ECE 5322
21st Century Electromagnetics

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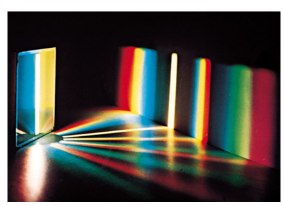
Lecture #9

Diffraction Gratings

Lecture Outline

- Fourier series
- Diffraction from gratings
- The plane wave spectrum
- Plane wave spectrum for crossed gratings
- The grating spectrometer
- Littrow gratings
- Patterned fanout gratings
- Diffractive optical elements



Fourier Series

Jean Baptiste Joseph Fourier

Born: March 21, 1768
in Yonne, France.

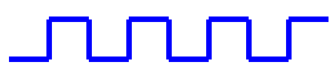
Died: May 16, 1830
in Paris, France.

1D Complex Fourier Series

If a function $f(x)$ is periodic with period Λ_x, it can be expanded into a complex Fourier series.

$$f(x) = \sum_{m=-\infty}^{\infty} a(m) e^{j\frac{2\pi m x}{\Lambda}}$$

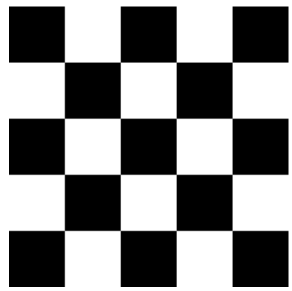
$$a(m) = \frac{1}{\Lambda} \int_{-\Lambda/2}^{\Lambda/2} f(x) e^{-j\frac{2\pi m x}{\Lambda}} dx$$



Typically, we retain only a finite number of terms in the expansion.

$$f(x) = \sum_{m=-M}^{M} a(m) e^{j\frac{2\pi m x}{\Lambda}}$$

2D Complex Fourier Series



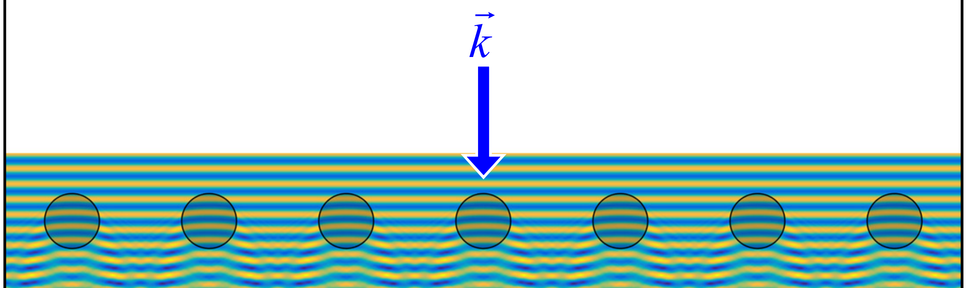
For 2D periodic functions, the complex Fourier series generalizes to

$$f(x,y)=\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}a(p,q)\,e^{j\left(\frac{2\pi px}{\Lambda_x}+\frac{2\pi qy}{\Lambda_y}\right)}\qquad a(p,q)=\frac{1}{A}\iint_A f(x,y)\,e^{-j\left(\frac{2\pi px}{\Lambda_x}+\frac{2\pi qy}{\Lambda_y}\right)}dA$$

Diffraction from Gratings

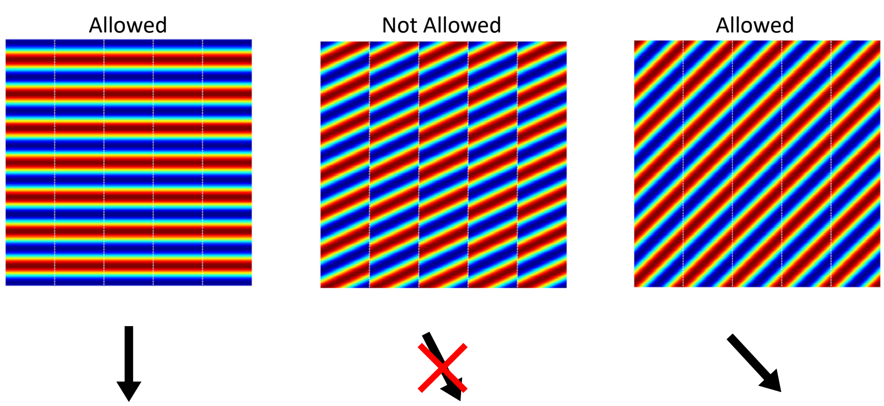
Fields in Periodic Structures

Waves in periodic structures take on the same periodicity as their host.



Diffraction Orders

The field must be continuous so only discrete directions are allowed.
The allowed directions are called the diffraction orders.
The allowed angles are calculated using the famous grating equation.



Field in a Periodic Structure

The dielectric function of a sinusoidal grating can be written as

$$\varepsilon_{\mathrm{r}}\left(\vec{r}\right)=\varepsilon_{\mathrm{r,avg}}+\Delta\varepsilon\cos\left(\vec{K}\bullet\vec{r}\right)$$

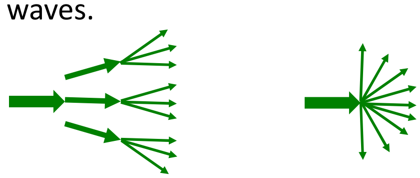
A wave propagating through this grating takes on the same symmetry.

$$\begin{aligned}E\left(\vec{r}\right)&=A\left(\vec{r}\right)e^{-j\vec{k}_{\mathrm{inc}}\bullet\vec{r}}\\&=A\left[\varepsilon_{\mathrm{r,avg}}+\Delta\varepsilon\cos\left(\vec{K}\bullet\vec{r}\right)\right]e^{-j\vec{k}_{\mathrm{inc}}\bullet\vec{r}}\\&\vdots\\&=\underbrace{A\varepsilon_{\mathrm{r,avg}}e^{-j\vec{k}_{\mathrm{inc}}\bullet\vec{r}}}_{\text{wave 1}}+\underbrace{\frac{A\Delta\varepsilon}{2}e^{-j\left(\vec{k}_{\mathrm{inc}}-\vec{K}\right)\bullet\vec{r}}}_{\text{wave 2}}+\underbrace{\frac{A\Delta\varepsilon}{2}e^{-j\left(\vec{k}_{\mathrm{inc}}+\vec{K}\right)\bullet\vec{r}}}_{\text{wave 3}}\end{aligned}$$

Grating Produces New Waves

The applied wave splits into three waves.

$$e^{-j\vec{k}_{\mathrm{inc}}\bullet\vec{r}}\rightarrow\begin{matrix}e^{-j\vec{k}_{\mathrm{inc}}\bullet\vec{r}}\\e^{-j\left(\vec{k}_{\mathrm{inc}}-\vec{K}\right)\bullet\vec{r}}\\e^{-j\left(\vec{k}_{\mathrm{inc}}+\vec{K}\right)\bullet\vec{r}}\end{matrix}$$



Each of those splits into three waves as well.

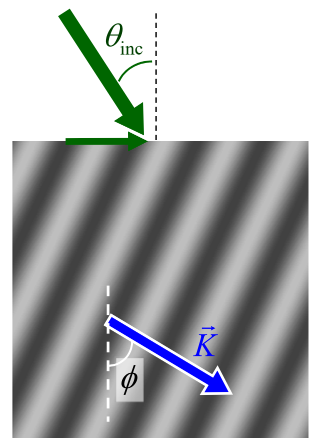
$$e^{-j\vec{k}_{\mathrm{inc}}\bullet\vec{r}}\rightarrow\begin{matrix}e^{-j\vec{k}_{\mathrm{inc}}\bullet\vec{r}}\\e^{-j\left(\vec{k}_{\mathrm{inc}}-\vec{K}\right)\bullet\vec{r}}\\e^{-j\left(\vec{k}_{\mathrm{inc}}+\vec{K}\right)\bullet\vec{r}}\end{matrix}\qquad e^{-j\left(\vec{k}_{\mathrm{inc}}-\vec{K}\right)\bullet\vec{r}}\rightarrow\begin{matrix}e^{-j\left(\vec{k}_{\mathrm{inc}}-\vec{K}\right)\bullet\vec{r}}\\e^{-j\left(\vec{k}_{\mathrm{inc}}-2\vec{K}\right)\bullet\vec{r}}\\e^{-j\vec{k}_{\mathrm{inc}}\bullet\vec{r}}\end{matrix}\qquad e^{-j\left(\vec{k}_{\mathrm{inc}}+\vec{K}\right)\bullet\vec{r}}\rightarrow\begin{matrix}e^{-j\left(\vec{k}_{\mathrm{inc}}+\vec{K}\right)\bullet\vec{r}}\\e^{-j\vec{k}_{\mathrm{inc}}\bullet\vec{r}}\\e^{-j\left(\vec{k}_{\mathrm{inc}}+2\vec{K}\right)\bullet\vec{r}}\end{matrix}$$

And each of these split, and so on.

$$\vec{k}\left(m\right)=\vec{k}_{\mathrm{inc}}-m\vec{K}\qquad m=-\infty,\ldots,-2,-1,0,1,2,\ldots,\infty$$

This equation describes the total set of allowed diffraction orders.

Wave Incident on a Grating



Boundary conditions required the tangential component of the wave vector be continuous.

$$k_{x,\text{trn}} \overset{?}{=} k_{x,\text{inc}}$$

The wave is entering a grating, so the phase matching condition is

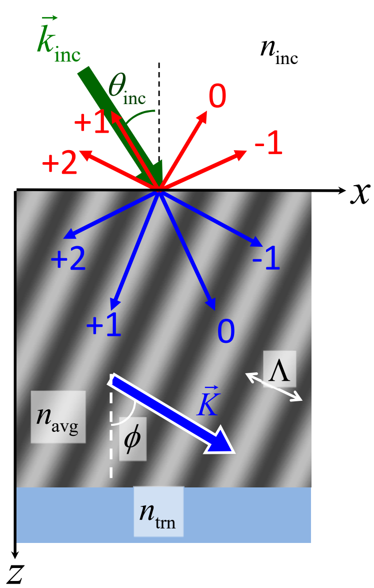
$$k_x(m) = k_{x,\text{inc}} - mK_x$$

The longitudinal vector component is calculated from the dispersion relation.

$$k_z^2(m) = \left(k_0 n_{\text{avg}}\right)^2 - k_x^2(m)$$

For large m, $k_{z,m}$ can actually become imaginary. This indicates that the highest diffraction-orders are evanescent.

The Grating Equation



The Grating Equation

$$\boxed{n_{\text{avg}} \sin\left[\theta(m)\right] = n_{\text{inc}} \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda}\sin\phi}$$

Note, this really is just

$$k_x(m) = k_{x,\text{inc}} - mK_x$$

Proof:

$$k_x(m) = k_{x,\text{inc}} - mK_x$$

$$k_0 n_{\text{avg}} \sin\left[\theta(m)\right] = k_0 n_{\text{inc}} \sin\theta_{\text{inc}} - m\frac{2\pi}{\Lambda_x}$$

$$\frac{2\pi}{\lambda_0} n_{\text{avg}} \sin\left[\theta(m)\right] = \frac{2\pi}{\lambda_0} n_{\text{inc}} \sin\theta_{\text{inc}} - m\frac{2\pi}{\Lambda_x}$$

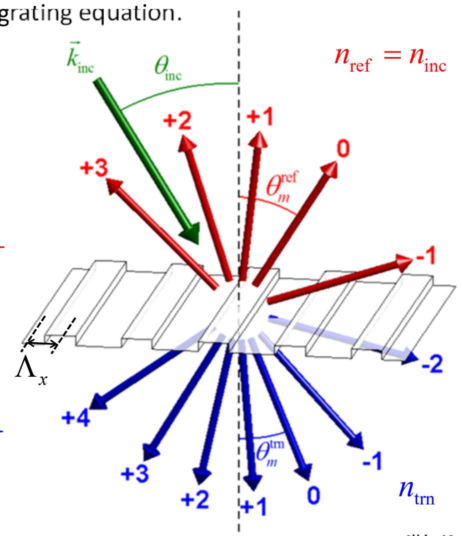
$$n_{\text{avg}} \sin\left[\theta(m)\right] = n_{\text{inc}} \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda_x}$$

$$n_{\text{avg}} \sin\left[\theta(m)\right] = n_{\text{inc}} \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda}\sin\phi$$

Grating Equation in Different Regions

The angles of the diffracted modes are related to the wavelength λ_0, refractive index, and grating period Λ_x through the grating equation.

The grating equation only predicts the directions of the modes, not how much power is in them.



Reflection Region

$$n_{\text{ref}} \sin\left[\theta(m)\right] = n_{\text{inc}} \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda_x}$$

Transmission Region

$$n_{\text{trn}} \sin\left[\theta(m)\right] = n_{\text{inc}} \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda_x}$$

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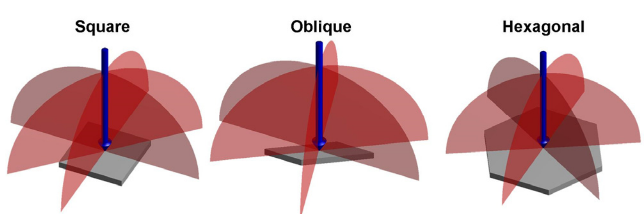
Diffraction in Two Dimensions

- We know everything about the direction of diffracted waves just from the grating period.



Diffraction tends to occur along the lattice planes.

- The grating equation says nothing about how much power is in the diffracted modes.
  - We need to solve Maxwell's equations for that!

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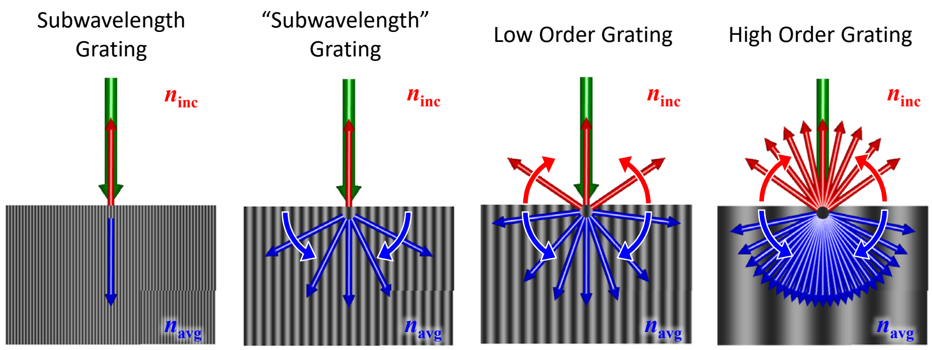
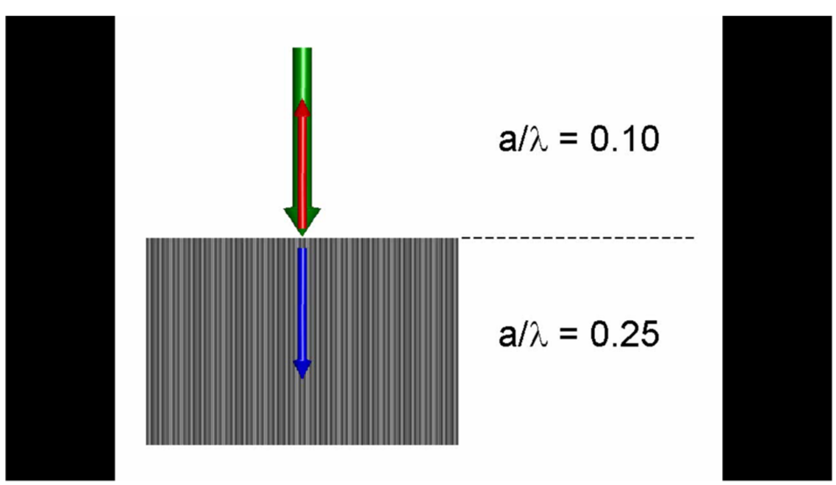
Effect of Grating Periodicity

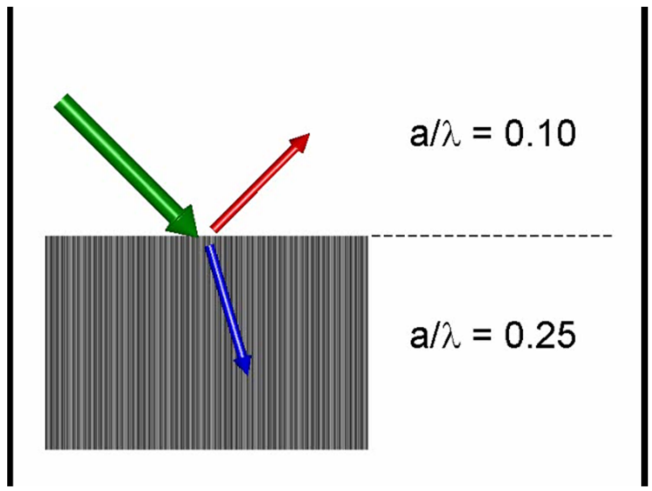


$\Lambda_x < \frac{\lambda_0}{n_{\mathrm{avg}}}$ $\qquad$ $\frac{\lambda_0}{n_{\mathrm{avg}}} < \Lambda_x < \frac{\lambda_0}{n_{\mathrm{inc}}}$ $\qquad$ $\Lambda_x > \frac{\lambda_0}{n_{\mathrm{inc}}}$ $\qquad$ $\Lambda_x >> \frac{\lambda_0}{n_{\mathrm{inc}}}$

Animation of Grating Diffraction at Normal Incidence



Animation of Grating Diffraction at an Angle of Incidence



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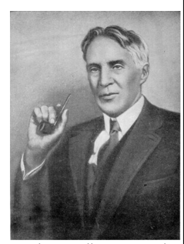
Wood's Anomalies

Robert W. Wood observed rapid variations in the spectrum of light diffracted by gratings which he could not explain. Later, Hessel explained them an identified two different types.



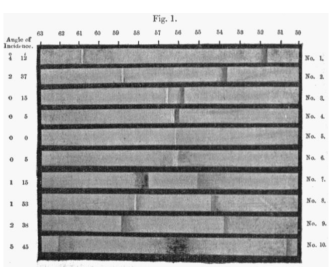
Robert Williams Wood
1868 - 1955

Type 1 – Rayleigh Singularities

Rapid variation in the amplitudes of the diffracted modes that correspond to the onset or disappearance of other diffracted modes.

Type 2 – Resonance Effects

A resonance condition arising from leaky waves supported by the grating. Today, we call this guided-mode resonance.



R. W. Wood, Phil. Mag. **4**, 396 (1902)

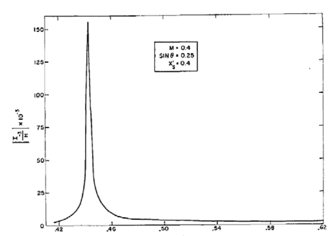


Fig. 23. Relative amplitude of the $n = -3$ resonant spectral order vs λ/d for the case of four propagating spectral orders.

A. Hessel, A. A. Oliner, "A New Theory of Wood's Anomalies on Optical Gratings," Appl. Opt., Vol. 4, No. 10, 1275 (1965).

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Grating Cutoff Wavelength

When θ_m becomes imaginary, the mth mode is evanescent and cut off.

Assuming normal incidence (i.e. $\theta_{\text{inc}} = 0°$), the grating equation reduces to

$$n \sin\left[\theta(m)\right] = -m\frac{\lambda_0}{\Lambda_x}$$

The first diffracted modes to appear are $m = \pm 1$.

The cutoff for the first-order modes happens when $\theta(\pm 1) = 90°$.

$$\theta(\pm 1) = 90°$$

$$\sin\left[90°\right] = 1 = \frac{\lambda_0}{n\Lambda_x}$$

$$\boxed{\Lambda_x = \frac{\lambda_0}{n}}$$

To prevent the first-order modes, we need

$$\Lambda_x < \frac{\lambda_0}{n} \quad \text{or} \quad \Lambda_x < \lambda$$

To ensure we have first-order modes, we need

$$\Lambda_x > \frac{\lambda_0}{n} \quad \text{or} \quad \Lambda_x > \lambda$$

Total Number of Diffracted Modes

Given the grating period Λ_x and the wavelength λ_0, we can determine how many diffracted modes exist.

Again, assuming normal incidence, the grating equation becomes

$$\sin\left[\theta(m)\right] = -\frac{m\lambda_0}{n_{\text{avg}}\Lambda_x} \quad \rightarrow \quad \left|\sin\left[\theta(m)\right]\right| = \left|\frac{m\lambda_0}{n_{\text{avg}}\Lambda_x}\right| < 1$$

Therefore, a maximum value for m is

$$m_{\max} = \frac{n_{\text{avg}}\Lambda_x}{\lambda_0}$$

The total number of possible diffracted modes M is then $2m_{\max}+1$

$$\boxed{M = \frac{2n_{\text{avg}}\Lambda_x}{\lambda_0} + 1}$$

Determining Grating Cutoff Conditions

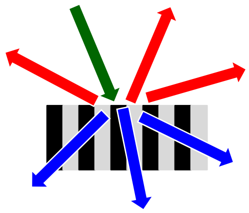
Condition	Requirements
0-order mode	Always exists unless there is total-internal reflection
No 1st-order modes	Grating period must be shorter than what causes $\theta(\pm 1) = 90°$
Ensure 1st-order modes	Grating period must be larger than what causes $\theta(\pm 1) = 90°$
No 2nd-order modes	Grating period must be shorter than what causes $\theta(\pm 2) = 90°$
Ensure 2nd-order modes	Grating period must be larger than what causes $\theta(\pm 2) = 90°$
No mth-order modes	Grating period must be shorter than what causes $\theta(\pm m) = 90°$
Ensure mth-order modes	Grating period must be larger than what causes $\theta(\pm m) = 90°$

Three Modes of Operation for 1D Gratings

Bragg Grating
Couples energy between counter-propagating waves.

Applications
- Thin film optical filters
- Fiber optic gratings
- Wavelength division multiplexing
- Dielectric mirrors
- Photonic crystal waveguides

Diffraction Grating
Couples energy between waves at different angles.



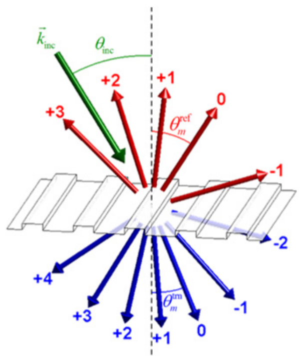
Applications
- Beam splitters
- Patterned fanout gratings
- Laser locking
- Spectrometry
- Sensing
- Anti-reflection
- Frequency selective surfaces
- Grating couplers

Long Period Grating
Couples energy between co-propagating waves.

Applications
- Sensing
- Directional coupling

Analysis of Diffraction Gratings

Direction of the Diffracted Modes



$$n \sin\left[\theta(m)\right] = n_{\text{inc}} \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda}\sin\phi$$

Diffraction Efficiency and Polarization of the Diffracted Modes

We must obtain a rigorous solution to Maxwell's equations to determine amplitude and polarization of the diffracted modes.

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E}$$

$$\nabla \bullet \left(\varepsilon\vec{E}\right) = 0$$

$$\nabla \bullet \left(\mu\vec{H}\right) = 0$$

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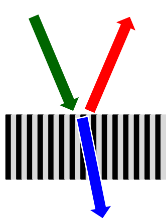
Applications of Gratings

Subwavelength Gratings

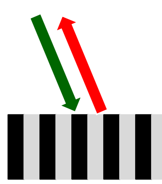
Only the zero-order modes may exist.



Applications
- Polarizers
- Artificial birefringence
- Form birefringence
- Anti-reflection
- Effective index media

Littrow Gratings

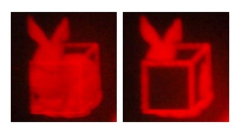
Gratings in the littrow configuration are a spectrally selective retroreflector.



Applications
- Sensors
- Lasers

Patterned Fanout Gratings

Gratings diffract laser light to form images.

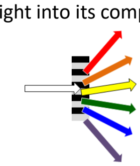


Holograms

Holograms are stored as gratings.

Spectrometry

Gratings separate broadband light into its component colors.



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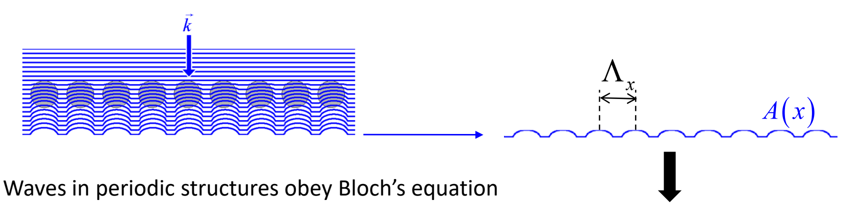
The Plane Wave Spectrum

Periodic Functions Can Be Expanded into a Fourier Series



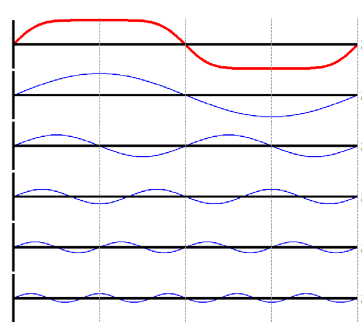
Waves in periodic structures obey Bloch's equation

$$E(x,y)=A(x)e^{j\vec{\beta}\bullet\vec{r}}$$

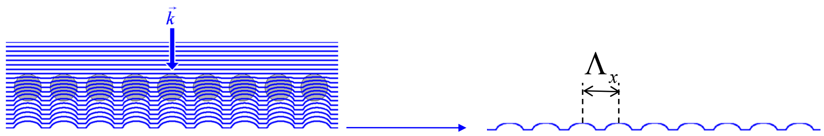
The envelop $A(x)$ is periodic along x with period Λ_x so it can be expanded into a Fourier series.

$$A(x)=\sum_{m=-\infty}^{\infty} S(m)e^{-j\frac{2\pi mx}{\Lambda_x}}$$

$$S(m)=\int_{\Lambda} A(x)e^{j\frac{2\pi mx}{\Lambda_x}}dx$$



Rearrange the Fourier Series (1 of 2)



A periodic field can be expanded into a Fourier series.

$$E(x,y) = A(x)e^{j\vec{\beta}\bullet\vec{r}}$$

$$= \left[\sum_{m=-\infty}^{\infty} S(m)e^{-j\frac{2\pi mx}{\Lambda_x}}\right]e^{j\vec{\beta}\bullet\vec{r}}$$

$$= \sum_{m=-\infty}^{\infty} S(m)e^{j\vec{\beta}\bullet\vec{r}}e^{-j\frac{2\pi mx}{\Lambda_x}}$$

Here the plane wave term $e^{j\beta_x x}$ is brought inside of the summation.

$$= \sum_{m=-\infty}^{\infty} S(m)e^{j\beta_x x}e^{j\beta_y y}e^{-j\frac{2\pi mx}{\Lambda_x}}$$

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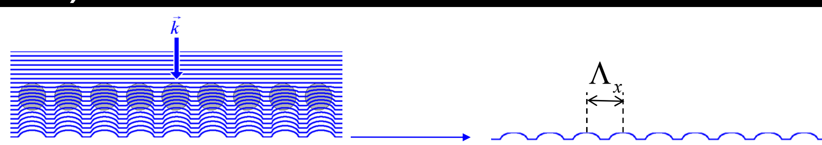
Rearrange the Fourier Series (2 of 2)



β_x can be combined with the last complex exponential.

$$E(x,y) = \sum_{m=-\infty}^{\infty} S(m)e^{j\beta_x x}e^{j\beta_y y}e^{-j\frac{2\pi mx}{\Lambda_x}}$$

$$= \sum_{m=-\infty}^{\infty} S(m)e^{j\beta_y y}e^{j\left(\beta_x - \frac{2\pi m}{\Lambda_x}\right)x}$$

Now let $k_{x,m} = \beta_x - \frac{2\pi m}{\Lambda_x}$ and $k_{y,m} = \beta_y$

$$E(x,y) = \sum_{m=-\infty}^{\infty} S(m)e^{j\vec{k}(m)\bullet\vec{r}}$$

$$\vec{k}(m) = \left(\beta_x - \frac{2\pi m}{\Lambda_x}\right)\hat{a}_x + \beta_y\hat{a}_y$$

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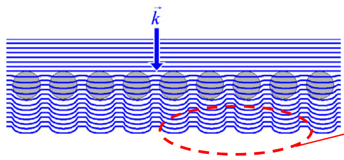
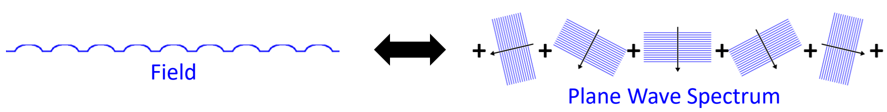
The Plane Wave Spectrum



$$E(x,y)=\sum_{m=-\infty}^{\infty} S(m)\, e^{j\vec{k}(m)\bullet\vec{r}}$$

We rearranged terms and now we see that a periodic field can also be thought of as an infinite sum of plane waves at different angles. This is the "plane wave spectrum" of a periodic field.

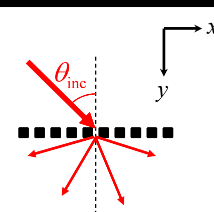


Longitudinal Wave Vector Components of the Plane Wave Spectrum

The wave incident on a grating can be written as

$$E_{\text{inc}}(x,y)=E_0 e^{j\left(k_{x,\text{inc}}x+k_{y,\text{inc}}y\right)}$$

$$k_{x,\text{inc}}=k_0 n_{\text{inc}}\sin\theta_{\text{inc}}$$
$$k_{y,\text{inc}}=k_0 n_{\text{inc}}\cos\theta_{\text{inc}}$$



Phase matching into the grating leads to

$$k_x(m)=k_{x,\text{inc}}-m\frac{2\pi}{\Lambda_x}\qquad m=\cdots,-2,-1,0,1,2,\cdots$$

Note: k_x is always real.

Each wave must satisfy the dispersion relation.

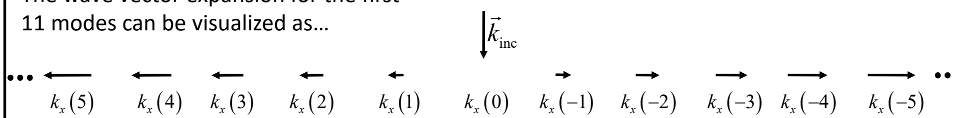
$$k_x^2(m)+k_y^2(m)=\left(k_0 n_{\text{grat}}\right)^2$$
$$\downarrow$$
$$k_y(m)=\sqrt{\left(k_0 n_{\text{grat}}\right)^2-k_x^2(m)}$$

We have two possible solutions here.

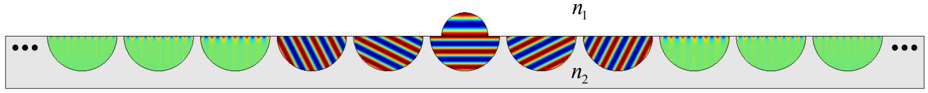
1. Purely real k_y
2. Purely imaginary k_y.

Visualizing Phase Matching into the Grating

The wave vector expansion for the first 11 modes can be visualized as…



Each of these is phase matched into material 2. The longitudinal component of the wave vector is calculated using the dispersion relation in material 2.

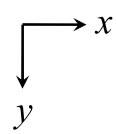


k_y is imaginary.
The field in material 2 is evanescent.

k_y is real.
A wave propagates into material 2.

k_y is imaginary.
The field in material 2 is evanescent.

Note: The "evanescent" fields in material 2 are not completely evanescent. They have a purely real k_x so power flows in the transverse direction.



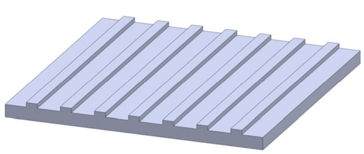
Conclusions About the Plane Wave Spectrum

- Fields in periodic media take on the same periodicity as the media they are in.
- Periodic fields can be expanded into a Fourier series.
- Each term of the Fourier series represents a spatial harmonic (plane wave).
- Since there are in infinite number of terms in the Fourier series, there are an infinite number of spatial harmonics.
- Only a few of the spatial harmonics are actually propagating waves. Only these can carry power away from a device. Tunneling is an exception.

Plane Wave Spectrum from Crossed Gratings

Grating Terminology

1D grating
Ruled grating



Requires a 2D simulation

2D grating
Crossed grating

Requires a 3D simulation

Diffraction from Crossed Gratings

Doubly-periodic gratings, also called crossed gratings, can diffract waves into many directions.

They are described by two grating vectors, $\mathbf{K}_x$ and $\mathbf{K}_y$.

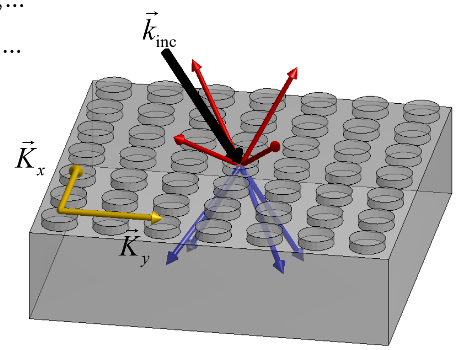
Two boundary conditions are necessary here.

$$k_x(m) = k_{x,\text{inc}} - mK_x \qquad m = \dots,-2,-1,0,1,2,\dots$$

$$k_y(n) = k_{y,\text{inc}} - nK_y \qquad n = \dots,-2,-1,0,1,2,\dots$$

$$\vec{K}_x = \frac{2\pi}{\Lambda_x}\hat{x}$$

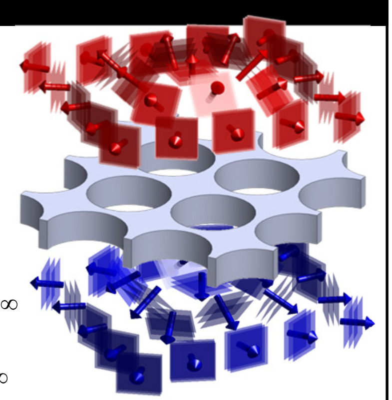
$$\vec{K}_y = \frac{2\pi}{\Lambda_y}\hat{y}$$



Transverse Wave Vector Expansion

Crossed gratings diffraction in two dimensions, x and y.

To quantify diffraction for crossed gratings, we must calculate an expansion for both k_x and k_y.



$$k_x(m) = k_{x,\text{inc}} - \frac{2\pi m}{\Lambda_x} \qquad m = -\infty,\cdots,-2,-1,0,1,2,\cdots\infty$$

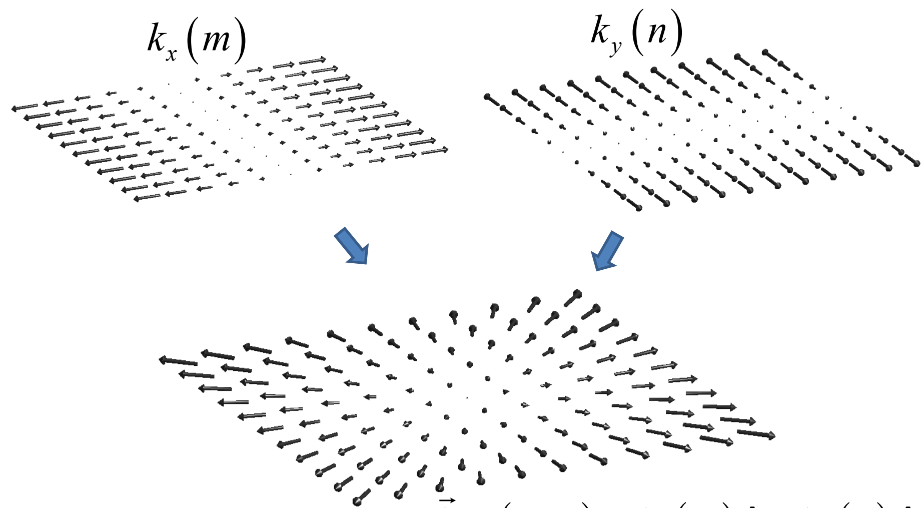
$$k_y(n) = k_{y,\text{inc}} - \frac{2\pi n}{\Lambda_y} \qquad n = -\infty,\cdots,-2,-1,0,1,2,\cdots\infty$$

$$\vec{k}_t(m,n) = k_x(m)\hat{x} + k_y(n)\hat{y}$$

```
% TRANSVERSE WAVE VECTOR EXPANSION
M  = [-floor(Nx/2):floor(Nx/2)]';
N  = [-floor(Ny/2):floor(Ny/2)]';
kx = kxinc - 2*pi*M/Lx;
ky = kyinc - 2*pi*N/Ly;
[ky,kx] = meshgrid(ky,kx);
```

We will use this code for 2D PWEM, 3D RCWA, 3D FDTD, 3D FDFD, 3D MoL, and more.

Visualizing the Transverse Wave Vector Expansion



$$\vec{k}_{\text{tran}}(m,n) = k_x(m)\,\hat{x} + k_y(n)\,\hat{y}$$

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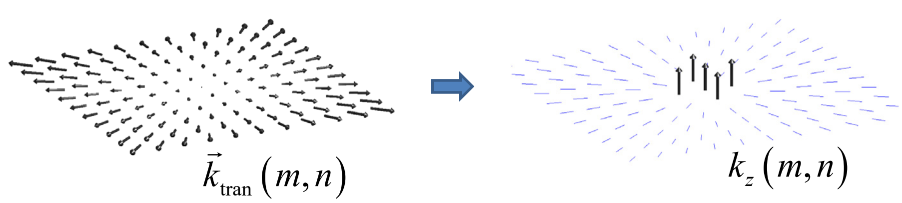
Longitudinal Wave Vector Expansion

The longitudinal components of the wave vectors are computed as

$$k_z^{\text{ref}}(m,n) = \sqrt{\left(k_0 n_{\text{ref}}\right) - k_x^2(m) - k_y^2(n)}$$
$$k_z^{\text{trn}}(m,n) = \sqrt{\left(k_0 n_{\text{trn}}\right) - k_x^2(m) - k_y^2(n)}$$

The center few modes will have real k_z's. These correspond to propagating waves. The others will have imaginary k_z's and correspond to evanescent waves that do not transport power.



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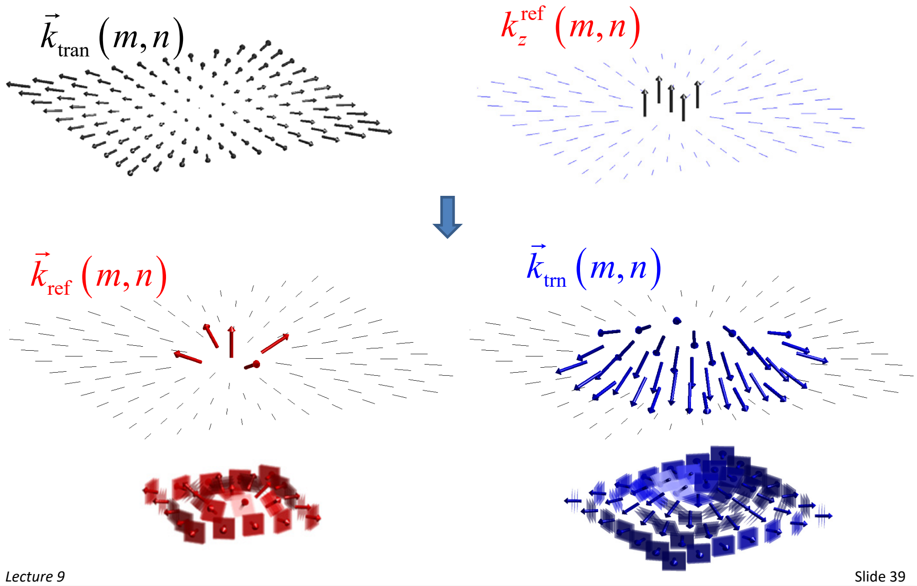
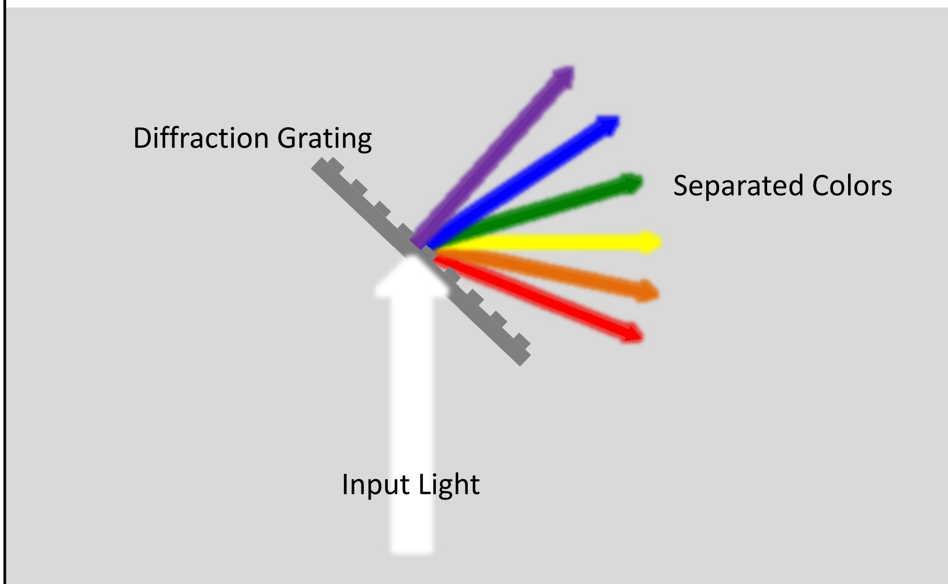
Visualizing the Overall Wave Vector Expansion



The Grating Spectrometer

What is a Grating Spectrometer



Spectral Sensitivity

We start with the grating equation.

$$n_{\text{avg}} \sin\left[\theta(m)\right] = n_{\text{inc}} \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda_x}$$

We define spectral sensitivity as how much the diffracted angle changes with respect to wavelength $\partial\theta(m)/\partial\lambda_0$.

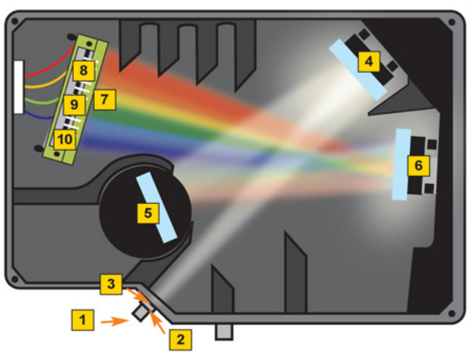
$$\frac{\partial\theta(m)}{\partial\lambda_0} = -\frac{m}{\Lambda_x n_{\text{avg}} \cos\theta_m} \qquad \boxed{\Delta\theta(m) \cong \frac{m}{\Lambda_x n_{\text{avg}} \cos\left[\theta(m)\right]}\Delta\lambda_0}$$

This equation tells us how to maximize sensitivity.

1. Diffract into higher order modes ($\uparrow m$).
2. Use short period gratings ($\downarrow \Lambda_x$).
3. Diffract into large angles ($\uparrow \theta(m)$).
4. Diffract into air ($\downarrow n_{\text{avg}}$).

Ocean Optics HR4000 Grating Spectrometer

http://www.oceanoptics.com/Products/benchoptions_hr.asp



Fiber Optic Input

1	SMA Connector	4	Collimating Mirror	7	Collection Lenses	10	Optional UV Detector
2	Entrance Slit	5	Diffraction Grating	8	Detector Array		
3	Optional Filter	6	Focusing Mirror	9	Optional Filter		

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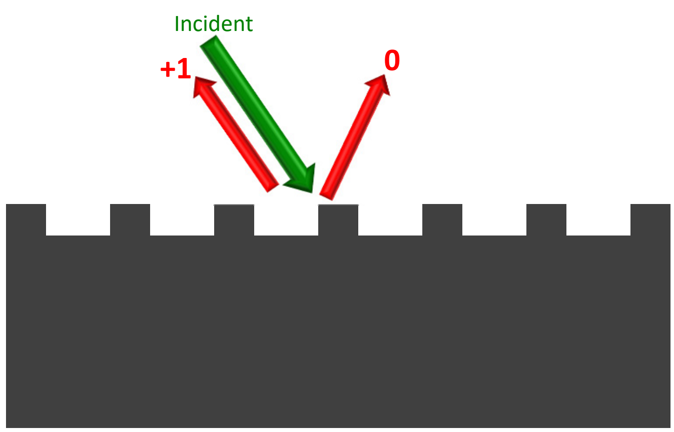
Littrow Gratings

Littrow Configuration

In the littrow configuration, the +1-order reflected mode is parallel to the incident wave vector. This forms a spectrally selective mirror.



Conditions for the Littrow Configuration

The grating equation is

$$n\sin\left[\theta(m)\right] = n\sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda_x}$$

The littrow configuration occurs when

$$\theta(+1) = -\theta_{\text{inc}}$$

The condition for the littrow configuration is found by substituting this into the grating equation.

$$\boxed{2n\sin\theta_{\text{inc}} = \frac{\lambda_0}{\Lambda_x}}$$

Spectral Selectivity

Typically only a cone of angles $\Delta\theta$ reflected from a grating is detected.

We wish to find $d\lambda/d\theta$ by differentiating our last equation.

$$\frac{d\lambda_0}{d\theta} = 2n\Lambda_x \cos\theta$$

Typically this is used to calculate the reflected bandwidth.

$$\boxed{\Delta\lambda_0 = 2n\Lambda_x \cos\theta \cdot \Delta\theta}$$

Linewidth (optics and photonics)

$$\boxed{\Delta f = \frac{2n\Lambda_x f^2 \cos\theta}{c_0}\Delta\theta}$$

Bandwidth (RF and microwave)

Example (1 of 2)

Design a metallic grating in air that is to be operated in the littrow configuration at 10 GHz at an angle of 45°.

Solution

Right away, we know that

$$n = 1.0$$
$$\theta_{\text{inc}} = 45°$$
$$\lambda_0 = \frac{c_0}{f} = \frac{3\times10^8 \ \frac{m}{s}}{10 \text{ GHz}} = 3.00 \text{ cm}$$

The grating period is then found to be

$$\Lambda_x = \frac{\lambda_0}{2n\sin\theta_{\text{inc}}} = \frac{3.00 \text{ cm}}{2(1.0)\sin(45°)} = 2.12 \text{ cm}$$

Example (2 of 2)

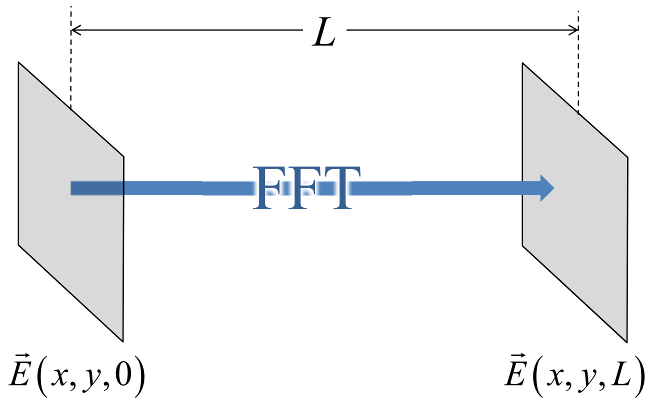
Solution continued
Assuming a 5° cone of angles is detected upon reflection, the bandwidth is

$$\Delta f = \frac{2(1.0)(2.12\ \text{cm})(10\ \text{GHz})^2 \cos(45°)}{3\times10^8\ \frac{m}{s}}\left(\frac{\pi}{180°}5°\right) = 0.87\ \text{GHz}$$

Patterned Fanout Gratings

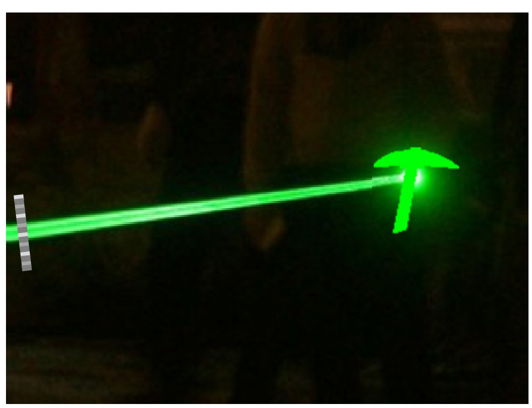
Near-Field to Far-Field

After propagating a long distance, the field within a plane tends toward the Fourier transform of the initial field.

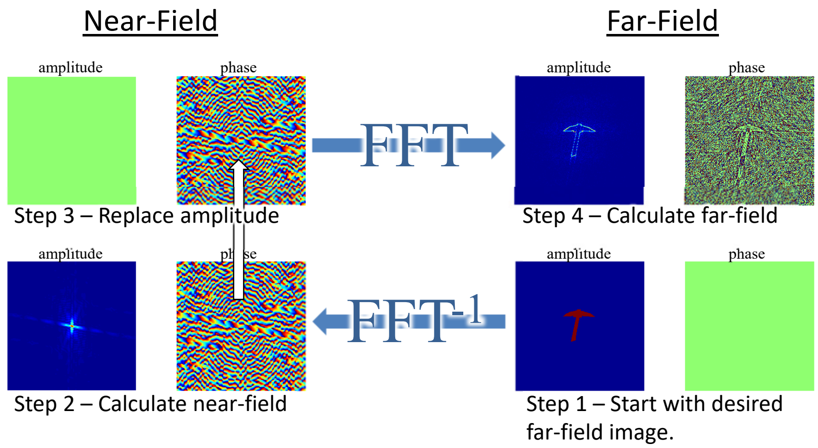


What is a Patterned Fanout Grating?

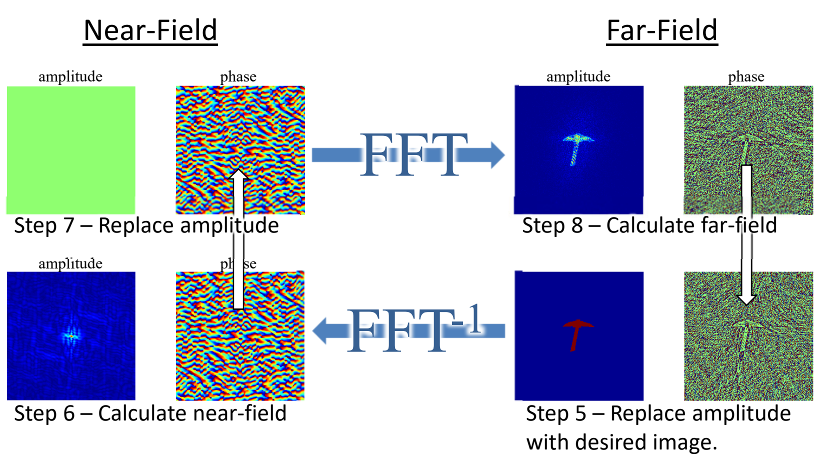
Diffraction grating forces the field to take on the profile of the inverse Fourier transform of an image. After propagating very far, the field takes on the profile of the image.



Gerchberg-Saxton Algorithm:
Initialization

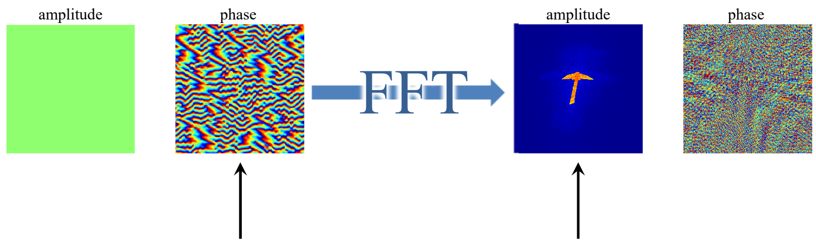


Gerchberg-Saxton Algorithm:
Iteration



Gerchberg-Saxton Algorithm: *End*

Near-Field | Far-Field

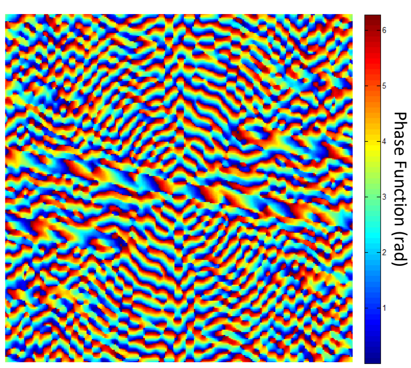


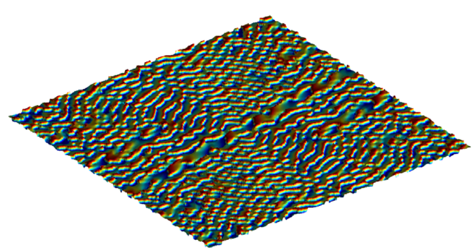
This is the phase function of the diffractive optical element.

This is what the final image will look like.

After several dozen iterations…

The Final Fanout Grating



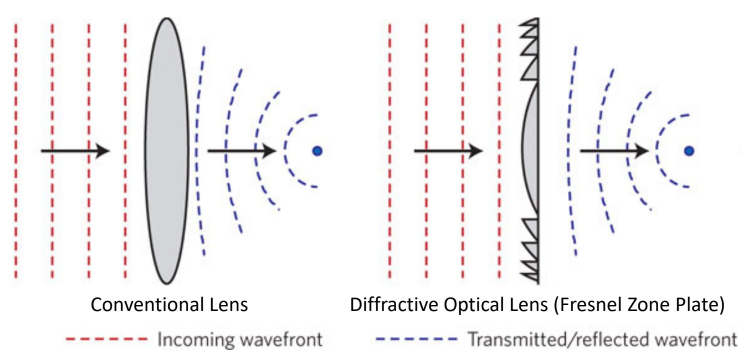


A surface relief pattern is etched into glass to induce the phase function onto the beam of light.

We could also print an amplitude mask using a high resolution laser printer.

Diffractive Optical Elements

What is a Diffractive Optical Element



If the device is only required to operate over a narrow band, devices can be "flattened."

The flattened device is called a diffractive optical element (DOE).

Lukas Chrostowski, "Optical gratings: Nano-engineered lenses," *Nature Photonics* **4**, 413-415 (2010).