

1. ~~Outer~~

fixed. so worst = best.

outer: $i = 0, \dots, n - (t-1) \cdot t$.

$$\therefore \text{last } i = k \quad k \leq n - (k-1) \cdot t$$

$$\therefore k(t+1) \leq n + t \leftarrow \sqrt{n}$$

$$\therefore k \leq \left(\frac{n + \sqrt{n}}{\sqrt{n} + 1} = \frac{\sqrt{n}(\sqrt{n} + 1)}{(\sqrt{n} + 1)} \right)$$

$\therefore \text{last } i = k = \sqrt{n}$.

Inner: When $i = m$ inner loop runs $n - (m-1) \cdot t$ times

\therefore total loop.

$$\sum_{m=1}^{\sqrt{n}} (n - (m-1) \cdot \sqrt{n}) = \text{number of series} \times \frac{\text{first} + \text{last}}{2}$$

$$= \sqrt{n} \left(\frac{n + n - (\sqrt{n}-1)\sqrt{n}}{2} \right) = \sqrt{n} \frac{(n + \sqrt{n})}{2}$$

$$\leq O(n\sqrt{n})$$

2. Outer 1: n

Outer 2: n

Worst case inner runs for only n times

$$\text{total runs} = \cancel{n^2 \times (1 + (n^2 - n))} +$$

$$= 1 \times (n^2 - n) + n \times (\text{inner loop})$$

$$m = 1, 2, 4 \dots 2^k$$

$$2^k \leq n$$

$$\therefore k \leq \log_2 n$$

Worst

$$\text{total runs} = (n^2 - n) + n \log n \leq O(n^2)$$

best.

no inner loop.

$$\text{total runs} = n^2 \geq \Omega(n^2)$$

3. **Worst**

the first for loop: $n \times (\text{runtime of insert})$

$$= n \log n$$

the second.

$$n \times (\text{find runtime} + \log n)$$

while $\frac{k}{2^{m-1}} < n$

$$\therefore m \leq \log n$$

find runtime $O(\text{size of } t_i)$

size of $t_i = n$ from the first loop.

and $k \leq n$ so t_i size doesn't change

\therefore find runtime = $\log n$

$$\therefore \text{total} = n \log n + 2n \log n \leq O(n \log n)$$

Best

$$n \log n + n \times (\text{find only})$$

" $\log n$ "

$$= 2n \log n \geq \Omega(n \log n)$$

4. Outer n

Best $n \leq 10$ just $n \geq \Omega(n)$

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inner loop for every $i = \text{size}$.

$$\text{size} = 10, 10 \cdot (1.5), 10 \cdot (1.5)^2, \dots$$

k assume all number of size.

$$\text{the last } 10 \cdot (1.5)^{k-1} \leq n$$

$$\therefore k = \log_{1.5} \left(\frac{n}{10} \right) + 1$$

\therefore for k times inner loop run

$$m^{\text{th}} \text{ size loop runtime} = 10 \cdot (1.5)^{m-1}$$

" m^{th} size."

\therefore total run

$$1 + \sum_{m=1}^k 10 \cdot (1.5)^{m-1}$$

$$\frac{10 \cdot (1.5)^k - 1}{1.5 - 1} = C_0 \cdot (1.5)^{\log_{1.5} \left(\frac{n}{10} \right) + 1} + C_1$$

$$= C_0' \cdot n + C_1$$

$$\therefore (n-k) + C_0' n + C_1 \leq O(n)$$