RBE 2002: Unified Robotics II
Force Sensing

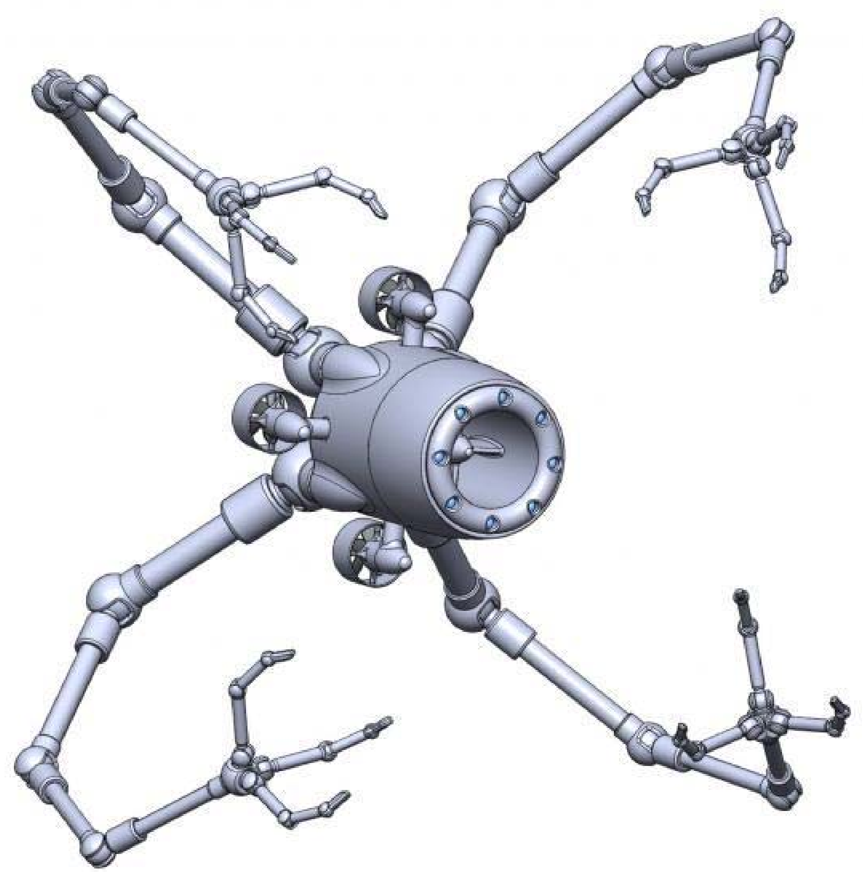
Prof. Putnam

Outline

- ✓ There are many ways to sense and measure forces
- ✓ We will examine one very popular and useful way – the Strain Gauge
- ✓ But before we do so, we need some grounding in the properties of materials as they react to the application of forces

Robot News

- ✓ [“One component in this tactile capability is a strain gauge.”, Marcus Maiwald]



Robot News

- ✓ A new approach was taken to develop a unique simplified six-axis load cell. It consists of a prismatic section of material with semiconductor strain gauges attached around the circumference.

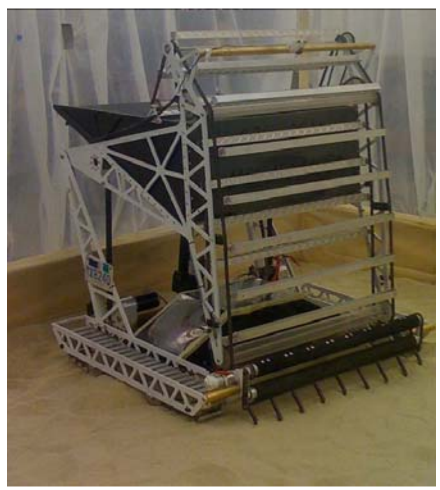
Materials

✓ What are robots made from?

✓ …and why?

Common robot materials:

- Aluminum
- Steel
- Polycarbonate
- Wood!?



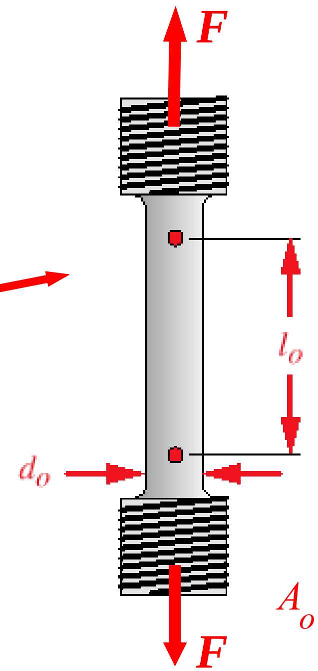
Materials Properties

- ✓ To begin to answer ‘Why?’, we need to examine the properties of various materials
- ✓ Obvious considerations are:
  - Weight (really density)
  - Cost
  - Availability
  - Safety
  - Ease of machining
- ✓ Another big consideration is:
  - Will the material perform well?
  - It is this question we will now explore…

Material Properties

✓ Tensile test machine

Photo courtesy of Instron, Corp.



$$A_o = \pi\left(\frac{d_o}{2}\right)^2$$

FIGURE 2-1

A Tensile Test Specimen

Stress: $\sigma = \frac{F}{A_o}$

Strain: $\varepsilon = \frac{l - l_o}{l_o}$

Modulus of elasticity:

$$E = \frac{\sigma}{\varepsilon}$$

Material Properties

✓ Tensile Test

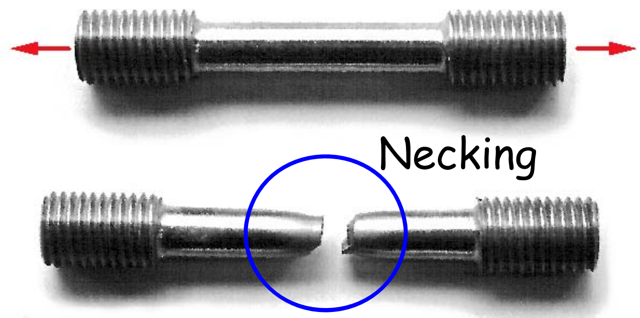


FIGURE 2-3

A Tensile Test Specimen of Mild, Ductile Steel After Fracture

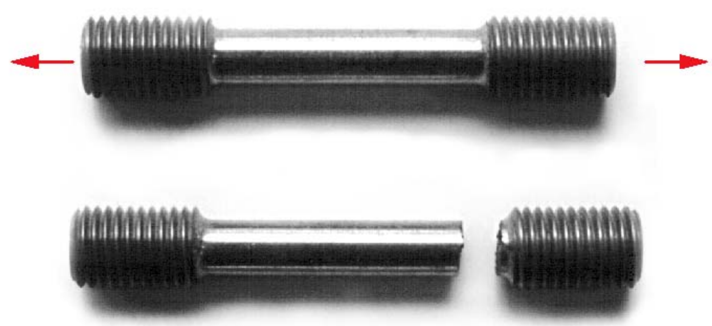
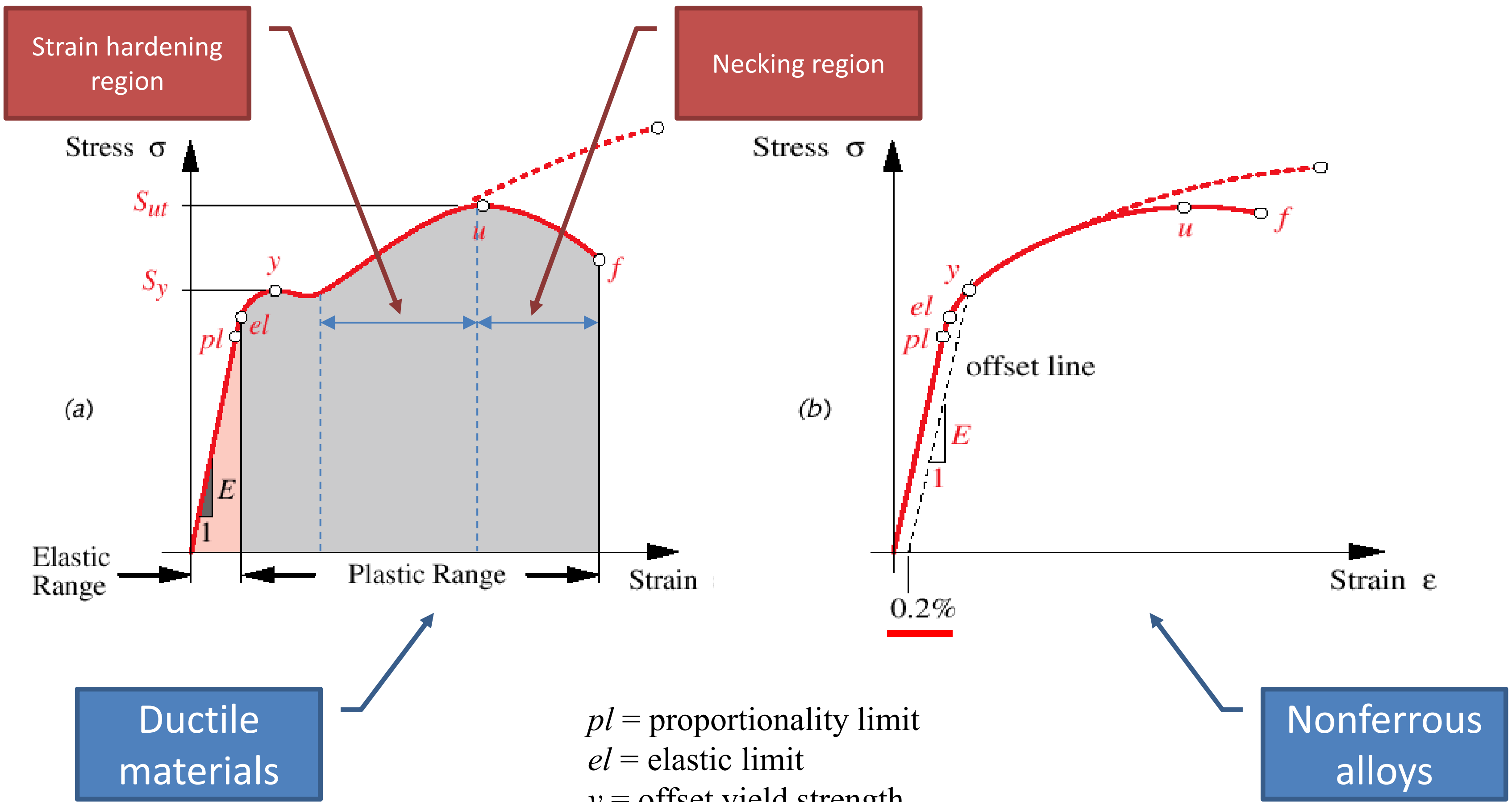


FIGURE 2-5

A Tensile Test Specimen of Brittle Cast Iron After Fracture

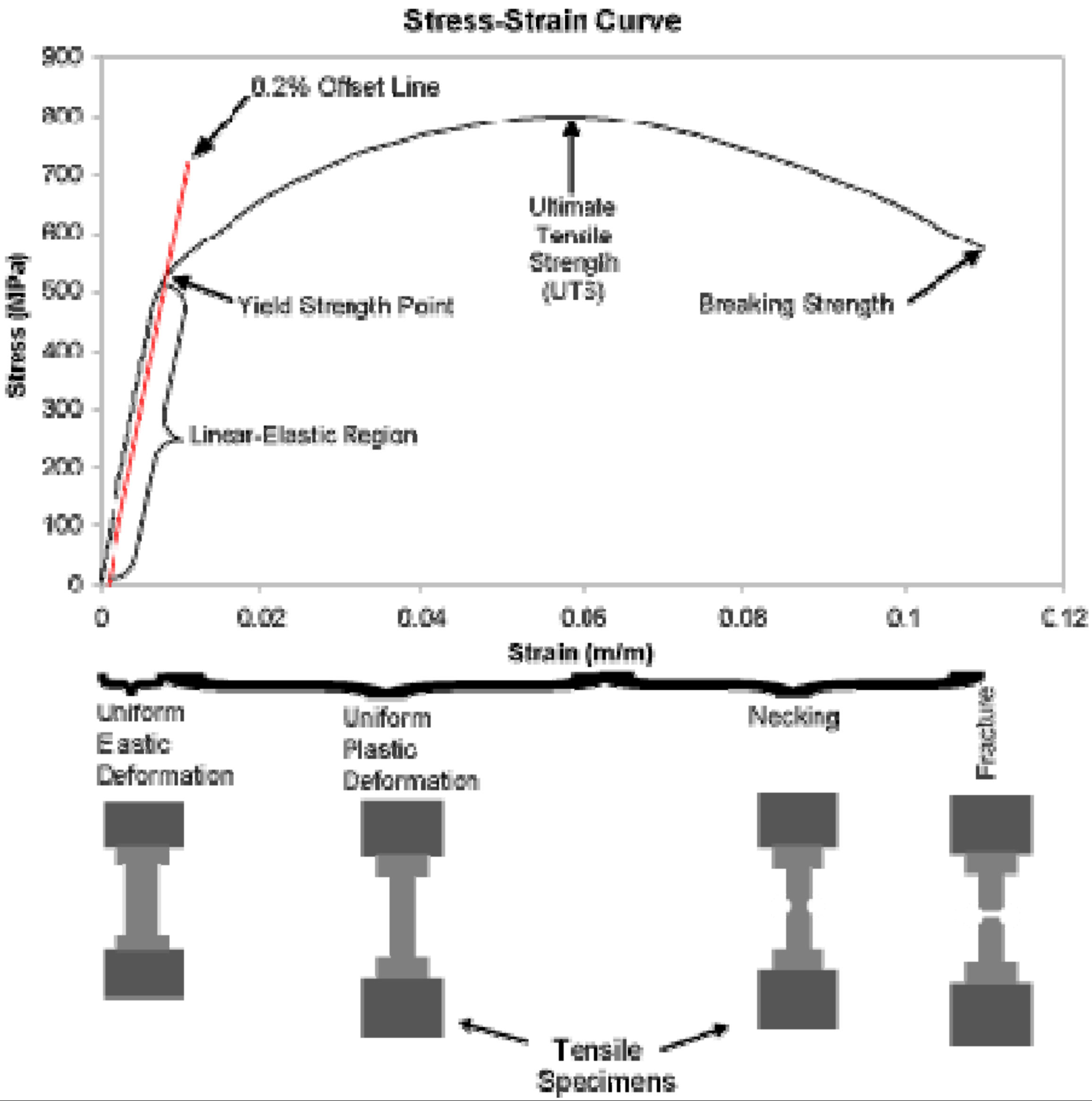
Material Properties

- ✓ Stress-strain diagrams



pl = proportionality limit
el = elastic limit
y = offset yield strength
u = ultimate strength
f = failure strength

Material Properties



Material Properties

✓ Compression Test

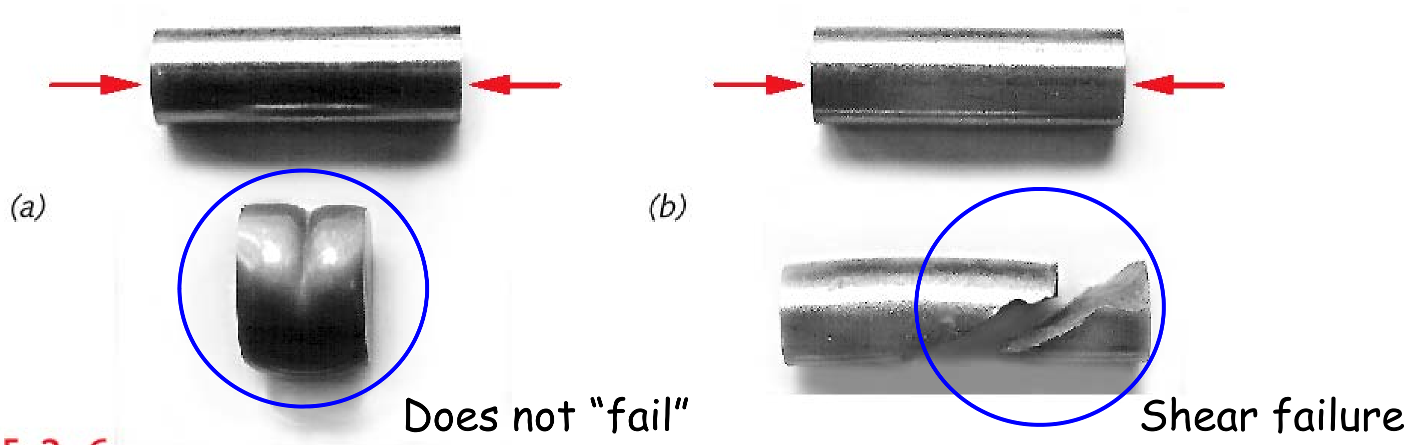


FIGURE 2-6

Compression Test Specimens After Failure *(a)* Ductile Steel *(b)* Brittle Cast Iron

Even materials: same behavior in tension and in compression.

Material Properties

✓ Bending Test

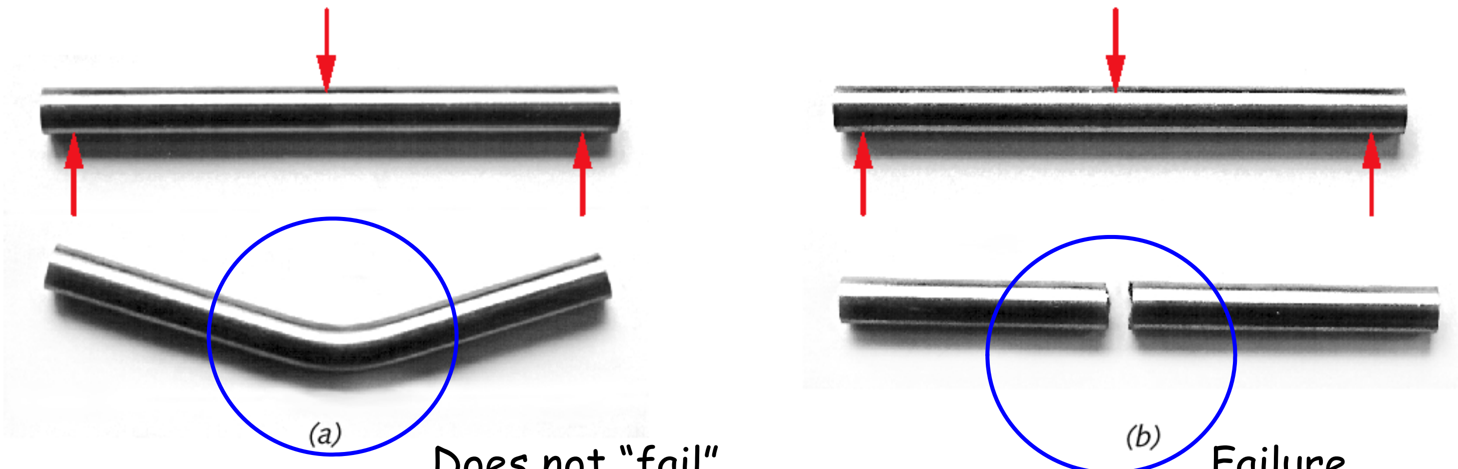


FIGURE 2-7

Bending Test Specimens After Failure *(a)* Ductile Steel *(b)* Brittle Cast Iron

Material Properties

✓ Torsion Test

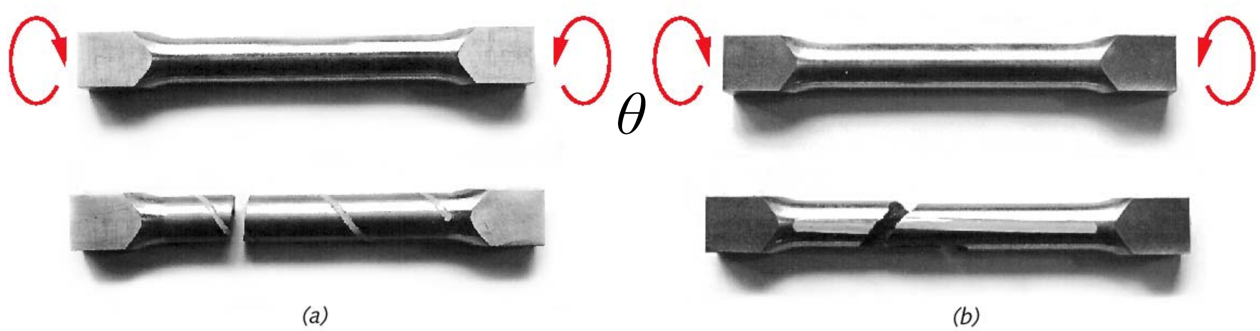


FIGURE 2-8

Torsion Test Specimens After Failure *(a)* Ductile Steel *(b)* Brittle Cast Iron

Stress-strain relation (torsion):

$$\tau = \frac{Gr\theta}{l_o}$$

Modulus of rigidity:

$$G = \frac{E}{2(1+\nu)}$$

Table 2-1
Poisson's Ratio ν

Material	ν
Aluminum	0.34
Copper	0.35
Iron	0.28
Steel	0.28
Magnesium	0.33
Titanium	0.34

Material Properties

✓ Fatigue strength S_f and endurance limit S_e

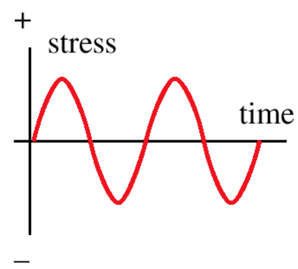
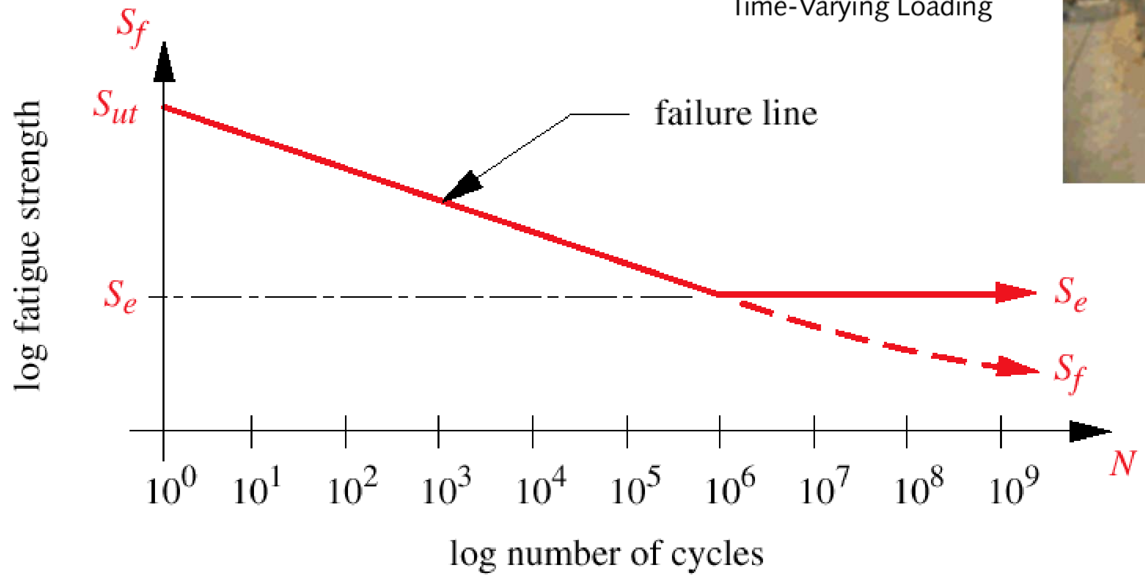


FIGURE 2-9

Time-Varying Loading

Testing of wind turbine blades



An endurance limit S_e exists for some ferrous metals and titanium alloys. Other materials show no endurance limit.

FIGURE 2-10

Wohler Strength-Life or *S-N* Diagram Plots Fatigue Strength Against Number of Fully Reversed Stress Cycles

Measuring Strain

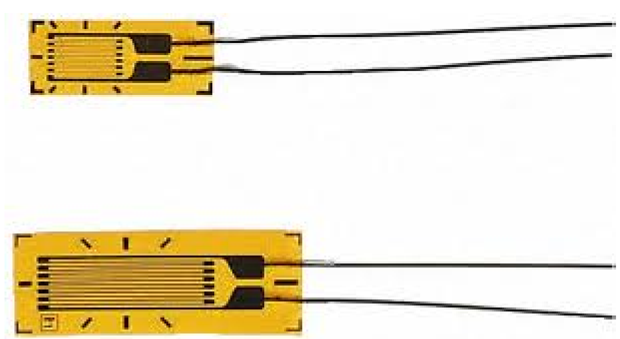
- ✓ Strain Gauges
  - What is a strain gauge and how does it work?
  - Properties
  - Parameters
- ✓ Measuring Strain
- ✓ Signal Conditioning
  - Wheatstone Bridge
  - Op Amps
  - Instrumentation Amplifiers

Have you used a strain gauge before?

1. Yes
2. No

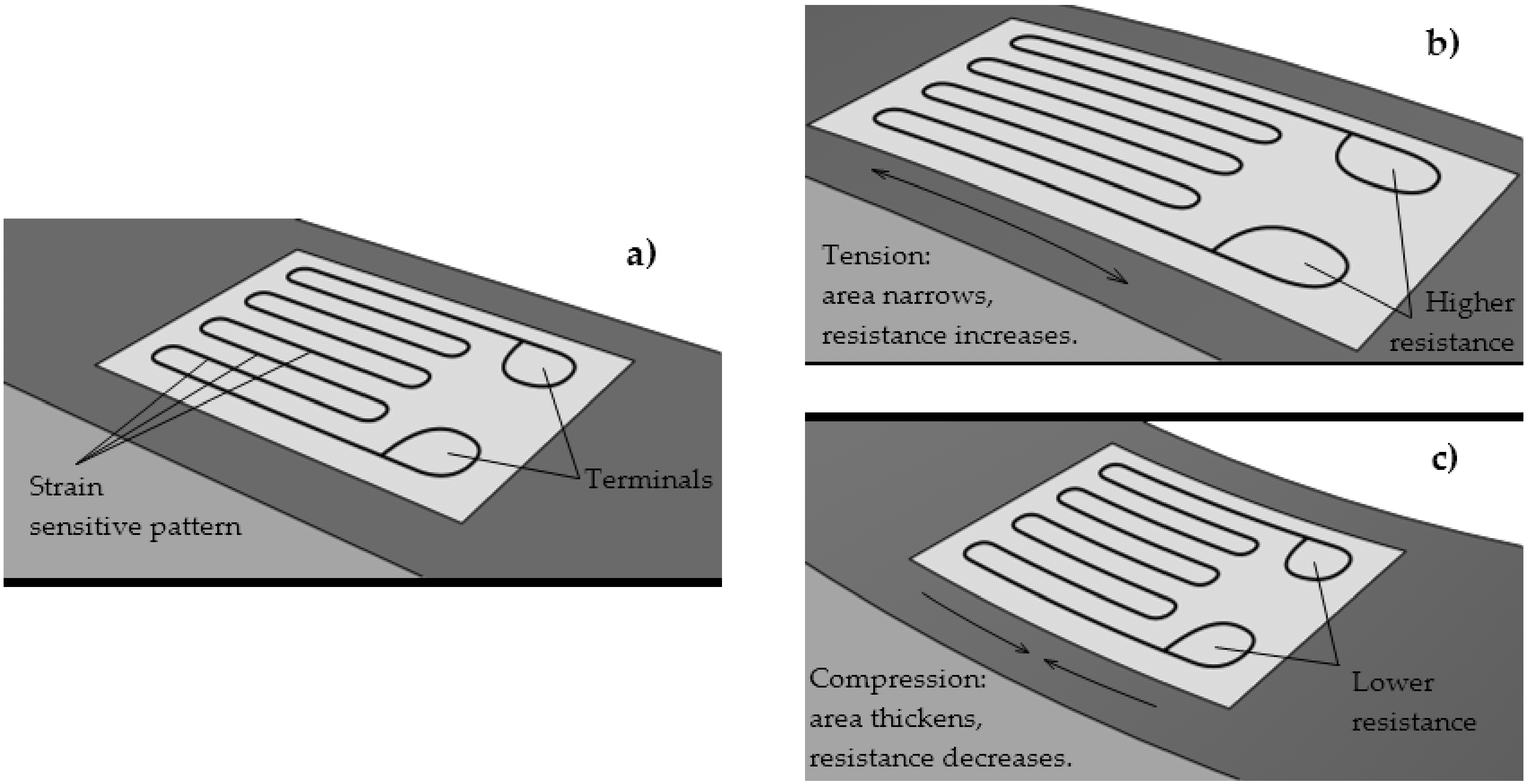
Strain Gauges

- ✓ What is a strain gauge and how does it work?
- ✓ As the name implies, it is a device used to measure strain
- ✓ Recall that strain is the change in length with respect to an initial reference length of a material that has a net force applied: $\varepsilon = \Delta l / l_o = (l - l_o)/l_o$
- ✓ The strain gauge measures strain indirectly, by generating tiny changes in electrical resistance



Strain Gauges

✓ How does it work?



Strain Gauges

- ✓ The resistance of a wire is given as:

$$R = \rho \frac{l}{A}$$

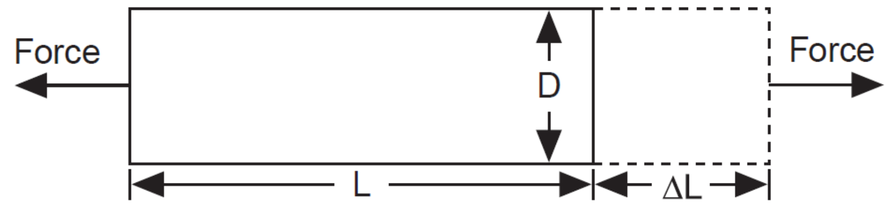
- ✓ When the wire is stressed longitudinally:

$$dR = \frac{\rho}{A} dl - \frac{\rho l}{A^2} dA + \frac{l}{A} d\rho \rightarrow \frac{dR}{R} = \frac{dl}{l} - \frac{dA}{A} + \frac{d\rho}{\rho}$$

Strain Gauges

- ✓ The stress is given by:

$$\sigma = E\varepsilon = E\frac{dl}{l}$$



where E = Young's Modulus

- ✓ We will bond the strain gauge to the material so that as the material is put into tension or compression, so is the strain gauge
- ✓ This means the strain in the gauge in the longitudinal direction is the same as the strain on the surface of the material in the same direction: $\varepsilon_l = dl/l$

Strain Gauges

- ✓ Suppose we are using a circular cross-section wire
- ✓ The normal strain along the radial direction is given by:

$$\varepsilon_r = \frac{dr}{r} = -v\varepsilon_l = -v\frac{dl}{l}$$

where v = Poisson's Ratio

- ✓ The rate-of-change of the cross-sectional area goes as twice the radial strain (when the strain is small)

$$\frac{dA}{A} = \left(1+\varepsilon_r\right)^2 - 1 = 2\varepsilon_r + \varepsilon_r^2 \cong 2\varepsilon_r$$

$$= -2v\frac{dl}{l}$$

Strain Gauges

✓ The resistance change rate therefore becomes:

$$\frac{dR}{R} = \frac{dl}{l} - \frac{dA}{A} + \frac{d\rho}{\rho} = \left(1+2v\right)\frac{dl}{l} + \frac{d\rho}{\rho}$$

$$= \left(1+2v\right)\varepsilon_l + \frac{d\rho}{\rho}$$

Strain Gauges

- ✓ The sensitivity of a strain gauge material, i.e., the rate-of-change in resistance given an applied strain is called the *Gauge Factor* and is given by:

$$GF = \frac{dR/R}{\varepsilon_l} = \left(1 + 2v\right) + \frac{d\rho/\rho}{\varepsilon_l}$$

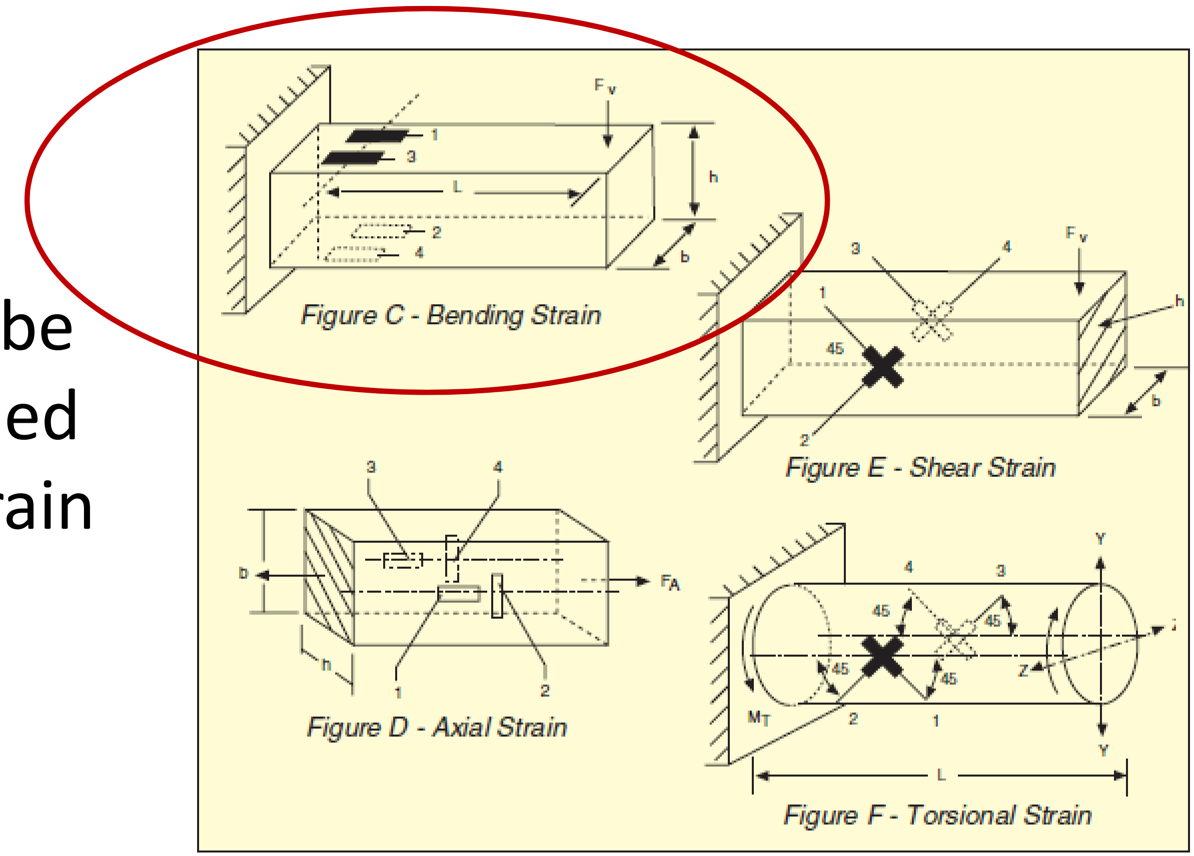
- ✓ This value is supplied by the strain gauge manufacturer and is usually in the range of 2 – 3
- ✓ Check with Joe St. Germaine for the GF value of the strain gauges you will be using in the upcoming strain gauge lab

Measuring Strain

- ✓ Strain Gauges
  - What is a strain gauge and how does it work?
  - Properties
  - Parameters
- ✓ **Measuring Strain**
- ✓ Signal Conditioning
  - Wheatstone Bridge
  - Op Amps
  - Instrumentation Amplifiers

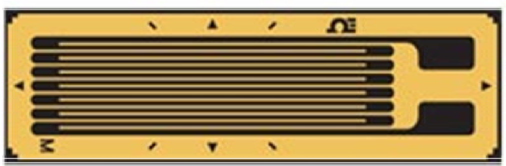
Measuring Strain

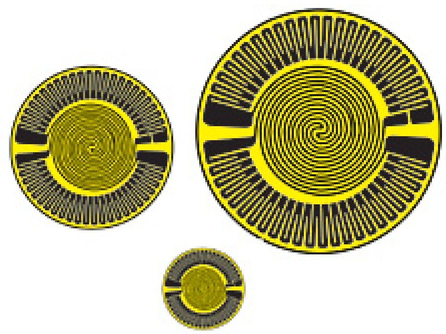
- ✓ The figure below shows how strain gauges can be used to measure different kinds of strain:
  - Bending
  - Axial
  - Shear
  - Torsional
- ✓ The lab you will be doing is concerned with bending strain

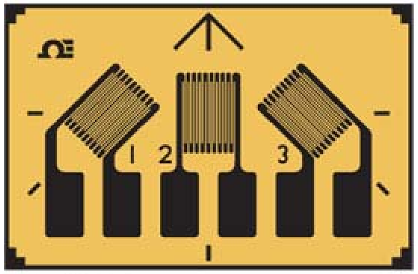


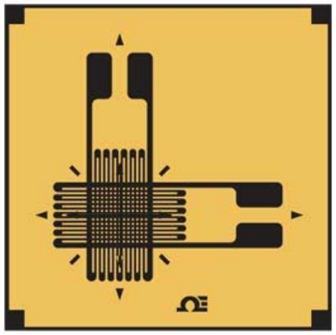
Measuring Strain

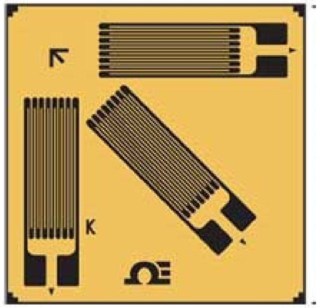
✓ Different kinds of strain gauges are used depending on the type of strain you are trying to measure (and other considerations…)

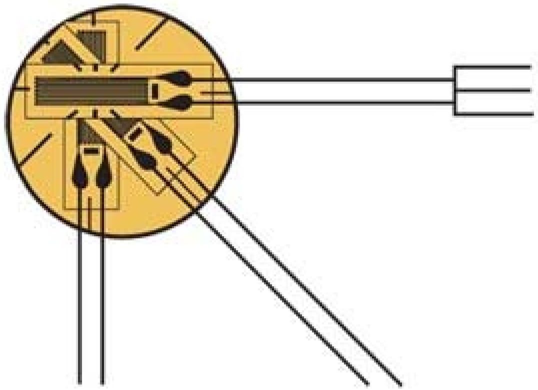












Images: www.omega.com

Measuring Strain

- ✓ The strain gauges you will be using in the lab look something like this:

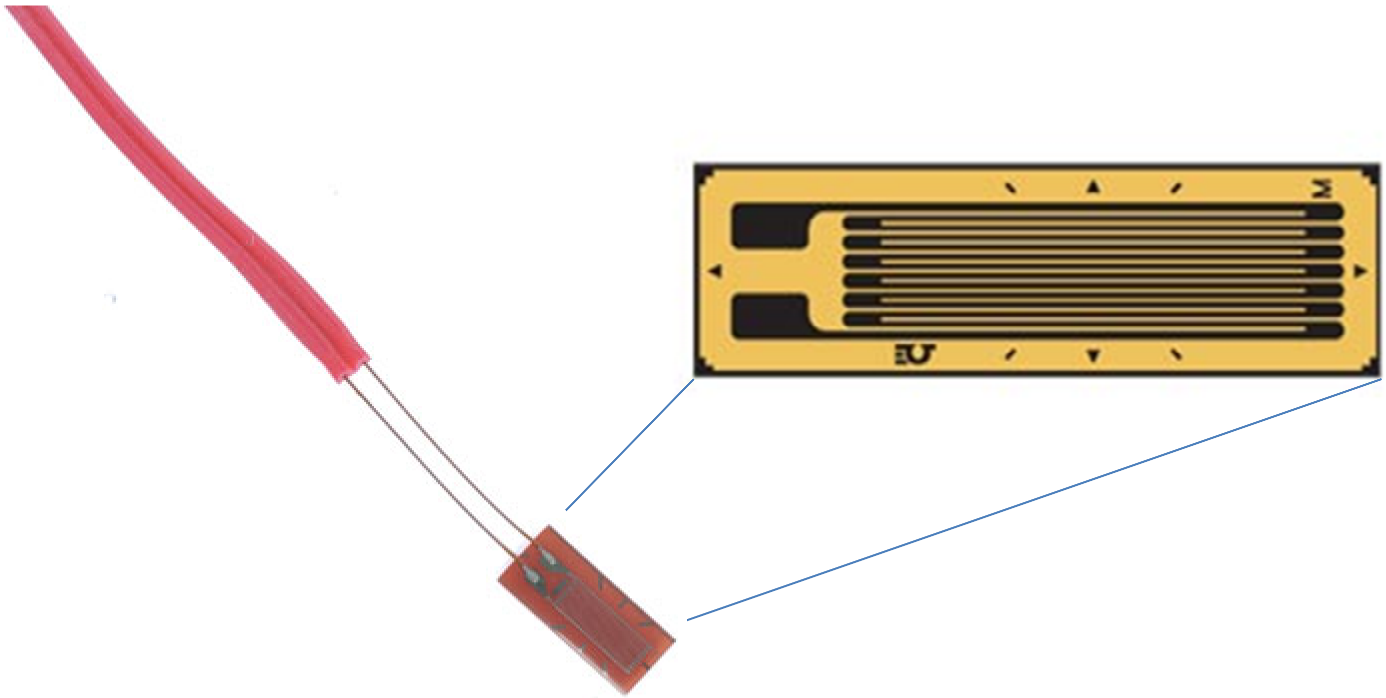


Image: www.omega.com

Measuring Strain

- ✓ You will be measuring bending strain using a cantilever beam
- ✓ Notice that the figure shows 4 strain gauges in use
- ✓ Why do that?

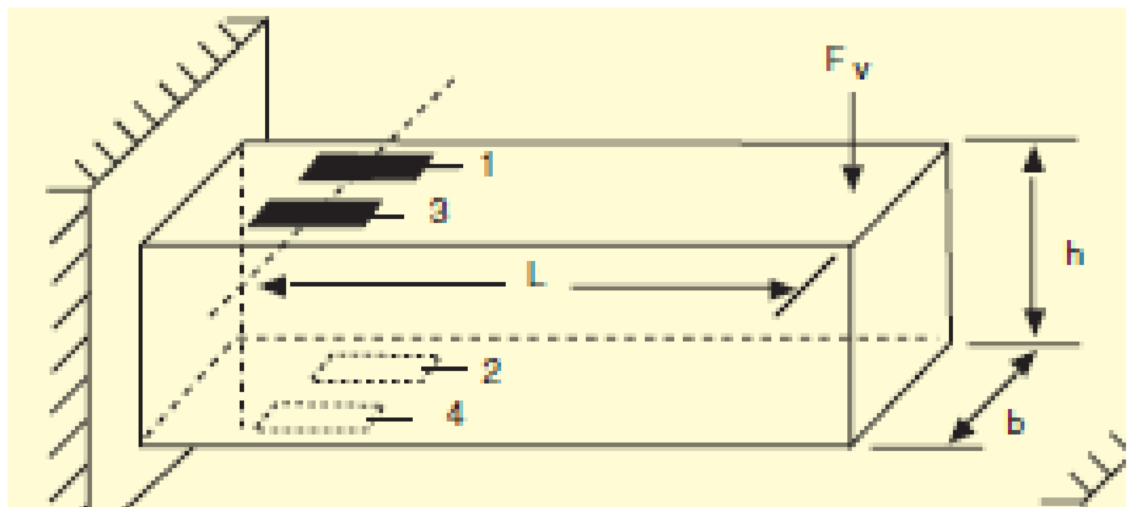


Figure C - Bending Strain

Measuring Strain

- ✓ The answer comes from what we are actually measuring with a strain gauge…
- ✓ …which is resistance
- ✓ …or more specifically, a <u>change</u> in resistance

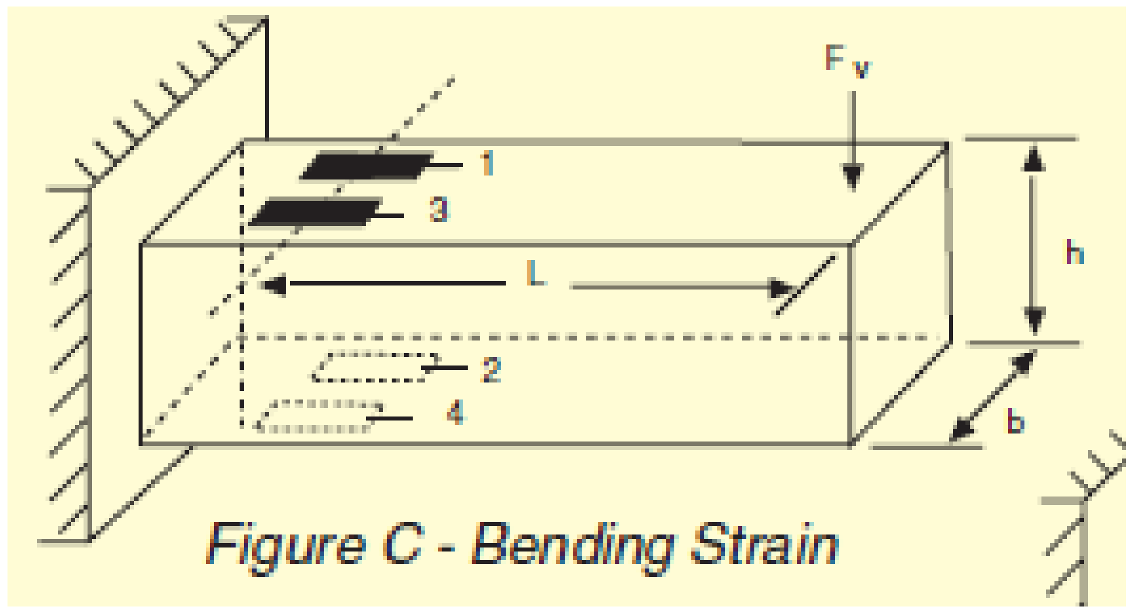


Figure C - Bending Strain

Measuring Strain

- ✓ The amount of strain typically measured does not exceed a few millistrain, i.e., $\varepsilon \times 10^{-3}$
- ✓ Given the small amount of strain, this means you will be measuring a small change in the resistance of the strain gauge

Measuring Strain

- ✓ Example: Suppose your strain gauge has a $GF = 2$ and experiences a strain of: $\varepsilon = 100 \times 10^{-6} = 0.1 \times 10^{-3}$
- ✓ This means that the change in resistance will be:

$$2\left(100 \times 10^{-6}\right) = 0.02\%$$

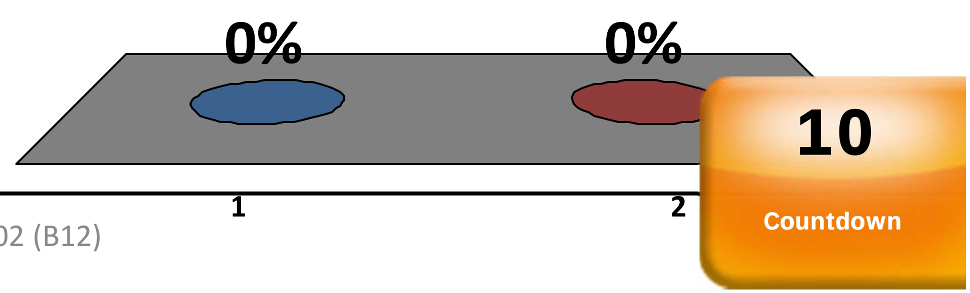
or, for a $350\ \Omega$ strain gauge, a change of $0.07\ \Omega$

Measuring Strain

- ✓ A change in resistance of $0.07\ \Omega$ is very small!
- ✓ Your typical ohmmeter will not measure changes in resistance that are this small
- ✓ Special methods are needed…
- ✓ Thankfully, a simple circuit makes it easy to measure very small changes in resistance
- ✓ It’s called a Wheatstone Bridge

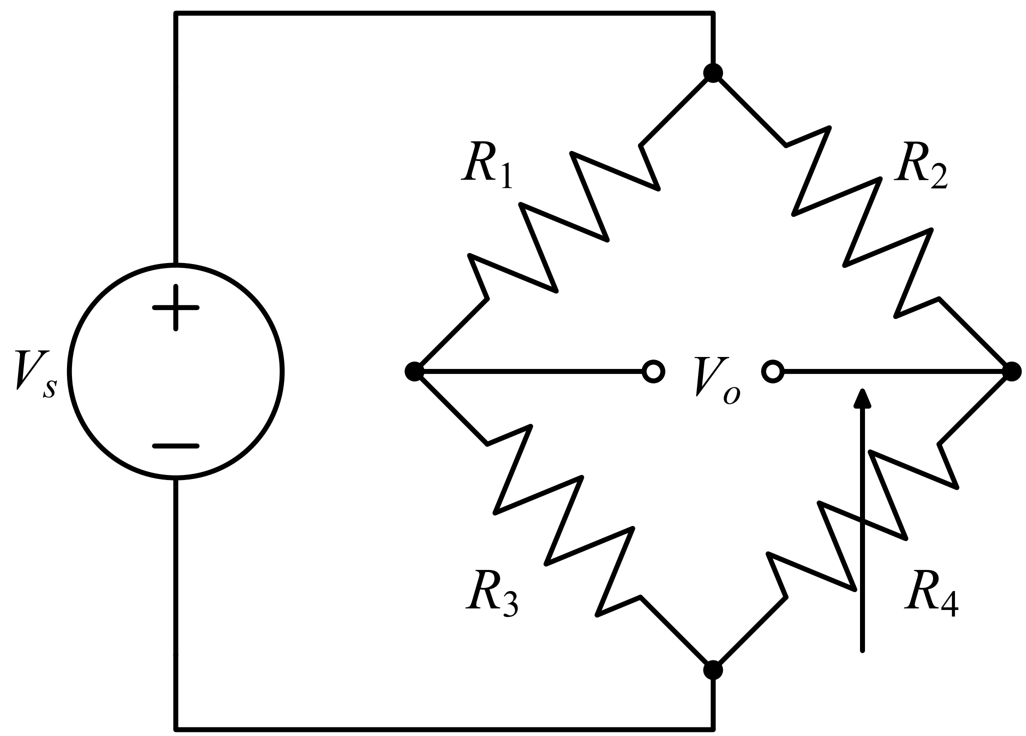
Have you used a Wheatstone Bridge before?

1. Yes
2. No



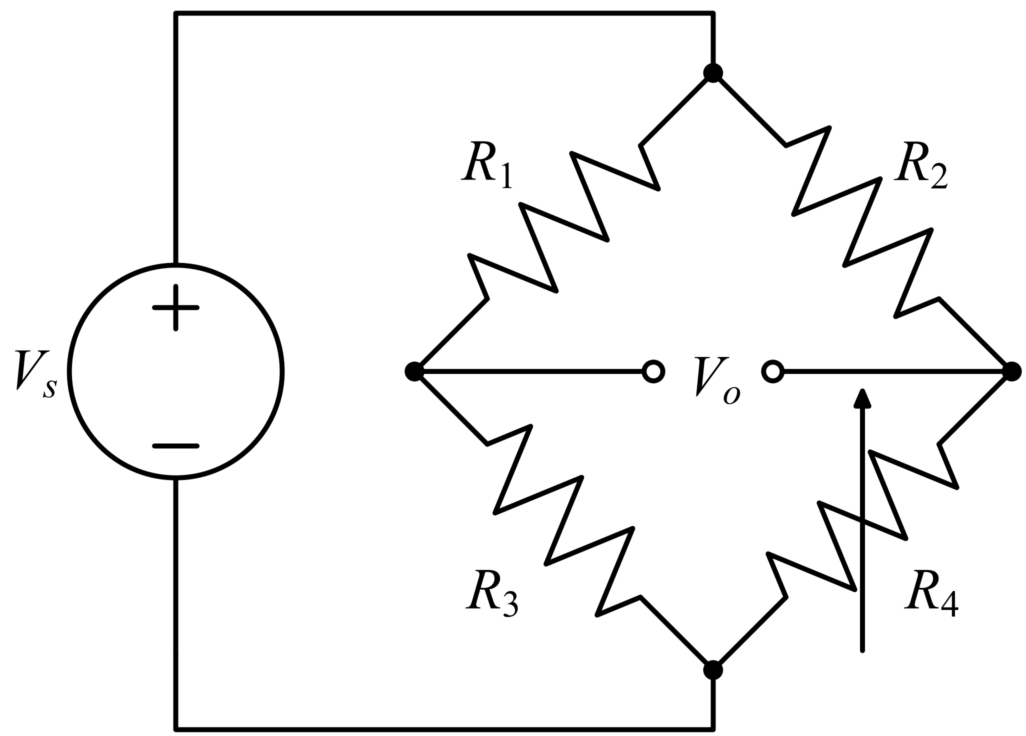
Wheatstone Bridge

- ✓ When the four resistances R_1, R_2, R_3, and R_4 are all equal, the voltage V_o will be zero



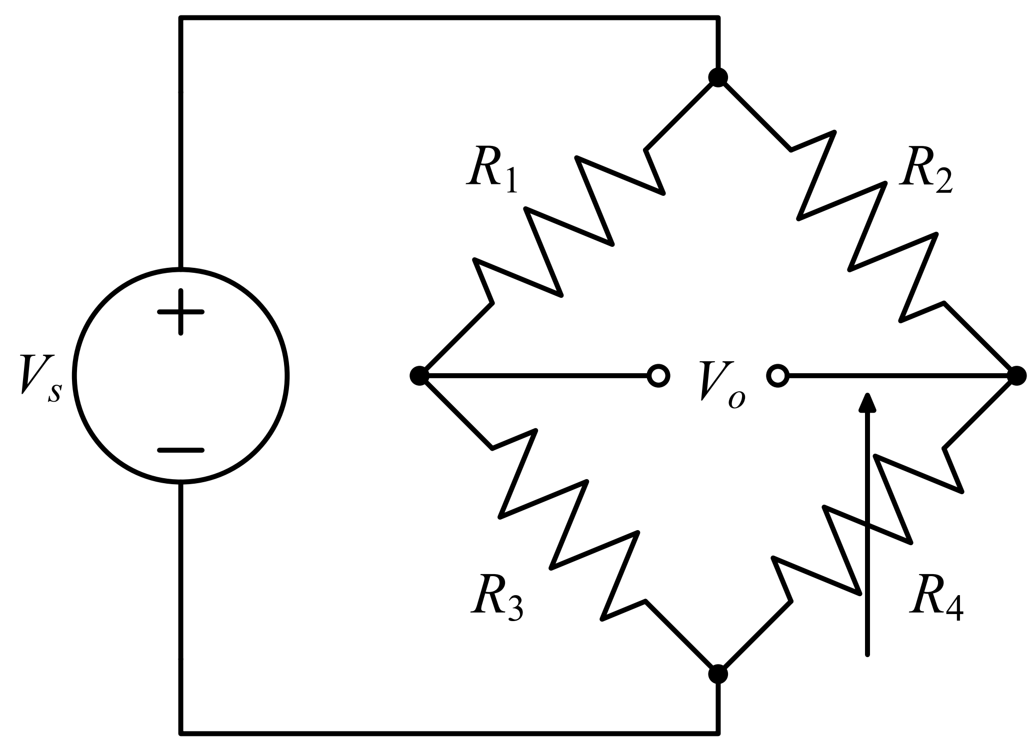
Wheatstone Bridge

- ✓ If the bridge becomes unbalanced, for example because R_4 has a different value from R_1, R_2, and R_3, then V_o is non-zero
- ✓ We can easily relate V_o to R_4 (and R_1, R_2, R_3 and V_S)



Wheatstone Bridge

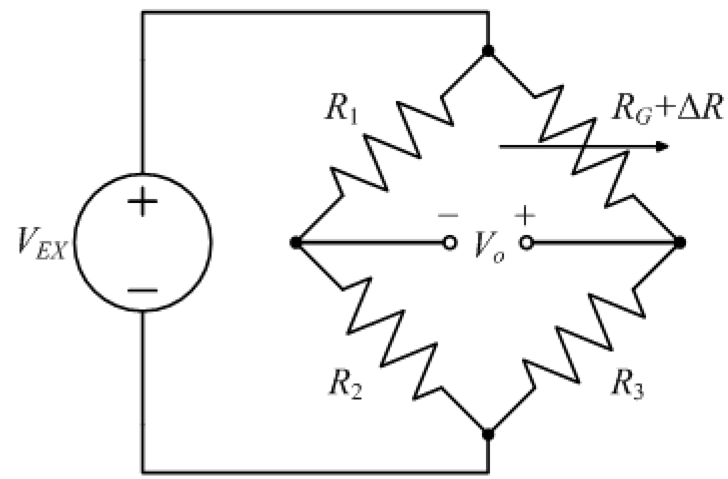
- ✓ We know that the change in R_4 will be small
- ✓ It turns out that V_o will also be small, but we can easily deal with that…



Wheatstone Bridge

- ✓ Let's assume that R_1, R_2, and R_3, are all equal to R_G and that the remaining resistor in the circuit is a strain gauge with an nominal value also equal to R_G
- ✓ The relationship between the output voltage V_o and the excitation voltage V_{EX} is given by:

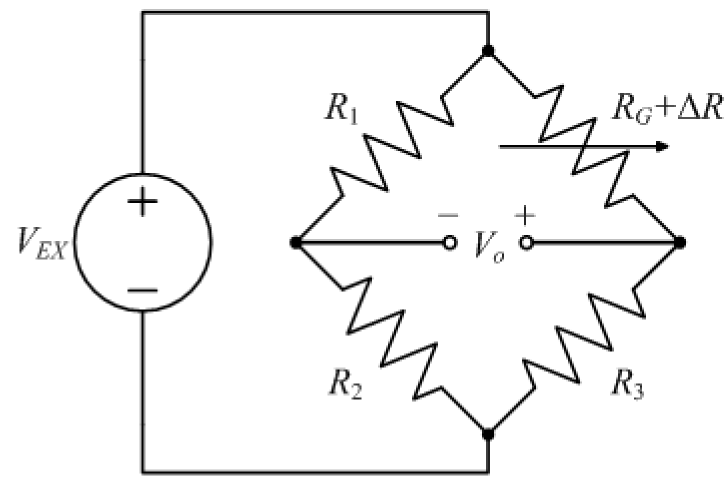
$$\frac{V_o}{V_{EX}} = -\frac{\mathrm{GF} \cdot \varepsilon}{4} \left[\frac{1}{1 + \mathrm{GF} \cdot \frac{\varepsilon}{2}} \right]$$



Wheatstone Bridge

- ✓ Going back to our earlier example, suppose that $V_{EX} = 12\text{V}$ and that we again have: $\varepsilon = 100 \times 10^{-6}$
- ✓ This results in an output voltage $V_o = 0.007199\ \text{V}$ – which is pretty small
- ✓ We know that we can use an op-amp to boost this up to a usable range, but it would be better if we had a larger signal to work with in the first place

$$\frac{V_o}{V_{EX}} = -\frac{\text{GF}\cdot\varepsilon}{4}\left[\frac{1}{1+\text{GF}\cdot\dfrac{\varepsilon}{2}}\right]$$



Other Considerations

- ✓ Before we look at ways to boost up the signal, there are also other problems to consider
- ✓ One problem has to do with the resistance of the leads used to connect the strain gauges to the measurement equipment
- ✓ If the leads are long and thin (which they may well be) that can add a lot of resistance to the legs of the bridge
- ✓ I'll leave it to you to research ways to deal with this…

Other Considerations

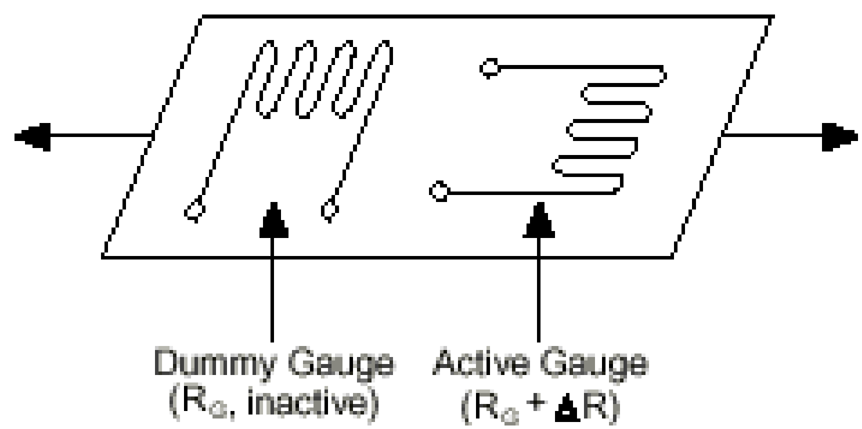
- ✓ Another consideration comes from the fact that we are bonding the strain gauge to some other material
- ✓ We need to deal with the fact that various materials have different coefficients of thermal expansion
- ✓ If the strain gauge and the material it is bonded to experience a temperature change – and have different coefficients of thermal expansion – then they will grow (or shrink) at different rates
- ✓ This will apply stress to the strain gauge and affect your results
- ✓ For this reason strain gauges are made for bonding to specific materials…

Other Considerations

- ✓ Another big problem has to do with the temperature coefficient of resistivity
- ✓ The issue here is that the resistance of the materials making up our Wheatstone Bridge are temperature sensitive
- ✓ Depending on how the bridge is constructed the various resistors may be at different locations and therefore at different temperatures!
- ✓ How do we deal with this?

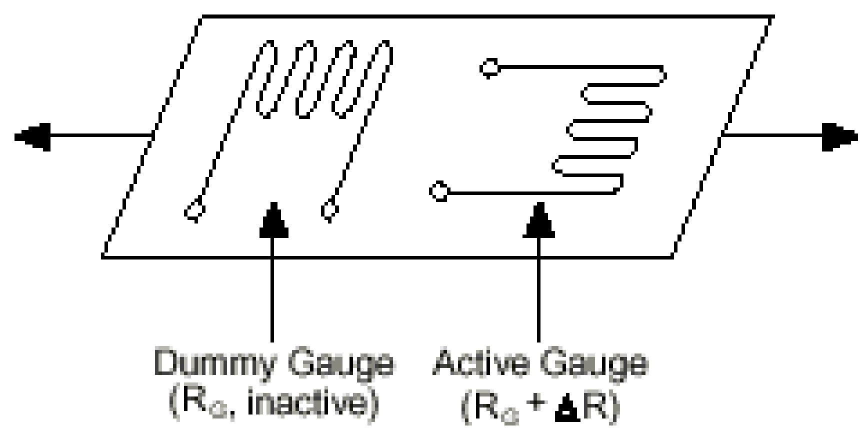
Temperature Compensation

- ✓ One way to deal with the temperature problem is to replace one of the bridge resistors with a dummy strain gauge (it must be in the same vertical leg of the Wheatstone bridge for this to work)
- ✓ The dummy gauge is placed at right angles to the active gauge at the same location



Temperature Compensation

- ✓ By placing the dummy gauge at right angles there is little effect from the strain
- ✓ The temperatures of the gauges will (presumably) be the same though, so the effects of temperature changes are minimized



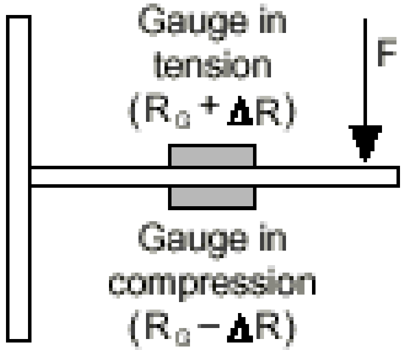
Half-Bridge Configuration

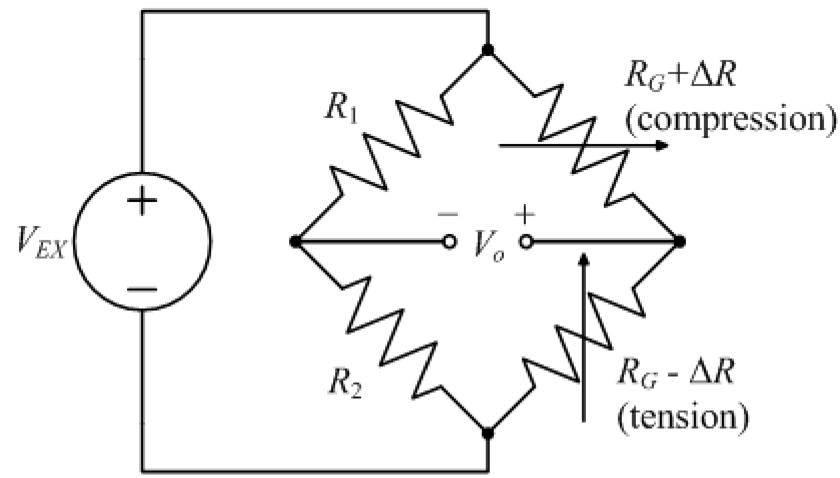
- ✓ But if you’re going to all the work of placing another strain gauge on the material, why not use it to measure strain (as well as compensating for temperature differences)?
- ✓ Doing this is called a half-bridge configuration
  [Using a single strain gauge is therefore called a quarter-bridge configuration]

Half-Bridge Configuration

- ✓ Here we see that the 2nd strain gauge has been placed on the bottom of a cantilever – so one strain gauge is in tension and the other is in compression
- ✓ It (the 2nd strain gauge) can still do temperature compensation, but in this case (because it is aligned with the strain) it also increases the signal output
- ✓ OK – so if we can do this on the right-hand side of the bridge…

$$\frac{V_o}{V_{EX}} = -\frac{\mathrm{GF}\cdot\varepsilon}{2}$$

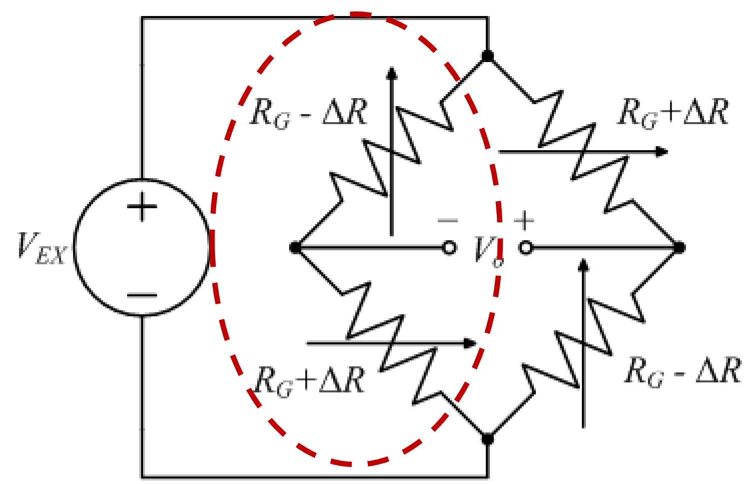




Full-Bridge Configuration

- ✓ Suppose we place two additional strain gauges on the top and bottom of the cantilever bar
- ✓ We still have temperature compensation, but now the signal output is <u>4 times</u> that of the original quarter-bridge configuration

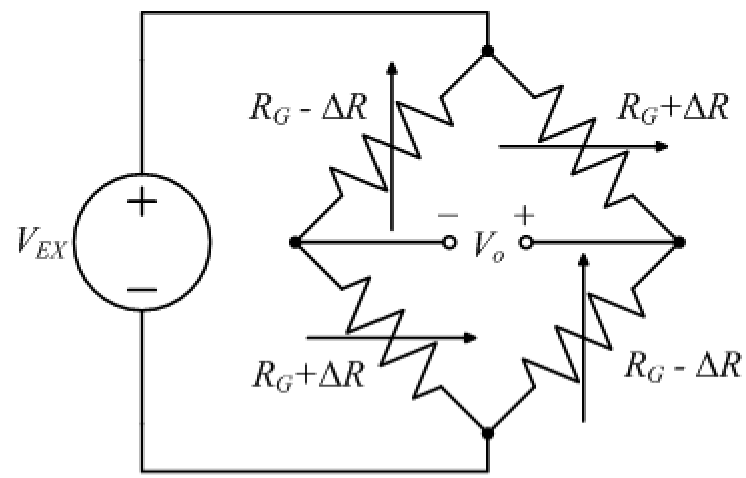
$$\frac{V_o}{V_{EX}} = -\mathrm{GF} \cdot \varepsilon$$



Full-Bridge Configuration

- ✓ As a result, we don't have to crank up the gain so high on the op-amp to get a usable signal
- ✓ What we have done is increase the signal-to-noise ratio (SNR) by a factor of 4
- ✓ This is always a good thing to do if possible!

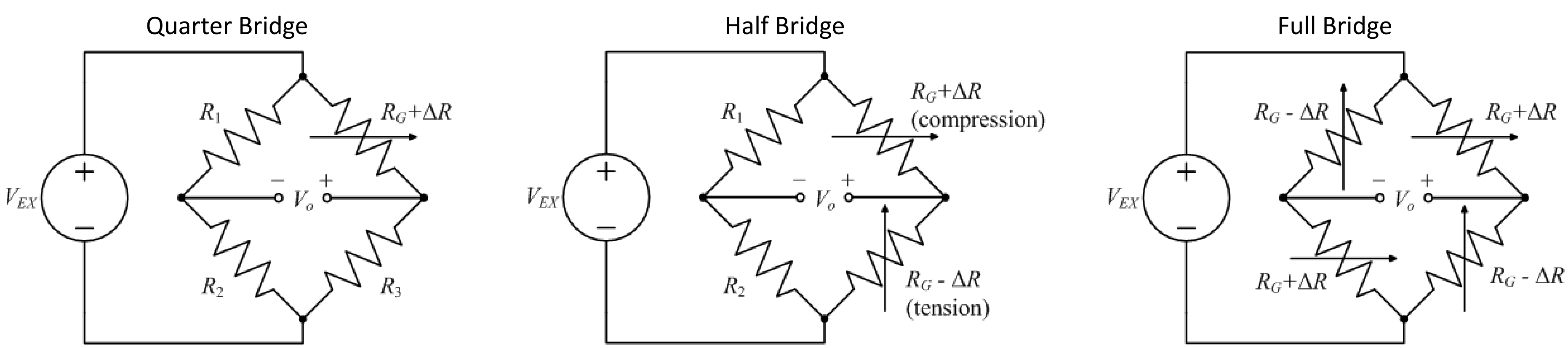
$$\frac{V_o}{V_{EX}} = -\mathrm{GF} \cdot \varepsilon$$



Comparison Summary

✓ Let's compare what we have so far

Type	Output	Temp Comp.
Quarter Bridge	$\frac{V_o}{V_{EX}} = -\frac{GF \cdot \varepsilon}{4}\left[\frac{1}{1+GF\cdot\frac{\varepsilon}{2}}\right]$	No
Half Bridge	$\frac{V_o}{V_{EX}} = -\frac{\mathrm{GF}\cdot\varepsilon}{2}$	Yes
Full Bridge	$\frac{V_o}{V_{EX}} = -\mathrm{GF}\cdot\varepsilon$	Yes



Measuring Strain

- ✓ So now we see why there were four strain gauges shown in Figure C below – they are assuming a full-bridge configuration
- ✓ Let's now examine this in a little more detail…

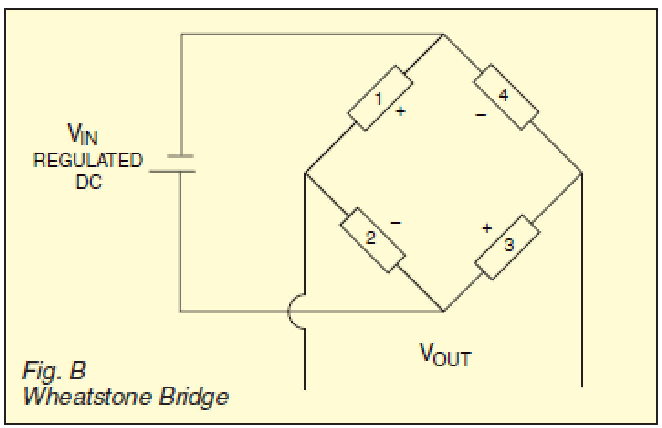


Fig. B
Wheatstone Bridge

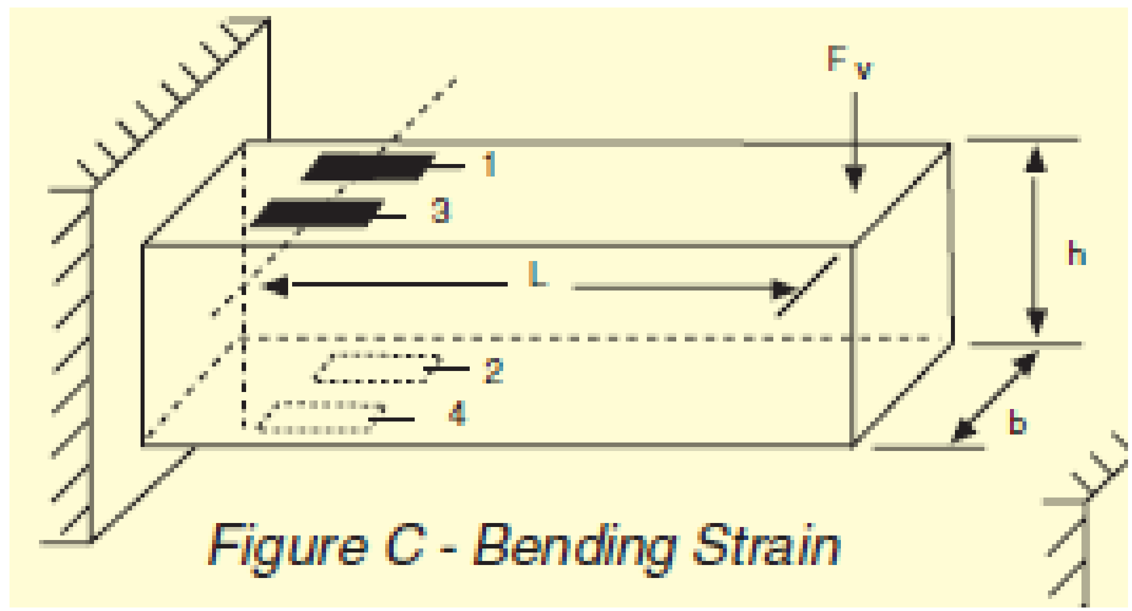


Figure C - Bending Strain

Measuring Strain In A Cantilever Bar

- ✓ Do the dimensions of the cantilever bar and the location of the strain gauges on the bar matter?
- ✓ If we measure the strain can we figure out what the applied force F_v is?
- ✓ The answers are Yes and Yes

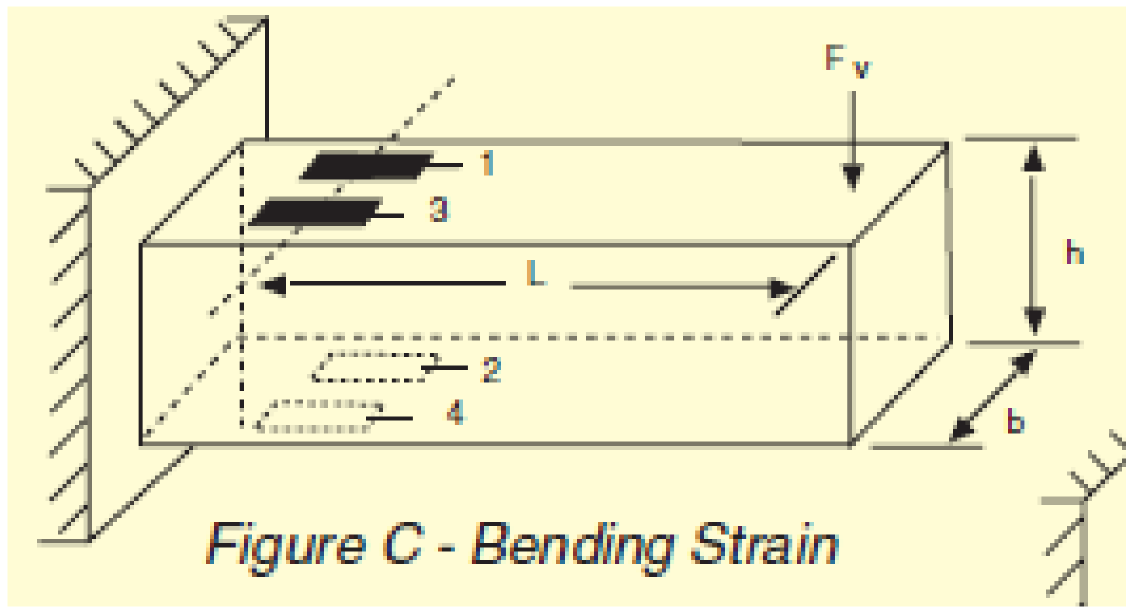


Figure C - Bending Strain

Measuring Strain In A Cantilever Bar

- ✓ Bending strain is equal to the bending or moment stress (O_B) divided by Young's Modulus of Elasticity (a property of the cantilever bar material)

$$\varepsilon_B = \frac{O_B}{E}$$

- ✓ The moment stress is equal to the bending moment ($F_v \times L$) divided by the sectional modulus (Z) – which is a property of the cross-sectional configuration of the cantilever bar

$$O_B = \frac{M_B}{Z} = \frac{F_v L}{Z}$$

Measuring Strain In A Cantilever Bar

- ✓ For rectangles only, the sectional modulus is given by:

$$Z = \frac{bh^2}{6}$$

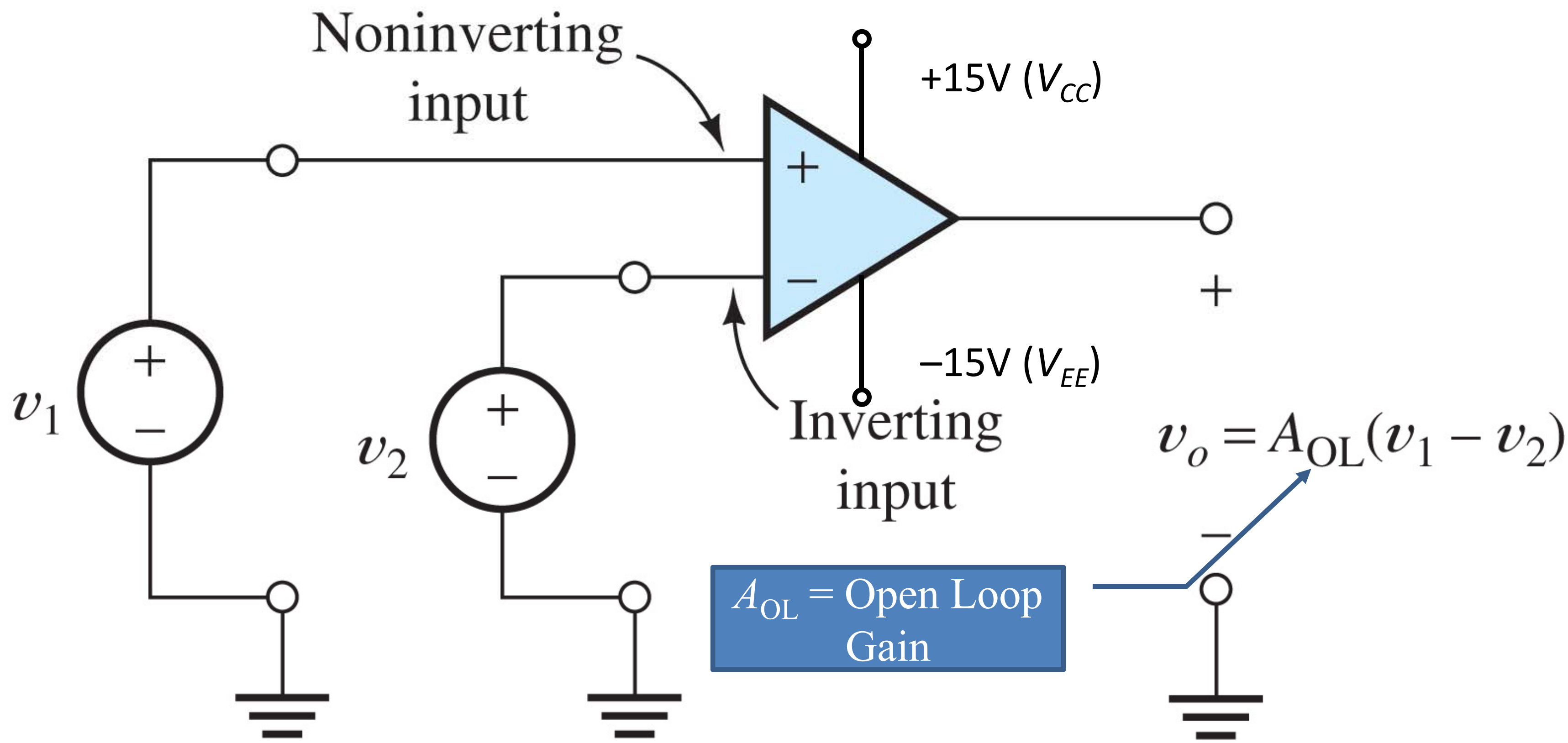
- ✓ We can now combine these to produce an equation for the applied force F_v in terms of the dimensions of the cantilever bar (b, h), the position of the strain gauges along the bar (L), the measured bending strain (ε_B), and the material properties of the bar (E)

$$F_v = E\varepsilon_B \frac{Z}{L} = E\varepsilon_B \frac{bh^2/6}{L}$$

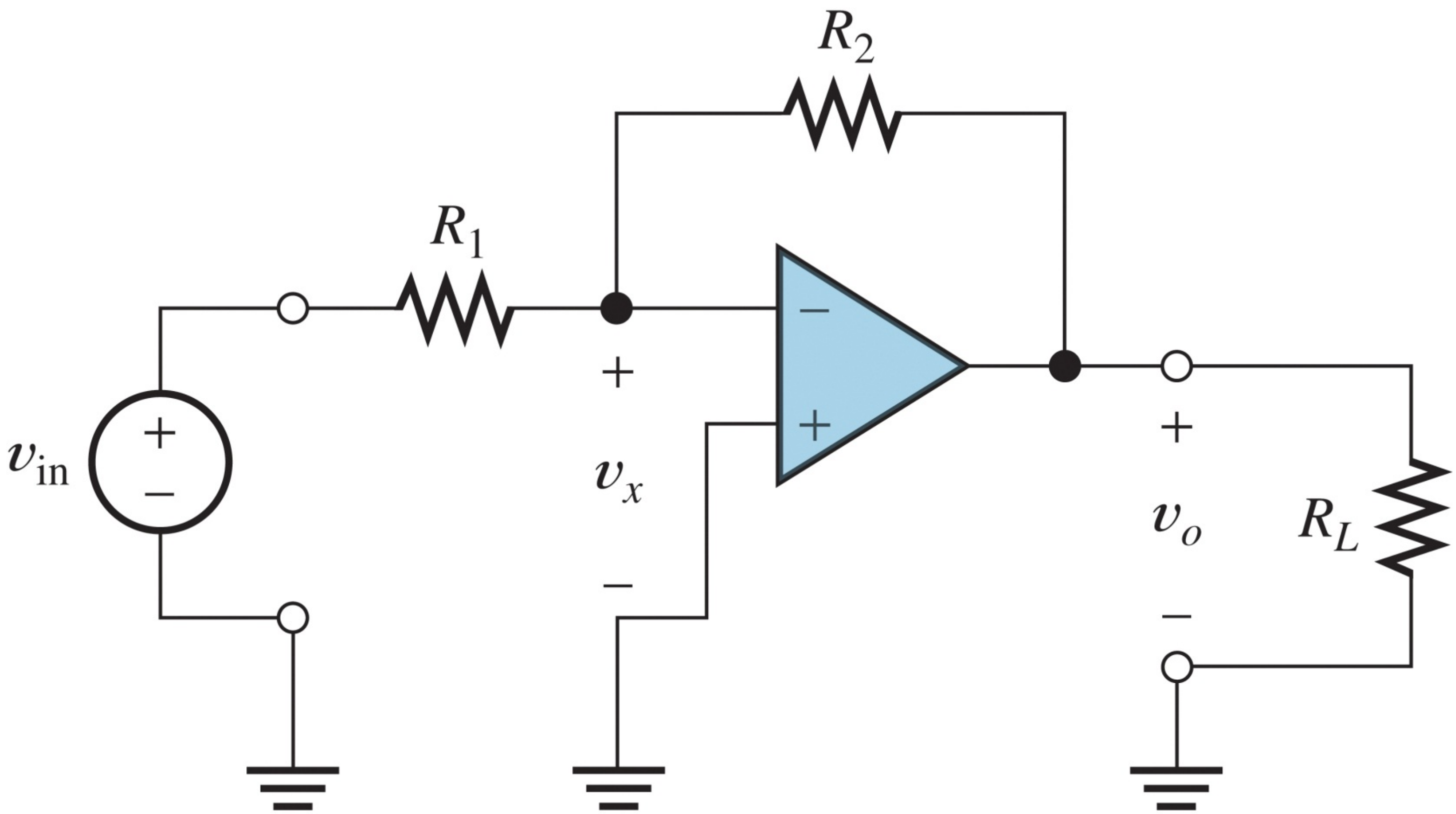
Measuring Strain

- ✓ Strain Gauges
  - What is a strain gauge and how does it work?
  - Properties
  - Parameters
- ✓ Measuring Strain
- ✓ Signal Conditioning
  - Op Amps
  - Wheatstone Bridge
  - Instrumentation Amplifiers

The Op Amp

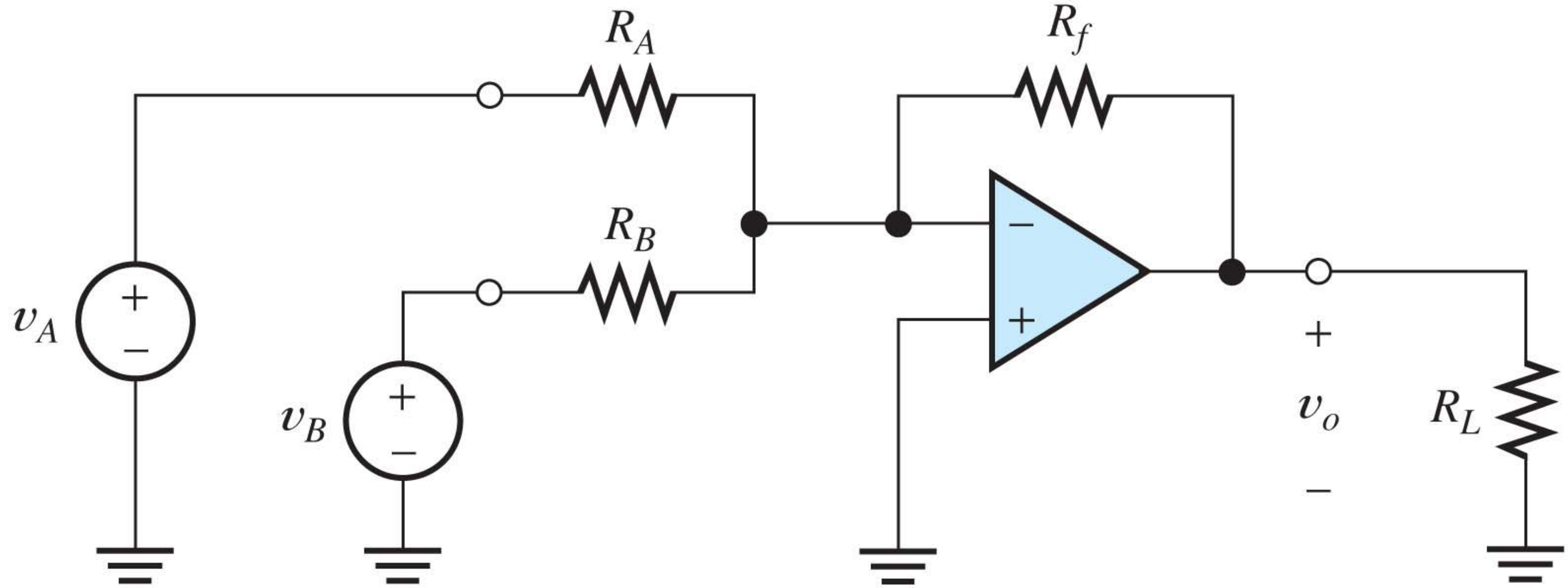

Noninverting
input
+15V (V_{CC})
−15V (V_{EE})
v_1
v_2
Inverting
input
$v_o = A_{OL}(v_1 - v_2)$
A_{OL} = Open Loop
Gain

Inverting Amplifier



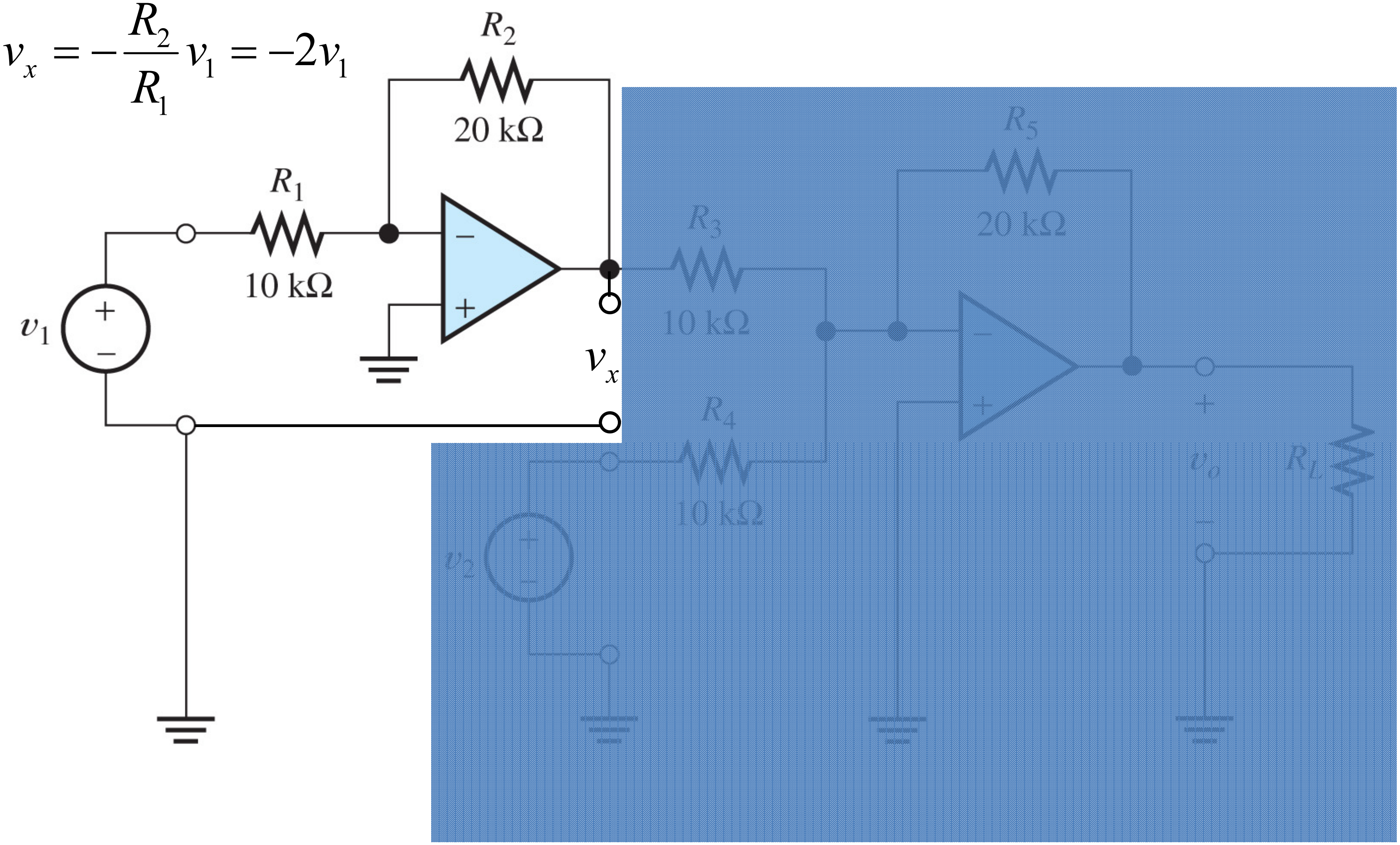
$$A_v = \frac{v_o}{v_{in}} = -\frac{R_2}{R_1}$$

Example 1: Find v_o in terms of v_A and v_B

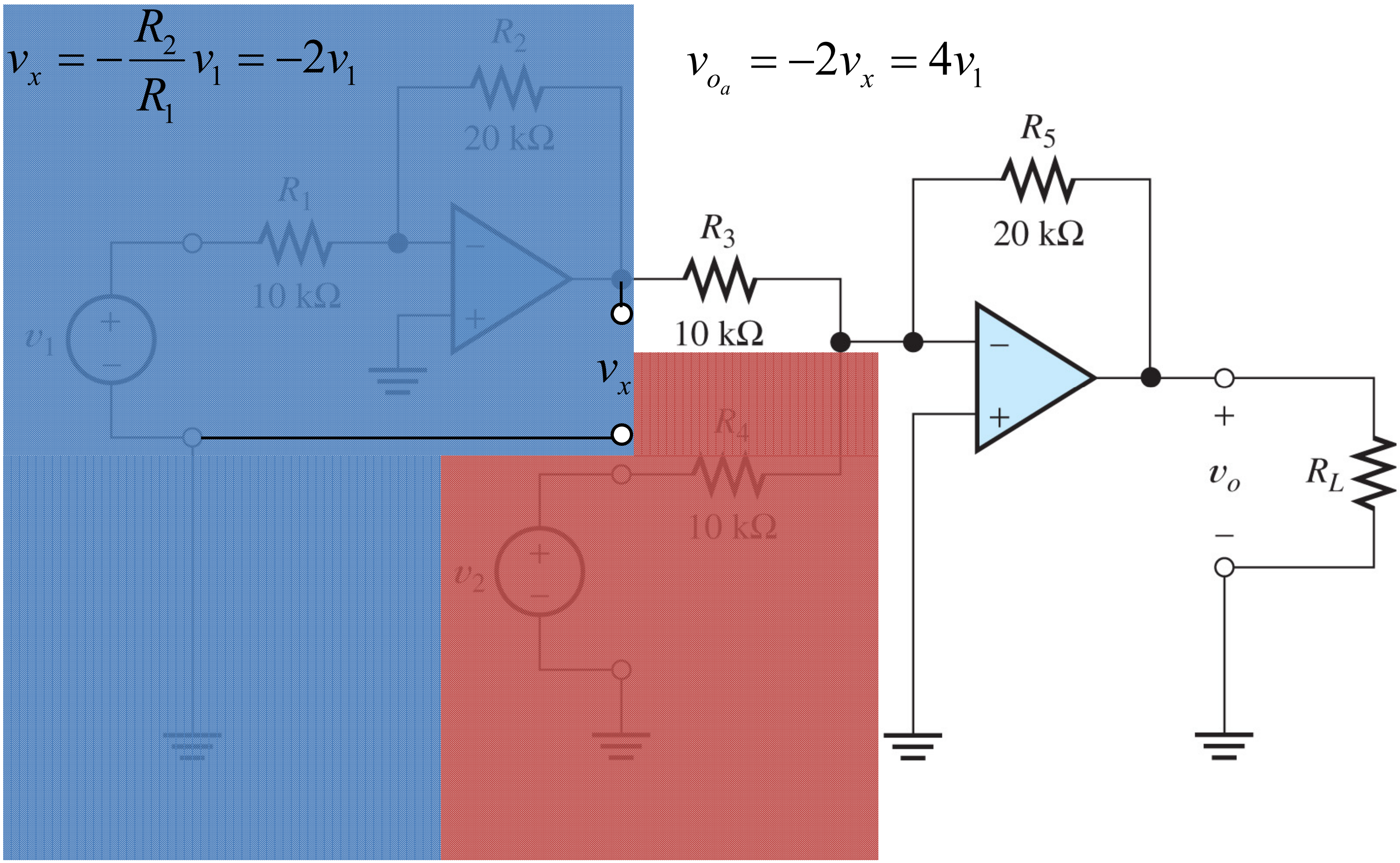


Example 2 – A little more complicated…

$$v_x = -\frac{R_2}{R_1} v_1 = -2v_1$$



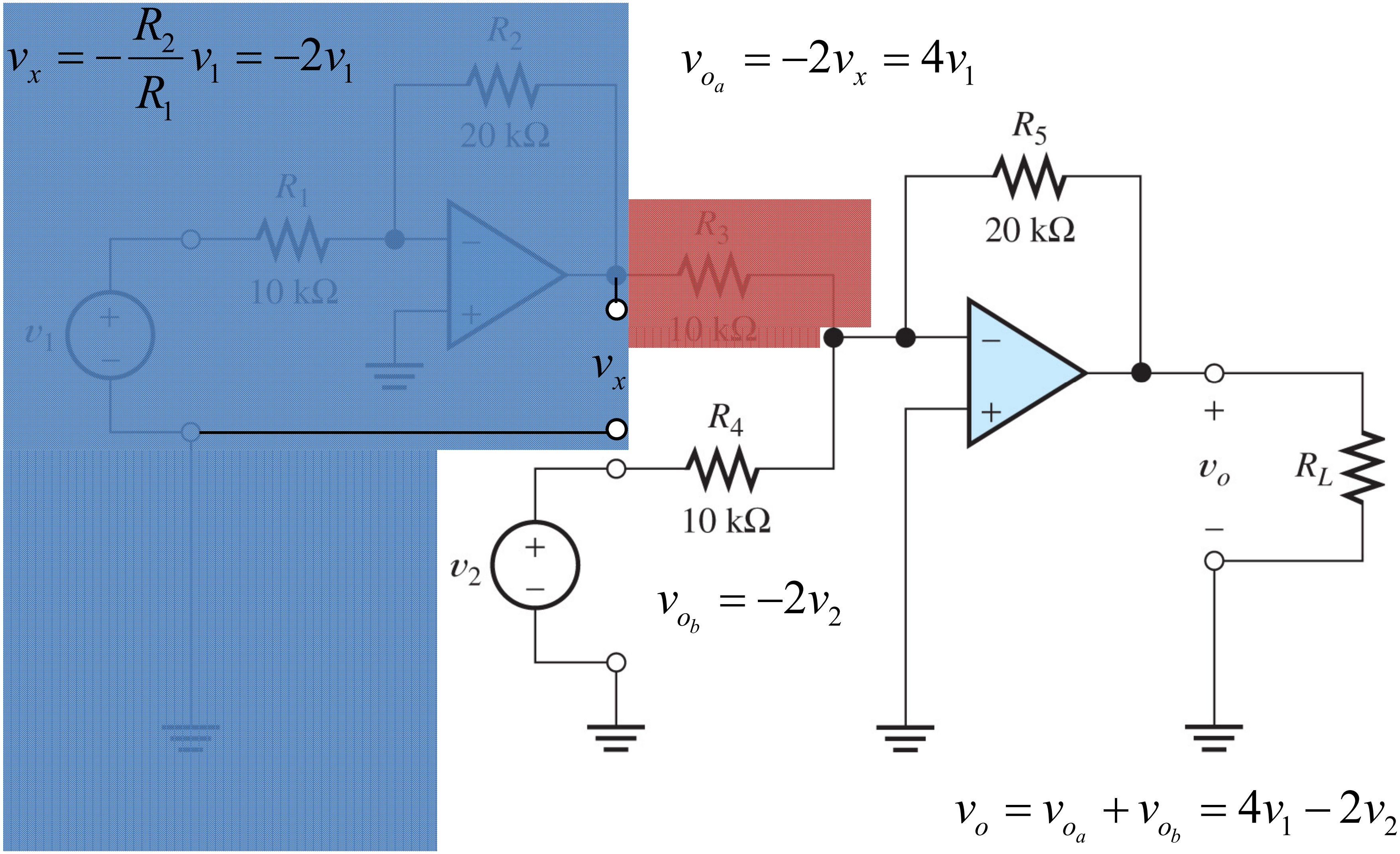
Example 2 – A little more complicated…



Example 2 – A little more complicated…

$$v_x = -\frac{R_2}{R_1} v_1 = -2v_1$$

$$v_{o_a} = -2v_x = 4v_1$$

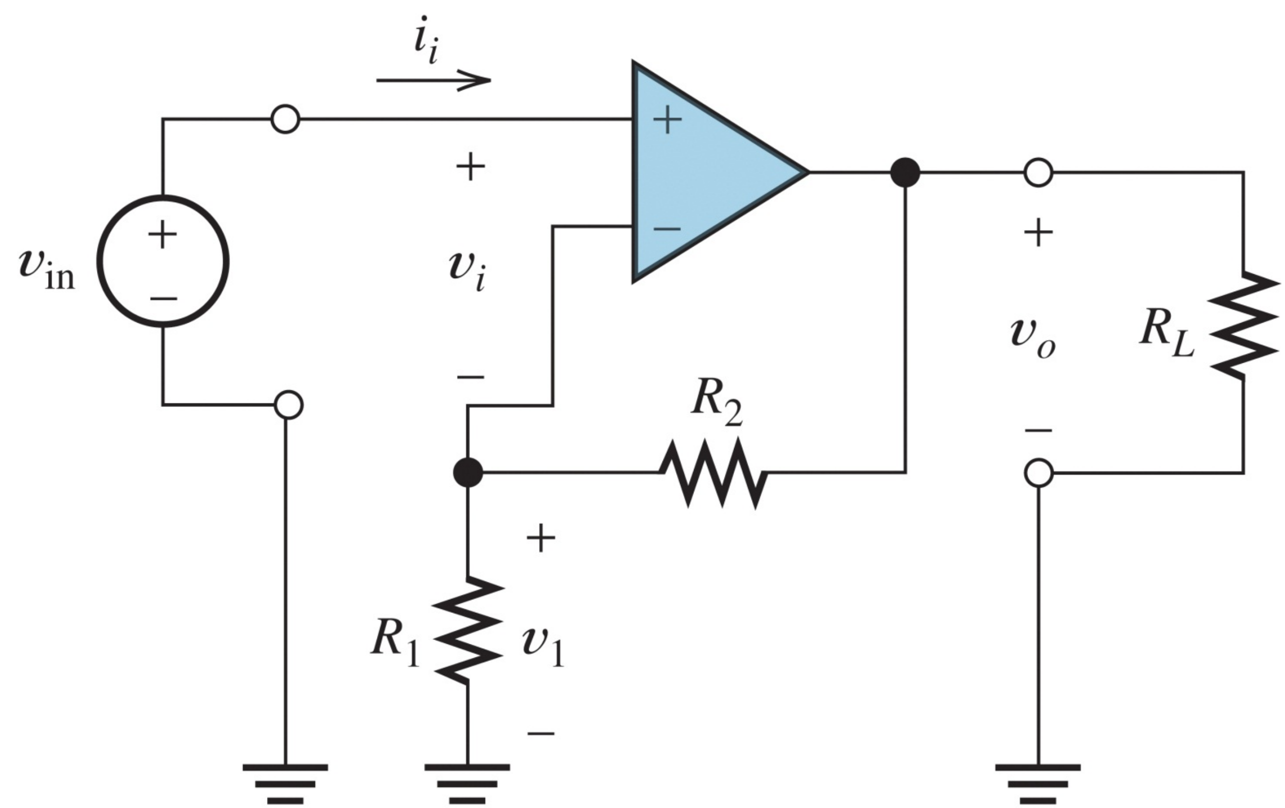


$$v_{o_b} = -2v_2$$

$$v_o = v_{o_a} + v_{o_b} = 4v_1 - 2v_2$$

Noninverting Amplifier

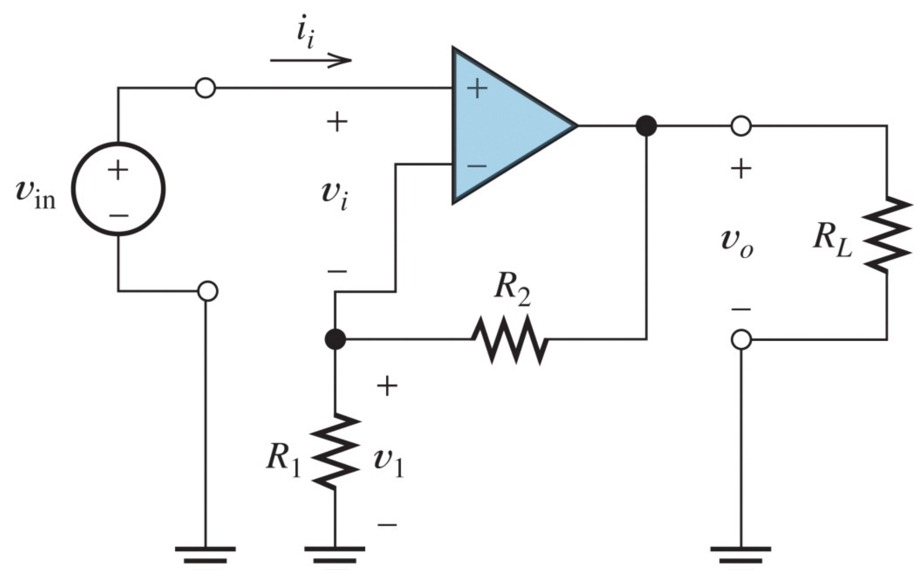
- ✓ So far we have looked at inverting amplifiers
- ✓ We now turn our attention to the noninverting case
- ✓ Consider the following circuit…



Noninverting Amplifier

✓ Is the feedback negative?

- Assume v_i is positive, thus v_o will be large and positive
- Part of v_o will appear across R_1 – call that v_1
- Since $v_i = v_{in} - v_1$, v_i will get smaller as v_o (and v_1) get larger
- Thus v_i is driven towards 0 and we have negative feedback
- What would happen if we had positive feedback???



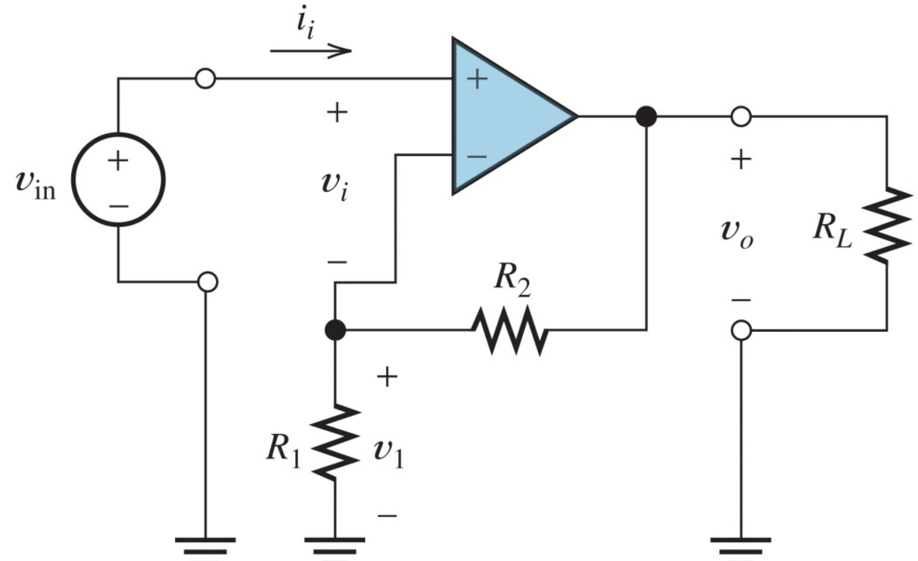
Noninverting Amplifier

- ✓ Now that we know the feedback is negative, we can use the summing-point constraint: v_i = 0 and i_i = 0
  - Applying KVL and v_i = 0 we get $v_{in} = v_1$
  - Since i_i = 0, R_1 and R_2 form a voltage divider giving us:

$$v_1 = \frac{R_1}{R_1 + R_2} v_o$$

  - Finally, we can see that:

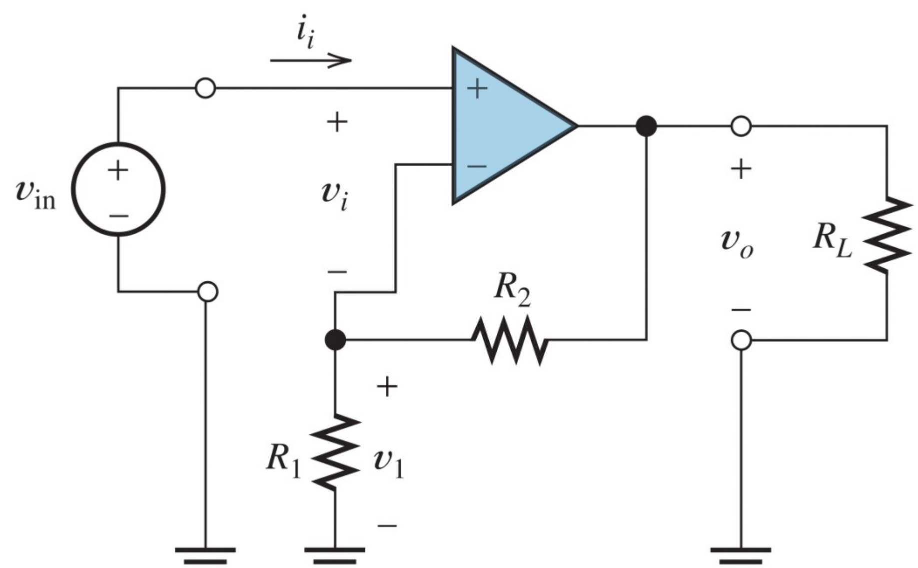
$$A_v = \frac{v_o}{v_{in}} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$



Noninverting Amplifier

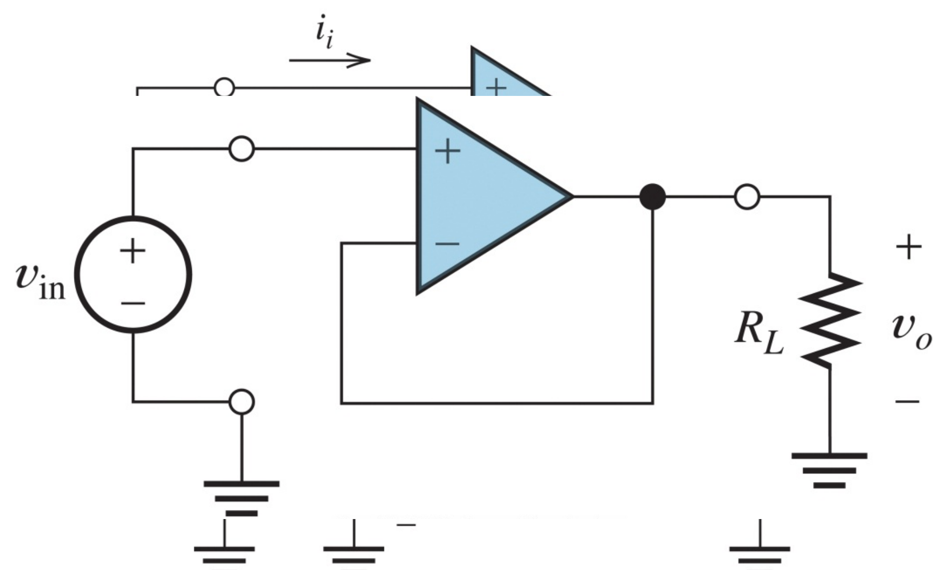
✓ Some interesting points:

- The input impedance of the circuit is theoretically infinite because i_i = 0
- Since the voltage gain is independent of the load resistance R_L, the output voltage is independent of the load resistance – therefore the output impedance is zero



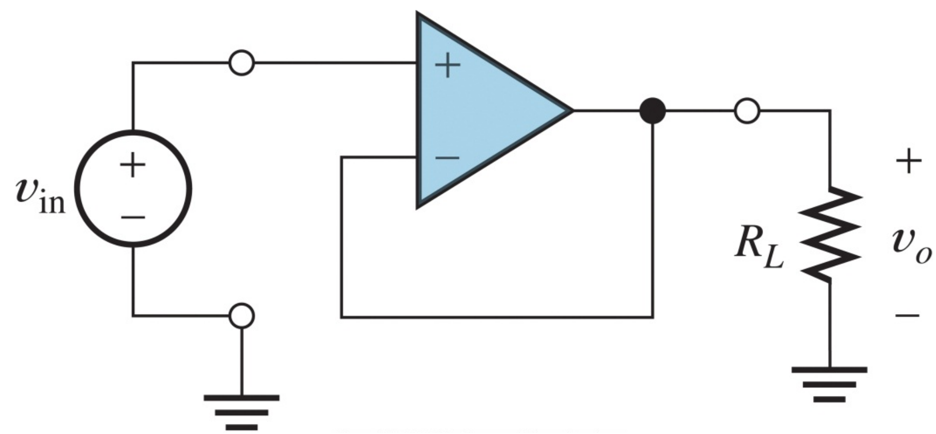
Application of Noninverting Amplifier

- ✓ Lets build on what we just did to produce another useful circuit
- ✓ The gain of the circuit is given by: $A_v = \frac{v_o}{v_{in}} = 1 + \frac{R_2}{R_1}$
- ✓ What do we need to do to have the gain approach unity?
- ✓ We could let R2 → 0
- ✓ If we do that, then we may as well let R1 → ∞



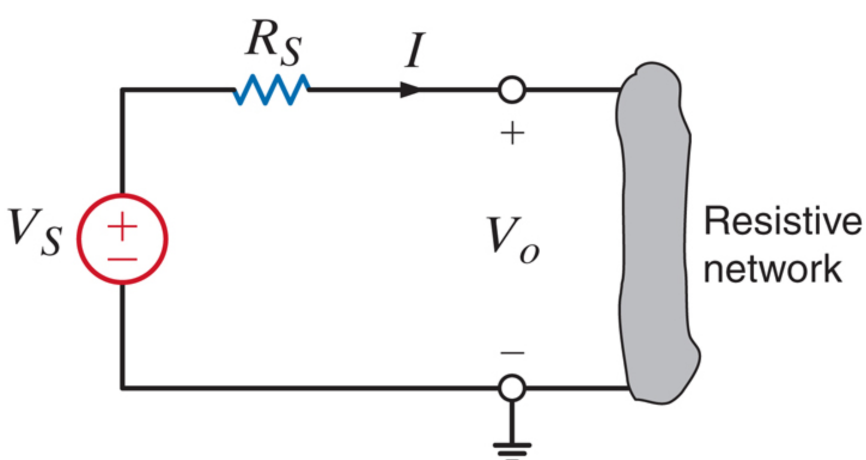
Voltage Follower

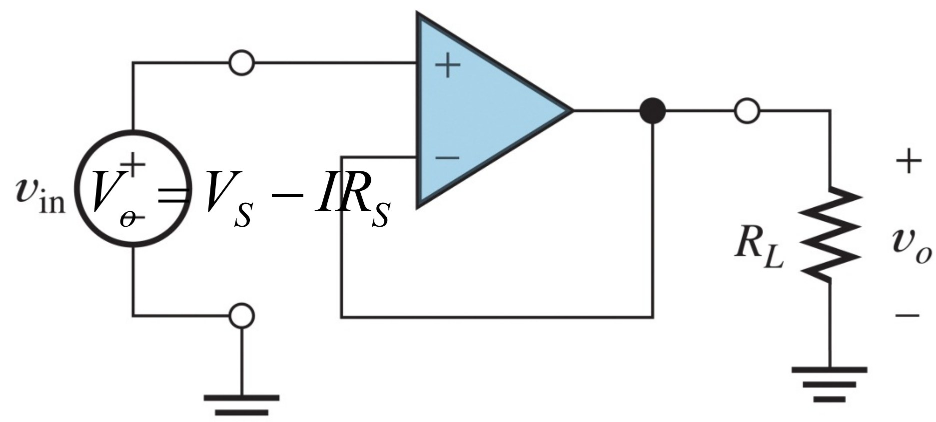
- ✓ What we end up with is called a voltage follower (also called a unity gain buffer)
- ✓ This is a <u>very</u> useful circuit – let's see why…



Voltage Follower

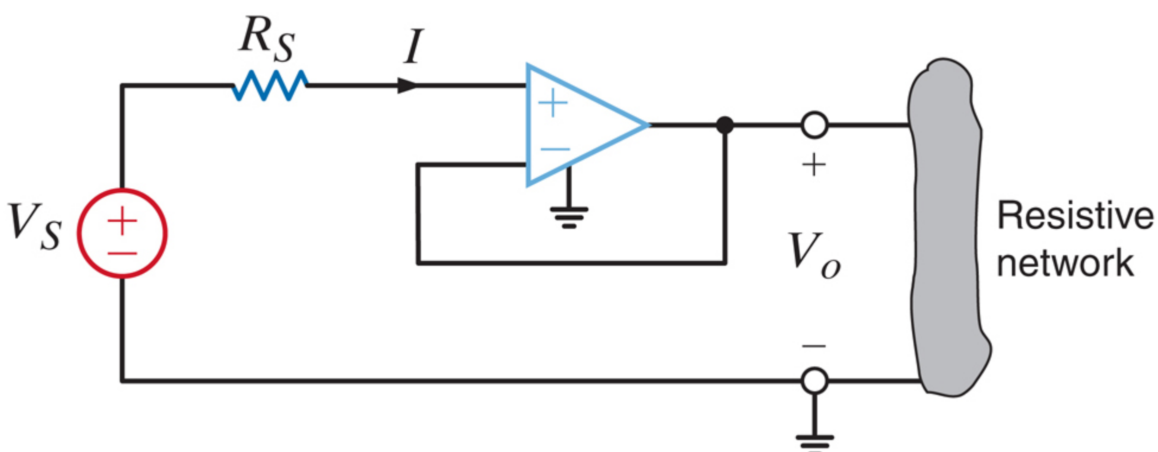
- ✓ If the gain of the op-amp circuit is 1, we can see that $v_o = v_{in}$ – so why bother with the op-amp?
- ✓ Wouldn't it be simpler to just dispense with the op-amp and wire things up as follows?
- ✓ First, we can see that $V_o \neq V_S$





Voltage Follower

- ✓ The problem is that the resistive network loads the source voltage V_S
- ✓ Inserting the unity gain buffer or voltage follower isolates the source from the resistive network
- ✓ The energy supplied to the resistive network comes from the power supplies that power the amplifier

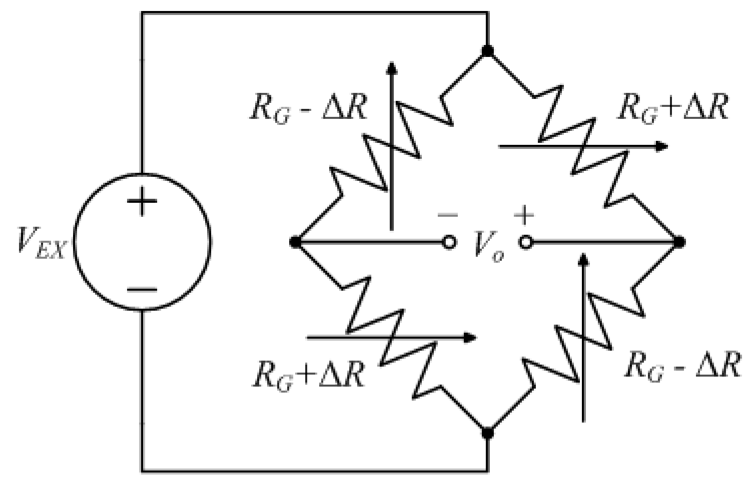


Signal Conditioning

- ✓ Now lets discuss how we are going to deal with the small voltage created by the bridge circuit
- ✓ We may have decided to use a full-bridge configuration, but even so the voltage from the Wheatstone Bridge will be very small
- ✓ We clearly need to amplify that voltage up to a more usable range and that sounds like a job for…
- ✓ An op-amp!

Signal Conditioning

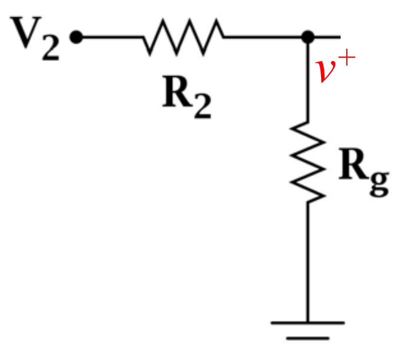
- ✓ We need to be careful though, as we have a difference in potentials at V_o in our bridge circuit, but that difference is not with respect to ground
- ✓ What we need is called a ***difference amplifier*** – so named because it amplifies the difference between two voltages



Difference Amplifier

- ✓ Here we have a simple difference amplifier
- ✓ We can use our circuit analysis tools to come up with an expression for V_{out} in terms of V_1, V_2 and the resistor values R_1, R_2, R_g and R_f
- ✓ Let's see what we get…

$$v^+ = \frac{R_g}{R_2 + R_g} V_2$$



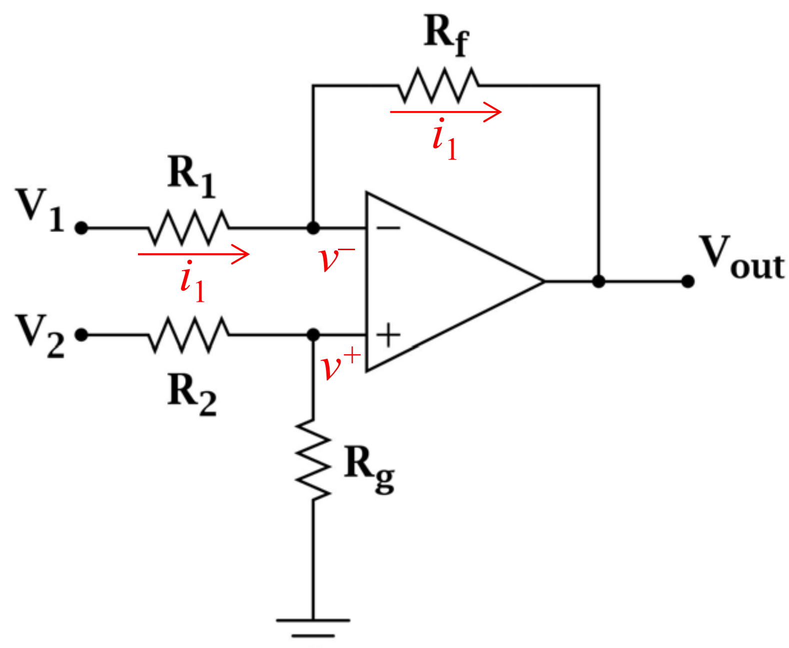
Difference Amplifier

- ✓ What can we say about the voltage v^-?

$$v^+ = v^-$$

- ✓ And what about i_1?

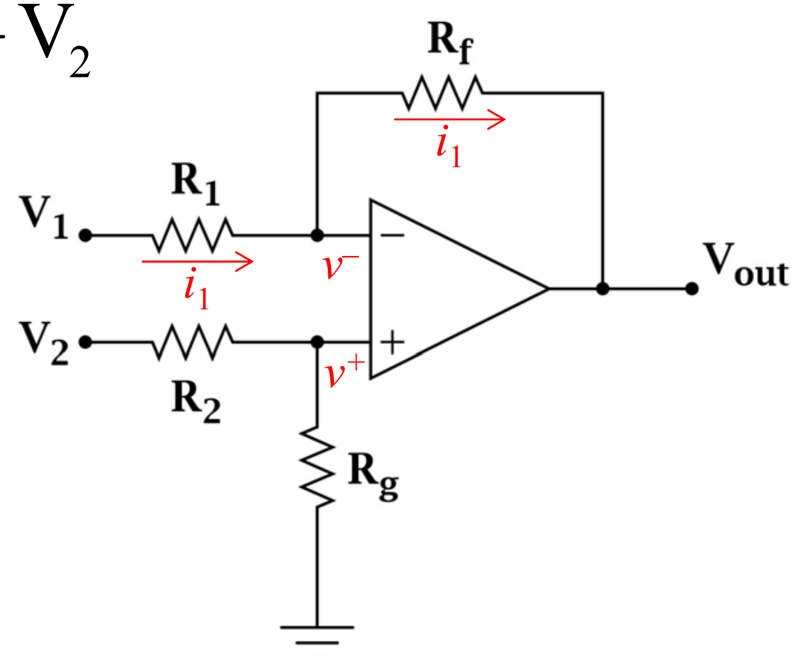
$$i_1 = \frac{V_1 - v^-}{R_1} = \frac{v_1 - \left[\frac{R_g}{R_2 + R_g} V_2\right]}{R_1}$$



Difference Amplifier

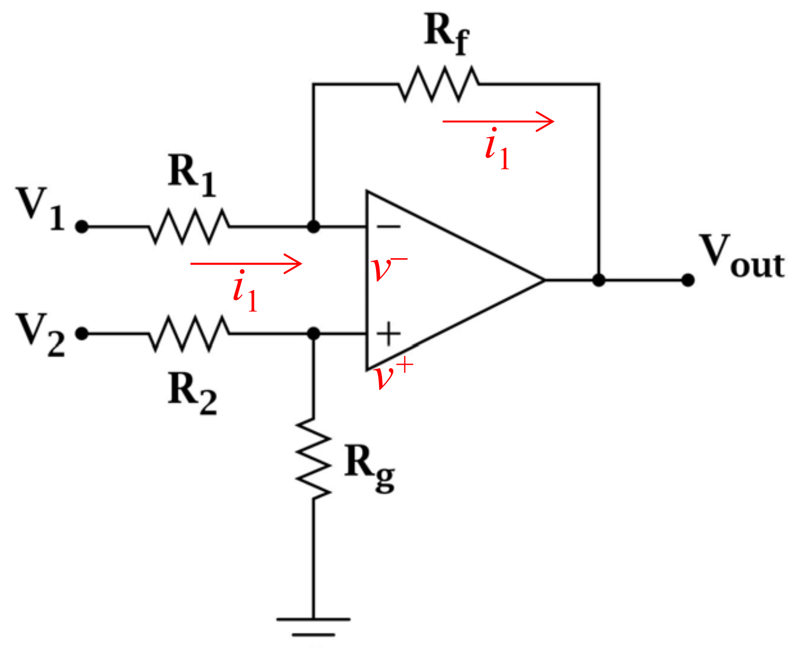
$$\frac{v^+ - V_{out}}{R_f} = \frac{V_1 - \dfrac{R_g}{R_2 + R_g} V_2}{R_1}$$

$$v^+ - V_{out} = \frac{V_1 R_f}{R_1} - \frac{R_g R_f}{\left(R_2 + R_g\right) R_1} V_2$$



Difference Amplifier

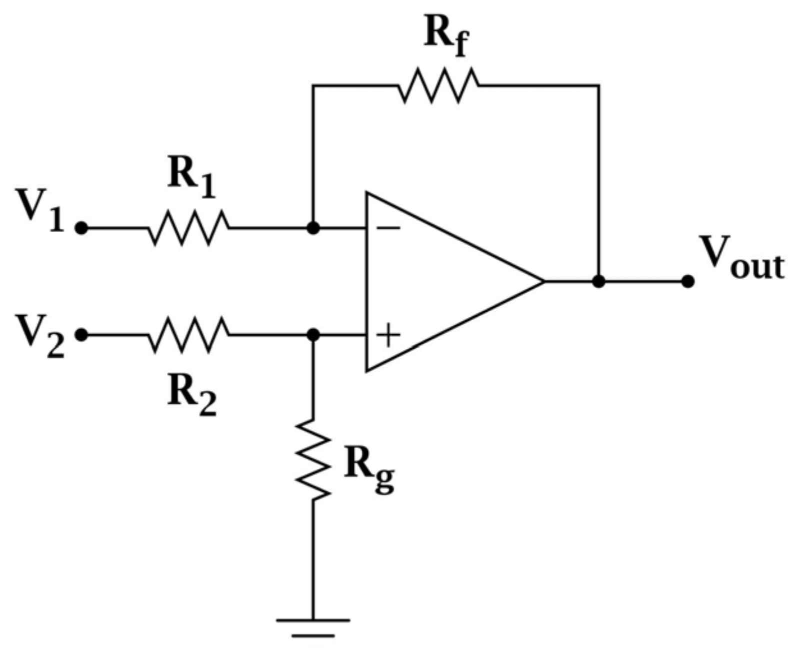
$$-V_{out} = \frac{V_1 R_f}{R_1} - \frac{R_g R_f}{\left(R_2 + R_g\right) R_1} V_2 - \frac{R_g R_1}{\left(R_2 + R_g\right) R_1^{2}} V_2 \, V_2$$



Difference Amplifier

✓ In the end, we get the following:

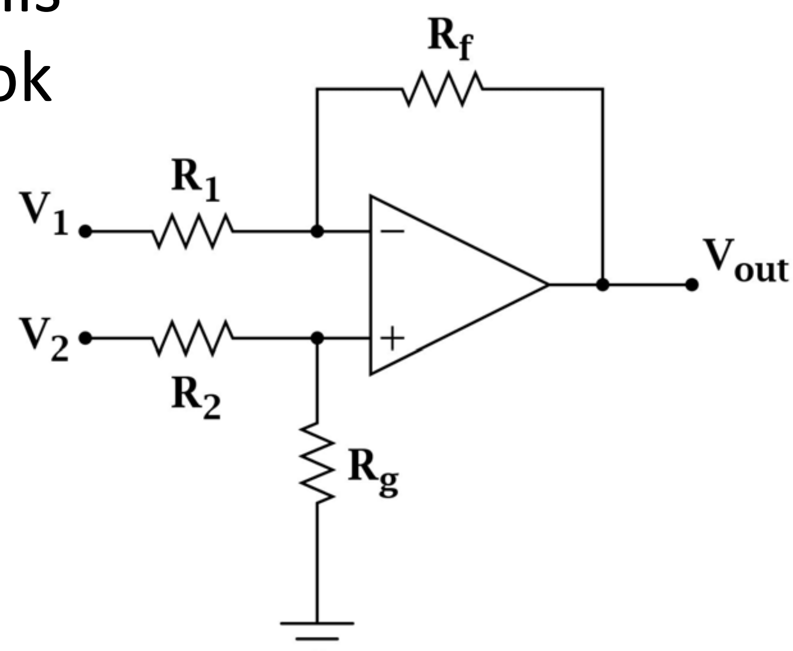
$$V_{out} = \frac{\left(R_f + R_1\right)R_g}{\left(R_g + R_2\right)R_1} V_2 - \frac{R_f}{R_1} V_1$$



Difference Amplifier

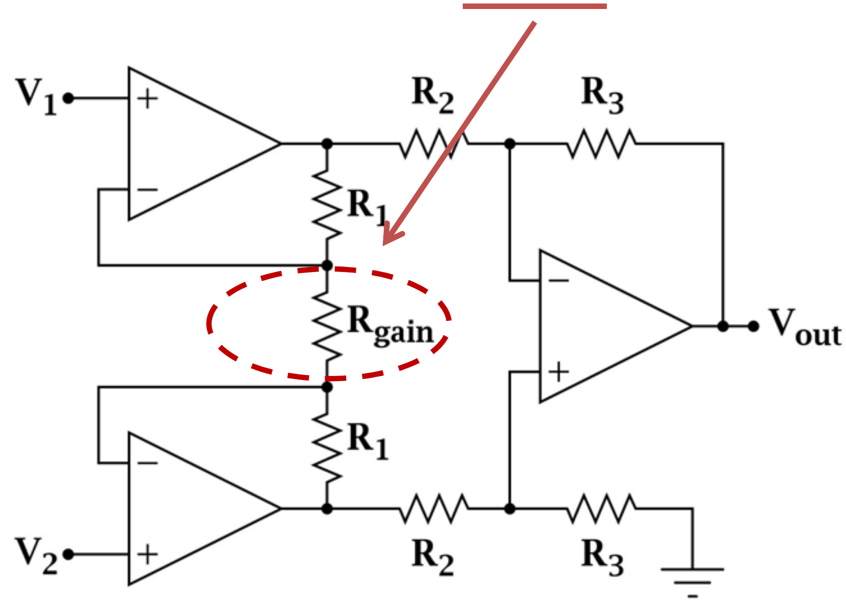
- ✓ In many cases it makes sense to make $R_1 = R_2$ and $R_f = R_g$
- ✓ The expression for the output voltage then becomes much simpler
- ✓ There are some issues with this circuit however, so we will look at another solution

$$V_{out} = \frac{R_f}{R_1}\left(V_2 - V_1\right)$$



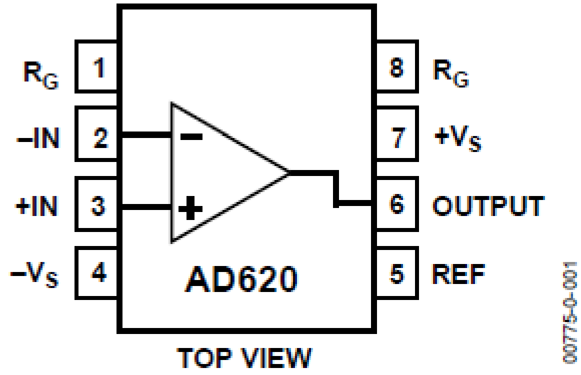
Instrumentation Amplifier

- ✓ Here we have a better circuit – called an instrumentation amplifier
- ✓ The advantage of this circuit is that it is more stable, is not as noisy, has a high CMR, and the gain can be set by adjusting the single external resistor R_{gain}
- ✓ You have an op-amp circuit of this type (an AD620) in your KOP



AD620

CONNECTION DIAGRAM



FEATURES

Easy to use

- **Gain set with one external resistor**
  - **(Gain range 1 to 10,000)**
- **Wide power supply range (±2.3 V to ±18 V)**
- **Higher performance than 3 op amp IA designs**
- **Available in 8-lead DIP and SOIC packaging**
- **Low power, 1.3 mA max supply current**

AD620 Gain Calculations

- ✓ The gain equation for the AD620 is given by:

$$G = \frac{49.4\ \text{k}\Omega}{R_G} + 1$$

- ✓ If you know the gain you need, then use:

$$R_G = \frac{49.4\ \text{k}\Omega}{G - 1}$$

AD620 Gain Calculations

Table 4. Required Values of Gain Resistors

1% Std Table Value of R_G(Ω)	Calculated Gain	0.1% Std Table Value of R_G(Ω)	Calculated Gain
49.9 k	1.990	49.3 k	2.002
12.4 k	4.984	12.4 k	4.984
5.49 k	9.998	5.49 k	9.998
2.61 k	19.93	2.61 k	19.93
1.00 k	50.40	1.01 k	49.91
499	100.0	499	100.0
249	199.4	249	199.4
100	495.0	98.8	501.0
49.9	991.0	49.3	1,003.0

AD620 Common Mode Rejection

- ✓ Instrumentation amplifiers, such as the AD620, offer high CMR, which is a measure of the change in output voltage when both inputs are changed by equal amounts
- ✓ These specifications are usually given for a full-range voltage change and a specified source imbalance

Force Sensing

- ✓ So far we have looked at one way of doing force sensing – using strain gauges
- ✓ We could characterize this as being a ‘high impedance’ method
- ✓ Depending on the materials involved, a fair amount of force may need to be used to cause enough deflection in the material the strain gauge is mounted on to register a useable output

Force Sensing

- ✓ Let's use an analogy: If we were talking about Hooke's Law, then the spring constant k would be large – meaning that a lot of force F is needed to get a small change in displacement x

$$F = kx$$

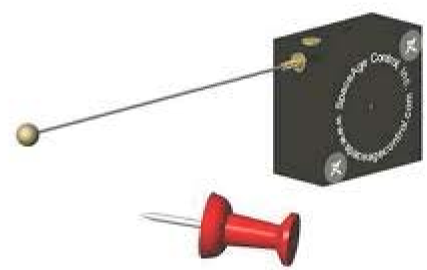
- ✓ Here's a good example of a mechanical system that would be high impedance…

Force Sensing

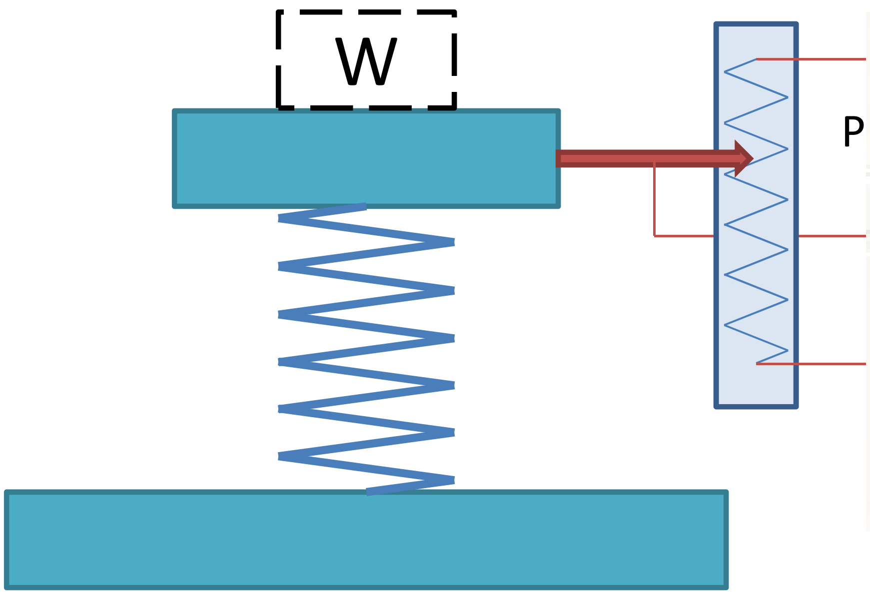
- ✓ So how else might we sense force?
- ✓ In particular, how might we sense force in a 'low impedance' fashion – where we don't need to apply so much force to get a reading?
- ✓ Hint: we were just talking about springs…
- ✓ Right – reduce the spring constant way down
- ✓ But we need a way to measure the displacement x

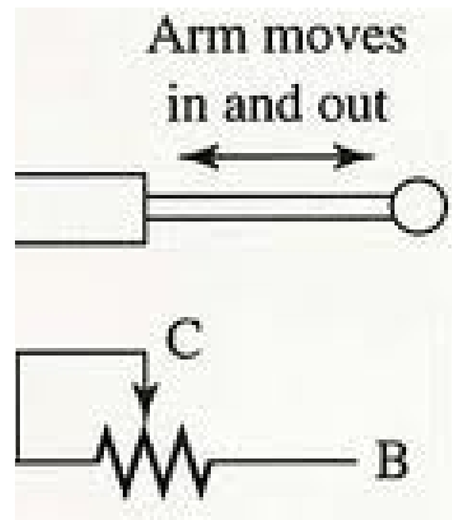
Force Sensing

- ✓ While there are many ways this can be accomplished, an easy solution is to use a linear potentiometer
- ✓ In this case, we are speaking about how the potentiometer mechanism is constructed as opposed to the 'taper' on the resistance
- ✓ So in this case the wiper moves linearly as opposed to being rotated as is more commonly encountered

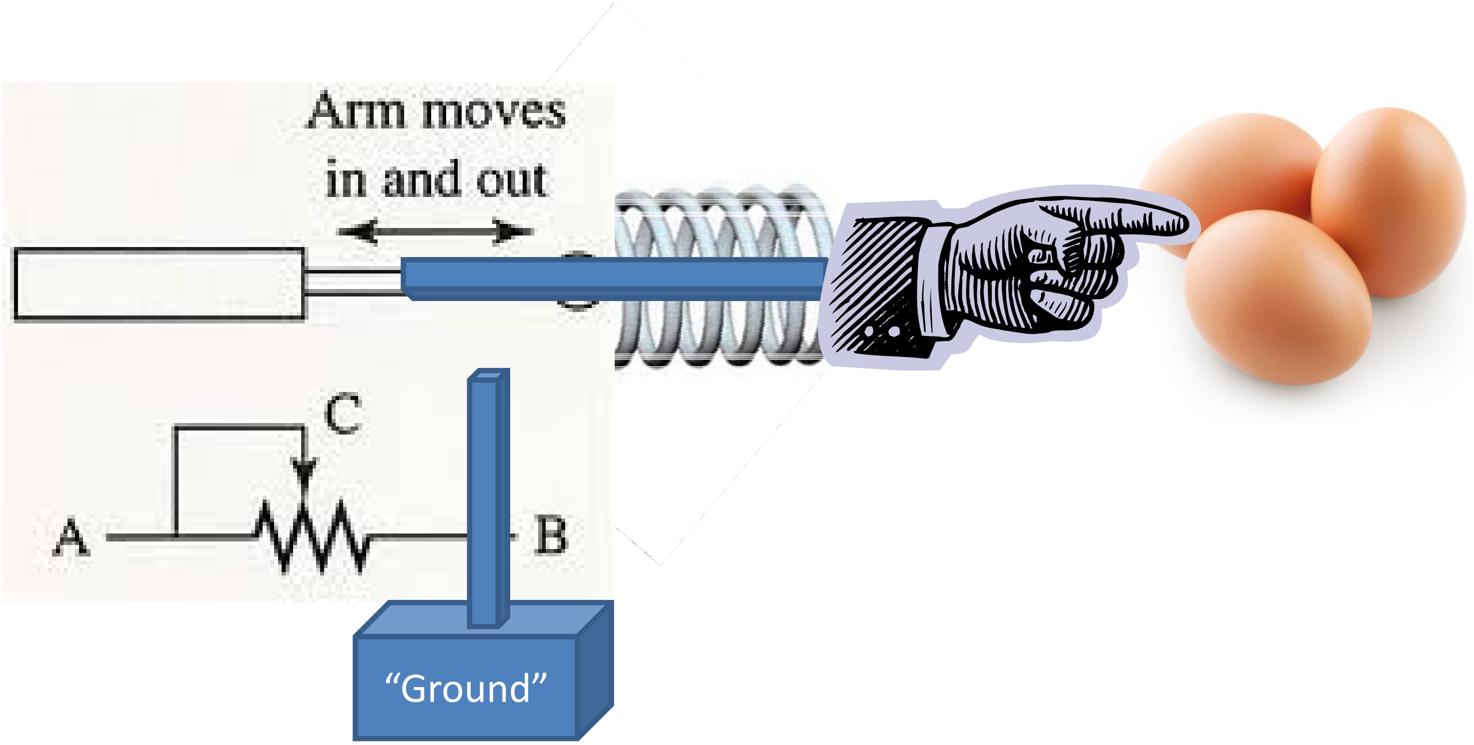


Low(er) Impedance Force Sensing





Low Impedance Force Sensing



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- ✓ *Positioning Strain Gauges*, Omega Engineering, Inc., http://www.omega.com/faq/pressure/pdf/positioning.pdf
- ✓ *AD620 Low Cost Low Power Instrumentation Amplifier (Rev. G)*, Analog Devices, http://www.analog.com/static/imported-files/data_sheets/AD620.pdf