Dynamic Auctions with Budget-Constrained Bidders: Evidence from the Online Advertising Market

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Abstract

When price discovery is necessary for time-sensitive goods, a common practice is to conduct an auction for each item sequentially, but dynamic incentives can lead to behavior distinct from static settings. We provide a novel empirical analysis of a large-scale sequential market that employs auctions to allocate objects to firms with budget constraints, leveraging a unique proprietary dataset of the online advertising market. In this market, because of their short-run budget constraints, participants face a tradeoff between winning auctions immediately or holding out for later opportunities. This dynamic incentive prompts them to adjust their entry rates and bidding strategies accordingly. We develop and estimate a finite-horizon dynamic game between bidders with heterogeneous budgets facing a sequence of simultaneous auctions to quantify this incentive and analyze its implication in competition and auction design. We find that a substantial markdown occurs due to the dynamic incentives arising from budget constraints, and this markdown varies significantly among bidders with different budgets. Using the estimated structural model, we provide a counterfactual simulation comparing the first-price and second-price formats. Unlike the standard environment, we find that dynamics and heterogeneous budgets lead to a significant disparity in the welfare distributions under them. This highlights that that even a seemingly simple mechanism choice can have competitive implications in such a dynamic environment.

Keywords: Dynamic Auctions, Online Advertising, Dynamic Game

JEL Codes: D44, C57, C73, L13, M37

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1 Introduction

A fundamental question in economics revolves around determining the optimal method for selling goods. Dynamic pricing has traditionally been used and studied for time-sensitive items, such as airline tickets. In contrast, auctions are often employed and analyzed when there is a need for price discovery, such as procurement and oil drilling rights. However, certain situations demand price discovery for time-sensitive goods, as seen in financial, electricity, and online advertising markets. Under such situations, practitioners often hold an independent auction for each good sequentially. While there is a growing empirical literature on sequential auctions, there is little empirical work on the effect of auction format in sequential auctions participated by budget-constrained bidders. This research question is crucial since many consumers and firms, when acting as buyers, face financial constraints. In particular, when firms have varied budgets due to market concentration, the impact of sequential auction mechanisms may carry significant competitive implications.

In this paper, we empirically analyze a large-scale sequential market that employs instantaneous auctions to allocate goods to firms with short-run budget constraints. Using a novel structural model, we simulate the differential impact of a counterfactual change in the auction format on buyers with varying budgets. To our knowledge, this is the first paper to provide a structural analysis of dynamic auctions with budget-constrained bidders. We develop and estimate a finite-horizon dynamic game between bidders with heterogeneous budgets facing a sequence of simultaneous auctions. We find that a significant amount of the markdown is driven by the dynamic incentives created by budget constraints, and it varies significantly across bidders with different budgets. Using the estimated structural model, we provide a counterfactual simulation comparing two auction mechanisms: first-price and second-price auctions. Unlike the standard auction environment, we find that dynamics and heterogeneous budgets lead to a significant disparity in the welfare distributions under the two mechanisms. This highlights that that even a seemingly simple choice of first-price or second-price can have competitive implications in such a dynamic environment.

Our empirical setting is the online display-advertising market, which is the market behind online banner and video advertisements, and we use a novel proprietary dataset of auctions hosted on Yahoo's ad exchange. In this market, a significant proportion of advertising opportunities are allocated through real-time auctions. When a user visits a website, it triggers an instantaneous auction where the user's characteristics are revealed to bidders. The highest bidder secures the privilege of displaying their ad on the user's screen once the page fully loads. These real-time auctions enable advertisers to effectively target users and ensure their ads are presented before users navigate away from the webpage. In this market, advertisers typically hire bidding agents who participate in these auctions on their behalf, and advertisers frequently impose specific campaign budgets on these agents, typically allocated on a daily basis. These bidding agents are frequently affiliated with major tech firms such as Google and Amazon, which tend to attract numerous advertisers, including those with substantial campaign budgets. One responsibility of these bidding agents is to strategize on how to effectively participate and bid within the continuous stream of instantaneous auctions while adhering to the daily budgets assigned to them.

We first document dynamic patterns in the data that are consistent with daily budget constraints. First, we observe a declining trend in both the entry rate and bid levels throughout the day, from morning to evening. This trend is in line with diminishing demand, likely caused by bidders exhausting their daily budgets. In fact, theoretical research has shown that sequential auctions can exhibit such a decreasing price pattern with unit-demand bidders (Engelbrecht-Wiggans, 1994; Bernhardt and Scoones, 1994; Gale and Hausch, 1994). Second, we also find that the entry rate and bid level are negatively affected by the frequency of auctions. In other words, when there is an increase in the number of auctions (from high supply), both the entry rate and bid levels decrease. This suggests that bidders may exercise caution by submitting less competitive bids when faced with a higher volume of auctions, aiming to preserve their future spending capacity and prevent exceeding their budget constraints by winning too many auctions. This relationship is robust to controlling for numerous bidder and auction characteristics, including time fixed effects.

Motivated by the institutional constraints and the dynamic pattern in the data, we propose a structural model of repeated, simultaneous auctions participated by dynamically constrained bidders. We model auctions hosted each day as a finite sequence of many simultaneous first-price auctions in which the number of auctions per period varies stochastically in a time-dependent way. We consider the bidding problem of a bidder imposed with a soft constraint over his total spending at the end of the sequence. By looking at the first-order condition of the model, we find that the dynamic constraint introduces another force to depress their bids in addition to the force from being in the first-price auction. There is a tradeoff between winning an auction now versus later, and this opportunity cost manifests as an additional markdown. In addition, the difference in the amount of remaining budget across bidders introduce asymmetry that is not present in the standard first-price auction.

Solving our model poses significant computational challenges due to its dynamic nature, exacerbated by several factors. These include the presence of continuous choice variables without closed-form expressions, a finite time horizon resulting in non-stationarity, a relatively large number of players (around thirty), and, most critically, a high-dimensional continuous state space with a continuous state variable (remaining budget) associated to each player. To tackle these complexities, we leverage the fact that bidders do not have access to information about their rivals' spending behaviors. In light of this information asymmetry, we adopt a large-market solution concept in which bidders rely on the equilibrium distribution of players' states for each period as their belief. Hence, bidders decide their entry and bid strategy conditional on the time period, public state (number of auctions), and its own remaining budget. By employing this approach, we effectively reduce the problem's dimensionality, enabling estimation and counterfactual simulation, while still allowing for meaningful analysis into dynamic bidding behaviors.

Furthermore, to avoid solving for equilibrium and simulating the equilibrium state distribution during estimation, we employ a two-step estimation method to estimate our structural model (Bajari et al., 2007; Aguirregabiria and Mira, 2007; Jofre-Bonet and Pesendorfer, 2003). In the first step, we estimate the time-dependent distribution of number of auctions and the reduced-form entry probability and bid distribution conditional on it. In the second step, we solve for bidders' entry and bid strategies as best responses to to rivals' estimated behaviors from the first stage and estimate the structural parameters within the framework of a single-agent finite-time horizon model.

The identification of our structural parameters, such as budgets and soft budget constraint parameter, relies on the exclusion restriction that bidders' valuations are independent of the state variables, which are the frequency of auctions and their remaining budgets. This exogeneity assumption is plausible for our market environment because advertisers and their bidding agents typically compute their valuations for impressions based on a combination of the probability of clicking/making a sale and their value of such events. This probability is computed based on the contextual and behavioral data of that user alone.

Applying our structural estimation method on a large-scale proprietary dataset of online banner-ad auctions from Yahoo reveals significant dynamic incentives arising from budget constraints. The markdown, representing the gap between valuations and bids, averages 83.5% of expected valuations. Our estimated model demonstrate that first-price auctions induce 59.4% shading, and dynamic budget constraints add an extra 24.2% shading. Our model also quantifies the impact on entry decisions, with an average entry probability of 19.4%, compared to a static entry probability of 45.1%. These findings underscore the significance of dynamic incentives in this context. Additionally, we observe a notable concentration of estimated budgets among bidders, which has a substantial impact on the heterogeneity in their bids and entry decisions.

Using our estimated structural model, we simulate a counterfactual scenario motivated by a significant institutional change that took place around 2018. During this period, the predominant auction mechanism in the online ad market shifted from the second-price format to the first-price auction. This transition was prompted by concerns within the industry that ad exchanges, serving as intermediaries between publishers and advertisers, were not actually implementing the second-price auction as claimed, leading to a loss of trust among market participants. In response to this industry-wide credibility crisis, market participants advocated for the first-price auction due to its transparency in revealing what winners pay. Motivated by this shift, which happened years before our sample period, we simulate the second-price auction format using the estimated structural model as a counterfactual scenario to analyze the revenue and welfare consequences.

Our counterfactual simulation reveals that the first-price auction yields slightly higher total revenue and total bidder surplus compared to the second-price auction. More importantly, we observe a substantial disparity in welfare distribution between these two auction formats. We find that bidders below the two largest budget holders face more favorable outcomes under the first-price auction. This suggests that, in addition to its transparency benefits, the first-price auction may offer a more robust competition in the presence of market concentration. This outcome can be attributed to the reduced price volatility under the first-price auction. Lower price volatility allows bidders to bid more aggressively, as it enables better control over their spending patterns. While bidders with intermediate-size and small budgets lower their entry rate in response to this increase in competition, the two bidders with the largest budgets keep a similar entry rate since they can afford to. Nevertheless, this increase in competition induces these top bidders to spend more rapidly, and leads to a decreased competition in later periods. Then, smaller bidders can enjoy this smaller competition and earn more surplus during this period. This difference leads bidders other than the top two bidders to be better off under the first-price auction. This results underscores that even a seemingly simple choice of first-price or second-price can have competitive implications when auctions are conducted sequentially and participated by bidders with heterogeneous budgets.

My paper contributes to the empirical literature that studies repeated auctions using structural models (Jofre-Bonet and Pesendorfer, 2003; Groeger, 2014; Kong, 2021; Bodoh-Creed et al., 2021; Backus and Lewis, 2023). While the existing works in this literature has focused on either unit-demand bidders or unlimited-demand bidders, we provide a novel structural analysis of bidders whose demand is limited by their short-run budget constraints. In addition, these studies have one auction per period, but our structural model features multiple auctions per period where then number of auctions is randomly drawn from a time-dependent distribution. In the online advertising market, the frequency of auctions is directly tied to web traffic, so many auctions can happen almost at the same time. Hence, we use a discrete-time approximation and model our it as a sequence of simultaneous auctions.

Auctions with budget-constrained bidders have received extensive attention in the theoretical auction literature (Palfrey, 1980; Che and Gale, 1998; Benoît and Krishna, 2001; Pitchik and Schotter, 1988; Pitchik, 2009). This literature has looked at one-shot auctions, simultaneous auctions, or sequential auctions. Our environment and structural model are substantially different from these theoretical works. We examine an environment that is well approximated by a finite sequence of simultaneous auctions, and it has a large number of auctions in every period (at least thousands) and relatively large number of bidders (around thirty). In contrast, theoretical studies on simultaneous or sequential auctions often focus on a small number of auctions and bidders (typically two for each) to investigate equilibrium existence and theoretical properties. Hence, the revenue and welfare ranking of auction formats in our environment is theoretically ambiguous. Our counterfactual exercise makes a novel finding that using the first-price format in this environment benefits bidders with smaller budgets by increasing the spending rate of bidders with large budgets.

2 Instituional Background

2.1 Display ad market

The recent online advertisement market employs the real-time bidding process to trade a large portion of impressions, which is the industry term for opportunities to display ads to visitors of websites. As the name suggests, through the real-time bidding (RTB) process, publishers of websites and advertisers trade impressions via auctions in real-time as consumers visit these websites. Each auction typically lasts only milliseconds. Hence, under the RTB process, impressions are sold impression-by-impression rather than via signing contracts in advance for bulks of impressions. Advertisers may display clickable banners or videos after purchasing impressions, and the content of these advertisements may reflect various characteristics of the impressions. For example, a retailer may attempt to retarget consumers by displaying products the consumers viewed in the past. One advantage of the RTB process is that it provides granularity to advertisers for targeting a specific audience. Rather than buying media or ad slots to a loosely targeted audience, the RTB process allows advertisers to target a particular audience directly.

Generally, the RTB process involves publishers, ad exchanges, demand-side platforms, and advertisers.[1] An *ad exchange* is an online server that hosts auctions. These auctions can have various formats, such as first-price and second-price auctions. Advertisers typically bid in ad auctions through demand-side platforms because it is technologically complex to target individual impressions and optimally bid for them. A *demand-side platform (DSP)* is an intermediary that assists advertisers in targeting and bidding for impressions in ad exchanges, it typically uses optimized bidding algorithms because of the fast-paced nature of ad auctions. The sequence of the RTB process roughly works as follows:

1. A user visits a webpage of a publisher and triggers an impression.
2. The publisher sends an ad request to an ad exchange containing the user information.
3. The ad exchange starts an auction for the impression and forwards the ad request to demand-side platforms (DSPs).
4. Each DSP decides whether to participate and which advertiser to allocate this impression among its clients, and then it bids on behalf of the chosen advertiser in the auction held in the ad exchange.
5. The advertiser represented by the winning DSP gets the impression.
6. Finally, the corresponding advertisement is displayed to the user.

2.2 Marketing campaign and budget settings

When an advertiser wants to start advertising a banner or video ad, they register a marketing campaign with a DSP. The advertiser sets various key marketing campaign parameters, such as performance goals (number of clicks, conversions, or impressions), targeting audience, campaign length, payment scheme, and budget. Advertisers may choose to pay DSPs a fee proportional to spending or a fee per click/conversion. The budget specifies how much the DSP can spend during the campaign period to purchase ad opportunities, and generally this budget is evenly split over days druing the campaign. See Figure 1 for an example of daily budget configuration of a marketing campaign. In practice, this daily budget constraint is soft since DSPs often underspend or overspend by a bit.

[1] In reality, there are also *supply-side platforms (SSPs)* that support publishers, but we omit them in our explanation for brevity. See Yuan et al. (2013) and Choi et al. (2020) for more detail.

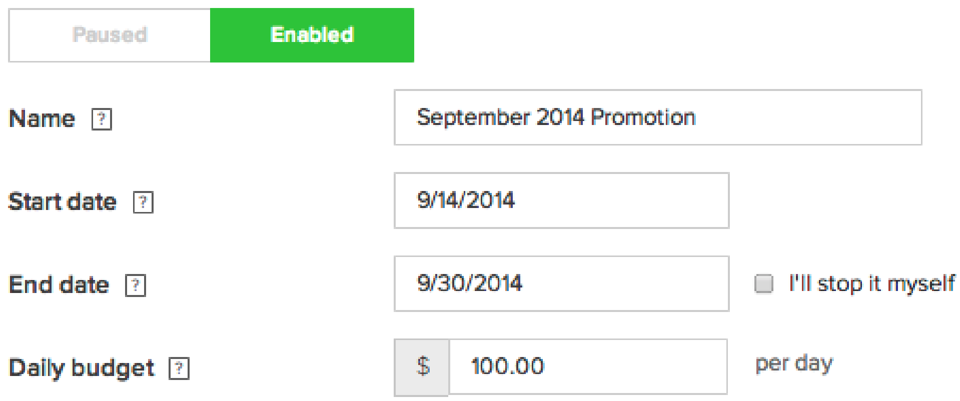

Figure 1: Settings of a marketing campaign on a major demand-side platform.

3 Data and Stylized Facts

3.1 Data description

This paper employs data of ad auctions held at the Yahoo Ad Exchange for ad opportunities on Yahoo's websites. Like other exchanges, the Yahoo Ad Exchange is a clearinghouse that facilitates transactions between publishers and advertisers (represented by DSPs), and it runs first-price auctions. This dataset is suitable for our study for two reasons. First, Yahoo is one of the most popular websites in the US[2], so the data provides a representative sample for our study. Yahoo is one of the most popular publishers that sell banner ad opportunities, and it also provides a diverse range of websites, such as Mail, News, and Finance. Therefore, although DSPs may be bidding for ad opportunities on multiple publishers and even in multiple exchanges, the data provides us a representative sample of ad auctions faced by advertisers who use banner ads.

Although social media websites are among other popular publishers, Yahoo's lack of social media services does not hurt the representativeness of our data. Popular social media websites usually sell their impressions through their own platforms and not RTB exchanges, and they host native ads rather than banner ads. Hence, marketing campaigns for such websites must be different from campaigns that qualify for impressions from Yahoo, indicating that they have different budgets.

3.2 Summary Statistics

Our dataset contains auctions for impressions generated in the US on Yahoo's websites during a week in the second quarter of 2021. Because there can easily be tens of millions of impressions on just one website per day, we sample our data at a rate of 0.08%. We restrict our attention on a specific popular banner format for simplification.

Table 1 provides summary statistics on the key variables in our dataset. We observe data on

[2] As of May 2021, Yahoo is ranked fourth in the US popularity by Alexa Rank (https://www.alexa.com/topsites/countries/US), which is an industry-standard website ranking.

variable	mean	std	min	median	max
timestamp (PDT)	Thu 00:06:05		Mon 00:00:04	Wed 18:01:42	Sun 23:59:57
Bid	1.000	1.682	0.061	0.577	369.070
Winning bid	2.294	3.441	0.061	1.182	369.070
Number of bidders	7.205	4.732	1.000	7.000	25.000
computer	0.953	0.212	0.000	1.000	1.000
optout	0.066	0.248	0.000	0.000	1.000
match_cookie_prop	0.628	0.336	0.000	0.778	1.000

Table 1: Summary statistics

8,856,603 bids from 1,229,300 auctions, each of which is for an impression triggered by a user[3]. For confidentiality reasons, we normalize bids to have a sample mean equal to 1, but we may use dollar signs for variables relating to bids in this paper. For each auction, we have the auction outcomes, winning bid (revenue), number of participants (DSPs), and impression characteristics. There are 33 unique DSPs bidding on behalf of 71,011 advertisers in the data; note that each DSP has at most one bid per auction in our dataset. The statistics for the number of participants indicates that these bidders (DSPs) enter only a subset of auctions, which suggests entry is an important behavior to investigate.

The `timestamp` variable provides the time in Eastern Daylight Time when Yahoo held the auction; this variable is central in our analysis as we use this variable to determine the temporal proximity between auctions. We have seven days' worth of data (Monday to Sunday) collected during the second quarter of 2021. The variable `computer` indicates whether the user is accessing from a computer or phone/tablet; it suggests that about 95% of impressions are from computers. Two variables correspond to the availability of the user's information. The variable `optout` is an indicator function of the user opting out from behavioral targeting. When a user opts out, advertisers can no longer target the user based on their personal information; nevertheless, they can still use the user's geographic location and contextual information. The variable `match_cookie_prop` is the proportion of DSPs that successfully matched the user with records in their databases via third-party cookies.[4] Thus, it is harder for DSPs to track a user with a small `match_cookie_prop`. Note that mechanically, we have `match_cookie_prop = 0` when the user opted out. Our data show that opt-out users trigger about 6.6% of impressions, and the average proportion of `match_cookie_prop` is about 63%.[5]

In addition, although we do not report their summary statistics for confidentiality, our dataset contains user characteristics drawn from Yahoo's database of user profiles, which are constructed based on users' cookies and Yahoo accounts (if they exist). Although bidders do not directly observe the content of this database, these variables are good proxies for user information bidders have access to. We have users' gender and age information. The gender variable is either Unknown,

[3]This indicates there were roughly 1.5 billion auctions on Yahoo during this period.
[4]We use an aggregate measure of cookie match since cookie-match information is unavailable for two DSPs.
[5]See Alcobendas, Kobayashi, Shi and Shum (2023) for how privacy protection measures impact ad auctions in this market.

variable	value	n
browser	Chrome	729484
	Edge	220771
	Safari	138794
	Firefox	103956
	(27 other browsers)	36295
sitename	Site-1	495951
	Site-2	243124
	Site-3	165875
	Site-4	113272
	Site-5	71922
	Site-6	35325
	Site-7	32163
	Site-8	22518
	Site-9	21725
	Site-10	9825
	Site-11	9275
	(5 other sites)	8325

Table 2: Frequency table for categorical variables

Male, and Female, and the age variable is either Unknown, 25 to 44, or 45 plus. The variable `seg_size` gives the number of market segments that the user belongs to; these segments predict the user's interests in particular topics, such as automobile and sports. The variables `total_rev`, `num_month_sold`, and `avg_month_sold` summarize the past monetization of impressions generated by the user. The variable `total_rev` is Yahoo's total revenue from selling the user's past impressions, which is standardized to have mean zero and variance one for confidentiality. The variable `num_month_sold` counts the number of months when Yahoo monetized the user, and `avg_month_rev` is the average revenue per month calculated with the two former variables. Finally, the variable `profile_length` measures in days how long the user profile existed in the database.

Table 2 provides the frequency table for two key categorical variables: browser and site name (anonymized for confidentiality). We observe that most impressions come from Chrome, followed by Edge, Safari, and Firefox. We also observe other browsers used by smartphones. The table also shows the range of websites provided by Yahoo, and we see that although each website has a significant number of observed impressions, there is a considerable variation in their total visits.

Figure 2 shows the geographic distribution of impressions graphed based on their geographic coordinates. We observe users accessing from a variety of regions, and many impressions come from cities with high population density, such as New York and Los Angeles. In addition to geographic coordinates, we observe state and city where impressions originate from; these variables are used as control variables in our reduced-form results.

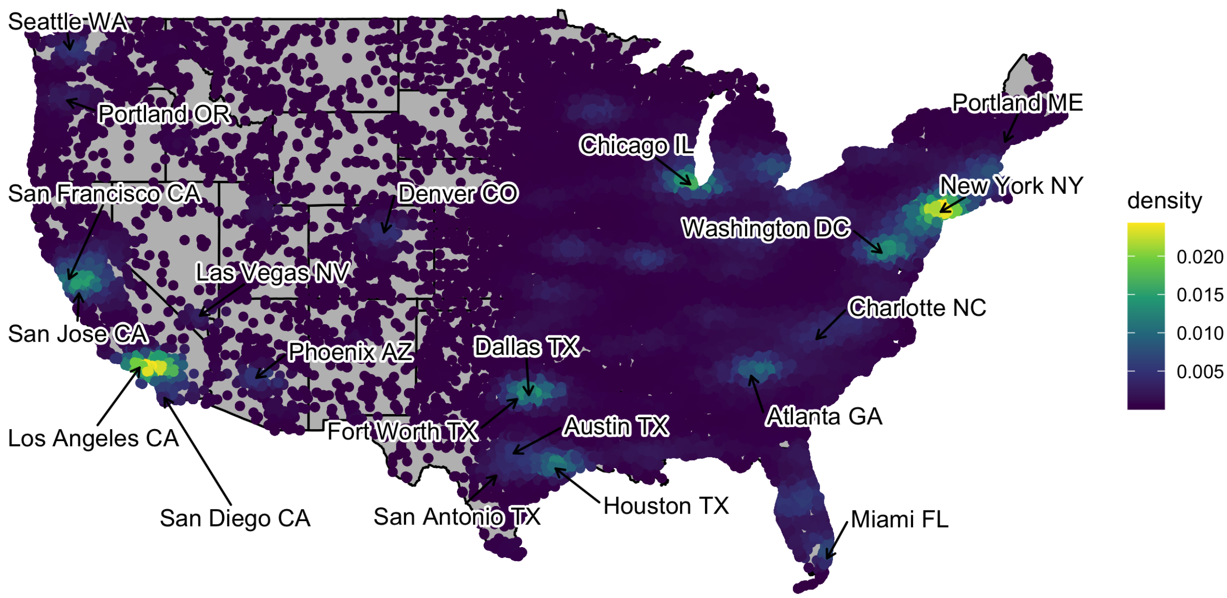

Figure 2: Geographic distribution of impressions. Impressions from Alaska and Hawaii are excluded from the figure. The labels are for the top 20 cities with the highest number of impressions.

3.3 Stylized Facts

We summarize some stylized facts in this market that are consistent with daily budget constraints.

Figure 3 shows the time-series plots of the number of auctions, average bid, average number of participants, and average winning bid per 5-minutes on a weekday. We find that these statistics show similar patterns on each day in our data. Since the supply of advertising opportunities is directly tied to the online traffic, the frequency of auctions achieves the highest around noon and the lowest around 3AM.

3.3.1 Observation 1: Declining Price

In Figure 3, we observe that the average bid, average number of bidders, and average winning bid (price) have a declining pattern. This is consistent with bidders having less purchasing power from spending their daily budgets. Because they have less remaining budgets as time goes on, they enter auctions at a lower rate and submit more conservative bids.

The literature on sequential auctions has studied declining price patterns in other settings extensively. A declining price in sequential auctions is often seen as an anomaly since it seemingly presents an arbitrage opportunity. Nevertheless, this literature has provided various mechanisms that lead

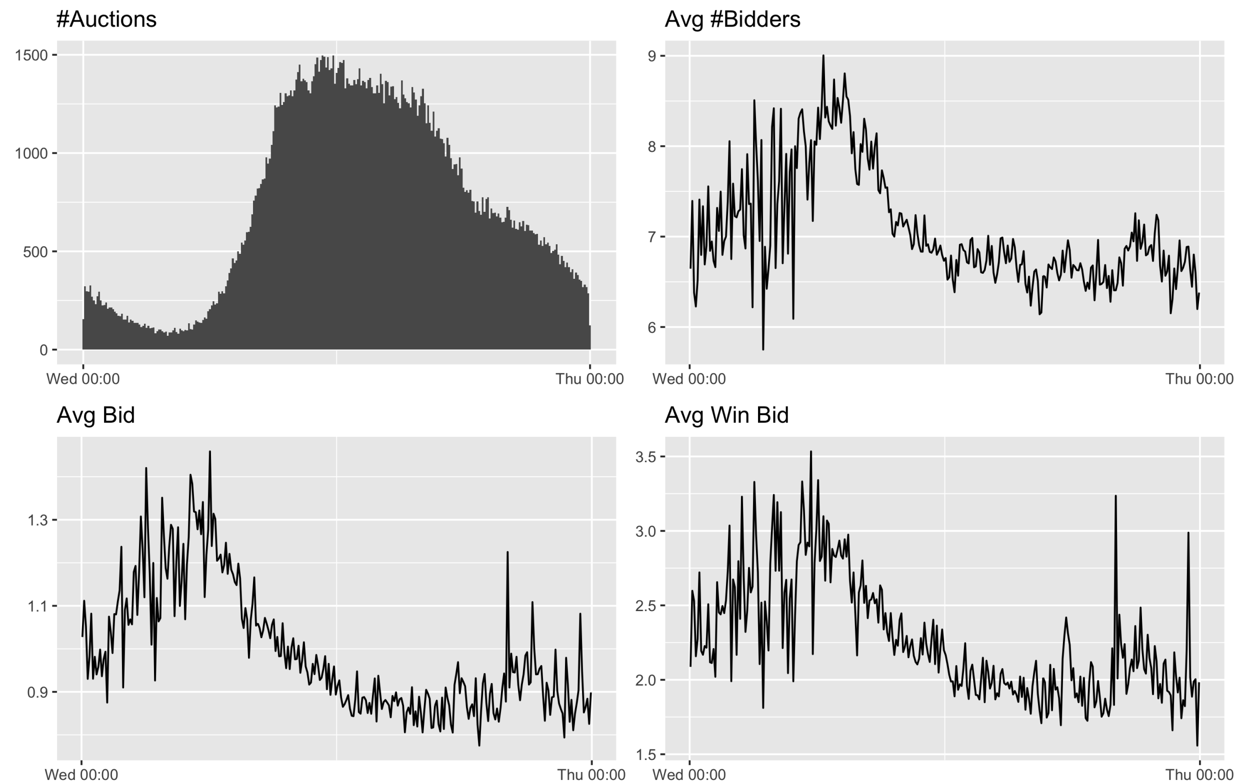

Figure 3: Time-series plots of the number of auctions (impressions), average bid, average number of participants, and average winning bid per 5-minute interval. The horizontal axis is the time in Eastern Daylight Time.

to such a phenomenon. In particular, Engelbrecht-Wiggans (1994); Bernhardt and Scoones (1994); Gale and Hausch (1994) find that a declining price can manifest in sequential auctions participated by unit-demand bidders whose valuations are random across objects. Our market environment is similar; bidders have a limited demand because of daily budget constraints, and there is a significant heterogeneity across impressions.

3.3.2 Observation 2: Price declines when the number of auctions is high

In Figure 4, we aggregate auctions in each 5-minute time interval, and we plot the number of auctions within the interval versus the average log bid, average number of participants, and average log winning bid within the interval. We fit local polynomial regression to each scatter plot to highlight the overall relationship. Figure 4 shows that when the number of auctions increases, the average bid, average number of entrants, and average winning bid decrease, and vice versa. This inverse relationship is consistent with budget constraints. When there is a large number of auctions, bidders risk hurting their future spending ability or violating budget constraint by winning too many auctions if they submit competitive bids. Thus, they need to depress their bids to mitigate this

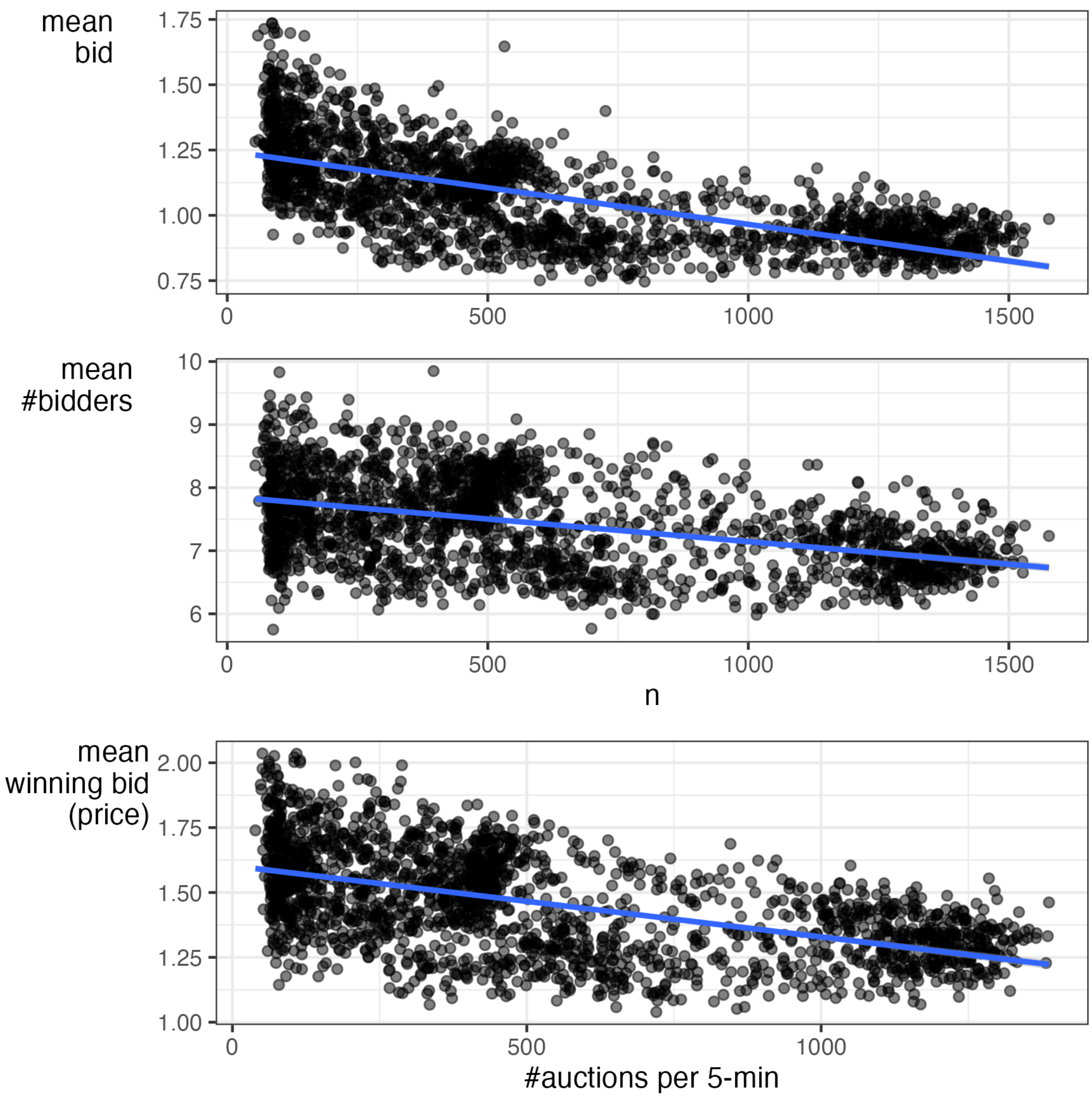

Figure 4: Scatter plots of the average bid, average number of participants, and average winning bid (price) versus the number of impressions per 5-minute interval. The blue curves correspond to linear regressions.

risk.

This relationship is robust to controlling for the rich observed heterogeneity of impressions and various fixed effects. Table 6 in Appendix A.1 shows the results from regressing bids and entry decisions on the number of auctions and control variables. As control variables, we include numerous impression characteristics and fixed effects for the websites, browsers, cities, day-hour, DSPs, and advertisers. We include Day-Hour FE to remedy any time-variant unobserved quality of impressions. The reduced-form results show that the coefficient of log of the number of auctions per 5-min interval is negative and statistically significant in both the bid and entry regressions. Hence, bidders become more conservative in their entry and bids when there are more auctions.

4 Structural Model

Motivated by the institutional settings and the stylized facts from the data, we formulate a structural model of forward-looking bidders making entry and bid decisions dynamically while facing a stream of first-price auctions.

There are two things we abstract away from. First, we treat each demand-side platform (DSP) representing multiple advertisers as one budget-constrained bidder. In reality, each advertiser has a separate campaign budget and DSPs need to make sure the constraint of each advertiser it represents is satisfied. However, capturing this would require modelling the relationship between DSPs and advertisers, which is beyond the scope of this paper, so we leave it for future research and a possible extension of our model.

Second, display-ad auctions occur continuously in the real market, but we discretize time and assume that multiple auctions occur at the same time. Hence, our model features a sequence of simultaneous first-price auction. We make this assumption because of tractability and often these auctions can even occur at the same time, and we want to abstract away from the micro-timing of auctions and rather focus on the frequency of auctions. We think this is a reasonable assumption as DSPs also use a discrete-time approximation and assume that auctions happening in the same time interval are happening at the same time.

We also add the restriction that a bid for one auction is determined only by the valuation for that auction and the number of auctions in that period and remaining budget. In other words, it is not influenced by valuations in other auctions happening in the same period. This is for tractability. We do not want to map multiple valuations into bids because the frequency of auctions can be high (1000s of auctions can happen in 5 minutes in our data). We implement this assumption by letting bidders choose the bidding strategy ex-ante before valuations are drawn in each period, and taking the constrained equilibrium approach of Armantier et al. (2008).

4.1 Model Setup

To focus on the intra-day dynamics coming from the daily budget constraint and cyclical supply, we model the market on each day as an isolated strategic environment. On each day, there are

$i = 1, \ldots, N$ bidders, and each bidder's initial budget W_{i1} is independently and privately drawn from F_W at the beginning of the day. These bidders face a sequence of simultaneous auctions for $t = 1, \ldots T$ periods. The last period T is determined and common knowledge; it corresponds to the final period before the end of the day.

At the beginning of each period t, bidders observe the number of auctions K_t, which is drawn from $F_K^{(t)}$. Before his entry costs and valuations are realized for these auctions, each bidder i commits to an entry threshold strategy $\tau_{it} \geq 0$ and bid strategy $b_{it} : \mathbb{R}_+ \to \mathbb{R}_+$ that are used for each auction $k = 1, \ldots, K_t$. For each auction k, an entry cost C_{ikt} is independently and privately drawn from F_C, and bidder i enters if $C_{ikt} \leq \tau_{it}$. If he enters, then valuation X_{ikt} is independently and privately drawn from F_X, and i submits $b_{it}(X_{ikt})$. To make the optimization problem tractable, we assume that bidders are constrained to use bid strategies of the form $b^\gamma(\cdot)$ for some $\gamma \in \mathbb{R}^J$ so that each bid strategy can be characterized by $\gamma \in \mathbb{R}^J$. In particular, we focus our attention to functions of the form $b^\gamma(x) = \sum_{j=1}^J \gamma_j h_j(x)$, which encompasses popular function approximations like polynomials and splines.

Given the submitted bids, the spot auction rule determines the winner and price for each auction k. With our institutional environment in mind, we suppose that the spot auction follows the first-price auction. Hence, for each auction k, the winner is the highest bidder and the price is his bid $\max_i B_{ikt}$ where $B_{ikt} = b(X_{ikt} \mid \gamma_{it})$. Then, each bidder receives the goods they won and earn $\sum_{k=1}^{K_t} \mathbb{1}\{C_{ikt} \leq \tau_{it}\} \mathbb{1}\{B_{ikt} > B_{-ikt}\} X_{ikt}$ and pays $S_{it} = \sum_{k=1}^{K_t} \mathbb{1}\{C_{ikt} \leq \tau_{it}\} \mathbb{1}\{B_{ikt} > B_{-ikt}\} B_{ikt}$. In sum, as the stage payoff, the bidder receives

$$\sum_{k=1}^{K_t} \mathbb{1}\{C_{ikt} \leq \tau_{it}\} \left(\mathbb{1}\{B_{ikt} > B_{-ikt}\} \left(X_{ikt} - B_{kt} \right) - C_{ikt} \right)$$

In addition, the bidder's budget for period $t+1$ is updated as $W_{it+1} = W_{it} - S_{it}$.

After period T, bidders suffer penalty $\eta Q(W_{iT+1})$, where $\eta > 0$. This represents the bidder's budget constraint.

The following summarizes the sequence of the game:

1. Each bidder i's initial budget W_{i1} is independently and privately drawn from F_W.

2. For each $t = 1, \ldots, T$,

   (a) Bidders observe the number of auctions K_t, which is drawn from $F_K^{(t)}$.

   (b) Each bidder i chooses bid strategy $b_{it} : \mathbb{R}_+ \to \mathbb{R}_+$ and entry threshold strategy $\tau_{it} \geq 0$.

   (c) For each auction $k = 1, \ldots, K_t$,

      i. i 's entry cost C_{ikt} is independently and privately drawn from F_C.

      ii. i enters if $C_{ikt} \leq \tau_{it}$

      iii. Each entrant's valuation X_{ikt} is independently and privately drawn from F_X.

      iv. Each entrant submits bid $B_{ikt} = b_{it}^\gamma(X_{ikt})$

v. The highest bidder gets the good and pays his own bid.

(d) Each bidder's remaining budget is subtracted by his spending, $W_{it+1} = W_{it} - S_{it}$.

3. Bidders suffer penalty $\eta Q(W_{iT+1})$

4.2 Best-Response Analysis

We begin our analysis by assuming the form of the bidder's belief over the strategic behavior of each other and analyzing the best-response problem. When bidders strategize, the key object that matters is the distribution of competing bids of each auction (accounting for entry). This distribution is influenced by the time period, the number of auctions, and the remaining budgets of competing players. In our empirical application of online banner-ad auctions held at Yahoo's ad exchange, the remaining budgets of players are not public, and when a bidder loses in an auction, they do not get the information of the identity of the winner and the price he paid. Hence, there is very little information a bidder has about other players' spending. Leveraging this asymmetric information, we assume that in period t with K_t auctions, bidders believe that the highest rival bid in each auction is independently drawn from distribution $\Psi_t(\cdot|K_t)$. In the next subsection, we endogenize this distribution by formulating our equilibrium concept.

The idea is that although bidders do not learn each other's private state variable (remaining budget), they can use the interactions from previous games to forecast the competition in each period t and how it changes with respect to the supply level (number of auctions). Not only this assumption is reasonable, it also greatly improves the tractability of the dynamic game. As our application has a relatively large number of bidders (around thirty), a fully rational belief with complete information over rivals' budgets would lead to a high-dimensiional state space, which would make both solving the best-response problem and solving for equilibrium computationally infeasible. The assumption we make over the belief over competing bids makes the best-response problem to a finite-time horizon dynamic problem with two state variables: the number of auctions and the bidder's own remaining budget.

Now given the belief, we look at a generic bidder's strategic problem while taking other bidders' strategies as given. This best-response problem provides us insights into the trade-offs bidders face, and it also forms the basis of our estimation method.

We proceed by backward induction and analyze the Bellman formulation. Given the number of auctions K_T and bidder i's remaining budget W_{iT}, his objective is

$$\begin{aligned}
&\max_{\gamma,\tau} E\left[\sum_{k=1}^{K_T} \mathbb{1}\{C_{ikT} \leq \tau_{iT}\}\left(\mathbb{1}\{b^\gamma(X_{ikT}) > B_{-ikT}\}\left(X_{ikT} - b^\gamma(X_{ikT})\right) - C_{ikT}\right)\right] \\
&\qquad + E\left[\eta Q(W_{iT+1}) \mid \gamma, \tau\right] \\
&= \max_{\gamma,\tau} K_T F_C(\tau)\left(E\left[\Psi_T(b^\gamma(X) \mid K_T)(X - b^\gamma(X))\right] - E[C \mid C \leq \tau]\right) + E\left[\eta Q(W_{iT+1}) \mid \gamma, \tau\right]
\end{aligned}$$

Denoting this maximized value as $V_T(K, W)$, the Bellman formulation of the objective of period

$t = 1, \ldots, T-1$ is given by

$$\max_{\gamma,\tau} K_t F_C(\tau) \left(E\left[\Psi_t(b^\gamma(X) \mid K_t)(X - b^\gamma(X))\right] - E[C \mid C \leq \tau] \right) + E\left[EV_{t+1}(W_{it+1}) \mid \gamma, \tau\right] \qquad (1)$$

where $EV_{t+1}(W) = E[V_{t+1}(K_{t+1}, W)]$ is the ex-ante value function in which the number of auctions is averaged out with distribution $F_K^{(t)}$. Given its similarity of the last period's objective, with a bit of abuse of notation, we denote $EV_{T+1}(W) = \eta Q(W)$ for the rest of our analysis.

Now we analyze the first-order necessary conditions for the bidding problem while assuming differentiablity. The one with respect to the bid function parameter γ is given by

$$E\Bigg[\underbrace{\left(X - \frac{\Psi_t(b^\gamma(X) \mid K_t)}{\Psi_t'(b^\gamma(X) \mid K_t)} - b^\gamma(X)\right)}_{\text{Static FOC}} \Psi_t'(b^\gamma(X) \mid K_t) \nabla_\gamma b^\gamma(X)\Bigg] + \underbrace{\frac{1}{K_t F_C(\tau)} \nabla_\gamma E\left[EV_{t+1}(W_{it+1}) \mid \gamma, \tau\right]}_{\text{Dynamic Tradeoff}} = 0$$

Meanwhile, the first-order condition with respect to the entry threshold τ is

$$\tau = \underbrace{E\left[\Psi_t(b^\gamma(X) \mid K_t)(X - b^\gamma(X))\right]}_{\text{Static Threshold}} + \underbrace{\frac{1}{K f_C(t)} \frac{\partial}{\partial \tau} E\left[EV_{t+1}(W_{it+1}) \mid \gamma, \tau\right]}_{\text{Dynamic Tradeoff}}$$

The first-order conditions mainly consist of the static component and dynamic component. The optimally condition for the bid strategy contains an expression that is typically found in static first-price auction models (Guerre et al., 2000), along with an additional element resulting from the dynamic budget constraint. In particular, we observe that the state variables (K_t, W_{it}) directly affect only the latter component. Similarly, the first-order condition for the participation strategy implies that the optimal threshold equals to the static entry threshold determined by the expected payoff from an auction (Li and Zheng, 2009) and an additional dynamic component.

The dynamic component illustrates how entry and bids are influenced by dynamic tradeoffs. Increasing the likelihood of participation and bids impact the continuation value by 1) making it more likely to win more auctions and 2) increasing the realized spending. Because having less remaining budget negatively affects his future surplus, the bidder must internalize this tradeoff and adjust his entry rate and bids further down from the statically optimal ones. Dynamic markdowns are a common feature in structural models that deal with sequential auctions, as demonstrated in prior research (Jofre-Bonet and Pesendorfer, 2003; Bodoh-Creed et al., 2021; Kong, 2021; Backus and Lewis, 2023). However, a notable departure in this study is the consideration that in earlier models, the option value was primarily influenced by whether one won an auction or not, while in our model, the option value is also impacted by the amount paid for a win.[6]

[6] As demonstrated in our counterfactual analysis, this feature makes the price distribution relevant in shaping

Now we consider how the optimal strategies reacts to changes in the state variables under this model. When the number of auctions K_t changes, it impacts the dynamic component. Specifically, a higher K_t tends to result in larger spending from this period while keeping the strategies same. Consequently, ceteris paribus, an increase in K_t introduces a force to make the strategies more conservative, consistent with the empirical pattern that bidders tend to be less aggressive when more auctions are present.

The current remaining budget W_{it} also appears in the first-order conditions only through the dynamic component. Ceteris paribus, decreasing W_{it} directly shifts down W_{it+1}. The ex-ante value function $EV_{t+1}(\cdot)$ typically exhibits a concave increasing pattern, as having a larger budget aids the bidder in securing more future opportunities, albeit at a diminishing rate. Consequently, the reduction in W_{it} amplifies the sensitivity of the continuation value to current-period spending, prompting the bidder to adopt more conservative strategies. The relationship between the optimal strategies and the remaining budget W_{it} highlights that the variability across bidders in their entry and bid decisions results from factors beyond random entry costs and valuations; it is also influenced by bidders' remaining budgets. This suggests that the size of each bidder's budget introduces heterogeneity in their behaviors.

The first-order necessary conditions highlight how this environment differs from the standard auction environment. If the budget constraint does not matter (i.e. if $\eta = 0$), the dynamic problem collapses into a series of static bidding problems, and the state variables K_t and W_{it} become irrelevant. However, with the budget constraint, the bidder needs to weigh the stage payoff and the option value from having more budget for the next period, and this tradeoff is influenced by the state variables.

4.3 Equilibrium

We now establish our solution concept by formalizing how a bidder's belief over competing bids is constructed. A pure strategy equilibrium of our model consists of time-dependent strategies $(\gamma_t(K, W), \tau_t(K, W))$ and bidders' beliefs regarding competing bids in each auction $\Psi_t(b \mid K)$ that satisfy the following conditions:

1. (Optimality) For each period t and state variables (K_t, W_{it}), $(\gamma_t(K, W), \tau_t(K, W))$ are a best response given the belief $\Psi_t(b \mid K)$, meaning they solve the problem specified in (1).

2. (Consistency)

$$\begin{aligned}\Psi_t(b \mid K) &= E\left[\prod_{j\neq i} \Pr(j \text{ does not enter, or } j \text{ ennters and submits } B_{jt} \leq b \mid K, W_{jt})\right] \\ &= E\left[\prod_{j\neq i} \big(1 - F_C(\tau_t(K, W_{jt})) + F_C(\tau_t(K, W_{jt}))F_X(b^{-1}(b \mid \gamma_t(K, W_{jt})))\big)\right]\end{aligned}$$

strategic behavior within our environment.

where the distribution of state variables (remaining budgets) $(W_{jt})_{j\neq i}$ is determined by the initial budget distribution F_W, the distribution of the number of auctions $\{F_K^{(s)}\}_{s=1,\ldots,t-1}$, the strategies employed by bidders $\{(\gamma_s(K,W),\tau_s(K,W))\}_{s=1,\ldots,t-1}$, and the state transition rule.

The first condition requires that bidders are acting optimal given their belief, and the second condition ensures that that current belief is consistent with the optimal strategies they have employed in previous periods.[7] If bidder i had the knowledge about other bidders' remaining budgets $(W_{jt})_{j\neq i}$, his fully rational belief over competing bids would be the expression inside the expectation in the second condition. However, given that bidders' initial budgets are private and they do not observe each other's spending, we require that their belief is averaged out with respect to the equilibrium state distribution of $(W_{jt})_{j\neq i}$. The rationale for this is that since a separate game occurs each day, bidders can rely on historical data regarding the intraday pattern of competition to formulate their participation and bidding strategies.[8]

In our analysis, we assume that a pure strategy equilibrium of the dynamic game exists and is unique given the model primitives, and we assume that each day in our dataset is independently sampled from this equilibrium.[9] The formal proof of equilibrium existence and uniqueness is left for future research, as the conventional approach of backward induction does not apply to our environment. This is due to the fact that the equilibrium strategy in period t depends on the strategies in periods 1 through $t-1$, given that these strategies determine the belief regarding competing bids in period t. In Appendix A.4, we present a computational algorithm for solving the dynamic game using our solution concept. This algorithm alternates between two steps: first, obtaining the belief Ψ_t by simulating the path of state variables using the given strategies, and second, obtaining the best response strategies given the belief through backward induction. Importantly, we find that our algorithm converges to the same equilibrium from various initial points, providing some support for our assumption of equilibrium existence and uniqueness.

5 Estimation

To avoid the computation of the equilibrium of the dynamic game during estimation, we take a two-step approach to estimate the model (Bajari et al., 2007; Jofre-Bonet and Pesendorfer, 2003; Aguirregabiria and Mira, 2007). In the first stage, we estimate bidders' entry probability and bid distribution conditional on the public state (number of auctions K_t), and we also estimate the time-dependent distribution of the number of auctions $F_K^{(t)}$. Then in the second stage, after

[7]Our solution concept is similar to the large market equilibrium concepts used in prior works that study dynamic games (Hopenhayn, 1992; Krusell and Smith, 1998; Weintraub et al., 2008; Bodoh-Creed et al., 2021; Backus and Lewis, 2023).

[8]One possible way to allow for firms learning each other's state within a game is to adapt the moment based Markov equilibrium proposed by Ifrach and Weintraub (2017) to our model. Their solution concept permits firms to track the state variables of a few dominant firms and form beliefs on the state variables of other firms conditional on their aggregate statistics. See also Fershtman and Pakes (2012); Asker et al. (2020) for tractable ways to model players learning each other's state when there is persistent private information.

[9]Our assumption of equilibrium existence and uniqueness parallels the existing empirical works on non-standard auction games (Fox and Bajari, 2013; Kim et al., 2014; Saini, 2012; Gentry et al., 2020).

deriving the probability of winning auction from the first-stage estimates, we frame each bidder's best-response problem as a single-agent dynamic problem with finite time-horizon, and estimate the model primitives. One advantage of our solution concept is that every bidder has the same ex-ante value function; hence, value function iteration via backward induction needs to happen only once for all bidders when evaluating the likelihood function.

One distinction with the two-step estimator of dynamic auction in Jofre-Bonet and Pesendorfer (2003) is that we do not combine the first-stage estimates and the first-order condition of the bidder's problem to obtain the ex-ante value function. This is because our dynamic problem relies crucially on the shape of the value function on the private state (remaining budget) that is very unlikely to happen in equilibrium and in data[10]. Specifically, we need the accurate continuation value for the situations in which the bidder runs out of budget so that he avoids bidding aggressively.

5.1 Econometric Specification

In the first stage, for tractability and numerical convenience, we use parametric forms to estimate the entry probability and bid distribution conditional on the number of auctions, which is the public state that is observed by everyone. For the entry probability, we use logistic regression of entry outcome on the number of auctions K_t with time-period fixed effect to estimate it, meaning that entry probability is logistic$(\alpha_t^{entry} + \beta_K^{entry} K_t)$. For the bid distribution, we assume $B_{ikt} \sim LogNormal(\alpha_t^{bid} + \beta_K^{bid} K_t, \sigma_t)$. We also estimate the distribution of the number of auctions in each period. We assume that the number of auctions comes from Negative Binomial with time-specific parameters.

The model primitives that need to be estimated are the entry cost distribution F_K, the valuation distribution F_X, the severity of the budget constraint penalty η, and finally the distribution of bidders' initial budgets F_W. To facilitate our estimation, we make parametric assumptions on F_C and F_X. We assume $C \sim TruncatedNormal(\mu_C, \sigma_C)$ and $X \sim LogNormal(\mu_X, \sigma_X)$. For bidders' budgets W_{i1}, we first estimate them as structural parameters for 33 bidders, and then use the empirical distribution of the estimated budgets as an estimate of F_W in our counterfactual exercises.[11]

5.2 Estimation Method

We use standard maximum likelihood estimators to obtain the first stage estimates: entry probability $\hat{p}_t(K)$, bid distribution $G_t(b \mid K_t)$, and the distribution of the number of auctions F_K. The second-stage estimation involves more complexities as it requires solving a single-agent continuous-choice dynamic problem with finite time-horizon. We describe our algorithm to solve the problem and estimate the structural parameters in this section.

There are two complexities involved in solving the dynamic problem. First, given that there is no closed-form solution for the entry threshold and bid strategy, we have to solve them numerically.

[10] See Bajari et al. (2007) for a relevant discussion.

[11] We obtained similar estimation results from using various parametric distributions for F_W.

Second, the non-stationarity (being finite time-horizon) of the problem requires us to solve it via backward induction, which adds another dimension to the optimization of the problem in addition to the state variables.

The state variables are the number of auctions K_t and bidder's remaining budget W_{it}. The central object of our dynamic programming is the ex-ante value function $EV_t(w) = E_{K_t}[V_t(K_t, w)]$. We approach this problem by solving the problem over a grid of the state variables K and W. We create the grid of the first state variable by taking random draws from the estimated distribution $\widehat{F}_K^{(t)}$ from the first stage; this grid is time-dependent. For the second state variable w, because this is a continuous variable, we create a grid by taking points in $[a, b]$ where $a < 0$ is a negative value that is unlikely to happen in equilibrium but nevertheless important for determining the shape of the value function, and $b > 0$ is a value above the maximum observed total spending. To evaluate the ex-ante value function outside of the grid and to obtain its derivative, we use a cubic spline with monotonicity constraint for interpolation. We solve bidder's optimization problem by solving the first-order conditions.

The above procedure provides us the entry thresholds $\tau_t(K, w)$ and bid strategies $b_t(\cdot \mid K, w)$ for each K, w, and t. Given these objects, we derive the entry probabilities and density of the bid distribution to compute the log-likelihood. The entry probability is expressed by

$$\tilde{p}_t(K_t, W_{it}; \theta) = 1 - F_C(\tau_t(K_t, W_{it}; \theta); \mu_C, \sigma_C)$$

and the bid density is

$$\tilde{g}_t(B \mid K_t, W_{it}; \theta) = f_X(b_t^{-1}(B \mid K_t, W_{it}; \theta); \mu_X, \sigma_X)(b_t'(B \mid K_t, W_{it}; \theta))^{-1}$$

where the right-most term comes from the change of variable from valuations to bids. Finally, we can calculate the likelihood from the observed data by using these objects.

5.3 Identification

Because our structural parameters enter the model non-linearly, especially the penalty parameter η, it is challenging to formalize non-parametric identification results. Nevertheless, we provide the intuition behind our identification strategy. The key exogeneity assumption is that valuations are independent with the state variables, the number of auctions and the remaining budget. This assumption allows us to use the correlation between the state variables and bidders' behavior to identify the dynamic incentives created from the soft budget constraint. For illustration, our assumption implies the following conditional moment conditions:

$$E[b_t^{-1}(B_{ikt} \mid K_t, W_{it}; \theta) - E[X_{ikt} \mid \theta] \mid Z_{it}] = 0$$

where Z_{it} is remaining budget W_{it} or number of auctions K_t. This essentially means that the valuations backed-out via the inverse bid function should not be correlated with our instruments.

Such an exclusion restriction with other instrumental variables have been used in the empirical auction literature to test a model (Haile et al., 2003) or identify structural parameters (Guerre et al., 2009; Gentry et al., 2020).

The exogeneity assumption is plausible for our market environment as typically demand-side platforms (bidders) and advertisers compute their valuations for impression based on a combination of the probability of clicking/making a sale and their value of such events, and this probability is computed based on the contextual and behavioral data of that user alone. Hence, the short-run supply level of impression (the number of auctions) and their current campaign budgets should not directly influence how they value advertising opportunities.

There are two potential sources of unobserved heterogeneity that might interfere with our identification, and we describe potential extensions to address them. First, valuations could be correlated with the number of auctions through a time-varying unobserved heterogeneity. For instance, the average user browsing the internet during the daytime could be different from the average online user at night, and the supply levels are different across these time periods. One possible remedy for this issue is to allow time-dependent valuation distribution $F_X^{(t)}$, analogous to introducing time fixed effects in standard econometric models. The second potential confounder is that valuations could be correlated with bidder's budgets through a bidder unobserved heterogeneity. Advertisers with larger budgets may also happen to have higher valuations for impressions. We can alleviate this issue by classifying bidders into groups and estimating structural parameters for each group separately.

6 Estimation Results

We now present the results from structural estimation on our data of online banner-advertising auctions. We focus on the estimated structural parameters from the second stage (see Appendix A.2 for the reduced-form estimates from the first stage).

Parameters	Estimate	SE
μ_X	0.9046	0.0007
σ_X	1.0950	0.0006
μ_C	-11.3776	0.0091
σ_C	7.2533	0.0062
η	0.6457	0.0084

Table 3: Estimates of Structural Parameters. The standard errors are computed using the White (sandwich) estimator using the numerical Hessian and Jacobian.

Table 3 presents estimates of the structural parameters, along with their estimated standard errors. To review, we assume valuations are drawn from LogNormal(μ_X, σ_X) and entry costs are drawn from TruncatedNormal$(\mu_C, \sigma_C; \underline{C} = 0)$. Finally, the penalty for violating the budget constraint is imposed after the last period by $\eta \max(0, W_{iT+1})^2$ where W_{iT+1} is the remaining budget of bidder i at the end of the day.

First, note that the estimate for η is positive and statistically-significantly different from 0. Since bidders' dynamic bidding problem collapses to a series of static bidding problems if $\eta = 0$, this confirms that bidders care about the budget constraint and hence act dynamically. The estimated model reveals that, within the dataset, the typical bidder exceeds their budget approximately 26% of the time. When such overspending occurs, it amounts to an average of around 8% of their budgets, indicating that these bidding agents occasionally exceed the budgets set by their clients (advertisers) in pursuit of maximizing their payoffs while avoiding excessive violations.

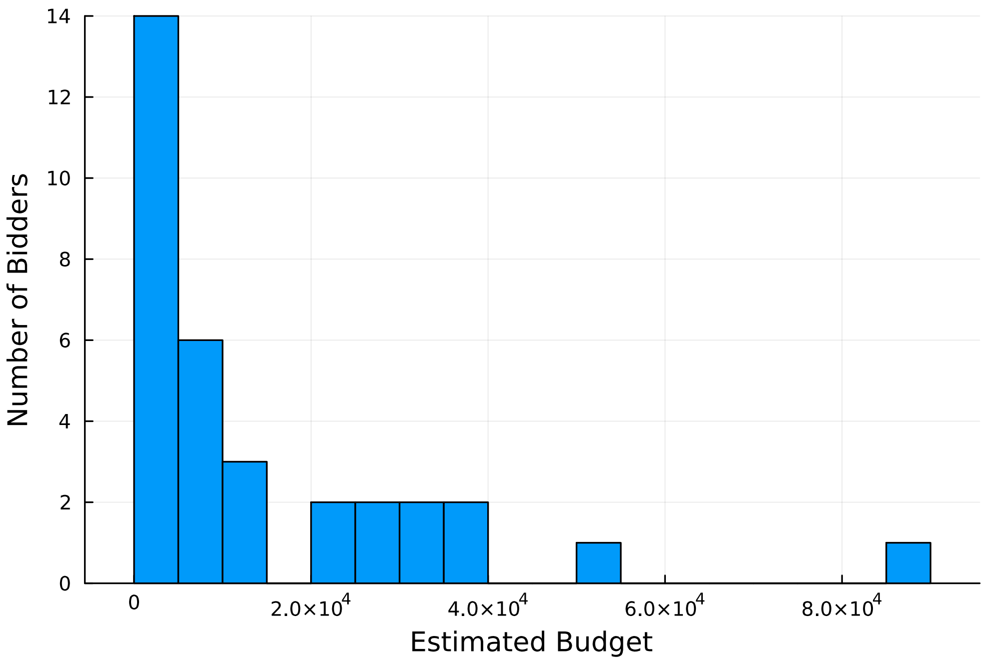

Figure 5: Histogram of etimated budgets

Figure 5 shows the histogram of the estimated daily budgets of bidders. The budget distribution is significantly skewed, and it reflects how the online advertising market is concentrated. In particular, the distribution shows that there are a few large players and many smaller players. The former type includes large tech companies like Google and Amazon.

6.1 Dynamic Incentive

Given the estimated structural model, we can analyze the magnitude of the dynamic incentives created by the budget constraint. First, we look at the markdown (valuation minus bid) obtained by the model. Averaging across time periods, bidders, and days, we find that the markdown is 3.76, which is 83.5% of the expected valuation (4.5). This markdown reflects both the fact that bidders face first-price auctions and dynamic incentives. To decompose these two different incentives, we simulate counterfactual static bids taking the probability of winning as in the data but removing the dynamics created by the budget constraint. We find that the counterfactual static markdown is 2.67 on average, and it is 59.4% of the expected valuation. This highlights that facing the first-price format for each auction leads bidders to shade their bids by 59.4% from valuations and the dynamic budget constraint leads them to shade further by 24.2% on average. This demonstrates that dynamic incentives in this market are significant for the bidders. Figure 6 shows the relationship between the daily budget and the average markdown. We see that the heterogeneity in budgets in turn leads to heterogeneity in how aggressive bidders are. We find in our counterfactual analysis (Section 7) that this competitive variation has a substantial welfare implication.

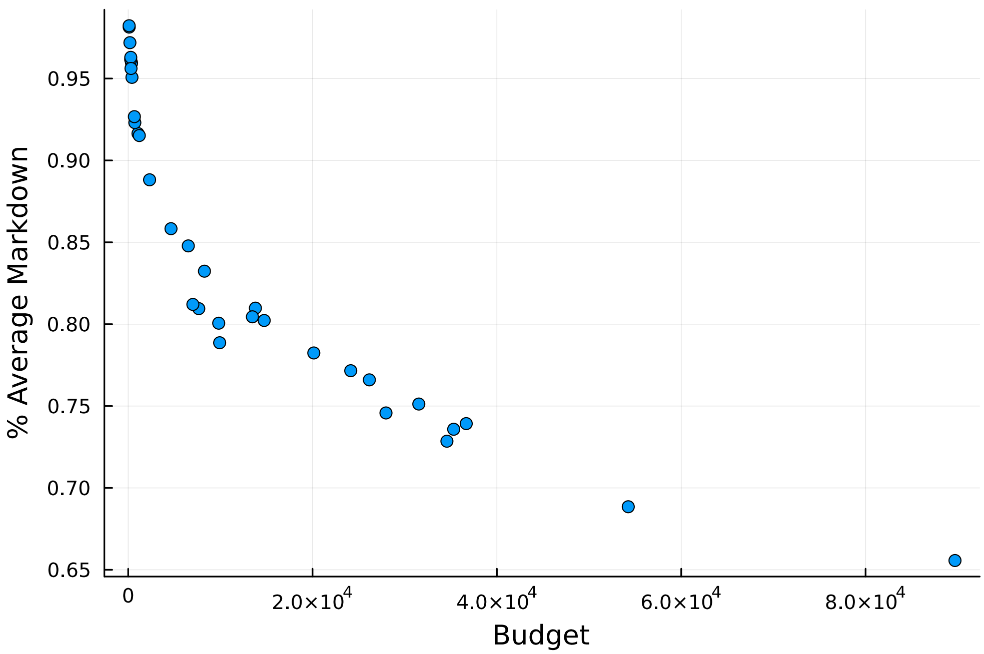

Figure 6: Budget vs %Average Markdown. Each point represents a biddder.

Our structural model also endogenizes entry, so it also allows us to quantify the effect of budget constraints on bidders' entry decisions. The static simulation above also provides us the counterfactual static entry probabilities, purely coming from stochastic entry costs. The average entry probability fitted by the model is 19.4%, and the average static entry probability is 45.1%, which again illustrates the importance of capturing the dynamic budget constraint to analyze bidders' behavior in this market. Figure 7 shows the relationship between the daily budget and the average entry probability, and again it shows that heterogeneity in budgets leads to heterogeneity in entry behaviors.

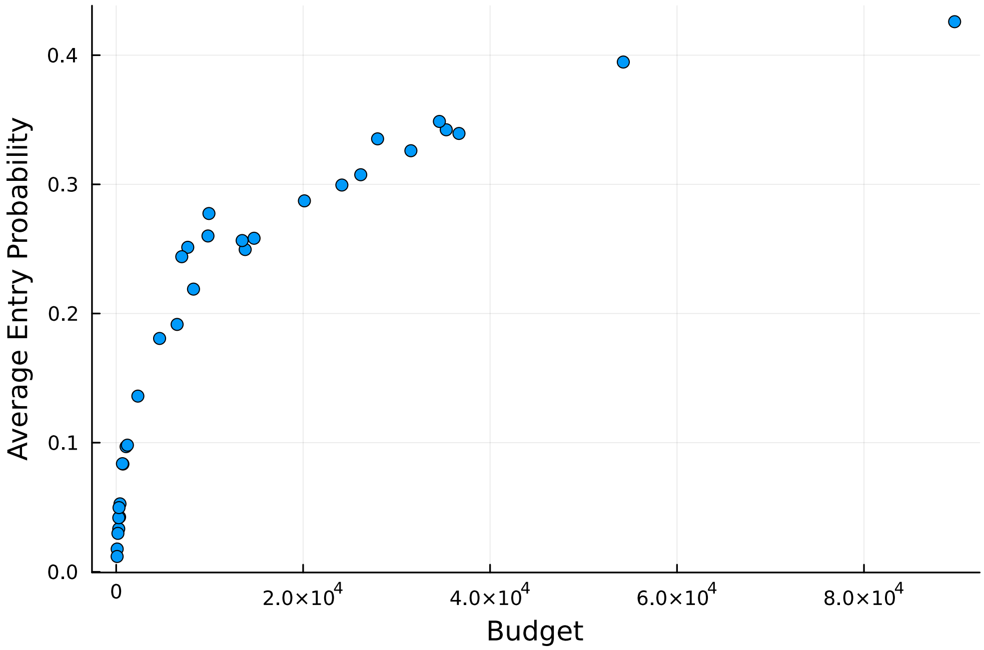

Figure 7: Budget vs Average Entry Probability. Each point represents a bidder.

Finally, we utilize the estimated structural model to decompose the dynamic incentive into two components: one arising from the diminishing budget and the other from approaching the terminal period. In the data, we observe that the average bid decreases over time due to diminishing budgets. However, as time progresses, bidders have fewer opportunities remaining, which should in principle make them less constrained. Therefore, what we observe in the data results from the interplay of these two effects: the diminishing budget effect and the diminishing remaining opportunities effect. In Figure **??**, we illustrate this by considering a bidder with a median budget (approximately $8000) and comparing their average bid as fitted to the data with the model-predicted average bid when their remaining budget is held constant, thereby isolating the effect of having fewer opportunities as time elapses. We observe that the bidder becomes more aggressive with a constant budget as time progresses. However, the diminishing budget effect ultimately dominates, leading to a declining bid path, as indicated by the fitted model.

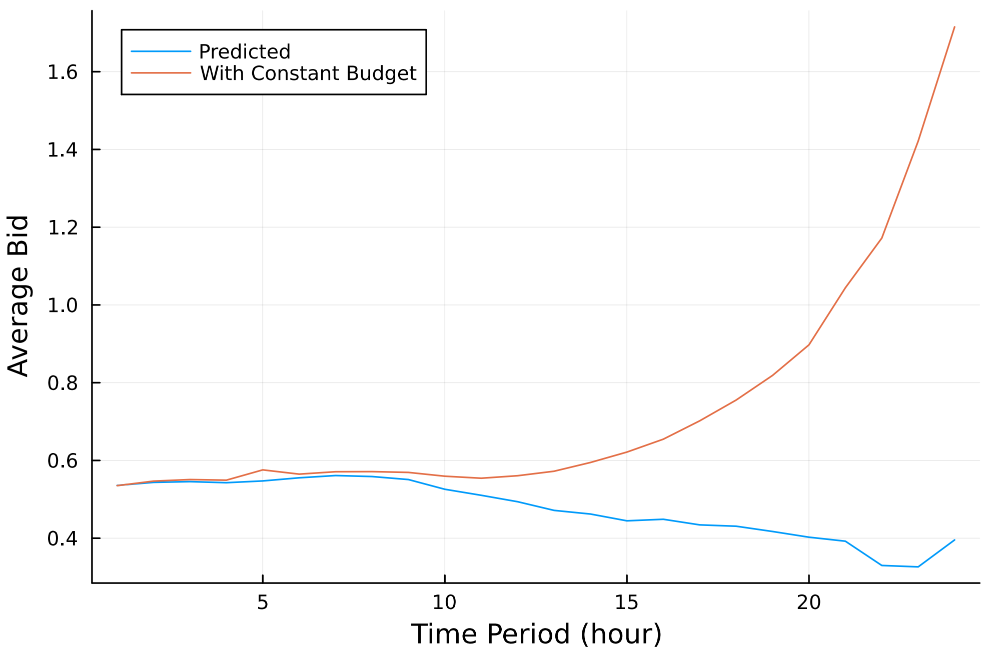

Figure 8: The median-budget bidder's predicted mean bid trajectory compared to the predicted path when keeping their remaining budget constant.

7 Counterfactuals

Using the estimated structural model, we simulate a counterfactual motivated by an institutional change that occurred several years ago. Although the current online ad market primarily uses the first-price auction mechanism, ad exchanges (auctioneers mediating publishers and advertisers) used the second-price format until around 2018. The shift from the second price to the first price was spurred by an industry-wise outcry that ad exchanges are charging something other than the second-price even though claiming to be running the second price auction. Hence, this industry-wide credibility loss of market makers led participants to demand the first-price auction for its transparency over what winners pay. Motivated by this shift, we simulate the second-price auction format using the estimated structural model as a counterfactual scenario to analyze the revenue and welfare consequences.

Although the theoretical auction literature has established that the first-price auction and the second-price auction provide the same revenue and welfare for the standard auction environment, it is ambiguous whether this holds for our environment. Alcobendas and Zeithammer (2023) and Goke et al. (2022) provide event-based analyses of this transition, and a prominent finding in their research is that bidding agents required an extended period, often spanning several months, to adapt their bidding strategies for the first-price auction format. As market conditions can drastically change in such a time span, it highlights that event-based approaches may be inadequate to provide an equilibrium analysis of the comparison between the two formats. Our structural framework provides a way to compare the long-run equilibrium outcomes from the first-price and second-price auctions.

Using the best-response iteration algorithm described in Appendix A.4, we solve both the benchmark scenario with the first-price auction (FPA) and the counterfactual scenario with the second-price auction (SPA) as continuous-action dynamic games with finite-time horizon. The best-response formulation for the second-price format is given in Appendix A.3.

Auction Format	First Price	Second Price
Price Average	\$2.364	\$2.362
Price Variance	1.1246	3.565
Expected Total Revenue	\$480,427.33	\$480,073.49
Expected Total Bidder Surplus	\$1,191,000	\$1,185,000

Table 4: Aggregate statistics of the simulated results under the first-price auction (status quo) and the second-price auction.

	First Price	Second Price	%(FPA - SPA)
Large Bidders	\$191,362.15	\$197,345.68	-3.03%
Medium Bidders	\$473,791.65	\$467,448.36	1.36%
Small Bidders	\$297,439.98	\$291,925.99	1.89%

Table 5: Total bidder surplus of each type of bidders.

Table 4 shows aggregate statistics of the two auction mechanisms. It shows that the total revenue and total bidder surplus are slightly better under the first-price format on average. We find that expected daily (total) revenue and expected total bidder surplus are slightly higher under the first-price format than the second-price format. They are both about 0.1% higher under the first-price format.

We find a more substantial difference when we analyze the difference in the welfare distribution among bidders. First, we classify bidders based on their estimated budgets. Based on the distribution of budgets in Figure 5, we classify two bidders with budgets ranging from \$50,000 to \$90,000 as 'Large,' eleven bidders with budgets between \$10,000 and \$50,000 as 'Medium,' and twenty bidders with budgets below \$10,000 as 'Small.' Table 5 shows the expected total utility obtained by each type of bidders under the two mechanisms, and it shows that the top two bidders with the largest budgets are better off under SPA while other bidders with smaller budgets are worse off. The combined welfare of the two large bidders is 3% higher in SPA than in FPA, and the one for the other bidders is 1% lower in SPA. This suggests that FPA has an interesting property in this environment that compared to SPA, it redistributes welfare from bidders with large budgets to those with smaller budgets. This suggests that the transition from SPA to FPA in the online display-ad market was a welfare improvement event for smaller players, in addition to the fact that they can enjoy the transparency of FPA.

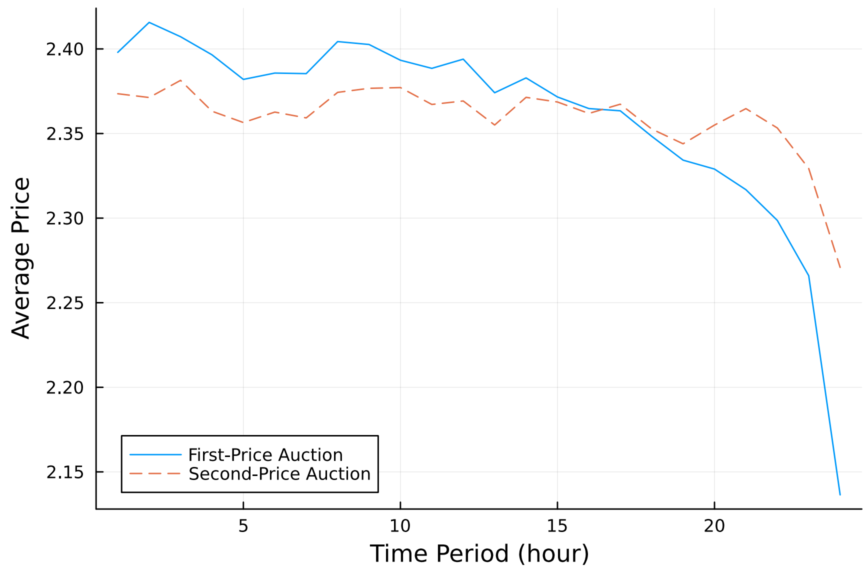

Figure 9: Average price per period

Now we compare the dynamic outcomes from FPA and SPA. Figure 9 shows the average price per period. Note that the price is the highest bid under FPA and second-highest bid under SPA. The average price under FPA, which is the mechanism used in the data, shows a declining pattern as we see in our descriptive results (Figure 3). As explained before, this is coming from bidders becoming conservative from decreasing budgets. Although the price path from SPA also shows a declining pattern, there is some distinctive difference between them. The figure shows that the average price from FPA is systematically higher than SPA until around 3PM, and then the relationship switches. This suggests that FPA generates more revenue until 3PM, and then SPA generates more revenue after that.

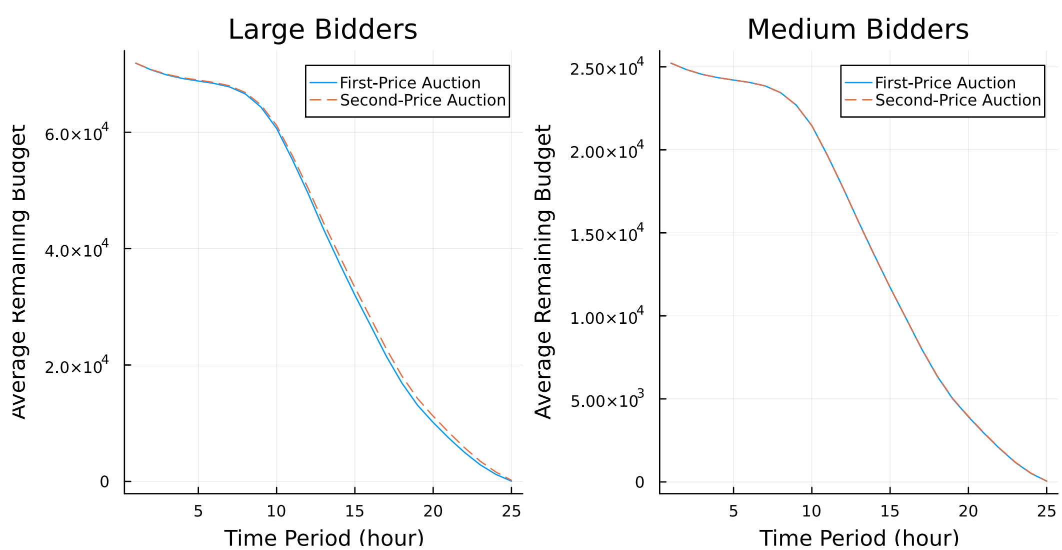

Figure 10: Average remaining budget per period for Large and Medium bidders

Analyzing the spending path of players reveals the critical difference that is driving the dynamic difference. Figure 10 shows the average remaining budget of Large and Medium bidders for each period. The spending path of Large bidders shows that in the afternoon, their remaining budgets tend to be lower under FPA. Meanwhile, the spending path of Medium bidders is relatively similar across the two auction formats. This suggests that the price difference between FPA and SPA after 3PM is primarily driven by the large bidders having tighter budgets in the afternoon under FPA.

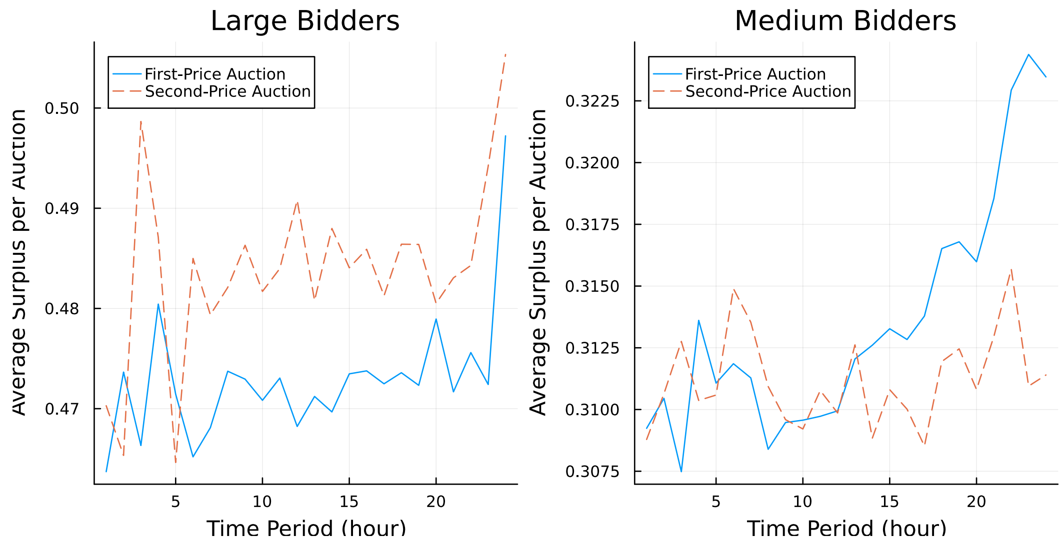

Figure 11: Average surplus per period for Large and Medium bidders

The difference in the spending speed of the large bidders across the two formats has a significant welfare consequence for smaller bidders. Figures 11 show the time series of average surplus per auction for Large and Medium bidders. They show that Medium bidders experience a larger surplus under FPA after around 3PM, when FPA becomes less competitive than SPA as shown in 9. Meanwhile, we do not see such a pattern for Large bidders. This dynamic difference suggests that the contrast in the welfare distribution is driven by smaller bidders enjoying less competition in the afternoon from the large bidders under FPA.

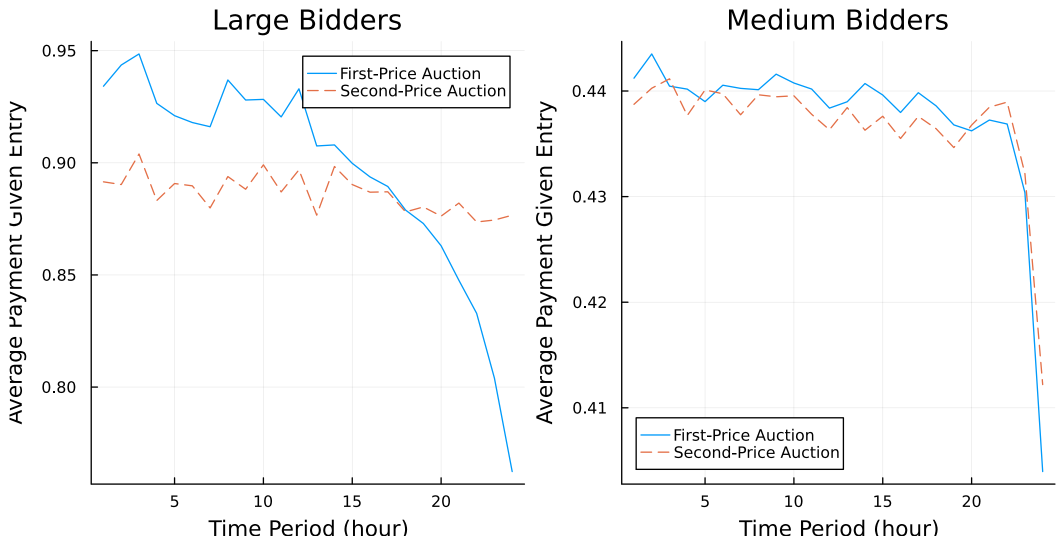

Figure 12: Average spending from entered auctions for Large and Medium bidders

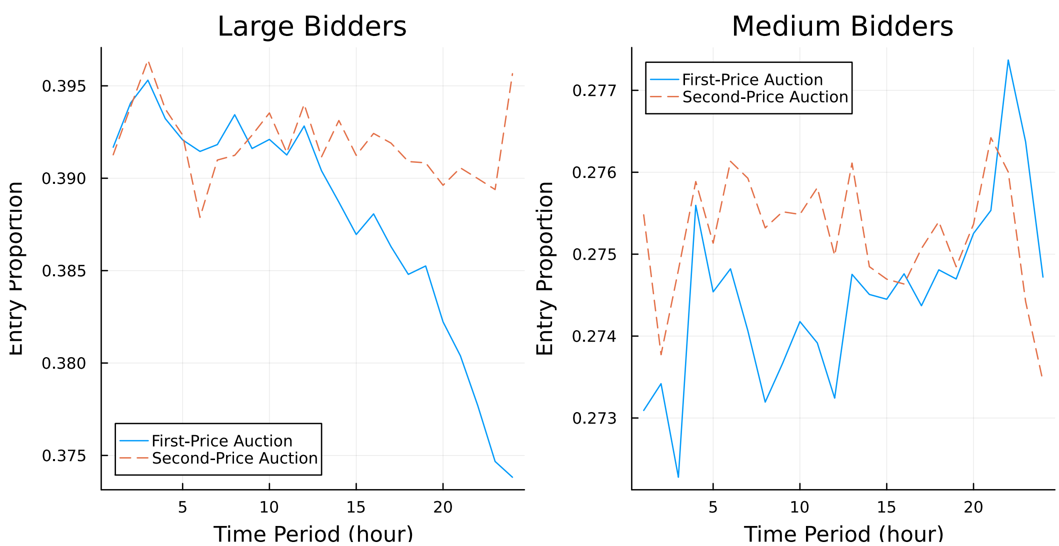

Figure 13: Average proportion of auctions entered for Large and Medium bidders

What is driving the large bidders to spend more rapidly under FPA? We analyze the entry and bidding behavior to understand this. First, in Figure 12, we find that FPA has a higher expected spending per auction conditional on entering for most of the time. This demonstrates that players bid more aggressively under FPA. However, in Figure 13, we also find that while Large bidders have similar entry patterns across the two formats, Medium and Small bidders enter auctions at lower rates under FPA. Since the number of entrants is not public, entry rates affect the probability of winning an auction. Hence, borrowing the terminology of Li and Zheng (2009), we find that for Large bidders' spending, the competition effect coming from all bidders submitting more aggressive bids under FPA dominates the entry effect from Medium and Small bidders entering less frequently.

Finally, we analyze why bidders submit more aggressive bids when they enter auctions under

FPA. In the standard auction environment, one key difference between the payment for FPA and SPA is that the variance is higher for SPA.[12] Similarly, Table 4 shows that this is also the case in our simulation. This difference can be crucial for bidders in our environment since it affects their ability to control their spending dynamically. In particular, there can be more "accidents" in which they end up paying more than they expected. Looking back at the bidder's bidding problem under FPA in (1) and SPA in (2), we see that the spending enters nonlinearly in the objective function through the continuation value,

$$E\left[EV_{t+1}(W_{it} - S_{it}) \mid \gamma, \tau\right]$$

where S_{it} is the total spending from the current period, given bid strategy γ and entry threshold τ. Note that $EV_t(\cdot)$ exhibits a concave increasing pattern in our estimated structural model because having a larger budget aids the bidder in securing more future opportunities, albeit at a diminishing rate. Intuitively, the continuation value makes bidders effectively risk averse; ceteris paribus, they dislike having a higher variance in their payment because of its concavity in the spending from the current period. Hence, bidders are more conservative under the second-price auction, resulting from the willingness to sacrifice some gain with a reduction in the variance.

The finance literature has extensively documented that financial constraints tend to induce risk aversion in firms (Froot et al., 1993; Opler et al., 1999). In particular, theoretical studies by Milne and Robertson (1996), Holt (2003), and Rochet and Villeneuve (2005) investigate the dynamic problem of a financially-constrained firm determining dividends and investment policies, and they consistently find that the concavity in the value function with respect to the cash holding leads the firm to exhibit risk aversion, which is in line with the findings in our model.

8 Conclusion

When price discovery is necessary for time-sensitive goods, it is common practice to conduct an auction for each item sequentially. These dynamic settings may lead to behaviors distinct from static environments and affect the revenue and welfare outcomes of various auction formats. This paper investigates how intertemporal budget constraints affect competition in the online advertising market. Furthermore, we examine how bidders with varying budgets face disparate welfare outcomes under different auction mechanisms.

We develop a finite-horizon dynamic game between bidders with heterogeneous budgets facing numerous auctions in each period. We estimate the model using a proprietary dataset of online ad auctions from Yahoo. Our estimation results show that bidders indeed exhibit behavior consistent with dynamic budget constraints, and there is a significant disparity in daily budgets among players, contributing to the heterogeneity observed in participation and bidding behaviors.

To gain insights into the strategic implications of dynamic incentives arising from intertemporal constraints, we conduct two counterfactual exercises. First, we simulate bidders' counterfactual

[12] In fact, the revenue from the second-price auction is a mean-preserving spread of the one from the first-price auction Krishna (2009).

entry and bidding behaviors if they were unconstrained. This exercise reveals that, on average, approximately 30% of the markdown can be attributed to dynamic constraints, which also lead to a reduction in participation probability by around 25 percentage points.

As our second counterfactual exercise, we compare first-price (the status quo) and second-price auction outcomes. Although both auction formats yield equivalent revenue and welfare outcomes in the standard auction environment with symmetric bidders, we discover that dynamics and heterogeneous budgets lead to substantial welfare differences between them. Intermediate and smaller budget bidders fare better in the first-price format due to reduced price variance, which allows for more aggressive bidding. This heightened competition prompts larger bidders to spend quickly, leading to diminished competition in later periods and ultimately benefiting smaller bidders overall. This highlights that even a seemingly simple mechanism choice can have competitive implications in such a dynamic environment.

The main contribution of this paper is to empirically analyze how budget constraints shape competition in auctions when held sequentially. Our approach involves introducing a novel structural framework for analyzing such an environment. The relevance of our findings and framework extends beyond the online advertising market. Sequential auctions are prevalent in various settings, encompassing online retail platforms, financial markets, and energy markets, where buyers often face financial constraints. Traditionally, these scenarios have been examined by treating individual auctions as isolated static events. However, our work reveals how dynamic constraints can interlink these sequential auctions, introducing nuanced insights into competition dynamics.

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A Appendix

A.1 Reduced-Form Evidence

Table 6: OLS Bid Regression and Logit Entry Regression

Model:	log(bidprice) OLS	entry Logit
log(#auctions per 5-min interval)	-0.0701*** (0.0114)	-0.1198*** (0.0246)
computer	-0.0888*** (0.0272)	-0.3689*** (0.1380)
optout	0.0626** (0.0242)	-0.3666 (0.2338)
match_cookie_prop	0.4880*** (0.0571)	2.087*** (0.4137)
gender = Male	-0.0551*** (0.0122)	-0.0130 (0.0323)
gender = Female	-0.0375*** (0.0112)	0.0090 (0.0332)
age = 25to44	0.0314*** (0.0099)	0.0262 (0.0211)
age = 45plus	0.0120 (0.0091)	0.0209 (0.0198)
seg_size	92.10*** (14.58)	642.1*** (109.9)
num_month_sold	-34.46** (16.02)	-272.8* (154.3)
total_rev	-26.59*** (4.851)	-60.62** (25.81)
avg_rev	-35.40*** (4.336)	-317.7*** (47.69)
profile_length	74.67*** (17.68)	586.1*** (210.4)
Site FE	Yes	Yes
Browser FE	Yes	Yes
City FE	Yes	Yes
Day-Hour FE	Yes	Yes
DSP FE	Yes	Yes
Advertiser FE	Yes	No
Observations	8,856,603	45,484,100
Adjusted/Pseudo R^2	0.44974	0.34448

Double-clustered (DSP & 5-min interval) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

A.2 First-step estimates

	$\hat{r}_t$	$\hat{p}_t$
$t = 1$	617.9 (440.6)	0.159 (0.095)
$t = 2$	2225.1 (2350.6)	0.498 (0.264)
$t = 3$	11055.2 (18673.2)	0.875 (0.185)
$t = 4$	1038.7 (910.5)	0.472 (0.218)
$t = 5$	238551.6 (288525.0)	0.995 (0.006)
$t = 6$	3933.6 (4070.4)	0.701 (0.217)
$t = 7$	822.3 (563.5)	0.202 (0.110)
$t = 8$	1079.0 (681.5)	0.151 (0.081)
$t = 9$	458.7 (298.2)	0.044 (0.027)
$t = 10$	421.5 (274.4)	0.028 (0.018)
$t = 11$	322.0 (197.4)	0.020 (0.012)
$t = 12$	1475.1 (1662.2)	0.080 (0.083)
$t = 13$	360.4 (227.6)	0.022 (0.013)
$t = 14$	232.0 (142.1)	0.015 (0.009)
$t = 15$	195.5 (125.0)	0.013 (0.008)
$t = 16$	199.2 (127.3)	0.013 (0.008)
$t = 17$	125.0 (78.1)	0.009 (0.006)
$t = 18$	154.5 (99.0)	0.014 (0.009)
$t = 19$	203.6 (131.6)	0.022 (0.014)
$t = 20$	108.1 (67.7)	0.013 (0.008)
$t = 21$	83.3 (52.9)	0.011 (0.007)
$t = 22$	109.6 (70.2)	0.016 (0.010)
$t = 23$	109.6 (70.5)	0.019 (0.012)
$t = 24$	232.6 (155.2)	0.053 (0.034)

Table 7: Estimated parameters for $F_K^{(t)} = \text{NegativeBinomial}(r_t, p_t)$, the distribution of the number of auctions for each period t.

	Entry Probability	Bid Distribution	
β_K	-3.627e-5 (5.485e-7)	-3.417e-5 (3.852e-7)	
	α_t	α_t	σ_t
$t = 1$	-1.236 (0.0038)	-0.2538 (0.003)	0.9009 (0.0019)
$t = 2$	-1.197 (0.0042)	-0.2488 (0.0033)	0.8935 (0.0022)
$t = 3$	-1.206 (0.0048)	-0.2375 (0.0038)	0.9028 (0.0027)
$t = 4$	-1.241 (0.0056)	-0.1851 (0.0047)	0.9477 (0.0033)
$t = 5$	-1.205 (0.0056)	-0.1577 (0.0046)	0.9446 (0.0033)
$t = 6$	-1.098 (0.0045)	-0.07127 (0.0038)	0.9509 (0.0026)
$t = 7$	-1.007 (0.0036)	-0.03548 (0.0028)	0.9152 (0.0018)
$t = 8$	-0.8971 (0.0041)	-0.01852 (0.0029)	0.8721 (0.0012)
$t = 9$	-0.7914 (0.0058)	0.05568 (0.0041)	0.838 (0.0009)
$t = 10$	-0.7135 (0.0081)	0.1483 (0.0057)	0.8232 (0.0008)
$t = 11$	-0.7149 (0.0089)	0.1626 (0.0063)	0.8085 (0.0008)
$t = 12$	-0.7107 (0.0094)	0.1652 (0.0066)	0.8003 (0.0007)
$t = 13$	-0.7675 (0.009)	0.1116 (0.0063)	0.7906 (0.0008)
$t = 14$	-0.7966 (0.0087)	0.08047 (0.0061)	0.7856 (0.0008)
$t = 15$	-0.8126 (0.0084)	0.06096 (0.0059)	0.7907 (0.0008)
$t = 16$	-0.8264 (0.0084)	0.03963 (0.0059)	0.7827 (0.0008)
$t = 17$	-0.8873 (0.0076)	-0.00469 (0.0053)	0.7829 (0.0008)
$t = 18$	-0.9607 (0.0064)	-0.07737 (0.0045)	0.7859 (0.0009)
$t = 19$	-1.054 (0.0054)	-0.1469 (0.0038)	0.793 (0.001)
$t = 20$	-1.079 (0.005)	-0.177 (0.0035)	0.7932 (0.0011)
$t = 21$	-1.066 (0.0047)	-0.1334 (0.0033)	0.8314 (0.0012)
$t = 22$	-1.097 (0.0044)	-0.2026 (0.0031)	0.825 (0.0012)
$t = 23$	-1.145 (0.004)	-0.26 (0.0029)	0.8308 (0.0014)
$t = 24$	-1.21 (0.0038)	-0.3232 (0.0028)	0.8372 (0.0016)

Table 8: Reduced-form estimates of entry probability logistic$(\alpha_t^{entry} + \beta_K^{entry} K_t)$ and bid distribution LogNormal$(\alpha_t^{bid} + \beta_K^{bid} K_t, \sigma_t)$

A.3 Bidders' objectives under the second-price auction

Let $\widetilde{\Psi}_t(\cdot \mid K)$ be the belief on rivals' highest bid P conditional on the current number of auction K under the SPA format. This belief represents the probability of winning an auction and also the distribution of the price in each auction. Given the ex-ante value function $\widetilde{EV}_{t+1}(W) = E[\widetilde{V}_{t+1}(K_{t+1}, W)]$ with $\widetilde{EV}_{T+1}(W) = \eta Q(W)$, the Bellman formulation of the objective is

$$\max_{\gamma,\tau} K_t F_C(\tau) \left[E\left[\int_0^{b^\gamma(X)} (X - p) d\widetilde{\Psi}_t(p \mid K_t) \right] - E[C \mid C \leq \tau] \right] + E\left[\widetilde{EV}_{t+1}(W_{it+1}) \mid \gamma, \tau \right] \quad (2)$$

Now we provide the first-order necessary conditions for the bidding problem under the second-price format while assuming differentiablity. The one with respect to the bid function parameter γ is given by

$$E\Big[\underbrace{\Psi_t'(b^\gamma(X) \mid K_t)(X - b^\gamma(X))}_{\text{Static FOC}}\nabla_\gamma b^\gamma(X)\Big] + \underbrace{\frac{1}{K_t F_C(\tau)}\nabla_\gamma E\left[\widetilde{EV}_{t+1}(W_{it+1}) \mid \gamma, \tau\right]}_{\text{Dynamic Tradeoff}} = 0$$

Meanwhile, the first-order condition with respect to the entry threshold τ is

$$\tau = \underbrace{E\left[\int_0^{b^\gamma(X)}(X-p)d\widetilde{\Psi}_t(p \mid K_t)\right]}_{\text{Static Threshold}} + \underbrace{\frac{1}{Kf_C(t)}\frac{\partial}{\partial\tau}E\left[\widetilde{EV}_{t+1}(W_{it+1}) \mid \gamma, \tau\right]}_{\text{Dynamic Tradeoff}}$$

Again, similarly to the first-order conditions under the first-price format for (1), we have both static and dynamic components in the optimality conditions here. For instance, in the condition for the optimal bid strategy, the static component encourages truthful bidding, but the dynamic component provides a counteracting force.

A.4 Algorithm for solving for an equilibrium

We setup the algorithm by first making a grid over the state space for (K_t, W_{it}). For the remaining budget, we make a grid over $[-M, M]$ where M is the upper bound of the initial budgets. For the number of auctions, we take Monte Carlo draws from $\widehat{F}_K^{(t)}$ for each period t. We set $\{(\gamma_t^{(0)}(K,W), \tau_t^{(0)}(K,W)\}_{t=1}^T$ as the initial strategies.

Then, we execute the following loop: For each $m = 1, \ldots,$

- Forward simulate numerous paths of $\{(K_t, W_{1t}, \ldots, W_{Nt})\}_{t=1}^T$ using $\{(\gamma_t^{(m-1)}(K,W), \tau_t^{(m-1)}(K,W)\}_{t=1}^T$ to numerically obtain the belief:

  $$\Psi_t^{(m)}(b \mid K) = E\left[\prod_{j\neq i}\Big(1 - F_C(\tau_t^{(m-1)}(K, W_{jt})) + F_C(\tau_t^{(m-1)}(K, W_{jt}))F_X(b^{-1}(b \mid \gamma_t^{(m-1)}(K, W_{jt})))\Big)\right]$$

  over the grid of K_t for each t.
- Obtain $\{(\gamma_t^{(m)}(K,W), \tau_t^{(m)}(K,W)\}_{t=1}^T$ over the grid of states that best respond given $\Psi_t^{(m)}(b \mid K)$ by solving (1) via backward induction.
- Break if $\|\{(\gamma_t^{(m)}(K,W), \tau_t^{(m)}(K,W)\}_{t=1}^T - \{(\gamma_t^{(m-1)}(K,W), \tau_t^{(m-1)}(K,W)\}_{t=1}^T\|$ is below some tolerance.