

part 1.

$$T(n) = 1 + 2 \cdot T\left(\frac{n}{2}\right)$$

$$= 1 + 2 \cdot \left(1 + 2 \cdot T\left(\frac{n}{2^2}\right)\right)$$

$$= 1 + 2 \cdot \left(1 + 2 \cdot \left(1 + \dots T\left(\frac{n}{2^h}\right)\right)\right)$$

$$= 1 + 2 + 2^2 + \dots + 2^h \quad \text{where } 2^h = n.$$

$$= 1 \cdot \frac{2^{h+1} - 1}{2 - 1} = 2 \cdot 2^h - 1$$

$$= 2 \cdot n - 1 \quad (\because 2^h = 2^{\log_2 n} = n)$$

part 2.

if $m \geq 0$

$$T(rf_2(m)) = 1 + T(rf_2, m-1) \\ = 1 + \dots + T(rf_2, 0) = m+1.$$

$$T(rf_1, n) = 1 + T(rf_2, n-1) + T(rf_1, n-1) \\ = (1+n) + ((1+n-1) + T(rf_1, n-2)) \\ = (1+n) + n + (n-1) + \dots + T(rf_1, 0) \\ = (1+n) + n + (n-1) + \dots + 1 \\ = \frac{(n+1)(n+2)}{2}.$$

Trace of data 1.

$n=5$	$(9, 3, 1, 7, 4) \rightarrow (3, 1, 7, 4, 9)$: 4 swaps, 5 runs
$n=4$	$(3, 1, 7, 4, 9) \rightarrow (1, 3, 4, 7, 9)$: 2 swaps, 4 runs
$n=3$	$(1, 3, 4, 7, 9) \rightarrow (1, 3, 4, 7, 9)$: 0 swaps, 3 runs
$n=2$	$(1, 3, 4, 7, 9) \rightarrow (1, 3, 4, 7, 9)$: 0 swaps, 2 runs
$n=1$	$(1, 3, 4, 7, 9) \rightarrow (1, 3, 4, 7, 9)$: 0 swaps, 1 run.
$n=0$	$(1, 3, 4, 7, 9)$: 0 swaps, 0 run

↓ denotes 'data' pointer. → denotes rf2 call

$$\text{total runs} = 6 + (1+2+\dots+5) = 21$$

$$T(rf_1, 5) = \frac{6 \cdot 7}{2} = 21.$$