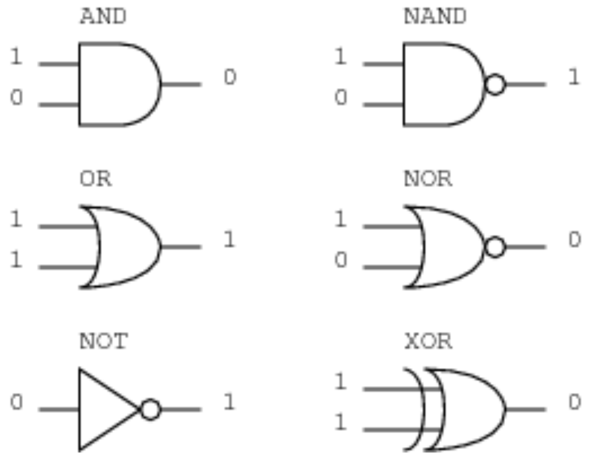
- Mathematically, they are 1-0
  - Binary logic

- Human is familiar with “decimal” systems; computer is m ore easily built in “binary” system

- Systems that use discrete numbers (2 or 10) for computati on is called digital systems

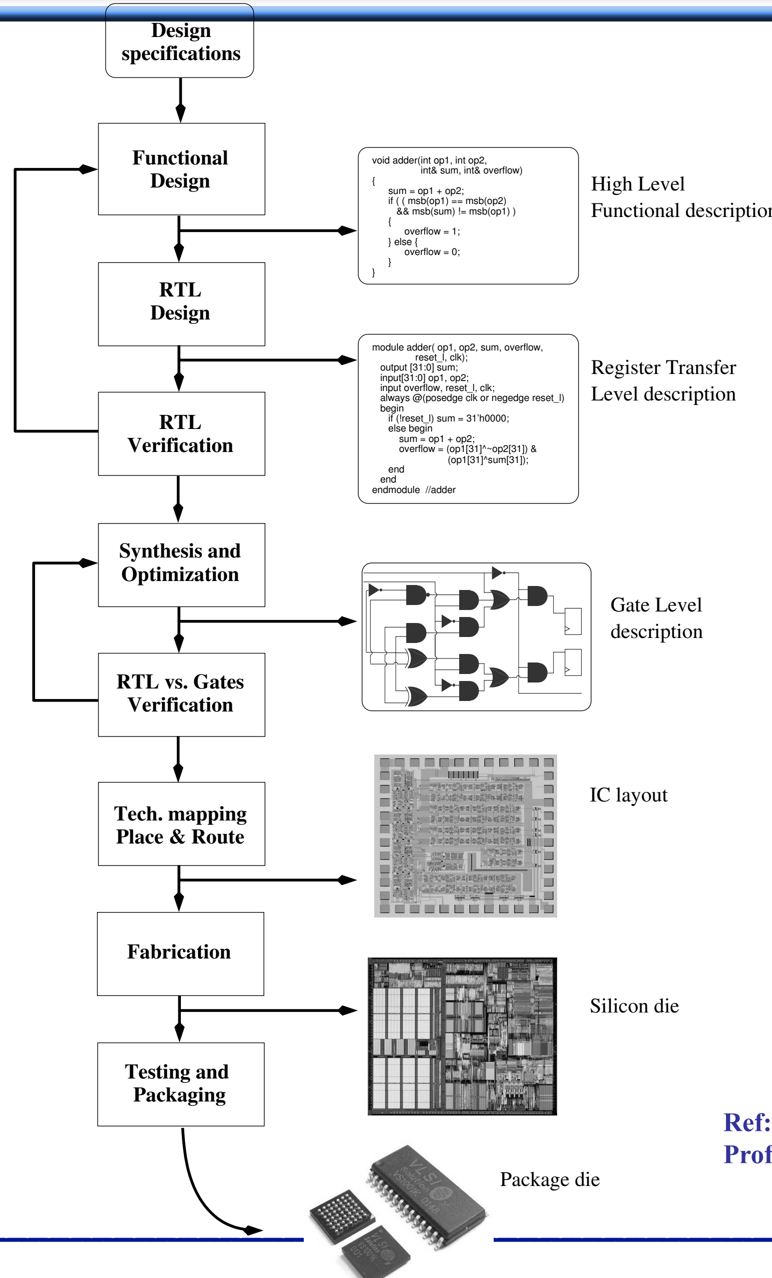
What do we learn digital logic design?

- Learn how computation is done based on 0-1 “logic” for d igital systems



- Learn basic arithmetic operations
  - AND. OR. NAND, NOR, XOR.
- Learn to optimize designs
  - How to build efficient hardware
    - less switches => less power, cheaper system
- Build systems
  - HDL (Hardware Description Language)

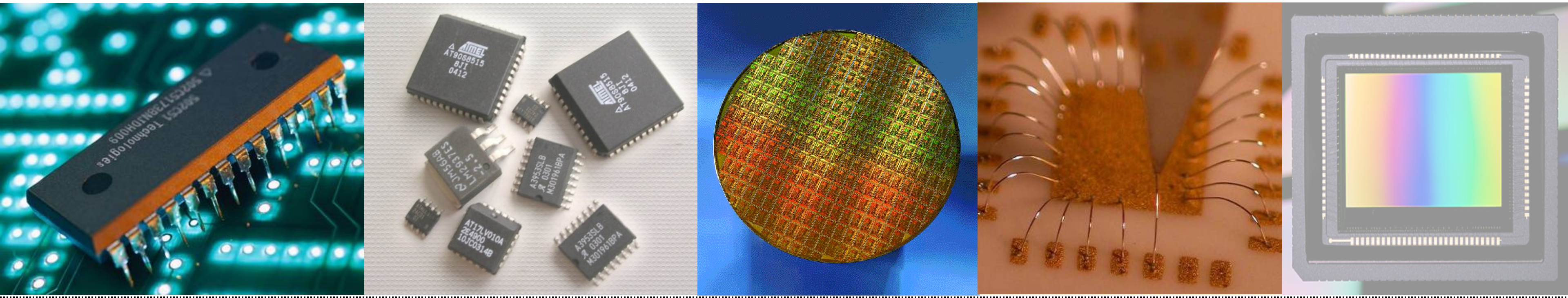
Integrated Circuit Design Flow



Ref: CAD and verification of Digital systems course, Prof. Bertacco, UMichgan

Course outline

- Binary number systems
- Logic gates
- Design optimization
- Combinational logic
- Sequential logic
- HDL (limited coverage..)
    - Verilog HDL



1. Digital System &Binary Numbers

1.1 Digital System

- Digital System
  - Communication, Business transaction, Traffic control , other commercial, industrial, scientific enterprises et c.
  - Discrete elements of information
- Signal
  - Discrete elements of information are represented in a digital system by physical quantities
- Binary Code
  - Discrete elements of information are represented wit h groups of bits
  - bit : binary + digit

- $a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3}$ (suppose this is r-ary number, $0 \le a_k < r$)
  $= a_n r^n + a_{n-1} r^{n-1} + \ldots + a_2 r^2 + a_1 r + a_0 + a_{-1} r^{-1} + a_{-2} r^{-2} + \ldots + a_{-m} r^{-m}$

$7392 = 7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$

$(11010.11)_2 = 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2_0 + 1\times2_{-1} + 1\times2_{-2}$

$= (26.75)_{10}$

2^{10} = 1Kilo

2^{20} = 1Mega

2^{30} = 1Giga

Table 1-1
Powers of Two

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096	20	1,048,576
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

1.2 Binary Number

Augend	101101
Addend	+100111
sum	1010100

Minuend:	101101
Subtrahend:	-100111
Difference:	000110

Multiplicand:	1011
Multiplier:	× 101
	1011
	0000
	1011
Product:	110111

addition may generate *carry*
subtraction may generate *borrow*

- Ex 1-1) Convert decimal 41 to binary.

	Integer Quotient		Remainder	Coefficient
41/2 =	20	+	½	$a_0 = 1$
20/2 =	10	+	0	$a_1 = 0$
10/2 =	5	+	0	$a_2 = 0$
5/2 =	2	+	½	$a_3 = 1$
2/2 =	1	+	0	$a_4 = 0$
1/2 =	0	+	½	$a_5 = 1$

Integer	Remainder
41	
20	1
10	0
5	0
2	1
1	0
0	1

Answer

=101001

answer : $(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$

1.3 Number Base Conversions

- Ex 1-2) Convert decimal 153 to octal.

153	
19	1
2	3
0	2

$= (231)_8$

- Ex 1-3) Convert $(0.6875)_{10}$ to binary.

	Integer		Fraction	Coefficient
0.6875*2 =	1	+	0.3750	$a_{-1} = 1$
0.3750*2 =	0	+	0.7500	$a_{-2} = 0$
0.7500*2 =	1	+	0.5000	$a_{-3} = 1$
0.5000*2 =	1	+	0.0000	$a_{-4} = 1$

Answer: $(0.6875)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.1011)_2$

1.4 Octal and Hexadecimal Numbers

Table 1-2
Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

$$(\underset{2}{\underline{10}}\ \underset{6}{\underline{110}}\ \underset{1}{\underline{001}}\ \underset{5}{\underline{101}}\ \underset{3}{\underline{011}}\ .\ \underset{7}{\underline{111}}\ \underset{4}{\underline{100}}\ \underset{0}{\underline{000}}\ \underset{6}{\underline{110}})_2 = (26153.7460)_8$$

$$(\underset{2}{\underline{10}}\ \underset{C}{\underline{1100}}\ \underset{6}{\underline{0110}}\ \underset{B}{\underline{1011}}\ .\ \underset{F}{\underline{1111}}\ \underset{2}{\underline{0010}})_2 = (2C6B.F2)_{16}$$

1.5 Complements – Diminished Radix Complement

- (r-1)'s complements of n-digit base-r number N is $(r^n-1)-N$

- r=10, r-1=9, 9'complements of N is $(10^n-1)-N$

  Ex) the 9's complements of 546700 is 999999-546700 = 453299

  the 9's complements of 012398 is 999999-012398 = 987601

- For binary number, r=2, r-1=1

  1'complements of N is $(2^n-1)-N$

  Ex) the 1's complements of 1011000 is 0100111 (bit-flip)

  the 1's complements of 0101101 is 1010010

1.5 Complements – Radix Complement

- The r's complements of n-digit base-r number N is
  r^n-N, N≠0
  0, N=0
- $r^n-N=[(r^n-1)-N]+1$
  → The r's complements is obtained by adding 1 to the (r-1)'s complements

- Ex) The 10's complements of 012398 is 987602
  The 10's complements of 246700 is 753300

  The 2's complements of 1101100 is 0010100 (bit-flip, then +1)
  The 2's complements of 0110111 is 1001001

- Subtraction operation is simplified using complements
- Also useful to represent negative numbers!

1.5 Complements – Subtraction with Complements

- Ex1-5) using 10's complement, subtract 72532-3250.

M =	72532
10's complement of N =	+ 96750
Sum =	169282
Discard end carry 10^5 =	-100000
Answer =	69282

- Ex1-6) Using 10's complement, subtract 3250-72532.

M =	03250
10's complement of N =	+27468
Sum =	30718

There is no end carry

Therefore, the answer is –(10's complement of 30718)=-69282

(minus number?-more later..)

1.5 Complements – Subtraction with Complements

- Ex1-7) X=1010100, Y=1000011, (a) X-Y, (b) Y-X

(a) X-Y

X =	1010100
2's complement of Y =	+0111101
Sum =	10010001
Discard end carry 2^7 =	-10000000
Answer: X-Y =	0010001

(b) Y-X

Y =	1000011
2's complement of X =	+0101100
Sum =	1101111

There is no carry.

The answer is Y-X = -(2's complement of 1101111)=-0010001

- Ex1-8) Repeat Example 1-7 using 1'complement.

(a) X-Y = 1010100-1000011

X =	1010100
1's complement of Y =	+0111100
Sum =	10010000
End-around carry =	+ 1
Answer: X-Y =	0010001

(b) Y-X = 1000011-1010100

Y =	1000011
1's complement of X =	+0101011
Sum =	1101110

There is no carry.

The answer is Y-X = -(1's complement of 1101110)=-0010001

1.6 Signed Binary Numbers

- Only use binary numbers to represent both positive and negative numbers
- Ex) The number 9 represented in binary with eight bit

  +9 : 00001001

  -9 : 10001001 (signed-magnitude representation)

  11110110 (signed-1's-complement representation)

  11110111 (signed-2's-complement representation)

Table 1-3
Signed Binary Numbers

Decimal	Signed-2's complement	Signed-1's complement	Signed magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

1.6 Signed Binary Numbers

- Arithmetic Addition
  - signed-magnitude system follows the rules of ordinary arithmetic.
  - signed-complement system requires only addition.

+6	00000110	-6	11111010
+13	00001101	+13	00001101
+19	00010011	+7	00000111

+6	00000110	-6	11111010
-13	11110011	-13	11110011
-7	11111001	-19	11101101

- Arithmetic Subtraction

$$(\pm A)-(+B) = (\pm A)+(-B)$$
$$(\pm A)-(-B) = (\pm A)+(+B)$$

- Only Adder is needed for 2's complement systems

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1.7 Binary Code-BCD code

- the 4-bit code for one decimal

$(185)_{10} = (0001\ 1000\ 0101)_{BCD} = (10111001)_2$

- BCD Addition

4	0100	4	0100	8	1000
+5	+0101	+8	+1000	+9	+1001
9	1001	12	1100	17	10001
			+0110		+0110
			10010		10111

Table 1-4
Binary Coded Decimal (BCD)

Decimal symbol	BCD digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

- if the binary sum is greater or equal to 1010, we add 0110 to obtain the corr ect BCD

1.7 Binary Code-Other Decimal Codes

Table 1-5
Four Different Binary Codes for the Decimal Digits

Decimal digit	BCD 8421	2421	Excess-3	8 4-2-1
0	0000	0000	0011	0 0 0 0
1	0001	0001	0100	0 1 1 1
2	0010	0010	0101	0 1 1 0
3	0011	0011	0110	0 1 0 1
4	0100	0100	0111	0 1 0 0
5	0101	1011	1000	1 0 1 1
6	0110	1100	1001	1 0 1 0
7	0111	1101	1010	1 0 0 1
8	1000	1110	1011	1 0 0 0
9	1001	1111	1100	1 1 1 1
	1010	0101	0000	0 0 0 1
Unused	1011	0110	0001	0 0 1 0
bit	1100	0111	0010	0 0 1 1
combi-	1101	1000	1101	1 1 0 0
nations	1110	1001	1110	1 1 0 1
	1111	1010	1111	1 1 1 0

1.7 Binary Code-Gray Code

Table 1-6
Gray Code

Gray code	Decimal equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

Only one bit changes as the number progresses

1.7 Binary Code- ASCII Character Code

Table 1-7
American Standard Code for Information Interchange (ASCII)

	$b_7b_6b_5$							
$b_4b_3b_2b_1$	**000**	**001**	**010**	**011**	**100**	**101**	**110**	**111**
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	“	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	‘	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	|
1101	CR	GS	−	=	M	]	m	}
1110	SO	RS	.	>	N	∧	n	~
1111	SI	US	/	?	O	−	o	DEL

1.7 Binary Code – Error-Detecting Code

- Error-Detecting Code

	With even parity	With odd parity
ASCII A = 1000001	01000001	11000001
ASCII T = 1010100	11010100	01010100

1.8 Binary Storage and Registers

- Registers – A register with n cells can store any discrete quantity of information that contains n bits.
- Register Transfer

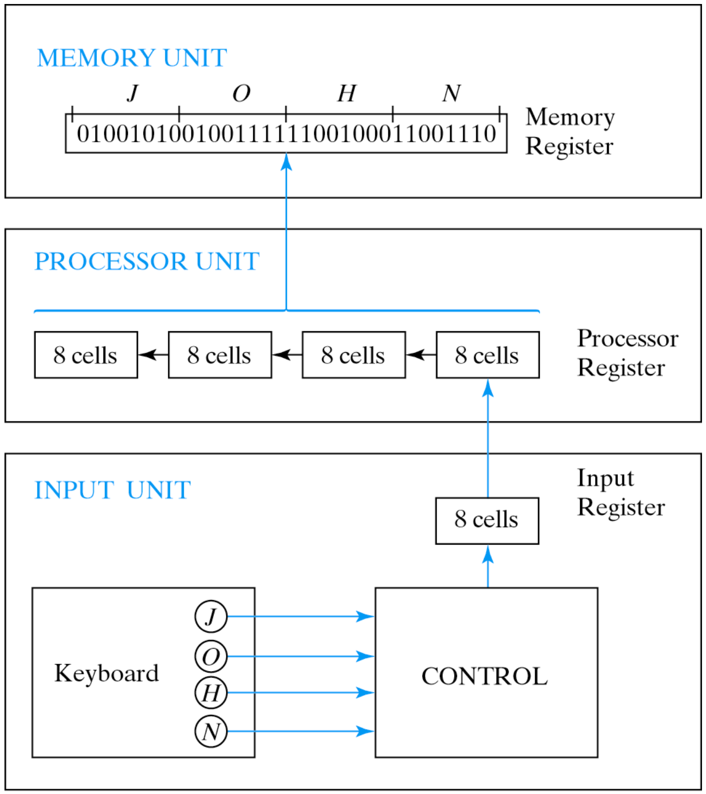


Fig. 1-1 Transfer of information with registers

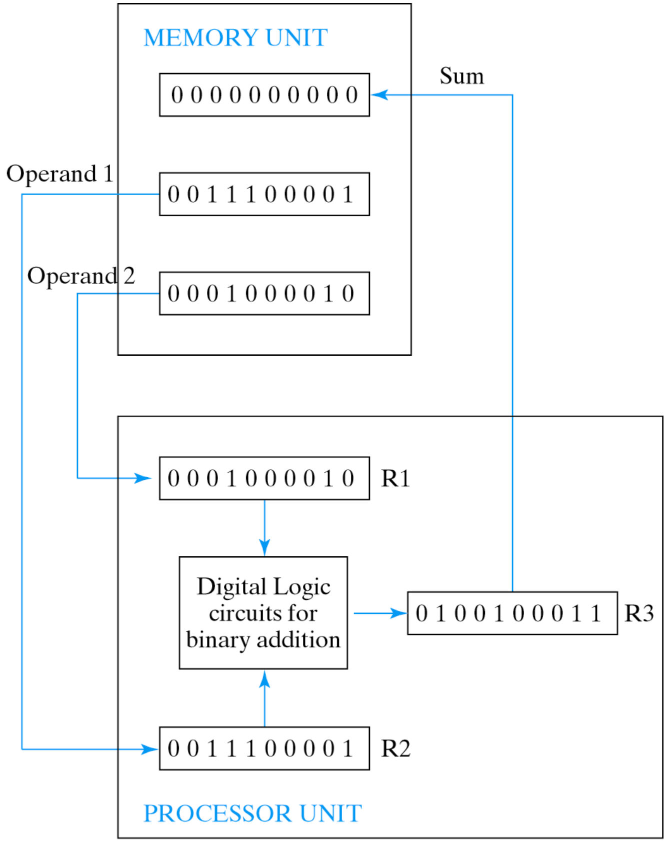


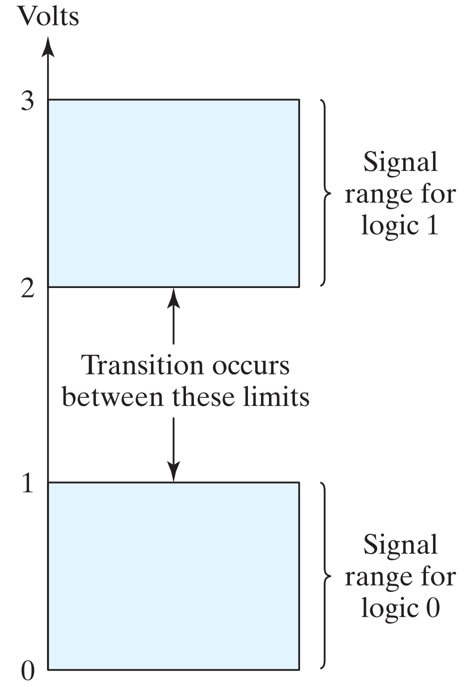
Fig. 1-2 Example of binary information processing

1.9 Binary Logic

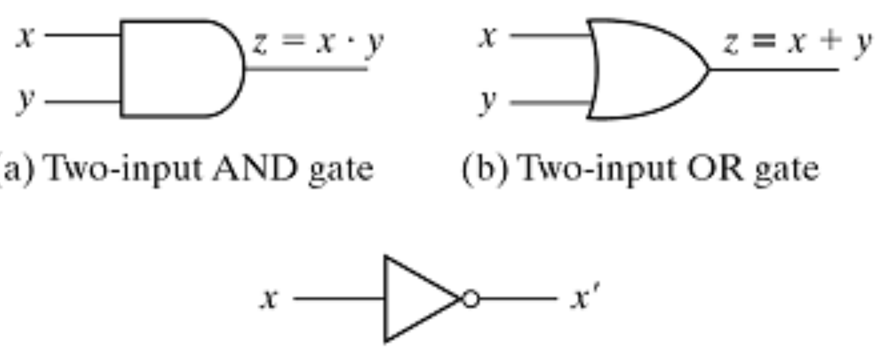
- Definition of Binary Logic – takes multiple binary inputs, produces output

Table 1-8
Truth Tables of Logical Operations

AND				OR				NOT	
x	y	$x \cdot y$		x	y	$x + y$		x	x'
0	0	0		0	0	0		0	1
0	1	0		0	1	1		1	0
1	0	0		1	0	1			
1	1	1		1	1	1			



- Logic Gates



(a) Two-input AND gate (b) Two-input OR gate

(c) NOT gate or inverter

Fig. 1-4 Symbols for digital logic circuits

1.9 Binary Logic

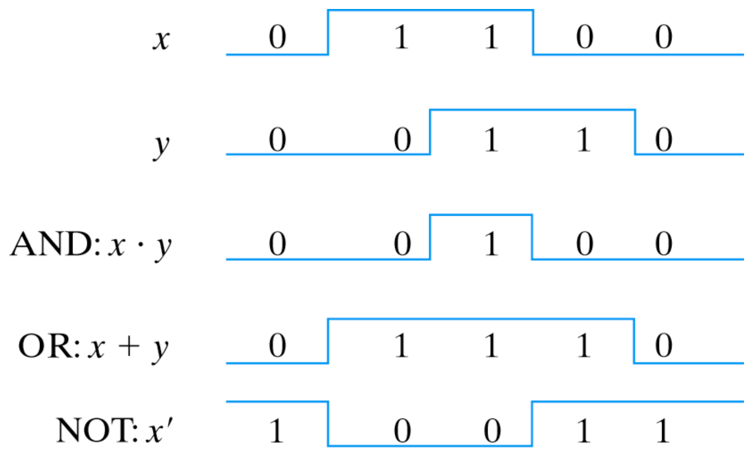
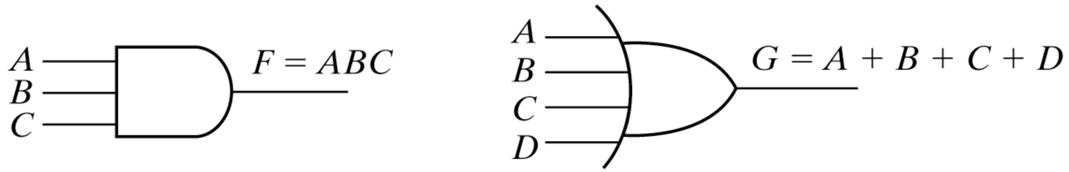


Fig. 1-5 Input-output signals for gates



(a) Three-input AND gate (b) Four-input OR gate

Fig. 1-6 Gates with multiple inputs