Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Lecture with Computer Exercises:
Modelling and Simulating Social Systems with MATLAB

Project Report

Careful Drivers in the Nagel-Schreckenberg Model

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Contents

1 Abstract

This study aims to develop and analyze an alternative model for cellular automaton based single lane traffic simulations by generalizing ideas from extensions of the classic Nagel-Schreckenberg Model, namely the Velocity Dependent Randomization Model. Specifically, we hope to gain some insight into the cause of traffic jam formation under a wide range of realistic circumstances by incorporating various new parameters such as the safety time.

2 Individual contributions

2.1 Jorit

I wrote big parts of the code and parts of the report.

2.2 Shin

I wrote big parts of the final report and parts of the code.

3 Introduction and Motivations

In densely populated areas with no possibility of extending the number of lanes in a highway, it is critically important to accurately model traffic dynamics in order to reduce the possibility of traffic jam formation and thus maximize the flow of traffic. The development of cellular automata (CA) models have made the simulation of complex dynamical systems much more straightforward than models using differential equations, since the behavior of the system boils down to a set of simple rules.

Numerous cellular automata-based traffic dynamic models have been developed over the last couple of decades incorporating traffic lights and ramps on the road [4], taking into account the drivers' reaction to braking lights of the cars in front of them [5], or extending the number of lanes to two [6]. One model even assumes that the drivers are able to anticipate the velocity of the car in front, in order to simulate the road conditions more realistically [3]. However, in order to narrow down our objective, this paper focuses particularly on single-lane highway traffic without external disturbances and with the only source of information for the drivers being the velocity of their cars and the distance between them.

Section 4 starts by introducing the Nagel-Schreckenberg (NaSch) model, detailing its effectiveness at describing single lane traffic flow as well as its limitations in reproducing different phases of traffic jams. It then presents the Velocity Dependent Randomization Model, and explains the advantages over the original model, mainly the emergence of hysteresis and metastable states. Finally, it proposes our model that generalizes and extends concepts from the Velocity Dependent Randomization Model.

Section 5 provides some details on the implementation of the model, including a discussion on the effect of the boundary conditions and the initial conditions on traffic jam formation. It also describes the tools used to compare simulation results with other models, for example the fundamental diagram.

Section 6 presents the simulation results for our model and compares them to those of the Velocity Dependent Randomization Model as found in literature. It then thoroughly explains the difference between the two models, and suggests the advantages and the limitations of using our model for traffic simulations as opposed to the Velocity Dependent Randomization Model.

4 Description of the Model

4.1 Standard Model Performance Measures

Before describing the models in detail, it is important to familiarize with the tools used to measure the performance of cellular automata based traffic dynamics models, so that our model can be qualitatively and quantitatively compared with existing models.

4.1.1 Space-time Diagram

A space-time diagram is a diagram that is used to easily visualize the formation and development of traffic jams on the road over time. The x-axis represents the cells on the road starting from the left and ending at the right, while the y-axis represents the time steps, advancing from bottom to top. The colored cells indicate the presence of a vehicle on that cell, and conversely the white cells indicate the absence of vehicles. Therefore, a traffic jam is represented by a congregation of colored cells, as seen in Figure 1. One can also see from the figure by tracing the red dot along the time steps, that the vehicles are traveling from left to right (in the absence of traffic jams).

An interesting phenomenon observed in this diagram is that traffic jams move in the opposite direction of the individual vehicles. This can be explained by the fact that the vehicles are not affected by the traffic behind them, which means that causality travels in the opposite direction [6]. It agrees with our intuition of traffic jams in form of start-stop waves: Cars at the front of the jam with a clear road ahead can continue their way and accelerate again, whereas cars coming from behind have to stop directly behind the traffic jam and thus extend the traffic jam in backwards direction.

Note that in the figure, the first 25 steps are use to show the initial distribution of the vehicles on the road. After that, the traffic is then updated according to our model in every timestep.

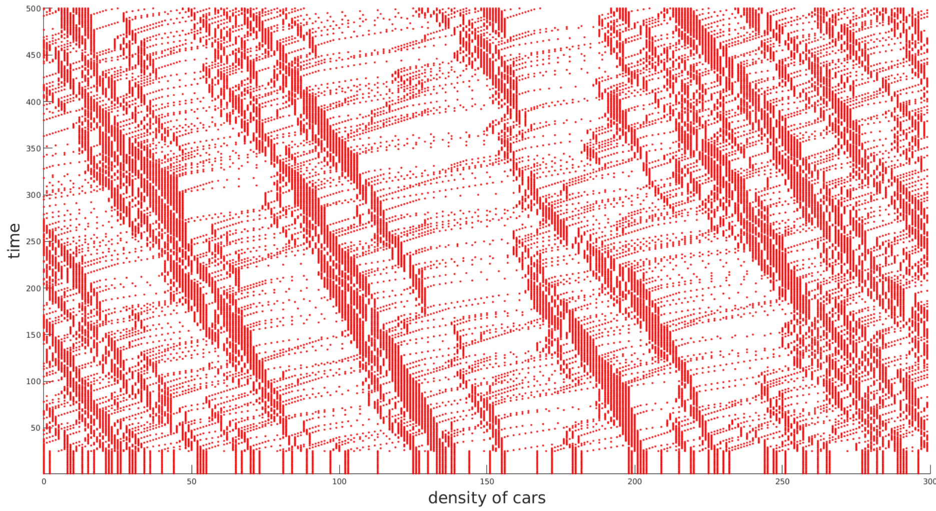

Figure 1: Space-time Diagram of the Nagel-Schreckenberg model. Red dots indicate the presence of a car, whereas white cells indicate the absence of a vehicle on that part of the road. The traffic is allowed to flow freely first after 25 timesteps. We can see the spontaneous emergence and vanishing of traffic jams, indicated by dense, red areas.

4.1.2 Fundamental Diagram

The standard tool used to visualize traffic dynamics simulation results is called the fundamental diagram within the traffic engineering community. This diagram is a plot of the global density of the cars vs. the global flow rate, in which the two variables are respectively defined as follows:

$$\rho = \frac{N}{L} = \frac{\text{Number of vehicles on the road}}{\text{Number of sites}} \tag{1}$$

$$J(\rho) = \frac{\text{Number of vehicles passing a point}}{\text{Number of time steps}} \tag{2}$$

Since the vehicles are initially randomly distributed, and the number of vehicles passing a particular point on the road is dependent upon the existence of traffic jams before that point, it is necessary to iterate over many time steps in order to minimize the variance between the values for the flow rate. After observing the variance for time steps ranging from 0 to 10,000 as shown in Figure 2, it was determined that a time step of 500 was sufficient to accurately approximate the global flow.

Note that the global density is constant after a small number of time steps, as the expected value of the number of vehicles on the road is constant for closed boundary conditions, as discussed in section 5.

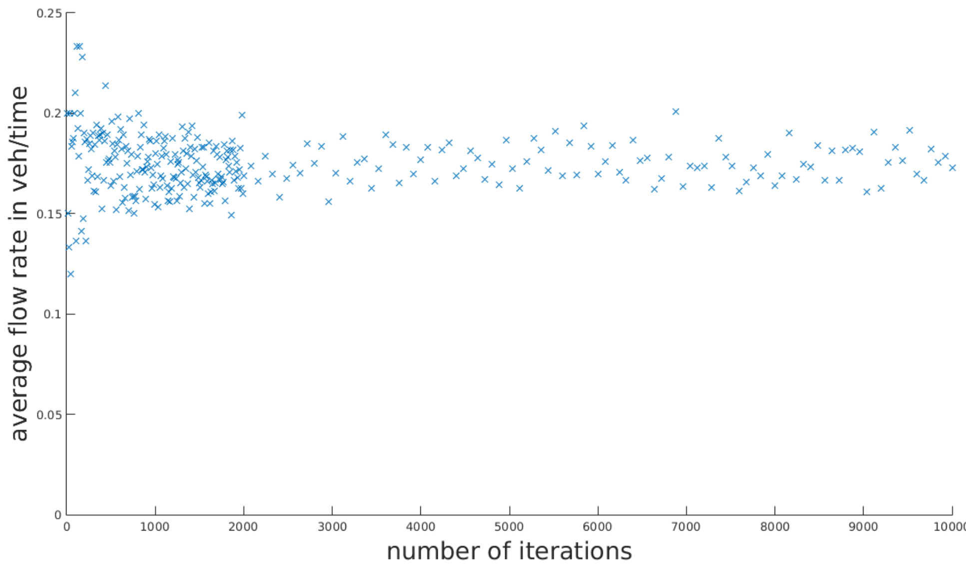

Figure 2: Number of iterations vs. global flow with an occupation probability of 0.3, VDR parameters of $p_0 = 0.75$ and $p = \frac{1}{64}$, road length = 200, maximum velocity = 33 $\frac{m}{s}$, and a unit velocity of 7.5 $\frac{m}{s}$. The parameters will be explained in section 5. We can see, that already for 500 iterations, we get a considerably smaller error in the flow rate.

Only for a small number of iterations, we get big effects in the average flow rate as seen in the following figure (figure 3).

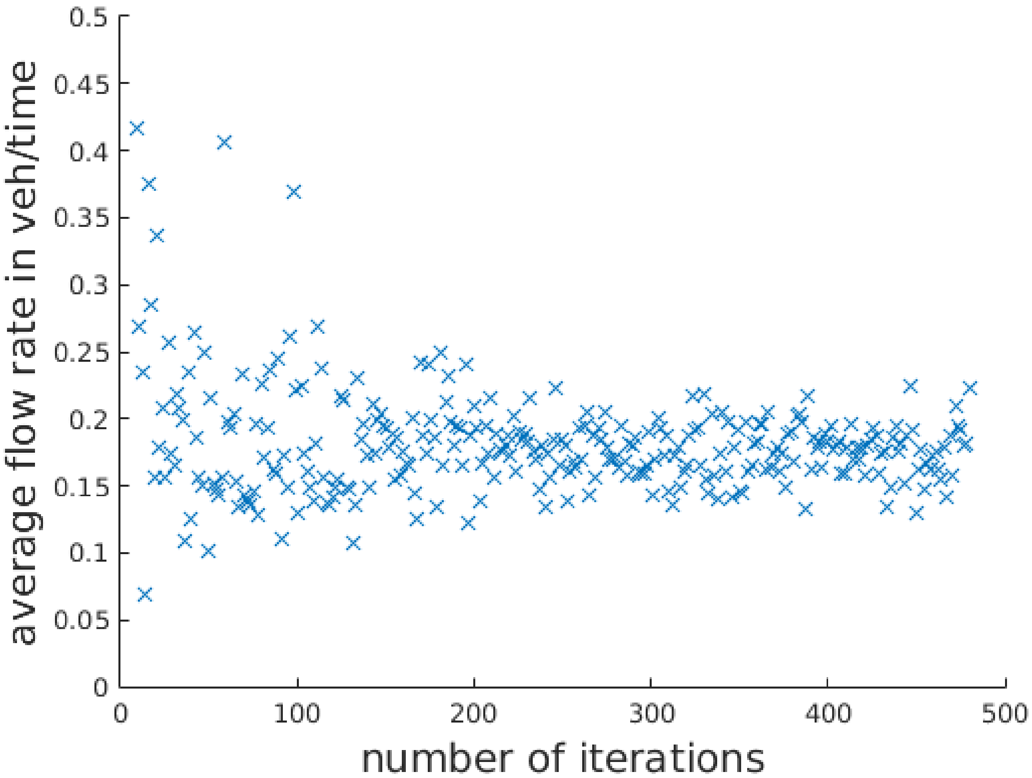

Figure 3: Number of iterations vs. global flow for small numbers of iterations with an occupation probability of 0.3, VDR parameters of $p_0 = 0.75$ and $p = \frac{1}{64}$, road length = 200, maximum velocity = 33 $\frac{m}{s}$, and a unit velocity of 7.5 $\frac{m}{s}$. The parameters will be explained in section 5. As shown here, only for less than 200 iteration steps, huge errors can occur.

Let us now analyze the fundamental diagram of the Nagel-Schreckenberg model as shown in Figure 4. Up until the critical density, where the flow rate is highest, the relationship between the flow rate and vehicle density is linear and positive. However, after the critical density is reached, this relationship becomes approximately linear but negative. This turning point denotes the emergence of traffic jams, which cause the decrease in flow rate. As the maximum velocity increases, the slope of the first part of the curve increases but the value of the critical density decreases. This means that at higher velocities, the flow of vehicles increases faster for the same density but traffic jams appear at lower densities. Once the critical density is exceeded, the second part of the curve is approximately equal for all maximum velocities.

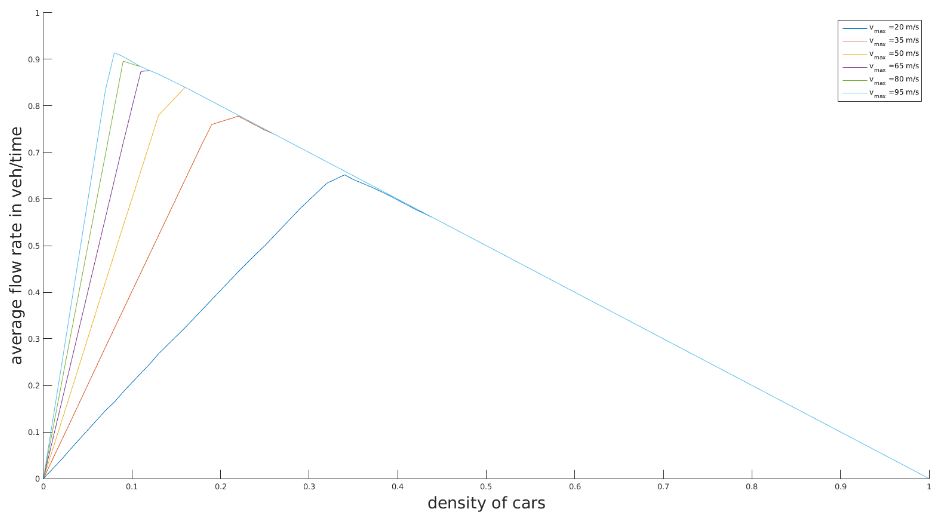

Figure 4: Fundamental Diagram of the basic Nagel-Schreckenberg model for different maximum velocities.

4.2 The Nagel-Schreckenberg Model

The first application of a cellular automaton based model to traffic simulations was realized by Nagel and Schreckenberg in 1992 by adding a probabilistic step to the famous one-dimensional binary cellular automaton rule called *Rule 184* in order to account for the randomness of the behavior of individual drivers on the highway. [1]. The model is defined on a one-dimensional array of L sites, representing the single lane highway. Each site may be either occupied by one vehicle or it may be empty. One update of the model consists of four steps that are performed for all vehicles on the road:

1. Acceleration: if the velocity v of a vehicle is lower than v_{max}, and if the distance to the next car ahead is larger than $v + 1$, the speed is advanced by one.
2. Slowing down: if a vehicle at site i sees the next vehicle at site j, and if $j \leq v$, it reduces its speed to $j - 1$.
3. Randomization: with probability p, the velocity of each vehicle is decreased by one.
4. Car motion: each vehicle is advanced v sites.

This set of steps describe general properties of human drivers in a highway:

- Step 1 is based on the drivers' tendency to drive at the maximum velocity of the highway.
- Step 2 ensures that collisions between vehicles are avoided.
- Step 3 takes into account the random deceleration of some drivers, and is the step that makes the formation of traffic jams possible. One can easily see that without this step, this model would become deterministic.
- Step 4 allows vehicles to move one time step along the road according to their respective speed.

This simple model can effectively simulate basic phenomena in real traffic like the spontaneous formation of traffic jams. On the other hand, it cannot accurately describe traffic dynamics at a microscopic level. For example, using this model, it is possible for a vehicle to decelerate from maximum velocity to zero velocity in one time step, which is extremely unlikely and physically almost impossible to happen considering the high kinetic energies of vehicles on highways.

4.3 The Velocity Dependent Randomization Model

The VDR model extends the original Nagel-Schreckenberg model by adding a *slow-to-start* rule. While the original model only has a constant braking probability p, the VDR model has a probability function $p_n(v_n)$ that depends on the velocity v_n of a vehicle n. This value is calculated before step 1 of the NaSch model, and is then used in step 3 as the randomization parameter. The rule is as follows:

0. Determination of the randomization parameter:

$$p_n(v_n) = \begin{cases} p_0 & \text{for } v_n = v_{threshold} \\ p & \text{for } v_n > v_{threshold} \end{cases}$$

In order for the stopped cars to be slower to accelerate than the cars that are already in motion, we restrict our parameters to be $p \leq p_0$. One can easily verify that for the case of $p_0 = p$, the NaSch model is recovered.

One advantage of the Velocity Dependent Randomization Model over the original model is that this model shows a phenomena that occurs in real traffic called hysteresis, where the global flow rate is different for the same critical density depending on whether the density is increased up to or decreased down to this critical value. This phenomena can be seen in the fundamental diagram of the VDR model in Figure 5, where the global flow suddenly drops as the critical density is surpassed before it decreases linearly.

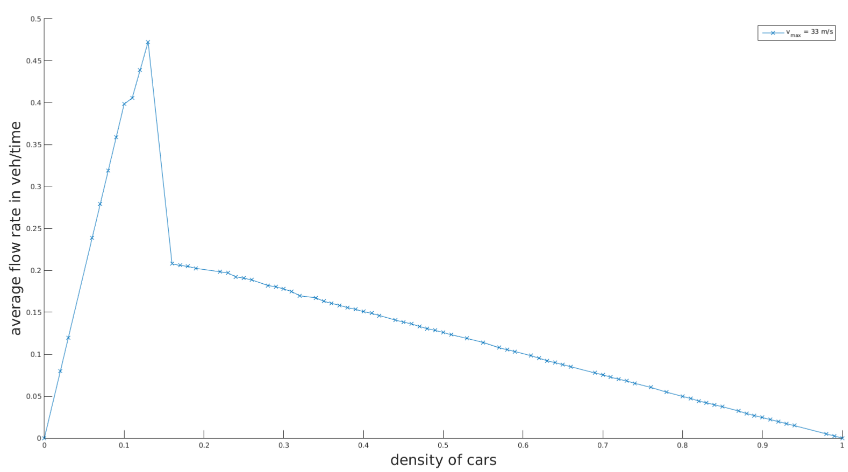

Figure 5: Fundamental Diagram of the VDR model with 10000 iterations per occupation probability. We can see a huge drop in the flow after passing the critical density.

4.4 The Careful Driver Model

4.4.1 Criticism of the original model and adaptation

As we mentioned before, in the original Nagel-Schreckenberg model, drivers are capable of stopping within one timestep representing one second and a distance $d = v_{unit} * 1s$ which is in our model ($v_{unit} = 7.5\frac{m}{s}$) $d = 7.5m$ even if going with v_{max}, which is in all of our experiments at least $v_{max} = 100\frac{km}{h}$.
On their way to a driver's licence, people learn, that they should keep a minimum safety distance depending on their own velocity of approximately $d_{safety} = \left(\frac{v}{10h}\right)^2$ which is $d_{safety} = 100m$ for $v = 100\frac{km}{h}$.
In most countries, speed limits on highways (if even existing) are well above $v = 100\frac{km}{h}$. That makes the Nagel-Schreckenberg model look a little bit too optimistic by assuming, that drivers can stop almost instantly.
Taking this fact into account, we extended the basic Nagel-Schreckenberg model first to a VDR model, and then added a *safety-distance*, depending on the velocities of the individual drivers. Every driver has to keep this minimum distance in form of a time between the driver's own vehicle and the car in front. If not otherwise specified, we defined

$$\Delta t_{safety} = 2s \rightarrow d_{safety} = v \cdot t_{safety} \tag{3}$$

If a car is going at say $v = 100\frac{km}{h} \approx 27\frac{m}{s}$, that would mean, that it had to keep a distance to the front car of at least

$$\Delta t_{safety} = 2s \tag{4}$$

$$\rightarrow d_{safety} = v \cdot t_{safety} = 27\frac{m}{s} \cdot 2s = 54m \tag{5}$$

We chose $v_{unity} = 7.5\frac{m}{s}$ in our model. Accordingly, since one time step corresponds to one second, which is approxematly the reaction time of a driver, a site in our model represents a space of 7.5 m in direction of the lane.
The above example would then be shown as free cells in our model:

$$d_{cells,free} = 54m/7.5m \approx 7cells \tag{6}$$

5 Implementation

5.1 Boundary Conditions

Since the road length is finite, we need to select its boundary conditions before running the simulation. The two possible boundary conditions are open and closed.

Open boundary conditions mean that the vehicles leaving the road are not the same as the vehicles entering the road. For this boundary condition, two parameters are necessary: one that controls the number of vehicles entering the road, and one for the vehicles exiting the road. Under this condition, the vehicles exiting the road will behave differently from the other vehicles because they are not subjected to the rules from the NaSch model (since they have no vehicles in front of them). Therefore, even if the two parameters controlling the number of vehicles are equal to each other, the simulation that uses this boundary condition will intrinsically be corrupted by finite size effects. One could iterate over many iterations in order to reduce the bias, but this would be computationally expensive.

Closed boundary conditions mean that the same vehicles leaving the road must enter the road in the next time step. For this boundary condition, only one parameter is needed to control the number of the vehicles on the road because the number of vehicles on the road is the same for all time steps. Under this condition, all vehicles will be subjected to the NaSch model so there will be no finite size effects regardless of the number of iterations.

Since the closed boundary condition significantly reduces the computation time over the open boundary conditions, we decided to use the closed boundary condition.

5.2 Initial Conditions

It is possible to have two extreme cases for the initial conditions for the cars:

- Homogenous starting state: a case where all the vehicles are in the cells next to each other, and the velocity of all the vehicles are zero.

- Inhomogenous starting state: a case where all the vehicles are placed equidistantly along the whole length of the road, and the velocity of all the vehicles are v_{max}

Depending on these two initial conditions, the global flow rate can take on two different states: for the homogenous case, a metastable state (absence of traffic jams) with a long life-time is formed, while for the inhomogenous case a phase separation between the jammed vehicles and the free vehicles is formed. [4]

5.3 Simulation Parameters

The simulation was carried out in MATLAB using the following default fixed parameters if not specified otherwise:

- Size of the road: $L = 500$ (corresponds to a road length of 3750 m, each cell being 7.5 m in length)
- Number of time steps: 500 (each time step corresponds to 1 second).
  First of all due to the big amount of computation time needed, we used this number of iterations.
- The initial velocity distribution was a normal distribution.

On the other hand, the following parameters were varied to simulate a variety of conditions:

- The probability of occupation of a cell by a vehicle was varied from 0 to 1 in steps of 0.01.
  This directly affects the densities of cars ρ, which is measured and plotted then.
- The maximum velocity was varied from $20\frac{m}{s}$ to $95\frac{m}{s}$ (72 to $342\frac{km}{h}$)
- The braking probability function $p_n(v_n)$. Default values used here were $p_0 = 0.75$ and $p = \frac{1}{64}$ if not specified otherwise.
  We chose those parameters in order to be able to compare our model to the literature, where most experiments use these parameters in the VDR model [2] [6] [4].
- The safety time constant Δt with a default value of $\Delta t = 2$ s.
- Distribution of occupied cells were set for special experiments. If not specified otherwise, it was completely random.

6 Simulation Results and Discussion

In this section, we present the simulation results for the Careful Driver model with the implementation as described in the previous section. The results are compared side by side with the results of the baseline VDR model, and the discrepancies between them are discussed in detail.

In Figure 6, we present the space-time diagram of the Careful Driver model with the vehicle locations initialized to a random distribution.

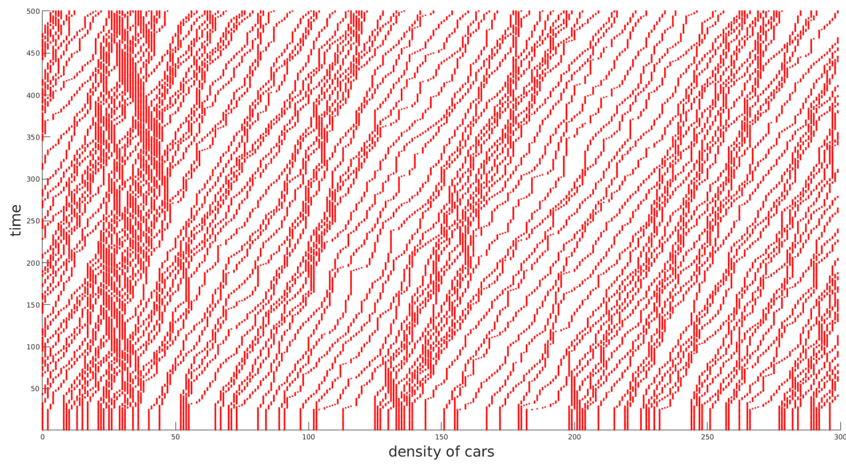

Figure 6: Space-time diagram of our careful driver model with same parameters as in figure 1. We see, that almost no traffic jams form anymore.

In Figure 7, we compare the space-time diagram of the VDR model with the Careful Driver model with homogenous starting states.

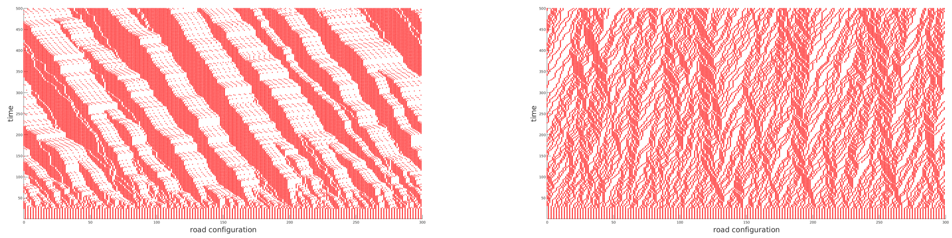

Figure 7: Space-time diagram of the VDR model (left) and our careful driver model (right) with the same parameters as in figure 1 with homogenous starting states. In the VDR case, jams are immediately created and their size grows with increasing time steps, while in our model the jams start at later time steps and do not grow significantly throughout the whole simulation.

In Figure 8, we compare the space-time diagram of the VDR model with the Careful Driver model with inhomogenous starting states.

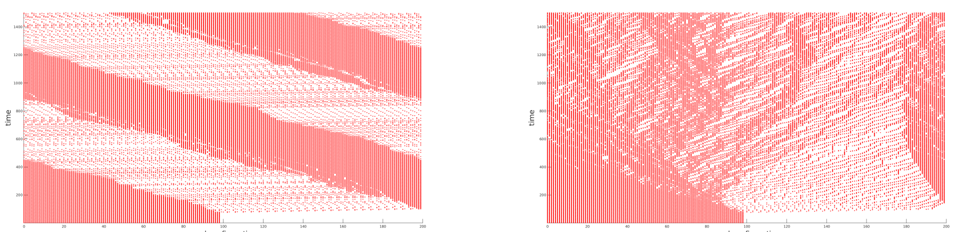

Figure 8: Space-time diagram of the VDR model (left) and our careful driver model (right) with the same parameters as in figure 1 with inhomogenous starting states. Note that in the VDR case, the initial jam persists throughout the simulation, while in our model the jam is dispersed by the end of the simulation.

In the fundamental diagram of the Careful Driver model shown in Figure 9, we see interesting phenomena that are not observed in the VDR model. First, the critical density depends very little on the maximum velocity, while in the VDR model the critical density decreases with increasing maximum velocity. This is because after a certain density, no vehicle can travel at the maximum velocity while simultaneously respecting the safety time due to the distance constraints. Similar to the VDR model, the curves with higher maximum velocities reach a higher flow rate at the critical density simply because the vehicles are traveling faster.

Once the critical density is surpassed, all the velocities follow the same curve until the maximum density. Interestingly, this curve momentarily climbs upwards with increasing density for a small range of densities, before falling back down again. One explanation for this phenomena is that the range of densities provides just the right distance between the vehicles so that they can travel at a specific velocity for the specified safety time without creating a jam. This effectively creates a local maxima in the diagram, where any density deviation results in a smaller flow rate.

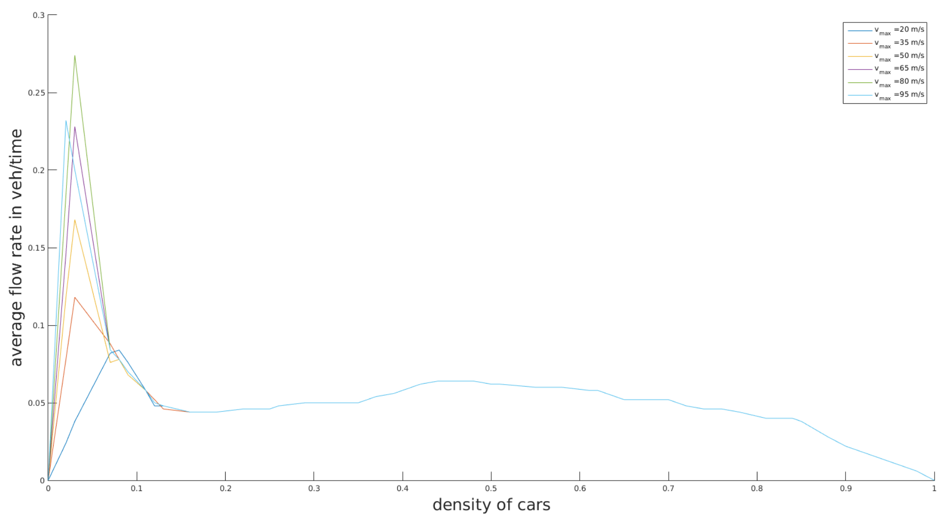

Figure 9: Fundamental diagram of our careful driver model with same parameters as in figure 4 for different maximum velocities. We see here that the flow rate actually increases for a range of densities after the critical density before decreasing again.

In Figure 10, we plot the fundamental diagram of the Careful Driver model for different safety times. The diagram shows that as the safety time is increased, the curves leading up to the critical density generally remains the same, with the same maximum flow rate. However, past the critical density, the flow rate of the curves with longer safety time drops off lower before all the curves converge at a higher density. In the limit as the safety time goes to zero, our model corresponds to the regular VDR model, shown in the figure by the blue curve. The high flow rate of the VDR model curve compared to the other curves for densities above the critical density suggests that when a traffic jam is formed, vehicles can travel faster as a whole by not keeping the safety time rule.

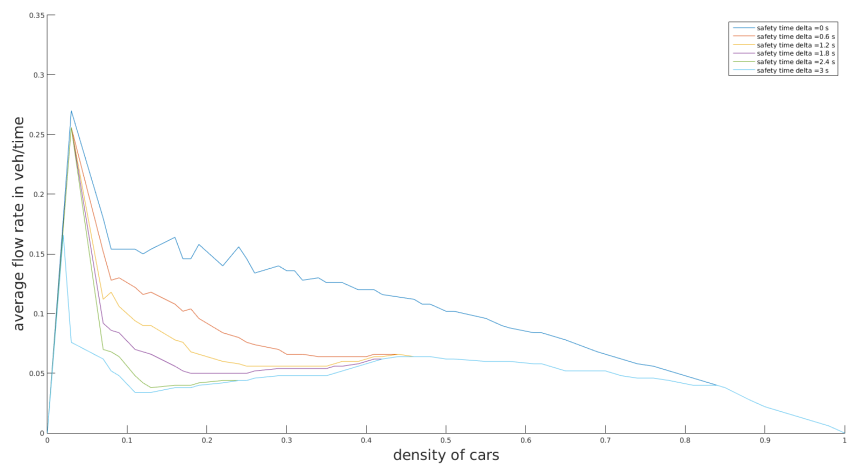

Figure 10: Fundamental diagram of our careful driver model with the same parameters as in figure 4 for different safety times, using a maximum velocity of 70 m/s. We see here that smaller safety times, the flow rate is higher for the same density.

A significant finding from the simulation is that although the fundamental diagram tells us the flow rate of our model is lower than for the VDR model for most density ranges, the space-time diagram tells us that the probability of traffic jams formation is much lower for our model than for the VDR model. In a real highway, this corresponds to the cars moving at a lower average velocity but avoiding traffic jams. This means that although the drivers will not be traveling faster by applying the safety time rule, they could save up on fuel economy. Traffic jams are a big cause of fuel consumption because vehicles waste unnecessary energy while the they are idling, or when they either accelerate getting out of a traffic jam or decelerate getting into one. Driving at the maximum velocity also leads to more fuel consumption because the aerodynamic drag scales to the power of two with increasing speed. Since in the Careful Driver model, most vehicles are driving under the maximum velocity and rarely come to complete stops, fuel savings can be compelling for some regions of the world due to financial or environmental concerns.

7 Summary and Outlook

In summary, we have successfully built a cellular automata based single lane traffic dynamics model as an extension of the original Nagel-Schreckenberg model, and have seen through our simulations that our model exhibits different dynamics than other extension models, namely the Velocity Dependent Randomization model. In particular, we observed that the Careful Driver model outputs lower global flow rates than the VDR model above the critical density, but it forms less traffic jams, leading to better fuel economy. In conclusion, we suggest that the introduction of the safety time rule could offer an environmental alternative to the more performance-driven VDR model.

Given the time and opportunity, we would like to compare our model with real traffic data to validate its accuracy. The data will most likely not be too similar to the model, because the assumed model parameters such as the safety time rule are not true in real traffic. Once the simulation results are verified, the model should be tested with test drivers on a small scale road in closed boundary conditions using the same parameters as in the simulation. If the model parameters such as safety time are precisely realizable by the drivers, the life time of the metastable state should increase and the occurrence of traffic jams can be delayed given homogenous starting states, just as we saw in the simulation results. In addition, the fuel economy data from these vehicles should be compared with the average fuel economy of the highway from which the traffic data was taken, so that we can verify whether the fuel economy was actually improved.

8 References

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